DISCUSSION PAPER SERIES

DP14120
(v. 2)

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Gaetano Gaballo and Ryan Chahrour

MONETARY ECONOMICS AND FLUCTUATIONS

ISSN 0265-8003

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Discussion Paper DP14120
First Published 13 November 2019
This Revision 14 November 2019

Centre for Economic Policy Research
33 Great Sutton Street, London EC1V 0DX, UK
Tel: +44 (0)20 7183 8801
www.cepr.org

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JEL Classification: D82, D83, E03

Keywords: Demand Shocks, House Prices, imperfect information, animal spirits

Gaetano Gaballo - gaballo@hec.fr
HEC Paris and CEPR

Ryan Chahrour - ryan.chahrour@bc.edu
Boston College

Learning from House Prices: Amplification and Business Fluctuations*

Ryan Chahrour[†] Gaetano Gaballo[‡]

Boston College HEC Paris and CEPR

November 14, 2019

Abstract

We provide a new theory of demand-driven business cycles based on learning from prices in an otherwise frictionless real model. In our model, house price increases caused by aggregate disturbances may be misinterpreted as a signal of improved local consumption prospects, leading households to demand more current consumption and housing. Higher demand reinforces the initial price increase in an amplification loop that drives comovement in output, labor, residential investment, and house prices even in response to aggregate supply shocks. The model's qualitative implications are consistent with observed business cycles, and it can explain apparently autonomous changes in sentiment without resorting to non-fundamental shocks.

Keywords: demand shocks, house prices, imperfect information, animal spirits.

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*The authors thank Jess Benhabib, Mehmet Ekmekci, Roger Guesnerie, Christian Hellwig, Peter Ireland, Jianjun Miao, Patrick Pintus, Kristoffer Nimark, Richard Tresch, Robert Ulbricht, Rosen Valchev, Laura Veldkamp, Venky Venkateswaran, Xavier Vives, and seminar participants at the Toulouse School of Economics, Cornell University, Barcelona GSE Summer Forum, Society for Economic Dynamics, and Boston Green Line Macro conferences for valuable suggestions and comments. Laura Veronika Gáti provided excellent research assistance. The research leading to these results has received financial support from the European Research Council under the European Community's Seventh Framework Program FP7/2007-2013 grant agreement No.263790. An earlier version of this paper circulated under the title "On the Nature and Stability of Sentiments".

[†]Department of Economics, Boston College, Chestnut Hill, MA 02467, U.S.A. Email: ryan.chahrour@bc.edu

[‡]Department of Economic Sciences, HEC Paris, 1 rue de la Liberation, 78350 Jouy en Josas. Email: gaballo@hec.fr

"*Housing* is *the Business Cycle.*"

Edward E. Leamer,
Jackson Hole Symposium, 2007.

1 Introduction

House prices provide valuable information about ongoing changes in economic activity, both at the aggregate and regional levels.[1] Over the last half century in the US, real house prices and output have moved together at least half of the time (Figure 1). However, people likely have very different real-time information about these variables. Precise information about *local* house prices is readily available and relevant to individual choices, while the earliest measures of GDP are imprecise, released with delay, and may be less relevant. For these reasons, people confronted with rising house prices may rationally raise their expectations about their economic prospects. Because of this learning channel, any factor driving house price movements may also drive waves of economic optimism or pessimism.

This paper proposes a new model of housing's *informational* role in generating and amplifying demand-driven business fluctuations. The essence of the model is a price-optimism feedback channel: higher house prices beget economic optimism, which begets even higher house prices, and so on. Since house prices reflect all economic developments, any aggregate shock can activate this loop, potentially driving comovement even in response to supply shocks. In this way, our learning channel offers a new source of amplification for fundamental shocks and breaks the strict dichotomy between disturbances to supply and demand.

We embed our learning mechanism within a neoclassical model with housing. Households are located on islands and consume an aggregate consumption good and local housing. Traded consumption is produced using labor from all islands, while local housing is produced using local land and labor, and a traded productive factor (commodity good) whose supply is fixed. Local house prices can move either because of an increase in the future product of local labor, or because of a current aggregate disturbance to housing production.

Most fluctuations in local house prices are driven by local labor productivity, so people

[1] Leamer (2007) and Leamer (2015) make the point forcefully for aggregates, while Campbell and Cocco (2007) and Miller et al. (2011) provide evidence at the regional level.

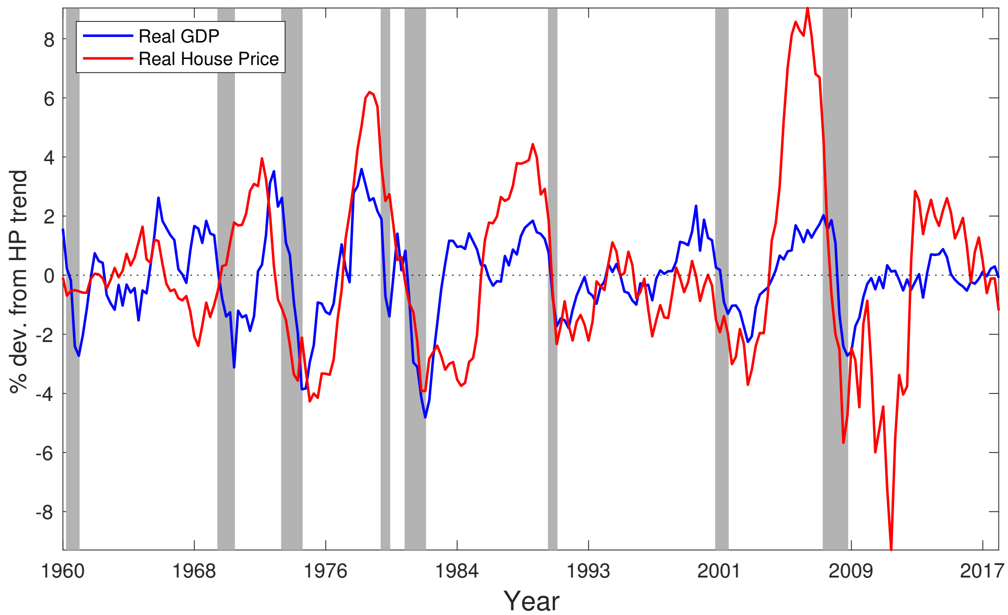


Figure 1: Real gross domestic product and the Shiller national house price index.

observing high house prices become optimistic about their own labor income prospects. However, a fall in the productivity (or availability) of the common productive factor can also drive an increase in house prices across islands. In this case, the increase in house prices is interpreted by households as a positive signal about future wages, increasing demand for both consumption and housing on all islands. Higher aggregate demand further increases house prices, and consequently the price of the common factor, reinforcing the initial price increase. In equilibrium, what started as a (possibly small) change in housing supply leads to an economy-wide increase in house prices, a boom in aggregate demand for both consumption and housing, and a spike in the price of the traded input factor.

Our model of learning from prices has several features that make it an appealing model of the business cycle. First, we embed our mechanism in a flexible price model with competitive markets. This means that fluctuations in housing demand, and their real effects, are not driven by competitive or nominal frictions, or by suboptimal monetary policy. Indeed, our real economy can be interpreted as a monetary economy with a fixed nominal price level. Hence, our model aligns with recent experience in developed economies, where real fluctuations have coincided with small, largely acyclical fluctuations in inflation (e.g. Angeletos et al., 2016).

Second, the signal structure faced by households is fully microfounded. All shocks are fundamental and we explicitly derive people's price signal as the outcome of competitive markets. In our context, aggregate fundamental shocks appear as aggregate noise in people's inference. Thus, the model explains how people's beliefs become coordinated rather than assuming coordination, as in the literature on sunspots (e.g. Cass and Shell, 1983).

Finally, the logic of our model extends to any local price and to other sorts of macroeconomic fundamentals. Hence, the mechanism we propose can be a general source of business cycle comovement, not just in response to a single shock. We show this generality by introducing a shock to consumption productivity, but also refer the reader to earlier drafts of this paper that demonstrate how the mechanism works for local consumption prices, and for shocks to the nominal money supply.[2]

The microfoundation of our signal structure as a *price* is crucial to our mechanism for two reasons. First, the fact that information comes from market prices, rather than from exogenously-specified signals, means that higher house prices can spur demand for both consumption and housing in our model. Hence, our learning channel can lead housing demand to be upward sloping, causing prices and quantities to comove.

Second, the feedback of the global factor price into local house prices allows the model to deliver strong amplification. For some calibrations, amplification can be so strong that aggregate prices and quantities exhibit sizable fluctuations in the limit of arbitrarily small aggregate shocks. To an econometrician, the fluctuations emerging at the limit of no aggregate shocks would appear to be driven by something akin to "animal spirits" (Shiller, 2007), "noise" (Gazzani, 2019), or "sentiment" (Angeletos and La'O, 2013; Benhabib et al., 2015).

After characterizing equilibrium, we examine the qualitative features of the economy. We show that the model implies positive comovement between output, employment, hours in both the consumption and housing sector, house prices, and land prices *for any calibration and any equilibrium* so long as aggregate shocks are small enough. Hence, the model provides a foundation for macroeconomic comovement across a wide range of variables.

We then enrich the model so that a portion of housing productivity is common knowledge. This allows the model to exhibit typical "supply-side" comovement in response to the common

[2]In Chahrour and Gaballo (2017) the leading price is the one of local consumption. There, we show that total factor productivity shocks can drive the business cycle and still be weakly correlated with business cycle variables, as is true in the data (Angeletos et al., 2014, 2016).

knowledge portion of the shock, but to continue to exhibit more "demand-like" fluctuations in response to the surprise component that blurs households' inference.

We show that a plausible calibration of the extended model delivers qualitatively realistic (i.e. positive but imperfect) correlations between real variables in the economy. Moreover, even when the model has a unique equilibrium, it delivers substantial amplification of housing market fluctuations and substantial fluctuations in consumption, which would disappear under full information. Indeed, amplification is strong enough that demand fluctuations dominate unconditional comovements even when the majority of productivity shocks are anticipated.

We augment our discussion of real comovements with some non-structural evidence favoring house prices as the source of people's economic learning. For this, we use Michigan Survey of Consumer Expectations data to show that people's past house price experiences are a far better predictor of their forecasts of their own income than are people's reports about aggregate economic news that they have heard. Moreover, house price experiences modestly lead income expectations, a timing that is consistent with information flowing from house prices to income expectations. While this evidence is only suggestive, we think it indicates that our model can help guide more structural interpretations of expectations survey data.

We conclude the analysis of the paper with several extensions that indicate the robustness of the mechanism. Among these, we show that aggregate consumption productivity shocks are isomorphic to exogenous housing demand shocks. When they are sufficiently small, consumption productivity delivers the same comovements as our baseline model.

Literature review

The expectation channel in our economy contrasts with how expectations shocks drive demand-like fluctuations in New-Keynesian models such as Lorenzoni (2009). Absent nominal rigidity, these models imply that optimism about the future drives increases in the real interest rate rather than in consumption (see Angeletos, 2018, for a nice discussion.) In our model, news about *local* productivity can affect local consumption because it cannot be offset by the *aggregate* real interest rate. Still, our learning mechanism is essential for generating aggregate demand fluctuations, as it serves to correlate forecasts of consumption across islands.

This paper is the first to demonstrate that rational learning from prices might play a central role in explaining business cycle comovements. Nevertheless, endogenous signal structures have appeared in macroeconomic contexts, starting with Lucas (1972). More recent examples include Amador and Weill (2010), Venkateswaran (2013), Benhima and Blengini (2017) and Benhima (2018). Most recently, Gaballo (2018) presents a learning-from-prices mechanism that can explain aggregate price rigidity in an otherwise frictionless monetary model, while Angeletos and Lian (2019) present a flexible price model where noisy observations of the intertemporal price lead discount rate shocks to drive real variables. Eusepi and Preston (2011) also show that adaptive learning can generate realistic business cycle comovements.

The informational role of prices has been studied in the finance literature since Grossman and Stiglitz (1976, 1980). Papers have shown that it can deliver price amplification or multiple equilibria, including Burguet and Vives (2000), Barlevy and Veronesi (2000), Albagli et al. (2014), Manzano and Vives (2011), and Vives (2014).[3] These papers usually include noise traders or exogenous shocks to information. By contrast, our model contains only fundamental shocks and we are the first to show extreme amplification in limit cases.

Our theory is consistent with a range of empirical evidence on housing and the business cycle. Early models of the housing market (e.g. Davis and Heathcote, 2005) struggle to explain price-quantity comovement and often introduce exogenous housing demand shocks to match these moments (e.g. Iacoviello and Neri, 2010). Though our model is similar to Davis and Heathcote (2005) and is also driven by housing productivity, our learning channel allows prices and quantities to positively comove.[4] Our model also qualitatively accounts for the high volatility of the price of land (Davis and Heathcote, 2007) and for its strong comovement with labor markets (Liu et al., 2016). Because of the demand-like effects of productivity, our model challenges conventional restrictions use to identify supply and demand shocks.

Our paper also contributes to a longstanding debate about the nature and size of house price wealth effects. Frictionless models typically imply that house prices should have no causal impact on consumption (e.g. Buiter, 2010) but empirical studies often suggest otherwise. For example, Muellbauer and Murphy (1990) argue that the spike in UK consumption

[3]The literature on price revelation in auction markets following Milgrom (1981) also features a dual informational/allocative role for prices. For recent examples, see Rostek and Weretka (2012); Lauermann et al. (2012); Atakan and Ekmekci (2014).

[4]Recently, Nguyen (2018) and Fehrle (2019) have also proposed particular types of segmentation in capital markets as solutions to these comovement challenges.

in the late 1980's was driven directly by rising house prices, while King (1990), Pagano (1990), and Attanasio and Weber (1994) argue consumption and house prices reflected people's changing perceptions of permanent income. In our model, these competing views coexist: high house prices drive increased consumption not because consumers expect to sell their houses at the high price, but because consumers interpret them as signaling higher permanent income.

Evidence from disaggregated data is also largely consistent with our theory. For example, Miller et al. (2011) find a positive effect of local house prices on local per capita growth in US metropolitan-level data. Campbell and Cocco (2007) find that a 1% increase in the value an individual's house is associated with a 1.22% increase in their real non-durable consumption in the UK. The recent studies by Mian et al. (2013) and Mian and Sufi (2014) also present regional evidence that falling house prices during the Great Recession are associated with consumption reductions at the ZIP code level.

Other theoretical mechanisms for a direct consumption effect of house prices have been proposed in the literature, including borrowing constraints (Iacoviello, 2005) and wealth heterogeneity with incomplete markets (Berger et al., 2017; Kaplan et al., 2017). The learning channel we formalize here offers a complimentary explanation. One difference is that our channel does not depend on actual new house sales or credit contracts, which might imply a longer delay between house prices and their effects on consumption.

2 A Housing Model with Learning from Prices

In this section, we present a simple real business cycle model with housing. We aim as much as possible to provide analytical results and make simplifying assumptions to this end. Most of these assumptions can be relaxed; we discuss when and how as we proceed.

