DISCUSSION PAPER SERIES

DP14281
(v. 2)

Measuring Trade in Value Added with Firm-Level Data

Rudolfs Bems and Ayumu Ken Kikkawa

INTERNATIONAL MACROECONOMICS AND FINANCE

ISSN 0265-8003

Measuring Trade in Value Added with Firm-Level Data

Rudolfs Bems and Ayumu Ken Kikkawa

Discussion Paper DP14281
First Published 06 January 2020
This Revision 26 January 2021

Centre for Economic Policy Research
33 Great Sutton Street, London EC1V 0DX, UK
Tel: +44 (0)20 7183 8801
www.cepr.org

Measuring Trade in Value Added with Firm-Level Data

Abstract

Global Value Chains have proliferated in economic policy debates. Yet a key concept—trade in value added—is likely mismeasured because of sectoral aggregation bias stemming from reliance on input-output tables. This paper uses comprehensive firm-level data on domestic and international transactions to study this bias. We find that sectoral aggregation leads to overstated trade in value added. The magnitude of the bias varies across countries—at 2-5 p.p. of gross exports for Belgium and 17 p.p. for China. We study how the interplay between within-sector heterogeneities in firms' import and export intensities and size determine the magnitude of the bias.

JEL Classification: E01, F14, L14

Keywords: global value chains, Input-Output Tables, Aggregation bias

Rudolfs Bems - rbems@imf.org
International Monetary Fund and CEPR

Ayumu Ken Kikkawa - ayumukikkawa@gmail.com
UBC Sauder School of Business

Acknowledgements
First version: May 2019. Part of the analyses was done when Kikkawa was at the National Bank of Belgium as a researcher. We thank Cedric Duprez, Rob Johnson, and Heiwai Tang for helpful comments on this paper, JaeBin Ahn for sharing firm-level data sources, Matilde Bombardini for providing the codes to clean the Chinese datasets, Masao Fukui, Parisa Kamali, and Isabelle Mejean for sharing statistics based on firm-level datasets. We would also like to thank Emmanuel Dhyne and the NBB for access to its datasets and for its assistance. Min Kyeong Baik, Yuting Chen, and Siying Wang provided excellent research assistance. This research has received funding from SSHRC Insight Development Grant (SSHRC 430-2019) and the Centre for Innovative Data at UBC. The views expressed in this study are the sole responsibility of the authors and should not be attributed to the International Monetary Fund, its Executive Board, or its management. The views expressed in this paper do not necessarily reflect the views of the NBB.

Measuring Trade in Value Added with Firm-Level Data*

Rudolfs Bems[a] and Ayumu Ken Kikkawa[b]

[a]International Monetary Fund and CEPR

[b]Sauder School of Business, University of British Columbia

This version: January 25, 2021

Abstract

Global Value Chains have proliferated in economic policy debates. Yet a key concept—trade in value added—is likely mismeasured because of sectoral aggregation bias stemming from reliance on input-output tables. This paper uses comprehensive firm-level data on domestic and international transactions to study this bias. We find that sectoral aggregation leads to overstated trade in value added. The magnitude of the bias varies across countries—at 2-5 p.p. of gross exports for Belgium and 17 p.p. for China. We study how the interplay between within-sector heterogeneities in firms' import and export intensities and size determine the magnitude of the bias.

*First version: May 2019. Part of the analyses was done when Kikkawa was at the National Bank of Belgium as a researcher. We thank Cedric Duprez, Rob Johnson, and Heiwai Tang for helpful comments on this paper, JaeBin Ahn for sharing firm-level data sources, Matilde Bombardini for providing the codes to clean the Chinese datasets, Masao Fukui, Parisa Kamali, and Isabelle Mejean for sharing statistics based on firm-level datasets. We would also like to thank Emmanuel Dhyne and the NBB for access to its datasets and for its assistance. Min Kyeong Baik, Yuting Chen, and Siying Wang provided excellent research assistance. This research has received funding from SSHRC Insight Development Grant (SSHRC 430-2019) and the Centre for Innovative Data at UBC. The views expressed in this study are the sole responsibility of the authors and should not be attributed to the International Monetary Fund, its Executive Board, or its management. The views expressed in this paper do not necessarily reflect the views of the NBB.

1 Introduction

The last two decades have seen a rise in global value chains (GVCs). The topic has proliferated in economic research both in international trade and macroeconomics. An important component of this research agenda has been the distinction between conventional gross exports and exports of value added, or the domestic value-added content of exports.[1] The wedge between these two trade concepts has been well documented: the decline in value-added exports relative to gross exports, has been the flip-side of the rise in GVC participation.[2] The distinction between the two trade measures matters because it feeds into a wide range of economic issues, from the size of the U.S.-China bilateral trade balance to the magnitude of cross-border exposures to Brexit.

However, research in this area is constrained by the quality of data that underpin it. At the level of the macroeconomy, what we know about a country's participation in GVCs and the trade in value added has been derived from sectoral input-output (I-O) tables. Reliance on I-O tables comes with stark assumptions that can significantly bias the widely used GVC-related statistics.[3]

This paper aims to shed light on the bias that is generated when GVCs and trade in value added are measured using I-O table data. We focus on one key issue: I-O tables aggregate out well-documented firm-level heterogeneities in cross-border trade. The empirical regularity of our primary concern is that more export-oriented firms tend to be more reliant on imports (see for example Bernard, Jensen, Redding, and Schott, 2012; Amiti, Itskhoki, and Konings, 2014; Blaum, 2019). To see how aggregating out such heterogeneities can bias I-O table based measures of trade in value added, consider a simple economy with two firms in Table 1, which contains the two key heterogeneities. First, there is heterogeneity in import intensity across firms, as the share of sales associated with imported inputs is higher for firm 1. Second, there is heterogeneity in export intensity, as the share of sales exported is higher for firm 1. We need both of these heterogeneities to create a discrepancy between the measure of import content of exports computed using aggregate and firm-level data.

Table 1: Economy with two firms

	Imports	Sales	Exports
Firm 1	50	100	50
Firm 2	10	100	10

An I-O table aggregates both firms and computes the import content of exports as (imported inputs)/(sales)=(50 + 10) / (100 + 100) = 0.3. Thus, import content of gross exports is 30% and the remaining 70% are exports of own value-added, i.e., domestic value-added share of gross exports is 0.7. When the same calculation is implemented at the firm-level, imported input share of sales is

[1] See OECD's Trade in Value Added (TiVA) database, which reports statistics on value-added exports.

[2] See Hummels, Ishii, and Yi (2001), Johnson and Noguera (2012), Johnson (2014), Koopman, Wang, and Wei (2014), Timmer, Erumban, Los, Stehrer, and de Vries (2014), and Pahl and Timmer (2019).

[3] See Johnson (2018) for a discussion of this topic.

$50/100 = 0.5$ for firm 1 and $10/100 = 0.1$ for firm 2. Weighting by firms' exports we arrive at the economy's import content in exports of $(50/60) * 0.5 + (10/60) * 0.1 = 0.43$, which is a larger number than the one obtained from the aggregated data. In this case, aggregation leads to understated import content of exports and overstated domestic value added in exports. This simple example illustrates a more general concern about the neglect of firm-level heterogeneities: measurement based on sectoral I-O tables may bias downwards the extent of a country's GVC involvement.

Our paper studies this bias at the level of the macroeconomy. Using an accounting framework, we show that the bias can be expressed in terms of three firm-level characteristics—import intensity, export intensity, and size—and can be decomposed into two components. First, the bias has a "direct" component that stems from firm-level heterogeneities in their import and export intensities, as shown in the two-firm example in Table 1. Second, there is also an "indirect" component associated with firm-level heterogeneities in their intensities of purchases from other domestic firms and their suppliers that import.

We apply the framework to Belgium. To do so, we use comprehensive data on firm-level cross-border trade and domestic firm-to-firm sales to measure trade in value added without relying on sectoral aggregation that underpins the construction of conventional I-O tables. We first establish that our dataset is representative of the macroeconomy. We then compute measures of trade in value added within our dataset, contrasting the case where aggregation has been imposed versus the case where the variables of interest are constructed directly from firm-level data. Biases are identified by comparing the two cases.

We find that sectoral I-O tables for Belgium overstate the share of domestic value added in exports by 2-5 percentage points. To put this result in a context, note that this bias amounts to 20-50% of the 10 percentage point decline in Belgium's domestic value-added share in exports generated by the rise of GVCs over the last two decades. The paper also finds large variation in the size of sector-by-sector biases, and shows that increased sectoral detail can only partially cure the bias. Importantly, results show that the "direct" component accounts for the majority of the aggregate bias throughout our sample period, while the "indirect component', the measurement of which requires hard-to-find data on firm-to-firm sales, plays a marginal role. Armed with these findings, we extend our exercise to other countries and find substantial heterogeneity in aggregation biases across countries, ranging from clost to zero in Chile to 17 percentage points of gross exports in China.

It is important to emphasize at the outset that this is a "measurement" paper with "model-free" accounting of cross-border and within-country flows and decomposition exercises that shed light on aggregation biases implicit in measures that are derived using sectorally aggregated data. We see this as an essential—and overdue—step towards an accurate measurement of trade in value added and GVC-related statistics more broadly. We should also stress that there is nothing unique about our focus on firms in assessing the aggregation bias. The same concern arises at any level of aggregation, when import and export intensities differ.[4] The relevant question is a quantitative one: is there a significant bias from (previously unobserved) heterogeneity in import and export intensities across firms (within sectors), on top of the (already accounted for) heterogeneity in import and export intensities across sectors?

Our paper builds on a literature that studies the aggregation bias by incorporating within-sector heterogeneities into I-O tables. Several papers have estimated the aggregation bias stemming from the presence of a processing trade sector, which represents an acute case of firm-level heterogeneity in openness to trade. Koopman, Wang, and Wei (2012) differentiate between import intensities of processing/non-processing economic activities and show that in the case of China measures based on the standard sectoral I-O table significantly understate import content of gross exports. De La Cruz, Koopman, Wang, and Wei (2011) report similar results for Mexico. The downward biases estimated by these papers are large, in the range of 10-25 percentage points of gross exports. More recently, statistical authorities in several countries have pursued similar analysis with a systematic treatment of heterogeneities for all sectors.[5] Kee and Tang (2016) go a step further: authors use firm-level data on sales and cross-border trade combined with I-O table data on sectoral linkages to study the trend and drivers of the domestic content of exports in China. They show that oversampling of large firms in China's I-O table in the presence of firm-level heterogeneities biases the domestic content of exports downwards.

The contribution of our paper relative to this literature is twofold. First, our dataset allows us to bring the measurement of domestic/import content of exports (and the implementation of the Hummels, Ishii, and Yi, 2001 VS measure in particular) fully into the realm of firm-level data, thus eliminating the sectoral aggregation bias entirely. In contrast, previous literature has at least partially relied on sectoral I-O tables. Second, our framework provides a joint treatment of heterogeneities in firm size, import intensities, and export intensities in generating the aggregation bias—three firm-level characteristics that can fully characterize the bias. Previous literature, instead, has focused on a specific dimension of heterogeneity (e.g., importer status, engagement in processing trade, firm size).

Our study complements a wider literature on measurement issues with GVC-related statistics that are derived from sectoral I-O tables. Feenstra and Jensen (2012) assess the "import proportionality" assumption that underlies I-O table construction and argue that this assumption can significantly mismeasure the extent of offshoring. While our paper does not directly address the distortions resulting from the imposition of the "import proportionality" assumption, the firm-level approach that

[4]Indeed, the heterogeneities could also be significant within firms at the product level, where our analysis implicitly imposes a proportionality assumption: both domestic sales and exports are assumed to use imported inputs in the same proportion. To our knowledge there are currently no data available to implement our exercise at the product level.

[5]Hambÿe, Hertveldt, and Michel (2018) for Belgium split I-O table's sectors based on export intensities and find a small (1 percentage point of gross exports) impact for heterogeneity on the import content of exports. Wu and Sabuhoro (2018) examine heterogeneities in firm size, export status, and ownership and find that standard I-O tables generate a 5 percentage point downward bias in the import content of exports for Canada. Yamano and Webb (2018) lay out a broader ongoing agenda for supply-use and input-output tables to better account for firm-level heterogeneities.

we develop does not require the assumption. Similar to our paper, de Gortari (2018) focuses on the aggregation as a source of the bias and shows that aggregating out heterogeneities in input use by export destination can bias conventional GVC statistics. Our paper, instead, focuses on an aggregation bias stemming from firm-level heterogeneities in overall import and export intensities, as opposed to destination-specific ones.[6]

The structure of the rest of the paper is as follows: Section 2 lays out the accounting framework for decomposing gross exports into exports of domestic and imported contents. The section also defines and decomposes the aggregation bias and discusses how firm-level heterogeneities can create the aggregation bias. Section 3 describes the Belgian data. Section 4 presents the paper's findings for Belgium. Section 5 extends our empirical results to other countries. Finally, Section 6 concludes and discusses implications for future research on the topic.

2 Framework for Measuring Trade in Value Added

Because we only have the detailed firm-level data for one country, the paper restricts attention to value-added trade statistics that can be derived from national I-O tables. We focus on the measure of "vertical specialization (VS)" introduced by (Hummels, Ishii, and Yi, 2001), that captures the share of imported contents in exports. The usage of I-O tables or in our case firm-to-firm transaction datasets in computing the VS allows one to measure both the "direct" import content—exporting firm's outlays on imports per dollar of its exports—and the "indirect" import content—exporting firm's outlays on imports per dollar of exports through its purchases of inputs from domestic suppliers and their suppliers that import. The difference between the import content of exports and gross exports is our proxy for the value-added trade, i.e., $1-\text{VS}$ measures the domestic value-added share in gross exports.[7]

More recent value-added trade measures utilize information from global I-O tables.[8] However, in practice, results based on the VS and the more full-fledged measures, such as the value-added to gross exports (VAX) ratio, are very similar. For example, Johnson and Noguera (2012) report that for Belgium, VAX in 2004 is 0.475 and 1-VS is 0.478. For other countries the difference is equally small.[9]

[6]Addressing both heterogeneities at the firm-level in a unified framework would require firm-to-firm domestic and cross-border transactions data for multiple countries, which at present are not available.

