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Consumers can acquire information through their own search efforts or through their social network. Information diffusion via word-of-mouth communication leads to some consumers free-riding on their "friends" and less information acquisition via active search. Free-riding also has an important positive effect, however, in that consumers that do not actively search themselves are more likely to be able to compare prices before purchase, imposing competitive pressure on firms. We show how market prices depend on the characteristics of the network and on search cost. For example, if the search cost becomes small, price dispersion disappears, while the price level converges to the monopoly level, implying that expected prices are decreasing for small enough search cost. More connected societies have lower market prices, while price dispersion remains even in fully connected societies.

JEL Classification: D43, D83, D85

Keywords: consumer search, Word-of-Mouth Communication, Social Networks

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Information Acquisition and Diffusion in Markets*

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Thursday 3rd October, 2019

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1 Introduction

Decentralized markets rely on how information in the hands of individual agents is aggregated (Hayek (1945)). Individual agents are, however, not endowed with a natural amount of information. Often, they have to spend resources, such as time, to search and acquire information. Accordingly, agents will only acquire information if the expected benefits exceed the opportunity cost of doing so. This has led Grossman and Stiglitz (1980) to pose that efficient markets cannot exist if arbitrage is costly. Information can, however, also be acquired in less costly ways, namely through word-of-mouth (WOM) communication via friends (see, e.g., Ellison and Fudenberg (1995) and Campbell (2013)).[1] WOM communication may come with a delay, however, as one has to wait for friends to communicate their information.

Costly information acquisition and diffusion (WOM communication) are clearly related. When few people acquire information themselves, little information will be diffused, while if information is disseminated efficiently, people may not have the incentive to spend resources to acquire information themselves. Thus, it is important to understand the interaction between the incentives to acquire information and the efficiency of the information diffusion process. This is especially so for online markets and online interaction through social networks, such as Facebook or LinkedIn. It is well documented[2] that online technologies have significantly reduced the search cost related to information acquisition and increased the possibilities of diffusion and it is important to understand how these developments affect market outcomes.

Information, whether acquired through costly search or through WOM, allows consumers to carefully compare product characteristics and prices of different firms. The better consumers are able to make comparisons across firms, the smaller firms' market power. In this paper we study the interaction between information acquisition, diffusion and market power, and explain the impact of changes in the connectedness of people (impacting diffusion of information through WOM) and search costs on market outcomes.

[1] Katz and Lazarsfeld (1955) is the classic study showing that information acquired through personal contacts is the prime reason why people buy a product.

[2] See, e.g., Brown and Goolsbee (2002), Jensen (2007) Aker (2010), and Aker and Mbiti (2010). On word-of-mouth communication, see, e.g., Godes and Mayzlin (2004), Chen et al. (2011) and Seiler et al. (2019).

We adopt a simple theoretical framework of a homogeneous goods market where firms set prices and consumers engage in costly sequential search to acquire information about prices before buying one unit of the good (Diamond (1971) and Stigler (1961)).[3] Consumers that have searched for prices themselves and bought the product spread this information through their network. Consumers who do not engage in search rely on their network of friends to get information on prices and where to buy. We discuss two versions of this general set-up, depending on whether or not consumers have better knowledge of the social network structure than firms before making price and search decisions.[4]

We find that independent of the acquisition (search) cost there always exists a no-trade equilibrium as no one will acquire information if firms set very high prices and setting high prices is optimal if no one acquires information.[5] In models that exhibit the Diamond paradox, no trade is the *only* equilibrium outcome: if all consumers have a positive search cost and costly search is their only source of information, firms will sell the same output at all prices that are smaller than or equal to a reservation price that is larger than the average price consumers expect in the market. This gives firms an incentive to raise prices above the expected price, contradicting the implicit assumption that there is an average trading price.[6]

Importantly, WOM resolves the Diamond paradox in that it creates additional equilibria with positive sales. Under WOM, consumers determine whether or not to acquire information themselves. The possibility to get information through their social network implies that in any equilibrium it cannot be the case that all consumers acquire information themselves. At a positive search cost, consumers have an incentive to free ride on their friends if all of them actively search as this also allows them to buy, possibly at the lowest price charged in the market without incurring the full search cost. Thus,

[3] Undoubtedly, in real world markets consumers also exchange information about product characteristics, such as quality, appearance and/or convenience. Yet, people also share information online about prices. In homogeneous goods markets, price communication is the only thing that matters.

[4] One may also distinguish between markets where consumers can or cannot credibly share information with friends *before* they have actually bought the product. We show, however, that in our set-up these two environments yield identical results.

[5] A no trade equilibrium exists in many simultaneous and sequential search models where the first search is costly for all consumers (see, e.g., Burdett and Judd (1983) and Diamond (1971))

[6] If individual consumers have downward sloping demand (or the first search is somehow free), then the Diamond paradox takes on a somewhat different form, namely that all firms charge the monopoly price.

an endogenously determined fraction of consumers is only informed through their friends. This *free-riding on the information acquisition of friends* has an important *positive effect* on the incentives of firms to compete in markets where consumers search sequentially. By free-riding, the non-searching consumers will be informed with positive probability about different prices and will buy at the lowest of these prices. This provides firms with an incentive to compete with each other. In particular, if a firm raises its price above the price of the competitor he will lose all free-riding consumers who are informed about both prices. Thus, WOM eliminates the source of the Diamond paradox and creates price dispersion, because a fraction of consumers is informed about one price only, whereas others are informed of more than one price.

The environment we study allows us to consider the impact of social networks and search costs on information acquisition and market power. In our model, a network is characterized by two features: (i) the number of friends different consumers are connected to, and (ii) the speed with which information flows through the network. In terms of the social network structure one would expect that prices are lower in societies where consumers are better connected. The reason is that more connections allow them to access more prices and the more consumers compare prices, the higher the competitive pressure on firms. Importantly, when the network gets very dense and many consumers have many connections, prices do *not* converge to marginal cost and price dispersion remains. What matters for price dispersion and also for the expected market price is the relative fraction of consumers that have access to only one price in equilibrium compared to the fraction that is better informed. This fraction is endogenously determined. As information from friends comes with a delay, the incentive to acquire information oneself remains and a positive fraction of consumers buy immediately after acquiring information. As passive consumers are likely to obtain more than one price quote and buy at the lowest price, price dispersion remains.

The speed of information diffusion is important in that it is a key determinant of the cost associated with waiting for information through WOM. A higher speed of information flow allows passive consumers to realize quicker whether or not they will receive any information via their social network. If they quickly realize their friends will not provide them with information, they can engage in information acquisition themselves without much delay. Thus, a higher speed of information diffusion in the population has a direct positive effect on the share of consumers who decide to wait for information from friends. This has a dampening effect on prices. There is also an indirect effect, however, namely that as prices and price dispersion decline, consumers have more incentives to become active searchers themselves, especially when the speed of information diffusion is low to begin with. Overall, the fraction of active consumers and firms' profits has an inverted U-shape with respect to the speed of information diffusion: when this speed is low to begin with, firms have an incentive to increase it as this will also speed up their sales, despite a decrease in prices. However, when the speed is already relatively large, the effect on expected price dominates and firms want to slow down information diffusion.

The impact of search cost is best illustrated by considering the case where the search cost becomes arbitrarily small. We show that in this case price dispersion disappears and almost all consumers become active themselves and buy immediately after they have searched themselves. As almost no consumer makes price comparisons, prices converge to monopoly levels. When the search cost increases, more consumers remain passive and a large fraction of consumers make price comparisons, resulting in lower prices. Thus, prices are decreasing in search cost in our model.

Galeotti (2010) is the paper that is closest to ours. There are three main differences between his paper and ours. First, in Galeotti (2010) consumers search for prices in a non-sequential fashion, whereas we have a sequential search framework. In most consumer retail markets, consumers observe the price at a firm before they decide whether to search another firm, making the sequential search paradigm more relevant. Second, Galeotti (2010) assumes the first search is free so that all consumers know at least one price. This guarantees that in his setting market outcomes tend to the perfectly competitive outcome when the number of connections each consumer has becomes large: the chance that at least one of a consumer's connections has observed a competitors' price tends to be close to 1. In our setting where the first search is also costly, consumers that do not search themselves may not be informed of any price in first instance.[7] If the number of connections becomes large, we show that price dispersion and market power remain. The positive impact, we find, of not searching on the equilibrium prices is not present in

[7] Janssen et al. (2005) were the first to study the impact of the first search being costly on the participation of consumers in the marketplace. In their setting (and in contrast to ours), search is, however, the only source of information acquisition.

Galeotti (2010) as under non-sequential search with the first search being free, consumers are informed about fewer prices when they do not search than when they do search.[8] Finally, where all consumers have the same number of links in the basic model studied in Galeotti (2010), we model the social network as a random graph.

There is also a literature on how WOM communication affects the pricing and advertising policy of a monopoly firm in the market (see, e.g., Arbatskaya and Konishi (2016), Biyalogorsky et al. (2001), Bloch (2016), Campbell (2013), Chuhay (2015), Fainmesser and Galeotti (2016), Galeotti and Goyal (2009), Jun and Kim (2008), and Kornish and Li (2010)).[9] There are several key differences between this literature and our paper. Instead of monopoly, we study a market where firms compete in prices. More importantly, whereas this literature assumes that consumers passively wait until they receive an advertisement from the firm, or they are informed through their network, we allow consumers to actively reach out and search for information, making the consumers important strategic players in the market. Whereas we provide more detail on how consumers acquire information, this literature is able to consider more detail on specific firm strategies, such as referrals and/or targeting.

The classic papers by Wolinsky (1986) and Stahl (1989) provide well-known solutions to the Diamond paradox. Wolinsky (1986) imposes that firms produce heterogeneous products, while Stahl (1989) imposes search cost heterogeneity among consumers, where some exogenously determined fraction of "shoppers" have zero search cost and compare all prices before buying. Unlike these two papers, we endow consumers with the possibility to acquire information through WOM in addition to their own information acquisition by means of search. In this way, we endogenize the fraction of price comparing consumers. In our model this fraction is endogenously determined in equilibrium.

The rest of the paper is organized as follows. The next section presents the model, while Section 3 presents preliminary findings, the most important of which is that any equilibrium with positive trade must be a reservation price equilibrium. Section 4 examines markets where consumers and firms are symmetrically (un)informed about the network structure, while Section 5 provides the comparative statics analysis of this model. Section

[8] Miegielsen (2014) adopts a sequential search framework but considers a model where somehow consumers possess information about prices before engaging in search and the amount of information that consumers have (and share with each other) is given exogenously.

[9] Fainmesser and Galeotti (2017) study an oligopolistic version of their 2016 paper.

6 continues the analysis by examining markets where consumers and firms are asymmetrically informed about the network: consumers know from how many friends they may acquire information before engaging in search activities, while firms only know aggregate network characteristics. We conclude with a discussion.

2 The Model

We consider a duopoly[10] market for a homogeneous good where firms compete in prices. The unit cost of production is constant and normalized to zero. As firms may choose mixed strategies, we denote the strategy of a firm i by $F_i(p)$, representing the probability that a firm charges a price not larger than p. The support of the price distribution is determined endogenously with $\underline{p}$ and $\overline{p}$ being the lower and upper bound of the support, with the possibility of some prices in the interior of the interval not being chosen.

On the demand side of the market, there is a countably infinite number of consumers, normalized to one, each with unit demand and a willingness to pay equal to v. A consumer buying at price p receives a pay-off of $v - p$. If a consumer does not consume, she receives a payoff of zero. Before making a purchase, a consumer needs to be informed about at least one price. We model the consumers' choice situation as follows, where Figure 1 illustrates the process. The basic idea behind the choice situation is that actively searching involves a search cost, while waiting for information involves a time delay.

In first instance, each consumer individually and simultaneously has to decide whether to acquire information by actively searching and visiting a firm herself at a cost s or to passively wait for information from friends via a social network. We follow the consumer search literature and assume that search is with perfect recall, i.e., once a firm is visited, consumers can come back to the firm without having to incur a return cost. We denote by $q(k)$ the fraction of consumers with k friends who choose to become active. If consumers do not or cannot condition their decision of becoming active searchers on the number of connections, q is independent of k. If a consumer decides to become active and searches herself, she has three options to choose from once she has observed a price: to buy, to

[10] With more than two firms, the characterization of the mixed strategy distribution in prices is more complicated and in this case, it is difficult to analyze the gains of search versus the gains of free riding. Galeotti (2010) also considers duopoly markets.

