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We analyze optimal strategic delay of bank resolution (\grq forbearance') and deposit insurance coverage. After bad news on the bank's assets, depositors fear for the uninsured part of their deposit and withdraw while the regulator observes withdrawals and needs to decide when to intervene. Optimal policy maximizes the joint value of the demand deposit contract and the insurance fund to avoid inefficient risk-shifting towards the fund while also preventing inefficient runs. Under low insurance coverage, the optimal intervention policy is never to intervene (laissez-faire). Optimal deposit insurance coverage is always interior. The paper sheds light on the differences between the U.S. and the European Monetary Union concerning their bank resolution policies.

JEL Classification: G28, G21, G33, D8, E6

Keywords: bank resolution, deposit insurance, global games, suspension of convertibility, bank run, mandatory stay, Forbearance, deposit freeze, Recovery Rates

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Optimal Forbearance of Bank Resolution

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December 7, 2018

Abstract

We analyze optimal strategic delay of bank resolution ('forbearance') and deposit insurance coverage. After bad news on the bank's assets, depositors fear for the uninsured part of their deposit and withdraw while the regulator observes withdrawals and needs to decide when to intervene. Optimal policy maximizes the joint value of the deposit contract and the insurance fund to avoid inefficient risk-shifting towards the fund while preventing inefficient runs. Under low insurance coverage, the optimal intervention policy is never to intervene. Optimal insurance coverage is always interior. I show that both E.M.U. and U.S regulations can be optimal.

Key words: Bank resolution, deposit insurance, global games, suspension of convertibility, bank run, mandatory stay, forbearance, deposit freeze, recovery rates

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1 Motivation

Banking is a highly regulated industry. Regulators not only set deposit insurance levels, but they also decide when to resolve banks (Martin et al., 2017). Once an institution is perceived as failing, the regulator through its resolution authority (RA) can intervene and organize a sale of the bank's assets. The delay of intervention ('forbearance') is at RA's discretion.[1] This paper studies the interaction between the level of deposit insurance and the degree of intervention delay. By examining this two-dimensional policy choice, the paper breaks new ground in the analysis of the regulator's double role and thus provides a novel perspective on this topic.

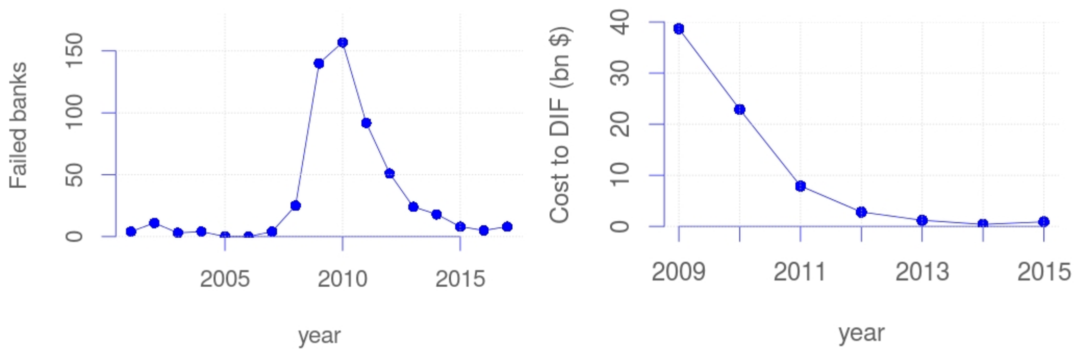


Figure 1: Left graph: Number of failed U.S banks under FDIC receivership, source: FDIC failed bank list. Right graph: Costs of bank failure to FDIC's deposit insurance fund (DIF), source: Bankrate.com

The question of how to resolve banks is vital since resolution procedures impose substantial losses on taxpayers and public funds, see Figure 1 and White and Yorulmazer (2014). Cases of bank resolution are common, not only during times of crises. Alone the FDIC's 'Failed Bank List' shows 553 entries of failed banks under U.S. FDIC supervision for the years 2001-2017. Prominent recent cases of bank resolution in Europe include the bail-out of Monte dei Paschi di Siena in Italy, the sale of Banco Popular in Spain, both in 2017, and the partial sale of

[1] The resolution procedure by the FDIC is initiated once a financial institution's chartering authority sends a Prompt Corrective Action Letter to the failing institution and advises that it is critically undercapitalized or insolvent, see the FDIC's Resolutions Handbook (FDIC RH). The FDIC either organizes a Purchase & Assumptions transaction or a deposit payoff to resolve banks. Both methods are comprised of the model outlined here. By FDIC RH 'Section 38 of the Federal Deposit Insurance Act (FDI Act) generally requires that an insured depository institution be placed in receivership within 90 days after the institution has been determined to be critically undercapitalized.'

Laiki Bank in 2013 during the Cypriot banking crises. Important differences exist between the European Monetary Union and the United States regarding their bank resolution policies. In the U.S., the Federal Deposit Insurance Corporation (FDIC) acts as RA and is appointed as the receiver if an FDIC insured depository institution or a non-deposit making, but systemically relevant institution becomes critically undercapitalized. The FDIC operates under the least cost resolution requirement to minimize net losses to the deposit insurance, regardless of factors such as maintaining market discipline, or prevention of contagion (Bennett, 2001). In contrast to the U.S., Article 31 of the European 'Bank Recovery and Resolution Directive' (BRRD) mentions competing objectives for bank resolution[2] such that the European resolution policy is potentially softer compared to the U.S. policy. To the best of our knowledge, there is no explanation for these differences in the literature so far. This paper sheds light on these differences and explains under what conditions the U.S approach to minimize public losses is desirable from a social perspective.

In our setting, a bank finances a risky asset with deposits where deposits are only partially insured at a level set by the regulator[3]. As in Goldstein and Pauzner (2005), depositors observe information about the fundamental of the bank and may decide to withdraw early. These withdrawals potentially impose losses on the deposit insurance fund. The RA observes withdrawals at the bank level. Should withdrawals exceed a critical level set beforehand by RA, RA intervenes. In that case, RA suspends convertibility of deposits such that depositors can no longer withdraw (mandatory stay). She seizes remaining bank assets which she then liquidates to evenly distribute proceeds to all depositors who were not served so far. If proceeds are below the insured amount of the deposit, the insurance fund is obliged to pay the difference.

RA's role as insurer interferes with her role as resolution authority. If RA intervenes later, she seizes a smaller proportion of the asset which diminishes the pro rata share to depositors under a resolution. If the pro rata share is below the insured fraction of the deposit, the insurance fund becomes liable. Thus losses to the insurance fund increase as RA intervenes later. On the other hand, as RA raises insurance coverage the exposure of the insurance fund increases which may

[2] .. 'to ensure continuity of critical functions' and 'to avoid a significant adverse effect on the financial system' in addition to the objective to protect depositors and public funds

[3] Despite the existence of (partial) deposit insurance in many countries, the possibility of bank runs persists since only about 59% of U.S. domestic deposits are insured as of 2016, see appendices (FDIC, 2016).

affect RA's forbearance policy to limit losses. Not only is RA's role as insurer intertwined with her role as resolution authority but also the bank's depositors are affected by and react to changes in deposit insurance coverage and timing of intervention in different ways.

The question we ask in this paper is, what is the welfare maximizing measure of withdrawals RA should tolerate before intervening ('forbearance policy') and how much insurance coverage should she provide. To the best of our knowledge, this is the first paper which considers a strategic resolution authority which fully internalizes the impact of her twofold policy on the endogenous probability that the bank is resolved. [4] Our analysis allows answering questions such as (i) given a cut in deposit insurance, how would RA need to adjust her intervention delay to keep the run probability constant (ii) given the authority wants to pursue a more lenient intervention policy, how does the insurance level need to change to maintain welfare at a particular level (iii) what are the welfare implications of full insurance coverage and how do they depend on the intervention delay?

As the main contribution, this paper points out hidden trade-offs and dependencies in resolving banks. In the unique equilibrium, late intervention imposes losses on the deposit insurance fund while early intervention increases the likelihood that the bank is resolved. The latter holds since depositors withdraw for smaller solvency shocks. The described trade-off crucially depends on the amount of deposit insurance coverage provided. As a first step, we show that independently of when RA intervenes, inefficiencies exist if deposit insurance is too high or too low. Under too low insurance coverage, inefficient runs may occur no matter when RA intervenes. The optimal forbearance policy by RA then is to walk away, that is never to intervene (laissez-faire), by this minimizing the likelihood of inefficient runs. This result means, even when inefficient runs occur and thus intervention was ex post optimal a stricter policy to intervene will alter depositors' behavior in a way that inefficient runs become more likely ex ante. RA fully anticipates this change in behavior and optimally commits never to intervene.

Under too high insurance coverage, however, the result flips. Inefficient invest-

[4]In Diamond and Dybvig (1983) for instance there is multiplicity of equilibria. Thus, marginal changes of run probability cannot be analyzed since the likelihood of runs cannot be determined from within the model unless the regulator sets a policy such that running is a dominated action. Thus, there is no feedback from depositors to the regulator unless the occurrence of a run can be excluded. In the paper here instead, marginal changes in resolution probability from within the model feedback into RA's objective function due to altered depositor behavior. The feedback loop between RA's policy and depositors' behavior allows in particular to analyze the interaction of the two policy parameters which has not been done before.

ment exists no matter the timing of intervention.[5] This holds since depositors' propensity to withdraw drops as insurance increases. Since in our model, a run with subsequent bank resolution is the only mechanism to enforce the liquidation of assets, under high insurance provision investment in high-risk assets is continued instead of interrupted.

Continuation of investment under high insurance however only shifts risk away from the demand deposit contract to the insurance fund which in return is financed by depositors. A certain propensity to liquidate investment by withdrawing is, therefore, socially desirable to eliminate excessive risk since depositors pay for losses shifted towards the insurance fund indirectly via taxation. This paper is to the best of our knowledge the first theory paper which demonstrates that as deposit insurance coverage increases, equilibrium outcomes shift from exhibiting inefficient runs to inefficient investment because depositors pay less attention to information on solvency shocks (gradual decline of market discipline).

These results also provide a theoretical foundation for the findings in Iyer et al. (2016, 2017) that propensity to run increases as insurance goes down.[6] In particular, under high insurance coverage, the optimal forbearance policy is to intervene as fast as possible, by this minimizing the likelihood of overinvestment. This result rationalizes the U.S. bank resolution policy to intervene fast.

One main implication of our results which should inform policymakers is, if RA does not fine-tune the amount of insurance coverage, inefficiencies may exist. If RA, however, jointly sets forbearance and insurance coverage, then for every forbearance level there exists a unique, interior, optimal level of insurance coverage which implements the first best outcome. As a consequence, full insurance coverage is never optimal, no matter the intervention delay. This result is essential for policymakers since in the United States and Europe we observe insurance levels which may imply full coverage.[7] The interior, optimal insurance coverage level is strictly monotone in RA's forbearance policy. To achieve optimality, RA manipulates information aggregation among depositors through her policy to balance prevention of both inefficient runs and inefficient investment.[8] If the RA liquidates equally or less efficient than the bank does, the first best policy is never to intervene ('laissez-faire') and in return provide low insurance coverage.

[5] The aggregate uncertainty in our model gives rise to efficient runs (see Allen and Gale (1998); Chari and Jagannathan (1988); Jacklin and Bhattacharya (1988)).

[6] See also Calomiris and Jaremski (2016); Goldberg and Hudgins (2002); Baer et al. (1986); Goldberg and Hudgins (1996).

[7] In the U.S. insurance coverage is $250,000 per account holder, in Europe, it is €100,000.

[8] In our setting, depositors are risk-neutral. Deposit insurance thus serves no risk sharing purpose but impacts welfare by modifying information aggregation.

If RA liquidates more efficiently than the bank does, the unique policy which implements the first best outcome implies immediate intervention combined with high insurance, which may justify the U.S. approach. Since the RA can always implement the first best outcome, she has no incentive to deviate from her announced policy, and there exists no time-inconsistency problem as for instance in Ennis and Keister (2009).

Since we allow for insurance coverage equal to zero, the paper also applies to non-deposit making institutions which are supervised by resolution authorities due to systemical relevance since Dodd-Frank and the inception of the European BRRD.

The papers closest to ours are Diamond and Dybvig (1983), Goldstein and Pauzner (2005), Keister and Mitkov (2016), Morris and Shin (2016) and Ennis and Keister (2009). We discuss the literature in detail at the end of this paper. The paper is structured as follows: section two describes the model, section three solves the interim stage of the three-period game, section four describes the frictions, explains the welfare concept and then solves the ex ante stage. Section five discusses the assumptions, some extensions, and deals with robustness. Section six discusses the literature, section seven concludes.

2 Model

The model extends the model set out by Goldstein and Pauzner (2005). There are three time-periods, $t = 0, 1, 2$ and no discounting. There are four kinds of agents, a bank, depositors, outside investors and a resolution authority (RA). The bank and outside investors are not strategic.[9] The bank invests in a risky and illiquid asset and finances her entire investment with short-term debt. There are constant returns to scale. Thus, we normalize the initial bank investment to one unit. There is free entry, such that the bank is in perfect competition with other banks and makes zero profit. Depositors are given by a continuum $[0, 1]$. They are risk-neutral, ex ante symmetric, and each endowed with one unit to invest at time zero. All depositors can consume at time one and two, i.e., they are patient.[10]

[9] This assumption shuts down moral hazard, as an additional channel to focus on changes in depositors' incentives, see Keister (2015). For the strategic case, please refer to Schilling (2017).

[10] Beginning with the seminal contribution of (Diamond and Dybvig, 1983), there is a large literature analyzing demand deposit contracts as a financial arrangement between an intermediary and two classes of agents, i.e., impatient and patient consumers. A crucial step in this literature is the analysis of the incentives of the patient consumers to withdraw early, while the analysis of the impatient consumers typically amounts to little more than stating their withdrawal at period 1. Given the substantial body of this literature, we shall, therefore, take it as given that

Investment and Financing For each unit invested at time zero, the risky asset pays off H at time two with probability θ and zero otherwise, where $\theta \sim U[0,1]$ is the unobservable, random state of the economy. Let $H > 2$ such that the asset has a positive net present value.

At time one, the asset yields no cash flow to the bank. Instead, the bank can use the asset as collateral to borrow in the money market from outside investors with deep pockets. As in Goldstein and Pauzner (2005), we assume that the bank can raise cash up to the fixed amount $l \in (0,1)$ against the asset.[11] Call l the asset's (funding) liquidity, see (Brunnermeier and Pedersen, 2009). The bank pays interest rate j equal to the high asset return H on the funds borrowed against the asset. This assumption captures that in the course of a run a bank has no bargaining power compared to outside investors since she needs to raise cash fast. In a generalization to partially debt-financed banks, we also generalize the interest rate j, see subsection 10.3.

To raise funds, in $t = 0$ the bank offers a demand deposit contract which for each initially invested unit, promises a depositor to pay a coupon of one unit if the contract is liquidated at time one ("withdraw"), by this the contract mimics storage. If the deposit is 'rolled over' until time two, the contract promises coupon H. The payment of the long-term coupon is contingent on the asset's payoff. The per period interest rate on collateralized borrowing exceeds the short-term coupon the bank pays to depositors, $j > 1$. By this, deposit financing is cheaper, and the bank does not select outside financing in the first place. We assume that the bank is prone to runs $l < 1$, that is overall debt claims at the interim period exceed the amount of cash the bank can raise by pledging the asset.

In subsection 10.3, we discuss the extension to general demand-deposit contracts which pay coupons (R_1, R_2).

Signals and actions (interim) Before depositors decide whether or not to withdraw they observe noisy, private signals about the state θ of the world, given by

$$\theta_i = \theta + \varepsilon_i \tag{1}$$

where the idiosyncratic noise is independent of state θ and iid distributed according to $\varepsilon_i \sim U[-\varepsilon, +\varepsilon]$. For ε small, signals become precise. The signal contains

banks only offer demand deposit contracts with the option to withdraw at time 1, and we shall feature patient agents only. Incorporating impatient depositors is straightforward but does not add to the main point we want to make here.

[11] Goldstein and Pauzner (2005) consider asset sales in $t = 1$. We instead analyze collateralized borrowing and later extend the model to accommodate asset sales. Morris and Shin (2009) treat the case of a state-dependent interim liquidation value.

information on how likely the asset pays off high return H at time t_2. Since signals are correlated through the state, each signal also conveys information on signals and beliefs of other agents. Depositors' strategies map their private signal θ_i to an action in the space {withdraw, roll over}.

Deposit Insurance Fund Each deposit is partially insured against the risk of bank illiquidity or insolvency. An insurance fund guarantees the endogenous fraction $\gamma \in (0, 1)$ of the interim face value of debt. Insurance is financed by lump-sum taxation of depositors. Each depositor $i \in [0, 1]$ is charged the same amount $\tau \in (0, 1)$ at the time she demands repayment from the bank. Withdrawing depositors are taxed at t_1, depositors who roll over are taxed at t_2. The tax immediately reduces a depositors' payoff from the contract. There are two ways to interpret this. Either, the regulator collects the tax through the bank. Alternatively, the regulator taxes the bank per depositor, and the bank, by the zero profit assumption, forwards the tax to depositors by reducing payoffs from the contract.[12] The budget of the insurance fund is $V_B = \int_0^1 \tau \, di$. The fund faces the maximum expenses $\int_0^1 \gamma \cdot 1 \, di$ if all depositors roll over their deposit and the asset fails to pay. The fund's budget constraint is $V_B \geq \int_0^1 \gamma \cdot 1 \, di$. To achieve that insurance is credible, we set $\tau = \gamma$.[13] Here, the amount taxed is independent of the realization of aggregate withdrawals and independent of when depositors withdraw. This fully symmetric taxation is simple and circumvents the problem that RA may not know about aggregate withdrawals when levying the tax. We discuss asymmetric taxation in subsection 6.3.

Resolution Authority (RA) Our paper adds new to the literature a strategic *resolution authority* (RA). The RA has two policy instruments, she provides deposit insurance and has the legal authority to protect the deposit insurance fund by intervention. Denote by (a, γ) RA's policy, and call $a \in (0, 1)$ the RA's *forbearance policy*. At time zero, RA sets and fully commits to her policy *before* depositors decide whether to roll over and before state θ realizes in $t = 0$. Her policy thus conveys no information on the state and is common knowledge among all agents. Let $n \in [0, 1]$ denote the endogenous equilibrium proportion and measure of depositors who withdraw at the interim period after observing RA's policy and their signals. In $t = 1$, RA observes aggregate withdrawals at

[12] In fact, in Germany, for instance, deposit insurance is financed by charging not depositors but banks a fraction of their total deposits.

[13] In case the insurance fund becomes liable, a depositor receives $\gamma - \tau$ which has to be non-negative. This will imply that in 'good times' the insurance fund builds up reserves. In particular, credible insurance requires the fund to build up reserves in some states since the funds budget has to be non-negative in all states of the world. Insurance which is budget balancing only in expectation is not credible to depositors.

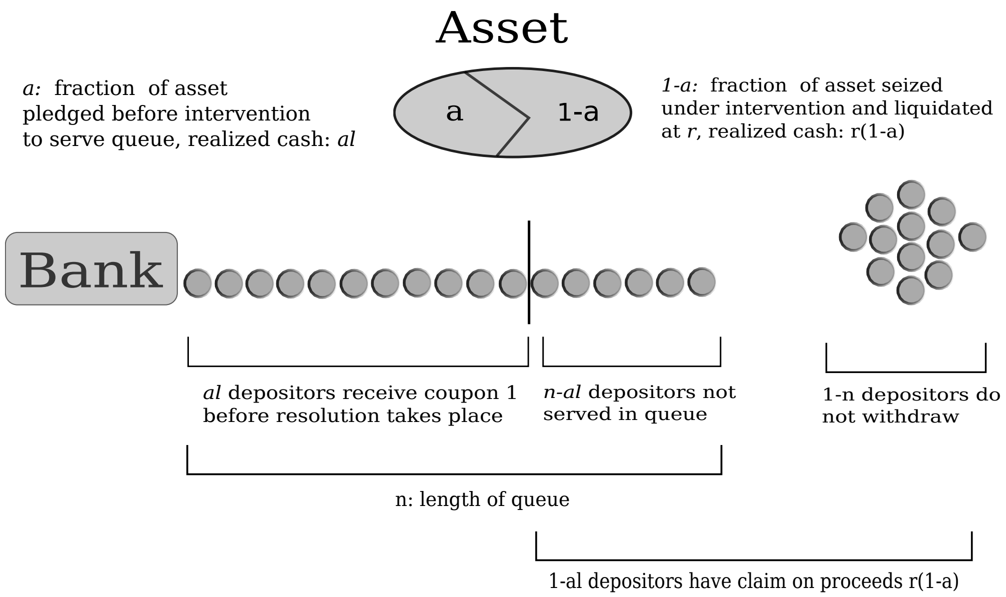


Figure 2: Forbearance-weighted liquidation procedure of assets: Forbearance determines the proportion of the asset liquidated during the run versus under bank resolution.

the bank level. If withdrawals exceed a particular threshold, which the RA had optimally set beforehand, she enforces bank resolution, i.e., she takes over control, imposes a mandatory stay for depositors, and by this stops the run on the bank (suspension on convertibility). More precisely, for a given forbearance policy a, the event 'bank resolution' is triggered if the measure of claimed funds n exceeds the critical level al of cash withdrawals RA tolerates.[14]

$$n \geq al \qquad \Leftrightarrow \qquad \{\text{Bank resolution}\} \tag{2}$$

Given intervention, the bank stops both the service of withdrawing depositors and the pledging of assets in the market. RA seizes and liquidates the remaining assets $1-a$ at an exogenous *recovery rate* $r \in (0,1)$ and evenly allocates the realized proceeds $r(1-a)$ among all remaining bank depositors of measure $1-la$ who were not paid so far. If this pro rata share to depositors

$$s(a) := \frac{r(1-a)}{1-la} \in (0,1) \tag{3}$$

is below the insured fraction of the deposit, the insurance fund becomes liable.

