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ROBOCALYPSE? IMPLICATIONS OF
DEMOGRAPHIC AND TECHNOLOGICAL
CHANGES**

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Abstract

Demographic changes and a new wave of innovation and automation are two main structural trends shaping the macroeconomy in the next decades. We present a general equilibrium model with a tractable life-cycle structure that allows the investigation of the main transmission mechanisms by which demography and technology affect economic growth. Due to a trade-off between innovation and automation, lower fertility and population ageing lead to a reduction in GDP per capita growth and the labour income share. Assuming different labour market configurations and scenarios for the integration of robots in economic activity only partially compensate for these effects.

JEL Classification: O31, O40, J11

Keywords: Population Ageing, automation, Innovation

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From Secular Stagnation to Robocalypse? Implications of Demographic and Technological Changes*

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Abstract

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1 Introduction

Demographic (baby boomers reaching retirement age, a fall in fertility, and the continuous rise in longevity) and technological changes (the new wave of automation brought by developments in robotics and in artificial intelligence) are two structural trends that will frame the macroeconomic context in the next decades. The implications of these trends for economic growth are subjects of much debate. On the one hand, population ageing is found to be associated with lower interest rates, less innovation activity, and lower output growth (Aksoy, Basso, Smith, and Grasl (2019), Gordon (2012) and Derrien, Kecskés, and Nguyen (2018)). On the other hand, Acemoglu and Restrepo (2017b, 2018a) argue that it may give incentives to automation and, hence, to higher productivity and growth, although it may also decrease employment and the labour income share.¹

We analyse the macroeconomic consequences of demographic and technological changes in a general equilibrium model in which both population dynamics and research and development (R&D) determine long-run growth. R&D comprises two activities: innovation, which involves the creation of new products, and automation, which is the development of procedures that allow robots to replace labour. Although demographic changes (lower fertility and mortality) boost automation, we find that they eventually lead to lower growth of GDP per capita.

The primary source of growth in our framework is innovation, that is the origination of new ideas (goods) that increase overall productivity (productivity effect). By creating new goods, innovation also creates new job opportunities (reinstatement effect). Eventually, as robots are more productive than labour, automation also increases productivity but destroys jobs (displacement effect).

How do demographic changes affect the economy? We identify three key chan-

¹The empirical literature on the employment and wage effects of automation is increasing rapidly, providing a wide range of estimates of the “number of jobs that will be lost to automation”. See, for instance Graetz and Michaels (2018), Acemoglu and Restrepo (2017a), Dauth, Findeisen, Südekum, and Woessner (2017) and Frey and Osborne (2017). On the determinants of the labour income share, and, in particular, its relation to technology and automation, see Martinez (2018) and Bergholt, Furlanetto, and Faccioli (2019).

nels. First, changes in labour supply affect factor prices (wages and the price of robots), altering the relative profitability of labour intensive and automated sectors, and, hence, the incentives to innovate and automate. Second, demographic changes affect savings and the interest rate, altering the amount of resources available for investment in capital accumulation, innovation, and automation. Third, insofar as the efficiency of R&D may depend on the age structure of population, the arrival rate of new goods is also affected.

Automation, by re-allocating production from a labour intensive sector to an automated sector, can eventually generate an imbalance between the labour and robot income shares, preventing the economy from reaching a balance growth path (*BGP*) — defined as a dynamic equilibrium with constant factor shares. We focus on dynamic equilibria under which structural changes do not generate divergent paths due to a restriction on the efficiency of robots production. This restriction eventually ensures the ratio of factor prices is constant.² Similarly to Aghion, Jones, and Jones (2017), we prevent a labour-free singularity by restricting the productivity gain of an essential input (robots), sustaining its relative price in terms of the final good. Ultimately, as the economy develops and gains in complexity, the robots that are capable of replacing labour in the production of an increasing variety of goods must also become harder to produce. Without this constraint, automation ultimately leads to full robotisation of production with the price of robots converging to zero.

We show analytically that if the economy is at a *BGP*, then a fall in fertility leads the economy to a new *BGP* with *lower* GDP per capita growth, a higher degree of automation, and a lower labour income share. Since in a *BGP* innovation and automation growth are matched, the creation of new goods, which increases labour productivity, is the main source of technological change. On the one hand, a fall in labour supply growth leads to a decrease in the incentive to innovate, pushing

²Another way to ensure existence of a *BGP* is to assume an exogenous labour supply adjustment as in Acemoglu and Restrepo (2018b). Such adjustment is not applicable in our framework since our interest is on the effects of demographics through changes in labour supply.

the growth rate down. On the other hand, lower labour supply growth increases the incentive to automate, and thus the robots production share of output increases and the labour income share falls. Higher longevity (holding population constant) increases the average age of the population. Unless R&D is severely harmed by the lack of young workers, the *BGP* of an economy with an older population entails lower interest rates, a higher degree of automation, and a lower labour income share.

Embedding the demographic projections for the United States (US) and Europe for the next decades into our model allows us to quantify the contribution of demographic changes to medium-run economic trends in these regions. Lower fertility and higher longevity lead to higher automation both in the US and in Europe, with a stronger effect in Europe, as observed in the available data on robot density. Despite the positive effects of automation, as resources are diverted from innovation, population changes lead to lower output growth in the medium run, even without assuming new ideas become harder to arise (as in Bloom, Jones, Reenen, and Webb (2017)). Our results indicate that demographics may also contribute to reinforce three observed trends in the past decades: the fall in real interest rates (Aksoy, Basso, Smith, and Grasl (2019) and Eggertsson, Mehrotra, and Robbins (2019)), the fall in labour income shares (Elsby, Hobijn, and Sahin (2013), and Karabarbounis and Neiman (2014)) and the fall in the price of robots (Graetz and Michaels (2018)).

We explore several extensions with alternative labour market configurations. We consider that *(i)* innovation no longer relies on labour input, eliminating labour supply constraints in R&D; *(ii)* workers move towards the R&D sector boosting labour supply in R&D after a fall in wage due to automation; *(iii)* workers alter labour supply at the intensive margin; and *(iv)* retirement age rises as longevity increases, boosting labour supply as a whole. In all cases the negative effect of population ageing on growth is only partially offset. When the retirement age increases to maintain the ratio of working life and retirement duration constant, demographic changes no longer generate an increase in automation.

We also consider different ways automation and robots influence economic activity. We allow (i) automation to also generate an increase in the relative productivity of robots; and (ii) robots to also replace labour in R&D. Once again the negative effects of demographic changes on per capita output growth are only partially offset. In the first case, due to the presence of intermediate inputs in both labour and robot intensive sectors, the higher relative productivity of robots increases total factor productivity (*TFP*) in both sectors, reducing automation in the medium run. In the second case automation is higher in the medium run since the negative effect of resource reallocation on innovation is mitigated by the use of robots in R&D.

A main feature of our analysis is a restriction on the efficiency of robots production that ensures factor prices and, ultimately factor income shares, do not diverge. We relax this assumption in the medium run (only enforcing it in the long run) allowing the efficiency of robots production to initially increase as the degree of automation increases. Under this scenario, the price of robots fall more significantly, diverging from the path of real wages. A fall in robot prices further boosts automation and, as a result, the share of output produced in the automated sector increases substantially while the labour income share falls. This happens at the cost of resources being diverted from innovation, which eventually leads to a sizeable fall in GDP per capita growth. Thus, a “robocalypse scenario”, resembling the immiseration equilibrium of Benzell, Kotlikoff, LaGarda, and Sachs (2015), may arise.

The key mechanism driving the results in all specifications is the trade-off between innovation and automation. Automation crowds out innovation, and, as automation is a subsidiary activity of innovation, automation cannot progress indefinitely without innovation. In this regard, the assumption that newly created goods need to necessarily be performed employing labour implies that labour constraints the creation of new goods, ultimately controlling productivity gains and growth.³

³This conclusion is analogous to the implications of proposition 4 in Acemoglu and Restrepo (2018b); a

Relaxing this assumption may generate demographic transitions in which the share of labour income falls and per-capita output growth increases. That may exacerbate the inequality between the production factor remunerations.

In what follows, we describe the model (Section 2), discuss the characteristics of the *BGP*, and present the comparative analysis results (Section 3). Section 4 focuses on the medium-run effects of demographic changes as predicted by population projections for the US and Europe. Section 5 provides concluding remarks.

2 The Model

The model economy consists of three sectors (goods production, R&D, and robot production) and households. The goods production sector comprises of a final good producer, who aggregates a continuum of intermediate differentiated goods $i \in Z_t$ produced by combining goods (intermediate inputs), capital, and either labour or robots.

The R&D sector comprises innovation and automation. Innovation creates new goods (Romer (1990) and Comin and Gertler (2006)) that are added to the set Z_t of intermediate inputs, and which, initially, can only be produced by labour. Automation develops procedures such that existing intermediate good i could be produced by robots. The set of goods produced by robots is denoted $A_t \subset Z_t$. Robots are machines used in production created in the robot production sector.

As in Gertler (1999), households, who supply labour, accumulate assets and consume the final good, face two stages of life, mature (working) and old (retirement). Thus, the framework has the flexibility to account for the three main drivers of population dynamics: fertility, longevity, and retirement age.

balanced growth equilibrium may be represented by a production function with purely labour-augmenting technological change.

2.1 Households

There are N_t households, divided amongst two age groups: workers (w) and retirees (r). $\omega_{t,t+1}^y N_t^w$ new households are born every period as workers. Workers (N_t^w) retire with a probability $1 - \omega^w$, and retirees (N_t^r) die with a probability $1 - \omega_{t,t+1}^r$. Thus,

$$N_{t+1}^w = \omega_{t,t+1}^y N_t^w + \omega^w N_t^w, \text{ and } N_{t+1}^r = (1 - \omega^w) N_t^w + \omega_{t,t+1}^r N_t^r. \quad (1)$$

Households face two idiosyncratic risks: i) loss of wage income at retirement and ii) time of death. There is a perfect annuity market allowing retirees to insure against time of death by turning their wealth over to perfectly competitive financial intermediaries that invest the proceeds and pay back a return of $R_t/\omega_{t-1,t}^r$ for surviving retirees. Households are risk neutral, so that uncertainty about employment tenure does not affect optimal choices. Nevertheless, there is consumption smoothing since preferences belong to the recursive utility family (Epstein and Zin (1989) and Farmer (1990)), such that risk neutrality coexists with a positive elasticity of intertemporal substitution.

For $z = \{w, r\}$, the household j selects consumption and asset holdings to maximise

$$V_t^{jz} = \{(C^{jz})^v + \beta_{t,t+1}^z (E_t[V_{t+1}^j | z]^v)\}^{1/v} \quad (2)$$

$$\text{subject to } C_t^{jz} + FA_{t+1}^{jz} = R_t^z FA_t^{jz} + W_t^j I^z + d_t^z \quad (3)$$

where $\beta_{t,t+1}^z$ is the discount factor, which is equal to β for workers and $\beta\omega_{t,t+1}^r$ for retirees, R_t^z is the return on assets, which is equal to the real rate R_t for workers and $R_t/\omega_{t-1,t}^r$ for retirees, W_t^j is the real wage for worker j , and I^z is an indicator function that takes the value of one when $z = w$ and zero otherwise. Thus, we assume that retirees do not work and worker's labour supply is fixed (we consider below an extension where workers alter labour supply in the intensive margin).

FA_t^{jz} and d_t^z denote, respectively, assets acquired and dividends from the financial intermediary.

A fixed share Sw_{RD} of new workers $\omega_{t,t+1}^y N_t^w$ are employed in $R\&D$ and the remaining $(1 - Sw_{RD})$ supplies labour to intermediate firms.⁴ At every period a fraction $drop_{RD}$ of R&D workers, who do not retire, are no longer able to work in this sector, and, thus, start supplying labour to firms in the production sector.⁵ Hence, employment in the R&D and labour intensive sectors are, respectively:

$$N_{t+1}^{wRD} = \omega_{t,t+1}^y N_t^w Sw_{RD} + (1 - drop_{RD}) \omega^w N_t^{wRD}, \text{ and} \quad (4)$$

$$N_{t+1}^{wL} = \omega_{t,t+1}^y N_t^w (1 - Sw_{RD}) + \omega^w N_t^{wL} + (drop_{RD}) \omega^w N_t^{wRD}. \quad (5)$$

with W_t^{RD} and W_t being respectively wages in the R&D and in the production sectors.

The resulting consumption functions of workers and retirees are:⁶

$$C_{w,t} = \varsigma_t [R_t FA_{w,t} + H_{w,t} + D_{w,t}], \text{ and } C_{r,t} = \varepsilon_t \varsigma_t [R_t FA_{r,t} + D_{r,t}]. \quad (6)$$

where, $H_{w,t}$ is the present value of human capital, $D_{z,t}$ is the present value of dividends for $z = \{w, r\}$. ς_t denotes the marginal propensity of consumption of workers and $\varepsilon_t \varsigma_t$ the one for retirees (where $\varepsilon_t > 1$). As marginal propensities to consume are different across ages, changes in the distribution of asset holdings as well as in the population age structure, affect aggregate demand. Moreover, the marginal propensities to consume are functions of fertility (ω^y), longevity (ω^r) and time of retirement (ω^w). Thus, through changes in savings, demographics affect the equilibrium interest rate.

Finally, labour supply is a function of fertility ($\omega_{t,t+1}^y$), the share of new workers entering the R&D sector (Sw_{RD}) and the retirement age (ω^w). In our benchmark

⁴We also consider below an extension where Sw_{RD} is determined endogenously.

⁵This is to reflect the fact that innovation productivity peaks during the first 10-15 years of a workers life (see Jones (2010)).

⁶The remaining equilibrium conditions of the household sector are described in the Appendix.

specification changes in labour supply will solely be a function of fertility. In different extensions we analyse the impact of variations in labour supply due to changes in the share of workers entering the *R&D* sector and the retirement age.

2.2 Production

A final producer combine intermediate goods (which are substitutes) according to

$$y_t = \left[\int_0^{Z_t} y_{i,t}^{\frac{\psi-1}{\psi}} di \right]^{\frac{\psi}{\psi-1}}, \text{ where } \psi > 1. \quad (7)$$

Each firm $i \in [0, Z_t]$ produces a specialised intermediate input that is sold to the final producer. For a subset $i \in A_t$ (the automated sector), they can be produced using intermediate inputs ($\Upsilon_{i,t}$), rented capital ($K_{i,t}$) and robots ($M_{i,t}$) or labour ($L_{i,t}$). Robots are more productive than labour, and, thus, if a good can be produced by robots, the firm selects to do so. For the remaining goods $i \in Z_t \setminus A_t$ (the labour intensive sector), production can only be done using intermediate inputs ($\Upsilon_{i,t}$), rented capital ($K_{i,t}$) and labour ($L_{i,t}$). Therefore,

$$\begin{cases} y_{i,t} = ((K_{i,t})^\alpha (\theta_t M_{i,t})^{1-\alpha})^{1-\gamma_I} \Upsilon_{i,t}^{\gamma_I} & \text{for } i \in A_t \\ y_{i,t} = ((K_{i,t})^\alpha (L_{i,t})^{1-\alpha})^{1-\gamma_I} \Upsilon_{i,t}^{\gamma_I} & \text{for } i \in Z_t \setminus A_t. \end{cases} \quad (8)$$

θ_t denotes the relative productivity of robots versus labour, and $\alpha, \gamma_I \in (0, 1)$ control the capital and intermediate input shares. The rental rate of capital is denoted $r_{k,t}$ and the relative price of robots, q_t . We initially set $\theta_t = \bar{\theta}$.⁷ As an extension, we also consider that the productivity of robots relative to labour increases as the economy automates more (θ_t increases as a function of A_t).

Since capital and labour/robots are complements, capital biased technological progress may increase labour productivity and wages. On the contrary, robot biased technological progress (automation) displaces labour and decreases wages by

⁷To ensure robots are more productive than labour we set $\bar{\theta}$ such that $W_t > q_t/\theta_t$ for all t .

substituting labour in the production of a subset of intermediate inputs. This is a crucial difference between automation and the previously observed technological revolutions, which introduced new forms of capital that were complementary to (some) labour inputs but did not displace all of them from the production process.

In this framework the ratio of profits ($\Pi_{i,t}$) of firms in the automated and labour intensive sector is given by

$$\frac{\Pi_{i \in A_t,t}}{\Pi_{i \in Z_t \setminus A_t,t}} = \left(\frac{W_t}{q_t/\theta_t} \right)^{(1-\alpha)(1-\gamma_I)(\psi-1)}, \quad (9)$$

thus, the higher the real wage is relative to the price of robots, the larger is the profit differential in favour of the automated sector.

