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Abstract

This paper studies mergers in markets where firms invest in a portfolio of research projects of different profitability and social value. The investment of a firm in one project imposes both a negative business-stealing and a positive business-giving externality on the rival firms. We show that when the project that is relatively more profitable for the firms appropriates a larger (smaller) fraction of the social surplus, a merger increases (decreases) consumer welfare by reducing investment in the most profitable project and increasing investment in the alternative project. The innovation portfolio effects of mergers may dominate the usual market power effects.

JEL Classification: L13, L22, O31, O32

Keywords: Horizontal mergers, innovation portfolios, R&D contests

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Mergers and Innovation Portfolios*

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1 Introduction

Innovation, an essential activity for economic growth and welfare, is often encouraged by public policies such as R&D subsidisation and intellectual property policy.[1] Recently, antitrust enforcers are actively taking further action by accounting for the effects of mergers on innovation. In fact, according to Gilbert and Greene (2015), during the 2004-2014 period, the US Antitrust Agencies invoked innovation-based concerns in about a third of the mergers they challenged. Likewise, the European Commission appealed to an innovation theory of merger harm in the recent Dow/DuPont, GSK Oncology/Novartis and General Electric/Alstom cases.

Starting with Schumpeter (1943) and Arrow (1962), an important and large theoretical literature in economics studies the relationship between competition and innovation. This work, however, does not take into account the specificities of merger activity and cannot readily be used to develop an innovation theory of merger harm. In particular, as demonstrated in the recent papers of Federico, Langus and Valletti (2017, 2018), Motta and Tarantino (2021), Bourreau, Jullien and Lefoulli (2021), Denicolò and Polo (2018) and Gilbert (2019), understanding the impact of merger activity on innovation necessitates a separate analysis because a merger cannot be understood as a mere reduction in the number of competitors in the market, or of the degree of product differentiation, but as a transaction that results in that the partner firms coordinate their strategic decisions. Moreover, the literature on innovation and competition has been somewhat inconclusive about how competition affects investment due to the variety of models analysed, with specific functional forms and modes of competition (see Vives (2008) and Schmutzler (2013)).[2]

The recently emerged literature on mergers and innovation focuses on how a merger impacts R&D investment of merging and non-merging firms. However, it is well known that firms engage themselves in multiple research projects with different chances of success and distinct returns to the innovators and society (see e.g. Mansfield (1981) and Cohen and Klepper (1996)). Typical examples include pharmaceutical firms, which engage in research to develop innovative medicines in distinct therapeutic areas such as ophthalmology, immunology, dermatology, oncology, etc.[3] Because the drugs they produce differ in profits and social value, the question that arises is how does merger activity shape firms' incentives to pursue the socially optimal composition of R&D portfolios.[4] To the best of our knowledge, this question has not been analysed so far. Our paper fills this gap by developing a model of investment portfolios to examine how mergers affect the portfolios chosen by merging and non-merging firms and assess the welfare effects of these choices.

To study how mergers affect equilibrium innovation portfolios, we consider a market where firms invest in two independent research projects. These projects vary in terms of three characteristics, namely, their profitability, difficulty and social value. By investing in a project, an individual firm engages in a contest with the rival firms.[5] In the baseline model, each contest is winner-take-all and the

[1] See e.g. Romer (1990), Aghion and Howitt (1998), and Grossman and Helpman (1994).

[2] Empirically, the relationship between competition and innovation has not reached a consensus either, with some authors finding it to be inverted-U shape (Aghion *et al.* (2005)) and others either increasing (Correa and Ornaghi (2014) and Beneito *et al.* (2017)) or even decreasing (Hashmi (2013)).

[3] One such company is *Novartis*; see its website `https://www.novartis.com/our-company/innovative-medicines`.

[4] In the European GSK Oncology/Novartis case, the Commission had concerns about a deleterious adjustment of investments post-merger. In particular, the Commission's assessed a potential cut in research oriented to treat skin cancer.

[5] In our model firms engage in a conflict involving multiple contests. This type of game was introduced by Borel (1921)

successful firm appropriates the full profits generated by the innovation, while the losing firms obtain zero rewards.[6] The cost of investment function exhibits decreasing returns in aggregate investment across the two projects, which implies that increasing investment in one project raises the marginal cost of investing in the other project. The existence of these negative externalities across the projects of a firm has the novel implication that the investment of a firm in a given project imposes on the rival firms both a *business-stealing* externality for that project and a *business-giving* externality for the alternative project.[7]

We show that firms hold an inefficient portfolio of investments for two reasons. First, because an individual firm ignores the business-stealing and business-giving externalities it imposes on the rest of the firms, it tends to invest excessively in the project with higher expected profitability and insufficiently in the alternative one. Second, because an individual firm fails to fully appropriate the social gains of an innovation, it tends to underinvest in the more socially desirable project and overinvest in the alternative one. This portfolio inefficiency of the market equilibrium leads us to explore how mergers affect the allocation of investment among these projects by the merging and non-merging firms.[8]

The merging firms internalize the two innovation externalities they exert on one another. Internalization of the business-stealing externality gives the merging firms incentives to lower investment in the more profitable project and raise it in the alternative project, while internalization of the business-giving externality incentivises them to adjust the portfolio of investments in the opposite direction. The tension between these two conflicting externalities determines the impact of a merger on the innovation portfolio of the partner firms. We refer to this new economic effect associated with mergers as the *innovation portfolio effect* of mergers.

When projects differ in expected profitability, a merger results in an increase in the investment of the merging firms in the less profitable project. Whether the non-merging firms increase or decrease their investment in the less profitable project depends on whether investments are strategic substitutes or strategic complements. With "linear" Tullock contest success functions, firms' investments are strategic substitutes and the non-merging firms adjust investment in the opposite direction compared to the merging firms. Despite this, we show that aggregate investment in the less profitable project increases after a merger and consumer welfare post-merger may be higher than pre-merger.[9] We derive a clear policy message: in the absence of any synergies, when the relatively less profitable project is also the project that appropriates a lower fraction of the social surplus, a merger raises consumer surplus by increasing

and is known as the Colonel Blotto game. For a survey of the literature see Kovenock and Roberson (2012). For a recent application, see Goyal and Vigier (2014).

[6]In Section 4.4 we extend our analysis to the case in which the winning and losing firms compete *à la* Cournot to sell products that are vertically differentiated. We show that the innovation portfolio effects of mergers may have a dominating influence over the usual price effects of mergers.

[7]Business-stealing and business-giving externalities are also at the heart of investment incentives in models with innovation spillovers (see e.g. Katz and Shapiro (1987) and Kamien *et al.* (1992)). These papers feature single-project firms and exogenous inter-firm spillovers. In our model, by contrast, the business-stealing and business-giving externalities originate from the portfolio problem the firms face.

[8]The inefficiency of the market portfolio of investments has also been investigated e.g. in the single-project papers of Klette and de Meza (1986), Bhattacharya and Mookherjee (1986) and Dasgupta and Maskin (1987). Cabral (1994) and section 5 of Bhattacharya and Mookherjee (1986) use a multi-project approach similar to ours. More recently, Hopenhayn and Squintani (2021), Bryan and Lemus (2017) and Chen *et al.* (2018) study the dynamic efficiency of the direction of innovation and Akcigit *et al.* (2017) present evidence of a dynamic misallocation in research by which there is overinvestment in applied research and underinvestment in basic research.

[9]This is in contrast with Farrell and Shapiro (1990) and Motta and Tarantino (2021), which show that in the absence of synergies mergers in single product markets cannot increase consumer welfare.

investment in it. Otherwise, when the project that appropriates a lower share of the social surplus is the one that is relatively more profitable, then a merger reduces welfare. This policy advice is easily implementable. Assuming constant marginal costs, for the family of ρ-linear demands (see Anderson and Renault, 2003), this policy prescription means that when the easier (more difficult) and more (less) profitable project is also the one with the more (less) concave demand, a merger increases consumer surplus. For the more special case of linear demands, all mergers are consumer welfare improving. Finally, for the family of isoelastic demands, a merger increases consumer welfare if the easier (more difficult) and more (less) profitable project corresponds to the market with the higher (lower) elasticity of demand.

We provide five extensions of our main analysis. First, our results carry over to the case of "non-linear" Tullock contest success functions provided that the success probabilities exhibit decreasing returns. With (moderate) increasing returns, the merged entity has an incentive to shut down the research facility of one of the constituent firms (cf. Denicolò and Polo (2018)). Numerical analysis nevertheless shows that our results still hold in such a setting, with the welfare implications being even stronger because of the *efficiency gains* that accrue from concentrating research in a single research facility. Second, we extend our analysis to the case of asymmetric firm research budgets and show that our results remain unchanged. Third, we show that our results also hold in situations where firms decide on the budget they spend on research and how they allocate it across projects. Allowing firms to choose how much to invest in research does not alter the basic intuition behind the innovation portfolio effects of mergers because investing in one project increases the marginal cost of investing in the other project. We show that while a merger results in a lower aggregate investment (as per the innovation theory of merger harm), this effect does not invalidate our results because the innovation portfolio effects of mergers have a dominating influence when the cost of effort is sufficiently convex. Fourth, we examine the case in which the product market is competitive and winning and losing firms engage in quantity competition to sell products of different quality. Firm competition in the product market causes an additional *market power effect* of mergers that may either operate counter to or in accordance with the innovation portfolio effect. We show that a merger may still be consumer welfare improving despite its price-increasing implications. Finally, we provide a model with general contest success functions for which firms' investment efforts may be strategic complements. We provide conditions under which mergers may be welfare improving and develop a graphical approach to show that the main insights of our baseline model hold for general success probabilities that allow for firms' investments to be strategic substitutes.

Related literature

Our paper adds to a recent literature on the impact of mergers on innovation. As mentioned above, this work has mainly focused on how mergers affect R&D expenditure. This literature has identified two channels through which a merger affects the innovation incentives of the partner firms. First, there is an internalization of negative innovation externalities. A firm that invests in R&D increases the likelihood with which it successfully innovates and this lowers the chance other firms appropriate the full gains from their innovation efforts. This channel corresponds to our business-stealing externality and tends to reduce the investment of the partner firms. Second, there is an internalization of negative quantity/pricing externalities through which a merger tends to soften competition in the product market. This weakening of competition may or may not increase the incentives to invest because the pre-merger profits may be more or less responsive than the post-merger profits to innovation effort. Whether a merger ultimately increases investment incentives depends on the relative strength of these two effects.

Motta and Tarantino (2017) study mergers in a deterministic R&D model with product market competition. They analyze a simultaneous process innovation and pricing game and show that absent spillovers or synergies, the reduction of output by the merged entity induces a further reduction of cost-reducing investment, which harms consumers. Federico *et al.* (2018) obtain a similar result in a two-stage model of product innovation and price competition despite the fact that in their model the reduction in the intensity of price competition following a merger favors innovation. Bourreau *et al.* (2021) study a demand-enhacing innovation model and show that the market power effect can be decomposed into two effects working in opposite directions, namely, a negative "margin expansion" effect and a positive "demand expansion" effect. They show that the overall impact of a merger on innovation incentives can be either positive or negative. However, they do not analyze consumer welfare implications. Finally, López and Vives (2018) use an approach based on cross-shareholding agreements between firms to argue that industry-wide mergers can be welfare improving in industries with sufficiently large R&D spillovers.[10]

Our work is closely related to Johnson and Rhodes (2019). They consider mergers in markets where firms choose their product lines and show that, when firms' product lines are asymmetric, mergers may raise consumer surplus due to a product-mix effect through which firms reposition their product lines after a merger. This product-mix effect is different from our innovation portfolio effect of mergers because in the paper of Johnson and Rhodes firms operate in a single market. Nocke and Schutz (2018) also study mergers in a multi-product setting. They show that standard merger results generalize to multi-product settings with nested CES or nested multinomial logit (NMNL) demand systems.

The multi-project setting of our study also brings our paper close to the recent contributions of Letina (2016) and Gilbert (2018), which, instead of looking at the effects of mergers on investment volumes, delve into the effects of mergers on the variety and diversity of R&D. Letina (2016) studies a model where firms can choose the number of projects they wish to activate, knowing that only one of the very many projects will turn out to be successful. He provides conditions under which the market opens too many research lines and there is too much duplication. A merger decreases the variety of developed projects and decreases the amount of duplication of research, which, depending on parameters, may increase or decrease welfare. Gilbert (2018) extends the model of Federico *et al.* (2017, 2018) by allowing the firms to invest in several research avenues to solve the same problem. He shows that, absent spillovers, mergers generally (but not always) decrease R&D diversity measured by the number of projects undertaken by the industry.

Finally, our paper is also related to studies on the bias of the market equilibrium research portfolios. Part of this literature uses models where firms choose the riskiness of their single research projects and compare the equilibrium with the social optimum (see e.g. Klette and de Meza (1986), Bhattacharya and Mookherjee (1986) and Dasgupta and Maskin (1987)). Klette and de Meza (1986) and Bhattacharya and Mookherjee (1986) find that the firms and the social planner choose the riskiest research program available. Dasgupta and Maskin (1987) introduce convex costs to induce interior solutions in a similar model with differentiated firms and find that the firms choose projects that are too risky from the

[10]Markets in which buyers have market power can be quite distinct. Loertscher and Marx (2018) show that in procurement markets a merger increases the incentives to invest of the non-merging suppliers and may also raise the incentives to invest of the merging firms.

point of view of social welfare. Bhattacharya and Mookherjee (1986) also study firm choice among two R&D approaches to solve the same hurdle. They show that firms maximally differentiate their R&D approaches, thereby maximising the riskiness and minimizing the correlation of the equilibrium firm portfolios. The market equilibrium is socially optimal. In Cabral (1994) firms also allocate funding across two projects of different riskiness. He shows that market competition results in a bias against the riskier project. Finally, Hopenhayn and Squintani (2021), Bryan and Lemus (2017) and Chen *et al.* (2018) present models of the choice of research direction. They show that firms pursue inefficient research lines, in particular they focus on relatively easy, safe and highly profitable projects. These papers relate to ours in terms of the externalities firms impose on one another and the resulting inefficiency of the investment portfolios held by firms. None of them studies the impact of mergers on social welfare. Their focus is on the merits of R&D subsidization and patent policy to improve directional efficiency.

The rest of the paper is organized as follows. We introduce our general model in Section 2. The pre-merger market equilibrium is analyzed in Subsection 2.1. The effect of mergers is discussed in Section 2.2. To deepen into the welfare effects of mergers in Section 3 we analyze the standard case of Tullock contests. In Section 4 we demonstrate the robustness of our results. In particular, in Subsection 4.3 we study a more general Tullock contest model where firms not only decide how to allocate funding across projects but also how much money to invest in total. Finally, some concluding remarks are provided in Section 5. All the proofs of the Propositions are relegated to the Appendix.

2 The model and preliminary intuition

We consider a market with $n \geqslant 3$ independent firms, indexed by i. The number of firms is exogenous, reflecting barriers to entry. The market develops over two stages. In the *pre-innovation* stage, stage one, firms race to introduce two types of innovations: A- and B-innovations. We model these innovation races as "winner-take-all contests". In the *post-innovation* stage, stage two, the winning firm obtains the monopoly profits from the market created by the innovation, while the losing firms make no profits.[11]

The two innovations, indexed by ℓ, differ in three aspects:

(i) the rewards, or profits, they generate for the sole winner of the innovation contest, denoted π_ℓ,

(ii) the intrinsic difficulty of successfully obtaining the innovation, denoted ϵ_ℓ, and

(iii) the social gains the innovation creates, denoted W_ℓ, $\ell = A, B$.

As usual, the social gains equal the sum of firm's profits and consumer surplus. The latter is denoted by S_ℓ, $\ell = A, B$. Summarizing, each innovation ℓ is characterised by a triplet $\{\pi_\ell, \epsilon_\ell, S_\ell\}$, $\ell = A, B$.

Firms pick their investments in the A- and B-innovation projects simultaneously. We assume that firms have a fixed mass of scientists (or a fixed budget), normalized to 1, and that they individually

[11] The winner-take-all assumption serves to focus the discussion on the *innovation portfolio* effects of mergers. Later in Section 4.4 we relax this assumption by allowing the winning and the losing firms to compete *á la* Cournot after they engage in a contest to introduce a product of higher quality than the existing product. In such an extension, in addition to the innovation portfolio effects of mergers, there are price effects of mergers. We show that the innovation portfolio effects of mergers have a dominating influence when the quality gap between the winning and losing firms is sufficiently large.

decide how to allocate them across the two projects. Let x_i^A denote the "investment" of a firm i in the A-project and, correspondingly, $x_i^B = 1 - x_i^A$ the "investment" of firm i in the alternative B-project.[12]

Let $p_i^\ell(x_i^\ell, \mathbf{x}_{-i}^\ell, \epsilon_\ell)$ denote the probability with which an individual firm i successfully introduces the ℓ-innovation, $\ell = A, B$. This probability depends on the firm i's investment, x_i^ℓ, the investments of the rival firms, denoted $\mathbf{x}_{-i}^\ell \equiv (x_1^\ell, x_2^\ell, ..., x_{i-1}^\ell, x_{i+1}^\ell, ..., x_n^\ell)$, and the intrinsic difficulty of the innovation ϵ_ℓ. The difficulty of the innovation ϵ_ℓ can be informally interpreted as the effort "Nature" puts to beat the firms' scientists. Correspondingly, let $p_{\mathcal{N}}^\ell(x_i^\ell, \mathbf{x}_{-i}^\ell, \epsilon_\ell)$ be understood as the probability with which Nature "wins" and no firm successfully introduces the ℓ-innovation.

We now impose standard assumptions on the success probabilities:

Assumption 1. *For $i = 1, 2, ..., n$ and $\ell = A, B$:*

(a) The function $p_i^\ell(\cdot)$ is a real-valued and twice differentiable function.

(b) $p_i^\ell(\cdot) \geqslant 0$, $p_{\mathcal{N}}^\ell(\cdot) \geqslant 0$, and $\sum_{i=1}^n p_i^\ell(\cdot) + p_{\mathcal{N}}^\ell(\cdot) = 1$.

(c) $\frac{\partial p_i^\ell(\cdot)}{\partial x_i^\ell} > 0$, $\frac{\partial p_i^\ell(\cdot)}{\partial x_j^\ell} < 0$, $\frac{\partial p_i^\ell(\cdot)}{\partial \epsilon_\ell} < 0$.

These assumptions are standard in innovation contests if we regard Nature as a non-strategic player (see e.g. Tullock, 1980; Dixit, 1987 and Skaperdas, 1996). Assumption (a) allows us to use calculus to address our research questions. Assumption (b) ensures that the contest success functions are proper probability distribution functions. Assumption (c) signifies that if a firm raises its investment in a research project, the probability that it succeeds increases and the probability that other firms succeed decreases. Further, the assumption implies that an increase in the difficulty of an innovation lowers the probability that any of the firms successfully innovates.

In addition to Assumption 1, we adopt the following:

Assumption 2. *For $i = 1, 2, ..., n$ and $\ell = A, B$:*

$$\frac{\partial^2 p_i^\ell(\cdot)}{(\partial x_i^\ell)^2} + \sum_{j \neq i}^n \left| \frac{\partial^2 p_i^\ell(\cdot)}{\partial x_i^\ell \partial x_j^\ell} \right| < 0.$$

This assumption implies that the dominant diagonal condition is satisfied for our payoff functions, which guarantees the existence and uniqueness of a Nash equilibrium in pure strategies (see Vives, 1999).

At this stage we do not impose any assumption on the sign of the second cross-partial derivative of the success probabilities with respect to own and rival firms' investments; this implies that the best-replies of the firms may be decreasing (in which case the game is one of strategic substitutes) or increasing (in which case the game is one of strategic complements).

[12] The fixed budget assumption, which is quite reasonable in the short-run because it often is very expensive to find and hire new scientists, serves the purpose of focussing the main body of our paper on the innovation portfolio effects of mergers, rather than on the effects of mergers on investment effort. Later in Section 4.3 we relax this assumption and allow the firms to choose the size of their research teams. We show that the main economic forces at play are similar provided that the costs of hiring additional scientists is convex. This will become clearer later but, in anticipation, the reason is simply that, when the cost is convex, allocating one more scientist to one project increases the marginal cost of allocating one more scientist to the alternative project. Our fixed budget assumption is an extreme version of this trade-off and comes out naturally when the degree of convexity of the cost function converges to infinity.

In what follows, because $x_i^B = 1 - x_i^A$, $i = 1, 2, ..., n$, we shall no longer write the super-index ℓ on a firm i's investment in project ℓ and write $p_i^A(x_i, \mathbf{x}_{-i}, \epsilon_A)$ and $p_i^B(1 - x_i, \mathbf{1} - \mathbf{x}_{-i}, \epsilon_B)$ for the success probabilities of a firm i, $i = 1, 2, ..., n$.

2.1 Pre-merger market equilibrium

The payoff to a firm $i = 1, 2, ..., n$ investing x_i in project A and $1 - x_i$ in project B is:

$$u_i(x_i; \mathbf{1} - \mathbf{x}_{-i}) = p_i^A(x_i, \mathbf{x}_{-i}, \epsilon_A)\pi_A + p_i^B(1 - x_i, \mathbf{1} - \mathbf{x}_{-i}, \epsilon_B)\pi_B. \quad (1)$$

As explained after Assumption 2, an equilibrium in pure strategies exists and is unique. Therefore, assuming the equilibrium is interior, it is given by the solution to the system of first order conditions (FOCs) for profits-maximization:[13]

$$\frac{\partial p_i^A(x_i, \mathbf{x}_{-i}, \epsilon_A)}{\partial x_i}\pi_A + \frac{\partial p_i^B(1 - x_i, \mathbf{1} - \mathbf{x}_{-i}, \epsilon_B)}{\partial x_i}\pi_B = 0, \; i = 1, 2, ..., n. \quad (2)$$

Equation (2) simply means that the marginal gains from investing in a project should be equal across projects. Note that $\partial p_i^B(\cdot)/\partial x_i < 0$ because the probability with which a firm i wins the contest for the B-innovation decreases with its effort in the A-innovation.

The solution to the FOC of a firm i in (2) gives the best-reply function of a firm i to the vector of rivals' investments $\mathbf{x}_{-i}$. As it is known from the work of Salant *et al.* (1983) and Deneckere and Davidson (1985), the strategic nature of the endogenous variables will be important later in our analysis of mergers. Applying the implicit function theorem to the FOC of a firm i in (2), we obtain:

$$\frac{\partial x_i}{\partial x_j} = -\frac{\frac{\partial^2 p_i^A(\cdot)}{\partial x_i \partial x_j}\pi_A + \frac{\partial^2 p_i^B(\cdot)}{\partial x_i \partial x_j}\pi_B}{\frac{\partial^2 p_i^A(\cdot)}{\partial x_i^2}\pi_A + \frac{\partial^2 p_i^B(\cdot)}{\partial x_i^2}\pi_B}.$$

When this derivative is negative (positive) for all $i, j = 1, 2, ...n$, $j \neq i$ we have a game of strategic substitutes (complements).

2.2 Mergers

Consider now that firms i and j merge and assume that it is optimal for the merged entity to keep the two labs of the constituent firms running.[14] In such a case, the merged entity chooses investments x_i and x_j in the A-project (and by implication $1 - x_i$ and $1 - x_j$ in the B-project) to maximise the (joint)

[13] Throughout the paper, we shall assume (or provide conditions under which) the equilibrium is interior. Note that when firms specialize and only invest in one of the projects, the notion of innovation portfolio, and by implication the innovation portfolio effects of mergers, has no meaning. We refer the reader to Section 4.5 for interiority conditions, and focus here on developing the main intuition behind the innovation portfolio effects of mergers.

[14] As shown by Denicolò and Polo (2018), for this to be the case the profits function of the merged entity must be strictly concave. Concavity of the success probabilities, which in our case is guaranteed by Assumption 2, is however not sufficient for this. Later in Section 4.1 we provide conditions under which this assumption is satisfied for the family of Tullock contest success functions.

payoff:

$$u_m(x_i, x_j; \cdot) = p_i^A(x_i, \mathbf{x}_{-i}, \epsilon_A)\pi_A + p_i^B(1 - x_i, \mathbf{1} - \mathbf{x}_{-i}, \epsilon_B)\pi_B + p_j^A(x_j, \mathbf{x}_{-j}, \epsilon_A)\pi_A$$
$$+ p_j^B(1 - x_j, \mathbf{1} - \mathbf{x}_{-j}, \epsilon_B)\pi_B. \tag{3}$$

Non-merging firms continue to maximise the payoff in (1).