[14] Since withdrawing depositors claim one unit each, n is also the realized measure of claimed funds at $t = 1$.

Every depositor involved in resolution obtains

$$s_\gamma(a) := \max\left(s(a), \gamma\right) = \begin{cases} s(a), & a \in (\underline{a}, \overline{a}) \quad \text{(early resolution)} \\ \gamma, & a \in (\overline{a}, 1] \quad \text{(late resolution)} \end{cases} \tag{4}$$

where we assume that the RA obeys a forbearance minimum $\underline{a} > 0$ which can be interpreted in the sense that RA observes withdrawals with a delay and cannot intervene immediately.[15] The bound $\overline{a}$ marks the critical forbearance level at which the insurance fund becomes liable. That is if the RA sets $a < \overline{a}$, she intervenes in a way such that she fully protects the insurance fund, the fund does not become liable. This bound may exist since

Lemma 2.1. *The pro rata share to depositors monotonically declines as RA grants more forbearance.*

The forbearance policy can, therefore, be understood as a reduced form of 'timing' of intervention in the sense that admitting few withdrawals corresponds to 'early' intervention while allowing many withdrawals corresponds to 'late' intervention. If RA intervenes 'later', she allows more depositors to withdraw their full deposit before triggering resolution proceedings. As a consequence, conditional on a resolution, the RA seizes a smaller proportion of the asset and the pro rata share to remaining depositors declines. If the RA intervenes sufficiently 'late', $a > \overline{a}$, the pro rata share falls below the guaranteed level of the deposit implying that the RA imposes losses on the insurance fund, given resolution takes place. Intervention guarantees a minimum pro rata share to depositors who roll over and prevents depositors from running at the expense of other depositors and the deposit insurance fund. A forbearance policy of $a = 1$ corresponds to the standard case where the bank is on her own when facing a run; there is no intervention (laissez-faire). Even in this case, the coordination problem among depositors, however, prevails since by assumption the asset is illiquid. For $a < 1$, RA intervenes and secures a strictly positive fraction $1 - a$ to the remaining depositors. see Figure 7.

Note, only if RA's recovery rate r exceeds the insurance coverage level γ, then

[15] A minimum forbearance level is required since otherwise the game structure changes because a single depositor becomes pivotal. The bound $\underline{a}$ can be arbitrarily close to but bounded away from zero. The imposition of a minimum forbearance level also has legal reasons. In the U.S., the FDIC has to obey a forbearance minimum, the asset to debt ratios has to be below a critical threshold otherwise interventions is not legally justified.

To give an example, in September 2017, bondholders of failed Banco Popular filed an appeal against Spain's banking bailout fund which followed European authorities (Single Resolution Board) and wiped out equity and junior bondholders before selling the bank to Banco Santander, see Bloomberg (2017) and Reuters (2017).

RA can set her forbearance policy in a way such that sufficiently early intervention prevents losses to the insurance fund.[16] The maximum forbearance level that the RA can grant such that the insurance runs no loss is given as

$$\overline{a}(\gamma) := \max\left(0, \frac{r-\gamma}{r-l\gamma}\right) \in [0,1) \tag{5}$$

Payoffs Depositors In $t=1$ aggregate withdrawals occur simultaneously and are perfectly observed by RA.[17] If withdrawals are below RA's tolerance threshold al, no resolution takes place. The bank finances all withdrawals at $t=1$ via pledging assets and the game proceeds to time two. At $t=2$, if the asset takes value zero, the bank defaults on both, the demand-deposit contract and the collateralized loan from outside investors.[18] Depositors who roll over receive only the insured fraction of their deposit. If the asset pays H, the bank can repay nj to the outside investor and gains back control of the pledged part of the asset. She earns return H on the entire asset and pro rates these returns to remaining depositors such that each depositor obtains $\frac{H-jn}{1-n}=H$ as promised in the contract.[19] If withdrawals exceed RA's tolerance threshold, RA randomly selects al out of n depositors who may receive the full coupon by the bank before RA takes over control for bank resolution. The remaining $n-al$ depositors are not served but enter the resolution proceedings where they are treated like depositors who rolled over, receiving s_γ. This procedure has the interpretation of a bank's sequential service constraint, as in Goldstein and Pauzner (2005). For withdrawing, depositors queue and are sequentially served the coupon of one unit. RA monitors the queue and shuts down withdrawals once the number of served depositors exceeds al. Depositors' positions in the queue are random. Given resolution, the likelihood of being served in the queue and obtaining the full deposit is $\frac{la}{n}$. With likelihood $1-\frac{la}{n}$, a withdrawing depositor is not served.

Additionally, depositors are taxed to finance deposit insurance. Depositors served in the queue are taxed in $t=1$ while depositors who enter resolution

[16] See Lemma 9.1 in the appendix.

[17] Since RA observes n, in equilibrium she could back out the state θ and set her policy depending on θ directly. We assume in the benchmark model that (due to the sequential nature of withdrawals) she cannot observe the state and thus does not set state-contingent policies. In subsection 6.3 we explain why a state-contingent policy does not help RA in achieving her objective.

[18] Here, participation by outside investors is exogenously given. For endogenous pricing of such collateralized loans to the bank, see Schilling (2017).

[19] The assumption $j=H$ achieves that the payoff becomes independent of n. In subsection 6.1 we discuss in detail why we make this assumption in the benchmark model and why the assumption is not restrictive once the bank is partially financed with equity.

proceedings or roll over are taxed in $t = 2$ to finance the insurance fund ex post. Given resolution takes place, the payoff from withdrawing always exceeds the payoff from rolling over, by this giving the incentive to withdraw if a resolution is anticipated. The payoff table after taxation is given as

Event/ Action	Withdraw	Roll-over
No resolution $n \in [0, la]$	$1 - \tau$	$\begin{cases} H - \tau & , p = \theta \\ \gamma - \tau & , p = 1 - \theta \end{cases}$
Bank resolution $n \in (la, 1]$	$\left(\frac{la}{n} \cdot 1 + (1 - \frac{la}{n}) s_\gamma(a)\right) - \tau$	$s_\gamma(a) - \tau$

By $\tau = \gamma$, all payoffs after taxation are non-negative.[20]

The ex post net value Γ of the insurance fund equals

Resolution	No resolution
$\int_0^1 \tau \, di - (1 - la) \max(0, \gamma - s_\gamma(a))$	$\begin{cases} \int_0^1 \tau \, di, & p = \theta \\ \int_0^1 \tau \, di - (1 - n)\gamma, & p = 1 - \theta \end{cases}$

Under no resolution, if all agents roll over and the asset does not pay off in $t = 2$, the insurance fund is budget balancing by $\tau = \gamma$. Otherwise, the net value is strictly positive which can be interpreted as reserves. The accumulation of reserves implies that under some conditions, depositors pay more into the insurance fund than they get out. This is, however, necessary for insurance to be credible. If the insurance fund was only budget balancing in expectation,i.e., here if all depositors roll over, but the asset fails to pay, the insurance fund could not pay γ to all agents who had a claim. Thus ex ante, since depositors are rational, they would act as if the insurer's payment was below γ. Given resolution, the fund's net value is strictly positive by $la > 0$.[21] If the RA intervenes early, $a < \bar{a}$, she fully protects the insurance fund from runs, $s_\gamma > \gamma$, and the fund has net value $\Gamma_r = \int_0^1 \tau \, di$.

Information structure We follow the information structure in Goldstein and Pauzner (2005) to obtain a unique equilibrium. We assume, there are states $\underline{\theta}$ and $\overline{\theta}$ which mark the bounds to dominance regions: For states in the range

[20] More intuitively, we can now rewrite the payoff from withdrawing as $s_\gamma(a) + \frac{la}{n} \cdot (1 - s_\gamma(a))$ where a withdrawing depositor receives $s_\gamma(a)$ for sure and with probability la/n she receives the haircut $1 - s_\gamma(a)$ on top.

[21] If RA could observe aggregate withdrawals in advance, then given a resolution of the bank she can lower the tax to a level below γ to achieve a balanced budget. By this she increases depositors' consumption conditional on a resolution, see our discussion in subsection 6.3. Due to aggregate risk, it is however not possible that the insurance fund runs a balanced budget in all circumstances. If the asset pays high, the fund builds up reserves absent resolution even if RA could tell aggregate withdrawals.

$[0, \underline{\theta}]$ withdrawing is dominant while for high states $[\overline{\theta}, 1]$ rolling over is dominant. Boundary $\underline{\theta}$ is defined via $H\underline{\theta} + \gamma(1 - \underline{\theta}) = 1$. That is,

$$\underline{\theta} = \frac{1-\gamma}{H-\gamma} \tag{6}$$

For the upper dominance region, as in Goldstein and Pauzner, we assume that for states $\theta > \overline{\theta}$ the asset pays off H for sure and already at time one.[22] We assume further that in this case the RA is not authorized to intervene $\underline{a} = 1$ since the bank is solvent for sure.[23] As a consequence, the coordination problem vanishes since bank resolution is never triggered and the bank can always repay all withdrawing depositors. As the support of noise ε vanishes, depositors can always infer from their signals whether the state is located in either of the dominance regions.

Timing At $t = 0$, RA sets her forbearance policy and deposit insurance coverage, the random state realizes unobservably, and depositors invest. At $t = 1$, all depositors observe private signals about the state, then decide whether to withdraw and aggregate withdrawals n realize. RA observes n and resolution occurs or not. In the case of resolution, payoffs realize accordingly, and the game ends. Absent resolution, withdrawing depositors are fully served, and the game proceeds until $t = 2$ where the asset may pay off or not.

Equilibrium Concept The equilibrium concept is perfect Bayes Nash. The analysis proceeds via backward induction. We first analyze the interim stage where depositors take as given RA's policy. For fixed (a, γ), a Bayesian equilibrium of the depositors' game is a strategy profile such that each depositor chooses the best action given her private signal and her belief about other players' strategies. Beliefs are inferred from Bayes rule. We analyze how depositors' equilibrium behavior alters as RA shifts her policy. At the ex ante stage, RA sets the socially optimal policy (a^*, γ^*) where RA takes as given the coordination behavior of depositors which follows in the subgame. All proofs can be found in the appendix.

3 Equilibrium coordination game - Interim stage

At the interim stage, depositors take RA's forbearance policy a and deposit insurance coverage γ as given when deciding whether to roll over their deposit. All following results are at the limit as noise ε vanishes. By the existence and

[22] This assumption is equivalent to a shift in interim liquidation value from l to H.

[23] The FDIC is only appointed as the receiver if a bank's capital to asset ratio falls below two percent (12 U.S. Code 1831o), i.e., the bank is close to insolvency.

uniqueness result in Goldstein and Pauzner (2005),

Proposition 3.1 (Existence and Uniqueness)
The game played by depositors has a unique equilibrium which is in trigger strategies. All depositors withdraw if they observe a signal below the threshold signal $\theta^*(a, \gamma)$ *and roll over otherwise.*

The complementarity of actions among depositors can lead to a self-fulfilling resolution of the bank. If a depositor believes that a group of other depositors will withdraw which is sufficiently large to trigger bank resolution, she will withdraw as well. If a large enough group of depositors believes bank resolution to occur, the entire group withdraws which *causes* the event 'bank resolution'.

Denote by $n(\theta, \theta^*)$ the endogenous equilibrium measure of withdrawn funds at state θ and trigger θ^*. Function $n(\theta, \theta^*)$ is pinned down by the measure of depositors who observe signals below θ^*, see (22). Bank resolution occurs if the measure of funds withdrawn by depositors exceeds the critical value al. Define the *critical state* θ_b implicitly by

$$n(\theta_b, \theta^*) = al \tag{7}$$

Bank resolution occurs if the true state realizes below the critical state. Since the random asset return is uniformly distributed, the probability that bank resolution occurs is just equal to θ_b. This fact motivates the following definition,

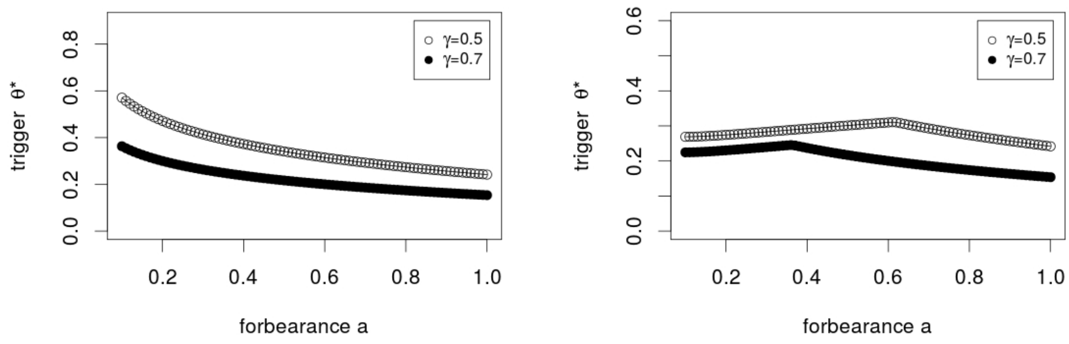
Definition 3.1. *We say bank stability increases if the ex ante probability of bank resolution* θ_b *goes down.*

Bank stability is directly related to depositors' propensity to withdraw θ^*.[24] We are interested in how a change in RA's forbearance policy affects depositors' behavior and thus bank stability.

Proposition 3.2 (Comparative statics: Stability and Forbearance)
Fix liquidity $l \in (0, 1)$ *and insurance coverage* $\gamma \in (0, 1)$.
(A) If $r \in (0, \gamma)$, *then stability improves in forbearance for all* $a \in (\underline{a}, 1]$.
(B) If $r \in (\gamma, 1)$, *then there exists* $\varepsilon > 0$ *such that*
(B1) If $r \in (0, l + \varepsilon) \cap (\gamma, 1)$, *bank stability monotonically improves in forbearance for all* $a \in (\underline{a}, 1]$.

[24] The critical state is linear in θ^*, via 22. Thus, as noise vanishes, θ_b and θ^* *and* their derivatives become indistinguishable. The change in θ^* directly describes the change in bank stability.

(B2) If $r \in (l + \varepsilon, 1) \cap (\gamma, 1)$: Bank stability becomes non-monotonic. For late interventions $a \in (\overline{a}(\gamma), 1]$ bank stability monotonically improves in forbearance. For early interventions $a \in (\underline{a}, \overline{a}(\gamma)]$ bank stability is non-monotone and decreases in forbearance for $r >> l$ when r approaches one.



(a) $r = 0.5 = l$ (b) $r = 0.9$

Figure 3: Monotonicity of the trigger varies in forbearance as recovery rate changes. For recovery rate r close to or below l, stability monotonically improves. For $r >> l$, stability deteriorates in forbearance given 'early intervention' but then improves in forbearance as the insurance fund becomes liable, which gives rise to the kink. Held fixed through all graphs: $H = 4$, $l = 0.5$

The results are depicted in Figure 3. Forbearance affects depositors' incentives in two ways. On the one hand, late intervention lowers depositors' pro rata share since the RA seizes fewer assets as she intervenes. The decline in the payoff for rolling over increases depositors' propensity to withdraw. As a second effect, however, as RA sets a higher forbearance policy, she alters strategic uncertainty among depositors. This property holds since as RA tolerates more withdrawals, runs need to be larger for triggering resolution, and 'withdrawing' is the optimal action if and only if resolution occurs.[25]

[25] Given resolution, by withdrawing a depositor has a shot at recovering her entire deposit while, if the depositor is late in the queue and withdrawing is not successful, she is treated just as well as if she had rolled over, she obtains the pro rata share. Moreover, the pro rata share can never exceed the face value of her deposit.In particular, there is no punishment to depositors who 'cause' bank resolution, in contrast to for instance Diamond and Dybvig (1983) where depositors who withdraw but are late in the queue lose their deposit. Given no resolution, rolling over is optimal since the bank can always pay the high coupon if the asset pays off. This property holds in the benchmark model since by pledging assets the bank avoids costly liquidation. If she can repay outside investors in $t = 2$, i.e., if the asset pays off high, the bank earns interest H also on the pledged fraction of the asset. This is a feature which changes under asset sales where withdrawing can be optimal absent resolution. Still, also under asset sales, withdrawing remains

Thus, as RA grants more forbearance, a depositor's belief about aggregate withdrawals needs to increase for her to respond optimally by withdrawing. Since the marginal investor who is indifferent between rolling over and withdrawing holds a uniform belief over aggregate withdrawals, her propensity to withdraw drops. Allover, depositors trade off the decline in pro rata share (increase in deviation loss) given a resolution against the drop in strategic uncertainty that the event bank resolution occurs. The relative strength of these two effects depends on RA's liquidation efficiency, recovery rate r, which is why its variation may alter the monotonicity of bank stability. In fact, if the RA provides high insurance coverage in excess of its recovery rate, $r < \gamma$, then depositors' payoffs under resolution equal the insured amount of the deposit, independently of when the RA intervenes, see Lemma 9.1 and the definition (5). This implies that the trade-off between the two described effects vanishes, forbearance solely ameliorates strategic uncertainty among depositors and does not affect payoffs. Thus, bank stability strictly improves in forbearance. If insurance coverage is lower $\gamma < r$, 'early' intervention can prevent losses to the insurance fund. Therefore, payoffs to depositors can exceed the insured level and may thus vary in forbearance. Consequently, the trade-off between strategic uncertainty and the change in payoffs exists.

The recovery rate determines the 'costs' which RA imposes on depositors by forbearing. While strategic uncertainty is independent of RA's recovery rate the pro rata share declines faster in forbearance as RA's recovery rate increases. The decline in pro rata share, therefore, dominates the drop in strategic uncertainty if RA's recovery rate is high, and the opposite is true if r is low.

In the following consider the case $\gamma < r$ such that payoffs vary in forbearance. If in addition $r \leq l$, the costs of liquidation decline as RA intervenes later since a larger proportion of the asset is liquidated by the bank as opposed to the RA.[26] One might now be tempted to believe that this cost reduction is the driver of our result that stability improves in forbearance for $r \leq l$. The case $r \in (l, l+\varepsilon)$ however proofs this intuition partially wrong. Here, depositors still prefer more forbearance even though RA liquidates more efficiently. The case shows that the drop in strategic uncertainty has a substantial effect on depositors' behavior. The case $r = l$ demonstrates that the drop in strategic uncertainty is the stronger effect as opposed to the decline in pro rata share. As RA's liquidation efficiency,

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optimal given resolution such that the main effects discussed here are robust, see subsection 10.5 of the supplementary appendix.

[26] The bank borrows against proportion a of the asset at value l, while RA liquidates proportion $1-a$ at value r.

however, exceeds $l + \varepsilon$, in fact, the monotonicity of stability changes since the costs RA imposes on depositors by forbearing become high. In the case, where the bank cannot pledge but sells assets to refinance withdrawals, similar results apply, see subsection 10.5 of the supplementary appendix for a detailed discussion.

In the case $\gamma < r$, for 'late' intervention the pro rata share falls below the insured part of the deposit, and the analysis becomes as in the case $\gamma > r$. Since the insurance fund becomes liable and pays the difference to the insured amount, the payoff for rolling over remains constant at the insured level as forbearance increases further. In particular, the payoff for rolling over becomes independent of forbearance. Thus, the trade-off between the two described effects vanishes.

Under late intervention, bank stability monotonically declines in forbearance for all higher forbearance levels, independently of RA's recovery rate, see the kink in Figure 3. Note, while under late intervention the payoff to depositors is constant in forbearance the insurance fund pays the costs of additional forbearing. Depositors incur this cost indirectly via the lump-sum tax.

