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JEL Classification: G11, G12

Keywords: Asset Pricing, Anomalies, capital allocation, General Equilibrium, factor-based investing, production economy

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A Supply and Demand Approach to Equity Pricing*

Sebastien Betermier, Laurent E. Calvet, and Evan Jo

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ABSTRACT

This paper presents a frictionless neoclassical model of financial markets in which firm sizes, stock returns, and the pricing kernel are all endogenously determined. The model parsimoniously specifies the supply and demand of financial capital allocated to each firm and provides general equilibrium sizes and returns in closed form. We show that the interaction of supply and demand can coherently explain a large number of asset pricing facts. The equilibrium security market line is flatter than the CAPM predicts and can be nonlinear or downward-sloping. The model also generates the size, profitability, investment growth, value, asymmetric volatility, betting-against-beta, and betting-against-correlation anomalies, while also fitting the cross-section of firm characteristics.

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"Finance takes the properties of cash flows as given, but we need to specify where they come from as well – betas are not exogenous. General equilibrium holds many surprises."
John Cochrane (2017)

1 Introduction

Over the past decades, researchers have uncovered hundreds of firm characteristics associated with persistent deviations from the Capital Asset Pricing Model, and have increasingly documented linkages between these characteristics (Cochrane 2011, Harvey, Liu, and Zhu 2016). Among classic predictors, high profitability, low investment growth, low market capitalization, and a high book-to-market ratio are known to be associated with higher long-run stock returns than the CAPM implies (Fama and French 1993, 2015, Hou, Xue, and Zhang 2015, Novy-Marx 2013). Perhaps more crucially, a stock's market beta is itself a source of deviation from the CAPM. Low-beta stocks generate on average positive alpha and high beta stocks generate negative alpha (Black, Jensen, and Scholes 1972, Fama and MacBeth 1973), which motivates the betting-against-beta ("BAB," Asness, Moskowitz, and Pedersen 2013, Frazzini and Pedersen 2014) and the betting-against-correlation ("BAC," Asness et al. 2019) factors.

This vast body of empirical findings has prompted both the rapid rise of factor-based investing (Ang 2014) and a deep reassessment of financial theory. Existing explanations relate deviations from the CAPM to omitted risk exposures driving differences in expected returns (Merton 1973), market frictions (Biais, Hombert, and Weil 2018, Black 1972, Frazzini and Pedersen 2014), and behavioral biases (Barberis and Thaler 2003, Hirshleifer 2015, Hong and Sraer 2016, Shleifer 2000).[1] Given the abundance of empirical risk factors and related explanations, financial economists are now reconsidering the nature of deviations from the CAPM. While empirical re-

[1]Examples of state risk include changes in investment opportunities (Campbell and Vuolteenaho 2004, Merton 1973), labor income risk (Campbell 1996, Cochrane 1999, Jagannathan and Wang 1996, Lettau and Ludvigson 2001, Mayers 1972, Santos and Veronesi 2006), long-run risks (Bansal and Yaron 2004, Hansen, Heaton, and Li 2008), liquidity risk (Acharya and Pedersen 2005, Pastor and Stambaugh 2003), systematic risk of assets in place (Berk, Green, and Naik 1999, Gomes, Kogan, and Zhang 2003), and operating leverage (Carlson, Fisher, and Giammarino 2004). Examples of frictions include margin constraints (Black 1972) and investment irreversibilities (Cooper 2006, Zhang 2005).

searchers are questioning the joint validity of the large number of factors,[2] theorists are asking if an integrated equilibrium model can explain anomalies by explicitly tying firm characteristics to corporate investment and production decisions (Berk 1995, Berk, Green, and Naik 1999, Kogan and Papanikolaou 2012).

A full theory of equity markets should jointly explain production decisions, investor portfolio decisions, and stock prices. This requirement is likely to be important because many anomalies relate stock prices to quantities, such as size or investment. A cohesive explanation of equity markets might therefore stem from endogenizing the trifecta of (i) production decisions, (ii) stock returns, and (iii) the pricing kernel, as is common in the general equilibrium (GE) literature (Arrow and Debreu 1954, Brock 1982, Geanakoplos 1987, Magill and Quinzii 1996).

The models most widely used in finance do not endogenize all elements of the trifecta. Investor-based models usually consider either endowment economies (Bansal and Yaron 2004, Campbell and Cochrane 1999, Lucas 1978, Rubinstein 1974), which endogenize asset returns and the pricing kernel but take firm sizes as given, or linear production technologies (Campbell et al. 2018, Merton 1973) where firm sizes are endogenous but stock returns are exogenous. Production-based models usually consider decreasing return to scale in production (Carlson Fisher and Giammarino 2004, Li, Livdan, and Zhang 2009, Zhang 2005), which allows for a true interaction between prices and quantities, but assume exogenous pricing kernels.[3] Full GE models, however, have been used to explain the size and value effects (Gomes, Kogan, and Zhang 2003, Garleanu, Kogan, and Panageas 2012, Kogan, Papanikolaou, and Stoffman 2013, Parlour and Walden 2011).[4] These papers suggest that going full GE bears considerable promise for asset pricing.

The present paper builds on these literatures and develops a simple GE approach in which the supply and demand of capital for each firm are available in closed form. Their intersection endogenously determines the size and return of each stock in GE, along with the pricing kernel. The foundations of the model are relatively standard. The supply of financial capital is specified by a representative investor with quadratic utility and exposure to state risk, which impacts nonfinancial

[2] Recent research considers the potential role of multicollinearity (Lewellen, Nagel, and Shanken 2010), data snooping, anomaly elimination (McLean and Pontiff 2016), measurement error (Andrei, Cujean, and Fournier 2019), and other statistical issues (Bryzgalova 2018, Harvey, Liu, and Zhu 2016, Hou, Xue and Zhang 2018).

[3] See also Balvers, Gu, and Huang (2017) and Kogan and Papanikolaou (2014). Cochrane (2008) and Campbell (2018) provide excellent reviews of these models.

[4] Other applications include volatility (Gourio 2011) and the durables anomaly (Gomes, Kogan, and Yogo 2009).

wealth such as labor income or private business holdings. Importantly, the representative investor faces no trading frictions. The demand for financial capital is generated by entrepreneurs who own firms and raise capital to purchase productive assets. Production technologies are characterized by decreasing returns to scale and risky profitabilities inducing cash flow risk. Firms exhibit both heterogeneous average profitabilities and heterogeneous exposures to state risk. We provide sufficient conditions under which an equilibrium exists in the absence of trading constraints and we develop an approximate closed-form solution that features linear relationships between the quantity of risk taken on by each firm, the price of risk, and underlying structural parameters. Throughout the paper, a stock's price of risk is naturally measured by its Sharpe ratio, the expected excess return earned for each unit of volatility risk.

In the most general version of the model, a firm's demand for capital is expressed as a function of its profitability, while its supply of capital depends on the characteristics of all firms. To gain further insights, we consider a symmetric version in which firms have identical cash-flow correlations. The supply of capital to a firm then only depends on its characteristics and cross-sectional constants. For every firm, the supply curve is upward sloping, the demand curve is downward sloping, and their intersection determines capital investment and the stock's Sharpe ratio.

These properties allow us to analyze the GE of financial markets by relying on standard supply and demand analysis. This approach is remarkable because, even though the GE of an economy with multiple firms is inherently a high-dimensional problem, the symmetric version of the model allows us to plot the supply and demand curves of each firm on the same plane. This graphical representation is inspired by the visual simplicity of standard equilibrium models such as IS-LM (Hicks 1937).

Our approach yields multiple insights. First, the model illustrates that the interplay of supply and demand has rich implications for the cross-section of risk prices and firm sizes and the composition of the market portfolio. The main driving forces are the firms' expected profitabilities, which drive up the demand schedule for financial capital, and firms' exposures to state risk, which drive down the supply schedule of financial capital. In GE, profitable firms tend to be larger and have high Sharpe ratios and expected returns. By contrast, firms with low state risk exposures are also larger but have low Sharpe ratios.

Second, we investigate how supply and demand drive the security market line (SML). As a starting point, consider an economy in which firms have zero state risk exposures. The supply schedule of financial capital by stock market investors is the same for all firms. Variation in demand curves due to heterogeneity in expected profitabilities produces equilibrium outcomes located along the common upward-sloping linear supply curve. This property directly implies the familiar positive linear CAPM relationship between systematic risk and expected return.

Third, in the presence of state risk, the equilibrium SML is flatter than the SML predicted by the CAPM, consistent with the empirical findings of Asness, Moskowitz, and Pedersen (2013), Black, Jensen, and Scholes (1972), Fama and MacBeth (1973), and many others. A firm uncorrelated to the market is compensated for its state risk exposure, so that the SML has a positive intercept. The market portfolio is exposed to state risk and is therefore less attractive than under the CAPM, so that the SML is not as steeply sloped as the CAPM predicts. Perhaps surprisingly, these results hold even when firms have identical exposures to state risk.

Fourth, if firms have heterogeneous state risk exposures, the SML is generally nonlinear and can be downward-sloping. The explanation is that the demand and supply schedules of firms can move freely with the firm's average profitability and state risk exposure, so that the cross-sectional relationship between risk and return is flexible in GE. The relationship between risk and return is upward-sloping if firms are primarily heterogeneous in average profitability, and downward-sloping if heterogeneity in state risk dominates. In the special case of endowment or linear economies, a downward-sloping SML cannot arise endogenously because either the quantity or the price of risk is fixed.

Fifth, our findings about the SML imply that a stock's alpha is not only driven by exposure to omitted state risk, as is commonly assumed in factor pricing research, but is also driven by the stock's correlation to the market portfolio, and in turn its beta. In our model, a stock's correlation to the market is endogenous and driven by a number variables, such as the state risk exposure, average profitability, and cash-flow volatility of every firm in the economy. The flatness of the SML implies that a stock with a low correlation to the market portfolio exhibits a positive alpha and a higher Sharpe ratio than the CAPM implies, and conversely for a high-correlation stock. These properties naturally generate the BAC factor. Thus BAC picks up the additional risk premia that CAPM fails to account for.

Sixth, we show that when technologies exhibit mild decreasing returns to scale, profitability and state risk impact a stock's market beta primarily through the stock's correlation to the market, while the impact on the stock's volatility tends to be more minor. The BAC factor thus naturally leads to the BAB factor, the return spread between low beta and high beta stocks. As such, BAB is likely to arise across asset classes, countries, and time, which is consistent with the empirical evidence that BAB is present everywhere (Asness, Moskowitz, and Pedersen 2013).

Seventh, the model has powerful implications for the relationship between stock returns and a set of firm characteristics. Technologies exhibiting slight but significant deviations from constant returns to scale can generate rich patterns in the cross-section of firms that are consistent with the data. For instance, the model implies that growth stocks generally have larger market capitalizations than value stocks, that the distribution of market values has a fatter upper tail than the distribution of book values, that the expected return and volatility decrease with firm size, and that the market correlation increases with firm size. All these predictions, which hold both in partial and general equilibrium, are supported by the data.

Eighth, in addition to BAC and BAB, the model coherently generates the size, value, and investment anomalies. In GE, firms with high state risk exposure have high discount rates and thus small market values, as Berk (1995) explains. The high discount rate implies a low market-to-book ratio and a reduction of productive investment, which further reduces firm size. The quantity channel therefore reinforces and broadens Berk's (1995) price channel, and it is consistent with recent production-based models relating firm investment decisions to discount rates (Belo, Xue, and Zhang 2013, Hou, Xue, and Zhang 2015, Zhang 2017). Our GE model closes the economy and connects firm characteristics directly to alpha.

Ninth, the model predicts that stocks that are larger, invest more, and have high market-to-book ratio should in turn have higher market correlation and higher beta. Therefore, BAC and BAB should be negatively related to the size, value, and investment growth anomalies. This prediction is exactly what is observed in the data (Bali et al. 2017, Frazzini and Pedersen 2014, De Giorgi, Post, and Yalçın 2017, Liu 2018).

Tenth, the model generates the gross profitability and asymmetric volatility anomalies. The data indicate that, if one compares stocks with the same market-to-book ratio, those with high gross profitability tend to generate positive alpha whereas stocks with low gross profitability generate negative alpha (Novy-Marx 2013, Hou, Xue, and Zhang 2015, Fama and French 2015). The data also indicate that the relation between the firm's volatility and its alpha is positive among firms that are underpriced by the CAPM and negative among overpriced firms (Stambaugh, Yu, and Yuan 2015). Both effects naturally arise in GE in the presence of state risk.

We draw several lessons from the model that can guide future research in asset pricing. Classic deviations from the CAPM arise naturally when firm sizes, asset returns, and the pricing kernel are jointly determined in GE. These deviations occur even in the absence of market frictions, but we anticipate that the mechanisms we are identifying would be strengthened by trading restrictions, such as short sales or leverage constraints. Our results only require that some form of state risk and cash flow risk be present. To emphasize the generality of our findings, we remain agnostic on the exact nature of the investor's state risk and the firms' cash-flow risks.

A key insight of the paper is that anomalies come as a pack in general equilibrium. In our model, the presence of state risk flattens the security market line, thereby generating the BAC and BAB factors. At the same time, state risk impacts the cost of capital and generates the size, investment, and profitability effects. Multiple factors should therefore be observed in GE across asset classes, countries, and time, consistent with the empirical evidence (Fama and French 1998, 2012, Asness, Moskowitz, and Pedersen 2013, Hawawini and Keim 2000).

Last but not least, alpha may not be tied to fundamental risk. In GE, alpha is driven not only by state risk but also by correlation with the market, which is itself driven by both supply and demand factors. Therefore, in contrast to the precepts commonly followed in multifactor asset pricing research, the factor zoo may be the natural consequence of a GE economy where supply and demand forces are both at play.

Our paper contributes to several strands of the literature. Campbell and Vuolteenaho (2004) decompose a stock's market beta into a good and a bad beta, which quantify the stock's comovement with, respectively, discount rate news and cash flow news on the market portfolio return.[5] Our model can be helpful to this literature because it endogenizes beta, which is driven by both state risk exposures driving discount rates and profitability effects driving cash flows.[6]

The framework developed in this paper provides a fresh perspective on the debate about whether

[5] See also Campbell, Polk, and Vuolteenaho (2010), Campbell, Giglio, and Polk (2013), and Campbell et al. (2018) for further work on multi-beta decompositions.

[6] We show that stocks with valuable hedging properties endogenously have higher beta and lower expected return, which an ICAPM of the type considered by Campbell and Vuolteenaho (2004) cannot do.

stock returns are driven by exposures to risk factors or firm-level characteristics (Daniel and Titman 1997, Berk 2000, Lin and Zhang 2013). We show that, in GE, both channels directly impact a stock's expected return and alpha. The firm's exposure to common forms of state risk is classified as a supply factor, whereas its profitability corresponds to a demand factor. In GE, both supply and demand factors impact the firm's size, expected return, and alpha. The debate about characteristics vs. risk exposures thus depends on whether cross-sectional heterogeneity mostly originates from supply or demand forces.

The paper is related to the growing literature that documents empirical linkages between stock prices and investors' supply of financial capital (Gompers and Metrick 2001, Koijen and Yogo 2019). Additions to a popular index trigger permanent stock price increases and conversely for deletions (Kaul, Mehrota, and Morck 2000, and Shleifer 1986), suggesting that supply shocks are priced.[7] Similarly, hedging by institutional investors and the resulting pricing effects have been widely reported (de Roon, Nijman, and Veld 2000, Garleanu, Pedersen, and Poteshman 2009, Klinger and Sundaresan 2019).[8] Recent research also documents hedging behavior among households and its ties to asset pricing factors (Betermier, Calvet, and Sodini 2017, Betermier et al. 2012, Bonaparte, Korniotis, and Kumar 2014).

Finally, we build on the influential literature that documents linkages between stock prices and firms' demand for financial capital. The volumes of initial public offerings are empirically linked to a firm's demand for financial capital (Lowry 2003) and tend to be large during times where expected market returns are low and expected aggregate profitability is high (Pastor and Veronesi 2005). Industry sectors that demand large amounts of capital typically see a significant increase in their capital cost (Pastor and Veronesi, 2009). Empirical tests of production-based models also reveal that firms align their investment policies with their cost of capital in ways that are consistent with theory (Andrei, Mann, and Moyen 2019, Liu, Whited, and Zhang 2009).

The structure of our paper is as follows. Section 2 discusses the supply of capital by stock market investors and the resulting asset pricing implications. Section 3 defines the demand for capital by entrepreneurs and verifies the empirical validity of the model's predictions for the cross-section of firms. Section 4 shows the existence of GE and provides a closed-form linear approximation.

[7] See also Cha and Lee 2001, Greenwood (2005), Hau, Massa, and Peress 2010, Petajisto (2009), and Wurgler and Zhuravskaya (2002).