[7]To simplify the expressions, our derivations focus on the import content of exports, VS, rather than the domestic value added contents in exports, $1-\text{VS}$.

[8]The main difference is that these measures account for third-country effects, which by construction are absent in a Belgium versus the rest of the world setting of a national I-O table.

[9]See Table 3 in June 2009 working paper version of Johnson and Noguera (2012).

2.1 Firm-Level Vertical Specialization Measure

We first define the VS measure allowing for heterogeneities at the firm-level and then illustrate how the VS measure can be decomposed into the direct and indirect components.[10]

Consider the following notations, with $n, m, l \in N$ denoting firms, $s, k \in S$ denoting sectors, and $i, j \in \{H, R\}$ denoting countries where H stands for Home country and R stands for the rest of the world. $\overset{N\times N}{\mathbf{x}}_{i,i}$ is a matrix of firm-to-firm sales flows within country i where rows correspond to suppliers and columns correspond to buyers, and $\overset{N\times 1}{y}_{i}$ is a vector of firm-level gross outputs in country i. Cross-border flows are denoted by $\overset{N\times 1}{x}_{H,R}$ or $\overset{N\times 1}{x}_{R,H}$, where the former represents Home firms' exports and the latter represents Home firms' imports. Denote by tilde the input flows expressed as a share of destination's gross output: $\overset{N\times N}{\widetilde{\mathbf{x}}}_{i,j} = \overset{N\times N}{\mathbf{x}}_{i,j} \oslash \iota \overset{N\times 1}{y}'_{j}$, where $\iota \equiv \overset{N\times 1}{1}$.

With these notations, the VS measure for Home can be expressed as

$$VS_H = \left[\overset{N\times 1}{\widetilde{\mathbf{x}}}'_{R,H} \left[\overset{N\times N}{\mathbf{I}} - \overset{N\times N}{\widetilde{\mathbf{x}}}_{H,H} \right]^{-1} \overset{N\times 1}{x}_{H,R} \right] \Big/ \left[\iota' \overset{N\times 1}{x}_{H,R} \right], \tag{1}$$

where $\left[\overset{N\times N}{\mathbf{I}} - \overset{N\times N}{\widetilde{\mathbf{x}}}_{H,H} \right]^{-1}$ is the Leontief inverse matrix. The VS measure is expressed relative to gross exports, so that the value always falls between 0 and 1. We alternatively also consider the measure in terms of nominal flows, in which we consider the numerator of the above,

$$\begin{aligned} XVS_H &= \iota' \overset{N\times 1}{x}_{H,R} VS_H \\ &= \overset{N\times 1}{\widetilde{\mathbf{x}}}'_{R,H} \left[\overset{N\times N}{\mathbf{I}} - \overset{N\times N}{\widetilde{\mathbf{x}}}_{H,H} \right]^{-1} \overset{N\times 1}{x}_{H,R}. \end{aligned} \tag{2}$$

To gain intuition about this measure, we decompose the nominal VS measure in the following way:

$$\begin{aligned} XVS_H &= \overset{N\times 1}{\widetilde{\mathbf{x}}}'_{R,H} \left[\overset{N\times N}{\mathbf{I}} + \overset{N\times N}{\widetilde{\mathbf{x}}}_{H,H} + \overset{N\times N}{\widetilde{\mathbf{x}}}_{H,H} \overset{N\times N}{\widetilde{\mathbf{x}}}_{H,H} + \ldots \right] \overset{N\times 1}{x}_{H,R} \\ &= \underbrace{\overset{N\times 1}{\widetilde{\mathbf{x}}}'_{R,H} \overset{N\times 1}{x}_{H,R}}_{XVS_H^{dir}} + \underbrace{\overset{N\times 1}{\widetilde{\mathbf{x}}}'_{R,H} \overset{N\times N}{\widetilde{\mathbf{x}}}_{H,H} \overset{N\times 1}{x}_{H,R} + \overset{N\times 1}{\widetilde{\mathbf{x}}}'_{R,H} \overset{N\times N}{\widetilde{\mathbf{x}}}_{H,H} \overset{N\times N}{\widetilde{\mathbf{x}}}_{H,H} \overset{N\times 1}{x}_{H,R} + \ldots}_{XVS_H^{indir}}. \end{aligned} \tag{3}$$

In a special case when the Leontief inverse matrix is an identity matrix, $\left[\overset{N\times N}{\mathbf{I}} - \overset{N\times N}{\widetilde{\mathbf{x}}}_{H,H} \right]^{-1} = \mathbf{I}$, no domestic inputs are used in production. Hummels, Ishii, and Yi (2001) denote this case as the "direct" case. This assumption simplifies the formula considerably. The resulting measure, XVS_H^{dir}, computes

[10] As explained in Section 3, in the data we only observe one primary sector for each firm. Therefore we assume throughout the paper that a firm's use of inputs in the production does not vary across its buyers.

gross exports that represent the *direct* import content of exports within each firm. For the more general cases of the Leontief inverse matrix when $\left[\overset{N\times N}{\mathbf{I}} - \overset{N\times N}{\widetilde{\mathbf{x}}}_{H,H}\right]^{-1} \neq \mathbf{I}$, the additional term XVS_H^{indir} takes into account the impact of domestic input linkages on the import content of exports as imports can be embedded in domestic inputs and exported *indirectly* across domestic firms.

The nominal VS measure, XVS_H, defined above and its direct and indirect components, XVS_H^{dir} and XVS_H^{indir}, are scalars when they are defined at the country-level. We will also work with the analogous measures defined at the sector-level, where $XVS_H = \sum_s XVS_s$, and study the aggregation bias in these sector-level measures. In particular, the direct component of the nominal VS measure for sector s, XVS_s^{dir}, is defined as

$$XVS_s^{dir} = \sum_{n\in N_s} \frac{x_{R,n}}{y_n} x_{n,R}, \tag{4}$$

where N_s denotes the set of firms in sector s. Analogously, the indirect component of the nominal VS measure for sector s is defined as

$$XVS_s^{indir} = \underbrace{\sum_{n\in N_s}\sum_m \frac{x_{R,m}}{y_m}\frac{x_{m,n}}{y_n}x_{n,R}}_{XVS_s^{indir1}} + \sum_{n\in N_s}\sum_m\sum_l \frac{x_{R,l}}{y_l}\frac{x_{l,n}}{y_m}\frac{x_{m,n}}{y_n}x_{n,R} + \cdots, \tag{5}$$

where we denote its first term coming from firms' first-link suppliers by XVS_s^{indir1}.

2.2 Aggregation Bias

Next we turn to the aggregation bias, deriving in turn its direct and indirect components. We define the aggregation bias of the nominal VS measure for each sector as the difference between the measure derived using firm-level data and the measure derived with sectoral aggregation: $XVS_s^{bias} = XVS_s - XVS_{IO,s}$. Analogous to the nominal VS measure, total bias for each sector has the direct and indirect components, $XVS_s^{bias} = XVS_s^{bias,dir} + XVS_s^{bias,indir}$. We define the "direct" bias as the difference between the measure of XVS_s^{dir} and its analogous measure but with sectoral aggregation, $XVS_s^{bias,dir} = XVS_s^{dir} - XVS_{IO,s}^{dir}$. The computation of the term $XVS_{IO,s}^{dir}$ requires aggregating firm-level variables (sales, imports, and exports) into sectors, in line with the conventional practice that utilizes I-O tables:

$$XVS_{IO,s}^{dir} = \frac{\sum_{n\in N_s} x_{R,n}}{\sum_{n\in N_s} y_n} \sum_{n\in N_s} x_{n,R}. \tag{6}$$

Firm-level studies have identified three key dimensions of firm-level heterogeneity: (i) size, (ii) intensity of imported inputs, and (iii) export intensity. The direct bias, $XVS_s^{bias,dir}$, can be further decomposed in terms of firm-level deviations from the "representative" sectoral firm along these three dimensions as follows:

$$XVS_s^{bias,dir} = \sum_{n \in N_s} \Delta\alpha_n^M \Delta\alpha_n^X y_n,$$
$$\Delta\alpha_n^M = \alpha_n^M - \widetilde{\alpha}_s^M$$
$$\Delta\alpha_n^X = \alpha_n^X - \widetilde{\alpha}_s^X \tag{7}$$

where $\alpha_n^M \equiv x_{R,n}/y_n$ denotes firm's import intensity and $\alpha_n^X \equiv x_{n,R}/y_n$ denotes export intensity.[11] For import and export intensities, we define sectoral weighted means as $\widetilde{\alpha}_s^M = \sum_n x_{R,n}/\sum_n y_n$ and $\widetilde{\alpha}_s^X = \sum_n x_{n,R}/\sum_n y_n$, and denote firm-level deviations from the means as $\Delta\alpha_n^M$ and $\Delta\alpha_n^X$. Equation (7) shows that if all firms in a sector had import intensity of $\widetilde{\alpha}_s^M$ and/or export intensity of $\widetilde{\alpha}_s^X$, then we obtain $XVS_s^{dir} = XVS_{IO,s}^{dir}$ and the sectoral aggregation would not generate any direct biases.

Equation (7) also implies that the direct bias is the numerator of the weighted covariance between the intensities α_n^M and α_n^X, with weights being firms' sales.[12]

$$XVS_s^{bias,dir} = \sum_{n \in N_s} y_n \frac{\sum_{n \in N_s} \Delta\alpha_n^M \Delta\alpha_n^X y_n}{\sum_{n \in N_s} y_n}$$
$$= N_s \bar{y}_s * cov^{w(y_n)}(\alpha_n^M, \alpha_n^X), \tag{8}$$

where we denote the unweighted mean of firms' sales in sector s by $\bar{y}_s$. If firms that are import intensive relative to the sectoral weighted average also tend to be export intensive, then the term $XVS_s^{bias,dir}$ becomes positive and the I-O table based VS measure would be downward biased.

The covariance term in equation (8) implies that the direct bias would be largest in an economy where a subset of firms specialize in international trade. That is, where firms with both import and export intensities close to one account for much of each sector's imports and exports. Consistent with this observation, when applying this framework to the Belgian data in Section 4, we show that including re-export activities of Belgian firms in the import and export statistics greatly magnifies the size of the direct bias.[13]

It is worth stressing that all three firm-level heterogeneities—size, import intensity, and export intensity—affect the magnitude of the direct bias in equation (8), as the covariance term is of firm's import and export intensities, with their size as denominators. In Appendix B.2 we decompose the direct bias into *unweighted* covariance terms that help to flash out in more detail how interactions between the three heterogeneities generate the aggregation bias. Further, the empirical results in Appendix C.2 show that the unweighted covariance between import and export intensities approximates well the direct bias.

[11] See Appendix B.1 for derivation.

[12] See Price (1972).

[13] Utilizing an unweighted covariance term that proxies well the weighted covariance term, Appendix B.3 demonstrates that the inclusion of re-exports systematically increases the aggregation bias.

We next turn to characterizing the aggregation bias of the indirect component of the nominal VS measure, $XVS_s^{bias,indir} = XVS_s^{indir} - XVS_{IO,s}^{indir}$, where $XVS_{IO,s}^{indir}$ is computed by first aggregating the firm-level variables for each sector. To preserve analytical tractability, we restrict attention to the bias in the indirect component of the nominal VS measure coming from firms' first-link suppliers, XVS_s^{indir1}.[14] This bias is defined as $XVS_s^{bias,indir1} = XVS_s^{indir1} - XVS_{IO,s}^{indir1}$, where

$$XVS_{IO,s}^{indir1} = \sum_{k\in S} \frac{\sum_{m\in N_k} x_{R,m}}{\sum_{m\in N_k} y_m} \frac{\sum_{n\in N_s}\sum_{m\in N_k} x_{m,n}}{\sum_{n\in N_s} y_n} \sum_{n\in N_s} x_{n,R}. \tag{9}$$

Similarly to the direct bias, the term XVS_s^{indir1} can be decomposed in terms of firm-level deviations from the "representative" sectoral firm along the import and export intensities and the intensities of purchases from other firms. Consider firm n in sector s, and firm m in sector k. Denoting firm n's share of purchases from firm m by $\alpha_{m,n} = \frac{x_{m,n}}{y_n}$, and sector s firms' weighted average share of purchases from a firm in sector k by $\tilde{\alpha}_{k,s} = \frac{1}{N_k}\frac{\sum_{n\in N_s}\sum_{m\in N_k} x_{m,n}}{\sum_{n\in N_s} y_n}$, and the firm-pair deviation from these weighted averages as $\Delta\alpha_{m,n} = \alpha_{m,n} - \tilde{\alpha}_{k,s}$, we obtain the following:[15]

$$\begin{aligned} XVS_s^{bias,indir1} =& \sum_{n\in N_s}\sum_{k\in S}\sum_{m\in N_k} \Delta\alpha_m^M \tilde{\alpha}_{k,s}\tilde{\alpha}_s^X y_n + \sum_{n\in N_s}\sum_{k\in S}\sum_{m\in N_k} \tilde{\alpha}_k^M \Delta\alpha_{m,n}\Delta\alpha_n^X y_n \\ &+ \sum_{n\in N_s}\sum_{k\in S}\sum_{m\in N_k} \Delta\alpha_m^M \Delta\alpha_{m,n}\tilde{\alpha}_s^X y_n + \sum_{n\in N_s}\sum_{k\in S}\sum_{m\in N_k} \Delta\alpha_m^M \Delta\alpha_{m,n}\Delta\alpha_n^X y_n. \end{aligned} \tag{10}$$

The first term in equation (10) comes from firm-level heterogeneity in import intensities, α_m^M. If large firms tend to be more import intensive within sectors, then the sum of the firm-level deviations in import intensities relative to the sector's weighted average, $\sum_m \left(\alpha_m^M - \tilde{\alpha}_k^M\right)$, becomes negative. The second term comes from the correlation between export intensities and the intensities of purchases from other firms. If firms that are more export intensive in a sector supply more intensively from other firms, then this term becomes positive. Similarly, the third term comes from the correlation between import intensities and the intensities of purchases from other firms. If firms supply more intensively from firms that are more import intensive, then this term becomes positive. Finally, the last term comes from the interaction between the three intensities. If export intensive firms in a sector supply more intensively from import intensive firms in the supplying sector, then this term becomes positive.

