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Abstract

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JEL Classification: E21, E62, H2, H21, G1

Keywords: taxation, Housing, Wealth

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Optimal Taxation with Homeownership and Wealth Inequality*

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Abstract

We consider optimal taxation in a model with wealth-poor and wealth-rich households, where wealth derives from business capital and homeownership, and investigate the consequences on these tax rates of a rising wealth inequality at steady state. The optimal tax structure includes some taxation of labor, zero taxation of financial and business capital, a housing wealth tax on the wealth-rich households and a housing subsidy on the wealth-poor households. When wealth inequality increases, the optimal balance between labor and housing wealth taxes depends on the source of the increasing wealth.

KEYWORDS: housing wealth, wealth inequality, optimal taxation.

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1 Introduction

There is mounting consensus, among scholars and commentators, that shifting taxation from labor to capital may be an optimal response to the increase in wealth–to–income ratios and wealth inequality that has been documented for many advanced economies over the last decades (Piketty and Saez, 2003; Piketty and Zucman, 2014; Saez and Zucman, 2016; Fagereng et al., 2016; Piketty et al., 2017). This policy is mostly motivated by distributional objectives and it is sometime credited as having small efficiency costs (Piketty et al., 2015; Saez and Zucman, 2019). In this paper we investigate the long-run social welfare effects of such tax reform under full commitment by considering a simple model where households accumulate different levels of wealth; the latter consisting in business capital, housing, and financial assets; and the government has access to a limited set of tax rates (a flat tax on wages, housing rents and wealth, the latter being possibly contingent on the types of wealth and on the households' net asset position). We show that an optimal tax structure implies heterogeneous tax rates/subsidies on housing wealth and no tax on financial and business capital.

In our model labor supply is inelastic and households can be lenders or borrowers, homeowners or renters. Wealth heterogeneity is based on the assumption that households have different time discount rates and face borrowing constraints, so that some households end up having zero net wealth. In this set up, the steady state distribution of wealth is perfectly polarized between a set of *wealth-rich* and *wealth-poor* households, although all of them may work and own some housing in different quantities. The only relevant difference between the two sets of households is that the wealth-poor are either renters, with zero homeownership, or homeowners, with the value of their home perfectly matched by mortgages. As in Gervais (2002), we endogenize the selection of poor households into renters and homeowners by assuming that homeownership is subject to a minimum size constraint. The supply side of the economy includes two produced goods; a perishable consumption good (also called *consumption*); and residential construction. The latter generates an evolving stock of housing subject to physical depreciation. Technologies employ labor and capital, although the housing sector also needs some flow of new land available for construction every period. All the revenues from the sale of land permits go to the government, either because it is the only land owner or because, despite land being privately owned, these revenues are fully taxed by the government.

We simulate the impact of the introduction a flat tax on the rich households' net wealth (or, equivalently, their total capital income including the net of depreciation housing value) at steady state and we characterize the optimal distortionary tax structure (for the given flow of positive public spending) when the Planner maximizes a weighted average of households' lifetime utility under full commitment. For both cases, we evaluate numerically how our results are affected at steady state when the level of the wealth-to-income ratio, a measure of wealth inequality, increases. We consider two alternative mechanisms to generate increasing aggregate wealth-to-income ratios. The first goes through a rising public debt, and the second through a drop in the real interest rate. Note that, under our parametrization of preferences and technologies, the second mechanism goes along with a stronger re-adjustment of all equilibrium variables, and, in particular, it generates a higher stock of capital (to compensate for the lower marginal productivity), a higher housing wealth (and prices) and a higher level of the poor households' mortgage debt[1].

Our main results are threefold. First, assuming Cobb-Douglas preferences and technologies, the introduction of a 1% tax on net wealth starting from zero (*i.e.*, from the case in which only labor is taxed) increases net wages modestly (by 2-3%), but has a strong positive effect on the user cost of housing faced by poor and rich households (*i.e.*, the effective price of housing services). We estimate that an income-equivalent welfare loss of this policy for poor households is in the 8-9% range, and these numbers are not

[1]These phenomena are broadly consistent with the experience of many advanced economies in the last decades (La Cava, 2016)

substantially affected when the aggregate wealth-to-income ratio increases. Second, in the optimal tax problem, we allow for tax rates on labor income, non-housing net wealth (financial and business capital), housing wealth (net of depreciation) and rental income. Although all taxes are expressed as flat rates non contingent on household types (as is standard in the optimal tax literature), we impose the constraint that negative net (non-housing) wealth (or capital income) is untaxed. Within this limited menu of taxes, we show that the Chamley-Judd's zero steady state tax on financial and business capital survives, whereas housing wealth is taxed at a non zero rate (Chamley, 1986; Judd, 1985). In particular, we identify a set of conditions under which it is optimal to impose a positive tax on rich households' housing wealth, and a subsidy on the user cost of housing (or rent) faced by poor households. For poor homeowners, this can be implemented as a negative tax on housing wealth or imputed rents. Somewhat surprisingly, optimal tax rates on all type of wealth (financial and housing) are zero when the rich households are "pure speculators" in the housing market, in the sense that they derive no utility from housing services. In the more general setting where all households enjoy housing services, the tax on the rich households' housing wealth is positive for all homogeneous utility functions and the housing subsidy is positive for all poor households whose marginal utility of consumption is sufficiently large. Finally, using again Cobb-Douglas preferences and technology, we evaluate numerically the impact on optimal tax rates of a rising aggregate wealth-to-income ratio generated by the two alternative mechanisms presented above: an increase in public debt or a fall in the real interest rate. We show that the behavior of the optimal tax rates changes dramatically according to which of the two mechanisms is in place. If wealth rises because of a rising public debt, then the optimal wage tax falls; the housing subsidy is flat at around 2%; and the housing tax on the rich households falls substantially. When, instead, aggregate wealth rises as a consequence of a falling real rate, then the optimal labor tax falls (by a small amount); the housing tax on the rich households rises strongly; and the housing subsidy falls by approximately two percentage points. In both scenarios, the housing subsidies are small, while the housing tax rates are large (between 30 to 60%). We additionally find that the tax on labor falls with wealth more strongly when the interest rate drops, compared to the first scenario, because the gross wage rises substantially, thereby generating a larger tax base.

The behavior of the housing tax rates and subsidies is related to the way in which a rising aggregate wealth affects the "general equilibrium elasticities" of consumption and housing services[2]: the larger is the former relative to latter, the larger are the efficiency gains from shifting taxation to housing wealth. For the Cobb-Douglas case, the elasticity of consumption is decreasing, and the elasticity of housing increasing, in the households' capital income per unit of net wage. Hence, other things equal, the higher is the share of income coming from wealth, the lower is the optimal housing wealth tax. This explains why a rising aggregate wealth-to-income ratio has no or little effects on housing subsidies (the poor households have zero net capital income), and why the housing taxes (on the rich) follow different patterns according to which mechanism generates a rising aggregate wealth-to-income. Namely, if a rising wealth-to-income ratio is obtained through a larger public debt that leaves the real interest rate unaltered, then rich households' capital income rises, so that the general equilibrium elasticity of housing grows relative to consumption and, then, it is optimal to decrease the housing wealth tax. If, on the other hand, a rising wealth-to-income is obtained through a falling real rate, then it is possible (as it happens in our simulations) that capital income falls relative to wages, so that the optimal housing wealth tax rises. Even though these findings are specific to the Cobb-Douglas case, they suggest that the way wealth taxes should respond to rising wealth and wealth inequality is far from obvious.

Our results depend on some strong assumptions. First, an inelastic labor supply

[2]The term "general equilibrium elasticities" is taken from Atkeson et al. (1999). These are expressions capturing the efficiency cost of taxing the corresponding good.

makes the model biased towards the idea that wealth should not be taxed, so that a positive taxation on housing should be fairly robust[3]. Second, deriving the wealth distribution from different subjective discount factors and debt limits has some limitations, although it is a very standard practice in neoclassical growth theory and, in some way, necessary to produce the stronger observed polarization in wealth than in income which is not easily reproducible in models with homogeneous preferences (Jones, 2015). Third, by concentrating the analysis on steady states we miss the analysis of the transition from low to higher tax rates, which is motivated by the need to focus on long-run phenomena.

Following the seminal contributions by Chamley (1986) and Judd (1985), the literature on optimal taxation has provided various arguments why wealth should be taxed, even in the long-run and under commitment, ranging from life-cycle considerations, precautionary savings and imperfect information. More recently, Piketty and Saez (2013) and Saez and Stantcheva (2018) have advanced the idea that the optimality of positive capital tax rates may emerge due to the *non-infinite elasticity* of the long-run supply of capital[4]. In turn, finite values for the elasticity of long-run wealth is obtained by assuming that the latter (or the services it generates) enters the individuals' utility function. In particular, Piketty and Saez (2013) consider a life-cycle model where households derive utility from bequests and Saez and Stantcheva (2018) assume that wealth enters the households utility function directly for various reasons, among which are "social status", "power", "philanthropy". In our model housing is both a store of value and an asset that generates utility services, whereas the supply of financial and business capital retains the property of being infinitely elastic in the

[3] Saez et al. (2009) argue that the estimated compensated elasticity of labor is small (close to zero for prime-age males).

[4] The reason why capital taxation is not optimal is that it implies exponentially growing distortions of investment over time, so that there are large benefits from shifting the tax burden from capital to labor of long-run capital or wealth. According to Saez and Stantcheva (2018) these growing distortions arise because long-run capital supply is infinitely elastic and taxing infinitely elastic bases is not desirable.

long run. This explains why, in our model, taxing housing wealth may be optimal and taxing financial wealth is not. In fact, housing taxation has been advocated in several studies, especially as a way to avoid a sub-optimal tax discrimination between factor inputs and sources of wealth, and many authors have highlighted the existence of substantial welfare gains from increased housing taxation, due to the failure to tax implicit rental income and because of mortgage interest deductibility characterizing existing tax codes in most advanced economies (see Poterba (1984), Gahvari (1984), Berkovec and Fullerton (1992), Auerbach and Hines (2002), Gervais (2002) and Mirrlees et al. (2011)). These distortions imply that housing investment crowds out business capital and generates excessive levels of homeownership. Furthermore, a heavier taxation of housing wealth may reduce inequality in economies where, because of capital market imperfections and indivisibilities, rental housing is concentrated among poor households (although Gervais (2002) finds that the distributional effects of eliminating housing tax incentives are quantitatively small). Our contribution differs from this literature because we are specifically interested in (differentiated) wealth taxation and the way it should evolve in response to increasing wealth inequality, instead of examining the welfare gains from reducing fiscal incentives on housing. Whereas the case for housing taxation is usually based on the unavailability of non distorting taxes, in our model housing taxes (and subsidies) survive despite the fact that labor taxes are non distortionary. The papers most related to ours are Alpanda and Zubairy (2016) and Bonnet et al. (2019). Alpanda and Zubairy (2016) consider a model with patient and impatient households, borrowers and lenders, homeowners and renters, and build a dynamic general-equilibrium model to study the transitional and steady-state effects of a large menu of taxes (mortgage interest deductions, taxation of imputed rental income, property tax rates and a reduction in depreciation allowance). Our model shares a similar environment and studies the optimal taxation with a smaller menu of taxes. Bonnet et al. (2019) consider an economy with heterogeneous wealth composition (business capital, housing and land) and heterogeneous households (capitalists/landlords and workers/tenants). Differently from our model, they assume that poor households have no wealth (in particular, no land and housing wealth) and obtain housing services by renting from rich households. In their model, capital should not be taxed and the first best allocation can be implemented by levying a tax on land. The optimality of a land tax follows from the Planner's preference for redistribution and the fact that land is a fixed factor (*i.e.*, a land tax is non-distortionary).