Under this production structure, economic growth is the result of i) the rise in the number of intermediate goods (Z_t grows), and ii) the introduction of robots that displace labour in the production of intermediate goods. These two forms of technological change come from R&D investment, described next.

2.3 Research and Development

R&D consists of the creation of goods (*innovation*), and the development of procedures that allow robots to be introduced in the production process (*automation*).

Let Z_t^p be the stock of goods for innovator p , who at each period spends S_t^p and employs labour ($L_{I,p,t}$) to invent $\varphi_t(S_{p,t})^{\kappa_{RD}}(L_{I,p,t})^{\kappa_L}$ new goods, where $\kappa_{RD}, \kappa_L \in [0, 1]$ represents the relative weight of investment and labour for R&D. Thus, the stock of goods, Z_{t+1}^p , is given by

$$Z_{t+1}^p = \varphi_t(S_{p,t})^{\kappa_{RD}}(L_{I,p,t})^{\kappa_L} + \phi Z_t^p, \quad (10)$$

where ϕ is the intermediate good survival rate. Following Comin and Gertler (2006) and Aksoy, Basso, Smith, and Grasl (2019) we set $\varphi_t \equiv \chi Z_t [\tilde{\Psi}^\rho(S_t)^{\kappa_{RD}-\rho}(N_t)^{\kappa_L}]^{-1}$. Since R&D productivity depends on the aggregate stock of goods (Z_t), there is a

positive spillover as in Romer (1990). There is also a congestion externality via the factor $[\tilde{\Psi}^\rho(S_t)^{\kappa_{RD}-\rho}(N_t)^{\kappa_L}]$ to ensure that a *BGP* exists.⁸ The R&D elasticity of new technology creation in equilibrium is ρ .

Innovators borrow S_t^p from the financial intermediary. Upon creation of a new good, they receive a fraction ϑ of the profits of the intermediate firm that produces it. Thus, the value of an invented good J_t is

$$J_t = \vartheta \Pi_{i,t} + (R_{t+1})^{-1} \phi E_t J_{t+1}, \text{ for } i \in Z_t \setminus A_t \quad (11)$$

where $\Pi_{i,t}$ for $i \in Z_t \setminus A_t$ is the profit of the intermediate good firm.

Innovator p will then invest $IS_{p,t} = (S_{p,t})^{\kappa_{RD}}(L_{I,p,t})^{\kappa_L}$ until the marginal cost equates the expected gain. Defining $\tau_{S,t}$ as the shadow price of $IS_{p,t}$, we have that $S_{p,t} = IS_{p,t} \tau_{S,t} \kappa_{RD}$, $L_{I,p,t} W_{RD,t} = IS_{p,t} \tau_{S,t} \kappa_L$ and $\phi E[J_{t+1}] = \frac{R_{t+1} \tau_{S,t}}{\varphi_t}$. Thus, using (10), we obtain

$$S_t = \kappa_{RD} R_{t+1}^{-1} \phi E_t J_{t+1} (Z_{t+1} - \phi Z_t). \quad (12)$$

The key mechanism driving the creation of new goods is through changes in the relative profitability of the labour intensive sector. If innovators expect $\Pi_{i,t}$ for $i \in Z_t \setminus A_t$ to increase, $E[J_{t+1}]$ goes up, increasing the incentives to invest ($S_{p,t}$) and to hire more labour ($L_{I,p,t}$). This leads to an increase in Z_t , and, eventually, in total output.

Automation investors (q) spend $\Xi_{q,t}$ and hires $L_{A,q,t}$ to transform a Z_t^q good into a A_t^q good, which then becomes part of the set of goods that can be produced by robots.⁹ This conversion process succeeds with probability $\lambda_t = \lambda \left(\frac{(Z_t^q - A_t^q)^{\kappa_{RD} + \kappa_L} \Xi_{q,A,t}}{\tilde{\Psi}_t^{\kappa_{RD}} N_t^{\kappa_L}} \right)$, with $\lambda'(\cdot) > 0$ and $\Xi_{q,A,t} = (\Xi_{q,t})^{\kappa_{RD}} (L_{A,q,t})^{\kappa_L}$. If unsuccessful, the good remains within the labour intensive sector. Once automation is successful the investor earns

⁸ N_t is included in the congestion factor since, as discussed in Jones (1995) and more recently Bloom, Jones, Reenen, and Webb (2017), models of endogenous growth where growing employment in R&D (due to population growth) generates faster steady state output growth are inconsistent with the data.

⁹We also consider an extension of the model in which robots can also be used as inputs in the automation of goods, which resembles the artificial intelligence model in Aghion, Jones, and Jones (2017).

a fraction ϑ of the profits of the robot intensive intermediate producer. Thus, the value of an automated good, V_t , is given by

$$V_t = \vartheta \Pi_{i,t} + (R_{t+1})^{-1} \phi E_t V_{t+1}, \text{ for } i \in A_t. \quad (13)$$

Let $\tau_{A,t}$ be the shadow price of $\Xi_{q,A,t}$, then automation investors solve

$$\max_{\Xi_{q,A,t}, \Xi_{q,t}, L_{A,q,t}} -\tau_{A,t} \Xi_{q,A,t} + (R_{t+1})^{-1} \phi E_t [\lambda_t V_{t+1} + (1 - \lambda_t) J_{t+1}]. \quad (14)$$

Assuming that the elasticity of λ_t to changes in its input is constant and smaller than one, then we define $\epsilon_\lambda \equiv \frac{\lambda'}{\lambda_t} \frac{(Z_t^q - A_t^q)^{\kappa_{RD} + \kappa_L} \Xi_{q,A,t}}{\bar{\Psi}_t^{\kappa_{RD}} N_t^{\kappa_L}} \frac{1}{\kappa_{RD}}$ and obtain

$$\Xi_{q,t} = \epsilon_\lambda \lambda_t R_t^{-1} \phi E_t [V_{t+1} - J_{t+1}], \text{ and } L_{A,q,t} W_{RD,t} = \Xi_{q,t} \frac{\kappa_L}{\kappa_{RD}}. \quad (15)$$

Since the stock of labour intensive goods at t for which automation is feasible, is $(Z_t^q - A_t^q)$, the flow of the stock of automated goods is given by

$$A_{t+1}^q = \lambda_t \phi (Z_t^q - A_t^q) + \phi A_t^q. \quad (16)$$

From (12) and (15) we obtain the ratio of aggregate investment in innovation and automation:¹⁰

$$\frac{S_t}{\bar{\Xi}_t} = \frac{E_t[\phi \kappa_{RD} (g_{Z,t+1} - \phi) J_{t+1}]}{E_t[\epsilon_\lambda (a_{z,t+1} g_{Z,t+1} - a_{z,t} \phi) (V_{t+1} - J_{t+1})]}, \text{ where, } g_{t+1}^Z = \frac{Z_{t+1}}{Z_t}, a_{z,t} = \frac{A_t}{Z_t} \quad (17)$$

This R&D set-up features trade-offs between automation and innovation. First, as the expected profits of firms in the automated sector increase relative to expected profits in the labour intensive sector (which using (11) and (13) implies that the ratio $E_t[J_{t+1}/(V_{t+1} - J_{t+1})]$ falls), automation investment increases relative to innovation investment. As the differential of profits is ultimately a function of factor

¹⁰Aggregate investment in automation is given by $\Xi_{q,t}(Z_t - A_t)$ and since innovators are of measure 1, $S_{p,t} = S_t$.

prices, automation and innovation respond to changes in real wages and the price of robots. Secondly, investment in automation is a negative function of $a_{z,t}$, the current level of the ratio of total number of automated goods to the total number of goods. As innovation decreases and this ratio falls, the pace of automation slows down. In this respect, automation is a subsidiary activity of innovation; without innovation, automation cannot progress indefinitely. Finally, innovators and automation investors compete for a limited supply of labour and loans to fund their activities.

2.4 Robots Production

Robot producers invest Ω_t final goods to produce $M_t = \varrho\Omega_t^\eta$ robots according to

$$\max_{\Omega,t} \Pi_{\Omega,t} = q_t M_t - \Omega_t \quad s.t. \quad M_t = \varrho\Omega_t^\eta. \quad (18)$$

where q_t is the relative price charged to intermediate good producers for each robot.

2.5 Financial Intermediary

A zero expected profit financial intermediary allocates assets among the household, and the production and R&D sectors and provides annuities to retired households. It sells assets to the households (FA_t^w, FA_t^r), owns capital (K_t) and rents it to firms and lends funds (B_{t+1}) to innovators and automation investors to finance their expenditures (given by S_t and Ξ_t , respectively). Finally, it also owns the innovation plants, robots and good producers and receives the corresponding dividends.

2.6 Market Clearing and Equilibrium

The market clearing conditions are: Final Good: $y_t = C_{w,t} + C_{r,t} + \int_0^{Z_t} \Upsilon_{i,t} di + \Omega_t + I_t$, Asset Flow Condition: $K_{t+1} = (1 - \delta)K_t + I_t$, Credit Markets: $B_{t+1} = S_t + \Xi_t$, Capital Markets: $K_t = \int_0^{Z_t} K_{i,t} di$, Inputs: $\Upsilon_t = \int_0^{Z_t} \Upsilon_{i,t} di$, Robots Markets:

$M_t = \int_0^{A_t} M_{i,t} di$, and Labour Markets: $N_t^{wR} = \int_q L_{A,q,t} di + \int_p L_{i,p,t} di$, and $N_t^{wL} = \int_0^{Z_t \setminus A_t} L_{i,t} di$.

The equilibrium consists of tuples of endogenous predetermined variables $\{FA_{t+1}^z, K_{t+1}, A_{t+1}, Z_{t+1}, B_{t+1}\}$ and of endogenous variables $\{C_t^z, H_t^w, d_t^z, D_t^z, N_t^{wR}, N_t^{wL}, y_t, y_{i,t}, y_{j,t}, M_t, \Omega_t, S_t, \Xi_t, L_{A,t}, S_t, L_{I,t}, V_t, J_t, \lambda_t, \Pi_t^i, \Pi_t^j, C_t, r_t^k, R_t, \Pi_t^{RD}, \Pi_t^A, W_t, W_{RD,t}, P_{i,t}, P_{j,t}, q_t, \varepsilon_t, \varsigma_t\}$ for $z = \{w, r\}$, $i \in A_t$, $j \in Z_t \setminus A_t$ such that:

- a. Workers and retirees maximise utility subject to their budget constraints;
- b. Intermediate and final firms maximise profits; c. Profits are also maximised in innovation, automation, and robot production ; d. The financial intermediary selects assets to maximise profits, and its profits are shared amongst retirees and workers according to their share of assets; and e. Consumption goods, capital, labour, robots, and asset markets clear.

3 Balanced Growth and Comparative Analysis

We define a *BGP* as an equilibrium in which the economy grows at a constant positive rate, greater than population growth, and factor shares, $(r_t^k + \delta)K_t/y_t$, $(W_t/P_t)L_t/y_t$ and $q_t M_t/y_t$, and the interest rate, R_t , are constant.

First, we set the efficiency of investment in the innovation sector ($\tilde{\Psi}_t$) such that investment in innovation does not diverge. Comin and Gertler (2006), in a similar model where the price of capital is determined at time t , assume that $\tilde{\Psi}_t$ equals the value of the stock of capital, and, hence, $\tilde{\Psi}_t$ fluctuates accordingly. In our model there is only one final good and thus the price of capital and the value of the capital stock are constant at t , which invalidates this choice of a scaling factor. Instead, we select the current value of automated goods as scaling factor, so that $\tilde{\Psi}_t \equiv V_t A_t$.¹¹

Second, in models with factor-augmenting technological progress, substitutability between factors of production may prevent the economy from reaching a *BGP*

¹¹We also verify the robustness of our results by setting $\tilde{\Psi}_t \equiv y_t$. Transition paths between *BGPs* are more persistent but the results are qualitatively similar.

(see Acemoglu (2003)). As such, automation may generate an imbalance between the labour and capital/robots income shares. To correct such an imbalance and to ensure the existence of a *BGP*, Acemoglu and Restrepo (2018b) assume a reduced form quasi-labour supply ($L^s(W/RK)$) that brings about a relationship between employment and the ratio of wages and capital income (capital times the interest rate).¹² As a consequence, whenever due to technological progress, capital accumulation grows at a different pace than wages, labour supply adjusts, creating a mechanism that ensures the economy reaches a dynamic equilibrium with constant factors shares. Following this approach would introduce an exogenous mechanism that directly offsets the demographic changes that alters labour supply. Thus, we need an alternative mechanism to ensure constant factor shares in the *BGP*, and, consequently, that the output shares of labour intensive and automated sectors, do not diverge. The introduction of a robot producing sector to determine the relative price of robots plays this role, as explained by the following proposition:

Proposition 1. *Let $g_{q,t} \equiv \frac{q_t}{q_{t-1}}$ be the growth rate of the relative price of robots, $g_t \equiv \frac{y_t}{y_{t-1}}$, the growth rate of output, $g_{n,t}$, the population growth and $g_t^Z \equiv \frac{Z_t}{Z_{t-1}}$, $g_t^A \equiv \frac{A_t}{A_{t-1}}$, the growth rates of varieties Z_t and A_t , respectively. Then, with*

$$\left\{ \begin{array}{l} \eta < 1, \text{ there exists a BGP under which } (g_t)^{\eta-1} g_{q,t} = 1 \text{ and } g_{q,t} = \frac{g_t}{g_{n,t}} > 1. \\ \eta = 1, \text{ the only equilibrium with constant shares has } g_t = g_{n,t} \text{ and } g_t^Z = g_t^A = 1 \\ \quad \text{(technological progress is enough only to offset the survival rate of goods } (\phi)). \\ \eta > 1, \text{ if there is an equilibrium with constant shares, then } g_t^A < 1, \text{ the number} \\ \quad \text{of goods with automated production asymptotically approaches zero, and} \\ \quad g_t < g_{n,t}, \text{ (technological progress is smaller than the survival rate).} \end{array} \right.$$

Proof

Under a *BGP*, g_t^A is constant and, thus, using (16), $a_{z,t} = \frac{A_t}{Z_t}$ is also constant.

Moreover, to keep factor shares constant, the output of the automated and the

¹²They show how such functional form could be rationalised by employing an efficiency wage framework in which workers can appropriate part of the firm's liquid assets (given by RK) while shirking.

labour intensive sectors, and total production ($\int_{i \in A_t} y_{i,t} di$, $1 \int_{i \in Z_t \setminus A_t} y_{i,t} di$, and y_t , respectively) must all grow at the same rate.

Aggregating the demand functions of intermediate goods across A_t and Z_t we obtain that relative prices in each sector — ($P_{i,t}/P_t$) for automated and ($P_{j,t}/P_t$) for labour intensive — grow at the same rate (see the Appendix for details on the de-trended system of equations that determine the equilibrium). Thus, $g_{pM,t} = (g_t^A)^{\psi-1} = (g_t^{ZA})^{\psi-1} = g_{pL,t}$, where $g_{pM,t} \equiv \frac{P_t^i/P_t}{P_{t-1}^i/P_{t-1}}$, $g_{pL,t} \equiv \frac{P_t^j/P_t}{P_{t-1}^j/P_{t-1}}$, $g_t^{ZA} \equiv \frac{Z_t - A_t}{Z_{t-1} - A_{t-1}}$.

Using marginal costs in each sector and demands for labour and robots, the growth rate of the price of robots is equal to the ratio between the growth rates of output and of the labour force. Furthermore, using the production function for robots we obtain

$$g_{pM,t} = g_{q,t}^{(1-\alpha)(1-\gamma_I)} = \left(\frac{g_t}{g_{n,t}} \right)^{(1-\alpha)(1-\gamma_I)} = g_{pL,t}, \quad (19)$$

$$\text{and } (g_t)^{\eta-1} g_{q,t} = 1 \quad (20)$$

Combining (19) and (20)

$$g_t^\eta = g_{n,t} \quad (21)$$

Thus, if $\eta < 1$ then as $g_{n,t} > 1$, it follows that $g_{q,t} > 1$ and $g_t > g_{n,t}$. Hence, the equilibrium has constant factor shares and output growth is greater than population growth.