2.1 Preferences and technology

The economy consists of a continuum of islands, indexed by $i \in (0, 1)$. Each island is inhabited by a continuum of price-taking households who consume local housing and a traded numeraire consumption good. Households provide local labor which is used in the production of both goods. On each island, a mass of competitive construction firms combine local labor and land with a traded commodity good to construct new houses, while an aggregate consumption sector combines all islands' labor to produce the traded consumption good.

Households

The representative household on island i chooses consumption, labor supply, and savings in a risk-free nominal bond to maximize the utility function:

$$U_{i0} \equiv \sum_{t=0}^{\infty} \beta^t \left\{ \log(C_{it}^{\phi} \mathcal{H}_{it}^{1-\phi}) - vN_{it} \right\}. \tag{1}$$

In the above utility function, C_{it} denotes household i's consumption of the tradable consumption good, $\mathcal{H}_{it}$ measures the total quantity of housing consumed, and N_{it} is the household's supply of labor. The household discount factor is $\beta \in (0,1)$, the share of housing in the consumption basket is $\phi \in (0,1)$, and v parameterizes the household's disutility of labor.[5]

We assume that housing consumption is composed of a sequence of housing vintages, $\Delta_{i\tau|k}$, constructed at time k and combined according to the Cobb-Douglas aggregator

$$\mathcal{H}_{it} \equiv \prod_{k=-\infty}^{t} \Delta_{it|k}^{(1-\psi)\psi^{t-k}}, \tag{2}$$

where $\psi \in (0,1)$. This formulation for housing utility adds a realistic dimension to the model, since housing vintages can have very different characteristics and are not perfect substitutes. More importantly for our purposes, however, this formulation in conjunction with log-utility implies that every housing vintage has an additive-separable impact on intertemporal utility, allowing us to analyze the dynamic model in closed form.

Each vintage of housing depreciates at a constant rate $d \in (0,1)$, so that

$$\Delta_{i\tau+1|k} = (1-d)\Delta_{i\tau|k}$$

for $\tau \geq k$ (while, of course, $\Delta_{i\tau|k} = 0$ for $\tau < k$). The aggregate housing stock, defined as $H_{it} = \sum_{k=-\infty}^{t} \Delta_{it|k}$, then evolves according to a standard evolution equation,

$$H_{it} = \Delta_{it|t} + (1-d)\, H_{it-1}.$$

Housing consumption can now be written $\mathcal{H}_{it} = \Delta_{it|t}^{1-\psi}(1-d)\mathcal{H}_{it-1}^{\psi}$, which we use going forward.

[5] We allow for convex disutility of labor in the Appendix.

The choices of the household are subject to the following budget constraint,

$$\mathcal{B}_{it} \equiv \frac{B_{it}}{R_t} + C_{it} + P_{it}\Delta_{it|t} - W_{it}N_{it} - B_{it-1} - \Pi_t^c - \Pi_{it}^h \leq 0 \tag{3}$$

for $t \in \{0, 1, 2...\}$ with $B_{i-1} = 0$. Household resources come from providing local labor at wage W_{it}, from past bond holdings, from profits Π_{it}^h of locally-owned housing firms, and from profits Π_t^c of homogeneous consumption firm, whose ownership is evenly held across islands. The household uses its funds to purchase numeraire consumption, to acquire new housing at price P_{it}, and to save in a zero-net-supply aggregate bond with a real risk-free return R_t.

We denote the price of the local housing vintages as $P_{it|k}$ and define the price of the total housing stock as $P_{it}^H = \sum_{k=-\infty}^{t} P_{it|k}\Delta_{it|k}/H_{it}$. Notice, however, that only the price of the current vintage, $P_{it} \equiv P_{it|t}$, shows up in the budget constraint in (3). This happens because we have anticipated an implication of market clearing: since the local household is the only potential buyer and seller of past vintages, trade in them can never generate net resources for the island. Hence, housing wealth is not wealth in the sense of Buiter (2010). The literature has proposed several strategies to break this irrelevance (see Introduction). Our goal is to describe a new, possibly complementary, channel through which house prices might have a causal effect on consumption.

Construction Firms

Construction firms produce new houses using a Cobb-Douglas technology,

$$\Delta_{it} = L_{it}^{1-\alpha} X_{it}^{\alpha}, \tag{4}$$

that combines land (L_{it}) with new residential structures (X_{it}) to generate new residential units $\Delta_{it} \equiv \Delta_{it|t}$ with structures share $\alpha \in (0, 1)$. Residential structures, in turn, are produced via a Cobb-Douglas production function

$$X_{it} = (N_{it}^h)^{\gamma}(e^{-\tilde{\zeta}_t} Z_{it})^{1-\gamma} \tag{5}$$

that combines local labor (N_{it}^h) with a traded commodity good (Z_{it}) according to the share parameter $\gamma \in (0, 1)$.

The construction firm maximizes profits,

$$\Pi_{it}^h \equiv P_{it}\Delta_{it} - W_{it}N_{it}^h - Q_t\left(Z_{it} - Z\right) - V_{it}L_{it}$$

subject to (4) and (5). In the above, V_{it} is the local price of land, W_{it} is the price of local labor, and Q_t is the price of the commodity good sold across islands. We assume that housing firms are endowed each period with Z units of the commodity good, which trades freely across islands at a common price and depreciates fully at the end of the period. Land supply is exogenous, as each period a fixed amount of residential land — normalized to one — becomes available to housing producers on the island.[6] Without any loss of generality, we assume that new land is endowed to local firms.

The only aggregate shock affecting our baseline economy is a shock to productivity of the commodity good, $\tilde{\zeta}_t$.[7] This shock evolves according to a random walk, $\tilde{\zeta}_t = \tilde{\zeta}_{t-1} + \zeta_t$, with i.i.d. innovation ζ_t distributed according to $N\left(0, \sigma_\zeta^2\right)$. We focus our presentation on this shock because it has no effect on consumption under full information. Still, other shocks could play a similar role: We consider an extension with an aggregate shock to consumption productivity in Section 5 and show that ζ_t is isomorphic to a shock to the endowment of Z in the Appendix.

Consumption Sector

The numeraire consumption good is traded freely across islands and is produced by a continuum of identical competitive firms. The representative consumption producer combines labor from all sectors to maximizes profits,

$$\Pi_t^c \equiv Y_t - \int W_{it} N_{it}^c di$$

subject to the production function,

$$Y_t = \left(\int e^{\tilde{\mu}_{it}/\eta} {N_{it}^c}^{1-\frac{1}{\eta}} di \right)^{\frac{1}{1-\frac{1}{\eta}}} . \tag{6}$$

The quantity of local labor used is denoted by N_{it}^c, and labor types can be substituted with elasticity $\eta > 0$. Island-specific labor productivity is a random walk, and evolves according to $\tilde{\mu}_{it} = \tilde{\mu}_{it-1} + \hat{\mu}_{it}$, where $\hat{\mu}_{it}$ is i.i.d. and drawn from the normal distribution $N\left(0, \hat{\sigma}_\mu\right)$.

[6] These assumptions do not imply that land supply grows over time. Provided an appropriate transformation of the depreciation rate, this formulation is equivalent to a model in which structures are placed on a fixed stock of land and existing land becomes free as those structures depreciate. See Davis and Heathcote (2005) for details.

[7] Notice that with our sign normalization in (5), a positive $\tilde{\zeta}$ corresponds to lower productivity.

Market Clearing

Clearing in the local land and labor markets requires

$$L_{it} = 1 \quad \text{and} \quad N_{it} = N_{it}^{c} + N_{it}^{h}. \tag{7}$$

Per the discussion above, we omit market clearing conditions for all past housing vintages, since their trade is irrelevant at the island level. Finally, clearing in the aggregate markets for bonds, consumption, and the commodity good requires

$$Y_t = \int C_{it} di, \quad 0 = \int B_{it} di, \quad \text{and} \quad Z = \int Z_{it} di. \tag{8}$$

2.2 Timing and equilibrium

The only friction that we introduce is uncertainty in households' demand. To model this in a parsimonious way, we use the family metaphor also adopted by Angeletos and La'O (2009) and Amador and Weill (2010). The household is composed of two types of agents: a shopper, who uses household resources to buy consumption and housing, and a worker-saver, who chooses the number of hours to supply and the quantity of bonds to buy.

Both family member types act in the interest of the household, but they cannot pool their information within a time period. Hence, choices of Δ_{it} and C_{it} are conditioned on the information set of shoppers, while N_{it} and B_{it} are conditioned on the full information set of workers. Each period is composed of four stages:

1. The household splits into shoppers and worker-savers.

2. Shocks realize, namely future local productivity innovations, $\{\hat{\mu}_{i,t+1}\}_{i\in(0,1)}$, and the current aggregate shock, ζ_t. The "best available" information set, $\Omega_t \equiv \{\{\hat{\mu}_{i,\tau}\}_0^{t+1}, \{\zeta_\tau\}_0^t\}$, is observed by firms and worker-savers, but not shoppers.

3. Production and trade take place. Shoppers and workers make their choices based on the information they have, which includes the competitive equilibrium prices in the markets in which they trade. Firms make production choices in light of realized productivity and input prices; and all markets clear.

4. Family members share information, revealing Ω_t to the shoppers.

Because shoppers do not immediately observe Ω_t, they make choices under uncertainty. However, they do observe the local price of housing in their island, P_{it}, which they use to make inference; shoppers' information set is therefore $\{P_{it}, \Omega_{t-1}\}$.[8] We derive the information about current conditions contained in P_{it} shortly.

The family metaphor is convenient but not essential. What is essential is that some agents have access to information about realized shocks: Prices cannot reveal information unless that information is already available, perhaps noisily, to some agents in the economy (Hellwig, 1980). We could have achieved the same effect by assuming that only a fraction of households on each island are informed in the spirit of Grossman and Stiglitz (1980). Nothing crucial about our results would change if did this, though the algebra is more cumbersome.[9]

The formal definition of equilibrium is the following.

Definition 1 (Equilibrium)**.** *Given initial conditions* $\left\{\{B_{i-1}, \mathcal{H}_{i-1}, \tilde{\mu}_{i0}\}_{i\in(0,1)}, \tilde{\zeta}_{i-1}\right\}$*, a rational expectations equilibrium is a set of prices,* $\{\{P_{it}, V_{it}, W_{it}\}_{i\in(0,1)}, Q_t, R_t\}_{t=0}^{\infty}$*, and quantities,* $\{\{B_{it}, N_{it}^c, N_{it}^h, N_{it}, C_{it}, H_{it}, \Delta_{it}, X_{it}, L_{it}, Z_{it}\}_{i\in(0,1)}, Y_t\}_{t=0}^{\infty}$*, which are contingent on the realization of the stochastic processes* $\{\{\tilde{\mu}_{it}\}_{i\in(0,1)}\}_{t=0}^{\infty}$ *and* $\{\tilde{\zeta}_t\}_{t=0}^{\infty}$*, such that for each* $t \geq 0$ *and* $i \in (0,1)$*:*

(a) Shoppers optimize, i.e. $\{C_{it}, \Delta_{it}\}$ *are solutions to* $\max_{C_{it},\Delta_{it}} E[U_{it}|P_{it}, \Omega_{t-1}]$ *subject to* $E[\mathcal{B}_{it}|P_{it}, \Omega_{t-1}] \leq 0$*;*

(b) Workers optimize, i.e. $\{N_{it}, B_{it}\}$ *are solutions to* $\max_{N_{it},B_{it}} E[U_{it}|\Omega_t]$ *subject to* $\mathcal{B}_{it} \leq 0$*;*

(c) Housing producers optimize, i.e. $\{N_{it}^h, Z_{it}, L_{it}, \Delta_{it}\}$ *are solutions to* $\max_{N_{it}^h,Z_{it},L_{it},\Delta_{it}} \Pi_{it}^h$ *subject to (4) and (5);*

(d) Consumption producers optimize, i.e. $\{N_{it}^c\}_{i\in(0,1)}$ *are solutions to* $\max_{\{N_{it}\}_{i\in(0,1)}} \Pi_t^c$ *subject to (6);*

(e) Markets clear, i.e. equations (7) - (8) hold.

Let Λ_{it}^c be the shopper's Lagrange multiplier associated with her expected budget constraint, and Λ_{it} be the multiplier associated with the problem of the worker. As the worker is perfectly informed, Λ_{it} is the actual shadow value of relaxing the household budget constraint.

[8]Shoppers also observe the price of old vintages for which trade does not occur in equilibrium. Nevertheless, these prices convey no new information to shoppers as these prices are a function of shoppers' local demand. We show this formally in our discussion before Proposition 3.

[9]We followed this track in our working paper, Chahrour and Gaballo (2017). Earlier drafts also showed that our mechanism could arise on the supply side of the economy, more like Lucas (1972).

Therefore, optimality on the side of the shopper requires $\phi C_{it}^{-1} = \Lambda_{it}^{c} = E[\Lambda_{it}|P_{it}]$. In the Appendix, we derive the full set of optimality conditions describing equilibrium.

2.3 Linearized Model

We now derive the model in terms of log-deviations from the steady state of the deterministic economy. Going forward, we refer to the shoppers' information set as p_{it}.

Shoppers demand consumption and housing goods according to the following:

$$c_{it} = -E[\lambda_{it}|p_{it}] \tag{9}$$

$$\delta_{it} = -E[\lambda_{it}|p_{it}] - p_{it} \tag{10}$$

where the operator $E[\cdot|p_{it}]$ represents the expectation of the shopper conditional on the market housing price p_{it} and λ_{it} is the actual marginal value of household i's resources (known by the worker but not by the shopper). In fact, p_{it}, is the only piece of information (jointly with common prior and past shock realizations) that a shopper has to infer the marginal value of consumption. The higher the perceived marginal valuation value of budget resources the lower the demand for consumption and housing by the shopper.

The solution to the worker-shopper problem is given by:

$$w_{it} = -\lambda_{it} \tag{11}$$

$$\lambda_{it} = E[\lambda_{it+1}|\Omega_t] + r_t. \tag{12}$$

The worker provides any quantity of labor demanded, so long as the offered wage equals the household Lagrangian, and purchases bonds until the interest rate reflects the difference between the current and the expected future marginal value of budget resources, which the worker-saver forecasts based on Ω_t, the full current information set.

Housing firm optimality conditions are standard:

$$z_{it} + q_t = p_{it} + \delta_{it}, \tag{13}$$

$$n_{it}^{h} + w_{it} = p_{it} + \delta_{it} \tag{14}$$

$$v_{it} = p_{it} + \delta_{it} \tag{15}$$

with production technology given by

$$\delta_{it} = \alpha\gamma n_{it}^{h} + \alpha(1-\gamma)\left(z_{it} - \tilde{\zeta}_t\right), \tag{16}$$

after imposing the fact that $l_{it} = 0$.

Consumption producer's optimal choices imply:

$$n_{it}^{c} = \tilde{\mu}_{it} - \eta(w_{it} - w_t) + n_t^{c} \tag{17}$$

$$y_t = n_t^{c} \tag{18}$$

$$w_t = 0 \tag{19}$$

where w_t denotes the average log-wage in the economy. Condition (17) captures firms' demand for island-specific labor. Firms demand more of a type of labor whenever its productivity is high or its wage is low compared to the average, or if they demand more labor overall. Nevertheless, the wage for the aggregate labor bundle is invariant as there is no change in aggregate productivity in the consumption sector; we relax this assumption in the Appendix.