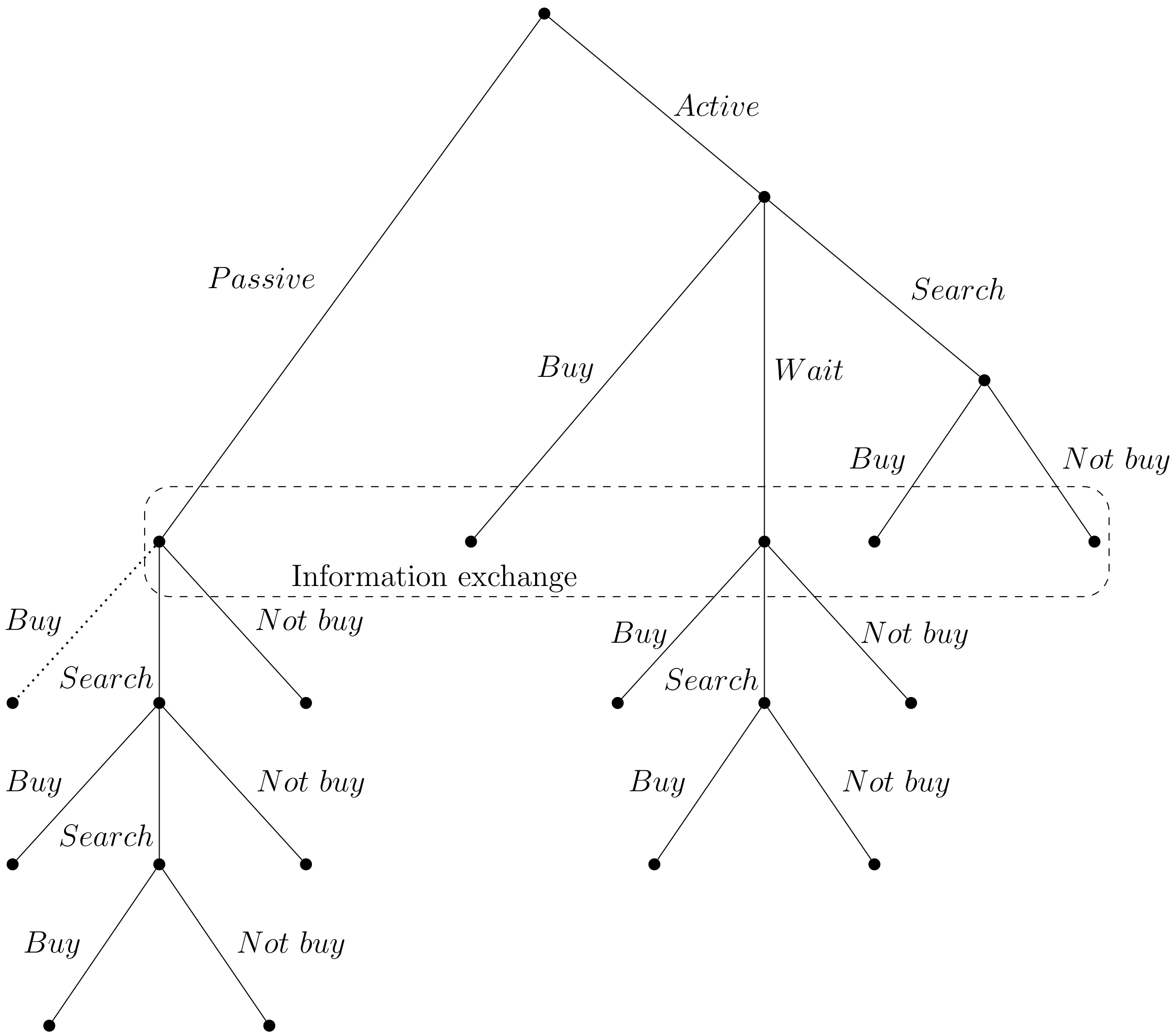

Figure 1: Consumer decision tree

continue searching for a rival firm or to wait. Thus, a consumer can wait at any moment in the decision process. All active consumers make their choices simultaneously without knowing the behavior of others.

Following the literature on observational learning (see, Kircher and Postlewaite (2008), Garcia and Shelegia (2015)), we initially consider an environment where consumers can credibly exchange information only after having purchased goods so that information exchange/diffusion takes place after all active consumers have finished their search and purchases. This case is certainly the relevant case to consider in markets where consumers may not be able to purchase the good due to sales restrictions (such as is the case in many online markets where firms may not ship to international consumers). In the next Section, we show however that our analysis continues to hold true if consumers also share information with friends after they have searched but before they have bought.

Diffusion takes time and the pay-offs of all consumers who decided to wait (either at the beginning or after having observed a price) are discounted by a factor $0 < \delta < 1$.[11] Thus, active consumers that decided to wait for more information via friends before making a purchase behave similarly to passive consumers and their pay-offs are also discounted by δ. If after having exchanged information with friends, consumers are still not fully informed about all prices, they may search at that stage themselves. This implies that, in principle, a passive consumer who has not received any information from friends may search two times, whereas active consumers who, in first instance, decided to wait for information from friends may search one more time. In Figure 1, the option for passive consumers of buying after information exchange is dashed to indicate that this option may not be available if they did not receive any information from friends. Consumers observing both prices buy at the lowest of these prices (if they buy at all).

The social network through which information is diffused is modeled by a given random graph. Consumers communicate only with those consumers whom they are connected to and provide all their immediate friends with information of the (lowest) price at which they have bought the product.[12] We take the point of view that consumers engage in their social network for many reasons, not only to exchange price information through friends. Thus, the social network is given and we do not study the incentives to form links. Also, as consumers are not (negatively) affected by others using the information they have acquired, we take it that all acquired information is exchanged.

The advantage of receiving information via friends instead of actively searching oneself is to economize on search cost and is modeled in the following way. The full cost c of searching for and buying from a firm is decomposed into two parts: a true cost of search $s > 0$ and a cost $b \geq 0$ of buying the good. A friend not only informs about the price at which she bought the product, but also of how to find the firm. Accordingly, a consumer

[11] We think of $\delta > 0$ as a measure of the speed of information communication. If δ is high, communication is fast and consumers that decided to wait quickly obtain information from active friends who purchased the good. If $\delta = 0$, information never gets disseminated so that the Diamond paradox arises.

[12] Formally, information decays after one step. This is also the setting studied in Bramoulle and Kranton (2007), Banerji and Dutta (2009) and Ellison and Fudenberg (1995). The assumption is, however, not crucial to our conclusions. In Section 3.2.2 of his paper, Galeotti (2010) shows how one can take into account that information may also flow from friends of your friends and decays only after a finite number of steps and a similar robustness check could be applied here.

who wants to follow a referral from a friend economizes on the true search cost and only incurs the purchasing cost b. The easiest way to understand the analysis is when $b = 0$. In that case, a consumer can simply follow a referral and purchase without incurring a cost.[13] When searching and buying online, the parameters s and b have a natural interpretation: s is the cost of reviewing an alternative; once a decision is made from which firm to buy, a consumer still has to go to the check-out page where billing information has to be filled out.[14]

The probability a consumer has k links is denoted by $t(k)$, $k \in \{1, 2, ...\} = O$ with $\sum_{k\in O} t(k) = 1$. For future use we define

$$\tau(x) = \sum_{k\in O} t(k)x^k$$

as the probability generating function for $0 \leq x \leq 1$. Two expressions will be of particular importance in our analysis: $\tau(1-q)$ represents the probability that a consumer has only friends that do not search themselves, if each of the friends searches with probability q, whereas $1-\tau\left(1-\frac{q}{2}\right)$ represents the probability an active consumer who has obtained one price quote herself and decided to wait obtains information concerning the competitor's price through the network of friends. It follows that

$$\sum_{k=1}^{N} t_k \sum_{j=0}^{N} \binom{k}{j} q^j (1-q)^{k-j} y^j = \tau(qy + (1-q)).$$

As $\tau(x)$ is a convex function with $\tau(1) = 1$ and $1 - \frac{q}{2} = (1-q+1)/2$, it follows that $1 + \tau(1-q) - 2\tau\left(1-\frac{q}{2}\right) > 0$ for all $q > 0$, a property we will use in the analysis below. Also, $\tau'(1) = \lim_{q\to 1} \tau(1-q)/(1-q) = t(1)$.

The timing of decisions is as follows. First, firms simultaneously set prices. Second, the network structure is realized (so that firms cannot condition their pricing strategy on the details of the network structure). Third, not knowing the prices, consumers simultaneously choose their strategies as described above. In different sections we consider two cases: one where consumers cannot condition their decision to become an active searcher

[13] Most of the search literature does not need to distinguish between s and b as active search is assumed to be the only way to acquire information.

[14] Note that in this static model, b is not a switching cost as in Wilson (2010) as consumers have to incur it at any firm they buy from.

on their position in the social network and one where they can.[15]

We inquire into the existence of symmetric perfect Bayesian equilibria (PBE) and their properties. A PBE is described by a set of firms' and consumers' strategies such that each is choosing optimally given beliefs and the strategies of the others.

3 Preliminary Results

Existence of equilibrium is not an issue. As the first search is costly, there always exists a trivial "no trade" equilibrium with $q(k) = 0$ for all k. In such an equilibrium, knowing they will not sell to anyone, firms set prices larger than $v - c$ and this pricing behavior rationalizes consumers' beliefs that it is not rational to search.

Proposition 1 *For any $s > 0$, there exists an equilibrium without sales where $q(k) = 0$.*

If $t(1) = 1$, *i.e.*, all consumers have only one friend, then only a no trade equilibrium exists as there would be no consumer who can make price comparisons.

In the sequel, we focus on symmetric equilibria with positive sales, where $t(1) < 1$. The pay-offs of consumers associated with buying and continuing to search after observing a price $\widetilde{p}$ are given by $v - \widetilde{p} - b$ and $v - (1 - F(\widetilde{p}))\widetilde{p} - F(\widetilde{p})E(p|p < \widetilde{p}) - b - s$, respectively. We denote by r the cut-off price such that consumers are indifferent between buying immediately and continuing to search, which using the expressions given above, is implicitly given by

$$F(r)(r - E[p|p \leq r]) = s. \tag{1}$$

It is easy to see that, if there is a solution to (1), it must be unique for any non-degenerate $F(p)$.

Similarly, we denote by ρ the price at which an active consumer is indifferent between the best option now (i.e. the better of buying and searching) and waiting for information from friends. In principle, there could be multiple ρ's. As firms never price below $\underline{p}$, ρ's below $\underline{p}$ are irrelevant and we let ρ_1 denote the smallest of those prices where $\rho_1 > \underline{p}$. As payoffs from the different options of buying, waiting and continuing to search after having observed the price $\underline{p}$, are given by $v - \underline{p} - b, v - \underline{p} - b - s$, and $\delta\left(v - \underline{p} - b\right)$, respectively,

[15] In both cases, firms set uniform prices and cannot price discriminate on the basis of the number of links a consumer has.

buying outright at $\underline{p}$ is strictly preferred for $s > 0$ and $\delta < 1$. By the continuity of payoffs with respect to p, it follows that $\min\{r, \rho_1, v - b\} > \underline{p}$ so that active consumers are willing to buy immediately if the price p is such that $\underline{p} \le p \le \min\{r, \rho_1, v - b\}$.

The uniqueness of r helps us to establish the following lemma. In terms of Figure 1, the lemma rules out all equilibria where after receiving some information, consumers decide to continue to search.

Lemma 1 *Any symmetric equilibrium where goods are bought has $F(r) = 1$.*

If an equilibrium with prices p larger than r would exist, a firm's price will always be compared with another price, which incentivizes the firms to undercut. Suppose that an active consumer observes $p > r$ at her first search. Clearly, she does not buy outright. She may either search or wait. If she searches, the consumer obviously compares p to another price. If she waits, the consumer may receive another firm's price from her friends or if she does not receive another firm's price quote, she herself searches again after the information exchange as $p > r$. In either case, the consumer compares prices. The same is true for passive consumers. As any price larger than r is always compared to one more price, consumers buy at the lowest of these prices and firms therefore have an incentive not to charge prices above r.

The next result establishes that in equilibrium it is not possible that there exists a ρ such that $\rho < r$. The proof essentially shows that a consumer, who is indifferent between being active and passive, would prefer to continue to search rather than wait after having observed a first price quote. The only reason for an active consumer to wait after having observed a first price quote is that she hopes to be able to economize on the search cost s by getting informed about the other price quote via a friend. However, the chance of being informed through the social network about another price is smaller than the chance of being informed about any price. Thus, intuitively it makes more sense to be passive right from the beginning, than to be active first and then wait. A similar argument holds if consumers strictly prefer to be active rather than be passive. Hence, it must be that consumers strictly prefer to search than to wait after having observed a first price quote.

Proposition 2 *In any equilibrium where there is active search, we have that $r \le \rho$.*

Thus, in what follows, we will focus on reservation price equilibria (RPE) where there is active search. These equilibria are characterized as follows.

Definition 1 *A symmetric RPE is characterized by a distribution of prices $F(p)$ and consumers' reservation price strategy $(q(k), r)$ such that given the strategies of the other players*

(*i*) *Each firm employs a price strategy $F(p)$ with $\overline{p} = r$ that maximizes its expected profit given the equilibrium pricing strategy of other firms and consumers' equilibrium search strategy;*

(*ii*) *Each consumer optimally chooses the probability of being an active searcher $q(k)$ and searches optimally according to the reservation price rule given by r, given her correct expectations concerning the equilibrium strategies of the firms and other consumers.*

It follows that in any equilibrium where there is active search and trade, it must be the case that a fraction of consumers is passive from the beginning and does not engage in active search, i.e., $q < 1$. The argument is akin to the consumer search literature and derives from the argument supporting the Diamond paradox (1971): if all consumers are active and search themselves, then no one compares prices and firms would charge a price equal to $v - b$. But then no one would be willing to invest the search cost in the first place. Thus, we have the following important result.

Proposition 3 *There does not exist an equilibrium where $q(k) = 1$ for all $k \in O$.*

Interestingly, it is the passive consumers that make an equilibrium with sales possible. Given that they wait for information from friends, there is a positive probability that they are informed about both prices without incurring the search cost s. A firm may then not have an incentive to marginally increase the price from $\underline{p}$ as he may lose these consumers to the competitor. This proposition also implies that the market must be characterized by price dispersion as some consumers do compare both prices whereas others do not.

In terms of Figure 1, this section has argued that in an equilibrium with active trade the equilibrium path in the consumer search problem is such that (*i*) some consumers are active, while others are passive, (*ii*) active consumers immediately buy and (*iii*) passive consumers who obtained information from friends, buy immediately after they have received this information, while passive consumers who did not obtain information from friends, search and then buy immediately. Given these results it is quite intuitive that the results of this section continue to hold in a modified model, where active consumers share price information even if they themselves have not bought the product.

Proposition 4 *If active consumers share information of prices they have observed without buying, then in any equilibrium we continue to have $r < \rho$ and $F(r) = 1$.*

The main difference between the two versions of the model is when active consumers have observed prices, but do not buy themselves. If in equilibrium, however, active consumers immediately buy, this situation does not arise and the two versions of the model are equivalent. Thus, the analysis that follows applies to both market environments.