Assuming that an interior equilibrium exists, the FOC for the maximisation of the profits of the merged entity with respect to x_i is given by:

$$\frac{\partial p_i^A(x_i, \mathbf{x}_{-i}, \epsilon_A)}{\partial x_i}\pi_A + \frac{\partial p_i^B(1 - x_i, \mathbf{1} - \mathbf{x}_{-i}, \epsilon_B)}{\partial x_i}\pi_B + \underbrace{\frac{\partial p_j^A(x_j, \mathbf{x}_{-j}, \epsilon_A)}{\partial x_i}\pi_A}_{\text{business-stealing externality on } j}$$
$$+ \underbrace{\frac{\partial p_j^B(1 - x_j, \mathbf{1} - \mathbf{x}_{-j}, \epsilon_B)}{\partial x_i}\pi_B}_{\text{business-giving externality on } j} = 0. \tag{4}$$

The FOC for profits maximisation of the merged entity with respect to x_j is similar and, to save space, omitted. The FOCs corresponding to the payoffs of the $n - 2$ non-merging firms are the same as those given in (2), excluding firms i and j.

The FOCs for profits maximisation with respect to x_i and x_j are central to the key contribution of this paper. The FOC (4) reveals that the merging unit i internalises *two* externalities (and similarly for the merging unit j). On the one hand, a merging firm's investment in a given project imposes a *business-stealing* externality on the partner firm for the same project. This is because when a merging firm, say i, increases its investment in the, say, A-project, it lowers the probability the partner entity j wins the contest for that innovation. On the other hand, a merging firm's investment in a given project imposes a *business-giving* externality on the partner firm for the alternative project. This is because when firm i increases its investment in the A-project, it consequently lowers its investment in the B-project and this therefore increases the probability the partner firm j wins the contest for the B-innovation.

Internalization of these two externalities leads to a new economic effect of mergers, which we call the *innovation portfolio effect of mergers*. The first, business-stealing, externality is negative, pushing the merged entity to reduce x_i relative to its investment in the pre-merger market.[15] The second, business-giving, externality is positive and works counter to the negative business-stealing externality. The net effect is thus ambiguous and, depending on parameters, may be towards more investment in the A-project and less in the B-project, or viceversa.

Whether the portfolio adjustment of the merging firms tends to benefit consumers or not depends on which of the two projects generates more surplus for them. If, for example, the internalization of the business-stealing and business-giving externalities results in the merging firms moving resources from the A-project to the B-project and consumer surplus from the B-project is higher than that from the A-project, then the merger will tend to increase consumer welfare.

Of course, even if a merger causes the merging firms to adjust their investment in a way that favours consumers, at this level of generality, it is not clear whether the post-merger market equilibrium

[15] The literature refers to this negative externality as the "innovation externality" and identifies it as an additional source of detrimental merger effects for consumers (cf. Federico *et al.*, 2018).

consumer welfare will be higher than pre-merger. For this, we also need to take into consideration the reaction of the non-merging firms, which depends on whether the game is one of strategic substitutes or one of strategic complements. If, compared to the pre-merger market equilibrium, the merged entity increases investment in project A (and so decreases investment in project B) and the game is of strategic substitutes, the non-merging firms will instead reduce investment in project A (and so increase it in project B). With strategic complements, the non-merging firms will adjust their investments in the same way as the merged entity.

To better understand the likely effects of a merger on welfare, let us now present the social optimum.

2.3 Welfare

We shall evaluate mergers from a consumer welfare perspective but for the moment let us adopt a social welfare standard not to lose generality.[16] Assuming that it is socially optimal to keep the labs of all the firms running,[17] the social planner picks a portfolio of investments $\mathbf{x}$ to maximize:

$$W(\mathbf{x}, \mathbf{1}-\mathbf{x}) = \left(\sum_{i=1}^{n} p_i^A(x_i, \mathbf{x}_{-i}, \epsilon_A)\right) W_A + \left(\sum_{i=1}^{n} p_i^B(1-x_i, \mathbf{1}-\mathbf{x}_{-i}, \epsilon_B)\right) W_B. \tag{5}$$

This welfare expression is equal to the sum across projects of the market probability with which an innovation is successfully obtained times the total surplus generated by the innovation. (If the planner maximizes consumer surplus, we simply replace $W_\ell = \pi_\ell + S_\ell$ by S_ℓ in (5), $\ell = A, B$.)

Assuming the social optimum exists and is interior, it is given by the solution of the following system of FOCs for social welfare maximization:

$$\frac{\partial W(\cdot)}{\partial x_i} = \left(\frac{\partial p_i^A(x_i, \mathbf{x}_{-i}, \epsilon_A)}{\partial x_i} + \underbrace{\sum_{j\neq i} \frac{\partial p_j^A(x_j, \mathbf{x}_{-j}, \epsilon_A)}{\partial x_i}}_{\text{negative externality}}\right) W_A$$
$$+ \left(\frac{\partial p_i^B(1-x_i, \mathbf{1}-\mathbf{x}_{-i}, \epsilon_B)}{\partial x_i} + \underbrace{\sum_{j\neq i} \frac{\partial p_j^B(1-x_j, \mathbf{1}-\mathbf{x}_{-j}, \epsilon_B)}{\partial x_i}}_{\text{positive externality}}\right) W_B = 0, \; i = 1, 2..., n. \tag{6}$$

It is interesting to compare the system of FOCs (6) with the equilibrium conditions in the pre- and post-merger markets to understand the difference between the social optimum and the market equilibria. Consider the case of the pre-merger market equilibrium first. The FOCs (2) and (6) differ in two important regards. First, when the social planner chooses its investment x_i, it cares about the market probability that the innovations are successful, and not just about the individual firms' success probabilities. This implies that the social planner takes into account *all* the externalities that the investment of an individual firm imposes on *all* other firms in the market. The fact that an individual

[16] EU and US competition authorities usually employ a consumer welfare standard to assess mergers.

[17] Alternatively, we may view this problem from a second-best perspective under which the planner can choose the allocation of scientists within firms but not across firms.

firm ignores all these externalities, some of which are negative business-stealing and some of which are positive business-giving externalities as indicated in equation (6), creates a source of inefficiency of the market equilibrium (unless they accidentally happen to cancel out). Second, the fact that the social planner also cares about the consumer value of an innovation S_ℓ and the firm only about the private reward π_ℓ, $\ell = A, B$, constitutes a second source of inefficiency of the market equilibrium (unless they coincidentally happen to be exactly equal).[18]

Comparing now the FOCs (4) and (6) reveals that while a merging firm does internalize the externalities on the partner firm, it fails to internalize the ones on the rest of the firms and consumers. No matter whether we adopt a consumer or a social welfare criterion, at this level of generality, it is very hard to tell how the post-merger market equilibrium will fare relative to the pre-merger market equilibrium in terms of welfare. To illustrate this, and without exhausting all possible cases, consider for example situations in which some firms over-invest in a given project while others under-invest in the same project compared to the social optimum. In those cases the impact of a merger will likely depend on the identity of the merging firms. Or for example consider cases in which all firms, say, underinvest in a given project compared to the social optimum. It is certainly possible that the merging firms and also, by strategic complementarity, the non-merging firms will adapt their portfolio of investments in such a way that welfare increases, in which case a merger would obviously be desirable, but it is also possible that they will adjust their portfolio of investments in the opposite way. Finally, consider cases in which firms' investments in a given project are strategic substitutes. In that case, even if the merging firms adapt their portfolio of investments in the right way, the non-merging firms will respond in an offsetting manner, making the desirability of a merger ambiguous.

In order to better understand when a merger is appealing, we thus need to introduce more structure into our model. In the next section, we use the well-known Tullock contest formulation to characterize consumer surplus increasing/reducing mergers and derive a clear-cut policy recommendation. We return to the general model in Section 4.5 and show that the insights obtained apply more generally once we apply symmetry.

3 Tullock contests

We analyze our model using the stylized ratio-form contest success function introduced by Tullock (1980). Specifically, let the probabilities with which a firm i succeeds to introduce A- and B-innovations be given

[18] Interestingly, with only one firm in the market there are no external effects and the first source of inefficiency disappears. Moreover, the second source of inefficiency disappears when the winning firm can perfectly price discriminate in the product market; or, alternatively, when the product market demand is rectangular. For these special cases, the winning firm extracts the whole surplus in the product market, i.e. $\pi_\ell = W_\ell$, $\ell = A, B$. Therefore, in our model the investment portfolio of a perfectly price discriminating monopolist is socially optimal. For a similar result in a related model of R&D portfolios see Cabral (1994).

by:[19]

$$p_i^A(x_i, \mathbf{x}_{-i}, \epsilon_A) = \frac{x_i}{\sum\limits_{k=1}^{n} x_k + \epsilon_A}, \tag{7}$$

$$p_i^B(1 - x_i, \mathbf{1} - \mathbf{x}_{-i}, \epsilon_B) = \frac{1 - x_i}{\sum\limits_{k=1}^{n} (1 - x_k) + \epsilon_B}, \; i = 1, 2, ..., n. \tag{8}$$

Moreover, we assume that firms are symmetric.[20] To ensure that our game of innovation portfolio choice is one of strategic substitutes, we assume that $\epsilon_\ell \geqslant 1$, $\ell = A, B$.[21]

3.1 Pre-merger market equilibrium

In the pre-merger market equilibrium, a firm i maximises the expression:

$$u_i(x_i, \mathbf{x}_{-i}) = \frac{x_i}{\sum\limits_{k=1}^{n} x_k + \epsilon_A}\pi_A + \frac{1 - x_i}{\sum\limits_{k=1}^{n} (1 - x_k) + \epsilon_B}\pi_B. \tag{9}$$

The FOC for an interior equilibrium is given by:

$$\frac{\sum_{k \neq i} x_k + \epsilon_A}{\left(\sum\limits_{k=1}^{n} x_k + \epsilon_A\right)^2}\pi_A - \frac{n - 1 + \epsilon_B - \sum_{k \neq i} x_k}{\left(n + \epsilon_B - \sum\limits_{k=1}^{n} x_k\right)^2}\pi_B = 0. \tag{10}$$

It is straightforward to verify that the second order condition for profits maximization holds.

The following result characterises the symmetric Nash equilibrium (SNE) of the portfolio investment game, and provides conditions on the parameters under which the SNE is interior.

Proposition 1. *Assume the parameters of the model satisfy the inequality*

$$\frac{\pi_A\epsilon_B - 2n\pi_B + \sqrt{\pi_A\epsilon_B\left(\pi_A\epsilon_B - 4\pi_B\right)}}{2\pi_B} < \epsilon_A < \frac{\pi_A(n + \epsilon_B)^2}{\pi_B(n - 1 + \epsilon_B)}. \tag{11}$$

Then there exists a unique interior SNE of the investment portfolio game. In equilibrium, each firm invests an amount $x^* \in (0, 1)$ in project A, where x^* is given by the unique solution to:

$$\frac{(n - 1)x^* + \epsilon_A}{(nx^* + \epsilon_A)^2}\pi_A - \frac{n + \epsilon_B - 1 - (n - 1)x^*}{(n + \epsilon_B - nx^*)^2}\pi_B = 0. \tag{12}$$

The rest of the budget, $1 - x^*$, is invested in project B. The equilibrium investment in project A, x^*, increases in π_A and ϵ_B, and decreases in ϵ_A and π_B.

[19]We adopt the "linear" version of the Tullock contest success function (see Tullock, 1980). In Section 4.1 we generalize our results to "concave" contest success functions, but we need to adopt a numerical approach for some of them. With sufficiently "convex" contest success functions an interior equilibrium does not exist; firms instead specialize and invest in one project only, which makes our study of portfolio effects of mergers uninteresting.

[20]In Section 4.2 we generalize our results to a setting in which firms have different research budgets. All our results hold.

[21]For a proof, see the Appendix. This condition ensures that the best-response of an individual firm is decreasing everywhere. For our results, however, it is sufficient that the best-replies are decreasing in a neighbourhood of the symmetric equilibrium, which holds for any $\epsilon_\ell > 0$, $\ell = A, B$.

The condition on the parameters of the model in the proposition ensures that corner solutions cannot be equilibria. This condition is always satisfied when n is sufficiently large. In what follows we assume that the parameters of the model satisfy the condition in (11) and focus on the interior equilibrium. The impact of the parameters of the model on the equilibrium innovation portfolio is intuitive: firms allocate more funding to project $\ell = A, B$ when such a project yields a higher reward and is easier to complete successfully.

Because we analyze mergers in an n-player game, it is illustrative to follow the graphical tool devised by Deneckere and Davidson (1985) and build the "pseudo" reaction functions corresponding to our setting. Deneckere and Davidson plot the joint reaction function of the potentially merging firms against the non-merging firms on the one hand, and the joint reaction of the non-merging firms against the potentially merging firms on the other hand. The crossing point between these two reaction functions depicts the pre-merger symmetric equilibrium.

Let firms i and j be the firms involved in a tentative merger. Let us define $X_m = x_i + x_j$ as the joint effort put by these firms in the A-project. Likewise, let $X_{nm} = \sum_{k \neq i,j} x_k$ be the corresponding joint effort of the non-merging firms. Using this notation, we can compute the equation that characterizes the joint best-response function of the potentially merging firms against the non-merging firms as follows. First write the FOCs for firms i and j in this way:

$$x_i \quad : \qquad \frac{x_j + X_{nm} + \epsilon_A}{(X_m + X_{nm} + \epsilon_A)^2}\pi_A - \frac{n - 1 + \epsilon_B - x_j - X_{nm}}{(n + \epsilon_B - X_m - X_{nm})^2}\pi_B = 0$$

$$x_j \quad : \qquad \frac{x_i + X_{nm} + \epsilon_A}{(X_m + X_{nm} + \epsilon_A)^2}\pi_A - \frac{n - 1 + \epsilon_B - x_i - X_{nm}}{(n + \epsilon_B - X_m - X_{nm})^2}\pi_B = 0$$

and sum them to obtain:

$$\frac{X_m + 2X_{nm} + 2\epsilon_A}{(X_m + X_{nm} + \epsilon_A)^2}\pi_A - \frac{2(n - 1 + \epsilon_B) - X_m - 2X_{nm}}{(n + \epsilon_B - X_m - X_{nm})^2}\pi_B = 0, \tag{13}$$

This expression defines implicitly the joint best-response function of firms i and j against the rest of the firms. By the implicit function theorem, the slope of this best-response function is easily shown to be negative, which reflects the strategic substitutability nature of the game.

We can construct the joint best-response of the non-merging firms in a similar way, that is, writing the FOC of a typical non-merging firm as:

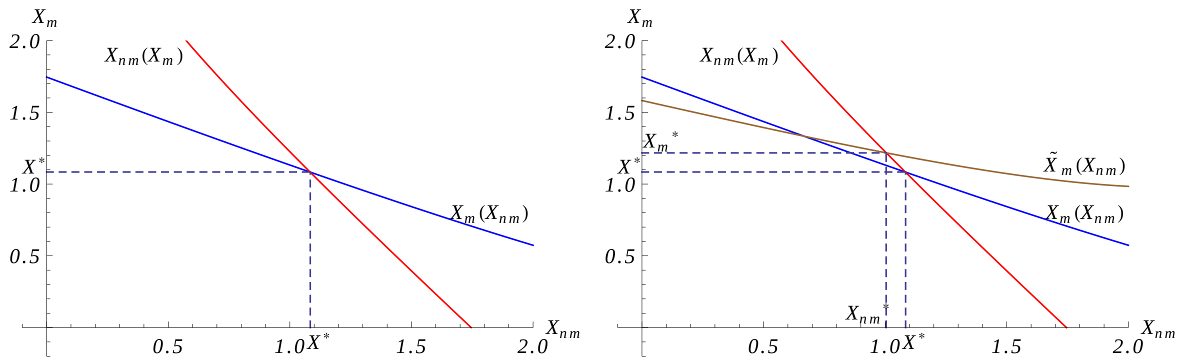
$$x_\ell : \qquad \frac{X_m + \sum_{k \neq i,j,\ell} x_k + \epsilon_A}{(X_m + X_{nm} + \epsilon_A)^2}\pi_A - \frac{n - 1 + \epsilon_B - X_m - \sum_{k \neq i,j,\ell} x_k}{(n + \epsilon_B - X_m - X_{nm})^2}\pi_B = 0,$$

and then summing across $\ell \neq i, j$ and dividing by $n - 2$ to obtain:

$$\frac{X_m + \frac{n-3}{n-2}X_{nm} + \epsilon_A}{(X_m + X_{nm} + \epsilon_A)^2}\pi_A - \frac{n - 1 + \epsilon_B - X_m - \frac{n-3}{n-2}X_{nm}}{(n + \epsilon_B - X_m - X_{nm})^2}\pi_B = 0. \tag{14}$$

This expression defines implicitly the joint best-response function of the non-merging firms against the potentially merging firms i and j. Using the implicit function theorem, it is readily seen that the slope of this pseudo best-response function is also negative for all $n \geqslant 4$. Therefore, for $n \geqslant 4$, the two pseudo best-response functions are decreasing and cross only once. The crossing point gives the pre-merger symmetric Nash equilibrium.

We represent these pseudo best-response functions in Figure 1(a). In this figure, we have the joint investment in project A of the potentially merging firms on the vertical axis, and the joint investment of the non-merging firms on the horizontal axis. The joint pseudo best-response function of the potentially merging firms against the non-merging firms is denoted $X_m(X_{nm})$ and the joint pseudo best-response function of the non-merging firms against the potentially merging firms by $X_{nm}(X_m)$. These best response functions are plotted for $n = 4$ and parameters $\epsilon_A = 4$, $\pi_A = 2$, $\epsilon_B = 1$ and $\pi_B = 1$. The point of intersection gives the pre-merger interior SNE: $X_m = X_{nm} = X^* = 1.084$. This signifies that each firm invests 54.2 percent of its funding in project A and the rest 45.8 percent in project B.



(a) Pre-merger (b) Post-merger

Figure 1: Pre- and post-merger market equilibrium ($\pi_A/\epsilon_A < \pi_B/\epsilon_B$)

3.2 Post-merger market equilibrium

We now study the effects of a merger on the equilibrium allocation of investments. Suppose firms i and j merge. After simplification, the payoff to the merged entity is:

$$u_m(x_i, x_j; \cdot) = \frac{x_i + x_j}{\sum_{k=1}^{n} x_k + \epsilon_A}\pi_A + \frac{2 - x_i - x_j}{\sum_{k=1}^{n}(1 - x_k) + \epsilon_B}\pi_B. \tag{15}$$

The "linearity" of the Tullock contest success function implies that the merged entity is indifferent between keeping the labs of the two constituent firms running or just one. Correspondingly, let $X_m = x_i + x_j$ denote the choice variable of the merged entity. As the payoff of the merged entity, as a function of X_m, satisfies Assumption 2, the existence of a unique equilibrium is guaranteed. As the non-merging firms are symmetric, they must play symmetric strategies in equilibrium.

The FOC necessary for an interior equilibrium for the merged entity is:

$$\frac{\sum_{k\neq i,j} x_k + \epsilon_A}{\left(X_m + \sum_{k\neq i,j}^{n} x_k + \epsilon_A\right)^2}\pi_A - \frac{n - 2 + \epsilon_B - \sum_{k\neq i,j} x_k}{\left(n + \epsilon_B - X_m - \sum_{k\neq i,j}^{n} x_k\right)^2}\pi_B = 0, \tag{16}$$

The FOC for the non-merging firms continues to be (12). As before, the second order conditions hold so the existence of a Nash equilibrium is established by the same arguments as in Proposition 1.

Applying symmetry for the non-merging firms and setting $X_{nm} = \sum_{k \neq i,j} x_k$, the FOC for the merging firms can be written as:

$$\frac{X_{nm} + \epsilon_A}{(X_m + X_{nm} + \epsilon_A)^2}\pi_A - \frac{n - 2 + \epsilon_B - X_{nm}}{(n + \epsilon_B - X_m - X_{nm})^2}\pi_B = 0. \tag{17}$$

This equation defines implicitly the new pseudo best-response of the merging firms after a merger. Let us denote such a best-response function as $\tilde{X}_m(X_{nm})$. The pseudo best-response of the non-merging firms against the merging firms continues to be implicitly defined by equation (14). The crossing point between these pseudo best-response functions gives the post-merger market equilibrium. Let (X_m^*, X_{nm}^*) denote the post-merger equilibrium aggregate investments of merging and non-merging firms.

Comparing the FOCs of the potentially merging firms before and after the merger we can prove that:

Proposition 2. *Suppose firms i and j merge. Then, the merged entity raises investment in project A (and so lowers it in project B) if and only if*

$$\frac{\pi_B}{\epsilon_B} > \frac{\pi_A}{\epsilon_A}. \tag{18}$$

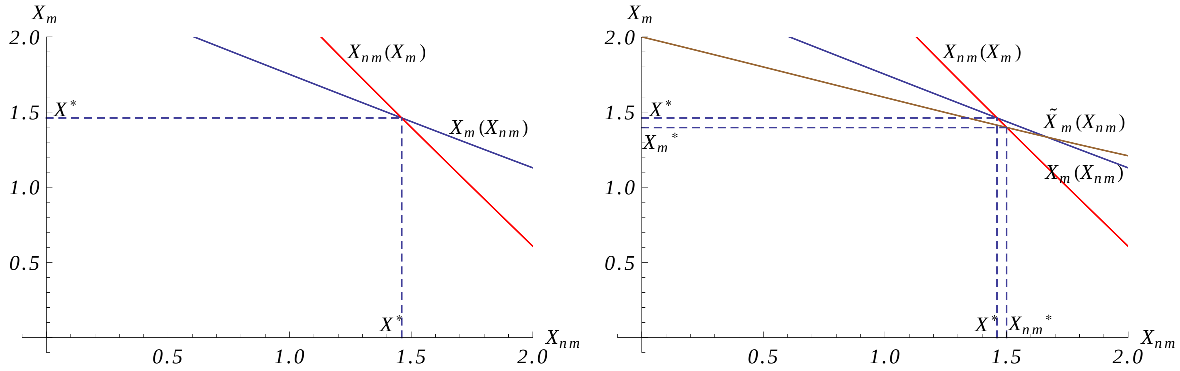
The non-merging firms, by strategic substitutability, reduce investment in project A (and therefore raise it in project B).

As explained in Section 2, when choosing how to adjust the funding allocated to the A- and B-projects, the merged entity internalizes the business-stealing and the business-giving externalities it imposes on the partner firm. The condition $\pi_B/\epsilon_B > \pi_A/\epsilon_A$ governs the relative strength of these two externalities. When π_A is small relative to π_B and/or ϵ_A is large relative to ϵ_B, investing in the A-project is relatively less attractive for the firms than investing in the B-project. This implies that the negative externalitity a firm imposes on another firm when investing in project A is weaker than the positive externality. The merged entity, internalizing these externalities, raises investment in project A and cuts investment in project B.

Using the notion of pseudo best-response functions, we illustrate Proposition 2 in Figure 1(b). Figure 1(b) adds to Figure 1(a) the pseudo best-response function of the merging firms after the merger, which is denoted by $\tilde{X}_m(X_{nm})$. For the chosen parameters (see above), the condition $\pi_B/\epsilon_B > \pi_A/\epsilon_A$ holds, so the best-response function of the merging firms after the merger lies above the best-response function of the merging firms before the merger in a neighbourhood of the pre-merger market equilibrium. As a result, the merged entity puts in a greater effort in the A-project and a lower effort in the B-project. Because the best-replies are decreasing, the non-merging firms do exactly the opposite.

When the parameters of the model violate the condition in the proposition so that $\pi_B/\epsilon_B < \pi_A/\epsilon_A$, the merger results in the opposite result. That is, the merged entity adjusts its investment portfolio by investing less in the A-project (and more in the B-project) and the non-merging firms do the reverse. This situation is illustrated in Figure 2. In this Figure we increase ϵ_B to 2.5; the rest of the parameters remain the same as in Figure 1.[22]

[22] Solving the model numerically, it can be seen that, despite the fact that firms' investments in the A project are strategic substitutes, mergers are incentive-compatible for sufficiently small ϵ_ℓ, $\ell = A, B$. This condition is not necessary in the richer models discussed in Section 4.



