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Abstract

We study how parliaments and other committees vote to select one out of several alternatives in situations where not all available options can be ordered along a \left-right" axis. Practically all democratic parliaments routinely use Sequential Binary Voting Procedures in or- der to select one of several alternatives. Which agendas are used in practice, and how should they be designed ? We assume that pref- erences are single-peaked on an arbitrary tree and we study convex agendas where, at each stage in the sequential, binary voting process, the tree of remaining alternatives is divided in two subtrees that are subjected to a binary Yes-No vote. In this wide class of situations we show that dynamic, strategic voting is congruent with sincere, unsophisticated voting even if agents are privately informed, and no matter what their beliefs about other voters are. We conclude the paper by illustrating the empirical implications of our results for two large case studies from Germany and from the UK.

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Abortions, Brexit and Trees

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November 27, 2019

Abstract

We study how parliaments and other committees vote to select one out of several alternatives in situations where not all available options can be ordered along a "left-right" axis. Practically all democratic parliaments routinely use Sequential Binary Voting Procedures in order to select one of several alternatives. Which agendas are used in practice, and how should they be designed ? We assume that preferences are single-peaked on an arbitrary tree and we study *convex* agendas where, at each stage in the sequential, binary voting process, the tree of remaining alternatives is divided in two subtrees that are subjected to a binary Yes-No vote. In this wide class of situations we show that dynamic, strategic voting is congruent with sincere, unsophisticated voting even if agents are privately informed, and no matter what their beliefs about other voters are. We conclude the paper by illustrating the empirical implications of our results for two large case studies from Germany and from the UK.

1 Introduction

We study how parliaments and other committees vote to select one out of several alternatives in situations where not all available options can be ordered along a "left-right" axis.

For example, in a well-known abortion legislation case from the German Bundestag , the main axis of conflict pitted the rights of women versus the

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rights of unborn life, but many of the eight proposed bills contained additional provisions about deadlines that need to be respected for legal abortions, possible punishments both for women and for doctors that perform illegal abortions, the need for counseling, psycho-social indications, etc.. Certain pairs of alternatives were easily ordered, while other pairs were not comparable along the main axis. The German Bundestag used a very particular agenda, and we offer here a theory that allows us to understand its rationale and the ensuing consequences on voting behavior and final outcome.

In another recent and dramatic case from the UK Parliament, the main conflict axis involved a "hard" vs. a "soft" (or no) Brexit. But, due to the great complexity of the question and the many potential post-Brexit arrangements, some of proposed bills were not easily comparable along this main conflict line. The voting agenda was unusual: Premier's May's possible calculations did not materialize and she was forced to resign.

As in the two cases mentioned above, practically all democratic parliaments routinely use *Sequential Binary Voting Procedures* in order to select one of several alternatives. At each stage in a sequence of votes, the set of remaining alternatives, starting with the full set, is divided in two strict subsets.[1] A binary Yes-No vote is taken on the two subsets. The subset that gains a majority of votes advances to the next stage, while the other subset is discarded. The process is repeated until a single alternative remains, and is formally elected. There is considerable variation in the choice of procedures and of the particular subsets that are put to vote. Well known, stylized representatives are:

1) The Amendment Procedure (AP), common in the Anglo-Saxon world. It works with a basic bill (proposed by the Government, say), amendments to that bills, amendments to amendments, etc.... At each stage, two alternatives (the original bill and an amended version, say) are pitted against each other, and the winner advances to the next stage that has a similar structure.

2) The Successive Procedure (SP) is common in continental Europe and usually works with independent, fully-formed bills. At each stage, a single bill is voted upon, and voting stops as soon as one alternative obtains a majority.

The agenda defining which subsets of alternatives are considered at each voting stage plays a crucial role in determining voting behavior and the ultimate outcome. Which agendas are used in practice, and how should they be designed?

In previous work, we identified a special class of carefully constructed agendas ensuring that sincere voting at each stage constitutes a very robust,

[1] These need not be disjoint.

dynamic equilibrium in any sequential binary voting procedure, as long as privately informed voters have single-peaked preferences on alternatives ordered on a line, e.g. when the underlying issue is one dimensional (see Kleiner and Moldovanu, [2017]). We also illustrated that such designed agendas are actually used in some (but not all) parliaments, and gave examples of observed strategic behavior in cases where the agenda was formed along different lines.

In the present paper we extend our analysis to the much larger class of preferences that are single-peaked on an arbitrary tree, introduced in an elegant paper by Demange [1988]. Single-peaked preferences on trees go well beyond the one-dimensional framework underlying single-peakedness on a line, but without going all the way to fully fledged multi-dimensional problems.[2] Demange showed that, although the induced majority dominance relation is not necessarily transitive, every profile of single-peaked preferences on a tree admits a Condorcet winner. This generalizes the classical insight, due to Black [1948], who showed that the peak of the median voter is a Condorcet winner for single-peaked preferences on a line.

In this paper we introduce *convex* agendas on trees: at each stage in the sequential, binary voting process, the tree of remaining alternatives is divided in two (connected) subtrees that are subjected to a binary Yes-No vote. Roughly speaking this says that each subset of considered alternatives is ideologically coherent (according to the underlying tree).

Assume that preferences of incompletely informed agents are single-peaked with respect to a an arbitrary tree, and that an arbitrary sequential binary procedure with an arbitrary convex agenda is used. Our main theoretical result shows that sincere, myopic voting at each stage in the sequence is an ex-post perfect equilibrium (and hence does not depend on beliefs), and that the Condorcet winner is elected in this equilibrium.

Thus, in a wide class of situations, equilibrium strategies in the dynamic voting game are congruent with sincere voting that is myopic and "unsophisticated". This holds even if agents are privately informed about their preferences, and no matter what their beliefs about other voters are.

We conclude the paper by illustrating the empirical implications of our results for the above mentioned case studies from Germany and from the UK. Those cases involved binary sequential voting, by more than 600 voters, on a relatively large number of alternatives. Therefore they represent complex strategic situations. Party discipline (or the "whip") - that would imply that parliamentarians have to vote according to a uniform party line - was either institutionally not imposed (Germany), or was not respected by many

[2]We note that complex multidimensional voting problems are often divided in several simpler ones. See also Poole [2005].

decisive voters (UK). As a consequence, in both cases the outcome was highly uncertain.