[14]This restriction has an empirical backing, as we find that for Belgium the first indirect term of the bias, $XVS^{bias,indir1}$, accounts for two-thirds of the overall indirect bias, $XVS^{bias,indir}$.

[15]See Appendix B.4 for derivation.

3 Data

This section describes the data that are used to apply the framework to Belgium, including data sources, variable definitions and macro representativeness of the dataset.

3.1 Data Sources

We use information from the National Bank of Belgium (NBB) Business-to-Business (B2B) transactions database, which is an administrative data for the Value-Added Tax (VAT) system. This database records VAT-ID to VAT-ID yearly transactions among all Belgian enterprises in the private non-financial sector in the form of a panel for years 2002 to 2014. As long as the value exceeds 250 Euro, all enterprises in Belgium are required to report their yearly sales to each individual VAT-ID. See Dhyne, Magerman, and Rubinova (2015) for the detailed construction of this dataset.

We combine this information with data from the annual account filings and the international trade dataset. The annual account filings contain the primary sector (NACE Rev. 2, 4-digit), total sales, value added, location (postcode) for each Belgian VAT-ID, and other standard balance sheet items. The international trade dataset combines information from the Belgian customs records and the intra-EU trade declarations, and records the values of imported and exported goods at the VAT-ID-country-product (CN 8-digit)-year level.[16]

We use the VAT-ID as the unit of observation. Papers such as Kikkawa, Dhyne, and Magerman (2018) and Dhyne, Kikkawa, Mogstad, and Tintelnot (2020) aggregate VAT-IDs up to the firm-level using ownership filings from the annual accounts and the foreign ownership filings in the Balance of Payments survey, as they focus on pricing decisions or transmission of shocks across firms. In this paper we focus on constructing VS measures using transaction data at the most granular level, and since the sectors of Belgian enterprises' primary economic activities are recorded for each VAT-ID, we keep the VAT-IDs as the unit of observation. That said, in this paper we use the terms enterprises, VAT-IDs, and firms interchangeably.

3.2 Construction of Variables

Here we explain how we construct the VS measure. From equation (2), we need measures of domestic firm-to-firm sales, $\overset{N\times N}{\mathbf{x}}_{H,H}$, firm-level measures on domestically absorbed imports, $\overset{N\times 1}{\mathbf{x}}_{R,H}$, exports, $\overset{N\times 1}{x}_{H,R}$, and sales, $\overset{N\times 1}{y}_{H}$.

Domestic firm-to-firm sales, $\overset{N\times N}{\mathbf{x}}_{H,H}$, are directly read-off from the firm-to-firm transactions data.[17] When we construct VS measures at sectoral levels, we use information on the primary sector of economic activity of each VAT-ID, and aggregate the domestic transactions to sector-to-sector level.

[16] See Appendix A.1 for more details of the international trade dataset.

[17] We use all the firm-to-firm transactions recorded in the data. Since we do not directly observe the types of goods that are transacted, we cannot exclude deliveries of capital or final goods in a consistent manner.

For value added, we use the values reported in the annual accounts in our baseline specification. Each enterprise's total output, $\overset{N\times 1}{y}_{H}$, is computed as the sum of value added, inputs purchased from other firms, and imports. Using this measure of total output, we also construct firms' outputs that are for capital formation or for final consumption. We identify these as the residual values of firms' output after subtracting their sales to other firms and their exports.

It is worth pointing out that the total output of firms calculated using the reported value added and purchased inputs does not necessarily match the total output that firms report in the annual accounts. Given the centrality of the firm-level output variable for our calculations, we also consider firm's reported output as an alternative measure.[18] In this case, the value added is computed as the residual of firms total output reported in the annual accounts, after subtracting the sum of purchases from other firms and imports. Sales for capital formation/final consumption in this case become the value of total output reported in the annual accounts less of their sales to other firms and their exports.[19]

Following the 2008 System of National Accounts (SNA), domestically absorbed goods imports, $\overset{N\times 1}{\mathbf{x}}_{R,H}$, exclude re-exports. We identify cross-border trade as re-exports if a Belgian VAT-ID imports and exports the same good in a given year.[20] Re-exports identified with this procedure amount to two-thirds of re-exports reported in the Belgiam I-O table, with a correlation between the two re-export series at the sectoral level of 0.95. Nevertheless, in later sections we present results based on both data with and without these identified re-exports. After identifying re-exports, we reclassify domestically absorbed imports from CN 8-digit level to NACE 4-digit level. We also classify these imports to either imported intermediate goods or to capital/consumption goods, using the Broad Economic Categories (BEC) classification.[21] When we construct VS measures at the sectoral level, domestically absorbed intermediate imports are aggregated up using the imported goods' NACE classification and the importers' NACE classification. Domestically absorbed capital/consumption goods are aggregated up using the imported goods' NACE classification.

Exports of goods, $\overset{N\times 1}{x}_{H,R}$, also exclude re-exports.[22] Analogous to imported goods, we also reclassify CN 8-digit codes to NACE 4-digit codes. When we construct VS measures at the sectoral level, these exports are aggregated using the exported goods' NACE classification and the exporters' NACE classification.

We consider VS measures constructed using VAT-IDs that report positive labor costs. This sample

[18]Our preferred measure of firms' output is the one computed from the reported value added and purchased inputs. This is because output in the annual accounts is only reported by large firms that are required to do so. See footnote 9 of Dhyne, Magerman, and Rubinova (2015) for the reporting thresholds.

[19]This alternative measure of sales to final demand require the total output to be reported in the annual accounts. For smaller firms that do not report the total output, we treat the sales to final demand as zero.

[20]Re-exports are a specific subcategory of gross exports and imports, which 2008 SNA defines as import/export activities, were the nature of the good does not change and there is change in ownership from a non-resident to resident, thus requiring some treatment by the national accounts and balance of payments statistics. According to 2008 SNA, re-exports should be excluded from gross exports and imports. In this procedure, re-exports are identified at the VAT-ID-CN8-product level, and defined as the minimum of the value of exports and the value of imports.

[21]See Appendix A.2 for details.

[22]For consistency, identified re-exports are also excluded from firms' sales.

selection removes VAT-IDs that are inactive or VAT-IDs that are mostly self-employed. This reduces the value-added covered in the sample by around 8 percent in 2010.

3.3 Macro Representativeness

This section examines macro representativeness of our data. Representativeness is essential, as we aim to study bias in the measurement of *aggregate* value-added trade. We start by comparing with national accounts the nominal levels of key macro aggregates that impact the VS measure, followed by their sectoral composition. Separately, we compare VS measures from a sectorally aggregated version of our firm-level dataset and national I-O tables. The 2016 vintage of the World Input-Output Database (WIOD), aggregated to Belgium and the-rest-of-the-world, is used as the national accounts' counterpart in all comparisons.

Aggregate variables

Figure 1 compares the four key variables that enter the VS calculation in equation (2): gross output, gross exports as well as intermediate domestic and imported non-service inputs. We report the comparison separately for non-service sectors and the aggregate economy.[23]

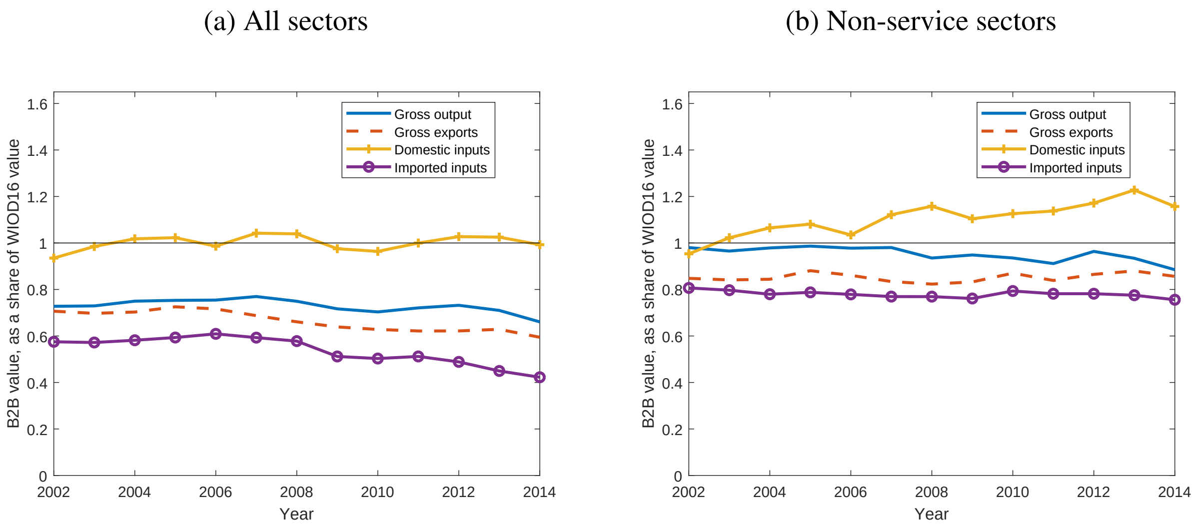
Results reveal that for non-service sectors we can broadly match the nominal levels of macro variables.[24] Any deviations are within a 20% range. When services are included, the fit deteriorates. This is to be expected, as our data does not cover cross-border trade in services and likely underreports aggregate economic activity in the service sector. We find that representativeness is broadly stable over time.[25]

[23] Re-exports are not separately identified in WIOD, which lumps them together with gross trade (see discussion on page 21 in Timmer et al., 2012 and Appendix B in Timmer, Los, Stehrer, and de Vries, 2016). Hence, to compare our firm-level data with WIOD, we add the identified re-exports back to the firm-level imports, exports and sales.

[24] We obtain similar results when using the alternative measure of value added based on the firms' reported output. See Appendix C.1.

[25] Intermediate (domestic and imported) input comparison for subsectors can in principle be based on the source or the destination of the input. In Figure 1 results for imported inputs are reported by source (i.e., imports of non-service inputs only), while for domestic inputs results are reported by destination (i.e., domestic inputs into non-service sectors only). The fit deteriorates for other subsets in the input matrix. This is to be expected, because of the more limited coverage of the service sector inputs and activities, and the lack of data on imports of services.

Figure 1: Comparison of key aggregate series



Notes: The figure displays how aggregate variables computed from the Belgian B2B database compare with the analogous variables in WIOD. The left panel shows the ratios of the Belgian B2B aggregates over those of WIOD for all sectors, and the right panels shows the same ratios for non-service sectors.

Sectoral shares

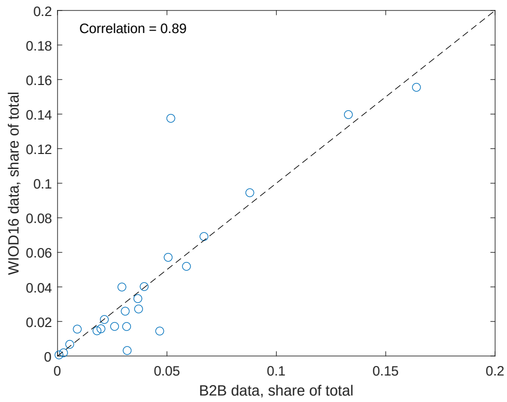
A representative aggregate series can potentially hide sectoral disparities. To address this concern, Figure 2 compares sectoral composition for non-service sectors of economic activity in the WIOD and our firm-level data for year 2010, using the most detailed sectoral breakdown in WIOD.[26] Despite significant deviations for some sectors, overall, we find that sectoral shares for the four variables of interest are highly correlated. Correlations for other sample years are equally strong, with the mean correlation of 0.88.[27]

[26] Firm-level data is aggregated accordingly by mapping NACE 2-digit sectors into WIOD's sectoral definitions.

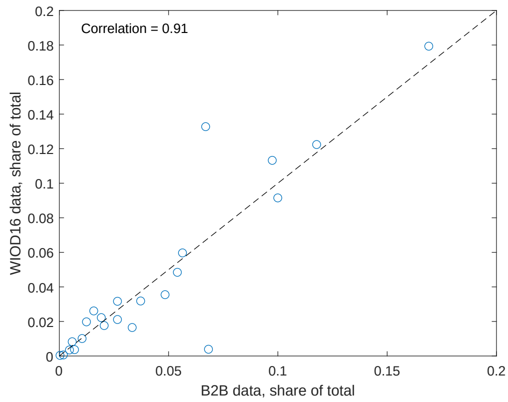
[27] Again, we obtain similar results when using alternative measures of value added, with results reported in Appendix C.1.