(a) Pre-merger (b) Post-merger

Figure 2: Pre- and post-merger market equilibrium ($\pi_B/\epsilon_B < \pi_A/\epsilon_A$)

As we will see later, irrespective of whether we use the consumer surplus or the social welfare standard, our welfare criterion depends on the aggregate investment level. Because investments of merging and non-merging firms move in opposite directions, the question that arises is what happens on aggregate. Our next result addresses this question.

Proposition 3. *Suppose firms i and j merge. Then, the industry-wide investment in project A increases (and so the industry-wide investment in project B decreases) if and only if condition (18) holds.*

Proposition 3 demonstrates that, on aggregate, the innovation portfolio adjustment of the merging firms has a dominating influence over that of the non-merging firms. That is, despite the fact that after a merger the non-merging firms adjust their portfolio of investments in a direction opposite to that of the merging firms, aggregate investment changes in the same direction as the investment of the merging firms. Therefore, the market as a whole will allocate more funding to the A-project (and hence less to the B project) if and only if the parameters of the model satisfy the condition in Proposition 2. The key issue here is that the best-response of the merged entity shifts in order to internalise the externalities the merging partners impose on one another. By contrast, the best-response of the non-merging firms does not change. Hence, the move from the pre- to the post-merger equilibrium is a move along the best-response of the non-merging firms. Because the marginal profits of the non-merging firms are more sensitive to own investment than to the merged entity's investment, their best-response has a slope lower than -1 and the result follows. In a sense, the impact of the changes in the merged entity's investments are of first order importance relative to the changes in the non-merging firms investments. Hence, changes in aggregate investment follow the changes in the merged entity's investments.

Now that we know how the market will adjust investments after a merger, we move on to examine the welfare effects of a merger. In doing so, we adopt a consumer welfare standard.

3.3 Consumer welfare effects of a merger

We define the social welfare criterion as the expected consumer surplus.[23] Using the winning probabilities (7) and (8) in the expression for social welfare (5) and replacing W_ℓ with S_ℓ, $\ell = A, B$, gives:

$$W = \frac{X}{X+\epsilon_A} S_A + \frac{n-X}{n+\epsilon_B - X} S_B, \tag{19}$$

where X is the aggregate industry investment in the A-project and $n - X$ the corresponding aggregate investment in the B-project. Pre-merger, this aggregate investment is equal to nx^*; post-merger, it is equal to $X_m^* + X_{nm}^*$. From Proposition 3 we know that $nx^* < X_m^* + X_{nm}^*$ if and only if the market parameters satisfy the condition $\pi_A/\epsilon_A < \pi_B/\epsilon_B$.

Note that the consumer welfare expression in (19) is strictly concave in X. Therefore, assuming an interior social optimum, the aggregate industry investment in project A that maximises consumer surplus is given by the solution to the FOC:

$$\frac{\epsilon_A}{(X+\epsilon_A)^2} S_A - \frac{\epsilon_B}{(n-X+\epsilon_B)^2} S_B = 0. \tag{20}$$

Let X^o be the consumer surplus maximizing investment portfolio. Solving the FOC for X^o gives:

$$X^o = \frac{n + \epsilon_B - \epsilon_A \sqrt{\frac{S_B \epsilon_B}{S_A \epsilon_A}}}{1 + \sqrt{\frac{S_B \epsilon_B}{S_A \epsilon_A}}}. \tag{21}$$

We say that the market under- (over)-invests in project A when the aggregate investment in such a project falls short of (resp. exceeds) the socially optimal investment. In the pre-merger market equilibrium, under- (over)-investment thus occurs when $nx^* < (>) X^o$. Likewise, in the post-merger market equilibrium under- (over)-investment occurs when $X_m^* + X_{nm}^* < (>) X^o$.

Our next result provides conditions under which the pre-merger market equilibrium investment portfolio is distorted relative to the consumer surplus maximizing investment portfolio. For this purpose, it is convenient to define the function:

$$f\big(S_A/S_B; n, \epsilon_A, \epsilon_B\big) \equiv \frac{S_A}{S_B} \left(\frac{(n-1)(n+\epsilon_A)\sqrt{\frac{S_B \epsilon_A}{S_A \epsilon_B}} + \epsilon_A \Big(1 + n\sqrt{\frac{S_B \epsilon_B}{S_A \epsilon_A}}\Big)}{(n+\epsilon_B)(n-1) + \epsilon_A \Big(n + \sqrt{\frac{S_B \epsilon_B}{S_A \epsilon_A}}\Big)} \right)$$

Proposition 4. *Suppose the social planner maximizes the expected consumer surplus. Then, in the pre-merger market equilibrium, there is under-investment in the A-project (and correspondingly over-investment in the B-project) if and only if:*

$$\frac{\pi_A}{\pi_B} < f\big(S_A/S_B; n, \epsilon_A, \epsilon_B\big). \tag{22}$$

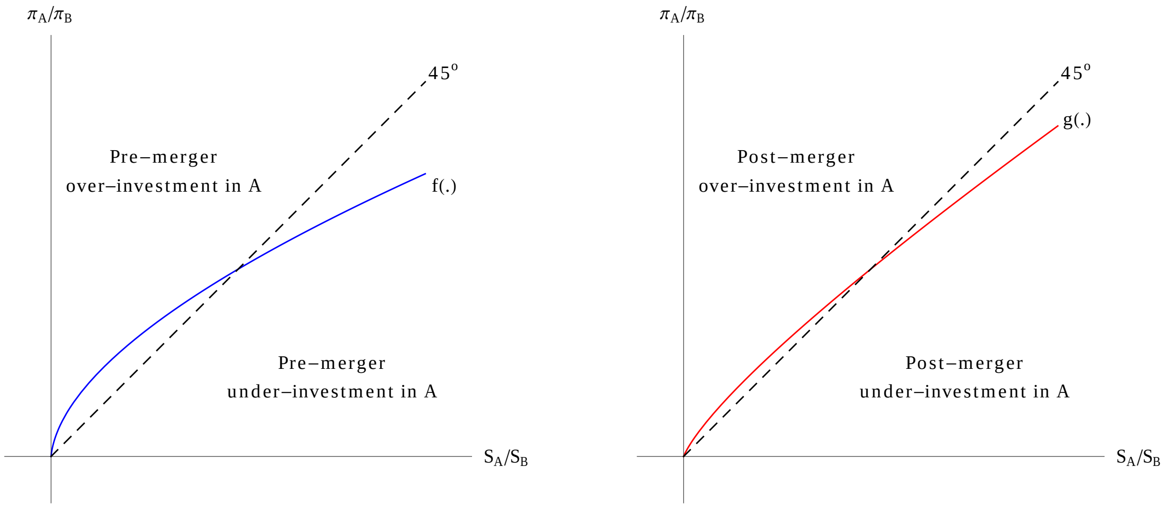
The pre-merger market over-invests in the A-project (and thus under-invests in the B-project) if and only if the above inequality is reversed.

[23] Our results can easily be extended to the alternative notion of social welfare that includes firms' profits and consumer surplus (see Section 2).

Proposition 4 characterizes the nature of the bias of the market investment portfolio in the pre-merger market equilibrium. As we discussed above, there are two sources of inefficiency in the portfolio of investments chosen by the firms. On the one hand, there is an inefficiency associated to competition because firms ignore the business-stealing and business-giving externalities generated by their investment portfolios. On the other hand, firms choose their investment portfolios based on the private profits they get, rather than on the benefits they bring to consumers.

These two sources of inefficiency are nicely captured by condition (22) of Proposition 4. To see this, notice that if we set $n = 1$ in condition (22) we shut down the inefficiency due to rivalry and then the condition only reflects the inefficiency due to the sole business focus of the firms. In fact, when $n = 1$ we have $f(S_A/S_B; n, \epsilon_A, \epsilon_B) = S_A/S_B$, which is the 45^o line in Figure 3(a), and the condition in the Proposition simplifies to $\pi_A/\pi_B < S_A/S_B$. Written as $\pi_A/S_A < \pi_B/S_B$, the Proposition then implies that a monopolist under-invests in the A-project so long as the share of total surplus the firm appropriates in the market created by the A-project is lower than that in the market created by the B-project.[24]

The insight that the market tends to allocate too little funding to the A-project and too much to the B-project when $\pi_A/S_A < \pi_B/S_B$ also applies to the case of oligopoly. In fact, we note that the function f is increasing and concave in S_A/S_B and takes on value zero when S_A/S_B is zero. Therefore, given the other parameters of the model, the market invests too much in the A-project whenever π_A/π_B is sufficiently large compared to S_A/S_B. To illustrate, Figure 3(a) represents the regions of parameters for which there is over- and under-investment in the A-project. (Parameters are set to $n = 3$, $\epsilon_A = 3/2$ and $\epsilon_B = 3/4$.)



(a) Pre-merger (b) Post-merger

Figure 3: Inefficiency of the market equilibrium

The effect of competition, n, on the market bias towards the more appropriable project is hard to disentangle analytically but numerical analysis of the RHS of condition (22) reveals that it depends on the ratio of consumer surpluses S_A/S_B. When S_A/S_B is small, then an increase in the number of firms tends to make the overinvestment problem in the more profitable project more likely. When S_A/S_B is

[24]Note that the condition $\pi_A/\pi_B > S_A/S_B$ is exactly equivalent to the condition $\pi_A/\pi_B > W_A/W_B$.

large, the result is opposite.

Following the same approach as above, we can derive conditions under which the post-merger market equilibrium investment portfolio is biased relative to the consumer surplus maximizing investment portfolio. For that purpose, consider the system of the FOCs of the non-merging and merging firms, equations (14) and (17). Summing them up we get:

$$\frac{\frac{n-2}{n-1}(X_m + X_{nm})\epsilon_A}{(X_m + X_{nm} + \epsilon_A)^2}\pi_A - \frac{\frac{n-2}{n-1}(n - X_m - X_{nm}) + \epsilon_B}{(n - X_m - X_{nm} + \epsilon_B)^2}\pi_B = 0 \tag{23}$$

Comparing this equation with the FOC that gives the socially optimal investment portfolio, our next result provides conditions under which the post-merger market equilibrium investment portfolio is distorted relative to the consumer surplus maximizing investment portfolio. For that purpose, we define the function:

$$g\big(S_A/S_B; n, \epsilon_A, \epsilon_B\big) \equiv \frac{S_A}{S_B}\left(\frac{(n-2)(n+\epsilon_A)\sqrt{\frac{S_B\epsilon_A}{S_A\epsilon_B}} + \epsilon_A\Big(1 + (n-1)\sqrt{\frac{S_B\epsilon_B}{S_A\epsilon_A}}\Big)}{(n+\epsilon_B)(n-2) + \epsilon_A\Big(n - 1 + \sqrt{\frac{S_B\epsilon_B}{S_A\epsilon_A}}\Big)}\right).$$

Proposition 5. *Suppose the social planner maximizes the expected consumer surplus. Then, in the post-merger market equilibrium, there is under-investment in the A-project and correspondingly over-investment in the B-project if and only if:*

$$\frac{\pi_A}{\pi_B} < g\big(S_A/S_B; n, \epsilon_A, \epsilon_B\big). \tag{24}$$

The post-merger market over-invests in the A-project and under-invests in the B-project if and only if the above inequality is reversed.

Proposition 5 shows that the post-merger market equilibrium investment portfolio will also in general deviate from the consumer surplus maximizing one due to competition and the sole business focus of the firms. Similarly to the pre-merger market equilibrium, if we set $n = 2$ in condition (24), we shut down the competition effect and we get that firms overinvest in the A-project (and hence underinvest in the B-project) so long as $\pi_A/S_A < \pi_B/S_B$. Note that for $n = 2$, $g(\cdot) = S_A/S_B$, which is the 45^o line in Figure 3(b). For arbitrary n, the condition in (24) is similar to that in (22): the RHS of (24) is an increasing and concave function of S_A/S_B, dividing the space of parameters π_A/π_B–S_A/S_B into a region for which there is over-investment in the A-project and a region for which there is under-investment. We illustrate this in Figure 3(b).

We are now ready to address the question whether a merger increases consumer surplus or not. Suppose, for example, that the parameters of the model are such that both pre-merger and post-merger there is under-investment in the A-project and hence over-investment in the B-project. If after a merger firms collectively increase their aggregate investment in the A-project, which, as shown in Proposition 3, happens when $\pi_A/\epsilon_A < \pi_B/\epsilon_B$, then a merger is welfare improving.

Thus, putting together Figures 3(a), 3(b) and the condition $\pi_A/\epsilon_A < \pi_B/\epsilon_B$, we can easily evaluate the welfare effects of a merger. We do this in Figure 4. Pairs of innovations above both curves f and g lead to over-investment in the A-project and hence under-investment in the B-project, both in the pre-and post-merger market equilibrium. Likewise, pairs of innovations below the curves f and g exhibit under-investment in the A-project and over-investment in the B- project. For pairs of innovations that are above one curve, but below the other curve, a merger changes the market distortion. Specifically, for pairs of innovations in between the two curves f and g and to the left of the crossing point between f and g there is under-investment in the A-project pre-merger but over-investment in the same project post-merger. The opposite happens to the right of the crossing point.

The horizontal line with intercept ϵ_A/ϵ_B divides the space into two regions. Above the line, project A is relatively more profitable than project B and, after a merger, aggregate investment in project A decreases. The opposite holds below the line. As a result, we can split the space of parameters into four main regions. These regions are depicted in Figure 4.

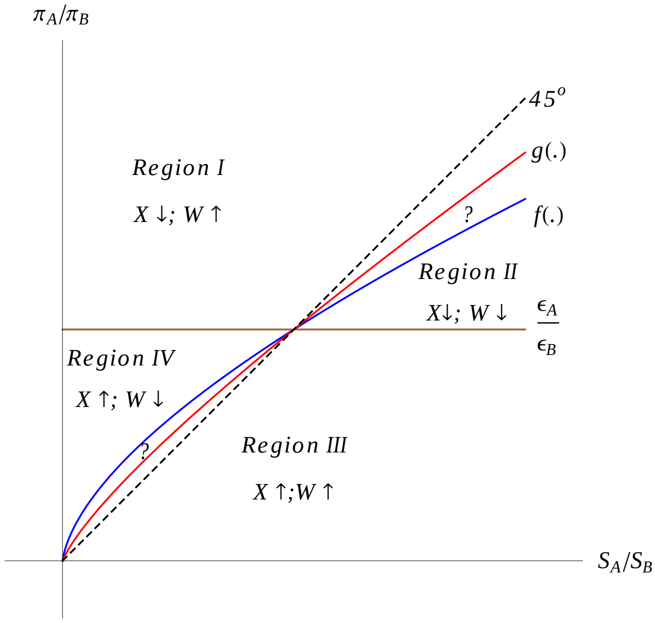


Figure 4: Consumer welfare effects of mergers

In Region I, the parameters satisfy the inequalities:

$$\text{Region I:} \qquad \frac{\pi_A}{\pi_B} > \frac{\epsilon_A}{\epsilon_B} \text{ and } \frac{\pi_A}{\pi_B} > g(\cdot).$$

For this region of parameters, both pre-merger and post-merger, there is over-investment in the A-project. Because the A-project is relatively more profitable than the B-project, by Proposition 3, aggregate investment in the A project post-merger is lower than pre-merger. This implies that in Region I consumer welfare post-merger is higher than pre-merger, hence a merger is beneficial for consumers.

In Region II, the parameters satisfy the inequalities:

$$\text{Region II:} \qquad \frac{\pi_A}{\pi_B} > \frac{\epsilon_A}{\epsilon_B} \text{ and } \frac{\pi_A}{\pi_B} < f(\cdot).$$

In this region, investment in the A-project is insufficient while it is excessive in the B-project. By Proposition 3, because the A-project is relatively more profitable than the B-project, investment in the A-project post-merger is lower than pre-merger. This aggravates the situation and post-merger welfare is thus lower than pre-merger.

In Region III, the parameters satisfy the inequalities:

$$\text{Region III:} \qquad \frac{\pi_A}{\pi_B} < \frac{\epsilon_A}{\epsilon_B} \text{ and } \frac{\pi_A}{\pi_B} < g(\cdot).$$

As in the previous parameter set, in this region the market allocates too little funding to the A-project and too much to the B-project, both before and after a merger. However, because the A-project is relatively less profitable than the B-project, by Proposition 3, investment in the A-project after a merger is higher than pre-merger. As a result, a merger increases consumer welfare.

Finally, the parameters in Region IV satisfy the inequalities:

$$\text{Region IV:} \qquad \frac{\pi_A}{\pi_B} < \frac{\epsilon_A}{\epsilon_B} \text{ and } \frac{\pi_A}{\pi_B} > f(\cdot).$$

For these parameters, investment in the A-project is excessive and hence insufficient in the B-project. Because the A-project is relatively less profitable than the B-project, investment in the A-project after a merger is higher than pre-merger. This aggravates the misallocation problem and so a merger decreases consumer welfare.

The following result summarises our findings:

Proposition 6. *Suppose the social planner maximizes the expected consumer surplus. For any fixed* n, ϵ_A *and* ϵ_B*, if the parameters of the model fall in:*
(i) Region I (resp. III): a merger increases welfare by reducing (resp. increasing) investment in project A and increasing (resp. decreasing) it in project B.
(ii) Region II (resp. IV): a merger decreases welfare by further reducing (resp. increasing) investment in project A and increasing (resp. reducing) it in project B.

We have not yet described the effects of a merger when the parameters of the model satisfy the inequalities:

$$\text{Ambiguity regions:} \qquad f(\cdot) > \frac{\pi_A}{\pi_B} > g(\cdot) \text{ and } f(\cdot) < \frac{\pi_A}{\pi_B} < g(\cdot).$$

When the parameters satisfy the first inequality and a merger occurs, the market evolves from suffering an under-investment in the A-project problem to an over-investment in the A-project problem. When the parameters satisfy the second inequality, it is the other way around. Without specifying further the values of the parameters, for these ambiguity regions it is not possible to say whether welfare post-merger is higher or lower than pre-merger.

The analysis above yields a clear policy message:

Corollary 1. *When the project that is relatively more profitable for the firms is also the one that appropriates a larger fraction of the social welfare, then a merger increases consumer surplus by reducing investment in the more profitable project and increasing investment in the alternative (less profitable) project. Otherwise, if the project that is relatively more profitable appropriates a lower fraction of social welfare, then a merger reduces consumer surplus.*

It is useful to provide a couple of examples to illustrate the power of Proposition 6.

Example 1: ρ-linear demand and constant marginal cost. Suppose the winning firms face ρ-linear demands in markets A and B, and constant marginal costs. The family of ρ-linear demands is $Q(p) = [1 + \rho(a - bp)]^{1/\rho}$, $a, b, \rho > 0$ (see Anderson and Renault, 2003). The parameter ρ captures the curvature of demand. When $0 < \rho < 1$ demand is convex, when $\rho = 1$ demand is linear and when $\rho > 1$ demand is concave. Standard derivations show that $\pi/S = 1+\rho$, indicating that consumer surplus appropriability increases as demand becomes more concave. It is then straightforward to conclude that when $\pi_A/\epsilon_A > (<)\pi_B/\epsilon_B$ and $\rho_A \geqslant (\leqslant)\rho_B$ a merger increases consumer welfare by reducing (raising) investment in A and increasing (decreasing) it in B. Likewise, when $\pi_A/\epsilon_A > (<)\pi_B/\epsilon_B$ and ρ_A is sufficiently small (large) compared to ρ_B, a merger decreases consumer welfare by reducing (raising) investment in A and increasing (decreasing) it in B. The case of linear demands is special in that $\rho_A = \rho_B = 1$. In that case, all the possible parameters lie in the 45 degrees line of Figure 4. This implies that with linear demands, mergers always increase consumer surplus.

Example 2: Isoelastic demand and constant marginal cost. Suppose the winning firms face constant elasticity demands in markets A and B, and constant marginal costs. The family of isoelastic demands is $Q = Kp^{-\eta}$, where $K > 0$, and $\eta > 1$. The parameter η is the elasticity of demand. Standard derivations show that $\pi/S = 1 - 1/\eta$. It is then straightforward to conclude that when $\pi_A/\epsilon_A > (<)\pi_B/\epsilon_B$ and $\eta_A \geqslant (\leqslant)\eta_B$ a merger increases consumer welfare by reducing (raising) investment in A and increasing (decreasing) it in B. Likewise, when $\pi_A/\epsilon_A > (<)\pi_B/\epsilon_B$ and η_A is sufficiently small (large) compared to η_B, a merger decreases consumer welfare by reducing (raising) investment in A and increasing (decreasing) it in B.

4 Extensions

In this section, we present a number of extensions of our model in Section 3. In particular, we consider the case of non-linear success probabilities, asymmetric firms, endogenous research budgets and strategic complements.

4.1 Non-linear Tullock

In the model of Section 3 we have considered the case of "linear" success probabilities. The linearity of the contest success functions has generated tractability for two main reasons. First, it has made the problems of the merged entity and the social planner unidimensional and, second, it has allowed us to derive the welfare effects of mergers by simply comparing pre- and post-merger aggregate investments in projects A and B with the socially optimal aggregate investments. Consider now that

$$p_i^A(x_i, \mathbf{x}_{-i}, \epsilon_A) = \frac{x_i^\beta}{\sum_{k=1}^n x_k^\beta + \epsilon_A},$$

$$p_i^B(1 - x_i, \mathbf{1} - \mathbf{x}_{-i}, \epsilon_B) = \frac{(1 - x_i)^\beta}{\sum_{k=1}^n (1 - x_k)^\beta + \epsilon_B}, \ i = 1, 2, ..., n,$$

where the parameter $\beta > 0$ captures the degree of non-linearity. When $0 < \beta < 1$ the success probabilities exhibit decreasing returns, while when $\beta > 1$ there are increasing returns. The case of constant returns, $\beta = 1$, is the one we have analyzed in Section 3. Unfortunately, when $\beta \neq 1$ the model becomes non-tractable for a complete set of analytical results. Nevertheless, we can report a number of observations.

We start with the case in which $0 < \beta < 1$. This case satisfies our Assumption 2 and a result similar to Proposition 1 on the existence and uniqueness of an interior pre-merger equilibrium can readily be established. More importantly, numerical results show that when $0 < \beta < 1$ the merged entity prefers to maintain the research facilities of the partner firms running. In such a case, the payoff of the merged entity in the post-merger market is given by:

$$u_m(x_i, x_j; \cdot) = \frac{x_i^\beta + x_j^\beta}{\sum_{k=1}^{n} x_k^\beta + \epsilon_A}\pi_A + \frac{(1 - x_i)^\beta + (1 - x_j)^\beta}{\sum_{k=1}^{n} (1 - x_k)^\beta + \epsilon_B}\pi_B, \tag{25}$$

Proposition 2 can then readily be extended and condition (18) is replaced by the similar condition:

$$\frac{\pi_B}{\epsilon_B^{1/\beta}} > \frac{\pi_A}{\epsilon_A^{1/\beta}}. \tag{26}$$

For details see the Appendix. Unfortunately, Proposition 3 cannot easily be extended in this setting because there is no natural notion of aggregate investment and the welfare analysis is more difficult because of the same reason. Nevertheless, numerical results show that our main results hold. In particular, mergers may be profitable and consumer surplus improving under similar conditions as in the case of Section 3.

Consider now the case in which $\beta > 1$. Here we must distinguish among two sub-cases. Suppose first that the success probabilities exhibit sufficiently large increasing returns (β much greater than 1). Then, the pre-merger market equilibrium is no longer interior. This means that firms only invest in a single project and our portfolio model then becomes uninteresting. Suppose now that the success probabilities exhibit increasing returns, but not large (β greater but close to 1). In this case the pre-merger equilibrium is interior, but the merged entity's payoff is no longer the expression in (25) because the merged entity prefers to shut down the research facility of one the constituent firms. This result is in line with Denicolò and Polo (2018). In such a case, the merged entity does maximize the payoff:

$$u_m(x_i, x_j; \cdot) = \frac{(x_i + x_j)^\beta}{(x_i + x_j)^\beta + \sum_{k \neq i,j} x_k^\beta + \epsilon_A}\pi_A + \frac{(2 - x_i - x_j)^\beta}{(2 - x_i - x_j)^\beta + \sum_{k \neq i,j} (1 - x_k)^\beta + \epsilon_B}\pi_B.$$

In this situation, in addition to the portfolio effects of mergers, there are also *efficiency gains* due to the shutting down of one of the research facilities. As expected, efficiency gains make mergers even more profitable and enlarge the set of parameters under which consumer surplus increases.