In each case, we first construct appropriate trees on which preferences were presumably single peaked. It is worth mentioning here that, if there exists a tree that renders a profile of preferences single-peaked, then, under a very mild richness condition, the tree is unique (Trick [1989]). We next derive the relatively few voting patterns (i.e., the individual sequences of Yes and No) that would be consistent with our theory given the employed agenda and the assumed preferences. Finally, we compare those predicted patterns with those observed in reality. This allows us to estimate the voters' preferences and whether the outcome was a Condorcet winner.

The paper is organized as follows: In the next subsection we review the related literature. In Section 2 we recall several fundamental definition and results about graphs that are trees. In Section 3 we introduce the social choice model, the sequential binary voting procedures under incomplete information, and their agendas. In Section 4 we prove our main theoretical result that connects sincere and strategic voting for convex agendas. In Section 5 we present several case studies form the German and UK parliaments. Section 6 concludes

1.1 Related literature

The study of (strategic) sequential binary voting has been pioneered by Farquharson [1969]. The literature has often assumed that agents are completely informed about the preferences of others (see Miller [1977], McKelvey and Niemi [1978] and Moulin [1979]). Under complete information, sophisticated voters can use backward induction: at each stage they foresee which alternative will be finally elected, essentially reducing each decision to a vote among two alternatives. Under simple majority, a Condorcet winner is selected by sophisticated voters whenever it exists, independently of the particular structure of the binary voting tree, and independently of its agenda. Thus, that body of theory cannot account for the use of carefully design agendas in those cases. If a Condorcet winner does not exist, then a member of the top Condorcet cycle is elected.[3]

Rasch [2000] surveys the employed voting and agenda-setting procedures in democratic parliaments. Poole and Rosenthal [2000] offer a masterly history of roll-call voting in the US Congress. Leininger [1993] and Pappi [1992]

[3]The influence of agenda manipulations on the outcome of binary, sequential voting under complete information has been emphasized by Ordeshook and Schwartz [?]. See Barbera and Gerber [2017] for a more recent contribution.

analyze the 1991 decision about the post-reunification location of the German capital, and attempt to reconstruct the legislators' preferences from the sequence of observed votes. Based of the inferred preferences they also conduct simulations with other, hypothetical voting procedures. Pappenberger and Wahl [1995] look at the regulation of abortion in 1992, which we also analyze here. Von Oertzen [2003] discusses several cases from the Bundestag. Ladha [1994] analyzes a large number of cases from the US Congress and focus on cases where the agenda followed a natural left-right order on a line.[4]

All above papers are based on the premise that parliamentarians vote sincerely: there is no attempt to investigate when and why is this assumption tenable, and what are the consequences when it is not. Moreover, their underlying guiding social choice intuitions are mostly gathered from the classical literature on binary, sequential voting under complete information.

An early analysis of strategic, sequential, binary voting under incomplete information is Ordeshook and Palfrey [1988], who constructed relatively complex Bayesian equilibria (that crucially depend on the agents' beliefs about others) for an *amendment* procedure with three alternatives and with three possible preference profiles that potentially lead to a Condorcet paradox. Gershkov, Moldovanu and Shi [2017] analyzed voting by qualified majority in the successive procedure in settings where agents are privately informed, and where preferences are assumed to be single-peaked on a line.[5] Kleiner and Moldovanu [2017] generalized their finding: under single-peaked, private-values preferences on a line, sincere voting constitutes an ex post perfect equilibrium in any sequential, binary voting procedure if the agenda is *convex*.[6] Kleiner and Moldovanu [2019] apply this theory to explain both the emergence and rarity of killer amendments, and illustrate this theory withe a case involving the Nazi party. Gershkov et al. (2019) also consider sequential voting with single-peaked preferences on a line, but assume that preferences are interdependent. In their model not all alternatives are fixed ex-ante: these authors study the emergence and location of compromise alternatives (e.g., the location of a compromise deal in the Brexit case and the emergence of the composite flag of the Weimar Republic).

[4] The implications for strategic vs. sincere voting are also discussed by Groseclose and Miljo[2010].

[5] Their focus was on finding the welfare maximizing procedure. This is achieved by varying the thresholds needed for the adoption of each alternative.

[6] Under a mild refinement, this equilibrium is unique.

1.2 Graphs and Trees

We recall here several basic definitions and results that will be useful for our analysis.

Definition 1 1. *A* graph *G on a set of nodes $\mathcal{A}$ with typical elements A, B, C, ..,is a set of unordered pairs of distinct elements of $\mathcal{A}$, called* edges*.*

2. *A* path *P of G is a sequence of distinct nodes $A_1, ..A_m$ such that (A_i, A_{i+1}) is an edge for $i = 1, 2, ..m - 1$.*

3. *A graph is* connected *if, for any pair of nodes A_i, A_j there is a path with initial node A_i and terminal node A_j.*

4. *A* cycle *(or circuit) is a path in which the initial node coincide with the terminal node.*

5. *The* degree *of vertex A_i, $d(A_i)$, is the number of edges having A_i as element.*

Definition 2 *A* tree *Ψ is a connected graph that contains no cycles. A node A is a* leaf *of tree Ψ if it has degree 1.*[7]

Theorem 1 *(see Berge, 1962) Any one of the following equivalent properties characterizes trees:*

1. *Ψ contains no cycles and has $k - 1$ edges (where k is the number of nodes).*

2. *Ψ is connected and has $k - 1$ edges.*

3. *Ψ contains no cycles, and if a new edge is added, one, and only one, cycle is formed.*

4. *Ψ is connected , but ceases to be so if any edge is deleted.*

5. *Any two nodes A and B in Ψ are linked by a unique path, denoted below by P_{AB}.*

The key to the proof of the above equivalences is the following elegant result:

[7] That is, there is exactly one edge of Ψ containing this node.

Theorem 2 *(see Berge, 1962) Consider a graph G with k nodes, m edges and p connected components. The* circuit rank *(or cyclomatic number) of G is $\nu(G) = m - k + p$. The graph G contains no cycle (a unique cycle) if and only if $\nu(G) = 0$ ($\nu(G) = 1$).*

Let us prove, for example, the result in Theorem 1-1: If Ψ is a tree, then it is connected, hence $p = 1$. It contains no cycles, hence $\nu(\Psi) = 0$. Thus, we obtain $0 = m - k + 1$, which implies that $m = k - 1$.The other points follow from Theorem 2 by similar arguments.

Theorem 3 1. *The number of trees with k vertices and degrees $d_1, ..d_k$ is*

$$N(k, d_1, ..d_k) = \binom{k-2}{d_1 - 1, d_2 - 1, ...d_k - 1}$$

2. *(Cayley's Formula, 1889) The number of trees with k vertices is k^{k-2}.*[8]

2 The Social Choice Model

Suppose that there are $2n + 1$ voters who need to select one alternative out of a finite set $\mathcal{A}$ with $k \geq 2$ elements. Each voter i is characterized by a preference relation $\succ_i$on $\mathcal{A}$, and this set is endowed with a tree structure Ψ. Preferences are private information.