4 Active Markets

We start by analyzing markets where all participants have imperfect information on the network structure when making their strategic decisions. Another way to interpret this is to say that even though consumers may know the number of friends they are connected to, they do not condition their search strategy on this information. We focus on equilibria with positive sales, i.e., $0 < q < 1$. As $F(r) = 1$, all active consumers make a purchase at the first search so that an active consumer's expected payoff is equal to

$$v - E[p] - s - b.$$

Given that a passive consumer will always follow the information they are provided with, their expected payoff is given by

$$\delta \sum_{k \in O} t(k) \left[v - b - \sum_{m=1}^{k} \binom{k}{m} q^m (1-q)^{k-m} \left(\tfrac{1}{2^{m-1}} E[p] - \left(1 - \tfrac{1}{2^{m-1}}\right) E_{\min}[p] \right) - (1-q)^k (E[p] + s) \right].$$

This expression can be understood as follows. A passive consumer always makes a purchase either because she gets informed about a price from friends or she searches herself. The probability that m of a consumer's k friends search is equal to $\binom{k}{m} q^m (1-q)^{k-m}$. They all visit the same firm with probability $1/2^{m-1}$, in which case she buys at the expected price. When the friends happen to search different firms, which happens with probability $1 - 1/2^{m-1}$, the consumer pays the lowest of the two prices, which in expected terms is

$E_{\min}[p]$. The probability that none of a consumer's k friends search is $(1-q)^k$, in which case the consumer searches herself incurring s and buys from the first visited firm paying $E[p]$. In all these cases, she incurs a purchase cost b. The entire payoff depends on how fast information arrives to the consumer, and thus the discount factor δ.

The probability q is determined such that the consumer is indifferent between being active and passive and, therefore, in equilibrium these two expressions have to be equal. Applying the probability generating function described in the beginning of Section 2, the indifference condition can be written as

$$(1-\delta)(v-E[p]-b) = (1-\delta\tau(1-q))s + \delta\widetilde{\tau}(q)\left(E[p]-E_{\min}[p]\right), \tag{2}$$

where $\widetilde{\tau}(q) \equiv 1+\tau(1-q)-2\tau\left(1-\frac{q}{2}\right)$ is the ex ante probability that a consumer's friends have observed two different prices.

We now turn to the determination of the equilibrium pricing strategy of the firms. Setting price p, an individual firm's expected profit is given by

$$\Pi(p) = \left(\frac{q}{2}+\delta(1-q)\left\{\sum_{k\in O} t(k)\sum_{m=1}^{k}\binom{k}{m}q^m(1-q)^{k-m}\left[\frac{1}{2^m}+\left(1-\frac{1}{2^{m-1}}\right)(1-F(p))\right]\right)+\frac{(1-q)^k}{2}\right\}p.$$

Clearly, a consumer is active with probability q and half of the times she visits the firm under question to buy outright. With probability $1-q$, a consumer is passive in which case she purchases only after the information arrives to her, thus speed of information diffusion δ. She definitely buys from the firm if all of her m (out of k) active friends happen to visit the firm, which happens with probability $1/2^m$. With probability $1-1/2^{m-1}$, these active friends happen to visit different firms, in which case the firm under question makes sales only if its price is lower than that of the rival firm, or $1-F(p)$. Finally, with probability $(1-q)^k$ none of her friends search, in which case a consumer searches herself and buys from the firm half of the time. Using probability generating functions, the profit expression can be rewritten as follows:

$$\Pi(p) = \left(\frac{q}{2}+\delta(1-q)\left[\tau\left(1-\frac{q}{2}\right)-\frac{\tau(1-q)}{2}+\widetilde{\tau}(q)(1-F(p))\right]\right)p.$$

Equating these expected profits with the profit of setting a price equal to the upper bound of the distribution gives the equilibrium price distribution as

$$F(p) = 1 + \eta - \eta \frac{\overline{p}}{p}, \text{ with support } [\underline{p}, \overline{p}] \tag{3}$$

where

$$\eta = \frac{\frac{q}{2} + \delta(1-q)\left(\tau\left(1-\frac{q}{2}\right) - \frac{\tau(1-q)}{2}\right)}{\delta(1-q)\widetilde{\tau}(q)} > 0, \tag{4}$$

and $\underline{p} = \frac{\eta}{1+\eta}\overline{p}$ solves $F(\underline{p}) = 0$, whereas $\overline{p} = \min\{r, v-b\}$. The fraction η is the ratio of consumers who do not compare prices to those that do compare prices (as in the traditional models of Varian (1980) and Stahl (1989)). Here, the fraction of consumers who are informed about only one price consists of the fraction of active consumers and those passive consumers who receive only one price quotation from friends, while the fraction of consumers who are informed about both prices consists of only passive consumers who receive through their social network information about the offerings of both firms.

Let us first discuss the extreme cases where $\delta = 0$ or $\delta = 1$. In these cases, the Diamond paradox emerges and only an equilibrium with no trade exists. If $\delta = 0$, there is no advantage to being passive and no consumer ever compares prices. The left-hand side of (2) reduces to $v - E[p] - b$ and the right-hand side to s. Consumers choose to be active if the expected price is below $v - c$ and drop out of the market if it is greater than $v - c$. From (4) it is clear that η (and thus expected price) become infinitely large. Hence, consumers prefer not to be active. On the other hand, if $\delta = 1$, there is no advantage to being active so that $q = 0$. If no consumer acquires information herself, no one compares prices and again η (and thus expected price) become infinitely large. In this case, this is because $\widetilde{\tau}(0) = 0$. Hence, passive consumers prefer not to search themselves in the market.

Proposition 5 *If $\delta = 0$ or $\delta = 1$, there does not exist an equilibrium with active trade.*

Having explained the different conditions that should hold in an RPE with positive sales, we are now able to provide the main result of this section, namely that for any $0 < \delta < 1$ an equilibrium exists if the search cost is sufficiently small.

Theorem 1 *For any given $0 \leq t(1) < 1$, $0 < \delta < 1$, $v > 0$ and $0 \leq b < v$, there exists a $\overline{s} \leq v - b$ such that an RPE exists if, and only if, $s \leq \overline{s}$. If an RPE exists it is determined by the triple $(q, r, F(p))$ solving (1), (2), and (3). Furthermore, as $s \to 0$ the optimal* *search probability q converges to 1, while price dispersion disappears with*

$$\overline{p} = \underline{p} = v - b.$$

The proof of the proposition is given in the appendix. The main intuition can be understood as follows. If a fraction $0 < q < 1$ of consumers actively search themselves, while the others are passive, then there is a strictly positive probability that some of the inactive consumers are informed about two prices so that the price distribution of firms is non-degenerate. For each such a non-degenerate price distribution one can define a reservation price r and a cutoff price ρ. The main challenge then is to find a value of q such that consumers are indifferent between being active and passive. The proof shows that when $s \leq \overline{s}$ there exists such a q. It is clear that $\overline{s}$ depends on the exogenous parameters and in particular (in line with Proposition 4) that $\overline{s}$ approaches 0 as δ approaches 0 or 1.

The interesting aspect is that if a fraction $0 < q < 1$ of consumers actively search themselves, while the others do not, then as long as some consumers have more than one connection, i.e., $t(1) < 1$, it is the fraction of the passive consumers who are free-riding that provide a positive service to the active consumers as they are the ones who are informed about two prices. Thus, the passive consumers play a crucial role here to resolve the Diamond paradox. If s becomes arbitrarily small, it is clear that the incentives to actively search are larger, reducing the incentives for firms to undercut and creating less price dispersion.

In the limit when s becomes arbitrarily small, price dispersion must vanish as consumers can obtain another price quote at virtually no additional cost. Then, the share of passive consumers must disappear in the limit as passive consumers pay almost the same price as active ones but incur a cost of waiting for $0 < \delta < 1$. This, however, raises the market power of firms as only passive consumers compare prices. In the limit when no one compares prices, firms obtain monopoly power, thus resulting in the monopoly price $v - b$. Interestingly, and different from the Diamond paradox, consumers choose to be active for $s > 0$.

Figure 2 illustrates the equilibrium construction. The horizontal axis represents the fraction q of active consumers, while the cost and the expected benefit of search are presented on the vertical axis. The solid curve represents the expected benefits, while

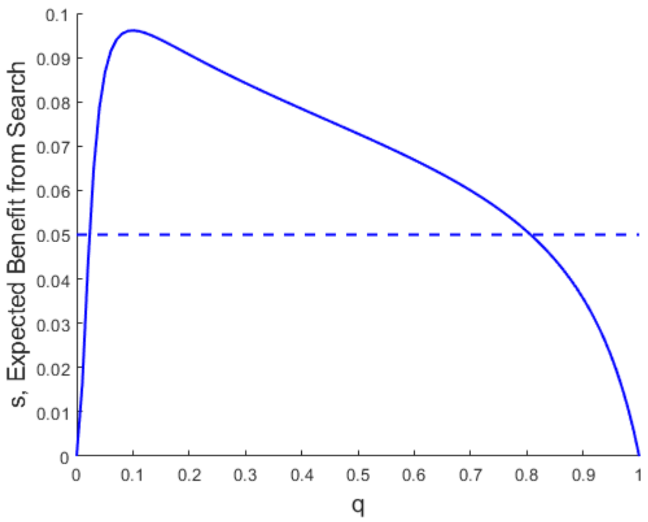

Figure 2: An illustration of existence of an RPE for $\delta = 0.9$, $b = s = 0.05$ and $t(k) = nk^{\gamma}$, where $\gamma = -1$, $\overline{k} = 100$ and $\sum_{k \in O} nk^{\gamma} = 1$.

the dashed horizontal line represents the cost of search. These parameters determine the shape of the bold curve representing the relative benefit of search. The proof shows what the figure presents, namely that when q approaches 0 or 1 the expected benefit of search approaches 0. As for interior values of q the expected benefits are positive and continuous in q, it must be the case that for small enough search cost values an equilibrium exists.

The figure shows that for (given) small enough values of s there are two intersection points and, hence, two equilibrium values of q where the market is active.[16] One may argue, however, as in other search models (see, e.g. Burdett and Judd (1983), Fershtman and Fishman (1992), Janssen and Moraga-Gonzalez (2004), and Honda (2015)) that the equilibrium corresponding to the higher search probability can be called a "stable" equilibrium in the sense that if the real search probability falls (slightly) short of the equilibrium value the expected benefit of search exceeds the cost so that consumers have an incentive to search more intensively. It follows that if the search cost asymptotically approaches zero, the optimal search probability in the active equilibrium approaches 1.[17] The comparative static analysis focuses on this "stable" equilibrium.

[16]For certain parameter configurations with high search costs, there may exist four equilibria with positive trade. For our comparative static analysis, we focus on sufficiently small search costs such that only two equilibria with active trade exist.

[17]The figure also shows there is a stable equilibrium where the market is inactive ($q = 0$) and (2) does not hold.

5 Comparative Statics

Given the equilibrium characterization, we now can provide insights into how market outcomes depend on exogenous parameters. We will first focus on the impact of network structure on equilibrium prices, before concentrating on the speed of communication in the network (represented by δ), and the impact of the cost of searching s and making the purchase b. A social network like Facebook has significantly increased the number of connections people have (although there remain a non-negligible fraction of consumers who do not use Facebook or other social networks) and the speed of information diffusion through the network. Online markets have significantly reduced search cost s, but not so much the cost b of making the purchase. In this section, we discuss the implications of these effects on market outcomes, especially the expected market price. Unless explicitly discussed otherwise, the changes in firms' profits is perfectly in line with expected price (as in an RPE eventually all consumers buy). In an RPE, the expected market price is proportional to s and given by $E[p] = s\left(\frac{\eta \ln\left(1+\frac{1}{\eta}\right)}{1-\eta \ln\left(1+\frac{1}{\eta}\right)}\right)$, which is increasing in η.

We investigate the limiting behavior when the network of consumers is dense and all consumers tend to be linked to each other. One may think that if all consumers potentially get information from many friends, competition would prevail and prices converge to marginal cost with price dispersion being eliminated. Surprisingly, however, as the next result shows, price dispersion remains an essential feature of any reservation price equilibrium.

Proposition 6 *For any $s > 0$, any RPE with active search is characterized by price dispersion even if all consumers tend to be linked to each other in the sense that $\lim_{k\to\infty} t(k) = 1$. Moreover, an RPE with active search exists if s is small enough.*

A consequence of the limiting price dispersion is that prices do not converge to marginal cost. The reason that price dispersion remains is that in any RPE without price dispersion, the reservation price, and hence, the expected price should be infinitely large. But then consumers are better off not being active. We show that if s is small, an equilibrium with active search and trade exists.

It is important to understand that for this result to hold the first search should be costly. If this were not the case, all consumers would search at least once and the probability of obtaining the second price from friends then goes to one as the number of links to each consumer in the population goes to infinity. As a consequence, market frictions would disappear and prices converge to marginal cost. This also explains why in Galeotti (2010) prices do converge to marginal cost if the network of consumers gets dense: it is the consequence of the first search being assumed to be free in his model.

By means of numerical simulations, we can analyze intermediate cases of network connectivity. We assume the social network can be described by a random graph that follows a power law[18] $t(k) = nk^{\gamma}$, as the probability a consumer has k links, where $\gamma \in \mathbb{R}$ represents the network density and larger γ values stand for denser networks where consumers tend to have more connections. In particular, comparing two networks with probabilities of connections being given by $t(k)$ and $\widetilde{t}(k)$, the network with probabilities $t(k)$ is generated by a higher γ (and thus, denser) if there exists a $\widetilde{k} \in O$ such that $t(k) < \widetilde{t}(k)$ for all $k < \widetilde{k}$, whereas $t(k) > \widetilde{t}(k)$ for all $k > \widetilde{k}$. In Figures 3 and 4,[19] we gradually increase γ from -2 to 2.

Figure 4 depicts the impact of network density on the expected price. It shows that the expected price is decreasing in γ and that certainly when the network is not very dense (γ is small), this impact is quite strong as the expected price may decrease by around 50% as γ increases from around -2 to 0. The main, direct impact can be understood by noting that if the network is described by the power law $t(k) = nk^{\gamma}$ it follows from (4) that the ratio η is given by

$$\eta = \frac{\frac{q}{2} + \delta(1-q)\sum_{k\in O} nk^{\gamma}\left[\left(1-\frac{q}{2}\right)^{k} - \frac{(1-q)^{k}}{2}\right]}{\delta(1-q)\left(1+\sum_{k\in O} nk^{\gamma}\left[(1-q)^{k} - 2\left(1-\frac{q}{2}\right)^{k}\right]\right)}.$$

As the term in square brackets in the numerator is positive and decreasing in k, while the term in square brackets in the denominator is negative and increasing in k, it follows that networks with higher γ's put relatively more weight on a higher number of connections, which implies that η is decreasing in γ. As lower η's reflect the fact that there are relatively more price comparing consumers in the market, the direct effect puts downward pressure on prices. There is, however, also an indirect effect through the consumers' search prob-

[18]Empirical analysis has demonstrated that many social networks can be described by a power-law of the form we assume here (see, for example, Price (1965)).

