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## LABOR MARKET TRENDS AND THE CHANGING VALUE OF TIME

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LABOUR ECONOMICS AND
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#### Abstract

During the past two decades, households experienced increases in their average wages and expenditures alongside with divergent trends in their wages, expenditures, and time allocation. We develop a model with incomplete asset markets and household heterogeneity in market and home technologies and preferences to account for these labor market trends and assess their welfare consequences. Using micro data on expenditures and time use, we identify the sources of heterogeneity across households, document how these sources have changed over time, and perform counterfactual analyses. Given the observed increase in leisure expenditures relative to leisure time and the complementarity of these inputs in leisure technology, we infer a significant increase in the average productivity of time spent on leisure. The increasing productivity of leisure time generates significant welfare gains for the average household and moderates negative welfare effects from the rising dispersion of expenditures and time allocation across households.


JEL Classification: D10, E21, J22
Keywords: time use, Consumption, Leisure Productivity, inequality
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# Labor Market Trends and the Changing Value of Time* 

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September 2019


#### Abstract

During the past two decades, households experienced increases in their average wages and expenditures alongside with divergent trends in their wages, expenditures, and time allocation. We develop a model with incomplete asset markets and household heterogeneity in market and home technologies and preferences to account for these labor market trends and assess their welfare consequences. Using micro data on expenditures and time use, we identify the sources of heterogeneity across households, document how these sources have changed over time, and perform counterfactual analyses. Given the observed increase in leisure expenditures relative to leisure time and the complementarity of these inputs in leisure technology, we infer a significant increase in the average productivity of time spent on leisure. The increasing productivity of leisure time generates significant welfare gains for the average household and moderates negative welfare effects from the rising dispersion of expenditures and time allocation across households.


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## 1 Introduction

Wages and expenditures increased substantially for the average household during the past two decades. At the same time, these gains were not distributed equally across households. ${ }^{1}$ The purpose of this paper is to develop a tractable framework that accounts quantitatively for both average and divergent trends in labor market outcomes and that allows us to assess the welfare consequences of the drivers that underlie these trends.

Our framework is a general equilibrium model with incomplete asset markets and household heterogeneity in market and home technologies and preferences. Households have access to various home technologies that, following Ghez and Becker (1975), combine expenditures and time as inputs to produce final consumption goods. In the home sector, households are heterogeneous with respect to their preferences across goods and their productivity of time. Home production is not tradeable and storable, meaning that in every instance home production must be consumed, and not insurable, meaning there are no assets that households can purchase to explicitly insure against differences that originate in the home sector. In the market sector, households are also heterogeneous with respect to their productivity. Following the approach of Heathcote, Storesletten, and Violante (2014), the structure of asset markets allows households to insure against transitory shocks in their market productivity but not against permanent productivity differences.

We apply our framework to married households surveyed by the Consumer Expenditure Survey (CEX) and the American Time Use Survey (ATUS) between 1995 and 2016. We split the home sector into a non-market sector in which expenditures and time are substitutes in production and a leisure sector in which expenditures and time are complements in production. The nonmarket sector includes expenditures such as food and household services and time uses such as housework and child care. The leisure sector includes expenditures such as telecommunication and entertainment and time uses such as television watching and other recreational activities.

A major advantage of the framework is the transparency and generality of the identification of

[^1]the sources of heterogeneity across households. The model retains tractability because it features a no-trade result with respect to certain assets. Therefore, we can characterize the allocations of expenditures and time across sectors in closed form. Following the same approach as in our earlier work (Boerma and Karabarbounis, 2019), we use the analytical solutions to invert the equilibrium allocations and identify the sources of heterogeneity across households that perfectly account for the household-level data in any given point of time. Our exercise is to then shut off particular aspects of the evolution of the sources of heterogeneity over time. This allows us to assess the drivers of trends in sectoral expenditures and time allocation for the average household, the drivers of trends in the dispersion of sectoral expenditures and time allocation across households, and the welfare consequences of these trends.

We reach two main conclusions regarding the sources of heterogeneity that characterize households and their evolution of time. First, we infer that mean productivity of leisure time more than doubles between the beginning and the end of the sample. The key feature of the data leading to this inference is the dramatic increase in leisure expenditures relative to leisure time for the average household. The increase in expenditures relative to time is larger than the one predicted only by the decline in the relative price of leisure goods. Given that expenditures and time are complements in the production of leisure goods, we infer that the productivity of leisure time must have been increasing.

Second, the dispersion of the productivity of non-market and leisure time is larger than the dispersion of market productivity across households. Our inference of large uninsurable differences in home productivity follows from the observation that in the cross-section of households time spent either on the non-market or the leisure sector is weakly correlated with sectoral expenditures and market productivity. As a result, home productivity needs to be significantly dispersed in order to rationalize the variation of these three observables. We document that the dispersion of the productivity of time inputs in home production has increased, paralleling the well-known increase in the dispersion of market productivity (wages) over time.

Our counterfactual analyses demonstrate the importance of market and home productivity and prices for the evolution of mean expenditures and market hours. Given the relative stability of
market hours over time, the increase in mean market productivity accounts for most of the increase in mean expenditures over time. The increase in the relative price of non-market goods induces households to substitute away from non-market expenditures toward non-market time and the decline in the relative price of leisure goods induces households to complement leisure expenditures with rising leisure time. Changes in relative prices generate roughly $11 \log$ points decline in market hours, with the majority of this decline accounted for by the increase in the relative price of nonmarket goods. This decline is offset by the rise of market and leisure productivities, which induce households to reallocate hours in the market sector.

To assess the welfare effects of trends in labor market outcomes, we calculate consumption equivalent changes that arise from changes in mean consumption and changes in the dispersion of consumption across households. By consumption we mean the final aggregator of the production process that involves aggregating sectoral goods produced with expenditures and time. A novel finding of our paper is to demonstrate that the rise of mean leisure productivity is quantitatively the most important driver of welfare changes over time. The increase in mean leisure productivity generates more than $30 \log$ points increase in mean consumption over time, whereas the increase in mean market productivity generates less than $10 \log$ points increase. At the same time, the increase in mean leisure productivity, which affects all households equally, moderates the rise of consumption dispersion across households induced by changes in the variance of market and home productivities over time. The contribution of mean leisure productivity to welfare through the dispersion channel is roughly $10 \log$ points of the consumption equivalent.

It is important to contrast our approach of assessing welfare effects through an equilibrium model to more descriptive approaches on the evolution of the dispersion of expenditures and time inputs. ${ }^{2}$ Similar to the distinction emphasized by Aguiar and Hurst (2005), in developing our

[^2]welfare metric we distinguish between expenditures that serve as an input in the production of final goods and consumption which is the result of a production process involving expenditures, time, and productivity. This distinction matters for our conclusions. For example, we find that the increase in the variance of the permanent component of market productivity is the most important factor accounting for the increase in the dispersion of total expenditures over time. However, this factor contributes significantly less to the welfare costs of dispersion once we recognize that these welfare costs are linked more closely to the consumption aggregator than to total expenditures.

We examine trends in labor market outcomes through the lens of a structural model, complementing earlier attempts to measure welfare effects from changes in the dispersion of observables. Attanasio and Davis (1996) is an early study that links the divergence of group wages to the divergence of group expenditures and argues that this departure from full insurance carries significant welfare costs. Heathcote, Storesletten, and Violante (2013) discuss the merits of structural approaches relative to statistical approaches when calculating welfare effects and estimate that, in response to the observed changes in the structure of wages, the welfare gains in terms of average consumption and leisure dominate the losses arising from increased dispersion. Relative to these papers, our paper incorporates multiple time uses and highlights the primary role of changes in leisure productivity in terms of understanding the welfare effects of recent labor market trends.

An emerging literature examines the role of shifts originating in the leisure sector for labor supply trends. Vandenbroucke (2009) adopts a quantitative Beckerian framework to study the driving forces behind the decline in working hours and their increased concentration over the first half of the 20th century. Accounting for the decline in market hours, he finds a primary role for increasing skilled wages and a limited role for the declining price of leisure goods. Bridgman (2016) develops a model with non-separable preferences that is able to accommodate the rise of average leisure and leisure inequality during the second half of the 20th century and Boppart and Ngai (2019) lay out conditions under which these trends are consistent with a balanced growth path. Like these papers, we are also interested in accounting for the evolution of the allocation of time. An important point of departure from this literature is that we incorporate micro-level data into our analysis of the heterogeneity in labor market trends across households.

Closest to our conclusions, Aguiar, Bils, Charles, and Hurst (2018) infer a significant increase in the technological progress of recreational time of young men. Their inference comes from the observed increase in recreational computer time in excess of the predicted increase along a leisure demand system. Similar to them, we find a significant increase in leisure productivity over time. Under our maintained assumption that expenditures and time are complements in leisure technology, our inference comes from the observed increase in leisure expenditures relative to time in excess of the increase predicted by the decline in the relative price of leisure goods. Aguiar, Bils, Charles, and Hurst (2018) do not map changes in leisure productivity to changes in welfare, whereas we uncover significant welfare effects from the rise of mean leisure productivity reflecting both an increase in average consumption and a moderation of consumption inequality.

## 2 Model

Our model of time allocation and expenditures is Beckerian (Becker, 1965; Ghez and Becker, 1975) in the sense that expenditures and time combine as inputs to produce final utility. We embed the Beckerian household production model into the tractable framework of incomplete asset markets and household heterogeneity developed by Heathcote, Storesletten, and Violante (2014). ${ }^{3}$ We first present the model and characterize its equilibrium in closed form. We then show how to infer the sources of heterogeneity across households such that the model accounts perfectly for cross-sectional data on sectoral expenditures and the allocation of time.

### 2.1 Environment

Demographics. The economy features perpetual youth demographics. We denote by $t$ the calendar year and by $j$ the birth year of a household. Households face a constant probability of survival $\delta$ in each period. Each period a cohort of mass $1-\delta$ is born, keeping the population size constant with a mass of one.

[^3]Household Technologies. Vector $\mathbf{c}$ collects goods, $\mathbf{x}$ collects (real) expenditure inputs, and $\mathbf{h}$ collects time inputs. We denote the market good by $c_{M}$ and home produced goods by $c_{K}$ where $K$ indexes different home goods. The difference between market and home goods is that the former are intensive in expenditures and do not use time as an input, $c_{M}=x_{M}$, whereas the latter use both expenditures and time to produce output, $c_{K}=c_{K}\left(x_{K}, h_{K}\right)$.

A household's technology in the market sector is characterized by its pre-tax earnings $y=$ $z_{M} h_{M}$, where $z_{M}$ denotes exogenous market productivity (wage) that varies across households and $h_{M}$ denotes hours worked in the market sector. A household's after-tax earnings are given by $\tilde{y}=\left(1-\tau_{0}\right) z_{M}^{1-\tau_{1}} h_{M}$, where parameter $\tau_{0}$ governs the level and parameter $\tau_{1}$ governs the progressivity of the tax system. A higher $\tau_{1}$ introduces more progressivity into the tax system as it compresses after-tax earnings relative to pre-tax earnings.

Home goods $c_{K}$ are produced by combining expenditures $x_{K}$ and time $h_{K}$ inputs according to CES aggregators:

$$
\begin{equation*}
c_{K}=\left(x_{K}^{\frac{\sigma_{K}-1}{\sigma_{K}}}+\left(z_{K} h_{K}\right)^{\frac{\sigma_{K}-1}{\sigma_{K}}}\right)^{\frac{\sigma_{K}}{\sigma_{K}-1}} \tag{1}
\end{equation*}
$$

where parameter $\sigma_{K} \geq 0$ denotes the elasticity of substitution between expenditures and time in the $K$ th home production technology and $z_{K}$ denotes exogenous productivity of time use $K$ (relative to expenditures) that varies across households. Home goods are consumed every period and cannot be stored or traded in a market. Households are endowed with one unit of time, $h_{M}+\sum h_{K}=1$.
Household Preferences. Households order sequences of goods by $\mathbb{E}_{j} \sum_{t=j}^{\infty}(\beta \delta)^{t-j} U\left(c_{t}\right)$, where $\beta$ is the discount factor and $c$ denotes a CES aggregator of goods. The period utility function is:

$$
\begin{equation*}
U(c)=\log \left(\omega_{M} x_{M}^{\frac{\phi-1}{\phi}}+\sum \omega_{K}\left(x_{K}^{\frac{\sigma_{K}-1}{\sigma_{K}}}+\left(z_{K} h_{K}\right)^{\frac{\sigma_{K}-1}{\sigma_{K}}}\right)^{\frac{\sigma_{K}}{\sigma_{K}-1} \frac{\phi-1}{\phi}}\right)^{\frac{\phi}{\phi-1}} \tag{2}
\end{equation*}
$$

where parameter $\phi \geq 0$ denotes the elasticity of substitution across goods and $\omega_{M}$ and $\omega_{K}$ govern preferences for goods that vary across households. We normalize the preference shifters such that $\omega_{M}+\sum \omega_{K}=1$ for each household and henceforth carry over in our notation only the $\omega_{K}$ 's.

Sources of Heterogeneity. Households are heterogeneous with respect to their market pro-
ductivity $z_{M}$, home productivities $z_{K}$, and preferences over goods $\omega_{K}$. Following Heathcote, Storesletten, and Violante (2014), we impose a random walk structure for market productivity that is important for obtaining the no-trade result underlying the analytical solutions. Households' $\log$ market productivity $\log z_{M}$ is the sum of a permanent component $\alpha$ and a more transitory component $\varepsilon$ :

$$
\begin{equation*}
\log z_{M, t}^{j}=\alpha_{t}^{j}+\varepsilon_{t}^{j} \tag{3}
\end{equation*}
$$

The permanent component follows a random walk, $\alpha_{t}^{j}=\alpha_{t-1}^{j}+v_{t}^{\alpha}$. The more transitory component, $\varepsilon_{t}^{j}=\kappa_{t}^{j}+v_{t}^{\varepsilon}$, equals the sum of a random walk component, $\kappa_{t}^{j}=\kappa_{t-1}^{j}+v_{t}^{\kappa}$, and an innovation $v_{t}^{\varepsilon}$. For any random walk, we use $v$ to denote innovations and $\Phi_{v_{t}}$ to denote distributions of innovations. We allow distributions of innovations to vary over time $t$.

Given the $\log$ preferences in equation (2), we are able to obtain the no-trade result with minimal structure on the processes that govern productivity and preferences in the home sectors. Home productivities follow $z_{K, t}^{j} \sim \Phi_{z_{K}, t}^{j}$ and preferences follow $\omega_{K, t}^{j} \sim \Phi_{\omega_{K}, t}^{j}$, where again we allow distributions to vary over time $t$. We assume that $z_{K, t}^{j}$ and $\omega_{K, t}^{j}$ are orthogonal to the innovations $\left\{v_{t}^{\alpha}, v_{t}^{\kappa}, v_{t}^{\varepsilon}\right\}$ and that all innovations are drawn independently from each other. The distribution of initial conditions of $\left(\omega_{K, j}^{j}, z_{K, j}^{j}, \alpha_{j}^{j}, \kappa_{j}^{j}\right)$ can be non-degenerate across households born at $j$ and can vary by birth year $j$. From now on, we identify a household $\iota$ by a sequence $\left\{z_{K}^{j}, \omega_{K}^{j}, \alpha^{j}, \kappa^{j}, v^{\varepsilon}\right\}$.

Asset Markets. We describe restrictions on asset markets using the definition of an island in the spirit of Heathcote, Storesletten, and Violante (2014). Islands capture insurance mechanisms available to households for smoothing more transitory shocks in the market sector. Households are partitioned into islands, with each island consisting of a continuum of households that are identical in terms of their productivity at home $z_{K}$, preferences $\omega_{K}$, permanent component of market productivity $\alpha$, and the initial condition of $\kappa$. More formally, household $\iota=\left\{z_{K}^{j}, \omega_{K}^{j}, \alpha^{j}, \kappa^{j}, v^{\varepsilon}\right\}$ lives on island $\ell$ consisting of $\iota$ 's with common initial state $\left(z_{K, j}^{j}, \omega_{K, j}^{j}, \alpha_{j}^{j}, \kappa_{j}^{j}\right)$ and sequences $\left\{z_{K, t}^{j}, \omega_{K, t}^{j}, \alpha_{t}^{j}\right\}_{t=j+1}^{\infty}$.

The structure of asset markets is as follows. Households cannot trade assets contingent on $z_{K, t}^{j}$ and $\omega_{K, t}^{j}$, but can trade one-period bonds $b^{\ell}\left(s_{t+1}^{j}\right)$ that pay one unit of market consumption
contingent on $s_{t}^{j} \equiv\left(\alpha_{t}^{j}, \kappa_{t}^{j}, v_{t}^{\varepsilon}\right)$ with households that live on their island $\ell$. Across islands, households can trade economy-wide one-period bonds $a\left(\zeta_{t+1}^{j}\right)$ that pay one unit of market consumption contingent on $\zeta_{t}^{j} \equiv\left(\kappa_{t}^{j}, v_{t}^{\varepsilon}\right)$.

As we discuss more formally below, differences in $\left(z_{K}, \omega_{K}, \alpha\right)$ across households remain uninsurable by the no-trade result that generates $a\left(\zeta_{t+1}^{j}\right)=0$ in equilibrium. The more transitory component of productivity $\varepsilon_{t}^{j}$ becomes fully insurable because households on an island are only heterogeneous with respect to $\zeta_{t}^{j}$ and can trade state-contingent bonds $b^{\ell}\left(\zeta_{t+1}^{j}\right)$. As a result, the island structure generates partial insurance with respect to market productivity differences. Anticipating these results, henceforth we call $\alpha$ the uninsurable permanent component of market productivity and $\varepsilon=\kappa+v^{\varepsilon}$ the insurable transitory component of market productivity. ${ }^{4}$

Household Optimization. We now describe the optimization problem of a particular household $\iota$ born in period $j$. The household chooses $\left\{\mathbf{c}_{t}, \mathbf{x}_{t}, \mathbf{h}_{t}, b^{\ell}\left(s_{t+1}^{j}\right), a\left(\zeta_{t+1}^{j}\right)\right\}_{t=j}^{\infty}$ to maximize the expected value of discounted flows of utilities in equation (2), subject to the home production technologies in equation (1), the time endowment $h_{M}+\sum h_{K}=1$, and the sequential budget constraints:

$$
\begin{equation*}
x_{M, t}+\sum p_{K, t} x_{K, t}+\int_{s_{t+1}^{j}} q_{b}^{\ell}\left(s_{t+1}^{j}\right) b^{\ell}\left(s_{t+1}^{j}\right) \mathrm{d} s_{t+1}^{j}+\int_{\zeta_{t+1}^{j}} q_{a}\left(\zeta_{t+1}^{j}\right) a\left(\zeta_{t+1}^{j}\right) \mathrm{d} \zeta_{t+1}^{j}=\tilde{y}_{t}^{j}+b^{\ell}\left(s_{t}^{j}\right)+a\left(\zeta_{t}^{j}\right) \tag{4}
\end{equation*}
$$

The market good $x_{M}$ is the numeraire good with a price of one in all periods. Denoting by $p_{K}$ the price of good $x_{K}$ relative to market good, the left-hand side of the budget constraint equals total expenditures on goods $(p x)_{t}=x_{M, t}+\sum p_{K, t} x_{K, t}$, island-level bonds $b^{\ell}\left(s_{t+1}^{j}\right)$ at prices $q_{b}^{\ell}\left(s_{t+1}^{j}\right)$, and economy-wide bonds $a\left(\zeta_{t+1}^{j}\right)$ at prices $q_{a}\left(\zeta_{t+1}^{j}\right)$. The right-hand side of the budget constraint consists of after-tax labor income $\tilde{y}_{t}^{j}$ and bond payouts.