Using the optimisation condition of the automated sector and robot production sectors, we obtain that under a *BGP*

$$g_t^A = \left(\frac{y_{m,t}}{y_{m,t-1}} \right)^{(1+(1-\eta)(\psi-1)(1-\alpha)(1-\gamma_I))} \left(\frac{r_t^k - \delta}{r_{t-1}^k - \delta} \right)^{\alpha(1-\gamma_I)(\psi-1)} (g_t)^{(1-\eta)(\psi-1)(1-\alpha)(1-\gamma_I)},$$

where $y_{m,t} \equiv \left(\int_{i \in A_t} y_{i,t} di \right) / y_t$

Thus, if $\eta = 1$, and since $\psi > 1$ and $\alpha, \gamma_I < 1$, with constant rates and factor shares,

which entail $\frac{y_{m,t}}{y_{m,t-1}} = 1$, then $g_t^A = 1$ and, using (21), $g_t = g_{n,t}$.

If $\eta > 1$, then in an equilibrium with constant growth rates and factor shares, a positive output growth implies $g_t^A < 1$, and, using (21), $g_t < g_{n,t}$. \square

This result is rather intuitive. As the economy grows faster than population, costs in the labour intensive sector (relative to the automated sector) increase. To obtain a higher output in the automated sector, the stock of produced robots M_t must increase. When $\eta > 1$, this does not require the same increase in the amount of investment (Ω_t). Hence, automation (greater g_t^A) may generate enough final goods to pay for the increase in robots production and also for the increase consumption. In this case $\frac{y_{m,t}}{y_{m,t-1}} > 1$, the robot factor share increases while the labour share decreases. Therefore, as final goods are transformed into robots more easily, to ensure that $\frac{y_{m,t}}{y_{m,t-1}} = 1$, the number of automated varieties must shrink, restricting the growth of M_t to be consistent with constant factor shares. In this scenario, the price of robots (q_t) asymptotically converges to zero.

In contrast, when $\eta < 1$, increases in the stock of produced robots M_t require greater investment. Thus, as the economy grows and robots become more abundant, it becomes relatively harder to transform a complex final good into robots, so that the marginal profitability of robot production falls and the differential in the output shares between the labour intensive and the automated sectors does not increase. As a result, the labour and robot income shares, $\left(l s_t = \frac{W_t L_t}{y_t}\right)$ and $\left(r s_t = \frac{q_t M_t}{y_t}\right)$, do not diverge in the long run. Therefore, holding wages constant, for robots to become more abundant than labour their relative price with respect to the final good must decrease. Eventually, as robots are extensively used in production to the detriment of labour, such price decreases are no longer feasible, preventing the economy from reaching an equilibrium where only robots are used in production.¹³

In sum, to avoid a labour-free singularity we restrict the efficiency gains in the transformation of final goods into robots. At a *BGP* the final good embeds an

¹³We extend the model to relax the assumption that robots fully depreciates in one period. Transitions between *BGP* are affected, but the restriction on η to ensure a *BGP* exists remains the same. Details are presented in the Appendix.

increasing set of intermediate goods, some produced employing the less efficient factor — labour. It is reasonable to assume that robots that are capable of replacing labour in production would also eventually increase in complexity to reflect the level of economic development. The robot producing sector with $\eta < 1$ reproduces such an environment by requiring that as the number of varieties and income increase, the transformation of the final good into robots does not become more efficient. This is similar to Aghion, Jones, and Jones (2017), who introduce a “bottleneck” to prevent a singularity by restricting the productivity gain of an essential input, sustaining its relative price in terms of the final good.¹⁴

We now turn to the comparative analysis, assessing the effects of demographic changes on output growth, automation and the labour income share under *BGPs*.

Proposition 2. *Starting from an initial BGP equilibrium, if population growth falls, the new BGP is characterised by lower per-capita output growth*

Proof

If the economy is initially at a *BGP*, $\eta < 1$. Then the result follows directly from (21). □

A lower growth of labour supply directly leads to lower output growth and also decreases incentives to innovate. This further reduces output growth such that output per capita growth in the new *BGP* falls. In sum, since labour is an essential input for innovation and the production of new goods, labour supply growth ends up determining output growth. In the words of Aghion, Jones, and Jones (2017), “growth may be constrained not by what we are good at but rather by what is essential and yet hard to improve”.

Before turning to the effects of a fall in population growth (proposition 3) and population ageing (proposition 4) on automation ($a_{z,t} = A_t/Z_t$) and on the labour

¹⁴Aghion, Jones, and Jones (2017) discuss different alternatives to prevent singularities. In one of the cases, which they frame as a form of Baumol’s cost disease, this is achieved by restricting the productivity gain of some tasks in a production framework where tasks are complementary. Thus, as income/output increases, the relative price of these tasks are sustained.

share ($l s_t$), we make two assumptions.¹⁵ (A1) At *BGP*, $a_z \leq \frac{\rho\phi}{\epsilon_\lambda}$ and (A2) $\psi > 1 + \frac{1}{(1-\eta)(1-\alpha)(1-\gamma_I)}$. Under our calibration (described in the next section) (A1) restricts the degree of automation such that no more than 75% of the existing varieties are produced in the automated sector, and (A2) restricts the intermediate good firm's mark-up to be smaller than 23%.¹⁶ We first look at the implication of population growth changes.

Proposition 3. *Starting from a BGP, when population growth falls, the change in the degree of automation and in the labour income share are respectively given by*

$$\frac{da_z}{a_z} = \frac{-\frac{1}{\eta} \left(d_1 \frac{\kappa_{RD}}{\kappa_L} \Gamma_2 + \frac{\rho}{\kappa_L} \Gamma_1 d_2 \right) \frac{dg_n}{g_n} - \left(d_1 \frac{\kappa_{RD}}{\kappa_L} + \frac{\rho}{\kappa_L} d_2 \right) \frac{dR}{R}}{c_2 d_1 - d_2 c_1} \quad (22)$$

$$\frac{dls}{ls} = (1 - y_L) \frac{\frac{1}{\eta} \left(c_1 \frac{\kappa_{RD}}{\kappa_L} \Gamma_2 + \frac{\rho}{\kappa_L} \Gamma_1 c_2 \right) \frac{dg_n}{g_n} + \left(c_1 \frac{\kappa_{RD}}{\kappa_L} + \frac{\rho}{\kappa_L} c_2 \right) \frac{dR}{R}}{c_2 d_1 - d_2 c_1} \quad (23)$$

where $y_L \equiv \left(\int_{i \in Z \setminus A} y_i di \right) / y$ is the share of output produced using labour, and $c_1, c_2, d_1, d_2 > 0$ are functions of the initial *BGP*. If A1 holds then $c_2 d_1 - d_2 c_1 > 0$ and if A2 holds then parameters $\Gamma_1, \Gamma_2 > 0$. As a result, a new *BGP* with lower population growth, in which interest rates are lower, is characterised by a higher degree of automation and a lower labour income share.

We now turn to the analysis of the effects of the change in the age composition of the population that results from a rise of the share of retirees relative to workers (population ageing), holding population growth constant.

Proposition 4. *Starting from a BGP, the changes in the degree of automation and*

¹⁵Both assumptions are sufficient but not necessary for propositions 3 and 4 to hold. Proofs of both propositions are shown in the Appendix.

¹⁶Since $g_t^Z = g_t^A = (g_t)^{(1-\eta)(\varphi-1)(1-\alpha)(1-\gamma_I)}$, if (A2) does not hold then an increase of one percent on the growth of varieties generates a more than 1% increase in output growth.

in the labour income share due to population ageing are respectively given by

$$\frac{da_z}{a_z} = \frac{\frac{dRDpop}{RDpop}(d_1 + d_2) - \left(d_1 \frac{\kappa_{RD}}{\kappa_L} + \frac{\rho}{\kappa_L} d_2\right) \frac{dR}{R}}{c_2 d_1 - d_2 c_1} \quad (24)$$

$$\frac{dls}{ls} = (1 - y_L) \frac{- \left(\frac{dRDpop}{RDpop}(c_1 + c_2) - \left(c_1 \frac{\kappa_{RD}}{\kappa_L} + \frac{\rho}{\kappa_L} c_2\right) \frac{dR}{R}\right)}{c_2 d_1 - d_2 c_1} \quad (25)$$

where $RDpop = \frac{N^{wRD}}{N}$, is the share of R&D workers in the population and $c_1, c_2, d_1, d_2 > 0$. Thus, the lower the weight of labour on R&D activity (κ_L) is, the more likely it is that in the BGP of a more aged economy, in which interest rates are smaller, is characterised by a higher degree of automation and a lower labour income share.

In both cases, automation and the labour income share in the new *BGP*s are directly affected, due to labour supply effects, and indirectly, due to changes in the equilibrium interest rate. A fall in the interest rate boosts greater investments in innovation and automation. However, as labour supply growth falls, relative costs in the labour intensive sector increase, giving incentives for R&D investment to be diverted to automation, which leads to a decrease in the labour share. In the case of ageing, the direct effect of labour supply lead to a fall of both innovation and automation.

We find that the positive productivity effects of the increase in automation are not able to fully compensate for the effects of the fall in labour supply growth once the economy converges to the new *BGP*. Acemoğlu and Restrepo (2018b) highlight a long-run productivity effect occurring when the pace of automation increases. As the economy converges back to a new *BGP*, the return on capital (interest rate), which initially increased together with automation, must fall back. As a result, capital accumulates, with the gains accruing to the relatively inelastic factor, namely, labour. In our model, capital is complementary to labour and robots. Thus, although capital accumulation also occurs after an ageing shock, the main driver is the saving effect due to the fall in marginal propensity to consume of households approaching retirement. In our setting, automation increases while interest rate

falls and capital accumulation increases, in line with the empirical evidence.¹⁷

4 Quantitative Analysis

We now describe the medium-run effects of a fall in fertility and an increase in longevity. Before presenting the results we briefly describe the choices of the parameter values.

4.1 Calibration

One period of the model corresponds to one year. Workers' age are between 20 and 65 years, and retirees are above 65 years old. Parameters controlling the law of motion of population are calibrated to match the average share of workers and retirees in total population in 1993 in the US, and the number of working years the individual may live before retiring (45 years). They turn out to be a birth rate of $\omega^y = 0.0265$, a probability of retirement of $1 - \omega^w = 0.022$, and a death probability of $1 - \omega^r = 0.07$.

The share of workers in innovation (Sw_{RD}) is set to match the share of R&D workers in US population, and $drop_{RD}$ is set to make the average age of R&D workers to be 40 (slightly lower than the average age of employed scientists reported in the Survey of Doctorate Recipients (SDR) of the National Science Foundation - 2013).

For parameters driving innovation, we closely follow Comin and Gertler (2006). Obsolescence (ϕ) and productivity in innovation (χ) are set so that growth of GDP per working age person is 0.016 (as in the US from 1970 onwards) and the share of innovation expenditures in total GDP is 0.012. The mark-up for intermediate goods is 15%. The elasticity of intermediate goods with respect to $R\&D$ (ρ) is 0.9.

The rate of automation is set to $\lambda = 0.1$. The elasticity of this rate to increasing

¹⁷Incorporating an explicit capital deepening effect as a result of automation in the long run as in Acemoglu and Restrepo (2018b) could further offset the negative effect of lower labour supply growth. Berg, Buffie, and Zanna (2018), employing a neoclassical growth model with robots, show that automation leads to higher growth and inequality. The key distinctions with ours is that technology in our case is endogenous, producing the trade-off between innovation and automation that leads to a decrease in growth.

intensity (ϵ_λ) is set to 1. Finally we set $\kappa_{RD} = 1$.

Regarding the link between demographics and innovation, which depends on the elasticity of invention to employed workers in *R&D* (κ_L), we follow Aksoy, Basso, Smith, and Grasl (2019) who, reflecting the changes in productivity of households of different ages described in Jones (2010), calibrate this parameter to $\kappa_L = 0.5$.

Finally we set the macro parameters in line with Comin and Gertler (2006). The discount factor $\beta = 0.96$; the capital share $\alpha = 0.33$; the yearly depreciation rate $\delta = 0.08$ the share of intermediate goods $\gamma_I = 0.5$. Following Gertler (1999) we set the intertemporal elasticity of substitution $(1/(1 - \nu)) = 0.25$. Given output and population growth and $(g_t)^{\eta-1}(g_t/g_{w,t}) = 1$, we obtain $\eta = 0.15$.¹⁸

4.2 Demographic Transition and Growth in Europe and in the US

We now use the model to analyse the consequences of demographic changes over the next decades predicted for the US and for Core Europe (defined as the aggregation of Germany, France, Italy and Spain).

From population shares of workers (age 20-65) and retirees (age above 65) in 1993 and 2055 for each country/region, taken from (United Nations (2016)), we recover the fertility and survival probability rates that are consistent with a stationary population distribution. We then simulate a transition path from 1993 to 2055 that closely matches the projected population changes. After discarding the first seven years of the simulated period to decrease the dependence of the simulation to the initial steady state, we depict results from 2000 up to 2040 (Figure 1).

Our interpretation is as follows. As mortality decreases, savings increase and the interest rate falls, providing more resources for innovation, automation, and capital investments. As fertility decreases, the new cohort of workers entering in the labour market also decreases, pushing wages up. Lower wages depress the profitability of

¹⁸A table listing all parameter values is in the Appendix.

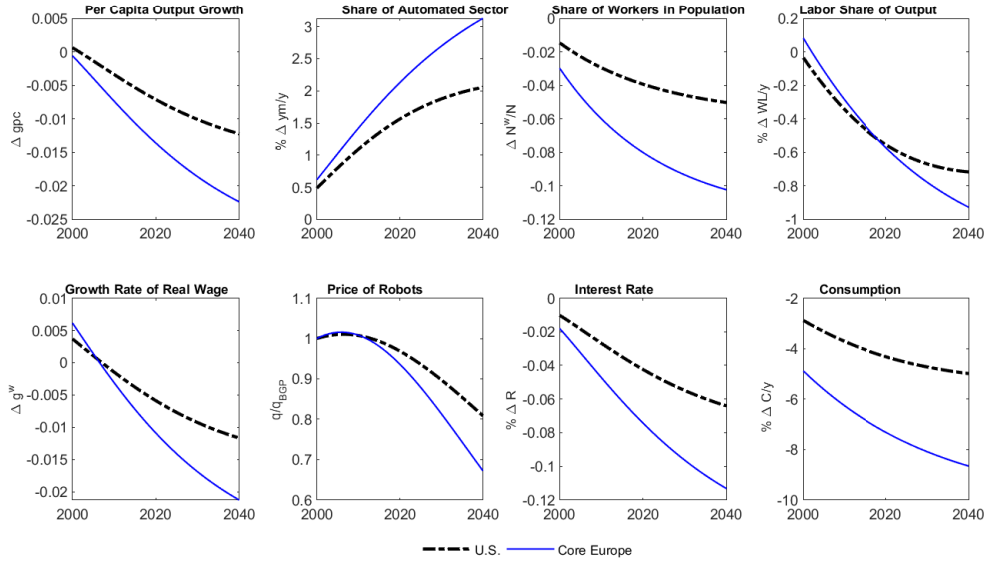


Figure 1: Demographic Transition: United States and Europe

Note: The figure plots the effects of the projected demographic changes in each region. For Per Capita Output growth = $(y_t/y_{t-1})(N_{t-1}/N_t)$, Share of Workers in Population - N_t^w/N_t and Growth Rate of real wage - W_t/W_{t-1} we show the Change relative to the initial *BGP*. For Share of Automated Sector - $y_{m,t}$, Labour Share of Output $W_t L_t/y_t$, R_t - Real Interest Rate and Consumption - C_t/y_t we show the percentage change relative to the initial *BGP*. For the price of robots we show the ratio of price of robots at transition and at *BGP* - q_t/q_{BGP} .

labour intensive sector, boosting automation. As robots are more productive than labour, productivity rises. Both of these mechanisms leads to an increase in output growth.¹⁹ However, as profits of the labour intensive sector fall the investment in innovation decreases. Moreover, a drop in fertility implies that the pool of workers available for innovation decreases.²⁰ As the growth of new goods Z_t decreases, overall growth is reduced, hampering the pace of automation in the future and, ultimately, delivering lower per capita growth. Thus, the initial effect of higher savings and lower interest rates wears off and the reduction in invention of new intermediate goods outweighs the productivity gains from automation.²¹

¹⁹The change in the growth rate from 1993 till 2000 is positive, we show results from 2000 in figure 1.

²⁰Using a cross-section data on patents and demographics, Acemoglu and Restrepo (2018a) document an increase in the number of patents related to robots and a decrease in those related to computer, software, nanotechnology and pharmaceuticals, supporting the trade-off between innovation and automation present in our model.