[8] See also Bongaerts, de Jong, and Driessen (2011), Greenwood and Vayanos (2010), and Zinna (2015).

Section 5 investigates the formation of the market portfolio and the cross-section of firm sizes and Sharpe ratios in GE. Section 6 and Section 7 respectively study the SML and alpha in GE. Section 8 concludes. The Appendix provides proofs and details of the empirical methodology.

2 The Supply of Financial Capital and Equity Pricing

2.1 Stock Market Investors and Financial Assets

We consider a simple economy with two periods $t \in \{0,1\}$ and a unique physical good. All quantities at dates 0 and 1 are expressed as units of this good. The economy is populated by two types of agents: stock market investors, who supply financial capital, and entrepreneurs, who own one firm each and demand financial capital to fund productive investment. We specify investors in the present section, and describe the behavior of entrepreneurs in Section 3.

The supply of financial capital is generated by a representative investor endowed with financial wealth W_0 at date 0. Since a representative investor can be constructed through the usual aggregation arguments in economies with heterogeneous agents, we will indifferently refer to the representative investor or investors in the remainder of the paper.

The representative investor is exposed to state risk in the form of an exogenous shock to non-financial wealth at $t = 1$. For convenience, we write the shock as $R_\theta W_0$, where R_θ is an exogenous random variable. State risk may stem from human capital, inflation, housing or a combination thereof.

The investor can trade at date 0 a risk-free asset and the stocks of N firms indexed by $n \in \{1,\cdots,N\}$. In contrast to many earlier models on anomalies (e.g., Black 1972, Frazzini and Pedersen 2014, Hong and Sraer 2016), the investor can freely purchase or sell short all financial assets. We denote by r_f the arithmetic risk-free rate and by $\boldsymbol{r} = (r_1,\cdots,r_n)'$ the vector of arithmetic returns on the stocks between dates 0 and 1. In order to better tie this section to the results of later sections, we consider that stock returns are defined as follows.

Firm n's equity generates at date 1 the cash flow

$$CF_n = \mu(CF_n) + \sigma(CF_n)\, z_n,$$

where z_n is an exogenous random shock with zero mean and unit variance, and the deterministic terms $\mu(CF_n)$ and $\sigma(CF_n)$ respectively denote the mean and standard deviation of the firm's cash flow. We stack firm shocks into the vector $\boldsymbol{z} = (z_1, \ldots, z_N)'$, which is random from the perspective of date 0 and is fully realized at date $t = 1$. Let

$$\boldsymbol{\rho}_{\mathbf{CF}} = \mathbb{E}(\boldsymbol{z}\boldsymbol{z}') = (\rho_{\mathrm{CF},i,j})_{1 \leq i,j \leq N} \tag{1}$$

denote the correlation matrix of $\boldsymbol{z}$. For every firm n, an important parameter is the covariance of the investor's state shock and the firm's shock:

$$\theta_{\mathrm{CF},n} = Cov(R_\theta, z_n).$$

We collect these covariances into the column vector $\boldsymbol{\theta}_{\mathbf{CF}} = (\theta_{\mathrm{CF},1}, ..., \theta_{\mathrm{CF},N})'$. The state shock R_θ, the random vector $\boldsymbol{z}$, the correlation matrix $\boldsymbol{\rho}_{\mathbf{CF}}$, and the vector of covariances $\boldsymbol{\theta}_{\mathbf{CF}}$ are exogenous throughout the paper, while the mean and standard deviation of cash flows are endogenized in Section 3.

Let V_n denote the market value of firm n's equity at date 0. The return on stock n is

$$r_n = \frac{CF_n}{V_n} - 1 = \mu_n + \sigma_n z_n, \tag{2}$$

where $\mu_n = \mu(CF_n)/V_n$ and $\sigma_n = \sigma(CF_n)/V_n$. We denote by $\boldsymbol{\mu} = (\mu_1, \cdots, \mu_N)'$ the vector of expected returns of the N stocks, $\boldsymbol{\Sigma}$ their variance-covariance matrix, and $\boldsymbol{\sigma}$ the diagonal matrix with elements $\sigma_1, ..., \sigma_n$.

The wealth available to the investor at $t = 1$ is:

$$W_1 = W_0 \left[1 + r_f + \boldsymbol{\omega}'(\boldsymbol{r} - r_f \mathbf{1}) + R_\theta\right], \tag{3}$$

where $\boldsymbol{\omega}$ is the $N \times 1$ vector of shares of wealth invested in the firms and $\mathbf{1} = (1, \ldots, 1)'$. As in Merton (1987), we consider that the investor has quadratic utility: $U = \mathbb{E}(W_1) - \gamma Var(W_1)/(2W_0)$. We verify in the Appendix that the investor optimally adopts the classic "two-fund rule":

$$\boldsymbol{\omega} = \frac{1}{\gamma} \boldsymbol{\Sigma}^{-1} (\boldsymbol{\mu} - r_f \mathbf{1}) - \boldsymbol{\Sigma}^{-1} \boldsymbol{\sigma} \boldsymbol{\theta}_{\mathbf{CF}}, \tag{4}$$

where γ is the coefficient of relative risk aversion, $\boldsymbol{\Sigma}^{-1} (\boldsymbol{\mu} - r_f \mathbf{1})$ is a portfolio of risky assets that achieves the highest Sharpe ratio, and $\boldsymbol{\Sigma}^{-1} \boldsymbol{\sigma} \boldsymbol{\theta}_{\mathbf{CF}}$ is the hedge portfolio that offsets the impact of state risk. As in Merton (1973), a stock represents a large share of the investor's portfolio $\boldsymbol{\omega}$ if it contributes to a high Sharpe ratio and has low exposure to state risk. We remain agnostic about the exact nature of state risk, since further specialization is not required for our main results.

2.2 Equilibrium Pricing Restrictions

We now show that in any stockmarket equilibrium, the expected return of a stock can be expressed as a function of the stock's univariate CAPM beta and the stock's exposure to state risk. This result departs from two widely used representations of the cross-section of expected returns: (i) one-factor representations, in which an asset's risk premium is driven by its loading on the *tangency* portfolio, and (ii) multi-factor representations, in which the factor loadings driving an asset's risk premium are defined by a *multivariate* regression of the asset's returns on the factors (Fama and French 1993, Ross 1976). Our representation is a variant of Merton (1987), who proposes a similar result in an economy with information costs.

We begin with some useful definitions. Let $V_{\mathrm{M}} = \sum_{n=1}^{N} V_n$ denote the aggregate value of the stock market. The market portfolio assigns the weight V_n/V_{M} to every stock n and earns the return $r_{\mathrm{M}} = \sum_{n=1}^{N} (V_n/V_{\mathrm{M}})\, r_n$. Let μ_{M} and σ_{M} denote the mean and standard deviation of the market portfolio return. The exposure of the market portfolio to state risk is defined by

$$\theta_{\mathrm{CF,M}} = \frac{Cov(R_\theta, r_{\mathrm{M}})}{\sigma_{\mathrm{M}}} = \frac{\sum_{n=1}^{N} \theta_{\mathrm{CF},n} V_n \sigma_n}{V_{\mathrm{M}} \sigma_{\mathrm{M}}}.$$

Stock n's correlation with the market is $\rho_{\mathrm{M},n} = Corr(r_{\mathrm{M}}, r_n)$, and the stock's beta to the market is:

$$\beta_n = \frac{Cov(r_{\mathrm{M}}, r_n)}{Var(r_{\mathrm{M}})} = \rho_{\mathrm{M},n} \frac{\sigma_n}{\sigma_{\mathrm{M}}}. \tag{5}$$

We emphasize that β_n is the univariate beta one would compute under the CAPM.

In equilibrium, the market share of every firm, V_n/V_{M}, is equal to its share in the portfolio of the representative investor, $\omega_n/(\sum_{\ell=1}^{N} \omega_\ell)$, given in equation (4). This implies:

Proposition 1 (Factor Representation) *The expected return of stock n satisfies*

$$\mu_n - r_f - c_n = \beta_n (\mu_{\mathrm{M}} - r_f - c_{\mathrm{M}}), \tag{6}$$

where

$$c_n = \gamma Cov(r_n, R_\theta) = \gamma \theta_{\mathrm{CF},n} \sigma_n \tag{7}$$

is the compensation for state risk exposure, and $c_{\mathrm{M}} = \gamma Cov(r_{\mathrm{M}}, R_\theta)$ *is the market portfolio's compensation for state risk exposure. Furthermore, the market portfolio satisfies:*

$$\mu_{\mathrm{M}} - r_f - c_{\mathrm{M}} = \gamma V_{\mathrm{M}} \sigma_{\mathrm{M}}^2 / W_0 > 0. \tag{8}$$

Hence conditional on the state cost c_n, the expected return of a stock, μ_n, is positively and linearly related to the stock's CAPM beta coefficient, β_n.

Equation (6) provides an equilibrium relationship between a stock's expected return, market beta, and state cost c_n. Given its proximity to related results in the literature, we will refer to it as the security market line (SML). Equation (8) implies that the share of risky assets in the agent's portfolio is $V_{\mathrm{M}}/W_0 = (\mu_{\mathrm{M}} - r_f - c_{\mathrm{M}})/(\gamma\sigma_{\mathrm{M}}^2)$, which illustrates how state risk is incorporated into the classic Merton portfolio selection rule. To facilitate the discussion, we consider that the exposures $\theta_{\mathrm{CF},n}$ are generally positive, so that the investor is positively compensated for bearing state risk exposure in stock n. Assets with high market beta must either have a high expected return or a low cost of state risk exposure to compensate investors for taking on the risks.

For future analysis, it is convenient to reexpress the factor representation developed in Proposition 1 in terms of the stock's Sharpe ratio, $\lambda_n = (\mu_n - r_f)/\sigma_n$.

Proposition 2 (Equilibrium Sharpe Ratio) *The Sharpe ratio of stock n satisfies:*

$$\lambda_n = \gamma\theta_{\mathrm{CF},n} + \rho_{\mathrm{M},n}\left(\lambda_{\mathrm{M}} - \gamma\theta_{\mathrm{CF,M}}\right) \tag{9}$$

for every $n \in \{1,\ldots,N\}$.

This result is obtained by normalizing the asset pricing relationship in (6) by the firm's volatility. Proposition 2 provides a decomposition of the stock's Sharpe ratio into a normalized compensation for state risk, $\gamma\theta_{\mathrm{CF},n}$, and a compensation for correlation with the market return, $\rho_{\mathrm{M},n}\left(\lambda_{\mathrm{M}} - \gamma\theta_{\mathrm{CF,M}}\right)$. We call the equilibrium relationship (9) the *normalized SML.*

In the absence of state risk ($\theta_{\mathrm{CF},n} = 0$ for all n), the CAPM security market line holds: $\mu_n - r_f = \beta_n\left(\mu_{\mathrm{M}} - r_f\right)$, and the normalized SML reduces to:

$$\lambda_n = \rho_{\mathrm{M},n}\lambda_{\mathrm{M}}. \tag{10}$$

An asset has the highest Sharpe ratio if it is perfectly correlated with the market. Importantly, the CAPM holds regardless of the exact specification of the production side of the economy.

In the presence of state risk, a stock's normalized compensation for state risk is the product of two parameters, γ and $\theta_{\mathrm{CF},n}$, that are exogenous in our setting. This property partly motivates the use of the normalized SML in the paper. More generally, the normalized SML is attractive because it provides condensed information about both the risk and expected return of the asset. This approach is consistent with the continuous-time literature, which focuses on the Sharpe ratio as the price of risk. Furthermore, we will obtain more straightforward relationships between stock returns and firm characteristics in our model once we scale returns by volatility.

2.3 Closing the Model

The asset-pricing restriction (6) provides useful information on the structure of equilibrium returns. It is a limited result, however, because the market portfolio and the beta coefficients of individual stocks relative to the market are taken as given. By contrast, a full theory of financial markets should explain firm sizes and the joint distribution of returns.

An extensive literature addresses this issue by considering endowment economies (Lucas 1978, Martin 2013, Rubinstein 1974), or linear technologies (Cox, Ingersoll, and Ross 1985, Merton 1973). Endowment economies provide endogenous asset returns but assume exogenous firm capital stocks. Conversely, linear technologies impose exogenous asset returns but provide endogenous firm sizes. We now discuss the asset-pricing implications of these restrictions, as well as the benefits of endogenizing both firm capital stocks and equity returns in GE.

Endowment economies, in which the productive capital of each firm is predetermined, have been widely used to develop consumption-based versions of the CAPM or for studying the pricing implications of state risk. By design, they cannot be used to explain the distribution of capital stocks and firm sizes, unless special assumptions are made. More generally, endowment economies are not well suited for studying the endogenous formation of the market portfolio and the beta coefficients of individual stocks relative to the market. In the next sections, we will also show that they cannot explain either pricing factors that have been recently documented in the empirical literature, such as the correlation, investment and profitability factors.

Linear production technologies are also commonly used to close GE asset pricing models. The exogenous productivities of the linear technologies determine asset returns. Investors allocate capital to firms depending on their productivities and exposures to state risk, along with their risk preferences. Linear technologies are therefore well-suited to study the formation of the market portfolio. However, linear technologies imply that a firm's market value is generally equal to its book value. As a result, they cannot explain the value factor. We will similarly show that the size, profitability, and investment factors do not arise endogenously in linear production models.

In order to address the many asset pricing puzzles in a single, coherent framework, it is therefore important to develop a model that endogenizes both asset returns and firm sizes. In the next section, we achieve this objective by developing a production-based asset pricing model in which firms exhibit decreasing returns to scale and raise capital from investors of the type considered in the present section.

3 Entrepreneurs, Firms, and the Demand for Financial Capital

3.1 A Model of Firms with Decreasing Returns to Scale

The behavior of the entrepreneurs defined in Section 2.1 is specified as follows. Each entrepreneur $n \in \{1, \cdots, N\}$ fully owns firm n, which is endowed with a specific decreasing return to scale technology.

An investment of I_n units of *uninstalled* capital in period $t = 0$ results in

$$K_n(I_n) = \left(\frac{\eta+1}{\eta} I_n\right)^{\frac{\eta}{\eta+1}} \tag{11}$$

units of *installed* capital in the same period, where $\eta \in (0, \infty)$ is a fixed parameter. A low value of η implies fast decreasing returns to scale, whereas a high value of η implies near constant returns to scale. Figure 1 plots the function $K_n(I_n)$ for various values of η.

The firm generates the cash flow (or operational profit)

$$CF_n(K_n) = \underbrace{(a_{\text{CF},n} + z_n)\,\sigma_{\text{CF},n}}_{ROA_n} K_n \tag{12}$$

at date $t = 1$. We note that $a_{\text{CF},n}\sigma_{\text{CF},n}$ and $\sigma_{\text{CF},n}$ represent, respectively, the mean and volatility of the firm's operational profit per unit of installed capital. These parameters are all exogenous.

We assume for simplicity that the entrepreneur has no upfront capital. She obtains external funding at date $t = 0$ by issuing a claim on the total cash flow CF_n, which is purchased by stock market investors. We assume that financial markets are competitive and that there exists a risk-adjusted probability measure $\mathbb{Q}$, which entrepreneurs and stock market investors use to value financial assets. We infer from equation (2) that the Sharpe ratio of stock n satisfies $\lambda_n = -\mathbb{E}^{\mathbb{Q}}(z_n)$. We will henceforth refer to λ_n as the unit price of risk associated with stock n. The market value of the firm's equity is therefore

$$V_n = \frac{\mathbb{E}^{\mathbb{Q}}(CF_n)}{1+r_f} = \left(\frac{a_{\text{CF},n} - \lambda_n}{1+r_f}\right) \sigma_{\text{CF},n} K_n \tag{13}$$

at date $t = 0$.

Optimal firm investment. The entrepreneur sets the firm's investment I_n to the level that maximizes her economic profit. Specifically, the entrepreneur receives V_n from stock market investors, allocates I_n to the firm in the form of uninstalled capital, and consumes the economic profit $V_n - I_n$ at $t = 0$. The optimal level of investment I_n therefore maximizes the net present value of the firm:

$$V_n - I_n = \frac{a_{\text{CF},n} - \lambda_n}{1+r_f} \sigma_{\text{CF},n} K_n(I_n) - I_n. \tag{14}$$

Since the function $K_n(\cdot)$ is concave, the optimal investment I_n is unique. The optimal level of installed capital is

$$K_n = \left[\frac{(a_{\text{CF},n} - \lambda_n)\,\sigma_{\text{CF},n}}{1+r_f}\right]^{\eta} \tag{15}$$

if $\lambda_n \leq a_{\text{CF},n}$ and zero otherwise. The entrepreneur invests more if the firm has a high profitability per unit of risk, $a_{\text{CF},n}$, and a low price of risk, λ_n. This result is intuitive because a higher value of λ_n imposes a higher hurdle for clearing projects. We henceforth focus on active firms for which $\lambda_n < a_{\text{CF},n}$. The installed capital stock K_n is the book value of the firm. We infer from (15) that η is the elasticity of the firm's book value, K_n, with respect to the risk-adjusted profitability of capital, $a_{\text{CF},n} - \lambda_n$.