All relations above obtain as exact log transformations. Only the island resource constraint needs to be log-linearized as follows:[10]

$$\beta b_{it} + C(c_{it} - c_t) = C(w_{it} - w_t) + C(n_{it}^{c} - n_t^{c}) - Qz_{it} + b_{it-1}. \tag{20}$$

In equation (20), C and Q represent the deterministic steady state values of C and Q used in the linearization. The equation states that higher than average consumption in one island must be financed by a higher local wage, by higher local labor supply, by selling the commodity good (i.e. using less z_{it} in production), or by decreased savings. As noted before, market clearing implies that the existing housing stock cannot be used to raise island level consumption. Market clearing conditions $0 = \int z_{it}di = \int b_{it}di$ and $n = \int n_{it}^{c}di + \int n_{it}^{h}di$ complete the description of equilibrium in the linear economy.

3 Learning from Prices

This section presents the main theoretical results regarding the inference problem of shoppers. We derive the value of household resources as a function of exogenous shocks, characterize

[10] We linearize bond holdings in levels because B_{it} can take negative values.

the shoppers' price signal, and then show the implications for equilibrium inference.

3.1 The marginal value of budget resources

The only friction in the economy is shoppers' uncertainty regarding the marginal value of household budget resources. Without this friction, the model is a standard real business cycle economy. Lemma 1 expresses the value of resources, λ_{it}, as a function of fundamentals.

Lemma 1. *In equilibrium,*

$$\lambda_{it} = E[\lambda_{it+\tau}|\Omega_t] = -\omega_\mu \hat{\mu}_{it+1} - \omega_b b_{it} \qquad \textit{and} \qquad r_t = 0 \tag{21}$$

for any $\tau \geq 0$ *and any* $i \in (0, 1)$*. In addition,* $\omega_\mu > 0$ *and* $\omega_b > 0$*, with* $\lim_{\beta \to 1} \omega_b = 0$*.*

Proof. Proofed in Appendix. □

Intuitively, the intertemporal arbitrage carried out by worker-savers allows them to equalize the marginal value of budget resources across time. One important implication is that the real interest rate does not react to the aggregate productivity shock in housing production. This is again a consequence of the fact that housing wealth cannot be sold across islands.

By contrast, local labor and bonds can be sold across islands in exchange for consumption. Therefore, islands with more productive labor or higher savings have better consumption prospects and a lower marginal value of resources. Thus, Lagrangian multipliers depends on the future labor productivity shock, $\hat{\mu}_{it+1}$, and on bond holdings at the end of the period, b_{it}.

As β tends to unity, however, λ_{it} becomes independent of bond holdings. This happens in this case because bond wealth generates no interest earnings and is rolled over indefinitely. To simplify our exposition going forward, we present analytical derivations in the case of β approaching one so that λ_{it} can be treated as entirely exogenous. However, our propositions are generic to $\beta \in (0, 1)$.

We conclude this section with a remark on the distinction between local and aggregate productivity in the consumption sector. Here, as in the standard real business cycle model, an *aggregate* shock to future productivity in the consumption sector would drive the future marginal value of resources and the real interest rate and in opposite directions, leaving λ_{it} and current consumption unchanged. This is why papers looking for business cycle effects of productivity news typically require nominal frictions along with suboptimal monetary policy, e.g. Lorenzoni (2009) (see also later Section 5.1). In our environment, however, *local* news has an equilibrium effect on λ_{it} because the real interest rate can only neutralize the aggregate component of news. However, transforming the effects of local news into an aggregate demand shock requires a friction, such as the information friction we describe below.

3.2 The local housing price

We now derive the signal that shoppers use to make their inferences about $\hat{\mu}_{it+1}$. To economize notation, we solve for equilibrium assuming that at time t, $\tilde{\mu}_t = \tilde{\zeta}_{t-1} = 0$; since past shocks are common knowledge, nothing in the description of equilibrium changes when we relax this.

Rearranging first order conditions from the housing sector, we recover the standard Cobb-Douglass result that the price is a linear combination of input costs weighted by their elasticity:

$$p_{it} = (1-\alpha)v_{it} + \alpha\gamma w_{it} + \alpha(1-\gamma)(\zeta_t + q_t). \tag{22}$$

We wish to rewrite (22) in terms of the exogenous variables and expectations thereof. As said, to simplify our presentation we consider $\beta \to 1$ so that $\omega_b = 0$, but later state general propositions. We substitute (21) into the local wage in (11) to conclude

$$w_{it} = \omega_\mu \hat{\mu}_{it+1} \equiv \mu_i, \tag{23}$$

where the rescaled local shock μ_i has variance σ_μ^2. Equation (23) says that when workers expect higher future local productivity, they demand higher current wages, which increases the price of new houses. Going forward, we use the definition of μ_i from (23) and drop time subscripts for contemporaneous relations.

Importantly, the price of local land only reflects shoppers' local housing demand, since equations (10), (15) and (21) can be combined to get $v_i = E[\mu_i|p_i]$. Hence, although shoppers' do not observe v_i, they can predict it exactly. By contrast, the price of the traded commodity good varies with the aggregate appetite for housing across islands, since market clearing for the commodity good and (13) together imply

$$q = \int v_i di = \int E[\mu_i|p_i]di. \tag{24}$$

Using (23), (24), and the fact that they infer v_i, shoppers' observation of the house price p_{it}

is informationally equivalent to observing the price signal:

$$s_i \quad = \quad \gamma\mu_i + (1-\gamma)\left(\zeta + \int E[\mu_i|p_i]di\right). \tag{25}$$

In this context, a shoppers' desired response to a price change depends on the reason that the price has changed. Yet, shoppers cannot directly observe why prices have changed. The shopper will attribute at least part of any price increase to higher μ_i, increasing her demand for consumption and housing. Yet, the increase could be driven by aggregate factors, either higher average expectations or a decrease in aggregate productivity, that are not related to the local conditions. Price changes driven by these aggregate factors will therefore trigger a common error, changing demand for consumption and housing on all islands.

3.3 Equilibrium

We now solve the shopper's inference problem. A key feature is that the precision of the price-signal depends on the volatility of the commodity price, itself an equilibrium outcome.

Following the related literature, we focus our analysis on linear equilibria. We therefore conjecture that the optimal individual expectation is linear in s_i and takes the form

$$E[\mu_i|p_i] = as_i = a\left(\gamma\mu_i + (1-\gamma)\left(\int E[\mu_i|p_i]di + \zeta\right)\right). \tag{26}$$

In (26), a measures the weight the shopper places on the her price signal in forming her forecast. Since the signal is *ex ante* identical for all shoppers, each uses a similar strategy. Integrating across the population and solving for the average expectation yields

$$\int E[\mu_i|p_i]di = a\,(1-\gamma)\,(E[\mu_i|p_i]di + \zeta)\,. \tag{27}$$

Equation (27) is useful for summarizing how changes in aggregate expectations are amplified by the endogenous signal structure: as the weight a grows from to zero towards $(1-\gamma)^{-1}$, initial changes in expectations experience increasingly strong amplification. The case where $a = (1-\gamma)^{-1}$ is particularly extreme, as any initial perturbation (i.e. by a non-zero productivity shock ζ) must lead to infinitely large fluctuations in $\int E[\mu_i|p_i]di$.

When a does not equal $(1-\gamma)^{-1}$, equation (27) can be solved for the average expectation,

$$\int E[\mu_i|p_i]di = \frac{a\,(1-\gamma)}{1-a\,(1-\gamma)}\zeta, \tag{28}$$

which is a nonlinear function of the weight a. The fact that the average expectation is normally distributed confirms the conjectured form of the optimal individual forecast.

Using (9) we have that aggregate consumption equals the average forecast, that is $c = \int E[\mu_i|p_i]di$. Hence, as long as the local housing price is informative about μ_i, aggregate consumption moves with aggregate productivity. The variance of consumption is given by

$$\sigma_c^2(a) = \left(\frac{a(1-\gamma)}{1-a(1-\gamma)}\right)^2 \sigma^2,$$

where $\sigma_c^2 \equiv var(\int E[\mu_i|p_i]di)/\sigma_\mu^2$ and $\sigma^2 \equiv \sigma_\zeta^2/\sigma_\mu^2$ are the variances of the average expectation and the aggregate shock after each is normalized by the variance of the idiosyncratic fundamental. Substituting (28) into the price signal described in equation (25), we get an expression for the local signal exclusively in terms of exogenous shocks:

$$s_i(a) = \gamma\mu_i + \frac{1-\gamma}{1-a(1-\gamma)}\zeta, \tag{29}$$

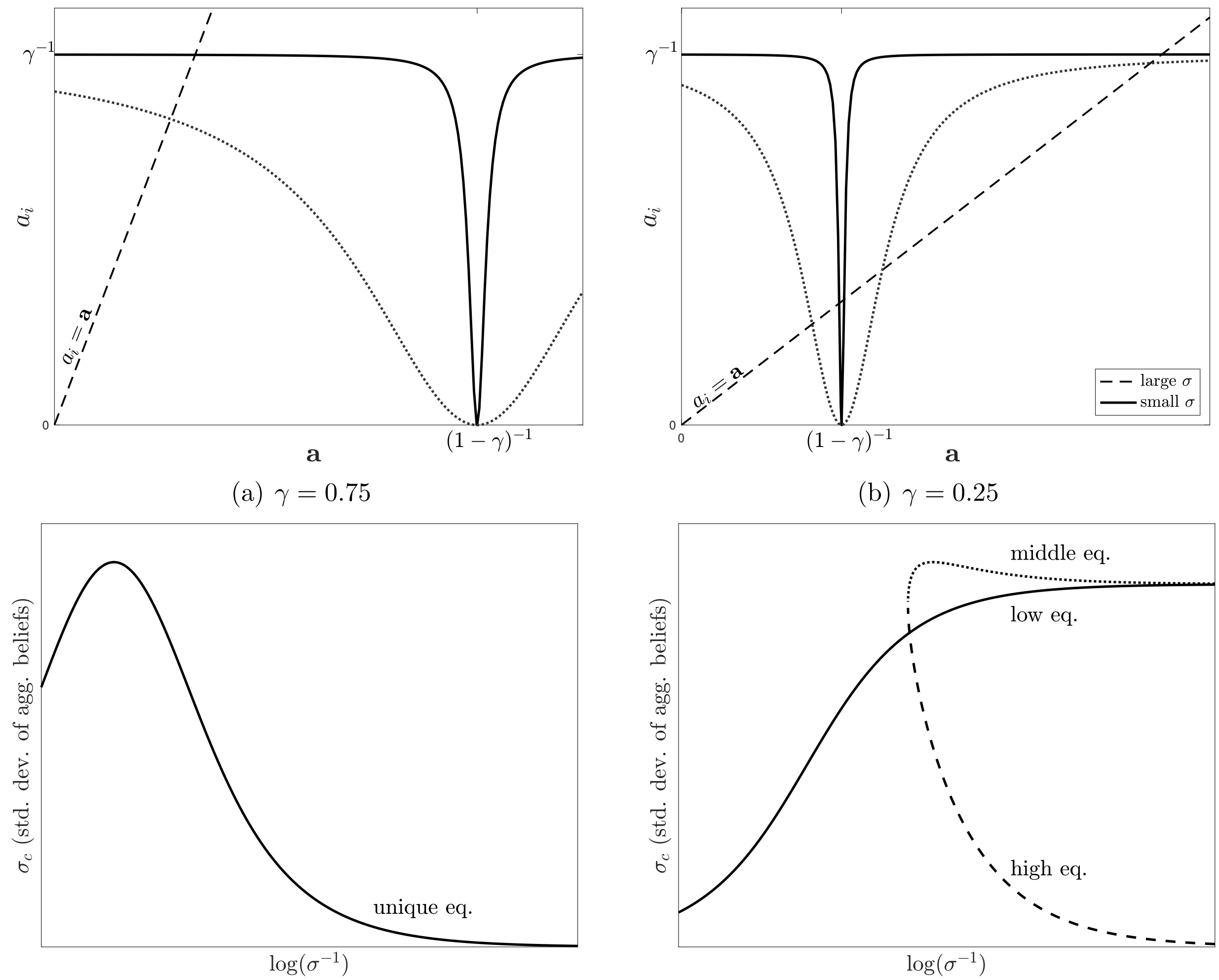
whose precision with regard to μ_i is given by

$$\tau(a) = \left(\frac{\gamma(1-a(1-\gamma))}{(1-\gamma)\sigma}\right)^2. \tag{30}$$

We are now ready to compute the shopper's optimal inference, taking the average weight of other households as given. We seek an a^* such that the covariance between the signal and forecast error is zero in expectation, i.e. $\mathrm{E}[s_i(a)(\mu_i - a^* s_i(a))] = 0$. This condition implies that information is used optimally. The individual best-response weight is thus given by

$$a^*(a) = \frac{1}{\gamma}\left(\frac{\tau(a)}{1+\tau(a)}\right). \tag{31}$$

We can interpret $a^*(a)$ in a game-theoretic fashion as an individual's best reply to the profile of others' actions. An equilibrium of the model is characterized by $\hat{a}$ such that $a^*(\hat{a}) = \hat{a}$, i.e. a fixed point of the individual best-weight mapping. In practice, there are as many equilibria as intersections between $a^*(a)$ and the 45° line. In the two top panels of Figure 2 we plot the best-response weight $a^*(a)$ for $\beta \to 1$ against the 45° line for two different values of σ; we illustrate the case $\gamma > 1/2$ in panel (a) and $\gamma < 1/2$ in panel (b). The Appendix shows that lower β strictly lowers the best-response weight. We now provide existence conditions for these equilibria and provide intuition for the different cases.



(a) $\gamma = 0.75$ (b) $\gamma = 0.25$

(c) $\gamma = 0.75$ (d) $\gamma = 0.25$

Figure 2: Top panels illustrate the best weight function $a^*(a)$ in a case with unique equilibrium (a) and with multiplicity (b) for two different values of σ. Bottom panels show the evolution of aggregate consumption volatility in a case with a unique equilibrium (c) and with multiplicity (d) as a function of an inverse measure of σ.

Unique Equilibrium

Our first proposition concerns the case in which local house prices respond relatively strongly to local conditions, i.e. the labor share in construction is greater than one half. In this case, the model always has a unique equilibrum.

Proposition 1. *For* $\gamma \geq 1/2$ *and any* $\beta \in (0,1)$*, there exists a unique REE equilibrium, which is characterized by* $a_u \in (0, \gamma^{-1})$*. Moreover,* $\lim_{\sigma\to\infty} a_u = 0$ *and* $\lim_{\sigma\to 0} a_u = \gamma^{-1}$ *with* $\partial a_u/\partial\sigma < 0$*.*

Proof. Given in Appendix A.5. □

The negative slope of the best response in the range $a \in [0, (1-\gamma)^{-1}]$ is crucial for understanding the forces behind the equilibrium. Negative slope entails *substitutability* in people's use of private information: a higher average response to the signal lowers the individually optimal weight. This happens because, as the equilibrium signal in (29) shows, higher a amplifies the effect of aggregate noise relative to private fundamentals, making s_i less informative. Our setting contrasts with the complementarity featured by other models, like Amador and Weill (2010), in which a higher average response increases the portion of signal variance driven by common fundamentals relative to private noise. Our finding of substitutability is also why, unlike Amador and Weill (2010), our model can deliver a unique equilibrium for any variance of the the aggregate shock.