Figure 2: Comparison of sectoral shares in non-service sectors, 2010

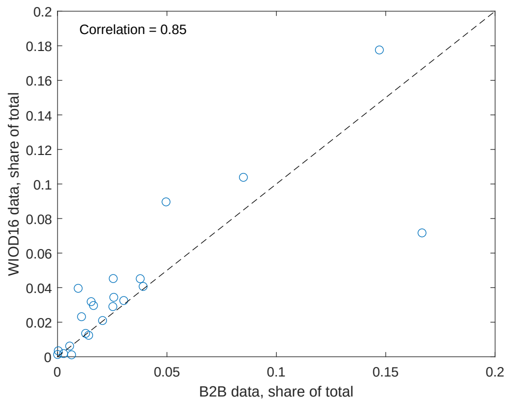
(a) Gross output



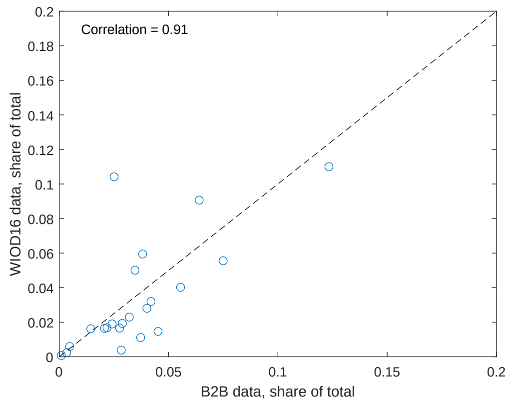
(b) Gross exports



(c) Imported non-service inputs



(d) Domestic inputs



Notes: The figure compares the distribution of sectoral shares for key macro variables—gross output, gross exports, imported inputs from non-service sectors and all domestic inputs—from the Belgian B2B database with the analogous variables from WIOD.

VS measure based on sectoral aggregates

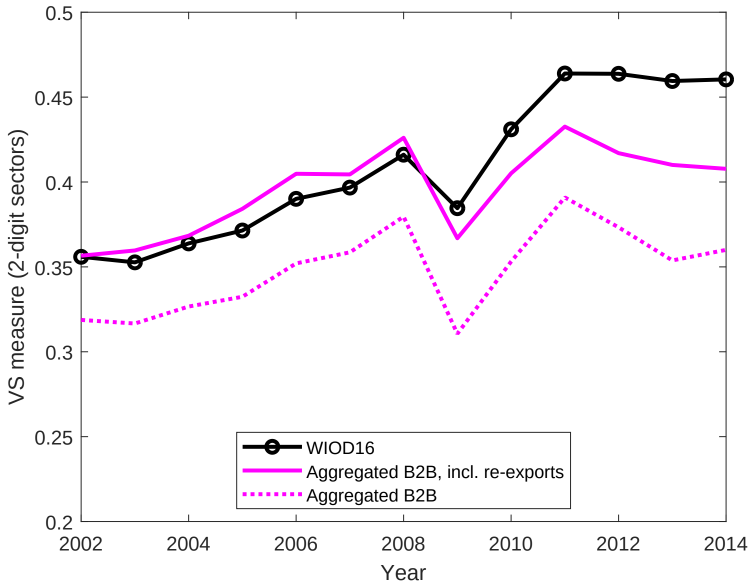
Figure 3 replicates the VS measure using our firm-level data by first aggregating data to the WIOD's 56 sectors and then calculating the VS measure for 2002-2014. The resulting comparison simultaneously relies on all input variables at the sectoral level and can be interpreted as a summary statistic for the macro representativeness of the firm-level dataset. As was the case with macro series in Figure 1, comparison of the VS measure with WIOD require that re-exports be added back to the firm-level imports, exports and sales.

Results show that, when sectorally aggregated, firm-level data capture the gradual rise in the WIOD-based VS measure during 2002-2008, as well as the relative stagnation after 2010, although some discrepancies emerge between the two series in the latter part of the sample. For completeness, the figure also reports the VA measure based on our baseline data that excludes re-exports, as required by 2008 SNA. As expected, re-exports shift the VS measure upwards level-wise, as by definition import content of re-exports is close to 100 percent (i.e., VS measure is close to 1).

It is worth stressing that the objective of the comparison made in Figure 3 is to establish representativeness. The identification of the aggregation bias in the VS measure, presented in the next section, is based exclusively on the firm-level dataset. Hence, the deviations in the VS measure between the WIOD and our aggregated firm-level data, as reported in Figure 3, do not impact the identified aggregation bias.

To summarize, we find that the firm-level dataset is representative of the macroeconomy and provides a suitable laboratory for studying the impact of the aggregation bias on the measurement of trade in value added.

Figure 3: Comparison of the VS measure (based on 2-digit sector data)



Notes: The figure compares three time-series of the VS measure: one constructed using the WIOD data and two using the Belgian B2B database aggregated at the 2-digit sector level with and without re-exports.

4 Results

This section applies the Belgian data to assess the size of the bias in the VS measure resulting from sectoral aggregation of firm-level data. The quantification is implemented using the value-added trade measurement framework of Section 2.

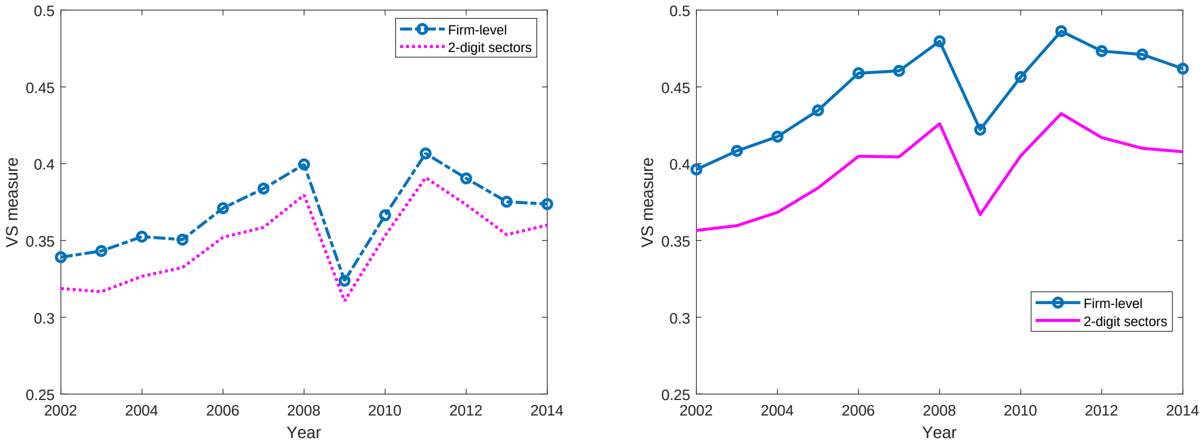
4.1 Aggregation Bias

Macro-level results for the role of the aggregation bias are reported in Figure 4, which compares the VS measure calculated directly from firm-level data (equation (1)) with the same measure computed by first aggregating firm-level data into sectors and then applying the VS formula.[28] As expected, we find that aggregating out firm-level heterogeneities leads to an *underestimate* of the import content of gross exports. In the figure this result is captured by the persistently lower VS measure when computed using 2-digit sectoral data. A key implication for our paper is that domestic value added in exports (i.e., 1-VS) is *overestimated*.[29]

In terms of magnitudes, estimates based on firm-level data suggest that on average over the sample period for each 1 euro of gross exports, 37 cents represent import content (see dashed line in left-side panel in Figure 4) and the remaining 63 cents are domestic value added in exports. Aggregating out firm level heterogeneities (dotted line), as done by I-O tables, on average, reduces the import content by 2 cents and, correspondingly, overstates the domestic value added in exports by 2 cents (i.e., an increase to 65 cents). The size of the bias is stable over time, varying in the range of 1.3-2.6 percentage points of gross exports during the 2002-2014 period.

Figure 4: Aggregation bias in the VS measure

(a) 2008 SNA (excl. re-exports) (b) 1993 SNA (incl. re-exports)



Notes: The figure displays the VS measures computed from the Belgian B2B database, one without aggregation (Firm-level) and another with sectoral aggregation (WIOD's 2-digit sectors). The left panel shows the time series of the VS measure when we exclude identified re-exports, and the right panel shows the time series of the VS measure when we include identified re-exports.

Two pertinent findings warrant a discussion. First, consistent with the discussion in Section 2.2 and Appendix B.3, the size of the bias increases when the identified re-exports are added back to firm-level imports, exports, and sales. While this modification deviates from the SNA 2008, the inclusion of re-exports is consistent with historical SNA 1993 national accounts data and I-O table compilation practices and therefore provides a more relevant estimate of the aggregation bias for I-O table based

[28] In line with conventional global I-O tables, NACE Rev.2 2-digit sectoral definitions (divisions) were used in the aggregation. To facilitate comparability with WIOD, the NACE 2-digit sectors were further aggregated to match the WIOD's 56 sectors.

[29] The convention used in this paper labels this bias as *positive*, based on the definitions of the bias in Section 2.

statistics on trade in value added. With re-exports included, the average bias increases from 2 to 5 cents for each euro of gross exports (see right-side panel in Figure 4). This finding suggests that the size of re-exports, which varies considerably across countries, is an important determinant of the magnitude of the bias.[30]

Second, one approach to reducing the bias would be to increase the sectoral detail. Our dataset allows us to examine the extent to which the bias in Figure 4 can be reduced via this avenue. We find that zooming in from 56 2-digit sectors used by WIOD to 267 3-digit NACE sectors reduces the aggregation bias by around 25%. A further breakdown of sectoral data into 600 4-digit NACE sectors reduces the aggregation bias in the Belgian data by 35-50% relative to the same WIOD 56-sector baseline.[31]

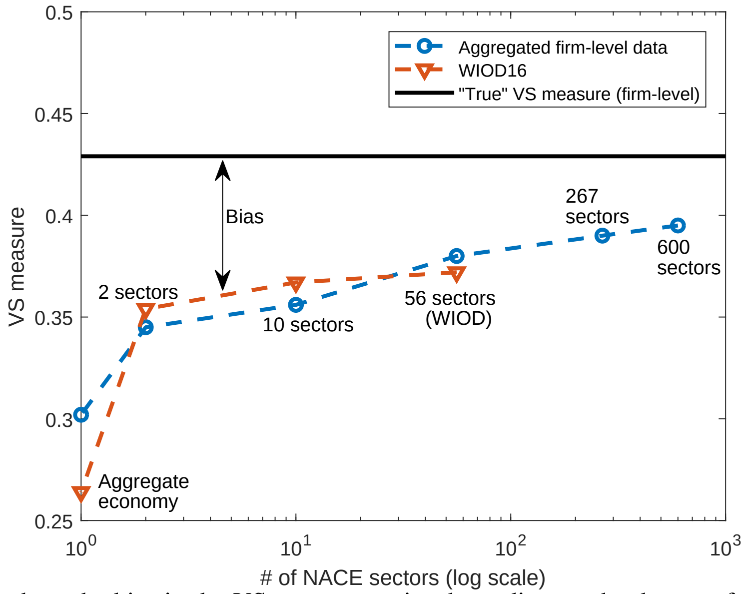
To put these magnitudes in a context, Figure 5 depicts the bias in the VS measure for varying degrees of sectoral aggregation, benchmarked against a computation done at the firm level. Where feasible, the figure also shows analogous aggregation results from the WIOD.[32] As one would expect, the figure reveals that greater sectoral detail helps to reduce the bias. Nevertheless, even with the 4-digit sectoral detail a sizable bias in the VS measure remains.

[30]2015 I-O tables for EU countries, published by the Eurostat, report re-export shares in gross exports varying from 2% in Italy to 50% in the Netherlands. While Belgium (at 28%), is on the high end, the EMU average of 13% suggests that re-exports are sizable for many economies, including at 16% in Germany.

[31]These findings refer to both cases: re-exports included and re-exports excluded.

[32]To enhance comparability, firm-level data in Figure 5 includes re-exports and results from both data sources refer to averages over the 2000-2007 period, for which there is a close match between the firm-level data and WIOD in Figure 3.

Figure 5: Aggregation bias in the VS measure by the degree of sectoral aggregation



Notes: The figure shows how the bias in the VS measure varies depending on the degree of sectoral aggregation, using the WIOD16 (red) and our firm-level data (blue). Left-most observations depict VS measures, when both datasets are aggregated to one-sector aggregate economy. Next observations to the right report results for a two-sector (tradable-nontradable) economy, followed by 10 1-digit NACE sectors, WIOD's 56 sectors, 267 3-digit NACE sectors and 600 4-digit NACE sectors. The solid black line depicts the "true" VS measure computed using firm-level data without any sectoral aggregation. To enhance comparability across the two datasets, firm-level data includes re-exports and for both datasets results refer to averages over the 2002-2007 period.