The remainder of this paper is organized as follows: section 2 presents the model; section 3 considers the optimal taxation problem and the results of our quantitative analysis; section 4 presents our conclusions.

2 The Model

In this section we present a model with two sectors: manufacturing and housing construction; different households, with preferences over consumption of a perishable manufacturing good and a durable good, which we call housing; and a government that uses a set of taxes to finance public spending.

2.1 Framework

We consider an economy with two sectors, *manufacturing* and *(housing) construction*; and a finite set $\mathcal{I}$ of households types indexed by i with preferences over consumption of the manufacturing good and housing services. The manufacturing good is a proxy for all non-construction consumption and the housing stock is a proxy for housing services. Household types have mass $m_i \in (0,1)$ per total population, with $\sum_{i\in\mathcal{I}} m_i = 1$, and belong to infinitely lived dynasties. Life time utilities are represented by

$$\mathcal{U}^i = \sum_{t=0}^{\infty} \beta_i^t U(c_t^i, z_t^i), \tag{1}$$

where $U(.)$ is the per period strictly increasing and strictly concave utility function (identical across types); $\beta_i \in (0,1)$ are the type-specific time discount rates; and c^i, z^i denote, respectively, household i's consumption of manufacturing goods and housing services. All households supply one unit of labor inelastically and have different labor productivities. In particular, we let $\epsilon^i \in (0,1)$ be the household i-specific contribution to production of a unit of labor and assume

$$\sum_{i\in\mathcal{I}} m_i \epsilon^i = 1.$$

Production takes place in the manufacturing (m) and housing (h) sector with heterogeneous neoclassical technologies. While the technology in manufacturing employs labor and capital only, production of new housing requires also land. In particular, technologies in the two sectors are defined by

$$y_t^m = f^m(k_t^m, l_t^m), \qquad y_t^h = f^h(k_t^h, l_t^h, x_t),$$

where k^j is the capital stock and l^j the amount of labor employed in sector j in efficiency units; x_t is the flow of new land available for housing construction. We think of the flow of new available land as "land permits" provided by the government on the basis of some physical constraint or environmental concern (Favilukis et al., 2017; Borri and Reichlin, 2018a). Both $f^m(.)$ and $f^h(.)$ are assumed to be increasing, strictly concave, to exhibit constant returns to scale, to be continuously differentiable and to verify Inada conditions. For simplicity, we assume that capital fully depreciate in one period and we let

$$c = \sum_{i\in\mathcal{I}} m_i c^i, \qquad z = \sum_{i\in\mathcal{I}} m_i z^i, \qquad k = k^h + k^m.$$

Then, for some given initial allocation of capital, k_0; and housing stock, h_0; a *feasible allocation* of individuals' consumption and sector specific capital and employment is a sequence $\{c_t^i, z_t^i, k_t^j, l_t^j, k_{t+1} h_{t+1}, ;\ i \in \mathcal{I},\ j = h, m\}_{t=0}^{\infty}$, satisfying, for all $t \geq 0$,

$$
\begin{aligned}
c_t + g_t + k_{t+1} &\leq f^m(k_t^m, l_t^m), &(2)\\
h_{t+1} &\leq f^h(k_t^h, l^h, x_t) + (1-\delta)h_t, &(3)\\
z_t &\leq h_t &(4)\\
l_t^m + l_t^h &\leq 1, &(5)\\
k_t^h + k_t^m &\leq k_t, &(6)
\end{aligned}
$$

where g_t is the total amount of public spending; $\delta \in (0, 1]$ is the housing depreciation rate; and $\{x_t\}_{t=0}^{\infty}$ is the given sequence of government provided flow of new land permits.

We let manufacturing be the *numeraire* good; q_t the unit price of housing; R_t the real gross interest rate; w_t the average real wage rate, with the i-specific wage rate being set at $\epsilon^i w_t$. Assuming perfect competition in both sectors, profit maximization and perfect labor mobility imply

$$
\begin{aligned}
R_t &= f_k^m(k_t^m, l_t^m) = q_t f_k^h(k_t^h, l_t^h, x_t), &(7)\\
w_t &= f_l^m(k_t^m, l_t^m) = q_t f_l^h(k_t^h, l_t^h, x_t). &(8)
\end{aligned}
$$

where f_k^j, f_l^j, f_x^j, for $j = h, m$, are the marginal products of capital, labor and land. Firms in the construction sector rebate any remaining profits to the government as a compensation for the use of land permits, and the government uses these resources to finance public spending. Then, the government revenue from land permits in units of labor efficiency is

$$\tau_t^L = q_t f_x^h(k_t^h, l_t^h, x_t) x_t. \tag{9}$$

In our model all tax revenues come from (possibly type-specific) income taxes, wealth taxes, and from the sale of land permits. Note that income, wealth, and housing taxes may be differentiated across types of wealth (*i.e.*, financial or housing) and made contingent on the households' net financial wealth position, *i.e.*, on whether a household is a net lender or a net borrower. Any household i, at all time $t \geq 0$, has access to some units, b^i_{t+1}, of a 1-period bond with gross pre-tax interest rate, R_{t+1}, and some units, h^i_{t+1}, of residential property with (before tax) unit price q_t. Housing services enjoyed at time t, z^i_t, come from rental housing, with a before tax unit rental price of s_t, or home ownership. We denote with $z^{r,i}_t$ the housing services from renting and the units of housing rented; and with $z^{o,i}$ the housing services from owner occupied housing as well as the units of housing property occupied by the owner. Hence, one unit of housing capital generates one unit of housing services. These two type of housing services are assumed to be perfect substitutes, so that

$$z^i_t = z^{r,i}_t + z^{o,i}_t.$$

We assume that housing capital is not perfectly divisible (Gervais, 2002). In particular, there exists a minimum size of owner occupied housing, $\bar{z}$, which also represents the smallest amount of housing services a homeowner (but not a renter) can consume. Hence, all households face the constraint:

$$z^{o,i}_t \geq \bar{z}. \tag{10}$$

The government can select tax rates, at all $t \geq 0$, from a menu, $(\tau^s_t, \tau^w_t, \tau^{k,i}_t, \tau^{h,i})$, representing, respectively, a tax rate on housing rent, labor income, financial and housing wealth. The per-period budget constraint of the household is then

$$b^i_{t+1}/R_{t+1} + c^i_t + q_t h^i_{t+1} + s_t z^{r,i}_t = \epsilon^i \hat{w}_t + \hat{s}_t(h^i_t - z^{o,i}_t) + (1 - \tau^{k,i}_t)b^i_t + (1 - \delta)\hat{q}^i_t h^i_t, \tag{11}$$

where $\hat{s}_t = (1 - \tau^s_t)s_t$ is the after tax housing rent (on landlords); $\hat{w}_t = (1 - \tau^w_t)w_t$ is the after tax wage rate per units of efficiency; and $\hat{q}^i_t = q_t(1 - \tau^{h,i}_t)$ is the housing price net of the housing tax. Note that the latter can be considered a tax on housing wealth or, equivalently, a sale tax on housing transactions. Later on, as it is common in most tax codes, we will impose that taxes on financial wealth may differ from zero if and only if the latter is positive, *i.e.*, debt is untaxed ($\tau_t^{k,i} = 0$ if $b_t^i \leq 0$). Now define households' before tax net assets as

$$a_{t+1}^i / R_{t+1} = b_{t+1}^i / R_{t+1} + q_t h_{t+1}^i \tag{12}$$

and the i-specific after tax net assets, $\hat{a}^i = (1 - \tau^{k,i}) a^i$, net interest rates, $\hat{R}^i = R(1 - \tau^{k,i})$, and present value prices, $\{p_t^i\}_{t=0}^{\infty}$, recursively from $p_t^i / p_{t+1}^i = \hat{R}_{t+1}^i$. Then, using (12), the t-period budget constraint becomes

$$p_{t+1}^i \hat{a}_{t+1}^i + p_t^i (c_t^i + s_t z_t^{r,i} + \hat{\pi}_t^i z_t^{o,i} + (\hat{\pi}_t^i - \hat{s}_t)(h_t^i - z_t^{o,i})) = p_t^i (\epsilon^i \hat{w}_t + \hat{a}_t^i), \tag{13}$$

where

$$\hat{\pi}_t^i = \hat{R}_t^i q_{t-1} - (1 - \delta) \hat{q}_t^i \tag{14}$$

is the after tax *user cost of housing*. The latter is a measure of the net of tax market price of housing services and it is equivalent to the present value of next period *imputed rents* from owner occupied housing. Finally, we assume that net assets must be non-negative at all periods,

$$\hat{a}_{t+1}^i \geq 0, \tag{15}$$

i.e., households debt must be fully collateralized by the housing wealth.

To close the model, we assume that the government, at all time t, issues one-period bonds in some amount b_{t+1}^g at the market interest rate, R_{t+1}. Then, the government budget constraint is

$$b_{t+1}^g / R_{t+1} \geq g_t + b_t^g - \mathcal{T}_t - \tau_t^w w_t - \tau_t^L, \tag{16}$$

where

$$\mathcal{T}_t = \sum_i m_i \left(\tau_t^{k,i} b_t^i + q_t(1-\delta)\tau_t^{h,i} h_t^i + \tau_t^s s_t (h_t^i - z_t^{o,i}) \right)$$

is the time-t revenue from wealth taxation.

2.2 Equilibrium

The following proposition provides a first order characterization of households' optimal choices at equilibrium. Appendix B contains the proof of the proposition.

Proposition 1. *Any equilibrium where some individuals are homeowners is such that*

$$\begin{aligned}
\textit{either} \quad z_t^{o,i} = 0 \quad & \textit{and } z_t^i = z_t^{r,i},\ h_t^i = 0, && (17)\\
\textit{or} \quad z_t^{r,i} = 0 \quad & \textit{and } z_t^i = z_t^{o,i} \geq \bar{z},\ h_t^i \geq z_t^i, && (18)\\
p_t^i \quad & \geq \quad \beta_i^t U_{1,t}^i p_0^i / U_{1,0}^i, && (19)\\
U_{1,t}^i s_t \quad & \geq \quad U_{2,t}^i, && (20)\\
U_{1,t}^i \hat{\pi}_t^i \quad & \geq \quad U_{2,t}^i, && (21)\\
s_t \geq \hat{\pi}_t^i \quad & \geq \quad \hat{s}_t && (22)\\
(\hat{\pi}_t^i - \hat{s}_t)(h_t^i - z_t^{o,i}) \quad & = \quad 0, && (23)
\end{aligned}$$

where $U_{j,t}^i \equiv U_j(c_t^i, z_t^i)$ and (20) and (21) hold with inequality only if $z^{r,i} = 0$ and $z_t^{o,i} = 0$, respectively. Furthermore, letting $p_t^{z,i} = \hat{\pi}_t^i$ if household-i is a homeowner and $p_t^{z,i} = s_t$ if household-i is a renter, the optimal choices of consumption and housing services, $\{c_t^i, z_t^i\}_{t=0}^\infty$, are subject to the following life-time present value budget constraint

$$p_0^i \hat{a}_0^i = \sum_{t=0}^{\infty} p_t^i (c_t^i + p_t^{z,i} z_t^i - \hat{w}_t \epsilon^i), \tag{24}$$

A *competitive equilibrium* is a sequence of quantities and prices,

$$\{c_t^i, z_t^i, b_{t+1}^i, h_{t+1}^i, k_t^j, l_t^j, k_{t+1}, h_{t+1}, q_t, w_t, R_{t+1}; i \in \mathcal{I},\ j = h, m\}_{t=0}^{\infty},$$

and a policy $\mathcal{P} = \{g_t, b_t^g, x_t, \tau_t^w, \tau_t^{h,i}, \tau_t^{k,i}, \tau_t^s; j = s, w, k, h, i \in \mathcal{I}\}_{t=0}^{\infty}$, satisfying the resource feasibility constraints (equations (2)–(6)); profit maximization, (equations (7)–(8)); utility maximization (equations (17)–(23)); the government budget constraint (equation (16)); and the asset markets equilibrium condition

$$\sum_i m_i b_t^i / R_t = k_t + b_t^g / R_t, \tag{25}$$

$$\sum_i m_i a_t^i / R_t = k_t + b_t^g / R_t + q_{t-1} h_t \tag{26}$$

for all $t \geq 0$ and some given initial stocks of capital, housing and public debt (k_0, h_0, b_0^g).