An important feature of our unique equilibrium is that the amplification of aggregate noise, ζ, grows as its relative variance, σ, shrinks. To see this, notice from equation (30) that, given average weight a, a lower variance of ζ implies an increase in signal precision. This increases the optimal weight that agents wish to put on the signal. But, equation (29) shows that higher a also increases the impact of a given realization of the aggregate shock. This happens both (i) because of the direct effect of the higher inference weight, and (ii) because higher a increases the feedback of average beliefs back into the signal.

Panel (c) of Figure 2 plots the variance of aggregate beliefs as a function of σ^{-1}. The figure shows that, even though the equilibrium weight on ζ grows as σ shrinks, the latter effect eventually dominates so that, in the limit $\sigma \to 0$, average beliefs exhibit no fluctuations. In this limit $a = \gamma^{-1} < (1-\gamma)^{-1}$ and the local price signal is perfectly informative about μ_i.

Multiple Equilibria

When local house prices respond strongly to aggregate conditions, i.e. the local labor share in construction is less than one half, the feedback loop between demand and commodity input prices can be so strong that multiple equilibria exist. Proposition 2 summarizes this result.

Proposition 2. *For $\gamma < 1/2$ there always exists a low REE equilibrium characterized by $a_- \in (0, (1-\gamma)^{-1})$; in addition, there exists a threshold $\bar{\sigma}^2(\beta)$ with $\partial \bar{\sigma}^2(\beta)/\partial \beta \geq 0$ such that, for any $\sigma^2 \in (0, \bar{\sigma}^2(\beta))$, a middle and a high REE equilibrium also exist characterized by $a_\circ$ and $= a_+$, respectively, both lying in the range $((1-\gamma)^{-1}, \gamma^{-1})$. In the limit $\sigma^2 \to 0$:*

i. the high equilibrium converges to a point with no aggregate volatility:

$$\lim_{\sigma^2 \to 0} a_+ = \min\left(\frac{1}{\gamma}, \frac{1}{1-\gamma}\right) \qquad \lim_{\sigma^2 \to 0} \sigma_c^2(a_+) = 0.$$

ii. the low and middle equilibria get the same value and exhibit non-trivial aggregate volatility:

$$\lim_{\sigma^2 \to 0} a_{\circ,-} = \frac{1}{1-\gamma} \qquad \lim_{\sigma^2 \to 0} \sigma_c^2(a_{\circ,-}) = \frac{\gamma(1-2\gamma)}{(1-\gamma)^2}. \tag{32}$$

Proof. Given in Appendix A.5. □

The best weight function in this case is plotted in panel (b) of Figure 2. It shows that the function yields three intersections with the 45° line provided the variance of productivity shocks σ is sufficiently low. We demonstrate in the proof that a lower β is isomorphic to considering a larger σ at any a, so $\beta \to 1$ turns out to be the case most favorable to multiplicity.

Importantly, the qualitative features of the unique equilibrium case still hold in the range $[0, (1-\gamma)^{-1}]$: (i) there is substitutability between individual and average weights; (ii) the equilibrium features larger amplification of aggregate shocks as they get smaller in variance; (iii) a single equilibrium exists in this range for any value of σ. However, since now $\gamma^{-1} > (1-\gamma)^{-1}$, the equilibrium in this range is necessarily distinct from the full-information equilibrium with $a = \gamma^{-1}$. Hence, the equilibrium lying in $[0, (1-\gamma)^{-1}]$ cannot approach full information and the economy must have multiple equilibria in the limit!

To have a limit equilibrium in the range $[0, (1-\gamma)^{-1}]$, it must be that responsiveness to the aggregate shock increases fast enough to offset its shrinking variance. Intuitively, this happens because the price signal in this case depends more on aggregate conditions than on local conditions. In particular, the "no information" threshold $a = (1-\gamma)^{-1}$ at which the signal becomes completely uninformative is now *less* than $a = \gamma^{-1}$, the weight the shoppers would place on the local price if it were perfectly informative. Since our low equilibrium approaches $(1-\gamma)^{-1}$ from below it necessarily reaches the "no information" threshold before it can reach the full-revelation weight.

The fact that multiplicity and imperfect revelation can persist in the limit contrasts with other signal structures studied in the literature. Since structures like Amador and Weill (2010) feature complementarity in information use, feedbacks from decreasing exogenous noise can only improve the precision of the signal and any limit equilibrium must be fully revealing. (Indeed, this is what happens in our high equilibrium.) With substitutability, by contrast, increasing feedback tends to offset the direct effect of reducing noise, potentially leading to noisy equilibria even in the limit.

Panel (d) of Figure 2 illustrates these equilibria. Consumption volatility in the "high" equilibrium case converges to zero as σ^{-1} goes to infinity. By contrast, consumption volatility in the "middle" and "low" equilibria converges to a positive, finite number. Surprisingly, the low and middle limit equilibria have the same stochastic properties as the extrinsic sentiment equilibrium described by Benhabib et al. (2015). In our case, however, fluctuations are driven by infinitesimally-small fundamental shocks, whose realizations coordinate sizable fluctuations in agents' expectations. We elaborate on this connection in Section 5.3.

4 Business cycle fluctuations

In this section, we show that many features of the business cycle can be explained by our model. Our analysis also suggests that the learning-from-prices mechanism can qualitatively change the comovement properties of fundamental shocks, implying that many common strategies for disentangling shocks may give misleading results if learning from prices is important.

4.1 Public News

Before proceeding to our analysis, we introduce an anticipated (common-knowledge) component of aggregate housing productivity. The decomposition of productivity into a common-knowledge and surprise component serves two purposes. First, it allows us to isolate the effects of the learning channel in our model, as the forecasted component of productivity transmits as a standard supply-side shock. Second, by combining the responses of the economy to forecasted and surprise productivity shocks, the model can generate a rich and realistic correlation structure among business cycle variables.

We assume that housing productivity is composed of two independent components

$$\zeta = \zeta^n + \zeta^s;$$

with $\zeta^n \sim (N, \sigma^2_{\zeta^n})$, $\zeta^s \sim (N, \sigma^2_{\zeta^s})$ and $\sigma^2_{\zeta^n} + \sigma^2_{\zeta^s} = \sigma^2_{\zeta}$. The first term ($\zeta^n$) is "news"; it corresponds to the common-knowledge component of productivity, and is known to all agents before they make consumption choices. The second term (ζ^s) is the "surprise"; it is unknown to shoppers and they seek to forecast it using their observation of prices.[11] For future reference, let $\sigma^2_n \equiv \sigma^2_{\zeta^n}/\sigma^2_\mu$, and $\sigma^2_s \equiv \sigma^2_{\zeta^s}/\sigma^2_\mu$ be the normalized variances of the news and surprise components of productivity respectively.

Only modest modifications are necessary to characterize equilibrium in this case. Shoppers refine the information contained in the price signal by "partialing-out" the known portion of productivity. We can thus rewrite households' expectations as

$$E[\mu_i|p_i] = a(s_i - (1-\gamma)\zeta^n), \tag{33}$$

where $s_i - (1-\gamma)\zeta^n$ capture the information available to the individual household, after she has controlled for the effect of ζ^n. The equilibrium values $\hat{a}$ and the conditions for their existence are isomorphic to the ones in the baseline economy once σ^2_s takes the place of σ^2.

For a given total variance of productivity, $\sigma^2 = \sigma^2_s + \sigma^2_n$, we can now span the space between two polar cases, one in which productivity occurs as a pure "surprise" to the case where the productivity shock is common-knowledge "news". Thus, overall comovements in the economy will represent a mix of demand and supply shocks. We note here that, because the

[11] Chahrour and Jurado (2018) show that this information structure is equivalent to assuming that agents observe a noisy aggregate signal, $s = \zeta + \vartheta$.

Table 1: Business Cycle Comovements

	GDP	**Cons**	**Hours**	**Res. Inv.**	**House Pr**	**Constr. Pr**	**Constr. TFP**
GDP	1.00	0.93	0.88	0.64	0.51	0.53	-0.17
Cons		1.00	0.80	0.65	0.47	0.47	-0.02
Hours			1.00	0.50	0.54	0.66	-0.35
Res. Inv.				1.00	0.62	0.37	-0.11
House Pr					1.00	0.81	-0.37
Constr. Pr						1.00	-0.43
Constr. TFP							1.00

Note: Data are real per-capita output, real per-capita consumption, per-capita hours in the non-farm business sector, real per-capita residential investment, Case-Schiller real house price index, real price of residential investment, and relative TFP in the construction sector from the World KLEMS databse (http://www.worldklems.net/data.htm). All data are annual log-levels, HP-detrended using smoothing parameter $\lambda = 10$. Date range: 1960 to 2018, except for construction TFP which ends in 2010.

two components of productivity transmit very differently in the economy, moments generated by projecting variables onto total productivity ζ could give very misleading inference on productivity's effects. Econometric identification of the distinct components of productivity represents a substantial empirical challenge, for which Chahrour and Jurado (2019) provide some guidance in related contexts.

4.2 Demand-driven Fluctuations

Table 1 summarizes unconditional correlations between business cycle variables in US data. Although these are simple raw statistics, the table summarizes several facts that have been documented by more sophisticated empirical analysis. In particular, the table demonstrates that business cycles are dominated by demand-like fluctuations with real quantities, house prices, and residential investment all substantially comoving. Meanwhile, construction productivity is at most weakly negatively related to any of these variables.

In the model, the emergence of demand fluctuations can be seen intuitively by analyzing the aggregate demand and aggregate supply schedules. Using equations (9), (10), (14) and (16), we can express aggregate demand and supply in the housing market as

$$\delta = c - p, \tag{34}$$

$$\delta = \frac{\alpha\gamma}{1-\alpha\gamma}p - \frac{\alpha(1-\gamma)}{1-\alpha\gamma}\zeta. \tag{35}$$

Moreover, because of the learning channel, we know that aggregate consumption shifts upwards in response to a correlated increase in price signals across island,

$$c = \int E[\mu_i|p_i]di = a(s - (1-\gamma)\zeta^n).$$

Note this expression implies c *does not* move with the news component of housing productivity, as ζ^n is being removed from the price signal.

To derive the implications of shopper inference for housing demand, use $p = (1-\alpha)v + \alpha s$ and $v = c$ to express $s = (p + (1-\alpha)a(1-\gamma)\zeta^n)/((1-\alpha)a + \alpha)$. Substituting the expression for c into (34) we get

$$\delta = \frac{\alpha(a-1)}{(1-\alpha)a+\alpha}p + \frac{\alpha a(1-\gamma)}{(1-\alpha)a+\alpha}\zeta^n. \tag{36}$$

When aggregate conditions do not feed into shoppers' beliefs ($a = 0$), equation (36) entails a standard downward-sloping aggregate demand relation in the housing market, and consumption and working hours that are invariant to housing sector productivity. In contrast, when learning from prices is sufficiently important—i.e. whenever a is larger than one—equation (36) shows that δ and p must comove in response to surprise shocks.

We can now solve for equilibrium consumption, residential investment, and the price of new housing as functions of shocks and the equilibrium inference coefficient:

$$c = \frac{a(1-\gamma)}{1-a(1-\gamma)}\zeta^s \tag{37}$$

$$p = \alpha(1-\gamma)\zeta + (1-\alpha\gamma)c \tag{38}$$

$$\delta = -\alpha(1-\gamma)\zeta + \alpha\gamma c. \tag{39}$$

The expressions above are useful for disentangling the direct effects of productivity from the learning channel. Equation (37) shows that a correlated mistake due to a surprise in aggregate productivity moves consumption. Equations (38) and (39) show how this change in beliefs transmits into the housing market, moving prices for new housing and residential investment in the same direction. The same equations also show that productivity shocks affect the housing market through a neoclassical channel, driving prices and quantities in opposite directions. As a result, consumption is correlated with the housing market *only* via the surprise component of productivity; on the other hand, prices and quantities in the housing market are also correlated via the news component.

With a few more lines of algebra, we have that

$$\int n_i^h di = \int \lambda_i - E[\lambda_i|p_i] di = c = \int n_i^c di. \tag{40}$$

Equation (40) implies that an increase in consumption corresponds to an increase in working hours in both sectors. In times of optimism, shoppers' spending increases but wages do not, so production increases.

Since empirical house price measures include both new and existing homes, we also derive the connection between p and the price of the total housing stock, p_t^H. In the Appendix, we show that the price of each vintage moves with shoppers' expected Lagrangian, $p_{it|k} = -E[\lambda_{it}|p_{it}]$. This is intuitive since the supply of past vintages cannot adjust, so that prices must absorb any change in expectations. We therefore find that $p^H = \kappa p + (1-\kappa)E[-\lambda_i|p_i]$ where $\kappa \in (0,1)$ is the steady state fraction of new houses in the total housing stock.

Collecting these results, it is straightforward to demonstrate the following:

Proposition 3. *For σ_s^2 sufficiently small, surprise aggregate productivity shocks drive positive comovement of consumption, employment (in both sectors), residential investment, prices for new and existing housing, commodity prices, and the price of land.*

Proof. The results follows from continuity of the best-response function, and the observation that $\lim_{\sigma_s \to 0} a > 1$ in the case of uniqueness ($a \to 1/\gamma$) or multiple equilibria ($a \to 1/(1-\gamma)$ or $1/\gamma$). □

In sum, our model exhibits comovements of aggregate business cycle variables in response to sufficiently small productivity shocks, *in any equilibrium* and for *any configuration of parameters*. To an outside observer, the economy would appear to be buffeted by recurrent shocks to aggregate demand.

Proposition 3 requires aggregate shocks to be "sufficiently small". Intuitively, this is needed because price signals must be informative enough that shoppers put substantial weight on them. Yet, Proposition 1 shows that for $\gamma \geq 1/2$ aggregate fluctuations still disappear in the limit $\sigma \to 0$. Taken together, these results raise the question: can the unique equilibrium model deliver comovement and realistically large business cycle fluctuations at the same time?

The answer is yes. As we show in the following section, even if the surprise component accounts for a small fraction of realized productivity, demand driven fluctuations may dominate unconditional comovements.

4.3 Business cycles under unique equilibrium

In this section, we discuss the model's business cycle properties when it has a unique equilibrium. We organize the discussion around three pictures illustrating its implications for business cycle comovements, amplification, and correlations with productivity. Our goal is to show that our model can qualitatively account for the empirical patterns reported in Table 1.