Lemma 3.1 (Decline of market discipline)**.** *Bank stability monotonically increases in deposit insurance coverage. As insurance coverage becomes full, depositors have a dominant strategy to roll over so bank runs do not occur in equilibrium.*

Deposit insurance coverage bounds the downside risk to the action of rolling over. As insurance coverage increases, the maximum loss a depositor faces under a resolution, the uninsured part of the deposit, declines while the upside, earning H, remains constant. The incentive to withdraw thus goes down. As a depositor becomes fully insured, she rolls over her deposit for every signal no matter how large the inferred solvency shock on the bank. Market discipline, exercised by withdrawing, collapses. Under full insurance coverage, investment in the risky asset is therefore always continued. The result provides a theoretic foundation for observations in Iyer et al. (2016) who show that less insured depositors are more prone to run than higher insured depositors.

4 Welfare - Ex ante stage

At the ex ante stage, the RA sets her policy to maximize welfare, taking as given depositors' behavior in the following period. Before we define welfare, we describe the frictions in the model and how they interact with RA's policy, the forbearance level, and insurance coverage.

4.1 Friction I: Direct Liquidation Efficiency

If withdrawals amount to a bank run, subsequent bank resolution liquidates the entire asset. The RA and the bank potentially liquidate assets at different efficiency levels. Given a run takes place, efficient liquidation requires that the institution with higher efficiency liquidates the entire asset. Given resolution, the *realized liquidation value* to depositors depends on RA's forbearance level and equals

$$T(a) = al + (1 - a)r \leq \max(l, r) \tag{8}$$

since the bank raises cash al until resolution while RA raises proceeds $(1-a)r$. The extent of forbearance determines the proportion of the asset 'liquidated' (pledged) by the bank versus the proportion liquidated by RA, given resolution takes place. If $r \neq l$, the *direct efficiency loss from liquidation* equals $\max(r, l) - T(a)$ when forbearing or intervening. This is since in either case, for every $a \in (0, 1)$ the institution with lower liquidation efficiency will still liquidate or pledge some fraction of the asset. If the bank liquidates more efficiently than RA, granting more forbearance reduces the direct efficiency loss, and the direct loss is zero if the bank pledges the entire asset, given resolution. In the opposite case, $l < r$, more forbearance increases the direct efficiency loss, and the direct loss is zero if RA intervenes as soon as feasible. If RA and the bank liquidate equally efficient, the direct loss or gain from granting forbearance is zero.

4.2 Friction II: Overinvestment and Inefficient Runs (indirect liquidation efficiency)

Friction I, the direct loss from liquidation, applies *given* resolution takes place. Friction II concerns the issue *whether* the occurrence of resolution is efficient or not. Since the asset is risky, asset liquidation is efficient if and only if the asset's continuation value realizes below its liquidation value.[27] Define the efficiency cut-off

$$\theta_e = \frac{\max(l, r)}{H} \tag{9}$$

as the state below which asset liquidation is efficient. As we will show below, the strategy 'asset liquidation if and only if the state realizes below the efficiency cut-off' is feasible by RA in the sense that there exists a policy which can implement this precise outcome. Similar to Allen and Gale (1998), one may imagine here an

[27] Here, with 'liquidation value' we mean the maximum amount of cash that can be raised against the asset at $t = 1$.

economic downturn which occurs naturally in the course of a business cycle, which impairs asset values. In our model, the only mechanism which enforces liquidation of investment is withdrawals by depositors with a subsequent run. Inefficiencies occur when depositors run inefficiently often or seldom. For state realizations below θ_e, bank runs are socially desirable due to excessive aggregate risk. Bank resolution, however, takes place only for state realizations below the critical state θ_b. Depending on RA's policy, there may exist a range of potential fundamental realizations (θ_b, θ_e) for which depositors do not withdraw, but asset liquidation was efficient. There is 'overinvestment.' RA can impact this inefficiency indirectly since her policy tools, forbearance and insurance coverage, manipulate depositors' incentives to run on the bank, by this changing the critical state and bank stability. In the case of overinvestment, a further stability improvement (decrease in critical bankruptcy state) lowers efficiency since inefficient continuation of investment becomes more pronounced. Higher propensity to run is socially desirable. If on the other hand, the critical state exceeds the efficient liquidation cut-off, (θ_e, θ_b), state realizations in this range cause 'inefficient runs' and more stability is desirable. The ocurrence of overinvestment or inefficient runs fundamentally depends on the amount of insurance coverage RA provides.

Lemma 4.1. *Let $r \in (0, 1)$ arbitrary. If deposit insurance is low, inefficient runs can occur for every forbearance policy: $(\theta_e, \theta_b(a))$ is non-empty for all $a \in (\underline{a}, 1)$. If deposit insurance is high, inefficient continuation of investment can occur for every forbearance policy: $(\theta_b(a), \theta_e)$ is non-empty for all $a \in (\underline{a}, 1)$.*

Intuitively, for low insurance coverage, depositors potentially face a full loss of their deposit. They pay much attention to their signals and therefore withdraw too often. For high insurance coverage, depositors face no losses when choosing the 'wrong' action and stop paying attention to their signals. They roll over their deposit also for large solvency shocks on the bank and investment is always continued. In particular, overinvestment or inefficient runs exist independently of forbearance if insurance coverage is high respectively low. Forbearance, however, plays a role in minimizing these inefficiencies. The main insight from Lemma 4.1 is, more bank stability is socially desirable only when inefficient runs exist. Otherwise, more stability is detrimental to efficiency. Second, RA's policy tools, insurance coverage and forbearance, strongly interfere with each other. To the best of our knowledge, the result that inefficient investment arises for high insurance coverage is new to the literature.[28]

[28] In both Diamond and Dybvig (1983) and Goldstein and Pauzner (2005) there is no ineffi-

4.3 Optimal policy

When RA sets her policy, she balances two things. On the one hand, she wants to keep the direct efficiency loss from liquidation small. On the other hand, she wants to design depositors' incentives in a way that neither overinvestment nor inefficient runs exist. For given policy (a, γ), define *welfare* as the ex ante value of the bank (investment) implied by the policy as

$$V(a, \gamma) = T(a)\, \theta_b(a, \gamma) + \int_{\theta_b(a,\gamma)}^{1} \theta H \, d\theta \tag{10}$$

To explain this definition, for state realizations below the critical state, runs trigger bank resolution. In this case, realized proceeds from liquidation equal $T(a)$, as defined in (8). For state realizations above the critical state, investment is continued which leads to the continuation value θH.

Define the *deadweight loss* at RA's policy (a, γ) as[29]

$$D(a, \gamma) := \underbrace{(\max(r, l) - T(a))}_{\substack{\text{direct loss} \\ \text{from liquidation}}} \theta_b(a, \gamma) + \underbrace{\int_{\theta_e}^{\theta_b(a,\gamma)} (\theta H - \max(r, l)) \, d\theta}_{\text{indirect liquidation inefficiency}} \tag{11}$$

In the first best case, two criteria need to be satisfied at the same time. First, forbearance needs to be such that given resolution the one institution with higher liquidation efficiency liquidates the entire asset. In that case, the direct loss from liquidation, mirrored in the first term of (11), is zero. Second, asset liquidation takes place if and only if the state realizes below θ_e such that neither inefficient runs nor overinvestment occurs. This holds if RA can set her policy such that the critical state matches the efficiency cut-off, putting the integral in (11) at zero. In the first best case, bank value is maximized respectively the deadweight loss is

cient investment since in Diamond and Dybvig the asset is safe while in Goldstein and Pauzner (2005) there is no deposit insurance. Lemma 4.1 is in contrast to Diamond and Dybvig (1983) where suspension of convertibility, i.e., setting a specific forbearance threshold, can prevent inefficient runs despite no provision of deposit insurance. The difference in results stems from two differences in the model. First, RA always liquidates investment while in Diamond and Dybvig (1983) she continues investment. Our assumption can be justified by considering that the asset here is risky but safe in Diamond and Dybvig (1983). RA's asset liquidation can be rationalized by considering that RA may not have the asset management skills to reap the same returns from investment as the bank does. Second, here, given resolution, depositors who withdraw and by this participate in causing the event bank resolution are at least as well off as those who roll over. In Diamond and Dybvig (1983), in contrast, depositors who withdraw but are not served in the queue are punished, they receive zero.

[29] Note: $\int_{\theta_e}^{\theta_b(a,\gamma)} d\theta = -\int_{\theta_b(a,\gamma)}^{\theta_e} d\theta$ if $\theta_b < \theta_e$

at zero.[30] For a given deposit insurance coverage, define the optimal forbearance policy $a^*(\gamma)$ as

$$a^*(\gamma) \in \arg\min D(a,\gamma) \qquad \text{subject to feasibility } a^*(\gamma) \in (\underline{a}, 1] \tag{12}$$

In general, if the bank's and RA's liquidation efficiency differ, RA may need to balance a trade-off between minimizing the direct loss from liquidation and the indirect inefficiency from runs and overinvestment. In the special case that RA liquidates as efficient as the bank does, $r = l$, the direct efficiency loss vanishes and the deadweight loss is minimized if RA can set her policy (a, γ) in a way such that depositors run on the bank to trigger bank resolution if and only if liquidation of investment is efficient.

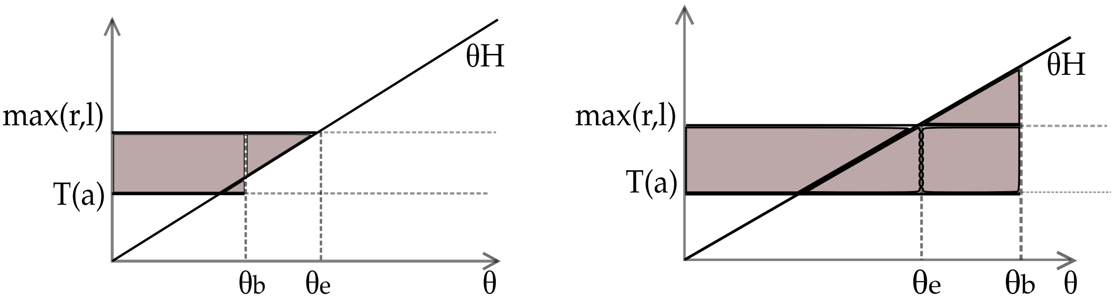


Figure 4: Left-hand side: Deadweight loss from overinvestment. Right-hand side: Deadweight loss from inefficient runs. In either case, the deadweight loss consists of two components. First, given resolution $\theta < \theta_b$, the direct liquidation loss $\max(r,l) - T(a)$ applies (rectangular region). Second, in the case of overinvestment $\theta_b < \theta_e$, there is a loss due to an inefficient continuation of investment (triangular region). In the case of inefficient runs, $\theta_b > \theta_e$, there is a loss due to inefficient liquidation of investment (triangular region). For $r = l$, it holds $T(a) = \max(r,l)$ and the direct liquidation loss is zero.

4.3.1 The Benchmark Case: $r \leq l$

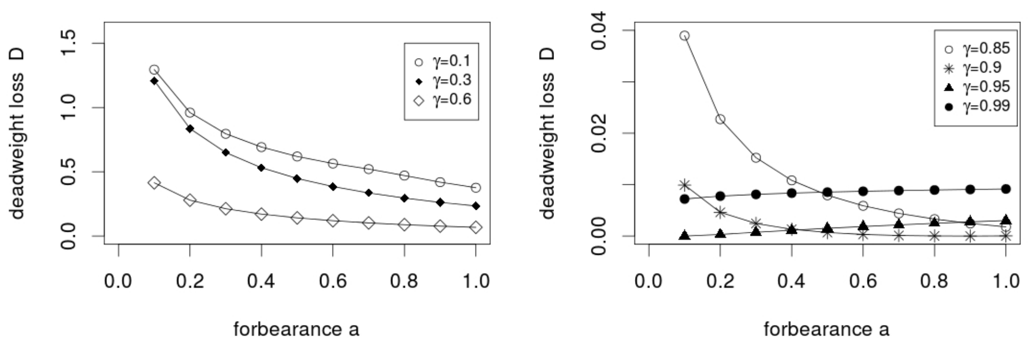
Assume, the bank is an investment expert in the sense of Diamond and Rajan (2001) and liquidates more efficiently than RA, $r \leq l$.[31] Note, that by Proposition 3.2, this case implies that stability monotonically improves in forbearance, independently of the relation of RA's liquidation efficiency r to the level of insurance

[30] The Modigliani Miller Theorem does not hold here by illiquidity of assets.

[31] One can imagine here, that as long as the bank remains solvent, she continues managing her investment, also the pledged proportion of the asset. Therefore, she can raise more cash by borrowing against the asset than by selling. RA, on the other hand, has to sell the asset since she lacks the bank's expert knowledge and $r \leq l$ obtains. Alternatively, consider Brunnermeier and Pedersen (2009) for why market and funding liquidity may differ.

coverage γ. The deadweight loss is in general not convex in forbearance. It, however, turns out that the loss is monotone in several interesting cases such that the optima are found at the boundaries $a^* \in \{\underline{a}, 1\}$.

Theorem 1 (Optimal Forbearance I)
Let $r \leq l$ arbitrary:
a) If deposit insurance is low, the deadweight loss monotonically decreases in forbearance and is minimized by never intervening $a^* = 1$ (laissez-faire is optimal).
b) If deposit insurance coverage is high and r close to l, the deadweight loss monotonically increases in forbearance and is minimized by intervening as soon as feasible $a^* = \underline{a}$.



(a) Low deposit insurance coverage (b) High deposit insurance coverage

Figure 5: Change of deadweight loss in forbearance for low and high insurance coverage. Right graph: As coverage increases, the deadweight loss changes its monotonicity in forbearance. Left graph: the curve for deadweight loss shifts down as coverage goes up, but in the right graph the curve starts shifting upwards as coverage becomes high. Parameters: $l = r = 0.3$, $H = 4$

The results are depicted in Figure 5. For intuition, first, consider the case $r = l$ such that a direct loss from liquidation does not arise. On (a), under low deposit insurance coverage, depositors are sensitive to bad news on the bank fundamental since they potentially face a full loss of their deposit when choosing the 'wrong' action. They withdraw too often such that inefficient runs may occur. The RA wants to make inefficient runs less likely from the ex ante perspective and therefore chooses a policy which lowers propensity to withdraw. Since given $r = l$, stability improves in forbearance, RA commits to intervene as late as possible, namely never. By this she imposes maximum losses on the deposit insurance fund should a run occur, but ex ante minimizes the likelihood of inefficient runs and maximizes welfare. If insurance coverage is 'high,' the result may revert. Under high coverage, depositors are insensitive to bad news on bank solvency and do not withdraw even for severe solvency shocks on the bank. Thus, investment is continued inefficiently often, and less stability in the form of a higher propensity to withdraw is desirable from a social perspective. Since stability improves in forbearance, RA intervenes as soon as possible to achieve a maximum propensity to run. In Figure 5, we see as insurance coverage approaches full coverage, the slope of the deadweight loss switches from negative to positive and fast intervention is desirable from a social perspective, by this minimizing public losses[32].

Now consider $r < l$ which implies an additional direct inefficiency from liquidation. Since now the bank liquidates more efficient than RA does, there exists a direct loss from intervening. This inefficiency declines as RA grants more forbearance. Therefore, under low insurance coverage,'walking away' remains the optimal forbearance policy since it minimizes both, the likelihood of inefficient resolution ex ante *and* the direct loss from liquidation should resolution occur. Under high insurance coverage, however, RA faces a trade-off between minimizing the likelihood of overinvestment versus reducing the direct loss. This holds since more forbearance shrinks the direct efficiency loss but makes overinvestment more likely. If RA and the bank, however, liquidate similarly efficient, the direct loss from forbearing is low, and RA minimizes the likelihood of inefficient investment ex ante by setting a policy to intervene as soon as feasible. By Lemma 4.1, if insurance coverage is too high or too low, RA cannot attain the first best outcome by solely choosing forbearance. We now allow RA to set the amount of deposit insurance coverage additionally. Define RA's optimal policy (a^*, γ^*) as

$$(a^*, \gamma^*) \in \arg\min D(a, \gamma) \qquad \text{subject to } a^* \in (\underline{a}, 1] \tag{13}$$

Theorem 2 (Optimal insurance coverage - Optimal Policy)
Let $r \leq l$. For every forbearance level $a \in (\underline{a}, 1]$ there exists a unique, interior level of insurance coverage $\gamma^*(a) \in (0, 1)$ which minimizes the deadweight loss. The pair is such that

$$\theta_b(a, \gamma^*(a)) = \frac{T(a)}{H} \tag{14}$$

and the deadweight loss strictly decreases in insurance coverage for $\gamma < \gamma^*(a)$ and increases in coverage for $\gamma > \gamma^*(a)$. The optimal insurance coverage level $\gamma^*(a)$ monotonically declines in forbearance.

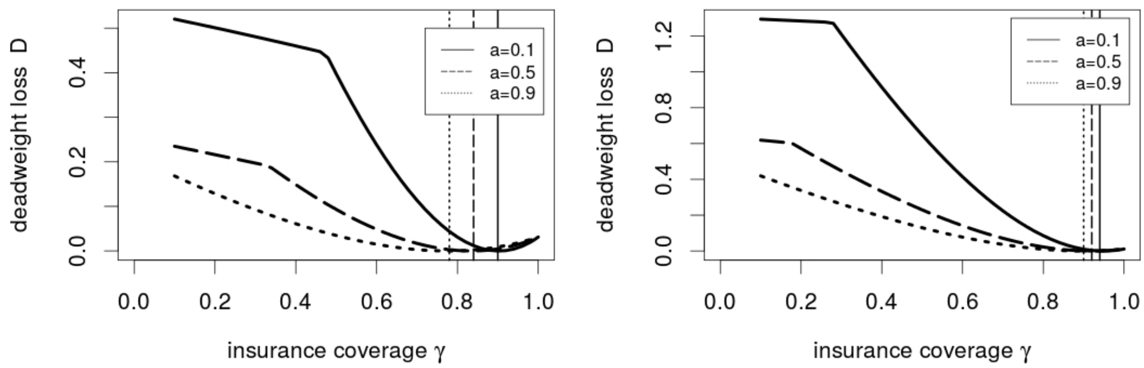
[32] We have $\underline{a} < \overline{a}$, thus the pro rata share recovered under $a^* = \underline{a}$ exceeds the insured fraction of the deposit.

First Best: If $r = l$, all pairs $(a, \gamma^*(a))$ achieve the first best outcome (multiplicity of optimal policy). That is, $\theta_b(a, \gamma^*(a)) = \theta_e$ and $T(a) = l = r$. If $r < l$, then among all optimal pairs $(a, \gamma^*(a))$, $a \in (\underline{a}, 1]$, only the pair $(1, \gamma^*(1))$ achieves first best with $T(a) = l$ (unique optimal policy).

The results are depicted in Figure 6. For intuition, first, consider the case $r = l$ which puts the direct loss from liquidation at zero. The optimal insurance coverage level exists and is unique since for a given forbearance level, depositors' propensity to withdraw strictly declines as RA provides more insurance and transitions from outcomes implying inefficient runs to outcomes implying overinvestment by Lemma (4.1). In particular, for every forbearance level, optimal insurance coverage is interior since, for too high coverage, inefficient investment may exist, but for too low coverage inefficient runs arise, see Figure 5. To argue why the optimal amount of insurance coverage decreases in forbearance, assume forbearance and insurance coverage are set such that the optimal outcome obtains. If RA marginally increases forbearance, depositors' propensity to withdraw goes down, and the critical state drops below the efficiency cut-off. Thus, overinvestment may occur which burdens the insurance fund with additional risk. To protect the insurance fund from excessive risk, RA needs to maintain depositors' propensity to withdraw at the target level. The deviation loss has to increase, RA needs to lower insurance coverage, see Figure 5.

Since the direct liquidation loss is zero in the benchmark case $r = l$, there are infinitely many pairs of forbearance and insurance coverage which achieve the first best outcome in which there are neither inefficient runs nor inefficient investment. Once RA liquidates less efficient than the bank, the multiplicity result breaks down, and there exists a unique policy which implements the first best outcome. This holds since under distinct liquidation efficiency, RA needs to trade off minimization of the direct loss from liquidation against minimization of ex ante likelihood of overinvestment or inefficient runs. For a given forbearance level, the optimal compromise for balancing these two effects is to set insurance coverage such that the critical state matches the state $T(a)/H$ at which realized liquidation value equals continuation value from investing. As a consequence, all optimal insurance coverage pairs $(a, \gamma^*(a))$ feature overinvestment unless RA never intervenes, see the left hand side of Figure 4.[33] Thus, never intervening together with low insurance coverage $\gamma^*(1)$ is the unique policy which achieves first best under direct liquidation inefficiency. The result that all optimal pairs feature overin-

[33] By $\theta_b(a, \gamma^*(a))H = T(a) < \max(r, l)$



(a) $l = r = 0.5,\ H = 4$ (b) $l = r = 0.3,\ H = 4$

Figure 6: Change of deadweight loss in insurance coverage for different degrees of forbearance. At each forbearance level a the deadweight loss is minimized and brought to zero at some unique, interior coverage level γ^*, marked by vertical lines. The minimizer γ^* decreases (moves to the left) as forbearance goes up, that is the vertical line moves to the left for higher a.

vestment is intuitive. Inefficient runs enforce asset liquidation and by this the occurrence of the direct efficiency loss inefficiently often. Under overinvestment, investment in assets is continued in too risky states which however implies that realization of the direct loss from liquidation occurs less often.