Definition 3 1. *An individual preference relation, $\succ_i$is an irreflexive, asymmetric , complete and transitive order on $\mathcal{A}$.*

2. *The preference $\succ_i$is* single-peaked *on the path P_{AC} of Ψ if, for any node B that lies on this path, it is not the case that both $A \succ_i B$ and $C \succ_i B$ hold.*

3. *The preference $\succ_i$is* single-peaked on the tree Ψ *if is single-peaked on every path P of Ψ.*[9]

Definition 4 *Given a preference profile $\{\succ_i\}_{i=1}^{2n+1}$, the Condorcet winner is an alternative $CW \in \mathcal{A}$ such that $\mid i \;/\; CW \succ_i A \mid > \mid i \;/\; A \succ_i CW \mid$ for any $A \neq CW$.*

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[8] Cayley's formula follows from the first statement by summing up all the relevant multinomials over $d_1 \geq 1, ...d_k \geq 1$ such that $\sum_k d_i = 2(k-1)$.

[9] This is equivalent to the following : If A is the peak of $\succ_i$and if B belongs to a path between A and C, then $B \succ_i C$.

The existence of a Condorcet winner for any profile of single-peaked preferences on a tree has been established by Demange [1988]. When a tree Ψ consists of a single path, we are in the classical case where alternatives can be ordered on a line, from "left" to "right".

Lemma 1 *Consider a tree Ψ and a subtree $\Psi' \subset \Psi$. If a preference relation $\succ_i$is singled-peaked with respect to Ψ, then its natural restriction is single-peaked on Ψ'. In particular, there is a Condorcet winner among the alternatives in Ψ'.*

Proof. Take any two nodes (alternatives) in Ψ', A and C. Since Ψ' is a tree and hence connected, there exists a path P in Ψ' that goes from A to C. Since P is also a path in Ψ, the result follows by single-peakedness with respect to Ψ. The last part follows from Demange's result. ■

2.1 Voting Procedures and Their Agendas

At each stage of a *Sequential Binary Voting Procedure*, the set of remaining alternatives (starting with the full set) is divided in two strict subsets (these need not be disjoint). Each voter approves one of the two subsets. The subset that gains a majority advances to the next stage, while the other subset is discarded. The process is repeated until a single alternative remains, and is formally elected.

Definition 5 *An* agenda *specifies for each stage of a binary sequential voting procedure the two subsets of alternatives whose union forms the set of alternatives that have not been yet rejected. An agenda is* convex *with respect to a tree Ψ if, at each stage of the voting process, it divides the set of remaining alternatives into two* ***subtrees****.*

The main ingredient in the above definition of convexity is the requirement that the division of alternatives at each voting stage is among two distinct (not necessarily disjoint) subsets that are connected. By the previous Lemma, the restricted preferences continue then to be single peaked on each subtree, and each binary division in a convex agenda is "ideologically coherent": if two alternatives A and B belong to one of the subtrees, all alternatives on the path P_{AB} must also belong to the same subtree. In other words, it cannot be the case that voters have to decide between, say, a subset of "centrist" alternative on the one hand, and a subset containing only "extreme right" and "extreme left" alternatives on the other. In such a case, the path connecting the extreme nodes may need to go via a centrist node, violating the requirement that each subset is a connected subtree.

Example 1 *1. Consider the successive voting procedure on $\mathcal{A}$. An agenda for this procedure is* convex *with respect to a tree Ψ if, at each stage, the alternative that is put to vote is a leaf of the subtree on the remaining alternatives. If alternative C is considered at a particular stage, the division in two subtrees is $[C,\ \mathcal{A}\diagdown\{C\}]$*

2. Consider voting by amendment on $\mathcal{A}$. An agenda for this procedure is convex *with respect to a tree Ψ if, at each stage, both alternatives that are put to vote against each other are leafs of the subtree induced on the remaining alternatives. If alternatives C, B are considered at a particular stage, the division in two subtrees is $[\mathcal{A}\diagdown\{C\}\ ,\ \mathcal{A}\diagdown\{B\}]$*

The above procedures are well-defined and convex by the following Lemma:

Lemma 2 *1. Any tree Ψ has at least two leaves.*

2. Let A be a leaf and denote by e the unique edge of Ψ that contains A. Then $\Psi\backslash\{e\}$ is a tree on $\mathcal{A}\backslash A$.

Proof. 1. *Let $P = \{A_1, ..A_m\}$ be the longest path in Ψ. Then A_1 and A_m must be leaves. Alternatively, note that the sum of degrees in any graph equals twice the number of edges. By Theorem 1 we obtain:*

$$\sum d(A_i) = 2(k-1) = 2k-2$$

If there are less than two leaves we obtain:

$$\sum d(A_i) \geq 2(k-1) + 1 = 2k-1 > 2k-2$$

a contradiction.

2. *Consider any two vertices $B, C \in \mathcal{A}\backslash A$. Then the unique path P_{BC} that connects them in Ψ is also the unique path that connects them in $\Psi\backslash\{e\}$.*
■

3 Sincere and Strategic Voting on Trees

We now study strategic voting in sequential, binary voting procedures where the set of alternatives has an underlying tree structure.

Definition 6 *A voting strategy for a sequential, binary voting procedure is* sincere *if, at each stage in the voting sequence, it prescribes voting Yes for the subset of alternatives that contains the most preferred alternative among all remaining ones. If that alternative is contained in both subsets that are put to vote at a certain stage, then a sincere voting strategy proceeds lexicographically (vote yes for the subset that contains the second-best alternative, and so on..)*

Our main theoretical result is:

Theorem 4 *Assume that preferences are single-peaked with respect to a tree* Ψ *, and that a sequential binary procedure with a convex agenda is used. Then, sincere voting is an ex-post perfect equilibrium, and the Condorcet winner is elected in this equilibrium.*[10]

Proof. Assume that all voters vote sincerely, and let CW be the Condorcet winner given the agents' preferences. We first show that CW must be elected under such a strategy profile.