²¹Gordon (2012) include demographic changes as one of the “headwinds” preventing economic growth. Aksoy, Basso, Smith, and Grasl (2019) estimate that from 2000 till 2015, population changes have lead to a reduction in output growth in the US of around 0.5 percentage points. The link between demographics and innovation is behind this negative effect. In a similar projection exercise, they report that demographic changes in the next decades lead to a more sizeable reduction in output than the results reported here. The key distinction of our analysis is the inclusion of automation, which has a positive offsetting effect on growth. Nonetheless, both analyses give support to the “headwind” effect of demographics on growth.

The share of workers in the European economies decreases faster (since fertility is considerably lower in Europe), boosting automation more than in the US. This result is consistent with the data. During the period 2000-2015, automation, measured as the stock of robots by thousand of employees, increased from 1.55 to 2.7 in the four core European countries, with an increase from 2.28 to 4.24 in Germany, 0.79 to 1.6 in Spain, 0.81 to 1.17 in France, and from 1.7 to 2.5 in Italy, while in the US it increased from 0.64 to 1.55 (International Federation of Robotics (2017)).

Graetz and Michaels (2018) show that during the period of 1990-2005 the price of robots fell by roughly 20% on average across developed countries. In our set-up, in a *BGP* the price of robots q must be increasing, ensuring it does not diverge from the growing real wages. However, during the demographic transition, as the degree of automation increases, the growth of the price of robots falls. Figure 1 depicts the price of robots, q , relative to its *BGP* path. By 2030, q would be 10% lower due to demographic changes. A more substantial fall would require the framework to account also for technological progress that increases the efficiency in robot production (we present an extension with these features in section 4.4).

Despite the initial increase in wages, as the economy becomes more automated, eventually the labour share of income decreases by around 0.6 percent from 2000 to 2020, due to both a fall in wages and employment. Karabarbounis and Neiman (2014) shows that from 2000 to 2015 the global labour income share fell around 4 percent (roughly from 0.615 to 0.59). The labour share has different drivers: price of capital, changes in goods and labour market structures, and automation, which we show are influenced by demographic changes.²² Finally, as in Aksoy, Basso, Smith, and Grasl (2019) and Eggertsson, Mehrotra, and Robbins (2019), demographic changes have significant impact in equilibrium real interest rates.

²²Bergholt, Furlanetto, and Faccioli (2019) show evidence that automation, or technological progress that leads to the substitution of labour by capital, may be behind the fall in labour income shares.

4.3 Extensions

In an attempt to account for different ways in which labour markets and technological progress may evolve, we modify labour supply, and the roles of workers in innovation and the integration of robots in economic activity.

4.3.1 Labour Market Configurations

We consider three alternative scenarios. First, we set $\kappa_L = 0$ so that the R&D sector does not need labour to transform the final good into a new or an automated good (*No Labour in R&D*). Second, we allow new workers with idiosyncratic inherited talent to select to which sector they will supply labour (*Labour Choice*). Once this decision is taken, workers drop from the R&D sector and cannot join the R&D sector during their working lives as in the benchmark case. Thus, the share of new workers that join the R&D sector, Sw_{RD} , is a function of the wage differential between the R&D and the production sectors (W_t^{RD}/W_t).²³ Third, workers may alter labour supply in the intensive margin (*Intensive Labour*). Comparisons of the effects of the demographic transition under these alternative scenarios with the benchmark for the US are in Figure 2.

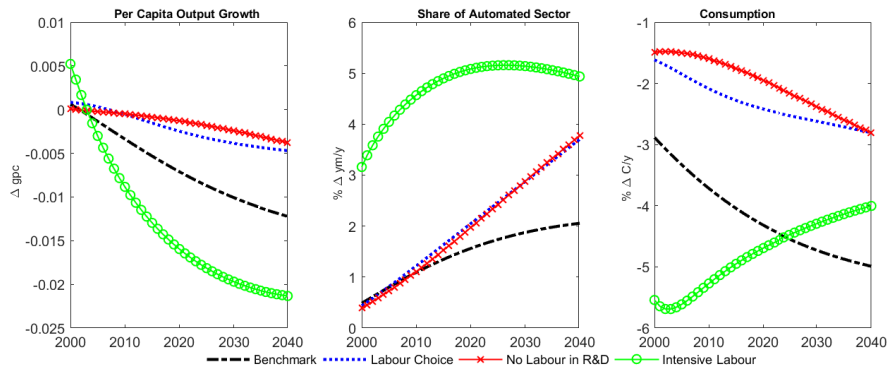


Figure 2: Demographic Transition: Labour in Innovation

Note: The figure plots the effects of the projected demographic changes under different specifications. For Per Capita Output growth = $(y_t/y_{t-1})(N_{t-1}/N_t)$ we show the Change relative to the initial *BGP*. For Share of Automated Sector - $y_{m,t}$ and Consumption - C_t/y_t we show the percentage change relative to the initial *BGP*.

As expected, excluding the labour input in innovation offsets negative effects of

²³Details on this extension are in the Appendix.

the demographic transition on growth, but still lower population growth ends up being detrimental (as discussed in proposition 2). Consumption decreases by less in this scenario and without a labour supply constraint demographic changes lead to more automation.

A similar outcome arises when entrant workers select their sector of activity. As automation peaks up, the wage in the production sector falls. Wages in the R&D sector, given the lack of substitutes, do not fall and thus Sw_{RD} increases. Labour employed in automation and innovation increases, with the former increasing more. The trade-off between innovation and automation is still present, but is less pronounced as the economy diverts their labour resources towards R&D.²⁴

Increasing labour supply in the intensive margin allows investment to be steered towards automation, and, hence, yields a higher increase of the automated sector than in the benchmark. Nonetheless, lower innovation investment eventually brings per capita output down. These results indicate reallocation of labour supply towards R&D is more effective in offsetting the effects of the demographic change on growth than when workers adjust labour supply.²⁵

Finally, we analyse the effects of delaying the retirement age. We alter the retirement age to keep the ratio between the durations of working life and of retirement approximately constant. Under the UN projections, in the US life expectancy will increase by 12,7 years from 1993 to 2055. Thus, we simulate the effects of rising retirement age by 8 years, roughly two thirds of the increase in life expectancy.

Delaying the retirement age obviously delivers a lower fall in the share of workers to total population. Incentives to automate are lower, and the fall in output growth is not as large (-0.4 pp instead of -1.2 pp) as in the benchmark (Figure 3). Although delaying retirement slows down the fall in working age population, it cannot avoid

²⁴A caveat is in order: as Bloom, Jones, Reenen, and Webb (2017) show, despite a sharp increase in labour employed in R&D, the production of new ideas is fairly constant; in their conclusion new ideas seem to be harder to find. If that is indeed the case migration of workers towards R&D might be less effective in dampening the effects of the demographic transition.

²⁵Consumptions fall more significantly in this extensions since the marginal propensity to consume (ς_t) becomes a function of the expected growth rate of wages. As wages are expected to fall, savings increase.

the negative impact of population ageing on innovation activity (less young workers involved in the creation of new goods depresses innovation). Wage growth is sustained since automation does not increase as much, and, as a result, the labour share is higher.²⁶ As such the interaction between demography and technology may depend on labour market institutions. Institutions leading to more inclusive employment and alignment of wages with productivity are more likely to deliver more incentive to innovation, less to automation, and, therefore, less wage stagnation and higher labour shares.²⁷

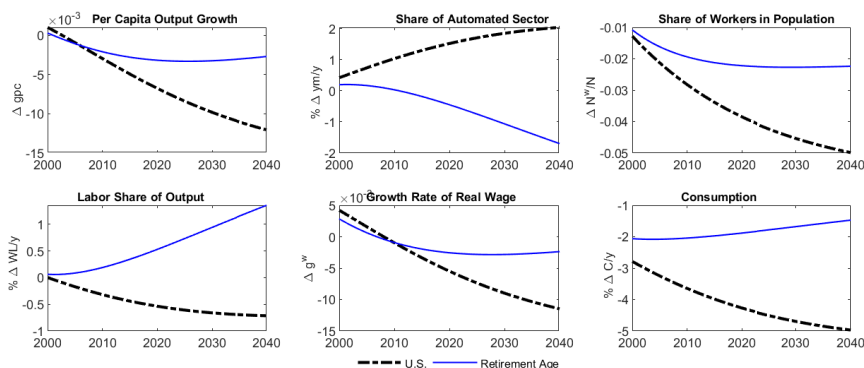


Figure 3: Longevity, Delay in Retirement Age, Automation and Growth

Note: The figure plots the effects of the projected demographic changes under different specifications. For Per Capita Output growth = $(y_t/y_{t-1})(N_{t-1}/N_t)$, Share of Workers in Population - N_t^w/N_t and Growth Rate of real wage - W_t/W_{t-1} we show the Change relative to the initial *BGP*. For Share of Automated Sector - $y_{m,t}$, Labour Share of Output $W_t L_t/y_t$, and Consumption - C_t/y_t we show the percentage change relative to the initial *BGP*.

4.3.2 Robots in Production and Innovation

In the benchmark model robots can only be used in production and their productivity relative to labour is constant ($\theta_t = \bar{\theta}$). We first consider the possibility (*Robots Productivity*) that the relative productivity of robots could rise as the automated sector (A_t) grows ($\theta_t = \bar{\theta} A_t^\mu$, $\mu = 0.1$). Second, we drop the assumption that robots fully depreciate at each period (*Depreciation of Robots*). Third, robots can also be used to innovate and automate (*Robots in R&D*). In this case, investment in

²⁶We do not explore whether the age structure within the working population has an effect on automation. If older workers are more/less replaceable relative to their younger counterparts our results may be altered. On this, see Acemoglu and Restrepo (2018a).

²⁷See also Lordan and Neumark (2017) for empirical evidence on rises of minimum wages boosting automation

innovation and automation are now, respectively, $IS_t = (S_{p,t})^{\kappa_{RD}}((1 - a_{z,t})L_{I,t}^{\xi_{LM}} + a_{z,t}M_{I,t}^{\xi_{LM}})^{\kappa_L/\xi_{LM}}$ and $\Xi_{A,t} = (\Xi_{q,t})^{\kappa_{RD}}((1 - a_{z,t})L_{A,t}^{\xi_{LM}} + a_{z,t}M_{A,t}^{\xi_{LM}})^{\kappa_L/\xi_{LM}}$, where $M_{I,t}$ and $M_{A,t}$ are robots used in *R&D*, produced by a similar robot production sector as in the benchmark model, and ξ_{LM} is the elasticity of substitution of robots and labour. Thus, as the economy becomes more automated ($a_{z,t}$ increases), robots replace a larger share of labour in innovation. This specification resembles the artificial intelligence model of Aghion, Jones, and Jones (2017), but restricting efficiency gains in robots production to ensure the economy converges to a *BGP*.

In all these extensions, the negative effects of lower population growth on GDP per capita growth and on consumption are of smaller magnitude (Figure 4). As intermediate inputs are used in production, higher robot productivity increases *TFP*, generating positive spillovers on the labour intensive sector. That partially offsets the negative impact of lower labour supply, reducing the incentive to divert resources from innovation such that the share of the automated sector is not as large as in the benchmark case. When robots are used in R&D, the negative effects of resource reallocation on innovation are also mitigated. Finally, under partial depreciation, during the transition the price of robots falls less than in the benchmark case, reducing the cost differential between the labour intensive and the automated sectors and also dampening the increase in automation.

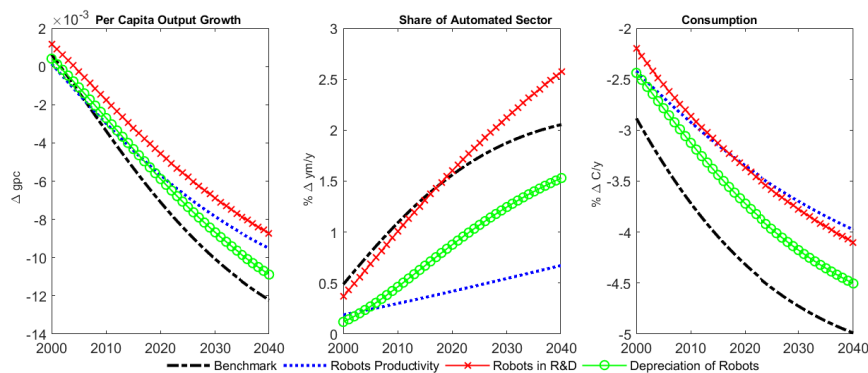


Figure 4: Demographic Transition: Robots vs Labour

Note: The figure plots the effects of the projected demographic changes under different specifications. For Per Capita Output growth = $(y_t/y_{t-1})(N_{t-1}/N_t)$ we show the Change relative to the initial *BGP*. For Share of Automated Sector - $y_{m,t}$ and Consumption - C_t/y_t we show the percentage change relative to the initial *BGP*.

4.4 Divergence and “Robocalypse”

Conceivably, it may be that robots are produced more efficiently as the economy becomes more automated. We incorporate this possibility by allowing TFP in the robots production sector to increase in the medium run (analogous to when $\eta > 1$). Eventually (after more than 150 years in our simulation) TFP in the robots sector converges to a constant, hence, in this scenario the efficiency restriction that ensures BGP convergence is in effect only in the long run.

Results are displayed in Figure 5. As demographics triggers automation, robots are produced more cheaply, further raising incentives to automate (from 2000 till 2020 the price of robots, q , falls by 40%). As TFP increases together with the ratio $a_{z,t} = (A_t/Z_t)$, most of the output is produced by the automated sector. Eventually, as innovation investment is compromised, output growth is negatively affected despite the efficiency gains in the production of robots. Thus, if robots cannot invent new intermediate goods, a demographic transition that generates automation, while processes to produce robots become more efficient, fails to increase output growth. Thus, a “robocalypse scenario”, resembling the immiseration equilibrium of Benzell, Kotlikoff, LaGarda, and Sachs (2015), may arise. This conclusion is robust to the case when both TFP of robots production and productivity of robots (θ_t) in good’s production both increase during the transition (*Robocalypse with Robots Productivity*).

5 Concluding remarks

Demographic changes are bound to shape the macroeconomic landscape of the next decades. Population ageing may affect the effectiveness of monetary and fiscal policies (Eggertsson, Mehrotra, and Robbins (2019) and Basso and Rachedi (2018)). In the medium run demographic changes may restrain economic growth (Aksoy, Basso, Smith, and Grasl (2019)) and promote automation (Acemoglu and Restrepo (2018a)).

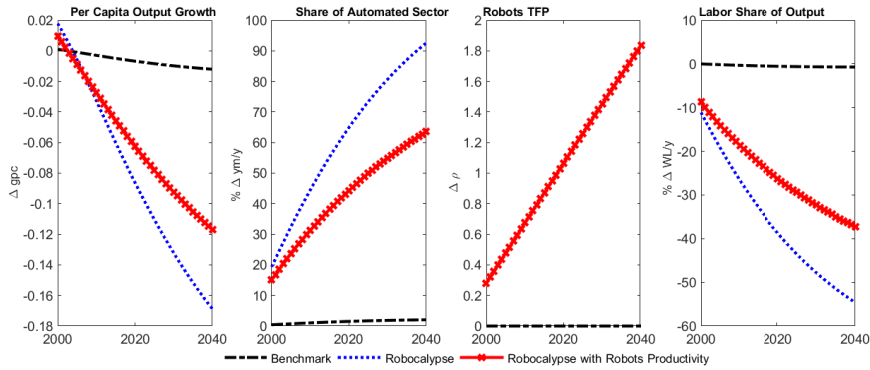


Figure 5: Demographics and Robocalypse

Note: The figure plots the effects of the projected demographic changes under different specifications. For Per Capita Output growth $= (y_t/y_{t-1})(N_{t-1}/N_t)$ and Robots $TFP - \varrho_t/\varrho_{t-1}$ we show the Change relative to the initial BGP . For Share of Automated Sector - $y_{m,t}$ and Labour Share of Output $(W_t L_t)/y_t$ we show the percentage change relative to the initial BGP .

We have analysed the main interactions between demographics and technology and their implications for economic growth. In our analysis, we stress the importance of considering that innovation and automation activities require resources, the trade-offs between the generation of new goods and the automation of the production of existing ones, and some likely consequences of population ageing for the productivity of R&D.

