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AVOIDING JUDGEMENT BY RECOMMENDING INACTION: BELIEFS MANIPULATION AND REPUTATIONAL CONCERNS

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Abstract

To evaluate an expert, the audience needs to compare the prediction of the expert with the realized outcome. But the prediction often affects the amount of public information about the outcome. The result is that the expert can manipulate her audience's ability to monitor the accuracy of her prediction. In a cheap-talk framework, we study how the endogenous nature of public information about the state of the world affects the information transmitted by an expert with reputational concerns. Our innovation consists in assuming that the precision of the public information on the realized state increases monotonically with the audience's interim beliefs. In addition to the conservatism bias found in the existing literature, our model predicts that (i) the expert is less communicative when the prior is low, and (ii) a higher initial reputation can make the expert less credible.(JEL D82, D83, L15)

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Avoiding Judgement by Recommending Inaction: Beliefs Manipulation and Reputational Concerns*

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November 23, 2019

Abstract

To evaluate an expert, the audience needs to compare the prediction of the expert with the realized outcome. But the prediction often affects the amount of public information about the outcome. The result is that the expert can manipulate her audience's ability to monitor the accuracy of her prediction. In a cheap-talk framework, we study how the endogenous nature of public information about the state of the world affects the information transmitted by an expert with reputational concerns. Our innovation consists in assuming that the precision of the public information on the realized state increases monotonically with the audience's interim beliefs. In addition to the conservatism bias found in the existing literature, our model predicts that (i) the expert is less communicative when the prior is low, and (ii) a higher initial reputation can make the expert less credible.(JEL D82, D83, L15)

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1 Introduction

To make better economic decisions, agents routinely rely on experts. But from the point of view of her audience, the ability of the expert is often uncertain. To decide whether to trust the expert, the audience can observe the recommendations that she has given in the past. Yet, whether the audience can assess the quality of a recommendation often depends on the actions taken on the basis of this recommendation.

For example, imagine that Sarah advises Jamal not to invest in Rajeev's start-up. If Rajeev remains unfunded, Jamal will never learn whether the start-up would have been profitable or whether he did well to follow Sarah's recommendation. Generally, potential investors cannot observe the accuracy of the analyst' s evaluation of an emerging firm unless enough agents invest in the firm.

Experience goods markets are another setting in which the expert's recommendation determines how much feedback the audience receives about the ability of the expert. The audience learns about the quality of an experience good through word-of-mouth or online reviews. The precision of the public information about the good's quality increases with the number of buyers. By advising consumers against buying a new product, the expert can prevent her audience from learning its true quality. The audience is then unable to assess the accuracy of the expert's recommendation.

A similar mechanism is at play in the market for academic research. To decide whether to publish an article, editors rely on referees' assessment of its scientific value. But the academic community can only appreciate this value if the editor publishes the article. If a referee recommends a rejection, he can ensure that a paper remains obscure and his recommendation will remain unquestioned.

In this paper, we develop a reputational cheap-talk model that features an expert with reputational concerns. Formally, we assume that the expert receives a private binary signal

($s \in \{0,1\}$) about a state of the world ($\theta \in \{0,1\}$). The expert's ability (t) determines the informativeness of her signal. After observing the signal, the expert sends a message (m) about the state to the audience. After observing the message, the audience updates its belief about the state. Ultimately the state is realized. The audience does not observe the realized state but observes an ex post public signal about it; the signal's precision depends on the interim beliefs. Crucially, we assume that the public signal is more precise when the audience expects that the high state $\theta = 1$ is more likely. The audience compares its posterior belief about the state of the world to the message in order to update its belief about the expert's ability. The audience's belief about the expert's ability – that is, the expert's posterior reputation – determines the payoff that the expert aims to maximize.

We first show that the expert will tend to distort information to prevent her audience from receiving accurate feedback about the realization of the state of the world (Proposition 2). This is because, in the absence of accurate information about the realized state, the audience is inclined to believe that the expert is right provided that her message conforms with the common prior belief. When the prior favors the low state $\theta = 0$, the expert might choose to report $m = 0$ even when she believes the high state to be more likely because such report will suppress the public information about the realized state.

Consider, for example, the case of a film critic reviewing the movie of a lesser known director. Even if the critic thinks highly of the movie, she might still prefer to write a bad review and dissuade her readers from watching it rather than take the risk of recommending a movie that her audience might find mediocre.

New products about which consumers are poorly informed depend on some form of quality assessment. Gill and Sgroi (2012) shows that firms that sell new products with unfavorable priors will choose the toughest test to certify their products if those tests are costless. But certification is usually costly, sometimes prohibitively so, which leads us to consider whether soft information provided by experts can help consumers find new innovative products. We show that reputational concerns actually increase the *Barrier to Entry* for new products: experts will favor products from well-established firms and reinforce pessimistic beliefs about new entrants.

We expect reputational concerns to create a more severe barrier to entry when the precision of the public information is highly dependent on the expert's recommendation. This implies that competition between experts or technologies that allow consumers to publish reviews can reduce informational distortion by providing the audience with additional sources of information about the state.

Our second main result is that a higher initial reputation can actually make the expert less credible. To understand this, observe that the audience will update its beliefs more strongly in the direction of the expert's report when the expert has a higher expected ability. This gives the expert more incentive to herd on prior beliefs that favor the low state of the world. We show that there is a range of prior beliefs for which a highly reputed expert cannot communicate credibly, while a less reputable expert can (Proposition 3). We use the term *Guru Effect*[1] to describe the negative impact of reputation on informative communication.

Our paper relates to Holmström (1999) which argues that career concerns play an important role in explaining the incongruence in risk preference between a manager and a firm: The manager's unobserved talent determines the probability of success of the investment. To avoid adversely affecting opinions about her talent, the manager has an incentive to pass on risky projects. A possible remedy for the firm is to hire consultants to obtain advice on potential investments. Our work shows that relying on external advisors for guidance may not help the firm invest optimally if those advisors also have career concerns.

Our paper also contributes to the literature on reputational cheap-talk games (see, e.g., Morris (2001), Ottaviani and Sørensen (2006)) by addressing cases where the expert can

[1]This term references Benabou and Laroque (1992) and Gossner and Melissas (2006) who present experts as "gurus" whom the media turn to for the perceived quality of their information.

influence the precision of the information that her audience relies on in order to assess the accuracy of her recommendation. To our knowledge Gentzkow and Shapiro (2006) were the first to consider reputational cheap-talk games in which the receivers may not receive feedback about the realization of the state of the world. They find that the expert is less communicative when her audience does not receive any feedback. We generalize their result in a setting in which the public information about the state is endogenous (see 6).

Closer to our work, Mariano (2012) and Rüdiger and Vigier (2019) identify the endogeneity problem that arises when the audience evaluates an expert based on some outcome that is influenced by the expert's recommendation. Mariano (2012) studies a simpler setting in which the correlation between the observed outcome and the underlying state depends solely on the report of the expert. By contrast, we assume that the audience's beliefs determine the precision of the public information about the state, which allows us to rationalize why highly reputed experts may behave differently than less reputed experts. In a dynamic game, Rüdiger and Vigier (2019) derive conditions under which an incompetent but highly reputed expert can maintain a good reputation. Lastly, Sanyal and Sengupta (2006) study a game in which the expert can induce the receiver to take a status quo action that does not reveal the state. They use a highly stylized information structure and focus on establishing conditions under which the expert can communicate information that increases the expected payoff of the receiver.

In section 2, we present the model. In section 3 , we characterize the most informative equilibrium of the game; we show how the endogeneity of the public information affects the reporting incentives of the expert and why the initial reputation of the expert matters. In section 4, we extend the model to allow for asymmetry of information about the expert's type and a more general functional form for the precision of the ex post signal. We also consider the impact of competition between experts and identify conditions under which competition can alleviate the barrier to entry. Section 5 concludes.

2 The Model

We consider a communication game played by an expert and an audience composed of multiple receivers. The state of the world is a binary random variable $\theta \in \{\theta_0, \theta_1\}$. In the following, we will refer to θ_0 as the low state and to θ_1 as the high state. The expert and her audience share common prior beliefs about the state $\mu_i \equiv \Pr(\theta = \theta_i)$, $\forall i \in \{0, 1\}$.

At the beginning of the game, the expert receives a private signal $s \in \mathbb{S} = \{s_0, s_1\}$ about the true state. The informativeness of the signal depends on the ability of the expert. An expert of ability t receives the correct signal about the state with probability $t \equiv \Pr(s = s_i|\theta = \theta_i)$, $\forall i \in \{0, 1\}$. The expert is either good, $t = g$, or bad, $t = b$. We assume that: $1/2 < b < g < 1$.

Assumption (A1). *Neither the expert nor the audience observes the expert's ability. Both have common prior beliefs about the expert's ability: they assign probability p_g to the expert being good and probability $p_b = 1 - p_g$ to the expert being bad. We denote by $\tilde{t}$ the prior expectation about the ability of the expert: $\tilde{t} = p_g g + p_b b$*

After receiving her signal, the expert forms a posterior belief about the state; $p(\theta_i|s_j) \equiv Pr(\theta = \theta_i|s = s_j)$, with $i, j \in \{0, 1\}$, denotes the expert's posterior on state θ_i following the signal s_j. She then sends a message $m \in \mathbb{M} = \{m_0, m_1\}$ to her audience. The expert's strategy takes the form of a mapping from signals into a probability distribution over messages: $\sigma : \mathbb{S} \times \mathbb{M} \to [0, 1]$, $\sigma(m|s) \equiv Pr(m|s)$. The audience uses the expert's message and its conjecture about the expert's strategy $\tilde{\sigma}$ in order to update its prior beliefs about the state. We denote the audience's updated prior by $\nu_i(m, \tilde{\sigma}) \equiv p^a(\theta_i|m, \tilde{\sigma})$[2].

The central idea of the paper is that the realization of the state of the world is generally not perfectly observed ex post. Instead, at the end of the game, all players observe a binary ex

[2] We will use the superscripts a to refer to the beliefs of the audience and e for the expert when we need this distinction to avoid confusion.

post signal $X \in \{X_0, X_1\}$. $X_i|\theta_i$ follows a binomial distribution whose probability of success τ is a function of the updated belief of the audience: $\tau(\nu_1) \equiv Pr(X = X_i|\theta = \theta_i) \in \left[\frac{1}{2}, 1\right]$.

The informativeness of the ex post signal depends on the interim beliefs of the audience through the following mechanism: Upon observing the expert's message, each receiver takes an action that delivers a state-dependent payoff. The most profitable action in the high state s_1 allows the receiver to observe an informative signal about the state while the most profitable action in the low state s_0 does not allow the receiver to observe any information. The aggregation of the receivers' individual signals forms the ex post signal. The ex post signal is more precise when more receivers choose the informative action. Since a receiver is more likely to choose the informative action when he expects the state to be high, higher audience's expectations on the high state increase the precision of the ex post signal.