4.2 Direct and Indirect Contributions

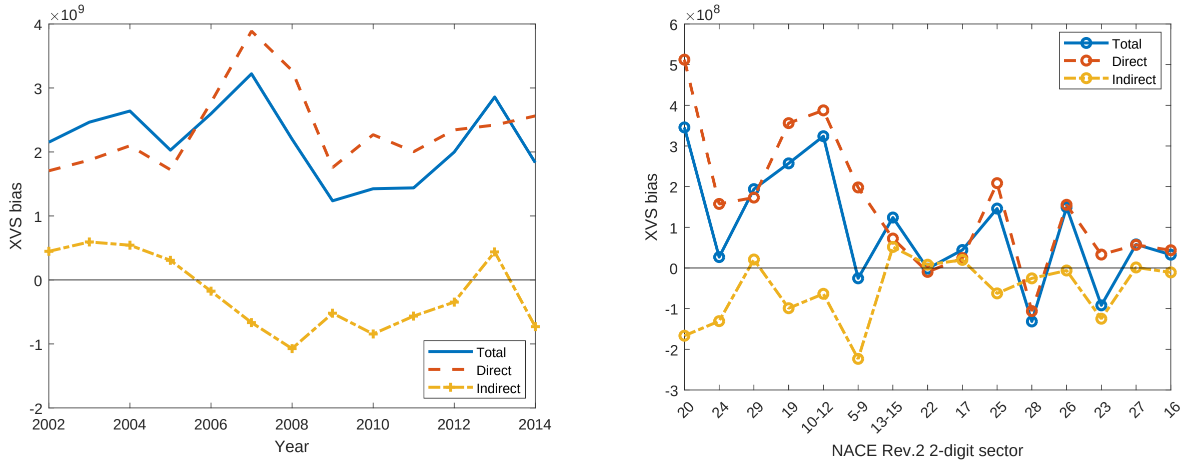
To shed more light on the bias, Figure 6 decomposes it into the direct and indirect components (see equation (3)), using the baseline data that excludes re-exports.[33] The bias plotted in the figure is identical to the one in Figure 4, but expressed in nominal terms, XVS_s. The left panel reports the evolution of the direct and indirect components of the bias over time, $\sum_s XVS_{s,t}^{bias,dir}$ and $\sum_s XVS_{s,t}^{bias,indir}$. The right panel shows the decomposition by 2-digit manufacturing sectors in a representative year, $XVS_{s,2010}^{bias,dir}$ and $XVS_{s,2010}^{bias,indir}$, with sectors ranked, left-to-right, by their contribution to the overall import content of gross exports in Belgium.

[33] Results are broadly similar when re-exports are included.

Figure 6: Contributions to the VS bias by "direct" and "indirect" components

(a) All sectors over time

(b) By sector, 2010



Notes: The left panel decomposes the bias of the nominal VS measure into the direct component, $\sum_s XVS_s^{bias,dir}$, and the indirect component, $\sum_s XVS_s^{bias,indir}$. The right panel shows the analogous decomposition for each sector, where sectors are ranked by their contribution to the overall import content of gross exports in Belgium.

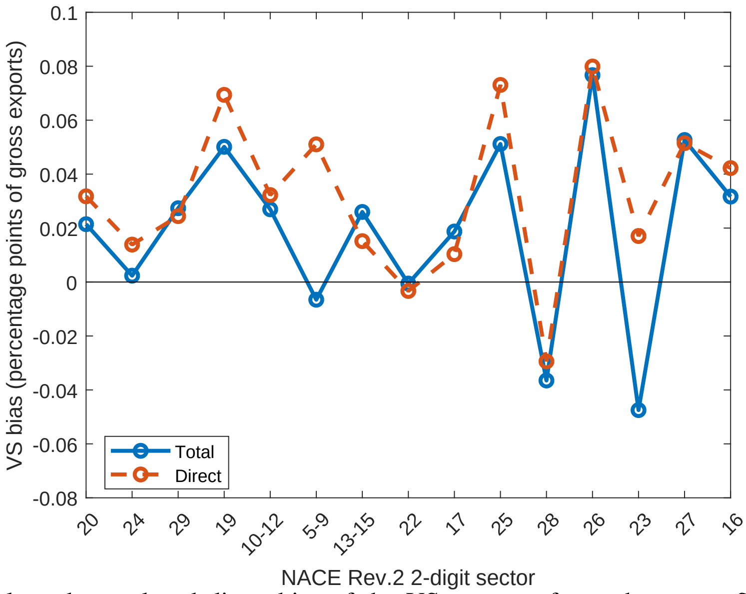
We find that the direct component of the bias closely mimics the overall bias. It accounts for 109% of the aggregate bias over the sample period, while the contribution of the indirect component, at -9%, is not systematically different from zero. The positive bias is present in the majority of sectors. Both in sectors that are the main generators of import content of gross exports in Belgium (for example, NACE Rev. 2 division 20: manufacture of chemicals), as well as in classic GVC sectors (i.e., NACE Rev. 2 division 29: manufacture motor vehicles), the dominant contribution stems from the direct component of the bias. For the median 2-digit manufacturing sector in 2010 the contribution of the direct component is 94%. The indirect component is not accounting for any sizable systematic bias across sectors, with a 6% contribution for the median manufacturing sector.

Characterizing the Direct Bias

These findings suggest that a further examination of the direct component can help our understanding of the overall bias. As shown in equation (8), the direct component of the overall bias is a sales-weighted covariance of firm-level import and export intensities. We show in Appendix B.2 that this covariance can be usefully decomposed further into (i) a simple *unweighted* covariance of import and export intensities and (ii) a set of residual terms that capture the impact of the interplay between firms' import/export intensities and firms' size on the bias. Further, in Appendix C.2 we demonstrate that the simple unweighted covariance of import and export intensities can closely capture the aggregate direct bias as well as the total bias.

Additional insights into the sectoral results can be obtained by recasting the bias in a more familiar form as percentage points of sectoral gross exports, thus emphasizing sectors with high VS measures rather than high nominal values of import content of gross exports (see Figure 7). We find that the aggregate bias (at 1.3 percentage points of gross exports in 2010) hides sizable sectoral variation. For example, 2-digit sectors that are commonly associated with supply chains (sectors 29, 26, 27) show elevated biases in the 3-8 percentage point range.

Figure 7: Direct bias by sector, 2010



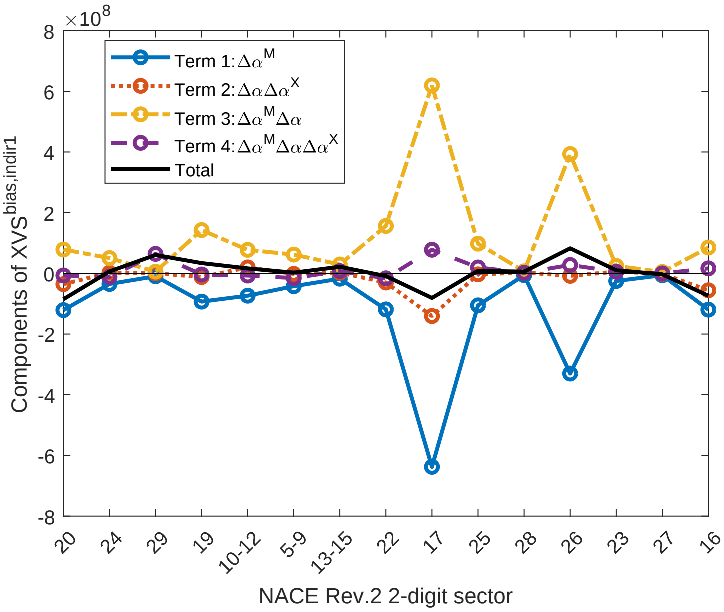
Notes: The figure displays the total and direct bias of the VS measure for each sector. Sectors are ranked by their contribution to the overall import content of gross exports in Belgium.

Characterizing the Indirect Bias

The indirect bias term accounts for a small fraction of the total bias. To characterize how this bias is generated, Section 2 focused on the first term of the indirect bias, and in Figure 8 we apply equation (10) to the data.[34] The four terms largely cancel each other out. Notably, the first term is always negative, as large firms tend to be import intensive on average (Dhyne, Kikkawa, Mogstad, and Tintelnot, 2020). We also find that across sectors, firms source more intensively from firms that are more import intensive as the third term is consistently positive. The second and the fourth terms tend to be smaller in magnitudes and exhibit both positive and negative signs, indicating that firms that are more export intensive are not always the ones sourcing more intensively from other firms.

[34]In Figure 12 of Appendix C.3 we plot the fraction that the first indirect terms explain out of the total bias.

Figure 8: Components of the first indirect term of the bias, 2010



Notes: The figure shows the decomposition of the first indirect term of the bias in each sector, $XVS_s^{bias,indir1}$. The four circled lines correspond to the four terms on the RHS of equation (10), and the solid black line corresponds to the sum of the four terms.

In sum, our key takeaways from this section are that (i) the bias in Belgium is 2-5 percentage points, depending on the treatment of re-exports, (ii) the direct bias accounts for the bulk of the overall aggregation bias and (iii) the bias can vary significantly across sectors.

5 Application to Other Countries

A key limitation of our findings so far is that they are limited to one country—Belgium. Even if Belgium is viewed as a representative small open economy, it would be important to know how the estimated bias differs with the extent and nature of a country's participation in global supply chains.

Unfortunately, the full-fledged approach implemented for Belgium is very data intensive and, to the best of our knowledge, comparable data on firm-level trade and firm-to-firm transactions are not publicly available for other countries of interest. Instead, this section leans on the finding that the direct component dominates the aggregation bias for Belgium. We assume that this is the case also for other countries and estimate the direct bias for a larger set of countries for which we could obtain firm-level data on sales, imports, exports, and the main sector of economic activity.[35]

[35] One needs data on all firm-to-firm transactions to verify whether the indirect bias is indeed small for other countries. Nevertheless, for Chile—one of the countries we use firm-level data for—Huneeus (2018) reports statistics on the firm-to-firm network that are comparable to those in Belgium (Kikkawa, Dhyne, and Magerman, 2018; Dhyne, Kikkawa, Mogstad, and Tintelnot, 2020).

Following the approach for Belgium, we first examine data of each country for macro representativeness in terms of key variables and the VS measure. The size of the direct component of the aggregation bias is then computed by applying equation (7). As we could not identify re-exports, results refer to the case with re-exports included. Furthermore, to ensure cross-country comparability, we focus on the manufacturing sector, as the firm-level data in some countries were limited to manufacturing firms.[36]

Findings for countries with available data are summarized in Table 2, which reports VS levels (columns 2-3) as well as the bias in percentage point terms (columns 4-5). To facilitate comparison of the bias across countries with widely differing VS levels, the last column of the table reports the bias in percent terms (column 6). We start by noting that results for Belgium are in line with findings of the previous section—limiting the focus to the manufacturing sector leaves the size of the direct bias, $VS^{bias,dir}$, broadly unchanged at 4 percentage points of gross exports, or 12% of the "true" direct VS. Beyond Belgium, results reveal considerable heterogeneity across countries. The very sizable aggregation bias for China (at 17 percentage points or 45% of the direct VS) is comparable to results reported in Koopman, Wang, and Wei (2012), where the authors, using a different methodology, estimate the direct aggregation bias for China to be in the 16-29 percentage point range.[37] For Vietnam, which in our sample exhibits the highest direct VS, the aggregation bias is 10 percentage points. Sizable bias is also found in France, especially when accounting for the relatively lower VS level. On the other end of the spectrum, based on a limited time frame of 2010-11, our estimates suggest close-to-zero bias in the case of Chile, which exports mostly commodities.

For the full set of countries included in Table 2 the correlation between the "true" direct VS and the aggregation bias is 0.76 in percentage point terms (0.62 in percent terms), indicating that the bias is systematically larger in countries that exhibit elevated import content of exports. To further examine the role of processing trade, we analyze data for China, which include the processing/non-processing breakdown of firm-level import and export flows. Results show that for China's manufacturing sectors the exclusion of processing import and export flows on average reduces the bias by 33% relative estimates in Table 2. This result is consistent with our earlier findings regarding the role of re-exports for Belgium – import/export intensive activities increase firm-level heterogeneities and the aggregation bias (see Appendix B.3).

Results for sector-level biases in column 5 reveal that the large sectoral heterogeneity reported for Belgium in Figure 7 is representative of other sample countries. Even in Chile, where the country-level aggregation bias is negligible, a quarter of manufacturing sectors exhibit a bias that exceeds 9 percentage points. These findings indicate that sectoral value-added trade statistics can be particularly prone to the aggregation bias.[38]

[36] See Appendix A.3 for the descriptions of data sources.

[37] See Table 2 in Koopman, Wang, and Wei (2012).

[38] Additional results on the size of sector-by-sector bias and the performance of a simple covariance statistic between import and export intensities as a proxy for the bias are available from the authors upon request. Details of the represen-

Table 2: The size of direct aggregation bias in select countries

	(1) Time frame	(2) "True" direct VS	(3) "Mis-measured" direct VS	(4) Aggregation bias, in p.p. [(2)-(3)]*100	(5) Interquartile range for sectoral biases, in p.p. (Q1;Q2;Q3)	(6) Aggregation bias, in % (4)/(2)
Belgium	2002-14	0.36	0.32	4.2	(1;3;7)	12
Chile	2010-11	0.09	0.10	-0.5	(-2;3;9)	-5
China	2000-07	0.39	0.21	17.3	(12;16;23)	45
France	2000-11	0.26	0.20	6.1	(3;6;9)	24
India	2014-15	0.18	0.14	3.3	(-2;2;7)	19
Indonesia	1992-96	0.17	0.14	2.7	–	16
Korea	2006-17	0.23	0.21	2.2	(1;3;6)	10
Latvia	2006-12	0.30	0.26	4.0	(2;4;7)	13
Vietnam	2011-13	0.45	0.35	10.2	(6;12;17)	23

Notes: VS abbreviates vertical specialization; Column 1 reports the time frame used for calculating the results; column 2 reports the average values of the "true" direct VS measure, VS^{dir}, which accounts for firm-level heterogeneity; column 3 reports the "mismeasured" average direct VS values, VS^{dir}_{IO}, based on aggregated data only, which mismeasures the direct VS; column 4 reports the average direct VS bias, $VS^{bias,dir}$, in percentage points, where a positive value implies that sectoral aggregation biases the direct VS measure downwards (see Section 2.2 for definition); column 5 reports average interquartile ranges (25%;50%;75%) for the bias at the level of 2-digit manufacturing sectors in percentage points (sufficient sectoral information not available for Indonesia); column 6 reports the average aggregation bias in percent terms. All calculations based on firm-level data.

Overall, results of this section suggest that firm-level heterogeneities can significantly bias the VS measure and other value-added trade statistics, more so in countries with a high import content of exports.