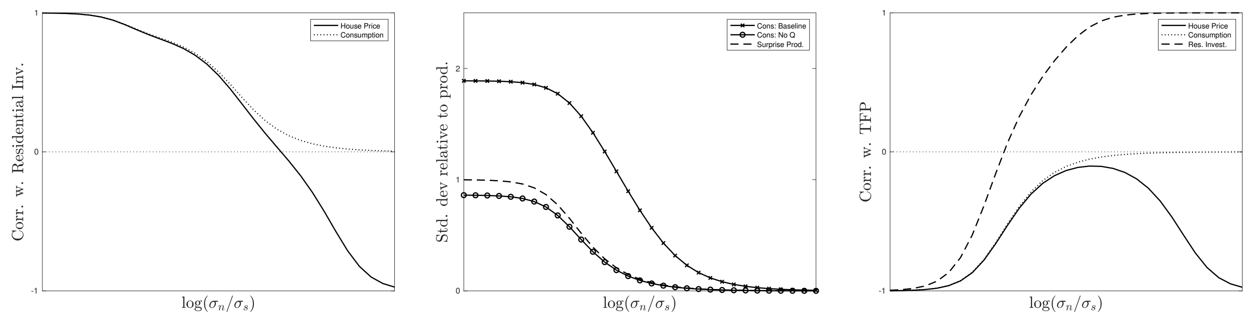
While we do not undertake a full quantitative evaluation of the model, we wish to demonstrate the mechanism can be very powerful for reasonable parameterizations of the model. To this end, we calibrate a set of parameters to standard values and/or long run targets in the data. We set the model period to one year. We set $\beta = 0.96$ consistent with an annual real interest rate of roughly 4%. We set $\phi = 0.66$, to be consistent with 2013-2014 CPI relative importance weight placed on shelter. Estimates of η, the elasticity of local labor demand, range in the literature from below one (Lichter et al., 2015) to above twenty (Christiano et al., 2005). We use $\eta = 2$ as a baseline, and note that the aggregate effects of changing η can be offset one-for-one by changing the volatility of local productivity.

For the housing sector, we follow Davis and Heathcote (2005) in fixing $\alpha = 0.89$ to match the evidence that land accounts about 11% of new home prices.[12] We pick the residential investment labor share parameter $\gamma = .526$ by computing the ratio of labor input costs to materials and energy costs in the construction sector, using Bureau of Labor Statistics data from 1997-2014. Finally, we select the volatility of local productivity shocks relative to aggregate shocks $\text{std}(\hat{\mu}_i)/\text{std}(\zeta) = 10$, implying $\sigma = 0.228$.

Comovement in business cycle variables

Figure 3 plots the unconditional correlations and volatilities of several variables in the economy. On the horizontal axis of each panel we vary the ratio between the forecastable and

[12] For existing homes, Davis and Heathcote (2007) find that land prices accounts for a larger portion of home prices.



(a) Comovements (b) Amplification (c) Relation with productivity

Figure 3: Panels illustrate unconditional correlation and volatility of business cycle variables as a function of the ratio between volatility of the forecastable and non forecastable component for the baseline case of $\gamma = 0.526$.

non-forecastable components of productivity, going from pure “surprise” on the left to pure “news” on the right, while holding the total variance σ constant.

Panel (a) of the figures plots the correlation of consumption and house prices with residential investment. Towards the left of the panel, when productivity is mostly unanticipated, our learning channel dominates: residential investment, house prices and consumption all perfectly comove. Given the results derived above, this also implies comovement in hours in both sectors, the average price of land, and the price of commodities.

By contrast, when productivity is largely commonly knowledge, prices and quantities in the housing market exhibit the negative correlation associated with supply-driven fluctuations, while consumption does not move. Therefore, the more housing productivity is anticipated, the more the economy behaves like a standard real business cycle model. In between these two extremes, the model generates positive but imperfect correlations, consistent with the data reported in Table 1.

Amplification

What is the role of the endogeneity of the signal in generating amplification? Panel (b) of Figure 3 plots the standard deviation of consumption relative to that of aggregate productivity, as a function of the share of productivity that is forecastable. The panel contrasts two cases (i) the baseline model and (ii) the counter-factual case in which the price signal, $\tilde{s}_i = \gamma\mu_i + (1-\gamma)\zeta^s$, excludes its dependence on q. This comparison is useful to evaluate the role of q in amplifying the impact of surprise shocks. To highlight this aspect we also draw the standard deviation of the surprise component of productivity, which by construction falls from one to zero going from left to right.

The comparison is striking. With a completely exogenous price signal, the volatility of consumption, while positive, would be strictly less than the volatility of the surprise component of productivity. This is not the case for our baseline calibration, when the signal is endogenous. The surprise component is amplified substantially, such that consumption remains more volatile than aggregate productivity even when more than 90% of productivity fluctuations are anticipated (near the middle of the horizontal axis)!

The source of amplification can also be seen in our analytical results via equation (37). That equation shows there is a range of parameters where aggregate consumption responds more than one-to-one to productivity shocks.[13] Note that this is a peculiar feature of having the price signal with endogenous precision and, in particular, of having the commodity price q entering in local house prices. One can easy verify that, with a constant q, the reaction of expectations to productivity shocks cannot exceed unity, provided $\gamma > 1/2$.[14]

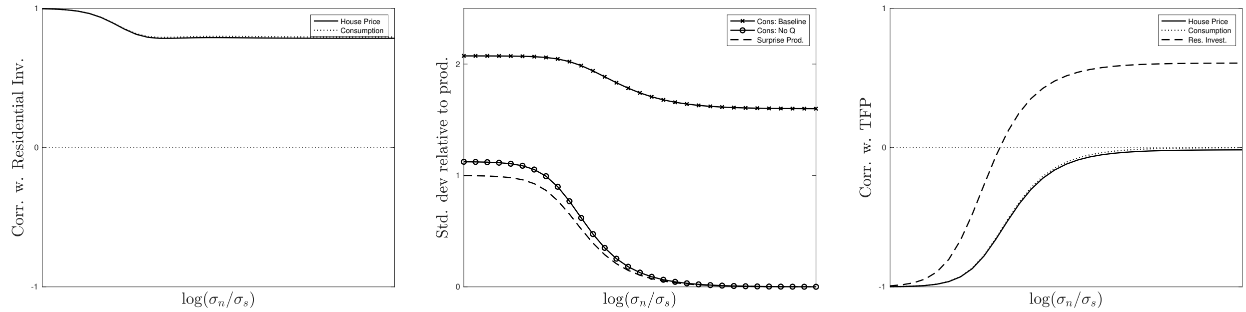
Relationship with construction TFP

In our model, the noise in people's inference comes from a fundamental shock: productivity in the housing sector. One major advantage of our approach to microfounding information is that it provides testable implications about how beliefs fluctuations should relate to measurable economic fundamentals. In this section, we explore this potential by showing that the data are generally consistent with the model's implications for one direct (i.e. model-independent) measure of a fundamental shock: construction TFP. Other shocks may play an important role in the cycle and, as we show in Section 5.1, can induce the same comovements via the learning channel. However, here emphasize how learning from prices qualitatively changes the transmission of supply shocks and offers one possible interpretation of TFP's contractionary effects.[15]

[13] This occurs when $\hat{a} \in (1/2(1-\gamma), 1/\gamma)$ with $\gamma \in (1/2, 2/3)$ then $\partial c/\partial \zeta^s > 1$.

[14] To see, suppose that q is fixed, so that the price signal corresponds to $s_i(0)$ in (29) having a precision $\tau(0)$. Then $E[\mu_i|s_i(0)] = \gamma^{-1}\tau(0)(1+\tau(0))^{-1}s_i(0)$, so that $\partial E[\mu_i|s_i(0)]/\partial\zeta = (1-\gamma)\gamma^{-1}\tau(0)(1+\tau(0))^{-1} < 1$.

[15] Galí (1999) and Basu et al. (2006) find that aggregate productivity is contractionary for hours, while Basu et al. (2014) find evidence that investment-specific productivity has contractionary effects across many vari-



(a) Comovements (b) Amplification (c) Relation with productivity

Figure 4: Panels illustrate correlation and the unconditional volatility of business cycle variables as a function of the ratio between volatility of the forecastable and non forecastable component for the case of $\gamma = 0.45$.

To this end, the last column of Table 1 reports business cycle correlations with relative productivity in the construction sector — the data analogue to ζ — using the USA KLEMS productivity data of Jorgensen et al. (2012). Overall, the column shows that this measure of housing-sector productivity is negatively, but weakly, correlated with business cycle variables. Most notably, residential investment is somewhat negatively correlated with this measure of productivity, a result that would be difficult to reproduce in a full information environment.

Panel (c) of Figure 3 illustrates the correlations of residential investment, the price of housing, and consumption as a function of the ratio between the volatilities of the news and surprise components of productivity. These correlations depend on the fraction of anticipated productivity and, as in the data, are generally not perfect. Correlations with total productivity are imperfect because the two components of productivity – surprise and news – are transmitted very differently in the economy. In particular, so long as a sufficient portion of productivity is unanticipated, all of these variables are negatively correlated with productivity. When instead productivity is mostly common knowledge, consumption and hours do not move while residential investment and house prices move in opposite directions.

4.4 Multiple equilibria: supply shocks or animal spirits?

In this section, we explore the properties of one of the equilibria when $\gamma < 1/2$ as an illustration of the virtually unbounded amplification power of our mechanism. We focus on the low

ables. Angeletos and La'O (2009) propose a different dispersed information mechanism by which employment can fall in response to positive productivity shocks.

equilibrium, characterized by a_- in Proposition 2, this equilibriums turns out to be learnable in the sense of the adaptive learning literature (see Section 5.4.)

In Figure 4 we present correlations and amplification plots for the case of the low equilibrium, changing only $\gamma = 0.45$ with respect to our baseline calibration. Panel (b) shows that, in contrast to our original calibration, consumption remains roughly twice as volatile as realized productivity even as the variance of its surprise component goes to zero. This happens because even infinitesimal surprises drive large fluctuations in beliefs. Note also that the endogeneity of the price signal is crucial to this result: if inference were based on the counter-factual signal $\tilde{s}_i$ that excludes q, the model could deliver large fluctuations in consumption, but these would disappear as σ_s shrinks.

The housing demand and supply relations in (35) and (36) provide an alternative perspective on this powerful amplification. As a approaches $1/(1-\gamma)$, the slope of the curves coincide, implying the two curves overlie one another. In this case, the model exhibits extreme amplification of vanishingly-small shocks, as any point along the coincident upward-sloping curves represents a market clearing allocation and equilibrium volatilities are pinned down by the conditions for optimal inference.

Since belief fluctuations do not disappear with σ_s in this parameterization of the model, it has very different implications for the correlation of consumption and house prices with residential investment. In particular, these variables remain positively correlated even when nearly all of realized productivity is anticipated. The relation of these variables with productivity is also affected. As more of productivity is anticipated, residential investment does not go full way to positive correlation as the expectation component continues to explain much of its volatility. More importantly, the correlation of consumption and the price of existing housing, which are driven by the expectational component, approaches zero as realized productivity becomes almost common knowledge. In other words, it is exactly when there is more public information about productivity, that productivity appears to be less correlated with consumption and house prices!

In the limit of a small surprise component, house prices and residential investment are moved by infinitesimal productivity surprises. An econometrician looking at the data generated by our model would be unable to measure such small revisions in productivity and would probably conclude that the housing market is moved by animal spirits in the vein of Burnside et al. (2016); Shiller (2007) or sentiments as in Angeletos and La'O (2013) and Benhabib et al. (2015). Our model shows how demand-driven waves can be the result of extreme amplification of small fundamental shocks sustained by the feedback loop of learning from prices. With respect to earlier models of sentiments, the different is sharp: the degree of optimism or pessimism in the economy in our model is fully determined by (potentially small) fundamental changes rather being totally erratic or "animal".

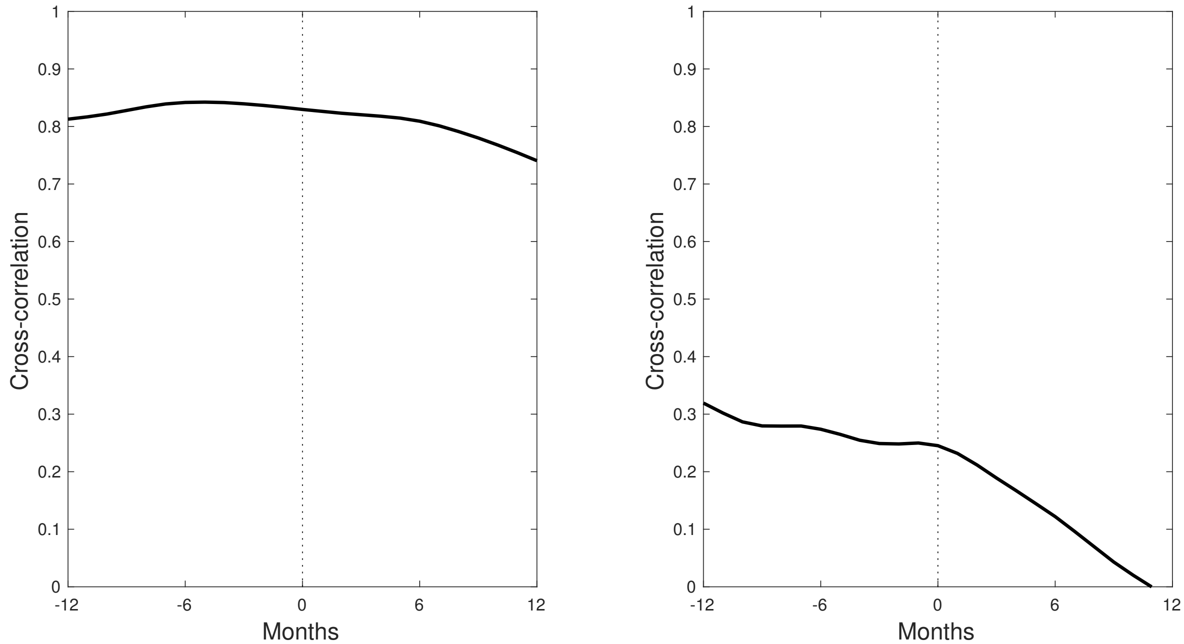
4.5 Evidence from survey data

The essential feature of our model is that people's expectations about their future prospects depend on their own market experiences, particularly housing. We provide here one piece of evidence from survey data that suggests this mechanism may be important in practice.

To this end, we use evidence from the Michigan Survey of Consumer Expectations. Survey participants are asked each month about (i) their perceptions of local house price growth over the last year (ii) what they expect regarding their real income growth over the coming year and (iii) whether they have heard good or bad news about overall economic conditions. The survey then produces index numbers from the answers to these questions, essentially subtracting those who experienced/expect/heard about negative outcomes from those who have experienced/expect/heard positive ones.

Panel (a) of Figure 5 plots the autocorrelation structure of people's current expectations about future income, with respect to their recent experiences in the housing market. Negative numbers on the horizontal axis reflect past responses to the housing experience question, while positive numbers reflect future responses. Panel (a) shows that the two series are *extremely* strongly correlated, with past housing experiences leading income expectations by roughly half a year (as measured by the peak correlation.) This result suggests a strong connection between people's past experiences in the housing market and their expectations about their own income, exactly as our model predicts.

By contrast, Panel (b) of the figure plots the correlation structure of people's current expectations of their own income with respect to what they report having heard about aggregate economic developments. The correlation in this picture is *much* smaller than in Panel (a), sug-



(a) Income expectations at time t vs house price experiences at time $t + h$.

(b) Income expectations at time t vs economic news heard $t + h$.

Figure 5: Auto-correlations of survey measure of own income expectations with respect to own house price experience (panel a) and with respect to news heard about the economy (panel b).

gesting that what people have heard about the aggregate economy (if they've heard anything) plays a much smaller role in forming people's expectations about their own prospects.