The firm's public equity is worth

$$V_n = \left[\frac{(a_{\text{CF},n} - \lambda_n)\,\sigma_{\text{CF},n}}{1+r_f}\right]^{\eta+1} = K_n^{1+\frac{1}{\eta}}, \tag{16}$$

as equations (13) and (15) imply. Since the entrepreneur invests more in the firm when it offers more valuable opportunities, the firm's market value V_n increases at a faster rate than the book value K_n. We classify a stock with a high market-to-book ratio, V_n/K_n, as a growth stock, and a stock with a low market-to-book ratio as a value stock.

Stock return. A stock market investor purchasing stock n earns the return defined in equation (2). Given the firm's cash flow and market value in equations (12) and (13), the mean μ_n and volatility σ_n of the return on stock n satisfy

$$\mu_n - r_f = \frac{a_{\text{CF},n}\,\sigma_{\text{CF},n}\,K_n}{V_n} - 1 - r_f = \frac{(1+r_f)\,\lambda_n}{a_{\text{CF},n} - \lambda_n}, \tag{17}$$

$$\sigma_n = \frac{\sigma_{\text{CF},n}\,K_n}{V_n} = \frac{1+r_f}{a_{\text{CF},n} - \lambda_n}. \tag{18}$$

The mean and standard deviation of the stock return are both low when the firm is highly profitable and has a low price of risk. The reason is that a high $a_{\text{CF},n}$ and a low λ_n both induce high market value relative to the expected value of future cash flows. The distribution of the firm's future returns is then scaled downwards, and the stock has low expected return and low volatility. In the extreme case where the market value is infinite, the investor is sure to lose her entire investment: $\mu_n = -100\%$ and $\sigma_n = 0$.

Since $\boldsymbol{r} = \boldsymbol{\mu} + \boldsymbol{\sigma} \boldsymbol{z}$, the variance-covariance matrix of returns satisfies

$$\boldsymbol{\Sigma} = \boldsymbol{\sigma}\,\boldsymbol{\rho}_{\mathbf{CF}}\,\boldsymbol{\sigma},$$

where $\boldsymbol{\rho}_{\mathbf{CF}}$ is the correlation matrix of cash flows defined in equation (1). The variance-covariance matrix of stock returns is driven by both the exogenous correlation matrix, $\boldsymbol{\rho}_{\mathbf{CF}}$, and the endogenous vector of stock return volatilities, $\boldsymbol{\sigma}$.

3.2 Risk Capital

In order to better connect the decisions of investors and entrepreneurs, it is convenient to express our results as a function of the risk capital:

$$Q_n = [Var(CF_n)]^{1/2} = \sigma_{\text{CF},n}\,K_n = \sigma_n\,V_n,$$

where the last equality follows from (18). The risk capital Q_n expresses the quantity of risk in the firm's cash flow, $[Var(CF_n)]^{1/2}$, and in the firm's public equity, $\sigma_n V_n$. The measure Q_n is therefore of interest to both investors and entrepreneurs. For this reason, we use it extensively throughout the paper. This approach is consistent with the risk management literature, which is increasingly focusing on the risk in future dollar amounts rather than the risk in proportional *changes* (see Artzner et. al. 1999).

Aggregate capital risk and the correlation of a stock and the market portfolio can be conveniently expressed as a function of the risk capital vector $\boldsymbol{Q} = (Q_1, \ldots, Q_N)'$. We verify in the Appendix that the dollar volatility of the market portfolio satisfies

$$Q_{\mathrm{M}} = \left[\mathrm{Var}\left(\sum_{n=1}^{N} CF_n\right)\right]^{1/2} = \sqrt{\boldsymbol{Q}'\boldsymbol{\rho}_{\mathbf{CF}}\boldsymbol{Q}} = \sigma_{\mathrm{M}} V_{\mathrm{M}}. \tag{19}$$

The correlation between the return on stock n and the market return satisfies

$$\rho_{\mathrm{M},n} = \frac{\sum_{i=1}^{N} \rho_{\mathrm{CF},n,i} Q_i}{Q_{\mathrm{M}}} = \frac{Q_n + \sum_{i \neq n} \rho_{\mathrm{CF},n,i} Q_i}{Q_{\mathrm{M}}}. \tag{20}$$

A firm has high correlation with the market if it has large risk capital Q_n or if it correlates with other firms that, due to high profitability or low price of risk, take up the lion's share of the economy's risk-bearing capacity.

3.3 Implications for Firm Sizes and Risk Pricing

Our simple production model explains several key empirical facts about the distribution of firm market capitalizations and risk pricing. These properties hold irrespective of the assumptions made in Section 2 about stockmarket investors.

Growth firms are larger than value firms. Fama and French (1992) and Loughran (1997) document that stocks with high market capitalizations have on average high market-to-book ratios. Jurek and Viceira (2011) also find that growth stocks have accounted for 70% to 85% of the U.S. aggregate stock market since 1927.

The dominance of growth firms in the market portfolio arises naturally in our model. By equations (15) and (16), the market-to-book ratio of firm n is

$$\frac{V_n}{K_n} = \frac{(a_{\mathrm{CF},n} - \lambda_n)\,\sigma_{\mathrm{CF},n}}{1 + r_f} = K_n^{\frac{1}{\eta}}. \tag{21}$$

The market-to-book ratio encodes information about the firm's prospects: a growth stock is highly profitable and has a low price of risk. Combining (15) and (21), we obtain that larger firms tend to be growth firms:

$$V_n = \frac{V_n}{K_n} K_n = \left(\frac{V_n}{K_n}\right)^{\eta+1}. \tag{22}$$

Since the entrepreneur invests more in firm n if it is more profitable and has a lower price of risk, the market value of a stock increases in its market-to-book ratio, consistent with Hou, Xue, and Zhang (2015) and Zhang (2015).

Market values have fatter upper tails than book values. Since growth firms are larger than value firms, the market value $V_n = (V_n/K_n)K_n$ is more dispersed then K_n. Furthermore, equation (16) implies that if the cross-sectional distribution of book values has a thick right tail with tail index α_K, that is $P(K_n > k) \sim c\,k^{-\alpha_K}$ as $k \to +\infty$, then the cross-sectional distribution of market values has a substantially fatter right tail with the lower index $\alpha_V = \alpha_K \eta/(1+\eta)$.

In Figure 2, we estimate the coefficient η by regressing the firms' log market values on their log book values,

$$\log(V_n) = \left(1 + \frac{1}{\eta}\right)\log(K_n). \tag{23}$$

We conduct the estimation at the daily frequency using CRSP data over the 1980 to 2018 period and report the distribution of the estimated η. The details of the procedure are provided in the Appendix. The analysis reveals that the slope coefficient $(1+1/\eta)$ is almost always significantly above unity, which confirms that firms exhibit decreasing returns to scale. The slope coefficient is 1.06 on average, which corresponds to a coefficient η of about 17. Thus, firms exhibit mild but significant departure from constant returns to scale.

Figure 3 illustrates the estimated densities of the firms' log market values and log book values on the last day of the sample (December 31, 2018). The densities are estimated non-parametrically via an Epanechnikov kernel with optimal bandwidth.[9] Because η is greater than one, the distribution of market values is shifted to the right. We also report the predicted density of the market value based on the observed book values, according to (23). The predicted and observed densities are remarkably similar. This result indicates that the coefficient η does not vary much across firms and that our production function accurately predicts the relationship between market and book values.

The mean and volatility of stock returns generally decrease with firm size. Small-cap stocks are known to be more volatile and have higher expected returns than large-cap stocks (Ang et.

[9] That is, we use $K(u) = 3/4(1-u^2)1_{\{|u|\leq 1\}}$. Since the Epanechnikov kernel has a finite support, the tail behavior of the density estimate is not predetermined by the shape of the kernel in the tails but is instead strongly determined by the data.

al. 2006, Chan, Chen, and Hsieh 1985, Fama and French 1996, Huberman, Kandel, and Karolyi 1987), which motivates the widespread use of size-related factors (Banz 1981, Fama and French 1993). As we have discussed, these properties hold in our model: low profitability and a high price of risk imply a low market value, a high mean return, and high volatility, as equations (16) to (18) imply. Panel A of Figure 4 confirms that the negative relationship between volatility and size holds strongly in our sample.

These results strengthen Berk's (1995) classic argument that size-related factors are bound to improve pricing performance because small market capitalizations reflect high discount rates. Consistent with the production-based literature, our model shows that a quantity effect amplifies Berk's discount rate effect. Firms that face a high discount rate choose to invest less than other firms, so that their low market values result from both low investment and high discounting.

The market correlation of stock returns generally increases with firm size. Small-cap stocks are known to have low correlation with the stock market (Ang and Chen 2002, Fama and French 1993, Roll 1988).[10] The positive relationship between market correlation and firm size arises naturally in the model. Indeed, high profitability and a low price of risk lead the firm to have a high level of risk capital Q_n and therefore a high correlation with the market $\rho_{\mathrm{M},n}$, as equation (20) implies. All else equal, this implies a positive relation between V_n and $\rho_{\mathrm{M},n}$.

In Panel B of Figure 4, we plot the firms' market correlation against their log market value on the last day of our sample. Market correlations are estimated from three-day returns over a period of five years, similar to Frazzini and Pedersen (2014). As the model predicts, stock market correlations are steadily increasing with firm size. These results show that the positive relationship between correlation and size is empirically very strong. It will play a key role in the theoretical developments of the next sections.

[10] Fama and French (1993) and Roll (1988) show that the R^2 from CAPM time-series regressions is decreasing with firm size. The R^2 from these regressions is by definition equal to the firm's squared market correlation.

4 Existence and Closed-Form Approximation of General Equilibrium

In this section, we combine the supply and demand of capital previously derived and solve for the general equilibrium determining the sizes and risk premia of all firms. In Section 4.1, we define the GE of the economy and provide sufficient conditions for existence. In Section 4.2, we provide an approximate linear equilibrium solution in closed form.

4.1 Equilibrium Risk Premia and Investments

General equilibrium results from the interaction of entrepreneurs and the stock market investor. The entrepreneurs' demand for capital, defined in Section 3, is driven by profitability and cash flow risk, as is standard in the production-based asset pricing literature (e.g., Cochrane 1991). The supply of capital by the stock market investor, defined in Section 2, is driven by both the tradeoff between volatility and expected return and the desire to hedge state risk, as in Merton (1973). Our classification of stock market investors as suppliers of financial capital may seem at odds with the traditional view that investors are on the demand side of the equity market. Both classifications are of course correct, but our terminology is preferable for this paper given the focus on the allocation of financial capital to firms. For simplicity, we assume that the supply of the risk-free asset is infinitely elastic, so that the risk-free rate r_f is exogenous.

We express general equilibrium as a function of the Sharpe ratio of each stock, λ_n, and the risk allocated to each firm, Q_n, defined in Sections 2.2 and 3.2. The supply and demand of risk capital invested in each firm are conveniently expressed in vector form.

The supply of risk capital follows directly from the optimal portfolio rule defined in Section 2.1. The investor supplies $W_0\,\omega_n$ units of financial capital and therefore the risk capital $Q_{S,n} = W_0\,\omega_n\,\sigma_n$ to firm n. We stack the risk capital supplied to each firm into the column vector $\boldsymbol{Q_S}$. By the two-fund portfolio rule (4), it satisfies:

$$\boldsymbol{Q_S} = W_0 \boldsymbol{\sigma\omega} = \frac{W_0}{\gamma}\left[\boldsymbol{\sigma\Sigma}^{-1}(\boldsymbol{\mu} - r_f\mathbf{1}) - \gamma\boldsymbol{\sigma\Sigma^{-1}\sigma\theta_{\mathrm{CF}}}\right].$$

Since $\boldsymbol{\Sigma} = \boldsymbol{\sigma\rho_{\mathrm{CF}}\sigma}$ and $\boldsymbol{\lambda} = \boldsymbol{\sigma^{-1}}(\boldsymbol{\mu} - r_f\mathbf{1})$, the vector of supply curves can be written as

$$\boldsymbol{Q_S}(\boldsymbol{\lambda}) = \boldsymbol{\Delta}_{\mathrm{S}}\left(\boldsymbol{\lambda} - \gamma\boldsymbol{\theta_{\mathrm{CF}}}\right), \tag{24}$$

where

$$\boldsymbol{\Delta}_{\mathrm{S}} = \frac{W_0}{\gamma}\boldsymbol{\rho}_{\mathbf{CF}}^{-\mathbf{1}}. \tag{25}$$

The vector of supply curves is linear in the vector of Sharpe ratios $\boldsymbol{\lambda}$ and in the vector of exposures $\boldsymbol{\theta}_{\mathbf{CF}}$. By equation (20), the vector of supply schedules (24) implies the normalized SML derived in Proposition 2, with correlation premium $\lambda_{\mathrm{M}} - \gamma\theta_{\mathrm{CF,M}} = \gamma Q_{\mathrm{M}}/W_0$.

The demand for risk capital is determined by the entrepreneur. Equation (15) implies that

$$Q_{D,n} = \sigma_{\mathrm{CF},n}^{\eta+1}(a_{\mathrm{CF},n} - \lambda_n)^{\eta} \tag{26}$$

for every n. It is convenient to denote by

$$\boldsymbol{Q_D}(\boldsymbol{\lambda}) = \left[\sigma_{\mathrm{CF},1}^{\eta+1}(a_{\mathrm{CF},1} - \lambda_1)^{\eta}, \ldots, \sigma_{\mathrm{CF},N}^{\eta+1}(a_{\mathrm{CF},N} - \lambda_N)^{\eta}\right]'$$

the column vector containing the demand for risk capital of each firm.

The general equilibrium of the economy is a vector of risk premia $\boldsymbol{\lambda}$ such that $\boldsymbol{Q_D}(\boldsymbol{\lambda}) = \boldsymbol{Q_S}(\boldsymbol{\lambda})$. In the Appendix, we use Brouwer's fixed point theorem to prove the existence of GE in the absence of trading constraints. The result applies to economies with $\eta > 1$, as is clearly the case in the data.

Proposition 3 (Existence of a Frictionless General Equilibrium) *Let $\boldsymbol{a}_{\mathbf{CF}} = (a_{\mathrm{CF},1}, \ldots, a_{\mathrm{CF},N})'$ be the $N \times 1$ vector of firm profitability per unit of risk. Under the sufficient condition that the components of the vector $\boldsymbol{\Delta}_{\mathbf{S}}(\boldsymbol{a}_{\mathbf{CF}} - \gamma\boldsymbol{\theta}_{\mathbf{CF}})$ are nonnegative, the economy without trading constraints has at least one general equilibrium $\boldsymbol{\lambda}^{*}$.*

The sufficient condition involves only exogenous parameters of the model. It focuses on the best possible scenario for stock market investors, under which they receive all the economic profits of the firm. If stock market investors are willing to invest positive amounts in all N firms under this best-case scenario, then a GE exists. The sufficient condition rules out cases where some firms are so unprofitable and highly exposed to state risk that, even under the most possibly generous conditions, investors would still like to short them.

Our model nests endowment economies and linear technologies as limiting special cases. Endowment economies correspond to the limiting case $\eta = 0$: the capital stock of each firm is fixed and inelastic to changes in risk-adjusted profitability. By construction, an endowment economy cannot explain firm sizes, nor can it tie firm sizes to firm characteristics, unless ad hoc assumptions are made. Linear technologies correspond to the limiting case $\eta = +\infty$ in our model. Each firm's demand schedule is infinitely elastic and the price of risk is set so that the firm generates zero economic profits. Risk prices, as well as the mean and volatility of the stock, are therefore only driven by the firm's own profit function, independently of other firms or the stock market investor's attitude toward risk. In the Appendix, we show that these special economies, which have either fixed quantities or fixed prices, cannot jointly explain the main pricing anomalies in equity data.

4.2 Linearized Equilibrium

We derive an approximate solution to the equilibrium condition for economies with a strictly positive and finite elasticity $\eta \in (0, +\infty)$. Recall that $\lambda_n < a_{\mathrm{CF},n}$ in order for a firm to be active. When the Sharpe ratio λ_n of each firm is small compared to profitability per unit of volatility $a_{\mathrm{CF},n}$, the demand for risk capital Q_n in (26) can be approximated as

$$Q_{D,n} \approx \delta_{D,n} \left(\frac{a_{\mathrm{CF},n}}{\eta} - \lambda_n \right), \tag{27}$$

where

$$\delta_{D,n} = \eta\, \sigma_{\mathrm{CF},n}^{\eta+1}\, a_{\mathrm{CF},n}^{\eta-1}. \tag{28}$$

Firm n's demand sensitivity, $\delta_{D,n}$, is driven by the firm's capital elasticity, cash flow volatility, and profitability. The $N \times 1$ vector of demand curves thus satisfies:

$$\boldsymbol{Q_D} \approx \boldsymbol{\Delta_\mathrm{D}} \left(\frac{\boldsymbol{a_\mathrm{CF}}}{\eta} - \boldsymbol{\lambda} \right), \tag{29}$$

where

$$\boldsymbol{\Delta}_\mathrm{D} = \mathrm{diag}(\delta_{D,1}, \ldots, \delta_{D,N}) \tag{30}$$

is the diagonal matrix with elements $\delta_{D,1}, \ldots, \delta_{D,N}$. Combining (24) and (29) yields linear expressions for equilibrium quantities and risk premia.