While keeping complementarities among inputs (intermediate goods, capital, and either labour and robots), we put at the front of our analysis the labour displacement effect of automation, and leave the creation of new job opportunities (the reinstatement effect of technological changes) only to innovation. This may be an extreme case but still a good starting point for the analysis of the consequences of demographic and technological changes.

Admittedly, it may be too early to conjecture how the new developments from robotics and artificial intelligence will change the production of goods and R&D activities. Thus, we have also considered several alternative specifications of how innovation and automation come about. The main conclusion is that, even though lower population growth and population ageing increase automation and, initially, raise productivity growth, in the medium run they are detrimental to economic growth. When using population forecasts for US and Europe, the model predicts a

fall in output per capita growth, an increase in automation, and a fall in the labour income share and in interest rates, reinforcing the economic trends already observed in the last decades.

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Appendix A. Equilibrium Conditions

We start by looking at the final and intermediate producers.

Goods Production Sector

Intermediate good firms $j \in A_t$ select capital, robots and inputs to minimise total costs, $TC = P_t q_t M_t^j + (r_t^k + \delta) K_t^j + P_t \Upsilon_t^j$ given a level of production $Y_t^j = [(K_t^j)^\alpha (\theta_t M_t^j)^{(1-\alpha)}]^{(1-\gamma_I)} [\Upsilon_t^j]^{\gamma_I}$.

Let ν_t^j be the real marginal cost for firm j . Then

$$\nu_t^j = \frac{(r_t^k + \delta)^{\alpha(1-\gamma_I)} q_t^{(1-\alpha)(1-\gamma_I)}}{(\alpha(1-\gamma_I))^{\alpha(1-\gamma_I)} \gamma_I^{\gamma_I} ((1-\alpha)(1-\gamma_I))^{(1-\alpha)(1-\gamma_I)}} \quad (\text{A.1})$$

$$K_t^j = \nu_t^j \frac{\alpha(1-\gamma_I)}{(r_t^k + \delta)} y_{j,t} \quad (\text{A.2})$$

$$\Upsilon_t^j = \nu_t^j \gamma_I y_{j,t} \quad (\text{A.3})$$

$$M_t^j = \nu_t^j \frac{(1-\alpha)(1-\gamma_I)}{q_t} y_{j,t} \theta_t \quad (\text{A.4})$$

And given the final good production function,

$$\frac{P_t^j}{P_t} = \frac{\psi - 1}{\psi} \nu_t^j \quad (\text{A.5})$$

$$y_{j,t} = \left(\frac{P_{j,t}^j}{P_t} \right)^\psi y_t \quad (\text{A.6})$$

$$\Pi_t^j = \left[\frac{P_t^j}{P_t} - \nu_t^j \right] y_{j,t} = \frac{1}{\psi - 1} \nu_t^j y_{j,t} \quad (\text{A.7})$$

Intermediate good firms $i \in Z_t \setminus A_t$ select capital, labour and inputs to minimise total costs, $TC = W_t L_t^i + (r_t^k + \delta) K_t^i + P_t \Upsilon_t^i$ given a level of production $Y_t^i = [(K_t^i)^\alpha (L_t^i)^{(1-\alpha)}]^{(1-\gamma_I)} [\Upsilon_t^i]^{\gamma_I}$.

Let ν_t^i be the real marginal cost for firm i . Then

$$\nu_t^i = \frac{(r_t^k + \delta)^{\alpha(1-\gamma_I)} (W_t)^{(1-\alpha)(1-\gamma_I)}}{(\alpha(1-\gamma_I))^{\alpha(1-\gamma_I)} \gamma_I^{\gamma_I} ((1-\alpha)(1-\gamma_I))^{(1-\alpha)(1-\gamma_I)}} \quad (\text{A.8})$$

$$K_t^i = \nu_t^i \frac{\alpha(1-\gamma_I)}{(r_t^k + \delta)} y_{i,t} \quad (\text{A.9})$$

$$\Upsilon_t^i = \nu_t^i \gamma_I y_{i,t} \quad (\text{A.10})$$

$$L_t^i = \nu_t^i \frac{(1-\alpha)(1-\gamma_I)}{(W_t)} y_{i,t} \quad (\text{A.11})$$

And given the final good production function,

$$\frac{P_t^i}{P_t} = \frac{\psi}{\psi - 1} \nu_t^i \quad (\text{A.12})$$

$$y_{i,t} = \left(\frac{P_{i,t}}{P_t} \right)^\psi y_t \quad (\text{A.13})$$

$$\Pi_t^i = \left[\frac{P_t^i}{P_t} - \nu_t^i \right] y_{i,t} = \frac{1}{\psi - 1} \nu_t^i y_{i,t} \quad (\text{A.14})$$

Robots Production Sector

Optimisation of robots producers imply

$$\Pi_{\Omega,t} = q_t P_t M_t - P_t \Omega_t \quad (\text{A.15})$$

$$M_t = \varrho \Omega_t^\eta \quad (\text{A.16})$$

$$q_t = \frac{\Omega_t}{M_t \eta} \quad (\text{A.17})$$

Innovation Process

One can easily determine the flow of the stock of goods (Z_t) and goods for which robots can be employed in the production process (A_t), which are given by

$$\frac{Z_{t+1}}{Z_t} = \chi \left(\frac{S_t}{\tilde{\Psi}_t} \right)^\rho (L_{I,t}/N_t)^{\kappa_L} + \phi, \text{ and} \quad (\text{A.18})$$

$$\frac{A_{t+1}}{A_t} = \lambda \left(\frac{(Z_t - A_t)^{\kappa_{RD} + \kappa_L} (\Xi_t)^{\kappa_{RD}} (L_{A,t})^{\kappa_L}}{\tilde{\Psi}_t^{\kappa_{RD}} N_t^{\kappa_L}} \right) \phi [Z_t/A_t - 1] + \phi \quad (\text{A.19})$$

Investment in R&D (S_t) and labour demand in product creation is determined by (12) which using (10) becomes

$$S_t = \kappa_{RD} R_{t+1}^{-1} \phi E_t J_{t+1} (Z_{t+1} - \phi Z_t). \quad (\text{A.20})$$

$$L_{I,t} W_{RD,t} = \frac{S_t \kappa_L}{\kappa_{RD}} \quad (\text{A.21})$$

Profits are given by the total gain in seeing the right to goods invented as a result of the previous period investment S_{t-1} to adopters minus the cost of borrowing for that investment. Thus,

$$\Pi_{RD,t} = \vartheta \int_{i \in Z_t \setminus A_t} \Pi_t^i di - S_{t-1} R_t - L_{I,t} W_{RD,t}$$

Investment in automation (Ξ_t) is determined by solving (14). We thus obtain

the following condition

$$\frac{(Z_t^q - A_t^q)^{\kappa_{RD} + \kappa_L}}{\tilde{\Psi}_t^{\kappa_{RD}} N_t^{\kappa_L}} \lambda' R_t^{-1} \phi[V_{t+1} - J_{t+1}] = 1 \quad (\text{A.22})$$

Assuming the elasticity of λ_t to changes in its input (denoted $\tilde{\epsilon}_\lambda$) is constant and smaller than one, we define $\epsilon_\lambda = \frac{\lambda'}{\lambda_t} \frac{(Z_t^q - A_t^q)^{\kappa_{RD} + \kappa_L} \Xi_{q,A,t}}{\tilde{\Psi}_t^{\kappa_{RD}} N_t^{\kappa_L}} \frac{1}{\kappa_{RD}} = \frac{\tilde{\epsilon}_\lambda}{\kappa_{RD}}$, then we obtain²⁸

$$\Xi_t = \epsilon_\lambda \lambda_t R_t^{-1} \phi[V_{t+1} - J_{t+1}] (Z_t - A_t) \quad (\text{A.23})$$

$$L_{A,t} W_{RD,t} = \Xi_t \frac{\kappa_L}{\kappa_{RD}} \quad (\text{A.24})$$

Finally, the value of labour intensive goods and automated goods are given by

$$J_t = \vartheta \Pi_t^j + (R_{t+1})^{-1} \phi E_t[J_{t+1}], \text{ and} \quad (\text{A.25})$$

$$V_t = \vartheta \Pi_t^i + (R_{t+1})^{-1} \phi E_t V_{t+1} \quad (\text{A.26})$$

Profits for adopters are given by the gain from marketing specialised intermediated goods net the amount paid to inventors to gain access to new goods and the expenditures on loans to pay for adoption intensity.

$$\Pi_{A,t} = \vartheta \int_{j \in A_t} \Pi_t^j dj - \Xi_{t-1} R_t - L_{A,t} W_{RD,t}$$

Household Sector

Retiree j decision problem is

$$\max V_t^{jr} = \{(C_t^{jr})^v + \beta \omega_{t,t+1}^r ([V_{t+1}^{jr}]^v)\}^{1/v} \quad (\text{A.27})$$

subject to

$$C_t^{jr} + F A_{t+1}^{jr} = \frac{R_t}{\omega_{t-1,t}^r} F A_t^{jr} + d_t^{jr}. \quad (\text{A.28})$$

The first order condition and envelop theorem are

$$(C_t^{jr})^{v-1} = \beta \omega_{t,t+1}^r \frac{\partial V_{t+1}^{jr}}{\partial F A_{t+1}^{jr}} (V_{t+1}^{jr})^{v-1}, \quad (\text{A.29})$$

$$\frac{\partial V_t^{jr}}{\partial F A_t^{jr}} = (V_t^{jr})^{1-v} (C_t^{jr})^{v-1} \frac{R_t}{\omega_{t-1,t}^r}. \quad (\text{A.30})$$

Combining these conditions above gives the Euler equation

$$C_{t+1}^{jr} = (\beta R_{t+1})^{1/(1-v)} C_t^{jr} \quad (\text{A.31})$$

²⁸We aggregate across automation investors to obtain $\Xi_t = X_{q,t}(Z_t - A_t)$ and $L_{A,t} = L_{A,q,t}(Z_t - A_t)$. Also note that $\tilde{\epsilon}_\lambda = \epsilon_\lambda \kappa_{RD} \leq 1$. We use this result in the proof of proposition 3 and 4.

Conjecture that retirees consume a fraction of all assets (including financial assets, profits from financial intermediaries), such that

$$C_t^{jr} = \varepsilon_t \varsigma_t \left[\frac{R_t}{\omega_{t-1,t}^r} F A_t^{rj} + D_t^{rj} \right]. \quad (\text{A.32})$$

Combining these and the budget constraint gives

$$F A_{t+1}^{jr} = \frac{R_t}{\omega_{t-1,t}^r} F A_t^{jr} (1 - \varepsilon_t \varsigma_t) + d_t^{jr} - \varepsilon_t \varsigma_t (D_t^{rj}).$$

Using the condition above the Euler equation and the solution for consumption gives

$$\begin{aligned} (\beta R_{t+1})^{1/(1-v)} \varepsilon_t \varsigma_t \left[\frac{R_t}{\omega_{t-1,t}^r} F A_t^{rj} + D_t^{rj} \right] = & \quad (\text{A.33}) \\ \varepsilon_{t+1} \varsigma_{t+1} \left[\frac{R_{t+1}}{\omega_{t,t+1}^r} \left(\frac{R_t}{\omega_{t-1,t}^r} F A_t^{jr} (1 - \varepsilon_t \varsigma_t) + d_t^{jr} - \varepsilon_t \varsigma_t D_t^{rj} \right) + D_{t+1}^{jr} \right]. \end{aligned}$$

Collecting terms we have that

$$1 - \varepsilon_t \varsigma_t = \frac{(\beta R_{t+1})^{1/(1-v)} \omega_{t,t+1}^r}{R_{t+1}} \frac{\varepsilon_t \varsigma_t}{\varepsilon_{t+1} \varsigma_{t+1}}, \quad (\text{A.34})$$

$$D_t^{jr} = d_t^{jr} + \frac{\omega_{t,t+1}^r}{R_{t+1}} D_{t+1}^{jr}. \quad (\text{A.35})$$

One can also show that $V_t^{jr} = (\varepsilon_t \varsigma_t)^{-1/v} C_t^{jr}$.

Worker j decision problem is

$$\max V_t^{jw} = \left\{ (C_t^{jw})^v + \beta [\omega^w V_{t+1}^{jw} + (1 - \omega^w) V_{t+1}^{jr}]^v \right\}^{1/v} \quad (\text{A.36})$$

subject to

$$C_t^{jw} + F A_{t+1}^{jw} = R_t F A_t^{jw} + W_t \xi_t + d_t^{jw} - \tau_t^{jw}. \quad (\text{A.37})$$

First order conditions and envelop theorem yield

$$(C_t^{jw})^{v-1} = \beta [\omega^w V_{t+1}^{jw} + (1 - \omega^w) V_{t+1}^{jr}]^{v-1} \left[\omega^w \frac{\partial V_{t+1}^{jw}}{\partial F A_{t+1}^{jw}} + (1 - \omega^w) \frac{\partial V_{t+1}^{jr}}{\partial F A_{t+1}^{jw}} \right],$$

$$\frac{\partial V_t^{jw}}{\partial F A_t^{jw}} = (V_{t+1}^{jw})^{1-v} (C_t^{jw})^{v-1} R_t, \quad \text{and} \quad (\text{A.38})$$

$$\frac{\partial V_t^{jr}}{\partial F A_t^{jw}} = \frac{\partial V_t^{jr}}{\partial F A_t^{jr}} \frac{\partial F A_t^{jr}}{\partial F A_t^{jw}} = \frac{\partial V_t^{jr}}{\partial F A_t^{jr}} \frac{1}{\omega_{t-1,t}^r} = (V_t^{jr})^{1-v} (C_t^{jr})^{v-1} R_t. \quad (\text{A.39})$$

$\frac{\partial F A_t^{jr}}{\partial F A_t^{jw}} = \frac{1}{\omega_{t-1,t}^r}$ since as households are risk neutral with respect to labour income they select the same asset profile independent of their worker/retiree status, adjusting only for expected return due to probability of death.

Combining these conditions above, and using the conjecture that $V_t^{jw} = (\varsigma_t)^{-1/v} C_t^{jw}$,

gives the Euler equation

$$C_t^{jw} = \left((\beta R_{t+1} \mathfrak{Z}_{t+1})^{1/(1-\nu)} \right)^{-1} [\omega^w C_{t+1}^{jw} + (1 - \omega^w) \varepsilon_{t+1}^{\frac{-1}{\nu}} C_{t+1}^{jr}] \quad (\text{A.40})$$

where $\mathfrak{Z}_{t+1} = (\omega^w + (1 - \omega^w) \varepsilon_{t+1}^{(v-1)/\nu})$.