Government. The government taxes labor income to finance public expenditures $G$. Its budget constraint is $G=\int_{\iota}\left[z_{M, t}(\iota)-\left(1-\tau_{0}\right) z_{M, t}(\iota)^{1-\tau_{1}}\right] h_{M, t}(\iota) \mathrm{d} \Phi(\iota)$, where $\Phi$ denotes the distribution function of households.

[^4]Production. Aggregate production is given by $Y=\int_{\iota} z_{M}(\iota) h_{M}(\iota) \mathrm{d} \Phi(\iota)$. The markets for labor and goods are perfectly competitive and the wage per efficiency unit of labor is one. Production $Y$ is transformed at a rate of one into market goods, $\int_{\iota} x_{M}(\iota) \mathrm{d} \Phi(\iota)+G$, and at rates $A_{K}^{-1}$ into expenditures of home goods $x_{K}$. Therefore, relative prices equal $p_{K}=A_{K}^{-1}$.

Equilibrium. Given a tax function $\left(\tau_{0}, \tau_{1}\right)$, an equilibrium consists of a sequence of allocations $\left\{\mathbf{c}_{t}, \mathbf{x}_{t}, \mathbf{h}_{t}, b^{\ell}\left(s_{t+1}^{j}\right), a\left(\zeta_{t+1}^{j}\right)\right\}_{\iota, t}$ and a sequence of prices $\left\{p_{K, t}\right\}_{t},\left\{q_{b}^{\ell}\left(s_{t+1}^{j}\right)\right\}_{\ell, t},\left\{q_{a}\left(\zeta_{t+1}^{j}\right)\right\}_{t}$ such that: (i) the allocations solve households' problems; (ii) asset markets clear:

$$
\begin{equation*}
\int_{\iota \in \ell} b^{\ell}\left(s_{t+1}^{j} ; \iota\right) \mathrm{d} \Phi(\iota)=0 \quad \forall \ell, s_{t+1}^{j}, \quad \text { and } \quad \int_{\iota} a\left(\zeta_{t+1}^{j} ; \iota\right) \mathrm{d} \Phi(\iota)=0 \quad \forall \zeta_{t+1}^{j} \tag{5}
\end{equation*}
$$

(iii) goods market clears:

$$
\begin{equation*}
\int_{\iota}\left(x_{M, t}(\iota)+\sum p_{K, t} x_{K, t}(\iota)\right) \mathrm{d} \Phi(\iota)+G=\int_{\iota} z_{M, t}(\iota) h_{M, t}(\iota) \mathrm{d} \Phi(\iota), \tag{6}
\end{equation*}
$$

(iv) the government budget constraint holds $G=\int_{\iota}\left[z_{M, t}(\iota)-\left(1-\tau_{0}\right) z_{M, t}(\iota)^{1-\tau_{1}}\right] h_{M, t}(\iota) \mathrm{d} \Phi(\iota)$, and (v) relative prices are pinned down by the relative efficiency of transforming production $p_{K, t}=A_{K, t}^{-1} .{ }^{5}$

### 2.2 Equilibrium Allocations

The model retains tractability because it features a no-trade result. This section explains the logic and usefulness of this result and Appendix A presents the proof. Our proof follows closely the proof presented by Heathcote, Storesletten, and Violante (2014) in an incomplete markets model with labor supply and further extended by Boerma and Karabarbounis (2019) to an incomplete markets model with multiple time uses.

We begin by guessing that the equilibrium features no trade across islands, that is $a\left(\zeta_{t+1}^{j} ; \iota\right)=$ $0, \forall \iota, \zeta_{t+1}^{j}$. Further, we postulate that an equilibrium allocation $\left\{\mathbf{c}_{t}(\iota), \mathbf{x}_{t}(\iota), \mathbf{h}_{t}(\iota)\right\}$ solves a sequence of static planning problems. The planner problems consist of maximizing average utility within each island, $\int_{\zeta_{t}^{j}} U\left(c_{t}(\iota) ; \iota\right) \mathrm{d} \Phi_{t}\left(\zeta_{t}^{j}\right)$, subject to households' home production technologies

[^5]in equation (1), households' time endowment $h_{M, t}(\iota)+\sum h_{K, t}(\iota)=1$, and the island-level resource constraint $\int_{\zeta_{t}^{j}}\left(x_{M, t}(\iota)+\sum p_{K, t} x_{K, t}(\iota)\right) \mathrm{d} \Phi_{t}\left(\zeta_{t}^{j}\right)=\int_{\zeta_{t}^{j}} \tilde{y}_{t}(\iota) \mathrm{d} \Phi_{t}\left(\zeta_{t}^{j}\right)$. We verify our guess by demonstrating that, at the postulated allocations, households solve their optimization problems and all asset and goods markets clear.

To understand the no-trade result, we observe that households on each island $\ell$ have the same marginal utility of market consumption because they are identical in terms of ( $z_{K}, \omega_{K}, \alpha$ ) and trade in state-contingent bonds allows them to perfectly insure against $\left(\kappa, v^{\varepsilon}\right)$. The island-level marginal utility of market consumption $\mu(\ell)$ in the no-trade equilibrium is:

$$
\begin{equation*}
\mu(\ell)=\frac{1}{x_{M}+\sum p_{K} x_{K}+\tilde{z}_{M} \sum h_{K}}=\frac{1}{\int_{\zeta} \tilde{z}_{M} \mathrm{~d} \Phi(\zeta)}=\frac{1}{\left.\exp \left(\left(1-\tau_{1}\right) \alpha\right)\right) \mathbb{C}} \tag{7}
\end{equation*}
$$

where for simplicity we have dropped the time subscript from all variables and the constant $\mathbb{C}=\int_{\zeta}\left(1-\tau_{0}\right) \exp \left(\left(1-\tau_{1}\right)(\varepsilon)\right) \mathrm{d} \Phi(\zeta)$ does not depend on $\ell$ and is common across all households. The no-trade result states that households do not trade bonds across islands, $a\left(\zeta_{t+1}^{j}\right)=0$. Given the random walk assumption on $\alpha$, equation (7) implies that the growth in marginal utility, $\mu_{t+1} / \mu_{t}$, does not depend on the state vector $\left(z_{K, t}^{j}, \omega_{K, t}^{j}, \alpha_{t}^{j}\right)$ that differentiates islands $\ell$. As a result, all households value bonds traded across islands identically in equilibrium and hence there are no mutual benefits from trading $a\left(\zeta_{t+1}^{j}\right) .{ }^{6}$

Solutions to standard general equilibrium models with uninsurable risk and self-insurance via a risk-free bond, such as Huggett (1993) and Aiyagari (1994), are obtained numerically. While the present model also allows households to trade a risk-free bond (by setting $a\left(\zeta_{t}^{j}\right)=1$ for all states $\zeta_{t}^{j}$ ), the assumptions on asset markets, stochastic processes, and preferences allow us to characterize equilibrium allocations in closed form without solving simultaneously for the wealth distribution. Dropping the time index for notational simplicity, we summarize the equilibrium

[^6]allocations in equations (8)-(11):
\[

$$
\begin{align*}
& \left.x_{M}=\exp \left(\left(1-\tau_{1}\right) \alpha\right)\right) \mathbb{C} \frac{1}{1+\sum\left(\frac{\omega_{K}}{\omega_{M}}\right)^{\phi} p_{K}^{1-\phi}\left(1+\left(\frac{z_{K} p_{K}}{\tilde{z}_{M}}\right)^{\sigma_{K}-1}\right)^{\frac{\phi-1}{\sigma_{K}-1}}},  \tag{8}\\
& \left.x_{K}=\exp \left(\left(1-\tau_{1}\right) \alpha\right)\right) \mathbb{C} \frac{p_{K}^{-\phi}\left(\frac{\omega_{K}}{\omega_{M}}\right)^{\phi}\left(1+\left(\frac{z_{K} p_{K}}{\tilde{z}_{M}}\right)^{\sigma_{K}-1}\right)^{\frac{\phi-\sigma_{K}}{\sigma_{K}-1}}}{1+\sum\left(\frac{\omega_{K}}{\omega_{M}}\right)^{\phi} p_{K}^{1-\phi}\left(1+\left(\frac{z_{K} p_{K}}{\tilde{z}_{M}}\right)^{\sigma_{K}-1}\right)^{\frac{\phi-1}{\sigma_{K}-1}}},  \tag{9}\\
& \left.h_{K}=\exp \left(\left(1-\tau_{1}\right) \alpha\right)\right) \mathbb{C} \frac{\left(\frac{p_{K}}{\tilde{z}_{M}}\right)^{\sigma_{K}} z_{K}^{\sigma_{K}-1} p_{K}^{-\phi}\left(\frac{\omega_{K}}{\omega_{M}}\right)^{\phi}\left(1+\left(\frac{z_{K} p_{K}}{\tilde{z}_{M}}\right)^{\sigma_{K}-1}\right)^{\frac{\phi-\sigma_{K}}{\sigma_{K}-1}}}{1+\sum\left(\frac{\omega_{K}}{\omega_{M}}\right)^{\phi} p_{K}^{1-\phi}\left(1+\left(\frac{z_{K} p_{K}}{\tilde{z}_{M}}\right)^{\sigma_{K}-1}\right)^{\frac{\phi-1}{\sigma_{K}-1}}},  \tag{10}\\
& h_{M}=1-\sum h_{K}, \tag{11}
\end{align*}
$$
\]

where all allocations and sources of heterogeneity $\left(z_{K}, \omega_{K}, \alpha, \varepsilon\right)$ are household-specific, prices $p_{K}$ are common across households, and the constant $\mathbb{C}=\int_{\zeta}\left(1-\tau_{0}\right) \exp \left(\left(1-\tau_{1}\right)(\varepsilon)\right) \mathrm{d} \Phi(\zeta)$ is common across households.

Starting with expenditures in equations (8) and (9), we first note that, holding constant relative productivities $\frac{z_{K}}{\tilde{z}_{M}}$, an increase in the permanent uninsurable component $\alpha$ of market productivity increases both $x_{M}$ and $x_{K}$ because all goods are normal. Holding constant relative productivities $\frac{z_{K}}{\bar{z}_{M}}$, expenditures do not depend on the transitory component of market productivity $\varepsilon$ because state-contingent assets insure against variation in $\varepsilon$.

Dividing equation (9) with equation (8) sheds light on the allocation of expenditures across sectors:

$$
\begin{equation*}
\frac{x_{K}}{x_{M}}=p_{K}^{-\phi}\left(\frac{\omega_{K}}{\omega_{M}}\right)^{\phi}\left(1+\left(\frac{z_{K} p_{K}}{\tilde{z}_{M}}\right)^{\sigma_{K}-1}\right)^{\frac{\phi-\sigma_{K}}{\sigma_{K}-1}} \tag{12}
\end{equation*}
$$

An increase in home productivity relative to the opportunity cost of time, $\frac{z_{K}}{\tilde{z}_{M}}$, has two effects on the allocation of expenditures. First, it makes good $c_{K}$ cheaper to produce relative to good $c_{M}$, which tends to increase $x_{K}$ relative to $x_{M}$. This effect is parameterized by the elasticity of substitution across goods $\phi$. Second, it makes input $x_{K}$ more expensive relative to input $h_{K}$ in the production of good $c_{K}$, which tends to decrease $x_{K}$. This effect is parameterized by the home production elasticity $\sigma_{K}$. When $\phi>\sigma_{K}$, the first effect dominates and $\frac{x_{K}}{x_{M}}$ is increasing in $\frac{z_{K}}{\tilde{z}_{M}}$. By contrast, the effect of an increase in the price $p_{K}$ is unambiguously negative because both the
substitution away from good $c_{K}$ toward good $c_{M}$ and the substitution away from expenditures $x_{K}$ toward time $h_{K}$ work in the same direction. ${ }^{7}$

For the allocation of time relative to spending, we use equations (9) and (10) to obtain:

$$
\begin{equation*}
\frac{h_{K}}{x_{K}}=\left(\frac{p_{K}}{\tilde{z}_{M}}\right)^{\sigma_{K}} z_{K}^{\sigma_{K}-1} \tag{13}
\end{equation*}
$$

The first term shows that an increase in the price of expenditures $p_{K}$ relative to the opportunity cost of time $\tilde{z}_{M}$ unambiguously increases time relative to expenditures in the production of good $c_{K}$. The second term shows that an increase in the relative productivity of time $z_{K}$ increases time relative to expenditures when the two inputs are substitutes $\left(\sigma_{K}>1\right)$.

### 2.3 Identification of Sources of Heterogeneity

Building on the methodology introduced by Boerma and Karabarbounis (2019), in this section we infer the sources of heterogeneity across households, $\left\{z_{K}, \omega_{K}, \alpha, \varepsilon\right\}_{\iota}$, such that the model accounts perfectly for any given cross-sectional data $\left\{x_{M}, x_{K}, h_{M}, h_{K}, z_{M}\right\}_{\iota}$ in any period. Given parameters $\left(\phi, \sigma_{K}, \tau_{0}, \tau_{1}\right)$, prices $p_{K}$, and cross-sectional data $\left\{x_{M}, x_{K}, h_{M}, h_{K}, z_{M}\right\}_{\iota}$, we invert the equilibrium allocations presented in equations (8)-(11) and the decomposition of market productivity into a permanent and transitory component, $\log z_{M}=\alpha+\varepsilon$, to obtain unique inferred sources of heterogeneity up to a constant (see Appendix B for more details):

$$
\begin{align*}
z_{K} & =\left(\frac{x_{K}}{h_{K}}\right)^{\frac{1}{1-\sigma_{K}}}\left(\frac{\tilde{z}_{M}}{p_{K}}\right)^{\frac{\sigma_{K}}{\sigma_{K}-1}},  \tag{14}\\
\omega_{M} & =\frac{1}{1+\sum p_{K}\left(\frac{x_{K}}{x_{M}}\right)^{\frac{1}{\phi}}\left(1+\left(\frac{z_{K} p_{K}}{\tilde{z}_{M}}\right)^{\sigma_{K}-1}\right)^{\frac{1}{\phi} \frac{\sigma_{K}-\phi}{\sigma_{K}-1}}},  \tag{15}\\
\omega_{K} & =\frac{p_{K}\left(\frac{x_{K}}{x_{M}}\right)^{\frac{1}{\phi}}\left(1+\left(\frac{z_{K} p_{K}}{\tilde{z}_{M}}\right)^{\sigma_{K}-1}\right)^{\frac{1}{\phi} \frac{\sigma_{K}-\phi}{\sigma_{K}-1}}}{1+\sum p_{K}\left(\frac{x_{K}}{x_{M}}\right)^{\frac{1}{\phi}}\left(1+\left(\frac{z_{K} p_{K}}{\tilde{z}_{M}}\right)^{\sigma_{K}-1}\right)^{\frac{1}{\phi} \frac{\sigma_{K}-\phi}{\sigma_{K}-1}}}  \tag{16}\\
\alpha & =\frac{1}{1-\tau_{1}}\left[\log \left(x_{M}+\sum p_{K} x_{K}+\tilde{z}_{M} \sum h_{K}\right)-\log \mathbb{C}\right]  \tag{17}\\
\varepsilon & =\log z_{M}-\frac{1}{1-\tau_{1}}\left[\log \left(x_{M}+\sum p_{K} x_{K}+\tilde{z}_{M} \sum h_{K}\right)-\log \mathbb{C}\right] . \tag{18}
\end{align*}
$$

[^7]The solution for $z_{K}$ in equation (14) comes from inverting equation (13) for the optimal allocation of expenditures and time inputs in the production of good $K$. Intuitively, when household expenditures $x_{K}$ increase relative to the time input $h_{K}$ and the two inputs are complements in production $\left(\sigma_{K}<1\right)$, it must be that household's time becomes more productive in the production of good $K$. Given an inferred $z_{K}$, equations (15) and (16) show how relative preferences for goods are pinned down by relative expenditures, prices, and productivities. Up to a constant which is common across households in a given period, the permanent component of log market productivity $\alpha$ in equation (17) equals the market value of total consumption which consists of the sum of expenditures $p x=x_{M}+\sum p_{K} x_{K}$ and the market value of time allocated to home production $\tilde{z}_{M} \sum h_{K}$. Finally, the transitory component of market productivity $\varepsilon$ equals the gap between $\log$ market productivity and its permanent component.

## 3 Data

For our baseline analyses we combine data from the Consumer Expenditure Survey (CEX), the American Time Use Survey (ATUS), and the national income and product accounts (NIPA). We consider married and cohabiting households with heads between 25 and 65 years old who are not students. We drop observations with market productivity below 3 dollars per hour in 2010 dollars, with market productivity above 300 dollars but working less than 20 hours per week, with expenditures at the top and bottom one percent, and with more than 105 reported hours per week in any of the time use categories we consider. In the ATUS we drop respondents during weekends and in the CEX we keep households that completed all four interviews. The final sample from CEX/ATUS contains 34,775 households between 1995 and 2016. In all our results, we weight households with the sample weights provided by the surveys.

For our quantitative results, we specialize the general model with $K+1$ goods to three goods. The market good $c_{M}=x_{M}$ is produced only with expenditures. The non-market good $c_{N}$ is produced with non-market time $h_{N}$ and non-market expenditures $x_{N}$. Finally, the leisure good $c_{L}$ is produced with leisure time $h_{L}$ and leisure expenditures $x_{L}$.

Data on expenditures come from CEX interview surveys. Our definition of expenditures is
closest to the one in Krueger and Perri (2006). It covers both expenditures on non-durables and expenditures on durables such as housing, vehicles, and furniture. Our measure of consumption reflects the flow of services in a given period. For housing services we use rent paid if the household rents and a self-reported imputed rent for households that own. For services generated by vehicles and furnishings, we use the imputation approach of Cutler and Katz (1991) since there is no direct information on the value of the stock of vehicles and furniture. ${ }^{8}$

We split total expenditures $p x$ between market expenditures $x_{M}$, non-market expenditures $p_{N} x_{N}$, and leisure expenditures $p_{L} x_{L}$ by mapping expenditures in 20 spending categories from the CEX to our three baskets. The logic underpinning our choice is to classify expenditures complementary to time as leisure (such as communication, entertainment, and reading), expenditures substitutable to time as non-market production (such as food, household services, and personal care), and expenditures that do not use a significant amount of time in the production of commodities as market goods. ${ }^{9}$

To obtain quantities $x_{M}, x_{N}$, and $x_{L}$, we deflate expenditures in each category with their corresponding price index. We construct the Fisher price index for each basket using the price indices and aggregate spending for the 20 CEX spending categories as provided in NIPA Table 2.5. ${ }^{10}$ For durable goods, we create corresponding price and spending series using user costs. ${ }^{11}$

From the CEX, we measure income as wage and salary income earned over the past 12 months and wages as income divided by hours usually worked in a year (the product of weeks worked with

[^8]usual hours worked per week). Because we focus on married or cohabiting households, we define household market hours $h_{M}$ as the sum of hours worked by spouses and market productivity $z_{M}$ as the average of wages of individual members weighted by their market hours.

The market good is the numeraire good and we deflate the price of non-market goods $p_{N}$, the price of leisure goods $p_{L}$, and market productivity $z_{M}$ with the price index for market goods. For consistency with the model in which aggregate expenditures $\int\left(x_{M, t}(\iota)+\sum p_{K, t} x_{K, t}(\iota)\right) \mathrm{d} \Phi(\iota)+G$ equal aggregate income $\int z_{M, t}(\iota) h_{M, t}(\iota) \mathrm{d} \Phi(\iota)$, for each household in the CEX we scale their quantities by a time-varying factor that aligns aggregate expenditures with aggregate income reported in the survey.