[19]The following parameter values have been used: $v = 1$, $s = 0.05$, $b = 0.04$, $\delta = 0.9$, and $\overline{k} = 100$.

ability q. A decrease in price levels is associated with a decrease in price dispersion, as measured by the difference $E[p] - E_{\min}[p]$. This makes it more attractive for consumers to become active as the main benefit of waiting, namely being informed about both prices and therefore being able to buy at the lowest of the prices, becomes smaller. In addition, as the expected price is lower, consumers would like to have the benefit of the purchase now rather than having to wait for information through friends. The associated increase in the share of active consumers as illustrated in Figure 3 increases η and therefore also increases expected price. This indirect effect is, however, smaller than the direct effect and thus, the overall impact on expected price is decreasing.

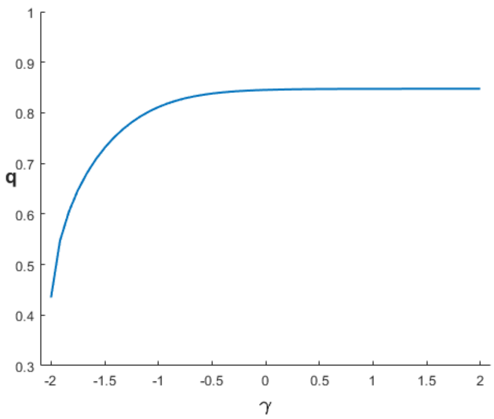

Figure 3: Impact of network density on the share of active consumers

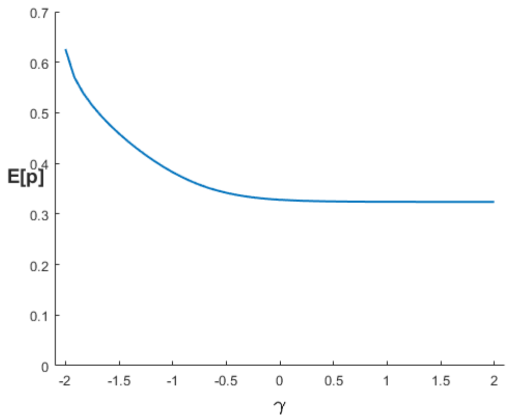

Figure 4: Impact of network density on the expected price

Note that the above result on the fraction of consumers actively acquiring information is strikingly different from Galeotti (2010) and Galeotti and Goyal (2010) where an agent's probability of actively acquiring information negatively correlates with the number of links she has. Intuitively, one might expect that the more connections a consumer has the less likely she is to become active as, all else being equal, the probability that she obtains information from friends rises. In our case, however, it is the combined price effect through a lower expected price and lower price dispersion as measured by $E[p] - E_{\min}[p]$ that overrides this intuitive effect.

Next, we investigate the impact of a change in δ. Online social media networks have significantly increased the speed with which consumers may share information, which in our model is measured by δ. We know that a higher δ, resulting in faster information transmission, permits a consumer to access information quickly from friends, but that

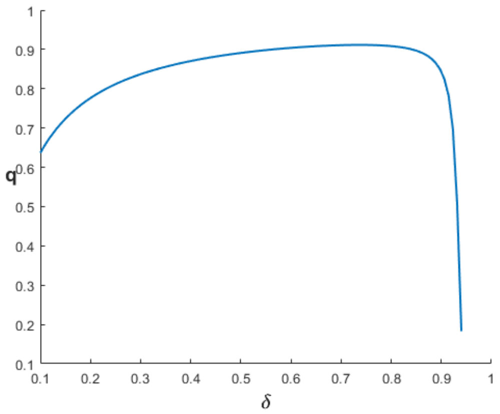

Figure 5: Impact of δ on the share of active consumers

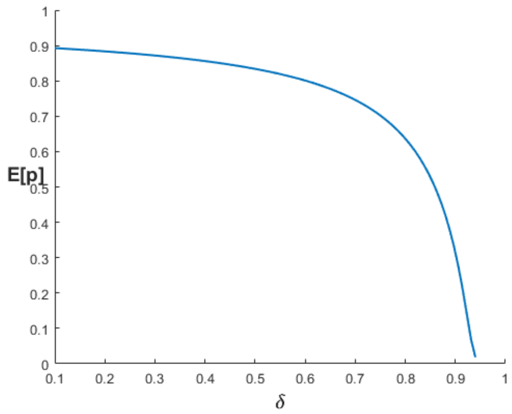

Figure 6: Impact of δ on expected price

this may have repercussions on the incentives to search. Proposition 4 establishes that at extreme values of δ an equilibrium with active search and trade does not exist. Numerical simulations in Figures 5 and 6 show the effect of an increase in δ on the share of active consumers and the expected price when we change the value of δ in an intermediate range (here from 0.1 to 0.94). For larger values of δ, an equilibrium with active search ceases to exist.[20] As before, there is a direct and an indirect effect of δ on prices. The direct effect can be seen by taking the derivative of η with respect to δ in (4). It is easy to see that this derivative is negative so that an increase in the speed of information processing in the population increases the share of price comparing (passive) consumers. This has a dampening effect on prices. There is also an indirect effect via q, however. For small values of δ the indirect effect is very similar to the indirect effect we mentioned in relation to the impact of γ, namely that as prices and price dispersion decline, consumers have more incentives to become active searchers themselves. In this case, there is, however, also a direct effect of δ on q (as shown in Equation (2)). As δ becomes large, there is almost no downside to waiting anymore and consumers massively change their behavior and become passive. As, focusing on stable equilibria, η is increasing in q this further strengthens the direct effect so that the total effect of δ on the expected price becomes more pronounced.

Interestingly, the speed of information diffusion also has a non-monotonic effect on firms' profits as shown in Figure 7. When the speed of information diffusion is relatively

[20] The Figures are drawn for the following parameter values: $v = 1$, $b = s = 0.05$, $t(k) = nk^{\gamma}$, $\overline{k} = 100$, and $\gamma = 0$.

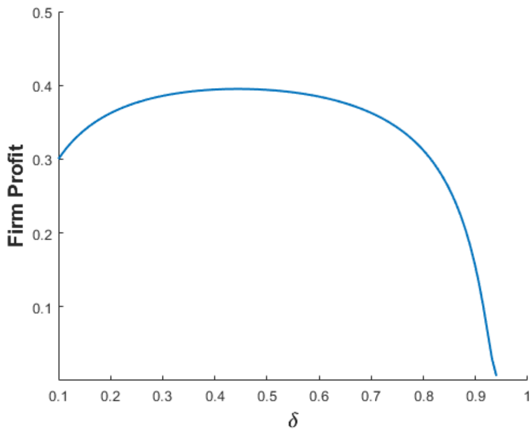

Figure 7: Impact of δ on firm profit

low, firms have an incentive to increase the number of active consumers as these consumers buy immediately in an RPE, whereas the sales to passive consumers are heavily discounted. Thus, even though expected price is monotonically decreasing in the speed of information diffusion, the fact that the fraction of active consumers is increasing offsets the decrease in revenue per consumer. On the other hand, when the speed of information diffusion is already relatively high to begin with, then firms would not want to increase it further. At some point, if the speed of information diffusion is very large, both firms and consumers suffer from a further increase in the diffusion speed as an equilibrium with active trade ceases to exist.

Finally, we proceed by studying the impact of a change of s and b. We already know that the optimal search probability goes to 1 in the limit when s goes to zero, suggesting that as s starts increasing from 0 the optimal search probability decreases. The next proposition shows that the underlying effects of changes in b and s are similar to each other and also hold true outside the region where s is close to 0.

Proposition 7 *In any stable RPE, the share of active consumers is decreasing in b and s.*

We know from Theorem 1 that in the limit when s is arbitrarily small, prices converge to the monopoly price $v - b$. This immediately implies that for small enough search cost, the expected price must be decreasing in s as more consumers make price comparisons and firms will compete for these consumers. As in the stable equilibria we consider η is increasing in q and expected price is increasing in η, it follows that the expected price is

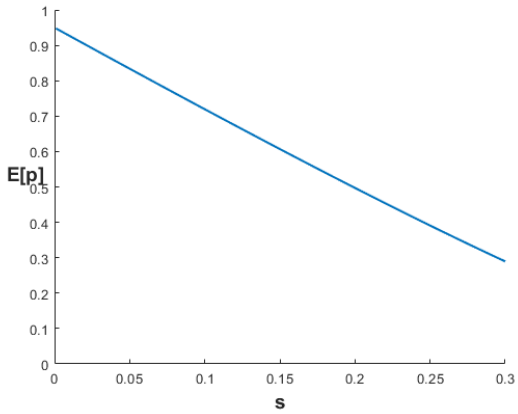

Figure 8: Impact of s on expected price

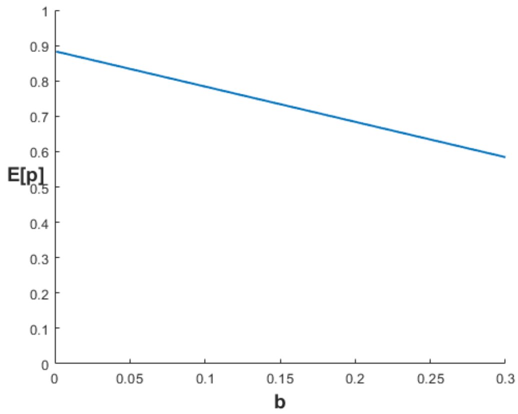

Figure 9: Impact of b on expected price

decreasing in s for larger values of s as well, as illustrated in Figure 8.[21] Not surprisingly, Figure 9 shows a similar pattern with respect to b.

6 Information Asymmetry about Connections

In many markets consumers know how well they are connected before deciding whether to search. In our model, this implies they can condition their search decision on the number of connections. Denote by $\overline{k}$ the largest number of links any consumer in the population has. Most firms, on the other hand, do not know the exact number of connections of each consumer, but may well know the distribution of the network structure, i.e., firms cannot price discriminate between consumers with different numbers of friends. Thus, in this section we analyze markets where consumers and firms have asymmetric knowledge about the relevant network structure and show that the qualitative results obtained so far continue to hold.

If consumers know the number of links they have before engaging in search, not all consumers search with the same probability. There may be consumers whose optimal search probability is between zero and one, and there will be ones who either definitely search or do not search at all. As consumers with more connections are more likely to obtain information via their social network than consumers with fewer links, the expected payoff from waiting is increasing in the number of connections. Thus, if a consumer with, say, $\widehat{k}$ connections is indifferent between becoming an active searcher and staying passive,

[21] We use the following parameter values for the both figures: $v = 1$, $t(k) = nk^{\gamma}$, $\overline{k} = 100$, $\gamma = 0, \delta = 0.5$. For Figure 8 we set $b = 0.05$, while for Figure 9 we fixed $s = 0.05$.

so that $0 < q(\widehat{k}) < 1$, then all consumers with more connections wait to obtain information through their social network, while those with fewer connections definitely search. If there is no consumer who is indifferent, then we can set $\widehat{k}$ to be the largest number such that $q(\widehat{k}) = 1$. Clearly, for any $s > 0$ it cannot be the case that $\widehat{k} = \overline{k}$ and $q(\overline{k}) = 1$ as then all consumers would search themselves and no one would compare prices.

Proposition 8 *Let $\widehat{k}, 1 \leq \widehat{k} \leq \overline{k}$ be as defined above. Consumers with a number of friends less than $\widehat{k}$ search with probability one and consumers with more than $\widehat{k}$ friends do not search.*

Thus, consumers with more connections are more inclined to not search themselves and wait for information from their friends. Given this result, the consumer behavior in any equilibrium can be characterized by $\widehat{k}$, $q(\widehat{k})$ and a reservation price r. In what follows we use q as a short-hand notation for $q(\widehat{k})$.

Correctly anticipating the optimal behavior of consumers, an individual firm's expected profit of setting a price equal to $p \leq r$ is equal to

$$\begin{aligned}
\Pi(p) =& \frac{1}{2}\widehat{w}p + \delta t(\widehat{k})(1-q)\left[\sum_{m=1}^{\widehat{k}}\binom{\widehat{k}}{m}w^m(1-w)^{\widehat{k}-m}\left(\frac{1}{2^m}\right) + \frac{(1-w)^{\widehat{k}}}{2}\right]p \\
&+ \delta t(\widehat{k})(1-q)\sum_{m=1}^{\widehat{k}}\binom{\widehat{k}}{m}w^m(1-w)^{\widehat{k}-m}\left(1-\frac{1}{2^{m-1}}\right)(1-F(p))p \\
&+ \delta\left[\sum_{k=\widehat{k}+1}^{\overline{k}} t(k)\sum_{m=1}^{k}\binom{k}{m}w^m(1-w)^{k-m}\left(\frac{1}{2^m}\right) + \frac{(1-w)^{\widehat{k}}}{2}\right]p \\
&+ \delta\sum_{k=\widehat{k}+1}^{\overline{k}} t(k)\sum_{m=1}^{k}\binom{k}{m}w^m(1-w)^{k-m}\left(1-\frac{1}{2^{m-1}}\right)(1-F(p))p,
\end{aligned} \tag{5}$$

where $\widehat{w} = \sum_{k=1}^{\widehat{k}-1} t(k) + t(\widehat{k})q$ is the average search probability of a consumer and $w = \frac{\sum_{k=1}^{\widehat{k}-1} t(k)k + t(\widehat{k})\widehat{k}q}{\sum_{k=1}^{\overline{k}} t(k)k}$ is the search probability of a consumer's neighbor (not knowing how many friends the neighbor has).