Proposition 4 (General Equilibrium) *The risk capital and Sharpe ratio of each firm in general equilibrium are given by:*

$$\boldsymbol{Q} = \left(\boldsymbol{\Delta}_\mathrm{S}^{-1} + \boldsymbol{\Delta}_\mathrm{D}^{-1}\right)^{-1} \left(\frac{\boldsymbol{a_\mathrm{CF}}}{\eta} - \gamma \boldsymbol{\theta_\mathrm{CF}} \right), \tag{31}$$

$$\boldsymbol{\lambda} = (\boldsymbol{\Delta}_\mathrm{S} + \boldsymbol{\Delta}_\mathrm{D})^{-1} \left(\boldsymbol{\Delta}_\mathrm{D} \frac{\boldsymbol{a_\mathrm{CF}}}{\eta} + \gamma \boldsymbol{\Delta}_\mathrm{S}\, \boldsymbol{\theta_\mathrm{CF}} \right), \tag{32}$$

where $\boldsymbol{a}_{\mathbf{CF}}$ denotes the vector of firm profitabilities per unit of risk, $\boldsymbol{\theta}_{\mathbf{CF}}$ is the vector of state risk exposures, and $\boldsymbol{\Delta}_{\mathrm{S}}$ and $\boldsymbol{\Delta}_{\mathrm{D}}$ are defined in equations (25) *and* (30).

The proposition expresses the firms' equilibrium quantities and Sharpe ratios as functions of exogenous parameters. Furthermore, if $\eta = 1$, the demand equation (27) and the equilibrium solution, which in general hold as approximations, are in fact exact.

The size and discount rate of each firm in GE depend on the characteristics of every other firm in the economy, as equations (31) and (32) show. This property is well summarized by Cochrane (2008): in GE each firm adds another state variable to the equilibrium. Our simple economy allows to obtain a joint solution for risk premia and firm sizes.

The general equilibrium (31)-(32) provides the main variables of interest for asset pricing. The vector of risk capitals $\boldsymbol{Q}$ in equation (31) provides the market capitalizations $V_n = (Q_n/\sigma_{\mathrm{CF},n})^{1+1/\eta}$ and therefore the market portfolio. The volatilities of individual stocks, the dollar volatility of the market portfolio, and the correlations of individual stocks with the market are given by equations (18) to (20), which imply the correlation premium $\lambda_{\mathrm{M}} - \gamma\theta_{\mathrm{CF,M}} = \gamma Q_{\mathrm{M}}/W_0$ of the normalized SML. We now investigate how these key variables are driven by supply and demand effects.

5 Firm Size and Sharpe Ratio in General Equilibrium

We now explore how supply and demand forces jointly impact the cross-section of firm sizes and risk pricing in general equilibrium. Section 5.1 considers firms with homogeneous cash-flow correlations and develops closed-form expressions for firm-level supply and demand curves. Section 5.2 shows how these curves shape the formation of the market portfolio.

5.1 Firm-Level Supply and Demand Curves

To provide additional insights about general equilibrium, we consider a symmetric economy in which firms have identical *cash-flow* correlations:

$$\rho_{\mathrm{CF},i,j} = \bar{\rho}_{\mathrm{CF}} \tag{33}$$

for every $i \neq j$. This hypothesis conveniently allows us to express a firm's size and risk premium as a function of its own characteristics and aggregate factors that are invariant in the cross section.

The supply schedule in equation (24) simplifies to

$$Q_{S,n} = \delta_S \left(\lambda_n - \gamma \theta_{\mathrm{CF},n} - \frac{\gamma}{W_0} \bar{\rho}_{\mathrm{CF}} N \bar{Q}_S \right), \tag{34}$$

where

$$\delta_S = \frac{W_0}{\gamma} \frac{1}{1 - \bar{\rho}_{\mathrm{CF}}} \tag{35}$$

and $\bar{Q}_S$ is the average quantity of risk supplied to all firms.[11] The stock market investor is strongly willing to supply risk capital to firm n if the Sharpe ratio is high and state risk exposure is low. As the linear coefficient δ_S in equation (35) indicates, the supply of capital $Q_{S,n}$ steeply increases with the firm's Sharpe ratio if the investor has high risk tolerance or if other firms are close substitutes for firm n. The cross-sectional constant in equation (34), $(\gamma/W_0)\bar{\rho}_{\mathrm{CF}} N \bar{Q}_S$, quantifies crowding out due to investment in other firms. The crowding out effect is large if the investor cannot afford to bear large amounts of risk or if firm n is highly correlated to other firms.

In Figure 5, we follow the economics tradition of plotting the inverse supply and demand curves:

$$\lambda_{S,n} = \gamma\, \theta_{\mathrm{CF},n} + \frac{\gamma}{W_0} \bar{\rho}_{\mathrm{CF}}\, N\, \bar{Q}_S + \frac{Q_{S,n}}{\delta_S}, \tag{36}$$

$$\lambda_{D,n} = \frac{a_{\mathrm{CF},n}}{\eta} - \frac{Q_{D,n}}{\delta_{D,n}}. \tag{37}$$

As the previous section shows and as equations (36) and (37) illustrate, the firm profitability $a_{\mathrm{CF},n}$ drives firm n's demand for capital, while the covariance of investor state risk with firm cash flow risk, $\theta_{\mathrm{CF},n} = Cov(R_\theta, z_n)$, drives their willingness to supply capital. For this reason, we will henceforth view profitability as a demand factor and state risk as a supply factor.

The inverse-supply curve (36) is pushed outward for firms with low state risk exposures, and is flatter if the investor has low risk-bearing capacity or if the firm strongly correlates with other

[11] By equation (24), the supply schedule satisfies $\boldsymbol{\rho_{\mathrm{CF}}} \boldsymbol{Q_S} = (W_0/\gamma)(\boldsymbol{\lambda} - \gamma \boldsymbol{\theta_{\mathrm{CF}}})$ and therefore

$$\bar{\rho}_{\mathrm{CF}} \sum_{i=1}^{N} Q_i + (1 - \bar{\rho}_{\mathrm{CF}})\, Q_n = \frac{W_0}{\gamma} (\lambda_n - \gamma \theta_{\mathrm{CF},n})$$

for every n. Since $\sum_{i=1}^{N} Q_i = N\bar{Q}$, we infer that equation (34) holds.

firms. The inverse-demand curve (37) is pushed outward by firm profitability and is flatter if the firm is highly profitable or has a highly elastic production technology. The intersection of the inverse supply and demand curves provides the equilibrium size and Sharpe ratio of firm n.

Proposition 5 (Firm Sizes and Sharpe Ratios) *If firms have homogeneous cash-flow correlations, the equilibrium risk capital, Q_n, and Sharpe ratio, λ_n, of firm n satisfy*

$$Q_n = \delta_n \left(\frac{a_{\mathrm{CF},n}}{\eta} - \gamma\theta_{\mathrm{CF},n} - \frac{\gamma}{W_0}\bar{\rho}_{\mathrm{CF}} N \bar{Q} \right), \tag{38}$$

$$\lambda_n = \frac{\delta_{D,n}}{\delta_{D,n}+\delta_S}\frac{a_{\mathrm{CF},n}}{\eta} + \frac{\delta_{S,n}}{\delta_{D,n}+\delta_S}\left(\gamma\theta_{\mathrm{CF},n} + \frac{\gamma}{W_0}\bar{\rho}_{\mathrm{CF}} N \bar{Q} \right), \tag{39}$$

where $\delta_{D,n}$ is defined by equation (28)*, δ_S is defined by equation* (35)*, and*

$$\delta_n = \left(\delta_{D,n}^{-1} + \delta_S^{-1} \right)^{-1}. \tag{40}$$

The mean quantity of risk, $\bar{Q} = N^{-1}\sum_{n=1}^{N} Q_n$, has an explicit solution provided in the Appendix.

The proposition allows us to obtain clean results on the cross-section of firms. We focus on a given general equilibrium and therefore take $\bar{Q}$ as given. By (38), a firm's risk capital Q_n increases with the firm's profitability (a demand factor) and decreases with the firm's state risk exposure (a supply factor). By (39), the Sharpe ratio λ_n increases in both profitability and state risk. Altogether, the cross-sectional relationships can be summarized by:

$$Q_n = Q_n(\underset{+}{a_{\mathrm{CF},n}}, \underset{-}{\theta_{\mathrm{CF},n}}), \qquad \lambda_n = \lambda_n(\underset{+}{a_{\mathrm{CF},n}}, \underset{+}{\theta_{\mathrm{CF},n}}), \tag{41}$$

where the $+$ and $-$ symbols refer to increasing and decreasing relationships, respectively.

The equilibrium sensitivity of the firm size Q_n to profitability and state risk is determined by the coefficient δ_n. By equation (40), δ_n is the harmonic sum of the slopes of the supply and demand schedules. As is well-known with harmonic sums, δ_n is driven more strongly by the smallest of the two slopes, so that the firm's equilibrium sensitivity δ_n is mainly driven by the least sensitive of the demand and supply schedules. The equilibrium sensitivities of the Sharpe ratio λ_n to supply and demand factors are quantified by the coefficients $\delta_S/(\delta_{D,n}+\delta_S)$ and $\delta_{D,n}/(\delta_{D,n}+\delta_S)$. A highly sensitive supply curve implies that the firm's price of risk is mainly driven by its exposure to state risk, whereas a highly sensitive demand curve implies that λ_n mostly stems from the firm's profitability.

5.2 Cross-Section of Equilibrium Firm Sizes and Risk Prices

Proposition 5 has rich implications for the equilibrium cross-sectional relationship between firm quantities and risk prices, which can be illustrated by standard supply and demand analysis. When the supply schedule is the same for all firms, variation in the demand schedule $Q_{D,n}(\lambda_{D,n})$ across firms induces a *positive* cross-sectional relationship between quantities and prices because the firms' outcomes are all located on the common supply curve. Similarly, variation in the supply of capital $Q_{S,n}(\lambda_{S,n})$ induces a *negative* relationship between quantities and prices if the demand schedule is common to all firms.

Panel A of Figure 6 illustrates the demand channel by comparing two firms that differ in their profitability but have identical parameters otherwise. The more profitable firm (dashed line) demands more risk capital from the stock market investor and is more sensitive to the Sharpe ratio than the less profitable firm (solid line). Because the investor requires compensation to supply the additional risk capital, the more profitable firm has a higher price of risk than the less profitable firm in equilibrium. We note that the increased sensitivity of the more profitable firm (i.e. higher $\delta_{D,n}$) accentuates both the quantity and price effects.

Panel B of Figure 6 illustrates the supply channel by comparing two firms that differ only in their state risk exposures. Because the investor is willing to supply greater risk capital to the firm with low exposure at a lower cost, the firm responds to the low cost of equity by increasing production. Consequently, the firm with low state risk exposure (solid line) has a higher quantity of risk but a lower price of risk than the firm with high exposure (dashed line).

To sum up, heterogeneity in firm profitability (a demand channel) tends to generate a positive cross-sectional relationship between firm size and the Sharpe ratio. By contrast, heterogeneity in firm state risk exposures (a supply channel) induces a negative relationship between firm size and the Sharpe ratio.

Formation of the market portfolio. In Section 3, we highlighted that firms with strong profitability and low price of risk choose to invest more and have high market-to-book ratios and market values. By combining (22) and (38), we now obtain the following relationship,

$$V_n = K_n^{1+\frac{1}{\eta}} = \Big[Q_n(\underset{+}{a_{\mathrm{CF},n}}, \underset{-}{\theta_{\mathrm{CF},n}}) / \sigma_{\mathrm{CF},n} \Big]^{1+\frac{1}{\eta}}. \tag{42}$$

All else equal, a firm has a large market capitalization if it is highly profitable and demands high levels of capital from investors. A firm also has a large market capitalization if it has low state risk exposure and investors are willing to provide it with high levels of capital at a low cost. Both scenarios imply that firms that dominate the market portfolio tend to invest heavily, have a high market-to-book ratio, and are therefore growth stocks.

6 The Security Market Line in General Equilibrium

Supply and demand analysis allows us to develop new insights on the security market line. Sections 6.1 investigates the normalized SML and Section 6.2 its nonnormalized counterpart. We show that general equilibrium naturally generates SMLs with a positive intercept and a lower slope than predicted by the CAPM, as in the empirical work of Black, Jensen, and Scholes (1972), Fama and MacBeth (1973), and many others. Perhaps surprisingly, this result holds even when all firms have the same state risk exposure. Under heterogeneous state risk exposures, the model can also generate downward-sloping or concave security market lines, consistent with the empirical findings of Fama and French (1992), De Giorgi, Post, and Yalçın (2017), and Hong and Sraer (2016).

6.1 Implications of GE for the Normalized Security Market Line

The normalized SML defined in Proposition 2 relates a firm's Sharpe ratio to its market correlation:

$$\lambda_n = \gamma\theta_{\mathrm{CF},n} + \rho_{\mathrm{M},n}\,(\lambda_{\mathrm{M}} - \gamma\theta_{\mathrm{CF,M}}), \tag{43}$$

where, under (33), the stock's correlation to the market in equation (20) simplifies to

$$\rho_{\mathrm{M},n} = [\bar{\rho}_{\mathrm{CF}}\, N\, \bar{Q}_S + (1-\bar{\rho}_{\mathrm{CF}})\, Q_{S,n}]/Q_{\mathrm{M}}. \tag{44}$$

Since risk capital and market correlation are related by a common affine relationship (44) for all firms, we can illustrate supply and demand in the plane of market correlation and Sharpe ratios, and thereby obtain the normalized SML. We successively discuss the normalized SML in general equilibrium for: (1) economies with zero state risk exposure that generate the CAPM; (2) economies in which firms have homogeneous state risk exposure, which implies a flatter SML, and (3) economies in which firms have heterogeneous state risk exposures, which can generate a downward-sloping or nonlinear SML.

Case 1: No exposure to state risk (CAPM) When firms have zero state risk exposures ($\theta_{\text{CF},n} = 0$ for all n), the CAPM holds, as Proposition 2 implies. This equilibrium is particular because by (36) and (44), all firms have the same inverse supply schedule:

$$\lambda_{S,n}^{CAPM} = \frac{\gamma}{W_0}[\bar{\rho}_{\text{CF}}\, N\, \bar{Q}_S + (1-\bar{\rho}_{\text{CF}})\, Q_{S,n}] = \frac{\gamma Q_{\text{M}}}{W_0}\, \rho_{\text{M},n}. \tag{45}$$

Firm equilibria therefore consist of different points along the same upward-sloping inverse supply curve (45). In Panel A of Figure 7, we plot the equilibrium outcomes of two firms with different profitabilities. Since firms share the same inverse supply curve, the cross-sectional relationship between price and quantity is positive. The Sharpe ratio of stock n in (45) is proportional to the correlation to the market, as the CAPM implies.

Case 2: Constant exposure to state risk (SML Flattening) If firms have homogeneous state risk exposures, $\theta_{\text{CF},n} = \bar{\theta}_{\text{CF}} > 0$, the normalized SML becomes

$$\lambda_n = \gamma\bar{\theta}_{\text{CF}} + \rho_{\text{M},n}\,(\lambda_{\text{M}} - \gamma\bar{\theta}_{\text{CF}}). \tag{46}$$

As Panel B of Figure 7 illustrates, the normalized SML differs from the predictions of the CAPM along two dimensions: (i) the intercept, $\gamma\bar{\theta}_{\text{CF}}$, is positive and (ii) the slope, $(\lambda_{\text{M}} - \gamma\bar{\theta}_{\text{CF}})$ is lower than the Sharpe ratio of the market. Since every firm is exposed to state risk, the Sharpe ratio required to invest in a firm uncorrelated to the market is positive and equal to the intercept $\gamma\bar{\theta}_{\text{CF}}$. The SML is also flatter than the CAPM predicts. Once the stock's reward for state risk exposure is taken into account, its incremental reward for market exposure, $\rho_{\text{M},n}\,(\lambda_{\text{M}} - \gamma\bar{\theta}_{\text{CF}})$, nets out the market portfolio's reward for state risk. Thus, in GE, the normalized SML naturally rotates in the presence of state risk, for reasons other than heterogeneity in firm exposures.