Data for non-market hours $h_{N}$ come from the ATUS waves between 2003 and 2017. Our definition of time spent on non-market production follows the one in Aguiar, Hurst, and Karabarbounis (2013) and includes housework, cooking, shopping, home and car maintenance, gardening, child care, and care for other household members. The CEX does not contain information on time spent on non-market production. To overcome this difficulty, we follow Boerma and Karabarbounis (2019) and impute time use data from the ATUS into the CEX. Our imputation procedure is to allocate to individuals in the CEX the mean non-market hours of matched individuals from the ATUS based on group characteristics that include work status, race, gender, age, family status, education, disability status, geography, hours worked, and wages. We first impute non-market hours to individuals and, similarly to market hours, then sum up these hours at the household level. Finally, we measure leisure residually as total disposable time, which we set to 105 hours per week, minus market hours and time spent on non-market production, $h_{L}=105-h_{M}-h_{N} .{ }^{12}$

In Figure 1 we present the time evolution of relative prices, $p_{N}$ and $p_{L}$, and means and variances of expenditure inputs $x_{M}, x_{N}$, and $x_{L}$, time inputs $h_{M}, h_{N}$, and $h_{L}$, total expenditures $p x=x_{M}+p_{N} x_{N}+p_{L} x_{L}$, and market productivity $z_{M}$. To obtain the time profiles for the means of variables, we regress each variable at the household level on age and time fixed effects. The

[^9]

Figure 1: Means and Variances of Observables
Figure 1 plots the evolution of the relative prices $p_{N}$, and $p_{L}$, means and variances of expenditure inputs $x_{M}, x_{N}$, and $x_{L}$, time inputs $h_{M}, h_{N}$, and $h_{L}$, total expenditures $p x=x_{M}+p_{N} x_{N}+p_{L} x_{L}$, and market productivity $z_{M}$.
plotted means are the coefficients on the time dummies and, therefore, correspond to the mean value of each variable using within-age variation over time. The variances refer to the variances of the residuals in a given year from these regressions and, similarly, reflect changes in within-age variances over time.

The top panel of Figure 1 shows that the relative prices of non-market and leisure goods move in opposite direction over time, with the relative price of non-market goods increasing by roughly 20 percent and the relative price of leisure falling by roughly 45 percent. The second row shows that while expenditures have increased for all three goods since the mid 1990s, the increase has been significantly larger in the leisure sector. Leisure time declines by almost 300 hours per year until the early 2000s, with this decline being offset by increases in both market and non-market hours. Since the 2000s, non-market time has declined whereas leisure time has returned to its 1995 level. These changes in the allocation of expenditures and time have been accompanied by a roughly 35 percent increase in average market productivity over time.

In the bottom rows of Figure 1, we document an increase in the variance of expenditures across households by roughly $5 \log$ points over time. The increase in the dispersion of expenditures is apparent in all three expenditure categories. The variation in leisure hours has been stable over our sample period, while the variation in non-market hours doubled over the same period. ${ }^{13}$ The variation in market hours has been relatively constant, with the exception of the period following the Great Recession. Finally, the variance of market productivity increases by roughly $4 \log$ points over time.

Table 1 displays unconditional cross-sectional correlations between observables. These correlations are obtained after we absorb time and age fixed effects by regressing each observable on both age and time dummies. Market productivity is positively correlated with expenditures (with a correlation of 0.5) but uncorrelated with all time inputs. Households with high levels of total expenditures also tend to spend more in each sector. By contrast, within sector $K$ expenditures

[^10]Table 1: Unconditional Correlations Between Observables

|  | $\log z_{M}$ | $\log (p x)$ | $\log x_{M}$ | $\log x_{N}$ | $\log x_{L}$ | $\log h_{M}$ | $\log h_{N}$ | $\log h_{L}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\log z_{M}$ | 1.00 | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ |
| $\log (p x)$ | 0.53 | 1.00 | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ |
| $\log x_{M}$ | 0.54 | 0.96 | 1.00 | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ |
| $\log x_{N}$ | 0.46 | 0.95 | 0.85 | 1.00 | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ |
| $\log x_{L}$ | 0.49 | 0.91 | 0.82 | 0.82 | 1.00 | $\cdot$ | $\cdot$ | $\cdot$ |
| $\log h_{M}$ | -0.07 | 0.23 | 0.27 | 0.18 | 0.17 | 1.00 | $\cdot$ | $\cdot$ |
| $\log h_{N}$ | 0.04 | -0.06 | -0.08 | -0.04 | -0.04 | -0.39 | 1.00 | $\cdot$ |
| $\log h_{L}$ | 0.02 | -0.18 | -0.20 | -0.15 | -0.15 | -0.47 | -0.37 | 1.00 |

$x_{K}$ and time $h_{K}$ are weakly correlated in the cross-section of households. Finally, higher time spent working in the market sector is offset by lower time in both the non-market and the leisure sector.

## 4 Quantitative Results

We begin by discussing the parameterization of the model. We estimate a progressivity parameter of $\tau_{1}=0.12$ based on a regression of $\log$ after-tax market productivity on log pre-tax market progressivity from the Current Population Survey between 2005 and 2015. We set the level parameter to $\tau_{0}=-0.34$ such that the average tax rate on labor income equals 0.10 which is the average ratio of personal current taxes to income from the national income and product accounts. For our baseline analyses, we set the elasticity of substitution across goods to $\phi=1$, the elasticity of substitution between expenditures and time in non-market technology to $\sigma_{N}=2.5$, and the elasticity of substitution between expenditures and time in leisure technology to $\sigma_{L}=0.5$. Our choice of $\sigma_{N}=2.5$ is motivated by previous estimates in the literature. For example, most estimates of Rupert, Rogerson, and Wright (1995) for couples fall between roughly 2 and 4, Aguiar and Hurst (2007a) obtain estimates of 1.8, and Boerma and Karabarbounis (2019) estimate a value of 2.3. The literature offers little guidance about the values of $\phi$ and $\sigma_{L}$. Consistent with our classification of expenditures and time in the three goods, we choose $\phi=1$ and $\sigma_{L}=0.5$ such
that the elasticity of substitution across goods is larger than the elasticity of substitution between expenditures and time in leisure technology and smaller than the elasticity of substitution in nonmarket technology. Some of our quantitative results are sensitive to these values of elasticities, so in Section 6 we present several analyses under alternative values.

Given parameter values, we identify the sources of heterogeneity using equations (14) to (18). Figure 2 presents the evolution of means and variances for each source of heterogeneity. Similarly to the means of observables discussed previously in Figure 1, in the upper four panels the plotted means are the coefficients on the time dummies from a regression of each source of heterogeneity on age and time fixed effects and the variances refer to the variances of the residuals in a given year from these regressions.

Beginning in the first panel, mean leisure productivity $\log z_{L}$ increases substantially over time and by the end of the sample reaches a level roughly $110 \log$ points higher than its 1995 level. To understand this pattern, equation (14) shows that leisure productivity $z_{L}$ increases in leisure expenditures relative to time $x_{L} / h_{L}$ and decreases in the relative input price $\tilde{z}_{M} / p_{L}$ when expenditures and time are complements $\left(\sigma_{L}<1\right)$. Quantitatively, the substantial increase in $x_{L} / h_{L}$ over time dominates the increase in $\tilde{z}_{M} / p_{L}$ and accounts for the increase in leisure productivity over time. ${ }^{14}$ Mean non-market productivity $\log z_{N}$ tracks mean market productivity $\log z_{M}$ until the mid 2000s, reflecting the growth of the relative input price $\tilde{z}_{M} / p_{N}$ and the fact that expenditures and time are substitutes in the non-market technology $\left(\sigma_{N}>1\right)$. After the mid 2000s, mean non-market productivity starts to decline, reflecting the increase in non-market expenditures relative to time $x_{N} / h_{N}$ and the flattening of $\tilde{z}_{M} / p_{N}$.

The second panel documents a decline in the mean preference for market goods $\omega_{M}$ relative to non-market and leisure goods. Given our choice of an elasticity of substitution $\phi=1$ across goods, preference weights equal the cost share of each good in the market value of total consumption, $\omega_{j^{\prime}}=\frac{p_{j^{\prime}} x_{j^{\prime}}+\tilde{z}_{M} h_{j^{\prime}}}{\sum_{j}\left(p_{j} x_{j}+\tilde{z}_{M} h_{j}\right)}$ for each good $j, j^{\prime}=\{M, N, L\}$. The decline in mean $\omega_{M}$, therefore, reflects the decline in market expenditures $x_{M}$ relative to the market value of total consumption $\sum_{j}\left(p_{j} x_{j}+\tilde{z}_{M} h_{j}\right)$. Finally, the third and fourth panels show an increase in the mean value of

[^11]

Figure 2: Means and Variances of Sources of Heterogeneity
Figure 2 plots the evolution of means and variances of productivities $z_{M}, z_{N}$, and $z_{L}$, preference weights $\omega_{M}, \omega_{N}$, and $\omega_{L}$, the uninsurable permanent component of market productivity $\alpha$, and the insurable transitory component of market productivity $\varepsilon$.
the uninsurable permanent component of $\log$ productivity $\alpha$ and the mean value of the insurable transitory component of productivity $\varepsilon$ over time. These increases reflect the growth of the market value of total consumption and market productivity over time. ${ }^{15}$

Moving to the bottom panels, we first observe that the (within-age) cross-sectional variances of non-market and leisure productivity are significantly larger than the variance of market productivity. ${ }^{16}$ To understand this result it is useful to once more refer to equation (14) that relates $\log z_{K}$ to $\log \tilde{z}_{M}, \log x_{K}$, and $\log h_{K}$. As discussed in Table 1 , time inputs are relatively uncorrelated with market productivity and expenditures in the cross-section of households and, as a result, the variance of $\log z_{K}$ cumulates the variances of these three observables and exceeds the variance of market productivity. The variance of market productivity rises somewhat over time. The variances of non-market and leisure productivity rise even more over time which, in addition to the increase in the variance of market productivity, reflects the increases in the variances of non-market production time $\log h_{N}$ and leisure expenditures $\log x_{L}$.

As Figure 2 shows, the cross-sectional variances of preference weights are relatively stable over time. The cross-sectional variance of the permanent component of market productivity $\alpha$ is large relative to the variance of the transitory component of market productivity $\varepsilon$. This reflects the fact that the cross-sectional variance of the market value of consumption is roughly equal to the variance of market productivity. The variance of $\alpha$ rises over time which reflects the increase in the cross-sectional variance of the market value of total consumption. By contrast, the variance of $\varepsilon$ is stable over time.

In Table 2 we present the cross-sectional correlations between sources of heterogeneity. Similar to the correlations of observables in Table 1, these correlations are obtained after absorbing time and age fixed effects in regressions of each source of heterogeneity on age and time fixed effects. We obtain a high and positive correlation between market productivity $z_{M}$ and non-

[^12]Table 2: Unconditional Correlations Between Sources of Heterogeneity

|  | $\log z_{M}$ | $\log z_{N}$ | $\log z_{L}$ | $\omega_{M}$ | $\omega_{N}$ | $\omega_{L}$ | $\alpha$ | $\varepsilon$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\log z_{M}$ | 1.00 | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ |
| $\log z_{N}$ | 0.80 | 1.00 | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ |
| $\log z_{L}$ | 0.08 | -0.17 | 1.00 | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ |
| $\omega_{M}$ | -0.33 | -0.59 | 0.60 | 1.00 | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ |
| $\omega_{N}$ | -0.06 | 0.26 | 0.27 | -0.03 | 1.00 | $\cdot$ | $\cdot$ | $\cdot$ |
| $\omega_{L}$ | 0.23 | 0.10 | -0.56 | -0.52 | -0.84 | 1.00 | $\cdot$ | $\cdot$ |
| $\alpha$ | 0.95 | 0.71 | 0.23 | -0.21 | 0.02 | 0.10 | 1.00 | $\cdot$ |
| $\varepsilon$ | 0.38 | 0.42 | -0.40 | -0.43 | -0.25 | 0.44 | 0.06 | 1.00 |

market productivity $z_{N}$ which, quantitatively, reflects the fact that expenditures and time are substitutes in the non-market technology $\left(\sigma_{N}>1\right)$. Market productivity is relatively uncorrelated with leisure productivity $z_{L}$, reflecting roughly offsetting effects from a strong correlation between $z_{M}$ and $x_{L}$ in the cross-section of households and the complementarity between expenditures and time in leisure technology $\left(\sigma_{L}<1\right)$. The correlation between preference weights $\omega_{K}$ and productivities $z_{K}$ are negative for the market and the leisure sector and positive for the nonmarket sector. Finally, the correlation between the two components of market productivity, $\alpha$ and $\varepsilon$, is essentially zero.

We conclude this section by presenting the evolution of welfare over time. Our measure of welfare is the consumption equivalent $\chi_{t}$ that leaves utilitarian welfare unchanged between the two allocations:

$$
\begin{equation*}
\sum \pi_{t}(\iota) \log \left(\left(1-\chi_{t}\right) c_{t}(\iota)\right)=\sum \pi_{0}(\iota) \log \left(c_{0}(\iota)\right), \tag{19}
\end{equation*}
$$

where $\pi_{t}(\iota)$ denote survey weights, the flow utility $\log \left(c_{t}(\iota)\right)$ is given by equation (2), and the right-hand side of the equation denotes the baseline allocation in some period 0 . A positive value for $\chi_{t}$ denotes an increase in welfare in period $t$ relative to period $0 .{ }^{17}$

Following Benabou (2002) and Floden (2001) who have emphasized that total welfare effects

[^13]

Figure 3: Welfare
Figure 3 plots the evolution of the level component of the consumption equivalent $\chi^{L}$, the dispersion component of the consumption equivalent $\chi^{D}$, the mean of $\log c$, and variance of $\log c$.
arise from level effects when aggregate allocations change and effects capturing changes in the dispersion of allocations across households, we break $\chi_{t}$ into a level component $\chi_{t}^{L}$ and a dispersion component $\chi_{t}^{D}$. We define the level component as:

$$
\begin{equation*}
\chi_{t}^{L}=1-\frac{\sum \pi_{0}(\iota) c_{0}(\iota)}{\sum \pi_{t}(\iota) c_{t}(\iota)} \tag{20}
\end{equation*}
$$

When the level component is positive, mean consumption is higher in the current allocation in period $t$ than in the baseline allocation in period 0 . Given this definition of $\chi_{t}^{L}$ we obtain the decomposition $\log \left(1-\chi_{t}\right)=\log \left(1-\chi_{t}^{L}\right)+\log \left(1-\chi_{t}^{D}\right)$, where the dispersion component is given by:

$$
\begin{equation*}
\log \left(1-\chi_{t}^{D}\right)=\sum \pi_{0}(\iota) \log \left(\frac{c_{0}(\iota)}{\sum \pi_{0}(\iota) c_{0}(\iota)}\right)-\sum \pi_{t}(\iota) \log \left(\frac{c_{t}(\iota)}{\sum \pi_{t}(\iota) c_{t}(\iota)}\right) \tag{21}
\end{equation*}
$$

When the dispersion component is negative, the consumption dispersion around its mean in the current allocation in period $t$ is higher than the consumption dispersion around its mean in the baseline allocation in period 0 . As a result, dispersion contributes negatively to welfare.

The first panel of Figure 3 shows that the level component of welfare (relative to 1995) grows
by roughly $40 \log$ points until the mid 2000 s and then stabilizes. In the second panel, we observe a roughly $4 \log$ points decline in welfare due to the dispersion component $\chi^{D}$ until 2000. After 2000, $\chi^{D}$ starts to rise and by the end of the sample it roughly goes back to its 1995 level. The lower panels of the figure demonstrate that changes in welfare due to level and dispersion effects are closely related to the mean of $\log c_{t}$ and the variance of $\log c_{t}$ over time. ${ }^{18}$

## 5 Counterfactuals

In this section we present counterfactual exercises in which we shut off the evolution of driving forces and assess their contribution to the evolution of observables and welfare. We begin in Table 3 by assessing the contribution of driving forces, presented in rows, to the mean values of observables, presented in columns. The first row documents the change of means between the end of the sample (2012-2016) and the beginning of the sample (1995-1999) in the baseline model which replicates the data perfectly. For example, the change in mean total expenditures $p x=x_{M}+p_{N} x_{N}+p_{L} x_{L}$ over that period is $24.6 \log$ points. Each other row represents a different experiment in which we shut off either the evolution of the mean or the evolution of the variance of driving forces. ${ }^{19}$ As an example, the second row shows that keeping the price of non-market goods $p_{N}$ constant at their lower initial level would generate an increase of 31.1 log points in mean total expenditures. Because mean total expenditures increased by $24.6 \log$ points in the baseline model which replicates the data perfectly, we conclude that the increase in $p_{N}$ over time

[^14]Table 3: Means: Counterfactuals

|  | $100 \times$ Difference between 2012-2016 and 1995-1999 |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Case | $\log (p x)$ | $\log x_{M}$ | $\log x_{N}$ | $\log x_{L}$ | $\log h_{M}$ | $\log h_{N}$ | $\log h_{L}$ |
| Baseline | 24.6 | 5.1 | 16.5 | 78.2 | -1.8 | -7.4 | 2.1 |
| Mean $p_{N}$ | 31.1 | 5.1 | 50.7 | 78.2 | 6.1 | -21.3 | 2.1 |
| Mean $p_{L}$ | 27.0 | 5.1 | 16.5 | 62.9 | 1.2 | -7.4 | 0.4 |
| Mean $\alpha+\varepsilon$ | -9.7 | -22.4 | -35.5 | 62.8 | -9.3 | 9.3 | 0.5 |
| Variance $\alpha$ | 26.4 | 6.5 | 18.9 | 79.0 | -1.3 | -8.6 | 2.2 |
| Variance $\varepsilon$ | 24.6 | 5.0 | 16.4 | 78.2 | -1.7 | -7.2 | 2.2 |
| Mean $\log z_{N}$ | 23.2 | 5.1 | 12.9 | 78.2 | -3.8 | -4.4 | 2.1 |
| Variance $\log z_{N}$ | 23.3 | 5.1 | 13.5 | 78.2 | -3.5 | -2.7 | 2.1 |
| Mean $\log z_{L}$ | 18.2 | 5.1 | 16.5 | 35.1 | -9.1 | -7.4 | 6.3 |
| Variance $\log z_{L}$ | 24.8 | 5.1 | 16.5 | 79.0 | -1.6 | -7.4 | 2.1 |
| Mean $\omega_{M}$ | 33.7 | 30.8 | 12.7 | 74.5 | 8.1 | -11.1 | -1.6 |
| Variance $\omega_{M}$ | 23.2 | 1.1 | 16.4 | 78.1 | -2.4 | -7.4 | 2.0 |
| Mean $\omega_{N}$ | 24.4 | 5.4 | 15.7 | 78.5 | -2.1 | -8.1 | 2.4 |
| Variance $\omega_{N}$ | 24.8 | 5.2 | 16.8 | 78.4 | -1.6 | -7.0 | 2.3 |
| Mean $\omega_{L}$ | 29.4 | 12.0 | 23.4 | 72.5 | 4.0 | -0.5 | -3.6 |
| Variance $\omega_{L}$ | 24.5 | 5.0 | 16.4 | 78.2 | -1.8 | -7.4 | 2.1 |

causes a $6.5 \log$ points decline in total expenditures.
The most important driver of the rise of total expenditures $p x$ is the increase in mean log market productivity (row "Mean $\alpha+\varepsilon$ "). The rise of market productivity is quantitatively important for the evolution of each expenditure input, $x_{M}, x_{N}$, and $x_{L}$. Among other driving forces, we note the role of the growth in the mean leisure productivity $\log z_{L}$, which accounts for a significant fraction of the increase in $p x$ and $x_{L}$ over time. ${ }^{20}$ The increase in the relative price of non-market goods $p_{N}$ significantly depresses the quantity of non-market expenditures $x_{N}$ and the decrease in the relative price of leisure goods $p_{L}$ contributes modestly to the increase in the quantity of leisure expenditures $x_{L}$ over time. The decline in the preference weight for

[^15]market goods $\omega_{M}$ offsets the increase in market productivity and moderates the rise of market expenditures $x_{M}$ over time.