This expression can be understood as follows. A fraction of $\sum_{k=1}^{\widehat{k}-1} t(k)$ consumers has less than $\widehat{k}$ links. They search themselves and visit the firm with probability 0.5. Since they do not compare prices they pay the price charged by the firm (if this is not larger than their reservation price). Consumers with $\widehat{k}$ links, who make a share of $t(\widehat{k})$ of the population, search with probability q and visit the firm in half of the cases, and these consumers along with those with fewer links give the first term in (5). With probability $(1-q)$, consumers with $\widehat{k}$ links do not search and can be informed by their friends about price(s). Each of their friends searches with an expected probability w. Then, consumers obtain information only about the firm's price if all of their searching friends, represented by m, happen to visit that firm, which occurs with probability $\frac{1}{2^m}\binom{\widehat{k}}{m} w^m (1-w)^{\widehat{k}-m}$. If none of her friends search, meaning $m = 0$, the consumer searches herself later and visits the firm half of the time. This gives the second term. The third term represents the probability that a consumer does not search and obtains information about both prices from her m searching friends. The probability of this event is equal to $\left(1-\frac{1}{2^{m-1}}\right)\binom{\widehat{k}}{m} w^m (1-w)^{\widehat{k}-m}$. In this case, a consumer buys from the firm if the other firm charges a higher price than p, which happens with probability $1-F(p)$. Finally, the last two terms in the profit function account for the share of the population with more than $\widehat{k}$ links. These expressions are similar to the ones for the consumers with $\widehat{k}$ links who do not search themselves. As all waiting consumers buy with a delay (after the information about prices arrives to them), the payoff from these consumers is discounted by δ.

The expected profit in (5) can be simplified as [22]

$$
\begin{aligned}
\Pi(p) = & \left[\frac{\widehat{w}}{2} + \delta t(\widehat{k})(1-q)\left(\left(1-\frac{w}{2}\right)^{\widehat{k}} - \frac{(1-w)^{\widehat{k}}}{2}\right) + \delta \sum_{k=\widehat{k}+1}^{\overline{k}} t(k)\left(\left(1-\frac{w}{2}\right)^{k} - \frac{(1-w)^{k}}{2}\right)\right] p \\
& + \delta\left[t(\widehat{k})(1-q)\left(1+(1-w)^{\widehat{k}} - 2\left(1-\frac{w}{2}\right)^{\widehat{k}}\right) + \sum_{k=\widehat{k}+1}^{\overline{k}} t(k)\left(1+(1-w)^{k} - 2\left(1-\frac{\widehat{w}}{2}\right)^{k}\right)\right](1-F(p))p,
\end{aligned}
$$

where, in the square brackets in the first line, we have the share of consumers who do not compare prices and in the square brackets in the second line, we have the share of consumers who compare prices.

Equating this expression to the profit that the firm expects to make by charging $\overline{p}$, we can derive the equilibrium pricing distribution function:

$$
F(p) = 1 + \widehat{\eta} - \widehat{\eta}\frac{\overline{p}}{p}, \text{ with support } \left[\underline{p}, \overline{p}\right], \tag{6}
$$

where (similar to the previous section) $\widehat{\eta}$ is the fraction of the share of consumers who do

[22] Unfortunately, we cannot use probability generating functions to simplify this expression further due to the fact that consumers search differently depending on the number of their connections.

not compare prices to the share of consumers who compare prices. As before, the upper bound of the distribution is not larger than the reservation price r and $v - b$, where the reservation price of a consumer is determined in (1) and $F(p)$ is given by (6).

Obviously, a full equilibrium analysis under asymmetric information on network structure is somewhat tedious. For a given $\widehat{k}$ and a $q(\widehat{k})$ one can derive the equilibrium price distribution, but given the equilibrium price distribution one should check whether the postulated behavior of consumers is indeed optimal and all consumers with $k > \widehat{k}$ prefer to be passive and all consumers with $k < \widehat{k}$ prefer to be active.

The proposition below states the main result for small enough values of s :

Proposition 9 *For any given $0 \leq t(1) < 1$, $0 < \delta < 1$, $v > 0$, $0 \leq b < v$, and sufficiently small search cost s, there exists an RPE given by the triple $(q, r, F(p))$ and the cutoff $\widehat{k}$, which are determined by (1), (6), and*

$$\frac{s}{v-b} \leq \frac{1-\delta}{1-\delta(1-w)^{\widehat{k}} + \frac{\widehat{\eta}}{1-\widehat{\eta}\ln\left(1+\frac{1}{\widehat{\eta}}\right)}\left[\delta\left(1+(1-w)^{\widehat{k}} - 2\left(1-\frac{w}{2}\right)^{\widehat{k}}\right)\left((1+2\widehat{\eta})\ln\left(\frac{1+\widehat{\eta}}{\widehat{\eta}}\right) - 2\right) + (1-\delta)\ln\left(\frac{1+\widehat{\eta}}{\widehat{\eta}}\right)\right]},$$

where the inequality holds with equality if $0 < q(\widehat{k}) < 1$. Furthermore, as $s \to 0$, the cutoff number of links $\widehat{k}$ equals $\overline{k}$ and the optimal search probability q converges to 1, while price dispersion disappears with

$$\overline{p} = \underline{p} = v - b.$$

Figure 10 provides an illustrative example of the Proposition.[23] Note that on the horizontal axis we represent w, the probability a random neighbor is active. The solid line representing the expected benefit from search has vertical elements. This is a consequence of the fact that for certain ranges of search costs, there is no indifferent consumer so that w, and the underlying decision of consumers whether or not to become active, does not change. This can be explained as follows. Suppose that the search cost is such that consumers with $\widehat{k} < \overline{k}$ links search with strictly positive probability, $0 < q(\widehat{k}) < 1$. This is a situation represented by one of the non-vertical parts of the stable, right-hand side of the Figure. From Proposition 8, it follows that consumers with more than $\widehat{k}$ connections still search with probability one. Suppose then that the search cost decreases

[23]The Figure is drawn for the following parameter values: $v = 1$, $b = 0.1$, $s = 0.025$, $\delta = 0.92$, $\overline{k} = 5$, and $\gamma = -2.5$.

gradually. This change in search cost first affects the equilibrium search probability $q(\widehat{k})$ of consumers with $\widehat{k}$ connections, until this search probability obtains an extreme value of 1. At this point, consumers with $\widehat{k}$ connections are indifferent between searching and not searching only when they search with probability one, while those with more than $\widehat{k}$ connections still search with probability zero. The difference in payoffs from being passive between consumers with $\widehat{k}$ friends and those with $\widehat{k}+1$ friends is related to the number of connections they have. This means that these two types of consumers cannot be indifferent between searching and not searching simultaneously. Then, obviously there is a range of search costs where consumers with $\widehat{k}$ connections strictly prefer to search, whereas those with $\widehat{k}+1$ links strictly prefer not to search (yet).

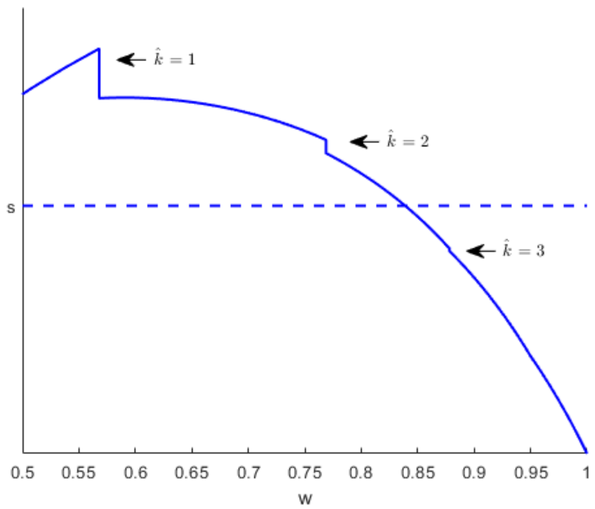

Figure 10: Illustration of existence of an RPE when consumers observe their number of links

We do not explicitly perform the comparative statics exercise for this asymmetric model of search and information acquisition, as it is similar to the one in the previous section. In particular, it remains true that for any positive (sufficiently) small cost s price dispersion remains if the number of connections becomes large as some consumers will be free riding.

7 Conclusion

In this paper we have analyzed how word-of-mouth (WOM) communication through social networks affects information acquisition and diffusion by consumers and how this impacts the market power of firms. Without WOM communication our model is prone to the

Diamond paradox where the market breaks down due to the fact that no consumer makes price comparisons. WOM communication overcomes the Diamond paradox. Consumers that do not actively search themselves and free-ride on their friends in the social network may well be informed about different prices. The price comparisons they make provide positive externalities to the rest of the consumer population that actively searches as firms compete to be able to also sell to them.

As some consumers do compare prices, while others do not, the market is characterized by price dispersion. The level of prices and the nature of price dispersion depends on the network architecture, the search cost and how quickly information is diffused in the social network. In the context of evaluating the impact of online markets and social networks it is important to know how expected price, price dispersion and firms' profits react to (i) a decrease in search cost, (ii) an increase in the connectivity of the social network and (iii) an increase in the speed of information diffusion in the network. We find that there are opposing effects as the increased connectivity and speed of information diffusion lower expected market prices, whereas the decrease in search cost increases them. Importantly, price dispersion does not disappear even if all consumers are very well connected.

We see our paper as making a first step in analyzing how WOM communication and sequential search interact with each other. There are obvious ways our work can be extended in different directions. When introducing WOM communication in a sequential search framework, one has to specify in detail at which stage in the search process consumers are able to exchange information. In this paper we have taken the view that active searchers search simultaneously and exchange information after they have all searched. Alternatively, one may consider different scenarios where some early adopters search first before others. Another direction that may be taken is to analyze markets with product differentiation a la Wolinsky (1986). In this case, consumers may not only communicate about prices, but also about the product match. In such markets, the degree of homophily (defined as the closeness of a consumer's preferences to those of his neighbors) will be important. Another direction for future research would be to model the incentives to share information directly. In some markets, consumers receive a financial benefit from firms for a successful referral and an important question is how consumers will react to such incentives and when such a financial incentive is optimal from a firm's perspective and to whom to give it.

8 Appendix A: Proofs

Before we prove Proposition 2, we first show that if there is some $\rho < r$, it must be unique. Clearly, if all ρ's other than ρ_1 are larger than r, there is a unique $\rho < r$. Thus, in what follows we rule out that there exists a $\rho_2 \neq \rho_1$ that is smaller than r.

Two cases need to be considered: at price p such that $\rho_1 < p < \rho_2$ (i) active consumers prefer waiting to buying, and (ii) active consumers prefer buying to waiting. In (i), prices in that interval are not exchanged. Let $\Pr[I, \widetilde{p}]$ be the probability that an active consumer who himself observes a price $\widetilde{p}$ has friends who share information about a price of the other firm which happens to price below $\widetilde{p}$. Then, it must be that $\Pr[I, \rho_1] = \Pr[I, \rho_2]$. The equations determining ρ_1 and ρ_2 are then

$$\begin{aligned}(1-\delta)(v-b-\rho_1) &= \delta \Pr[I, \rho_1](\rho_1 - E[p \leq \rho_1]),\\ (1-\delta)(v-b-\rho_2) &= \delta \Pr[I, \rho_1](\rho_2 - E[p \leq \rho_1]),\end{aligned}$$

respectively. It is easy to see that these equations cannot hold simultaneously.

In (ii), given that a share q of consumers is active, we have $\Pr[I, \rho_1] = \left(1 - \tau(1 - \frac{q}{2})\right) F(\rho_1)$. The equation determining ρ_1 can then be written as

$$(1-\delta)(v-b-\rho_1) = \delta \left(1 - \tau(1 - \frac{q}{2})\right) F(\rho_1)(\rho_1 - E[p \leq \rho_1]).$$

However, the LHS of the equation is strictly decreasing in ρ_1 while the RHS is increasing, which means that at prices slightly above ρ_1 active buyers prefer waiting to buying, a contradiction. Thus, there is at most one $\rho < r$.

Next, we present two lemmas that are relevant for the case where r would be larger than ρ. First, in that case observe that for any price in region (ρ, r) consumers cannot be indifferent between searching and waiting. We know that there is a unique ρ (if it exists) such that for $p > \rho$ consumers (strictly) prefer waiting to buying. Also uniqueness of r implies that consumers strictly prefer buying to searching for any $p < r$. These two facts imply that buyers strictly prefer waiting to searching as waiting is preferred to buying and buying is preferred to searching for $p \in (\rho, r)$.

Lemma 2 *For $r > \rho$, consumers strictly prefer waiting to searching for any $p \in (r, \rho)$.*

An immediate consequence of the lemma is, that in a symmetric price dispersed equilibrium with $\rho < r$ and $F(r) = 1$, $F(p)$ must be a continuous and strictly increasing function of p for $p \in [\underline{p}, \rho]$,where $\underline{p}$ is the lower bound of the support of $F(p)$. Notice that there is a discontinuity in the demand of a firm at ρ. If a firm prices at ρ, it certainly sells to active consumers who searches it first. Yet if it prices at $\rho + \epsilon$, its demand from active consumers who buy outright vanishes—these consumers choose to wait and may be informed of a lower price and therefore do not buy from the firm. Thus, there must be a $\underline{r}$ with $\rho < \underline{r} \leq r$ such that $F(p)$ is flat for $p \in (\rho, \underline{r})$. However, we can go one step further as prices larger than ρ never get shared as active consumers who observe those prices do not make purchase, but wait. Therefore, firms' demand is the same for any price larger than ρ and smaller than r. Thus, setting r dominates setting any price between ρ and r.

Lemma 3 *If $\rho < r$, it must be that $\underline{r} = r$ and that there a mass point at r.*

We are now ready to prove Proposition 2.

Proof of Proposition 2

Given that $F(r) = 1$, it is clear that any $\rho_i > r$ is irrelevant for our analysis as firms do not price above r. Then, if $\rho_1 > r$, all ρ's are irrelevant for our analysis. Thus, we suppose that $\rho < r$ and prove the proposition by contradiction. We consider two cases: $F(\rho) = 1$ and $F(\rho) < 1$.