Finally, given the audience's conjecture about the strategy played by the expert $\tilde{\sigma}$, the audience computes the posterior reputation of the expert by comparing the message that she sent with the ex post signal. We denote the audience's posterior beliefs that the expert is good $p_g(m, X)$. The expert's preferences over perceived abilities are represented by a strictly increasing von Neumann-Morgenstern utility function $v(t)$. The reputational payoff from sending the message m when the realization of the ex post signal is X is:

$$\pi(m, X) = v(g)p_g(m, X) + v(b)(1 - p_g(m, X))$$

Note that the message that maximizes the payoff of the expert is the message that leads to the highest posterior reputation $p_g(m, X)$. Figure 1 summarizes the timing of the game.

2.1 Reputation Formation

Before determining the information transmitted in the equilibrium of this game, it is necessary to describe how the audience computes the posterior reputation of the expert. After

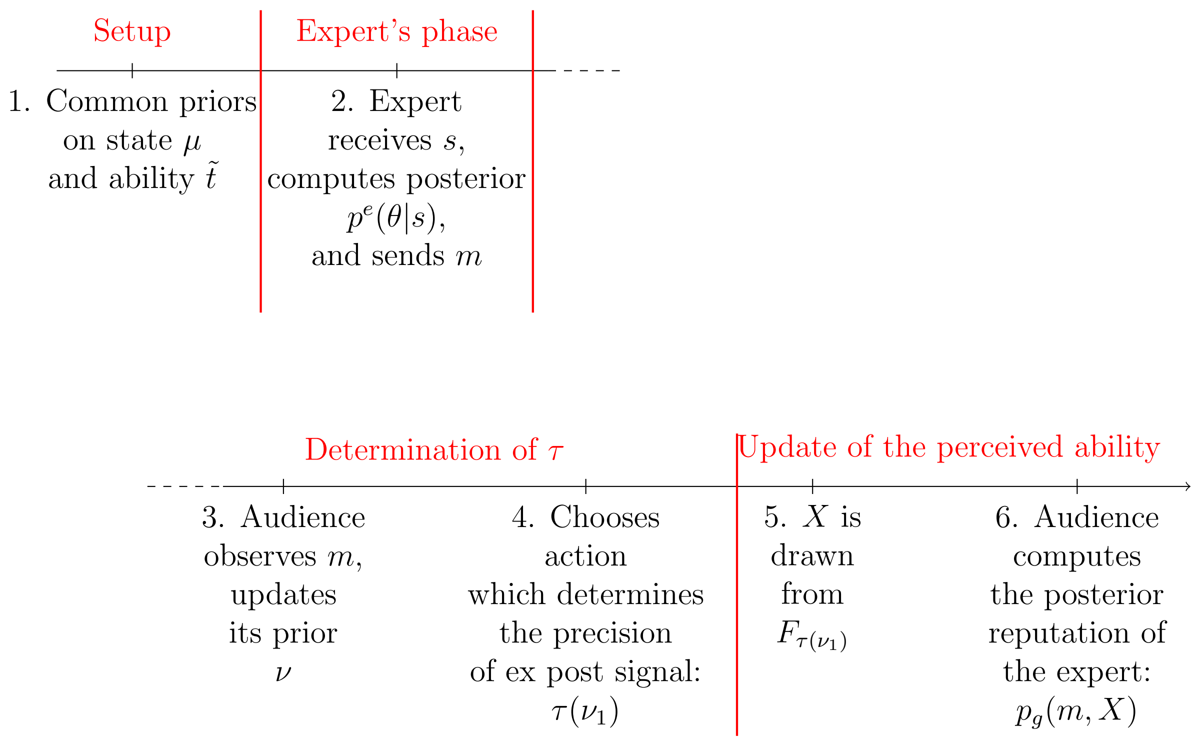


Figure 1: Timing of the game

receiving the message sent by the expert, the Bayesian audience forms a belief about the state that depends on the perceived ability of the expert $\tilde{t}$ and the conjecture that the audience makes about her strategy $\tilde{\sigma}$. The updated prior beliefs of the audience write as:

$$\nu_i(m, \tilde{\sigma}) = \sum_{j \in \{0,1\}} p(\theta_i | s_j) p^a(s_j | m, \tilde{\sigma}) \quad \forall i \in \{0, 1\} \tag{2.1}$$

$p(\theta_i|s_j)$ coincides with the expert's posterior beliefs:

$$p(\theta_i | s_j) = \begin{cases} \frac{\mu_i \, \tilde{t}}{\mu_i \, \tilde{t} + (1-\mu_i)\,(1-\tilde{t})} & \text{if } i = j \\ \frac{\mu_i \,(1-\tilde{t})}{\mu_i \,(1-\tilde{t}) + (1-\mu_i)\,\tilde{t}} & \text{if } i \neq j \end{cases} \tag{2.2}$$

$p^a(s_j|m, \tilde{\sigma})$ is the audience's belief that the expert received the signal s_j after observing the message m. When the expert reports m with some positive probability under strategy $\tilde{\sigma}$, we use Bayes rules to derive $p^a(s_j|m, \tilde{\sigma})$:

$$p^a(s_j | m, \tilde{\sigma}) = \frac{\tilde{\sigma}(m|s_j)}{\tilde{\sigma}(m|s_j) + \tilde{\sigma}(m|s_i)}$$

Upon observing the realization of the ex post signal X and the message m, and given some conjecture about the strategy played by the expert, $\tilde{\sigma}$, the audience forms posterior beliefs on the state that are given by:

$$p^a(\theta_i | X_j, m) = \begin{cases} \frac{\tau(\nu_1)\nu_i}{\tau(\nu_1)\nu_i + (1-\tau(\nu_1))\nu_{-i}} & \text{if } i = j \\ \frac{(1-\tau(\nu_1))\nu_i}{(1-\tau(\nu_1))\nu_i + \tau(\nu_1)\nu_{-i}} & \text{if } i \neq j \end{cases} \tag{2.3}$$

where ν_i stands for $\nu_i(m, \tilde{\sigma})$.

If the audience had perfect information about the true state, it would confront the message of the expert with the observed state in order to assess the ability of the expert:

$$p(g|m,\theta) = p_g \frac{p(m|\theta,g)}{p(m|\theta)},$$

where

$$p(m|\theta,g) = \sum_{i\in\{0,1\}} \tilde{\sigma}(m|s_i)p^a(s_i|\theta,g),$$

and

$$p(m|\theta) = E_t(p(m|\theta,t)).$$

However, the audience can only observe a noisy signal about the realized state, X. To compute the posterior reputation of the expert, the audience compares the message with its ex post beliefs about the true state:

$$p_g(m,X) = \sum_{i\in\{0,1\}} p(g|m,\theta_i)p^a(\theta_i|X).$$

In deciding what message to send, the expert aims at maximizing her expected reputation. The observation that drives our results is that the audience observes a less informative ex post signal when it believes the low state to be the most likely upon receiving the expert's message. This has two implications.

First, the expert has an incentive to lie and report the low message in order to garble the ex post signal. To understand this, consider a situation in which the low state is initially expected to be more likely, that is $\mu_0 > 1/2$. Observe that the expert's own expectation about her ability is higher than her initial reputation $\tilde{t}$ when she observes a signal that conforms to the prior and lower than $\tilde{t}$ when she receives a signal that contradicts the prior: $E^e(t|s_0) > \tilde{t} > E^e(t|s_1)$. Suppose that the expert actually receives the high signal. Assume that upon receiving s_1, the expert expects the high state to be the more likely, $p^e(\theta_1|s_1) \geq 1/2$. Additionally, assume that the precision of the ex post signal $\tau(\nu_1)$ is such that the signal is uninformative when ν_1 is below 1/2 and perfectly informative otherwise.

If the expert is truthful, the ex post signal will be perfectly informative. In this case, the audience will receive strictly more information about the state than the expert. The expert's expected reputation from truthtelling is equal to her own expectation about her ability, having received s_1, $(E^e(t|s_1))$.[3] To the contrary, if the expert deviates from truthtelling and sends m_0, the audience will not receive any feedback about the realized state. In this case, the audience has no more information than the expert. Having assumed truthtelling, the audience's expectation about the expert's ability corresponds to the expert's expectation about her ability, having received s_0, $(E^e(t|s_0))$. It follows that the expert will misreport her signal even though she believes θ_1 to be more likely given her private information.

Second, the initial reputation of the expert affects the information she can transmit. Consider the reporting behavior of two experts: a highly reputed expert with initial reputation $\overline{p_g}$ and a less reputed expert with reputation $\underline{p_g} < \overline{p_g}$. As above, we assume that the low state is initially more likely, $\mu_0 > 1/2$. Upon receiving the high signal, both experts expect the high state to be the more likely, $p^e(\theta_1|s_1, p_g) \geq 1/2 \; \forall p_g \in \{\underline{p_g}, \overline{p_g}\}$. Suppose that the precision of the ex post signal $\tau(\nu_1)$ is such that the signal is uninformative when $\nu_1 \leq \bar{\nu} \equiv p^a(\theta_1|m_0, \sigma^T, \overline{p_g})$ and perfectly informative otherwise. This means that the high reputation expert can suppress feedback about the state by credibly sending the low message. In this case we know that the expert will deviate from truthtelling. However, since $p^a(\theta_1|m_0, \sigma^T, \overline{p_g}) < p^a(\theta_1|m_0, \sigma^T, \underline{p_g})$, the audience always observes the realization of the state perfectly when the expert is less reputed. The low-reputation expert cannot do better than announcing the state that she believes to be more likely and will therefore report s_1 truthfully.

[3]This is because the expected posterior probability equals the prior

3 Equilibrium Reporting

We turn to the formal analysis of the equilibrium of the game. Cheap-talk games typically admit multiple equilibria, including a pooling equilibrium wherein the expert does not transmit any credible information to her audience. Here we will characterize the most informative equilibrium of the game and study how the prior on the state and the reputation of the expert affect the quantity of information that can be transmitted in this most informative equilibrium.