6 Conclusions

This paper studies the aggregation bias in widely used GVC measures. It would be preferable for measures such as trade in value added to be constructed using firm-level data. In practice, however, sectorally aggregated I-O tables are used. The problem with this practice is that aggregated data can bias the GVC measures.

We start by defining and deriving the aggregation bias conceptually. Focusing on the VS measure (Hummels, Ishii, and Yi, 2001), we show that the bias has a direct component—capturing firms' "direct" import and export engagements—and an indirect component—capturing firms' "indirect" exposure to imports via other domestic firms that import. The direct component is a weighted covariance of import and export intensities, which can be further decomposed into a simple covariance and a residual that accounts for heterogeneities in firm size and the interaction between trade intensities

tativeness tests are also available.

and firm size. A decomposition of the indirect component is also provided.

The paper then applies these concepts to Belgium by comparing the VS measure derived from firm-level data with the same measures derived from firm-level data that has been sectorally aggregated. We find that in the case of Belgium the bias in the VS measure, computed from sectorally aggregated data, is at 2-5 percentage points of gross exports. Furthermore, the bias is mostly stemming from its direct component.

Our paper offers several takeaways for future work on this topic. First, we show that the indirect component of the aggregate VS bias does not contribute sizably to the overall bias in the case of Belgium, including in classic supply chain sectors. We see this as an encouraging news for the measurement of supply chains directly from firm-level data, as it suggests that firm-level imports, exports and sales are sufficient to derive a good proxy for the VS measure. In the penultimate section of the paper we pursue this approach and compute the "direct" biases for eight additional countries and find sizable heterogeneity, with the biases ranging from 0 percentage points in Chile to 17 percentage points in China. This proposed approach offers several advantages over the current practice based on I-O tables. Yearly firm-level microdata are becoming increasingly available in many countries, and our approach allows one to compute more up-to-date estimates of the VS measure relative to the time lags that come with I-O tables. The approach avoids sectoral aggregation biases, and might be preferable to the alternative of incorporating firm-level heterogeneities into the I-O tables. An extension of our analysis to other countries is needed to assess the broader applicability of this proposal.

Another takeaway from the paper is that aggregation biases might be particularly concerning for sectoral GVC measures. We find that for Belgium, despite the moderate overall bias, biases in sectoral VS measures can be sizable. Thus, one needs to be particularly careful when interpreting empirical findings that are based on sectoral GVC measures.

Next, we shed light on the role that import/export intensive activities, and re-exports in particular, play in generating the aggregation bias. Results for Belgium reveal that countries with sizable re-export activities can be expected to exhibit a larger aggregation bias. A related implication from our findings is that, as countries transition from the SNA 1993 to the SNA 2008 standards, which instruct statistical offices to exclude re-exports from trade statistics, the aggregation bias can be expected to decrease. Without taking into account the impact of the change in the treatment of re-exports on the aggregation bias, researchers can mislabel a reduction in the aggregation bias as a boost to the import content of exports.

Finally, our results indirectly speak to the role that "import proportionality" assumption (see Feenstra and Jensen, 2012) plays in distorting trade in value added and other GVC statistics. The VS measure derived from our firm-level data is by construction free of distortions induced by the 'import proportionality" assumption. We find that firm-level data, when aggregated, can match the I-O table based VS measure reasonably well, as reported in Figure 3. This suggests that, at least for Belgium, the use of "import proportionality" assumption in the I-O table is not a significant source of concern for GVC statistics.

References

Amiti, M., O. Itskhoki, and J. Konings (2014): "Importers, exporters, and exchange rate disconnect," *American Economic Review*, 104, 1942–1978.

Bernard, A. B., J. B. Jensen, S. J. Redding, and P. K. Schott (2012): "The empirics of firm heterogeneity and international trade," *Annual Review of Economics*, 4, 283–313.

Blaum, J. (2019): "Global Firms in Large Devaluations," *mimeo*.

de Gortari, A. (2018): "Disentangling Global Value Chains," *mimeo*.

De La Cruz, J., R. B. Koopman, Z. Wang, and S.-J. Wei (2011): "Estimating Foreign Value-Added in Mexico's Manufacturing Exports," *USITC Office of Economics Working Paper*.

Dhyne, E., A. K. Kikkawa, M. Mogstad, and F. Tintelnot (2020): "Trade and Domestic Production Networks," *Review of Economic Studies*.

Dhyne, E., G. Magerman, and S. Rubinova (2015): "The Belgian production network 2002-2012," *National Bank of Belgium Working Paper Series*, 288.

di Giovanni, J., A. A. Levchenko, and I. Mejean (2014): "Firms, Destinations, and Aggregate Fluctuations," *Econometrica*, 82, 1303–1340.

Feenstra, R. C. and J. B. Jensen (2012): "Evaluating estimates of materials offshoring from US manufacturing," *Economics Letters*, 117, 170–173.

Hambÿe, C., B. Hertveldt, and B. Michel (2018): "Value Chain Integration of Export-Oriented and Domestic Market Manufacturing Firms," *Federal Planning Bureau Working Paper*.

Hummels, D., J. Ishii, and K. M. Yi (2001): "The nature and growth of vertical specialization in world trade," *Journal of International Economics*, 54, 75–96.

Huneeus, F. (2018): "Production network dynamics and the propagation of shocks," *mimeo*.

Johnson, R. C. (2014): "Five Facts about Value-Added Exports and Implications for Macroeconomics and Trade Research," *Journal of Economic Perspectives*, 28, 119–142.

——— (2018): "Measuring Global Value Chains," *Annual Review of Economics*, 10, 207–236.

Johnson, R. C. and G. Noguera (2012): "Accounting for intermediates: Production sharing and trade in value added," *Journal of International Economics*, 86, 224–236.

Kamali, P. (2019): "Indirect Exporter Dynamics and the Impact of Trade Liberalization," *mimeo*.

Kee, H. L. and H. Tang (2016): "Domestic Value Added in Exports: Theory and Firm Evidence from China," *American Economic Review*, 106, 1402–1436.

Kikkawa, A. K., E. Dhyne, and G. Magerman (2018): "Imperfect Competition in Firm-to-Firm Trade," *mimeo*.

Koopman, R., Z. Wang, and S. J. Wei (2012): "Estimating domestic content in exports when processing trade is pervasive," *Journal of Development Economics*, 99, 178–189.

——— (2014): "Tracing value-added and double counting in gross exports," *American Economic Review*, 104, 459–494.

Ma, Y., H. Tang, and Y. Zhang (2014): "Factor Intensity, product switching, and productivity: Evidence from Chinese exporters," *Journal of International Economics*, 92, 349–362.

Pahl, S. and M. P. Timmer (2019): "Patterns of Vertical Specialisation in Trade: Long-run Evidence for 91 Countries," *Review of World Economics*, 155, 459–486.

Price, G. R. (1972): "Extension of covariance selection mathematics," *Annals of Human Genetics*, 35, 485–490.

Timmer, M. P., A. Erumban, R. Gouma, B. Los, U. Temurshoev, G. J. de Vries, I. Arto, A. Genty, F. Neuwahl, J. M. Rueda-Cantuche, A. Villanueva, J. Francois, O. Pindyuk, J. Poschl, R. Stehrer, and G. Streicher (2012): "The World Input-Output Database (WIOD): Contents, Sources and Methods," *mimeo*.

Timmer, M. P., A. A. Erumban, B. Los, R. Stehrer, and G. J. de Vries (2014): "Slicing Up Global Value Chains," *Journal of Economic Perspectives*, 28, 99–118.

Timmer, M. P., B. Los, R. Stehrer, and G. J. de Vries (2016): "An Anatomy of the Global Trade Slowdown based on the WIOD 2016 Release," *GGDC Research Memorandum*, 162.

Wu, W. and J. B. Sabuhoro (2018): "Extended Supply-Use Tables and Trade in Value Added Measures for Canada: Preliminary Estimates, Issues and Challenges," *mimeo*.

Yamano, N. and C. Webb (2018): "Future Development of the Inter-Country Input-Output (ICIO) Database for Global Value Chain (GVC) and Environmental Analyses," *Journal of Industrial Ecology*, 22, 487–488.

Yu, M. and W. Tian (2012): "China's Processing Trade: A Firm-level Analysis," in *Rebalancing and Sustaining Growth in China*, ed. by H. McKay and L. Song, ANU Press.

A Data appendix

A.1 International trade dataset

The international trade dataset contains information from the Belgian customs records which cover extra-EU trade, and the intra-EU trade declarations. The customs records cover all extra-EU imports and exports at the VAT-ID level with values higher than 1,000 Euro or with weights bigger than 1,000kg. As for the intra-EU trade, for years prior to 2006, the dataset covers all imports and exports by Belgian enterprises whose combined imports from other EU countries that are more than 250,000 Euro a year. For years 2006 onward, the thresholds for exports and imports changed to 1,000,000 Euro and 400,000 Euro, respectively. The reporting threshold for intra-EU imports was changed to 700,000 Euro per year in 2010.

A.2 Classifying imported goods

The raw international trade dataset comes at the CN 8-digit level. We map these codes to NACE Rev. 2, 4-digit level, and also to either intermediate goods or capital/consumption goods.

To map CN 8-digit classifications to NACE 4-digit classifications, we first convert the CN codes to Harmonized System (HS) 6-digit codes. The CN codes are constructed so that their first 6 digits are identical to the contemporary HS codes. We then convert these HS codes to Classification of Products by Activity (CPA) 2008 codes, and then final these CPA codes to NACE Rev. 2 codes using the fact that CPA 2008 codes are identical to NACE Rev.2 codes up to the first 4 digits.

To map CN 8-digit classifications to intermediate or capital/consumption goods, we use the Broad Economic Categories (BEC) classification. First we convert the contemporary CN 8-digit codes to HS 2002 6-digit codes, and reclassify them to BEC Rev. 4 codes. The BEC Rev. 4 codes provide us with a broad classification of whether the good is used as intermediate goods, or for capital formation, or for final consumption. Using this information, we classify each CN 8-digit code to goods for intermediate goods, or to goods for capital/consumption goods.

A.3 Data sources of Section 5

Here we briefly describe the data sources for the analyses done in Section 5. We obtain firm-level data on sales, imports, exports, and their main sector at the 2-digit level. We focus our attention to manufacturing firms, since some datasets only provide representative sample for the manufacturing sectors. Results on the representativeness tests for these countries are available from the authors upon request.

Chile

Our data source for Chile is the plant-level panel data from the Encuesta Nacional Industrial Anual (ENIA). The ENIA contains survey data on all manufacturing establishments of more than 10 employees, including data on imports and exports. We use the ISIC Revision 3 for sectoral classification.

China

For China we merge two datasets. One is the manufacturing survey data from the National Bureau of Statistics (NBS) that covers all state-owned firms and non-state-owned firms with sales above 5 million Chinese yuan. This data contains information on firms' sales, export values, and their main sector (Classification of National Economy Industries). However, the data does not record firms' imports. Therefore we merge the NBS survey with the Chinese customs data, using information on firms' names as identifiers.

As is well-known (see for example, Yu and Tian, 2012 and Ma, Tang, and Zhang, 2014), the merged data contains some discrepancies, which we address through two additional adjustments. First, we drop firms that report positive exports in the NBS survey but are not matched with the customs data. Second, for firms that are matched in the two datasets but show different total export values in the NBS survey and the customs data, we take the export values in the NBS survey, and recover their import values as the export values in the NBS survey multiplied by the ratio of imports over exports in the customs data.

France

The data source for our French analysis is the combination of firm-level dataset from the fiscal administration in which we take firms' sales and sector information, and customs dataset in which we take information of firms' import and export values. We use the French Classification of Activities (Nomenclature d'Activités Française, NAF) for firms' sectoral classification. See di Giovanni, Levchenko, and Mejean (2014) for further details.

India

We use the Annual Survey of Industries (ASI) for India. The ASI survey covers all manufacturing establishments with over 100 workers, and smaller establishments are typically surveyed every three to five years. We adjust for this sample selection issue by using sampling weights for smaller establishments. We use India's National Industrial Classification (NIC) codes for establishments' sectoral classification.

Indonesia

The data for Indonesia comes from the Manufacturing Survey of Large and Medium-sized firms (Survei Industri). The survey covers all large and medium manufacturing firms with twenty or more employees. We use the Indonesia Standard Commodity Classification (Klasifikasi Komoditi Indonesia, KBKI) for firms' sectoral classification.

Korea

The Korean data is from the Survey of Business Activities (SBA) from Statistics Korea. The SBA covers all firms with 50 or more employees and 300 million won or greater capital, each year. The SBA follows the Korean Standard Industry Classification for industry classification.

Latvia

Results are based on merged data covering firms' financial statements and customs activities, obtained from the National Statistical Office. Resulting dataset includes a universe of manufacturing firms.

Vietnam

Firm-level data is from the Vietnamese Enterprise Survey, conducted by the General Statistics Office of the ministry of Planning and Investment of Vietnam. The survey includes data on cross-border trade flows. All firms that are at least 50% state-owned as well as all foreign-owned firms are included. For other private enterprises, all firms with at least 20 employees are included, while a random sample is chosen for firms with less that 20 employees. For further details see Kamali (2019).