In the remainder of the paper we assume that households' subjective discount rates may take one of two values. In particular, there exist two time discount rates only, β^H, β^L, with $\beta^H > \beta^L$, and a partition $(\mathcal{R}, \mathcal{P})$ of $\mathcal{I}$, such that $\beta_i = \beta^H$ if $i \in \mathcal{R}$ and $\beta_i = \beta^L$ if $i \in \mathcal{P}$. As we know from the the literature studying infinitely lived households with heterogeneous discount rates, the equilibrium real interest rate typically converges to the rate of time preference of the most patient household. Hence, defining as r the asymptotic real rate, it is convenient to set

$$\beta^H = 1/(1 + r).$$

We refer to $\mathcal{R}$ as the set of (wealth) *rich* households and $\mathcal{P}$ as the set of (wealth) *poor* households. Hence, rich households are relatively patient and, at any equilibrium around a steady state, they are lenders with respect to the rest of the economy; whereas poor households end up with zero net wealth asymptotically. Motivated by these considerations, in what follows we concentrate only on equilibria such that the debt limits are binding only for poor households, so that $\hat{a}_t^i = 0$ for all $i \in \mathcal{P}$ and $\hat{a}^i \geq 0$ for all $i \in \mathcal{R}$. We also assume that rich households have enough wealth to be homeowners, *i.e.*, to overcome the minimum home size $\bar{z}$ at all existing market prices. On the contrary, poor households can either be homeowners and borrowers, or renters. Under this simple partition, taxes on financial assets fall on the type $i \in \mathcal{R}$ only, *i.e.*, $\tau_t^{k,i} = 0$ for all $i \in \mathcal{P}$. On the other hand, by allowing $\tau^{h,i}$ to be contingent on types, we consider the possibility of a subsidy on the housing wealth backed by mortgages. Since tax rates can only be contingent on whether a household is a lender or a borrower, there is no ambiguity in setting $\tau^{k,i} = \tau^k$, $\hat{R}^i = \hat{R}$, $p_t^i = p_t$ and, with some abuse of notation, we set

$$(\tau^{h,i}, \hat{\pi}^i) = (\tau^{h,r}, \hat{\pi}^r) \quad \text{for all } i \in \mathcal{R},$$
$$(\tau^{h,i}, \hat{\pi}^i) = (\tau^{h,p}, \hat{\pi}^p) \quad \text{for all } i \in \mathcal{P}.$$

Note that the first order condition from the households' problem is

$$p_t^i / p_{t+1}^i = \hat{R}_{t+1} = (1+r) U_{1,t}^i / U_{1,t+1}^i \qquad \text{for all } i \in \mathcal{R} \tag{27}$$

so that, at a steady state equilibrium, equation (27) provides the following characterization of the gross interest rate and marginal products of capital

$$f_k^m(k^m, l^m) = q f_k^h(k^h, l^h, x) = R = (1+r)/(1-\tau^k), \tag{28}$$

which can only be verified for $\tau^k < 1 - (1+r)\beta^L$. Observe also that, at steady state, a positive financial tax raises the user cost of housing faced by poor households.

Specifically, we have

$$\hat{\pi}^r = q\left(r + \delta + (1-\delta)\tau^{h,r}\right), \tag{29}$$

$$\hat{\pi}^p = q\left(r + \delta + (1-\delta)\tau^{h,p} + (1+r)\frac{\tau^k}{1-\tau^k}\right), \tag{30}$$

so that

$$\hat{\pi}^p \geq \hat{\pi}^r \quad \Leftrightarrow \quad \tau^k \geq \frac{(1-\delta)(\tau^{h,r} - \tau^{h,p})}{(1+r) + (1-\delta)(\tau^{h,r} - \tau^{h,p})}.$$

Therefore, with a positive financial tax rate, poor households may end up paying a higher user cost of housing than rich households unless the latter face a high enough housing wealth tax (higher than that faced by poor households). In particular, if taxes on housing wealth cannot be made contingent on types, then equation (22) and $\tau^k \geq 0$ imply

$$s \geq \hat{\pi}^p \geq \hat{\pi}^r \geq \hat{s}.$$

Observe that, by the first order conditions and the complementary slackness condition (23), if $h^i > z^i$, *i.e.*, if rich households are landlords, then it must be that $\hat{\pi}^r = \hat{s}$, so that the existence of poor homeowners, *i.e.*, $s \geq \hat{\pi}^p$, implies

$$\frac{q\hat{R} - (1-\delta)\hat{q}^r}{1-\tau^s} \geq qR - (1-\delta)\hat{q}^p$$

at steady state. By rearranging terms and recalling that, at steady state, $\hat{R} = (1+r)$, the above implies that, if there is a uniform tax on housing property irrespective of wealth, *i.e.*, $\tau^h = \tau^{h,r} = \tau^{h,p}$, then the coexistence of rich landlords and poor homeowners requires the taxation of rents, *i.e.*, $\tau^s > 0$. If, on the other hand, $\tau^s = 0$ and there is a uniform wealth tax on the rich ($\tau^{h,r} = \tau^k$), then the above is only verified if the poor households' homeownership is subsidized, *i.e.*, $\tau^{h,p} < 0$.

2.3 Effects of Introducing a General Wealth Tax

In this section we evaluate the quantitative effects of introducing a general (flat) tax on net wealth. Specifically, we compare two scenarios: a benchmark scenario where (housing and financial) wealth is untaxed, and an alternative scenario characterized by a flat 1% tax rate, τ^k, on total net wealth, which is comparable to the rates we observe in existing tax codes (Jacobsen et al., 2017; Seim, 2017; Brülhart et al., 2019). This implies that the steady state user costs of housing are

$$\hat{\pi}^p = q\left(r + \delta + (1+r)\frac{\tau^k}{1-\tau^k}\right), \qquad \hat{\pi}^r = q\left(r + \delta + (1-\delta)\tau^k\right).$$

To reduce the dimensionality of the problem (from the point of view of the effects on the distribution of income and wealth), we only consider the case where the wealth-poor face the same cost of housing services, *i.e.*, we assume that the rent tax is such that $\hat{\pi}^p = s$. Therefore, we are limiting the degree of inequality across households below the level that could be otherwise achieved (*i.e.*, for $s \geq \hat{\pi}^p$).

Here and in the following numerical exercises we use a very parsimonious parametrization of the model based on Cobb-Douglas preferences and technologies

$$U(c,z) = c^{1-\theta}z^{\theta}, \tag{31}$$

$$f^m(k^m,l^m) = (k^m)^{\alpha_k^m}(l^m)^{\alpha_l^m}, \tag{32}$$

$$f^h(k^h,l^h,x) = (k^h)^{\alpha_k^h}(l^h)^{\alpha_l^h}x^{\alpha_x^h} \tag{33}$$

where $\sum_{j=k,l}\alpha_j^m = \sum_{j=k,l,x}\alpha_j^h = 1$. Note that, since the utility function is linearly homogeneous and the cost of housing services faced by the poor households are assumed to be identical, the consumption-to-housing ratio are equalized across this set of households, although individuals' labor productivities could be heterogeneous and are left unspecified. We calibrate the model by borrowing some of the parameter values from existing literature and setting the others in order to match some moments of the data. All the details about this calibration exercise and the specific parameter values are reported in Appendix A of the appendix along some robust checks,. We are especially interested in evaluating the effects of the introduction of a flat wealth tax for different levels of wealth and wealth inequality. In the model, we generate different levels of the wealth-to-income ratio, by assuming two scenarios. In the first, we exogenously generate an increasing level of the wealth-to-income ratio by increasing the level of government debt (b^g), while keeping all the other parameters unchanged. In the second scenario, we endogenously generate an increasing level of the wealth-to-income ratio by reducing the level of the real interest rate (r). To guarantee comparability between the two scenarios, we pick values for the real interest rates to match the exogenous wealth ratios obtained under the first scenario.

Table 1 presents the results of a comparison at the steady state of the benchmark model with zero wealth tax and the model with the 1% wealth tax for different levels of the wealth ratios. Panel A refers to the first scenario, in which higher wealth is associated to higher public debt; while panel B to the second scenario, in which higher wealth is associated to a lower real interest rate. Although they have zero net wealth, poor households are affected by the wealth tax because of the general equilibrium effect on prices. Specifically, poor and rich households face different net user costs of housing services (equations (29) and (30)). We summarize the results as follows. First, in both scenarios, the introduction of the wealth tax increases the user costs of housing services for both poor and rich households, and more so for poor households. Specifically, the user cost of housing services for poor households, after the introduction of the 1% flat wealth tax and for a medium wealth-to-income ratio, increases by approximately 15% in the first scenario, and by 14% in the second scenario. The user costs of housing services for rich households increase, respectively, by approximately 14% and 13%. Second, the introduction of the wealth tax, under both scenarios, increases the net wage and, conversely, decreases the wage tax. Specifically, under the first scenario and for a medium wealth-to-income ratio, the net wage increases by approximately 2% and the wage tax decreases by approximately 1.5%. The effects on the net wage and on the wage tax are higher under the second scenario: the net wage increases by approximately 3% and the wage tax decreases by approximately 2%. Third, the equivalent income loss for poor households of introducing the wealth tax is large and approximately equal to 8% for both scenarios. Fourth, while the effects on the net wage, wage tax, and user costs of housing, are similar for the different levels of the wealth-to-income ratio under the first scenario, they are increasing with the wealth-to-income ratio under the second scenario. For example, the net user cost of housing for poor households, after the introduction of a 1% wealth tax, increases by approximately 10% for a low level of the wealth-to-income ratio, and by 20% for a high level of the wealth-to-income ratio.

3 Optimal Tax Rates

In this section we consider the optimal taxation problem under commitment. We first present the theoretical framework, and then analyze the quantitative effects of a model considering two mechanisms that generate an increasing wealth-to-income ratio.