Definition 1 (Perfect Bayesian Nash Equilibrium). *A strategy σ^* constitutes a perfect Bayesian Nash equilibrium of the game if the following conditions are met:*

1. *σ^* maximizes the expected payoff of the expert:*

$$\sigma^*(m_i|s) = \arg\max_{\eta\in[0,1]} \eta E^e[p_g(m_i, X)|s] + (1-\eta)E^e[p_g(m_{-i}, X)|s] \quad \forall i \in \{0,1\} \quad \forall\, s \in \{s_0, s_1\}.$$

2. *The audience updates its belief about the ability of the expert using Bayes' rule:*

$$p_g(m, X) = \sum_{i\in\{0,1\}} p(g|m, \theta_i)p^a(\theta_i|X, m) \quad \textit{if} \sum_{i\in\{0,1\}} \tilde{\sigma}(m|s_i) > 0.$$

   If the audience expects both types of expert to be as likely to send a zero-probability message:

$$p_g(m, X) = p_g \quad \textit{if} \sum_{i\in\{0,1\}} \tilde{\sigma}(m|s_i) = 0.$$

3. *The audience correctly forecasts the expert's strategy: $\tilde{\sigma}(.|s) = \sigma^*(.|s)\,\forall s$.*

To characterize the most informative equilibrium, we start by fixing the reputation p_g and by assuming that the audience believes that the expert is truthful ($\sigma = \sigma^T$). Given those beliefs, we identify the priors for which the expert finds it optimal to tell the truth. After receiving the signal s_i, the expert will have an incentive to report her signal truthfully if her expected reputation is higher when she tells the truth than when she lies. Mathematically this condition translates in equation 3.1.

$$E^e[p_g(m_i, X)|s_i] \geq E^e[p_g(m_{-i}, X)|s_i] \tag{3.1}$$

We can show that a signal s_i satisfies condition 3.1 whenever, assuming truthtelling, the audience's posterior about θ_i given m_i *as expected by the expert* is higher than the audience's posterior about θ_{-i} given m_{-i}. The expected posterior of the audience is given by:

$$E^e[p^a(\theta_i|X, m_i)|, s_i] = \tag{3.2}$$

$$\sum_{k\in\{0,1\}} p^a(\theta_i|X_k, m_i) \sum_{j\in\{0,1\}} p(\theta_j|s_i)[\tau(\nu_1(m_i, \sigma^T))\mathbb{1}\{j = k\} + (1 - \tau(\nu_1(m_i, \sigma^T))\mathbb{1}\{j \neq k\}]. \tag{3.3}$$

Lemma 1. *3.1 is verified if and only if:*

$$E^e[p^a(\theta_i|X, m_i)|s_i] \geq E^e[p^a(\theta_{-i}|X, m_{-i})|s_i]. \tag{3.4}$$

(See proof in Appendix A.)

Solving equation 3.4 for both signals allows us to derive the incentive compatible set that is the range of prior beliefs for which the expert has an incentive to play truthfully.

Definition 2. *The incentive compatible set, denoted $ICS_{\tilde{t}}$*[4][5]*, is defined as:*

$$ICS_{\tilde{t}} \equiv \{\mu_1 \in [0,1] \mid E^e[p^a(\theta_i|X, m_i)|s_i] \geq E^e[p^a(\theta_{-i}|X, m_{-i})|s_i],\ \forall i\}\,,$$

which is:

$$ICS_{\tilde{t}} = [\underline{\mu}_1^T(\tilde{t}), \overline{\mu}_1^T(\tilde{t})],$$

with

$$\underline{\mu}_1^T(\tilde{t}) \equiv \mu_1 \mid E^e[p^a(\theta_1|X, m_1)|s_1] = E^e[p^a(\theta_0|X, m_0)|s_1], \tag{3.5}$$

and

$$\overline{\mu}_1^T(\tilde{t}) \equiv \mu_1 \mid E^e[p^a(\theta_0|X, m_0)|s_0] = E^e[p^a(\theta_1|X, m_1)|s_0]. \tag{3.6}$$

$\underline{\mu}_1^T(\tilde{t})$ is the lowest prior belief on the high state for which there exists a separating equilibrium. $\overline{\mu}_1^T(\tilde{t})$ is the highest prior belief on the high state for which there exists a separating equilibrium.

Proposition 1. *In the most informative equilibrium, the expert is truthful for any prior within the incentive compatible set $ICS_{\tilde{t}}$: For all $\mu_1 \in ICS_{\tilde{t}}$, $\sigma(m_1|s_1) = \sigma(m_0|s_0) = 1$. The expert plays a pooling equilibrium for any prior falling outside the Incentive Compatible Set: For all $\mu_1 \notin IC_{\tilde{t}}$, $\sigma(m|s_i) = \sigma(m|s_j)$, for all $i, j \in \{0,1\}$.*

Proof. By definition, truthtelling is an equilibrium inside the incentive compatible set. Ottaviani and Sørensen (2001) shows that the only equilibrium outside the incentive compatible set is the pooling equilibrium. □

[4]Notice that $E^e[p^a(\theta|X)|m, s]$ depends on the expected ability of the expert $\tilde{t}$ via $p(\theta|s)$ rather than her true ability since the expert does not know her type. We index the incentive compatible set by $\tilde{t}$ to reflect this dependence.

[5]Note that $\mu_1 = 1/2$ always belong to the ICS so that $\underline{\mu}_1^T(\tilde{t}) \leq 1/2$ and $\overline{\mu}_1^T(\tilde{t}) \geq 1/2$.

3.0.1 Special Case: the All-or-Nothing Signal

Consider an ex post signal that is uninformative when the updated belief on the state being high, ν_1, is below a given threshold χ and perfectly informative otherwise. The *All-or-Nothing* precision function of this ex post signal is described below:

$$\tau = \begin{cases} \frac{1}{2} & \text{if } \nu_1(m) < \chi \\ 1 & \text{if } \nu_1(m) \geq \chi \end{cases}$$

Note that when the audience believes the expert to be truthful, $\nu_1(m_i) = p(\theta_1|s_i)$, $\forall i\{0,1\}$. Assuming truthtelling, the corresponding expectation of the expert on the posterior belief of the audience depends on the updated prior ν_1 and the threshold χ:

$$\text{For any } i,j,k \in \{0,1\}, \quad E^e[p^a(\theta_i|X,m_j)|s_k] = \begin{cases} p(\theta_1|s_j) & \text{if } p(\theta_1|s_j) < \chi \\ p^e(\theta_i|s_k) & \text{if } p(\theta_1|s_j) \geq \chi \end{cases}$$

To study the reporting incentives of the expert, we partition the prior space into the three following intervals:

(A) Define $\underline{\mu}_1 \equiv \mu_1 | p(\theta_1|s_1) = \chi$. Since $\forall i \in \{0,1\}$, $p(\theta_1|s_i)$ is a strictly increasing function of μ_1, for any prior $\mu_1 \in [0, \underline{\mu}_1)$, $p(\theta_1|s_1) < \chi$. When the prior on the high state is sufficiently low, the ex post signal is uninformative regardless of the recommendation given by the expert. The audience uses only its updated prior to update its belief about the expert's ability. As a result, the expert has incentives to pretend to have observed the realization of the state associated with the highest prior. The expert is truthful only if the low state is initially perceived as exactly as likely as the high state. Formally, truthtelling is incentive compatible if and only if:

$$p(\theta_i|s_i) \geq p(\theta_{-i}|s_i)\, \forall i \quad \Leftrightarrow \quad \mu_1 = \frac{1}{2}.$$

(B) Define $\overline{\mu}_1 \equiv \mu_1 | p(\theta_1|s_0) = \chi$. For all prior $\mu_1 \in [\overline{\mu}_1, 1]$, $p(\theta_1|s_0) \geq \chi$. When the prior on the high state is sufficiently large, the ex post signal is perfectly informative regardless of the recommendation given by the expert. At the end of the game, the audience always observes perfectly the true state of the world. This case is similar to the analysis developed in Ottaviani and Sørensen (2001, 2006). Upon observing her signal, the expert has incentive to report the state to which she attached the highest posterior belief. Formally, the expert has the incentive to report her signal truthfully if and only if:

$$p(\theta_i|s_i) \geq p(\theta_{-i}|s_i) \forall i \Leftrightarrow 1 - \tilde{t} \leq \mu_1 \leq \tilde{t}.$$

(C) When the expert is thought to be truthful, for all prior $\mu_1 \in [\underline{\mu}_1, \overline{\mu}_1)$, $p(\theta_1|s_0) \leq \chi \leq p(\theta_1|s_1)$. For intermediate values of the prior on the high state, the precision of the ex post signal depends on the recommendation given by the expert: following a bad recommendation, the ex post signal is uninformative; following a good recommendation, the ex post signal is perfectly informative. The expert has incentives to report the low signal truthfully if and only if the low state is *ex ante* perceived as the most likely. She has incentive to report the high signal truthfully if and only if the high state is *ex post* perceived as the most likely. Formally, the expert has incentive to report both signals truthfully if and only if:

$$\frac{1}{2} \leq \mu_1 \leq \tilde{t}.$$

3.1 The Barrier to Entry Effect

Compared to the benchmark, where the audience perfectly observes the realization of the state ex post, truthful reporting of the high signal is supported over a more narrow range

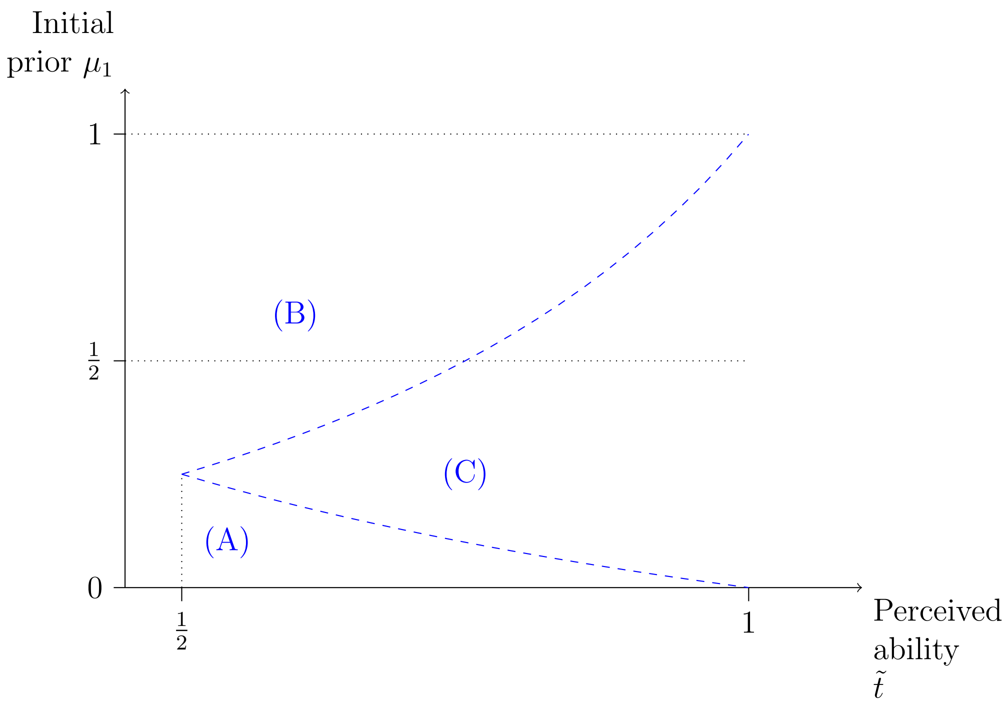
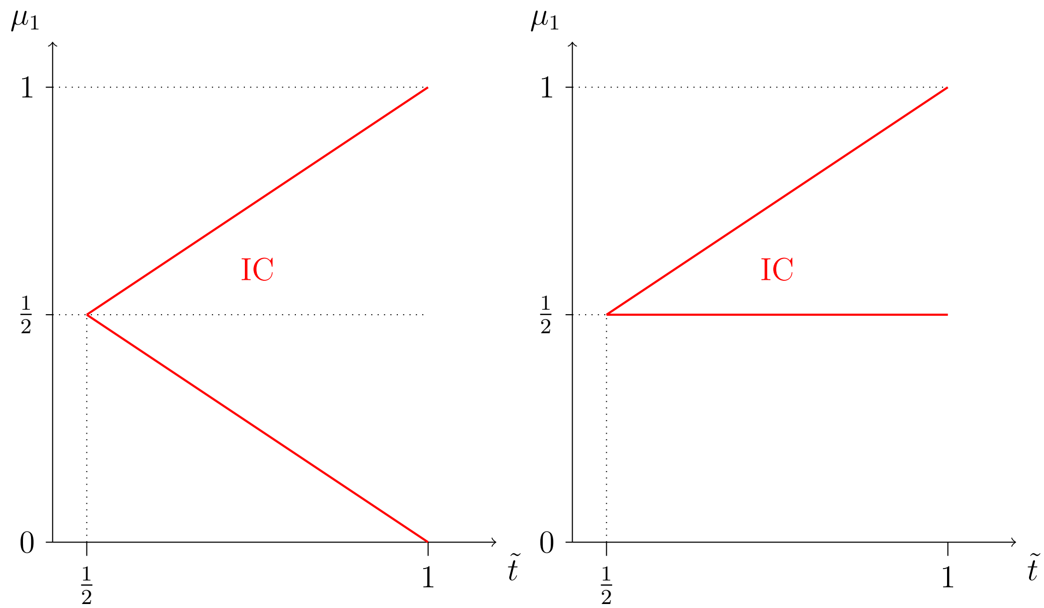