4.2 Asymmetric firms

In the model of Section 3, we have assumed that firms are symmetric in every respect, including their endowed research budget. In this section, we report on the case in which firms vary in their research budgets. Let y_i denote the research budget of a firm i, $i = 1, 2, ..., n$. For the proofs of the claims we make in this section see the Appendix.

In the pre-merger market, a firm i with budget y_i chooses its investment portfolio to maximize:

$$u_i(x_i, \mathbf{x}_{-i}) = \frac{x_i}{\sum_{k=1}^{n} x_k + \epsilon_A}\pi_A + \frac{y_i - x_i}{\sum_{k=1}^{n}(y_k - x_k) + \epsilon_B}\pi_B, \quad i = 1, 2, ..., n. \tag{27}$$

To ensure that our game of innovation portfolio choice is one of strategic substitutes, we assume that $\epsilon_\ell \geqslant \max\{y_1, y_2 ... y_n\}$ where $\ell \in \{A, B\}$. There exists a unique Nash equilibrium of the portfolio investment game denoted by $\mathbf{x}^* = \{x_1^*, x_2^*, ..., x_n^*\}$ that satisfies the FOCs:

$$\frac{\sum_{k\neq i} x_k^* + \epsilon_A}{(x_i^* + \sum_{k\neq i} x_k^* + \epsilon_A)^2}\pi_A - \frac{\sum_{k\neq i}(y_k - x_k^*) + \epsilon_B}{(y_i - x_i^* + \sum_{k\neq i}(y_k - x_k^*) + \epsilon_B)^2}\pi_B = 0 \quad i = 1, 2, ..., n. \tag{28}$$

The above system of FOCs can be re-arranged so as to show that for any $i, j = 1, 2, ..., n$, $i \neq j$:

$$x_j^* = \lambda_{ji}^1 + \lambda_{ji}^2 x_i^* \quad \text{and} \quad y_j - x_j^* = \lambda_{ji}^1 + \lambda_{ji}^2(y_i - x_i^*), \tag{29}$$

where

$$\lambda_{ji}^1 = \frac{(y_j - y_i)\epsilon_A}{(n-1)y_i + \epsilon_A + \epsilon_B} \quad \text{and} \quad \lambda_{ji}^2 = \frac{(n-1)y_j + \epsilon_A + \epsilon_B}{(n-1)y_i + \epsilon_A + \epsilon_B}.$$

This implies that the difference in the investments of two arbitrary firms i and j depends on the difference in the budgets of only these two firms. Moreover, firms with larger budgets invest more in both projects A and B. Lastly, this equilibrium $\mathbf{x}^*$ is interior, i.e. $y_i > x_i^* > 0$ for all $i \in N$, only if the parameters of the model satisfy the inequality

$$\underline{\pi} < \frac{\pi_A}{\pi_B} < \bar{\pi}, \tag{30}$$

where the exact values of $\underline{\pi}$ and $\overline{\pi}$ are given in the Appendix. We assume that this condition is satisfied for the remainder of this section.

As in Section 3, when two firms i and j merge, the merged entity chooses X_m to maximize:

$$u_m(x_i, x_j, \mathbf{x}_{-i,j}) = \frac{X_m}{X_m + \sum_{k\neq i,j} x_k + \epsilon_A}\pi_A + \frac{y_i + y_j - X_m}{y_i + y_j - X_m + \sum_{k\neq i,j}(y_k - x_k) + \epsilon_B}\pi_B. \tag{31}$$

The non-merging firms continue to maximize the payoff in (27). The FOC for the maximization of the profits of the merged entity with respect to X_m is:

$$\frac{\sum_{k\neq i,j} x_k + \epsilon_A}{(X_m + \sum_{k\neq i,j} x_k + \epsilon_A)^2}\pi_A - \frac{\sum_{k\neq i,j}(y_k - x_k) + \epsilon_B}{(y_i + y_j - X_m + \sum_{k\neq i,j}(y_k - x_k) + \epsilon_B)^2}\pi_B = 0. \tag{32}$$

Evaluating this FOC at the pre-merger market equilibrium as we did in Section 3, we can show that the merged entity will increase its investment in project A (and decrease it in project B) if and only if $y_i\epsilon_A - x_i^*(\epsilon_A + \epsilon_B) > 0$, which holds under the condition in Proposition 2. Although the magnitudes of the business-stealing and the business-giving externalities a firm i causes on its partner firm j depend on the budget of firm i, whether the business-stealing or the business-giving externality has a dominant influence does not. Therefore, Proposition 2 does not change, that is, no matter the identity of the merging firms, the merged entity's investment in project A increases (and that in project B decreases) if and only if project A is relatively less profitable than project B. Proposition 3 also remains unchanged.

The welfare implications of mergers are also similar. Note that the socially optimal portfolio of investments follows from maximizing

$$W(x_i, \mathbf{x}_{-i}) = \frac{X}{X + \epsilon_A} S_A + \frac{Y - X}{Y - X + \epsilon_B} S_B \quad \text{where} \quad X = \sum_{j=1}^{n} x_j \quad \text{and} \quad Y = \sum_{i=1}^{n} y_i.$$

Taking the FOC with respect to X and solving gives:

$$X^o = \frac{Y + \epsilon_B - \epsilon_A \sqrt{\frac{S_B \epsilon_B}{S_A \epsilon_A}}}{1 + \sqrt{\frac{S_B \epsilon_B}{S_A \epsilon_A}}}. \tag{33}$$

Following the same steps as in Section 3, that is, evaluating the marginal profits of the firms in the pre- and post-merger market at the consumer surplus maximizing investment portfolio, we show that firms under-invest in project A (and correspondingly over-invest in project B) in the pre- and post-merger market equilibria if and only if, respectively, the following inequalities are satisfied:

$$\frac{\pi_A}{\pi_B} < f_a(S_A/S_B; n; \pi_A; \pi_B; Y) \quad \text{and} \quad \frac{\pi_A}{\pi_B} < g_a(S_A/S_B; n; \pi_A; \pi_B; Y) \tag{34}$$

Firms over-invest in the A-project (and thus under-invest in the B-project) in the pre-merger and post-merger market equilibria if and only if, respectively, the inequalities are reversed. Note that with asymmetric budgets the condition that determines over/under investment involves the industry's aggregate budget. If the budget is symmetric, i.e $y_i = 1$ for all $i \in N$, then $Y = n$.

4.3 Endogenous research budget

So far we have restricted the analysis to a situation where firms are research-budget constrained. This restriction has served to focus our discussion solely on the innovation portfolio effects of mergers. In this section, we allow the firms to decide not only how they allocate their research budgets across the two projects but also how much to invest in total. We next show that when the research budget is endogenous and the cost of research effort is convex, a similar tension as in the main model of Section 3 exists between investment in project A and investment in project B. Relatedly, we show that we get our main model in the limit when the degree of convexity goes to infinity. Furthermore, despite the fact that a merger results in less aggregate investment, we give conditions under which the merged entity tends to increase investment in A (B) and decrease it in B (A), or decrease investment in both projects.

Let x_i be the investment of a firm i in the A-project, and y_i its investment in the B-project. The total research effort of the firm is thus $x_i + y_i$. Let the cost of research effort be denoted $c(x_i + y_i)$ and assume $c', c'' > 0$; specifically, it will be instructive to use the parametric cost function $c(x_i + y_i) = \frac{1}{\alpha}(x_i + y_i)^\alpha$, with $\alpha > 1$.

Let $(\mathbf{x}^*, \mathbf{y}^*)$ denote the pre-merger market symmetric equilibrium. The characterization of the pre-merger market symmetric equilibrium follows the same steps as in our main model. Given the other firms' investments, a firm i chooses x_i and y_i to maximize the payoff:

$$u_i(x_i, y_i; \mathbf{x}^*_{-i}, \mathbf{y}^*_{-i}) = \frac{x_i}{x_i + (n-1)x^* + \epsilon_A}\pi_A + \frac{y_i}{y_i + (n-1)y^* + \epsilon_B}\pi_B - \frac{1}{\alpha}(x_i + y_i)^{\alpha}. \tag{35}$$

At an interior equilibrium, (x_i, y_i) must satisfy the FOCs for the maximization of firm i's profits:

$$\frac{(n-1)x^* + \epsilon_A}{(x_i + (n-1)x^* + \epsilon_A)^2}\pi_A = \frac{(n-1)y^* + \epsilon_B}{(y_i + (n-1)y^* + \epsilon_B)^2}\pi_B = (x_i + y_i)^{\alpha-1}. \tag{36}$$

Recall that in the main model of the paper budget-constrained firms choose the optimal portfolio to equalize the marginal profits across innovations. When the firms are not budget-constrained, in addition to this requirement, the marginal profits from either project must equal the marginal cost of investment.[25]

The trade-off that a firm faces when deciding how much to invest in either project can be seen by examining the implicit relation, denoted $y_i(x_i)$, between investment in the B-project and investment in the A-project given by the second FOC in (36). Applying the implicit function theorem to such a relation, and simplifying, gives:

$$\frac{\partial y_i}{\partial x_i} = -\frac{\alpha - 1}{\frac{2(x_i+y_i)}{y_i+(n-1)y^*+\epsilon_B} + (\alpha - 1)}$$

Observe that for convex cost functions ($\alpha > 1$), everything else constant, an increase in investment in the A-project is paired with a decrease in investment in the B-project. This, which is in line with the main model of our paper, is quite intuitive because when the cost function is convex, an increase in investment in the A-project raises the marginal cost of investing in the B-project. In fact, in the limit when $\alpha \to \infty$ we get $\partial y_i/\partial x_i = -1$ no matter the strategy of the rest of the firms, which brings us back to the setting in the main model of our paper where, by assumption, we had $y_i = 1 - x_i$. This derivative also shows that when $\alpha \to 1$ the projects become independent of one another, i.e. $\partial y_i/\partial x_i = 0$.

Consider now that firms i and j merge. Let (x^*_{nm}, y^*_{nm}) denote the investments of each non-merging firm in the post-merger market equilibrium. Taking as given (x^*_{nm}, y^*_{nm}), and assuming for the moment that the merged entity keeps the research facilities of the constituent firms running, in the post-merger market equilibrium the merged entity chooses (x_i, y_i, x_j, y_j) to maximize the payoff:

$$\begin{aligned} u_m(x_i, y_i, x_j, y_j; \mathbf{x}^*_{nm}, \mathbf{y}^*_{nm}) = &\frac{x_i + x_j}{x_i + x_j + (n-2)x^*_{nm} + \epsilon_A}\pi_A + \frac{y_i + y_j}{y_i + y_j + (n-2)y^*_{nm} + \epsilon_B}\pi_B \\ &- \frac{1}{\alpha}(x_i + y_i)^{\alpha} - \frac{1}{\alpha}(x_j + y_j)^{\alpha}. \end{aligned} \tag{37}$$

At an interior equilibrium, the merged entity's investments must satisfy the FOCs for the maximization

[25] An analysis of the 2x2 Hessian matrix corresponding to the payoff in (35) reveals that the payoff function of a firm i is strictly concave in (x_i, y_i) for convex cost functions (for details see the Appendix). This means that there exists a unique SNE. The case of linear cost functions ($c'' = 0$) is special in that the problem of a firm is separable across projects. This means that the multi-project firm problem is not different from the single-project problem.

of its profits with respect to (x_i, y_i, x_j, y_j), which are given by:

$$\begin{aligned}
\frac{\partial u_m}{\partial x_i} &= \frac{(n-2)x^*_{nm} + \epsilon_A}{(x_i + x_j + (n-2)x^*_{nm} + \epsilon_A)^2}\pi_A - (x_i + y_i)^{\alpha-1} = 0 \\
\frac{\partial u_m}{\partial y_i} &= \frac{(n-2)y^*_{nm} + \epsilon_B}{(y_i + y_j + (n-2)y^*_{nm} + \epsilon_B)^2}\pi_B - (x_i + y_i)^{\alpha-1} = 0 \\
\frac{\partial u_m}{\partial x_j} &= \frac{(n-2)x^*_{nm} + \epsilon_A}{(x_i + x_j + (n-2)x^*_{nm} + \epsilon_A)^2}\pi_A - (x_j + y_j)^{\alpha-1} = 0 \\
\frac{\partial u_m}{\partial y_j} &= \frac{(n-2)y^*_{nm} + \epsilon_B}{(y_i + y_j + (n-2)y^*_{nm} + \epsilon_B)^2}\pi_B - (x_j + y_j)^{\alpha-1} = 0
\end{aligned} \tag{38}$$

It is straightforward to see that profits maximization requires the merged entity to invest an equal amount of funds in each of its research units, i.e. $x_i + y_i = x_j + y_j$. Using this observation, an analysis of the 4x4 Hessian matrix corresponding to the payoff of the merged entity in (37) reveals that it is negative semidefinite for convex cost functions (for details see the Appendix). Hence, the merged entity's objective function is concave, the solution to the system of FOCs in (38) is a local maximum and the merged entity indeed keeps both its research facilities running i.e. $x_i + y_i = x_j + y_j > 0$.[26]

Inspection of the system of FOCs in (38) reveals that one of the equations is redundant. This implies that the system of FOCs is underdetermined and that it has multiple solutions. We have analyzed the system of FOCs numerically and observed that the symmetric solution gives the same payoff as any other non-symmetric solution. Hence, from now on, we focus on the symmetric solution. Let (x^*_m, y^*_m) denote the symmetric post-merger market equilibrium investments of each of the merging firms. This symmetric portfolio of investments has to satisfy the following FOCs:

$$\frac{(n-2)x^*_{nm} + \epsilon_A}{(2x^*_m + (n-2)x^*_{nm} + \epsilon_A)^2}\pi_A - (x^*_m + y^*_m)^{\alpha-1} = 0, \tag{39}$$

$$\frac{(n-2)y^*_{nm} + \epsilon_B}{(2y^*_m + (n-2)y^*_{nm} + \epsilon_B)^2}\pi_B - (x^*_m + y^*_m)^{\alpha-1} = 0. \tag{40}$$

Meanwhile, the FOCs corresponding to the payoffs of the non-merging firms are:

$$\frac{2x^*_m + (n-3)x^*_{nm} + \epsilon_A}{(2x^*_m + (n-2)x^*_{nm} + \epsilon_A)^2}\pi_A = (x^*_{nm} + y^*_{nm})^{\alpha-1}, \tag{41}$$

$$\frac{2y^*_m + (n-3)y^*_{nm} + \epsilon_B}{(2y^*_m + (n-2)y^*_{nm} + \epsilon_B)^2}\pi_B = (x^*_{nm} + y^*_{nm})^{\alpha-1}. \tag{42}$$

A symmetric equilibrium of the post-merger portfolio investment game with endogenous budget is a solution to the system of four equations (39)-(42).

A complete analytical comparison between the pre- and post-merger equilibrium is beyond the scope of this extension section. However, following Jullien and Lefouili (2018) and Federico *et al.* (2018), we can examine the so-called *initial impetus* of a merger, which captures the change in the behaviour of

[26]When $\alpha < 1$ the cost function is concave and the Hessian matrix of the payoff is no longer negative semidefinite and, as in Denicolò and Polo (2018), the merged entity prefers to shut down the research facility of one of the merging firms. Intuitively, when the cost function is sufficiently concave, cost synergies imply that it is cost-efficient to shut down one of the research facilities and concentrate research efforts in the remaining lab. When $\alpha = 1$, the merged entity is indifferent between keeping the two research facilities running and closing down one of them.

the merged entity under the assumption that the behaviour of non-merging firms is not affected by the merger. A study of the initial impetus of a merger provides useful information because the change in the behaviour of the non-merging firms is a second-order effect relative to the initial impetus and hence unlikely to reverse it. (We have confirmed this for our model solving it numerically.)

Using the same procedure as in Proposition 2 we now show that, keeping constant the investment portfolio of the non-merging firms at the pre-merger market equilibrium, the merged entity reduces its aggregate investment and, depending on parameters, either reduces investment in project A and increases it in project B, or increases investment in project A and reduces it in project B, or decreases investment in each of the projects.

Proposition 7. *In the model with endogenous research budget, there exists a unique symmetric pre-merger market equilibrium* $(\mathbf{x}^*, \mathbf{y}^*)$ *which is given by the solution to (36) after setting* $x_i = x^*$ *and* $y_i = y^*$*. Further, keeping the behaviour of the non-merging firms fixed, if a merger occurs the merged entity's portfolio of investments* (x_m^*, y_m^*) *is such that aggregate investment decreases, i.e.* $x_m^* + y_m^* < x^* + y^*$*. Furthermore, if the parameters of the model satisfy the condition:*

(i) $\frac{\pi_A}{\pi_B} > \frac{(nx^*+\epsilon_A)^2((n-2)y^*+\epsilon_B)}{((n-2)x^*+\epsilon_A)(2y_m^2(x^*)+(n-2)y^*+\epsilon_B)^2}$, *then the merged entity increases investment in A and decreases it in B;*

(ii) $\frac{((n-2)y^*+\epsilon_B)(2x_m^1(y^*)+(n-2)x^*+\epsilon_A)^2}{((n-2)x^*+\epsilon_A)(ny^*+\epsilon_B)^2} > \frac{\pi_A}{\pi_B}$, *then the merged entity decreases investment in A and increases it in B;*

(iii) otherwise, the merged entity decreases investment in both projects,

where $x_m^1(y^*)$ *solves the FOC (39) after setting* $y_m^* = y^*$ *and* $y_m^2(x^*)$ *solves the FOC (40) after setting* $x_m^* = x^*$*.*

This result shows that the insights from Proposition 2 on how the merged entity adjusts its portfolio of investments remain valid when the firms can also decide on the level of their investments. Moreover, in the limit when $\alpha \to \infty$ we have $y_m^1(x^*), y_m^2(x^*), y^* \to 1 - x^*$ so that the thresholds in Proposition 7(i) and (ii) coincide and equal

$$\frac{(nx^* + \epsilon_A)^2((n-2)(1-x^*) + \epsilon_B)}{((n-2)x^* + \epsilon_A)(n(1-x^*) + \epsilon_B)^2} = \frac{((n-1)x^* + \epsilon_A)((n-2)(1-x^*) + \epsilon_B)\pi_A}{((n-2)x^* + \epsilon_A)((n-1)(1-x^*) + \epsilon_B)\pi_B},$$

where the equality follows from using the FOC (12). Therefore, condition in Proposition 7(i) becomes

$$1 > \frac{((n-1)x^* + \epsilon_A)((n-2)(1-x^*) + \epsilon_B)}{((n-2)x^* + \epsilon_A)((n-1)(1-x^*) + \epsilon_B)},$$

which holds whenever $x^* < \epsilon_A/(\epsilon_A + \epsilon_B)$, which is exactly the condition in Proposition 2 (for details, see the proof of Proposition 2). In conclusion, for α sufficiently large, the main results of our paper hold. Moreover, numerical results show that even if α is small so that a merger results in a lower aggregate investment (as per the innovation theory of merger harm), the innovation portfolio effects of mergers may dominate and still make a merger socially desirable.[27]

[27] The numerical analysis also suggests that the region of parameters for which mergers are profitable and socially desirable enlarges as α decreases.

4.4 Price effects of mergers: a model of quality innovation and quantity competition

So far, we have examined the portfolio effects of mergers in a two-period model with winner-take-all contests. In the first period, the *pre-innovation* market, firms chose investment efforts for the A- and B-projects. In the second period, the *post-innovation market*, after the outcome of the firms' research efforts became known, the winning firm in each market put its product on sale and obtained monopoly profits. The simplicity of the model has served to isolate the novel issues that arise when examining firms' investment portfolios at the expense of ignoring other relevant aspects of innovation and mergers. In particular, and most importantly, the winner-take-all contest formulation has *de facto* ruled out any price effect of mergers (Salant *et al.*, 1983; Davidson and Deneckere, 1985).

In order to demonstrate that the portfolio effects of mergers can have a dominating influence over the usual and well understood price effects of mergers, we now extend our model to a richer situation in which firms compete in quantities to sell differentiated products. To be precise, we adopt the well-known model of vertical and horizontal product differentiation of Sutton (1997, 1998), further studied e.g. by Symeonidys (2003). To serve our purpose, it will be sufficient to assume that there is competition in one of the markets.

Specifically, we examine the following two-stage game:

- In stage 1, firms engage in the research contests for the A- and B-innovations. In market A, the winning firm produces a good of high quality; the losing firms produce goods of low quality. If no firm succeeds, all firms produce goods of low quality. The market for the B-innovation continues to be a winner-take-all market as in Section 2.
- In stage 2, in the market generated by the A-innovation winning and losing firms engage in Cournot competition and choose quantities to maximize their profits. In the market created by the B-innovation the winning firm produces the monopoly quantitiy and gets a profit of π_B; the losing firms obtain zero profits.

In market A, thus, there are n firms selling differentiated products. Let us denote the quantity-quality combination produced by firm i by $\{q_i, s_i\}$, and the price received p_i. The representative consumer's utility in market A is given by

$$U^A = \sum_{i=1}^{n} \left[q_i - \left(\frac{q_i}{s_i} \right)^2 \right] - \sigma \sum_{i<j} \frac{q_i}{s_i} \frac{q_j}{s_j} - \sum_{i=1}^{n} p_i q_i, \tag{43}$$

where

$$s_i = \begin{cases} \overline{s} \text{ if } i \text{ wins contest for } A\text{-innovation} \\ \underline{s} \text{ otherwise.} \end{cases}$$

The corresponding system of inverse demands for the products of the firms in market A is:

$$p_i = 1 - \frac{2q_i}{s_i^2} - \frac{\sigma}{s_i} \sum_{j \neq i} \frac{q_j}{s_j}, \; i = 1, 2, ..., n. \tag{44}$$

The parameter $\sigma \in [0, 2]$ is an inverse measure of the degree of horizontal product differentiation. When $\sigma \to 2$, unless they are differentiated in terms of quality, products are homogeneous. When $\sigma \to 0$, products become independent. In order to present a model as close as possible to our baseline model of Section 3, we set $\sigma = 2$ in what follows. In such a case, we are back to our baseline model in the limiting case when $\underline{s} = 0$. When $\underline{s} = 0$ only the winner of the contest obtains positive profits and when all firms lose the contest they all make zero profits. This is the case that isolates the portfolio effects of mergers. As we increase $\underline{s}$ (relative to $\overline{s}$), the product of the losing firms becomes more attractive for the consumers. Yet, if $\underline{s}$ is positive but sufficiently low, the winner of a contest can monopolize the market and therefore losing firms only obtain positive profits when no firm wins the contest and therefore they all produce low quality. Here mergers have price effects. When $\underline{s}$ becomes sufficiently large, losing firms make positive profits even if there is a winner of the contest. We also normalize the marginal cost of production to zero. In this situation, mergers have even stronger price effects.[28]

Pre-merger market

We first consider the portfolio problem in the pre-merger market corresponding to the A-project. To characterize the equilibrium of the two-stage game, we proceed by backwards induction. That is, we first solve the continuation game firms play in the post-innovation market. Then, folding the game backwards, we examine firms' decisions in the pre-innovation market.

Stage 2, post-contests markets

In market A there are two types of subgames. In the first type of subgame, one of the firms has succeeded at innovating and produces the good of high quality $\overline{s}$; the rest of the firms all produce a good of low quality $\underline{s}$. In the second subgame, all firms fail at obtaining the innovation and produce a good of low quality $\underline{s}$. In market B there are also two such subgames but, because of the winner-take-all feature of the B-innovation, the continuation payoffs are the same as in the main model of Section 2.

Therefore, we now focus on market A. Consider first the subgame in which one of the firms innovates and sells a high-quality product, $\overline{s}$, while the rest of the firms sell a low-quality product, $\underline{s}$. Standard derivations yield the Cournot equilibrium:

$$\overline{q}_A = \begin{cases} \frac{\overline{s}^2}{4} & \text{if } \underline{s} < \overline{s}/2 \\ \frac{\overline{s}[n(\overline{s}-\underline{s})+\underline{s}]}{2(n+1)} & \text{otherwise} \end{cases} \tag{45}$$

$$\underline{q}_A = \begin{cases} 0 & \text{if } \underline{s} < \overline{s}/2 \\ \frac{\underline{s}(2\underline{s}-\overline{s})}{2(n+1)} & \text{otherwise} \end{cases} \tag{46}$$

where $\overline{q}_A$ is the quantity put in the market by the high-quality seller and $\underline{q}_A$ the quantity sold by each of the low-quality sellers.