B Derivations

B.1 Characterizing the "direct" bias

Arranging equation (4) gives

$$XVS_s^{dir} = \sum_{n \in N_s} \alpha_n^M \alpha_n^X y_n,$$

where $\alpha_n^M = \frac{x_{R,n}}{y_n}$ and $\alpha_n^X = \frac{x_{n,R}}{y_n}$. We can rewrite these components of XVS_s^{dir} in terms of deviations from sectoral means:

$$\begin{aligned} XVS_s^{dir} &= \sum_{n \in N_s} \left(\widetilde{\alpha}_s^M + \Delta\alpha_n^M\right)\left(\widetilde{\alpha}_s^X + \Delta\alpha_n^X\right) y_n \\ &= \widetilde{\alpha}_s^M \widetilde{\alpha}_s^X \sum_{n \in N_s} y_n + \widetilde{\alpha}_s^M \sum_{n \in N_s} \Delta\alpha_n^X y_n + \widetilde{\alpha}_s^X \sum_{n \in N_s} \Delta\alpha_n^M y_n + \sum_{n \in N_s} \Delta\alpha_n^M \Delta\alpha_n^X y_n. \end{aligned}$$

If all firms in the sector are identical, then we obtain the measure with sectoral aggregation in equation (6):

$$\begin{aligned} XVS^{dir}_{IO,s} &= \tilde{\alpha}^M_s \tilde{\alpha}^X_s \sum_{n\in N_s} y_n \\ &= \tilde{\alpha}^M_s \tilde{\alpha}^X_s \sum_{n\in N_s} \bar{y}_s, \end{aligned}$$

where $\bar{y}_s$ is the mean firm-level sales in sector s. Taking the difference, we obtain

$$XVS^{bias,dir}_s = \widetilde{\alpha}^M_s \sum_{n\in N_s} \Delta\alpha^X_n y_n + \widetilde{\alpha}^X_s \sum_{n\in N_s} \Delta\alpha^M_n y_n + \sum_{n\in N_s} \Delta\alpha^M_n \Delta\alpha^X_n y_n.$$

This expression can be simplified further. Using

$$\begin{aligned} \widetilde{\alpha}^M_s \sum_{n\in N_s} \Delta\alpha^X_n y_n &= \widetilde{\alpha}^M_s \sum_{n\in N_s} \left(\frac{x_{n,R}}{y_n} - \frac{\sum_{n\in N_s} x_{n,R}}{\sum_{n\in N_s} y_n} \right) y_n \\ &= \widetilde{\alpha}^M_s \left(\sum_{n\in N_s} x_{n,R} - \frac{\sum_{n\in N_s} x_{n,R}}{\sum_{n\in N_s} y_n} \sum_{n\in N_s} y_n \right) \\ &= 0, \end{aligned}$$

and similarly $\widetilde{\alpha}^X_s \sum_{n\in N_s} \Delta\alpha^M_n y_n = 0$, we obtain

$$XVS^{bias,dir}_s = \sum_{n\in N_s} \Delta\alpha^M_n \Delta\alpha^X_n y_n.$$

B.2 Decomposing the "direct" bias

The direct bias characterized in equation (8) involves a weighted covariance between firms' import and export intensities. Here we decompose the direct bias into unweighted covariance terms that contain only two of the three firm-level heterogeneities at a time and, hence, are easier to interpret. We obtain

$$\begin{aligned} XVS^{bias,dir}_s =& \bar{y}_s \sum_{n\in N_s} \left(\alpha^M_n - \bar{\alpha}^M_s\right)\left(\alpha^X_n - \bar{\alpha}^X_s\right) + \left(\bar{\alpha}^X_s - \tilde{\alpha}^X_s\right) \sum_{n\in N_s} \left(\alpha^M_n - \bar{\alpha}^M_s\right)(y_n - \bar{y}_s) \\ &+ \left(\bar{\alpha}^M_s - \tilde{\alpha}^M_s\right) \sum_{n\in N_s} \left(\alpha^X_n - \bar{\alpha}^X_s\right)(y_n - \bar{y}_s) + N_s \left(\bar{\alpha}^M_s - \tilde{\alpha}^M_s\right)\left(\bar{\alpha}^X_s - \tilde{\alpha}^X_s\right)\bar{y}_s \\ &+ \sum_{n\in N_s} \left(\alpha^M_n - \bar{\alpha}^M_s\right)\left(\alpha^X_n - \bar{\alpha}^X_s\right)(y_n - \bar{y}_s) \\ =& N_s \bar{y}_s cov(\alpha^M_n, \alpha^X_n) + N_s \left(\bar{\alpha}^X_s - \tilde{\alpha}^X_s\right) cov(\alpha^M_n, y_n) + N_s \left(\bar{\alpha}^M_s - \tilde{\alpha}^M_s\right) cov(\alpha^X_n, y_n) \\ &+ N_s \bar{y}_s \left(\bar{\alpha}^M_s - \tilde{\alpha}^M_s\right)\left(\bar{\alpha}^X_s - \tilde{\alpha}^X_s\right) + \sum_{n\in N_s} \left(\alpha^M_n - \bar{\alpha}^M_s\right)\left(\alpha^X_n - \bar{\alpha}^X_s\right)(y_n - \bar{y}_s), \end{aligned} \tag{11}$$

where the terms $\bar{\alpha}_s^M$ and $\bar{\alpha}_s^X$ represent the sector's unweighted average import and export intensities. The first term of equation (11) contains the unweighted covariance between import and export intensities, the second term relates to the unweighted covariance between import intensity and firms' size, and the third term relates to the unweighted covariance between export intensity and firms' size. The fourth term is a constant, and the last term is a residual coming from the triple covariance term.

Equation (11) demonstrates how the three dimensions of firm heterogeneity—firm size and import and export intensities—interact with each other to create the direct bias. The first term—unweighted covariance of import and export intensities—accounts for the contribution to the bias from systematic firm-level differences in import/export exposures, but switches off interactions with the third heterogeneity—variation in firm size. The remaining four terms of equation (11) account for the impact of the variation in firm size on the bias. These four additional terms contain two key adjustments.

First, simple import/export covariance needs to be adjusted for interactions with firm size. This is done via the $cov(\alpha_n^M, y_n)$, $cov(\alpha_n^X, y_n)$ and the triple covariance terms. Second, in the presence of firm size heterogeneities, the simple import/export covariance needs to be adjusted for differences between simple and weighted average export and import intensities. The terms $\bar{\alpha}_s^X - \tilde{\alpha}_s^X$ and $\bar{\alpha}_s^M - \tilde{\alpha}_s^M$ adjust for these distortions.

For example, if firm-level trade intensities are positively correlated with firm size, one would observe $cov(\alpha_n^M, y_n)$, $cov(\alpha_n^X, y_n) > 0$, and an unweighted sectoral average for export and import intensities will understate the sectoral average trade intensities used in the covariance calculations (i.e., $\bar{\alpha}_s^X - \tilde{\alpha}_s^X < 0$ and $\bar{\alpha}_s^M - \tilde{\alpha}_s^M < 0$). In this example, the two adjustments have opposing impacts on $XVS_s^{bias,dir}$, through negative second and third terms and positive fourth and fifth terms in equation (11).

In Appendix C.2, we show empirically that the unweighted covariance term, $N_s \bar{y}_s cov(\alpha_n^M, \alpha_n^X)$, approximates well the aggregate size of the direct bias, $XVS_s^{bias,dir}$.

B.3 Simple covariance when including re-exports

Here we focus on the simple unweighted covariance term between firms' import and export intensities to study how the inclusion of re-exports impacts the aggregation bias. Let us consider a sector with $N+1$ firms with initial import and export intensities α_n^M and α_n^X for $n \in \{1, \cdots, N, N+1\}$, with unweighted means of $\bar{\alpha}^M$ and $\bar{\alpha}^X$.

We explore how a sector's simple covariance of the two trade intensities changes when we include re-exports for the firm indexed with $N+1$. That is, we change the import and export intensities of the firm indexed with $N+1$ to $\hat{\alpha}_{N+1}^M$ and $\hat{\alpha}_{N+1}^X$, where $\hat{\alpha}_{N+1}^M > \alpha_{N+1}^M$ and $\hat{\alpha}_{N+1}^X > \alpha_{N+1}^X$. We denote the differences between the new and old intensities by $\hat{M} = \hat{\alpha}_{N+1}^M - \alpha_{N+1}^M$ and $\hat{X} = \hat{\alpha}_{N+1}^X - \alpha_{N+1}^X$, and the new unweighted means by $\hat{\bar{\alpha}}^M$ and $\hat{\bar{\alpha}}^X$.

The numerator of the simple covariance after the inclusion of re-exports can be written as

$$\sum_{n=1}^{N}\left(\alpha_n^M-\hat{\bar{\alpha}}^M\right)\left(\alpha_n^X-\hat{\bar{\alpha}}^X\right)+\left(\hat{\alpha}_{N+1}^M-\hat{\bar{\alpha}}^M\right)\left(\hat{\alpha}_{N+1}^X-\hat{\bar{\alpha}}^X\right). \tag{12}$$

Since the new unweighted means can be written as

$$\begin{aligned}\hat{\bar{\alpha}}^M &= \frac{1}{N+1}\left(\sum_{n=1}^{N}\alpha_n^M+\alpha_{N+1}^M+\hat{M}\right)\\ &= \bar{\alpha}^M+\frac{1}{N+1}\hat{M}\\ \hat{\bar{\alpha}}^X &= \bar{\alpha}^X+\frac{1}{N+1}\hat{X},\end{aligned}$$

we can rearrange equation (12) as follows:

$$\begin{aligned}&\sum_{n=1}^{N}\left(\alpha_n^M-\hat{\bar{\alpha}}^M\right)\left(\alpha_n^X-\hat{\bar{\alpha}}^X\right)+\left(\hat{\alpha}_{N+1}^M-\hat{\bar{\alpha}}^M\right)\left(\hat{\alpha}_{N+1}^X-\hat{\bar{\alpha}}^X\right)\\ &=\sum_{n=1}^{N}\left(\alpha_n^M-\bar{\alpha}^M\right)\left(\alpha_n^X-\bar{\alpha}^X\right)+\left(\alpha_{N+1}^M-\bar{\alpha}^M\right)\left(\alpha_{N+1}^X-\bar{\alpha}^X\right)+\frac{N}{N+1}\left(\hat{M}\left(\alpha_{N+1}^X-\bar{\alpha}^X\right)+\hat{X}\left(\alpha_{N+1}^M-\bar{\alpha}^M\right)+\hat{M}\hat{X}\right),\end{aligned}$$

where the first two terms in the last equation correspond to the numerator of the initial covariance term.

If the initial import and export intensities of the firm indexed with $N+1$ are larger than the sector's average, $\alpha_{N+1}^M > \bar{\alpha}^M$ and $\alpha_{N+1}^X > \bar{\alpha}^X$, then the simple covariance will increase after the inclusion of re-exports.

B.4 Characterizing the "first indirect" bias

Arranging the term XVS_s^{indir1} in equation (5) and expressing the terms in terms of deviations from sectoral means gives

$$\begin{aligned}XVS_s^{indir1} &= \sum_{n\in N_s}\sum_{k\in S}\sum_{m\in N_k}\alpha_m^M\alpha_{m,n}\alpha_n^X y_n\\ &= \sum_{n\in N_s}\sum_{k\in S}\sum_{m\in N_k}\left(\tilde{\alpha}_k^M+\Delta\alpha_m^M\right)\left(\tilde{\alpha}_{k,s}+\Delta\alpha_{m,n}\right)\left(\tilde{\alpha}_s^X+\Delta\alpha_n^X\right)y_n,\end{aligned}$$

where $\alpha_{m,n} = \frac{x_{m,n}}{y_n}$, and $\tilde{\alpha}_{k,s} = \frac{1}{N_k}\frac{\sum_{n\in N_s}\sum_{m\in N_k} x_{m,n}}{\sum_{n\in N_s} y_n}$. If all firms in each sector are identical, then we obtain the measure with sectoral aggregation in equation (9):

$$\begin{aligned} XVS_{IO,s}^{indir1} &= \sum_{n\in N_s}\sum_{k\in S}\sum_{m\in N_k} \tilde{\alpha}_k^M \tilde{\alpha}_{k,s} \tilde{\alpha}_s^X y_n \\ &= \sum_{n\in N_s}\sum_{k\in S}\sum_{m\in N_k} \tilde{\alpha}_k^M \tilde{\alpha}_{k,s} \tilde{\alpha}_s^X \bar{y}_s. \end{aligned}$$

Taking the difference, we obtain

$$\begin{aligned} XVS_s^{bias,indir1} = &\sum_{n\in N_s}\sum_{k\in S}\sum_{m\in N_k} \Delta\alpha_m^M \tilde{\alpha}_{k,s}\tilde{\alpha}_s^X y_n + \sum_{n\in N_s}\sum_{k\in S}\sum_{m\in N_k} \tilde{\alpha}_k^M \Delta\alpha_{m,n}\Delta\alpha_n^X y_n \\ &+ \sum_{n\in N_s}\sum_{k\in S}\sum_{m\in N_k} \Delta\alpha_m^M \Delta\alpha_{m,n}\tilde{\alpha}_s^X y_n + \sum_{n\in N_s}\sum_{k\in S}\sum_{m\in N_k} \Delta\alpha_m^M \Delta\alpha_{m,n}\Delta\alpha_n^X y_n, \end{aligned}$$

from the fact that

$$\begin{aligned} \sum_{n\in N_s}\sum_{k\in S}\sum_{m\in N_k} \tilde{\alpha}_k^M \tilde{\alpha}_{k,s}\Delta\alpha_n^X y_n &= \sum_{n\in N_s} \Delta\alpha_n^X y_n \sum_{k\in S} N_k \tilde{\alpha}_k^M \tilde{\alpha}_{k,s} \\ &= \sum_{n\in N_s}\left(\frac{x_{n,R}}{y_n}y_n - \frac{\sum_{n\in N_s} x_{n,R}}{\sum_{n\in N_s} y_n}y_n\right)\sum_{k\in S} N_k \tilde{\alpha}_k^M \tilde{\alpha}_{k,s} \\ &= 0, \end{aligned}$$

$$\begin{aligned} \sum_{n\in N_s}\sum_{k\in S}\sum_{m\in N_k} \Delta\alpha_m^M \tilde{\alpha}_{k,s}\Delta\alpha_n^X y_n &= \sum_{n\in N_s} \Delta\alpha_n^X y_n \sum_{k\in S}\sum_{m\in N_k} \Delta\alpha_m^M \tilde{\alpha}_{k,s} \\ &= 0, \end{aligned}$$

and

$$\begin{aligned} \sum_{n\in N_s}\sum_{k\in S}\sum_{m\in N_k} \tilde{\alpha}_k^M \Delta\alpha_{m,n}\tilde{\alpha}_s^X y_n &= \tilde{\alpha}_s^X \sum_{n\in N_s}\sum_{k\in S} \tilde{\alpha}_k^M \sum_{m\in N_k}\left(\frac{x_{m,n}}{y_n} - \frac{1}{N_k}\frac{\sum_{n\in N_s}\sum_{m\in N_k} x_{m,n}}{\sum_{n\in N_s} y_n}\right) y_n \\ &= \tilde{\alpha}_s^X \sum_{k\in S} \tilde{\alpha}_k^M \left(\sum_{n\in N_s}\sum_{m\in N_k} x_{m,n} - \sum_{n\in N_s} y_n \sum_{m\in N_k} \frac{1}{N_k}\frac{\sum_{n\in N_s}\sum_{m\in N_k} x_{m,n}}{\sum_{n\in N_s} y_n}\right) \\ &= 0. \end{aligned}$$