Figure 2: Partition of the prior space for $\chi = \frac{1}{4}$

of priors. However, truthful reporting of the low signal is supported over the same range of priors. It follows that the incentive compatible set for the expert is asymmetric, as shown by the bounds derived in section 3.0.1. If we interpret the state of the world to be the unknown quality of an experience good newly launched onto the market, and a high signal as a recommendation to buy the good, this shift of the incentive compatible set towards higher prior beliefs translates into a *barrier to entry effect*: The reputational concern of the expert coupled with the dependence of the precision of the ex post signal on the beliefs of the audience prevents the expert from making credible recommendations about lower prior products.

Definition 3. *There is a barrier to entry if and only if* $\underline{\mu}_0^T(\tilde{t}) < \underline{\mu}_1^T(\tilde{t})$.

Proposition 2. *For all* χ *and* $\tilde{t}$ *such that* $\nu_1(m_0) \leq \chi \leq \nu_1(m_1)$*, the most informative equilibrium of the game displays a barrier to entry effect.*



(a) Incentive compatible set - Benchmark model

(b) Incentive compatible set - Noisy ex post Signal

Figure 3: Barrier to entry effect

Proof. For all χ and $\tilde{t}$ such that $\nu_1(m_0) \leq \chi \leq \nu_1(m_1)$, $\underline{\mu}_0^T(\tilde{t}) = 1 - \tilde{t} < 1/2 = \underline{\mu}_1^T(\tilde{t})$. □

Figure 3 shows the asymmetric incentive compatible set for the expert when the precision of the aggregate signal increases from $\frac{1}{2}$ to 1 when the expert changes her message from 0 to 1 and compares this set with the incentive compatible set of the expert in the benchmark model.

3.2 The Guru Effect

The dependence of the precision of the ex-post signal on the expert's message implies that the expert's initial reputation affects whether she can transmit her private information credibly. There is a range of prior beliefs for which, by sending a bad recommendation, a high reputation expert can render the ex-post signal uninformative, while a low reputation expert cannot. We say that the endogeneity of the precision of the ex-post signal creates a "guru effect": a high reputation expert has incentive to misreport a high signal in order to prevent her audience from updating their beliefs about her ability, while a low reputation expert has no incentive to deviate from truthtelling. We give a formal definition for the guru effect.

Definition 4. *guru effect:*

$$\exists\, \mu_1 \in [0,1] \;\; and \; \tilde{t}_1, \tilde{t}_2 \in [b, g] \; with \; \tilde{t}_1 < \tilde{t}_2, \;\; s.t. \;\; \mu_1 \in IC_{\tilde{t}} \quad \forall \tilde{t} \in \left[\tilde{t}_1, \tilde{t}_2\right] \;\; but \; \mu_1 \notin IC_{\tilde{t}'} \quad \forall \tilde{t}' > \tilde{t}_2.$$

Proposition 3. *The guru effect arises if and only if $\mu_1 < \frac{1}{2}$ and $\chi < \frac{1}{2}$.*

Proof. Consider a prior $\mu_1 \geq \frac{1}{2}$ and assume that the expert with perceived ability $\tilde{t}_1$ is truthful. It must be that $\nu_1(m_1, \tilde{t}_1) \geq \chi$, otherwise the expert would prefer to pretend that she received s_1 when she observed s_0. This implies that $\mu_1 \geq \underline{\mu}_1(\tilde{t}) = \frac{1}{2}$. For such a prior, truthtelling is incentive compatible if and only if $\mu_1 \leq \tilde{t}_1$. Any expert with an ability greater than $\tilde{t}_1$ will therefore be truthful. Furthermore, when $\mu_1 = \frac{1}{2}$, experts of all abilities are truthful. It follows that the guru effect can only arise for $\mu_1 < \frac{1}{2}$.

Now consider a prior $\mu_1 < \frac{1}{2}$ and assume that the expert with perceived ability $\tilde{t}_1$ is truthful. It must be that $\nu_1(m_0, \tilde{t}_1) \geq \chi$, otherwise $\mu_1 < \underline{\mu}_1(\tilde{t}_1) = \frac{1}{2}$ and the expert would prefer to pretend that she received s_0 when she observed s_1. $\nu_1(m_0, \tilde{t}_1) \geq \chi$ implies that $\mu_1 < \overline{\mu}_1(\tilde{t_1}) = \tilde{t_1}$. Note that $\nu_1(m_0, \tilde{t}_1) < \mu_1 < \frac{1}{2}$ when the expert is thought to be truthful. So χ must be strictly lower than $\frac{1}{2}$. Since $\nu_1(m_0, \tilde{t})$ is a strictly decreasing function of $\tilde{t}$, with $\nu_1(m_0, \tilde{t}) = 0$ when $\tilde{t} = 1$, there exists a value $\tilde{t}_2$ such that $\nu_1(m_0, \tilde{t}) < \chi \, \forall \tilde{t} > \tilde{t}_2$. For such threshold value, we have $\mu_1 < \underline{\mu}_1(\tilde{t}_1) = \frac{1}{2} \, \forall \tilde{t} > \tilde{t}_2$.

□

Figure 4 shows the incentive compatible set (ICS) when $\chi = \frac{1}{4}$.

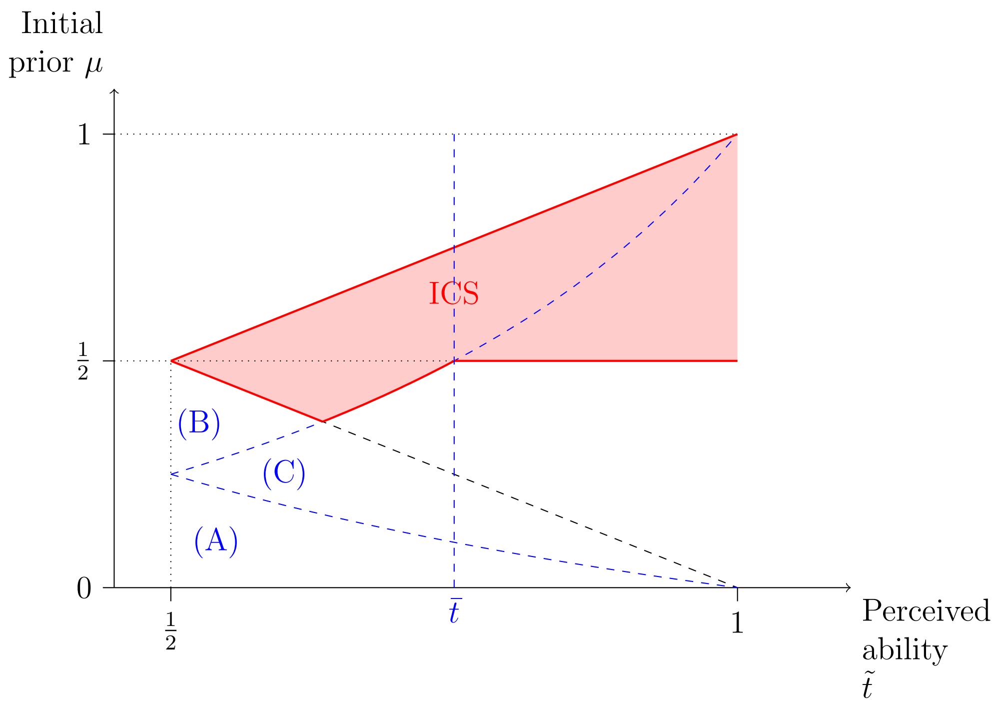


Figure 4: Guru effect: experts with perceived abilities $\tilde{t} > \bar{t}$ cannot be truthful for any prior μ_1 below $\frac{1}{2}$, while experts with abilities $\tilde{t} < \bar{t}$ can. (Here $\chi = \frac{1}{4}$.)

4 Extensions

4.1 Asymmetry of Information about the Ability of the Expert

Assumption (A1*). *The expert privately observes her ability. The audience believes that the expert is good with probability p_g.*

In this section, we add another dimension of asymmetry of information about the expert's ability. The expert conditions her reporting strategy to her ability, t, such that $\sigma(m|s,t) \equiv p(m|s,t)$. The incentive compatible set of this expert depends on her privately observed ability t as well as her perceived ability $\tilde{t}$:

$$ICS_{t,\tilde{t}} = \{\mu_1 \in [0,1] \mid E^e[p_g(m_i, X)|s_i, t] \geq E^e[p_g(m_{-i}, X)|s_i, t], \ \forall i\}.$$

Before characterizing the equilibrium of the game, it is useful to note that the incentive compatible sets for experts with lower abilities are included in the incentive compatible sets for experts with higher abilities. In other words, when a bad expert has no incentive to deviate from truthtelling neither does a good expert.