3.1 Framework

The Planner maximizes a weighted average of per period utilities across households types at competitive equilibrium allocations by choosing appropriate values of the available tax rates. In order to obtain the steady state allocation as a possible solution to the optimal policy we assume that per period utilities are discounted at the same rate, $(1+r)^{-1}$, *i.e.*, the discount rate of the most patient households. Note that this type of social welfare function implies that the impatient households will be saving more than they would if the Planner was discounting utilities at the (heterogeneous) subjective

Table 1: The Effects of Introducing a Flat 1% Wealth Tax

Panel A: increase in public debt (Δb^g)			
wealth-to-income	low v/y	medium v/y	high v/y
Δ% net wage ($\hat{w}$)	1.94	1.99	2.07
Δ wage tax (τ^w)	-1.58	-1.57	-1.56
Δ% net user cost of housing poor ($\hat{\pi}^p$)	15.10	15.11	15.11
Δ% net user cost of housing rich ($\hat{\pi}^r$)	13.95	13.95	13.96
Δ% equivalent income loss poor households	-8.55	-8.44	-8.34
Panel B: decrease in real interest rate (Δr)			
wealth-to-income	low v/y	medium v/y	high v/y
Δ% net wage ($\hat{w}$)	2.31	2.85	3.55
Δ wage tax (τ^w)	-1.72	-2.07	-2.53
Δ% net user cost of housing poor ($\hat{\pi}^p$)	10.33	14.08	20.10
Δ% net user cost of housing rich ($\hat{\pi}^r$)	9.23	12.94	18.90
Δ% equivalent income loss poor households	-8.29	-8.53	-8.69

Notes: This table reports the change in the net wage; wage tax; net user costs of housing for poor and rich households, between the scenario with a flat 1% wealth tax and the scenario with a zero wealth tax, for different levels of total wealth-to-income ratio v/y. For the wage tax we report the difference in percentage points. For all other variables we report percentage changes. In addition, the table reports the equivalent income loss of poor households, in percentage, determined by the introduction of the flat 1% wealth tax. The equivalent income loss is equal to $1 - (\hat{\pi}_0^p/\hat{\pi}_1^p)^\theta$, where we denote with "0" the scenario with zero wealth tax and with "1" the scenario with the flat 1% wealth tax. Panel A corresponds to a "increase in public debt" scenario, in which the change in wealth is determined exogenously by changing government debt (b^g); panel B corresponds to a "decrease in real interest rate" scenario, in which the change in wealth is determined endogenously by changing the level of the real interest rate (r). We change r in order to exactly match the wealth-to-income ratios obtained under the "ncrease in public debt" scenario. Parameters are from Table 2. The values for the wealth-to-income ratios are $v/y = 1$ (low); $v/y = 1.5$ (medium); and $v/y = 2$ (high). Refer to Appendix A for details on the numerical solution of the model.

discount rates. However, if the equilibrium generated by the Planner's policies implies binding debt limits for the impatient households at all $t \geq 0$, then replacing their subjective discount rate with the higher value, $(1+r)^{-1}$, has no consequences on these households' net wealth, which is going to be zero in both cases. Social welfare functions with welfare weights reflecting the Planner's (or society's) preferences have been widely used in the literature. For instance, Saez and Stantcheva (2016) propose to evaluate tax reforms using "generalized social marginal welfare weights" to capture society's concerns for fairness without being necessarily tied to individual utilities. We simplify the Planning problem by selecting the poor and the rich households' welfare weights in $\{1, \eta\}$, where $\eta \geq 0$ is the one attached to the rich households' utility, so that the social welfare function is

$$\mathcal{W} = \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \left(\eta \sum_{i\in\mathcal{R}} m_i U(c_t^i, z_t^i) + \sum_{i\in\mathcal{P}} U(c_t^i, z_t^i)\right). \tag{34}$$

For simplicity, we restrict our attention to equilibria such that the minimum home size constraint (10) is non binding for all i and such that the renters are sufficiently poor (ϵ^i low enough) and $\bar{z}$ large enough to be better-off being renters than homeowners. Finally, we concentrate on equilibria where the households' partition is as specified in section 2, *i.e.*, the patient households' debt limits are non binding at all times and $a_t^i = 0$ for all $t \geq 0$ and $i \in \mathcal{P}$, a condition that is certainly verified if the economy is sufficiently close to the steady state.

To set up the planner's problem we start by exploiting the market clearing conditions in the good, housing and asset markets, as well as profit maximization, to restate the t-period government budget constraint (16) as follows

$$\sum_i m_i \left(\frac{\hat{a}_{t+1}^i}{\hat{R}_{t+1}^i} + c_t^i + \hat{\pi}_t^i h_t^i + (s_t - \hat{s}_t)(h_t^i - z_t^i) - \hat{w}_t - \hat{a}_t^i\right) \geq 0. \tag{35}$$

Equation (35) can be simplified under the assumed household's partition. Specifically, recall that $\hat{a}_t^i = 0$ for all $i \in \mathcal{P}$ and $t \geq 0$. Furthermore, letting $\mathcal{S}$ be the set of poor-renters, note that

$$\sum_{i\in\mathcal{P}\setminus\mathcal{S}} (c_t^i + \hat{\pi}^p h^i - \epsilon^i \hat{w}_t) = \sum_{i\in\mathcal{S}} (c_t^i + s_t z_t^i - \epsilon^i \hat{w}_t) = 0, \tag{36}$$

and

$$\sum_{i\in\mathcal{R}} m_i (h_t^i - z_t^i) = \sum_{i\in\mathcal{S}} m_i z_t^i.$$

Then, using the above into equation (35) and exploiting the no arbitrage condition

(23), the latter is equivalent to

$$\sum_{i \in \mathcal{R}} m_i \left(\frac{\hat{a}^i_{t+1}}{\hat{R}^i_{t+1}} + c^i_t + \hat{\pi}^r z^i_t - \epsilon^i \hat{w}_t - \hat{a}^i_t \right) \geq 0. \tag{37}$$

The available menu of (proportional) tax rates on housing, rents and financial wealth is unrestricted. To find the optimal mix of tax rates, we follow the primal approach (Lucas and Stokey, 1983; Atkeson et al., 1999; Chari and Kehoe, 1999). Since the rich are never financially constrained,

$$p_t = \frac{U^i_{1,t} p_0}{(1+r)^t U^i_{1,0}} \qquad \text{for all } i \in \mathcal{R}.$$

Then, using the first order conditions from utility maximization, (20)-(23); the complementary slackness conditions; and the assumption that the minimum home size constraint is non binding; we can rewrite equations (36), (37) as

$$\sum_{i \in \mathcal{R}} m_i \left(\frac{U^i_{1,t+1} \hat{a}^i_{t+1}}{1+r} + H^i(c^i_t, z^i_t, \hat{w}_t) - U^i_{1,t} \hat{a}^i_t \right) \geq 0, \tag{38}$$

$$H^i(c^i_t, z^i_t, \hat{w}_t) = 0 \qquad i \in \mathcal{P} \tag{39}$$

where

$$H^i(c^i, z^i, \hat{w}) \equiv U_1(c^i, z^i) c^i + U_2(c^i, z^i) z^i - U_1(c^i, z^i) \epsilon^i \hat{w}_t.$$

Equations (38), (39) are the *implementability conditions* and define the households' budget constraints in terms of first order conditions, instead of prices. Finally, by (24), equation (38) can be iterated forward from period zero to provide the following present value representation of the government budget constraint

$$\sum_{t=0}^{\infty} \left(\frac{1}{1+r} \right)^t \sum_{i \in \mathcal{R}} m_i H^i(c^i_t, z^i_t, \hat{w}_t) \geq \sum_{i \in \mathcal{R}} m_i U_1(c^i_0, z^i_0) \hat{a}^i_0, \qquad \text{for all } i \in \mathcal{R}. \tag{40}$$

Any sequence $\{c^i_t, z^i_t, \hat{w}_t; i = l, d, r\}^{\infty}_{t=0}$ satisfying conditions (39), (40) together with the resource feasibility constraints (equations (2)–(6)), for all $t \geq 0$, and for some initial aggregate wealth (verifying (26)),

$$\sum_{i \in \mathcal{R}} m_i \hat{a}_0^i = \hat{R}_0(k_0 + b_0^g/R_0 + q_{-1}h_0),$$

is a competitive equilibrium implemented by some set of implicit individual specific tax rates.

Now define the *pseudo welfare function*

$$\tilde{U}_t = \eta \sum_{i \in \mathcal{R}} m_i U(c_t^i, z_t^i) + \sum_{i \in \mathcal{P}} U(c_t^i, z_t^i) + \mu \sum_{i \in \mathcal{R}} m_i H^i(c^i, z^i, \hat{w}), \tag{41}$$

where the multiplier μ is positive if the Planner needs distortionary taxation to finance public spending. This multiplier represents a "bonus to date-t allocations that brings in extra government revenues, thereby relieving other periods from distortionary taxation, and the same term imposes a penalty in the opposite situation" (Erosa and Gervais, 2001). Then, for a given policy, $\{g_t, x_t\}_{t=0}^{\infty}$, the Planner's decision variables are defined by the sequence

$$\mathbf{d} = \{c_t^i, z_t^i, k_t^j, h_{t+1}, l_t^j, k_t; i = \mathcal{I},\ j = h, m\}_{t=0}^{\infty},$$

and we define the optimal taxation problem as follows

$$\max_{(\mathbf{d},\mu) \geq 0} \sum_{t=1}^{\infty} \left(\frac{1}{1+r}\right)^t \tilde{U}_t - \mu W_0 \qquad \text{s.t. equations (2)–(6) at all } t \geq 1, \tag{PP}$$

where

$$W_0 = \sum_{i \in \mathcal{R}} m_i U_1(c_0^i, z_0^i) \hat{a}_0^i, \qquad \sum_{i \in \mathcal{R}} m_i a_0^i = R_0 k_0 + b_0^g + q_{-1} h_0.$$

Note that, since we only consider the case of full commitment, the planner is unable to revise the given initial tax rates, so that W_0 is a predetermined initial condition in the planning problem.

3.2 Wealth-Rich as Pure Speculators

To gain intuition about the optimal tax structure (which we derive in the next section), it is useful to start with a restricted version of the present model which is directly comparable to the literature on optimal taxation with heterogeneous households (rich and poor). In his seminal work, Judd (1985) considers an economy with a single good produced by a constant-returns-to-scale production function with capital and labor as inputs, populated by a capitalist (with capital as the only source of income) and a worker. The Planner must select two distortionary tax rates on labor and capital to finance a stream of lump-sum expenditures. Judd (1985) shows that the Planner would not use tax rates for redistribution (at least asymptotically), *i.e.*, the capital tax rate is zero at steady state, even if the social welfare function is totally biased toward the worker ($\eta = 0$, with our notation). A standard interpretation is that the inefficiency of capital taxation grows extremely large over time due to the infinite elasticity of the supply of capital (Saez and Stantcheva, 2018).

To get our model closer to the framework adopted in Judd (1985), in this section we assume that the rich households are acting as "pure speculators" in the housing market, in the sense that they derive no utility from housing services, and we study the optimal tax structure as their labor income goes to zero. In our model, Judd's zero capital tax result holds. However, it is optimal to introduce a subsidy on housing services specifically targeted to poor households as long as the rich derive some income from work. Intuitively, since we allow for homeownership among the poor, this subsidy may be interpreted as a negative capital income tax as well as a negative tax on imputed rents. In the next section, when we consider the unrestricted version of the model, we will show that a positive housing wealth tax on the rich is optimal under some robust assumptions on rich households' utility function.