For $F(\rho) = 1$, it is easy to see that r solves

$$r = E[p] + s. \tag{7}$$

If to the contrary of what is claimed in the proposition, $r > \rho$, then after observing r a consumer prefers waiting to buying, which results in the following inequality:

$$v - b - r < \delta\left(v - b - r + \left(1 - \tau(1 - \frac{q}{2})\right)(r - E[p])\right).$$

Taking the terms containing $v-b-r$ to the LHS and employing (7) simplifies the inequality as

$$(1-\delta)(v - b - E[p] - s) < \delta\left(1 - \tau(1 - \frac{q}{2})\right)s,$$

or

$$(1-\delta)(v - b - E[p]) < \left(1 - \delta\tau(1 - \frac{q}{2})\right)s. \tag{8}$$

Next, in an equilibrium with active search, consumers should at least weakly prefer being active. An active consumer's expected payoff is equal to

$$v - E[p] - s - b,$$

while (always following the information they are provided with), a passive consumer's expected payoff is given by

$$\delta \sum_{k\in O} t(k)\left[v - b - \sum_{m=1}^{k}\tbinom{k}{m}q^m(1-q)^{k-m}\left(\tfrac{1}{2^{m-1}}E[p] - \left(1 - \tfrac{1}{2^{m-1}}\right)E_{\min}[p]\right) - (1-q)^k(E[p]+s)\right].$$

Weakly preferring being active implies that

$$(1-\delta)(v - b - E[p]) \geq (1 - \delta\tau(1-q))s + \delta\widetilde{\tau}(q)(E[p] - E_{\min}[p]),$$

where $\widetilde{\tau}(q) = 1 + \tau(1-q) - 2\tau(1-q/2)$. As $\tau(1-q/2) > \tau(1-q)$, it is easy to see that this inequality and (8) cannot hold simultaneously.

We next consider the case where $F(\rho) < 1$. Given Lemma 3 there must be a mass point at r and no probability mass in (ρ, r). Then, the reservation price r is determined by $r = E[p \leq \rho] - \frac{s}{F(\rho)}$, or

$$s = F(\rho)(r - E[p \leq \rho]). \tag{9}$$

As at price r buyers strictly prefer waiting to buying, which results in the following inequality

$$v - b - r < \delta\left\{v - b - r + F(\rho)\left(1 - \tau(1 - \frac{q}{2})\right)(r - E[p \leq \rho])\right\}.$$

Employing (9) we obtain

$$v-b-r<\delta\left\{v-b-r+\left(1-\tau(1-\frac{q}{2})\right)s\right\},$$

or

$$(1-\delta)(v-b-r)<\delta\left(1-\tau(1-\frac{q}{2})\right)s. \tag{10}$$

Now, consider that consumers should at least weakly prefer being active. The payoff of being active is

$$F(\rho)(v-b-E[p\leq\rho])+\delta(1-F(\rho))\left(v-b-r+F(\rho)\left(1-\tau(1-\frac{q}{2})\right)(r-E[p\leq\rho])\right)-s,$$

or adding $F(\rho)(r-r)=0$ to the first term and employing (9) we get

$$F(\rho)(v-b-r)+\delta(1-F(\rho))\left(v-b-r+\left(1-\tau(1-\frac{q}{2})\right)s\right).$$

Finally, adding and subtracting $v-b-r$, we obtain

$$v-b-r-(1-F(\rho))\left[(1-\delta)(v-b-r)-\delta\left(1-\tau(1-\frac{q}{2})\right)s\right]$$

Next, the payoff of being passive is

$$\begin{aligned}&\delta\left\{v-b-\left[F^2(\rho)+2F(\rho)(1-F(\rho))\left(1-\frac{\tau(1-\frac{q}{2})}{2}\right)\right]E[p\leq\rho]\right.\\&-\tau(1-\frac{q}{2})F(\rho)(1-F(\rho))r-(1-F(\rho))^2r+F^2(\rho)\widetilde{\tau}(q)(E[p\leq\rho]-E_{\min}[p\leq\rho])\\&\left.-\left[F^2(\rho)\tau(1-q)+2F(\rho)(1-F(\rho))\tau(1-\frac{q}{2})+(1-F(\rho))^2\right]s\right\},\end{aligned}$$

or

$$\begin{aligned}&\delta\left\{v-b-E[p\leq\rho]-\left[1-F(\rho)-F(\rho)(1-F(\rho))\left(1-\tau(1-\frac{q}{2})\right)\right](r-E[p\leq\rho])\right.\\&+F^2(\rho)\widetilde{\tau}(q)(E[p\leq\rho]-E_{\min}[p\leq\rho])\\&\left.-\left[F^2(\rho)\tau(1-q)+2F(\rho)(1-F(\rho))\tau(1-\frac{q}{2})+(1-F(\rho))^2\right]s\right\}\\=\;&\delta\left\{v-b-r+F(\rho)\left[1+(1-F(\rho))\left(1-\tau(1-\frac{q}{2})\right)\right](r-E[p\leq\rho])\right.\\&+F^2(\rho)\widetilde{\tau}(q)(E[p\leq\rho]-E_{\min}[p\leq\rho])\\&\left.-\left[F^2(\rho)\tau(1-q)+2F(\rho)(1-F(\rho))\tau(1-\frac{q}{2})+(1-F(\rho))^2\right]s\right\}.\end{aligned}$$

Employing (9), we obtain

$$\begin{aligned}&\delta\left\{v-b-r+\left[1+(1-F(\rho))\left(1-\tau(1-\frac{q}{2})\right)\right]s+F^2(\rho)\widetilde{\tau}(q)(E[p\leq\rho]-E_{\min}[p\leq\rho])\right.\\&\left.-\left[F^2(\rho)\tau(1-q)+2F(\rho)(1-F(\rho))\tau(1-\frac{q}{2})+(1-F(\rho))^2\right]s\right\}\end{aligned}$$

It is easy to verify that the multiplicative terms of s can be simplified so that we have

$$\delta\left\{v-b-r+\left[(1+F(\rho))\left(1-\tau(1-\tfrac{q}{2})\right)-F^2(\rho)\widetilde{\tau}(q)\right]s+F^2(\rho)\widetilde{\tau}(q)(E[p\leq\rho]-E_{\min}[p\leq\rho])\right\}.$$

Hence, the weak inequality that consumers should weakly prefer being active can be written as

$$\begin{aligned}&v-b-r-(1-F(\rho))\left[(1-\delta)(v-b-r)-\delta\left(1-\tau(1-\frac{q}{2})\right)s\right]\\&\geq\delta\left\{v-b-r+\left[(1+F(\rho))\left(1-\tau(1-\frac{q}{2})\right)-F^2(\rho)\widetilde{\tau}(q)\right]s+F^2(\rho)\widetilde{\tau}(q)(E[p\leq\rho]-E_{\min}[p\leq\rho])\right\}.\end{aligned}$$

Transferring all terms containing $v-b-r$ to the LHS and those containing s to the RHS and simplifying yields

$$\begin{aligned}(1-\delta)(v-b-r)\quad\geq\quad&\delta\left(1-\tau(1-\frac{q}{2})+1-\tau(1-\frac{q}{2})-F(\rho)\widetilde{\tau}(q)\right)s\\&+\delta F(\rho)\widetilde{\tau}(q)(E[p\leq\rho]-E_{\min}[p\leq\rho]).\end{aligned}$$

Observe that the LHS of the inequality is equal to the LHS of (10). The RHS of the equation is certainly larger than the RHS of (10) if

$$\begin{aligned}1-\tau(1-\frac{q}{2})-F(\rho)\widetilde{\tau}(q)\quad&>\quad0,\\1-\tau(1-\frac{q}{2})-F(\rho)\left(1-\tau(1-\frac{q}{2})+\tau(1-q)-\tau(1-\frac{q}{2})\right)\quad&>\quad0,\\(1-F(\rho))\left(1-\tau(1-\frac{q}{2})\right)+F(\rho)\left(\tau(1-\frac{q}{2})-\tau(1-q)\right)\quad&>\quad0,\end{aligned}$$

which is clearly true. Then, the weak inequality that consumers should weakly prefer being active and (10) cannot hold simultaneously. This completes the proof.

Proof of Proposition 4

Consider a market environment where active consumers share all prices they observe whether or not they buy themselves at these prices. Let $q>0$ be the share of active consumers. Then, an active consumer who waits after searching the first firm receives a price quote of the other firm from friends with probability $1-\tau(1-\frac{q}{2})$. Thus, the payoff of buying and searching after observing price $\widetilde{p}$ are $v-b-\widetilde{p}$ and $v-b-(1-F(\widetilde{p}))\widetilde{p}-F(\widetilde{p})E[p|p<\widetilde{p}]-s$, respectively. It is easy to see that r is determined by (1). If there exists a solution to (1), it must be unique. Moreover, at a price around $\underline{p}$, it is easy to see that active buyers prefer buying to both searching and waiting.

Lemma 4 *If active buyers share information of all prices they observe, then $F(r)=1$ must hold in any equilibrium.*

Proof. For the proof, it suffices to show that consumers who observe $p>r$ always compare prices. An active buyer who observes $p>r$ never buys outright. Searching the other firm may be optimal for the buyer, in which case she clearly compares prices. It may also be that waiting is optimal for the buyer, in which case she either receives the second price quote from active friends or does not receive any information and, thus, searches herself. In either case, she compares prices. This establishes that any price greater than r is compared to another price. ■

It follows that we can write the payoff of an active buyer who observes price $\widetilde{p}$ and waits as $\delta\left[v-b-\left(1-\tau(1-\frac{q}{2})\right)F(\widetilde{p})E[p|p<\widetilde{p}]-\left[1-\left(1-\tau(1-\frac{q}{2})\right)F(\widetilde{p})\right]\widetilde{p}\right]$. Then, ρ is implicitly given by

$$(1-\delta)\left(v-\rho-b\right)=\delta\left(1-\tau(1-\frac{q}{2})\right)F(\rho)\left(\rho-E[p|p<\rho]\right),$$

or

$$\rho=\frac{(1-\delta)\left(v-b\right)+\delta\left(1-\tau(1-\frac{q}{2})\right)F(\rho)E[p|p<\rho]}{(1-\delta)+\delta\left(1-\tau(1-\frac{q}{2})\right)F(\rho)}. \tag{11}$$

Thus, ρ is a weighted average of $v-b$ and $E[p|p<\rho]$. It is easy to establish that there must be a unique ρ that satisfies (11) if a solution exists. Note that, depending on the parameter values, r and/or ρ may be outside the relevant domain of prices $[0,v-b]$.

As active buyers are willing to buy immediately at prices around $\underline{p}$, they prefer buying outright for p such that $\underline{p}<p\leq\min\{r,\rho\}\leq v-b$. This follows from the continuity in p of the payoffs of buying, searching, and waiting.

Lemma 5 *In equilibrium, $F(p)$ cannot have atoms.*

Proof. Suppose that in equilibrium there is an atom at $\widetilde{p}$, such that $\underline{p}\leq\widetilde{p}\leq r$. As the share of active consumers is strictly positive, this price $\widetilde{p}$ gets compared to another price with strictly positive probability. Put differently, there is a strictly positive share of consumers that compare any price in the support of $F(p)$. Then, undercutting $\widetilde{p}$ would be beneficial as this yields a discontinuous increase in the demand. This means that undercutting is profitable for firms, a contradiction. ■

Notice that ρ is irrelevant for $\rho>r$. Therefore we focus consider $\rho<r$ in order to show this cannot be the case.

Observe that at $p\in(\rho,r)$, active consumers prefer waiting to searching (which is equivalent to Lemma 2). Then, in a symmetric price dispersed equilibrium with $\rho<r$ and $F(r)=1$, $F(p)$ must be a continuous and strictly increasing function of p for $p\in[\underline{p},\rho]$. Notice that there is a discontinuity in the demand of a firm at ρ. If a firm prices at ρ, it certainly sells to active consumers who searches it first. Yet if it prices at $\rho+\epsilon$, its demand from active consumers who buy outright vanishes—these consumers choose to wait and may be informed of a lower price and therefore do not buy from the firm. Thus, there must be a $\underline{r}$ with $\rho<\underline{r}\leq r$ such that $F(p)$ is flat for $p\in(\rho,\underline{r})$.