Market hours $\log h_{M}$ fall moderately between the beginning and the end of the sample. Movements in the relative prices of goods, $p_{N}$ and $p_{L}$, generate a significant decline in market hours over time. To understand this result, we refer to equation (10) which shows that an increase in $p_{N}$ leads to an increase in $h_{N}$ since expenditures and time are substitutes in non-market production and a decline in $p_{L}$ leads to an increase in $h_{L}$ since expenditures and time are complements in leisure production. The increase in the relative price of non-market goods generates 8 log points decline in market hours, whereas the decline in the relative price of leisure goods generates $3 \log$ points decline in the market hours. ${ }^{21}$ The other significant contributor to the decline in mean hours is the decline in the preference for market goods $\omega_{M}$ relative to non-market and leisure goods. As Table 3 shows, the decline in market hours generated by changes in relative prices and preference weights is offset by the rise of mean market productivity, $\alpha+\varepsilon$, and leisure productivity, $\log z_{L}$.

Next, Figure 4 assesses the welfare effects of shutting off the evolution of driving forces. In the left panels, we plot the time path of the level component $\chi^{L}$ in the baseline model and in various counterfactual exercises. In the right panels, we plot the time paths of the dispersion component $\chi^{D}$. The main takeaway from Figure 4 is that the growth in mean leisure productivity is the most important driver for welfare and this influence is apparent in both the level and the dispersion components of welfare. The rise of mean leisure productivity generates more than 30 log points welfare gain in terms of mean consumption. To set a benchmark for comparisons, the rise of mean market productivity generates less than 10 log points gain. Further, mean leisure productivity moderates the rise of inequality over time. The increase of mean leisure productivity generates additional $10 \log$ points of welfare gain in terms of lower consumption dispersion and offsets the negative welfare effects that arise from increases in the variances of market and leisure productivity over time.

To understand the importance of the rise of mean leisure productivity for welfare, we use the

[^16]

Figure 4: Welfare: Counterfactuals
Figure 4 plots the evolution of the level component of the consumption equivalent $\chi^{L}$ (left panels) and the dispersion component of the consumption equivalent $\chi^{D}$ (right panels). In each panel we present the evolution in the baseline path (solid line) together with the evolution in counterfactuals (dashed lines) in which we shut off particular aspects of the evolution of the heterogeneity across households.
close relationship between $\chi^{L}$ and $\chi^{D}$ and the mean and variance of $\log$ consumption in Figure 3. Using our analytical solutions under the parametric restriction $\phi=1$, we express log consumption for every household $\iota$ as a function of the primitive sources of heterogeneity:

$$
\begin{align*}
\log c & =\left(1-\tau_{1}\right) \alpha+\log \mathbb{C}+\omega_{M} \log \left(\omega_{M}\right)  \tag{22}\\
& +\sum \omega_{K}\left[\log \left(\frac{\omega_{K}}{p_{K}}\right)+\frac{1}{\sigma_{K}-1} \log \left(1+\left(\frac{z_{K} p_{K}}{\tilde{z}_{M}}\right)^{\sigma_{K}-1}\right)\right] .
\end{align*}
$$

In the first two terms, a higher permanent component of market productivity $\alpha$ or aggregate transitory productivity encoded in the function $\mathbb{C}=\int_{\zeta}\left(1-\tau_{0}\right) \exp \left(\left(1-\tau_{1}\right)(\varepsilon)\right) \mathrm{d} \Phi(\zeta)$ raise the consumption of all three goods, $c_{M}, c_{N}$, and $c_{L}$, and lead to higher $\log c$. As expected, higher price of expenditures $p_{K}$ lowers $\log c$ and higher productivity of time $z_{K}$ increase $\log c$. Given this result and the significant growth of mean $z_{L}$ over time, it is not surprising that the level component of welfare $\chi^{L}$ increases significantly in response to the increase in mean leisure productivity.

To understand the result that higher mean leisure productivity lowers the welfare cost of dispersion, we parameterize leisure productivity as $z_{L}=\gamma_{L} \hat{z}_{L}$ where $\gamma_{L}$ is the common component of $z_{L}$ across households in each period and $\hat{z}_{L}$ is the idiosyncratic component. Next, we express the variance of log consumption in equation (22) as the sum of the variance of the term that involves $\gamma_{L}$, and other variances and covariances:

$$
\begin{equation*}
\operatorname{Var}(\log c)=\left(\frac{1}{1-\sigma_{L}}\right)^{2} \operatorname{Var}\left(\log \left(1+\gamma_{L}^{\sigma_{L}-1}\left(\frac{\hat{z}_{L} p_{L}}{\tilde{z}_{M}}\right)^{\sigma_{L}-1}\right)\right)+\operatorname{Var}(.)+\ldots+\operatorname{Cov}(.) \tag{23}
\end{equation*}
$$

In Appendix C we prove that, holding constant the other variances and covariances in equation (23), the variance of $\log$ consumption is decreasing in mean leisure productivity $\gamma_{L}$ if and only if $\sigma_{L}<1 .{ }^{22}$ The key term $1+\gamma_{L}^{\sigma_{L}-1}\left(\frac{\tilde{z}_{L} p_{L}}{\tilde{z}_{M}}\right)^{\sigma_{L}-1}$ that appears in equation (23) is related to the consumption of the leisure good $c_{L}$ after factoring out the contribution of leisure expenditures $x_{L}$ which is already accounted for through terms that involve $\left(1-\tau_{1}\right) \alpha+\log \mathbb{C}, \omega_{L}$, and $p_{L}$ in equation (22). This key term equals the constant 1 and a term that denotes the contribution of the time input $h_{L}$ to consumption $c_{L}$. When mean leisure productivity $\gamma_{L}$ increases, the relative

[^17]contribution of the time input to the sum becomes smaller given that $\sigma_{L}<1$, the key term approaches the constant, and the variance of log consumption across households declines.

Before concluding this section, it is worth contrasting our welfare costs of dispersion to alternative approaches we discussed in the introduction that describe the dispersion of observables and its evolution over time. Figure 5 evaluates the effects of shutting off the evolution of driving forces on the variance of total expenditures $\log (p x)$ (in the left panels) and the variance of market hours $\log h_{M}$ (in the right panels). An important difference between the welfare-based measures of dispersion shown in the right panels of Figure 4 and the variances of observables shown in Figure 5 is that the latter fail to capture the welfare effects of an increase in mean leisure productivity in terms of lowering consumption dispersion. In Figure 5, the increase in variance of the permanent component of market productivity $\alpha$ generates most of the increase in the variance of total expenditures and the decline in the mean preference for market goods $\omega_{M}$ generates most of the increase in the variance of market hours. However, as Figure 4 shows these factors are less important quantitatively than mean leisure productivity for the evolution of the welfare costs of dispersion. The welfare costs of dispersion are associated more closely with the dispersion of the consumption aggregator and less with the dispersion of expenditures or market hours.

## 6 Sensitivity Analyses

In this section we discuss sensitivity analyses. Here we summarize the most important results and present the detailed tables and figures underlying our analyses in Appendix D. For each sensitivity analysis, we repeat the identification of the sources of heterogeneity as in Section 4 and then perform the same counterfactuals as in Section 5.

We begin by varying the elasticities of substitution between expenditures and time in each sector. Increasing $\sigma_{N}$ from 2.5 in the baseline to 3.5 magnifies the negative impact of the price of non-market goods $p_{N}$ on market hours $h_{M}$ from $8 \log$ points to $13 \log$ points. Lowering $\sigma_{N}$ to 1.5 mitigates the negative impact of $p_{N}$ to $3 \log$ points. Similarly, lowering $\sigma_{L}$ from 0.5 in the baseline to 0.2 magnifies the negative impact of the price of leisure goods $p_{L}$ on market hours to $5 \log$ points and increasing $\sigma_{L}$ to 0.8 mitigates the negative impact of $p_{L}$ to $1 \log$ point.


Figure 5: Variances: Counterfactuals
Figure 5 plots the evolution of the variance of $\log$ expenditures $p x$ (left panels) and the variance of $\log$ market hours $h_{M}$ (right panels). In each panel we present the evolution in the baseline path (solid line) together with the evolution in counterfactuals (dashed lines) in which we shut off particular aspects of the evolution of the heterogeneity across households.

The impact of the increase in mean leisure productivity on welfare remains relatively robust across all these parameterizations. In the baseline parameterization, the increase in mean leisure productivity contributes to welfare $35 \log$ points through an increase in mean consumption and 9 $\log$ points through a decline in consumption dispersion. Under $\sigma_{N}=3.5$, we obtain contributions of 35 and $9 \log$ points and under $\sigma_{N}=1.5$ we obtain contributions of 36 and $10 \log$ points. Under $\sigma_{L}=0.2$, we obtain contributions of 31 and $5 \log$ points and under $\sigma_{L}=0.8$ we obtain contributions of 51 and $27 \log$ points. In all cases mean leisure productivity is the most important factor driving welfare trends over time. ${ }^{23}$

Our quantitative results on the role of mean leisure productivity in increasing mean consumption and decreasing consumption dispersion are not sensitive to perturbations of the progressivity parameter $\tau_{1}$ to 0.06 and to 0.18 . By contrast, the results are sensitive to the value of the elasticity of substitution across goods $\phi$. We have experimented with many values of $\phi$ and concluded that $\phi$ changes in a non-monotonic way the contributions of mean leisure productivity. In all cases, however, the contributions are positive both in terms of the level and the dispersion components of welfare. ${ }^{24}$

Next, we perform sensitivity analyses with respect to the measurement of key variables underlying our analysis. To address potential measurement error in expenditures in the CEX, for each of the 20 spending categories underlying the construction of our three baskets we use the estimated expenditure elasticity in Aguiar and Bils (2015) together with households' spending share in the cross section and construct an alternative measure of household spending. ${ }^{25}$ Our results are almost unchanged using this alternative measure of expenditures. For example, the increase in the mean leisure productivity contributes to welfare $33 \log$ points through an increase in mean consumption and 9 log points through a decline in consumption dispersion.

[^18]In our baseline analyses, we define non-market hours directly from the survey data and leisure residually as total disposable time minus market hours and time spent on non-market production, $h_{L}=105-h_{M}-h_{N}$. We examine the sensitivity of this choice by repeating our analyses when defining leisure hours directly from the survey data and non-market hours residually as $h_{N}=105-h_{M}-h_{L}$. We find that our welfare results and counterfactuals are robust to the measurements of non-market and leisure time. For example, using this alternative definition of leisure, the increase in the mean leisure productivity contributes to welfare $27 \log$ points through an increase in mean consumption and 8 log points through a decline in consumption dispersion.

## 7 Conclusion

The purpose of this paper is to account for recent trends in labor market outcomes and understand their welfare consequences. To do so, we develop a model with incomplete asset markets and household heterogeneity in market and home technologies and preferences. Using micro data on expenditures and time use, we identify the sources of heterogeneity across households, document how these sources have changed over time, and perform counterfactual analyses.

Our most important finding is to document the substantial increase of leisure productivity over time. This follows from the observation that, for the average household, leisure expenditures relative to leisure time increases dramatically more than predicted from the decline in the relative price of leisure goods. We demonstrate that the increasing productivity of leisure time is associated with significant welfare gains. The increase in mean productivity of leisure time generates significantly larger gains in terms of mean consumption than the increase in mean wages. Additionally, the increase in mean leisure productivity induces significant welfare gains by lowering the dispersion of consumption across households.

Finally, we wish to highlight the importance of taking into account the allocation of time and expenditures across sectors in evaluating welfare effects of trends in labor market outcomes. We demonstrate that the distinction between expenditures and consumption matters for the conclusions one draws from trends in labor market outcomes. While the increase in the variance of the permanent component of wages is the most important factor accounting for the increase
in the dispersion of expenditures over time, this factor contributes significantly less than leisure productivity to the welfare costs of dispersion. This is because these welfare costs are linked more closely to consumption than to expenditures.

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# Labor Market Trends and the Changing Value of Time Online Appendix 

Job Boerma and Loukas Karabarbounis

## A Equilibrium Allocations

In this appendix, we derive the equilibrium allocations presented in Section 2. We proceed in three steps. First, in anticipation of the no-trade result, we solve the planner problems. Second, we postulate equilibrium allocations and prices using the solutions to the planner problems. Third, we establish that the postulated equilibrium allocations and prices indeed constitute an equilibrium as defined in Section 2.

## A. 1 Preliminaries

In what follows, we define the following state vectors. The idiosyncratic shifters that differentiate households within each island $\ell$ is given by the vector $\zeta^{j}$ :

$$
\begin{equation*}
\zeta_{t}^{j}=\left(\kappa_{t}^{j}, v_{t}^{\varepsilon}\right) \in Z_{t}^{j} \tag{A.1}
\end{equation*}
$$

Households can trade bonds within each island contingent on the vector $s^{j}$ :

$$
\begin{equation*}
s_{t}^{j}=\left(\alpha_{t}^{j}, \kappa_{t}^{j}, v_{t}^{\varepsilon}\right) \tag{A.2}
\end{equation*}
$$

We define a household $\iota$ by a sequence of all dimensions of heterogeneity:

$$
\begin{equation*}
\iota=\left\{z_{K}^{j}, \omega_{K}^{j}, \alpha^{j}, \kappa^{j}, v^{\varepsilon}\right\} . \tag{A.3}
\end{equation*}
$$

Finally, we denote the history of all sources of heterogeneity up to period $t$ with the vector:

$$
\begin{equation*}
\theta_{t}^{j}=\left(z_{K, t}^{j}, \omega_{K, t}^{j}, \alpha_{t}^{j}, \kappa_{t}^{j}, v_{t}^{\varepsilon}, \ldots, z_{K, j}^{j}, \omega_{K, j}^{j}, \alpha_{j}^{j}, \kappa_{j}^{j}, v_{j}^{\varepsilon}\right) \tag{A.4}
\end{equation*}
$$

We denote conditional probabilities by $f^{t, j}(. \mid$.$) . For example, the probability that we observe \theta_{t}^{j}$ conditional on $\theta_{t-1}^{j}$ is $f^{t, j}\left(\theta_{t}^{j} \mid \theta_{t-1}^{j}\right)$ and the probability that we observe $s_{t}^{j}$ conditional on $s_{t-1}^{j}$ is $f^{t, j}\left(s_{t}^{j} \mid s_{t-1}^{j}\right)$.

We use $v$ to denote innovations to the processes and $\Phi_{v}$ to denote the distribution of the innovation. We allow the distributions of innovations to vary over time, $\left\{\Phi_{v_{t}^{\alpha}}, \Phi_{v_{t}^{\kappa}}, \Phi_{v_{t}^{\varepsilon}}, \Phi_{z_{K, t}}^{j}, \Phi_{\omega_{K, t}}^{j}\right\}$, and the initial distributions to vary over cohorts $j,\left\{\Phi_{\alpha, j}^{j}, \Phi_{\kappa, j}^{j}\right\}$. We assume that $z_{K, t}^{j}$ and $\omega_{K, t}^{j}$ are orthogonal to the innovations $\left\{v_{t}^{\alpha}, v_{t}^{\kappa}, v_{t}^{\varepsilon}\right\}$ and that these innovations are drawn independently from each other.

## A. 2 Planner Problem

In every period $t$ and in every island $\ell$, the planner solves a static problem that consists of finding the allocations that maximize average utility for households on the island subject to a resource constraint and household-specific home production technologies. We omit $t$ and $\ell$ from the notation for convenience. The planner chooses $\left\{x_{M}(\iota), h_{M}(\iota), x_{K}(\iota), h_{K}(\iota)\right\}$ to maximize:

$$
\int_{Z} \log \left(\omega_{M}(\iota) x_{M}(\iota)^{\frac{\phi-1}{\phi}}+\sum \omega_{K}(\iota)\left(x_{K}(\iota)^{\frac{\sigma_{K}-1}{\sigma_{K}}}+\left(z_{K}(\iota) h_{K}(\iota)\right)^{\frac{\sigma_{K}-1}{\sigma_{K}}}\right)^{\frac{\sigma_{K}}{\sigma_{K}-1} \frac{\phi-1}{\phi}}\right)^{\frac{\phi}{\phi-1}} \mathrm{~d} \Phi_{\zeta}(\zeta)
$$

subject to an island resource constraint for market goods:

$$
\begin{equation*}
\int_{Z}\left(x_{M}(\iota)+\sum p_{K} x_{K}(\iota)\right) \mathrm{d} \Phi_{\zeta}(\zeta)=\int_{Z} \tilde{z}_{M}(\iota) h_{M}(\iota) \mathrm{d} \Phi_{\zeta}(\zeta) \tag{A.5}
\end{equation*}
$$

and individual-specific time constraints:

$$
\begin{equation*}
1=\sum h_{K}(\iota)+h_{M}(\iota) \tag{A.6}
\end{equation*}
$$

Denoting by $\mu\left(z_{K}, \omega_{K}, \alpha\right)$ the multiplier on the island resource constraint and by $\chi(\iota)$ the multipliers on the household's time constraint, the solution to this problem is characterized by the following first-order conditions (for every household $\iota$ ):

$$
\begin{align*}
& {\left[x_{M}(\iota)\right]: \mu\left(z_{K}, \omega_{K}, \alpha\right)=\frac{1}{\mathcal{C}(\iota)} \omega_{M}(\iota) x_{M}(\iota)^{-\frac{1}{\phi}}}  \tag{A.7}\\
& {\left[x_{K}(\iota)\right]: \mu\left(z_{K}, \omega_{K}, \alpha\right)=\frac{1}{\mathcal{C}(\iota)} \frac{\omega_{K}(\iota)}{p_{K}}\left(x_{K}(\iota)^{\frac{\sigma_{K}-1}{\sigma_{K}}}+\left(z_{K}(\iota) h_{K}(\iota)\right)^{\frac{\sigma_{K}-1}{\sigma_{K}}}\right)^{\frac{\sigma_{K}(\phi-1)}{\left.\sigma_{K}-1\right) \phi}-1} x_{K}(\iota)^{-\frac{1}{\sigma_{K}}}}  \tag{A.8}\\
& {\left[h_{M}(\iota)\right]: \chi(\iota)=\tilde{z}_{M}(\iota) \mu\left(z_{K}, \omega_{K}, \alpha\right)}  \tag{A.9}\\
& {\left[h_{K}(\iota)\right]: \chi(\iota)=\frac{z_{K}(\iota)}{\mathcal{C}(\iota)} \omega_{K}(\iota)\left(x_{K}(\iota)^{\frac{\sigma_{K}-1}{\sigma_{K}}}+\left(z_{K}(\iota) h_{K}(\iota)\right)^{\frac{\sigma_{K}-1}{\sigma_{K}}}\right)^{\frac{\sigma_{K}}{\sigma_{K}-1} \frac{\phi_{-1}}{\phi}-1}\left(z_{K}(\iota) h_{K}(\iota)\right)^{-\frac{1}{\sigma_{K}}}} \tag{A.10}
\end{align*}
$$

where

$$
\mathcal{C}(\iota)=\omega_{M}(\iota) x_{M}(\iota)^{\frac{\phi-1}{\phi}}+\sum \omega_{K}(\iota)\left(x_{K}(\iota)^{\frac{\sigma_{K}-1}{\sigma_{K}}}+\left(z_{K}(\iota) h_{K}(\iota)\right)^{\frac{\sigma_{K}-1}{\sigma_{K}}}\right)^{\frac{\sigma_{K}}{\sigma_{K}-1} \frac{\phi-1}{\phi}} .
$$

Combining (A.7) to (A.10), we obtain effective labor input relative to expenditures for each home sector:

$$
\begin{equation*}
\frac{z_{K}(\iota) h_{K}(\iota)}{x_{K}(\iota)}=\left(\frac{z_{K}(\iota) p_{K}}{\tilde{z}_{M}(\iota)}\right)^{\sigma_{K}} \tag{A.11}
\end{equation*}
$$

Using this equation, we note that the home production aggregator simplifies to:

$$
x_{K}(\iota)^{\frac{\sigma_{K}-1}{\sigma_{K}}}+\left(z_{K}(\iota) h_{K}(\iota)\right)^{\frac{\sigma_{K}-1}{\sigma_{K}}}=x_{K}(\iota)^{\frac{\sigma_{K}-1}{\sigma_{K}}}\left(1+\left(\frac{z_{K}(\iota) p_{K}}{\tilde{z}_{M}(\iota)}\right)^{\sigma_{K}-1}\right)
$$

Using this expression, we relate home production expenditures to market expenditures by (A.7) and (A.8):

$$
\begin{equation*}
x_{K}(\iota)=\left(\frac{\tilde{\omega}_{K}(\iota)}{\omega_{M}(\iota) p_{K}}\right)^{\phi} x_{M}(\iota), \tag{A.12}
\end{equation*}
$$

where the transformed preference shifter on good $k$ is

$$
\tilde{\omega}_{K}(\iota) \equiv \omega_{K}(\iota)\left(1+\left(\frac{z_{K}(\iota) p_{K}}{\tilde{z}_{M}(\iota)}\right)^{\sigma_{K}-1}\right)^{\frac{\sigma_{K}}{\sigma_{K}-1} \frac{\phi-1}{\phi}-1}
$$

Substituting into equation (A.7), we derive:

$$
\begin{equation*}
x_{M}(\iota)=\frac{1}{\mu\left(z_{K}, \omega_{K}, \alpha\right)} \frac{1}{1+\sum\left(\frac{\omega_{K}(\iota)}{\omega_{M}(\iota)}\right)^{\phi} p_{K}^{1-\phi}\left(1+\left(\frac{z_{K}(\iota) p_{K}}{\tilde{z}_{M}(\iota)}\right)^{\sigma_{K}-1}\right)^{\frac{\phi-1}{\sigma_{K}-1}}} . \tag{A.13}
\end{equation*}
$$

This expression, combined with the relation between home production and market expenditures (A.12), the relation between effective labor inputs and home production expenditures (A.11), and the household time constraint (A.6), yield solutions for $\left\{x_{M}(\iota), h_{M}(\iota), x_{K}(\iota), h_{K}(\iota)\right\}$ given a multiplier $\mu\left(z_{K}, \omega_{K}, \alpha\right)$.