Assume, by contradiction that CW is not elected under sincere voting. Consider then the first stage in the voting process where the majority approved subtree is Ψ' such that $CW \notin \Psi'$. Then, there exist $m \geq n+1$ agents whose preferred alternative among the remaining ones is in Ψ'. Denote those most preferred alternatives by $A_1, A_2, ..A_m$, respectively (these need not be distinct). If $A_1 = A_2.. = A_m$ then there are $m \geq n+1$ agents that prefer A_1 to CW, which is impossible by the definition of CW. Assume then w.l.o.g that $A_1 \neq A_2$. Because Ψ' is a tree, there exists a unique path, $P_{A_1 A_2}$, entirely contained in Ψ', that connects these two nodes. In particular, CW cannot be on this path since $CW \notin \Psi'$. Consider next the uniquely defined paths P_{CWA_1} and P_{CWA_2} in Ψ. Then there must exist an alternative, denoted by B, such that B belongs to P_{CWA_1}, P_{CWA_2} and $P_{A_1A_2}$. Otherwise, the concatenation of P_{CWA_1}, $P_{A_1A_2}$ and P_{CWA_2} yields a cycle, contradicting the assumption that Ψ is a tree.

By single-peakedness, we obtain that **all** agents whose most preferred alternative is **either** A_1 or A_2 prefer alternative B to CW. Arguing in the same manner for $A_3, ..A_m$ shows that there must be an alternative in Ψ' that is preferred by $m \geq n+1$ agents to CW, which is impossible. Thus, CW can never be eliminated, and will thus be elected.

We now argue that sincere voting is an ex-post perfect equilibrium. Fix an arbitrary preference profile, and an arbitrary voter i. We show that, given sincere behavior by all other voters, i has no profitable deviation from sincere voting. Consequently, sincere voting is an ex-post perfect equilibrium.

Observe first that sincere voting is a best response if only two alternatives remain. Consider a voting stage where the decision is between the two subtrees Ψ' and Ψ'' and assume that sincere voting is a best response after this stage. Hence, if Ψ' gains a majority at this stage, it follows from the first part that the final outcome will be the Condorcet winner among the

[10] Once trivial equilibria (that always exist in voting situations) are discarded, this equilibrium is essentially unique.

alternatives in Ψ', which we denote by C'. Similarly, if Ψ'' gains a majority, the final outcome will be the Condorcet winner among alternatives in Ψ'', denoted by C''.

To obtain a contradiction, suppose w.l.o.g. that i's peak is $A \in \Psi'$ but that he is strictly better-off to vote for Ψ''. Then, there must be at least n other voters with peak in Ψ'' and it must hold that $C'' \succ_i C'$. Because $\Psi' \cup \Psi''$ is also a tree (that has been approved at the previous stage) there exists an alternative B that satisfies $B \in P_{AC'}$, $B \in P_{AC''}$ and $B \in P_{C'C''}$. Since $A, C' \in \Psi'$ and since Ψ' is a tree, it must also hold that $B \in \Psi'$. Also, because alternative A is i's peak and because $B \in P_{AC''}$, single-peakedness implies $B \succeq_i C'' \succ_i C'$. Hence, $B \neq C'$.

We now consider two cases:

(1) Suppose that $B \notin \Psi''$. Since Ψ'' is a tree and since $B \in P_{C'C''}$, $C' \notin \Psi''$. Also, for all $D \in \Psi''$, it must be the case that $B \in P_{C'D}$ (if not, then the concatenation of $P_{C'C''}$, $P_{C''D}$ and $P_{DC'}$ contains a cycle). By single-peakedness, every voter with a peak in Ψ'' prefers alternative B to C'. Since at least n other voters have a peak in Ψ'' and since $B \succ_j C'$ for all such voters, we obtain a contradiction to the assumption that C' is the Condorcet winner among alternatives in Ψ'.

(2) Suppose that $B \in \Psi''$.[11] Since C'' is the Condorcet winner among the alternatives in Ψ'', if $C'' \neq B$ then at least $n+1$ voters prefer C'' to B and hence, by single-peakedness, prefer B to C', contradicting that C' is the Condorcet winner among alternatives in Ψ'. Hence, $C'' = B$. Since C' is the Condorcet winner in Ψ', at least $n+1$ other voters prefer C' to C'', and hence $C' \notin \Psi''$. Since at least n other voters have a peak in Ψ'', there is a voter with peak in Ψ'' who prefers C' to C''. Denote his peak by D. Then, there is an alternative $E \neq C', C''$ such that $E \in P_{DC'}$, $E \in P_{DC''}$ and $E \in P_{C'C''}$. Since Ψ'' is a tree, $E \in \Psi''$. Since Ψ' is a tree and $C', C'' \in \Psi'$, $E \in \Psi'$. Therefore, $n+1$ voters prefer C' to E and hence E to C'', contradicting the assumption that C'' is the Condorcet winner among alternatives in Ψ''. ■

Recall that, if the set of alternatives $\mathcal{A}$ has cardinality k, then there are k^{k-2} distinct trees on $\mathcal{A}$. For example, in the abortion law case discussed below there were 8 alternatives, and hence $8^6 = 262.140$ trees on which preferences could have been, at least theoretically, single-peaked. Since in real-life, complex cases the tree structure is almost never made explicit, an important criterion for assessing the power of the subsequent empirical analysis is: How arbitrary is the analyst's choice of a tree with respect to which preferences are potentially single-peaked? The rather surprising answer is:

[11] This case cannot occur in procedures where the binary decision is among disjoint subsets of alternatives e.g., in the successive procedure.

Proposition 1 *(Trick [1989]) Fix a profile of individual preferences such that each alternative in $\mathcal{A}$ is the peak of some voter. Then, there exists at most one tree Ψ such that this profile of preferences is single peaked on Ψ.*

Proof. For the sake of completeness, we reproduce here the simple proof. Assume that the preferences are single-peaked on two distinct trees, Ψ and Ψ'. Then Ψ has an edge $e = (A, B)$ that is not contained in Ψ'. Consider then any node C on the path between A and B in Ψ', and its respective placement in Ψ. There must be such a node because, by assumption, $e = (A, B) \notin \Psi'$. There are two cases: either the path from A to C in Ψ contains B, or the path from B to C in Ψ contains A. In the first case, consider a voter i that has a peak on A. Then, we must have $A \succ_i B \succ_i C$. But, then $i's$ preferences cannot be single-peaked on Ψ'. The other case is similar, and this yields a contradiction. ■

4 Case Studies

In this section we apply our analysis to two real cases from the German Bundestag and from the UK Parliament.

4.1 Abortion law after the German reunification

Prior to the 1992 reunification, abortions were relatively strictly regulated in the Federal Republic of Germany, while the former Democratic Republic of Germany had a more liberal law. The German reunification treaty required new, uniform legal foundations. After a long debate, 7 proposals for a new formulation of the law were put up for vote in the Bundestag. The proposals covered a wide range of opinions and details, and there was considerable uncertainty about how many members of parliament supported each proposal.