Assume that rich households' preferences are described by a utility function $U^r(c)$, increasing and strictly concave. There is a mass m_r of identical rich households whose labor productivity is ϵ^r, and a mass m_p of identical poor households whose labor productivity is ϵ^p. The model can accomodate for the existence of both homeowners and renters under the condition $\pi_t^p = s_t$ for all $t \geq 0$. This is compatible with individual optimality when capital and housing taxes are all zero or, alternatively, when poor households' home ownership is subsidized. Since rich and poor households are identical we restrict the set $\mathcal{I}$ to $\{r, p\}$. For simplicity, we also assume that business capital is zero, so that the consumption good is produced only with labor and the construction good is produced with labor and a fixed flow of land per period. Since $k = 0$ and the manufacturing sector exhibits constant returns to scale, the production functions in manufacturing and construction are

$$f^m(l^m) = wl^m, \qquad f^h(l^h, x) \equiv f(l^h),$$

where $w > 0$ is a productivity parameter and $f_l^h(L^h) > 0$, $f_{ll}^h(L^h) < 0$. These assumptions imply the following profit maximization condition

$$q_t = w/f_l^h(l_t^h), \tag{42}$$

and the first order conditions from utility maximization

$$\begin{aligned}
U_1^r(c_t^r)/U_1^r(c_{t+1}^r) &= \hat{R}_{t+1}/(1+r), && (43)\\
U_2^p(c_t^p, h_t)/U_1^p(c_t^p, h_t) &= \hat{\pi}_t^p = s_t. && (44)\\
q_t &= ((1-\delta)\hat{q}_{t+1}^r + \hat{s}_{t+1})/\hat{R}_{t+1}. && (45)
\end{aligned}$$

By exploiting the above restrictions; the assumption $\hat{\pi}^p = s$; and recalling that rich households derive no utility from housing services (so that the poor households' consumption of housing services is equal to the stock of housing) we can derive the t-period government budget constraint as

$$q_t h_{t+1} + m_p(c_t^p + s_t h_t - \epsilon^r \hat{w}_t) + m_r(c_t^r - \epsilon^r \hat{w}_t) \geq \hat{R}_t q_{t-1} h_t. \tag{46}$$

The above is the equivalent of equation (37) under this restricted version of our model. It shows that any extra unit of net wage reduces the government's revenue by one unit. Therefore, given the initial outstanding net capital income, $\hat{R}_t q_{t-1} h_t$, any extra unit of net wage must be compensated by some extra value of next period wealth or households' consumption. However, note also that, since poor households are financially constrained, *i.e.*,

$$c_t^p + s_t h_t - \epsilon^r \hat{w}_t = 0,$$

we can rewrite (46) as

$$q_t h_{t+1} + m_r(c_t^r - \epsilon^r \hat{w}_t) \geq \hat{R}_t q_{t-1} h_t. \tag{47}$$

Then, an extra unit of the net wage, $\hat{w}_t$, has two separate effects. First, it reduces poor households' total expenditure by $m_p \epsilon^p$, because poor households are financially constrained. Second, it generates a tax revenue shortfall equal to $m_r \epsilon^r$, which can only be compensated by a change in rich households' wealth or consumption. These additional resources have a cost in terms of incremental distortions. To derive the optimal tax rates, we can replace $\hat{R}_t$ using the first order condition (43), and then substitute q_t and q_{t-1} with the no arbitrage condition (45) in equation (47). Then, we can rewrite (47) as

$$\frac{U_1^r(c_{t+1}^r)}{1+r}((1-\delta)\hat{q}_{t+1}^r + \hat{s}_{t+1}) + U_1^r(c_t^r) m_r(c_t^r - \epsilon^r \hat{w}_t) \geq U_1^r(c_t^r)((1-\delta)\hat{q}_t^r + \hat{s}_t) h_t. \tag{48}$$

By iterating forward (48) we obtain the *long-run implementability constraint* (equivalent to (40) in the unrestricted model):

$$\sum_{t=0}^{\infty}\left(\frac{1}{1+r}\right)^{t} U_1^r(c_t^r)m_r(c_t^r-\epsilon^r\hat{w}_t)\geq W_0, \tag{49}$$

where $W_0 \equiv U_1^r(c_0^r)(b_0^g+q_{-1}h_0)$. Since we are looking at the optimal tax problem under commitment, the term W_0 is given exogenously.

In the remaining part of this section, we assume that rich households' utility function, $U^r(c^r)$, exhibits a constant relative degree of risk aversion, $\sigma > 0$, and that poor households' utility function, $U^p(c^p, z^p)$ is Cobb-Douglas as in (31). This specification of preferences has the advantage of simplifying the characterization of the optimal tax structure and provides additional intuition. In particular, by the poor households' preference representation, we obtain the constant expenditure shares

$$c_t^p = (1-\theta)\hat{w}_t, \qquad s_t z_t^p = s_t h_t/m_p = \theta\hat{w}_t.$$

Then the planning problem (PP) boils down to the maximization of the function

$$\mathcal{L} = \sum_{t=0}^{\infty}\left(\frac{1}{1+r}\right)^{t}[\eta m_r U^r(c_t^r) + m_p U^p\left((1-\theta)\hat{w}_t, h_t/m_r\right) + \mu U_1^r(c_t^r)m_r(c_t^r-\epsilon^r\hat{w}_t)$$

$$+\lambda_t^m(wl_t^m - m_r c_t^r - m_p(1-\theta)\hat{w}_t - g_t) + \lambda_t^h(h_{t+1} - f(l_t^h) - (1-\delta)h_t) + \xi_t(1-l_t^h-l_t^m)],$$

with respect to $\{c_t^r, \hat{w}_t, h_{t+1}, l_t^h, \lambda_t^m, \lambda_t^h, \xi_t\}_{t=1}^{\infty}$, where $(\lambda_t^m, \lambda_t^h, \xi_t)$ are the (discounted) Lagrange multipliers associated to the resource feasibility constraints in manufacturing, housing and labor market (*i.e.*, the shadow prices of consumption, housing and labor, respectively).

Now consider an interior solution and observe that optimality requires that the net marginal benefit of increasing the rich households' consumption, c_t^r, and the net wage, $\hat{w}_t$, at time t, must be equal to the shadow price of consumption in manufacturing. In particular, these conditions can be stated as follows

$$
\begin{aligned}
\eta U_1^r(c_t^r) &= \lambda_t^m + \mathcal{D}_t^r && (50) \\
U_1^p(c_t^p, z_t^p) &= \lambda_t^m + \mathcal{D}_t^p, && (51)
\end{aligned}
$$

where

$$
\mathcal{D}_t^r \equiv \mu U_1^r(c_t^r)\left(\sigma\left(1 - \epsilon^r \hat{w}_t / c_t^r\right) - 1\right), \qquad \mathcal{D}_t^p = \mu U_1^r(c_t^r) m_r \epsilon^r \hat{w}_t / m_p c_t^p
$$

are the net cost of the fiscal distortions generated by a rise in the rich and poor households' consumption, respectively. Hence, the left hand sides of equations (50) and (51) represent the direct benefit on social welfare of increasing the consumption of rich and poor households, and the right hand side is the sum of two costs: the shadow price of consumption, λ_t^m, and the net cost of the fiscal distortions, $\mathcal{D}_t^r$, $\mathcal{D}_t^p$. Note that these are proportional to the multiplier μ, which represents the gain from relaxing the government budget constraint; and they have different size and, possibly, sign. In particular, $\mathcal{D}_t^r$ can be positive or negative depending on the elasticity of the marginal utility, $1/\sigma$, and the rich household's wage-to-consumption ratio. If $\sigma > 1$, *i.e.*, marginal utility is relatively inelastic, the fiscal distortions related to a rise in the rich households' consumption is positive.

Crucially, we note that the fiscal distortion caused by a rising poor households' consumption (or wage), $\mathcal{D}^p$, is always positive for $\epsilon^r > 0$, whereas $\mathcal{D}_t^r = \mathcal{D}_t^p = 0$ if $\epsilon^r = 0$, *i.e.*, if rich households are not working. The intuition is as follows. If rich households are working, a drop in the labor tax, or, equivalently, a rising net wage, is only partly compensated by a rising consumption by poor households. But this is not enough to compensate for the total lost revenue because part of it comes from the labor tax on the rich. Then, some other revenue compensation must be generated by other sources of the rich households' income, implying some additional distortions. Then, the social cost of raising the poor households' consumption is larger than the shadow price of consumption.

Because $\mathcal{D}_t^p > 0$, then the poor households' marginal rate of substitution between housing and consumption at the planning optimum is lower than the shadow relative price of housing services, *i.e.*, housing must be subsidized. In particular, turning to the first order conditions related to housing and labor, we obtain

$$
\begin{aligned}
(1+r)\lambda_{t-1}^h &= U_2(c_t^p, z_t^p) + (1-\delta)\lambda_t^h, && (52)\\
\lambda_t^h/\lambda_t^m &= w/f'(l_t^h), && (53)
\end{aligned}
$$

implying that $\lambda_t^h/\lambda_t^m = q_t$. Now consider a steady state of the optimal allocation. In this case, equations (51) and (52) imply

$$
\frac{U_2^p}{U_1^p} = \left(1 - \frac{\mathcal{D}^p}{U_1^p}\right) q(r+\delta). \tag{54}
$$

Recalling the definition of $\hat{\pi}^p$ given in (30), the above optimal condition can be implemented in steady state through the assumed menu of tax rates by setting

$$
\tau^k = 0, \qquad \tau^{h,p} = -\left(\frac{r+\delta}{1-\delta}\right)\frac{\mathcal{D}^p}{U_1^p} \leq 0.
$$

3.3 The Unrestricted Model

We now consider the unrestricted model of section 2. To derive the Planner's problem, we use equation (39) to express the poor households' consumption as a function of housing demand. In particular, note that, for $j = 1, 2$,

$$
H_j^i(c^i, z^i, \hat{w}) = U_j(c^i, z^i, \hat{w})\left(1 + g_j(c^i, z^i, \hat{w})\right), \qquad H_3(c^i, z^i, \hat{w}) = -U_1(c^i, z^i)\epsilon^i,
$$

where

$$g_j^i = \frac{U_{1,j}^i c^i + U_{2,j}^i z^i}{U_j} - \frac{U_{1,j}^i c^i}{U_j^i} \left(\frac{\epsilon^i \hat{w}}{c^i} \right) \tag{55}$$

are the *general equilibrium elasticities* related, respectively, to the tax rates on capital and housing (Chari and Kehoe, 1999; Atkeson et al., 1999). These elasticities capture the extent to which a fall in the corresponding tax rates is reducing distortions. In particular, the higher is g_2 relative to g_1, the higher are the efficiency costs from taxing housing. We now discuss two properties of these elasticities in the following proposition. Appendix B contains the proof.

Proposition 2. *If $U(c^i, z^i)$ is concave, the following properties hold for all $i \in \mathcal{P}$:*

$$\begin{aligned} g_1^i &= z^i \frac{\partial}{\partial c^i} \left(\frac{U_2^i}{U_1^i} \right) \geq 0, && (56) \\ g_1^i &\geq g_2^i && (57) \end{aligned}$$

with strict inequalities if $U(c^i, z^i)$ is strictly concave.