4.3.2 The Case $r > l$

We discuss this case in detail in subsection 10.2 of the supplementary appendix but name the highlights here. To motivate the case $r > l$, one can imagine that the bank has to raise cash fast during the run. The RA may have more time to find a buyer with a high valuation for the asset or high asset management skills. Three major changes occur. First, the efficiency cut-off switches from $\frac{l}{H}$ to $\frac{r}{H}$. Second, and as a consequence, the direct inefficiency loss from liquidation now declines as RA grants more forbearance. Third, for $r >> l$ bank stability becomes non monotone in forbearance by Proposition 3.2 (B2) if $r > \gamma$. Therefore, this case needs to be discussed more carefully, and the results are not as clear-cut as in the case $r < \ell$. We can show that Theorem 1 is robust if RA's recovery rate is close to funding liquidity l, since then either the case (B1) or (A) of Proposition 3.2 applies, stability is monotone in forbearance. Under low insurance coverage 'never intervene' remains the optimal forbearance policy. For insurance coverage large, immediate intervention remains optimal. This robustness result shows that not RA's lower liquidation efficiency but the drop in strategic uncertainty is the main driver of Theorem 1. For $r >> l$ and $r > \gamma$ and r close to one, the results of Theorem 1 change and the deadweight loss becomes non-monotonic in forbearance. The results of Theorem 2 change since the efficiency cut-off switches to $\frac{r}{H}$. Several properties of the optimal policy, however, remain robust. As in the case $l \leq r$, for every forbearance level, the RA can find a unique, interior optimal insurance coverage level which is characterized by matching the critical state to $T(a)/H$. As before, all pairs of forbearance and optimal insurance coverage are such that overinvestment occurs.[34] That is, *every* optimal policy excludes inefficient runs, independently of the relation of r to l. Under $r > l$, however, the unique optimal policy which implements the first best outcome changes. Immediate intervention $a^* = \underline{a}$ is the unique forbearance level which minimizes the direct loss from liquidation. The optimal insurance coverage level $\gamma^*(a)$ is in general non-monotone. This holds since the target level for the critical state $T(a)/H$ declines in forbearance by $r > \ell$ while the critical state is non-monotone in forbearance. The non-monotonicity of $\gamma^*(a)$ in the case $r > l$ implies that there may exist distinct forbearance levels $a_1 \neq a_2$ for which the levels of optimal insurance coverage coincide $\gamma^*(a_1) = \gamma^*(a_2)$.

4.4 Construction of optimal deposit insurance levels

We next construct optimal insurance levels to demonstrate the various effects at play. Fix $a = \underline{a}$. We want to find $\gamma^*(\underline{a})$. Calculate $s(\underline{a})$ for $s(a) = \frac{r(1-a)}{1-\ell a}$ and determine $T(\underline{a})/H$. At the limit, if the insurance fund is not liable given resolution, the trigger is given as

$$\theta_A(a, \gamma) = \frac{(1-\gamma) - (1-s(a))\ln(\ell a)}{H - \gamma} \tag{15}$$

If the insurance fund is liable given resolution, the trigger is given as

$$\theta_B(a, \gamma) = \frac{(1-\gamma)(1-\ln(\ell a))}{H - \gamma} \tag{16}$$

For fixed forbearance level a, it holds $\theta_A(a, \gamma) < \theta_B(a, \gamma)$ if and only if $\gamma < s(a)$, i.e. if the insurance fund is not liable. Thus, the smaller trigger is the equilibrium

[34] By $\theta_b = T(a)/H < \theta_e$. Note, this result also holds due to the redefinition of the efficiency cut-off from l/H to r/H.

trigger. To find $\gamma^*(\underline{a})$, solve for $\gamma_A^*(\underline{a})$ and $\gamma_B^*(\underline{a})$ via the implicit functions

$$\theta_A(\underline{a}, \gamma_A^*) = \frac{T(\underline{a})}{H}, \quad \text{and} \qquad \theta_B(\underline{a}, \gamma_B^*) = \frac{T(\underline{a})}{H} \tag{17}$$

The optimal insurance level in $a = \underline{a}$ equals

$$\gamma^*(\underline{a}) = \min(\gamma_A^*(\underline{a}), \gamma_B^*(\underline{a})) = \begin{cases} \gamma_A^*, & \gamma_A^*, \gamma_B^* < s(\underline{a}) \\ \gamma_B^*, & \gamma_A^*, \gamma_B^* > s(\underline{a}) \end{cases} \tag{18}$$

Note, the trigger functions coincide only at the insurance level $\gamma = s(a)$ where they cross. Therefore, either $s(a)$ exceeds or undercuts both γ_A^* and γ_B^* for any forbearance level one may consider. Intuitively, for a fixed forbearance level, the bound $s(a)$ marks the insurance level at which the fund becomes liable.

To analyze a change in the optimal insurance level γ^*, consider a marginal change in forbearance. As forbearance increases, the curve $\theta_B(\gamma)$ decreases pointwise for all γ. Further, the bound $s(a)$ declines. In the case $r \leq \ell$, also the curve $\theta_A(\gamma)$ decreases pointwise in a for all γ and $T(a)/H$ increases. Therefore, both candidates γ_A^* and γ_b^* drop, the optimal $\gamma^*(a)$ has to decline. The case is less

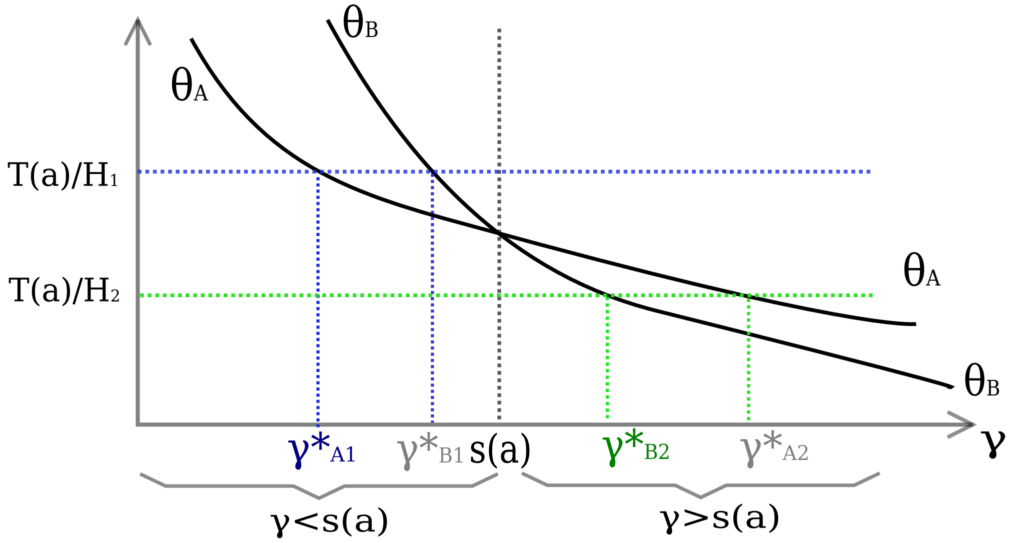


Figure 7: Construction of optimal insurance at fixed forbearance level a. Depending on the bound $T(a)/H$, the candidates γ_A^* and γ_B^* both lie either above or below the threshold $s(a)$. In either case, the smaller candidate is the optimal insurance level. If the candidates are located above $s(a)$, both candidates impose losses on the insurance fund given a resolution. If they are located below $s(a)$, the insurance fund does not become liable.

clear-cut if $r > \ell$. The curve $\theta_B(\gamma)$ still decreases pointwise for all γ and thus the candidate θ_B^* declines in a first step. On the other hand, the curve $\theta_A(\gamma)$ now monotonically increases in a for all γ if r is close to one.Thus, the candidate θ_A^* increases. If $T(a)/H$ is such that both candidates exceed $s(a)$, the upwards shift in $\theta_A(\gamma)$ may not be relevant since θ_B^* is the important candidate. In a second effect, however, $T(a)/H$ declines which strengthens the increase of the candidate θ_A^* but opposes the change in candidate θ_B^*. Therefore, the optimal insurance level γ^* may increase or decrease in forbearance.

5 Discussion of Results and Policy Implications

Strength of policy tools Our results for the case where the bank liquidates as efficient as RA does, demonstrate that deposit insurance coverage is the stronger policy parameter than the forbearance level: By Theorem 2, RA can achieve first best for every *arbitrarily* fixed forbearance level by finetuning the amount of insurance coverage. By Lemma (4.1) the opposite is not true. Theorem 1 in combination with Lemma (4.1) states that if RA cannot finetune deposit insurance coverage, inefficiencies may exist. If coverage is low, not intervening is optimal but inefficient runs will persist. If coverage is high, immediate intervention is optimal, but inefficient investment remains possible.

Time-inconsistency When RA can set both, the amount of insurance coverage and forbearance, she can achieve the first best outcome, independently of the relation of r to l. As a consequence, the time-inconsistency problem discussed in Ennis and Keister (2009) vanishes. That is, ex post, given a run is on the way, RA has no incentive to deviate from her announced policy since the run is efficient. Crucial for this result to obtain is that RA can commit to walking away.

In the case where RA cannot finetune deposit insurance coverage and coverage is low, inefficient runs still occur. That is, RA may have an incentive to deviate from her policy and stop the run. Our results, however, say that to minimize the likelihood of inefficient runs ex ante, it is optimal to commit never to intervene. That is, even though intervention is ex post optimal when inefficient runs occur, a stricter intervention policy, i.e., stop the run at some point, will alter depositors' behavior only in a way that inefficient runs become more likely ex ante. That is, given RA cannot finetune deposit insurance coverage and coverage is low, our results coincide with Allen and Gale (1998). In a setting without deposit insurance, they show that a laissez-faire regime cannot achieve first best under costly liquidation. On the other side, however, we show, if the RA can finetune insurance, then for every $r < l$, RA can achieve the first best outcome via laissez-faire (not intervening) when providing insurance $\gamma^*(1)$.

Policy and Applications When it comes to resolving banks, the objectives of European regulators potentially differ from those in the United States. The FDIC operates under the least cost resolution requirement to minimize losses to the deposit insurance fund. In Europe, on the other hand, the BRRD also mentions the prevention of contagion to other institutions and maintenance of market discipline as objectives. The U.S. approach foresees fast intervention while the European approach is potentially softer.[35] Under the assumption that the bank and RA liquidate equally efficient, both the U.S and the European forbearance policy can, however, achieve the first-best outcome, by this realizing equal levels of welfare. For this to obtain, Europe and the U.S. need to provide distinct levels of insurance coverage. If European interventions were slower than U.S. interventions in the sense that Europe allowed more withdrawals before shutting down banks, the European level of deposit insurance coverage would need to be lower compared to the U.S. coverage level. Under the premise that the regulator and the bank liquidate at distinct efficiency levels, this equivalence result, however, breaks down. If the regulator liquidates less efficient, laissez-faire (never intervene) combined with low insurance coverage is the only policy to implement the first best outcome. If the regulator, on the other hand, liquidates more efficiently than the bank does, immediate intervention combined with high insurance coverage is the unique policy to implement first best. In particular, the first best policy implies extreme intervention, either immediately or never. The reason for optimality of such extreme intervention is the direct liquidation efficiency which arises as additional friction as soon as the bank and the regulator differ in their liquidation efficiency levels. Only extreme intervention puts the direct liquidation inefficiency $\max(r, \ell) - T(a)$ at zero since it imposes that the institution with higher liquidation efficiency liquidates the entire asset.[36] 'Mild' intervention $a \in (\underline{a}, 1)$, on the other hand, allows the less efficient institution to liquidate some fraction of the asset which results in a loss, see also the discussion in section 4.1.

While the extremes of immediate intervention or never intervening can be optimal under the right amount of insurance coverage, by Theorem 2 and 4 in the appendix, there exists no intervention policy under which full insurance coverage of 100% or zero coverage 0% is optimal. Optimality requires partial coverage no matter the intervention policy, independently of whether $r \leq l$ or $r > l$.

The case of low or zero deposit insurance coverage also applies to systemically

[35] This also holds by the possibility to delay the resolution of banks in the framework of the BRRD via veto by the European Commission and the Council.

[36] By the lower bound on $a \geq \underline{a}$, in the case $r > \ell$, the regulator liquidates the entire asset under immediate intervention only asymptotically as $\underline{a} \to 0$.

relevant investment or shadow banks. Such institutions do not offer deposit insurance but are prone to runs by their investors, for instance, uninsured money market investors. Systemically relevant non-deposit making institutions are supervised by the FDIC since the inception of the Dodd-Frank Act. Thus, the FDIC may intervene while not providing insurance coverage. By Lemma 4.1, these institutions suffer from inefficient runs, independently from when the FDIC decides to intervene. If the U.S. regulator liquidates less efficient than the according financial institution, never intervening minimizes the likelihood of inefficient runs ex ante and maximizes welfare by Theorem 1. If, however, the regulator liquidates more efficient, immediate intervention is optimal by Theorem 3 in the appendix.

Efficient Runs It is important to understand that given our welfare concept, runs on the bank can be efficient, namely for state realizations below the efficiency cut-off, see also (Allen and Gale, 1998) and (Eisenbach, 2017). Bank runs can be socially desirable because the insurance fund is financed by depositors. Losses born by the insurance fund are ultimately imposed on depositors via taxation. Second, as opposed to the setting provided in Diamond and Dybvig (1983), here, the asset is risky. Thus, the insurance fund becomes liable not only in the case of bank resolution but also absent resolution if the asset fails to pay. High insurance prevents runs and thus resolution but does not affect the aggregate risk. Since absent resolution, investment in the asset is continued, high insurance leads to a risk-shift away from the deposit contract towards the insurance fund. Losses born by depositors if the asset does not pay off are reimbursed by the insurance fund if insurance coverage is high. Depositors pay for this risk-shift through the back door via taxation unless excessive aggregate risk is eliminated through asset liquidation enforced by runs, see also Cooper and Ross (2002). Provision of partial insurance such that runs occur if and only if it is efficient to liquidate the asset leads to a Pareto improvement compared to the case of full insurance coverage since it balances risk-shifting while limiting inefficient runs.

Optimal contracts and the Welfare concept The paper assumes an exogenously given demand-deposit contract which may not be optimal. Instead, the regulator maximizes welfare via her policy tools. In fact, in this model, solely maximizing the contract value is not reasonable. The contract value is simply maximized by setting deposit insurance at 100%. This, however, implies complete free riding at the expense of the insurance fund and neglects that depositors finance the fund themselves. Excessive costs imposed on the insurance fund in the form of high insurance coverage are paid indirectly by depositors via the lump-sum tax τ, see above. Payments between the insurance fund and depositors are not solely transfers. Insurance coverage and intervention delay affect information aggregation of depositors, by this affecting when the insurance fund becomes liable. If depositors trigger asset liquidation by running on the bank, they may protect the insurance fund from excessive aggregate risk, by this increase the value of the fund. Vice versa, if the fund provides too little insurance, depositors withdraw too often by this reducing the value of the demand-deposit contract.

Value maximization to depositors, therefore, requires to maximize the joint utility of the contract and the value of the insurance fund. Following the well-known accounting identity[37], we can show that the RA maximizes this joint value when minimizing the deadweight loss[38] :

Lemma 5.1. *Welfare equals the combined value of debt and the insurance fund.*

A bank has no control over the insurance coverage level and can therefore hardly maximize this joint value when setting the contract only. This is particularly apparent by Lemma 4.1 which states that inefficient runs persist under low insurance, independently of when intervention takes place, see also (Goldstein and Pauzner, 2005). While the bank cannot set the insurance level, the RA cannot set the contract directly. The fact that RA can achieve the first best outcome, however, implies that the contract $(1, H)$ is *the* optimal contract under RA's optimal policy. That is, under the first best policy, the contract $(1, H)$ achieves that depositors' joint value from the contract and the insurance fund is at its maximum. We further show in the appendix that our optimality results extend to more general contracts. Our approach to optimize via the policy maker's tools has an additional advantage. Under partial debt financing, banks no longer set contracts to maximize the value of debt but the value of equity. Welfare in this context, however, equals the joint value of equity, the contract, and the insurance fund. The objectives of the bank and the regulator thus no longer coincide. In the appendix, section 10.3, we show how our setting easily extends to partially debt-financed banks.

Our welfare concept nests those from the previous literature where welfare is defined as the value depositors infer from the demand deposit contract but where the setting either imposes zero deposit insurance (Goldstein and Pauzner, 2005;

[37]The bank's value equals the value of the banks' liabilities and equity, given no deposit insurance. For the all debt-financed bank, equity is zero, and the value of debt equals the value inferred from the demand-deposit contract.

[38]The definitions of the values of the debt contract and the insurance fund can be found in the proof of Lemma 5.1. We have not included them here since we state Lemma 5.1 for the purpose to justify our welfare concept.

Allen and Gale, 1998) or a safe asset (Diamond and Dybvig, 1983).[39] Imposing zero insurance coverage, RA's objective here becomes equivalent to maximizing the value of the given demand deposit contract since the bank is all debt-financed. RA, however, differs in the tools she uses. In Diamond and Dybvig (1983); Goldstein and Pauzner (2005); Allen and Gale (1998) the planner optimizes via the contract directly, while here, RA takes the demand-deposit contract as given by the bank and uses forbearance and insurance coverage as tools to alter outcomes. If RA's policy implements the first best outcome, then, nevertheless, the contract is the optimal contract given her policy. RA's objective can, therefore, be understood as to design depositors' incentives through her policy in a way which makes the bank-given contract the optimal contract given her policy. Note, under partial debt financing, RA's objective to minimize the deadweight loss is equivalent to maximizing the combined value of the bank (debt and equity), and the insurance fund, see subsection 10.3 of the supplementary appendix for details.

6 Robustness and Discussion of Assumptions

The following section briefly discusses model assumptions and summarizes extensions to the model. Subsection 10.4 discusses robustness when the bank pledges assets, not to outside investors but the lender of last resort (ELA). Subsection 10.5 discusses how our results extend to the case of asset sales.

6.1 Transition to the general setting

The benchmark model assumes that the bank is all debt-financed. Further, the model assumes an exogenously fixed contract $(1, H)$ to depositors and an interest rate $j = H$ for borrowing from outside investors. The benchmark model thus features that the asset return H is also the long-term coupon to depositors and the interest rate payable to outside investors for borrowing short-term for one period. We first explain why we make these assumptions and then explain, why these assumptions are generally not restrictive. In subsection 10.3 of the supplementary appendix, we give the technical details on the generalization. The idea of the article is to provide a simple analysis which however also applies to partially debt-financed banks. Accommodating partial debt financing into the benchmark model involves adding two additional parameters. First, a *fixed* long-term coupon R_2

[39] In Diamond and Dybvig (1983), all runs are inefficient, $\theta_e = 0$. In particular, high insurance coverage cannot shift asset risk towards the insurance.

for rolling over the deposit which is distinct from the asset's return H. Second, a debt ratio $\delta \in (0,1)$ which pins down the proportion of the initial bank investment financed with debt and the proportion $1-\delta$ financed with equity. One way of circumventing the explicit treatment of these two additional parameters is by assuming an all debt-financed bank. This assumption alone is however not sufficient. Once the bank is all debt-financed, depositors own the bank. Following the models of Diamond and Dybvig (1983) and Goldstein (2010), depositors who roll over their deposit will share the residual value from the investment. The payoff from rolling over is thus 'soft' and depends on the equilibrium measure of depositors who withdraw, n. As the bank substitutes some debt with equity, by this leaving the setting of Diamond and Dybvig (1983) and Goldstein (2010), debt contracts to depositors however typically offer *fixed* payments instead of pro rata shares, also to depositors who roll over.[40] In short, depositors of an all debt-financed bank have in general different preferences and thus incentives compared to depositors of a partially debt-financed bank. Here the assumption $j = H$ comes into play. When combining the assumption of an all debt-financed bank with the assumption $j = H$, the pro rata share $\frac{H-nj}{1-n}$ to depositors who roll over becomes constant and therefore debt-like. We achieve a parameter reduction while maintaining depositors' incentives. Our treatment not only allows a straightforward extension of the benchmark model to the more general case of partial debt financing but the results developed remain robust. In subsection 10.3, we extensively discuss the general setting for contracts (R_2, R_2), general capital structures and interest rates j. There, we also explain in detail why more general repo rates $j \neq H$ do not affect depositors' behavior. In short, given the bank is not resolved and solvent, depositors obtain their fixed payments since the interest rate j eats into equity investors' profits, not depositors'. As a consequence, for fixed long-term coupons, changes in the interest rate leave depositors' incentives unchanged.