While these results are far from dispositive on the merits of our mechanism, we think they provide some initial evidence that learning from prices is plausible in the context of housing.

5 Extensions

This section presents several extensions that demonstrate the mechanism is robust to various modeling details. In Section 5.1, we explore the impact of contemporaneous and future aggregate shocks to consumption production. In Section 5.2, we allow households to observe additional private information about local conditions and show that our results do not rely on excluding exogenous sources of information. In Section 5.3, we explore whether extrinsic noise may drive fluctuations jointly with aggregate productivity and conclude that this is never the case. Finally, to address concerns about the plausibility of learning from prices equilibria, Section 5.4 studies the issue of stability under adaptive learning for the various equilibria of the baseline model.

5.1 Aggregate shocks in consumption production

For this extension, we modify the production function of the consumption sector to allow for aggregate shocks to labor productivity,

$$Y_t = \tilde{\zeta}_t^c \left(\int e^{\tilde{\mu}_{it}/\eta} N_{it}^{c\,1-\frac{1}{\eta}} di \right)^{\frac{1}{1-\frac{1}{\eta}}} . \tag{41}$$

Consumption productivity is a random walk with i.i.d. disturbance $\zeta_t^c \sim N(0, \sigma_\zeta)$. To simplify our exposition, we focus on time t and assume that workers in island i, but not shoppers, know $\{\zeta_{t+1}^c, \zeta_t^c, \mu_{it+1}\}$ and abstract from the presence of other aggregate shocks. A few lines of algebra shows that

$$\lambda_{it} = -\omega_\mu \mu_{it+1} - \omega_b b_{it} - \zeta_t^c \tag{42}$$

We note immediately that a contemporaneous productivity shock in consumption is equivalent to an increase in consumption spending (measured in consumption units). Given the properties of log utility, an increase in consumption spending induces an increase in housing spending as well. In other words, a productivity shock to consumption production is equivalent to an exogenous demand shock in the housing sector.

Including the future realization of aggregate productivity helps to clarify that the model cannot generate demand shocks in the form of news about aggregate productivity as in Lorenzoni (2009). To see this, notice that

$$r_t = \lambda_t - \lambda_{t+1} = -\zeta_t^c + \zeta_{t+1}^c. \tag{43}$$

Thus, the real interest rate adjusts to equalize the return on savings in the two periods. Therefore, the anticipation of higher productivity in the future has no effect on consumption choices today. This is a feature that our model shares with frictionless real economies, as Angeletos (2018) clarifies. By contrast, news about future productivity creates a demand shock in Lorenzoni (2009) because of the presence of nominal rigidities and monetary policy that is suboptimal. A corollary to this result is that no current variable in the economy, other than the real interest rate moves with anticipated aggregate consumption shocks, so shoppers will not be able to learn about them in advance.

Contemporaneous consumption productivity shocks, by contrast, decrease the marginal value of households resources, pushing up the real wages demanded by workers. In the Appendix we show that the price signal in this case is:

$$s_{it} = \gamma(\mu_{it+1} + \zeta_t^c) + (1-\gamma) \int E[\mu_{it+1} + \zeta_t^c | s_{it}] di, \tag{44}$$

where again we have present the case $\lim_{\beta \to 1} \omega_\mu = 0$ with $\tilde{\mu}_{it+1}$ normalize by ω_μ.

One again, correlated fundamentals generate confusion between the idiosyncratic and common components of the signal. As before, the individual expectation of a household is formed according to the linear rule $E[\mu_{it+1} + \zeta_t^c | s_{it}] = a s_i$. Hence, the signal embeds the average expectation, which causes the precision of the signal to depend on the average weight a. Following our earlier analysis, the realization of the price signal can be rewritten as

$$s_i = \gamma \mu_{it+1} + \frac{\gamma}{1 - a(1-\gamma)} \zeta_t^c, \tag{45}$$

where a represents the average weight placed on the signal by other shoppers. The average expectation is given by

$$\int E[\mu_{it+1} + \zeta_t^c | s_{it}] di = \frac{\gamma a}{1 - a(1-\gamma)} \zeta_t^c, \tag{46}$$

which is slightly different from (3.3). The shopper's best response function is now given by

$$a^*(a) = \frac{1}{\gamma} \left(\frac{(1 - a(1-\gamma))^2 + (1 - a(1-\gamma))\, \sigma^2}{(1 - a(1-\gamma))^2 + \sigma^2} \right). \tag{47}$$

While the best-response function in equation (47) is slightly different than in (31), the characterization of the limit equilibria is identical.

Proposition 4. *In the limit $\sigma_\mu^2 \to 0$, the equilibria of the economy converge to the same points as the baseline economy. For $\gamma > 1/2$: there exists a unique equilibrium $\hat{a}$ such that $\lim_{\sigma^2_{\zeta^c} \to 0} a^\mu = \gamma^{-1}$ with $\lim_{\sigma^2_{\zeta^c} \to 0} \sigma_c^2 = 0$. For $\gamma < 1/2$ instead three equilibria exist such that*

$$\lim_{\sigma^2_{\zeta^c} \to 0} \hat{a} \in \{a_-, a_\circ, a_+\} \qquad \textit{with} \lim_{\sigma^2_{\zeta^c} \to 0} \sigma_c^2(\hat{a}) \in \{\sigma_c^2(a_-), \sigma_c^2(a_\circ), \sigma_c^2(a_+)\}.$$

Proof. Follows from the fact that the best response in (47) converges to the best response in (31). □

The proposition has a straightforward intuition. In the limit of small productivity shocks, it does not matter if perturbations emerge from the consumption or housing sector. Hence, Proposition 3 follows identically, and the proof proceeds in parallel with only obvious algebraic substitutions.

The important difference with respect to our baseline model is that, in this case, our mechanism is amplifying an otherwise smaller demand driven fluctuation in the housing market. In other words, under perfect information a shock to consumption productivity would already translate into a smaller, but still correlated, movement in business cycle variables. To see this, rewrite aggregate consumption of residential investment and consumption in the case of perfect information: $c = \zeta_t^c$ and $\delta_t = -\lambda_t - p = (1-\gamma)\zeta_t^c$, which says that residential investment, the price of new housing and consumption move together even under perfect information. Therefore, having focused our main discussion on the case of aggregate productivity shocks in the housing markets has the merits of showing that, not only our mechanism is able to generate high amplification of fundamental shocks, but also can dramatically affect the transmission of shocks in the economy.

5.2 Signal extraction problem with private signals

Here we show that the signal extraction problem, and corresponding equilibria, are not qualitatively affected by the availability of a private signal about the local shock. Instead, the addition of private information maps into our analysis of Section 3.3 as an increase in the relative variance of aggregate shocks.

Let us assume that a household $j \in (0,1)$ in island i has a private signal

$$\omega_{ij} = \mu_i + \eta_{ij} \tag{48}$$

where $\eta_{ij} \sim N(0, \sigma_\eta)$ is identically and independently distributed across households and islands. In this case, households form expectations according to

$$E[\mu_i|p_i, \omega_{ij}] = a\left(\gamma\mu_i + (1-\gamma)\left(\int E[\mu_i|p_i,\omega_{ij}]di - \zeta\right)\right) + b\left(\mu_i + \eta_{ij}\right),$$

where b measures the weight given to the additional private signal. Averaging out the relation above and solving for the aggregate expectation gives

$$\int E[\mu_i|p_i, \omega_{ij}]di = -\frac{a\left(1-\gamma\right)}{1-a\left(1-\gamma\right)}\zeta,$$

which is identical to (28). However, now we need two optimality restrictions to determine a and b. These are

$$E[p_i(\mu_i - E[\mu_i|p_i,\omega_{ij}])] = 0 \;\Rightarrow\; \gamma\sigma_\mu - a\left(\gamma^2\sigma_\mu + \frac{(1-\gamma)^2}{(1-a(1-\gamma))^2}\sigma_\zeta\right) - b\gamma\sigma_\mu = 0,$$
$$E[\omega_{ij}(\mu_i - E[\mu_i|p_i,\omega_{ij}])] = 0 \;\Rightarrow\; \sigma_\mu - a\gamma\sigma_\epsilon - b(\sigma_\mu + \sigma_\eta) = 0,$$

which identify the equilibrium a and b such that each piece of information is orthogonal with the forecast error. Solving the system for a, we get a fix point equation written as

$$a = \frac{\gamma}{\gamma^2 + \frac{(1-\gamma)^2}{(1-a(1-\gamma))^2}\frac{\sigma_\mu+\sigma_\eta}{\sigma_\eta}\frac{\sigma_\zeta}{\sigma_\mu}}. \tag{49}$$

For $\sigma_\eta \to \infty$, the right-hand side of the relation above matches (31). In particular, it follows that a lower σ_η in (49) is equivalent to considering a larger σ_ζ in (31). The analysis of the baseline model thus applies directly to this generalization, and small amounts of exogenous private information do not qualitatively change any of our earlier results.

5.3 Relation with Sentiments

Authors such as Benhabib et al. (2015) have found that *extrinsic* (non-fundamental) sentiment shocks may emerge in environments with endogenous signals. A natural question, given the results in Proposition 2, is whether any equilibria exist in which errors are driven by extrinsic shocks in addition or instead of productivity. The next proposition states that, in fact, *extrinsic* sentiments are always crowded-out by common shocks to productivity.

Proposition 5. *Suppose that*

$$\int E[\mu_i|p_i]di = \phi_\zeta\zeta + \phi_\varepsilon\varepsilon,$$

where ϕ_ε is the equilibrium effect of an extrinsic sentiment shock, $\varepsilon \sim N(0,\sigma_\varepsilon^2)$, not related to fundamentals. Then, $\phi_\varepsilon = 0$ for any $\sigma^2 > 0$.

Proof. Given in Appendix A.5. □

Fundamental shocks always dominates extrinsic shocks because the former have two channels — one endogenous and one exogenous — through which they influences people's information. Intuitively, conjecture that the average action reflects a response to both fundamental and extrinsic shocks. In equilibrium, agents respond to the average expectation, and therefore proportionally to the conjectured endogenous coefficients for each shock. But agents also respond to the exogenous component of the fundamental that appears in the price signal. Thus, any equilibrium must depend somewhat more-than-conjectured on the fundamental relative to the extrinsic shock. This guess and update procedure cannot converge unless the weight on the extrinsic shock is zero.

This logic highlights the fragility of the extrinsic version of sentiments, which are coordinated by endogenous signal structures. For, any shock which tends to coordinate actions for exogenous reasons will also benefit from the self-reinforcing nature of learning, thereby absorbing the role of belief shock for itself. Indeed, the same results emerge if *local* shocks μ_i have any common component, as we consider in Section 5.1.

5.4 Stability analysis

Here, we examine the issue of out-of-equilibrium convergence, that is, whether or not an equilibrium is a rest point of a process of revision of beliefs in a repeated version of the static economy. We suppose that agents behave like econometricians. At time t they set a weight $a_{i,t}$ that is estimated from the sample distribution of observables collected from past repetitions of the economy.

Agents learn about the optimal weight according to an optimal adaptive learning scheme:

$$a_{i,t} = a_{i,t-1} + \gamma_t \; S_{i,t-1}^{-1} \; p_{i,t} \left(\mu_{i,t} - a_{i,t-1} p_{i,t} \right) \tag{50}$$

$$S_{i,t} = S_{i,t-1} + \gamma_{t+1} \left(p_{i,t}^2 - S_{i,t-1} \right), \tag{51}$$

where γ_t is a decreasing gain with $\sum \gamma_t = \infty$ and $\sum \gamma_t^2 = 0$, and matrix $S_{i,t}$ is the estimated variance of the signal. A rational expectations equilibrium $\hat{a}$ is a locally learnable equilibrium if and only if there exists a neighborhood $\digamma(\hat{a})$ of $\hat{a}$ such that, given an initial estimate $a_{i,0} \in \digamma(\hat{a})$, then $\lim_{t \to \infty} a_{i,t} \overset{a.s}{=} \hat{a}$; it is a globally learnable equilibrium if convergence happens for any $a_{i,0} \in \mathbb{R}$.

The asymptotic behavior of statistical learning algorithms can be analyzed by stochastic approximation techniques (see Marcet and Sargent, 1989a,b; Evans and Honkapohja, 2001, for details.) Below we show that the relevant condition for stability is $a_i'(a) < 1$, which can easily checked by inspection of Figure 2.

Proposition 6. *For $\gamma > 1/2$ the unique equilibrium a_u is globally learnable. For $\gamma < 1/2$ the low and the high equilibrium, a_- and a_+, respectively, are always locally learnable, whereas the middle equilibrium $a_\circ$ is never.*

Proof. Given in Appendix A.5. □

It turns out that the unique equilibrium is globally learnable: revisions will lead agents to coordinate on the equilibrium regardless of initial beliefs. With multiplicity, the high and low equilibrium are locally learnable, whereas the middle equilibrium is not. Instead, the middle equilibrium works as a frontier between the basins of attraction of the low and high equilibria.

6 Conclusion

Learning from prices has played an important role in our understanding of financial markets since at least Grossman and Stiglitz (1980). Yet, learning from prices appeared even earlier in the macroeconomics literature, including in the seminal paper of Lucas (1972). Nevertheless, that channel gradually disappeared from models of the business cycle, in large part because people concluded that fundamental shocks would be effectively revealed before incomplete knowledge about them could influence relatively slow-moving macroeconomic aggregates.

In this paper we have shown that, even if aggregate shocks are *nearly* common knowledge, learning from prices may still play a crucial role driving fluctuations in beliefs. In fact, the feedback mechanism we described may be strongest precisely when the aggregate shock is almost, but not-quite-fully, revealed. Endogenous information structures can deliver strong multipliers on small common disturbances, and thus offer a foundation for coordinated, expectations-driven economic fluctuations that are entirely rational. Moreover, the key feature of our theory is also a feature of reality: agents observe and draw inference from prices that are, themselves, influenced by aggregate conditions.

We have applied this idea to house prices, because these are among the most salient prices in the economy. Even if the economy is driven only by productivity shocks, we have shown that this mechanism captures several salient features of business cycles and its close correlation with the housing market. Our approach is consistent both with the evidence that productivity and endogenous outcomes are weakly correlated and our results suggest that the relationship between supply and demand shocks is more subtle than typically assumed in the empirical literature. Future empirical work may wish to take in account the implications of price-based learning.

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A Model

A.1 The extended version

In this section, we introduce and solve the model in its extended version. The representative Household living in island i has the following utility function

$$\sum \beta^t \Theta_t \left(\log \left(C_{it}^{\phi} \mathcal{H}_{it}^{1-\phi} \right) - v^c \frac{\left(N_{it}^c\right)^{1+\chi_c}}{1+\chi_c} - v^h \frac{\left(N_{it}^h\right)^{1+\chi_h}}{1+\chi_h} \right)$$

and faces the budget constraint,

$$\frac{1}{R_t} B_{it} + C_{it} + P_{it}\Delta_{it} = W_{it}^c N_{it}^c + W_{it}^h N_{it}^h + B_{it-1} + \Pi_{it}^c + \Pi_t^h.$$

In this new version we consider the case the household may be hot by a "demand" shock Θ_t such that $\log \Theta_t \equiv \theta \sim N(0, \sigma_\theta)$. We also allow for convex disutility of labor in each sector, with curvature parametrized by χ_c and χ_h. Since workers are imperfectly mobile across sectors, we track sector-specific wages W_{it}^c and W_{it}^h and drop the labor market clearing condition.