By (56) we can use equation (39) to express the poor households' consumption as a function of housing demand and the net wage, *i.e.*,

$$c^i = \psi^i(z_t^i, \hat{w}_t).$$

Under the maintained assumptions, the above is a continuously differentiable function such that

$$\psi_1^i = -\frac{H_2^i}{H_1^i} = -\frac{U_2^i(1 + g_2^i)}{U_1^i(1 + g_1^i)}, \qquad \psi_2^i = -\frac{H_3^i}{H_1^i} = \frac{\epsilon^i}{(1 + g_1^i)}. \tag{58}$$

It follows that the pseudo welfare function $\tilde{U}_t$ in (41) is

$$\tilde{U}_t = \eta \sum_{i \in \mathcal{R}} m_i U(c_t^i, z_t^i) + \sum_{i \in \mathcal{P}} U(\psi^i(z_t^i, \hat{w}_t), z_t^i) + \mu \sum_{i \in \mathcal{R}} m_i H^i(c^i, z^i, \hat{w}).$$

To characterize the planning optimum, let $\{\lambda_t^m, \lambda_t^h\}_{t=0}^{\infty}$ be the non-negative discounted Lagrange multipliers associated to the resource constraints in the manufacturing sector, (2), in the construction sector, (3), and in the labor market, (5), respectively; f_t^j the time-t output per unit of labor efficiency; and $f_{s,t}^j$, for $j = h, m$ and $s = k, l, x$, the time-t marginal products of capital, labor and land in the two sectors. Then, we can split the first order characterization of the optimal taxation problem into two sets of conditions. The first concerns the optimal allocation of capital, labor and land across sectors, consumption of manufacturing, and housing:

$$\lambda_t^h/\lambda_t^m = f_{l,t}^m = (\lambda_t^h/\lambda_t^m) f_{l,t}^h, \tag{59}$$

$$(1+r)\lambda_t^m/\lambda_{t+1}^m = f_{k,t+1}^m = (\lambda_t^h/\lambda_t^m) f_{k,t+1}^h. \tag{60}$$

Note that, by the profit maximization conditions, (7), (8), the above imply

$$\lambda_t^h/\lambda_t^m = q_t, \qquad (1+r)\lambda_{t-1}^m/\lambda_t^m = R_t.$$

The second set of conditions concerns the optimal allocation of consumption, housing and labor across households. Letting

$$\pi_t = q_{t-1}R_t - (1-\delta)q_t,$$

then the optimal allocation of consumption and housing services across rich households, *i.e.*, for all $i \in \mathcal{R}$, is defined as

$$\lambda_t^m = U_{1,t}^i(\eta + \mu(1 + g_{1,t}^i)), \tag{61}$$

$$\lambda_t^m \pi_t = U_{2,t}^i(\eta + \mu(1 + g_{2,t}^i)), \tag{62}$$

which provide an interpretation of λ^m and $\lambda^m \pi_t$ as the shadow prices of consumption and housing services, respectively. Note that, by (60), equation (61) implies that, at steady state,

$$f_k^m(k^m, l^m) = R \equiv (1+r), \tag{63}$$

which establishes the Chamley-Judd zero capital tax rate result at steady state. Now note that, by (61) and (62), we get the marginal rate of substitution between housing services and consumption

$$\frac{U_{2,t}^i}{U_{1,t}^i} \equiv \hat{\pi}_t^i = \pi_t \left(\frac{\eta + \mu(1+g_{1,t}^i)}{\eta + \mu(1+g_{2,t}^i)} \right) \qquad \text{for all } i \in \mathcal{R}. \tag{64}$$

Hence, $g_1^i - g_2^i$ is a measure of the social benefit from housing taxation. It is worth noticing, at this point, that the composition of the households' budget constraint plays a role in determining the size of these elasticities. To get more intuition, consider a steady state where assets, net wages and consumption of manufacturing and housing services are time independent, and let the household i's steady state net income be defined as $m^i = \hat{w} + \omega^i$, where ω^i is the (steady state) value of household i's net capital income. Furthermore, let

$$\sigma_c^i = -U_{1,1}^i c^i / U_1^i \tag{65}$$

be the elasticity of the marginal utility of consumption and assume that the households' utility is linearly homogeneous. Then, we derive

$$1 + g_1^i = 1 + \sigma_c \left(\frac{\epsilon^i \hat{w}}{c^i} \right), \qquad 1 + g_2^i = 1 - \sigma_c \left(\frac{\epsilon^i \hat{w}}{\hat{\pi}^i z^i} \right).$$

Letting θ (respectively, $1-\theta$) be the share of the steady state income spent for housing services (respectively, consumption), we obtain

$$1 + g_1^i = 1 + \frac{\sigma_c/(1-\theta)}{1 + \omega^i/\epsilon^i \hat{w}}, \tag{66}$$

$$1 + g_2^i = 1 - \frac{\sigma_c/\theta}{1 + \omega^i/\epsilon^i \hat{w}}. \tag{67}$$

Then, the higher is the household i's *capital to wage income*, $\omega^i/\epsilon^i\hat{w}$, the lower is the general equilibrium elasticity $1+g_1^i$ relative to $1+g_2^i$, and, then, the lower is the scope for taxing the household i's housing wealth. To make this statement more precise, assume that rich households are all identical, so that we can identify them with the unique subscript r. Note that their steady state budget constraint is

$$c^r + \hat{\pi}^r z^r = \epsilon^r \hat{w} + \left(\frac{r}{1+r}\right)\hat{a}^r.$$

By the asset market equilibrium,

$$m_r \hat{a}^r/(1+r) = k + qh + b^g/(1+r) \equiv v,$$

where v denotes the aggregate net wealth at a steady state equilibrium. It follows that

$$\frac{\omega^r}{\epsilon^r \hat{w}} = \frac{rv}{m_r \epsilon^r \hat{w}}.$$

In light of the above discussion, this implies that the rich household's housing wealth tax should diminish with the ratio $rv/\hat{w}$, *i.e.*, the ratio between the returns from aggregate wealth and the net wage. This observation can be used to explain the numerical simulations of the next section, where we verify the effects on the optimal tax rates of increasing aggregate wealth-to-income ratios.

We turn now to the first order conditions for the optimal allocation of poor households' housing services, using (58) we get

$$\lambda_t^m \pi_t = U_{2,t}^i \left(1 - \left(\frac{1+g_{2,t}^i}{1+g_{1,t}^i}\right)\left(1 - \frac{\lambda_t^m}{U_{1,t}^i}\right)\right) \qquad \text{for all } i \in \mathcal{P}, \tag{68}$$

where the left hand side is the shadow price of housing services and the right hand side is the utility gain from an extra unit of housing services net of the (possible) reduction in consumption that follows from the budget constraint and the feasibility constraint. Finally, the first order condition related to the optimal allocation of the net wages, $\hat{w}_t$, can be stated as follows

$$\underbrace{\left(\sum_{i\in\mathcal{P}} m_i \frac{U^i_{1,t}\epsilon^i}{1+g^i_{1,t}}\right)}_{\text{extra consumption}} = \mu \underbrace{\left(\sum_{i\in\mathcal{R}} m_i U^i_{1,t}\epsilon^i\right)}_{\text{extra distortions}} + \lambda^m_t \underbrace{\left(\sum_{i\in\mathcal{P}} m_i \frac{\epsilon^i}{1+g^i_{1,t}}\right)}_{\text{reduced resources}}. \tag{69}$$

Note that (69) equates the gain from any extra unit of net wage due to poor households' extra consumption to the sum of two different costs: the cost of the additional distortions following from the fall in the tax revenue plus the cost of the fall in the available resources. The last two costs are weighted, respectively, by the Lagrange multiplier μ (representing the gain from a fall in distortionary taxation) and the shadow price of consumption, λ^m_t. Now observe that, by (64),

$$\hat{\pi}^i_t > \pi_t \qquad \Leftrightarrow \qquad g^i_{1,t} > g^i_{2,t} \qquad \forall i \in \mathcal{R}, \tag{70}$$

i.e., the cost of housing services for rich households must be taxed if the gain in efficiency from a fall in the (implicit) tax on consumption exceeds the gain from a fall in the tax on housing. Note that, if $U(c, z)$ is homogeneous of degree $\zeta \geq 0$, we have

$$(U^i_{1,1}c^i + U^i_{1,2}z^i)/U^i_1 = (U^i_{1,2}c^i + U^i_{2,2}z^i)/U^i_2 = \zeta - 1,$$

so that, for all $i \in \mathcal{I}$,

$$1 + g^i_1 = \zeta - \frac{U^i_{1,1}c^i}{U^i_1}\left(\frac{\epsilon^i\hat{w}}{c^i}\right), \qquad 1 + g^i_2 = \zeta - \frac{U^i_{1,2}c^i}{U^i_2}\left(\frac{\epsilon^i\hat{w}}{c^i}\right),$$

and, by strict concavity, $g^i_1 > g^i_2$ for all $i \in \mathcal{I}$. In this case, by (70), it follows that $U^i_{2,t}/U^i_{1,t} > \pi_t$ for all $i \in \mathcal{R}$. In particular, since $U^i_{1,1} < 0 < U^i_{1,2}$, the difference $(g^i_1 - g^i_2)$ is increasing in the ratio between wage income and consumption, $\epsilon^i\hat{w}/c^i$. The latter is constant and equal to $1/(1-\theta)$ for the poor, and it depends on net assets for rich households. Note that, at steady state, $c^i > (1-\theta)\epsilon^i \hat{w}$ and, then, $\epsilon^i \hat{w}/c^i$ is decreasing in the size of wealth for all $i \in \mathcal{R}$. Hence, the difference $(g_1^i - g_2^i)$, *i.e.*, the scope for housing taxation, is decreasing in the size of wealth for all $i \in \mathcal{R}$.

A second important observation is that, by (68), and since $g_{1,t}^i \geq g_{2,t}^i$ for all $i \in \mathcal{P}$,

$$\frac{U_{2,t}^i}{U_{1,t}^i} \equiv \hat{\pi}_t^i = \pi_t \left(\frac{(1+g_{1,t}^i)\lambda_t^m}{U_{1,t}^i(g_{1,t}^i - g_{2,t}^i) + (1+g_{2,t}^i)\lambda_t^m} \right) \qquad \forall i \in \mathcal{P}. \tag{71}$$

implying that, for all $i \in \mathcal{P}$,

$$U_{2,t}^i / U_{1,t}^i < \pi_t \qquad \Leftrightarrow \qquad \lambda_t^m < U_{1,t}^i \qquad \forall i \in \mathcal{P}. \tag{72}$$

In other words, the cost of housing services for the worker must be subsidized if her marginal utility of consumption exceeds the shadow price of consumption. To understand the circumstances under which this condition holds, define the "weights"

$$\xi_t^i = \frac{m_i \epsilon^i / (1+g_{1,t}^i)}{\sum_{j\in\mathcal{P}} m_j \epsilon^i / (1+g_{1,t}^j)},$$

and notice that, by rearranging the terms in equation (69), we obtain

$$\lambda_t^m = \sum_{i\in\mathcal{P}} \xi_t^i U_{1,t}^i - \mu \left(\frac{\sum_{i\in\mathcal{R}} m_i \epsilon^i U_{1,t}^i}{\sum_{j\in\mathcal{P}} m_j \epsilon^i / (1+g_{1,t}^j)} \right). \tag{73}$$

Since the weights, ξ_t^i, are positive and they sum up to one and $\mu > 0$, then λ_t^m is strictly smaller than a convex linear combination of the poor households' marginal utilities of consumption. Namely, the shadow price of consumption falls short of an average of the poor households' marginal utility of consumption because of the extra-distortions implied by shifting taxation from labor to housing. This implies that

$$\lambda_t^m < \max_{i \in \mathcal{P}} U_{1,t}^i. \tag{74}$$

Then, by (72), the user cost of housing faced by poor households whose marginal utility of consumption is relatively large must be subsidized. We summarize these findings in the following proposition.