6.2 Asset Sales

Now assume, we change the model such that the bank no longer refinances withdrawals by pledging but by selling assets. In the case of pledging assets, the bank avoids liquidation costs absent resolution, even if withdrawals occur in $t = 1$. Interest accrued on the pledged part of the asset goes to the bank if she can repay the outside investor in $t = 2$, i.e., if the asset pays off. In the case of asset sales instead, the interest goes to the new owner of the asset. Therefore, under asset

[40] This is since equity investors are the ones to earn the residual value.

sales, withdrawals are costly to the bank in terms of foregone profits even absent resolution while no liquidation cost applies in case of asset pledging.

In the case of asset pledging, as the bank forbears more, ex ante, a smaller liquidation cost applies. This holds since it takes more depositors to cause resolution. Thus, liquidation costs apply only for larger aggregate withdrawals. One might believe that this feature, the avoidance of liquidation costs, drives the result that stability can improve in forbearance. By considering asset sales instead of asset pledging, we, however, argue that this is not the case. Lemma 10.2 in the appendix 10.5 shows that stability can improve in forbearance under asset sales although RA imposes the costs of liquidation on depositors. The result demonstrates that the primary driver of stability improvements in forbearance is the drop in strategic uncertainty as RA intervenes later. As long as forbearance is sufficiently low, rolling over is optimal absent resolution, despite liquidation costs. As forbearance becomes high, liquidation costs do however play a more decisive role to depositors. This holds since under asset sales, liquidation costs for repaying withdrawing depositors eat into remaining bank investment. If remaining investment at $t = 1$ is insufficient, the bank cannot repay depositors who roll over entirely, and the bank is not resolved but insolvent in $t = 1$. In this case, depositors who roll over only obtain a pro rata share of returns on investment. Given insolvency, this pro rata share from rolling over may undercut the payoff from withdrawing early if RA intervenes late. Thus, under asset sales, 'withdraw' can be the optimal action even absent resolution while under asset pledging, 'withdraw' is optimal if and only if a resolution occurs. We show that robustness fails as forbearance goes to the maximum: stability declines in forbearance because liquidation costs which are potentially imposed on depositors absent resolution become high. The destabilizing effect of higher liquidation costs dominates the stabilizing effect of a drop in strategic uncertainty. The case of asset sales also shows that in general, depositors face a trade-off between increased liquidation cost versus the drop in uncertainty, as the bank forbears more.[41] For low forbearance, liquidation costs are small which explains Lemma 10.2. For high forbearance, costs are large which explains why robustness then fails. For details, refer to subsection 10.5 of the supplementary appendix.

[41] To be more precise, under asset pledging there is a trade-off between a decline in the pro rata share versus the drop in strategic uncertainty. Under asset sales, the liquidation costs occur on top, and the trade-off exists with the decline in the pro rata share and the increase in the liquidation costs on the one hand versus the drop in strategic uncertainty on the other hand.

6.3 Discussion

As in Goldstein and Pauzner (2005), our analysis assumes that the state θ is uniformly distributed on $[0, 1]$. Their model additionally assume a return likelihood function $p(\theta)$ which allows to pin down alternative distribution functions of the state. Since $p(\cdot)$ is strictly increasing, incorporating the same function into our model is straightforward and will yield the same results.

We assume in the paper that RA does not know the state while depositors observe information via signals and act upon them. Assume depositors could report their signals to RA and RA could pool signals to learn the state perfectly. Then, depositors will not necessarily report truthfully. If a depositor observes a signal from which she infers that the state lies in the lower dominance region, a truthful report will cause RA to resolve the bank for liquidation. The depositor obtains a pro rata share in return. If she reports a high signal instead but withdraws, RA may not resolve the bank, and the depositor has a shot at recovering her entire deposit. As a consequence, RA cannot rely on reports.

In the benchmark model, RA commits to her policy at time zero before the state realizes. In particular, in $t = 0$, she commits to allowing a critical measure of withdrawals in $t = 1$ before intervening. In equilibrium, however, RA could correctly learn the state from observing withdrawals and act upon the state realization directly, by this potentially deviating from her announced policy. First, in the benchmark model where RA can set both insurance coverage and forbearance, she can always achieve the first best outcome. Thus, acting upon the state directly cannot improve welfare as opposed to the benchmark case where RA commits to her policy. Now assume instead, RA can only alter forbearance for given insurance coverage level. Thus, inefficiencies may exist. Consider the case $r = l$ and assume insurance coverage is low. Then inefficient runs may occur, $I := (\theta_e, \theta_b)$ is non empty. The optimal forbearance policy is never to intervene, which shrinks the interval as much as possible. If RA could observe the state, she may only want to deviate from her policy for state realizations in I. RA would want to prevent depositors from withdrawing to continue investment, but intervention always liquidates the investment. Thus, intervention does not help. Further, under rational expectations, depositors know that RA deviates for state realizations in I. The interval I might widen as an equilibrium response by depositors since withdrawing is optimal given resolution.[42] Therefore, commitment is crucial to our results.

[42] In fact, the policy 'never intervene' for $\theta \notin (\theta_e, \theta_b)$ but 'intervene as soon as possible' for $\theta \in (\theta_e, \theta_b)$ implies a different game to depositors compared to the benchmark case since forbearance is now state dependent, and thus random.

Limited commitment of a regulator has for instance been studied in Keister and Mitkov (2016) and Ennis and Keister (2009).

Acting upon the state may not increase welfare since RA has no tool for protecting the bank's investment. This feature is in contrast to Diamond and Dybvig (1983) where RA continues investment as she intervenes. In their model, however, the asset is risk-free. If RA could continue investment despite intervention in our set up, one also needs to consider that RA may not have the same level of asset management expertise as the bank does. As a consequence, the returns on the risky asset may drop as RA takes over. Given RA could partially protect the investment by intervention, as long as depositors' payoffs under resolution undercut the face value of the contract, 'withdraw' remains the optimal action given a resolution. Thus, our results may be robust even though RA continues investment when intervening.

In our model, both the bank and outside investors are non-strategic. The bank has either access to refinancing via asset sales or asset pledging, not both. Outside investors offer a given amount of funds at a fixed interest rate i. For a setting where the bank strategically selects her refinancing instrument and where the repo market competitively prices the interest rate, refer to (Schilling, 2017).

In the real world, we typically observe that funds are insured up to a fixed limit. The model assumes instead partial insurance as a percentage of the deposit. This assumption is without loss of generality. Assume the interim face value of debt is R_1. Assume, insurance covers the maximum amount of x. Then, for $x > R_1$, the depositor is fully insured, $\gamma = 1$.[43] We discuss this case as a limit result and show that depositors become unresponsive. If depositors hold deposits more than the fixed insured amount, $x \in (0, R_1)$, the depositor is partially insured at fraction $\gamma = \frac{x}{R_1}$ and our setting fully applies. In a setting where some depositors are fully insured while others are only partially so, fully insured deposits are like equity to partially insured depositors. We have analyzed this case in an extension as well.[44]

Our model assumes that insurance is financed lump-sum via symmetric taxation of depositors. This kind of taxation does not alter incentives. Here symmetry is with respect to timing and aggregate withdrawals. If RA can tell aggregate withdrawals in advance, as in Diamond and Dybvig (1983), she can anticipate the maximum liability of the insurance fund and tax contingent on aggregate withdrawals. This feature allows her to lower taxation for some realizations of withdrawals while still

[43] Note, our results show that fully insured depositors are unresponsive to news.

[44] Voellmy (2017) provides an application of this feature. If bank assets are safe, insured deposits can deter self-fulfilling runs up to certain bank balance sheet size. This feature allows Voellmy (2017) to study the optimal size of the competing shadow banking sector.

guaranteeing the same level of insurance coverage. Ultimately, taxation contingent on aggregate withdrawals allows depositors to increase consumption. RA can however only lower taxation given resolution takes place. In this case, RA anticipates that la depositors are served in the queue who will not require insurance. If RA sets her forbearance level such that the fund does not become liable, given resolution, she can, in fact, set the tax to zero while guaranteeing γ. If RA sets a forbearance level such that given resolution the insurance fund becomes liable, RA can still lower the tax to $\tau = (1 - la)(\gamma - s(a))$ per depositor. By aggregate risk, absent resolution RA still needs to tax the full amount $\tau = \gamma$. This is, since if all depositors roll over and the asset fails, the fund faces claims by all depositors. As a consequence, RA cannot help but build up reserves absent resolution if the asset pays off high. Given resolution, she can always run a balanced budget. Also, under withdrawal-contingent taxation, depositors who withdraw at time one and time two are taxed the same amount. Thus, our results on depositor behavior remain identical.[45] Also, our welfare results remain valid. This is since the tax is only a transfer from depositors to the insurance fund. As long as a change in tax guarantees the same level of insurance γ, value remains constant.

If taxation asymmetry is introduced by levying different taxes on depositors who withdraw at time one and time two, relative payoffs and thus our results change. This asymmetry adds a degree of freedom and may help RA to overcome inefficiencies, see Keister (2015) for Pigouvian taxation.

7 Discussion of the Literature

Our paper is connected to the literature stand on bank runs, liquidity risk and self-fulfilling beliefs. In a seminal paper, Diamond and Dybvig (1983) show that risk sharing through demand-deposit contracts gives rise to multiple equilibria. In their model, tax-financed deposit insurance or adequate suspension of convertibility can eradicate the bad bank run equilibrium, by this allowing the implementation of optimal risk sharing. Our paper instead focuses on the interaction between the two instruments. Concerning the model, we are closest to Goldstein and Pauzner (2005) who use global games methods to analyze the optimality of risk sharing via demand deposit contracts in the context of bank runs. As opposed to Goldstein and Pauzner (2005) we add a strategic resolution authority who can intervene and (partially) insured deposits. Keister and Mitkov (2016)

[45] Our results transfer, since to depositors, only payoff differences conditional on aggregate withdrawals matter. Under symmetric taxation across time, the tax cancels out of the difference.

study flexible resolution policies where banks may set state-contingent contracts. They show how the regulator's lack of commitment to bail out rules, and her delay in pinpointing weak banks during crises gives the incentive and opportunity for intermediaries to delay efficient bail-ins. As opposed to Keister and Mitkov (2016), in our setting the regulator's delay until intervention is strategic. Similar to Allen and Gale (1998), runs in our setting can be first best efficient namely if aggregate risk realizes high. Allen and Gale (1998) however do not consider deposit insurance. Dávila and Goldstein (2016) analyze optimal deposit insurance under moral hazard from the bank side. Cooper and Ross (2002) analyze risk shifting of banks through deposit insurance. As opposed to Dávila and Goldstein (2016) and Cooper and Ross (2002), our model features a non-strategic bank to consider changes in depositors' incentives instead. Also, we add strategic delay. Similar to Ennis and Keister (2009), we analyze how anticipation of intervention can generate and affect depositors' incentives to participate in the run. In a Diamond and Dybvig type model, Ennis and Keister (2009) focus on ex post efficient intervention during runs. As opposed to their paper and Diamond and Dybvig (1983), our model features aggregate uncertainty. Runs can be efficient, and depositors may run inefficiently seldom (overinvestment) which impacts optimal intervention policies. As opposed to Diamond and Dybvig (1983), Ennis and Keister (2009) and Dávila and Goldstein (2016), we use a global games approach to obtain a unique equilibrium such that we can analyze how the propensity to run changes and feeds back into RA's objective function as she varies intervention delay and insurance coverage. Similar to Calomiris and Kahn (1991), in our paper demandable debt serves as a disciplining device. However, while in Calomiris and Kahn (1991), depositors discipline the bank which faces a moral hazard problem, here RA finetunes depositors' incentives to prevent the insurance fund from bearing excessive risk. Our paper further adds to the literature strand on bank resolution frameworks. Colliard and Gromb (2018) study how austerity of bank resolution frameworks affect incentives of bank stakeholders to restructure debt. Bolton and Oehmke (2018) analyze the efficiency of transnational bank resolution mechanisms when bank regulators have national objectives. Under rational expectations, deposit insurance acts similar to anticipated, and publicly financed bailouts for deterring runs. In this context, Keister (2015) analyzes under what conditions the restriction of bailouts is socially desirable when bailouts can deter runs but distort incentives of intermediaries to privately provision for crises. Relatedly, Li (2016) studies this problem under bank portfolio choice. Walther and White (2017) study optimality of ex ante commitment to policy rules when the revelation of private information through the regulator's policy choice may trigger adverse behavior by creditors. To obtain an equilibrium selection this paper uses global games technique (Carlsson and Van Damme, 1993; Morris and Shin, 2001). In this respect, our paper is similar to Morris and Shin (2016), Rochet and Vives (2004) and Eisenbach (2017) who consider credit risk, respectively interventions by a lender of last resort or efficiency of asset liquidation, all in a global games context.

Further related in this respect are Allen et al. (2017), Ahnert and Kakhbod (2017) and Matta and Perotti (2017) who analyze government guarantees, amplification mechanism of financial crises respectively secured repo funding under rollover risk using global games. In Green and Lin (2003), Peck and Shell (2003), Diamond and Dybvig (1983) and Goldstein and Pauzner (2005), depositors can be of two types, patient and impatient. The regulator, who cannot observe types, sets the deposit contract to balance the benefits of risk-sharing and the risk of runs caused by patient types who pretend to be impatient. Here instead, all depositors are patient. However, deterrence of runs can be inefficient since the asset is risky and the level of deposit insurance is not state contingent. In our framework, depositors take their rollover decisions simultaneously, but withdrawals occur gradually. As opposed to Green and Lin (2003), here depositors cannot anticipate their position in the queue upon withdrawing. Consequently, the action to withdraw is strictly optimal when a resolution is anticipated, as in He and Manela (2016); Peck and Shell (2003). In a model without endogenous roll-over risk, Kareken and Wallace (1978) show that banks react to deposit insurance by risk-shifting through altering their investment portfolios. Our paper here abstracts from portfolio choice to isolate the interaction between depositors and the regulator. Allowing for additional portfolio choice by the bank would be interesting. Since we impose rational expectations and sequential rationality, depositors will however perfectly anticipate the bank's risk-shifting, even if risk-shifting is not directly observable. As a consequence, depositors will adapt their run behavior and may punish the bank for risk-shifting by withdrawing more often ex ante such that the bank becomes illiquid more often. Since the bank is rational too, she may refrain from risk-shifting despite increasing deposit insurance. Alternatively, the bank's incentive to shift risk may be mitigated through depositors' threat to run, see also Deidda and Panetti (2018) for portfolio choice under endogenous roll-over risk without a regulator (deposit insurance and intervention). Similar to Eisenbach (2017), this paper discusses efficiency of asset liquidation through bank runs under endogenous roll-over risk. Eisenbach (2017) considers the privately implemented liquidation policy and how the efficiency achieved interacts with aggregate risk in a general equilibrium setting. In contrast, this paper considers the liquidation policy implemented by a regulatory authority and focuses on the rich interactions of the regulatory instruments of forbearance and deposit insurance coverage.

8 Conclusion

In the U.S. and Europe, the regulator, through her resolution authority (RA), has a double role when it comes to resolving banks. On the one hand, RA provides deposit insurance. On the other hand, RA monitors withdrawals of depositors at the bank level. If withdrawals are 'abnormally high' due to a solvency shock, RA has the authority to suspend conversion of deposits by putting the bank into receivership and seizing bank assets to protect the deposit insurance fund. This paper analyzes how these roles interfere with each other. We analyze optimal strategic delay of bank resolution in combination with a provision of deposit insurance.

We show, if RA can only set the forbearance level (intervention delay), inefficient runs or inefficient investment may exist, depending on the level of provided insurance coverage.

If RA can, however, set both the intervention threshold and insurance coverage, she can always implement the first best outcome. This means she can steer depositors' incentives to withdraw in a way that bank resolution occurs if and only if asset liquidation is efficient. As a consequence, the joint value of the demand deposit contract and the insurance fund is maximized.

9 Appendix

Proof. [Proposition 3.1] We first show equivalence of this game to a version of the game in Goldstein Pauzner: Conditional on resolution, the payoff difference from rolling over versus withdrawing equals

$$\Delta = (s_\gamma(a) - \tau) - \left[\frac{la}{n} \cdot 1 + (1 - \frac{la}{n})s_\gamma(a) - \tau\right] = -\frac{la}{n}(1 - s_\gamma(a)) = -\frac{f(a,\gamma)}{n} \tag{19}$$

for $f(a,\gamma) = la\ (1 - s_\gamma(a)) > 0$. Conditional on no resolution, the payoff difference equals

$$\Delta_2 = (H - \tau)\theta + (\gamma - \tau)(1 - \theta) - (1 - \tau) = H\theta + \gamma(1 - \theta) - 1 \tag{20}$$

Thus, for $\tau = \gamma$ but in particualar also for $\tau \neq \gamma$, the benchmark model is equivalent to a game which has close similarity to the model analyzed in Goldstein and Pauzner (2005)

Event/ Action	Withdraw	Roll-over
no Run $n \in [0, la]$	1	$\begin{cases} H & ,p = \theta \\ \gamma & ,p = 1 - \theta \end{cases}$
Run $n \in (la, 1]$	$\frac{f}{n}$	0

The proof is identical to the proof in Goldstein and Pauzner (2005). For details, please see subsection 10.1 of the supplementary appendix. □

Lemma 9.1. *For $\gamma > r$, it holds $s_\gamma = \gamma$ for all $a \in (0, 1]$. For $\gamma < r$, it holds $s_\gamma = r(1-a)/(1-\ell a)$ for $a \in (0, (r-\gamma)/(r-\ell\gamma)]$ and $s_\gamma = \gamma$ for $a \in ((r-\gamma)/(r-\ell\gamma), 1)$.*

Proof. [Lemma 9.1] Let $\gamma > r$. Remember, $a \in (\underline{a}, 1] \subset (0, 1]$ and $\ell \in (0, 1)$. Thus, $\ell\gamma - r$ can be negative or positive. It holds

$$\frac{r(1 - a)}{1 - \ell a} > \gamma \tag{21}$$

if and only if $a(\ell\gamma - r) > \gamma - r$. Thus, if $\ell\gamma - r$ is negative, (21) can never hold for any $a \in (0, 1]$. Thus, $s_\gamma = \gamma$ for all $a \in (0, 1]$. If $\ell\gamma - r > 0$, then $a(\ell\gamma - r) \leq \ell\gamma - r < \gamma - r$ by $\ell \in (0, 1)$. Again, $s_\gamma = \gamma$ for all $a \in (0, 1]$. In particular, s_γ is independent of a.