Proposition 4. $ICS_{b,\tilde{t}} \subseteq ICS_{g,\tilde{t}}$ *for* $\frac{1}{2} < b < g \leq 1$.

We detail the proof of this result in Appendix B. The proof does not rely on the assumption we made about the functional form of τ.

4.1.1 Equilibrium Reporting

Although the game admits multiple equilibria, we characterize the most informative equilibrium:

Proposition 5. *With privately observed abilities, the most informative equilibrium of the game is characterized by the following strategy:*

- *The good expert is truthful over the entire prior space.*

- *The bad expert is truthful over her incentive compatible set* $ICS_{b,\tilde{t}} = \left[\underline{\mu}_1^T(b,\tilde{t}), \overline{\mu}_1^T(b,\tilde{t})\right]$. *With*

$$\underline{\mu}_1^T(b,\tilde{t}) \equiv \mu_1 \mid E^e[p^a(\theta_1|X,m_1)|s_1,l] = E^e[p^a(\theta_0|X,m_0))|s_1,l)$$

And

$$\overline{\mu}_1^T(b,\tilde{t}) \equiv \mu_1|E^e[p^a(\theta_0|X,m_0)|s_0,l] = E^e[p^a((\theta_1|X,m_1)|s_0,l]$$

For $\mu_1 < \underline{\mu}_1^T(b,\tilde{t})$ *the expert sends* m_0 *after observing* s_0 *and sends* m_1 *with some positive probability after observing* s_1. *For* $\mu_1 > \overline{\mu}_1^T(b,\tilde{t})$, *the expert sends* m_1 *after observing* s_1 *and sends* m_0 *with some positive probability after observing* s_0.

Here both expert types report signals that they know not to be the most likely. To understand why this is possible, consider a prior $\mu_1 > \overline{\mu}_1^T(b,\tilde{t})$. If the audience expects the good expert to be truthful and therefore to send both messages with positive probability, the bad expert will be compelled to randomize and send m_0 with some positive probability upon observing s_0 in order to mimic the expected behavior of the good expert. Whenever the bad expert is just indifferent between sending m_0 and m_1, the good expert is confident enough to report s_0 truthfully.

We give a detailed proof in Appendix B. The proof does not rely on the assumption we made about the functional form of τ. This result is similar to Ottaviani and Sørensen (2006) who show that in the presence of asymmetry of information about the expert type, the good expert is always truthful while the bad expert is truthful for a range of priors centered around 1/2 and garbles the signal for more extreme priors.

4.1.2 Barrier to Entry and Guru Effect

In the most informative equilibrium of the game, the good expert is truthful for all priors. Therefore, we restrict our attention to the equilibrium reporting strategy of the bad expert to analyze the robustness of our main results to the introduction of asymmetry of information about the expert's ability. When the bad expert observes her ability, the range of priors for which she can be truthful is:

$$\frac{1}{1+\sqrt{\frac{b}{1-b}\frac{1-\tilde{t}}{\tilde{t}}}} \leq \mu_1 \leq \frac{1}{1+\sqrt{\frac{1-b}{b}\frac{1-\tilde{t}}{\tilde{t}}}}$$

The lower bound of this interval $\underline{\mu}_1^T(b,\tilde{t})$ is strictly higher than $\frac{1}{2}$, which means that the bad expert cannot report truthfully a high signal even when the state of the world ex ante is as likely to be low as to be high. It follows that truthful reporting of the high signal by the low type expert is supported for a more narrow range of priors than truthful reporting of the low signal. In other words, the reporting strategy of the bad expert induces a barrier to entry effect. Since the lower bound of the incentive compatible set of the bad expert increases with her perceives ability $\tilde{t}$, the reporting strategy of the expert is also characterized by a guru effect. We note that the upper bound of the incentive compatible set is now increasing in the reputation of the expert. This is because the added noise in the ex post signal following a bad recommendation from a more reputable expert means the expert is more likely to expect her audience to believe the state to be ω_0 when she sends the message m_0. It follows that truthful reporting of the low signal is supported over a wider range of priors.

Figure 5 represents the incentive compatible set of the bad expert.

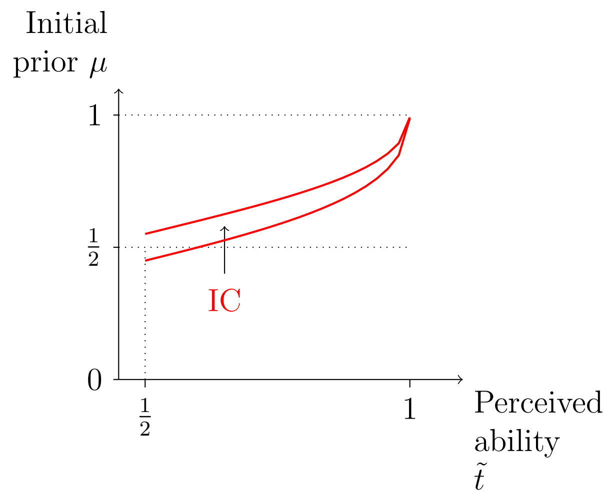


Figure 5: Incentive compatible set of the bad expert, $l = 0.6$.

4.2 General Precision Function

In this section we show that our results extend to a more general functional form for the informativeness of the ex post signal.

Assumption (A2*). *$\tau()$ is continuously differentiable and nondecreasing in the audience's updated prior on the high state, $\tau'(\nu_1) \geq 0$.*

Under assumptions (A1) and (A2*) we show that the equilibrium is more informative–i.e. truthtelling is supported over a larger range of priors–when the ex post signal is more precise. This means that efficient mechanisms of information propagation within the audience yield a higher degree of information transmission from the expert to her audience. This finding is reminiscent of Gentzkow and Shapiro (2006) who show that the sender is more truthful when her audience is more likely to receive exogenous feedbacks about the realized state. Here we show that this result continues to hold when the expert can influence the amount of feedback received by the audience.

We first show in Lemma 2 that we can write the expected posterior of the audience as a weighted average of the audience's interim beliefs and the expert's posterior:

Lemma 2. *For all functions $\tau : [0,1] \to [1/2,1]$ and all strategy $\tilde{\sigma}$, there exists a function $\alpha : [1/2,1] \times \{0,1\} \times [0,1] \to [0,1]$ such that:*

$$E^e[p^a(\theta_i|X,m)|s] = (1-\alpha(\tau,m,\mu_1))\nu_i(m,\tilde{\sigma}) + \alpha(\tau,m,\mu_1)p(\theta_i|s).$$

For all $m \in \{0,1\}$ and $\mu_1 \in [0,1]$, $\alpha(1/2,m,\mu_1) = 0,\ \alpha(1,m,\mu_1) = 1,$ and $\frac{\partial\alpha}{\partial\tau}(\tau,m,\mu_1) \geq 0$.

See the proof in Appendix C.

Lemma 3. *When presumed truthful, the expert has no incentive to misreport s_i if and only if:*

$$p(\theta_i|s_i) \geq (1-\alpha(\tau(\nu_1(m_{-i},\sigma^T),m_{-i},\mu_1))p(\theta_{-i}|s_{-i}) + \alpha(\tau(\nu_1(m_{-i},\sigma^T),m_{-i},\mu_1)p(\theta_{-i}|s_i). \tag{4.1}$$

Proof. Follows directly from Lemma 2. □

Proposition 6. *The incentive compatibility set is increasing in the precision of the ex post signal: Take two functions $\tau_1, \tau_2 : [0,1] \to [1/2,1]$, such that $\forall x \in [0,1]$, $\tau_1(x) < \tau_2(x)$, then $ICS_{\tilde{t},\tau_1} \subseteq ICS_{\tilde{t},\tau_2}$.*

We detail the proof in Appendix C.

This leads to the following result:

Proposition 7. *The incentive compatibility set in the game with perfect observation of the state of the world contains the incentive compatibility set with imperfect information about the state of the world : $ICS_{\tilde{t},\tau=1} \supseteq ICS_{\tilde{t},\tau} \quad \forall \quad \tau \neq 1$.*

Proof. Direct from proposition 6. □

Proposition 8 (Barrier to entry with general precision function)**.** *Under assumption (A2*), there is a barrier to entry in the most informative equilibrium of the game that is:*

$$\underline{\mu}_0^T(\tilde{t}) < \underline{\mu}_1^T(\tilde{t}).$$

Proof. We begin by remarking that 4.1 can be rewritten as a condition on the prior μ_i: (i) $p(\theta_i|s_i) \geq p(\theta_{-i}|s_{-i}) \Leftrightarrow \mu_i \geq 1/2$; (ii) $p(\theta_i|s_i) \geq p(\theta_{-i}|s_i) \Leftrightarrow \mu_i \geq 1-\tilde{t}$; inequalities (i) and (ii) together with 4.1 imply that truthful reporting of s_1 is incentive compatible if and only if:

$$\mu_1 \geq (1-\alpha(\tau_{m_0}, m_0, \mu_1))1/2 + \alpha(\tau_{m_0}, m_0, \mu_1)(1-\tilde{t}).$$

Where τ_{m_0} stands for $\tau(\nu_1(m_0, \sigma^T))$. Let $g_1 : [0,1] \to [1-\tilde{t}, 1/2]$, with:

$$g_1(\mu) = (1-\alpha(\tau_{m_0}, m_0, \mu_1))1/2 + \alpha(\tau_{m_0}, m_0, \mu_1)(1-\tilde{t}).$$

Note that $0 < g_1(0) = g_1(1) = 1/2 < 1$, $g_1'() < 0$ for all $\mu < 1/2$, and $g_1'() > 0$ for all $\mu > 1/2$. Hence, $g_1()$ admits at least one fixed point. $\underline{\mu}_1^T = \max\{\mu_1 \,|\, g_1(\mu_1) = \mu_1\}$. Similarly we can show that $\underline{\mu}_0^T$ is the largest fixed point of $g_0(\mu_1) = (1-\alpha(\tau_{m_1}, m_1, 1-\mu_1))1/2 + \alpha(\tau_{m_1}, m_1, 1-\mu_1)(1-\tilde{t})$.

For fixed $\bar{\tau}$ and $\bar{\mu_1}$, $\alpha(\bar{\tau}, m_0, \bar{\mu_1}) = \alpha(\bar{\tau}, m_1, 1-\bar{\mu_1})$. Since $\frac{\partial \alpha}{\partial \tau} > 0$, it follows that $\alpha(\tau_{m_0}, m_0, \bar{\mu_1}) < \alpha(\tau_{m_1}, m_1, 1-\bar{\mu_1})$. In turn, we get that $\underline{\mu}_0^T < \underline{\mu}_1^T$. $\square$

Let us denote $\underline{\mu_1}$ and $\overline{\mu}_1$ respectively as the infimum and the supremum of $ICS_{\tilde{t},\tau}$. It is easy to show that $\underline{\mu_1} \leq 1/2 \leq \overline{\mu}_1$.