Case 3: Varying exposure to state risk (Nonlinear or Downward-Sloping SML) If firms have varying exposures to state risk, the inverse supply curve given by (36) is no longer the same across stocks. As a result, the normalized SML is generally nonlinear and can even slope down.

As Figure 7 shows, the shape of the normalized SML depends on the characteristics along which stocks are sorted. The SML is upward-sloping among stocks that have different profitability but similar exposures to state risk (Panel B), because the cross-section produces equilibrium outcomes along the common supply curve. However, it is downward-sloping among stocks that have similar profitability but different exposures to state risk (Panel C), because firm outcomes are now located along the common demand curve. If firms differ in both profitability and state risk exposure (Panel D), the slope of the SML is driven by the dominant source of heterogeneity.

In linear or endowment economies, a downward-sloping SML cannot arise endogenously and can only exist in GE if it is hard-wired into the model, as Figure 8 illustrates. To generate a downward-sloping SML in an endowment economy (Panel A), one needs to assume that the low-supply firm has a lower market correlation than the high-supply firm. Similarly, in a linear production setting (Panel B), one needs to assume that the low-supply firm has a higher Sharpe ratio than the high-supply firm. By contrast, a downward-sloping SML arises naturally when quantities and prices of risk are both flexible (Panel C).

6.2 Equilibrium Relationship between Beta and Expected Return

We now investigate the cross-sectional relationship between beta and expected return in general equilibrium. The endogeneity of stock return volatilities implies that the usual SML has slightly richer properties than the normalized SML studied in the previous section.

Drivers of beta The firm's beta, defined in (5), is driven by market correlation and volatility:

$$\beta_n = \rho_{\mathrm{M},n}(\underset{+}{a_{\mathrm{CF},n}}, \underset{-}{\theta_{\mathrm{CF},n}})\ \ \sigma_n(\underset{-}{a_{\mathrm{CF},n}}, \underset{+}{\theta_{\mathrm{CF},n}})\ /\ \sigma_{\mathrm{M}}. \tag{47}$$

A firm with high profitability or low state risk has high market value and high market correlation, as discussed in previous section. This *correlation* channel tends to generate a high market beta. However, there is also a countervailing *volatility* channel, which induces stocks with high profitability and low state risk to have low volatility,

$$\sigma_n = \sigma_{\mathrm{CF},n}^{1+\frac{1}{\eta}} \left[Q_n(\underset{+}{a_{\mathrm{CF},n}}, \underset{-}{\theta_{\mathrm{CF},n}}) \right]^{-\frac{1}{\eta}}, \tag{48}$$

and therefore a lower market beta.

We illustrate both channels in the top row of Figures 9 and 10. Because the correlation and volatility channels offset each other, the firm's beta is generally non-monotonic in $a_{\text{CF},n}$ and $\theta_{\text{CF},n}$. This property is consistent with the empirical evidence in Asness et al. (2019) that beta is driven by both the correlation and volatility channels.

When technologies exhibit mildly decreasing returns to scale ($\eta > 1$), the correlation channel generally dominates, as the various panels of Figure 10 show.[12] This property is consistent with the data. In Table 1, we run a quick empirical exercise and separately regress the firms' log market beta on their log market correlation and their log volatility on the last day of our sample. The R^2 of the market correlation regression is substantially higher (46.5%) than the R^2 of the volatility regression (6.4%). Because the correlation channel generally dominates, high-beta firms generally consist of larger firms with high profitability and low state risk exposure. The relationships are not strictly monotonic, however, so that high-beta firms also include highly volatile small firms.

Drivers of expected returns and the SML A stock's expected excess return is the product of its Sharpe ratio and return volatility:

$$\mu_n - r_f = \lambda_n(\underset{+}{a_{\text{CF},n}}, \underset{+}{\theta_{\text{CF},n}})\ \sigma_n(\underset{-}{a_{\text{CF},n}}, \underset{+}{\theta_{\text{CF},n}}). \tag{49}$$

State risk exposure unambiguously increases the firm's expected return through its positive effects on the Sharpe ratio and volatility. By contrast, high profitability induces a high Sharpe ratio but a low volatility, so the overall effect on the stock's expected return is generally ambiguous. Intuitively, the more profitable firm demands more capital from investors who then require compensation for taking a more concentrated bet. But as the firm becomes larger, its volatility decreases and the overall impact of profitability on the firm's expected return is unclear.

When firms feature mildly decreasing returns to scale, the volatility channel is weak and the Sharpe ratio channel generates a positive relationship between profitability and expected return, consistent with the empirical evidence (Fama and French 2006, Haugen and Baker 1996, Novy-Marx 2013). We illustrate this property in the bottom rows of Figures 9 and 10. The relationship

[12] In the extreme case where firms have linear technologies ($\eta \to \infty$), the volatility effect disappears and the correlation channel drives beta for virtually all firms. This is consistent with the well-known result that the stock's volatility converges to the exogenously specified cash flow volatility $\sigma_{\text{CF},n}$.

between the firm's expected return and its beta is therefore similar to that between Sharpe ratio and market correlation. Specifically, cross-sectional variation in profitability leads to an upward-sloping SML, whereas cross-sectional variation in state risk exposure leads to a downward-sloping SML. The slope of the SML depends on the characteristics of the stocks that are being compared.

7 Alpha in General Equilibrium

7.1 Drivers of Alpha and the Correlation and Beta Anomalies

The deviation from the expected return predicted by the CAPM, $\alpha_n = \mu_n - r_f - \beta_n(\mu_M - r_f)$, satisfies

$$\frac{\alpha_n}{\sigma_n} = \gamma \left[\theta_{CF,n} - \theta_{CF,M}\, \rho_{M,n}(\underset{+}{a_{CF,n}}, \underset{-}{\theta_{CF,n}}) \right], \tag{50}$$

as Proposition 2 implies. The alpha-to-volatility ratio α_n/σ_n is a variant of the firm's Information Ratio (Treynor and Black 1973).

By equation (50), the alpha-to-volatility ratio of a stock is driven by (i) the direct effect of state risk on the Sharpe ratio, $\gamma\theta_{CF,n}$, and (ii) the stock's correlation to the market, $\rho_{M,n}$. Market correlation, given in equation (44), is driven by firm size and therefore represents a source of cross-sectional variation of the alpha-to-volatility ratio. Any variable affecting firm size, such as profitability or state risk exposure, can therefore impact the alpha-to-volatility ratio via the correlation channel. This mechanism stems from the flatness of the normalized SML and therefore arises in a broad set of production economies. It does not operate, however, in endowment economies with fixed firm sizes.

The correlation channel has a simple graphical interpretation. As noted in Section 5.2, the slope of the normalized SML is flatter than the slope predicted by the CAPM. Thus if one mistakenly uses the CAPM to price firms, one overprices firms that strongly correlate with the market, and conversely. A portfolio with unit market correlation and the same state risk as the market is correctly priced.

The market correlation channel generates the Betting-Against-Correlation factor (BAC) recently documented by Asness et al. (2019). The BAC factor goes long stocks with low market-correlation and shorts stocks with high correlations, and is known to generate positive alpha. This effect immediately arises in GE since alpha decreases with the stock's market correlation.

In addition to BAC, there is evidence that portfolios that are short high-beta stocks and long low-beta stocks tend to generate positive alpha (Black 1993, Black, Jensen, and Scholes 1972, Hong and Sraer 2016). This observation prompted the development of betting-against-beta ("BAB") strategies (Asness, Moskowitz, and Pedersen 2013, Bali et al. 2017, Frazzini and Pedersen 2014). In our setting, a stock's alpha is given by

$$\alpha_n = \gamma(\theta_{\mathrm{CF},n}\,\sigma_n - \theta_{\mathrm{CF,M}}\,\sigma_{\mathrm{M}}\beta_{\mathrm{M},n})\,. \tag{51}$$

Section 6.2 shows that, when technologies exhibit mildly decreasing returns to scale, endogenous variation in volatility plays only a minor role. A stock with a high beta tends to have a negative alpha, and conversely. Thus, BAB and BAC arise naturally in general equilibrium.

We emphasize that we obtain BAB and BAC in an economy in which capital is allocated by a representative investor facing no trading constraints. The present theory therefore contrasts with earlier explanations of BAB and BAC based on market frictions or investor heterogeneity. For example, Black (1993) and Frazzini and Pedersen (2014) generate BAB via a leverage constraint, while Hong and Sraer (2016) consider both heterogeneous beliefs and short-sales constraints. In our model, BAC and BAB are inevitable outcomes of GE in the presence of state risk, which hold regardless of market frictions.

7.2 Equilibrium Relationships between Alpha and Firm Characteristics

In addition to BAB and BAC, the model coherently generates a number of firm-based anomalies.

Size, value, investment and profitability anomalies. The evidence on firm-based characteristics reveals that stocks with small-capitalization, low market-to-book ratio (value), and high investment persistently generate positive alpha, whereas stocks with large-capitalization, high market-to-book ratio (growth), and low investment generate negative alpha (see Fama and French 1993 and Cooper, Gulen, and Schill 2008, among many others). There is also evidence of a profitability anomaly conditional on the market-to-book ratio (Novy-Marx 2013).

We obtain all these relationships in the model. As we discuss in Section 5.2, stocks with strong profitability and low state risk exposure tend to be large growth stocks with high investment. Equation (50) shows that such stocks tend to generate negative alpha. By contrast, stocks with low profitability and high exposure to state risk tend to be small value stocks with low investment and positive alpha.

Because stocks that are larger, invest more, and have high market-to-book ratio tend to have higher market correlation and higher beta, the model predicts that the BAC and BAB factors should be negatively related to size, value, and investment growth anomalies. This prediction of the model is exactly what is observed in the data (Bali et al. 2017, Frazzini and Pedersen 2014, De Giorgi, Post, and Yalçın 2017, Liu 2018).

Novy-Marx (2013) documents a gross profitability premium unexplained by the CAPM that co-exists with the value premium (see also Fama and French 2015, and Hou, Xue, and Zhang 2015). The evidence indicates that, if one compares stocks with the same market-to-book ratio, those with high gross profitability tend to generate positive alpha whereas stocks with low gross profitability generate negative alpha.

The gross profitability premium naturally arises in the model. By (22), a firm's market to book ratio is equal to $V_n/K_n = [Q_n(a_{\mathrm{CF},n}, \theta_{\mathrm{CF},n})/\sigma_{\mathrm{CF},n}]^{\frac{1}{\eta}}$, where the risk capital Q_n is known to increase with profitability and decrease with state risk. Assume for simplicity that all firms have the same cash flow volatility: $\sigma_{\mathrm{CF},n} = \sigma_{\mathrm{CF},1}$. If one compares several firms with the *same* market-to-book ratio, $V_n/K_n = V_1/K_1$, the more profitable firms have *high* exposures to state risk while the less profitable firms bear low state risk. Sorting these firms by profitability is thus equivalent to sorting them by state risk exposure.[13]

Moreover, because firms with the same market-to-book ratio, V_1/K_1, also have the same market correlation, $\rho_{\mathrm{M},n} = \rho_{\mathrm{M},1}$, this particular sort neutralizes the impact of the market correlation channel on alpha:

$$\alpha_n = \gamma \sigma_{\mathrm{CF},1} \frac{K_1}{V_1} \left(\theta_{\mathrm{CF},n} - \theta_{\mathrm{CF,M}}\, \rho_{\mathrm{M},1}\right). \tag{52}$$

Thus, the alpha of each firm is solely driven by its exposure to state risk. Because the less profitable

[13]Hou, Xue, and Zhang (2015) and Zhang (2017) obtain a similar result. Controlling for the firm's level of investment, firms with high profitability have higher expected returns because they must also have a higher discount rate.

firm is also less exposed to state risk, it must have lower alpha than the more profitable firm. Consequently, a strategy that tilts toward the more profitable stocks and away from the less profitable stocks generates positive alpha, consistent with the gross profitability anomaly.

Asymmetric volatility anomaly Stambaugh, Yu, and Yuan (2015) document that the relation between the firm's volatility and its alpha is positive among firms that are underpriced by the CAPM and negative among overpriced firms. We obtain this relation in the model. The firm's alpha can be decomposed as,

$$\alpha_n = \frac{\alpha_n}{\sigma_n}(\underset{-}{a_{\mathrm{CF},n}}, \underset{+}{\theta_{\mathrm{CF},n}}) \times \sigma_n(\underset{-}{a_{\mathrm{CF},n}}, \underset{+}{\theta_{\mathrm{CF},n}}). \tag{53}$$

The firm's alpha-to-volatility ratio and its volatility tend to move together as they are similarly driven by the firm's profitability and state risk exposure. However, because of its multiplicative nature, the volatility effect has an asymmetric impact on the firm's alpha. When the firm has negative alpha, a drop in volatility leads to an increase in alpha toward zero. Conversely, when the firm has positive alpha, a drop in volatility leads to a decrease in alpha toward zero. The relationship between alpha and volatility is asymmetric, consistent with the empirical evidence.

7.3 Take-Aways for Future Research

Anomalies come as a pack. The first takeaway is that the size, value, investment growth, BAB, and profitability effects all jointly result from firm-level variation in profitability and state risk. These effects endogenously arise as equilibrium outcomes of market-clearing mechanisms, irrespective of the exact nature of the state risks. As such, these effects are likely to be observed across asset classes, countries, and time, which is consistent with the empirical evidence (Fama and French 1998, 2012, Asness, Moskowitz, and Pedersen 2013, Hawawini and Keim 2000).

As discussed in Section 3, our study reinforces and broadens the point made by Berk (1995) that size-related anomalies should not be regarded as anomalous. On the contrary, it is the absence of these effects that should be puzzling. Our model shows that this argument applies not only to size-related anomalies, but also to investment growth, BAB, and profitability because of the quantity effect that comes along with the price effect. These anomalies *all* proxy for the additional forms of risk premium not captured by misspecified asset pricing models.

Firm characteristics correlated with alpha need not reflect underlying risk exposures The fact that firm-level variation in profitability generates the anomalies underlines a critical point. Cross-sectional variation in firm characteristics that is empirically associated with alpha does not necessarily reflect variation in the firms' underlying risk exposures. For example, as (50) illustrates, a variable like the firm's profitability affects its market correlation and, consequently, its alpha, even though it has nothing to do with the firm's exposure to state risk. This could potentially explain why hundreds of empirical factors related to firm characteristics have been documented to this day (Harvey, Liu, and Zhu 2015).

Anomalies should be observed across and within industries. The model predicts that if one focuses on firms with similar state risk exposures, such as firms in the same industry, variation in profitability still generates deviations from the CAPM, such as BAB and value. The empirical work of Asness, Frazzini, and Pedersen (2014), Baker, Bradley, and Taliaferro (2014) and Banko, Conover, and Jensen (2006) indeed shows that anomalies exist both across and within industries.

8 Conclusion

Over the past several decades, the investor-based and production-based literatures have produced numerous insights about the properties of investor supply and firm demand for financial capital and their respective implications for equity pricing. Building on the promise of earlier full GE models, we have developed an approach that explicitly studies the equilibrium interactions of supply and demand. The GE has a closed-form linear approximate solution in the general case. When cash-flow correlations are homogeneous across firms, the cross-section of equilibrium outcomes can be obtained by plotting the supply and demand curves of each firm in the same plane. Our approach thus brings to equity pricing the simplicity and convenience of familiar GE models, such as the Edgeworth box or IS-LM.

We confirm Cochrane's (2017) prediction that general equilibrium is full of surprises, and these surprises are mostly good. A frictionless closed-form GE model can coherently explain some of the leading anomalies in asset pricing, such as size, profitability, investment growth, value, asymmetric volatility, BAB and BAC, while also fitting the cross-section of firm characteristics. The main driving force of our model is that firm sizes, beta coefficients, and Sharpe ratios are endogenously driven by fundamental firm-level characteristics, such as expected profitability and state risk exposure.

In the presence of state risk, the security market line is flatter than is predicted by the CAPM, consistent with the empirical evidence. Any variable driving the size of a firm impacts its correlation with the market portfolio and therefore its alpha. As a consequence, a stock's alpha can be driven by a number of supply and demand factors that are not necessarily tied to fundamental risk.

Our model also shows that negative relationships between beta and expected return can arise in general equilibrium among stocks with different state risk exposures. This result is noteworthy because, historically, the fact that some low-beta stocks generate higher returns than high-beta stocks has been viewed as a core violation of the trade-off between risk and return. In our model, a downward-sloping SML arises whenever heterogeneity in state risk dominates heterogeneity in average profitability even though agents are fully rational and face no stock trading constraints.

The paper suggests several directions for future research. Consistent with the central role of the Sharpe ratio in the CAPM and continuous-time finance, we obtain more straightforward relationships between firm returns and their characteristics once we scale them by volatility. This theoretical insight is consistent with the BAC approach of Asness et al. (2019) and, more broadly, may help identify cleaner asset pricing relationships in future work.