In the competitive consumption sector we introduce the possibility of an aggregate productivity shock and decreasing return to scale. The new technological constraint is given by

$$Y_t = e^{\tilde{\zeta}_t^c} (N_t^c)^{\alpha_c},$$

and

$$N_t^c \equiv \left(\int e^{\tilde{\mu}_{it}/\eta} N_{it}^{c\,1-\frac{1}{\eta}} di \right)^{\frac{1}{1-\frac{1}{\eta}}}$$

where $\tilde{\zeta}_t^c = \tilde{\zeta}_{t-1}^c + \zeta_t^c$ where ζ_t^c is an iid innovation drawn from a normal distribution $N\left(0, \sigma_{\zeta^c}\right)$, and $\alpha_c \in (0,1)$ measures economies of scale. We denote by W_t^c the price of N_t^c such that $W_t^c N_t^c = \int W_{it}^c N_{it}^c di$. The rest of the model is as in the main text. The version presented in the main text obtains fixing $\chi_c = \chi_h = 0$, $v^c = v^h$, $\sigma_{\zeta^c} = 0$ and $\alpha_c = 1$.

A.2 Complete list of equilibrium conditions

We list here all the equilibrium conditions under full information at a given time t. The first order conditions for household are:

$$\begin{aligned} \Lambda_{it} &= \beta \Lambda_{i,t+1} R_t \\ \Theta_t \phi C_{it}^{-1} &= \Lambda_{it}, \\ \Theta_t W_{it}^c &= \Lambda_{it}^{-1} N_{it}^{c\,\chi_c} \end{aligned}$$

Optimality in the production of non-durable consumption requires

$$\begin{aligned} N_{it}^c &= e^{\tilde{\mu}_{it}} \left(\frac{W_{it}^c}{W_t^c} \right)^{-\eta} N_t^c \\ N_t^c W_t^c &= \alpha_c Y_t \\ Y_t &= e^{\zeta_t^c} (N_t^c)^{\alpha_c}. \end{aligned}$$

One can easily check that

$$\frac{\partial U_{i0}}{\partial \Delta_{it}} = (1-\psi)(1-\phi)\sum_{\tau=t}^{\infty}((1-d)\beta\psi)^{\tau-t}\Delta_{it}^{-1} = \frac{(1-\psi)(1-\phi)}{1-(1-d)\beta\psi}\Delta_{it}^{-1}. \tag{A.1}$$

The first order conditions for the housing market are then

$$\begin{aligned}
(1-\psi)(1-\phi)(1-(1-d)\beta\psi)^{-1}\Delta_{it}^{-1} &= \Lambda_{it}P_{it}, \\
\left(N_{it}^{h}\right)^{\chi_h} &= \Lambda_{it}W_{it}^{h} \\
Z_{it}Q_t &= \alpha(1-\gamma)P_{it}\Delta_{it}, \\
N_{it}^{h}W_{it}^{h} &= \gamma\alpha P_{it}\Delta_{it} \\
V_{it}L_{it} &= (1-\alpha)P_{it}\Delta_{it}
\end{aligned}$$

where technology is given by

$$\Delta_{it} = L_{it}^{1-\alpha}\left(\left(N_{it}^{h}\right)^{\phi}\left(e^{-\tilde{\zeta}_t}Z_{it}\right)^{1-\phi}\right)^{\alpha}$$

with a market clearing condition $\int Z_{it}di = Z_t$. The budget constraint

$$\frac{1}{R_t}B_{it} + C_{it} + P_{it}\Delta_{it} = W_{it}^{c}N_{it}^{c} + W_{it}^{h}N_{it}^{h} + B_{it-1} + \Pi_t^{c} + \Pi_{it}^{h}$$

must hold as an equality and the transversality condition

$$\lim_{\tau\to\infty} R^{-1-\tau}B_{it+\tau} = 0,$$

must hold at the individual level. Finally market clearing for the endowment reads as:

$$Z_t = \int Z_{it}di, \tag{A.2}$$

where keeping track of Z_t will help us making clear that productivity shock in housing production could be interpreted equivalently in changes in the supply of raw capital. Market clearing conditions (7) - (8) complete the list of equilibrium conditions.

A.3 Linearized Model

In the following subsection we will introduce log-linear relations. At any time t, we will keep distinct the expectations of shoppers – denoted by $E_t^{i,c}[\cdot]$ – and the ones of workers – denoted by $E_t^{i,w}[\cdot]$ – to demonstrate some interesting properties of the model. Let us list first the equations at the island level. The first order conditions for the consumption sector and bond holdings are:

$$E_t^{i,w}[\lambda_{it}] = E_t^{i,w}[\lambda_{it+1}] + r_t \tag{A.3}$$

$$-c_{it} = E_t^{i,c}[\lambda_{it}] \tag{A.4}$$

$$\chi_c n_{it}^{c} = E_t^{i,w}[\lambda_{it}] + w_{it}^{c} \tag{A.5}$$

$$n_{it}^{c} = \tilde{\mu}_{it} - \eta\left(w_{it}^{c} - w_t^{c}\right) + n_t^{c} \tag{A.6}$$

$$n_t^{c} + w_t^{c} = y_t \tag{A.7}$$

$$y_t = \tilde{\zeta}_t^{c} + \alpha_c n_t^{c}. \tag{A.8}$$

The first order conditions for the housing market are

$$\begin{aligned}
-\delta_{it} &= E_t^{i,c}[\lambda_{it}] + p_{it}, && \text{(A.9)}\\
\chi_h n_{it} &= E_t^{i,w}[\lambda_{it}] + w_{it}^h && \text{(A.10)}\\
z_{it} + q_t &= p_{it} + \delta_{it}, && \text{(A.11)}\\
n_{it}^h + w_{it}^h &= p_{it} + \delta_{it} && \text{(A.12)}\\
v_{it} = &= p_{it} + \delta_{it} && \text{(A.13)}
\end{aligned}$$

where technology is given by

$$\delta_{it} = (1-\alpha)l_i + (\alpha\gamma)n_{it}^h + \alpha\left(1-\gamma\right)\left(-\tilde{\zeta}_t + z_{it}\right). \tag{A.14}$$

We log-linearize the budget constraint here. At the individual level we have

$$\frac{1}{R_t}B_{it} + C_{it} + P_{it}\Delta_{it} + P_{it}H_{it-1} =$$
$$= W_{it}^c N_{it}^c + W_{it}^h N_{it}^h + \underbrace{Y_t - W_t^c N_t^c}_{\Pi_t^c} + \underbrace{P_{it}H_{it} - W_{it}^h N_{it}^h - Q_t\left(Z_{it} - Z_t\right) + V_{it}}_{\Pi_{it}^h} + P_{it}H_{it-1} + B_{it-1}$$

$$\frac{1}{R_t}B_{it} + (C_{it} - Y_t) - (W_{it}^c N_{it}^c - W_t^c N_t^c) = -Q_t\left(Z_{it} - Z_t\right) + B_{it-1}$$

We consider a linearization computed from the situation of the economy at time $t = 0$ before shock realize, in which $B_{it} = 0$ for all i, hence we linearize around B_{it} and log-linearize for other variables.

In such a steady-state, all of the terms in parenthesis above are zero, so that the linearization is

$$\beta b_{it} + C(c_{it} - c_t) = C(w_{it}^c - w_t^c) + C(n_{it}^c - n_t^c) - Q(z_{it} - z_t) + b_{it-1}, \tag{A.15}$$

where capital letters denote steady states values. Finally market clearing conditions read as: $z_t = \int z_{it}di$, $\int b_{it}di = 0$ and $c_t = y_t$.

A.4 Solution

This section shows the analytical solution of the model in this extended version. We also generalise our shock structure by introducing a news about future aggregate productivity. To demonstrate some properties of our model, we focus on time t and we assume that in the second stage the worker-saver i knows the current housing productivity, current and future consumption productivity and local productivity, i.e. $\Omega_t = \{\zeta_t, \zeta_t^c, \zeta_{t+1}^c, \mu_{it+1}\}$. We also assume that shoppers only observe p_{it} at time t and share the information set of the worker-saver at time $t+1$.

A.4.1 Solution from $t+1$ onwards

Derivation of λ_{it+1}. Manipulating first order conditions, one finds that:

$$\begin{aligned} p_{i,t}+\delta_{it} &= -E_t^{i,c}[\lambda_{it}] = c_{it}, \\ q_t &= -\int E_t^{i,c}[\lambda_{it}]di - z_t = c_t - z_t, \\ z_{it}-z_t &= c_{it}-c_t, \end{aligned}$$

for any t, which we will use in the following. The transversality condition at the individual level requires that we focus on the stationary solution $b_{it+1}=b_{it}$, with b_{it} being predetermined in the t period. We first use the budget constraint to characterise the solution as follows:

$$(C+Q)(c_{it+1}-c_{t+1})-(1-\beta)b_{it} = C(\mu_{it+1}-\eta\left(w_{it+1}^c-w_{t+1}^c\right))+C(w_{it+1}^c-w_{t+1}^c)$$

or

$$(C+Q)(c_{it+1}-c_{t+1})-(1-\beta)b_{it} = C\mu_{it+1}+(1-\eta)C(w_{it+1}^c-w_{t+1}^c).$$

We use relations at the aggregate level to get $c_{t+1}=w_{t+1}^c=\zeta_{t+1}^c$, $E_{t+1}^{i,w}[\lambda_{it+1}]=E_{t+1}^{i,c}[\lambda_{it+1}]=-c_{it+1}$ and $n_{t+1}=0$ to establish

$$(C+Q)(c_{it+1}-\zeta_{t+1}^c)-(1-\beta)b_{it} =$$

$$= C\mu_{it+1}+(1-\eta)C\left(\underbrace{\frac{\chi_c}{1+\eta\chi_c}\mu_{it+1}+\frac{1}{1+\eta\chi_c}c_{it+1}+\frac{\eta\chi_c}{1+\eta\chi_c}\zeta_{t+1}^c}_{w_{it+1}}-\zeta_{t+1}^c\right)$$

that becomes

$$\left(C\frac{\eta(1+\chi_c)}{1+\eta\chi_c}+Q\right)(c_{it+1}-\zeta_{t+1}^c) = C\left(1+\frac{(1-\eta)\chi_c}{1+\eta\chi_c}\right)\mu_{it+1}+(1-\beta)b_{it}.$$

So that we finally get (remember $b_{it+1}=b_{it}$)

$$\lambda_{it+1} = -c_{it+1} = -\omega_\mu\mu_{it+1}-\omega_b b_{it}-\zeta_{t+1}^c,$$

where

$$\omega_\mu = \frac{C\left(1+\frac{(1-\eta)\chi_c}{1+\eta\chi_c}\right)}{C\frac{\eta(1+\chi_c)}{1+\eta\chi_c}+Q} > 0, \quad \text{and} \quad \omega_b = \frac{1-\beta}{C\frac{\eta(1+\chi_c)}{1+\eta\chi_c}+Q} > 0.$$

As stated in the main text, notice that $\lim_{\beta\to 1}\omega_b = 0$.

A.4.2 Solution at time t

Derivation of λ_{it}. The first step is finding out an expression for λ_t. One can use: $\chi_c n_t = \lambda_t + w_t$ and $w_t = \zeta_t^c + (\alpha_c-1)n_t$ to get

$$(1-\alpha_c+\chi_c)n_t = \lambda_t+\zeta_t^c$$

and then $c_t=\zeta_t^c+\alpha_c n_t$ to get a relation between the actual aggregate lambda and shoppers' expectations

$$\lambda_t = -\frac{1-\alpha_c+\chi_c}{\alpha_c}\int E_t^{i,c}[\lambda_{it}]di - \frac{1+\chi_c}{\alpha_c}\zeta_t^c.$$

Note that this expression is valid also for future times. In fact, under the assumption that uncertainty vanishes after the first period, i.e. $\int E_{t+1}^{i,c}[\lambda_{it+1}]di = -\zeta_{t+1}^c$, we have that $\lambda_{t+1} = -\zeta_{t+1}^c$ which is consistent with what we have found above. In this case, the Euler equation implies,

$$r_t = \lambda_t - \lambda_{t+1} = -\frac{1-\alpha_c+\chi_c}{\alpha_c}\int E_t^{i,c}[\lambda_{it}]di - \frac{1+\chi_c}{\alpha_c}\zeta_t^c + \zeta_{t+1}^c.$$

Note that in the quasi linear case ($\chi_c = 0$ and $\alpha_c = 1$) actual lambda is independent of consumers' expectations, and the above equation reduces to (21).

Given the Euler equation must hold at the local level, we have the following

$$\lambda_{it} = \underbrace{-\omega_\mu \mu_{it+1} - \omega_b b_{it} - \zeta_{t+1}^c}_{=\lambda_{it+1}} + r_t = -\omega_\mu \mu_{it+1} - \omega_b b_{it} + \frac{1-\alpha_c+\chi_c}{\alpha_c}c_t - \frac{1+\chi_c}{\alpha_c}\zeta_t^c \tag{A.16}$$

The equation above shows that the anticipation of future aggregate productivity does not affect the marginal valuation of current consumption. This is a standard finding in real business cycle model, where real interest rates neutralize the effect of anticipated aggregate productivity news. On the other hand current productivity moves the current marginal valuation of current consumption. The case explored in the main text obtains in the quasi-linear case $\alpha_c = 1$ and $\chi_c = 0$.

Derivation of price for new housing. Here we derive the expression for the equilibrium price of new housing. By simple algebra we get

$$\begin{aligned} p_{it} + \delta_{it} &= -E_t^{i,c}[\lambda_{it}], \\ z_{it} &= -E_t^{i,c}[\lambda_{it}] - q_t, \\ n_{it}^h &= \frac{1}{1+\chi_h}\left(\lambda_{it} - E_t^{i,c}[\lambda_{it}]\right) \end{aligned}$$

And the housing price gets

$$\begin{aligned} p_{it} &= -E_t^{i,c}[\lambda_{it}] - \alpha\gamma n_{it} - \alpha(1-\gamma)(-\zeta_t + z_{it}), \\ p_{it} &= -E_t^{i,c}[\lambda_{it}] - \alpha\gamma\left(\frac{1}{1+\chi_h}\left(\lambda_{it} - E_t^{i,c}[\lambda_{it}]\right)\right) - \alpha(1-\gamma)\left(-\zeta_t - q_t - E_t^{i,c}[\lambda_{it}]\right), \\ p_{it} &= \left(1 - \alpha(1-\gamma) - \frac{\alpha\gamma}{1+\chi_h}\right)E_t^{i,c}[-\lambda_{it}] + \alpha\underbrace{\left(\gamma\left(\frac{1}{1+\chi_h}(-\lambda_{it})\right) + (1-\gamma)(\zeta_t + q_t)\right)}_{=s_i}. \end{aligned}$$

The final step is substituting $q_t = -\int E_t^{i,c}[\lambda_{it}]di - z_t$ in it to clearly see that an increase in productivity (negative ζ_t) in the housing sector is isomorphic to an increase in the endowment in raw capital (positive z_t). The case in the text obtains for $\chi_h = 0$.