Lemma 6 *If $\rho<r$, it must be that $F(p)$ is flat in $(\rho,\underline{r})$.*

It follows that the reservation price r is determined by

$$\begin{aligned} s &= r-E[p] \\ &= r-E[\underline{r}\leq p\leq r]+F(\rho)(E[\underline{r}\leq p\leq r]-E[p\leq\rho]). \end{aligned} \tag{12}$$

As at price r buyers prefer waiting to buying, we can write that

$$(1-\delta)(v-b-r)<\delta\left(1-\tau(1-\frac{q}{2})\right)s. \tag{13}$$

In equilibrium, an individual buyer is indifferent between being active and passive. The payoff from being active is

$$\begin{aligned} F(\rho)(v-b-E[p\le\rho]) + \delta(1-F(\rho)) \\ \times\left(v-b-E[\underline{r}\le p\le r] + F(\rho)\left(1-\tau(1-\frac{q}{2})\right)(E[\underline{r}\le p\le r]-E[\rho\le p])\right) - s. \end{aligned}$$

Use (12) to expand the first term and simplify to obtain

$$\begin{aligned} F(\rho)(v-b-E[\underline{r}\le p\le r]) - (r-E[\underline{r}\le p\le r]) + \delta(1-F(\rho)) \\ \times\left(v-b-E[\underline{r}\le p\le r] + F(\rho)\left(1-\tau(1-\frac{q}{2})\right)(E[\underline{r}\le p\le r]-E[p\le\rho])\right). \end{aligned}$$

Add and subtract $v-b$ and simplify to obtain

$$\begin{aligned} v-b-r-(1-\delta)(1-F(\rho))(v-b-E[\underline{r}\le p\le r]) \\ +\delta F(\rho)(1-F(\rho))\left(1-\tau(1-\frac{q}{2})\right)(E[\underline{r}\le p\le r]-E[p\le\rho]). \end{aligned}$$

The payoff from being passive is

$$\begin{aligned} \delta\Bigg\{v-b-\left[F^2(\rho)+2F(\rho)(1-F(\rho))\left(1-\frac{\tau(1-\frac{q}{2})}{2}\right)\right]E[p\le\rho] \\ -\left[\tau(1-\frac{q}{2})F(\rho)(1-F(\rho))+(1-F(\rho))^2\right]E[\underline{r}\le p\le r] + F^2(\rho)\widetilde{\tau}(q)(E[\underline{r}\le p\le r]-E[p\le\rho]) \\ -\left[F^2(\rho)\tau(1-q)+2F(\rho)(1-F(\rho))\tau(1-\frac{q}{2})+(1-F(\rho))^2\right]s\Bigg\} \end{aligned}$$

or

$$\begin{aligned} \delta\Bigg\{v-b-E[\underline{r}\le p\le r]+F(\rho)\left[1+(1-F(\rho))\left(1-\tau(1-\frac{q}{2})\right)\right](E[\underline{r}\le p\le r]-E[p\le\rho]) \\ +F^2(\rho)\widetilde{\tau}(q)(E[\underline{r}\le p\le r]-E[p\le\rho]) - \left[F^2(\rho)\tau(1-q)+2F(\rho)(1-F(\rho))\tau(1-\frac{q}{2})+(1-F(\rho))^2\right]s\Bigg\}. \end{aligned}$$

Use (12) for the third term in the curly brackets and simplify it as

$$\begin{aligned} \delta\Bigg\{v-b-r+F(\rho)\left[1+F(\rho)\tau(1-q)-(1+F(\rho))\tau(1-\frac{q}{2})\right](E[\underline{r}\le p\le r]-E[p\le\rho]) \\ +\left[1-F^2(\rho)\tau(1-q)-2F(\rho)(1-F(\rho))\tau(1-\frac{q}{2})-(1-F(\rho))^2\right]s\Bigg\}. \end{aligned}$$

Thus, the indifference condition of buyers is

$$\begin{aligned} (1-\delta)(v-b-r) = (1-\delta)F(\rho)(v-b-E[\underline{r}\le p\le r]) \\ +\delta(E[\underline{r}\le p\le r]-E[p\le\rho]) + \delta\left[2(1-\tau(1-\frac{q}{2}))-F(\rho)\widetilde{\tau}(q)\right]s. \end{aligned}$$

The LHS of the equation is equal to the LHS of (13). The RHS of the indifference equation is certainly greater than the RHS of (13) if

$$\begin{aligned}
\delta\left[2(1-\tau(1-\frac{q}{2}))-F(\rho)\widetilde{\tau}(q)\right] &> \delta\left(1-\tau(1-\frac{q}{2})\right), \\
1-\tau(1-\frac{q}{2}) &> F(\rho)\left(1-\tau(1-\frac{q}{2})-\left[\tau(1-\frac{q}{2})-\tau(1-q)\right]\right), \\
(1-F(\rho))\left(1-\tau(1-\frac{q}{2})\right) &> -F(\rho)\left(\tau(1-\frac{q}{2})-\tau(1-q)\right).
\end{aligned}$$

which is clearly true. Then, however, the indifference condition of consumers and (13) cannot hold simultaneously.

Proof of Theorem 1.

We use the following facts to rewrite the consumer's indifference condition. First, $E[p] = \overline{p} - \int_{\underline{p}}^{\overline{p}} F(p)dp = \eta\overline{p}\ln\left(1+\frac{1}{\eta}\right)$. Second, $\overline{p} = r = \frac{s}{1-\eta\ln\left(1+\frac{1}{\eta}\right)}$. Third, $E[\min\{p_1,p_2\}] = \overline{p} - 2\int_{\underline{p}}^{\overline{p}} F(p)dp + \int_{\underline{p}}^{\overline{p}} F^2(p)dp$ so that

$$\begin{aligned}
E[p] - E[\min\{p_1,p_2\}] &= \int_{\underline{p}}^{\overline{p}} F(p)dp - \int_{\underline{p}}^{\overline{p}} F^2(p)dp \\
&= \eta\overline{p}\left((1+2\eta)\ln\left(1+\frac{1}{\eta}\right)-2\right).
\end{aligned}$$

Thus, the consumers' indifference condition can be rewritten as

$$\begin{aligned}
(1-\delta)(v-b) &= (1-\delta\tau(1-q))s + \delta\widetilde{\tau}(q)\frac{\eta\left((1+2\eta)\ln\left(1+\frac{1}{\eta}\right)-2\right)}{1-\eta\ln\left(1+\frac{1}{\eta}\right)}s \\
&\quad + (1-\delta)s\frac{\eta\ln\left(1+\frac{1}{\eta}\right)}{1-\eta\ln\left(1+\frac{1}{\eta}\right)},
\end{aligned}$$

or

$$\frac{s}{v-b} = \frac{1-\delta}{1-\delta\tau(1-q)+\frac{\eta}{1-\eta\ln\left(\frac{1+\eta}{\eta}\right)}\left[\delta\widetilde{\tau}(q)\left((1+2\eta)\ln\left(\frac{1+\eta}{\eta}\right)-2\right)+(1-\delta)\ln\left(\frac{1+\eta}{\eta}\right)\right]}, \tag{14}$$

We know that η is a function of q and that $\lim_{q\uparrow 1}\eta(q) = \infty$.

As the RHS of (14) is continuous in $0 < q < 1$, to prove the existence of an RPE for small enough values of s it is sufficient to show that the RHS of (14) is positive and approaches 0 zero as $q \uparrow 1$. As $\eta\ln\left(\frac{1+\eta}{\eta}\right) < 1$, the denominator is clearly positive if $\ln\left(\frac{1+\eta}{\eta}\right) > \frac{2}{1+2\eta}$. As for $\eta \downarrow 0$ this latter inequality clearly holds, while the LHS and the RHS both approach 0 as $\eta \to \infty$, this inequality holds for all η if the derivative of the LHS is more negative than that of the RHS. The derivate of the LHS is $-\frac{1}{\eta(1+\eta)}$, while the derivate of the RHS is $-\frac{2}{(1+2\eta)^2}$. It is easy to see that the former derivate is smaller than the latter. Thus, as both numerator and denominator of (14) are positive, the whole expression is clearly positive.

Also that $1-\delta \leq 1-\delta\tau(1-q)$ means that the numerator is smaller than the denominator of the RHS of (14) for $0 < q < 1$. Thus, there exists an upper bound $\overline{s}$ on the search cost for which an RPE may exist.

To demonstrate the RHS of (14) converges to zero as $q \to 1$, we employ the following evaluations:

$$\lim_{\eta\to\infty} \eta \ln\left(1+\frac{1}{\eta}\right) = \lim_{z\downarrow 0} \frac{\ln(1+z)}{z} \overset{\text{l'Hopital}}{=} \lim_{z\downarrow 0} \frac{1}{1+z} = 1, \tag{15}$$

$$\lim_{\eta\to\infty} \frac{\eta\ln\left(1+\frac{1}{\eta}\right)}{1-\eta\ln\left(1+\frac{1}{\eta}\right)} = \frac{\lim\limits_{\eta\to\infty}\eta\ln\left(1+\frac{1}{\eta}\right)}{1-\lim\limits_{\eta\to\infty}\eta\ln\left(1+\frac{1}{\eta}\right)} = \infty, \tag{16}$$

and

$$\begin{aligned}
\lim_{\eta\to\infty} \frac{\eta\left((1+2\eta)\ln\left(\frac{1+\eta}{\eta}\right)-2\right)}{1-\eta\ln\left(\frac{1+\eta}{\eta}\right)} &= \lim_{z\downarrow 0} \frac{\frac{1}{z}\left(\left(1+\frac{2}{z}\right)\ln(1+z)-2\right)}{1-\frac{\ln(1+z)}{z}} \\
&= \lim_{z\downarrow 0} \frac{(2+z)\ln(1+z)-2z}{z^2-z\ln(1+z)} \\
&\overset{\text{l'Hopital}}{=} \lim_{z\downarrow 0} \frac{\ln(1+z)+\frac{2+z}{1+z}-2}{2z-\ln(1+z)-\frac{z}{1+z}} \\
&= \lim_{z\downarrow 0} \frac{(1+z)\ln(1+z)-z}{z(1+2z)-(1+z)\ln(1+z)} \\
&\overset{\text{l'Hopital}}{=} \lim_{z\downarrow 0} \frac{\frac{z}{(1+z)^2}}{\frac{z(3+2z)}{(1+z)^2}} = \lim_{z\downarrow 0} \frac{1}{3+2z} = \frac{1}{3}.
\end{aligned} \tag{17}$$

Finally, noting $\lim\limits_{q\uparrow 1} \widetilde{\tau}(q) = 1-2\tau(1/2)$, we can see that the denominator of the RHS of (14) increases unboundedly as $q \uparrow 1$. This means that the RHS of (14) converges to zero as $q \uparrow 1$.

Now, we show the limiting price for $s \downarrow 0$. We know that s approaching zero is associated with $q \uparrow 1$, or $\eta \to \infty$ implying that price dispersion vanishes. Then, it suffices to evaluate the limiting value of r. We note that

$$\begin{aligned}
r &= \frac{s}{1-\eta\ln\left(1+\frac{1}{\eta}\right)} \\
&= \frac{(1-\delta)(v-b)}{(1-\delta\tau(1-q))\left(1-\eta\ln\left(1+\frac{1}{\eta}\right)\right)+\eta\left[\delta\widetilde{\tau}(q)\left((1+2\eta)\ln\left(\frac{1+\eta}{\eta}\right)-2\right)+(1-\delta)\ln\left(\frac{1+\eta}{\eta}\right)\right]}.
\end{aligned}$$

Notice that the numerator of r does not depend on s. Recalling (15), we can see that the first term in the denominator converges to zero as $q \uparrow 1$ (with associated $\eta \to \infty$). As

$$\begin{aligned}
\lim_{\eta\to\infty} \eta\left((1+2\eta)\ln\left(\frac{1+\eta}{\eta}\right)-2\right) &= \lim_{z\downarrow 0}\frac{\left((1+\frac{2}{z})\ln(1+z)-2\right)}{z} \\
&= \lim_{z\downarrow 0}\frac{((z+2)\ln(1+z)-2z)}{z^2} \\
&\overset{\text{l'Hopital}}{=} \lim_{z\downarrow 0}\frac{\ln(1+z)+\frac{2+z}{1+z}-2}{2z} \\
&= \lim_{z\downarrow 0}\frac{(1+z)\ln(1+z)-z}{2z(1+z)} \\
&\overset{\text{l'Hopital}}{=} \lim_{z\downarrow 0}\frac{\ln(1+z)+1-1}{2+4z} = 0,
\end{aligned}$$

it follows that the second term in the denominator approaches $1-\delta$ as $\eta \to \infty$. Then, the entire term converges to $(1-\delta)(v-b)/(1-\delta) = v-b$ as $q \uparrow 1$. This completes the proof.

Proof of Proposition 6

The proof that price dispersion must remain in the limit as $t(\infty) \to 1$ is by contradiction. Suppose to the contrary that price dispersion vanishes in the limit, or alternatively, that $\eta \to \infty$.

As

$$\lim_{\eta\to\infty} r = \lim_{\eta\to\infty}\frac{s}{1-\eta\ln(1+\frac{1}{\eta})} = \infty,$$

it follows that in an RPE $E[p]$ must also increase without bound. But this would imply that the pay-off of becoming active is negative, which cannot be the case in active markets. Hence, price dispersion cannot vanish as $k \to \infty$ (or alternatively $t(\infty) \to 1$) and $\lim_{t(\infty)\to 1} q = \overline{q}$, where $0 < \overline{q} < 1$.

We next consider the question whether an active market exists in the limit as $t(\infty) \to 1$. To this end, rewrite the consumers' indifference condition

$$v-b-E[p]-s = \delta\Big(v-b-E[p]+\widetilde{\tau}(q)(E[p]-E_{\min}[p])-\tau(1-q)s\Big),$$

as

$$\begin{aligned}
\frac{s}{v-b} &= \frac{1-\delta}{1-\delta\tau(1-q)+\left[\delta\widetilde{\tau}(q)\frac{E[p]-E_{\min}[p]}{s}+(1-\delta)\frac{E[p]}{s}\right]} \\
&= \frac{1-\delta}{1-\delta\tau(1-q)+\left[\frac{\eta}{1-\eta\ln\left(1-\frac{1}{\eta}\right)}\left\{\delta\widetilde{\tau}(q)\left((1+2\eta)\ln\left(1+\frac{1}{\eta}\right)-2\right)+(1-\delta)\ln(1+\frac{1}{\eta})\right\}\right]},
\end{aligned}$$

where the LHS is the normalized cost of being active, while the RHS represents the incremental benefit of being so. Observe that the denominator of the RHS consists of three terms, the third one of which is the sum in the large square brackets. The first and the third terms are positive, while the second term is negative. As the second term converges to zero and $\widetilde{\tau}(q)$ to 1 (when $t(\infty) \to 1$), the limiting indifference condition is approximately

$$\begin{aligned} \frac{s}{v-b} &= \frac{1-\delta}{1+\frac{1}{s}\left[\lim\limits_{t(\infty)\to 1} E[p] - \delta \lim\limits_{t(\infty)\to 1} E_{\min}[p]\right]} \\ &= \frac{1-\delta}{1+\lim\limits_{t(\infty)\to 1} \frac{\eta}{1-\eta\ln\left(1-\frac{1}{\eta}\right)}\left[\ln(1+\frac{1}{\eta}) - 2\delta\left(1-\eta\ln\left(1+\frac{1}{\eta}\right)\right)\right]}. \end{aligned} \tag{18}$$

As the RHS approaches 0 if $\eta \to \infty$ it is clear this equation can always be satsified if s is small enough.

Proof of Proposition 7

Observe that changes in b and s affect the LHS of (14) only. In particular, the LHS is increasing in both b and s. As the RHS of the indifference equation (14) must be decreasing in q in a stable RPE, the optimal search probability q must be decreasing in both b and s.