The multiplier is equal to the inverse of the market value of consumption:

$$
\begin{equation*}
x_{M}(\iota)+\sum p_{K} x_{K}(\iota)+\tilde{z}_{M}(\iota) \sum h_{K}(\iota)=\frac{1}{\mu\left(z_{K}, \omega_{K}, \alpha\right)} \tag{A.14}
\end{equation*}
$$

which is derived by substituting the solutions given a multiplier $\mu\left(z_{K}, \omega_{K}, \alpha\right)$ into the expression on the left-hand side.

Substituting the individuals' time constraint into the island resource constraint to eliminate market hours, we write:

$$
\int_{Z}\left(x_{M}(\iota)+\sum p_{K} x_{K}(\iota)+\tilde{z}_{M}(\iota) \sum h_{K}(\iota)\right) \mathrm{d} \Phi_{\zeta}(\zeta)=\int_{Z} \tilde{z}_{M}(\iota) \mathrm{d} \Phi_{\zeta}(\zeta)
$$

which by substitution of the expression for the multiplier $\mu\left(z_{K}, \omega_{K}, \alpha\right)$ in (A.14) yields a closedform characterization of this multiplier:

$$
\begin{equation*}
\mu\left(z_{K}, \omega_{K}, \alpha\right)=\left(\int_{Z} \tilde{z}_{M}(\iota) \mathrm{d} \Phi_{\zeta}(\zeta)\right)^{-1} \tag{A.15}
\end{equation*}
$$

The denominator is an expected value independent of $\zeta$. Thus $\mu$ is independent of $\zeta$. Note that $\mu\left(z_{K}, \omega_{K}, \alpha\right)=\mu(\alpha)$. The marginal utility from market spending is independent of non-market productivity and preference shifters. Given this solution for $\mu\left(z_{K}, \omega_{K}, \alpha\right)$, we obtain the solutions

$$
\begin{align*}
& x_{M}(\iota)=\left(\int_{Z} \tilde{z}_{M}(\iota) \mathrm{d} \Phi_{\zeta}(\zeta)\right) \frac{1}{1+\sum\left(\frac{\omega_{K}(\iota)}{\omega_{M}(\iota)}\right)^{\phi} p_{K}^{1-\phi}\left(1+\left(\frac{z_{K}(\iota) p_{K}}{\tilde{z}_{M}(\iota)}\right)^{\sigma_{K}-1}\right)^{\frac{\phi-1}{\sigma_{K}-1}}}  \tag{A.16}\\
& x_{K}(\iota)=\left(\int_{Z} \tilde{z}_{M}(\iota) \mathrm{d} \Phi_{\zeta}(\zeta)\right) \frac{\left(\frac{\omega_{K}(\iota)}{\omega_{M}(\iota)}\right)^{\phi} p_{K}{ }^{1-\phi}\left(1+\left(\frac{z_{K}(\iota) p_{K}}{\tilde{z}_{M}(\iota)}\right)^{\sigma_{K}-1}\right)^{\frac{\phi-\sigma_{K}}{\sigma_{K}-1}}}{1+\sum\left(\frac{\omega_{K}(\iota)}{\omega_{M}(\iota)}\right)^{\phi} p_{K}^{1-\phi}\left(1+\left(\frac{z_{K}(\iota) p_{K}}{\tilde{z}_{M}(\iota)}\right)^{\sigma_{K}-1}\right)^{\frac{\phi-1}{\sigma_{K}-1}} \frac{1}{p_{K}}}  \tag{A.17}\\
& h_{K}(\iota)=\frac{\left(\int_{Z} \tilde{z}_{M}(\iota) \mathrm{d} \Phi_{\zeta}(\zeta)\right)}{z_{K}(\iota)} \frac{\left(\frac{z_{K}(\iota) p_{K}}{\tilde{z}_{M}(\iota)}\right)^{\sigma_{K}}\left(\frac{\omega_{K}(\iota)}{\omega_{M}(\iota)}\right)^{\phi} p_{K}^{-\phi}\left(1+\left(\frac{z_{K}(\iota) p_{K}}{\tilde{z}_{M}(\iota)}\right)^{\sigma_{K}-1}\right)^{\frac{\phi-\sigma_{K}}{\sigma_{K}-1}}}{1+\sum\left(\frac{\omega_{K}(\iota)}{\omega_{M}(\iota)}\right)^{\phi} p_{K}^{1-\phi}\left(1+\left(\frac{z_{K}(\iota) p_{K}}{\tilde{z}_{M}(\iota)}\right)^{\sigma_{K}-1}\right)^{\frac{\phi-1}{\sigma_{K}-1}}} \frac{1}{p_{K}}  \tag{A.18}\\
& h_{M}(\iota)=1-\sum h_{K}(\iota) \tag{A.19}
\end{align*}
$$

## A. 3 Postulating Equilibrium Allocations and Prices

We postulate an equilibrium in four steps.

1. We postulate that the equilibrium features no trade between islands, $a\left(\zeta_{t}^{j} ; \iota\right)=0$.
2. We postulate that the solutions $\left\{x_{M}(\iota), h_{M}(\iota), x_{K}(\iota), h_{K}(\iota)\right\}$ to the planner problem in Section A. 2 constitute components of the equilibrium.
3. We use the sequential budget constraints to postulate equilibrium holdings for the bonds $b^{\ell}\left(s_{t}^{j} ; \iota\right)$ that are traded within islands:

$$
\begin{equation*}
b^{\ell}\left(s_{t}^{j} ; \iota\right)=\mathbb{E}\left[\sum_{n=0}^{\infty}(\beta \delta)^{n} \frac{\mu_{t+n}\left(\alpha_{t+n}^{j}\right)}{\mu_{t}\left(\alpha_{t}^{j}\right)}\left(x_{M, t+n}(\iota)+\sum p_{K} x_{K, t+n}(\iota)-\tilde{y}_{t+n}(\iota)\right)\right] \tag{A.20}
\end{equation*}
$$

where $\tilde{y}=\tilde{z}_{M} h_{M}=\left(1-\tau_{0}\right) z_{M}^{1-\tau_{1}} h_{M}$ denotes after-tax labor income.
4. We use the intertemporal marginal rates of substitution implied by the planner solutions to postulate asset prices for $b^{\ell}\left(s_{t+1}^{j} ; \iota\right)$ and $a\left(\zeta_{t+1}^{j} ; \iota\right)$ :

$$
\begin{align*}
& q_{b}^{\ell}\left(s_{t+1}^{j}\right)=\beta \delta \exp \left(-\left(1-\tau_{1}\right) v_{t+1}^{\alpha}\right) \\
& \times\left[\frac{\int \exp \left(A v_{t}^{\varepsilon}\right) \mathrm{d} \Phi_{v_{t}^{\varepsilon}}\left(v_{t}^{\varepsilon}\right)}{\int \exp \left(A v_{t+1}^{\kappa}\right) \mathrm{d} \Phi_{v_{t+1}^{\kappa}}\left(v_{t+1}^{\kappa}\right) \int \exp \left(A v_{t+1}^{\varepsilon}\right) \mathrm{d} \Phi_{v_{t+1}^{\varepsilon}}\left(v_{t+1}^{\varepsilon}\right)}\right] f^{t+1, j}\left(s^{t+1, j} \mid s^{t, j}\right)  \tag{A.21}\\
& q_{a}\left(Z_{t+1}\right)=\beta \delta \int \exp \left(-\left(1-\tau_{1}\right) v_{t+1}^{\alpha}\right) \mathrm{d} \Phi_{v_{t+1}^{\alpha}}\left(v_{t+1}^{\alpha}\right) \\
& \times\left[\frac{\int \exp \left(A v_{t}^{\varepsilon}\right) \mathrm{d} \Phi_{v_{t}^{\varepsilon}}\left(v_{t}^{\varepsilon}\right)}{\int \exp \left(A v_{t+1}^{\kappa}\right) \mathrm{d} \Phi_{v_{t+1}^{\kappa}}\left(v_{t+1}^{\kappa}\right) \int \exp \left(A v_{t+1}^{\varepsilon}\right) \mathrm{d} \Phi_{v_{t+1}^{\varepsilon}}\left(v_{t+1}^{\varepsilon}\right)}\right] \mathbb{P}\left(\left(v_{t+1}^{\kappa}, v_{t+1}^{\varepsilon}\right) \in Z_{t+1}\right) \tag{A.22}
\end{align*}
$$

where $A \equiv 1-\tau_{1}$.

## A. 4 Verifying the Equilibrium Allocations and Prices

We verify that the equilibrium postulated in Section A. 3 constitutes an equilibrium by showing that the postulated equilibrium allocations solve the households' problem and that all markets clear.

## A.4.1 Household Problem

The problem for a household $\iota$ born in period $j$ is described in the main text. We denote the Lagrange multiplier on the household's budget constraint by $\tilde{\mu}_{t}$. We drop $\iota$ from the notation for simplicity.

Using the correspondence between the planner and the household first-order conditions to
relate the multipliers $\tilde{\mu}_{t}$ and $\mu\left(\alpha_{t}^{j}\right)$, we write the optimality conditions directly as:

$$
\begin{align*}
& \omega_{M} x_{M}^{-\frac{1}{\phi}}=\frac{\omega_{K}}{p_{K}}\left(x_{K}^{\frac{\sigma_{K}-1}{\sigma_{K}}}+\left(z_{K} h_{K}\right)^{\frac{\sigma_{K}-1}{\sigma_{K}}}\right)^{\frac{\sigma_{K}}{\sigma_{K}-1} \frac{\phi-1}{\phi}-1} x_{K}^{-\frac{1}{\sigma_{K}}}  \tag{A.23}\\
& \omega_{M} x_{M}^{-\frac{1}{\phi}}=\omega_{K}\left(x_{K}^{\frac{\sigma_{K}-1}{\sigma_{K}}}+\left(z_{K} h_{K}\right)^{\frac{\sigma_{K}-1}{\sigma_{K}}}\right)^{\frac{\sigma_{K}}{\sigma_{K}-1} \frac{\phi-1}{\phi}-1}\left(z_{K} h_{K}\right)^{-\frac{1}{\sigma_{K}}} \frac{z_{K}}{\tilde{z}_{M}}  \tag{A.24}\\
& q_{b}^{\ell}\left(s_{t+1}^{j}\right)=\beta \delta \int \frac{\mu\left(\alpha_{t+1}^{j}\right)}{\mu\left(\alpha_{t}^{j}\right)} f^{t+1, j}\left(\theta^{t+1, j} \mid \theta^{t, j}\right) \mathrm{d} \omega_{K, t+1} \mathrm{~d} z_{K, t+1}  \tag{A.25}\\
& q_{a}\left(\zeta_{t+1}^{j}\right)=\beta \delta \int \frac{\mu\left(\alpha_{t+1}^{j}\right)}{\mu\left(\alpha_{t}^{j}\right)} f^{t+1, j}\left(\theta^{t+1, j} \mid \theta^{t, j}\right) \mathrm{d} v_{t+1}^{\alpha} \mathrm{d} \omega_{K, t+1} \mathrm{~d} z_{K, t+1} \tag{A.26}
\end{align*}
$$

## A.4.2 Euler Equations

We next verify that the Euler equations are satisfied at the postulated equilibrium allocations and prices.

Using the marginal utility of market consumption of the planner problem $\mu\left(\alpha_{t}^{j}\right)$, we write the Euler equation for the bonds $b^{\ell}\left(s_{t+1}^{j}\right)$ at the postulated equilibrium as:

$$
\begin{align*}
q_{b}^{\ell}\left(s_{t+1}^{j}\right) & =\beta \delta \int \frac{\mu_{t+1}\left(\alpha_{t+1}^{j}\right)}{\mu_{t}\left(\alpha_{t}^{j}\right)} f^{t+1, j}\left(\theta^{t+1, j} \mid \theta^{t, j}\right) \mathrm{d} \omega_{K, t+1} \mathrm{~d} z_{K, t+1}  \tag{A.27}\\
& =\beta \delta \int \frac{\left(\int \tilde{z}_{M, t+1}^{j}\left(\alpha_{t+1}^{j}, \varepsilon_{t+1}\right) \mathrm{d} \Phi_{\zeta_{t+1}^{j}}\left(\zeta_{t+1}^{j}\right)\right)^{-1}}{\left(\int \tilde{z}_{M, t}^{j}\left(\alpha_{t}^{j}, \varepsilon_{t}\right) \mathrm{d} \Phi_{\zeta_{t}^{j}}\left(\zeta_{t}^{j}\right)\right)^{-1}} f^{t+1, j}\left(\theta^{t+1, j} \mid \theta^{t, j}\right) \mathrm{d} \omega_{K, t+1} \mathrm{~d} z_{K, t+1}
\end{align*}
$$

where the second line follows from equation (A.15).
To simplify the fraction in $q_{b}^{\ell}\left(s_{t+1}^{j}\right)$ we use that:

$$
\begin{equation*}
\tilde{z}_{M, t+1}^{j}=\left(1-\tau_{0}\right) \exp \left(\left(1-\tau_{1}\right)\left(\alpha_{t}^{j}+v_{t+1}^{\alpha}+\kappa_{t}^{j}+v_{t+1}^{\kappa}+v_{t+1}^{\varepsilon}\right)\right) \tag{A.28}
\end{equation*}
$$

Given $A=\left(1-\tau_{1}\right)$, the expectation over the random variables in the numerator is given by:

$$
\begin{align*}
& \int \exp \left(A\left(\kappa_{t}^{j}+v_{t+1}^{\kappa}+v_{t+1}^{\varepsilon}\right)\right) \mathrm{d} \Phi_{\zeta_{t+1}^{j}}\left(\zeta_{t+1}^{j}\right) \\
= & \int \exp \left(A \kappa_{t}^{j}\right) \mathrm{d} \Phi_{\kappa_{t}^{j}}\left(\kappa_{t}^{j}\right) \int \exp \left(A v_{t+1}^{\kappa}\right) \mathrm{d} \Phi_{v_{t+1}^{\kappa}}\left(v_{t+1}^{\kappa}\right) \int \exp \left(A v_{t+1}^{\varepsilon}\right) \mathrm{d} \Phi_{v_{t+1}^{\varepsilon}}\left(v_{t+1}^{\varepsilon}\right), \tag{A.29}
\end{align*}
$$

where the final equality follows from the assumption that the innovations are drawn independently. Similarly, the expectation over the random variables in the denominator equals:

$$
\begin{equation*}
\int \exp \left(A \kappa_{t}^{j}\right) \mathrm{d} \Phi_{\kappa^{j}, t}\left(\kappa_{t}^{j}\right) \int \exp \left(A v_{t}^{\varepsilon}\right) \mathrm{d} \Phi_{v_{t}^{\varepsilon}}\left(v_{t}^{\varepsilon}\right) \tag{A.30}
\end{equation*}
$$

As a result, the price $q_{b}^{\ell}\left(s_{t+1}^{j}\right)$ is:

$$
q_{b}^{\ell}\left(s_{t+1}^{j}\right)=\beta \delta\left(\frac{\exp \left(-\left(1-\tau_{1}\right) v_{t+1}^{\alpha}\right) \int \exp \left(A v_{t}^{\varepsilon}\right) \mathrm{d} \Phi_{v^{\S}, t}\left(v_{t}^{\varepsilon}\right)}{\int \exp \left(A v_{t+1}^{\kappa}\right) \mathrm{d} \Phi_{v^{\kappa}, t+1}\left(v_{t+1}^{\kappa}\right) \int \exp \left(A v_{t+1}^{\varepsilon}\right) \mathrm{d} \Phi_{v^{\S}, t+1}\left(v_{t+1}^{\varepsilon}\right)}\right) f^{t+1, j}\left(s^{t+1, j} \mid s^{t, j}\right)
$$

where $f^{t+1, j}\left(s^{t+1, j} \mid s^{t, j}\right)=f\left(v_{t+1}^{\alpha}\right) f\left(v_{t+1}^{\kappa}\right) f\left(v_{t+1}^{\varepsilon}\right)$. This confirms our guess in equation (A.21). The key observation is that the distributions for next-period innovations are independent of the current period state and, therefore, the term in brackets is independent of the state vector that differentiates islands $\ell$. As a result, all islands $\ell$ have the same bond prices, $q_{b}^{\ell}\left(s_{t+1}^{j}\right)=$ $Q_{b}\left(v_{t+1}^{B}, v_{t+1}^{\alpha}\right)$.