From expression (45)-(46), note that the firms that lose the contest and sell quality $\underline{s}$ put a positive quantity in the market only if the quality disadvantage vis-á-vis the winning firm is not excessive. If this condition is violated, the winning firm monopolizes the market.

[28] We have performed the analysis for an arbitrary degree of product differentiation and the insights remain the same. Later we shall report some results for cases in which the firms' products are horizontally differentiated.

The winner's and losers' profits, denoted $\overline{\pi}$ and $\underline{\pi}$ respectively, are equal to:

$$\overline{\pi}_A = \left(\frac{\overline{q}_A}{\overline{s}}\right)^2, \underline{\pi}_A = \left(\frac{\underline{q}_A}{\underline{s}}\right)^2, \tag{47}$$

while consumer surplus is given by

$$\overline{S}_A = \left(\frac{(n-1)\overline{s}\underline{q}_A + \underline{s}\overline{q}_A}{\overline{s}\underline{s}}\right)^2 \tag{48}$$

The other type of subgame is one in which no firm succeeds at innovating in the pre-innovation stage, in which case all firms sell a product of quality $\underline{s}$. Standard derivations yield the symmetric Cournot equilibrium:

$$q_A^* = \frac{\underline{s}^2}{2(n+1)}. \tag{49}$$

In this situation, all firms earn profits

$$\pi_A^* = \left(\frac{q_A^*}{\underline{s}}\right)^2,$$

and consumer surplus is given by

$$S_A^* = \left(n\frac{q_A^*}{\underline{s}}\right)^2 = \left(\frac{n\underline{s}}{2(n+1)}\right)^2.$$

It is obvious that for $\overline{s} > \underline{s}$, the inequalities $\overline{\pi}_A > \pi_A^* > \underline{\pi}_A$ and $\overline{S}_A > S_A^*$ hold.

Stage 1

We are now ready to fold the game backwards and look at the pre-innovation stage of the pre-merger market. In this stage, anticipating the payoffs from the continuation subgames, firms choose their portfolio of investments to maximize expected profits. Using the profits notation introduced above, the expected profits of a firm i investing an amount x_i in project A and an amount $1 - x_i$ in project B are:

$$\begin{aligned} u_i(x_i, \mathbf{x}_{-i}) &= p_i^A(x_i, \mathbf{x}_{-i}, \epsilon_A)\overline{\pi}_A + \sum_{j\neq i} p_j^A(x_j, \mathbf{x}_{-j}, \epsilon_A)\underline{\pi}_A + \Big(1 - \sum_{k=1}^{n} p_k^A(x_k, \mathbf{x}_{-k}, \epsilon_A)\Big)\pi_A^* \\ &+ p_i^B(1 - x_i, \mathbf{1} - \mathbf{x}_{-i}, \epsilon_B)\pi_B, \end{aligned} \tag{50}$$

where the success probabilities are the same as the Tullock probabilities given in (7)-(8).

The first line of this expression is the expected payoff from investing in the A-project. With probability $p_i^A(\cdot)$, firm i wins the contest and obtains the profits corresponding to a high quality seller in the Cournot market; with probability $\sum_{j\neq i} p_j^A(\cdot)$, some other firm wins the contest so firm i obtains the profits corresponding to a low-quality seller; finally, with probability $1 - \sum_{k=1}^{n} p_k^A(\cdot)$, no firm wins the contest and firm i gets the symmetric equilibrium profits π_A^*. The second line of the payoff in (50) is the expected profit from investing in the B-project. Firm i only obtains rents when winning the contest, which happens with probability $p_i^B(\cdot)$ and the rents are equal to π_B.

In terms of the incentives of the firms to invest in the A- and B-projects, the new firm's payoff (50) represents a similar trade-off as that in the basic model of Section 2. It is then straightforward to extend the existence result in Proposition 1 to this setting, which we omit to save on space. To derive the pre-merger equilibrium investment, we take the FOC, apply symmetry $x_i = x^*$, $i = 1, 2, ..., n$ and solve for x^*.

We shall compare the welfare generated pre-merger to that after a merger. In the pre-merger market, using the consumer welfare standard, social welfare is given by the expression:

$$W(x_i, x_{-i}) = \sum_{i=1}^{n} p_i^A(x_i, \mathbf{x}_{-i}, \epsilon_A)\overline{S}_A + \Big(1 - \sum_{i=1}^{n} p_i^A(x_i, \mathbf{x}_{-i}, \epsilon_A)\Big) S_A^* + \sum_{i=1}^{n} p_i^B(1 - x_i, \mathbf{1} - \mathbf{x}_{-i}, \epsilon_B) S_B.$$

In the above formula, the first two terms represent the expected consumer benefits from firm participation in the contest for introducing the A-innovation. With probability $\sum_{i=1}^{n} p_i^A(\cdot)$, the A-innovation is obtained by one of the firms, in which case consumer surplus is $\overline{S}_A$; with the remaining probability, no firm wins the contest for the A-innovation and the surplus generated is S_A^*. The last term is the value to consumers arising from the B-innovation.

Plugging the symmetric equilibrium x^* into this consumer surplus expression gives the pre-merger market social welfare level $W(x^*, \mathbf{x}^*)$.

Post-merger Market

We now present the analysis for the post-merger market. Suppose firms i and j merge. As in the basic model of Section 2, firms will choose their portfolio of investments to maximize the sum of the partners' pre-innovation profits. What is new in this richer model is that a merger brings detrimental price effects in market A. Compared to the situation before the merger, the equilibrium quantities of the merging firms in market A will reflect the internalization of the (negative) quantity externalities they impose on one another (cf. Salant *et al.*, 1983).

Stage 2

Post-merger, in the market created by the A-project, there are three types of subgames. We label these subgames as I, II and III. In the first type of subgame, subgame I, one of the merging firms wins the contest for the innovation. In such a case, the two partner firms produce a product of high quality $\overline{s}$, while the rest of the firms produce a good of low quality $\underline{s}$. In the second type of subgame, subgame II, one firm other than the partners to the merger obtains the innovation and produces a good of high quality $\overline{s}$; the rest of the firms, including the merging firms, produce a good of low quality $\underline{s}$. In the third type of subgame, subgame III, all firms, merging and non-merging, fail to innovate and all produce a good of low quality $\underline{s}$. In all these subgames, the merging firms coordinate their quantities to maximize the joint post-innovation profits and this coordination generates detrimental price effects of mergers.

Consider the first type of subgame. The merged firms produce high quality and coordinate their production; the rest of the firms sell low quality and operate independently. Standard derivations yield the Cournot equilibrium:

$$\overline{q}_A^{m,I} = \begin{cases} \frac{\overline{s}^2}{8} & \text{if } \underline{s} < \overline{s}/2 \\ \frac{\overline{s}[(n-1)\overline{s} - (n-2)\underline{s}]}{4n} & \text{otherwise} \end{cases} \tag{51}$$

$$\underline{q}_A^{nm,I} = \begin{cases} 0 & \text{if } \underline{s} < \overline{s}/2 \\ \frac{\underline{s}(2\underline{s}-\overline{s})}{2n} & \text{otherwise} \end{cases} \tag{52}$$

where $\overline{q}^{m,I}$ denotes the quantity produced by each of the merging firms and $\overline{q}^{nm,I}$ the quantity produced by each of the non-merging firms. Note that the assumption $\overline{s} < 4\underline{s}$ suffices for the low-quality non-merging firms to supply a positive quantity in the market.

The equilibrium profits of the merging and non-merging firms in this subgame are

$$\overline{\pi}_A^{m,I} = 8\left(\frac{\overline{q}^{m,I}}{\overline{s}}\right)^2, \; \underline{\pi}_A^{nm,I} = 2\left(\frac{\underline{q}^{nm,I}}{\underline{s}}\right)^2.$$

Consumer surplus is given by

$$S_A^I = \left(\frac{(n-2)\overline{s}\underline{q}_A^{nm,I} + 2\underline{s}\overline{q}_A^{m,I}}{\overline{s}\underline{s}}\right)^2.$$

In the second type of subgame, denoted II, a non-merging firm, say k, successfully introduces the A-innovation, in which case it produces high quality and commercializes it independently. The rest of non-merging firms sell low quality and also operate independently. Finally, the merging firms also sell low-quality but coordinate their quantities to maximize profits. Standard derivations yield the Cournot equilibrium:

$$\underline{q}_A^{m,II} = \begin{cases} 0 & \text{if } \underline{s} < \overline{s}/2 \\ \frac{\underline{s}(2\underline{s}-\overline{s})}{4n} & \text{otherwise} \end{cases} \tag{53}$$

$$\overline{q}_A^{nm,II} = \begin{cases} \frac{\overline{s}^2}{4} & \text{if } \underline{s} < \overline{s}/2 \\ \frac{\overline{s}[(n-1)\overline{s}-(n-2)\underline{s}]}{2n} & \text{otherwise} \end{cases} \tag{54}$$

$$\underline{q}_A^{nm,II} = \begin{cases} 0 & \text{if } \underline{s} < \overline{s}/2 \\ \frac{\underline{s}(2\underline{s}-\overline{s})}{2n} & \text{otherwise} \end{cases} \tag{55}$$

where $\underline{q}_A^{m,II}$ is the equilibrium quantity of the low-quality merging firms, $\overline{q}_A^{nm,II}$ is the quantity of the high-quality non-merging firm and $\underline{q}_A^{nm,II}$ is the quantity of the rest of the non-merging firms, which also sell low quality.

The corresponding profits are:

$$\underline{\pi}_A^{m,II} = 8\left(\frac{\underline{q}_A^{m,II}}{\underline{s}}\right)^2, \; \overline{\pi}_A^{nm,II} = 2\left(\frac{\overline{q}_A^{nm,II}}{\overline{s}}\right)^2, \; \underline{\pi}_A^{nm,II} = 2\left(\frac{\underline{q}_A^{nm,II}}{\underline{s}}\right)^2.$$

In this second type of subgame, consumer surplus is given by

$$S_A^{II} = \left(\frac{(n-3)\overline{s}\underline{q}_A^{nm,II} + 2\overline{s}\underline{q}_A^{m,II} + \underline{s}\overline{q}_A^{nm,II}}{\overline{s}\underline{s}}\right)^2.$$

Finally, in the last type of subgame, no firm introduces the A-innovation and all the firms produce a low-quality product. Non-merging firms operate independently while merging firms coordinate their quantities. Standard derivations yield the Cournot equilibrium quantities:

$$\underline{q}_A^{m,III} = \frac{\underline{s}^2}{4n}, \ \underline{q}_A^{nm,III} = \frac{\underline{s}^2}{2n}.$$

The profits firms obtain are given by:

$$\pi_A^{m,III} = 8\left(\frac{\underline{q}_A^{m,III}}{\underline{s}}\right)^2, \ \pi_A^{nm,III} = 2\left(\frac{\underline{q}_A^{nm,III}}{\underline{s}}\right)^2.$$

Consumer surplus is given by

$$S_A^{III} = \left(\frac{2\underline{q}_A^{m,III} + (n-2)\underline{q}_A^{nm,III}}{\underline{s}}\right)^2 = \frac{(n-1)^2\underline{s}^2}{4n^2}.$$

Stage 1

In the pre-innovation stage of the post-merger market, anticipating the payoffs in the continuation subgames I, II, and III, merging and non-merging firms choose their portfolio of investments to maximize their expected payoffs.

The expected payoff of the merged entity is given by:

$$\begin{aligned} u_m(x_i, \mathbf{x}_{-i}) &= \Big(p_i^A(x_i, \mathbf{x}_{-i}, \epsilon_A) + p_j^A(x_j, \mathbf{x}_{-j}, \epsilon_A)\Big)\overline{\pi}_A^{m,I} + \sum_{k \neq i,j} p_k^A(x_k, \mathbf{x}_{-k}, \epsilon_A)\underline{\pi}_A^{m,II} \\ &+ \Big(1 - \sum_{k=1}^{n} p_k^A(x_k, \mathbf{x}_{-k}, \epsilon_A)\Big)\underline{\pi}_A^{m,III} + p_i^B(1 - x_i, \mathbf{1} - \mathbf{x}_{-i}, \epsilon_B)\pi_B, \end{aligned}$$

where the success probabilities are again the Tullock's ones as in (7)-(8).

Likewise, the expected payoff of a non-merging firm is given by

$$\begin{aligned} u_{nm}(x_k, x_{-k}) &= p_k^A(x_k, \mathbf{x}_{-k}, \epsilon_A)\overline{\pi}_A^{nm,II} + \Big(p_i^A(x_i, \mathbf{x}_{-i}, \epsilon_A) + p_j^A(x_j, \mathbf{x}_{-j}, \epsilon_A)\Big)\underline{\pi}_A^{nm,I} \\ &+ \sum_{l \neq i,j,k} p_l^A(x_l, \mathbf{x}_{-l}, \epsilon_A)\underline{\pi}_A^{nm,II} + \Big(1 - \sum_{h=1}^{n} p_h^A(x_h, \mathbf{x}_{-h}, \epsilon_A)\Big)\underline{\pi}_A^{nm,III} \\ &+ p_k^B(1 - x_k, \mathbf{1} - \mathbf{x}_{-k}, \epsilon_B)\pi_B \end{aligned}$$

To derive the equilibrium of the investment game, we take the FOC for the maximization of $u_m(x_i, \mathbf{x}_{-i})$ with respect to x_i as well as the FOC for the maximization of $u_{nm}(x_k, x_{-k})$ with respect to x_k, apply symmetry for the merging and non-merging firms' investments, and derive the equilibrium vector of investments post-merger.

We evaluate the merger in terms of consumer welfare, which again depends on whether the innovations are obtained or not, as well as on whether the successful firms are merging or non-merging firms. Taking into account the probability with which the different events occur, consumer welfare is given by:

$$\begin{aligned} W_m(x_i, x_{-i}) &= \Big(p_i^A(x_i, \mathbf{x}_{-i}, \epsilon_A) + p_j^A(x_j, \mathbf{x}_{-j}, \epsilon_A)\Big)S_A^I + \sum_{k=i,j}^{n} p_k^A(x_k, \mathbf{x}_{-k}, \epsilon_A)S_A^{II} \\ &+ \Big(1 - \sum_{l=1}^{n} p_l^A(x_l, \mathbf{x}_{-l}, \epsilon_A)\Big)S_A^{III} + \sum_{h=1}^{n} p_h^B(1 - x_h, \mathbf{1} - \mathbf{x}_{-h}, \epsilon_B)S_B \end{aligned}$$

In the above expression, the first two terms represent the expected benefit to consumers from introducing the A-innovation depending on who innovates: the merging firm or one of the non-merging firms. The third term is the expected benefit when all firms fail to introduce the A-innovation in the market. The last term corresponds to the market generated by the B-innovation.

Results

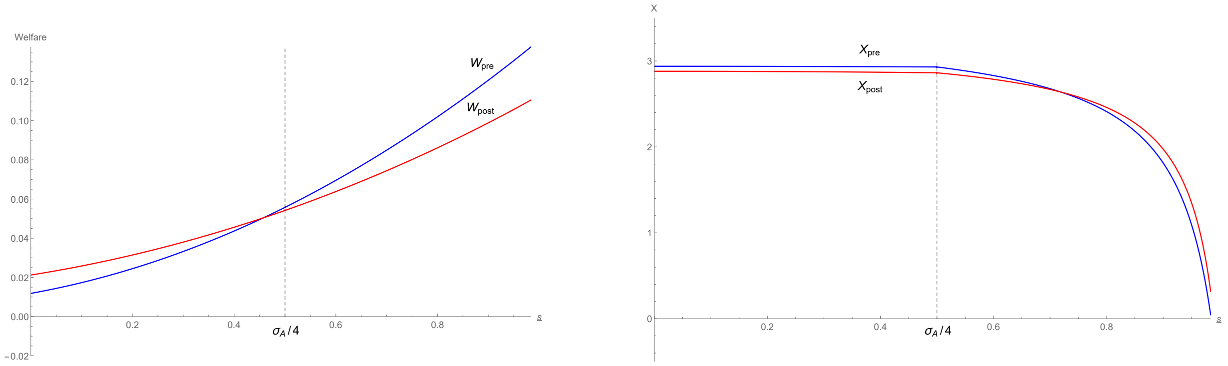
In the basic model of this paper, we have analyzed the impact of mergers on equilibrium research portfolios and concluded that mergers align the incentives of the merging firms to those of the planner when the relatively more profitable project is also the project that appropriates a larger share of the social surplus. In that analysis, standard market power effects of mergers (price increases) did not play a role because of the winner-take-all feature of the contests. The present model incorporates the detrimental price effects of mergers because merging firms are allowed to coordinate their prices after merger. The question that arises is whether the positive portfolio effects of mergers can have a dominating influence over the negative price effects of mergers. We address this question in the remaining of this section.

An analytical approach to the above question is more difficult than in the baseline model of Section 3 because of the richness of subgames in stage 2, specially after a merger. We thus proceed to a numerical analysis that shows that mergers may improve consumer welfare, despite the detrimental price effects of mergers.

In Figure 5 we represent pre- and post-merger welfare levels and aggregate investment in the A project as a function of $\underline{s}$. In these graphs we set the parameters of the model as follows. For the A project, we set $\epsilon_A = 1$, $\overline{s} = 1$ and let $\underline{s}$ vary from 0 to 1. For the B-project, which is winner-take-all, we just set $\epsilon_B = 1/2$, $\pi_B = 1/72$ and $S_B = 1/144$.

We emphasize that when $\underline{s} = 0$, we are back to our model of Section 3 in which mergers have no price effects at all. As shown earlier in the paper, if the relatively more profitable project is also the relatively more appropriable, then a merger increases consumer welfare by reducing investment in the more profitable project. This is exactly what we see in Figure 5. It is straightforward to verify that, in a neighbourhood of $\underline{s} = 0$, the inequality $\overline{\pi}_A/\epsilon_A > \pi_B/\epsilon_B$ holds. Therefore, a merger increases welfare (see Figure 5(a)) by reducing investment (see Figure 5(b)) in the more profitable project. Note further that when $\underline{s} = 0$ demands are linear (and marginal costs are constant) so all mergers are profitable.

As we increase $\underline{s}$ away from zero, in addition to the portfolio effects of mergers, the price effects of mergers start playing a role. In the graphs we depict the threshold value corresponding to the condition $\underline{s} = \overline{s}/2$ (see e.g. the equilibrium quantities (45)-(46)), which is $\sigma_A/4 = 1/2$. When $\underline{s} < 1/2$, the winner of the contest for the A-project monopolizes the post-innovation market. However, if no firm obtains the innovation, then all the firms, merging and non-merging, produce low quality and compete. This competition is the source of detrimental price effects when firms i and j merge in this region of



(a) Consumer welfare

(b) Aggregate investment in A

Figure 5: Pre- and post-merger welfare and aggregate investment in the A-project

parameters. As $\underline{s}$ increases towards $1/2$, the price effects remain similar (they do not depend on $\underline{s}$) while the profitability of the A-project decreases relative to that of the B-project. (Investment in A pre-and post-merger correspondingly decreases, though this is hardly perceptible in the graph.) This weakens the portfolio effect of mergers while keeping the price effects similar. Eventually, as Figure 5(a) shows, as the degree of quality differentiation vanishes ($\underline{s} \to \overline{s}$), the merger becomes detrimental for consumers.

Beyond the threshold value $\underline{s} > 1/2$, much of the same occurs but note that the price effects of mergers in this region are much larger than when $\underline{s} < 1/2$ because they also arise when one of the firms wins the contest. Observe also that when $\underline{s}$ approaches $\underline{s} = 1$, the market associated to the A-project turns extremely competitive because the products are virtually identical (neither vertical nor horizontal differentiation whatsoever). In this situation price effects are strongest (like in a Cournot market for homogeneous products) and moreover the profitability of the A-project ends up being below that of project B so that a merger results in an increase in investment in the A-project rather than in a decrease.

While constructing Figure 5 we have set $\sigma_A = 2$, which implies that products are not horizontally differentiated at all. In the Appendix we report Figure 9, which corresponds to the case in which $\sigma_A = 1$. As explained in the Appendix, the insights we obtain are similar: for low $\underline{s}$, a merger increases welfare by reducing investment in the relatively more profitable project, while for high $\underline{s}$ a merger reduces welfare because the price effects are quite strong.

We close this section with a comment on the incentive-compatibility of mergers. In the baseline model of Section 3, mergers are not always incentive-compatible. This result is reminiscent of mergers in Cournot settings. As shown above, investments are strategic substitutes so that when the merged entity, say, reduces investment in the relatively more profitable project, rival firms increase it. This reaction of the rival firms lowers the probability the merged entity wins the contest for the relatively more profitable project, which makes merger profitability more difficult. When project A is not winner-take-all and firms sell horizontally differentiated products, mergers are more often incentive-compatible. In the Appendix we report the profits of the merging and non-merging firms for the case in which $\sigma_A = 1$ in Figure 10. It can be seen that mergers increase the profits of the merging and non-merging firms; moreover, the profits of the latter increase to a lower extent when $\underline{s}$ is relatively low, while the opposite holds when $\underline{s}$ is relatively high. Note that with $\sigma_A = 1$ mergers are profitable even if $\underline{s} \to 0$. The reason is that products are horizontally differentiated and in such a case the quantity effects of a merger increase the continuation game payoff of the merged entity. Even though the investments in the A-project are strategic substitutes and merger profitability is compromised, the former effect dominates and merging becomes incentive-compatible. However, when $\underline{s} \to \overline{s}$, firms' products are not anymore vertically differentiated and, like in the merger paradox, the internalization of the price effects of mergers tend to lower the profitability of the insiders and favor the outsiders.

4.5 General success probabilities and strategic complements

In Section 3, we have assumed the specific Tullock functional form for the success probabilities. In this section we return to the general formulation of Section 2 with symmetric firms and show that the main insights obtained from the Tullock model continue to hold. Two important differences should however be highlighted. First, with general success probabilities, the game may either be one of strategic substitutes or strategic complements. Second, with general success probabilities, the welfare function depends on the vector of investments in the A-project, rather than on the aggregate level of investment in such project. Despite these two differences, the mechanism by which mergers may increase/decrease welfare is the same and we next develop a graphical approach to demonstrate it.

Consider the general model of Section 2 and assume firms are symmetric.

Proposition 8. *In the general game of portfolio choice, there exists a unique pre-merger SNE denoted* $\mathbf{x}^* = (x^*, x^*, ..., x^*)$*. The SNE is interior provided that*

$$\frac{\partial p_i^A(0, \mathbf{0}_{-i}, \epsilon_A)}{\partial x_i}\pi_A > \left|\frac{\partial p_i^B(1, \mathbf{1}_{-i}, \epsilon_B)}{\partial x_i}\right|\pi_B \text{ and } \frac{\partial p_i^A(1, \mathbf{1}_{-i}, \epsilon_A)}{\partial x_i}\pi_A < \left|\frac{\partial p_i^B(0, \mathbf{0}_{-i}, \epsilon_B)}{\partial x_i}\right|\pi_B,$$

in which case it is given by the x^* *that solves:*

$$\frac{\partial p_i^A(x^*, \mathbf{x}^*, \epsilon_A)}{\partial x_i}\pi_A + \frac{\partial p_i^B(1-x^*, \mathbf{1}-\mathbf{x}^*, \epsilon_B)}{\partial x_i}\pi_B = 0. \tag{56}$$

Let us assume that interiority conditions also hold in what follows. Consider now that firms i and j merge and assume that the payoff (3) is strictly concave. Given our assumptions, there exists a unique NE of the post-merger market where all the non-merging firms choose the same investment portfolio. Likewise the merged entity chooses to invest symmetrically across the research facilities of the constituent firms. Let x_m denote the investment of each of the merging firms in project A; likewise, let x_{nm} be the investment of each of the non-merging firms in project A. The post-merger market equilibrium is obtained after applying symmetry in the FOCs (2) and (4) and solving for x^*_m and x^*_{nm}. Let $\mathbf{x}^M = (\mathbf{x}^*_m, \mathbf{x}^*_{nm})$ denote the post-merger market equilibrium vector of investments in project A.