In ethical decisions it is customary to free members of the Bundestag from party discipline. Our assumption of incomplete information becomes then salient: support for various alternatives crosses party lines, and members of the same party vote in favor of different alternatives, introducing real uncertainty about the outcome.[12]

Following the Standing Orders of the Bundestag, voting proceeded according to the successive voting procedure where single alternatives are put

[12] For example, in a recent case from 2018, Chancellor Merkel and a majority of legislators belonging to her governing party lost a landmark case that legalized gay marriage.

to vote, one after the other, until one is elected. The procedural agenda formation rule in those Standing Orders order implicitly assume that the issue is one-dimensional (i.e., the alternatives can be ordered on a line) and calls for voting on extreme alternatives first.

4.1.1 The proposed bills

The Elders' Council, headed by the Bundestag's president, suggested a very specific agenda. We very briefly describe here the proposed laws according to the order in which they were actually put up for vote, from **A** to **G**. The status quo is denoted by **H**.

A The Greens' proposal was very liberal and basically allowed any abortion.

B Similarly, the proposal by the Left party would allow any abortion, and there were several minor differences compared to proposal **A**.

C This proposal, coming from a subgroup of very conservative parliamentarians, led by Werner, was very restrictive: it allowed an abortion only if the life of the mother was otherwise at stake.

D The Liberals proposed that abortions should be legal in the first 12 weeks of pregnancy, but only if the mother takes part in pregnancy counseling. Moreover, the proposal demanded punishment for women aborting after the first 12 weeks.

E The Social Democrats suggested instead that any abortion within the first 12 weeks should be legal, but without enforcing punishments for later abortions.

F The main proposal brought forward by conservatives and supported by the leaders of the ruling CDU/CSU, allowed abortions only under restrictive regulations: even early abortions would remain legal only under medical and/or psycho-social indications.[13] Both woman and treating doctor would be punished for an abortion after the first 12 weeks. This proposal effectively handed the final decision to the doctor, who also had to record and explain the decision in writing.

[13] This effectively handed the final decision to the doctor, who also had to explain the decision in writing.

G The so-called Group proposal was suggested by a group of legislators that crossed party lines: it was meant as a compromise between proposals **E** and **F**. An abortion within the first 12 weeks would not be punished. The woman needs to take part in a pregnancy counseling and the abortion must be performed by a doctor, but the ultimate decision stays with the woman.

H The status quo in the former Democratic Republic allowed an abortion in the first 12 weeks. In contrast, in the Federal Republic, an abortion required the presence of several criteria (or "indications") that were not easy to fulfill.

4.1.2 Assumptions about preferences

The simplest hypothesis is that preferences were single peaked with respect to a linear order that goes from an emphasis on the free decisions for women on the one side, to an emphasis on the protection of unborn life on the other. Given the above described alternatives, there is, however, only one order for which single-peaked preferences seem a more or less reasonable, approximate assumption: this is the linear order **A-B-E-G-D-H-F-C**, also suggested by Pappenberger and Wahl [1995].

Note that, however, alternative **D** was **not** a leaf for the tree **E-G-D-H-F** consisting of the alternatives remaining when **D** was put up for vote. In particular, such an agenda would also contradict the general rule to vote on extreme alternatives first. Therefore, the theoretical results provide no foundation for using the particular agenda suggested by the Elders' Council: if any single-peaked preference with respect to the order **A-B-E-G-D-H-F-C** is feasible, then sincere voting in the game induced by the actually employed agenda **ABCDEFGH** is not a robust equilibrium, and the Condorcet winner is not necessarily elected (see Kleiner and Moldovanu [2017].)

In order to estimate the true preferences of the legislators, Pappenberger and Wahl conducted a survey after the voting took place. Out of 72 legislators who reported a complete preference ranking in this survey, only 4 of them reported a preference that is single-peaked according to the above linear order. On the other hand, it seems reasonable to assume that preferences were single-peaked with respect to the tree shown in Figure 1. Indeed, a majority of the available reported preferences of more than 70 legislators were indeed single-peaked with respect to this tree, and for no other tree more reported preference profiles were single-peaked. Therefore, we use this tree for our analysis.

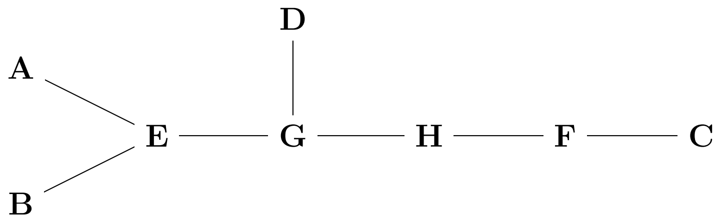

Figure 1: Preference tree

4.1.3 Analysis

Note that, when they were put up for vote according to the agenda **ABCDE-FGH,** all alternatives were leafs of the above described tree! Hence, the chosen agenda was convex and our theoretical result predicts that sincere voting constitutes a robust equilibrium, and that the Condorcet winner will be elected in this equilibrium.[14]

The following table summarizes the actual voting results in the sequence of binary votes:

	Yes	**No**	**Abstain**	**Total**
A	17	632	6	655
B	17	633	3	653
C	104	492	57	653
D	74	575	4	653
E	236	402	16	654
F	272	369	16	657
G	**355**	**283**	**16**	**654**

Alternative **G**, the compromise among the main alternatives supported by the big parties, was elected in the final vote.

We can now use the available records (see the archives of Deutscher Bundestag [1992]) of individual voting profiles to test our predictions. We should mainly observe voting profiles that are consistent with sincere voting according to single-peaked preferences on the assumed tree. In contrast, if we observe large numbers of voters using voting profiles that are inconsistent with sincere voting (for example, legislators voting Yes for the very liberal proposal **A**, but also voting Yes for the very conservative proposal **C**), we would have to reject the hypothesis that voting was sincere.

[14]Hence, the critique of this voting procedure that was put forward by some legislators has no theoretical foundation.

Abstract for the moment from abstentions and assume that voters can only vote Yes or No at each stage. This yields $2^7 = 128$ possible individual voting profiles. In the successive procedure with a convex agenda, each alternative is a leaf of the tree remaining at the time it is voted upon. Therefore sincere voting prescribes to vote Yes if the current proposal is the most preferred among the remaining alternatives, and No otherwise. This feature shows that that the location of the peak completely determines the corresponding sincere voting strategy, i.e., this strategy is independent of how exactly alternatives are ranked below the peak. This implies that, out of the 128 possible voting profiles, only 8 are consistent with sincere voting according to strict and single-peaked preferences on the above tree.