Proposition 3. *Assume that $U(c, z)$ is a homogeneous function and let $\mathcal{P}^* \subset \mathcal{P}$ such that*

$$U_1(c_t^i, z_t^i) \geq U_1(c_t^j, z_t^j) \qquad \text{for all } i \in \mathcal{P}^* \text{ and } j \in \mathcal{P}.$$

Then, the optimal tax structure is such that

$$\hat{\pi}_t^i < \pi_t < \hat{\pi}_t^j \qquad \text{for all } i \in \mathcal{P}^* \text{ and } j \in \mathcal{R}.$$

By (29) and (30), the implicit tax rates derived in this section can be implemented through the tax instruments considered in section 2. Namely, letting the steady state implicit optimal tax rate be

$$t^{h,i} = \frac{\hat{\pi}^i}{\pi} - 1,$$

we derive

$$\tau^{h,i} = \left(\frac{r + \delta}{1 - \delta}\right) t^{h,i}.$$

3.4 Numerical Simulation

To provide a better analytical representation of the optimal tax structure, assume that rich households are all identical (in terms of labor productivities and initial asset holdings) and utility is Cobb-Douglas (31). Assume, also, that $\eta = 0$ and use the index

$i = r$ to identify rich households' decisions. In this case, for all $i \in \mathcal{I}$,

$$(1 + g_1^i) = 1 + \theta \epsilon^i \hat{w}/c^i, \qquad (1 + g_2^i) = 1 - (1-\theta)\epsilon^i \hat{w}/c^i.$$

Note that, for all $t \geq 0$ and $i \in \mathcal{P}$,

$$c_t^i = (1-\theta)\epsilon^i \hat{w}_t. \tag{75}$$

Then,

$$(1 + g_{1,t}^i) = 1/(1-\theta), \qquad (1 + g_{2,t}^i) = 0 \qquad \text{for all } t \geq 0 \text{ and } i \in \mathcal{P}.$$

On the other hand, by exploiting the asset market equilibrium condition, the rich households' general equilibrium elasticities are

$$1 + g_{1,t}^r = \frac{(1-\theta)r + (1-\bar{\epsilon}^p)\hat{w}/v}{(1-\theta)(r + (1-\bar{\epsilon}^p)\hat{w}/v)}, \qquad 1 + g_{2,t}^r = \frac{(1-\theta)r}{r + (1-\bar{\epsilon}^p)\hat{w}/v}.$$

Hence, the rich households' general equilibrium elasticities depend on the net wage-to-wealth ratio, $\hat{w}/v$, and the scope for taxing housing, as measured by the gap $g_1^r - g_2^r$, is increasing in this ratio. Note, also, that, by (68),

$$U_{2,t}^i = \lambda_t^m \pi_t$$

for all $i \in \mathcal{P}$, and, by the linear homogeneity of U, this implies that all marginal utilities are equalized across the set of poor households. Hence, with some abuse of notation, we set

$$U_{j,t}^i = U_{j,t}^p \qquad \text{for all } i \in \mathcal{P}.$$

This implies that (73) becomes

$$\lambda_t^m = U_{1,t}^p - \frac{\mu}{(1-\theta)}\left(\frac{1-\bar{\epsilon}^p}{\bar{\epsilon}^p}\right)U_{1,t}^r, \tag{76}$$

where $\bar{\epsilon}^p = \sum_{i\in\mathcal{P}} m_i \epsilon^i$. Now we derive the value of μ by equating (61) to (76) to obtain

$$\mu = \frac{U_{1,t}^p}{U_{1,t}^r}\left(\frac{(1-\theta)\bar{\epsilon}^p}{(1-\theta)\bar{\epsilon}^p(1+\theta\epsilon^r\hat{w}_t/c_t^r)+(1-\bar{\epsilon}^p)}\right). \tag{77}$$

The above is a measure of the cost of the additional distortions following from the fall in the tax revenue due to a higher net wage, $\hat{w}$. Finally, by the above value of μ and the assumption $\eta = 0$, the optimal user costs of housing are

$$\hat{\pi}_t^i = \pi_t(1+t_t^i),$$

where

$$t_t^r = \frac{\epsilon^r \hat{w}_t}{c_t^r - (1-\theta)\epsilon_t^r \hat{w}_t}, \qquad t_t^p = -\frac{(1-\bar{\epsilon}^p)}{(1-\bar{\epsilon}^p)+(1-\theta)\bar{\epsilon}^p(1+\theta\epsilon^r\hat{w}/c^r)}. \tag{78}$$

Now consider a steady state, and denote aggregate net wealth as

$$v = k + qh + b^g/(1+r)$$

Then, recalling the asset market equilibrium condition (26), the steady state consumptions of the rich and poor households are

$$c^r = (1-\theta)(\epsilon^r\hat{w} + rv/m_r). \tag{79}$$

The above implies that

$$\frac{\epsilon^r \hat{w}}{c^r} = \frac{(1-\bar{\epsilon}^p)\hat{w}/rv}{(1-\theta)((1-\bar{\epsilon}^p)\hat{w}/rv+1)}$$

and, then, using the above in (78), we obtain

$$t_t^r = \left(\frac{1-\bar{\epsilon}^p}{1-\theta}\right)\frac{\hat{w}}{rv}, \qquad t_t^p = -\frac{(1-\bar{\epsilon}^p)}{1-\frac{\theta\bar{\epsilon}^p}{1+(1-\bar{\epsilon}^p)\hat{w}/rv}}. \tag{80}$$

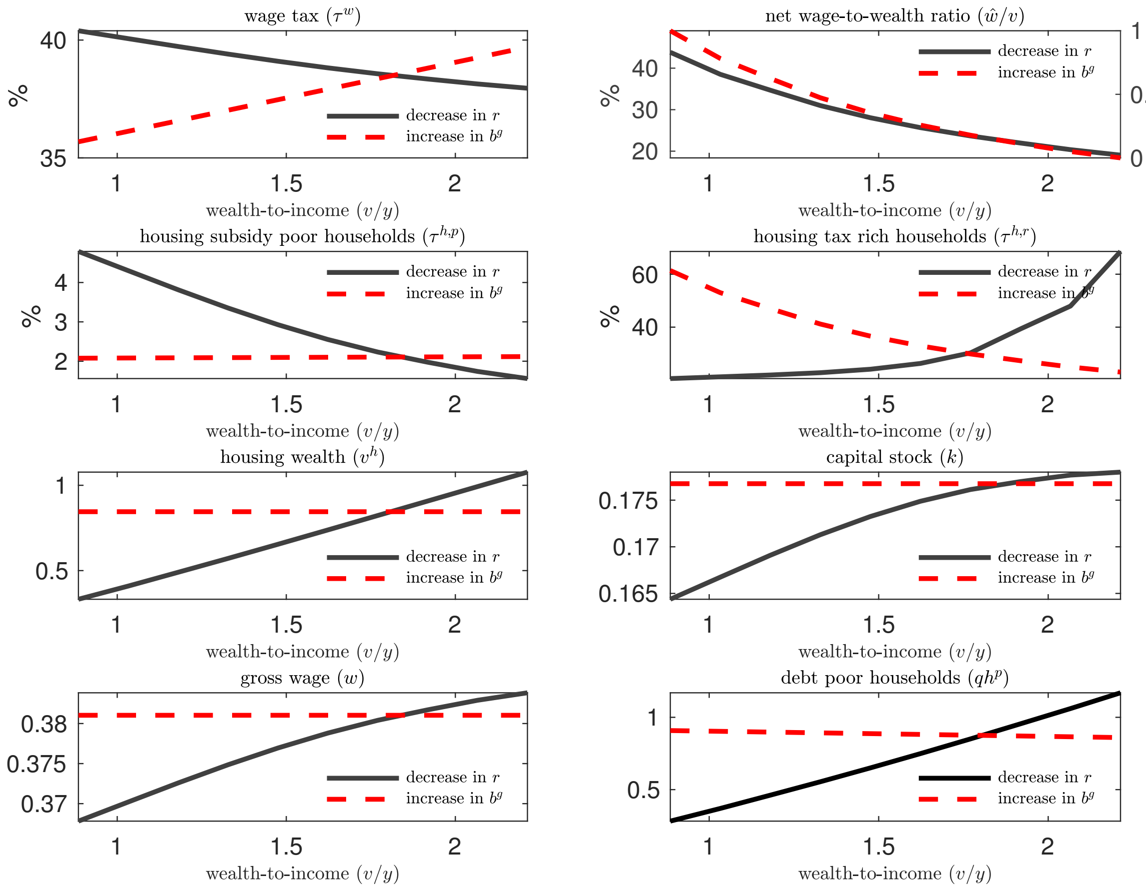
Therefore, the implicit taxation of housing for rich households is decreasing in the net wage-to-wealth ratio, $\hat{w}/v$, and the implicit subsidy on housing for the poor households is increasing in the same ratio. This result follows from the fact that a higher net wage-to-wealth ratio makes the rich households' housing demand more elastic, *i.e.*, it lowers the spread between the general equilibrium elasticities, $g_1^i - g_2^i$.

We now consider the quantitative results of the optimal taxation problem. We consider two scenarios that generate a path of increasing wealth inequality. In the first, we generate different levels of the wealth-to-income ratio by exogenously changing the level of government debt (b^g). In the second, we generate different levels of the wealth-to-income ratio by changing the level of the real interest rate. Specifically, we pick the level of interest rates to match the exogenous wealth ratios obtained under the fiscal scenario. Intuitively, wealth is decreasing in the level of the real interest rate. We solve for the steady state of the model by solving the system described in detail in **??** and setting $\tau^k = 0$. The model and the parameters are the same as those presented in section 2.3. Figure 1 plots the steady state values for the wage tax (τ^w); the net wage-to-wealth ratio ($\hat{w}/v$); the housing subsidy to poor households ($\tau^{h,p}$); the housing tax on rich households ($\tau^{h,r}$); the housing wealth (qh); the capital stock (k); the gross wage (w); the debt of poor households (qh^p); for different levels of the wealth-to-income ratio (v/y). We denote with a red dashed-line the first scenario, in which a higher wealth ratio is associated to higher public debt, and with the black solid line the second scenario, in which a higher wealth ratio is associated to a lower real interest rate. We summarize our results as follows. First, when the wealth ratio increases, the wage tax increases from approximately 36% to 40% under the first scenario, while it decreases by approximately 40% to 38% under the second scenario. Second, under both scenarios, wealth inequality, proxied by the net wage-to-wealth ratio, decreases with the level of the wealth ratio. Third, when the wealth ratio increases, the housing subsidy to poor households is approximately unchanged and equal to 2% under the first scenario, while it decreases from 5% to 1% under the second scenario. Fourth, when the wealth ratio increases, the housing tax on rich households decreases from 60% to 20% in the first scenario, while it increases from 20% to 60% in the second scenario. Fifth, the housing wealth and capital stock are approximately unchanged under the first scenario, while they are increasing in wealth, and decreasing in the real interest rate, in the second scenario. Note that the second scenario is "less neutral" relative to the first scenario, but more appealing, in terms of some important stylized facts that have characterized the experience of most advanced economies in the past thirty years. In particular, the rise in aggregate wealth went along with rising housing prices and falling real interest rates (Bonnet et al., 2019), and the latter may have been responsible for the increase in the value of housing property through a rise in the demand of housing (due to the fall in the cost of housing services) and a rise in the demand of housing mortgages (La Cava, 2016).