Now, let $r > \gamma$, then also $r > \ell\gamma$ and (21) holds if and only if $a < (r - \gamma)/(r - \ell\gamma) \in (0, 1)$. In that case, $s_\gamma = r(1 - a)/(1 - \ell a)$ in the pro rata share depends on

a. For $a \in ((r-\gamma)/(r-\ell\gamma), 1)$, $s_\gamma = \gamma$ and the pro rata share in independent of a. $\square$

Proof. [Proposition 3.2] We derive the trigger directly. By uniqueness of a trigger equilibrium the proportion of withdrawing depositors n is a deterministic function of the state and is given by

$$n(\theta,\theta^*) = \mathbb{P}(\theta_i < \theta^*|\theta) = \mathbb{P}(\varepsilon_i < \theta^* - \theta|\theta) = \begin{cases} \frac{1}{2} + \frac{\theta^*-\theta}{2\varepsilon}, & \theta_i \in [\theta^*-\varepsilon, \theta^*+\varepsilon] \\ 1, & \theta_i < \theta^* - \varepsilon \\ 0, & \theta_i > \theta^* + \varepsilon \end{cases} \tag{22}$$

Given signal θ_i a depositors posterior on θ is uniform on $[\theta_i - \varepsilon, \theta_i + \varepsilon]$. The expected payoff difference at a signal θ_i equals

$$0 = \frac{1}{2\varepsilon}\int_{\theta_i-\varepsilon}^{\theta_i+\varepsilon} (H\,\theta + (1-\theta)\gamma - 1)\mathbf{1}_{\{n\in[0,la]\}} - \frac{la}{n}\,(1-s_\gamma(a))\mathbf{1}_{\{n\in[la,1]\}}d\theta$$

Substituting using the function $n(\theta,\theta^*)$, this is equivalent to

$$0 = \int_0^{la} ((H-\gamma)\theta(n,\theta^*) - (1-\gamma))\,dn - (1-s_\gamma(a))\int_{la}^{1}\frac{la}{n}\,dn$$

where

$$\theta(n,\theta^*) = \theta^* + \varepsilon(1-2n), \theta^* \in [\underline{\theta}-\varepsilon, \overline{\theta}+\varepsilon] \tag{23}$$

is the inverse of the function $n(\theta,\theta^*)$. Plugging in $\theta(n,\theta^*)$ and canceling terms yields

$$\theta^* = \frac{(1-\gamma) - (1-s_\gamma(a))\,\ln(la)}{H-\gamma} - \varepsilon(1-la) \tag{24}$$

Since the noise term enters linearly, we can take partial derivatives directly from the limit of the trigger. By (23), $\theta_b = \theta^* + \varepsilon(1-2\frac{la}{\delta})$. Thus, at the limit $\varepsilon \to 0$, we have $\theta_b = \theta^*$ and also the partial derivatives coincide. Set $n^* := la$. With (24),

$$\frac{\partial}{\partial a}\theta^* = -\frac{1}{H-\gamma}\left(\frac{\partial}{\partial a}(1-s_\gamma(a))\,\ln(n^*) + (1-s_\gamma(a))\frac{1}{n^*}\frac{\partial n^*}{\partial a}\right) \tag{25}$$

Further,

$$\frac{\partial n^*}{\partial a} = l \tag{26}$$

thus, the second term in the bracket is positive by

$$(1 - s_\gamma(a))\frac{1}{n^*}\frac{\partial n^*}{\partial a} = \frac{1}{a}(1 - s_\gamma(a)) > 0 \tag{27}$$

By definition, for $a \in (\underline{a}, \overline{a})$, $s_\gamma(a) = \frac{r(1-l)}{1-la}$ while for $a \in (\overline{a}, 1)$, $s_\gamma(a) = \gamma$. Immediately, for $a \in (\overline{a}, 1)$: $\frac{\partial}{\partial a}(1 - s_\gamma(a)) = 0$ and

$$\frac{\partial}{\partial a}\theta^* = -\frac{1}{H-\gamma}\left((1 - s_\gamma(a))\frac{1}{n^*}\frac{\partial n^*}{\partial a}\right) < 0 \tag{28}$$

By Lemma 9.1, for $r < \gamma$ it holds $\overline{a} = 0$, thus stability monotonically improves in forbearance for all $a \in (\underline{a}, 1) \subset (0, 1)$ which yields result (A). In what follows, consider $r > \gamma$ such that $\overline{a} \in (0, 1)$. By the argument above, for $a > \overline{a}$, stability monotonically improves in forbearance on $a \in (\overline{a}, 1)$ and we obtain the first part of result (B2). Now consider $a \in (\underline{a}, \overline{a})$, then $(1 - s_\gamma(a)) = 1 - \frac{r(1-l)}{1-la}$ and

$$\frac{\partial}{\partial a}(1 - s_\gamma(a)) = -\frac{-r(1-la) + lr(1-a)}{(1-la)^2} = \frac{r(1-l)}{(1-la)^2} > 0 \tag{29}$$

However, $\ln(n^*) < 0$, thus in general, the change of the trigger in forbearance can be non-monotone. From (25), using the logarithm inequality $\ln(1+x) > x/(x+1)$,

$$\frac{\partial}{\partial a}\theta^* = -\frac{1}{H-\gamma}\left(\frac{r(1-l)}{(1-la)^2}\ \ln(la) + \frac{1}{a}(1 - \frac{r(1-a)}{1-la})\right) \tag{30}$$

$$< -\frac{1}{H-\gamma}\left(\frac{r(1-l)}{(1-la)^2}\ \frac{la-1}{la} + \frac{1}{a}(1 - \frac{r(1-a)}{1-la})\right) \tag{31}$$

$$= -\frac{1}{H-\gamma}\frac{1}{a}\left(1 - \frac{r}{l}\right) < 0 \tag{32}$$

if $r \leq l$. Next consider $r >> l$. From (30) and since $\ln(1+x) < x$,

$$\frac{\partial}{\partial a}\theta^* = -\frac{1}{H-\gamma}\frac{1-l}{1-la}\left(r\left(\frac{1}{(1-la)}\ \ln(la) - \frac{(1-a)}{a(1-l)})\right) + \frac{1}{a}\frac{1-la}{1-l}\right) \tag{33}$$

$$> -\frac{1}{H-\gamma}\frac{1-l}{1-la}\left(r\left(-1\ \ - \frac{(1-a)}{a(1-l)})\right) + \frac{1}{a}\frac{1-la}{1-l}\right) \tag{34}$$

$$= -\frac{1}{H-\gamma}\frac{1}{a}\ \ (1-r) \tag{35}$$

This lower bound is negative for all $r \in (0, 1)$ but approaches zero as $r \to 1$. Thus, $\frac{\partial}{\partial a}\theta^* \geq 0$ for r sufficiently close to one. Last, we show that the cross derivative of the trigger with respect to forbearance and recovery rate is positive.

$$\begin{aligned}\frac{\partial}{\partial r}\frac{\partial\theta^*}{\partial a} &= -\frac{1}{H-\gamma}\left[\left(\frac{\partial}{\partial r}\frac{\partial}{\partial a}(1-s_\gamma(a))\right)\cdot\ln(n^*)+\frac{1}{a}\left(\frac{\partial}{\partial r}(1-s_\gamma(a))\right)\right] \\ &= -\frac{1}{H-\gamma}\left[\frac{(1-l)}{(1-la)^2}\cdot\ln(n^*)+\frac{1}{a}\left(-\frac{1-a}{1-la}\right)\right]>0\end{aligned}$$

since $\ln(n^*)<0$. Altogether, since $\frac{\partial\theta^*}{\partial a}$ is continuous and strictly increasing in r with $\frac{\partial\theta^*}{\partial a}<0$ for $r\leq l$ and $\frac{\partial\theta^*}{\partial a}>0$ for r close to one, the derivative $\frac{\partial\theta^*}{\partial a}$ crosses zero only once. That is, there exists $\varepsilon>0$ such that for all $r\in(0,l+\varepsilon)$ it holds $\frac{\partial\theta^*}{\partial a}\leq 0$ and for all $r\in(l+\varepsilon,1)$, $\frac{\partial\theta^*}{\partial a}>0$. □

Proof. [Lemma 3.1]

$$\frac{\partial\theta^*}{\partial\gamma}=\frac{(-1+\frac{\partial s_\gamma}{\partial\gamma}\ln(la))(H-\gamma)+(1-\gamma)-(1-s_\gamma(a))\ln(n^*)}{(H-\gamma)^2} \tag{36}$$

Plugging in the equilibrium condition (24), at the limit we obtain

$$\frac{\partial\theta^*}{\partial\gamma}=\frac{(H-\gamma)(\theta^*-1+\frac{\partial s_\gamma}{\partial\gamma}\ln(la))}{(H-\gamma)^2}<0 \tag{37}$$

Since either $\frac{\partial s_\gamma}{\partial\gamma}=1$ or zero and $\ln(la)<0$, $\theta^*<1$. □

Proof. [Lemma 4.1] We have for every $r,l\in(0,1)$,

$$\lim_{\gamma\to 0}\theta^*=\frac{1-(1-s_\gamma(a))\ \ln(la)}{H}>\frac{1}{H}>\frac{\max(r,l)}{H}=\theta_e \tag{38}$$

since $-\ln(la)>0$, and $\max(l,r)<1$. On the other hand, for every recovery rate $r\in(0,1)$,

$$(1-s_\gamma(a))\leq 1-\gamma \tag{39}$$

Thus, by $-\ln(la)>0$ and $k>1$,

$$\theta^*=\frac{(1-\gamma)+(1-s_\gamma(a))\ (-\ln(la))}{H-\gamma}\leq\frac{(1-\gamma)(1-\ln(la)}{H-\gamma}\to 0 \qquad \text{as } \gamma\to 1 \tag{40}$$

Thus, we have found an upper majorant for the trigger which converges to zero. By the sandwich lemma therefore $\lim_{\gamma\to 1}\theta^*=0<\max(r,l)/H=\theta_e$. □

Proof. [Lemma 5.1] Since the bank is all debt-financed, define the value of debt as the net value of the debt contract DC:

$$DC(a,\gamma) = \int_0^{\theta_b} n(\theta)\,\left(\frac{la}{n}\cdot 1 + (1-\frac{la}{n})s_\gamma(a)\right) + (1-n(\theta))\,s_\gamma(a)\,d\theta \tag{41}$$

$$+\int_{\theta_b}^1 n(\theta)\cdot 1 + (1-n(\theta))\,(\theta H + (1-\theta)\gamma)\;d\theta - \int_0^1 \gamma\,d\theta \tag{42}$$

$$= \int_0^{\theta_b} (la + s_\gamma(a)(1-la))\;d\theta + \int_{\theta_b}^1 n(\theta) + (1-n(\theta))\,(\theta H + (1-\theta)\gamma)\;d\theta \tag{43}$$

$$-\int_0^1 \gamma\,d\theta$$

Define the net value of the insurance fund as

$$\Gamma(a,\gamma) := \int_0^1 \gamma\,d\theta - (1-la)\;\max(0,\gamma - s(a))\int_0^{\theta_b} d\theta - \int_{\theta_b}^1 (1-\theta)\,\gamma\,d\theta$$

This value holds, since all depositors finance the insurance fund via taxation. The fund pays out in two cases. If resolution occurs and the pro rata share $s(a)$ falls below the insured amount, the fund pays to those depositors which were not served. In addition, absent resolution, in case the asset fails to pay the fund pays all depositors.[46] It is straightforward to show that the value of investment equals the value of debt plus the value of the insurance fund, in either case, $s_\gamma(a) = s(a) = \frac{r(1-a)}{1-la}$ or $s_\gamma(a) = \gamma$, by (i) using the definition of $s_\gamma(a)$ and $T(a)$ and (ii) since for $\varepsilon \to 0$ it holds $n = 0$ for $\theta > \theta_b$. □

Proof. [Theorem 1] The change in deadweight loss is given by

$$\frac{\partial}{\partial a}D(a,\gamma) = \underbrace{-T'(a)\,\theta_b}_{\substack{\text{direct change}\\ \text{in liquidation}\\ \text{efficiency}}} + \underbrace{(\max(r,l) - T(a))\,\frac{\partial\theta_b}{\partial a}}_{\substack{\text{indirect change in}\\ \text{liquidation efficiency due}\\ \text{to change in run behavior}}} + \underbrace{\frac{\partial\theta_b}{\partial a}\cdot(\theta_b H - \max(l,r))}_{\substack{\text{change in efficiency due}\\ \text{to more/less overinvestment}\\ \text{or inefficient runs}}} \tag{44}$$

where the first two terms equal zero in the case $r = l$. As forbearance increases, a larger proportion of the asset is pledged by the bank before a bank resolution is triggered. In return, RA seizes a smaller proportion of the asset given resolution. This change in direct liquidation efficiency is described by the first term in (44), and is positive (increases the deadweight loss), if and only if RA liquidates more

[46] As noise vanishes, absence of resolution implies that all depositors roll over. Thus, if the asset does not pay the insurance fund pays to all depositors.

efficiently than the bank does. Second, forbearance impacts the likelihood of the event bank resolution θ_b and by this alters how often the direct efficiency loss realizes. The second term is positive if and only if depositors run more often as RA grants more forbearance. The third term is the most interesting and concerns two things. First, the bracket is positive if inefficient runs occur with positive likelihood but is negative otherwise, if inefficient investment may occur. As depositors alter their run behavior, inefficient runs or inefficient investment becomes more or less likely. If overinvestment can occur, an increase in stability raises the deadweight loss while a decline in stability lowers the deadweight loss. If inefficient runs are possible instead, an increase in stability lowers the deadweight loss while a decline in stability raises the deadweight loss. If the bank and RA liquidate equally efficient, $r = l$, the first two terms are zero. Let $r \leq l$. Then, from (44), with $T(a) = al + (1-a)r \leq l$,

$$\frac{\partial}{\partial a} D(a, \gamma) = -(l-r)\,\theta_b + (l - T(a))\,\frac{\partial \theta_b}{\partial a} + \frac{\partial \theta_b}{\partial a} \cdot (\theta_b H - l) \tag{45}$$

The first term is negative, the second term is negative since stability improves in forbearance for $r \leq l$ by Proposition 3.2. Both terms together capture the direct efficiency gain from showing forbearance since the bank liquidates more efficient than RA. The sign of the third term depends on the level of insurance coverage provided. (a) If insurance coverage is low, we know $\theta_b > \theta_e = l/H$ by Lemma 4.1. Thus, the bracket of the third term is positive which makes the third term allover negative. All three terms are negative and $a^* = 1$ is optimal. (b) If insurance coverage is low, the third term is positive while the first two terms are negative. If r is close to l, $T(a) \to l$ and the first two terms are close to zero such that the positive third term dominates and $a^* = \underline{a}$ is optimal. □

Proof. [Theorem 2] Let $r \leq l$ and $a \in (\underline{a}, 1]$ arbitrary. Then $T(a) = al + (1-a)r \leq l$ and $\theta_e = \frac{l}{H}$. From (11),

$$\frac{\partial}{\partial \gamma} D = (l - T(a))\frac{\partial \theta_b}{\partial \gamma} + \frac{\partial \theta_b}{\partial \gamma}(\theta_b H - l) = (\theta_b H - T(a))\frac{\partial \theta_b}{\partial \gamma} \tag{46}$$

The critical state monotonically decreases in coverage $\frac{\partial \theta_b}{\partial \gamma} < 0$ by Lemma 3.1. The sign of the bracket depends on the size of insurance coverage and satisfies strict single-crossing: By Lemma 4.1, for any $a \in (\underline{a}, 1]$, θ_b goes to zero for insurance coverage to one, but exceeds $\theta_e > 0$ for coverage to zero where $\theta_e > T(a)/H$. In addition, the critical state declines monotonically and is continuous in γ. Thus, for given $a \in (\underline{a}, 1]$, there exists a unique , interior $\gamma^*(a) \in (0, 1)$ such that

$$\theta_b(a, \gamma^*(a)) = \frac{T(a)}{H} \tag{47}$$

and thus $\frac{\partial}{\partial \gamma} D = 0$. The function $\gamma^*(a)$ describes the minimizers of the deadweight loss: For $\gamma < \gamma^*(a)$, it holds $\theta_b > \frac{T(a)}{H}$, that is the bracket in (46) is positive and $\frac{\partial}{\partial \gamma} D < 0$, the deadweight loss declines in insurance coverage for all $\gamma \in (0, \gamma^*(a))$. For $\gamma > \gamma^*(a)$, it holds $\theta_b < \frac{T(a)}{H}$ and $\frac{\partial}{\partial \gamma} D > 0$ and the deadweight loss monotonically increases in insurance coverage over the range $\gamma \in (\gamma^*(a), 1)$. That is, the deadweight loss is minimized in $\gamma = \gamma^*(a)$. To determine the change of $\gamma^*(a)$ in forbearance for $a \in (\underline{a}, 1]$, consider the total derivative. As a increases, γ^* has to change in a way such that the total change in θ_b equals the change in $T(a)/H$

$$\frac{d}{da}\theta_b(a, \gamma^*(a)) = \frac{\partial \theta_b}{\partial a} + \frac{\partial \theta_b}{\partial \gamma}\frac{\partial \gamma^*}{\partial a} = \frac{T'(a)}{H} \tag{48}$$

which implies $\frac{\partial \gamma^*}{\partial a} = \frac{\frac{T'(a)}{H} - \frac{\partial \theta_b}{\partial a}}{\frac{\partial \theta_b}{\partial \gamma}}$. We know $\frac{\partial \theta_b}{\partial \gamma} < 0$. Further, since $r \leq l$, we have $\frac{\partial \theta_b}{\partial a} < 0$ by Proposition 3.2, $T'(a) = l - r \geq 0$ and thus $\frac{\partial \gamma^*}{\partial a} < 0$. Let $r < l$. Then, among all optimal pairs $(a, \gamma^*(a))$ only $(1, \gamma^*(1))$ achieves the first best outcome: This is since all optimal pairs feature overinvestment by $\theta_b(a, \gamma^*(a)) = \frac{T(a)}{H} \leq \frac{l}{H} = \theta_e$, except for $(1, \gamma^*(1))$ by $T(1) = l$. And the direct efficiency loss is always positive by $l - T(a) \geq 0$. Thus, only the pair $(1, \gamma^*(1))$ achieves $D(1, \gamma^*(1)) = 0$. If instead $r = l$, then all optimal pairs achieve the first best outcome and there is a multiplicity of optimal policy. This is since (i) the direct efficiency loss $l - T(a)$ is zero by $T(a) = l = r$ and (ii) $T(a)/H = l/H = \theta_e$. □

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10 Supplementary Appendix - Not for Publication

10.1 Proof: Existence and Uniqueness

Proof. A: Existence and uniqueness of a trigger equilibrium Closely following Goldstein and Pauzner (2005): For fixed contract $(1, H)$, recovery rate r, forbearance policy a, and insurance coverage γ, a Bayesian equilibrium is a strategy profile such that each investor chooses the best action given her private signal and her beliefs about other players strategies. In equilibrium, an investor decides to withdraw when her expected payoff from rolling over versus withdrawing given her signal is negative, decides to roll over when it is positive and is indifferent if the expected payoff is zero. Since investors are identical ex ante, investors strategies can only differ at signals that make an investor indifferent between rolling over and withdrawing.

In a trigger equilibrium around trigger signal θ^*, all investors withdraw when they observe signals below θ^* and roll over if they observe signals above θ^*. If investors directly observe θ^*, they are indifferent, and we specify here that they will roll over. A threshold equilibrium around trigger θ^* exists if and only if given that all other investors use a trigger strategy around signal θ^* an investor finds it optimal to also use a trigger strategy around trigger θ^*.

If all investors follow the same strategy, the proportion of investors who withdraw at each state is deterministic. Define $n(\theta, \theta^*)$ as the proportion of investors who observe signals below signal θ^* and thus withdraw if the state is θ, $n(\theta, \theta^*) = \mathbb{P}(\theta_i < \theta^* | \theta)$. We can explicitly calculate $n(\theta, \theta^*)$ using the distribution function of noise as given in (22). Note that if a continuum of investors but one single investor follow the same strategy, this result continues to hold. Denote by $D(\theta_i, n(\cdot, \theta^*))$ the expected payoff difference from rolling over versus withdrawing when the investor observes signal θ_i, and other investors follow a trigger strategy around θ^*. Since a run is triggered if the measure of withdrawing depositors n exceeds al, we have

$$D(\theta_i, n(\cdot, \theta^*)) = \frac{1}{2\varepsilon} \int_{\theta_i - \varepsilon}^{\theta_i + \varepsilon} (H\,\theta + \gamma(1-\theta) - 1)\ \mathbf{1}_{\{n(\theta,\theta^*) \leq la\}} - \frac{f}{n(\theta, \theta^*)}\,\mathbf{1}_{\{n(\theta,\theta^*) > la\}} d\theta \tag{49}$$

For existence of a trigger equilibrium we need to show

$$D(\theta_i, n(\cdot, \theta^*)) < 0 \qquad \text{for all } \theta_i < \theta^* \tag{50}$$

$$D(\theta_i, n(\cdot, \theta^*)) > 0 \qquad \text{for all } \theta_i > \theta^* \tag{51}$$

and existence and uniqueness of a signal θ^* for which an investor is indifferent between rolling over and withdrawing (payoff indifference equality)

$$0 = D(\theta^*, n(\cdot,\theta^*)) = \frac{1}{2\varepsilon}\int_{\theta^*-\varepsilon}^{\theta^*+\varepsilon} (H\,\theta + \gamma(1-\theta) - 1)\ \mathbf{1}_{\{n(\theta,\theta^*)\le la\}} - \frac{f}{n(\theta,\theta^*)}\ \mathbf{1}_{\{n(\theta,\theta^*)>la\}} d\theta \tag{52}$$

The function $D(\theta^*, n(\cdot,\theta^*))$ is continuous in θ^*. By existence of dominance regions, $D(\theta^*, n(\cdot,\theta^*)) < 0$ for $\theta^* < \underline{\theta} - \varepsilon$ and $D(\theta^*, n(\cdot,\theta^*)) > 0$ for $\theta^* > \overline{\theta} + \varepsilon$. By the Intermediate value Theorem there exists at least one $\theta^* \in [\underline{\theta} - \varepsilon, \overline{\theta} + \varepsilon]$ for which (52) holds. To see uniqueness, since all other agents use a threshold strategy around θ^*, substitute for $n(\theta,\theta^*) = \frac{1}{2} + \frac{\theta^*-\theta}{2\varepsilon}$ and derive

$$D(\theta^*, n(\cdot,\theta^*)) = \int_0^{la} (H\,\theta(n,\theta^*) + \gamma(1-\theta(n,\theta^*)) - 1)\ dn - \int_{la}^{1} \frac{f}{n}\ dn \tag{53}$$

where $\theta(n,\theta^*) = \theta^* + \varepsilon(1-2n), \theta^* \in [\underline{\theta} - \varepsilon, \overline{\theta} + \varepsilon]$ is the inverse of the function $n(\theta,\theta^*)$. For uniqueness, $D(\theta^*, n(\cdot,\theta^*))$ is strictly increasing in signal θ^* for $\theta^* < \overline{\theta} + \varepsilon$ which gives single-crossing. Next, show (50): Following Goldstein and Pauzner (2005), let $\theta_i < \theta^*$. Decompose the intervals $[\theta_i - \varepsilon, \theta_i + \varepsilon]$ and $[\theta^* - \varepsilon, \theta^* + \varepsilon]$ over which the integrals $D(\theta_i, n(\cdot,\theta^*))$ and $D(\theta^*, n(\cdot,\theta^*))$ are calculated into a potentially empty common part $c = [\theta_i - \varepsilon, \theta_i + \varepsilon] \cap [\theta^* - \varepsilon, \theta^* + \varepsilon]$ and the disjoint parts $d^i = [\theta_i - \varepsilon, \theta_i + \varepsilon] \setminus c$ and $d^* = [\theta^* - \varepsilon, \theta^* + \varepsilon] \setminus c$. Then,

$$D(\theta_i, n(\cdot,\theta^*)) = \frac{1}{2\varepsilon}\int_{\theta\in c} v(\theta, n(\theta,\theta^*))\ d\theta + \frac{1}{2\varepsilon}\int_{\theta\in d^i} v(\theta, n(\theta,\theta^*))\ d\theta \tag{54}$$

$$D(\theta^*, n(\cdot,\theta^*)) = \frac{1}{2\varepsilon}\int_{\theta\in c} v(\theta, n(\theta,\theta^*))\ d\theta + \frac{1}{2\varepsilon}\int_{\theta\in d^*} v(\theta, n(\theta,\theta^*))\ d\theta \tag{55}$$

Considering (55), the integral $\int_{\theta\in c} v(\theta, n(\theta,\theta^*))\ d\theta$ has to be negative since by (52) $D(\theta^*, n(\cdot,\theta^*)) = 0$ and since the fundamentals in range d^* are higher than in c. This is, since we assumed $\theta_i < \theta^*$ and because in interval $[\theta^*-\varepsilon, \theta^*+\varepsilon]$ the payoff difference $v(\theta,n)$ is positive for high values of θ, negative for low values of θ and satisfies single-crossing. In addition, the function $n(\theta,\theta^*)$ equals one over the interval d^i, since d^i is below $\theta^* - \varepsilon$ and thus all other investors withdraw. Therefore, the integral $\int_{\theta\in d^i} v(\theta, n(\theta,\theta^*))\ d\theta$ is negative too which with (54) implies that $D(\theta_i, n(\cdot,\theta^*))$ is negative. The proof for $\theta_i > \theta^*$ proceeds analogous.