In reality, members of parliament can choose not to cast a vote on a specific proposal, or to formally abstain. 658 voters participated in at least one vote, while 638 voters participated in all votes in the sequence and we focus our analysis on the latter. More than 100 of these voters abstained at least once, which is why we include these voters in our analysis and treat an abstention as an expression of indifference.

Our main finding is that 601 out of 638 profiles, a vast majority, used a strategy that is consistent with sincere voting!

As explained above, we can infer the most preferred alternative of each legislator: for example, a legislator voting Yes at the first vote has a peak on **A**, a legislator who votes Yes for the first time at the second vote has a peak on **B**, and so on... Based on the record of voting profiles, the following table shows, for each alternative, how many legislators have a peak on that alternative.[15]

Peak	A	B	C	D	E	F	G	H	other profiles
Number (min)	2	0	93	71	206	126	30	1	37
Number (max)	6	2	144	73	225	179	48	7	37

Although only a small minority had a peak on the elected alternative **G**, it turns out that, under the inferred possible distributions of peaks, this alternative is indeed the Condorcet winner. Thus, our analysis also implies that the Bundestag's president and the Elders' Council managed to intuitively choose an agenda that made strategic voting unnecessary, and that ensured the election of the Condorcet winner (who did not have much direct support) . In other words, the employed agenda consistently extended the

[15]Due to abstentions, we cannot precisely identify the peak of some legislators. We therefore display for each alternative a lower and an upper bound on the number of legislators that have a peak on this alternative.

traditional “extremes first” doctrine from a line to a more complex tree that remained implicit - no mean feat in this complex situation.

4.2 The Brexit Voting Marathon

A voting marathon consisting of a sequence of eight binary votes was conducted by the British Parliament between March 12 and March 14, 2019. At stake was the shape and even the future of Brexit - UK’s separation form the European Union - that was supposed to formally take place just two weeks later, on March 29, 2019.

We precisely describe below the complex, non-convex agendas that were used. It is worth mentioning here that at least one explicit deviation from convexity was actually part of premier’s May strategy in order to get her deal through. Here is what the Economist wrote about it:[16]

> [...] Mrs May’s plan is to hold yet another vote on her deal and to cudgel Brexiteers into supporting it by threatening them with a long extension that she says risks the cancellation of Brexit altogether. At the same time she will twist the arms of moderates by pointing out that a no-deal Brexit could still happen, because avoiding it depends on the agreement of the EU, which is losing patience. It is a desperate tactic from a prime minister who has lost her authority. It forces MPs to choose between options they find wretched when they are convinced that better alternatives are available. [...]

Prior to the voting marathon, May’s negotiated Brexit deal with the EU has been rejected by a very large margin of 230 votes on January 15, 2019.[17] Nevertheless, it was put to vote again, **before** the more “extreme” alternatives such as a no-deal Brexit or a new referendum (or, say, an arrangement whereby the UK remains in the EU common market and customs union) were formally discarded.[18] As the Economist explains, her hope was that both Leavers and Remainers would finally unite behind her deal because each group perceived one of the remaining, extreme alternatives still on the table (and thus also a “lottery” among them) as catastrophic from their

[16] “Whatever next?” Lead Article, *The Economist* March 16th 2019, page 11.

[17] This was the largest defeat for a sitting government in history.

[18] The same strategy has been pursued by May’ successor, Boris Johnson. It was repeatedly countered by a majority in Parliament who refused to vote for a deal while a no-deal Brexit was still an option (the Benn and Letwin amendments).

point of view.[19] Such an agenda, where a compromise is voted upon before the extremes, clearly violates convexity.

The UK parliament has 649 members. Since Sinn Fein's 7 MPs do not take their seats, a majority of 322 was needed to pass legislation. The Tory (Conservative) government, supported by the North-Irish DUP had a very thin, theoretical majority of 324, but was facing many rebel members in favor of a hard Brexit. Thus, the outcomes of various votes were highly uncertain and the entire situation was rather dramatic.

4.2.1 The Motions, Agendas and Outcomes

The UK Parliament used a relatively complicated sequential, binary agenda that mixed elements of the Amendment Procedure (AP) and the Successive Procedure (SP). This was necessary because some of the bills (such as May's negotiated deal) were complete pieces of legislation while others were partial amendments, etc.

The First Voting Sequence The first sequence of votes involved decisions about alternative courses of actions up to the official Brexit date on March 29, 2019. It consisted of 4 binary votes involving 5 alternatives:[20]

- 0 We denote by 0 a no deal Brexit on March 29. Implicitly, this was the legal status quo unless further action was taken, and this was mentioned as such in May's motion 1.
- 1 May's deal with the EU.
- 2 May's no Brexit without a deal on March 29.
- 3 Malthouse: An alternative to May's deal (1) that would execute Brexit on March 29.[21]
- 4 Spelman: No Brexit without deal, ever (amendment to 2).

The voting agenda is illustrated in Figure 2. The first vote was on May's motion 1, according to SP: voting would have stopped in case of acceptance.

[19] Zeckhauser [1969] shows that introducing lotteries may destroy single-peakedness. Lotteries become relevant when the agenda is not convex because the anticipated outcome depends then on beliefs about others' preferences.

[20] Many other proposals were ultimately not put to a vote - the ultimate agenda setting power lied with the powerful Speaker John Bercow.

[21] This was procedurally presented as an amendment to 2, but logically represented an altogether independent course of action.