4 Conclusions

We have studied optimal tax rates when households accumulate different levels of wealth; the latter consisting in business capital, housing, and financial assets. The Chamley-Judd's zero steady state tax on financial and business capital survives, whereas housing wealth is taxed at a non zero rate. In particular, we identify a set of conditions

Figure 1: Steady State: Main Variables



Notes: This figure plots the steady state values for the wage tax (τ^w); the net wage-to-wealth ratio ($\hat{w}/v$); the housing subsidy to poor households ($\tau^{h,p}$); the housing tax on rich households ($\tau^{h,r}$); the housing wealth ($v^h = qh$); the capital stock (k); the gross wage (w); the debt of poor households (qh^p); for different values of the total wealth-to-income ratio (v/y). The wage and housing tax rates, the housing subsidy, and the net wage-to-wealth ratio, are reported in percentage. We consider two scenarios that generate a path of increasing wealth inequality. In the first (dashed-red line), we exogenously change the level of govern debt (b^g). In the second (solid black line), we change the level of the real interest rate. Parameters are from Table 2. Refer to Appendix A for details on the numerical solution of the model.

under which it is optimal to impose a positive tax on rich households' housing wealth, and a subsidy on the user cost of housing (or rent) faced by poor households. Finally, using again Cobb-Douglas preferences and technology, we evaluate numerically the impact on optimal tax rates of a rising aggregate wealth-to-income ratio. We find that the mechanism used to generate different wealth levels matters for the optimal tax rates. While the results of a positive housing subsidy on poor households and housing tax on rich households is very robust, the evolution of the other optimal tax rates changes depending on the source of the increasing wealth.

Our results depend on two assumptions that can be relaxed in future research. First, we assume that income taxes cannot be contingent on types or increase progressively with households' income. Although this is possibly an interesting extension, restricting the attention to flat tax rates on labor income and types of wealth (or sources of capital income) has the advantage of providing results that are more comparable with the existing literature on optimal taxation and to generate simpler (and more easily implementable) policy proposals. For example, it could be difficult to implement income tax rates contingent on households' wealth size and composition. Second, we have excluded direct taxation (or subsidization) of imputed rents, but including this instrument has no consequences on our results. Note that, since we concentrate on steady state equilibria, any distinction between housing wealth taxes and indirect taxes on housing services is somewhat artificial, and the user cost of housing (a proxy for imputed rents) is proportional to the value of housing property. More generally, it is often noted that a tax on imputed rental income could be approximated through an annual recurrent tax on property since imputed rents are typically proportional to property values. However, taxing imputed rents for owner-occupied housing is difficult in practice, and in fact it is rarely implemented, as it involves some practical difficulties such as properly evaluating depreciation and capital gains[5]. The Mirrlees' Review suggests that a tax related to the consumption value of a property bears some resemblance with the British *council tax*, which is essentially a locally collected property tax based on a limited set of brackets (*bands*) for the property values (Mirrlees et al., 2011). Similar tax systems for housing wealth are applied in almost all advanced economies. The Mirrlees' Review also claims that the council tax is generally regressive relative to its base and should be replaced by a *housing service tax*, *i.e.*, a flat percentage of

[5] In a recent report, Fatica and Prammer (2017) claim that, "while imputed rents are generally not taxed, all the euro area countries in the HFCS survey - except Malta - levy recurrent taxes on real estate property".

the rental value of property, whether it is rented or owner-occupied. However, our findings suggest that, with an inequality averse Planner, this tax should not be flat, but contingent on the size of individuals' net wealth.

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Appendix

A Calibration

All the parameters used in the quantitative analysis are reported in Table 2. We set the baseline consumption preference parameter $\theta = 0.2$ in order to match the U.S. households expenditure on housing services (approximately 15% of 2015 GDP according to the BEA NIPA Table 2.3.5). The time discount parameter of patient households (*i.e.*, rich households) is set to $\beta^H = 0.98$, implying a steady state real interest rate of 2%. Impatient households (*i.e.*, poor households) have a lower value for the time discount parameter, which we set to $\beta^L = 0.95$. The annual depreciation of the housing stock is set equal to $\delta = 4\%$ as in Iacoviello and Neri (2010). We use O'Mahony and Timmer (2009)'s KLEMS data to have rough estimates of the capital factor shares in construction and manufacturing in the US over the 1970–2010 period and, accordingly, set $\alpha_k^m = 1/3$ and $\alpha_k^h = 1/5$. These numbers are in line with those in Valentinyi and Herrendorf (2008) who set the capital share in manufacturing and construction respectively to 0.4 and 0.2. We set the weight attached to the land input to $\alpha_x^h = 1/10$, which is in line with the value used by Davis and Heathcote (2005). Finally, we set the government expenditure g to 0.15 to match the U.S. Federal expenditure as fraction of GDP; and the share of patient (*i.e.*, rich) households to 10%, and we consider a constant flow of new land permits. Robustness of our results with respect to changes in these parameter values is verified in the online appendix to this paper[6].

B Proofs

Proof of proposition 1.

To characterize the households' optimal choices, it is convenient to start by solving an *auxiliary problem*, corresponding to the problem faced by a household that is "forced" to buy an amount of housing at least as large as the minimum amount $\bar{z}$ at all $t \geq 0$. Whether this solution is optimal will be verified *ex-post* by confronting it with the solution of the problem faced by the same household when setting $z_t^o = h_t = 0$ at all $t \geq 0$. The solution to the *auxiliary problem* faced by household i follows from the

[6] We do not directly calibrate the housing wealth as a fraction of total wealth. In the simulations, this share is approximately equal to 80% and higher then in the data. For example, Iacoviello (2010) reports a value of approximately 50% for the U.S., where a large fraction of housing wealth (80 percent) is made up by the stock of owner-occupied homes).

Table 2: Model Parameters

Preferences		
consumption expenditure share (baseline):	$1-\theta$	0.80
housing expenditure share (baseline):	θ	0.20
discount rate rich households:	β^H	0.98
discount rate poor households:	β^L	0.95
Technology		
Housing depreciation:	δ	0.04
capital share manufacturing:	α_k^m	0.33
capital share construction:	α_k^h	0.10
housing share construction:	α_x^h	0.10
Economy structure		
Government expenditure:	g	0.15
Share rich households:	m_r	0.10
Share poor households:	m_p	0.90

Notes: This table reports all the parameters used to simulate the model. The model is simulated for different values of total wealth under two scenarios. In the first, which we label "fiscal contraction" scenario, we exogenously generate an increasing level of wealth by increasing the level of government debt (b^g) while keeping all the other parameters unchanged. In the second, which we label "preference shock" scenario, we endogenously generate an increasing level of wealth by reducing the level of the real interest rate (r). To guarantee comparability between the two scenarios, we pick values for the real interest rates to match the exogenous wealth levels obtained under the "fiscal contraction" scenario. The utility function u is Cobb-Douglas and described in equation (31). The production functions are Cobb-Douglas.

maximization of the utility function, (1), subject to the the budget constraints (13), the debt limits (15) and

$$z_t^{r,i} \geq 0, \tag{A1}$$

$$z_t^{o,i} \geq \bar{z}, \tag{A2}$$

$$h_t \geq z_t^{o,i}. \tag{A3}$$

Using a standard Lagrange method, we let $\{\eta_t^i, \mu_t^i, \xi_t^i, \lambda_t^i\}_{t=0}^{\infty}$ be a sequence of (non-negative) Lagrange multipliers related to the constraints, (15), (A1), (10), (A3), respectively, and, assuming strict positivity of (c_t^i, z_t^i, l_t^i) for all $t \geq 0$, we state the home-owners's first order conditions as

$$U_1(c_t^i, z_t^i, l_t^i)/\hat{R}_{t+1}^i - \beta_i U_1(c_{t+1}^i, z_{t+1}^i, l_{t+1}^i) = \eta_t^i, \tag{A4}$$

$$U_1(c_t^i, z_t^i, l_t^i)(\hat{\pi}_t^i - \hat{s}_t^i) = \lambda_t^i, \tag{A5}$$

$$U_1(c_t^i, z_t^i, l_t^i)s_t - U_2(c_t^i, z_t^i, l_t^i) = \mu_t^i, \tag{A6}$$

$$U_1(c_t^i, z_t^i, l_t^i)\hat{\pi}_t^i - U_2(c_t^i, z_t^i, l_t^i) = \xi_t^i, \tag{A7}$$

together with the complementary slackness conditions

$$\eta_t^i a_{t+1}^i = \lambda_t^i (h_t^i - z_t^{o,i}) = \mu_t^i z_t^{r,i} = \xi_t^i (z_t^{o,i} - \bar{z}) = 0 \tag{A8}$$

and the transversality condition

$$\lim_{t\to\infty} \beta_i^t U_1(c_t^i, z_t^i) a_{t+1}^i = 0. \tag{A9}$$

If, on the other hand, the household is forced to be a renter, the first order conditions are only those specified in equations (A4), (A6), whereas the values of z_t^o and h_t are zero at all $t \geq 0$ and the transversality condition remains as specified in (A9). It is clear that, a necessary condition for having an equilibrium with home-owners is that conditions (A5) and (A7) are verified for some i. Now note that conditions (A5) and (A8) imply the right hand side inequality in (22) and condition (23) for all $t \geq 0$. Using the transversality condition (A9), we can derive the life-time present value representation of the individuals budget constraints (24). By (A4) and the definition of present value prices, we get (19). Finally, by the form of the budget constraints (24), it is clear that a necessary condition for the existence of some home-owners is the left hand side inequality in (23) for all $t \geq 0$. In fact, suppose that $\hat{\pi}_t^i > s_t$ for all $t \geq 0$. Then, a home-owner can buy the same amount of consumption of manufacturing goods, housing services and leisure at the given prices by spending strictly less than her initial wealth if she was becoming a renter, and this is incompatible with optimality. It follows that (23) must hold in any equilibrium with some home-owner.

Proof of proposition 2.

Since workers are hand-to-mouth, $U_1^i c^i + U_2^i z^i = U_1^i \epsilon^i \hat{w}$, and, then, we have

$$g_1^i = \frac{z^i}{(U_1^i)^2}\left(U_{2,1}^i U_1^i - U_{1,1}^i U_2^i\right) = z^i \frac{\partial}{\partial c^i}\left(\frac{U_2^i}{U_1^i}\right),$$

which, by (strict) concavity, is a (strictly) positive value. Now notice that

$$g_1^i - g_2^i = \left(\frac{U_{1,2}^i}{U_2^i} - \frac{U_{1,1}^i}{U_1^i}\right)(\epsilon^i \hat{w} - c^i) + \left(\frac{U_{1,2}^i}{U_1^i} - \frac{U_{2,2}^i}{U_2^i}\right) z^i$$

$$= \left(\frac{U_{1,2}^i}{U_2^i} - \frac{U_{1,1}^i}{U_1^i}\right)\frac{U_2^i z^i}{U_1^i} + \left(\frac{U_{1,2}^i}{U_1^i} - \frac{U_{2,2}^i}{U_2^i}\right) z^i$$

$$= \frac{z^i}{(U_1^i)^2 U_2^i} \left(2U_1^i U_2^i U_{1,2}^i - U_{1,1}^i (U_2^i)^2 - U_{2,2}^i (U_1^i)^2 \right).$$

By (strict) concavity the above is (strictly) positive.