C Additional empirical results

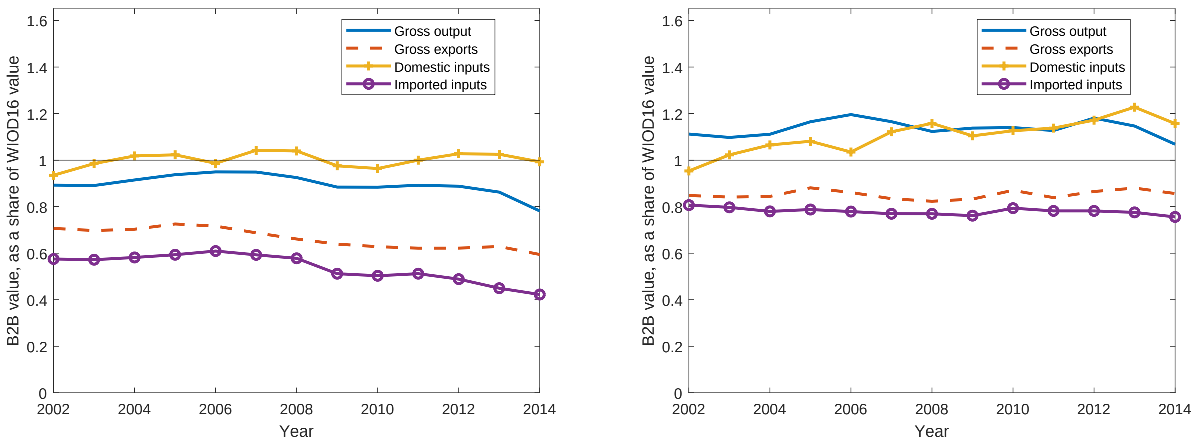
C.1 Alternative value added measures

In our baseline, our measure of value added is what firms report in their annual accounts. As an alternative, we also consider an alternative measure of value added that is consistent with the output reported in the annual accounts. Value added in this case, is computed as the residual of firms total output reported in the annual accounts, after subtracting the sum of purchases from other firms and imports. Using these measures, we plot in Figures 9 and 10 the analogous figures from Figures 1 and 2 in the main text.

Figure 9: Comparison of key aggregate series (alternative value added measure)

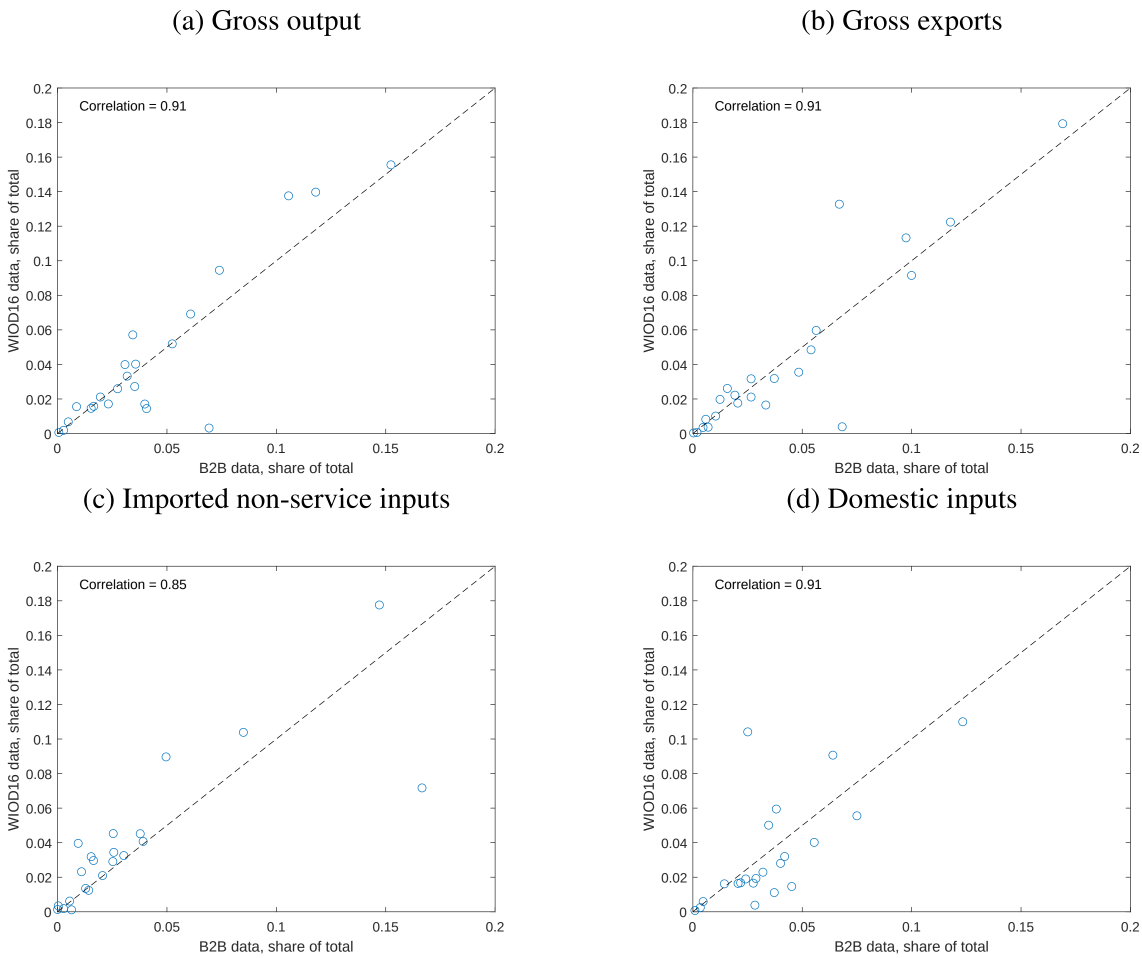
(a) All sectors

(b) Non-service sectors



Notes: Analogous to Figure 1, the figure displays how aggregate variables computed from the Belgian B2B database compare with the analogous variables in WIOD. The gross output series are different from those of Figure 1 as here we compute an alternative measure of value added that is consistent with the output reported in the annual accounts.

Figure 10: Comparison of sectoral shares in non-service sectors, 2010 (alternative value added measure)



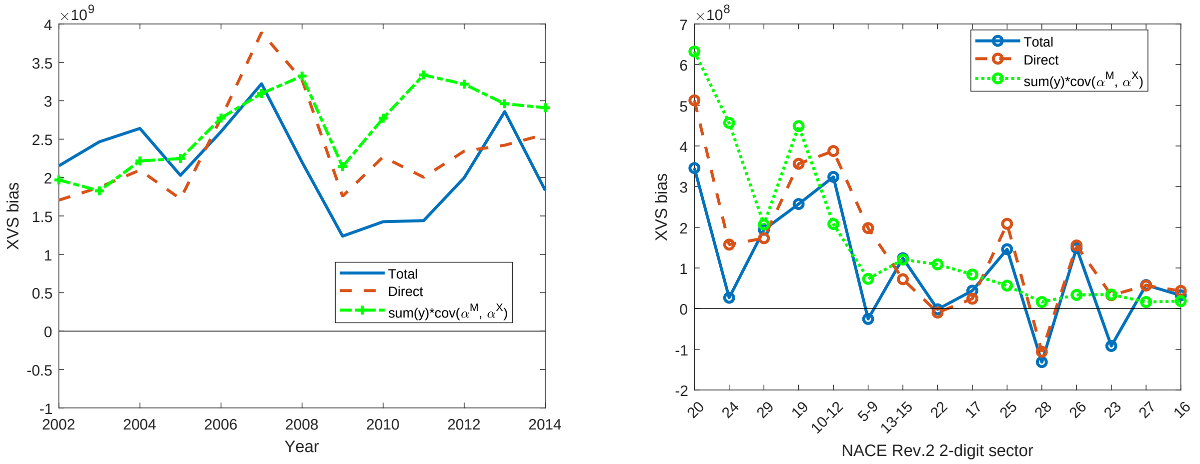
Notes: Analogous to Figure 2, this figure displays how sector-level variables computed from the Belgian B2B database compare with the analogous variables in WIOD. The gross output series are different from those of Figure 2 as here we compute an alternative measure of value added that is consistent with the output reported in the annual accounts.

C.2 Decomposition of the "direct" bias

Appendix B.2 shows that the weighted covariance term characterizing the direct bias can be usefully decomposed further into (i) a simple *unweighted* covariance of import and export intensities and (ii) a set of residual terms that capture the impact of the interplay between firms' import/export intensities and firms' size on the bias (equation (11)). Here we show in Figure 11 the extent to which the simple covariance term can approximate the direct bias term. As a benchmark, the figure also reports the size of the total bias.

Figure 11: Weighted and unweighted covariance terms

(a) All sectors over time (b) By sector, 2010



Notes: This figure plots the first term in equation (11), $sum(y) * cov(\alpha^M, \alpha^X)$, in addition to the Total and Direct bias plotted on Figure 6.

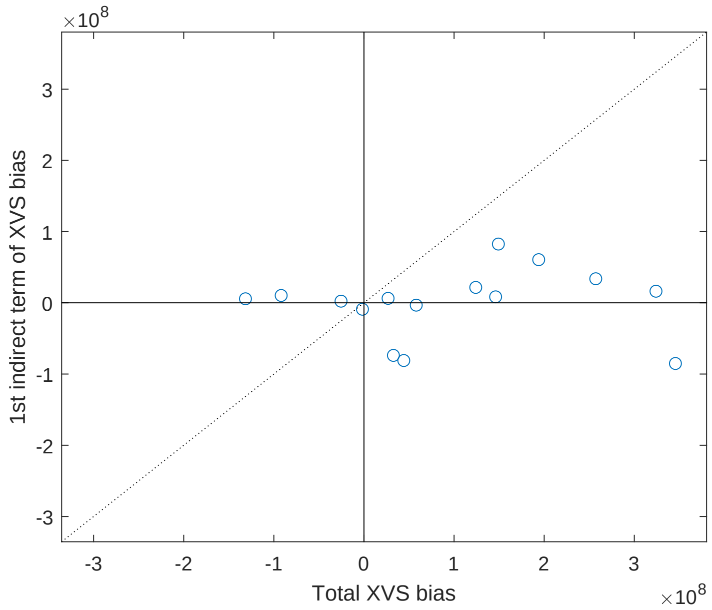
We find that the simple firm-level covariance of import and export intensities can closely capture the *aggregate* direct bias, as well as the total bias (left-side figure). However, it does less well at capturing the variation of the bias *across sectors* (right-side figure). The differences between the direct bias term and the simple covariance term across sectors (i.e., gaps between red and greed lines in right-side figure) are driven by two countering forces captured by the residual terms in equation (11), and stemming from the positive covariance between trade intensities and firm size. On the one hand, when firm size co-varies with trade intensities, the bias increases. Not only exporters import more, but they are also larger in size. Sectoral aggregation averages out these three co-varying heterogeneities—import intensity, export intensity, and firm size. In the decomposition this effect is captured by the positive covariance terms between trade intensities and firm size.

On the other hand, the *simple* unweighted covariance term between import and export intensities understates actual import and export intensities of a sector. The residual terms in equation (11) correct for this problem by using sales-weighted trade intensities instead, which are larger when firm size co-varies positively with trade intensities ($\bar{\alpha}_s^M < \tilde{\alpha}_s^M$ and $\bar{\alpha}_s^X < \tilde{\alpha}_s^X$). Higher sectoral trade intensities, in turn, increase import content as a share of gross exports (i.e., the VS measure), reducing the bias, or even reversing it for some sectors.

C.3 The share of the first term of the indirect bias

Figure 12 plots the first indirect term of the bias against the total bias for 15 manufacturing sectors with the highest shares in total import content of gross exports for Belguim. There is a positive relationship between the two sectoral biases, but the first indirect term accounts for only a fraction of the total sectoral bias. This findings is consistent with the direct bias term being the main driver of the total bias.

Figure 12: Comparison of total and first indirect biases for 2-digit manufacturing sectors



Notes: This figure plots the bias of the first indirect term for each sector, $XVS_s^{bias,indir1}$, against each sector's total bias, XVS_s^{bias}.