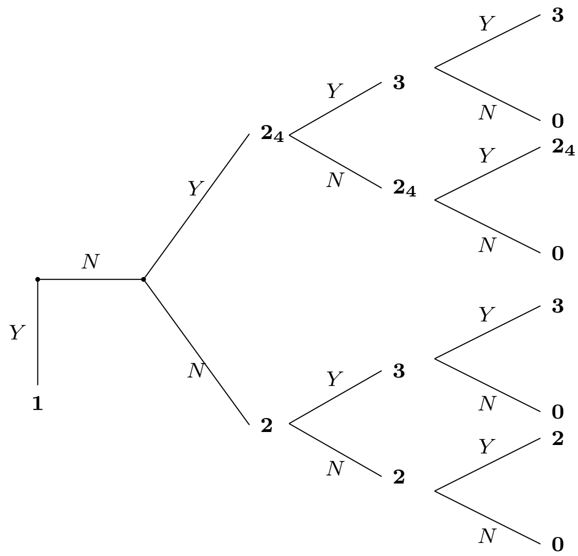

Figure 2: Voting procedure

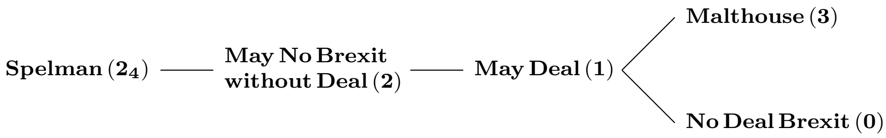

Figure 3: Preference tree underlying the first votes on Brexit

But motion 1 was defeated by 391 to 242 votes,[22] and a more traditional sequence according to AP followed. First, the Spelman amendment 4 narrowly passed. In other words, the original motion 2 was defeated against the amended version by 312 to 308 votes. Hence, motion 2 amended by 4, denoted here by 2_4, became the standing motion. Then, the Malthouse proposal 3 was defeated by 374 to 164 votes. Finally, the still standing motion 2_4 passed against the status quo 0 by 312 to 278 votes.

We suggest that the five motions in this part of the voting marathon can be arranged on a tree as shown in Figure 3. We assume below that preferences were single-peaked on this tree, and check whether the actually observed voting profiles are consistent with sincere voting according to such preferences **given** the employed, non-convex agenda. In the concluding Section we discuss the sincerity assumption in this context.

[22]Note that this was tighter than the original defeat by 230 votes. An even tighter outcome was obtained at a later, third vote on the same issue. Thus, May's strategy, described by the Economist, might have worked to some extent.

Out of the 120 possible rankings over the five alternatives, only 24 are compatible with single-peakedness on the above tree. Moreover, while there are $2^4 = 16$ possible Yes-No sequences on the 4 votes actually taken, only 10 of them can be induced by single-peaked preferences on the tree according to sincere voting given the agenda. Thus, we expect to observe at most 10 basic voting patterns among the 649 individual voting records.

Let us also briefly comment on the construction of the tree: By Cayley's formula there are $5^3 = 125$ potential trees here. Since all motions were proposed by a member of parliament and since the motions selected by the speaker Bercow were those that seemed to have some chance of success, we assume that the requirement for Trick's Theorem was satisfied.[23]

The following table summarizes the most frequently observed profiles and the single-peaked preference order on the tree that would generate each of the observed profiles given sincere voting, and given the agenda used.[24] We denote indifference between alternatives 1 and 2 by $1 \sim 2$, and the notation $1 \succ (2,3)$ summarizes that the preference could be either $1 \succ 2 \succ 3$ or $1 \succ 3 \succ 2$.

Profile	Observations	Implied single-peaked ranking
NYNY	310	$2_4 \succ 2 \succ 1 \succ (3,0)$
YNYN	94	$1 \succ (3,0,2) \succ 2_4$
YNAN	68	$1 \succ (2,0) \succ 3 \sim 2_4$
NNYN	65	$2 \succ 1 \succ (3,0) \succ 2_4$, $3 \succ 1 \succ (2,0) \succ 2_4$, $0 \succ 1 \succ (3,2) \succ 2_4$
YNNN	32	$1 \succ (0,2) \succ 2_4 \succ 3$
YANN	16	$1 \succ 0 \succ 2 \sim 2_4 \succ 3$
NNAN	11	$0 \succ 1 \succ 2 \succ 2_4 \sim 3$, $2 \succ 1 \succ 0 \succ 2_4 \sim 3$
AAAA	11	$1 \sim 2 \sim 2_4 \sim 3 \sim 0$
YAAN	7	$1 \succ 0 \succ 2 \sim 2_4 \sim 3$
YYNY	5	None
NNNN	5	$0 \succ 1 \succ 2 \succ 2_4 \succ 3$, $2 \succ 1 \succ 0 \succ 2_4 \succ 3$
Others	25	Diverse (including peaks on 2)

Table 1: Individual vote profiles for the first sequence of Brexit votes.

It follows from the above table that, with the exception of one profile that was observed just five times (YYNY), all common profiles are indeed consistent with our assumption that voting was sincere according to single-peaked

[23]In particular, two Labor voters voted against the Spelman amendment at the second vote, but for May's motion 2 amended by 4 at the last vote. Assuming sincere voting, their peak was on alternative 2.

[24]We show all vote profiles that were cast by at least 5 voters.

preferences on the constructed tree.[25] Under this assumption, alternative 2_4 that was selected was indeed the Condorcet winner because it won the direct vote against alternative 2, the only other close contender.

The Second Voting Sequence The second sequence of votes can be seen as determining how to precisely continue the process, and how to implement the previous decision of not leaving the EU without a deal by March 29, 2019. The motions were:

5 Corbyn: extend Article 50[26] + new Brexit approach (amendment to 8)

6 Wollaston: Hold a new referendum (amendment to 8)

7 Benn: Hold indicative votes (amendment to 8)[27]

8 May: Motion to delay the Brexit date.

9 We denote by 9 the status quo, a no deal Brexit on March 29. Although Parliament has just excluded a no-deal Brexit "forever", without further legislative steps, including the approval of the EU, a Brexit on March 29 was still the legal default.[28]

The voting agenda is depicted in Figure 4. The agenda for this sequence was again a combination of SP and AP, but with a more pronounced SP component.

May's basic motion 8 asked for a delay in the Brexit process that would give the parliament more time to approve a deal. The first vote was on amendment Wollaston 6 (new referendum). If accepted, the only other vote would be on May's motion 8 amended by 6, denoted by 8_6, pitted against the status quo. Wolllaston was defeated by 85 to 334 votes. The second vote was on Benn's amendment 7. If accepted, the only other vote would be on motion 8_7 pitted against the status quo. Benn's amendment was narrowly

[25]After alternative 1 was defeated by a large majority, the problematic profile YYNY is consistent with single-peaked preferences with a peak on 2_4. Out of the rare profiles that were used by 25 voters and that we didn't list, 14 voters cast profiles that are inconsistent with our assumption.

[26]This was the legal step announcing the intention to leave the EU, including the deadline of March 29.

[27]The purpose was to find a deal that can be approved by a majority. For simplicity we ignore here the Powell amendment to this amendment, which would hold indicative votes while specifying a precise Brexit date of June 30.

[28]This has also been emphasized by the EU's leadership in the summit that followed the defeat of May's deal. The legal conundrum stemming from this status quo continued also after Brexit's delay and Johnson's premiership.