Conjecture that retirees consume a fraction of all assets (including financial assets, human capital and profits from financial intermediaries), such that

$$C_t^{jw} = \varsigma_t [R_t F A_t^{jw} + H_t^{jw} + D_t^{jw}]. \quad (\text{A.41})$$

Following the same procedure as before we have that

$$\begin{aligned} & \varsigma_t [R_t F A_t^{jw} + H_t^{jw} + D_t^{jw}] (\beta R_{t+1} \mathfrak{Z}_{t+1})^{1/(1-\nu)} = \\ & \omega^w \varsigma_{t+1} [R_{t+1} (R_t F A_t^{jw} (1 - \varsigma_t) + W_t \xi_t + d_t^{jw} - \varsigma_t (H_t^{jw} + D_t^{jw})) + H_{t+1}^{jw} + D_{t+1}^{jw}] + \\ & \varepsilon_{t+1}^{\frac{-1}{\nu}} (1 - \omega^w) \varepsilon_{t+1} \varsigma_{t+1} [R_{t+1} (R_t F A_t^{jw} (1 - \varsigma_t) + W_t \xi_t + d_t^{jw} - \varsigma_t (H_t^{jw} + D_t^{jw})) + D_{t+1}^{jr}]. \end{aligned} \quad (\text{A.42})$$

Collecting terms and simplifying we have that

$$\varsigma_t = 1 - \frac{\varsigma_t}{\varsigma_{t+1}} \frac{(\beta R_{t+1} \mathfrak{Z}_{t+1})^{1/(1-\nu)}}{R_{t+1} \mathfrak{Z}_{t,t+1}} \quad (\text{A.43})$$

$$H_t^{jw} = (W_t^j) + \frac{\omega^w}{R_{t+1} \mathfrak{Z}_{t,t+1}} H_{t+1}^{jw} \text{ and} \quad (\text{A.44})$$

$$D_t^{jw} = d_t^{jw} + \frac{\omega^w}{R_{t+1} \mathfrak{Z}_{t,t+1}} D_{t+1}^{jw} + \frac{(1 - \omega^w) \varepsilon_{t+1}^{(v-1)/\nu}}{R_{t+1} \mathfrak{Z}_{t,t+1}} D_{t+1}^{jr}. \quad (\text{A.45})$$

Aggregation across households

Assume that for any variable X_t^{jz} we have that $X_t^z = \int_0^{N_t^z} X_t^{jz}$ for $z = \{w, r\}$, then

$$L_t = N_t^{wL}, \quad (\text{A.46})$$

$$L_{I,t} + L_{A,t} = N_t^{wRD}, \quad (\text{A.47})$$

$$H_t^w = (W_t) N_t^{wL} + (W_t^{RD}) N_t^{wRD} + \frac{\omega^w}{R_{t+1} \mathfrak{Z}_{t,t+1}} H_{t+1}^w \frac{N_t^w}{N_{t+1}^w}, \quad (\text{A.48})$$

$$D_t^w = d_t^w + \frac{\omega^w}{R_{t+1} \mathfrak{Z}_{t,t+1}} \frac{D_{t+1}^w N_t^w}{N_{t+1}^w} + \frac{(1 - \omega^w) \varepsilon_{t+1}^{(v-1)/\nu}}{R_{t+1} \mathfrak{Z}_{t,t+1}} \frac{D_{t+1}^r N_t^w}{N_{t+1}^w}, \quad (\text{A.49})$$

$$C_t^w = \varsigma_t [R_t F A_t^w + H_t^w + D_t^w - T_t^w], \quad (\text{A.50})$$

$$D_t^r = d_t^r + \frac{\omega^r}{R_{t+1}} D_{t+1}^r \frac{N_t^r}{N_{t+1}^r}, \quad (\text{A.51})$$

$$C_t^r = \varepsilon_t \varsigma_t [R_t F A_t^r + D_t^r]. \quad (\text{A.52})$$

Note that $\omega_{t,t+1}^r$ is not shown in the last equation due to the perfect annuity market for retirees, allowing for the redistribution of assets of retirees who died at the end of the period.

Financial Intermediary

The profits of the financial intermediary are

$$\begin{aligned} \Pi_t^F = & [r_t^k + 1]K_t + R_t B_t - R_t(FA_t^w + FA_t^r) - K_{t+1} - B_{t+1} + FA_{t+1}^w + FA_{t+1}^r + \\ & + (\Pi_{A,t} + \Pi_{RD,t} + (1 - \vartheta) \left(\int_{j \in A_t} \Pi_t^j dj + \int_{i \in Z_t \setminus A_t} \Pi_t^i di \right) + \Pi_{\Omega,t}), \end{aligned} \quad (\text{A.53})$$

where $B_{t+1} = S_t + \Xi_t$ and $FA_t = FA_t^w + FA_t^r$.

The financial intermediaries selects capital and bonds such that it maximise profits and thus we obtain the standard arbitrage conditions whereby all assets must pay the same expected return, thus

$$E_t [r_{t+1}^k + 1] = R_t. \quad (\text{A.54})$$

Also note that under a perfect foresight solution, by ensuring the financial intermediary behaves under perfect competition, this equality holds without expectations, $\Pi_t^F = 0$ and thus $d_t^r = d_t^w = 0$. If $\Pi_t^F \neq 0$, then we assume profits are divided based on the ratio of assets. As such, $d_t^r = \Pi_t^F \frac{FA_t^r}{FA_t^r + FA_t^w}$ and $d_t^w = \Pi_t^F \frac{FA_t^w}{FA_t^r + FA_t^w}$.

The flow of capital is then given by

$$K_{t+1} = K_t(1 - \delta) + I_t. \quad (\text{A.55})$$

Where I_t is the investment in capital made by the financial intermediary.

Asset Markets

Asset Market clearing implies

$$FA_{t+1} = FA_{t+1}^w + FA_{t+1}^r = K_{t+1} + B_{t+1} \quad (\text{A.56})$$

Finally, the flow of assets are given by

$$FA_{t+1}^r = R_t FA_t^r + d_t^r - C_t^r + (1 - \omega^w)(R_t FA_t^w + W_t \xi_t L_t + d_t^w - C_t^w - \tau_t) \quad (\text{A.57})$$

$$FA_{t+1}^w = \omega^w (R_t FA_t^w + W_t \xi_t L_t + d_t^w - C_t^w - \tau_t) \quad (\text{A.58})$$

Clearing conditions

$$y_t = C_{w,t} + C_{r,t} + \Upsilon_t + \Omega_t + I_t \quad (\text{A.59})$$

$$K_{t+1} = (1 - \delta)K_t + I_t \quad (\text{A.60})$$

$$K_t = \int_{j \in A_t} k_t^j dj + \int_{i \in Z_t \setminus A_t} k_t^i di \quad (\text{A.61})$$

$$\Upsilon_t = \int_{j \in A_t} \Upsilon_t^j dj + \int_{i \in Z_t \setminus A_t} \Upsilon_t^i di \quad (\text{A.62})$$

$$M_t = \int_{j \in A_t} M_t^j dj \quad (\text{A.63})$$

$$N_t^{wR} = \int_q L_{A,q,t} di + \int_p L_{i,q,t} di N_t^{wL} = \int_{i \in Z_t \setminus A_t} L_t^i di \quad (\text{A.64})$$

$$(\text{A.65})$$

Appendix B. Detrended equilibrium conditions

This section shows the detrended equilibrium conditions. Note that \bar{x} denotes the steady state of variable x_t .

$$w_t = ls_t + li_t + la_t \quad (\text{A.66a})$$

$$h_t^w = w_t + \frac{\omega^w}{R_{t+1}\mathfrak{Z}_{t,t+1}} \frac{g_{t+1}h_{t+1}^w}{g_{t+1}^w} \text{ where } h_t^w = \frac{H_t^w}{Y_t}, g_{t+1} = \frac{Y_{t+1}}{Y_t}, g_{t+1}^w = \frac{N_{t+1}^w}{N_t^w} \quad (\text{A.66b})$$

$$\tilde{D}_t^r = \tilde{d}_t^r + \frac{\omega_{t,t+1}^r}{R_{t+1}} g_{t+1} \frac{\tilde{D}_{t+1}^r \zeta_t^r}{\zeta_{t+1}^r g_{t+1}^w} \text{ where } \tilde{D}_t^r = \frac{D_t^r}{Y_t}, \tilde{d}_t^r = \frac{d_t^r}{Y_t} \quad (\text{A.66c})$$

$$\tilde{D}_t^w = \tilde{d}_t^w + \frac{\omega^w}{R_{t+1}\mathfrak{Z}_{t,t+1}} \frac{g_{t+1}\tilde{D}_{t+1}^w}{g_{t+1}^w} + \frac{(1 - \omega^w)\varepsilon_{t+1}^{(v-1)/v}}{R_{t+1}\mathfrak{Z}_{t,t+1}} \frac{g_{t+1}\tilde{D}_{t+1}^r}{\zeta_{t+1}^r g_{t+1}^w} \text{ where } \tilde{D}_t^w = \frac{D_t^w}{Y_t}, \tilde{d}_t^w = \frac{d_t^w}{Y_t} \quad (\text{A.66d})$$

$$c_t^w = \varsigma_t [R_t \frac{fa_t^w}{g_t} + h_t^w + \tilde{D}_t^w] \text{ where } fa_t^w = \frac{FA_t^w}{Y_{c,t-1}}, c_t^w = \frac{C_t^w}{Y_t} \quad (\text{A.66e})$$

$$c_t^r = \varepsilon_t \varsigma_t [R_t \frac{fa_t^r}{g_t} + \tilde{D}_t^r] \text{ where } fa_t^r = \frac{FA_t^r}{Y_{c,t-1}}, c_t^r = \frac{C_t^r}{Y_t} \quad (\text{A.66f})$$

$$1 - \varepsilon_t \varsigma_t = \frac{(\beta R_{t+1})^{1/(1-v)} \omega_{t,t+1}^r}{R_{t+1}} \frac{\varepsilon_t \varsigma_t}{\varepsilon_{t+1} \varsigma_{t+1}} \quad (\text{A.66g})$$

$$\varsigma_t = 1 - \frac{\varsigma_t}{\varsigma_{t+1}} \frac{(\beta R_{t+1}\mathfrak{Z}_{t,t+1})^{1/(1-v)}}{R_{t+1}\mathfrak{Z}_{t,t+1}} \quad (\text{A.66h})$$

$$\mathfrak{Z}_{t+1} = (\omega^w + (1 - \omega^w)\varepsilon_{t+1}^{(v-1)/v}) \quad (\text{A.66i})$$

$$g_{t+1}^w = \omega^w + (1 - \omega^y)\zeta_t^y \quad (\text{A.67a})$$

$$n_{t,t+1} = \frac{\zeta_{t+1}^y}{\zeta_t^y} (\omega^w + \zeta_t^y (1 - \omega^y)) \quad (\text{A.67b})$$

$$\zeta_{t+1}^r = ((1 - \omega^w) + \omega_{t,t+1}^r \zeta_t^r) (\omega^w + (1 - \omega^y) \zeta_t^y)^{-1} \text{ and} \quad (\text{A.67c})$$

$$g_{t+1}^n = (n_{t,t+1} \zeta_t^y) + (\omega^w + (1 - \omega^y) \zeta_t^y) + ((1 - \omega^w) + \omega_{t,t+1}^r \zeta_t^r) (1 + \zeta_t^r + \zeta_t^y)^{-1} \text{ where } g_{t+1}^n = \frac{N_{t+1}}{N_t} \quad (\text{A.67d})$$

Note that all firms $j \in A_t$ take the same decisions, then $\int_{j \in A_t} k_t^j dj = A_t k_t^j$. A similar argument holds for firms $i \in Z_t \setminus A_t$.

$$k_{m,t} = \frac{\alpha(1 - \gamma_I) \psi - 1}{(r_t^k + \delta) \psi} y_{m,t} g_t \text{ where } k_{m,t} = \frac{A_t k_t^j}{Y_{t-1}}, y_{m,t} = \frac{(P_t^j / P_t) y_t^j A_t}{Y_t} \quad (\text{A.68a})$$

$$\Upsilon_{m,t} = \gamma_I \frac{\psi - 1}{\psi} y_{m,t} \text{ where } \Upsilon_{m,t} = \frac{A_t \Upsilon_t^j}{Y_t} \quad (\text{A.68b})$$

$$m_t = (1 - \alpha)(1 - \gamma_I) \frac{\psi - 1}{\psi} y_{m,t} \text{ where } m_t = \frac{A_t m_t^j q_t}{Y_t} = \frac{q_t M_t}{Y_t} \quad (\text{A.68c})$$

$$g_{pm,t} = \left(\frac{(r_t^k + \delta)}{(r_{t-1}^k + \delta)} \right)^{\alpha(1-\gamma_I)} \left(\frac{\theta_{t-1}}{\theta_t} \right)^{(1-\alpha)(1-\gamma_I)} g_{q,t}^{(1-\alpha)(1-\gamma_I)} \text{ where } g_{pm,t} = \frac{(P_t^j / P_t)}{(P_{t-1}^j / P_{t-1})}, g_{q,t} = \frac{q_t}{q_{t-1}} \quad (\text{A.68d})$$

$$\frac{y_{m,t}}{y_{m,t-1}} = g_t^A g_{pm,t}, \text{ where } g_t^A = \frac{A_t}{A_{t-1}} \quad (\text{A.68e})$$

$$\pi_{m,t} = \frac{1}{\psi} y_{m,t} \text{ where } \pi_{m,t} = \frac{A_t \Pi_t^j}{Y_t} \quad (\text{A.68f})$$

$$k_{L,t} = \frac{\alpha(1 - \gamma_I) \psi - 1}{(r_t^k + \delta) \psi} y_{L,t} g_t \text{ where } k_{L,t} = \frac{(Z_t - A_t) k_t^i}{Y_{t-1}}, y_{L,t} = \frac{(P_t^i / P_t) y_t^i (Z_t - A_t)}{Y_t} \quad (\text{A.68g})$$

$$\Upsilon_{L,t} = \gamma_I \frac{\psi - 1}{\psi} y_{L,t} \text{ where } \Upsilon_{L,t} = \frac{(Z_t - A_t) \Upsilon_t^i}{Y_t} \quad (\text{A.68h})$$

$$l_{s,t} = (1 - \alpha)(1 - \gamma_I) \frac{\psi - 1}{\psi} y_{L,t} \text{ where } l_{s,t} = \frac{(W_t) N_t^{wL}}{Y_t} \quad (\text{A.68i})$$

$$l_{s,t} / l_{s,t-1} = l_{spop,t} / l_{spop,t-1} (g_t^{wg} g_{t-1}^n) / g_t \text{ where } g_t^{wg} = \frac{W_t}{W_{t-1}} \quad (\text{A.68j})$$

$$g_{pL,t} = \left(\frac{(r_t^k + \delta)}{(r_{t-1}^k + \delta)} \right)^{\alpha(1-\gamma_I)} \left(\frac{l_{s,t}}{l_{s,t-1}} \right)^{(1-\alpha)(1-\gamma_I)} \left(\frac{g_t}{g_{w,t}} \right)^{(1-\alpha)(1-\gamma_I)} \text{ where } g_{pL,t} = \frac{(P_t^i / P_t)}{(P_{t-1}^i / P_{t-1})} \quad (\text{A.68k})$$

$$\frac{y_{L,t}}{y_{L,t-1}} = g_t^{ZA} g_{pL,t}, \text{ where } g_t^{ZA} = \frac{(Z_t - A_t)}{(Z_{t-1} - A_{t-1})} \quad (\text{A.68l})$$

$$\pi_{L,t} = \frac{1}{\psi} y_{L,t} \text{ where } \pi_{L,t} = \frac{(Z_t - A_t) \Pi_t^i}{Y_t} \quad (\text{A.68m})$$

$$m_t = \frac{\tilde{\Omega}_t}{\eta} \text{ where } \tilde{\Omega}_t = \frac{\Omega_t}{Y_t} \quad (\text{A.68n})$$

$$\pi_{\Omega,t} = m_t - \tilde{\Omega}_t \text{ where } \pi_{\Omega,t} = \frac{\Pi_{\Omega,t}}{Y_t} \quad (\text{A.68o})$$

$$\frac{m_t}{m_{t-1}} = \left(\frac{\tilde{\Omega}_t}{\tilde{\Omega}_{t-1}} \right)^\eta (g_t)^{\eta-1} g_{q,t} \quad (\text{A.68p})$$

$$g_{t+1}^Z = \chi \left(\frac{s_t}{\Psi_t} \right)^\rho (lipop_t)^{\kappa_L} + \phi \text{ where } g_t^Z = \frac{Z_t}{Z_{t-1}}, s_t = \frac{S_t}{Y_t}, \Psi_t = \frac{\tilde{\Psi}_t}{Y_t}, lipop_t = \frac{L_{I,t}}{N_t} \quad (\text{A.69a})$$

$$g_{t+1}^A = \lambda_t \phi [1/a_{z,t} - 1] + \phi \text{ where } a_{z,t} = \frac{A_t}{Z_t} \quad (\text{A.69b})$$

$$g_t^{ZA} = g_t^Z \frac{1 - a_{z,t}}{1 - a_{z,t-1}} \quad (\text{A.69c})$$

$$az_t = az_{t-1} \frac{g_t^A}{g_t^Z} \quad (\text{A.69d})$$

$$s_t = \kappa_{RD} g_{t+1} R_{t+1}^{-1} \phi j_{t+1} \left(\frac{g_{t+1}^Z - \phi}{g_{t+1}^Z (1 - a_{z,t+1})} \right) \text{ where } j_t = \frac{J_t(Z_t - A_t)}{Y_t} \quad (\text{A.69e})$$

$$li_t = s_t \frac{\kappa_L}{\kappa_{RD}} \text{ where } li_t = \frac{L_{I,t} W_{RD,t}}{Y_t} \quad (\text{A.69f})$$

$$li_t/li_{t-1} = lipop_t/lipop_{t-1} (g_t^{wrd} g_{t-1}^n)/g_t \text{ where } g_t^{wrd} = \frac{W_{RD,t}}{W_{RD,t-1}} \quad (\text{A.69g})$$

$$v_t = \vartheta \pi_{m,t} + (R_{t+1})^{-1} \phi \frac{g_{t+1}}{g_{t+1}^A} v_{t+1} \text{ where } v_t = \frac{V_t A_t}{Y_t} \quad (\text{A.69h})$$