In this paper, we have purposely remained agnostic about the sources of state risk. Future work may be either more precise about state risk or consider alternative sources of investor supply. For instance, variation in supply may stem from human capital, investor preference for socially responsible investing, or differences in beliefs. A full quantitative assessment of our approach would also be of interest, for instance to explain the magnitude of risk premia and the slope of the SML. We leave these questions for future research.

APPENDIX

Data Construction

The empirical exercises presented in Sections 3.3 and 6.2 are based on merged data from CRSP and Compustat.[14] We calculate book equity exactly as in Hou, Xue, and Zhang (2015). Book equity corresponds to shareholders' equity, plus deferred taxes and investment tax credit, minus preferred stock. Firms with negative book equity are dropped from the data. Our sample goes from 1980-01-02 to 2018-12-31 and includes 2,906 firms as of Dec 2018. These firms represent about 40% of the firms in the CRSP universe and 60% of the total market capitalization. Information about daily market risk premia and risk-free rates comes from Ken French's data library.

Stock-level measures of historical volatilities and market correlations are estimated for the last day of the sample (2018-12-31). Market correlation are calculated using overlapping 3-day sum of log returns over 1259 trading days and require 750 trading days of non-missing data. Volatilities are calculated 1-day log returns over 251 trading days and require 120 trading days of non-missing data. A stock's beta is then calculated as the product of its volatility and market correlation, divided by the volatility of the market portfolio.

Appendix to Section 2 (Investors and Asset Pricing)

Optimal Portfolio

Let $r_w = r_f + \boldsymbol{\omega}'(\boldsymbol{r} - r_f\mathbf{1}) + R_\theta$. Maximizing expected utility,

$$U = \mathbb{E}\left[W_0\left(1 + r_w\right)\right] - \frac{\gamma}{2W_0} Var[(1 + r_w)W_0], \tag{54}$$

is equivalent to maximizing the criterion

$$\mathbb{E}(r_w) - \frac{\gamma}{2} Var(r_w) = r_f + \boldsymbol{\omega}'(\boldsymbol{\mu} - r_f\mathbf{1}) + \mathbb{E}(R_\theta) - \frac{\gamma}{2}\left[\boldsymbol{\omega}'\boldsymbol{\Sigma}\boldsymbol{\omega} + 2\boldsymbol{\omega}'\boldsymbol{\sigma}\boldsymbol{\theta}_{\mathbf{CF}} + Var(R_\theta)\right]. \tag{55}$$

[14]The procedure that is used to merge both datasets is standard. We use the link table `crsp.ccmxpf_lnkused` and select observations for which `usedflag = 1` and flag types are `LC`, `LU`, or `LS`. For firms with multiple share classes, we select the security that has the largest market capitalization. The other securities are dropped from the data and their market capitalizations are attributed to the main security. The Compustat record is then merged onto the main security.

We differentiate this equation with respect to $\boldsymbol{\omega}$:

$$0 = \boldsymbol{\mu} - r_f \mathbf{1} - \gamma \left(\boldsymbol{\Sigma\omega} + \boldsymbol{\sigma\theta}_{\mathbf{CF}} \right),$$

and conclude that equation (4) holds.

Proof of Proposition 1

The proof directly results from the optimal investor portfolio rule. Equation (4) implies

$$\boldsymbol{\Sigma\omega} = \frac{1}{\gamma}(\boldsymbol{\mu} - r_f \mathbf{1}) - \boldsymbol{\sigma\theta}_{\mathbf{CF}}. \tag{56}$$

We denote by $\boldsymbol{v} = (V_1/V_{\mathrm{M}}, \ldots, V_N/V_{\mathrm{M}})'$ the $N \times 1$ vector of firm shares in the market portfolio. In equilibrium, $V_{\mathrm{M}}\, \boldsymbol{v} = W_0\, \boldsymbol{\omega}$ or equivalently

$$\boldsymbol{\Sigma v} = \frac{W_0}{V_{\mathrm{M}}} \left[\frac{1}{\gamma} \left(\boldsymbol{\mu} - r_f \mathbf{1} \right) - \boldsymbol{\sigma\theta}_{\mathbf{CF}} \right]. \tag{57}$$

The variance of the market portfolio, $\sigma_{\mathrm{M}}^2 = \boldsymbol{v}'\boldsymbol{\Sigma v}$, is therefore:

$$\sigma_{\mathrm{M}}^2 = \frac{W_0}{V_{\mathrm{M}}} \left[\frac{1}{\gamma} \boldsymbol{v}' \left(\boldsymbol{\mu} - r_f \mathbf{1} \right) - \boldsymbol{v}'\boldsymbol{\sigma\theta}_{\mathbf{CF}} \right] = \frac{W_0}{\gamma V_{\mathrm{M}}} \left(\mu_{\mathrm{M}} - r_f - c_{\mathrm{M}} \right), \tag{58}$$

which implies that (8) holds.

The vector of market betas, $\boldsymbol{\beta} = \sigma_{\mathrm{M}}^{-2} \boldsymbol{\Sigma v}$, satisfies:

$$\boldsymbol{\beta} = \frac{W_0}{V_{\mathrm{M}}} \frac{1}{\gamma \sigma_{\mathrm{M}}^2} \left(\boldsymbol{\mu} - r_f \mathbf{1} - \gamma \boldsymbol{\sigma\theta}_{\mathbf{CF}} \right). \tag{59}$$

Combining (58) and (59), we conclude that equation (6) holds.

Appendix to Section 3 (Entrepreneurs)

Optimal Investment. Entrepreneurs choose the level of investment I_n that maximizes the net present value of the firm defined in equation (14). The optimal investment level solves the first-order condition

$$\frac{\sigma_{\mathrm{CF},n} \left(a_{\mathrm{CF},n} - \lambda_n \right)}{1 + r_f} \frac{\partial K_n}{\partial I_n}(I_n) = 1. \tag{60}$$

Under specification (11), the investment level satisfying the first-order-condition (60) is

$$I_n = \frac{\eta}{\eta+1}\left[\frac{\sigma_{\mathrm{CF},n}\,(a_{\mathrm{CF},n}-\lambda_n)}{1+r_f}\right]^{\eta+1}. \tag{61}$$

The optimal level of installed capital is therefore given by (15).

Correlation between the Stock and the Market. The covariance between the return on stock n and the return on the market portfolio satisfies

$$Cov(R_n, R_{\mathrm{M}}) \;=\; \sum_{i=1}^{N} \frac{V_i}{V_{\mathrm{M}}} Cov(R_n, R_i) = \sum_{i=1}^{N} \frac{V_i}{V_{\mathrm{M}}} \rho_{\mathrm{CF},n,i}\,\sigma_n\,\sigma_i.$$

The correlation between the stock and the market is therefore:

$$Corr(R_n, R_{\mathrm{M}}) = \frac{\sum_{i=1}^{N} V_i\,\rho_{\mathrm{CF},n,i}\,\sigma_i}{V_{\mathrm{M}}\,\sigma_{\mathrm{M}}}.$$

We infer from (18) that $\sigma_i V_i = Q_i$ and conclude that (20) holds.

Appendix to Section 4 (General Equilibrium)

Proof of Proposition 3 (Existence)

Consider the vector $\boldsymbol{p} = \boldsymbol{a}_{\mathbf{CF}} - \boldsymbol{\lambda} \in \mathbb{R}_+^N$ and the excess demand function:

$$\boldsymbol{Z}(\boldsymbol{p}) = \boldsymbol{Q_D}(\boldsymbol{a}_{\mathbf{CF}} - \boldsymbol{p}) - \boldsymbol{Q_S}(\boldsymbol{a}_{\mathbf{CF}} - \boldsymbol{p}).$$

The demand vector $\boldsymbol{Q_D}(\boldsymbol{a}_{\mathbf{CF}} - \boldsymbol{p})$ has components

$$Q_{D,i}(\boldsymbol{a}_{\mathbf{CF}} - \boldsymbol{p}) = \sigma_{\mathrm{CF},i}^{\eta+1}\, p_i^{\eta}\, 1_{\{p_i \geq 0\}}.$$

The expression for $\boldsymbol{Q_D}$ hinges on the first-order condition of each firm. It is only valid if $p_i = a_{\mathrm{CF},i} - \lambda_i \geq 0$, and we henceforth focus on this case. The supply demand function satisfies:

$$-\boldsymbol{Q_S}(\boldsymbol{a} - \boldsymbol{p}) = \frac{W_0}{\gamma}\mathbf{min}[\boldsymbol{\rho}_{\mathbf{CF}}^{-1}(\boldsymbol{p} - \boldsymbol{a}_{\mathbf{CF}} + \gamma\boldsymbol{\theta}_{\mathbf{CF}}); 0].$$

We know that if $p_i = 0$, then $Z_i(\boldsymbol{p}) \leq 0$. Furthermore if $\eta > 1$, the convex demand curve dominates the linear supply curve for large values of $\boldsymbol{p}$. In particular, there exists M such that $p_i = M$ and $\max_j |p_j| \leq M$ implies that $Z_i(\boldsymbol{p}) > 0$.

The set $C = [0;M]^N$ is convex and compact. We define the auxiliary function $\boldsymbol{G} : C \to C$ by

$$G_j(\boldsymbol{p}) = \psi[p_j - Z_j(\boldsymbol{p})] \text{ for all } j,$$

where $\psi(x) = \min[\max(x,0),M]$. The function $\boldsymbol{G}$ is continuous on C. By Brouwer's fixed point theorem, $\boldsymbol{G}$ has a fixed point $\boldsymbol{p^*}$, which satisfies

$$p_j^* = \psi[p_j^* - Z_j(\boldsymbol{p^*})] \tag{62}$$

for all j.

Consider $j \in \{1,...,N\}$. If $p_j^* = 0$, then $Z_j(\boldsymbol{p^*}) \leq 0$, as explained above. The fixed point equation (62) therefore implies that $Z_j(\boldsymbol{p^*}) = 0$. If $p_j^* = M$, then $Z_j(\boldsymbol{p^*}) > 0$, and the fixed point equation cannot hold. If $p_j^* \in (0,M)$, then

$$p_j^* = p_j^* - Z_j(\boldsymbol{p^*}),$$

and therefore $Z_j(\boldsymbol{p^*}) = 0$. We therefore know that $Z_j(\boldsymbol{p^*}) = 0$. Since this results holds for every j, we conclude that $\boldsymbol{p^*}$ is a general equilibrium.

Proof of Proposition 4 (Linear Solution)

The demand for capital of firm n can be approximated with a first order Taylor expansion around $\lambda_n = 0$:

$$Q_{D,n} = \sigma_{\mathrm{CF},n}^{\eta+1} \left(a_{\mathrm{CF},n} - \lambda_n\right)^{\eta} \approx \eta\, \sigma_{\mathrm{CF},n}^{1+\eta}\, a_{\mathrm{CF},n}^{\eta-1} \left(\frac{a_{\mathrm{CF},n}}{\eta} - \lambda_n\right). \tag{63}$$

We observe that the approximation is exact if $\eta = 1$.

The system of market clearing equations $\boldsymbol{Q_S(\lambda)} = \boldsymbol{Q_D(\lambda)}$ can therefore be rewritten as:

$$\boldsymbol{\Delta}_S \left(\boldsymbol{\lambda} - \gamma \boldsymbol{\theta}_{\mathbf{CF}}\right) = \boldsymbol{\Delta}_D \left(\frac{\boldsymbol{a}}{\eta} - \boldsymbol{\lambda}\right). \tag{64}$$

Equation (64) implies that

$$(\boldsymbol{\Delta}_S + \boldsymbol{\Delta}_D)\boldsymbol{\lambda} = \boldsymbol{\Delta}_S\, \gamma \boldsymbol{\theta}_{\mathbf{CF}} + \boldsymbol{\Delta}_D \boldsymbol{\lambda},$$

and the vector of Sharpe ratios $\boldsymbol{\lambda}$ is therefore given by equation (32).

The matrix $\boldsymbol{\Delta} = \left(\boldsymbol{\Delta_S^{-1}} + \boldsymbol{\Delta_D^{-1}}\right)^{-1}$ is the $N \times N$ matrix harmonic sum of the demand and supply slopes. It satisfies:

$$\boldsymbol{\Delta} = \boldsymbol{\Delta}_S \left(\boldsymbol{\Delta}_S + \boldsymbol{\Delta}_D\right)^{-1} \boldsymbol{\Delta}_D \tag{65}$$

$$= \boldsymbol{\Delta}_D \left(\boldsymbol{\Delta}_S + \boldsymbol{\Delta}_D\right)^{-1} \boldsymbol{\Delta}_S. \tag{66}$$

We also note that

$$\boldsymbol{\Delta}_{\mathrm{S}} \left(\boldsymbol{\Delta}_{\mathrm{S}} + \boldsymbol{\Delta}_{\mathrm{D}}\right)^{-1} \boldsymbol{\Delta}_{\mathrm{S}} - \boldsymbol{\Delta}_{\mathrm{S}} = \boldsymbol{\Delta}_{\mathrm{S}} \left(\boldsymbol{\Delta}_{\mathrm{S}} + \boldsymbol{\Delta}_{\mathrm{D}}\right)^{-1} \left(\boldsymbol{\Delta}_{\mathrm{S}} + \boldsymbol{\Delta}_{\mathrm{D}}\right) - \boldsymbol{\Delta} - \boldsymbol{\Delta}_{\mathrm{S}} = -\boldsymbol{\Delta}. \tag{67}$$

The equilibrium quantity of risk $\boldsymbol{Q} = \boldsymbol{\Delta_S}(\boldsymbol{\lambda} - \gamma \boldsymbol{\theta_{CF}})$ satisfies

$$\boldsymbol{Q} = \boldsymbol{\Delta}_S \left[\left(\boldsymbol{\Delta}_{\mathrm{S}} + \boldsymbol{\Delta}_{\mathrm{D}}\right)^{-1} \boldsymbol{\Delta}_{\mathrm{D}} \frac{\boldsymbol{a}}{\eta} + \left(\boldsymbol{\Delta}_{\mathrm{S}} + \boldsymbol{\Delta}_{\mathrm{D}}\right)^{-1} \boldsymbol{\Delta}_{\mathrm{S}} \gamma \boldsymbol{\theta_{CF}} - \gamma \boldsymbol{\theta_{CF}} \right]. \tag{68}$$

We infer from (65) and (67) that the equilibrium quantity of risk satisfies (31).

Special Cases

Our model nests endowment economies and linear technologies as limiting special cases.

Endowment Economies

Endowment economies correspond to the limiting case $\eta = 0$: the capital stock of each firm is fixed and inelastic to changes in risk-adjusted profitability. The demand for financial capital is specified by a fixed vector $\boldsymbol{Q_D} \in \mathbb{R}_+^N$.[15] By construction, an endowment economy cannot explain firm sizes, nor can it tie firm sizes to firm characteristics, unless ad hoc assumptions are made.

Endowment economies generate the value and profitability effects. By Proposition 1, the vector of equilibrium Sharpe ratios, $\boldsymbol{\lambda} = (\gamma/W_0)\boldsymbol{\rho_{CF}} \boldsymbol{Q_D} + \gamma \boldsymbol{\theta_{CF}}$. A stock has a high Sharpe ratio if (i) the firm is highly correlated with other firms with large risk capital and (ii) the firm has high state risk exposure. A stock has a low market to book ratio, $V_n/K_n = (a_{\mathrm{CF},n} - \lambda_n)\sigma_{\mathrm{CF},n}/(1+r_f)$,[16] if (i) and (ii) hold, so that it has high Sharpe ratio, if (iii) the firm has low profitability per unit of risk or

[15] Most of the results of Section 3 stemming from the optimality of investment do not apply.

[16] The value of the firm is $V_n = (a_{\mathrm{CF},n} - \lambda_n)Q_n/(1+r_f)$, a direct consequence of the definitions of cash flows and the Sharpe ratio.

if (iv) the firm has low cash flow volatility. The valuation equation $V_n = (a_{\mathrm{CF},n} - \lambda_n) Q_n / (1 + r_f)$ implies that small stocks have high Sharpe ratios conditional on profitability, installed capital, and cash flow volatility.

Linear Technologies

The linear technology setup corresponds to the limiting case $\eta = +\infty$ in our model. Each firm's demand schedule is infinitely elastic and the price of risk is set to that the firm generates zero economic profits: $\sigma_{\mathrm{CF},n} (a_{\mathrm{CF},n} - \lambda_n) = 1 + r_f$. Risk prices, as well as the mean and volatility of the stock, are therefore only driven by the firm's own profit function, independently of other firms or the stock market investor's attitude toward risk. The market-to-book ratio of each firm is equal to unity, so that the model cannot generate the value premium. The capital allocated to each firm is determined by the supply schedule, $\boldsymbol{Q_S}(\boldsymbol{\lambda})$, in equation (24). Therefore linear economies have limited explanatory power for the cross-section of stock returns.