Proposition 9. *Assume that firms* i *and* j *merge in the general game of innnovation portfolio choice with symmetric firms. Assume also that the merged entity prefers to keep the research facilities of the merging firms running. Then, compared to the pre-merger market equilibrium, the merged firms invest more in project A and less in project B, that is,* $x^*_m > x^*$*, if and only if:*

$$\frac{\pi_A}{\pi_B} < -\frac{\frac{\partial p_j^B(\mathbf{1}-\mathbf{x}^*, \epsilon_B)}{\partial x_i}}{\frac{\partial p_j^A(\mathbf{x}^*, \epsilon_A)}{\partial x_i}}. \tag{57}$$

The non-merging firms invest less (more) in project A, that is $x^*_{nm} < (>) x^*$ and more (less) in project B if firms' investments are strategic substitutes (complements).

Proposition 9 states that, compared to the situation before a merger, the merging firms invest more in project A and less in project B if and only if condition (57) holds. This condition, which for the Tullock model of Section 3 boils down to (18), ensures that after a merger the best-response surface of the merged entity shifts up. As before, this condition tends to be satisfied when the rewards from market A relative to those from market B are low enough.[29]

We evaluate the performance of mergers using the welfare function in (5), but adopting a consumer surplus standard. Our next result characterizes the inefficiency of the pre-merger market equilibrium. To present it, it is useful to denote the gradient of the social welfare function evaluated at the pre-merger market equilibrium as:[30]

$$\nabla W(\mathbf{x}^*) = \left(\frac{\partial W(\mathbf{x}^*)}{\partial x_1}, \frac{\partial W(\mathbf{x}^*)}{\partial x_2}, ..., \frac{\partial W(\mathbf{x}^*)}{\partial x_n} \right),$$

where $\partial W(\cdot)/\partial x_i$ is given by (6). Let $\mathbf{x}^o = (x^o, x^o, ..., x^o)$ denote the socially optimal investment portfolio. To obtain x^o, apply symmetry in the FOC (6) and solve for x^o.

Proposition 10. *In the general game of innovation portfolio choice with symmetric firms, the pre-merger market equilibrium investment in project A is insufficient (and, correspondingly, excessive in project B) from the point of view of social welfare, that is,* $\mathbf{x}^* < \mathbf{x}^o$*, if and only if*

$$\nabla W(\mathbf{x}^*) > 0. \tag{58}$$

When this inequality is reversed, the market invests too much in project A and too little in project B.

Our next result provides conditions under which a merger is a corrective device. Before presenting it, let us denote the gradient of the social welfare function evaluated at the post-merger market equilibrium

[29] Formally, this follows from the following observations. An increase in π_A (or a decrease in π_B) raises the LHS of (57). At the same time, using the implicit function theorem in (2) immediately reveals that the pre-merger market equilibrium investment increases. Because of Assumption 2, this decreases the numerator and increases the denominator of the RHS of (57), making the inequality more difficult to hold.

[30] Each component of the gradient evaluated at the pre-merger market equilibrium is given by the expression:

$$(S_A - \pi_A) \frac{\partial p_i^A(\mathbf{x}^*, \epsilon_A)}{\partial x_i} + S_A \sum_{j \neq i}^{n} \frac{\partial p_j^A(\mathbf{x}^*, \epsilon_A)}{\partial x_i}$$
$$+ (S_B - \pi_B) \frac{\partial p_i^B(\mathbf{1} - \mathbf{x}^*, \epsilon_B)}{\partial x_i} + S_B \sum_{j \neq i}^{n} \frac{\partial p_j^B(\mathbf{1} - \mathbf{x}^*, \epsilon_B)}{\partial x_i} > 0, \ i = 1, 2, ..., n,$$

where we have used the fact that x^* solves the FOC (2).

by:[31]

$$\nabla W(\mathbf{x}^M) = \left(\frac{\partial W(\mathbf{x}^M)}{\partial x_i}, \frac{\partial W(\mathbf{x}^M)}{\partial x_j}, ..., \frac{\partial W(\mathbf{x}^M)}{\partial x_n} \right) \tag{59}$$

Proposition 11. *In the general game of portfolio choice with symmetric firms, suppose the parameters of the model are such that firms' investments are strategic complements. Then:*

a. If conditions (57), (58) hold and the gradient (59) is strictly positive, then a merger increases welfare by raising innovation in the A-project.

b. If neither condition (57) nor (58) hold and the gradient (59) is strictly negative, then a merger increases welfare by raising innovation in the B-project.

c. If condition (57) does not hold, condition (58) holds, and the gradient (59) is strictly positive, a merger reduces welfare by decreasing innovation in the A-project.

d. If condition (57) holds, condition (58) does not hold and the gradient (59) is strictly negative, a merger reduces welfare by decreasing innovation in the B-project.

Proof. See the Appendix.

The result in Proposition 11 is intuitive. If there is too little funding allocated to project A in the pre-merger market equilibrium and all the firms invest more in such a project in the post-merger market equilibrium, then it is likely that welfare post-merger is higher than welfare pre-merger.[32] To ensure that welfare increases after a merger, it suffices to require condition (59) on the gradient of the welfare function evaluated at the post-merger market equilibrium in Proposition 11. This condition ensures that post-merger there is still under-investment and therefore the unique global maximum of the welfare function has not been reached yet.

It is useful to illustrate the result in Proposition 11 graphically. In Figure 6 we have represented an arbitrary family of iso-welfare curves in the (x_{nm}, x_m) space. In the vertical axis we have the investment in the A-project of a representative merging firm, x_m, while in the horizontal axis we place the corresponding investment of a representative non-merging firm, x_{nm}. Assume the rest of the firms

[31]To be sure, the n components of the gradient evaluated at the post-merger market equilibrium are:

$$(S_A - \pi_A)\frac{\partial p_i^A(\mathbf{x}^M, \epsilon_A)}{\partial x_i} + (S_A - \pi_A)\frac{\partial p_j^A(\mathbf{x}^M, \epsilon_A)}{\partial x_i} + S_A \sum_{k \neq i,j}^{n} \frac{\partial p_k^A(\mathbf{x}^M, \epsilon_A)}{\partial x_i}$$

$$+ (S_B - \pi_B)\frac{\partial p_i^B(\mathbf{1} - \mathbf{x}^M, \epsilon_B)}{\partial x_i} + (S_B - \pi_B)\frac{\partial p_j^B(\mathbf{1} - \mathbf{x}^M, \epsilon_B)}{\partial x_i} + S_B \sum_{k \neq i,j}^{n} \frac{\partial p_k^B(\mathbf{1} - \mathbf{x}^M, \epsilon_B)}{\partial x_i} > 0 \text{ for } i, j.$$

and

$$(S_A - \pi_A)\frac{\partial p_k^A(\mathbf{x}^M, \epsilon_A)}{\partial x_k} + S_A \sum_{h \neq k}^{n} \frac{\partial p_h^A(\mathbf{x}^M, \epsilon_A)}{\partial x_k}$$

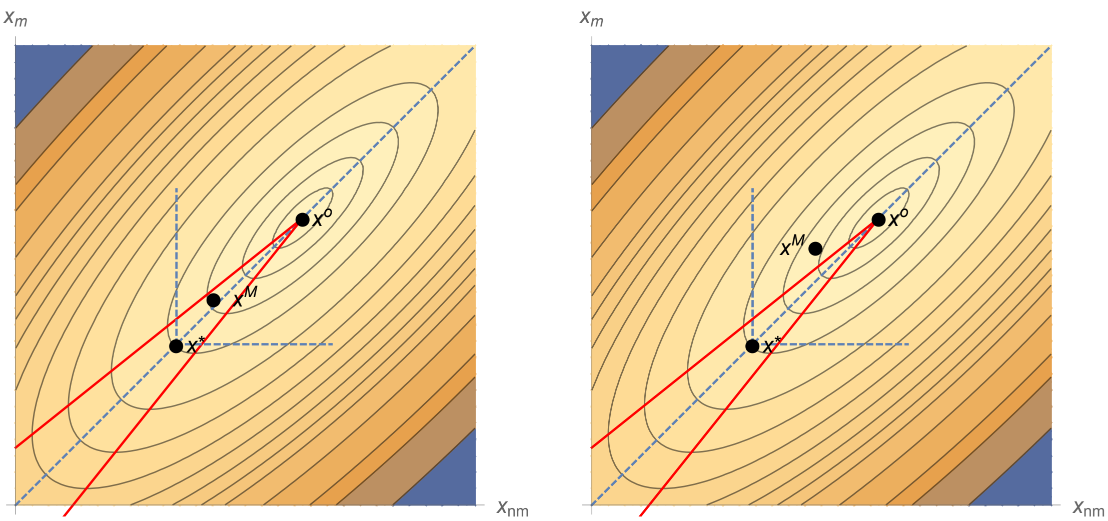
$$+ (S_B - \pi_B)\frac{\partial p_k^B(\mathbf{1} - \mathbf{x}^M, \epsilon_B)}{\partial x_k} + S_B \sum_{h \neq k}^{n} \frac{\partial p_h^B(\mathbf{1} - \mathbf{x}^M, \epsilon_B)}{\partial x_k} > 0, \text{ for } k \neq i, j,$$

where we have used the fact that the post-merger market equilibrium vector of investments satisfies the corresponding FOCs.

[32]More investment in project A post-merger does not guarantee that welfare increases even if pre-merger there is under-investment in such a project. One reason for this is that the market equilibrium may move from an under-investment situation to an over-investment one.

are in the background playing the same strategies as the representative firms. Lighter colours indicate higher welfare levels. The social optimum is represented by the point $\mathbf{x^o}$, while the pre-merger market equilibrium by $\mathbf{x}^*$. The space in between the red lines going through the social optimum determines the area for which the social welfare function has a positive gradient.

The left graph of Figure 6 shows the situation described by Proposition 11a. At the pre-merger market equilibrium, firms invest insufficiently in the A-project. Post-merger, all the firms invest more in project A than pre-merger and because the gradient at the post-merger market equilibrium is positive, welfare surely increases. That the gradient is positive is a sufficient condition but not necessary for welfare to increase after all firms raise their investments in project A. This can be seen in the right graph, where we have plotted a situation where welfare increases but the component of the gradient corresponding to x_m is negative.



(a) Merger raises welfare (gradient positive) (b) Merger raises welfare (gradient non-positive)

Figure 6: Pre- and post-merger market equilibrium and welfare (strategic complements)

When firms' investments are strategic substitutes, the non-merging firms change their investment in opposite direction to the merging firms. Because of this, the result in Proposition 11 is invalid. Even if the gradient is positive at the post-merger market equilibrium, the fact that the non-merging firms cut their investment may result in a decrease in social welfare. Nevertheless, with strategic substitutes it is possible that welfare increases. We illustrate these two points in Figure 7. On the left graph, we show a case in which welfare falls, even if the gradient is positive. On the right graph welfare increases.

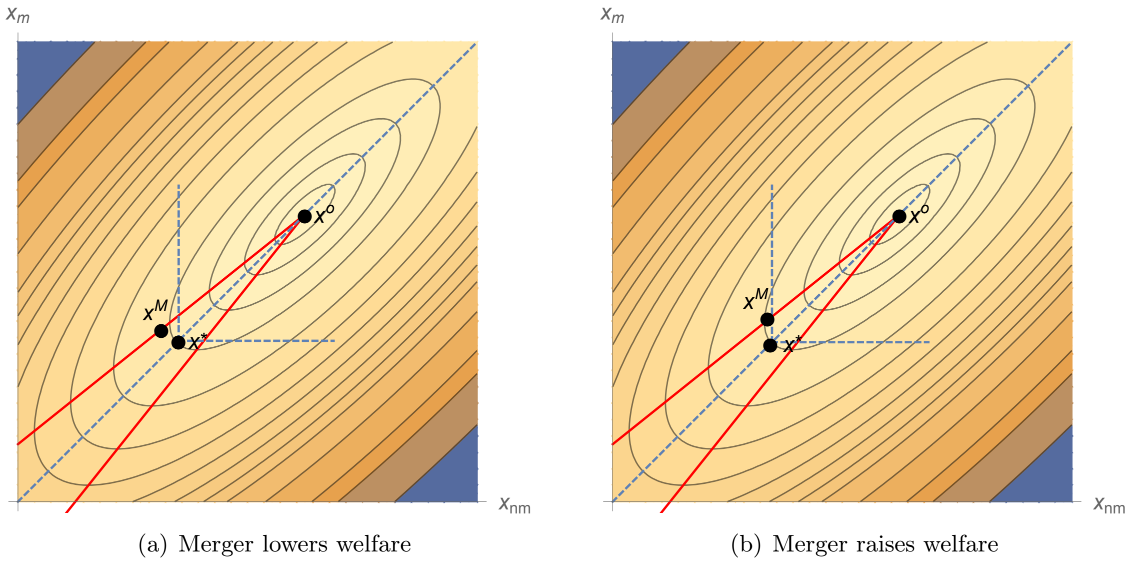


Figure 7: Pre- and post-merger market equilibrium and welfare (strategic substitutes)

5 Conclusions

There is a growing consensus that innovation should be protected by economic policy and a literature has recently emerged on the effects of mergers on innovation. We have contributed to this literature by presenting a model of mergers in which firms pursue an array of projects. While the focus of the existing literature has been on how mergers affect investment effort, our paper has paid attention to the question how mergers affect the portfolio of projects firms invest in.

We have studied a market in which firms can invest in two independent research projects. The projects vary in terms of profitability and surplus appropriation. By investing in a project a firm engages in a contest with the rival firms. The portfolio nature of the firms' problem leads to new insights about firm interaction in the market. In settings in which investing in a project raises the marginal cost of investing in another project, putting effort in one project creates both a negative business-stealing externality and a positive business-giving externality on the rival firms. The negative externality relates to the project in which a firm raises investment and is the usual innovation externality by which the effort of a firm decreases the chance rival firms innovate. The positive externality is novel and goes via the alternative project. Because raising investment in one project increases the marginal cost of investing in the alternative project, rival firms are conferred a positive externality in the competition for that project.

We have shown that, compared to the optimal portfolio, the market portfolio is typically biased due to two reasons. First, ignoring the business-stealing and business-giving externalities, firms tend to allocate too much funding to the more profitable project and too little to the less profitable one. Second, because firms only care about the profits generated by the projects and not about consumer surplus, they typically tend to put too little effort on projects that generate a large social surplus.

We have characterized the impact of a merger on the portfolio of the merging and non-merging firms. Merging firms adjust their portfolio of investments to internalize the business-stealing and business-giving externalities they impose on one another. The strength of these externalities depend on the relative profitability of the projects and after merging they cut investment in the more profitable project and raise it in the less profitable one. In the absence of synergies, even if the non-merging firms adjust their portfolio in the opposite direction, we have seen that a merger may increase consumer welfare. This occurs when the relatively more profitable project is also the project for which the firm appropriates a larger fraction of the social value. In such a case, a merger increases welfare by reducing investment in the more profitable and more appropriable project and increasing it in the less profitable project but also less appropriable project. In the opposite case where the more profitable project is the one which appropriates a lower fraction of the social surplus, a merger reduces welfare by further misaligning the market and the social incentives.

We have argued that our results carry over to cases in which, in addition to innovation portfolio effects, there are price effects of mergers. Moreover, we have seen that the results hold if firms not only choose how to allocate funding across projects but also how much money they spend on research. Finally, our results also hold if firms have asymmetric research budgets.

Appendix

Proof that the innovation portfolio game with Tullock success probabilities is a game of strategic substitutes.

To show that $\epsilon_\ell \geqslant 1$, $\ell = A, B$, suffices for strategic substitutability, we need to prove that the marginal benefit a firm obtains from its investment in the A-project decreases as a rival firm's investment in the same project rises. We thus compute the second cross partial derivative of the payoff of a firm i with respect to own and rival investment:

$$\begin{aligned}\frac{\partial u(x_i, x_j)}{\partial x_i \partial x_j} &= -\frac{\left(\sum_{i=1}^n x_i + \epsilon_A - 2x_i\right)\pi_A}{\left(\sum_{i=1}^n x_i + \epsilon_A\right)^3} - \frac{\left(n - \sum_{i=1}^n x_i - 2 + 2x_i + \epsilon_B\right)\pi_B}{\left(n - \sum_{i=1}^n x_i + \epsilon_B\right)^3} \\ &= -\frac{\left(\sum_{j\neq i}^n x_j + \epsilon_A - x_i\right)\pi_A}{\left(\sum_{i=1}^n x_i + \epsilon_A\right)^3} - \frac{\left(n - 1 - \sum_{j\neq i}^n x_j - (1 - x_i) + \epsilon_B\right)\pi_B}{\left(n - \sum_{i=1}^n x_i + \epsilon_B\right)^3}. \qquad (60)\end{aligned}$$

It is now straightforward to see that $\epsilon_A \geqslant 1$ guarantees that the first summand of this expression is negative and that $\epsilon_B \geqslant 1$ ensures that the second summand is also negative. Hence, the investments of the firms in the A-project are strategic substitutes. ■

Proof of Proposition 1.

In the pre-merger market equilibrium, a firm i maximises the expression:

$$u_i(x_i, \mathbf{x}_{-i}) = \frac{x_i}{\sum\limits_{k=1}^n x_k + \epsilon_A}\pi_A + \frac{1 - x_i}{\sum\limits_{k=1}^n (1 - x_k) + \epsilon_B}\pi_B. \qquad (61)$$

The FOC for an interior equilibrium is given by:

$$\frac{\sum_{k\neq i} x_k + \epsilon_A}{\left(\sum\limits_{k=1}^n x_k + \epsilon_A\right)^2}\pi_A - \frac{n - 1 - \sum_{k\neq i} x_k + \epsilon_B}{\left(n - \sum\limits_{k=1}^n x_k + \epsilon_B\right)^2}\pi_B = 0. \qquad (62)$$

It is straightforward to verify that the second order condition for profits maximization holds.[33] Because the payoff is strictly concave in firm i's own investment and continuous in x_{-i}, the existence of equilibrium follows from the Debreu-Glicksberg-Fan theorem. Assumption 2 implies that the equilibrium is unique. Because the game is symmetric, there exists a unique SNE where every firm invests an amount $x_i = x^*$, $i = 1, 2, ..., n$ (Hefti, 2017). Equation (12) follows from applying symmetry in the FOC (62).

Let us denote the LHS of (12) as $h(x^*)$ and first notice that h is continuous and monotone decreasing in x^*. Moreover, note that

$$h(0) = \frac{\pi_A}{\epsilon_A} - \frac{n + \epsilon_B - 1}{(n + \epsilon_B)^2}\pi_B > 0 \text{ if and only if } \epsilon_A < \frac{\pi_A(n + \epsilon_B)^2}{\pi_B(n - 1 + \epsilon_B)}$$

[33]The second order condition is given by

$$-\frac{2\left(\sum_{k\neq i} x_k + \epsilon_A\right)}{\left(\sum\limits_{k=1}^n x_k + \epsilon_A\right)^3}\pi_A - \frac{2\left(n - 1 - \sum_{k\neq i} x_k + \epsilon_B\right)}{\left(n - \sum\limits_{k=1}^n x_k + \epsilon_B\right)^3}\pi_B < 0.$$

and that

$$h(1) = \frac{\pi_A(n-1+\epsilon_A)}{(n+\epsilon_A)^2} - \frac{\pi_B}{\epsilon_B} < 0 \text{ if and only if } \epsilon_A > \frac{\pi_A\epsilon_B - 2n\pi_B + \sqrt{\pi_A\epsilon_B\,(\pi_A\epsilon_B - 4\pi_B)}}{2\pi_B}$$

Therefore, the unique equilibrium is interior provided that (11) holds.

To show that equilibrium investment x^* increases in π_A and ϵ_B and decreases in ϵ_A and π_B, we apply implicit differentiation to Equation (12). We then obtain:

$$\begin{aligned}\frac{\partial x^*}{\partial \pi_A} &= -\frac{\partial h/\partial \pi_A}{\partial h/\partial x^*} > 0,\ \frac{\partial x^*}{\partial \epsilon_A} = -\frac{\partial h/\partial \epsilon_A}{\partial h/\partial x^*} < 0 \\ \frac{\partial x^*}{\partial \pi_B} &= -\frac{\partial h/\partial \pi_B}{\partial h/\partial x^*} < 0,\ \frac{\partial x^*}{\partial \epsilon_B} = -\frac{\partial h/\partial \epsilon_B}{\partial h/\partial x^*} > 0,\end{aligned}$$

where we have used the derivatives

$$\frac{\partial h}{\partial x^*} = -\frac{n((n-1)x^*+\epsilon_A)+\epsilon_A}{(nx^*+\epsilon_A)^3}\pi_A - \frac{n(n-1)(1-x^*)+(n+1)\epsilon_B}{n^2(1-x^*)^2}\pi_B < 0$$

$$\frac{\partial h}{\partial \pi_A} = \frac{(n-1)x^*+\epsilon_A}{(nx^*+\epsilon_A)^2} > 0,\ \frac{\partial h}{\partial \epsilon_A} = -\frac{(n-2)x^*+\epsilon_A}{(nx^*+\epsilon_A)^3}\pi_A < 0,$$

$$\frac{\partial h}{\partial \pi_B} = -\frac{(n-1)(1-x^*)+\epsilon_B}{(n(1-x^*)+\epsilon_B)^2} < 0, \text{ and } \frac{\partial h}{\partial \epsilon_B} = \frac{(n-2)(1-x^*)+\epsilon_B}{(n(1-x^*)+\epsilon_B)^3}\pi_A < 0.$$

■

Proof of Proposition 2.

As explained in the text, the merged entity chooses X_m to maximize the payoff:

$$u_m(X_m;\cdot) = \frac{X_m}{X_m + \sum\limits_{k\neq i,j}^{n} x_k + \epsilon_A}\pi_A + \frac{2-X_m}{2-X_m+\sum\limits_{k\neq i,j}^{n}(1-x_k)+\epsilon_B}\pi_B. \tag{63}$$

The FOC necessary for an interior equilibrium for the merged entity is:

$$\frac{\sum_{k\neq i,j} x_k + \epsilon_A}{\left(X_m + \sum\limits_{k\neq i,j}^{n} x_k + \epsilon_A\right)^2}\pi_A - \frac{n-2-\sum_{k\neq i,j} x_k + \epsilon_B}{\left(n - X_m - \sum\limits_{k\neq i,j}^{n} x_k + \epsilon_B\right)^2}\pi_B = 0. \tag{64}$$

The FOC for the non-merging firms continues to be (62). As before, the second order conditions hold so the existence and uniqueness of a Nash equilibrium is established by the same arguments as in Proposition 1. In the unique SNE, the non-merging firms must play symmetric strategies.

Applying symmetry for the non-merging firms and using the notation $X_{nm} = \sum_{k\neq i,j} x_k$, the FOC for the merging firms can be written as:

$$\frac{X_{nm}+\epsilon_A}{(X_m+X_{nm}+\epsilon_A)^2}\pi_A - \frac{n-2+\epsilon_B-X_{nm}}{(n+\epsilon_B-X_m-X_{nm})^2}\pi_B = 0. \tag{65}$$

Evaluating this FOC at the pre-merger market equilibrium and using the FOC (12) to simplify gives:

$$\frac{\partial u_m}{\partial x_i}(x^*) = -\frac{x^*}{(nx^* + \epsilon_A)^2}\pi_A + \frac{(1 - x^*)}{(n + \epsilon_B - nx^*)^2}\pi_B. \tag{66}$$

Next, rearranging the FOC (12) we get that:

$$\frac{\pi_A}{(nx^* + \epsilon_A)^2} = \frac{n + \epsilon_B - 1 - (n-1)x^*}{(n + \epsilon_B - nx^*)^2((n-1)x^* + \epsilon_A)}\pi_B. \tag{67}$$

Using (67) in (66) yields

$$\frac{\partial u_m}{\partial x_i}(x^*) = \frac{\pi_B((1 - x^*)\epsilon_A - x^*\epsilon_B)}{((n-1)x^* + \epsilon_A)(n(1 - x^*) + \epsilon_B))^2}$$

This is positive whenever $(1 - x^*)\epsilon_A - x^*\epsilon_B > 0$ or, rearranging, when

$$x^* < \frac{\epsilon_A}{\epsilon_A + \epsilon_B}. \tag{68}$$

We now show that this inequality holds whenever $\pi_B/\epsilon_B > \pi_A/\epsilon_A$. This follows from the observation that the FOC (12) is a decreasing function of x^* and when we evaluate it at $\epsilon_A/(\epsilon_A + \epsilon_B)$ we get

$$-\frac{(\epsilon_A + \epsilon_B)(n - 1 + \epsilon_A + \epsilon_B)(\epsilon_A\pi_B - \epsilon_B\pi_A)}{\epsilon_A\epsilon_B(n + \epsilon_A + \epsilon_A\pi_B)^2},$$

which is negative provided that $\pi_B/\epsilon_B > \pi_A/\epsilon_A$. Therefore, x^*, which makes the FOC (12) equal to zero, must be to the left of $\epsilon_A/(\epsilon_A + \epsilon_B)$. This proves that (66) is positive, which implies that the merged entity increases investment in the A-project. ■

Proof of Proposition 3.