$$j_t = \vartheta \pi_{L,t} + (R_{t+1})^{-1} \phi \frac{g_{t+1}}{g_{t+1}^{ZA}} j_{t+1} \quad (\text{A.69i})$$

$$\varpi_t = \epsilon_\lambda \lambda_t R_{t+1}^{-1} \phi g_{t+1} \left[\frac{v_{t+1}}{g_{t+1}^A} [1/a_{z,t} - 1] - \frac{j_{t+1}}{g_{t+1}^{ZA}} \right] \text{ where } \varpi_t = \frac{\bar{\Xi}_t}{Y_t} \quad (\text{A.69j})$$

$$la_t = \varpi_t \frac{\kappa_L}{\kappa_{RD}} \text{ where } la_t = \frac{L_{A,t} W_{RD,t}}{Y_t} \quad (\text{A.69k})$$

$$la_t/la_{t-1} = lapop_t/lapop_{t-1} (g_t^{wrd} g_{t-1}^n)/g_t \quad (\text{A.69l})$$

$$\lambda_t = \lambda \left(\left(\frac{\varpi_t}{\Psi_t} \right)^{\kappa_{RD}} lapop_t^{\kappa_L} \right) \approx \bar{\lambda} \left(1 + \epsilon_\lambda \left(\kappa_{RD} \frac{\varpi_t - \bar{\varpi}}{\bar{\varpi}} - \kappa_{RD} \frac{\Psi_t - \bar{\Psi}}{\bar{\Psi}} + \kappa_L \frac{lapop_t - lapop}{lapop} \right) \right) \quad (\text{A.69m})$$

$$\pi_t^A = \vartheta \pi_{m,t} - R_t \varpi_{t-1}/g_t - li_t \quad (\text{A.69n})$$

$$\pi_t^{RD} = \vartheta \pi_{L,t} - R_t s_{t-1}/g_t - la_t \quad (\text{A.69o})$$

where ϵ_λ is the elasticity of $\lambda(\cdot)$

$$r_{t+1}^k + 1 = R_{t+1} \quad (\text{A.70a})$$

$$\tilde{d}_t^r = \pi_t^F \frac{f a_t^r}{f a_t} \text{ where } \pi_t^F = \frac{\Pi_t^F}{Y_t} \quad (\text{A.70b})$$

$$\tilde{d}_t^w = \pi_t^F \frac{f a_t^w}{f a_t} \quad (\text{A.70c})$$

$$b_{t+1} = s_t + \varpi_t \text{ where } b_{t+1} = \frac{B_{t+1}}{Y_t} \quad (\text{A.70d})$$

$$\pi_t^F = (r_t^k + 1) \frac{k_t}{g_t} + \frac{R_t}{g_t} b_t - \frac{R_t}{g_t} (fa_t) - k_{t+1} - b_{t+1} + (fa_{t+1}) + \pi_t^A + \pi_t^{RD} + (1 - \vartheta)(\pi_{m,t} + \pi_{L,t}) \quad (\text{A.70e})$$

$$lspop = \frac{\zeta_t^{wL}}{1 + \zeta_t^y + \zeta_t^r} \text{ where } \zeta_t^{wL} = \frac{N_t^{wL}}{N_t^w} \quad (\text{A.71a})$$

$$lipop_t + lapop_t = \frac{\zeta_t^{wRD}}{1 + \zeta_t^y + \zeta_t^r} \text{ where } \zeta_t^{wRD} = \frac{N_t^{wRD}}{N_t^w} \quad (\text{A.71b})$$

$$k_{t+1} = (1 - \delta) \frac{k_t}{g_t} + i_t \text{ where } i_t = \frac{I_t}{Y_t} \quad (\text{A.71c})$$

$$k_t = k_{m,t} + k_{L,t} \quad (\text{A.71d})$$

$$\tilde{\Upsilon}_t = \Upsilon_{m,t} + \Upsilon_{L,t} \quad (\text{A.71e})$$

$$1 = y_{m,t} + y_{L,t} \quad (\text{A.71f})$$

$$1 = c_t + i_t + s_t + \varpi_t + \tilde{\Omega}_t + \tilde{\Upsilon}_t \text{ where } c_t = \frac{C_t}{Y_t} \quad (\text{A.71g})$$

$$c_t = c_t^w + c_t^r \quad (\text{A.71h})$$

$$fa_{t+1}^w + fa_{t+1}^r = k_{t+1} + b_{t+1} \quad (\text{A.71i})$$

$$fa_{t+1}^r = \frac{R_t}{g_t} fa_t^r + \tilde{d}_t^r - c_t^r + (1 - \omega^w) \left(\frac{R_t}{g_t} fa_t^w + w_t + \tilde{d}_t^w - c_t^w \right) \quad (\text{A.71j})$$

$$fa_{t+1} = fa_{t+1}^w + fa_{t+1}^r \quad (\text{A.71k})$$

$$\Psi_t = v_t \quad (\text{A.71l})$$

$$fa_{t+1}^w = \omega^w \left(\frac{R_t}{g_t} fa_t^w + w_t + \tilde{d}_t^w - c_t^w \right)$$

Appendix C. Comparative Analysis

In this section of the appendix we present the proofs of Proposition 3 and 4. For both propositions we use the main detrended equilibrium conditions from firms, innovators and automation investors optimization problems depicted above.

Total differentiation around the *BGP* equilibrium of the core equilibrium conditions for R&D, (A.69), using $\Psi_t = v_t$ and (A.68a) to replace for π_L and π_m we obtain

$$\frac{dg^Z}{g^Z + \phi} = \rho \frac{ds}{s} - \rho \frac{dv}{v} + \kappa_L \frac{dlipop}{lipop} \quad (\text{A.72a})$$

$$\frac{ds}{s} = -\frac{dR}{R} + \frac{dj}{j} + \frac{a_z}{(1 - a_z)} \frac{da_z}{a_z} + \frac{dg}{g} + \frac{\phi}{g^Z} \frac{dg^Z}{g^Z + \phi} \quad (\text{A.72b})$$

$$\frac{dg^Z}{g^Z + \phi} = \frac{d\lambda}{\lambda} - \frac{1}{(1 - a_z)} \frac{da_z}{a_z} \quad (\text{A.72c})$$

$$\frac{d\lambda}{\lambda} = \epsilon_\lambda \kappa_{RD} \left(\frac{d\varpi}{\varpi} - \frac{dv}{v} \right) + \epsilon_\lambda \kappa_L \frac{dlapop}{lapop} \quad (\text{A.72d})$$

$$\frac{d\varpi}{\varpi} = \frac{d\lambda}{\lambda} - \frac{dR}{R} + \frac{dg}{g} + \frac{dg^Z}{g^Z} + \frac{v[1/a_z - 1]}{v[1/a_z - 1] - j} \frac{dv}{v} - \frac{j}{v[1/a_z - 1] - j} \frac{dj}{j} - \frac{v[1/a_z - 1]}{v[1/a_z - 1] - j} \frac{1}{1 - a_z} \frac{da_z}{a_z} \quad (\text{A.72e})$$

$$\frac{dv}{v} = -\frac{dy_L}{(1-y_L)} - \Gamma \frac{dR}{R} - \Gamma \frac{dg^Z}{g^Z} + \Gamma \frac{dg}{g}, \text{ where } \Gamma = \frac{\frac{\phi g}{g^Z R}}{\left(1 - \frac{\phi g}{g^Z R}\right)^2} \quad (\text{A.72f})$$

$$\frac{dj}{j} = \frac{dy_L}{y_L} - \Gamma \frac{dR}{R} - \Gamma \frac{dg^Z}{g^Z} + \Gamma \frac{dg}{g} \quad (\text{A.72g})$$

$$\frac{dlapop}{lapop} - \frac{dlipop}{lipop} = \frac{d\varpi}{\varpi} - \frac{ds}{s} \quad (\text{A.72h})$$

$$\frac{dg}{g} = \frac{1}{\eta} \frac{dg_n}{g_n} \quad (\text{A.72i})$$

$$\frac{dg^z}{g^z} = \frac{(1-\eta)(\varphi-1)(1-\alpha)(1-\gamma_I)}{\eta} \frac{dg_n}{g_n} \quad (\text{A.72j})$$

$$dlapop + dlipop = dRDpop \quad (\text{A.72k})$$

where $RDpop \equiv \frac{N_t^{wRD}}{N_t}$

Proof of Proposition 3:

Proposition 3 focuses on changes on population growth (dg_n), maintaining demographic structure (age shares) constant and thus $dRDpop = 0$. Combining (A.72) we obtain two conditions linking labour output share and the degree of automation with changes in population growth and changes in interest rates

$$\begin{aligned} -\frac{1}{\eta} \frac{\rho}{\kappa_L} \Gamma_1 \frac{dg_n}{g_n} - \frac{\rho}{\kappa_L} \frac{dR}{R} &= -c_1 \frac{da_z}{a_z} - d_1 \frac{dy_L}{y_L(1-y_L)} \\ -\frac{1}{\eta} \frac{\kappa_{RD}}{\kappa_L} \Gamma_2 \frac{dg_n}{g_n} - \frac{\kappa_{RD}}{\kappa_L} \frac{dR}{R} &= c_2 \frac{da_z}{a_z} + d_2 \frac{dy_L}{y_L(1-y_L)} \end{aligned}$$

where

$$\begin{aligned} c_1 &\equiv \left(\frac{lapop}{RDpop} \frac{(v+j)}{v[1/a_z-1]-j} + \frac{\rho}{\kappa_L} \frac{a_z}{(1-a_z)} \right) > 0 \\ c_2 &\equiv \left(\frac{(RDpop-lapop)}{RDpop} \frac{(v+j)}{v[1/a_z-1]-j} + \frac{1}{\epsilon_\lambda \kappa_L} \left(1 + \epsilon_\lambda \kappa_{RD} \frac{j}{v[1/a_z-1]-j} \right) \frac{1}{(1-a_z)} \right) > 0 \\ d_1 &\equiv \left(\frac{lapop}{RDpop} \frac{v[1/a_z-1]}{v[1/a_z-1]-j} + \frac{\rho}{\kappa_L} \right) > 0 \\ d_2 &\equiv \left(\frac{\epsilon_\lambda \kappa_{RD}}{\epsilon_\lambda \kappa_L} \frac{j}{v[1/a_z-1]-j} + \frac{(RDpop-lapop)}{RDpop} \frac{v[1/a_z-1]}{v[1/a_z-1]-j} \right) > 0 \\ \Gamma_1 &\equiv \left(\frac{\frac{g^Z}{\phi} - \rho}{(g^Z + \phi)} (1-\eta)(\varphi-1)(1-\alpha)(1-\gamma_I) - 1 \right) > 0 \\ \Gamma_2 &\equiv \left(\frac{\frac{g^Z}{\epsilon_\lambda \kappa_{RD}} - \phi}{(g^Z + \phi)} (1-\eta)(\varphi-1)(1-\alpha)(1-\gamma_I) - 1 \right) > 0 \end{aligned}$$

The first four inequalities follow from the fact that at any *BGP*, $\varpi > 0 \Rightarrow [v[1/a_z-1]-j] > 0$, and $a_z < 1$ and the last two since $\phi, \kappa_{RD} \leq 1$ and from assumption 2 (A2).

Then, as the labour income share is given by $l_{s_t} = (1 - \alpha)(1 - \gamma_I) \frac{\psi-1}{\psi} y_{L,t}$ we have that

$$\frac{da_z}{a_z} = \frac{-\frac{1}{\eta} \left(d_1 \frac{\kappa_{RD}}{\kappa_L} \Gamma_2 + \frac{\rho}{\kappa_L} \Gamma_1 d_2 \right) \frac{dg_n}{g_n} - \left(d_1 \frac{\kappa_{RD}}{\kappa_L} + \frac{\rho}{\kappa_L} d_2 \right) \frac{dR}{R}}{c_2 d_1 - d_2 c_1} \quad (\text{A.73})$$

$$\frac{dl_s}{l_s} = (1 - y_L) \frac{\frac{1}{\eta} \left(c_1 \frac{\kappa_{RD}}{\kappa_L} \Gamma_2 + \frac{\rho}{\kappa_L} \Gamma_1 c_2 \right) \frac{dg_n}{g_n} + \left(c_1 \frac{\kappa_{RD}}{\kappa_L} + \frac{\rho}{\kappa_L} c_2 \right) \frac{dR}{R}}{c_2 d_1 - d_2 c_1} \quad (\text{A.74})$$

To conclude the proof of Proposition 3 we need to ensure the denominator is positive. From the definitions of $c_1, c_2, d_1,$ and d_2 and as $\epsilon_\lambda \kappa_{RD} \leq 0$ we have that

$$\begin{aligned} c_2 d_1 &> \tilde{c}_2 d_1 = \frac{1}{\epsilon_\lambda \kappa_L (1 - a_z)} \left(\frac{v[1/a_z - 1]}{v[1/a_z - 1] - j} \right) \frac{\rho}{\kappa_L} + \frac{(RDpop - lapop)}{RDpop} \frac{(v + j)}{v[1/a_z - 1] - j} \frac{\rho}{\kappa_L} \\ &+ \frac{1}{\epsilon_\lambda \kappa_L (1 - a_z)} \left(\frac{v[1/a_z - 1]}{v[1/a_z - 1] - j} \right) \frac{lapop}{RDpop} \frac{v[1/a_z - 1]}{v[1/a_z - 1] - j} \\ &+ \frac{(RDpop - lapop)}{RDpop} \frac{(v + j)}{v[1/a_z - 1] - j} \frac{lapop}{RDpop} \frac{v[1/a_z - 1]}{v[1/a_z - 1] - j} \\ c_1 d_2 &= \frac{\rho}{\kappa_L} \frac{a_z}{(1 - a_z)} \frac{\epsilon_\lambda \kappa_{RD}}{\epsilon_\lambda \kappa_L} \frac{j}{v[1/a_z - 1] - j} + \frac{\rho}{\kappa_L} \frac{a_z}{(1 - a_z)} \frac{(RDpop - lapop)}{RDpop} \frac{v[1/a_z - 1]}{v[1/a_z - 1] - j} \\ &+ \frac{lapop}{RDpop} \frac{(v + j)}{v[1/a_z - 1] - j} \frac{(RDpop - lapop)}{RDpop} \frac{v[1/a_z - 1]}{v[1/a_z - 1] - j} \\ &+ \frac{lapop}{RDpop} \frac{(v + j)}{v[1/a_z - 1] - j} \frac{\epsilon_\lambda \kappa_{RD}}{\epsilon_\lambda \kappa_L} \frac{j}{v[1/a_z - 1] - j} \end{aligned}$$

Note that $\frac{(RDpop - lapop)}{RDpop} = \frac{lipop}{RDpop} = \frac{lapop}{RDpop} \frac{lipop}{lapop} = \frac{lapop}{RDpop} \frac{s}{\varpi}$.

As $\frac{s}{\varpi} = \frac{\kappa_{RD} g R^{-1} \phi j \left(\frac{g^Z - \phi}{g^Z (1 - a_z)} \right)}{\epsilon_\lambda \frac{g^Z - \phi}{\phi [1/a_z - 1]} R^{-1} \phi \frac{g}{g^Z} [v[1/a_z - 1] - j]} = \frac{\kappa_{RD} \phi j}{\epsilon_\lambda a_z [v[1/a_z - 1] - j]}$ then $\frac{(RDpop - lapop)}{RDpop} = \frac{lapop}{RDpop} \frac{\phi j}{\epsilon_\lambda a_z [v[1/a_z - 1] - j]}$, and thus

$$\begin{aligned} \tilde{c}_2 d_1 - c_1 d_2 &= \frac{\rho}{\epsilon_\lambda \kappa_L^2 (1 - a_z)} \left(\frac{v[1/a_z - 1] - a_z j}{v[1/a_z - 1] - j} \right) \\ &+ \frac{1}{a_z \epsilon_\lambda \kappa_L} \frac{lapop}{RDpop} \left(\frac{v(v[1/a_z - 1]) - \epsilon_\lambda \kappa_{RD} a_z j}{(v[1/a_z - 1] - j)^2} \right) \\ &+ \frac{\kappa_{RD}}{\epsilon_\lambda \kappa_L} \frac{lapop}{RDpop} \left(\frac{\rho \phi}{a_z} - \epsilon_\lambda \right) \frac{j^2}{(v[1/a_z - 1] - j)^2} \end{aligned}$$

As $v[1/a_z - 1] - j > v[1/a_z - 1] - a_z j > 0$ it is sufficient that $a_z \leq \frac{\rho \phi}{\epsilon_\lambda}$ to ensure $c_2 d_1 - c_1 d_2 > \tilde{c}_2 d_1 - c_1 d_2 > 0$. Note that given that the first two terms are positive, and the first increases as a_z increases, even when A1 does not hold, and the third term is negative the denominator may still be positive. \square

Proof of Proposition 4:

Proposition 4 assumes population growth is keep constant, $dg_n = 0$ (which im-

plies $dg = dg^Z = 0$), and focuses on changes in the demographic structure particularly considering an increase the share of retirees (ageing) and thus $dRDpop < 0$.