Appendix to Section 5

Proof of Proposition 5

We aggregate up (38) across firms:

$$N\bar{Q} + \frac{\gamma}{W_0} \rho N \bar{Q} \sum_{n=1}^{N} \delta_n = \sum_{n=1}^{N} \delta_n \left(\frac{a_{\mathrm{CF},n}}{\eta} - \gamma \theta_{\mathrm{CF},n} \right).$$

The average value of Q_n in GE is therefore

$$\bar{Q} = \frac{1}{N} \sum_{n=1}^{N} Q_n = \frac{\sum_{n=1}^{N} \delta_n \left(\frac{a_{\mathrm{CF},n}}{\eta} - \gamma \theta_{\mathrm{CF},n} \right)}{N \left(1 + \frac{\gamma}{W_0} \rho \sum_{n=1}^{N} \delta_n \right)}. \tag{69}$$

Note that this expression only depends on exogenous parameters.

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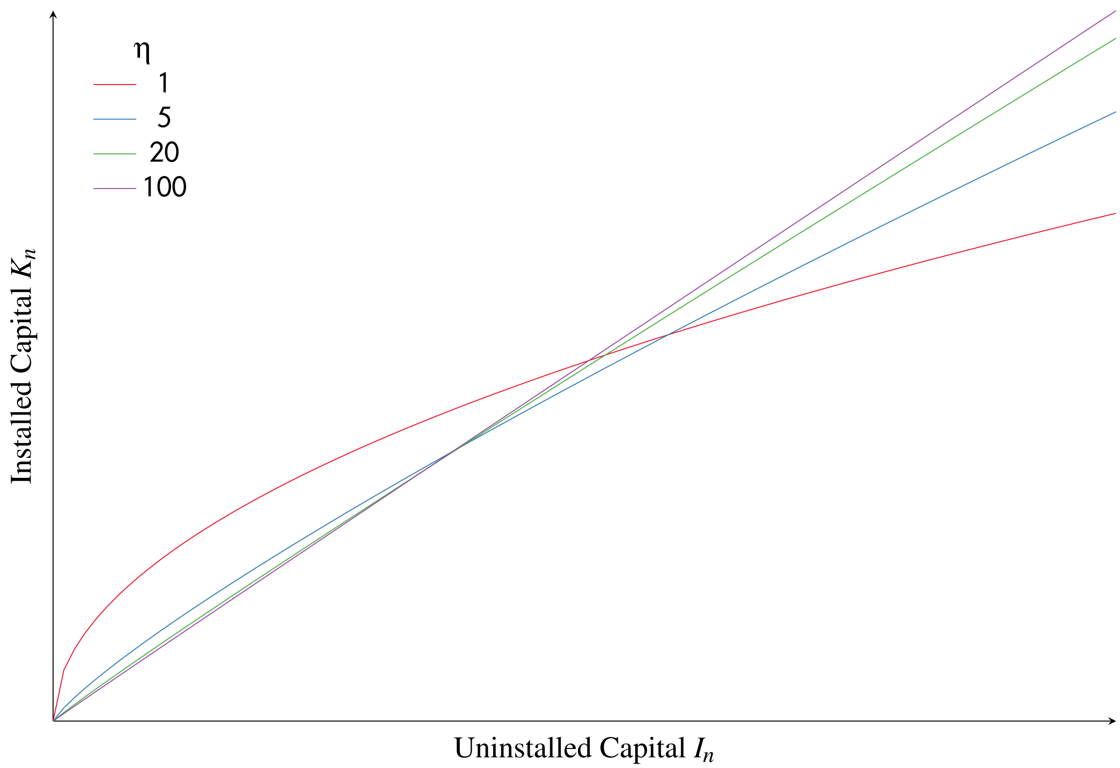


Figure 1
Decreasing Return to Scale Production Technology. This figure plots a firm's installed capital, K_n, as a function of uninstalled capital, I_n, for various values of the production elasticity coefficient η.

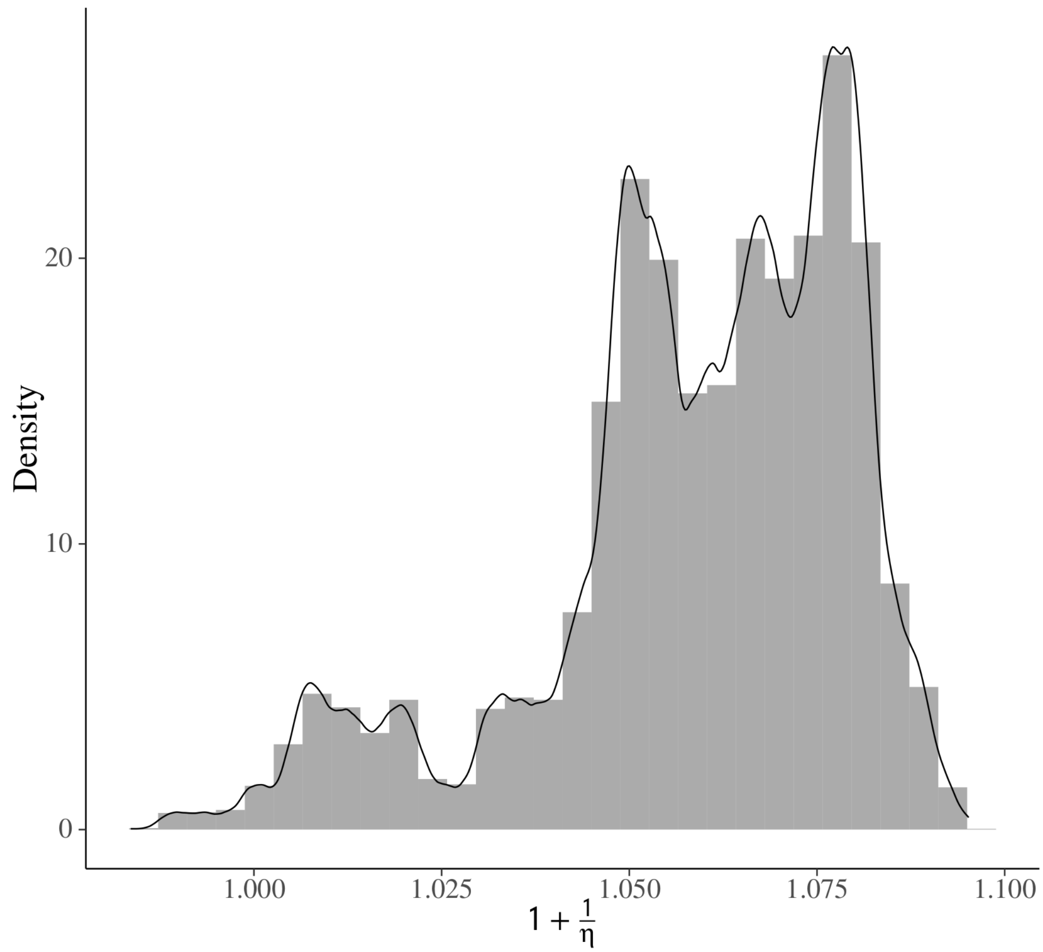


Figure 2
Distribution of Production Elasticity Coefficient η. This figure plots the distribution of η coefficients estimated each trading day from 1980 to 2018 across the entire cross-section of U.S. stocks. The density function is obtained via an Epanechnikov density kernel with optimal bandwidth (Sheather-Jones estimator). The η coefficient is estimated daily from a cross-sectional regression of firms' log market values against their log book values,

$$\log\left(V_n\right) = \left(1 + \frac{1}{\eta}\right) \log\left(K_n\right).$$

All variables are described in the Appendix.

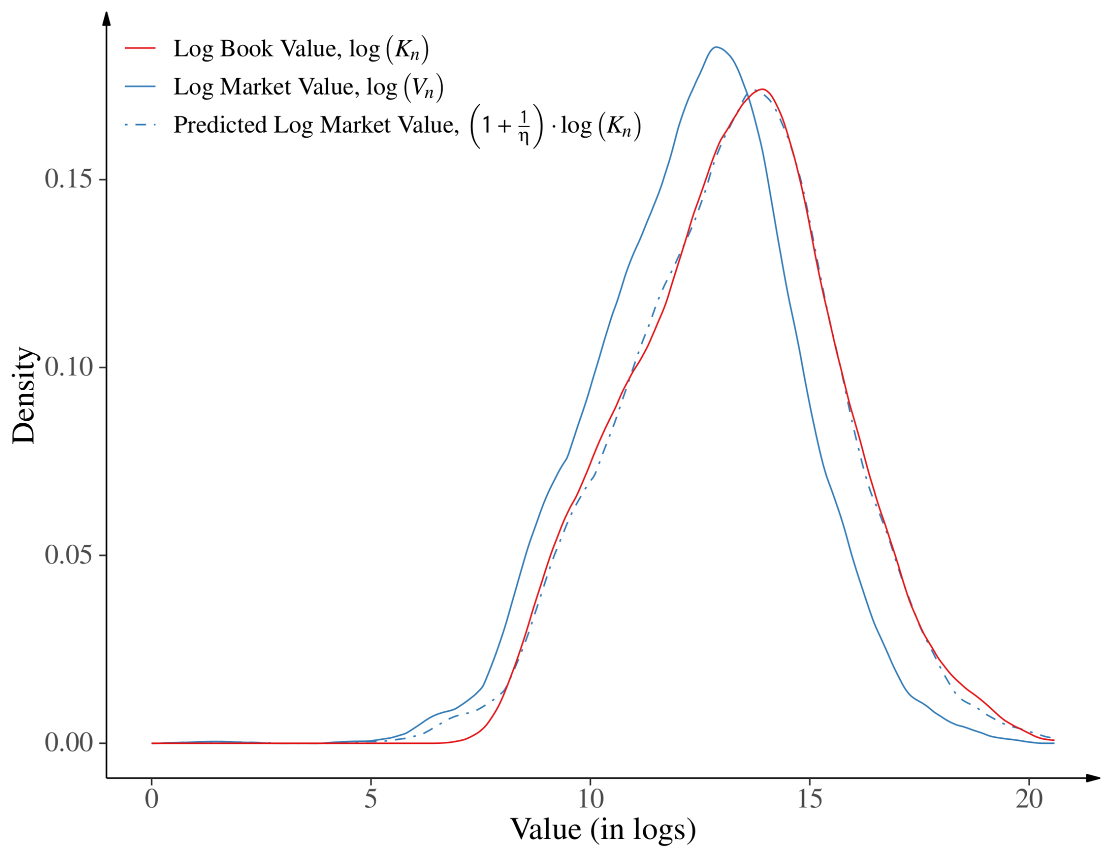
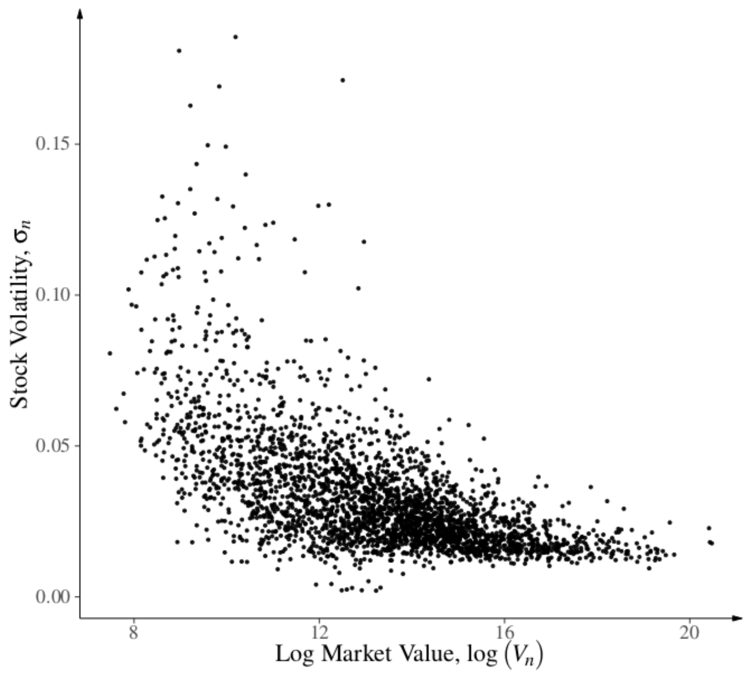
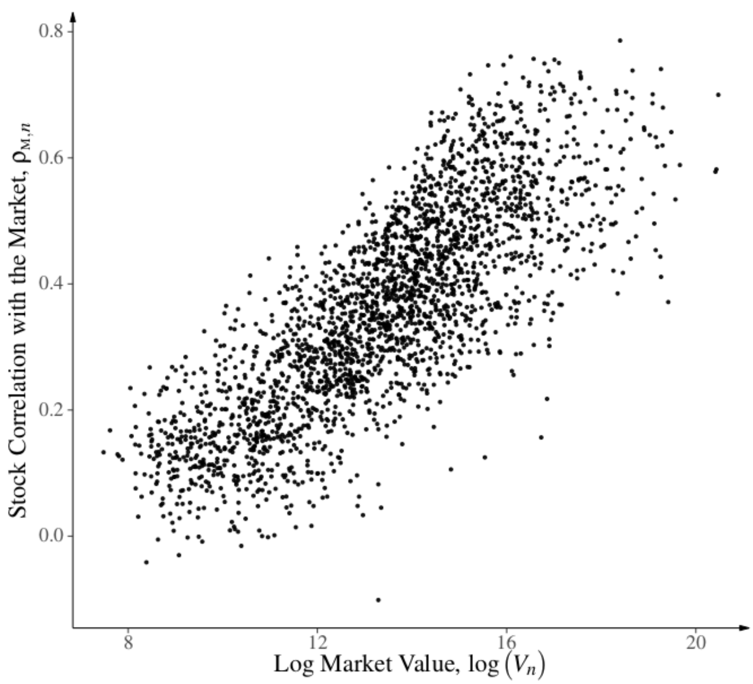


Figure 3
Cross-Section Densities of Firm Book and Market Values. This figure plots the density functions of firms' log book values (blue) and log market values (red) estimated on the last day of our sample (Dec 31, 2018). The density functions are obtained via an Epanechnikov density kernel with optimal bandwidth (Sheather-Jones estimator). The dotted blue line corresponds to the density function of $\left(1+\frac{1}{\eta}\right)\log(K_n)$, where the coefficient η is estimated from a cross-sectional regression of firms' log market values against their log book values that day. All variables are described in the Appendix.



(a) **Volatility vs. Size**



(b) **Market Correlation vs. Size**

Figure 4

Relationships between Stock Volatility, Market Correlation, and Size. This figure plots the volatility (Panel A) and market correlation (Panel B) of firms' stock returns against their log market value on the last day of our sample (Dec 31, 2018). All variables are described in the Appendix.