To prove this result, we make use of the pseudo best-response functions analysed above. In the pre-merger symmetric equilibrium, this total investment is equal to nx^*, where x^* solves (12). In the post-merger market equilibrium, we have denoted the total investment in the A-project by $X_m + X_{nm}$. The change from nx^* to $X_m + X_{nm}$ after the merger is determined by the slope of the joint best-response function of the non-merging firms, given by the expression (14).

Under the condition $\pi_B/\epsilon_B > \pi_A/\epsilon_A$, we know that X_m increases compared to $2x^*$ after the merger while X_{nm} decreases compared to $(n - 2)x^*$. Consequently, if the slope of the pseudo best-response function of the non-merging firms is smaller than -1, then the increase in X_m is larger than the reduction in X_{nm}. Hence, aggregate investment in the A-project increases after the merger.

Using the implicit function theorem in equation (14), the slope of the pseudo best-response function of the non-merging firms is given by:

$$\frac{\partial X_{nm}}{\partial X_m} = -\frac{-\frac{(n-2)X_m+(n-4)X_{nm}+(n-2)\epsilon_A}{(X_m+X_{nm}+\epsilon_A)^3}\pi_A - \frac{(n-2)(n-2+\epsilon_B)-(n-2)X_m-(n-4)X_{nm}}{(n-X_m-X_{nm}+\epsilon_B)^3}\pi_B}{-\frac{(n-1)X_m+(n-3)X_{nm}+(n-1)\epsilon_A}{(X_m+X_{nm}+\epsilon_A)^3}\pi_A - \frac{n(n-3)+(n-1)+4-(n-1)X_m-(n-3)X_{nm}}{(n-X_m-X_{nm}+\epsilon_B)^3}\pi_B}. \tag{69}$$

This slope is smaller than -1 provided that

$$\frac{(n-1)X_m+(n-3)X_{nm}+(n-1)\epsilon_A}{(X_m+X_{nm}+\epsilon_A)^3}\pi_A+\frac{n(n-3)+(n-1)\epsilon_B+4-(n-1)X_m-(n-3)X_{nm}}{(n-X_m-X_{nm}+\epsilon_B)^3}>$$
$$\frac{(n-2)X_m+(n-4)X_{nm}+(n-2)\epsilon_A}{(X_m+X_{nm}+\epsilon_A)^3}\pi_A+\frac{(n-2)(n-2+\epsilon_B)-(n-2)X_m-(n-4)X_{nm}}{(n-X_m-X_{nm}+\epsilon_B)^3}\pi_B.$$

After simplification, this condition is equivalent to

$$\frac{1}{(X_m+X_{nm}+\epsilon_A)^2}\pi_A>-\frac{1}{(n-X_m-X_{nm}+\epsilon_B)^2}\pi_B,$$

which is always satisfied because the RHS is negative. As a result, total investment X_m+X_{nm} increases after a merger. ■

Proof of Proposition 4.

We start by rewriting the FOC (12) in terms of aggregate investment X in the A-project as follows:

$$\frac{\partial u_i(\cdot)}{\partial x_i}=\frac{\frac{n-1}{n}X+\epsilon_A}{(X+\epsilon_A)^2}\pi_A-\frac{\frac{n-1}{n}(n-X)+\epsilon_B}{(n-X+\epsilon_B)^2}\pi_B. \tag{70}$$

At the pre-merger market equilibrium $X=nx^*$, this expression is equal to zero. We now evaluate it at the socially optimal aggregate investment, X^o:

$$\left.\frac{\partial u_i(\cdot)}{\partial x_i}\right|_{X=X^o}=n\Big(1+\frac{\epsilon_A}{\epsilon_B}+\frac{n-1}{\epsilon_B}\Big)\Big(1-\frac{\frac{\pi_B}{S_B}}{\frac{\pi_A}{S_A}}\sqrt{\frac{S_B\epsilon_A}{S_A\epsilon_B}}\Big)-\Big(1+\frac{\epsilon_A}{\epsilon_B}\frac{\frac{\pi_B}{S_B}}{\frac{\pi_A}{S_A}}\Big)\Big(1-\sqrt{\frac{S_B\epsilon_A}{S_A\epsilon_B}}\Big). \tag{71}$$

If the above expression is negative, it implies that $nx^*<X^o$; in different words, the market under-invests in the A-project and correspondingly over-invests in the B-project. Solving the inequality

$$\left.\frac{\partial u_i(\cdot)}{\partial x_i}\right|_{X=X^o}<0$$

in π_A/π_B gives condition (22) in the proposition. If (71) is instead positive, then the market over-invests in the A-project and under-invests in the B-project. ■

Proof of Proposition 5.

The socially optimal investment in the A-project is given by (21). From equation (23) we have the following FOC:

$$\frac{\frac{n-2}{n-1}X+\epsilon_A}{(X+\epsilon_A)^2}\pi_A-\frac{\frac{n-2}{n-1}(n-X)+\epsilon_B}{(n-X+\epsilon_B)^2}\pi_B=0. \tag{72}$$

Following the approach in the proof of Proposition 4, we evaluate the LHS of the above expression at the socially optimal aggregate investment X^o and then solve for π_A/π_B that makes the LHS of the FOC (72) negative. This gives condition (24). ■

The "non-linear" Tullock model.

In the pre-merger market, the payoff of a firm i is:

$$u(x_i, \mathbf{x}_{-i}) = \frac{x_i^\beta}{\sum\limits_{k=1}^{n} x_k^\beta + \epsilon_A}\pi_A + \frac{(1-x_i)^\beta}{\sum\limits_{k=1}^{n}(1-x_k)^\beta + \epsilon_B}\pi_B, \ i = 1, 2, ..., n.$$

The FOC for the maximization of the profits in the pre-merger market equilibrium is given by:

$$\frac{\beta x_i^{\beta-1}(\sum_{k\neq i} x_k^\beta + \epsilon_A)}{(x_i^\beta + x_j^\beta + \sum_{k\neq i} x_k^\beta + \epsilon_A)^2}\pi_A - \frac{\beta(1-x_i)^{\beta-1}(\sum_{k\neq i}(1-x_k)^\beta + \epsilon_B)}{((1-x_i)^\beta + (1-x_j)^\beta + \sum_{k\neq i}(1-x_k)^\beta + \epsilon_B)^2}\pi_B = 0.$$

In an interior symmetric equilibrium, every firm invests x^* in project A and $1 - x^*$ in project B, where x^* is the solution to:

$$\frac{\beta x^{*\beta-1}((n-1)x^{*\beta} + \epsilon_A)}{(nx^{*\beta} + \epsilon_A)^2}\pi_A - \frac{\beta(1-x^*)^{\beta-1}((n-1)(1-x^*)^\beta + \epsilon_B)}{(n(1-x^*)^\beta + \epsilon_B)^2}\pi_B = 0. \tag{73}$$

Assuming the merged entity keep the research facilities of the constituent firms running, the merged entity's payoff is given by (25). The FOC for the maximization of the profits of the merged entity with respect to x_i is given by:

$$\frac{\beta x_i^{\beta-1}(\sum_{k\neq i,j} x_k^\beta + \epsilon_A)}{(x_i^\beta + x_j^\beta + \sum_{k\neq i,j} x_k^\beta + \epsilon_A)^2}\pi_A - \frac{\beta(1-x_i)^{\beta-1}(\sum_{k\neq i,j}(1-x_k)^\beta + \epsilon_B)}{((1-x_i)^\beta + (1-x_j)^\beta + \sum_{k\neq i,j}(1-x_k)^\alpha + \epsilon_B)^2}\pi_B = 0. \tag{74}$$

The FOC for the maximization of the profits of the merged entity with respect to x_j is similar.

Evaluating (74) at the pre-merger market equilibrium gives:

$$\frac{\beta x^{*\beta-1}((n-2)x^{*\beta} + \epsilon_A)}{(nx^{*\beta} + \epsilon_A)^2}\pi_A - \frac{\beta(1-x^*)^{\beta-1}((n-2)(1-x^*)^\beta + \epsilon_B)}{(n(1-x^*)^\beta + \epsilon_B)^2}\pi_B.$$

Using the FOC (73), this can be simplified to:

$$-\frac{\beta x^{*2\beta-1}}{(nx^{*\beta} + \epsilon_A)^2}\pi_A + \frac{\beta(1-x^*)^{2\beta-1}}{(n(1-x^*)^\beta + \epsilon_B)^2}\pi_B.$$

Using the FOC (73) again, we can write this expression as follows:

$$-\frac{\beta^2 x^{*2\beta-1}(1-x^*)^{\beta-1}((n-1)(1-x^*)^\beta + \epsilon_B)}{(n(1-x^*)^\beta + \epsilon_B)^2 \beta x^{*\beta-1}((n-1)x^{*\beta} + \epsilon_A)}\pi_B + \frac{\beta(1-x^*)^{2\beta-1}}{(n(1-x^*)^\beta + \epsilon_B)^2}\pi_B =$$
$$-\frac{\beta^2 x^{*\beta-1}(1-x^*)^{\beta-1}\pi_B\left[x^{*\beta}((n-1)(1-x^*)^\beta + \epsilon_B) - (1-x^*)^\beta((n-1)x^{*\beta} + \epsilon_A)\right]}{(n(1-x^*)^\beta + \epsilon_B)^2 \beta x^{*\beta-1}((n-1)x^{*\beta} + \epsilon_A)}.$$

This expression is positive if and only if

$$x^{*\beta}\epsilon_B - (1-x^*)^\beta \epsilon_A < 0.$$

This condition is basically the same as condition (68) in the Proof of Proposition 2. Following the same steps, it is easy to show that $x^{*\beta}\epsilon_B - (1-x^*)^\beta\epsilon_A < 0$ whenever (26) holds. ■

The model with asymmetric firms.

Game of strategic substitutes: If $\epsilon_\ell \geqslant \max\{y_1, y_2, ..., y_n\}$, where $\ell = A, B$, then the game of innovation portfolio choice is one of strategic substitutes.

Proof. The cross partial derivative of a firm i's payoff with respect to its own and firm j's investment is:

$$\frac{x_i - \sum_{j \neq i} x_j - \epsilon_A}{(x_i + \sum_{j \neq i} x_j + \epsilon_A)^3}\pi_A + \frac{y_i - x_i - \sum_{j \neq i}(y_j - x_j) - \epsilon_B}{(y_i - x_i + \sum_{j \neq i}(y_j - x) + \epsilon_B)^2}\pi_B. \tag{75}$$

As the budget is fixed, the maximum value that investment levels x_i and $y_i - x_i$ can achieve is y_i. Therefore, (75) is negative if $\epsilon_A, \epsilon_B \geqslant \max\{y_1, y_2, ..., y_n\}$. ■

Pre-merger market equilibrium: In the pre-merger market, there exists a unique Nash equilibrium of the portfolio investment game denoted by $\mathbf{x}^* = \{x_1^*, x_2^*, ..., x_n^*\}$ where

$$x_j^* = \frac{(y_j - y_i)\epsilon_A}{(n-1)y_i + \epsilon_A + \epsilon_B} + \frac{(n-1)y_j + \epsilon_A + \epsilon_B}{(n-1)y_i + \epsilon_A + \epsilon_B}x_i^* \quad \text{for any} \quad i, j \in N, i \neq j.$$

This is an interior equilibrium if and only if $\underline{\pi} < \frac{\pi_A}{\pi_B} < \overline{\pi}$, where $\underline{\pi}$ and $\overline{\pi}$ are provided in the proof below.

Proof. In the pre-merger market equilibrium, a firm i maximises the expression:

$$u_i(x_i, \mathbf{x}_{-i}, y_i, \mathbf{y}_{-i}) = \frac{x_i}{\sum_{k=1}^{n} x_k + \epsilon_A}\pi_A + \frac{y_i - x_i}{\sum_{k=1}^{n}(y_k - x_k) + \epsilon_B}\pi_B. \tag{76}$$

The FOC for an interior equilibrium is given by (28), which can be manipulated to write is as:

$$\frac{\sum_{j \neq i} x_j^* + \epsilon_A}{(x_i^* + \sum_{j \neq i} x_j^* + \epsilon_A)^2}\pi_A = \frac{\sum_{j \neq i}(y_j - x_j^*) + \epsilon_B}{(y_i - x_i^* + \sum_{j \neq i}(y_j - x_j^*) + \epsilon_B)^2}\pi_B, \quad i = 1, 2, ..., n. \tag{77}$$

It is straightforward to verify that the second order condition for profits maximization holds. As the payoff is strictly concave in firm i's own investment and continuous in x_{-i}, the existence of equilibrium follows from the Debreu-Glicksberg-Fan theorem. By Assumption 2 we know that there exists a unique NE.

Next, dividing the FOC of a firm i in (77) by that of the firm $i+1$, we convert the system of n FOCs into a system of $n-1$ linear equations:

$$\frac{x_{i+1}^* + \sum_{k \neq i, i+1} x_k^* + \epsilon_A}{x_i^* + \sum_{k \neq i, i+1} x_k^* + \epsilon_A} = \frac{(y_{i+1} - x_{i+1}^*) + \sum_{k \neq i, i+1}(y_j - x_j^*) + \epsilon_B}{(y_i - x_i^*) + \sum_{k \neq i, i+1}(y_k - x_k^*) + \epsilon_B}, \quad i = 1, 2, ..., n-1. \tag{78}$$

Through algebraic manipulations, we can rewrite the above system as follows:

$$x_{i+1}^* = \frac{(y_{i+1}-y_i)\left(\sum\limits_{j\neq i,i+1} x_j^* + \epsilon_A\right) + \left(y_{i+1} + \sum\limits_{k\neq i,i+1} y_k + \epsilon_A + \epsilon_B\right) x_i^*}{y_i + \sum\limits_{k\neq i,i+1} y_k + \epsilon_A + \epsilon_B}, \quad i = 1,2,...,n-1.$$

Solving the above system of equations gives:

$$x_j^* = \frac{(y_j - y_i)\epsilon_A}{(n-1)y_i + \epsilon_A + \epsilon_B} + \frac{(n-1)y_j + \epsilon_A + \epsilon_B}{(n-1)y_i + \epsilon_A + \epsilon_B} x_i^* \quad \text{for any} \quad i, j \in N, i \neq j. \tag{79}$$

Using these relationships, the system of FOCs in (77) can be rewritten as follows:

$$\begin{aligned} h(x_i^*) \quad \equiv \quad & \frac{(n-1)x_i^* + \epsilon_A}{\left(\epsilon_A\left(\sum\limits_{j\neq i} y_j + \epsilon_A + \epsilon_B\right) + x_i^*\left((n-1)y_i + (n-1)\sum\limits_{j\neq i} y_j + n(\epsilon_A+\epsilon_B)\right)\right)^2}\pi_A \\ & - \frac{(n-1)(y_i - x_i^*) + \epsilon_B}{\left((y_i - x_i^*)\left((n-1)y_i + (n-1)\sum\limits_{j\neq i} y_j + n\epsilon_A\right) + \left(n(y_i - x_i^*) + \sum\limits_{j\neq i} y_j + \epsilon_A\right)\epsilon_B + \epsilon_B^2\right)^2}\pi_B \end{aligned} \tag{80}$$

Observe that $h(x_i^*)$ is univariate, continuous and monotone decreasing in x_i^*.

From (79) we can conclude that $x_i^* > x_j^*$ if and only if $y_i > y_j$. Hence, if $y_j = \underline{y}$, where $\underline{y} = \min\{y_1, y_2, ..., y_n\}$ then $x_j^* = \underline{x}^* = \min\{x_1^*, x_2^*, ..., x_n^*\}$. If $h(x_j^* = 0) > 0$, then $\underline{x}^* > 0$ which implies that $x_i^* > 0$ for all $i \in N$. Moreover, if $h(x_j^* = y_j) < 0$, then $x_j^* < y_j$. The condition $h(x_j^* = 0) > 0$ can be simplified to:

$$\frac{\pi_A}{\pi_B} > \underline{\pi} \quad \text{where} \quad \underline{\pi} = \frac{\epsilon_A\left((n-1)y_j + \epsilon_B\right)\left(\sum\limits_{k\neq j} y_k + \epsilon_A + \epsilon_B\right)^2}{\left((n-1)y_j\left(\sum\limits_k y_k\right) + ny_j\epsilon_A + \left(ny_j + \sum\limits_{k\neq j} y_k + \epsilon_A\right)\epsilon_B + \epsilon_B^2\right)^2}.$$

Similarly, the condition $h(x_j^* = y_j) < 0$ can be written as:

$$\frac{\pi_A}{\pi_B} < \bar{\pi}_j \quad \text{where} \quad \bar{\pi}_j = \frac{\left(\left((n-1)y_j + \epsilon_A\right)\left(\sum\limits_k y_k + \epsilon_A\right) + (ny_j + \epsilon_A)\epsilon_B\right)^2}{\epsilon_B\left((n-1)y_j + \epsilon_A\right)\left(\sum\limits_{k\neq j} y_j + \epsilon_A + \epsilon_B\right)^2}.$$

Therefore, the unique equilibrium is interior provided

$$\underline{\pi} < \frac{\pi_A}{\pi_B} < \bar{\pi} \quad \text{where} \quad \bar{\pi} = \min\{\bar{\pi}_1, \bar{\pi}_2 ... \bar{\pi}_n\}$$

■

Effect of a merger on the merged entity's innovation portfolio: The merged entity's investment in project A increases if and only if

$$\frac{\pi_A}{\epsilon_A} < \frac{\pi_B}{\epsilon_B}. \tag{81}$$

Consequently, the merged entity's investment in project B decreases.

Proof. This proof follows the same steps as Proposition 2, but for asymmetric budgets. The merged entity chooses X_m to maximize the payoff in (31). We evaluate the FOC (32) at the pre-merger market equilibrium using the relation (29) to rewritte all equilibrium investments in terms of x_i^*:

$$\frac{\partial u_m}{\partial x_i}(\mathbf{x}^*) = \frac{((n-1)y_i+\epsilon_A+\epsilon_B)^2\left(y_i\epsilon_A - x_i^*(\epsilon_A+\epsilon_B)\right)}{((n-1)x_i^*+\epsilon_A)\left((y_i-x_i^*)\left((n-1)\sum_j y_j + n\epsilon_A\right) + \left(n(y_i-x_i^*)+\sum_{k\neq i} y_k+\epsilon_A\right)\epsilon_B+\epsilon_B^2\right)^2}\pi_B \tag{82}$$

This is positive whenever $y_i\epsilon_A - x_i^*(\epsilon_A+\epsilon_B) > 0$ or, rearranging, when

$$x_i^* < \frac{y_i\epsilon_A}{\epsilon_A+\epsilon_B}. \tag{83}$$

We now show that this inequality holds whenever $\pi_B/\epsilon_B > \pi_A/\epsilon_A$.

Because the pre-merger FOC in (80) is a decreasing funciton of x_i^*, when we evaluate it at $(y_i\epsilon_A)/(\epsilon_A+\epsilon_B)$ we get

$$-\frac{(\epsilon_A+\epsilon_B)\left(\sum_{k\neq i} y_k+\epsilon_A+\epsilon_B\right)(\epsilon_A\pi_B-\epsilon_B\pi_A)}{\epsilon_A\epsilon_B\left(\sum_{j=1}^n y_j+\epsilon_A+\epsilon_A\pi_B\right)^2},$$

which is negative provided that $\pi_B/\epsilon_B > \pi_A/\epsilon_A$. Therefore, x_i^*, which makes the FOC (80) equal to zero, must be to the left of $y_i\epsilon_A/(\epsilon_A+\epsilon_B)$. This proves that (82) is positive, which implies that the merged entity increases investment in the A-project. ■

Effect of a merger on the industry's aggregate innovation portfolio: The aggregate industry investment in project A increases if and only if

$$\frac{\pi_A}{\epsilon_A} < \frac{\pi_B}{\epsilon_B}. \tag{84}$$

Consequently, the aggregate industry investment investment in project B decreases.

Proof. To prove this result, we make use of the pseudo best-response functions introduced in Section 3. In the pre-merger market equilibrium, total investment is equal to $\sum_{j=1}^n x_j^*$, where $\{x_1^*, x_2^*, ..., x_n^*\}$ solves the system of equations (28). In the post-merger market equilibrium, assuming firms i and j merge, we have denoted the total investment in the A-project by X_m+X_{nm}. The change from $\sum_{j=1}^n x_j^*$ to X_m+X_{nm} after the merger is determined by the slope of the joint best-response function of the non-merging firms, which is given implicitly by the expression:

$$\frac{X_m+\frac{n-3}{n-2}X_{nm}+\epsilon_A}{(X_m+X_{nm}+\epsilon_A)^2}\pi_A - \frac{Y_m-X_m+\frac{n-3}{n-2}(Y_{nm}-X_{nm})+\epsilon_B}{(n+\epsilon_B-X_m-X_{nm})^2}\pi_B = 0. \tag{85}$$

where $Y_m = y_i + y_j$ and $Y_{nm} = \sum_{k \neq i,j} y_k$. This equation is the same as (14) when $y_i = 1$, for all i. Using the implicit function theorem as in Proposition 3, the result follows. ■

Effects of a merger on consumer welfare: In the pre-merger and post-merger market equilibrium, there is under-investment in the A-project (and correspondingly over-investment in the B-project) if and only if the respective inequalities are satisfied:

$$\frac{\pi_A}{\pi_B} < f_a(S_A/S_B; n; \pi_A; \pi_B; \mathbf{y}) \text{ and } \frac{\pi_A}{\pi_B} < g_a(S_A/S_B; n; \pi_A; \pi_B; \mathbf{y}) \text{ where } \mathbf{y} = \{y_1, y_2, ..., y_n\}.$$

The pre-merger and post-merger market over-invest in the A-project (and thus under-invests in the B-project) if and only if the respective inequalities are reversed.