B No existence of non-monotone equilibria

See Goldstein and Pauzner, proof of Theorem 1, first page of part C □

10.2 Extension: CASE $r > l$

Theorem 3 (Optimal Forbearance II)

Assume $r \in (l, l+\varepsilon)$, where ε stems from Proposition 3.2.

b1) If insurance coverage is low, the deadweight loss monotonically decreases and maximum forbearance (no intervention) is optimal $a^* = 1$.
b2) For insurance coverage high, the deadweight loss monotonically increases in forbearance and immediate intervention is optimal $a^* = \underline{a}$.
Assume $r >> l$ *with* r *close to one:*
a1) Let deposit insurance be low: Then the deadweight loss monotonically increases in forbearance over the range of low forbearance levels $a \in (\underline{a}, \overline{a})$ *and is locally minimized by intervening as soon as possible* $a^* = \underline{a}$. *The deadweight loss is non-monotonic in forbearance over the range* $a \in (\overline{a}, 1]$.
a2) Let deposit insurance be high: Then the deadweight loss can be non-monotonic for $r > \gamma$ *and is minimized for some forbearance level in the set* $(\underline{a}, \overline{a}]$. *If* $\gamma > r$, *immediate intervention is optimal.*

On (b), even though RA liquidates more effectively than the bank, if RA's efficiency advantage when liquidating is only small, the results are equivalent to the results in the case $r \leq l$ of Theorem (1). This is since the first two terms in (44) are close to zero and stability improves in forbearance for $r > l$ but r close to l, independently of whether r exceeds γ or not, see Proposition 3.2.

For (a) with $r >> l$, $r > \gamma$ and r close to one, the results change fundamentally. There now exists $\overline{a} \in (0, 1)$ such that stability declines in forbearance for $a \in (\underline{a}, \overline{a})$ since r is close to one. On $a \in (\overline{a}, 1)$ however, stability remains decreasing in forbearance since the pro rata share is constant in a. Under low insurance coverage, inefficient runs exist. On the set $a \in (\underline{a}, \overline{a})$, the likelihood of inefficient runs is now minimized by intervening as soon as possible. On the set $a \in (\overline{a}, 1)$ the likelihood of inefficient runs is minimized by forbearing as much as possible. Overall, the global minimizer of inefficient runs is therefore located at the boundary $\underline{a}$ or 1. In addition, by $r > \ell$, the direct efficiency loss strictly increases as RA forbears more. Altogether, immediate intervention is optimal on the set $a \in (\underline{a}, \overline{a})$ while for $a \in (\overline{a}, 1)$, the optimal forbearance policy is not clear since the RA needs to trade-off the effects of reducing inefficient runs versus lowering the direct liquidation loss. Considering the entire set of possible forbearance levels $a \in (\underline{a}, 1)$, the overall change of the deadweight loss is potentially non-monotonic in forbearance.

Under high insurance coverage, there is inefficient investment. If insurance coverage exceeds r, we are back in the case of Theorem (1) by Proposition 3.2 (A). If insurance coverage is high but below r (e.g. r close to one): By the same argument as above, the overall effect of forbearing on the deadweight loss is non-monotonic. On the set $a \in (\underline{a}, \overline{a})$, stability declines in forbearance, thus the likelihood of overinvestment is minimized by maximum delay $a = \overline{a}$. On the set $a \in (\overline{a}, 1)$, stability improves in forbearance, and the likelihood of overinvestment is minimized by fast intervention $a = \overline{a}$ within the considered set $(\overline{a}, 1)$. Thus, the interior forbearance level $a = \overline{a}$ globally minimizes the likelihood of overinvestment. In addition, however, the direct liquidation loss strictly increases as RA forbears more. Thus, all forbearance levels in $(\overline{a}, 1)$ cannot be optimal. The deadweight loss is potentially non-monotonic on $(\underline{a}, \overline{a}]$ but contains the optimal level. On this set, the RA trades off the loss due to a direct liquidation efficiency versus the reduction in overinvestment when raising forbearance.

Proof. [Theorem 3] Let $r > l$. From (44), with $T(a) = al + (1-a)r \leq r$,

$$\frac{\partial}{\partial a} D(a, \gamma) = -(l-r)\,\theta_b + (r - T(a))\,\frac{\partial \theta_b}{\partial a} + \frac{\partial \theta_b}{\partial a} \cdot (\theta_b H - r) \tag{56}$$

The first term is always positive.

(b) Let $r > l$ with r close to l. We show the results are exactly as in the case $r \leq l$. Independently of the size of γ, for $r < \ell + \varepsilon$, stability improves in forbearance by Proposition 3.2. Thus, the second term is negative but small since r close to l. The first term is positive but small due to r close to l. Thus all over, only the third term is important.

(b1) For insurance coverage low, the bracket of the third term is positive. Since stability improves in forbearance, the third term is negative and $a^* = 1$ is optimal.

(b2) For insurance coverage high, the bracket of the third term is negative. Thus, the third term is positive and $a^* = \underline{a}$ is optimal.

(a) Let $r >> l$, r close to one, $r > \gamma$. Then, $\overline{a} \in (0, 1)$ exists. For $a \in (\underline{a}, \overline{a})$, stability declines in forbearance by Proposition 3.2 and the second term is positive. For $a \in (\overline{a}, 1]$, however, stability improves in forbearance and the second term is negative. The sign of the third term depends on the level of insurance coverage provided.

(a1) If insurance coverage is low, it holds $\theta_b > \theta_e = r/H$ by Lemma 4.1. Thus, the bracket of the third term is positive. Stability declines in forbearance for $a \in (\underline{a}, \overline{a})$, and the third term and therefore all terms are positive for $a \in (\underline{a}, \overline{a})$. Thus, $a^* = \underline{a}$ is locally optimal among all $a \in (\underline{a}, \overline{a})$. For $a \in (\overline{a}, 1]$, stability improves in forbearance. Term two becomes negative and, combined with low deposit insurance, term three is negative. Term one remains positive such that the overall change in deadweight loss is undetermined over the range $a \in (\overline{a}, 1]$.

(a2) If insurance coverage is high, and insurance coverage exceeds r, we are back in the case of Theorem (1) by Proposition 3.2 (A). Let $r > \gamma$, then for high insurance, there is inefficient investment. Thus, the bracket of the third term is negative. Thus, for $a \in (\underline{a}, \overline{a})$, since stability declines in forbearance, the third term is now negative while the second and first term is positive. The overall change in deadweight loss is undetermined over the range $a \in (\underline{a}, \overline{a})$. For $a \in (\overline{a}, 1]$, stability improves in forbearance. Thus, the third term is positive as is the first term but the second term is negative. Again, the change of deadweight loss is undetermined and can be non-monotonic.

□

Theorem 4 (Optimal insurance coverage - Optimal Policy II)
Let $r > l$. For every forbearance policy $a \in (\underline{a}, 1]$ there exists a unique interior level of insurance coverage $\gamma^*(a) \in (0,1)$ which minimizes the deadweight loss. The pair is such that

$$\theta_b(a, \gamma^*(a)) = \frac{T(a)}{H} \tag{57}$$

and the deadweight loss strictly decreases in insurance for $\gamma < \gamma^*(a)$ and increases in insurance for $\gamma > \gamma^*(a)$. For $r > l$, the optimal insurance coverage level $\gamma^*(a)$ can be non-monotone in forbearance. Among all optimal insurance coverage pairs $(a, \gamma^*(a))$, $a \in (\underline{a}, 1]$, the optimal policy is given by the pair $(\underline{a}, \gamma^*(\underline{a}))$ and asymptotically achieves first best as $\underline{a} \to 0$.

Bank stability improves with insurance coverage, independently of the relation between r, ℓ and γ. Thus, the first part of the Theorem is identical to the case $r \leq \ell$.

Proof. [Theorem 4] Let $r > l$. Then $\theta_e = r/H$. As before

$$\frac{\partial}{\partial \gamma} D = (\theta_b H - T(a)) \frac{\partial \theta_b}{\partial \gamma} \tag{58}$$

As in the proof of Theorem 2, since the monotonicity of the critical state in insurance coverage is unchanged, for every $a \in (\underline{a}, 1]$ there exists a unique, interior insurance level $\gamma^*(a) \in (0,1)$ such that

$$\theta_b(a, \gamma^*(a)) = \frac{T(a)}{H} \tag{59}$$

which minimizes the deadweight loss. For $\gamma < \gamma^*(a)$, $\theta_b(a, \gamma) > \frac{T(a)}{H}$ and thus the deadweight loss is decreasing on $(0, \gamma^*(a))$, $\frac{\partial D}{\partial \gamma} < 0$, while for $\gamma > \gamma^*$ the deadweight loss is increasing. As before in the case $r \leq \ell$, as forbearance a increases in $(\underline{a}, 1]$, $\gamma^*(a)$ has to change such that the total change of the critical state equals the change in $T(a)/H$, i.e., $\frac{\partial \gamma^*}{\partial a} = \frac{\frac{T'(a)}{H} - \frac{\partial \theta_b}{\partial a}}{\frac{\partial \theta_b}{\partial \gamma}}$. Bank stability improves with insurance coverage, independently of the relation between r, ℓ and γ, thus, as before, $\frac{\partial \theta_b}{\partial \gamma} < 0$. The case $r > l$ differs from the case $r \leq l$ regarding the monotonicity of $\gamma^*(a)$ since $\frac{\partial \theta_b}{\partial a} \geq 0$ for $r \to 1$ as long as the insurance fund does not become liable and since $T'(a) = \ell - r < 0$. The function γ^* weakly increases in forbearance for small values of a, if in $a = \underline{a}$ the implied $\gamma^*(\underline{a})$ is such that $\gamma^*(\underline{a}) < \frac{r(1-a)}{1-\underline{a}\ell}$, that is if $\underline{a} < \overline{a}(\gamma)(\underline{a})$ holds. If this is the case, then as forbearance increases, the critical state increases in a, and $\frac{T'(a)}{H} - \frac{\partial \theta_b}{\partial a}$ is negative, thus γ^* goes up. As a second effect however, the threshold value $\overline{a}(\gamma^*)$ at which the insurance becomes liable declines from above towards the 45 degree line. When forbearance a and thus $\gamma^*(a)$ increase to the level where $\overline{a}(\gamma^*)(a) = a$, by Proposition 3.2, the critical state θ_b changes its monotonicity and declines in forbearance. In that case, the sign of $\frac{T'(a)}{H} - \frac{\partial \theta_b}{\partial a}$

becomes ambiguous. Therefore, the case $r > \ell$ with r close to ℓ is also ambiguous: It holds $\frac{T'(a)}{H} < 0$ but $-\frac{\partial \theta_b}{\partial a} > 0$. Allover, $\gamma^*(a)$ can be non-monotone.

□

10.3 Robustness: General Setting

In this subsection we explain how the model extends to general contracts (R_1, R_2) and general interest rates j: In the model we have fixed demand deposit contract coupons at $(1, H)$ and the refinancing interest rate at $j = H$ to obtain a parameter reduction. For general interest rate $j \in (1, H/l)$, contract (R_1, R_2), $l \leq R_1 < R_2 < H$ and debt ratio $\delta \in (0, 1)$ the bank is prone to runs if and only if $\delta R_1 > l$. We maintain this assumption from here on. Let again n denote the proportion of withdrawing depositors. Then the measure of withdrawn funds equals $\delta n R_1$. Let $a \in (\underline{a}, 1]$ RA's forbearance policy. Resolution takes place if and only if $\delta R_1 n > la$. The total measure of taxes to be raised is $\delta \gamma R_1$. Each depositor is taxed $\gamma R_1 \in (0, 1)$ at the point in time of repayment of the bank. The payoff table becomes

Event/ Action	Withdraw	Roll-over
No resolution $n \in [0, a \cdot \frac{l}{\delta R_1}]$	$R_1 - \gamma R_1$	$\begin{cases} R_2 - \gamma R_1 & , p = \theta \\ \gamma R_1 - \gamma R_1 & , p = 1 - \theta \end{cases}$
Bank resolution $n \in (a \cdot \frac{l}{\delta R_1}, 1]$	$\frac{la}{\delta R_1 n} \cdot R_1 + (1 - \frac{la}{\delta R_1 n}) s_\gamma(a) - \gamma R_1$	$s_\gamma(a) - \gamma R_1$

with

$$s_\gamma(a) = \max\left(\gamma R_1, \frac{r(1-a)}{\delta - la/R_1}\right) = \max\left(\gamma R_1, \frac{\frac{r}{\delta}(1-a)}{1 - a \cdot (\frac{l}{\delta R_1})}\right) \tag{60}$$

since the measure of depositors served before resolution is la/R_1 thus the measure of depositors involved in resolution is $\delta - la/R_1$. For the payoff table to be consistent, the bank has to be able to repay depositors who roll over conditional on no resolution taking place and the asset paying off high. In other words equity value has to be positive. At repo rate j, the net return on equity equals

$$H - j\delta R_1 n - (1-n)\delta R_2 \geq 0 \qquad \text{for all } n > \frac{la}{\delta R_1} \tag{61}$$

Assume that per period refinancing via outside investors is more expensive than financing via deposits,

$$j > R_2/R_1 \tag{62}$$

This assumption is reasonable, otherwise, the bank could choose outside financing in the first place at $t = 0$ respectively has an incentive to replace deposits with outside financing in $t = 1$. Condition $jR_1 > R_2$ is an incentive condition on the bank and says that the bank cannot make money by encouraging withdrawals by depositors in t_1. Then, condition (61) holds if

$$\frac{H}{\delta R_1} \geq j \tag{63}$$

In that case, the game is consistent and can be analyzed as before. Note, that our benchmark game with $\delta = 1$, $R_1 = 1$, $j = H$ satisfies condition (63). Define $\tilde{l} = \frac{l}{\delta R_1}$, recovery rate $\tilde{r} = \frac{r}{\delta R_1}$, $\tilde{\gamma} = \gamma$, $\tilde{R}_2 = \frac{R_2}{R_1}$. Since incentives are robust under rescaling of payoffs, the game above is equivalent to the game

Event/ Action	Withdraw	Roll-over
No resolution $n \in [0, a \cdot \tilde{l}]$	$1 - \tilde{\gamma}$	$\begin{cases} \tilde{R}_2 - \tilde{\gamma} & ,p = \theta \\ \tilde{\gamma} - \tilde{\gamma} & ,p = 1 - \theta \end{cases}$
Bank resolution $n \in (a\tilde{l}, 1]$	$\frac{\tilde{l}a}{n} \cdot 1 + (1 - \frac{\tilde{l}a}{n}) s_{\gamma,R_1}(a) - \tilde{\gamma}$	$s_{\gamma,R_1}(a) - \tilde{\gamma}$

where

$$s_{\gamma,R_1}(a) = \max\left(\tilde{\gamma}, \frac{\tilde{r}(1-a)}{1 - a\tilde{l}}\right) \tag{64}$$

Thus, all results from previous sections go through under the renamed parameters. Concerning the adaption of the welfare concept: The value of the bank, by definition, equals the value of debt and equity. In the case of the partially debt-financed bank, we can show

Lemma 10.1. *The value of investment equals the value of the bank plus the value of the insurance fund.*

Thus, RA's objective to minimize the deadweight loss is equivalent to maximizing the joint value of the bank and the insurance fund.

Proof. [Lemma 10.1] This proof draws on notation introduced in the general setting in the supplementary appendix in subsection 10.3. Since the bank is partially debt-financed, define the value of debt as δ times the net value of the debt contract DC, for general debt ratio $\delta \in (0, 1)$:

$$DC(a,\gamma) = \delta \int_0^{\theta_b} n(\theta) \left(\frac{la}{\delta n R_1} \cdot R_1 + (1 - \frac{la}{\delta n R_1}) s_\gamma(a) \right) + (1 - n(\theta))\, s_\gamma(a)\, d\theta \tag{65}$$

$$+ \delta \int_{\theta_b}^1 n(\theta) \cdot R_1 + (1 - n(\theta))\, (\theta R_2 + (1-\theta)\gamma R_1)\, d\theta - \delta \int_0^1 \gamma R_1\, d\theta \tag{66}$$

for

$$s_\gamma(a) = \max\left(\gamma R_1, \frac{r(1-a)}{\delta - la/R_1}\right) = \max\left(\gamma R_1, \frac{\frac{r}{\delta}(1-a)}{1 - a \cdot (\frac{l}{\delta R_1})}\right) \tag{67}$$

Equity value equals

$$E(a,\gamma) = \int_{\theta_b}^1 \theta\, (H - i\delta R_1 n - (1-n)\delta R_2)\, d\theta \tag{68}$$

The net value of the insurance fund becomes

$$\Gamma(a,\gamma) = \int_0^1 \delta R_1 \gamma \, d\theta - (1-la)\,\delta \max(0, \gamma R_1 - s_\gamma(a)) \int_0^{\theta_b} d\theta - \delta \int_{\theta_b}^1 (1-\theta)\,\gamma R_1 \delta \, d\theta \quad (69)$$

Note, for $\varepsilon \to 0$ it holds

$$\int_{\theta_b}^1 n(\theta)\,\delta R_1 (1-\theta i)\, d\theta \to 0 \quad (70)$$

since $n(\theta) \to 0$ for $\theta \in (\theta_b, 1)$ (depositors do not withdraw for states above the critical state). Thus, the convergence to zero follows by the dominated convergence theorem by $n \leq 1$. As a consequence, at the limit $\varepsilon \to 0$,

$$DC(a,\gamma) + E(a,\gamma) + \Gamma(a,\gamma) = \int_0^{\theta_b} T(a)\, d\theta + \int_{\theta_b}^1 \theta H \, d\theta \quad (71)$$

□

10.4 Robustness: Emergency Liquidity Assistance (ELA) and the lender of last resort

A different situation compared to the benchmark model emerges if the bank taps ELA instead of borrowing from outside investors directly. In Europe, ELA is paid by the national central bank to banks which are illiquid but solvent. In return, the bank has to provide assets as collateral. If the bank accesses emergency liquidity assistance, the institution with whom the bank pledges assets *is* the resolution authority, since ELA is paid under the supervision of the European Central Bank. In that case, our results change since in the moment when RA seizes proportion a of the asset, RA is already in possession of the remaining proportion $1-a$ since RA acts as the counterparty in the money market. RA, therefore, liquidates the entire asset and the pro rata share to depositors becomes

$$s_{\text{ELA}}(a) = \frac{r}{1-al} \quad (72)$$

As opposed to the benchmark case, this pro rata share has the novel feature to be increasing in forbearance. Therefore, our results will only partially change: The change in strategic uncertainty and the shift in pro rata share affect incentives in the same direction. The trigger θ^* monotonically declines in forbearance for arbitrary r. As a consequence, all previous results for the case $r \leq l$ and $r > l$ with r close to l are robust, in particular Thereom 1 and Theorem 2. The previous results on $r >> l$ were influenced by the flip in monotonicity of the trigger θ^*. Under ELA, the trigger remains monotonically declining in forbearance, and the deadweight loss becomes non-monotonic in forbearance. Statements on optimal forbearance policies cannot be derived.