Proof of Proposition 8

We prove the proposition with the help of two claims.

Claim 1 *If consumers with $1 \leq \widehat{k} \leq \overline{k}$ number of links search with strictly positive probability, all consumers with numbers of links less than $\widehat{k}$ links (if there are such) search with probability one.*

Proof. As w represents the probability a consumer assigns to a neighbor actively searching, a consumer with $\widehat{k}$ friends searches with positive probability only if doing so is weakly better than not searching, i.e.,

$$\delta\left(v-b-E[p]+\left(1+(1-w)^{\widehat{k}}-2\left(1-\tfrac{w}{2}\right)^{\widehat{k}}\right)(E[p]-E_{\min}[p])-(1-w)^{\widehat{k}}s\right) \leq v-E[p]-b-s.$$

For consumers with less than $\widehat{k}$ links, searching yields the same payoff as the RHS of the inequality, whereas not searching yields a payoff strictly smaller than the LHS of the inequality as the LHS is increasing in $\widehat{k}$ for $0 < w < 1$. Hence, consumers with less than $\widehat{k}$ connections search for sure. ■

Claim 2 *If consumers with $1 \leq \widehat{k} \leq \overline{k}$ number of links search with positive probability less than 1, all consumers with numbers of links greater than $\widehat{k}$ links (if there are such) do not search.*

Proof. The proof is analogous to the proof of Claim 1. ■

If a consumer with $\widehat{k}$ links is indifferent between searching and not searching, her optimal search probability lies between zero and one. Then, from the above two claims it follows that consumers with lower than $\widehat{k}$ search for sure, whereas those with greater than $\widehat{k}$ links do not search at all.

Proof of Proposition 9

Some parts of the proof are similar to the proof of Theorem 1. To avoid repetition, we omit some details here. It is clear that to obtain the equilibrium distribution function in (6), we should have

$$\widehat{\eta} = \frac{\frac{\widehat{w}}{2} + \delta t(\widehat{k})(1-q)\left(\left(1-\frac{w}{2}\right)^{\widehat{k}} - \frac{(1-w)^{\widehat{k}}}{2}\right) + \delta \sum_{k=\widehat{k}+1}^{\overline{k}} t(k)\left(\left(1-\frac{w}{2}\right)^{k} - \frac{(1-w)^{k}}{2}\right)}{\delta t(\widehat{k})(1-q)\left(1+(1-w)^{\widehat{k}} - 2\left(1-\frac{w}{2}\right)^{\widehat{k}}\right) + \delta \sum_{k=\widehat{k}+1}^{\overline{k}} t(k)\left(1+(1-w)^{k} - 2\left(1-\frac{\widehat{w}}{2}\right)^{k}\right)}.$$

It is clear that $\widehat{\eta}$ is a function of $\widehat{k}$ and q and that $\lim_{q\to 1} \widehat{\eta}(\widehat{k} = \overline{k}, q) = \infty$.

If a consumer with $\widehat{k}$ friends is indifferent between being passive and active, then it must be the case that the pay-off of being passive

$$\delta\left(v - b - E[p] + \left(1+(1-w)^{\widehat{k}} - 2\left(1-\frac{w}{2}\right)^{\widehat{k}}\right)(E[p] - E_{\min}[p]) - (1-w)^{\widehat{k}} s\right)$$

is equal to the pay-off $v - E[p] - b - s$ of being active. We can rewrite this indifference equation as

$$\left(1 - \delta(1-w)^{\widehat{k}}\right) s + \delta W(E[p] - E_{\min}[p]) = (1-\delta)(v - E[p] - b),$$

where $W = \left[1 + (1-w)^{\widehat{k}} - 2\left(1-\frac{w}{2}\right)^{\widehat{k}}\right]$ and, using the expressions for $E[p]$, $E[p] - E_{\min}[p]$ and $\overline{p}$ developed in the beginning of the proof of Theorem 1, express this condition further as

$$\left(1 - \delta(1-w)^{\widehat{k}}\right) s + s\delta W \frac{\widehat{\eta}\left((1+2\widehat{\eta})\ln\left(1+\frac{1}{\widehat{\eta}}\right) - 2\right)}{1 - \widehat{\eta}\ln\left(1+\frac{1}{\widehat{\eta}}\right)} = (1-\delta)\left(v - b - \frac{\widehat{\eta}\ln\left(1+\frac{1}{\widehat{\eta}}\right)}{1-\widehat{\eta}\ln\left(1+\frac{1}{\widehat{\eta}}\right)} s\right).$$

Bringing all the terms with s on one side and re-arranging gives the condition mentioned in the Proposition.

We now show that if $s \to 0$, $\widehat{k} = \overline{k}$ and $q(\widehat{k}) \uparrow 1$ hold in equilibrium. It is clear that if $s \to 0$ price dispersion must disappear as otherwise active consumers would have an incentive to continue searching. Suppose then that if $s \to 0$, $\widehat{k} < \overline{k}$. This would imply that for consumers with $\overline{k}$ friends the pay-off of waiting is strictly larger than the pay-off of actively searching. However, with price dispersion disappearing if $s \to 0$, for all $0 < \delta < 1$ and for every consumer (no matter how many friends she has) this cannot be the case. If $q(\widehat{k})$ would not converge to 1 if $s \to 0$, then $\widehat{\eta}$ would converge to a finite number and price dispersion would persist, a contradiction.

Finally, we focus on the price level to which the price distribution converges if $s \to 0$. As $\widehat{k} = \overline{k}$, $\widehat{\eta}$ reduces to

$$\widehat{\eta} = \frac{\frac{\widehat{w}}{2} + \delta t(\widehat{k})(1-q)\left(\left(1-\frac{w}{2}\right)^{\widehat{k}} - \frac{(1-w)^{\widehat{k}}}{2}\right)}{\delta t(\widehat{k})(1-q)W}$$

so that $\lim_{q\uparrow 1}\widehat{\eta} = \infty$ as both w and $\widehat{w}$ converge to 1. Since search behavior of consumers with $\overline{k}$ links is of interest in the limit, we can write

$$\begin{aligned}\lim_{s\to 0} r &= \lim_{s\to 0} \frac{s}{1-\widehat{\eta}\ln\left(\frac{1}{\widehat{\eta}}+1\right)} \\ &= \lim_{q\uparrow 1} \frac{(1-\delta)(v-b)}{\left(1-\delta(1-w)^{\widehat{k}}\right)\left(1-\widehat{\eta}\ln\left(1+\frac{1}{\widehat{\eta}}\right)\right)+\widehat{\eta}\left[\delta W\left((1+2\widehat{\eta})\ln\left(\frac{1+\widehat{\eta}}{\widehat{\eta}}\right)-2\right)+(1-\delta)\ln\left(\frac{1+\widehat{\eta}}{\widehat{\eta}}\right)\right]}.\end{aligned} \tag{19}$$

To evaluate the limit, we undertake similar steps as when we evaluated the limiting r in the proof of Theorem 1. Note that the numerator of the expression is independent of q. In the denominator, the first two term converges to zero, following (15). The second term in the denominator converges to $1-\delta$ since

$$\begin{aligned}\lim_{\widehat{\eta}\to\infty}\widehat{\eta}\left((1+2\widehat{\eta})\ln\left(\frac{1+\widehat{\eta}}{\widehat{\eta}}\right)-2\right) &= \lim_{z\downarrow 0}\frac{\left((1+\frac{2}{z})\ln(1+z)-2\right)}{z} \\ &= \lim_{z\downarrow 0}\frac{((z+2)\ln(1+z)-2z)}{z^2} \\ &\overset{\text{l'Hopital}}{=} \lim_{z\downarrow 0}\frac{\ln(1+z)+\frac{2+z}{1+z}-2}{2z} \\ &= \lim_{z\downarrow 0}\frac{(1+z)\ln(1+z)-z}{2z(1+z)} \\ &\overset{\text{l'Hopital}}{=} \lim_{z\downarrow 0}\frac{\ln(1+z)+1-1}{2+4z} = 0.\end{aligned}$$

Summing up, the limiting r converges to $\frac{(1-\delta)(v-b)}{1-\delta} = v-b$. The proof is complete.

References

Aker, J. (2010). "Information from Markets Near and Far: Mobile Phones and Agricultural Markets in Niger". *American Economic Journal: Applied Economics*, 2(3), pp. 46–59.

Aker, J. and Mbiti, I. (2010). "Mobile Phones and Economic Development in Africa". *Journal of Economic Perspectives*, 24(3), pp. 207–232.

Arbatskaya, M. and Konishi, H. (2016). "Consumer Referrals". *International Journal of Industrial Organization*, 48, pp. 34–58.

Banerji, A. and Dutta, B. (2009). "Local Network Externalities and Market Segmentation". *International Journal of Industrial Organization*, 27, pp. 605–614.

Biyalogorsky, E.; Gerstner, E.; and Libai, B. (2001). "Customer Referral Management: Optimal Reward Programs". *Marketing Science*, 20(1), pp. 82–95.

Bramoulle, Y. and Kranton, R. (2007). "Public Goods in Networks". *Journal of Economic Theory*, 135, pp. 478–494.

Brown, R., J. and Goolsbee, A. (2002). "Does the Internet Make Markets More Competitive? Evidence from the Life Insurance Industry". *Journal of Policitcal Economy*, 110(3), pp. 481–507.

Burdett, K. and Judd, K. (1983). "Equilibrium Price Dispersion". *Econometrica*, 51(4), pp. 955–969.

Campbell, A. (2013). "Word-of-Mouth Communication and Percolation in Social Networks". *American Economic Review*, 103(6), pp. 2466–2498.

Chen, Y.; Wang, Q.; and Xie, J. (2011). "Online Social Interactions: A Natural Experiment on Word of Mouth Versus Observational Learning". *Journal of Marketing Research*, 48(2), pp. 238–254.

Chuhay, R. (2015). "Pricing Innovation in the Presence of Word-of-Mouth Communication". *Working Paper*. Working Paper.

Diamond, P. (1971). "A Model of Price Adjustment". *Journal of Economics Theory*, 3, pp. 156–168.

Ellison, G. and Fudenberg, D. (1995). "Word-of-Mouth Communication and Social Learning". *The Quarterly Journal of Economics*, 110, pp. 93–125.

Fainmesser, I. and Galeotti, A. (2016). "Pricing Network Effects". *The Review of Economic Studies*, 83, pp. 165–198.

Fainmesser, I. and Galeotti, A. (2017). "Pricing Network Effects: Competition". *Working Paper*.

Fershtman, C. and Fishman, A. (1992). "Price Cycles and Booms: Dynamic Search Equilibrium". *The American Economic Review*, 82(5), pp. 1221–1233.

Galeotti, A. (2010). "Talking, Searching, and Pricing". *International Economic Review*, 51(4), pp. 1159–1174.

Galeotti, A. and Goyal, S. (2009). "Influencing the Influencers: a Theory of Strategic Diffusion". *RAND Journal of Economics*, 40(3), pp. 509–532.

Galeotti, A. and Goyal, S. (2010). "The Law of the Few". *American Economic Review*, 100(4), pp. 1468–1492.

Garcia, D. and Shelegia, S. (2015). "Consumer Search with Observational Learning". *Forthcoming.*

Godes, D. and Mayzlin, D. (2004). "Using Online Conversations to Study Word-of-Mouth Communication". *Marketing Science*, 23(4), pp. 545–560.

Grossman, S. J. and Stiglitz, J. E. (1980). "On the Impossibility of Informationally Efficient Market". *The American Economic Review*, 70(3), pp. 393–408.

Hayek, F. (1945). "The Use of Knowledge in Society". *The American Economic Review*, 35(4), pp. 519–530.

Honda, J. (2015). "Intermediary Search for Suppliers in Procurement Auctions". *Working Paper.*

Janssen, M. and Moraga-Gonzalez, J. L. (2004). "Strategic Pricing, Consumer Search and the Number of Firms". *Review of Economic Studies*, 71, pp. 1089–1118.

Janssen, M.; Moraga-Gonzalez, J. L.; and Wildenbeest, M. R. (2005). "Truly Costly Sequential Search and Oligopolistic Pricing". *International Journal of Industrial Organization*, 23, pp. 451–466.

Jensen, R. (2007). "The Digital Provide: Information (Technology), Market Performance, and Welfare in South Indian Fisheries Sectior". *The Quarterly Journal of Economics*, CXXII, pp. 879–924.

Jun, T. and Kim, J.-Y. (2008). "A Theory of Consumer Referral". *International Journal of Industrial Organization*, 26, pp. 662–678.

Katz, E. and Lazarsfeld, P. F. (1955). *Personal Influence.* New York: Free Press.

Kircher, P. and Postlewaite, A. (2008). "Strategic Firms and Endegenous Consumer Emulation". *The Quarterly Journal of Economics*, 123.

Kornish, L. and Li, Q. (2010). "Optimal Referral Bonuses with Asymmetric Information: Firm-Offered and Interpersonal Incentives". *Marketing Science*, 29(1).

Miegielsen, S. (2014). "Consumer Information Networks". *Discussion Paper Series. DPS14.14.*

Price, D. (1965). "Networks of Scientific Papers". *Science*, 149(3683), pp. 510–515.

Seiler, S.; Yao, S.; and Wang, W. (2019). "Does Online Word of Mouth Increase Demand? (And How?) Evidence from a Natural Experiment". *Marketing Science*, 36(6).

Stahl, D. O. (1989). "Oligopolistic Pricing with Sequential Consumer Search". *The American Economic Review*, 79(4), pp. 700–712.

Stigler, G. (1961). "The Economics of Information". *The Journal of Political Economy*, 69, pp. 213–225.

Varian, H. R. (1980). "A Model of Sales". *The American Economic Review*, 70(4), pp. 651–659.

Wilson, C. (2010). "Ordered Search and Equilibrium Obfuscation". *International Journal of Industrial Organization*, 28, pp. 496–506.

Wolinsky, A. (1986). "True Monopolistic Competition as a Result of Imperfect Information". *Quarterly Journal of Economics*, 101(3), pp. 493–512.