Proof. We first calculate the socially optimal investment in the A-project, X^o, by maximizing equation the planner's payoff

$$W(x_i, \mathbf{x}_{-i}) = \frac{X}{X + \epsilon_A} S_A + \frac{Y - X}{Y - X + \epsilon_B} S_B \text{ where } X = \sum_{j=1}^{n} x_j \text{ and } Y = \sum_{i=1}^{n} y_i.$$

to obtain

$$X^o = \frac{Y + \epsilon_B - \epsilon_A \sqrt{\frac{S_B \epsilon_B}{S_A \epsilon_A}}}{1 + \sqrt{\frac{S_B \epsilon_B}{S_A \epsilon_A}}}. \tag{86}$$

We now rewrite the FOC (77) in terms of aggregate investment X in the A-project using the relation (79 as follows

$$\frac{\partial u_i(\cdot)}{\partial x_i} = \frac{n \left(\sum_{j \neq i} y_j + \epsilon_A + \epsilon_B \right)}{((n-1)Y + n(\epsilon_A + \epsilon_B))} \left(\frac{\frac{n-1}{n} X + \epsilon_A}{(X + \epsilon_A)^2} \pi_A - \frac{\frac{n-1}{n}(Y - X) + \epsilon_B}{(Y - X + \epsilon_B)^2} \pi_B \right) \tag{87}$$

At the pre-merger market equilibrium X, this expression is equal to zero. We now evaluate it at the socially optimal aggregate investment, X^o:

$$\left.\frac{\partial u_i(\cdot)}{\partial x_i}\right|_{X = X^o} = \left((n-1)(Y + \epsilon_B) + \epsilon_A \left(n + \sqrt{\frac{S_B \epsilon_B}{S_A \epsilon_A}}\right)\right) \pi_A S_B \epsilon_B - \left((n-1)(Y + \epsilon_A)\sqrt{\frac{S_B \epsilon_B}{S_A \epsilon_A}} + \epsilon_B \left(1 + n\sqrt{\frac{S_B \epsilon_B}{S_A \epsilon_A}}\right)\right) \pi_B S_A \epsilon_A \tag{88}$$

If the above expression is negative, it implies that $nx^* < X^o$; in different words, the market under-invests in the A-project and correspondingly over-invests in the B-project. Solving the inequality

$$\left.\frac{\partial u_i(\cdot)}{\partial x_i}\right|_{X = X^o} < 0$$

in π_A/π_B gives the following condition:

$$\frac{\pi_A}{\pi_B} < \frac{\left((n-1)(Y + \epsilon_A)\sqrt{\frac{S_B \epsilon_B}{S_A \epsilon_A}} + \epsilon_B \left(1 + n\sqrt{\frac{S_B \epsilon_B}{S_A \epsilon_A}}\right)\right) S_A \epsilon_A}{\left((n-1)(Y + \epsilon_B) + \epsilon_A \left(n + \sqrt{\frac{S_B \epsilon_B}{S_A \epsilon_A}}\right)\right) S_B \epsilon_B} \equiv f_a(S_A/S_B; n; \pi_A; \pi_B; Y) \tag{89}$$

If (88) is instead positive, then the market over-invests in the A-project and under-invests in the B-project. Similarly, we can derive $g_a(S_A/S_B; n; \pi_A; \pi_B; Y)$ for the post-merger market equilibrium. The welfare results described in Section 3 are then similar in this model with asymmetric budgets. ■

Concavity of payoffs in the endogenous research budget model.

We analyze first the conditions under which the payoff of a firm i in the pre-merger market given in (35) is strictly concave in x_i and y_i for arbitrary investment efforts of the rival firms. For this, we compute the Hessian matrix of the payoff function:

$$\mathbf{H}_u = \begin{bmatrix} \frac{-2\left(\sum_{k\neq i} x_k + \epsilon_A\right)}{\left(\sum\limits_{k=1}^{n} x_k + \epsilon_A\right)^3}\pi_A - c''(x_i+y_i) & -c''(x_i+y_i) \\ -c''(x_i+y_i) & \frac{-2\left(\sum_{k\neq i} y_k + \epsilon_B\right)}{\left(\sum\limits_{k=1}^{n} y_k + \epsilon_B\right)^3}\pi_B - c''(x_i+y_i) \end{bmatrix}.$$

The determinants of the leading principal minors are

$$|L_1| = \frac{-2\left(\sum_{k\neq i} x_k + \epsilon_A\right)}{\left(\sum\limits_{k=1}^{n} x_k + \epsilon_A\right)^3}\pi_A - c''(x_i+y_i),$$

$$|L_2| = \frac{4\left(\sum_{k\neq i} x_k + \epsilon_A\right)\left(\sum_{k\neq i} y_k + \epsilon_B\right)\pi_A\pi_B}{\left(\sum\limits_{k=1}^{n} x_k + \epsilon_A\right)^3\left(\sum\limits_{k=1}^{n} y_k + \epsilon_B\right)^3} + 2c''(x_i+y_i)\left(\frac{\left(\sum_{k\neq i} x_k + \epsilon_A\right)\pi_A}{\left(\sum\limits_{k=1}^{n} x_k + \epsilon_A\right)^3} + \frac{\left(\sum_{k\neq i} y_k + \epsilon_B\right)\pi_B}{\left(\sum\limits_{k=1}^{n} y_k + \epsilon_B\right)^3}\right).$$

Note that

$$|L_1| < 0 \Leftrightarrow c''(x_i+y_i) > -\frac{2\left(\sum_{k\neq i} x_k + \epsilon_A\right)}{\left(\sum\limits_{k=1}^{n} x_k + \epsilon_A\right)^3}\pi_A \text{ and} \tag{90}$$

$$|L_2| > 0 \Leftrightarrow c''(x_i+y_i) > -\frac{2\left(\sum_{k\neq i} x_k + \epsilon_A\right)\left(\sum_{k\neq i} y_k + \epsilon_B\right)\pi_A\pi_B}{\left(\sum_{k\neq i} x_k + \epsilon_A\right)\left(\sum\limits_{k=1}^{n} y_k + \epsilon_B\right)^3\pi_A + \left(\sum_{k\neq i} y_k + \epsilon_B\right)\left(\sum\limits_{k=1}^{n} x_k + \epsilon_A\right)^3\pi_B}.$$

When the conditions in (90) are satisfied, the Hessian $\mathbf{H}_u$ is negative definite and the firm's payoff is strictly concave in own investments. This definitely holds for convex and linear cost functions, i.e., when $c''(\cdot) \geqslant 0$, but may also hold for slightly concave cost functions.[34] In such cases, as mentioned above, a symmetric equilibrium in pure strategies exists. When the equilibrium is interior, it is given by solution to the system of FOCs for the maximization of firm i's payoff.

We now examine the conditions under which the payoff of the merged entity given in (37) is concave in x_i x_j, y_i and y_j for arbitrary investment efforts of the rival firms. For this, we calculate the 4x4 Hessian matrix of the payoff function of the merged entity. As mentioned above, the FOCs imply that the merged entity invests symmetrically across the research facilities of the constituent firms, i.e. $x_i + y_i = x_j + y_j$

[34] For the parametric cost function given by $c(x_i+y_i) = \frac{1}{\alpha}(x_i+y_i)^{\alpha}$, this holds when $\alpha \geqslant 1$.

and therefore $c''(x_i + y_i) = c''(x_j + y_j) = c''$. The Hessian matrix of the payoff function of the merged entity is:

$$\mathbf{H}_{u_m} = \begin{bmatrix} -A-c'' & -A & -c'' & 0 \\ -A & -A-c'' & 0 & -c'' \\ -c'' & 0 & -B-c'' & -B \\ 0 & -c'' & -B & -B-c'' \end{bmatrix},$$

where

$$A \equiv \frac{2\left(\sum_{k\neq i,j} x_k + \epsilon_A\right)}{\left(\sum_{k=1}^{n} x_k + \epsilon_A\right)^3}\pi_A, \quad B \equiv \frac{2\left(\sum_{k\neq i,j} y_k + \epsilon_B\right)}{\left(\sum_{k=1}^{n} y_k + \epsilon_B\right)^3}\pi_B.$$

It is easy to show that the determinants of the leading principal minors are as follows:

$$\begin{aligned} |L_1| &= -A-c'', \\ |L_2| &= 2Ac'' + \left(c''\right)^2, \\ |L_3| &= -\left(A+c''\right)^2\left(B+c''\right) + 0 + 0 - \left(c''\right)^2\left(-A-c''\right) - 0 - A^2\left(-B-c''\right) \\ &= -2ABc'' - (A+B)\left(c''\right)^2, \\ |L_4| &= \left(A+c''\right)c''\left[Bc'' + Ac'' + 2AB\right] - Ac''\left[2AB + Ac'' + Bc''\right] - \left(c''\right)^2\left[Bc'' + Ac'' + 2AB\right] = 0. \end{aligned}$$

Since $|L_4| = 0$ we cannot conclude that the Hessian is negative definite. Hence, we cannot conclude that the payoff function is strictly concave.

Further analysis of the determinants of all the principal minors shows that the Hessian matrix is negative semidefinite when $c'' > 0$.[35] This implies that for convex costs the merged entity's objective function is concave and an interior solution to the FOCs is a local maximum. When the cost function is concave, the signs of the determinants of the principal minors do not follow the correct pattern and therefore we cannot conclude that the objective function is concave. ■

Proof of Proposition 7.

We start by showing that the gradient of the payoff of the merged entity in (37) evaluated at pre-merger symmetric equilibrium is negative. In fact,

$$\begin{aligned} \left.\frac{\partial u_m}{\partial x_m}\right|_{x_m = x_{nm} = x^*, y_m = y_{nm} = y^*} &= \frac{(n-2)x^* + \epsilon_A}{(nx^* + \epsilon_A)^2}\pi_A - (x^* + y^*)^{\alpha-1} = -\frac{x^*\pi_A}{(nx^* + \epsilon_A)^2} < 0 \\ \left.\frac{\partial u_m}{\partial y_m}\right|_{x_m = x_{nm} = x^*, y_m = y_{nm} = y^*} &= \frac{(n-2)y^* + \epsilon_B}{(ny^* + \epsilon_B)^2}\pi_B - (x^* + y^*)^{\alpha-1} = -\frac{y^*\pi_B}{(ny^* + \epsilon_B)^2} < 0, \end{aligned}$$

where we have used the FOCs in (36) to simplify.

[35] For the Hessian to be negative semidefinite it must be the case that $(-1)^k\Delta_k \geqslant 0$, where Δ_k are the determinants of all the principal minors of order k of the Hessian matrix. The principal minors of a $n \times n$ matrix H of order k are obtained by deleting $n-k$ rows and the corresponding $n-k$ columns. For example, Δ_2^{13} is the principal minor of H of order 2 obtained by deleting the first and the third rows and columns from the original 4x4 matrix. Then, it is straightforward to show that when $c'' > 0$ we have $\Delta_1^{hij} \leqslant 0$ for all h, i, j, $\Delta_2^{ij} \geqslant 0$ for all i, j and $\Delta_3^i \leqslant 0$ for all i. In particular, $\Delta_3^1 = -\left(A+c''\right)\left(B+c''\right)^2 + \left(c''\right)^2\left(B+c''\right) + B^2\left(A+c''\right) \leqslant 0$, $\Delta_3^2 = -\left(A+c''\right)\left(B+c''\right)^2 + B^2(A+c'') + \left(c''\right)^2\left(B+c''\right) \leqslant 0$, $\Delta_3^3 = -\left(A+c''\right)^2\left(B+c''\right) + \left(c''\right)^2\left(A+c''\right) + A^2\left(B+c''\right) \leqslant 0$, and $\Delta_3^4 = |L_3| \leqslant 0$.

Our second observation concerns the best-response of the merged entity, which is given by the solution to the system of equations in (39)-(40). Setting $(x_{nm}, y_{nm}) = (x^*, y^*)$, the FOC in (39) gives a relationship:

$$y_m^1(x_m) = \left(\frac{(n-2)x^* + \epsilon_A}{(2x_m + (n-2)x^* + \epsilon_A)^2} \pi_A \right)^{\frac{1}{\alpha-1}} - x_m. \tag{91}$$

This relationship is downward sloping and has a slope smaller than -1. This can be seen by computing

$$\frac{\partial y_m^1}{\partial x_m} = -1 - \frac{2\pi_A^{\frac{1}{\alpha-1}}}{\alpha - 1} \left(\frac{(n-2)y^* + \epsilon_A}{(2x_m + (n-2)x^* + \epsilon_A)^2} \right)^{\frac{1}{\alpha-1}} \frac{1}{2x_m + (n-2)x^* + \epsilon_A} < -1.$$

The function $y_m^1(x_m)$ is invertible and we denote its inverse as $x_m^1(y_m)$.

Likewise, the FOC in (40) defines a relationship:

$$x_m^2(y_m) = \left(\frac{(n-2)y^* + \epsilon_B}{(2y_m + (n-2)y^* + \epsilon_B)^2} \pi_B \right)^{\frac{1}{\alpha-1}} - y_m. \tag{92}$$

The function $x_m^2(y_m)$ is downward sloping. We denote its inverse as $y_m^2(x_m)$. The function $y_m^2(x_m)$ has a slope greater than -1. This follows from applying the inverse function theorem in (92):

$$\frac{\partial y_m^2}{\partial x_m} = \left(\frac{\partial x_m^2}{\partial y_m} \right)^{-1} = \left(-1 - \frac{2\pi_B^{\frac{1}{\alpha-1}}}{\alpha - 1} \left(\frac{(n-2)y^* + \epsilon_B}{(2y_m + (n-2)y^* + \epsilon_B)^2} \right)^{\frac{1}{\alpha-1}} \frac{1}{2y_m + (n-2)y^* + \epsilon_A} \right)^{-1} > -1,$$

where the inequality follows from the fact that the expression in parenthesis is smaller than -1. Because the slope of $y_m^1(x_m)$ is smaller than -1 and that of $y_m^2(x_m)$ is greater than -1, the function $y_m^1(x_m)$ intersects the function $y_m^2(x_m)$ from above.

We are now ready to show that, keeping fixed the portfolio of investments of the non-merging firms, the merged entity adjusts its portfolio of investments so that $x_m + y_m < x^* + y^*$. To see this, we make use of Figure 8. In this figure, we have represented the functions $y_m^1(x_m)$ and $y_m^2(x_m)$. The intersection between these functions gives the best-response of the merged entity, (x_m^*, y_m^*), keeping the investment of the non-merging firms at the pre-merger symmetric equilibrium. Because we have shown that both components of the gradient of the payoff of the merged entity at the pre-merger symmetric equilibrium are negative, we know that the pre-merger symmetric equilibrium (x^*, y^*) is located somewhere in the coloured regions, denoted regions A, B and C.

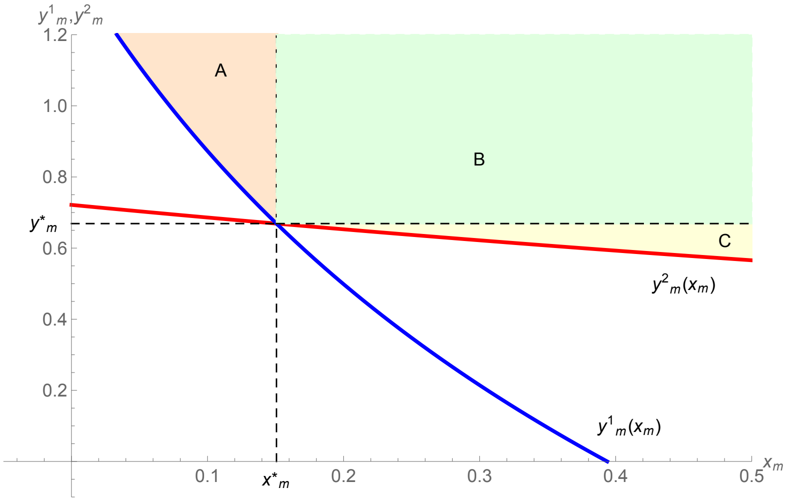


Figure 8: Model with endogenous efforts

It is obvious that if (x^*, y^*) lies in region B, $x_m + y_m < x^* + y^*$, moreover $x_m < x^*$ and $y_m < y^*$. Suppose now that (x^*, y^*) lies in region A. In this case $x_m > x^*$ and $y_m < y^*$ but because the slope of $y_m^1(x_m)$ is smaller than -1, it must be the case that $x_m + y_m < x^* + y^*$. Finally, suppose (x^*, y^*) lies in region C. In this case $x_m < x^*$ and $y_m > y^*$ but because the slope of $y_m^2(x_m)$ is greater than -1, it must be the case that $x_m + y_m < x^* + y^*$.

To conclude we provide the characterization of the regions A, B and C. When (x^*, y^*) lies in region A, because $y_m^1(x_m)$ intersects the function $y_m^2(x_m)$ from above, we must have $y_m^1(x^*) > y_m^2(x^*)$. Using (91) and (92), we can write:

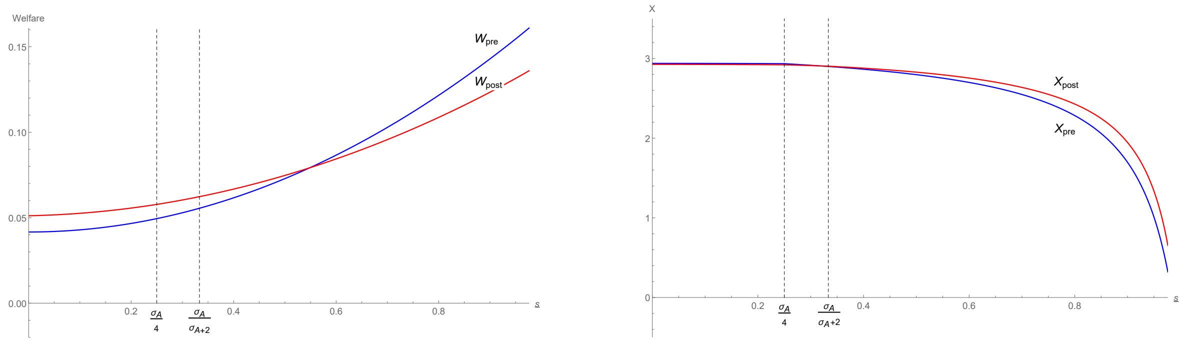
$$
\begin{aligned}
& y_m^1(x^*) > y_m^2(x^*) \Leftrightarrow \\
& \left(\frac{(n-2)x^* + \epsilon_A}{(nx^* + \epsilon_A)^2}\pi_A\right)^{\frac{1}{\alpha-1}} - x^* > \left(\frac{(n-2)y^* + \epsilon_B}{(2y_m^2(x^*) + (n-2)y^* + \epsilon_B)^2}\pi_B\right)^{\frac{1}{\alpha-1}} - x^* \Leftrightarrow \\
& \frac{\pi_A}{\pi_B} > \frac{(nx^* + \epsilon_A)^2((n-2)y^* + \epsilon_B)}{((n-2)x^* + \epsilon_A)(2y_m^2(x^*) + (n-2)y^* + \epsilon_B)^2},
\end{aligned}
$$

which is the condition in Proposition 7(i).

Likewise, when (x^*, y^*) lies in region C, because $y_m^1(x_m)$ intersects the function $y_m^2(x_m)$ from above, we must have $x_m^1(y^*) < x_m^2(y^*)$. Proceeding similarly as above, we obtain the condition in Proposition 7(ii). Finally, when (x^*, y^*) lies in region B, parameters must be such that $x_m^2(y^*) < x_m^1(y^*)$ and $y_m^1(x^*) < y_m^2(x^*)$. ■

Price effects of mergers when the products of the winning and losing firms are horizontally differentiated.

In Figure 9 we report results for the case in which $\sigma_A = 1$. The rest of the parameters are exactly the same as in Section 4.4. In the graphs of Figure 9, we depict two threshold values. The first, $\sigma_A/4$, corresponds to the pre-merger market equilibrium. When $\underline{s} < 1/4$, the winning firm monopolizes the market and therefore price effects only arise when no firm wins the contest for the A-innovation. The second threshold, $\sigma_A/(\sigma_A + 2)$, corresponds to the post-merger. When $\underline{s} < 1/3$, the winning firm monopolizes the market.



(a) Consumer welfare

(b) Aggregate investment in A

Figure 9: Pre- and post-merger welfare and aggregate investment in the A-project

The figure shows that introducing product differentiation does not change the main insights. For low $\underline{s}$ a merger increases consumer welfare by (slightly) reducing investment in the A-project. As we increase $\underline{s}$ the price effects of a merger become stronger, eventually turning a merger undesirable for consumers.

Finally, in Figure 10 we report the profits of merging and non-merging firms. It can be seen that mergers are incentive compatible. For low levels of the quality parameter $\underline{s}$, the non-merging firms benefit less than the merging firms. However, when $\underline{s}$ is quite large and the price effects are quite strong, the non-merging firms benefit more than the merging firms.

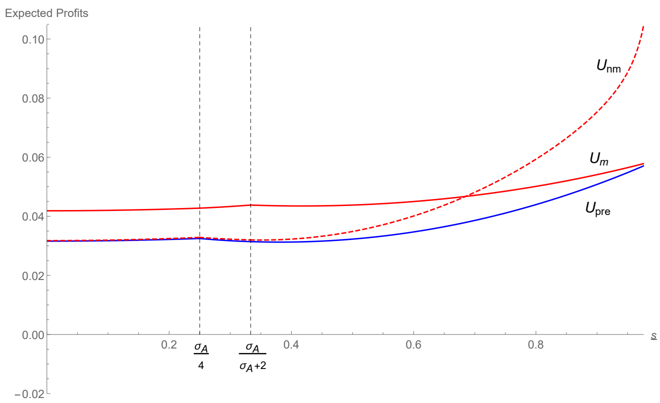


Figure 10: Pre- and post-merger profits of merging and non-merging firms.

Proof of Proposition 8.

Note that the payoff function (1) is strictly concave in firm i's own investment because the second derivatives $\partial^2 p_i^A(x_i, \mathbf{x}_{-i}, \epsilon_A)/\partial x_i^2$ and $\partial^2 p_i^B(1 - x_i, \mathbf{1} - \mathbf{x}_{-i}, \epsilon_B)/\partial x_i^2$ are negative. The existence of equilibrium then follows from the Debreu-Glicksberg-Fan theorem because the strategy spaces are compact and convex sets, and the payoff functions are strictly concave in x_i and continuous in x_{-i}. Because our game is symmetric, there exists a SNE $x_i = x^*$ (see Hefti, 2017).

Uniqueness follows from Assumption 2. Rewrite equation (56) as follows:

$$\frac{\partial p_i^A(x^*, \mathbf{x}^*, \epsilon_A)}{\partial x_i}\pi_A = -\frac{\partial p_i^B(1 - x^*, \mathbf{1} - \mathbf{x}^*, \epsilon_B)}{\partial x_i}\pi_B. \tag{93}$$

Note now that Assumption 2 implies that the LHS of (93) is strictly decreasing in x^*, while the RHS of (93) is strictly increasing in x^*. Hence, given the conditions in the Proposition, the LHS and RHS surely intersect once and only once at x^*.

Finally, the comparative statics of x^* with respect to the parameters of the model follow from a straightforward application of the implicit function theorem to equation (56). For the signs, we employ again Assumption 2. ■

Proof of Proposition 9.

For any vector of investments in project A of the non-merging firms $\mathbf{x}_{-ij}$, the best-response surface of the merged entity is given by the solution to the system of equations given by FOC (4) and the symmetric FOC corresponding to the decision variable x_j (which has been omitted to save on space). Evaluating the LHS of the FOCs of the merged entity at the pre-merger symmetric equilibrium $\mathbf{x}^*$ gives:

$$\frac{\partial p_j^A(\mathbf{x}^*, \epsilon_A)}{\partial x_i}\pi_A + \frac{\partial p_j^B(\mathbf{1} - \mathbf{x}^*, \epsilon_B)}{\partial x_i}\pi_B,$$
$$\frac{\partial p_i^A(\mathbf{x}^*, \epsilon_A)}{\partial x_j}\pi_A + \frac{\partial p_i^B(\mathbf{1} - \mathbf{x}^*, \epsilon_B)}{\partial x_j}\pi_B,$$

where we have used the fact that x^* satisfies the FOC (2). Because of symmetry, the above expressions are positive if and only if condition (57) holds, in which case the payoff of the merged entity increases at $\mathbf{x}^*$. This implies that, relative to the pre-merger situation, the best-reply surface of the merged entity shifts up so that the merged entity invests more in project A and less in project B. Regarding the non-merging firms, when the game is of strategic substitutes (decreasing best-replies), they cut investment in the A-project and correspondingly increase it in the B-project. When the game is of strategic complements (increasing best-replies), the non-merging firms also raise investment in the A-project and thus cut it in the B-project. ■

Proof of Proposition 10.

Evaluating the LHS of the social planner's FOC (6) at the pre-merger market equilibrium $\mathbf{x}^*$ gives the LHS of (58), where we have used the fact that x^* solves (2) and $W_\ell = \pi_\ell + S_\ell$, $\ell = A, B$. When this expression is positive, the payoff of the social planner increases in x_i at $\mathbf{x}^*$. This implies that, relative to the planner's choice, the market puts too little funding in the A-project and, correspondingly, too much in the B-project. ■

Proof of Proposition 11.

The following arguments prove result a. Suppose conditions (57) and (58) hold. Note first that condition (57) means that the merging firms allocate more funding to the A-project in the post-merger market equilibrium than in the pre-merger one. Because of strategic complementarity, so do the non-merging firms. Hence, all the firms invest more in the A-project post-merger than pre-merger. Observe now that condition (58) means that, from the point of view of social welfare maximization, in the pre-merger market equilibrium firms under-invest in the A-project. Therefore, if the gradient of the social welfare function at the post-merger market equilibrium is strictly positive, which is ensured by condition

(59), then we are sure that the welfare level increases after a merger. Results *b*, *c* and *d* follow from similar arguments and we omit the details to save on space. ■

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