We next calculate the bond price for a set of states $\mathcal{V}_{t+1} \subseteq \mathbb{V}_{t+1}$ :

$$
\begin{aligned}
q_{b}^{\ell}\left(\mathcal{V}_{t+1}\right) & =\beta \delta \int_{\mathcal{V}^{\alpha}} \exp \left(-\left(1-\tau_{1}\right) v_{t+1}^{\alpha}\right) \mathrm{d} \Phi_{v_{t+1}^{\alpha}}\left(v_{t+1}^{\alpha}\right) \\
& \times\left(\frac{\int \exp \left(A v_{t+1}^{\kappa}\right) \mathrm{d} \Phi_{v_{t+1}^{\kappa}}\left(v_{t+1}^{\kappa}\right) \int \exp \left(A v_{t+1}^{\varepsilon}\right) \mathrm{d} \Phi_{v_{t+1}^{\varepsilon}}\left(v_{t+1}^{\varepsilon}\right)}{\int \exp \left(A v_{t}^{\varepsilon}\right) \mathrm{d} \Phi_{v_{t}^{\varepsilon}}\left(v_{t}^{\varepsilon}\right)}\right)^{-1}
\end{aligned}
$$

Similarly, all islands face the same price $q_{b}^{\ell}\left(\mathcal{V}_{t+1}\right)=Q_{b}\left(\mathcal{V}_{t+1}\right)$.
Finally, we calculate the price for a claim that does not depend on the realization of $\left(v_{t+1}^{\alpha}\right)$ :

$$
\begin{aligned}
q_{b}^{\ell}\left(\mathbb{V}_{t+1}\right) & =\beta \delta \int_{\mathbb{V}^{\alpha}} \exp \left(-\left(1-\tau_{1}\right) v_{t+1}^{\alpha}\right) \mathrm{d} \Phi_{v_{t+1}^{\alpha}}\left(v_{t+1}^{\alpha}\right) \\
& \times\left(\frac{\int \exp \left(A v_{t+1}^{\kappa}\right) \mathrm{d} \Phi_{v_{t+1}^{\kappa}}\left(v_{t+1}^{\kappa}\right) \int \exp \left(A v_{t+1}^{\varepsilon}\right) \mathrm{d} \Phi_{v_{t+1}^{\varepsilon}}\left(v_{t+1}^{\varepsilon}\right)}{\int \exp \left(A v_{t}^{\varepsilon}\right) \mathrm{d} \Phi_{v_{t}^{\varepsilon}}\left(v_{t}^{\varepsilon}\right)}\right)^{-1}
\end{aligned}
$$

All islands face the same price $q_{b}^{\ell}\left(\mathbb{V}_{t+1}\right)=Q_{b}\left(\mathbb{V}_{t+1}\right)$.
By no arbitrage, the price of an inter-island claim equals the price of the same within-island claim. The price of a claim traded across islands for set $Z$ gives:

$$
q_{t+1}^{a}\left(Z ; s^{t, j}\right)=\mathbb{P}\left(\left(v_{t+1}^{\kappa}, v_{t+1}^{\varepsilon}\right) \in Z\right) q_{b}^{\ell}\left(\mathbb{V}_{t+1}\right)
$$

This concludes the discussion on asset prices.
By no arbitrage, the prices of bonds $a$ and $b$ that are contingent on the same set of states must be equalized. Therefore, the price of a claim traded across islands for some set $Z_{t+1}$ is equalized across islands at the no-trade equilibrium and given by:

$$
\begin{equation*}
q_{a}\left(Z_{t+1}\right)=\mathbb{P}\left(\left(v_{t+1}^{\kappa}, v_{t+1}^{\varepsilon}\right) \in Z_{t+1}\right) Q_{b}\left(\mathbb{V}_{t+1}\right) \tag{A.31}
\end{equation*}
$$

where $\mathbb{P}\left(\left(v_{t+1}^{\kappa}, v_{t+1}^{\varepsilon}\right) \in Z_{t+1}\right)$ denotes the probability of $\left(v_{t+1}^{\kappa}, v_{t+1}^{\varepsilon}\right)$ being a member of $Z_{t+1}$. The expression for $q_{a}\left(Z_{t+1}\right)$ confirms our guess in equation (A.22).

## A.4.3 Household's Budget Constraint

We now verify our guess for the bond positions $b_{t}^{\ell}\left(s_{t}^{j}\right)$ and confirm that the household budget constraint holds at the equilibrium allocations that we postulated. We define the deficit term by $d_{t} \equiv x_{M, t}+\sum p_{K} x_{K, t}-\tilde{y}_{t}$. Using the expression for the price $q_{b}^{\ell}\left(s_{t+1}^{j}\right)$ in equation (A.25), the budget constraint at the no-trade equilibrium is given by:

$$
b_{t}^{\ell}\left(s_{t}^{j}\right)=d_{t}+\beta \delta \iiint \frac{\mu\left(\alpha_{t+1}^{j}, \omega_{K, t+1}^{j}, z_{K, t+1}^{j}\right)}{\mu\left(\alpha_{t}^{j}, \omega_{K, t}^{j}, z_{K, t}^{j}\right)} b_{t+1}^{\ell}\left(s_{t+1}^{j}\right) f^{t+1}\left(\sigma_{t+1}^{j} \mid \sigma_{t}^{j}\right) \mathrm{d} s_{t+1}^{j} \mathrm{~d} z_{K, t+1}^{j} \mathrm{~d} \omega_{K, t+1}^{j}
$$

By substituting forward using equation (A.25), we confirm the guess for $b_{t}^{\ell}\left(s_{t}^{j}\right)$ in equation (A.20) and show that the household budget constraint holds at the postulated equilibrium allocations.

## A.4.4 Goods Market Clearing

Aggregating the resource constraints in every island, we obtain that the allocations that solve the planner problems satisfy the aggregate goods market clearing condition:

$$
\int_{\iota}\left(x_{M, t}(\iota)+\sum A_{K}^{-1} x_{K, t}(\iota)\right) \mathrm{d} \Phi(\iota)+G=\int_{\iota} z_{M, t}(\iota) h_{M, t}(\iota) \mathrm{d} \Phi(\iota) .
$$

## A.4.5 Asset Market Clearing

We now confirm that asset markets clear. The asset market clearing conditions $\int_{\iota} a\left(\zeta_{t}^{j} ; \iota\right) \mathrm{d} \Phi(\iota)=0$ hold trivially in a no-trade equilibrium with $a\left(\zeta_{t}^{j} ; \iota\right)=0$. Next, we confirm that asset markets within each island $\ell$ also clear, that is $\int_{\iota \in \ell} b^{\ell}\left(s_{t}^{j} ; \iota\right) \mathrm{d} \Phi(\iota)=0, \forall \ell, s_{t}^{j}$.

Omitting the household index $\iota$ for simplicity, we substitute the postulated bond holdings in equation (A.20) into the asset market clearing conditions:

$$
\begin{aligned}
\int b^{\ell}\left(s_{t}^{j}\right) \mathrm{d} \Phi(\iota) & =\int \mathbb{E}\left[\sum_{n=0}^{\infty}(\beta \delta)^{n} \frac{\mu\left(\alpha_{t+n}^{j}, \omega_{K, t+n}, z_{K, t+n}^{j}\right)}{\mu\left(\alpha_{t}^{j}, \omega_{K, t}, z_{K, t}^{j}\right)} d_{t+n}\right] \mathrm{d} \Phi(\iota) \\
& =\sum_{n=0}^{\infty}(\beta \delta)^{n} \iint \frac{\mu\left(\alpha_{t+n}^{j}, \omega_{K, t+n}, z_{K, t+n}^{j}\right)}{\mu\left(\alpha_{t}^{j}, \omega_{K, t}, z_{K, t}^{j}\right)} d_{t+n} f\left(\theta_{t+n}^{j} \mid \theta_{t-1}^{j}\right) \mathrm{d} \theta_{t+n}^{j} \mathrm{~d} \Phi(\iota) .
\end{aligned}
$$

For simplicity we omit conditioning on $\theta_{t-1}^{j}$ and write the density function as $f\left(\theta_{t+n}^{j} \mid \theta_{t-1}^{j}\right)=$ $f\left(\left\{v_{t+n}^{\alpha}\right\}\right) f\left(\left\{v_{t+n}^{\kappa}\right\}\right) f\left(\left\{v_{t+n}^{\varepsilon}\right\}\right) f\left(\left\{z_{K, t+n}, \omega_{K, t+n}\right\}\right)$. Further, we denote the marginal utility growth by $\mathcal{Q}\left(v_{t+n}^{\alpha}\right) \equiv \frac{\mu\left(\alpha_{t+n}^{j}\right)}{\mu\left(\alpha_{t}^{j}\right)}$. Hence, we write aggregate bond holdings $\int b^{\ell}\left(s_{t}^{j}\right) \mathrm{d} \Phi(\iota)$ as:

$$
\begin{aligned}
& \sum_{n=0}^{\infty}(\beta \delta)^{n} \iint \mathcal{Q}\left(v_{t+n}^{\alpha}\right) d_{t+n} f\left(\left\{v_{t+n}^{\alpha}\right\}\right) f\left(\left\{v_{t+n}^{\kappa}\right\}\right) f\left(\left\{v_{t+n}^{\varepsilon}\right\}\right) f\left(\left\{z_{K, t+n}, \omega_{K, t+n}\right\}\right) \ldots \\
& \ldots \mathrm{d}\left\{v_{t+n}^{\alpha}\right\} \mathrm{d}\left\{v_{t+n}^{\kappa}\right\} \mathrm{d}\left\{v_{t+n}^{\varepsilon}\right\} \mathrm{d}\left\{z_{K, t+n}, \omega_{K, t+n}\right\} \mathrm{d} \Phi(\iota) \\
&= \sum_{n=0}^{\infty}(\beta \delta)^{n} \int d_{t+n} f\left(\left\{v_{t+n}^{\kappa}\right\}\right) f\left(\left\{v_{t+n}^{\varepsilon}\right\}\right) \mathrm{d}\left\{v_{t+n}^{\kappa}\right\} \mathrm{d}\left\{v_{t+n}^{\varepsilon}\right\} \mathrm{d} \Phi(\iota) \\
& \times \int \mathcal{Q}\left(v_{t+n}^{\alpha}\right) f\left(\left\{v_{t+n}^{\alpha}\right\}\right) f\left(\left\{z_{K, t+n}, \omega_{K, t+n}\right\}\right) \mathrm{d}\left\{v_{t+n}^{\alpha}\right\} \mathrm{d}\left\{z_{K, t+n}, \omega_{K, t+n}\right\} .
\end{aligned}
$$

Recalling that the deficit terms equal $d_{t}=x_{M, t}+\sum p_{K} x_{K, t}-\tilde{y}_{t}$, the bond market clearing condition holds because the first term is zero by the island-level resource constraint.

## B Inference of Sources of Heterogeneity

In this appendix we show how to derive the sources of heterogeneity $\left\{z_{K, t}, \omega_{K, t}, \alpha_{t}, \varepsilon_{t}\right\}_{\iota}$ presented in Section 2.3. Our strategy is to invert the equilibrium allocations presented in Section 2.2 and solve for the unique sources of heterogeneity that lead to these allocations. We note that the identification is defined up to a constant because the constant $\mathbb{C}$ that appears in the equations of Section 2.3 depends on the $\varepsilon$ 's.

The solution for $z_{K}$ in equation (14) comes from inverting equation (13) for the optimal allocation of expenditures and time inputs in the production of good $K$. Next, we use the solution for $x_{K}$ in equation (9) together with the solution for $x_{M}$ in equation (8) and invert these solutions to solve for the preference weight for good $K$ relative to $M$ :

$$
\frac{\omega_{K}}{\omega_{M}}=p_{K}\left(\frac{x_{K}}{x_{M}}\right)^{\frac{1}{\phi}}\left(1+\left(\frac{z_{K} p_{K}}{\tilde{z}_{M}}\right)^{\sigma_{K}-1}\right)^{\frac{\sigma_{K}-\phi}{\sigma_{K}-1} \frac{1}{\phi}}
$$

Using this equation together with the normalization $\omega_{M}+\sum \omega_{K}=1$ yields solutions for $\omega_{M}$ in equation (15) and $\omega_{K}$ in equation (16) in the text. Finally, we infer the permanent component of market productivity $\alpha$ by inverting equation (7) in the text that defines the marginal utility of market consumption. The transitory part of labor productivity is then given by $\varepsilon=\log z_{M}-\alpha$.

## C Variance of Log Consumption

In this section we prove the statement that the variance of log consumption in equation (??)) is decreasing in $\gamma_{L}$. To proof the statement, let $f\left(x ; \kappa, \gamma_{L}, \sigma_{L}\right)=\log \left(\kappa+\gamma_{L}^{\sigma_{L}-1} x\right)$, where we restrict $\kappa, \gamma_{L} \geq 0, \sigma_{L} \geq 1$, and $x \geq 0$. This implies that the derivative of $f$ is increasing in $x$, and increasingly so for larger values of $\gamma_{L}$,

$$
\begin{aligned}
f_{x}\left(x ; \kappa, \gamma_{L}, \sigma_{L}\right) & =\frac{1}{\kappa+\gamma_{L}^{\sigma_{L}-1} x} \gamma_{L}^{\sigma_{L}-1}=\left(\kappa \gamma_{L}^{1-\sigma_{L}}+x\right)^{-1} \geq 0 \\
f_{x \gamma_{L}}\left(x ; \kappa, \gamma_{L}, \sigma_{L}\right) & =-\left(\kappa \gamma_{L}^{1-\sigma_{L}}+x\right)^{-2} \kappa\left(1-\sigma_{L}\right) \gamma_{L}^{-\sigma_{L}} \geq 0
\end{aligned}
$$

The cross-derivative is the key component of the proof.
To establish the result, it is useful to write the variance as:

$$
\begin{aligned}
\operatorname{Var}\left(f\left(x ; \kappa, \gamma_{L}, \sigma_{L}\right)\right) & =\int\left(f\left(x ; \kappa, \gamma_{L}, \sigma_{L}\right)-\mathbb{E} f\left(x ; \kappa, \gamma_{L}, \sigma_{L}\right)\right)^{2} f(x) \mathrm{d} x \\
& =\int\left(\int_{\bar{x}\left(\gamma_{L}\right)}^{x} f_{x}\left(x ; \kappa, \gamma_{L}, \sigma_{L}\right)\right)^{2} f(x) \mathrm{d} x
\end{aligned}
$$

where $\bar{x}\left(\gamma_{L}\right)$ is such that $f\left(\bar{x}\left(\gamma_{L}\right) ; \kappa, \gamma_{L}, \sigma_{L}\right)=\mathbb{E} f\left(x ; \kappa, \gamma_{L}, \sigma_{L}\right)$.
Because $f_{x}$ is increasing in $\gamma_{L}$, we know that for any $\tilde{\gamma}_{L} \leq \gamma_{L}$.

$$
\int\left(\int_{\bar{x}\left(\gamma_{L}\right)}^{x} f_{x}\left(x ; \kappa, \gamma_{L}, \sigma_{L}\right)\right)^{2} f(x) \mathrm{d} x \geq \int\left(\int_{\bar{x}\left(\gamma_{L}\right)}^{x} f_{x}\left(x ; \kappa, \tilde{\gamma}_{L}, \sigma_{L}\right)\right)^{2} f(x) \mathrm{d} x
$$

Second, we know that the mean minimizes the variance, that is,

$$
\begin{equation*}
\mathbb{E} f\left(x ; \kappa, \gamma_{L}, \sigma_{L}\right)=\underset{\nu}{\arg \min } \int\left(f\left(x ; \kappa, \gamma_{L}, \sigma_{L}\right)-\nu\right)^{2} f(x) \mathrm{d} x \tag{A.32}
\end{equation*}
$$

We let $\bar{x}\left(\tilde{\gamma}_{L}\right)$ such that $f\left(\bar{x}\left(\tilde{\gamma}_{L}\right) ; \kappa, \tilde{\gamma}_{L}, \sigma_{L}\right)=\mathbb{E} f\left(x ; \kappa, \tilde{\gamma}_{L}, \sigma_{L}\right)$.
As a result, we know that

$$
\int\left(\int_{\bar{x}\left(\gamma_{L}\right)}^{x} f_{x}\left(x ; \kappa, \tilde{\gamma}_{L}, \sigma_{L}\right)\right)^{2} f(x) \mathrm{d} x \geq \int\left(\int_{\bar{x}\left(\tilde{\gamma}_{L}\right)}^{x} f_{x}\left(x ; \kappa, \tilde{\gamma}_{L}, \sigma_{L}\right)\right)^{2} f(x) \mathrm{d} x
$$

and hence that

$$
\begin{equation*}
\int\left(\int_{\bar{x}\left(\gamma_{L}\right)}^{x} f_{x}\left(x ; \kappa, \gamma_{L}, \sigma_{L}\right)\right)^{2} f(x) \mathrm{d} x \geq \int\left(\int_{\bar{x}\left(\tilde{\gamma}_{L}\right)}^{x} f_{x}\left(x ; \kappa, \tilde{\gamma}_{L}, \sigma_{L}\right)\right)^{2} f(x) \mathrm{d} x \tag{А.33}
\end{equation*}
$$

which is what we wanted to show.

## D Sensitivity Analyses

In this appendix we present the details underlying our sensitivity analyses. For every sensitivity analysis, we regenerate the sources of heterogeneity and then perform the counterfactual analyses.

- $\sigma_{N}=3.5$ : Table A. 1 assesses the contribution of driving forces, presented in rows, to the mean values of observables, presented in columns. Figure A. 1 presents the welfare effects of shutting off the evolution of driving forces.
- $\sigma_{N}=1.5$ : Table A. 2 assesses the contribution of driving forces, presented in rows, to the mean values of observables, presented in columns. Figure A. 2 presents the welfare effects of shutting off the evolution of driving forces.
- $\sigma_{L}=0.2$ : Table A. 3 assesses the contribution of driving forces, presented in rows, to the mean values of observables, presented in columns. Figure A. 3 presents the welfare effects of shutting off the evolution of driving forces.
- $\sigma_{L}=0.8$ : Table A. 4 assesses the contribution of driving forces, presented in rows, to the mean values of observables, presented in columns. Figure A. 4 presents the welfare effects of shutting off the evolution of driving forces.
- $\tau_{1}=0.06$ : Table A. 5 assesses the contribution of driving forces, presented in rows, to the mean values of observables, presented in columns. Figure A. 5 presents the welfare effects of shutting off the evolution of driving forces.
- $\tau_{1}=0.18$ : Table A. 6 assesses the contribution of driving forces, presented in rows, to the mean values of observables, presented in columns. Figure A. 6 presents the welfare effects of shutting off the evolution of driving forces.
- $\phi=0.5$ : Table A. 7 assesses the contribution of driving forces, presented in rows, to the mean values of observables, presented in columns. Figure A. 7 presents the welfare effects of shutting off the evolution of driving forces.
- $\phi=2.0$ : Table A. 8 assesses the contribution of driving forces, presented in rows, to the mean values of observables, presented in columns. Figure A. 8 presents the welfare effects of shutting off the evolution of driving forces.
- Adjusted consumption measures: Table A. 9 assesses the contribution of driving forces, presented in rows, to the mean values of observables, presented in columns. Figure A. 9 presents the welfare effects of shutting off the evolution of driving forces.
- Singles: Table A. 10 assesses the contribution of driving forces, presented in rows, to the mean values of observables, presented in columns. Figure A. 10 presents the welfare effects of shutting off the evolution of driving forces.
- Alternative measure of leisure: Table A. 11 assesses the contribution of driving forces, presented in rows, to the mean values of observables, presented in columns. Figure A. 11 presents the welfare effects of shutting off the evolution of driving forces.