Combining (A.72) we obtain two conditions linking labour output share and the degree of automation with changes in demographic structure and changes in interest rates. The system of equation, using the definitions of c_1, c_2, d_1 and d_2 , becomes

$$\begin{aligned} \frac{dRDpop}{RDpop} - \frac{\rho}{\kappa_L} \frac{dR}{R} &= -c_1 \frac{da_z}{a_z} - d_1 \frac{dy_L}{y_L(1-y_L)} \\ \frac{dRDpop}{RDpop} - \frac{\kappa_{RD}}{\kappa_L} \frac{dR}{R} &= c_2 \frac{da_z}{a_z} + d_2 \frac{dy_L}{y_L(1-y_L)} \end{aligned}$$

As the labour income share is given by $l_{s_t} = (1 - \alpha)(1 - \gamma_I) \frac{\psi-1}{\psi} y_{L,t}$ we have that

$$\frac{da_z}{a_z} = \frac{\frac{dRDpop}{RDpop}(d_1 + d_2) - \left(d_1 \frac{\kappa_{RD}}{\kappa_L} + \frac{\rho}{\kappa_L} d_2\right) \frac{dR}{R}}{c_2 d_1 - d_2 c_1} \quad (\text{A.75})$$

$$\frac{dl_s}{l_s} = (1 - y_L) \frac{- \left(\frac{dRDpop}{RDpop}(c_1 + c_2) - \left(c_1 \frac{\kappa_{RD}}{\kappa_L} + \frac{\rho}{\kappa_L} c_2\right) \frac{dR}{R}\right)}{c_2 d_1 - d_2 c_1} \quad (\text{A.76})$$

As the denominator is positive that concludes the proof of proposition 4. \square

Appendix D. More on Calibration

This Section reports the values of the set of parameters of the model.

Table A.1: Calibration

Parameter	Value	Target/Source
Time Discount Factor	$\beta = 0.96$	Standard Value
Elasticity Intertemporal Substitution	$\nu = -3$	EIS = 0.25 (Gertler(1999))
Capital Depreciation Rate	$\delta = 0.08$	Standard Value
Capital Share in Production	$\alpha = 0.33$	Standard Value
Intermediate Share in Production	$\gamma_I = 0.5$	Comin and Gertler(2006)
Elasticity Substitution of Varieties	$\psi = 8$	Standard Value
Obsolescence	$\phi = 0.85$	Growth per Working age person
Productivity Innovation	$\chi = 5.67$	Share of innovation expenditure in GDP
Elasticity of Investment to Innovation	$\rho = 0.9$	Comin and Gertler (2006)
Elasticity of Final Goods to R&D Investment	$\kappa_{RD} = 1$	Comin and Gertler (2006)
Elasticity of Labour to R&D Investment	$\kappa_L = 0.5$	Aksoy et al. (2018)
Rate of Automation	$\lambda = 0.1$	Share of Automated Varieties
Robots Production Function	$\eta = 0.15$	Balanced Growth
Birth Rate	$\omega_n = 0.0265$	Share of Workers in Population
Probability Transition from Mature to Old	$1 - \omega_w = 0.022$	Avg. Number of Years as Worker: 45y
Death Probability of Old Agents	$1 - \omega_o = 0.07$	Share of Old in Population
Share of Workers in R&D	$Sw_{RD} = 0.07$	Share of R&D workers in Population
Probability Workers leaves R&D	$drop_{RD} = 0.07$	Average age of R&D workers

Appendix E. Model Extensions

E.1 Extension - Labour Choice Model

Under this extension, $Sw_{RD,t}$, the share of new workers that enter the economy and work in the R&D sector, is endogenous. In order to obtain that we assume a household, when entering her working life selects in which labour market (R&D or intermediate good production) to participate. At entry she is randomly assigned an efficiency level in R&D activity, denoted $\xi \tilde{\nu}_t^i$, where $\tilde{\nu}_t^i$ is drawn from a Pareto distribution with shape parameter $\epsilon > 1$ and support $[1, \infty)$. We denote the cumulative distribution by $F(\nu)$. The household then compares the human capital gain under the R&D sector (H_t^{RD}) which is a function of the wage W^{RD} and the average efficiency of workers in the sector, denoted $\nu_{m,t}$, and the human capital gain in the production sector (H_t , which is a function of the wage W) and selects in which labour market to be active in.

There exists a cut-off point ν_t^* such that given H_t^{RD} and H_t the household is indifferent between choosing each sector. Then, the share of households in R&D is given by

$$Sw_{RD,t} = \int_{\nu_t^*}^{\infty} dF(\nu) = \int_{\nu_t^*}^{\infty} \frac{\epsilon 1^\epsilon}{\nu^{\epsilon+1}} d\nu = \int_{\nu_t^*}^{\infty} \epsilon \nu^{-(\epsilon+1)} = (\nu_t^*)^\epsilon$$

The average efficiency of entrants in the R&D labour market is

$$\nu_{E,t} = \frac{\int_{\nu_t^*}^{\infty} \xi \nu dF(\nu)}{1 - F(\nu_t^*)} = \frac{\int_{\nu_t^*}^{\infty} \xi \epsilon \nu^{-(\epsilon)} d\nu}{1 - F(\nu_t^*)} = \xi \frac{\epsilon}{\epsilon - 1} \nu_t^*$$

The average efficiency of all workers in the R&D sector is then given by

$$\nu_{m,t} = \frac{Sw_{RD,t}\omega_{t,t+1}^y N_t^w}{N_{t+1}^{wRD}} \nu_{E,t} + (1 - drop_{RD})\omega^w N_t^{wRD} N_{t+1}^{wRD} \nu_{m,t-1}$$

Defining

$$H_t^{jw} = (W_t) + \frac{\omega^w}{R_{t+1}\mathfrak{Z}_{t,t+1}} H_{t+1}^{jw}, \text{ where } j \text{ works in production}$$

$$H_t^{iwRD} = (\nu_{m,t} W_t^{RD}) + \frac{\omega^w}{R_{t+1}\mathfrak{Z}_{t,t+1}} H_{t+1}^{iwRD}, \text{ where } i \text{ works in R\&D}$$

And since $\nu_{m,t}$ is a function of ν_t^* , ν_t^* is such that $H_t^{jw} = H_t^{iwRD}$. Finally, we calibrate ϵ and ξ to obtain the same effective wage in R&D and Sw_{RD} at steady state as in the benchmark model.

E.2 Labour Supply - Intensive Margin

In this extension we assume that all households also decide how much labour to supply (we allow retirees to also supply labour, although

Retiree j decision problem is

$$\max V_t^{jr} = \left\{ (C_t^{jr})^{\nu\mu_L} (\chi_r - l_t^{jr})^{\nu(1-\mu_L)} + \beta\omega_{t,t+1}^r ([V_{t+1}^{jr}]^\nu) \right\}^{1/\nu}$$

subject to

$$C_t^{jr} + FA_{t+1}^{jr} = \frac{R_t}{\omega_{t-1,t}^r} FA_t^{jr} + \xi W_t l_t^{jr} + d_t^{jr}.$$

Following similar steps as in the benchmark model we get

$$C_t^{jr} = \varepsilon_t \varsigma_t \left[\frac{R_t}{\omega_{t-1,t}^r} FA_t^{jr} + H_t^{jr} + D_t^{jr} \right]$$

$$1 - \varepsilon_t \varsigma_t = \frac{\left(\beta R_{t+1} \left(\frac{W_t}{W_{t+1}} \right)^{(1-\mu_L)\nu} \right)^{1/(1-\nu)} \omega_{t,t+1}^r}{R_{t+1}} \frac{\varepsilon_t \varsigma_t}{\varepsilon_{t+1} \varsigma_{t+1}}$$

$$D_t^{jr} = d_t^{jr} + \frac{\omega_{t,t+1}^r}{R_{t+1}} D_{t+1}^{jr}.$$

$$H_t^{jr} = \xi W_t l_t^{jr} + \frac{\omega_{t,t+1}^r}{R_{t+1}} H_{t+1}^{jr}.$$

$$(\chi_r - l_t^{jr}) = \frac{\mu_L C_t^{jr}}{\xi W_t (1 - \mu_L)}$$

$$V_t^{jr} = (\varepsilon_t \varsigma_t)^{-1/\nu} C_t^{jr} (\chi_r - l_t^{jr})$$

With endogenous labour supply wages affect the marginal propensity to consume. As a result we can no longer solve a single problem for all workers.

Production workers j decision problem is

$$\max V_t^{jw} = \left\{ (C_t^{jw})^{\mu_L\nu} (\chi_w - l_t^{jw})^{\nu(1-\mu_L)} + \beta[\omega^w V_{t+1}^{jw} + (1 - \omega^w) V_{t+1}^{jr}]^\nu \right\}^{1/\nu}$$

subject to

$$C_t^{jw} + FA_{t+1}^{jw} = R_t FA_t^{jw} + W_t l_t^{jw} + d_t^{jw}$$

Following the same procedure as before we have that

$$\begin{aligned} C_t^{jw} &= \varsigma_t [R_t FA_t^{jw} + H_t^{jw} + D_t^{jw}] \\ \varsigma_t &= 1 - \frac{\varsigma_t}{\varsigma_{t+1}} \frac{\left(\beta R_{t+1} \mathfrak{Z}_{t+1} \left(\frac{W_t}{W_{t+1}} \right)^{(1-\mu_L)v} \right)^{1/(1-v)}}{R_{t+1} \mathfrak{Z}_{t,t+1}} \\ H_t^{jw} &= (W_t l_t^{jw}) + \frac{\omega^w}{R_{t+1} \mathfrak{Z}_{t,t+1}} H_{t+1}^{jw} + \frac{(1-\omega^w)(1/\xi)^{1-\mu_L} \varepsilon_{t+1}^{(v-1)/v}}{R_{t+1} \mathfrak{Z}_{t,t+1}} H_{t+1}^{jw} \\ D_t^{jw} &= d_t^{jw} + \frac{\omega^w}{R_{t+1} \mathfrak{Z}_{t,t+1}} D_{t+1}^{jw} + \frac{(1-\omega^w)(1/\xi)^{1-\mu_L} \varepsilon_{t+1}^{(v-1)/v}}{R_{t+1} \mathfrak{Z}_{t,t+1}} D_{t+1}^{jr} \\ (\chi_w - l_t^{jw}) &= \frac{\mu_L C_t^{jw}}{W_t(1-\mu_L)} \\ V_t^{jw} &= (\varepsilon_t \varsigma_t)^{-1/v} C_t^{jw} (\chi_w - l_t^{jw}) \\ \mathfrak{Z}_{t+1} &= (\omega^w + (1-\omega^w) \varepsilon_{t+1}^{(v-1)/v}). \end{aligned}$$

R&D workers j decision problem is

$$\begin{aligned} \max V_t^{jwR} &= \left\{ (C_t^{jwR})^{\mu_L v} (\chi_{wR} - l_t^{jwR})^{v(1-\mu_L)} + \beta [\omega^w (1 - drop_{RD}) V_{t+1}^{jwR} \right. \\ &\quad \left. + \omega^w (drop_{RD}) V_{t+1}^{jr} + (1-\omega^w) V_{t+1}^{jr}]^v \right\}^{1/v} \end{aligned}$$

subject to

$$C_t^{jwR} + FA_{t+1}^{jwR} = R_t FA_t^{jwR} + W_t^{RD} l_t^{jwR} + d_t^{jwR}$$

Following the same procedure as before we have that

$$\begin{aligned} C_t^{jwR} &= \varsigma_t o_t [R_t FA_t^{jwR} + H_t^{jwR} + D_t^{jwR}] \\ \frac{1 - o_t \varsigma_t}{o_t} &= 1 - \frac{\varsigma_t}{\varsigma_{t+1}} \frac{\left(\beta R_{t+1} \mathfrak{Z}_{t+1}^{RD} \left(\frac{W_t^{RD}}{W_{t+1}^{RD}} \right)^{(1-\mu_L)v} \right)^{1/(1-v)}}{R_{t+1} \mathfrak{Z}_{t,t+1}^{RD}} \\ H_t^{jwR} &= (W_t^{RD} l_t^{jwR}) + \frac{\omega^w}{R_{t+1} \mathfrak{Z}_{t,t+1}^{RD}} H_{t+1}^{jwR} + \frac{(1-\omega^w)(1/\xi)^{1-\mu_L} \varepsilon_{t+1}^{(v-1)/v}}{R_{t+1} \mathfrak{Z}_{t,t+1}^{RD}} H_{t+1}^{jwR} \\ &\quad + \frac{(1-\omega^w) drop_{RD} \left(\frac{W_{t+1}^{RD}}{W_{t+1}^{RD}} \right)^{(1-\mu_L)v} o_{t+1}^{(v-1)/v}}{R_{t+1} \mathfrak{Z}_{t,t+1}^{RD}} H_{t+1}^{jwRD} \\ D_t^{jwR} &= (W_t^{RD} l_t^{jwR}) + \frac{\omega^w}{R_{t+1} \mathfrak{Z}_{t,t+1}^{RD}} D_{t+1}^{jwR} + \frac{(1-\omega^w)(1/\xi)^{1-\mu_L} \varepsilon_{t+1}^{(v-1)/v}}{R_{t+1} \mathfrak{Z}_{t,t+1}^{RD}} D_{t+1}^{jwR} \\ &\quad + \frac{(1-\omega^w) drop_{RD} \left(\frac{W_{t+1}^{RD}}{W_{t+1}^{RD}} \right)^{(1-\mu_L)v} o_{t+1}^{(v-1)/v}}{R_{t+1} \mathfrak{Z}_{t,t+1}^{RD}} D_{t+1}^{jwRD} \end{aligned}$$

$$\begin{aligned}
(\chi_{wR} - l_t^{wR}) &= \frac{\mu_L C_t^{jwR}}{W_t(1 - \mu_L)} \\
V_t^{jwR} &= (\varepsilon_t \varsigma_t)^{-1/v} C_t^{jwR} (\chi_{wR} - l_t^{jwR}) \\
\mathfrak{Z}_{t+1}^{\mathfrak{R}\mathfrak{D}} &= ((1 - \omega^w) drop_{RD} o_{t+1}^{(v-1)/v} \left(\frac{W_{t+1}}{W_{t+1}^{RD}} \right)^{1-\mu_L} \omega^w drop_{RD} + (1 - \omega^w) \varepsilon_{t+1}^{(v-1)/v}).
\end{aligned}$$

Finally, in order to ensure unique transition path we assume innovators and automation investors pay a cot to adjust labour demand given by $\frac{\varepsilon}{2}(L_{X,t} - g_n L_{X,t-1})^2$, for $X = I, A$.

E.3 Depreciation of Robots

We assume at every period robots producers start with $(1 - \delta_R)M_t$ amount of robots and invest Ω_t and get $I_t^R = \varrho(\Omega_t)^\eta$. Robots are rented to firms at a price q_t . Problem of robots producers is

$$\max_{\Omega, t} \sum_{t=0}^{\infty} \beta^t \Pi_{\Omega, t} = q_t M_t - \Omega_t \quad s.t. \quad M_t = \varrho \Omega_t^\eta + (1 - \delta_R) M_{t-1}. \quad (\text{A.77})$$

Maximisation conditions are

$$\frac{\eta \varrho q_t}{\Omega_t^{1-\eta}} = 1 - (1 - \delta_R) \frac{\Omega_{t+1}^{1-\eta}}{\Omega_{t+1}^{1-\eta}} \quad (\text{A.78})$$

$$M_t = \varrho \Omega_t^\eta + (1 - \delta_R) M_{t-1} \quad (\text{A.79})$$

If (A.78) holds then at a *BGP*, $(g_t)^{\eta-1} g_{q,t} = 1$ and $\frac{\Omega_t}{y_t}$ is constant. Thus, (20) in proposition 2 continues to hold and thus restriction on η to ensure *BGP* exists is unchanged in this extension.