10.5 Robustness: Selling assets

Now assume, the bank refinances withdrawals at the interim period not by pledging but by selling assets. Assume, the bank can raise the maximum amount l by liquidating her entire investment. Let $\delta < 1$ the bank's debt ratio and (R_1, R_2) the debt contract with $\delta R_1 > l$ such that the bank is prone to runs. Define again the measure n^* of withdrawals RA tolerates before intervening

$$\delta R_1 n^* = al \tag{73}$$

For $n < n^*(a) = \frac{al}{\delta R_1}$ no resolution takes place and the bank can finance all interim withdrawals. The main difference compared to asset pledging is that the bank incurs a liquidation cost even absent resolution at $t = 1$ which diminishes remaining investment.[47]

If remaining investment at $t = 1$ is insufficient to earn returns high enough to repay depositors who roll over entirely, the bank is not resolved but insolvent in $t = 1$. Define n^{**} as the critical proportion of depositors who need to withdraw to put the bank on the edge of insolvency: n^{**} solves

$$\frac{H(1 - n\frac{R_1\delta}{l})}{(1-n)\delta} = R_2 \tag{74}$$

For $n > n^{**}$, return on remaining investment undercuts remaining debt claims, the bank cannot repay R_2 to all depositors who roll over. In this case, depositors who roll over obtain a pro rata share of remaining investment $\frac{H(1-n\frac{R_1\delta}{l})}{(1-n)\delta} < R_2$. Note, n^{**} is independent of both forbearance and insurance coverage.[48] If this share is below the insured amount, the insurance fund becomes liable. In particular, given insolvency the pro rata share from rolling over may undercut the payoff from withdrawing early if RA intervenes late. Thus, withdrawing can be the optimal action even absent resolution. This is in contrast to asset pledging where withrawing is optimal if and only if resolution occurs. Define

$$\tilde{R}_2(n) = \min\left(R_2, \max\left(\gamma R_1, \frac{H(1 - n\frac{R_1\delta}{l})}{(1-n)\delta}\right)\right) \tag{76}$$

where

$$\tilde{R}_2(n) = \begin{cases} R_2, & n < n^{**} \\ \max\left(\gamma R_1, \frac{H(1-n\frac{R_1\delta}{l})}{(1-n)\delta}\right), & n > n^{**} \end{cases} \tag{77}$$

[47] More precisely: She foregoes profits on sold assets and incurs liquidation costs which diminish returns on remaining investment. Under pledging, the bank earns returns also on pledged assets absent resolution if the asset pays off high. No liquidation costs arise but instead the interest rate i applies on borrowed funds.

[48] Here, we impose the incentive condition that the bank cannot make money by encouraging depositors to withdraw early

$$R_1\frac{H}{l} > R_2 \tag{75}$$

This incentive condition takes the same role as condition (62) in case of pledging assets.

Allover, the payoff table becomes

Event/ Action	Withdraw	Roll-over
No resolution $n \in [0, n^*(a)]$	R_1	$\begin{cases} \tilde{R}_2(n) & ,p=\theta \\ \gamma R_1 & ,p=1-\theta \end{cases}$
Bank resolution $n \in (n^*(a), 1]$	$\frac{la}{\delta R_1 n} \cdot R_1 + (1 - \frac{la}{\delta R_1 n}) s_\gamma(a)$	$s_\gamma(a)$

$$s_\gamma(a) = \max(\gamma R_1, \frac{r(1-a)}{\delta - la/R_1}) \tag{78}$$

Under asset sales, RA's forbearance policy has an additional effect as opposed the the case of asset pledging. By intervening, RA not only protects depositors who roll over given resolution, but also absent resolution. Define implicitly $a_b \in (\underline{a}, 1]$ such that[49]

$$n^*(a_b) = n^{**} \tag{79}$$

Threshold a_b is the maximum level of forbearance RA can grant such that for all $a \in (\underline{a}, a_b)$ she can guarantee payoff R_2 to depositors who roll over if the asset pays off, absent bank resolution. That is, for $a \in (\underline{a}, a_b)$ it holds $n^*(a) < n^{**}$. If RA intervenes later $a \in (a_b, 1]$, RA no longer guarantees return R_2 in the absence of intervention. Depositors who roll over may earn the lower pro rata share $\frac{H(1-n\delta R_1/l)}{\delta(1-n)}$ if withdrawals at the interim period are low enough to not trigger resolution but substantial enough to cause insolvency. That is, for $a \in (a_b, 1]$ there exists a range of withdrawals $n \in [n^{**}, n^*(a)]$ for which the bank is not resolved but insolvent in $t = 1$.

Under asset sales, for $a \in (a_b, 1)$ the bank is insolvent given resolution but not necessarily solvent absent resolution even if the asset pays high. In contrast, under pledging of assets for refinancing it holds $a_b = 1$. Thus, the bank is solvent absent resolution if the asset pays high for all forbearance levels $a \in (\underline{a}, 1)$, the range $[n^{**}, n^*(a)]$ is empty. More concrete, for $a \in (\underline{a}, a_b)$, the payoff table in the case of asset sales is as in the case of asset pledging

Event/ Action	Withdraw	Roll-over
No resolution $n \in [0, n^*(a)]$	R_1	$\begin{cases} R_2 & ,p=\theta \\ \gamma R_1 & ,p=1-\theta \end{cases}$
Bank resolution $n \in (n^*(a), 1]$	$\frac{la}{\delta R_1 n} \cdot R_1 + (1 - \frac{la}{\delta R_1 n}) s_\gamma(a)$	$s_\gamma(a)$

while for $a \in (a_b, 1)$, the payoff table under asset sales has an additional row

[49] Threshold a_b has to exist if $\underline{a}$ is sufficiently small since $n^*(a)$ increases in forbearance, by $n^*(1) > n^{**}$ and $n^*(0) < n^{**}$.

Event/ Action	Withdraw	Roll-over
No resolution and solvent $n \in [0, n^{**}]$	R_1	$\begin{cases} R_2, & p = \theta \\ \gamma R_1, & p = 1-\theta \end{cases}$
No resolution and insolvent $n \in [n^{**}, n^*(a)]$	R_1	$\begin{cases} \max\left(\gamma R_1, \frac{H(1-n\frac{R_1\delta}{l})}{\delta(1-n)}\right), & p = \theta \\ \gamma R_1, & p = 1-\theta \end{cases}$
Bank resolution $n \in (n^*(a), 1]$	$\frac{la}{\delta R_1 n} \cdot R_1 + (1 - \frac{la}{\delta R_1 n}) s_\gamma(a)$	$s_\gamma(a)$

Assume deposit insurance is zero to purely focus on the contrast between selling assets as opposed to pledging when forbearing. In the case of pledging assets, the bank avoids liquidation costs absent resolution even if withdrawals arise. One might believe that this feature drives the result that stability can improve in forbearance. As RA forbears more, it takes more depositors to cause resolution thus liquidation costs apply only for larger aggregate withdrawals. The following results show that the avoidance of liquidation costs do not drive the result that stability can improve in forbearance.

Under asset sales, liquidation costs also apply absent resolution once withdrawals occur at the interim period. First note, for all $a \in (\underline{a}, a_b)$, the bank can fully repay depositors who roll over since liquidation costs are born by equity investors.[50] On $a \in (\underline{a}, a_b)$, depositors' preferences in the cases of pledging and selling assets are thus indentical. Thus, payoff difference functions coincide, and the trigger behaves as in the benchmark case. From Proposition 3.2,

Corollary 10.1
Under asset sales, there is $\varepsilon > 0$ such that for all $r \in (0, l + \varepsilon)$ stability improves in forbearance for $a \in (\underline{a}, a_b)$ but declines in forbearance for $a \in (\underline{a}, a_b)$ if $r > l + \varepsilon$.

One may think that this result holds since equity investors bear the costs of liquidation. The next result, however, shows, stability can improve as RA intervenes later even if RA sets forbearance levels which impose the costs of liquidation on depositors, absent resolution.

Lemma 10.2. *Let $r = l$ and consider asset sales. There exists $\delta > 0$ such that stability improves in forbearance over $[\underline{a}, a_b + \delta)$.*

As a consequence, under low deposit insurance, forbearing in excess of the insolvency threshold a_b is optimal. In particular, immediate intervention is inefficient.

As RA forbears in excess of a_b, she no longer guarantees payoff R_2 to depositors who roll over, absent resolution even if the asset pays off high. In the 'worst-best case',

[50] They receive the residual value $H(1 - n\delta R_1/l) - \delta(1-n)R_2$.

if the bank is just not resolved $n \nearrow n^*(a)$, depositors only obtain the pro rata share $\frac{H(1-n^*(a)\delta R_1/l)}{\delta(1-n^*(a))} = \frac{H(1-a)}{\delta - la/R_1}$ which strictly declines in forbearance.

The result demonstrates that avoidance of liquidation costs under asset pledging is not the main driver of stability improvements in forbearance. Instead, the main driver is the drop in strategic uncertainty: As long as forbearance is sufficiently low, rolling over remains the optimal action absent resolution although forbearing eats into depositors' pro rata share. As RA forbears more, aggregate withdrawals need to be higher to trigger resolution, thus withdrawing is the optimal action less often. The propensity to withdraw drops.

In the case of asset sales, liquidation costs do however play a more decisive role to depositors, in particular as forbearance becomes high. For high forbearance levels, the liquidation costs depositors bear when rolling over can become substantial.[51] As a consequence, withdrawing can be the optimal response by depositors even absent resolution. Therefore,

Lemma 10.3. *Under asset sales, as $a \to 1$, stability declines in forbearance.*

This result is in contrast to the case of asset pledging. Under low deposit insurance and asset sales, the optimal forbearance level is interior in $(a_b, 1)$

10.5.1 Proofs: Assets sales for refinancing

Preliminary As before in the case of asset pledging, the change of stability in forbearance is fundamentally influenced by how the expected payoff difference from rolling over versus withdrawing alters in forbearance. The payoff difference now equals

$$\Delta(\theta^*, a) = \int_0^{n^*(a)} (\tilde{R}_2(n)\theta(\theta^*, n) + (1 - \theta(\theta^*, n))\gamma R_1 - R_1)\, dn \tag{80}$$

$$+ \int_{n^*(a)}^1 (s_\gamma(a) - [\frac{la}{\delta R_1 n} \cdot R_1 + (1 - \frac{la}{\delta R_1 n})s_\gamma(a)])\, dn \tag{81}$$

Again, the zeroes of the payoff difference Δ yield the equilibrium trigger. To obtain the change in the equilibrium trigger as a alters, we have as before

$$\frac{\partial}{\partial \theta^*}\Delta > 0 \tag{82}$$

and now for general $a \in (\underline{a}, 1]$,

$$\frac{\partial}{\partial a}\Delta = \frac{\partial n^*}{\partial a} \left((\tilde{R}_2(n^*(a)) - \gamma R_1)\theta(\theta^*, n) + (\gamma R_1 - s_\gamma(a)) \right) \tag{83}$$

$$- \int_{n^*(a)}^1 \frac{\partial}{\partial a} \left[\frac{n^*(a)}{n} \cdot (R_1 - s_\gamma(a)) \right] dn \tag{84}$$

[51] $\frac{H(1-a)}{\delta - la/R_1}$ goes to zero for a to one.

Proof. [Lemma 10.2] Now consider $a \in (a_b, 1)$ which implies $n^{**} < n^*(a)$. We want to show $\lim_{a \to a_b, a \in (a_b, a_c)} \frac{\partial \Delta}{\partial a} > 0$. Taking the limit of the payoff difference function yields

$$\Delta(\theta^*, a) \to n^{**} R_2 \theta^* + n^*(a) \left((1-\theta^*)\gamma R_1 - R_1\right) \\ + \theta^* \int_{n^{**}}^{n^*(a)} \max\left(\gamma R_1, \frac{H(1 - n\frac{\delta R_1}{l})}{\delta(1-n)}\right) dn + n^*(a) \ln(n^*(a)) \cdot (R_1 - s_\gamma(a))$$

In equilibrium, the trigger solves $\Delta(\theta^*, a) = 0$. Dividing by $n^*(a)$, the payoff difference at the limit in equilibrium satisfies

$$0 = \frac{n^{**}}{n^*(a)} R_2 \theta^* + \left((1-\theta^*)\gamma R_1 - R_1\right) \tag{85}$$

$$+ \theta^* \frac{1}{n^*(a)} \int_{n^{**}}^{n^*(a)} \max\left(\gamma R_1, \frac{H(1 - n\frac{\delta R_1}{l})}{\delta(1-n)}\right) dn + \ln(n^*(a)) \cdot (R_1 - s_\gamma(a)) \tag{86}$$

Consider the derivative of the payoff difference function, away from the limit

$$\frac{\partial}{\partial a}\Delta = \frac{\partial n^*}{\partial a} \left((\max\left(\gamma R_1, \frac{H(1-a)}{\delta - la/R_1}\right) - \gamma R_1)\theta(\theta^*, n) + (\gamma R_1 - s_\gamma(a) + (R_1 - s_\gamma(a))\ln(n^*(a)) \right) \tag{87}$$

$$- \int_{n^*(a)}^{1} \frac{n^*(a)}{n} \frac{\partial}{\partial a} \left[R_1 - s_\gamma(a)\right] dn \tag{88}$$

For $a \in (a_b, 1)$ but $a \to a_b$ it holds $a \in (a_b, a_c)$, thus $\max\left(\gamma R_1, \frac{H(1-a)}{\delta - la/R_1}\right) = \frac{H(1-a)}{\delta - la/R_1}$. Further, since $\frac{H(1-n\delta R_1/l)}{\delta(1-n)}$ is decreasing in n, with $\frac{H(1-n^{**}\delta R_1/l)}{\delta(1-n^{**})} = R_2 > \gamma R_1$ and $\frac{H(1-n^*(a)\delta R_1/l)}{\delta(1-n^*(a))} = \frac{H(1-a)}{\delta - la/R_1}$,

$$\int_{n^{**}}^{n^*(a)} \max(\gamma R_1, \frac{H(1 - n\delta R_1/l)}{\delta(1-n)})\, dn = \int_{n^{**}}^{n^*(a)} \frac{H(1 - n\delta R_1/l)}{\delta(1-n)}\, dn \tag{89}$$

In $\frac{\partial}{\partial a}\Delta$, we replace the term $\ln(n^*(a)) \cdot (R_1 - s_\gamma(a))$ by substitution, using the equilibrium condition (85):

$$\frac{\partial}{\partial a}\Delta = \frac{\partial n^*}{\partial a} \left(\max\left(\gamma R_1, \frac{H(1-a)}{\delta - la/R_1}\right) \theta(\theta^*, n) + (R_1 - s_\gamma(a)) \right) \\ - \frac{\partial n^*}{\partial a} \left(\frac{n^{**}}{n^*(a)} R_2 \theta^* + \theta^* \frac{1}{n^*(a)} \int_{n^{**}}^{n^*(a)} \max\left(\gamma R_1, \frac{H(1 - n\frac{\delta R_1}{l})}{\delta(1-n)}\right) dn \right) \\ - \int_{n^*(a)}^{1} \frac{n^*(a)}{n} \frac{\partial}{\partial a} \left[R_1 - s_\gamma(a)\right] dn$$

Integration by parts yields,

$$\int_{n^{**}}^{n^*(a)} \frac{H(1-n\frac{\delta R_1}{l})}{\delta(1-n)}\,dn = n^*(a)\,\frac{H(1-a)}{\delta - la/R_1} - n^{**}\,R_2 - \int_{n^{**}}^{n^*(a)} n\cdot\frac{\partial}{\partial n}\frac{H(1-n\frac{\delta R_1}{l})}{\delta(1-n)}\,dn \tag{90}$$

Plugging the integration result into $\frac{\partial}{\partial a}\Delta$ using that that for $a \in (a_b, a_c)$, it holds $\max\left(\gamma R_1, \frac{H(1-a)}{\delta - la/R_1}\right) = \frac{H(1-a)}{\delta - la/R_1}$,

$$\frac{\partial}{\partial a}\Delta = \frac{\partial n^*}{\partial a}\,(R_1 - s_\gamma(a)) - \frac{\partial n^*}{\partial a}\,\theta^*\,\frac{1}{n^*(a)}\left(-\int_{n^{**}}^{n^*(a)} n\cdot\frac{\partial}{\partial n}\frac{H(1-n\frac{\delta R_1}{l})}{\delta(1-n)}\,dn\right) \tag{91}$$

$$-\int_{n^*(a)}^{1}\frac{n^*(a)}{n}\frac{\partial}{\partial a}\left[R_1 - s_\gamma(a)\right]\,dn \tag{92}$$

Comparing (92) versus the derivative of the payoff difference function for $a \in (\underline{a}, a_b)$, we see that for $a > a_b$ the derivative $\frac{\partial}{\partial a}\Delta$ has the extra term

$$-\frac{\partial n^*}{\partial a}\,\theta^*\,\frac{1}{n^*(a)}\left(-\int_{n^{**}}^{n^*(a)} n\cdot\frac{\partial}{\partial n}\frac{H(1-n\frac{\delta R_1}{l})}{\delta(1-n)}\,dn\right) \tag{93}$$

This term is negative by $n^* = \frac{la}{\delta R_1}$, $\frac{\partial}{\partial n}\frac{H(1-n\frac{\delta R_1}{l})}{\delta(1-n)} < 0$ and $\delta R_1 > l$. The term however goes to zero for $a \to a_b$, since then $n^*(a) \to n^{**}$ and since the integrand is bounded:

$$\left|\frac{\partial}{\partial n}\frac{H(1-n\frac{\delta R_1}{l})}{\delta(1-n)}\right| = \left|\frac{H(1-\frac{\delta R_1}{l})}{\delta(1-n)^2}\right| < c \text{ for } n \in (n^{**}, n^*(a)) \tag{94}$$

For $a > a_b$ and $a \to a_b$, since the negative term vanishes, the limit $a \to a_b$ for $a \in (a_b, a_c)$ (limit from the right) of the derivative of the payoff difference function coincides with the limit $a \to a_b$ on $a \in (\underline{a}, a_b)$ (limit from the left) of the derivative of the payoff difference function (no jump). From the latter we know that it is positive for all $a \in (\underline{a}, a_b)$ when $r = l$. Thus, $\lim_{a\to a_b, a\in(a_b,a_c)} \frac{\partial\Delta}{\partial a} > 0$. By continuity, there exists $\delta > 0$ such for all $a \in (\underline{a}, a_b + \delta)$, $\frac{\partial\Delta}{\partial a} > 0$ and stability improves in forbearance in excess of the insolvency threshold a_b. □

Proof. [Lemma 10.3] It holds $a_b \in (0,1)$, thus for a close to one, we have $a \in (a_b, 1)$. Let $\gamma = 0$, then plugging in for $\tilde{R}_2(a)$, the derivative of the payoff difference function equals, away from the limit

$$\frac{\partial}{\partial a}\Delta = \frac{\partial n^*}{\partial a}\left(\frac{H(1-a)}{\delta - la/R_1}\,\theta(\theta^*, n) - s_\gamma(a) + (R_1 - s_\gamma(a))\ln(n^*(a))\right) \tag{95}$$

$$-\int_{n^*(a)}^{1}\frac{n^*(a)}{n}\frac{\partial}{\partial a}\left[R_1 - s_\gamma(a)\right]\,dn \tag{96}$$

where $s_\gamma = \frac{r(1-a)}{\delta - la/R_1}$. It holds $\lim_{a\to 1} s_\gamma = \frac{r(1-a)}{\delta - la/R_1} = 0$ and $\lim_{a\to 1} \frac{H(1-a)}{\delta - la/R_1} = 0$. Thus, the derivative further simplifies to

$$\frac{\partial}{\partial a}\Delta = \frac{\partial n^*}{\partial a} \; (R_1) \ln(n^*(1))) - \int_{n^*(1)}^{1} \frac{n^*(1)}{n} \left(\lim_{a\to 1} \frac{\partial}{\partial a} \left[R_1 - s_\gamma(a)\right] \right) dn < 0 \tag{97}$$

by $n^*(a) < 1$ and since s_γ declines in a. □