Table A.1: Means: Counterfactuals $\left(\sigma_{N}=3.5\right)$

|  | $100 \times$ Difference between 2012-2016 and 1995-1999 |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Case | $\log (p x)$ | $\log x_{M}$ | $\log x_{N}$ | $\log x_{L}$ | $\log h_{M}$ | $\log h_{N}$ | $\log h_{L}$ |
| Baseline | 24.6 | 5.0 | 16.5 | 78.2 | -1.8 | -7.4 | 2.1 |
| Mean $p_{N}$ | 35.3 | 5.0 | 59.7 | 78.2 | 10.9 | -31.6 | 2.1 |
| Mean $p_{L}$ | 27.0 | 5.0 | 16.5 | 62.9 | 1.2 | -7.4 | 0.4 |
| Mean $\alpha+\varepsilon$ | -15.6 | -22.4 | -54.0 | 62.8 | -16.8 | 18.3 | 0.5 |
| Variance $\alpha$ | 26.7 | 6.5 | 19.5 | 79.0 | -0.5 | -9.4 | 2.2 |
| Variance $\varepsilon$ | 24.5 | 5.0 | 16.4 | 78.2 | -1.8 | -7.1 | 2.2 |
| Mean $\log z_{N}$ | 25.0 | 5.0 | 17.3 | 78.2 | -1.5 | -8.2 | 2.1 |
| Variance $\log z_{N}$ | 23.5 | 5.0 | 14.0 | 78.2 | -3.6 | -3.3 | 2.1 |
| Mean $\log z_{L}$ | 18.2 | 5.0 | 16.5 | 35.1 | -9.1 | -7.4 | 6.3 |
| Variance $\log z_{L}$ | 24.8 | 5.0 | 16.5 | 79.0 | -1.6 | -7.4 | 2.1 |
| Mean $\omega_{M}$ | 33.7 | 30.8 | 12.7 | 74.4 | 8.1 | -11.1 | -1.6 |
| Variance $\omega_{M}$ | 23.2 | 1.1 | 16.4 | 78.1 | -2.4 | -7.5 | 2.0 |
| Mean $\omega_{N}$ | 24.3 | 5.4 | 15.7 | 78.5 | -2.1 | -8.1 | 2.4 |
| Variance $\omega_{N}$ | 24.8 | 5.2 | 16.8 | 78.4 | -1.6 | -7.0 | 2.3 |
| Mean $\omega_{L}$ | 29.4 | 11.9 | 23.3 | 72.5 | 4.0 | -0.5 | -3.6 |
| Variance $\omega_{L}$ | 24.5 | 5.0 | 16.4 | 78.2 | -1.8 | -7.4 | 2.1 |

Table A.2: Means: Counterfactuals $\left(\sigma_{N}=1.5\right)$

|  | $100 \times$ Difference between 2012-2016 and 1995-1999 |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Case | $\log (p x)$ | $\log x_{M}$ | $\log x_{N}$ | $\log x_{L}$ | $\log h_{M}$ | $\log h_{N}$ | $\log h_{L}$ |
| Baseline | 24.6 | 5.1 | 16.5 | 78.3 | -1.8 | -7.4 | 2.1 |
| Mean $p_{N}$ | 26.8 | 5.1 | 41.0 | 78.3 | 1.1 | -11.8 | 2.1 |
| Mean $p_{L}$ | 27.0 | 5.1 | 16.5 | 62.9 | 1.3 | -7.4 | 0.4 |
| Mean $\alpha+\varepsilon$ | -3.6 | -22.4 | -18.7 | 62.9 | -2.3 | -1.4 | 0.5 |
| Variance $\alpha$ | 26.1 | 6.5 | 18.3 | 79.1 | -1.7 | -7.8 | 2.2 |
| Variance $\varepsilon$ | 24.6 | 5.1 | 16.5 | 78.3 | -1.6 | -7.3 | 2.2 |
| Mean $\log z_{N}$ | 21.3 | 5.1 | 8.3 | 78.3 | -5.6 | -0.9 | 2.1 |
| Variance $\log z_{N}$ | 21.7 | 5.1 | 9.7 | 78.3 | -5.7 | 0.8 | 2.1 |
| Mean $\log z_{L}$ | 18.3 | 5.1 | 16.5 | 35.2 | -9.0 | -7.4 | 6.3 |
| Variance $\log z_{L}$ | 24.8 | 5.1 | 16.5 | 79.0 | -1.5 | -7.4 | 2.0 |
| Mean $\omega_{M}$ | 33.7 | 30.8 | 12.8 | 74.5 | 8.1 | -11.1 | -1.6 |
| Variance $\omega_{M}$ | 23.2 | 1.2 | 16.4 | 78.2 | -2.4 | -7.5 | 2.0 |
| Mean $\omega_{N}$ | 24.4 | 5.4 | 15.8 | 78.6 | -2.0 | -8.2 | 2.4 |
| Variance $\omega_{N}$ | 24.9 | 5.3 | 16.9 | 78.4 | -1.6 | -7.1 | 2.3 |
| Mean $\omega_{L}$ | 29.5 | 12.0 | 23.4 | 72.5 | 4.0 | -0.5 | -3.6 |
| Variance $\omega_{L}$ | 24.6 | 5.0 | 16.5 | 78.2 | -1.7 | -7.5 | 2.1 |

Table A.3: Means: Counterfactuals ( $\sigma_{L}=0.2$ )

|  | $100 \times$ Difference between 2012-2016 and 1995-1999 |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Case | $\log (p x)$ | $\log x_{M}$ | $\log x_{N}$ | $\log x_{L}$ | $\log h_{M}$ | $\log h_{N}$ | $\log h_{L}$ |
| Baseline | 24.6 | 5.1 | 16.5 | 78.2 | -1.8 | -7.4 | 2.1 |
| Mean $p_{N}$ | 31.1 | 5.1 | 50.8 | 78.2 | 6.2 | -21.3 | 2.1 |
| Mean $p_{L}$ | 28.6 | 5.1 | 16.5 | 69.9 | 3.1 | -7.4 | -0.8 |
| Mean $\alpha+\varepsilon$ | -8.0 | -22.4 | -35.5 | 70.0 | -7.4 | 9.3 | -0.7 |
| Variance $\alpha$ | 26.3 | 6.5 | 18.9 | 78.7 | -1.4 | -8.6 | 2.3 |
| Variance $\varepsilon$ | 24.6 | 5.1 | 16.4 | 78.3 | -1.7 | -7.2 | 2.1 |
| Mean $\log z_{N}$ | 23.2 | 5.1 | 12.9 | 78.2 | -3.8 | -4.4 | 2.1 |
| Variance $\log z_{N}$ | 23.3 | 5.1 | 13.5 | 78.2 | -3.5 | -2.7 | 2.1 |
| Mean $\log z_{L}$ | 16.6 | 5.1 | 16.5 | 19.7 | -11.0 | -7.4 | 7.3 |
| Variance $\log z_{L}$ | 24.9 | 5.1 | 16.5 | 79.6 | -1.4 | -7.4 | 2.0 |
| Mean $\omega_{M}$ | 33.7 | 30.8 | 12.8 | 74.5 | 8.1 | -11.1 | -1.6 |
| Variance $\omega_{M}$ | 23.2 | 1.1 | 16.4 | 78.2 | -2.4 | -7.5 | 2.0 |
| Mean $\omega_{N}$ | 24.4 | 5.4 | 15.7 | 78.6 | -2.0 | -8.1 | 2.4 |
| Variance $\omega_{N}$ | 24.8 | 5.3 | 16.8 | 78.4 | -1.6 | -7.0 | 2.3 |
| Mean $\omega_{L}$ | 29.5 | 12.0 | 23.4 | 72.5 | 4.0 | -0.5 | -3.6 |
| Variance $\omega_{L}$ | 24.6 | 5.0 | 16.4 | 78.2 | -1.7 | -7.4 | 2.1 |

Table A.4: Means: Counterfactuals ( $\sigma_{L}=0.8$ )

|  | $100 \times$ Difference between 2012-2016 and 1995-1999 |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Case | $\log (p x)$ | $\log x_{M}$ | $\log x_{N}$ | $\log x_{L}$ | $\log h_{M}$ | $\log h_{N}$ | $\log h_{L}$ |
| Baseline | 24.6 | 5.1 | 16.5 | 78.2 | -1.8 | -7.4 | 2.1 |
| Mean $p_{N}$ | 31.1 | 5.1 | 50.8 | 78.2 | 6.1 | -21.3 | 2.1 |
| Mean $p_{L}$ | 25.5 | 5.1 | 16.5 | 55.8 | -0.6 | -7.4 | 1.5 |
| Mean $\alpha+\varepsilon$ | -11.3 | -22.4 | -35.5 | 55.6 | -11.2 | 9.3 | 1.5 |
| Variance $\alpha$ | 26.5 | 6.5 | 18.9 | 79.4 | -1.3 | -8.6 | 2.1 |
| Variance $\varepsilon$ | 24.6 | 5.0 | 16.4 | 78.2 | -1.7 | -7.2 | 2.2 |
| Mean $\log z_{N}$ | 23.2 | 5.1 | 12.9 | 78.2 | -3.8 | -4.4 | 2.1 |
| Variance $\log z_{N}$ | 23.3 | 5.1 | 13.5 | 78.2 | -3.5 | -2.7 | 2.1 |
| Mean $\log z_{L}$ | 20.2 | 5.1 | 16.5 | 50.3 | -6.5 | -7.4 | 5.1 |
| Variance $\log z_{L}$ | 24.7 | 5.1 | 16.5 | 78.1 | -1.7 | -7.4 | 2.1 |
| Mean $\omega_{M}$ | 33.7 | 30.8 | 12.8 | 74.5 | 8.1 | -11.1 | -1.6 |
| Variance $\omega_{M}$ | 23.2 | 1.1 | 16.4 | 78.1 | -2.4 | -7.5 | 2.0 |
| Mean $\omega_{N}$ | 24.4 | 5.4 | 15.7 | 78.5 | -2.1 | -8.1 | 2.4 |
| Variance $\omega_{N}$ | 24.8 | 5.2 | 16.9 | 78.4 | -1.6 | -7.0 | 2.3 |
| Mean $\omega_{L}$ | 29.5 | 11.9 | 23.4 | 72.5 | 4.0 | -0.5 | -3.6 |
| Variance $\omega_{L}$ | 24.5 | 5.0 | 16.4 | 78.2 | -1.7 | -7.4 | 2.1 |

Table A.5: Means: Counterfactuals $\left(\tau_{1}=0.06\right)$

|  | $100 \times$ Difference between 2012-2016 and 1995-1999 |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Case | $\log (p x)$ | $\log x_{M}$ | $\log x_{N}$ | $\log x_{L}$ | $\log h_{M}$ | $\log h_{N}$ | $\log h_{L}$ |
| Baseline | 26.7 | 7.2 | 18.6 | 80.4 | -1.8 | -7.4 | 2.1 |
| Mean $p_{N}$ | 33.2 | 7.2 | 52.8 | 80.4 | 6.2 | -21.4 | 2.1 |
| Mean $p_{L}$ | 29.1 | 7.2 | 18.6 | 65.0 | 1.3 | -7.4 | 0.4 |
| Mean $\alpha+\varepsilon$ | -9.8 | -22.2 | -36.9 | 63.9 | -10.1 | 10.5 | 0.3 |
| Variance $\alpha$ | 28.6 | 8.7 | 21.1 | 81.2 | -1.3 | -8.7 | 2.2 |
| Variance $\varepsilon$ | 26.7 | 7.2 | 18.5 | 80.4 | -1.9 | -7.2 | 2.2 |
| Mean $\log z_{N}$ | 25.9 | 7.2 | 16.5 | 80.4 | -3.0 | -5.6 | 2.1 |
| Variance $\log z_{N}$ | 25.3 | 7.2 | 15.4 | 80.4 | -3.9 | -2.4 | 2.1 |
| Mean $\log z_{L}$ | 20.2 | 7.2 | 18.6 | 36.3 | -9.5 | -7.4 | 6.5 |
| Variance $\log z_{L}$ | 26.9 | 7.2 | 18.6 | 81.0 | -1.6 | -7.4 | 2.1 |
| Mean $\omega_{M}$ | 35.8 | 32.8 | 14.9 | 76.6 | 8.2 | -11.1 | -1.6 |
| Variance $\omega_{M}$ | 25.3 | 2.9 | 18.5 | 80.3 | -2.4 | -7.5 | 2.0 |
| Mean $\omega_{N}$ | 26.5 | 7.6 | 17.8 | 80.7 | -2.1 | -8.2 | 2.5 |
| Variance $\omega_{N}$ | 27.0 | 7.4 | 19.0 | 80.5 | -1.6 | -7.0 | 2.3 |
| Mean $\omega_{L}$ | 31.5 | 14.0 | 25.4 | 74.6 | 4.1 | -0.6 | -3.6 |
| Variance $\omega_{L}$ | 26.7 | 7.1 | 18.6 | 80.3 | -1.7 | -7.4 | 2.1 |

Table A.6: Means: Counterfactuals $\left(\tau_{1}=0.18\right)$

|  | $100 \times$ Difference between 2012-2016 and 1995-1999 |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Case | $\log (p x)$ | $\log x_{M}$ | $\log x_{N}$ | $\log x_{L}$ | $\log h_{M}$ | $\log h_{N}$ | $\log h_{L}$ |
| Baseline | 22.4 | 2.9 | 14.3 | 76.1 | -1.8 | -7.3 | 2.1 |
| Mean $p_{N}$ | 29.0 | 2.9 | 48.7 | 76.1 | 6.1 | -21.2 | 2.1 |
| Mean $p_{L}$ | 24.8 | 2.9 | 14.3 | 60.7 | 1.2 | -7.3 | 0.4 |
| Mean $\alpha+\varepsilon$ | -9.5 | -22.6 | -33.9 | 61.8 | -8.7 | 8.1 | 0.6 |
| Variance $\alpha$ | 24.1 | 4.2 | 16.5 | 76.8 | -1.4 | -8.4 | 2.2 |
| Variance $\varepsilon$ | 22.4 | 2.9 | 14.3 | 76.1 | -1.8 | -7.2 | 2.1 |
| Mean $\log z_{N}$ | 20.3 | 2.9 | 9.1 | 76.1 | -4.3 | -3.2 | 2.1 |
| Variance $\log z_{N}$ | 21.2 | 2.9 | 11.6 | 76.1 | -3.4 | -2.9 | 2.1 |
| Mean $\log z_{L}$ | 16.2 | 2.9 | 14.3 | 34.0 | -8.5 | -7.3 | 6.1 |
| Variance $\log z_{L}$ | 22.6 | 2.9 | 14.3 | 77.0 | -1.5 | -7.3 | 2.0 |
| Mean $\omega_{M}$ | 31.5 | 28.7 | 10.6 | 72.3 | 8.1 | -11.1 | -1.6 |
| Variance $\omega_{M}$ | 21.1 | -0.9 | 14.2 | 76.0 | -2.5 | -7.4 | 2.0 |
| Mean $\omega_{N}$ | 22.2 | 3.2 | 13.6 | 76.3 | -2.1 | -8.0 | 2.4 |
| Variance $\omega_{N}$ | 22.6 | 3.1 | 14.6 | 76.2 | -1.6 | -7.0 | 2.3 |
| Mean $\omega_{L}$ | 27.3 | 9.8 | 21.2 | 70.3 | 4.1 | -0.4 | -3.6 |
| Variance $\omega_{L}$ | 22.4 | 2.8 | 14.2 | 76.0 | -1.8 | -7.4 | 2.1 |

Table A.7: Means: Counterfactuals $(\phi=0.5)$

|  | $100 \times$ Difference between 2012-2016 and 1995-1999 |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Case | $\log (p x)$ | $\log x_{M}$ | $\log x_{N}$ | $\log x_{L}$ | $\log h_{M}$ | $\log h_{N}$ | $\log h_{L}$ |
| Baseline | 24.6 | 5.1 | 16.5 | 78.2 | -1.8 | -7.4 | 2.1 |
| Mean $p_{N}$ | 30.4 | 6.4 | 47.5 | 79.6 | 5.3 | -24.6 | 3.5 |
| Mean $p_{L}$ | 26.5 | 4.2 | 15.6 | 63.7 | 0.6 | -8.3 | 1.2 |
| Mean $\alpha+\varepsilon$ | -5.7 | -13.0 | -34.3 | 60.1 | -5.2 | 10.5 | -2.2 |
| Variance $\alpha$ | 26.2 | 6.0 | 18.7 | 79.2 | -1.6 | -8.7 | 2.4 |
| Variance $\varepsilon$ | 24.6 | 5.0 | 16.4 | 78.2 | -1.7 | -7.2 | 2.2 |
| Mean $\log z_{N}$ | 23.1 | 5.4 | 12.1 | 78.6 | -4.0 | -5.3 | 2.5 |
| Variance $\log z_{N}$ | 23.2 | 5.4 | 12.8 | 78.5 | -3.6 | -3.4 | 2.4 |
| Mean $\log z_{L}$ | -2.3 | -21.9 | -10.4 | 51.3 | -32.2 | -34.3 | 22.5 |
| Variance $\log z_{L}$ | 25.1 | 5.6 | 17.0 | 78.7 | -1.3 | -6.9 | 1.8 |
| Mean $\omega_{M}$ | 29.3 | 21.1 | 12.9 | 74.5 | 2.5 | -6.9 | -0.3 |
| Variance $\omega_{M}$ | 23.9 | 2.9 | 17.3 | 79.1 | -2.6 | -7.7 | 2.4 |
| Mean $\omega_{N}$ | 24.6 | 5.6 | 15.5 | 79.0 | -1.5 | -8.0 | 2.4 |
| Variance $\omega_{N}$ | 23.8 | 6.2 | 13.3 | 79.4 | -2.5 | -9.6 | 3.9 |
| Mean $\omega_{L}$ | 60.7 | 47.7 | 58.2 | 71.5 | 35.6 | -11.6 | -27.5 |
| Variance $\omega_{L}$ | 19.3 | 2.7 | 13.6 | 63.6 | -2.0 | 9.1 | -1.8 |

Table A.8: Means: Counterfactuals $(\phi=2.0)$

|  | $100 \times$ Difference between 2012-2016 and 1995-1999 |  |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Case | $\log (p x)$ | $\log x_{M}$ | $\log x_{N}$ | $\log x_{L}$ | $\log h_{M}$ | $\log h_{N}$ | $\log h_{L}$ |  |
| Baseline | 24.2 | 4.6 | 16.2 | 77.6 | -1.9 | -7.3 | 2.1 |  |
| Mean $p_{N}$ | 32.2 | 1.7 | 56.8 | 74.7 | 7.8 | -14.9 | -0.8 |  |
| Mean $p_{L}$ | 27.6 | 6.4 | 17.9 | 60.7 | 2.5 | -5.5 | -1.2 |  |
| Mean $\alpha+\varepsilon$ | -17.9 | -42.1 | -38.8 | 67.1 | -18.2 | 6.4 | 5.4 |  |
| Variance $\alpha$ | 26.4 | 7.0 | 18.8 | 78.1 | -0.8 | -8.2 | 1.9 |  |
| Variance $\varepsilon$ | 24.2 | 4.6 | 16.1 | 77.7 | -1.9 | -7.2 | 2.2 |  |
| Mean $\log z_{N}$ | 23.0 | 3.9 | 14.2 | 76.9 | -3.6 | -2.7 | 1.4 |  |
| Variance $\log z_{N}$ | 23.1 | 4.0 | 14.5 | 77.0 | -3.4 | -1.3 | 1.5 |  |
| Mean $\log z_{L}$ | 48.6 | 41.7 | 53.2 | -14.6 | 24.1 | 29.8 | -42.8 |  |
| Variance $\log z_{L}$ | 23.7 | 3.6 | 15.1 | 78.9 | -1.7 | -8.4 | 2.6 |  |
| Mean $\omega_{M}$ | 19.8 | -34.4 | 17.8 | 79.3 | -6.9 | -5.6 | 3.8 |  |
| Variance $\omega_{M}$ | 23.3 | 4.8 | 16.7 | 78.2 | -2.9 | -6.8 | 2.7 |  |
| Mean $\omega_{N}$ | 17.4 | 18.5 | -81.9 | 91.5 | -9.0 | -105.3 | 16.0 |  |
| Variance $\omega_{N}$ | 27.5 | -1.2 | 22.9 | 71.8 | 0.8 | -0.6 | -3.7 |  |
| Mean $\omega_{L}$ | -3.0 | -59.8 | -48.3 | 94.0 | -27.4 | -71.7 | 18.5 |  |
| Variance $\omega_{L}$ | 27.2 | 8.1 | 19.6 | 74.6 | -0.5 | -3.9 | -0.9 |  |