Proposition 9 (Guru effect with general precision function)**.** *Under assumption (A2*), the incentive compatibility set converges to a singleton as the expected ability tends to 1.* $\lim_{\tilde{t}\to 1} \underline{\mu}_1^T = \lim_{\tilde{t}\to 1} \underline{\mu}_0^T = \frac{1}{2}$.

Proof. For all $i \in \{0,1\}$, $\lim_{\tilde{t}\to 1} \alpha(\tau_{m_i}, m_i, \mu_1) = 0$. It follows that $\lim_{\tilde{t}\to 1} \underline{\mu}_i^T = 1/2\, \forall i \in \{0,1\}$. $\square$

4.3 Competition between experts

In this section, we examine the impact of competition between experts. Ottaviani and Sørensen (2006) show that when the true state of the world is perfectly observed ex post, competition does not matter. They argue that for competition to change the amount of information transmitted by the expert, one would need to introduce correlation between experts' private signals or assume that the expert does not have a von Neumann-Morgenstern utility function. In this section, we show that competition between experts generally changes the equilibrium when the precision of the ex post signal is endogenous. This result holds even when private signals are conditionally independent and when the expert has a von Neumann-Morgenstern utility.

Consider a communication game similar to the one described in Section 2 with two experts. Both experts have the same initial reputation $\tilde{t}$. Experts do not have private information about their own ability or the ability of their opponent. Upon receiving her private signal $s^n \in \mathbb{S}^n = \{s_0, s_1\}$, the expert $n \in \{1,2\}$ sends a message $m^n \in \mathbb{M}^n\{m_0, m_1\}$ according to strategy $\sigma_n : \mathbb{S}^n \times \mathbb{M}^n \to [0,1]$ with $\sigma_n(m^n|s^n) = p(m^n|s^n)$. We denote by $\sigma = (\sigma_1, \sigma_2)$ a strategy profile. Experts' signals are conditionally independent: $p(s^n|\theta, s^{-n}) = p(s^n|\theta)$. Both experts send their messages simultaneously. The interim beliefs of the audience depend on both expert messages and strategies:

$$\nu_i(m^1, m^2, \tilde{\sigma}) = \sum_{j\in\{0,1\}} \sum_{k\in\{0,1\}} p(\theta_i|s_j, s_k) p^a(s_j|m^1, \tilde{\sigma}_1) p^a(s_k|m^2, \tilde{\sigma}_2) \quad \forall i \in \{0,1\}, \tag{4.2}$$

with:

$$p(\theta_i|s_j, s_k) = \begin{cases} \mu_i & \text{if } j \neq k, \\ \frac{p(\theta_i|s_i)\tilde{t}}{p(\theta_i|s_i)\tilde{t}+(1-p(\theta_i|s_i))(1-\tilde{t})} & \text{if } j = k = i, \\ \frac{p(\theta_i|s_{-i})(1-\tilde{t})}{p(\theta_i|s_{-i})(1-\tilde{t})+(1-p(\theta_i|s_{-i}))\tilde{t}} & \text{if } j = k \neq i. \end{cases}$$

The payoff of expert n, $p_g^n(m^n, m^{-n}, X)$ depends on the message of expert $-n$ insofar as this message affects the posterior beliefs of the audience about the realized state:

$$p_g^n(m^n, m^{-n}, X) = \sum_{i\in\{0,1\}} p(g|\theta_i, m^n)p^a(\theta_i|X, m^n, m^{-n}).$$

This is the key difference with the benchmark model where the expert reputational payoff depends only on $p(g|\theta, m)$.

Definition 5 (Perfect Bayesian Nash Equilibrium with two experts)**.** *A strategy profile (σ_1^*, σ_2^*) constitutes a perfect Bayesian Nash equilibrium of the game if:*

1. *For all $n \in \{1,2\}$, for all $i \in \{0,1\}$, and for all $s^n \in \mathbb{S}^n$, σ_n^* maximizes the expected reputational payoff of the expert:*

$$\sigma_n^*(m_i|s^n) = \arg\max_{\eta\in[0,1]} \eta E^e_{\tilde{\sigma}_{-n}}[p_g(m_i, m^{-n}, X)|s^n] + (1-\eta)E^e\tilde{\sigma}_{-n}[\pi_n(m_{-i}, m^{-n}, X)|s^n],$$

with:

$$E^e_{\tilde{\sigma}_{-n}}[p_g(m^n, m^{-n}, X)|s^n] = \sum_{k\in\{0,1\}}\sum_{j\in\{0,1\}} E^e[p_g^n(m^n, m_j, X)|s^n]\tilde{\sigma}_{-n}(m_j|s_k)p(s_k|s^n).$$

2. *The audience updates its belief about the ability of the expert using Bayes' rule:*

$$p_g^n(m^n, m^{-n}, X) = \sum_{i\in\{0,1\}} p(g|\theta_i, m^n)p^a(\theta_i|X, m^n, m^{-n})$$

$$\textit{if} \sum_{i\in\{0,1\}} \tilde{\sigma_n}(m^n|s_i) > 0 \quad \textit{and} \sum_{i\in\{0,1\}} \tilde{\sigma}_{-n}(m^{-n}|s_i) > 0.$$

If the audience receives a zero-probability message, it does not update the expert's reputation:

$$p_g^n(m^n, m^{-n}, X) = p_g \quad \textit{if} \sum_{i\in\{0,1\}} \tilde{\sigma_n}(m^n|s_i) = 0.$$

A zero-probability message does not affect the audience beliefs about the state:

$$p(\theta_i^n|m^n) = \mu_i \quad \textit{if} \sum_{i\in\{0,1\}} \tilde{\sigma_n}(m^n|s_i) = 0$$

3. *The audience and the competing expert correctly forecast the expert's strategy:*

$$\tilde{\sigma_n} = \sigma_n^* \quad \forall\, n \in \{0,1\}.$$

As shown in Section 3, when presumed truthful and upon receiving the signal s_i, the expert has no incentive to deviate from truthtelling if and only if:

$$E^e_{\tilde{\sigma}_{-n}}[p_g(m_i, m^{-n}, X)|s_i] > E^e_{\tilde{\sigma}_{-n}}[p_g(m_{-i}, m^{-n}, X)|s_i]. \tag{4.3}$$

We can generalize Lemma 1 and show that Inequality 4.3 is equivalent to:

$$E^e_{\tilde{\sigma}_{-n}}[p^a(\theta_i|X, m_i, m^{-n})|s_i] \geq E^e_{\tilde{\sigma}_{-n}}[p^a(\theta_{-i}|X, m_{-i}, m^{-n})|s_i].$$

Is the expert more or less communicative when she faces competition? The answer depends on whether the expert expects the message of her opponent to increase or decrease the precision of the ex post signal.

Let us develop this argument in the special case of the All-or-Nothing Signal defined in Section 3.0.1 with:

$$\tau = \begin{cases} \frac{1}{2} & \text{if } \nu_1 < \chi \\ 1 & \text{if } \nu_1 \geq \chi \end{cases}$$

Definition 6 (Incentive compatible set under competition). *Assuming truthtelling for both experts, the set of prior beliefs under which expert n has no incentive to deviate from truthtelling is:*

$$ICS_{\tilde{t}}(2) = \left\{\mu_1 \in [0,1] \mid E^e_{\sigma^T_{-n}}[p^a(\theta_i|X, m_i, m^{-n})|s_i] \geq E^e_{\sigma^T_{-n}}[p^a(\theta_{-i}|X, m_{-i}, m^{-n})|s_i],\ \forall i \in \{0,1\}\right\}.$$

We start by showing how competition affects the incentive for the expert to misreport the signal *if the expert believes her opponent to be truthful.*

Lemma 4. *If $p(\theta_1|s_0) < \chi \leq \mu_1$, facing an opponent who is believed to be truthful strictly increases the range of prior beliefs for which the expert has no incentive to deviate from truthtelling. More precisely, $ICS_{\tilde{t}} \subset ICS_{\tilde{t}}(2)$*

See proof in Appendix D.

Proposition 10. *If $p(\theta_1|s_0) < \chi \leq \mu_1$, there exists a symmetric competitive equilibrium in which each expert is truthful for a larger range of priors than in the most informative equilibrium of the single-expert game.*

Proof. Lemma 4 shows that when the expert expects her opponent to be truthful, she has no incentives to deviate from truthtelling for any $\mu_1 \in ICS_{\tilde{t}}(2)$. Therefore truthtelling for both experts is an equilibrium for any $\mu_1 \in ICS_{\tilde{t}}(2)$. Since $ICS_{\tilde{t}}(2) \supset ICS_{\tilde{t}}$, truthtelling is supported at equilibrium for a larger range of priors under competition. □

When $\tau()$ is such that a single expert can suppress feedback on the realization of the state by sending m_0, competition alleviates the barrier to entry effect. This is because, upon receiving s_1, the expert expects her competitor to send m_1. Competition creates greater scope for the audience to learn the true state and hence limits the value of misreporting in order to decrease the precision of the ex post signal.

However when the recommendation of a single expert cannot suppress feedback on the state, competition actually hinders communication. This is because the message of the opponent can only induce the ex post signal to be less informative.

Lemma 5. *If $p(\theta_1|s_0, s_0) < \chi \leq p(\theta_1|s_0)$, facing an opponent who is believed to be truthful strictly decreases the range of prior beliefs for which the expert has no incentive to deviate from truthtelling. More precisely, $ICS_{\tilde{t}}(2) \subset ICS_{\tilde{t}}$*

See proof in Appendix D.

5 Conclusion

Our model shows how the endogenous nature of the information about the realization of the state gives additional incentive to the expert to bias her report. The expert will tend to dismiss private information that she believes to be accurate and send a recommendation that suppresses public information about the state. This implies that experts who review experience goods exacerbate the barrier to entry for new products. We predict that the expert has less incentive to transmit her information when the audience faces a higher cost of observing the true state of the world, which is precisely when the audience would benefit the most from the honest advice of an expert. Competition can discipline the experts to be truthful upon receiving favorable information when the competition is expected to provide the audience with more precise ex post public information. Camara and Dupuis (2014) use the model developed in this paper to build a structural estimation approach that allows us to quantify the abilities and the strategic biases of experts.

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A Equilibrium Reporting

Proof of Lemma 1. We first introduce some notation. Let $\Delta(m_i, m_{-i}|s_i)$ be the difference in the expected reputational payoff of reporting m_i rather than m_{-i} upon observing s_i for an expert of perceived ability $\tilde{t}$ who is believed truthful. $\Delta(m_i, m_{-i}|s_i) \equiv E^e[p_g(m_i, X)|s_i] - E^e[p_g(m_{-i}, X)|s_i]$. We denote by p_g^+ the audience's belief about the expert's ability when the recommendation matches the realization of the state and the expert is believed to be truthful. We denote by p_g^- the belief about the expert's ability when the recommendation does not match the realization of the state and the expert is believed to be truthful.