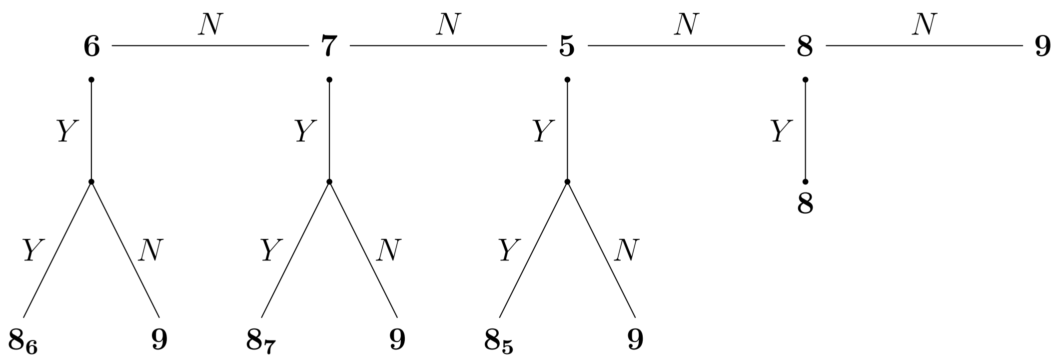

Figure 4: Agenda

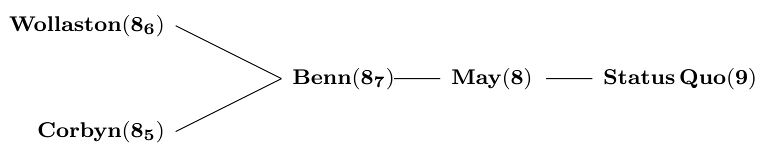

Figure 5: Preference tree underlying the second Brexit vote

defeated by 312 to 314 votes. The third vote was on Corbyn's amendment 5. If accepted, the only other vote would be motion 8_5 pitted against the status quo. Corbyn's amendment lost by 302 to 318 votes. Finally, as none of the amendments was successful, the unamended motion 8 was pitted against the status quo, and passed by 413 to 202 votes.

We now assume that preferences for the second sequence were single-peaked on the tree shown in Figure 5. Analogously to the tree of the first sequence, only ten voting profiles are consistent with sincere voting according to single-peaked preferences on the tree in the agenda employed.

Profile	Observed Number	Implied single-peaked preference relation
AYYY	202	$8_6 \sim 8_7 \succ 8_5 \succ 8 \succ 9$
NNNN	200	$9 \succ 8 \succ 8_7 \succ (8_5, 8_6)$
NNNY	103	Any with peak on 8
YYYY	83	$8_6 \succ 8_7 \succ 8_5 \succ 8 \succ 9$
AAAA	14	$8_6 \sim 8_7 \sim 8_5 \sim 8 \sim 9$
NYYY	10	$8_7 \succ (8_6, 8_5) \succ 8 \succ 9,\ 8_7 \succ 8_5 \succ 8 \succ (8_6, 9)$
NNNA	8	$9 \sim 8 \succ 8_7 \succ (8_5, 8_6)$
NYNY	6	$8_7 \succ 8 \succ (8_5, 8_6, 9),\quad 8_7 \succ 8_6 \succ 8 \succ (8_5, 9)$
Others	23	Diverse

Table 2: Individual vote profiles for the second sequence of Brexit votes.

Table 2 summarizes all common profiles and the single-peaked preference orders that would generate each of these observed profiles given sincere voting and given the agenda. All common profiles are indeed consistent with our assumptions,[29] but the identification of the Condorcet winner is here more complex: either alternative 8_7 (Benn) or alternative 8 (May) could have been it. Alternative 8 very narrowly won against 8_7 by 314 to 312 votes, suggesting at first sight that 8 was the Condorcet winner. But, note that at that point in the voting sequence, alternative 8_5 (Corbyn) was still in play. For a voter with a peak on 8_5, sincere voting prescribes a vote **against** 8_7 even though he/she prefers 8_7 to 8. Since we do not have direct information on how many voters had a peak on 8_5, it is not completely clear which alternative was the Condorcet winner. On the other hand, the second vote in the sequence clearly pitted 8_7 vs. 8, so a home-style argument a la Fenno (see discussion below) might actually speak here against sincere voting and thus reinforce the view that alternative 8 (May) was the Condorcet winner. The identification difficulty described here is typical of non-convex agendas.

5 Discussion

The employed agendas in the Brexit case were not convex, partly by design. Thus, sincere voting need not constitute a strategic equilibrium. Nevertheless, we have shown that sincere voting based on single-peaked preferences on a tree yields relatively precise predictions that agree well with the data. Why would legislators vote sincerely?

An important force behind sincere, straightforward voting is the need to explain behavior and make it transparent to constituents (see Fenno [1]).[30] For example, we observe a high correlation between MP's hawkish voting behavior on Brexit and the percentage in favor of Leave in their constituency at the 2016 Referendum. Thus, an MP from a strong Leave constituency may find it difficult, if not impossible, to opportunistically approve a soft-Brexit alternative even if it yields some strategic gain. This disciplining effect seems to be particularly relevant in the UK, where each member of parliament is individually elected (first past the post) in relatively small constituencies of about 70-80.000 people each.[31]

[29] Among the rare profiles cast by 23 voters, only 5 voters behaved inconsistently with our assumption.

[30] But note that in non-convex procedures sincere voting might not always be the easiest behavior to explain one's constituents.

[31] For example, Prime Minister Boris Johnson was elected to Parliament by gathering 29.000 votes in his constituency

This should be contrasted with Germany, where a majority of legislators are elected on statewide party lists (proportional representation), and are therefore not directly accountable to a local community. Moreover, the directly elected legislators represent much larger, possibly more diverse constituencies of about 250.000 people each. Thus, if sincere, transparent voting is a desideratum, a well designed agenda seems relatively more important in Germany than in the UK. But even if sincere voting is being enforced via motives that lie outside the immediate scope of this paper, we strongly believe that having procedural rules that induce convex agendas ensure a much smoother process both at the agenda setting stage and at the voting stage. We recommend their use.

We conclude by noting that our general method of inquiry can be extended to obtain a more robust inference of preferences even for non-convex agendas. Rather than assuming sincere voting one could compute equilibrium strategies and use these to infer preferences. However, equilibrium computation is very complex, and inferences are sensitive to the exact beliefs voters hold, which are not observable. For future work, we propose instead to determine the strategies that survive iterated elimination of weakly dominated strategies and to base inference on these strategies.

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