Table A.9: Means: Counterfactuals (Adjusted Consumption Measures)

|  | $100 \times$ Difference between 2012-2016 and 1995-1999 |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Case | $\log (p x)$ | $\log x_{M}$ | $\log x_{N}$ | $\log x_{L}$ | $\log h_{M}$ | $\log h_{N}$ | $\log h_{L}$ |
| Baseline | 24.1 | 6.1 | 15.1 | 78.3 | -1.8 | -7.4 | 2.1 |
| Mean $p_{N}$ | 30.6 | 6.1 | 49.3 | 78.3 | 6.2 | -21.4 | 2.1 |
| Mean $p_{L}$ | 26.4 | 6.1 | 15.1 | 63.0 | 1.2 | -7.4 | 0.4 |
| Mean $\alpha+\varepsilon$ | -10.3 | -21.4 | -36.8 | 62.9 | -9.3 | 9.4 | 0.5 |
| Variance $\alpha$ | 26.1 | 7.7 | 17.8 | 79.2 | -1.2 | -8.7 | 2.2 |
| Variance $\varepsilon$ | 24.0 | 6.0 | 15.1 | 78.3 | -1.7 | -7.2 | 2.1 |
| Mean $\log z_{N}$ | 23.0 | 6.1 | 12.5 | 78.3 | -3.2 | -5.1 | 2.1 |
| Variance $\log z_{N}$ | 22.7 | 6.1 | 12.0 | 78.3 | -3.6 | -2.5 | 2.1 |
| Mean $\log z_{L}$ | 18.0 | 6.1 | 15.1 | 35.1 | -8.7 | -7.4 | 6.3 |
| Variance $\log z_{L}$ | 24.4 | 6.1 | 15.1 | 79.9 | -1.4 | -7.4 | 2.0 |
| Mean $\omega_{M}$ | 32.8 | 29.8 | 11.4 | 74.6 | 8.0 | -11.0 | -1.6 |
| Variance $\omega_{M}$ | 22.3 | 1.9 | 15.0 | 78.2 | -2.1 | -7.4 | 2.0 |
| Mean $\omega_{N}$ | 23.9 | 6.2 | 14.8 | 78.4 | -2.0 | -7.6 | 2.2 |
| Variance $\omega_{N}$ | 24.3 | 6.2 | 15.3 | 78.5 | -1.5 | -7.1 | 2.3 |
| Mean $\omega_{L}$ | 29.1 | 13.0 | 22.1 | 72.4 | 4.3 | -0.4 | -3.8 |
| Variance $\omega_{L}$ | 24.0 | 6.0 | 15.0 | 78.2 | -1.8 | -7.5 | 2.1 |

Table A.10: Means: Counterfactuals (Singles)

|  | $100 \times$ Difference between 2012-2016 and 1995-1999 |  |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Case | $\log (p x)$ | $\log x_{M}$ | $\log x_{N}$ | $\log x_{L}$ | $\log h_{M}$ | $\log h_{N}$ | $\log h_{L}$ |  |
| Baseline | 17.6 | -1.3 | 9.7 | 66.8 | -3.0 | -18.2 | 8.2 |  |
| Mean $p_{N}$ | 22.2 | -1.3 | 39.7 | 66.8 | 3.2 | -36.5 | 8.2 |  |
| Mean $p_{L}$ | 20.0 | -1.3 | 9.7 | 51.3 | 0.4 | -18.2 | 6.4 |  |
| Mean $\alpha+\varepsilon$ | -10.8 | -25.7 | -30.6 | 52.9 | -7.3 | 2.4 | 6.6 |  |
| Variance $\alpha$ | 20.5 | 1.2 | 13.4 | 68.2 | -2.5 | -20.7 | 8.3 |  |
| Variance $\varepsilon$ | 17.7 | -1.3 | 9.7 | 66.8 | -4.4 | -17.9 | 8.3 |  |
| Mean $\log z_{N}$ | 14.9 | -1.3 | 3.1 | 66.8 | -6.1 | -8.6 | 8.2 |  |
| Variance $\log z_{N}$ | 17.5 | -1.3 | 9.5 | 66.8 | -3.3 | -17.3 | 8.2 |  |
| Mean $\log z_{L}$ | 13.1 | -1.3 | 9.7 | 37.5 | -8.2 | -18.2 | 11.5 |  |
| Variance $\log z_{L}$ | 14.2 | -1.3 | 9.7 | 45.6 | -7.6 | -18.2 | 12.8 |  |
| Mean $\omega_{M}$ | 27.2 | 26.2 | 5.2 | 62.2 | 7.1 | -22.7 | 3.7 |  |
| Variance $\omega_{M}$ | 16.0 | -5.8 | 9.6 | 66.7 | -2.4 | -18.3 | 8.1 |  |
| Mean $\omega_{N}$ | 18.9 | -3.2 | 15.5 | 64.9 | -1.5 | -12.4 | 6.3 |  |
| Variance $\omega_{N}$ | 16.9 | -1.6 | 8.3 | 66.6 | -3.9 | -19.6 | 8.0 |  |
| Mean $\omega_{L}$ | 27.1 | 11.8 | 22.9 | 57.0 | 8.6 | -5.1 | -1.4 |  |
| Variance $\omega_{L}$ | 16.5 | -2.5 | 8.6 | 65.8 | -3.8 | -19.3 | 7.2 |  |

Table A.11: Means: Counterfactuals (Alternative Measure of Leisure)

|  | $100 \times$ Difference between 2012-2016 and $1995-1999$ |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Case | $\log (p x)$ | $\log x_{M}$ | $\log x_{N}$ | $\log x_{L}$ | $\log h_{M}$ | $\log h_{N}$ | $\log h_{L}$ |
| Baseline | 24.5 | 4.9 | 16.4 | 78.1 | -1.8 | 12.0 | -4.0 |
| Mean $p_{N}$ | 32.6 | 4.9 | 54.2 | 78.1 | 8.0 | 1.6 | -4.0 |
| Mean $p_{L}$ | 26.8 | 4.9 | 16.4 | 62.3 | 1.2 | 12.0 | -6.2 |
| Mean $\alpha+\varepsilon$ | -11.6 | -22.5 | -40.5 | 62.3 | -11.4 | 23.8 | -6.1 |
| Variance $\alpha$ | 26.4 | 6.4 | 19.1 | 78.9 | -1.1 | 11.1 | -3.9 |
| Variance $\varepsilon$ | 24.4 | 4.9 | 16.3 | 78.1 | -1.7 | 12.1 | -4.0 |
| Mean $\log z_{N}$ | 28.5 | 4.9 | 25.5 | 78.1 | 2.9 | 7.2 | -4.0 |
| Variance $\log z_{N}$ | 27.9 | 4.9 | 23.6 | 78.1 | 3.4 | 5.0 | -4.0 |
| Mean $\log z_{L}$ | 17.6 | 4.9 | 16.4 | 30.8 | -9.5 | 12.0 | 1.8 |
| Variance $\log z_{L}$ | 29.8 | 4.9 | 16.4 | 97.6 | 4.2 | 12.0 | -9.5 |
| Mean $\omega_{M}$ | 33.6 | 30.6 | 12.7 | 74.3 | 8.1 | 8.3 | -7.7 |
| Variance $\omega_{M}$ | 23.1 | 1.1 | 16.3 | 78.0 | -2.4 | 11.9 | -4.1 |
| Mean $\omega_{N}$ | 24.5 | 9.2 | 10.1 | 82.4 | -1.7 | 5.7 | 0.3 |
| Variance $\omega_{N}$ | 24.4 | 4.9 | 16.2 | 78.0 | -1.8 | 11.8 | -4.1 |
| Mean $\omega_{L}$ | 25.4 | 6.3 | 17.7 | 76.4 | -0.6 | 13.3 | -5.7 |
| Variance $\omega_{L}$ | 24.5 | 4.9 | 16.4 | 78.3 | -1.7 | 12.0 | -3.8 |



Figure A.1: Welfare: Counterfactuals $\left(\sigma_{N}=3.5\right)$


Figure A.2: Welfare: Counterfactuals $\left(\sigma_{N}=1.5\right)$


Figure A.3: Welfare: Counterfactuals $\left(\sigma_{L}=0.2\right)$


Figure A.4: Welfare: Counterfactuals $\left(\sigma_{L}=0.8\right)$


Figure A.5: Welfare: Counterfactuals $\left(\tau_{1}=0.06\right)$


Figure A.6: Welfare: Counterfactuals $\left(\tau_{1}=0.18\right)$


Figure A.7: Welfare: Counterfactuals $(\phi=0.5)$


Figure A.8: Welfare: Counterfactuals $(\phi=2.0)$








Figure A.9: Welfare: Counterfactuals (Adjusted Consumption Measures)


Figure A.10: Welfare: Counterfactuals (Singles)


Figure A.11: Welfare: Counterfactuals (Alternative Measure of Leisure)


[^0]:    *The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

[^1]:    ${ }^{1} \mathrm{~A}$ large literature has documented the rise of the dispersion of wages, expenditures, and time allocation across households. For example, Autor, Katz, and Kearney (2008), Heathcote, Perri, and Violante (2010), and Attanasio, Hurst, and Pistaferri (2015) discuss several empirical facts underlying the evolution of heterogeneity in labor market outcomes.

[^2]:    ${ }^{2}$ For example, Krueger and Perri (2006) and Aguiar and Bils (2015) measure the evolution of dispersion of expenditures over time, Aguiar and Hurst (2007b) document the rise of leisure inequality between the 1965 and the early 2000s, and Attanasio, Hurst, and Pistaferri (2015) and Han, Meyer, and Sullivan (2018) provide statistics of the evolution of dispersion of expenditures jointly with dispersion of time use. In their study of increasing inequality, Krueger and Perri (2003) conduct welfare experiments by essentially varying allocations that enter directly into the utility function. The difference with our approach is that we develop an equilibrium model that solves for arguments of the utility function as a function of more primitive productivity shifters, preference shifters, and policy parameters and, therefore, our counterfactuals account for equilibrium responses when conducting welfare analyses.

[^3]:    ${ }^{3}$ In Boerma and Karabarbounis (2019) we also extended the framework of Heathcote, Storesletten, and Violante (2014) to a model with home production. The difference is that here we use the Beckerian framework in which expenditures and time are inputs in the production of goods that enter directly into utility whereas in Boerma and Karabarbounis (2019) time spent working at the market and at home enters directly into utility as in Gronau (1986). While the two versions of the household production model share many predictions, in this paper we prefer the former because it allows us to model more directly changes in the price of leisure expenditures and returns to leisure time.

[^4]:    ${ }^{4}$ The framework accommodates implicit insurance against $\alpha$ differences because households can substitute expenditures and time across sectors. Apart from explicit asset markets, some examples of mechanisms that insure $\varepsilon$ shocks include family and government transfers.

[^5]:    ${ }^{5}$ We allow relative prices $p_{K, t}$ to vary over time. For the no-trade theorem, we do not need to impose restrictions on the stochastic processes of $A_{K, t}$. Henceforth, we treat the prices $p_{K, t}$ as exogenous with the understanding that there is a unique mapping from sectoral productivity to prices $p_{K, t}=A_{K, t}^{-1}$ that we could use to rationalize any path of prices we observe in the data. We also note that productivity changes in the market sector are implicitly subsumed into a common time component of $z_{M, t}$ across households.

[^6]:    ${ }^{6}$ For this result we note the importance of log preferences with respect to the consumption aggregator. Log preferences generate a separability between the marginal utility of market consumption and $z_{K}$ and $\omega_{K}$ and, thus, the no-trade result holds irrespective of the value of the elasticity of substitution across sectors $\phi$, the elasticity of substitution within sector $\sigma_{K}$, and further stochastic properties of $z_{K}$ and $\omega_{K}$. In Boerma and Karabarbounis (2019) we show that the no-trade result holds in the Gronau (1986) version of the home production model when the disutility of total hours enter additively into the utility function and sectoral hours are perfect substitutes in the disutility.

[^7]:    ${ }^{7}$ The elasticity of $\frac{x_{K}}{x_{M}}$ with respect to $p_{K}$ is a weighted average of the two substitution elasticities and equals $-\frac{1}{1+\left(\frac{z_{K} p_{K}}{\frac{\varepsilon_{M}}{M}}\right)^{\sigma_{K}-1}} \phi-\frac{\left(\frac{z_{K} p_{K}}{\varepsilon_{M}}\right)^{\sigma_{K}-1}}{1+\left(\frac{z_{K} p_{K}}{\varepsilon_{M}}\right)^{\sigma_{K}-1}} \sigma_{K}$.

[^8]:    ${ }^{8}$ For households that report spending on vehicles or furniture, we regress their durables spending on a quadratic in household expenditures (excluding vehicles and furniture), income, age, sex, and education of the household head. The imputed expenditure of vehicles or furniture is the predicted value of spending from this regression multiplied by the user cost of each durable (for vehicles we also multiply by the number of vehicles owned).
    ${ }^{9}$ Market expenditures $x_{M}$ include clothing and footwear, utilities and fuels, health, vehicles, public transport, motor vehicle operations, education, insurance, tobacco, and professional services. Non-market expenditures $p_{N} x_{N}$ include food and beverages (home and away), household services, and personal care. Leisure expenditures $p_{L} x_{L}$ include communication, entertainment, reading, and personal items. Housing, furniture and household equipment are allocated proportional to the expenditure shares of the three types of goods.
    ${ }^{10}$ Table 2.5 .4 provides the price indices and Table 2.5 .5 gives the corresponding aggregate spending levels. To illustrate our approach, we use the price index for communication in Table 2.5.4 as the price for the CEX category communication. In constructing the price index for leisure goods, we weight the price index for communication by aggregate spending on communication as documented in Table 2.5.5.
    ${ }^{11}$ To calculate the user cost for durable consumption goods we use the price index for the spending category from Table 2.5, the interest rate on the five-year constant maturity Treasury for the cost of capital, and NIPA fixed assets accounts to construct the depreciation rate. We calculate the depreciation rate as current-cost depreciation over the current-cost net stock plus half of investments using Tables 8.1, 8.4, and 8.7.

[^9]:    ${ }^{12}$ We define leisure residually to ensure that all households have the same endowment of time. The cross-sectional correlation between this definition of leisure and the direct ATUS measure of leisure defined in Aguiar, Hurst, and Karabarbounis (2013) is 0.5 . Given the imperfect correlation, in our sensitivity analyses we reverse the definitions by using leisure directly from the ATUS and defining non-market hours residually as $h_{N}=105-h_{M}-h_{L}$. As reported in Section 6, our inferences on the role of leisure productivity and counterfactuals are not sensitive to this alternative measurement of time uses.

[^10]:    ${ }^{13}$ This result is sensitive to identifying leisure time as the residual time given observed market and non-market hours. The direct measure of leisure from the ATUS (that includes activities such as television watching, socializing, exercising, playing sports, reading, computer time, and listening to music) displays an increasing dispersion over time. However, as reported in Section 6, our inferences on the role of leisure productivity and counterfactuals are not sensitive to this discrepancy.

[^11]:    ${ }^{14}$ The inferred increase in mean leisure productivity becomes larger as $\sigma_{L}$ increases toward one. For $\sigma_{L}=0, z_{L}$ equals $x_{L} / h_{L}$ and grows by roughly $90 \log$ points.

[^12]:    ${ }^{15}$ In equation (17), $\left(1-\tau_{1}\right) \alpha$ equals the difference between the market value of total consumption and a moment of the transitory component of productivity $\exp (\varepsilon)$ and, in equation (18), $\varepsilon$ equals the difference between market productivity $\log z_{M}$ and $\alpha$. As a result, the plotted means depend on an arbitrary choice of means in some initial period. We choose to attribute half of the level of $\log z_{M}$ to $\alpha$ and half to $\varepsilon$ and, so, both rise by roughly the same amount over time. Our inferences of the other sources of heterogeneity, welfare effects, and our counterfactuals are not sensitive to this normalization.
    ${ }^{16}$ A similar finding is documented by Boerma and Karabarbounis (2019) in the context of a model with a non-market technology only.

[^13]:    ${ }^{17}$ We have confirmed that all our conclusions are similar if we consider a consumption equivalent that leaves an unborn household indifferent over its life-cycle between two allocations. The difference between the two welfare measures is that the life-cycle measure discounts future utilities more than the utilitarian measure.

[^14]:    ${ }^{18}$ We drop from the sample an additional 0.1 percent of observations ( 34 observations) with extreme levels of consumption $c_{t}$ to improve the visibility of this figure. The mean of $\log c_{t}$ deviates slightly from the level effect $\chi_{t}^{L}$ since the mean of $\log c_{t}$ reflects within-age variation over time, whereas the level effect does not correct for differences in the age structure over time. We also observe a close (negative) association between the dispersion effect $\chi_{t}^{D}$ and the variance of $\log$ consumption. If $\log$ consumption follows a normal distribution, then $\chi_{t}^{D}$ and the variance of $\log$ consumption are related by $\log \left(1-\chi_{t}^{D}\right)=-\operatorname{Var}\left(\log c_{t}\right) / 2$. While $\log$ consumption is not exactly normally distributed in our economy, this equation still provides a useful approximation in thinking about the dispersion effect.
    ${ }^{19}$ Let $x_{t}(\iota)$ be the $\log$ of a source of heterogeneity in the baseline and $x_{t}^{c}(\iota)$ be the counterfactual which keeps either the mean or the variance of the source of heterogeneity constant at its 1995 value. When we shut off the evolution of the mean of a source of heterogeneity, we set $x_{t}^{c}(\iota)=x_{t}(\iota)-\mathbb{E} x_{t}(\iota)+\mathbb{E} x_{95}(\iota)$, so that in all periods we retain the same dispersion across households as in the baseline. When we shut off the evolution of the variance of a source of heterogeneity, we set $x_{t}^{c}(\iota)=\lambda_{t}^{0}+\lambda_{t}^{1} x_{t}(\iota)$ and solve for $\lambda_{t}^{0}$ and $\lambda_{t}^{1}$ such that in all periods the variance equals its 1995 value and in all periods we retain the same mean across households as in the baseline. In conducting counterfactuals with a particular preference weight, we adjust the other weights such that the weights sum up to one. For example, when we shut off the decline in the mean value of $\omega_{M}$, we allocate proportionally to $\omega_{N}$ and $\omega_{L}$ the difference relative to the baseline path of $\omega_{M}$.

[^15]:    ${ }^{20}$ As can be seen from equations (9) and (10), with unitary elasticity $\phi=1$ across goods cross-price effects are zero and $z_{K}$ and $p_{K}$ do not affect $x_{-K}$ and $h_{-K}$.

[^16]:    ${ }^{21}$ Vandenbroucke (2009) emphasizes the small effects of the decline in leisure prices between 1900 and 1950 on market hours.

[^17]:    ${ }^{22} \mathrm{To}$ gain some insight of why this is true, consider a function $f\left(x ; \gamma_{L}\right)=\log \left(1+\gamma_{L}^{\sigma_{L}-1} x\right)$. To a first-order approximation, we obtain $\operatorname{Var}\left(f\left(x ; \gamma_{L}\right)\right)=\left(\frac{\gamma_{L}^{\sigma_{L}{ }^{-1}} \bar{x}}{1+\gamma_{L}^{\sigma_{L}-1} \bar{x}}\right) \operatorname{Var}(x / \bar{x})$, where $\bar{x}$ is an approximation point. This formula shows that the variance is decreasing in $\gamma_{L}$ if and only if $\sigma_{L}<1$. Our proof in the appendix does not rely on approximations.

[^18]:    ${ }^{23}$ It may appear surprising that mean leisure productivity becomes more important as $\sigma_{L}$ approaches closer to one. Equation (14) shows that, for a given increase in leisure expenditures relative to time, the inferred increase in mean leisure productivity is larger as $\sigma_{L}$ approaches one.
    ${ }^{24}$ For values of $\phi=\{0.2,0.5,0.7,1.0,1.3,2.0,3.0\}$ the contribution of mean leisure productivity to welfare through an increase in mean consumption is $\{57,30,21,35,20,35,67\} \log$ points. The contribution to welfare through a decline in consumption dispersion is $\{1,1,2,9,3,6,14\} \log$ points.
    ${ }^{25}$ Let $\beta_{K}$ be the estimated elasticity in Aguiar and Bils (2015), $\overline{p x}$ be mean total expenditures in the cross section, and $p x(\iota)$ be total expenditures of household $\iota$. We allocate total spending in each category to households in proportion to their spending share $p_{K} x_{K}(\iota)=\frac{\left(\frac{p x(\iota)}{p x}\right)^{\beta} K}{\sum_{\iota}\left(\frac{p x(\iota)}{p \bar{x}}\right)^{\beta}{ }^{\beta}} p_{K} X_{K}$, where $p_{K} X_{K}$ is a cross-sectional measure of expenditures in category $K$.