- $\forall i,\ p_g^+ \equiv p(g|m_i, \theta_i)$
- $\forall i,\ p_g^- \equiv p(g|m_i, \theta_{-i})$

$p_g^+ \geq p_g^-$ with strict inequality for $\mu_1 \in (0, 1)$

Notice that:

$$E^e[p_g(m_i, X)|s_i] = (p_g^+ - p_g^-)E^e[p^a(\theta_i|X, m_i)|s_i] + p_g^-,$$

and

$$E^e[p_g(m_{-i}, X)|s_i] = (p_g^+ - p_g^-))E^e[p^a(\theta_{-i}|X, m_{-i})|s_i] + p_g^-.$$

It follows that:

$$\Delta(m_i, m_{-i}|s_i) = (p_g^+ - p_g^-)\{E^e[p^a(\theta_i|X, m_i)|s_i] - E^e[p^a(\theta_{-i}|X, m_{-i})|s_i]\}.$$

Inequality 3.1 is verified if and only if:

$$\Delta(m_i, m_{-i}|s_i) \geq 0.$$

That is:

$$E^e[p^a(\theta_i|X, m_i)|s_i] \geq E^e[p^a(\theta_{-i}|X, m_{-i})|s_i]. \tag{A.1}$$

Inequality A.1 says that the expert has the incentive to be truthful if and only if she expects to be more likely to be perceived as correct when truthfully reporting her signal than when choosing to misreport. Formally, after receiving s_i, the expert must expect that reporting m_i will induce higher audience's posterior beliefs on θ_i than the audience's posterior on θ_{-i} induced by m_{-i}.

$\square$

B Extension – Known Ability

Proof of Proposition 4. Let $\Delta(m_i, m_{-i}|s_i, t)$ be the difference in the expected reputational payoff of reporting m_i rather than m_{-i} for an expert of ability t who is believed to be truthful and who observes the signal s_i. By Lemma 1 the net payoff from truthtelling as:

$$\Delta(m_i, m_{-i}|s_i, t) \equiv E^e[p^a(\theta_i|X, m_i)|s_i, t] - E^e[p^a(\theta_{-i}|X, m_{-i})|s_i, t].$$

The signal s_i belongs to the incentive compatible set $ICS_{t,\tilde{t}}$ if and only if:

$$\Delta(m_i, m_{-i}|s_i, t) \geq 0. \tag{B.1}$$

For any $\theta \in \{\theta_0, \theta_1\}$ and for any $m \in \{m_0, m_1\}$:

$$E^e[p^a(\theta_i|X, m_i)|s_i] = \sum_{k\in\{0,1\}} p^a(\theta_i|X_k, m_i) \sum_{j\in\{0,1\}} p(\theta_j|s_i, t)p^e(X_k|\theta_j, m_i), \tag{B.2}$$

with

$$p^e(X_k|\theta_j, m_i) = [\tau(\nu_1(m_i, \sigma^T))\mathbb{1}\{j = k\} + (1 - \tau(\nu_1(m_i, \sigma^T))\mathbb{1}\{j \neq k\}].$$

For any $i \in \{0, 1\}$, define:

$$p_\theta^+(m_i) \equiv p^a(\theta_i|X_i, m_i),$$
$$p_\theta^-(m_i) \equiv p^a(\theta_i|X_{-i}, m_i).$$

Condition B.1 is verified whenever:

$$[p_\theta^+(m_i) - p_\theta^-(m_i)] \sum_{j\in\{i,-i\}} p(\theta_j|s_i, t)p^e(X_i|\theta_j, m_i) + p_\theta^-(m_i) \geq \tag{B.3}$$
$$[p_\theta^+(m_{-i}) - p_\theta^-(m_{-i})] \sum_{j\in\{i,-i\}} p(\theta_j|s_i, t)p^e(X_{-i}|\theta_j, m_{-i}) + p_\theta^-(m_{-i}).$$

Notice that $p^e(\theta_i|s_i, g) > p^e(\theta_i|s_i, b)$ for any $\mu_1 \in (0, 1)$. Moreover, $p^e(X_i|\theta_i, m_i) > p^e(X_i|\theta_{-i}, m_i)$ since X is an informative signal. For the same reason, $p_\theta^+(m_i) > p_\theta^-(m_i)$. It follows that the left-hand side of Inequality B.3 is strictly increasing in t. Symmetrically $p_\theta^+(m_{-i}) > p_\theta^-(m_{-i})$ and $p^e(X_{-i}|\theta_i, m_{-i}) < p^e(X_{-i}|\theta_{-i}, m_{-i})$. It follows that the right-hand side is strictly decreasing in t. We conclude that:

$$\Delta(m_i, m_{-i}|s_i, b) \geq 0 \quad \Rightarrow \quad \Delta(m_i, m_{-i}|s_i, g) > 0.$$

$\square$

Proof of Proposition 5. By construction of the incentive compatible set, for $\mu_1 \in ICS_{b,\tilde{t}}$, truthtelling is incentive compatible for both types which implies that truthtelling is an equilibrium. When $\mu_1 > \overline{\mu}_1^T(b,\tilde{t})$, we will show that there is an equilibrium in which the good expert is truthful and the bad expert truthfully reports s_1 and randomizes when she observes s_0. A similar reasoning applies to show that, when $\mu_1 < \underline{\mu}_1^T(b,\tilde{t})$, there is an equilibrium in which the good expert is truthful and the bad expert truthfully reports s_0 and randomizes when she observes s_1.

Let us introduce some notation. We denote by $\breve{\sigma}$ the strategy described in Proposition 5. We denote by ξ the probability with which the bad expert truthfully reports s_0: $\xi \equiv p(m_0|s_0,\breve{\sigma})$. The bad expert randomizes after receiving s_0 if and only if:

$$\Delta(m_0,m_1|s_0,b,\breve{\sigma}(\xi)) = 0,$$

where:

$$\begin{aligned}\Delta(m_0,m_1|s_0,b,\breve{\sigma}(\xi)) = \sum_{j\in\{0,1\}} \{ & p(g|m_0,\theta_j,\breve{\sigma}(\xi))E^e[p^a(\theta_j|X,m_0,\breve{\sigma}(\xi))|s_0,b] \\ & - p^a(g|m_1,\theta_j,\breve{\sigma}(\xi))E^e[p^a(\theta_j|X,m_1,\breve{\sigma}(\xi))|s_0,b]\},\end{aligned} \tag{B.4}$$

with:

$$p^a(g|m_i,\theta_j,\breve{\sigma}(\xi)) = \begin{cases} \frac{gp_g}{gp_g+\xi bp_b} & \text{if } (i,j)=(0,0) \\ \\ \frac{(1-g)p_g}{(1-g)p_g+\xi(1-b)p_b} & \text{if } (i,j)=(0,1) \\ \\ \frac{gp_g}{gp_g+[b+(1-b)(1-\xi)]p_b} & \text{if } (i,j)=(1,1) \\ \\ \frac{(1-g)p_g}{(1-g)p_g+[(1-b)+b(1-\xi)]p_b} & \text{if } (i,j)=(1,0) \end{cases} \tag{B.5}$$

- When the audience believes that the bad expert chooses $\xi = 1$, by construction, since $\mu_1 > \overline{\mu}_1^T(b,\tilde{t})$, we have $\Delta(m_0, m_1|s_0, b, \breve{\sigma}(\xi)) < 0$.

- When the audience believes that the bad expert chooses $\xi = 0$, $p^a(g|m_0,\theta_0,\breve{\sigma}(\xi)) = p^a(g|m_0,\theta_1,\breve{\sigma}(\xi)) = 1$. For any $p_g < 1$ and for all $j \in \{0,1\}$, $p^a(g|m_1,\theta_j,\breve{\sigma}(\xi)) < 1$. It follows that $\Delta(m_0, m_1|s_0, b, \breve{\sigma}(\xi)) > 0$.

By continuity of $\Delta(m_0, m_1|s_0, b, \breve{\sigma}(\xi))$ in ξ, there must exist $\bar{\xi} \in (0,1)$ such that $\Delta(m_0, m_1|s_0, b, \breve{\sigma}(\bar{\xi})) = 0$.

We now need to check that it is optimal for the high type expert to follow $\breve{\sigma}(\xi)$, which is to report both signals truthfully. As shown in B, for any ξ such that $\Delta(m_0, m_1|s_0, b, \breve{\sigma}(\xi)) = 0$, $\Delta(m_0, m_1|s_0, g, \breve{\sigma}(\xi)) > 0$, which means that reporting s_0 truthfully is incentive compatible for the good expert. □

C Extension – General Precision Function

Proof of Lemma 2. First, let us rewrite $E^e[p^a(\theta_i|X,m)|s]$ as a weighted average of $p(\theta_i|s)$ and $\nu_i(m,\tilde{\sigma})$. To simplify notation, we replace $\tau(\nu_1(m,\tilde{\sigma}))$ by τ:

From 3.2 we show that:

$$
\begin{aligned}
E^e[p^a(\theta_i|X,m)|s] =& p(\theta_i|s)(2\tau-1)\left[p^a(\theta_i|X_i) - p^a(\theta_i|X_{-i})\right] \\
&+ (1-\tau)p^a(\theta_i|X_i) + \tau p^a(\theta_i|X_{-i}) \\
\\
=& p(\theta_i|s)(2\tau-1)\left[\frac{\tau\nu_i(m,\tilde{\sigma})}{p^a(X_i|m)} - \frac{(1-\tau)\nu_i(m,\tilde{\sigma})}{p^a(X_{-i}|m)}\right] \\
&+ \nu_i(m,\tilde{\sigma})(1-\tau)\tau\left[\frac{1}{p^a(X_i|m)} + \frac{1}{p^a(X_{-i}|m)}\right] \\
\\
=& p(\theta_i|s)(2\tau-1)\frac{\nu_i(m,\tilde{\sigma})\nu_{-i}(m,\tilde{\sigma})\left[\tau^2-(1-\tau)^2\right]}{p^a(X_i|m)p^a(X_{-i}|m)} \\
&+ \nu_i(m,\tilde{\sigma})\tau(1-\tau)\left[\frac{p^a(X_i|m)+p^a(X_{-i}|m)}{p^a(X_i|m)p^a(X_{-i}|m)}\right] \\
\\
=& p(\theta_i|s)(2\tau-1)^2\frac{\nu_i(m,\tilde{\sigma})\nu_{-i}(m,\tilde{\sigma})}{p^a(X_i|m)p^a(X_{-i}|m)} + \nu_i(m,\tilde{\sigma})\frac{\tau(1-\tau)}{p^a(X_i|m)p^a(X_{-i}|m)},
\end{aligned}
$$

where $p^a(X_i|m) = \tau\nu_i(m,\tilde{\sigma}) + (1-\tau)\nu_{-i}(m,\tilde{\sigma})$.