Derivation of price of the stock of housing. In analogy with (A.1) we can derive the price in consumption units $P_{it|v}^h$ of housing vintage $\Delta_{it|v}$ as

$$P_{it|v}^h = E[\Lambda_{it}|p_{it}]^{-1}\frac{\partial U_{i0}}{\partial \Delta_{it|v}} = \frac{(1-\psi)(1-\phi)\psi^{t-v}}{1-(1-d)\beta\psi}\Delta_{it|v}^{-1}E[\Lambda_{it}|p_{it}]^{-1}, \tag{A.17}$$

for any $v \leq t$. Therefore the price of the total stock of housing is given as

$$P_{it}^h = \sum_{v=-\infty}^{t} \frac{P_{it|v}^h \Delta_{it|v}}{H_{it}} = \frac{1-\phi}{1-(1-d)\psi\beta} E[\Lambda_{it}|p_{it}]^{-1} H_{it}^{-1} \tag{A.18}$$

which in log-terms gives

$$p_{it}^h = -E[\lambda_{it}|p_{it}] - \kappa\delta_{it} = (1-\kappa)E[-\lambda_{it}|p_{it}] + \kappa p_{it} \tag{A.19}$$

where $\kappa = \bar{\Delta}/\bar{H}$ defines the steady state share of residential investment over existing housing stock.

Derivation of b_{it}. To compute b_{it} (bear in mind that we are assuming $b_{it-1} = 0$ and $\mu_{it} = 0$ here) we re-consider the same step leading to (A.16) at time t where now $\mu_{it} = b_{it-1} = 0$ to get:

$$(C+Q)(c_{it} - c_t) + \beta b_{it} = (1-\eta)C(w_{it}^c - w_t^c)$$

or

$$(C+Q)\left(-E_t^{i,c}[\lambda_{it}] + \int E_t^{i,c}[\lambda_{it}]di\right) + \beta b_{it} = (1-\eta)C(-\lambda_{it} + \lambda_t).$$

Use the fact $-E_t^{i,c}[\lambda_{it}] = as_i$, where s_i is defined as above, to get

$$\begin{aligned}
(C+Q)\left(a\frac{\gamma}{1+\chi_h}(\omega_\mu\mu_{it+1} + \omega_b b_{it})\right) + \beta b_{it} &= (1-\eta)C(\omega_\mu\mu_{it+1} + \omega_b b_{it}) \\
\left((C+Q)a\frac{\gamma}{1+\chi_h}\omega_b - (1-\eta)C\omega_b + \beta\right) b_{it} &= \left((1-\eta)C - (C+Q)a\frac{\gamma}{1+\chi_h}\right)(\omega_\mu\mu_{it+1})
\end{aligned}$$

and finally

$$b_{it} = \frac{-(1+\chi_h)(\eta-1)C - (C+Q)a\gamma}{(C+Q)a\gamma\omega_b + (1+\chi_h)(\eta-1)C\omega_b + (1+\chi_h)\beta}(\omega_\mu\mu_{it+1}),$$

so that

$$\begin{aligned}
\lambda_t - \lambda_{it} &= \omega_\mu\mu_{it+1} + \omega_b b_{it} = \\
&= \omega_\mu\mu_{it+1} + \omega_b \frac{-(1+\chi_h)(\eta-1)C\omega_\mu - (C+Q)a\gamma\omega_\mu}{(C+Q)a\gamma\omega_b + (\eta-1)(1+\chi_h)C\omega_b + (1+\chi_h)\beta}\omega_\mu\mu_{it+1} \\
&= \frac{(1+\chi_h)\beta}{\underbrace{(C+Q)a\gamma\omega_b + (1+\chi_h)(\eta-1)C\omega_b}_{\equiv f(a,\beta)} + (1+\chi_h)\beta}\omega_\mu\mu_{it+1}.
\end{aligned} \tag{A.20}$$

Remark: Given that ω_b is a decreasing function of β, we can conclude that a higher a or lower β strictly increases $f(a, \beta)$, and so it strictly decreases the volatility of the idiosyncratic component of λ_{it}, namely $Var(\lambda_t - \lambda_{it})$. This remark will be useful in the following proof.

A.5 Proofs of Propositions

Proof of Proposition 1. Let us first solve the case for which σ is exogenous and fixed which corresponds to the limit case $\beta \to 1$. The fix point equation reads as

$$a^*(a) = \frac{1}{\gamma}\frac{\tau(a)}{1+\tau(a)} = \frac{1}{\gamma}\frac{1}{1+\left(\frac{(1-\gamma)\sigma}{\gamma(1-a(1-\gamma))}\right)^2} = \frac{\gamma\left(1-a\left(1-\gamma\right)\right)^2}{\gamma^2\left(1-a\left(1-\gamma\right)\right)^2+(1-\gamma)^2\sigma^2} \tag{A.21}$$

To prove uniqueness for $\gamma \geq 1/2$, observe that the function $a^*(a)$ is continuous, bounded above by γ^{-1}, and monotonically decreasing in the range $(0,(1-\gamma)^{-1})$. From $\gamma \geq 1/2$, we have $(1-\gamma)^{-1} > \gamma^{-1}$. Thus $a^*(a)$ intersects the 45-degree line a single time.

To prove the existence of a_-, notice that $\lim_{a\to -\infty} a^* = \gamma^{-1}$ and $a^*((1-\gamma)^{-1}) = 0$. By continuity, an equilibrium $a_- \in (0,(1-\gamma)^{-1})$ must always exist. Moreover a_- must be monotonically decreasing in σ^2 as a^* is monotonically decreasing in σ^2.

We now assess the conditions under which additional equilibria may also exist. Because $\lim_{a\to\infty} a^* = \gamma^{-1}$, the existence of a second equilibrium (crossing the 45-degree line in Figure 2) implies the existence of a third. Thus, we must determine whether the difference $a^*(a)-a$ is positive anywhere in the range $a > (1-\gamma)^{-1}$. Such a difference is positive if and only if

$$\Phi\left(\sigma\right) \equiv \gamma\left(1-a\left(1-\gamma\right)\right)^2\left(1-\gamma a\right) - a\left(1-\gamma\right)^2\sigma^2 > 0, \tag{A.22}$$

which requires $a < \gamma^{-1}$ as a necessary condition. Therefore, if two other equilibria exist they must lie in $(\left(1-\gamma\right)^{-1}, \gamma^{-1})$. Fixing $a \in (\left(1-\gamma\right)^{-1}, \gamma^{-1})$, $\lim_{\sigma\to 0}\Phi\left(\sigma\right)$ is positive, implying that there always exists a threshold $\bar{\sigma}$, and so a threshold $\bar{\sigma_\zeta}$, such that two equilibria $a_+, a_\circ \in (\left(1-\gamma\right)^{-1}, \gamma^{-1})$ exist with $a_+ \geq a_\circ$ for $\sigma^2 \in (0, \bar{\sigma}^2)$.

Let us now consider β less than one. In this case, $\omega_b \neq 0$ and the variance of the idiosyncratic portion of λ_{it} is also endogenous to a, as captured by the function $f(a,\beta)$ in equation (A.20). Since $\eta > 1$ and $\omega_b > 0$, it follows that $f(a,\beta)$ is strictly positive and increasing in a for all $\beta < 1$. In this case, we must replace σ with the endogenous variance $\sigma(a,\beta)$ in the fixed-point equation (A.21). Since the $\sigma(a,\beta) > \sigma$ and is increasing in a, $a^*(a,\beta)$ is weakly below $a^*(a)$ and any intersection (fixed point) $a^*(\beta)$ must lie strictly to the left of the value a^* for the model with $\beta \to 1$. Hence, if the economy has a unique equilibrium when $\beta \to 1$ it must also have a unique equilibrium $\beta < 1$. Moreover, since $\sigma(a,\beta)$ increases with β, it must be true that the threshold $\bar{\sigma}$ for a multiplicity falls along with β.

□

Proof of Proposition 2. To prove the limiting statement for $\gamma \geq 1/2$, consider any point $a_\delta = \frac{1-\delta}{1-\gamma}$ such that $\delta > 0$. We then have

$$a^*(a_\delta) = \frac{\gamma\delta^2}{\gamma^2\delta^2+\sigma^2(1-\gamma)^2.} \tag{A.23}$$

Since $\lim_{\sigma^2\to 0} a^*(a_\delta) = \frac{1}{\gamma}$ for any δ, the unique equilibrium must converge to the same point. That the variance of this equilibrium approaches zero follows from equation (28).

To prove the limiting statement for $\gamma < 1/2$, recall the monotonicity of $a^*(a)$ on the range $(0,(1-\gamma)^{-1})$. Following the logic of Proposition 1, for any point a_δ in that range, $\lim_{\sigma^2\to 0} a^*(a_\delta) = \gamma^{-1}$, while $a^*((1-\gamma)^{-1}) = 0$. Thus, the intersection defining a_- must approach $(1-\gamma)^{-1}$. An analogous argument for the point just to the right of $(1-\gamma)^{-1}$

establishes that a_- converges to the same value. Finally, the bounded monotonic behavior of $a^*(a)$ establishes that $\lim_{\sigma^2\to 0} a_+ = \gamma^{-1}$ for the high equilibrium.

That the output variance of the high equilibrium in the limit $\sigma \to 0$ is zero follows from equation (3.3). The limiting variance of the two other limit equilibria can be established by noticing that (A.21) implies

$$\frac{(1-\gamma)^2 a^2 \sigma^2}{(1-a(1-\gamma))^2} = a\gamma(1-a\gamma) \tag{A.24}$$

which gives (32) for $a \to (1-\gamma)^{-1}$. □

Proof of Proposition 5. Suppose not, i.e. suppose that

$$\int E[\mu_i|p_i]di = \phi_\zeta \zeta + \phi_\varepsilon \varepsilon,$$

where ϕ_ε is the equilibrium effect of an extrinsic sentiment shock, ε, not related to fundamentals. Then, the price signal is equivalent to

$$p_i = \gamma\mu_i + (1-\gamma)\left((\phi_\zeta + 1)\zeta + \phi_\varepsilon \varepsilon\right)$$

Using the conjectured weights a^*, we have

$$\int a^* p_i di = a(1-\gamma)(\phi_\zeta + 1)\zeta + a(1-\gamma)\phi_\varepsilon \varepsilon$$

implying that

$$\phi_\zeta = a(1-\gamma)(\phi_\zeta + 1)$$
$$\phi_\varepsilon = a(1-\gamma)\phi_\varepsilon$$

which cannot both be true unless $\phi_\varepsilon = 0$. Notice that, differently from the case with multiple sources of signals studied by Benhabib et al. (2015) (section 2.8 page 565), in our case an aggregate shock (our productivity shock) shows up directly in the signal, which ensures determinacy of the average expectation. An implication of this theorem is that the analysis in Benhabib et al. (2015) is not robust to the introduction of correlation (no matter how small) in the v_{jt} shocks appearing in their endogenous signals. □

Proof of Proposition 6. To check local learnability of the rational expectations equilibrium, suppose we are already close to the resting point of the system. That is, consider the case $\int \lim_{t\to\infty} a_{i,t}\, di = \hat{a}$, where $\hat{a}$ is one of the equilibrium points $\{a_-, a_\circ, a_+\}$, and so

$$\lim_{t\to\infty} S_{i,t} = \sigma_s^2(\hat{a}) = \gamma^2\sigma_\mu^2 + \frac{(1-\gamma)^2}{(1-\hat{a}(1-\gamma))^2}\sigma_\zeta^2. \tag{A.25}$$

According to stochastic approximation theory, we can write the associated ODE governing the stability around the equilibria as

$$\begin{aligned}\frac{\mathrm{d}a}{\mathrm{d}t} &= \int \lim_{t\to\infty} \mathrm{E}\left[S_{i,t-1}^{-1} p_{i,t}\left(\mu_{i,t} - a_{i,t-1}p_{i,t}\right)\right] di \\ &= \sigma_s^2\left(\hat{a}\right)^{-1} \int \mathrm{E}\left[p_{i,t}\left(\mu_{i,t} - a_{i,t-1}p_{i,t}\right)\right] di \\ &= \sigma_s^2\left(\hat{a}\right)^{-1}\left(\gamma\sigma_\mu^2 - a_{i,t-1}\left(\gamma^2\sigma_\mu^2 + \frac{\left(1-\gamma\right)^2}{\left(1-a_{t-1}\left(1-\gamma\right)\right)^2}\sigma_\zeta^2\right)\right) \\ &= a_i\left(a\right) - a. \end{aligned} \tag{A.26}$$

For asymptotic local stability to hold, the Jacobian of the differential equation in (A.26) must be less than zero at the conjectured equilibrium. The derivative of $a_i(a)$ with respect to a is:

$$a_i'(a) = -\frac{2\gamma\left(1-\gamma\right)^3\left(1-\left(1-\gamma\right)a\right)\sigma^2}{\left(\left(1-\gamma\right)^2\sigma^2 + \left(1-\left(1-\gamma\right)a\right)^2\gamma^2\right)^2}, \tag{A.27}$$

which is positive whenever $a > (1-\gamma)^{-1}$. Then, necessarily, $a_i'(a_\circ) > 1$, $a_i'(a_+) \in (0,1)$, $a_i'(a_-) < 0$ and $a_i'(a_u) < 0$. i.e. the low and unique equilibrium are respectively locally and globally learnable. □

A.6 Data Definitions

Unless otherwise noted, variables are download from the FRED database maintained by the Federal Reserve Bank of St. Louis. FRED variable codes are provided in parenthesis when available. Annual variables are averaged over all observations of the given year.

For per-capita variables, our measure of population is the civilian non-institutional population over the age of 16 (CNP16OV). Our measure of real per-capita gross domestic product is seasonally-adjusted nominal quarterly GDP (GDP) deflated by the GDP price deflator (GDPDEF) and population. Our measure of real per-capita consumption is nominal personal consumption expenditure (PCEC) deflated by the GDP price deflator and population. Our measure of per-capita hours in the seasonally-adjusted hours of all persons in the non-farm business sector (HOANBS) divided by population.

We compute real per-capita residential investment using nominal private residential fixed investment (PRFI) deflated by the chain-type price index for the real private fixed investment in the residential sector (B011RG3Q086SBEA) and population. We use the same deflator (B011RG3Q086SBEA) divided by the GDP deflator price index for the real price of residential construction. For real house prices, we used the index of Shiller (downloaded from http://www.econ.yale.edu/ shiller/data.htm). Construction and aggregate productivity are taken from the World KLEMS database (downloaded from http://www.worldklems.net/data.htm).

Finally, for our measures expectations, we use data from the Survey of Consumers at the University of Michigan (downloaded from https://data.sca.isr.umich.edu/charts.php). For real income expectations we use their index generated from question 14, "Expected Change in Real Income During the Next Year." For past house price experience, we use the index generated from question 45, "Change in Home Values During the Past Year." Finally, for the measure of economic news heard, we use the index generated from question 23 "News Heard of Recent Changes in Business Conditions."