Let $\alpha(\tau,m,\mu_1) \equiv (2\tau-1)^2\frac{\nu_1(m,\tilde{\sigma})\nu_0(m,\tilde{\sigma})}{p^a(X_1|m)p^a(X_0|m)}$.

To complete the proof, we need to show:

(a) $1-\alpha(\tau,m,\mu_1) = \frac{\tau(1-\tau)}{p^a(X_1|m)p^a(X_0|m)}$

(b) $\alpha(1/2,m,\mu_1) = 0$ and $\alpha(1,m) = 1$, for all $m \in \{0,1\}$

(c) $\frac{\partial\alpha}{\partial\tau}(\tau,m,\mu_1) \geq 0$

(a) is equivalent to proving:

$$
(2\tau-1)^2\nu_1(m,\tilde{\sigma})\nu_0(m,\tilde{\sigma}) = p^a(X_1|m)p^a(X_0|m) - \tau(1-\tau),
$$

which is indeed the case:

$$
\begin{aligned}
& p^a(X_1|m)p^a(X_0|m) - \tau(1-\tau) \\
=& \tau(1-\tau)\left[\nu_1(m,\tilde{\sigma})^2 + \nu_0(m,\tilde{\sigma})^2 - 1\right] + \nu_1(m,\tilde{\sigma})\nu_0(m,\tilde{\sigma})\left[\tau^2 + (1-\tau)^2\right] \\
=& \tau(1-\tau)\left[2\nu_1(m,\tilde{\sigma})^2 + 1 - 2\nu_0(m,\tilde{\sigma}) - 1\right] + \nu_1(m,\tilde{\sigma})\nu_0(m,\tilde{\sigma})\left[\tau^2 + (1-\tau)^2\right] \\
=& \tau(1-\tau)\left[-2\nu_1(m,\tilde{\sigma})\nu_0(m,\tilde{\sigma})\right] + \nu_1(m,\tilde{\sigma})\nu_0(m,\tilde{\sigma})\left[\tau^2 + (1-\tau)^2\right] \\
=& \nu_1(m,\tilde{\sigma})\nu_0(m,\tilde{\sigma})\left[\tau - (1-\tau)\right]^2.
\end{aligned}
$$

(b) Direct as $\alpha(1/2, m, \mu_1) = 0 = (1 - \alpha(1, m, \mu_1))$ for all $m \in \{0, 1\}$.

(c)

$$
\begin{aligned}
& \frac{\partial \alpha}{\partial \tau}(\tau, m, \mu_1) \geq 0 \\
\Leftrightarrow & \, 4(2\tau-1)p^a(X_1|m)p^a(X_0|m) - (2\tau-1)^2 \frac{\partial p^a(X_1|m)p^a(X_0|m)}{\partial \tau} \geq 0 \\
\Leftrightarrow & \, 4p^a(X_1|m)p^a(X_0|m) \\
& - (2\tau-1)(2\nu_1(m,\tilde{\sigma})-1)\left[(2\tau-1)(1-\nu_1(m,\tilde{\sigma})) + (1-2\tau)\nu_1(m,\tilde{\sigma})\right] \geq 0 \\
\Leftrightarrow & \, 4p^a(X_1|m)p^a(X_0|m) + (2\tau-1)^2(2\nu_1(m,\tilde{\sigma})-1)^2 \geq 0
\end{aligned}
$$

. □

Proof of Proposition 6. It suffices to notice that the left-hand side of equation 4.1 is invariant with τ, but that the right-hand side is increasing in the precision of the ex post signal: For all $\tau_1 < \tau_2 \in [0,1]$, $\alpha(\tau_1, m_{-i}, \mu_1) < \alpha(\tau_2, m_{-i}, \mu_1)$ and $p(\theta_{-i}|s_{-i}) \geq p(\theta_{-i}|s_i)$, which implies that $(1-\alpha(\tau_2, m_{-i}, \mu_1))p(\theta_{-i}|s_{-i}) + \alpha(\tau_2, m_{-i}, \mu_1)p(\theta_{-i}|s_i) \leq (1-\alpha(\tau_1, m_{-i}, \mu_1))p(\theta_{-i}|s_{-i}) + \alpha(\tau_1, m_{-i}, \mu_1)p(\theta_{-i}|s_i)$. We conclude that if $\mu_1 \in IC_{\tilde{t},\tau_1}$, then $\mu_1 \in IC_{\tilde{t},\tau_2}$. □

D Extensions – Competition

Proof of Lemma 4. In the single-expert model, $p(\theta_1|s_0) < \chi \leq \mu_1 < p(\theta_1|s_1)$ implies that if the audience assumes that the expert is truthful, the ex post signal is perfectly informative when the expert sends the message m_1 and uninformative when the experts sends m_0. We have shown in Section 3.0.1 shows that $ICS_{\tilde{t}} = [\frac{1}{2}, \tilde{t}]$.

In the two-expert model, $p(\theta_1|s_0) < \chi \leq \mu_1$ implies that, assuming both experts are truthful, the ex post signal is perfectly informative as soon as at least one expert sends m_1 and uninformative otherwise. Consider the reporting behavior of, say, expert 1, who faces a truthful expert 2. If $m^1 = s^1$, note that:

$$
\begin{aligned}
E^e_{\sigma_2^T}[p^a(\theta|X, m^1, m^2)|s^1] &= \sum_{j\in\{0,1\}} p(\theta_j|s^1) \sum_{k\in\{0,1\}} \sum_{i\in\{0,1\}} \{p^a(\theta|X_k, m^1, m_i)\, p^e(X_k|\theta_j, m^1, m_i)\, p^e(m_i|\theta_j)\} \\
&\text{(since both experts are presumed truthful:)} \\
&= \sum_{j\in\{0,1\}} p(\theta_j|s^1) \sum_{k\in\{0,1\}} \sum_{i\in\{0,1\}} \{p^a(\theta|X_k, s^1, s_i)\, p^e(X_k|\theta_j, s^1, s_i)\, p^e(s_i|\theta_j)\} \\
&= \quad p(\theta|s^1).
\end{aligned}
$$

$$
\begin{aligned}
E^e_{\sigma_2^T}[p^a(\theta_0|X, m_0, m^2)|s_1] &= \sum_{j\in\{0,1\}} p(\theta_j|s_1) \sum_{k\in\{0,1\}} \sum_{i\in\{0,1\}} \{p^a(\theta_0|X_k, s_0, s_i)\, p^e(X_k|\theta_j, s_0, s_i)\, p^e(s_i|\theta_j)\} \\
&= p^e(s_0|s_1)p(\theta_0|s_0, s_0) + p^e(s_1|s_1)p(\theta_0|s_1, s_1).
\end{aligned}
$$

$$
\begin{aligned}
E^e_{\sigma_2^T}[p^a(\theta_1|X, m_0, m^2)|s_0] = &\sum_{j\in\{0,1\}} p(\theta_j|s_0) \sum_{k\in\{0,1\}} \sum_{i\in\{0,1\}} \{p^a(\theta_1|X_k, s_1, s_i)\, p^e(X_k|\theta_j, s_1, s_i)\, p^e(s_i|\theta_j)\} \\
=&p^e(s_0|s_0)p(\theta_1|s_0, s_0) + p^e(s_1|s_0)p(\theta_1|s_0, s_1) \\
=&p(\theta_1|s_0).
\end{aligned}
$$

$\underline{\mu}_1^T(2)$ is the lowest μ_1 for which the expert can be truthful in the two-expert model when the opponent is expected to be truthful:

$$\underline{\mu}_1^T(2) \equiv \mu_1 | E^e_{\sigma_2^T}[p^a(\theta_1|m_1, m^2)|s_1] = E^e_{\sigma_2^T}[p^a(\theta_0|m_0, m^2)|s_1],$$

which is

$$\underline{\mu}_1^T(2) \equiv \mu_1 | p(\theta_1|s_1) = p(s_1|s_1)p(\theta_0|s_0, s_1) + p(s_0|s_1)p(\theta_0|s_0, s_0).$$

Note that $p(s_1|s_1)p(\theta_0|s_0, s_1) + p(s_0|s_1)p(\theta_0|s_0, s_0) < p(\theta_0|s_0)$ since $p(s_1|s_1) > p(s_1|s_0)$. Thus, $\underline{\mu}_1^T(2) < \frac{1}{2}$.

$\overline{\mu}_1^T(2)$ is the largest μ_1 for which the expert can be truthful in the two-expert model when the opponent is expected to be truthful:

$$\overline{\mu}_1^T(2) \equiv \mu_1 | E^e_{\sigma_2^T}[p^a(\theta_0|m_0, m^2)|s_0] = E^e_{\sigma_2^T}[p^a(\theta_1|m_1, m^2)|s_0],$$

which is

$$\overline{\mu}_1^T(2) \equiv \mu_1 | p(\theta_0|s_0) = p(\theta_1|s_0),$$

$$\overline{\mu}_1^T(2) = \tilde{t}.$$

The set of priors for which the expert can be truthful under competition is: $ICS_{\tilde{t}}(2) = [\underline{\mu}_1^T(2), \tilde{t}] \supset ICS_{\tilde{t}}$. □

Proof of Lemma 5. In the single-expert model, $\chi \leq p(\theta_1|s_0)$ implies that the ex post signal is perfectly informative regardless of the message sent by the expert. We have shown in Section 3.0.1 that $ICS_{\tilde{t}} = [1 - \tilde{t}, \tilde{t}]$. In the two-expert model $p(\theta_1|s_0, s_0) < \chi \leq p(\theta_1|s_0)$ implies that, assuming both experts are truthful, the ex post signal is perfectly informative as soon as at least one expert sends m_1 and uninformative otherwise.

$$\underline{\mu}_1^T(2) \equiv \mu_1|p(\theta_1|s_1) = p(s_1|s_1)p(\theta_0|s_1, s_1) + p(s_0|s_1)p(\theta_0|s_0, s_0).$$

Note that $p(s_1|s_1)p(\theta_0|s_1, s_1) + p(s_0|s_1)p(\theta_0|s_0, s_0) > p(\theta_0|s_1)$ since $p(\theta_0|s_0, s_0) > p(\theta_0|s_1, s_0)$. Thus, $\underline{\mu}_1^T(2) > 1 - \tilde{t}$.

$$\overline{\mu}_1^T(2) \equiv \mu_1|p(\theta_0|s_0) = p(\theta_1|s_0).$$

Thus $ICS_{\tilde{t}}(2) = [\underline{\mu}_1^T(2), \tilde{t}] \subset ICS_{\tilde{t}}$.

$\square$