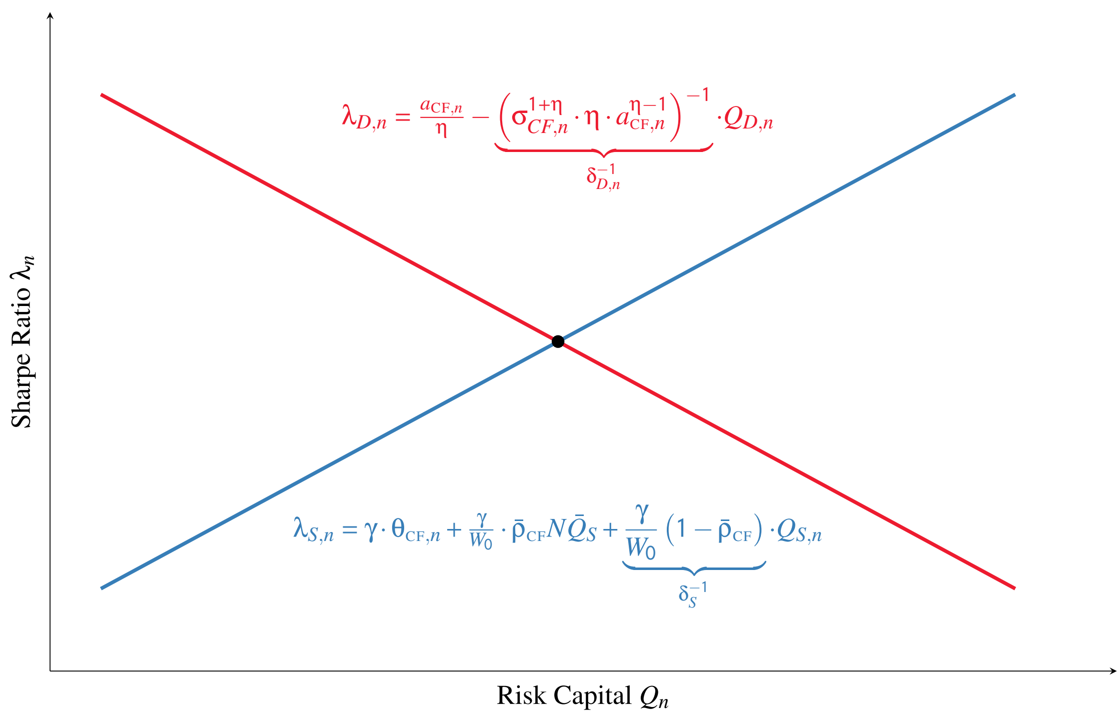
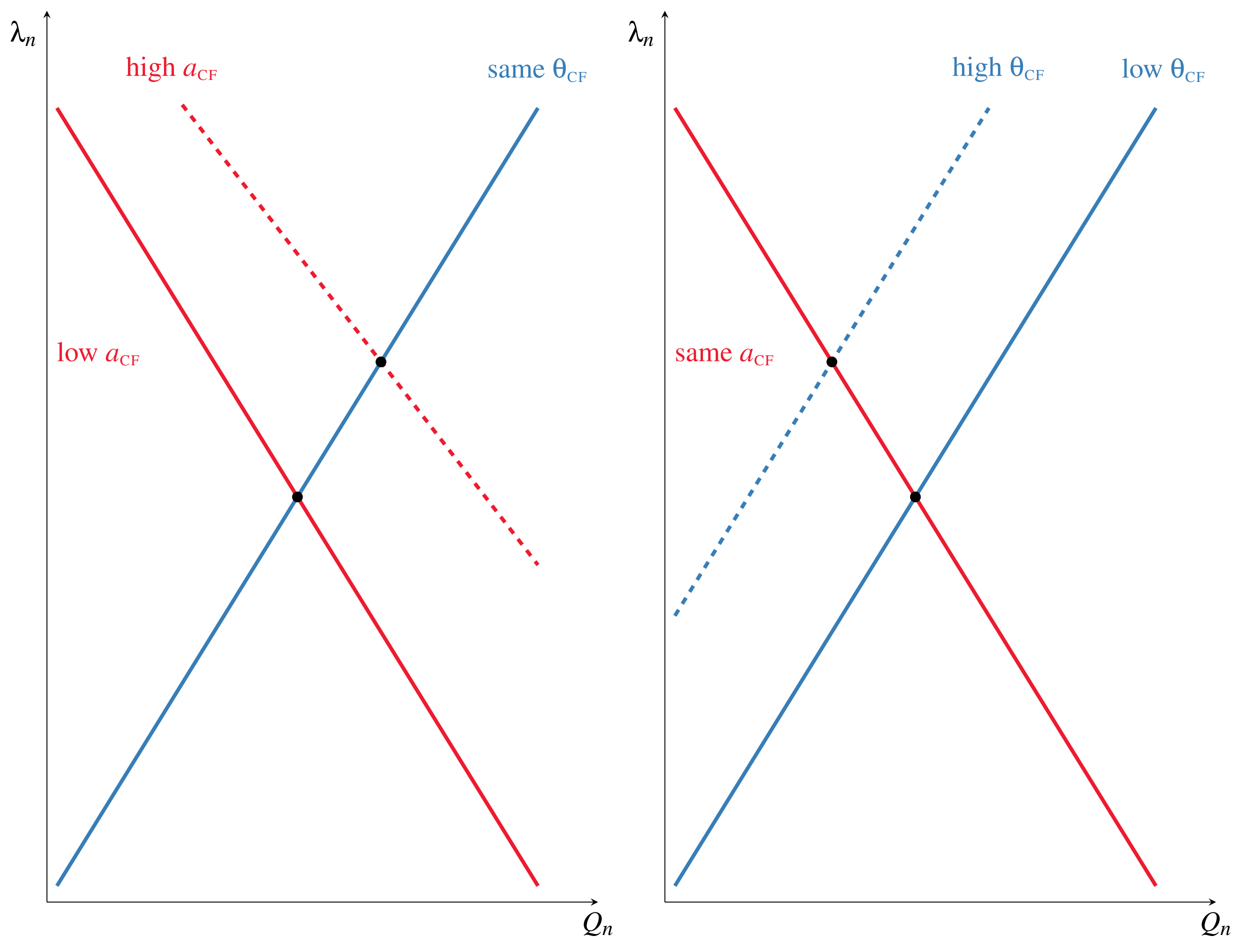


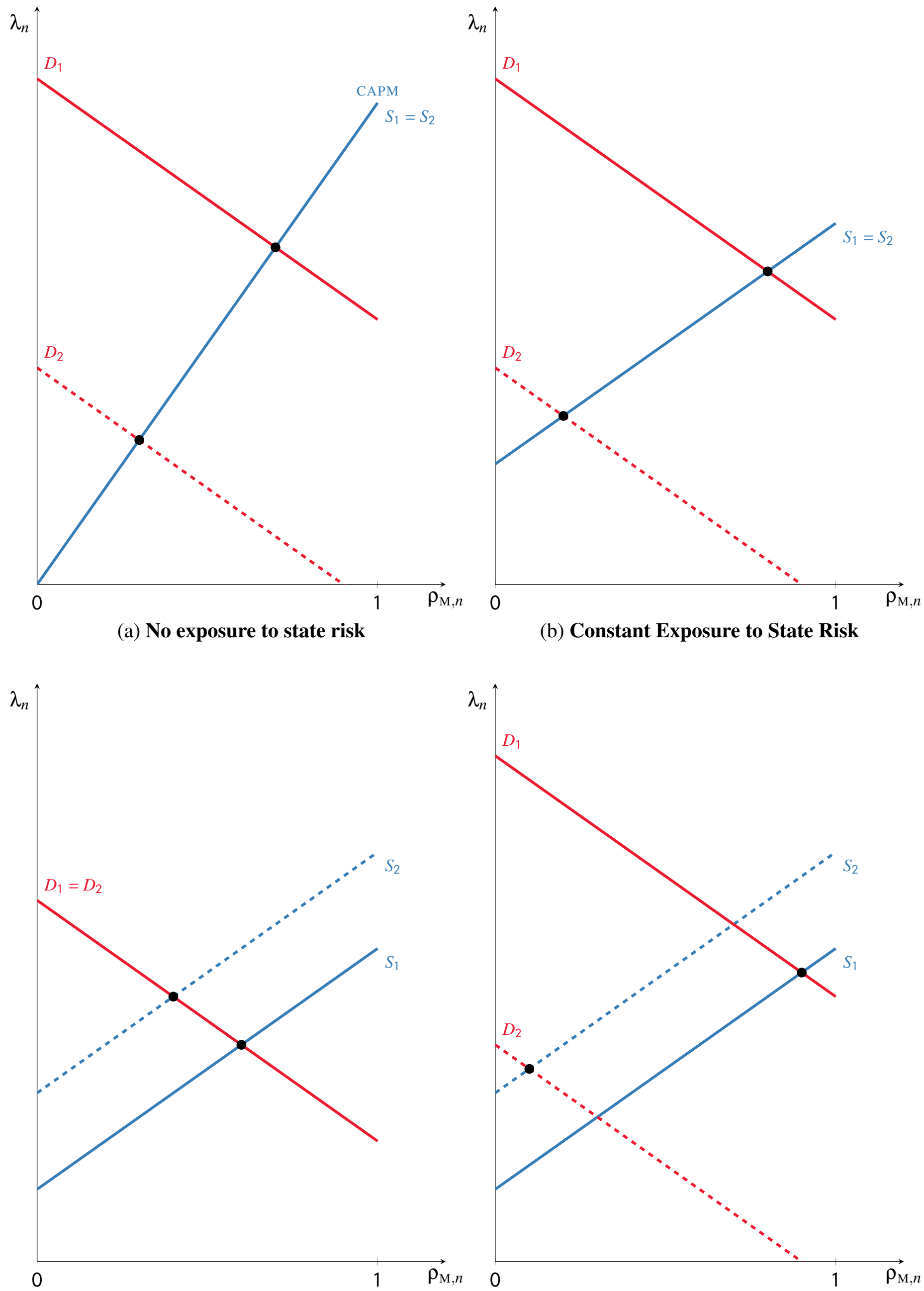
Figure 5
Equilibrium Inverse Supply and Demand Curves for Firm n. This figure plots the inverse demand (red) and supply (blue) curves for firm n. The variables Q_n and λ_n correspond to the firm's risk capital and Sharpe ratio. The figure is based on the symmetric version of the model in which firms have equal cash-flow correlation.



(a) **Variation in Profitability**

(b) **Variation in State Risk Exposure**

Figure 6
Cross-Sectional Equilibrium Variation in Firm Sizes and Risk Prices. This figure illustrates the inverse supply and demand schedules for two firms that either differ only in their profitabilities (Panel A) and or in their exposures to state risk (Panel B). The variables Q_n and λ_n correspond to each firm's risk capital and Sharpe ratio. The figure is based on the symmetric version of the model in which firms have equal cash-flow correlation.



(a) **No exposure to state risk**

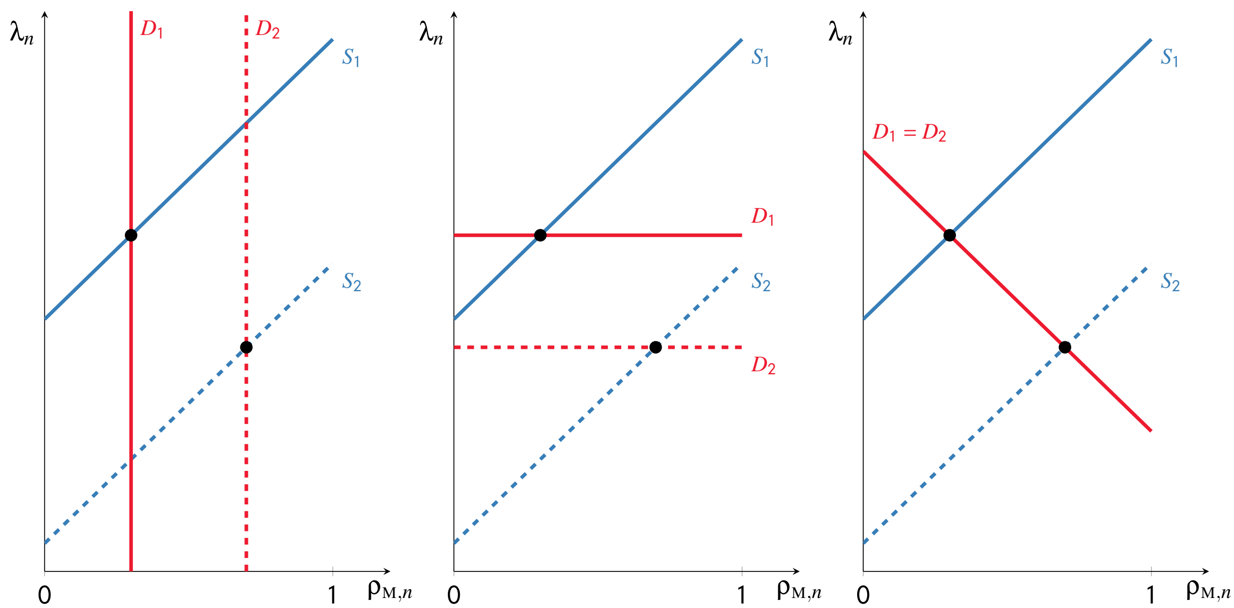
(b) **Constant Exposure to State Risk**

(c) **Different Exposure to State Risk Constant Profitability**

(d) **Different Exposure to State Risk Different Profitability**

Figure 7

Normalized SML. This figure illustrates the cross-sectional equilibrium relationship between firms' Sharpe ratio, λ_n, and their correlation to the stock market, $\rho_{M,n}$. The panels consider various scenarios for two firms that differ either in their profitabilities, $a_{CF,n}$ (red inverse demand curves), and/or in their state risk exposures, $\theta_{CF,n}$ (blue inverse supply curves). The figure is based on the symmetric version of the model in which firms have equal cash-flow correlation.



(a) **Endowment Economy** (b) **Linear Economy** (c) **Full GE**

Figure 8

Downward-Sloping Normalized SML. This figure illustrates the case of two firms for which cross-sectional equilibrium relationship between the Sharpe ratio, λ_n, and correlation to the stock market, $\rho_{M,n}$ is negative. The figure shows how the downward-sloping normalized SML can be generated in (a) an endowment economy, (b) a linear economy, and (c) full GE. The figure is based on the symmetric version of the model in which firms have equal cash-flow correlation.

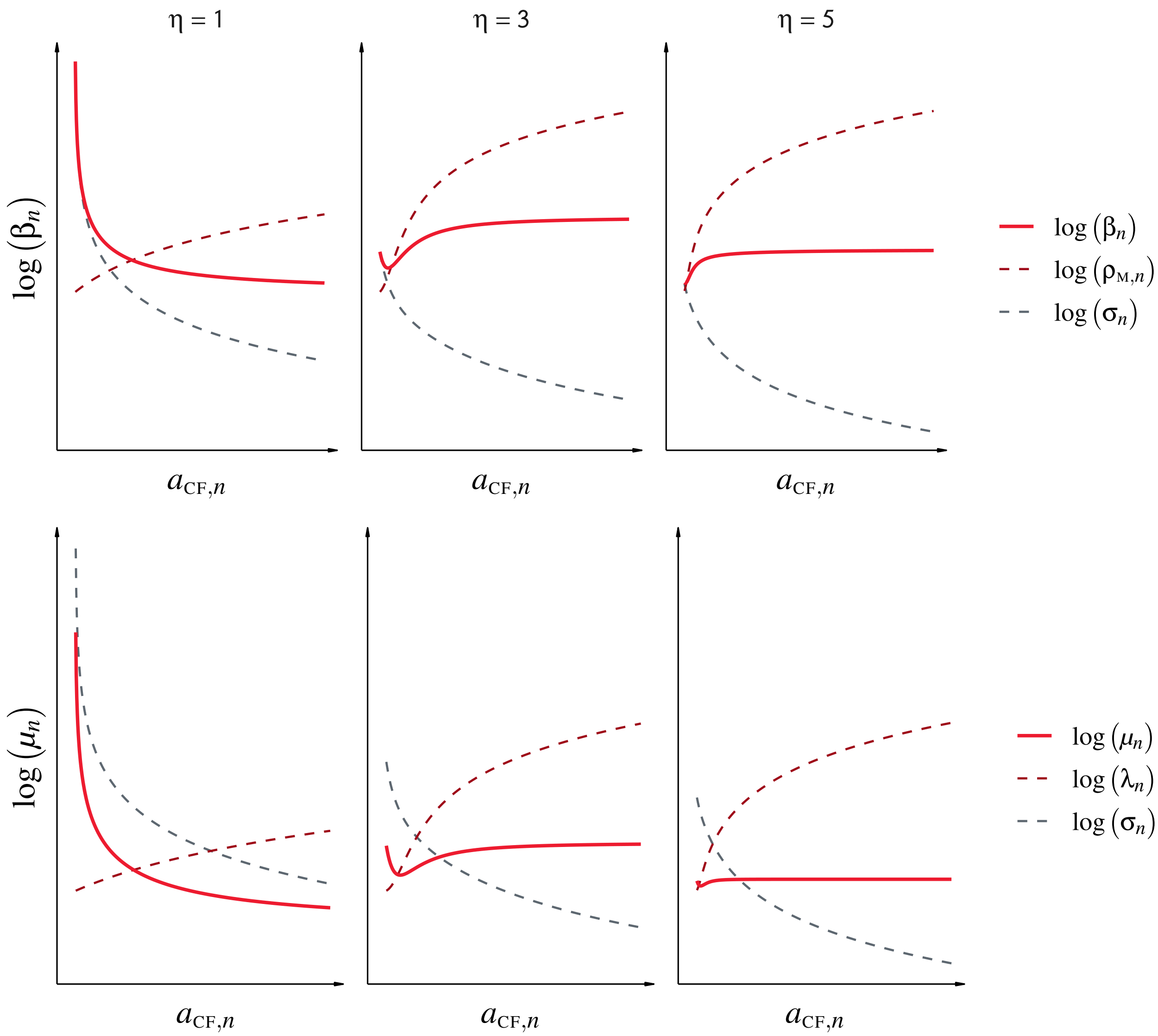


Figure 9
Relationships between Beta, Expected Return, and Profitability. This figure relates a firm's log beta (top row) and log expected return (bottom row) to its profitability, $a_{\text{CF},n}$. Each column corresponds to a specific value of the production elasticity coefficient η. The charts in the top row decompose log beta into log market correlation and log volatility. The charts in the bottom row decompose log expected return into log Sharpe ratio and log volatility. All y-values are re-scaled linearly in order to fit on the same panel. The figure is based on the symmetric version of the model in which firms have equal cash-flow correlation.

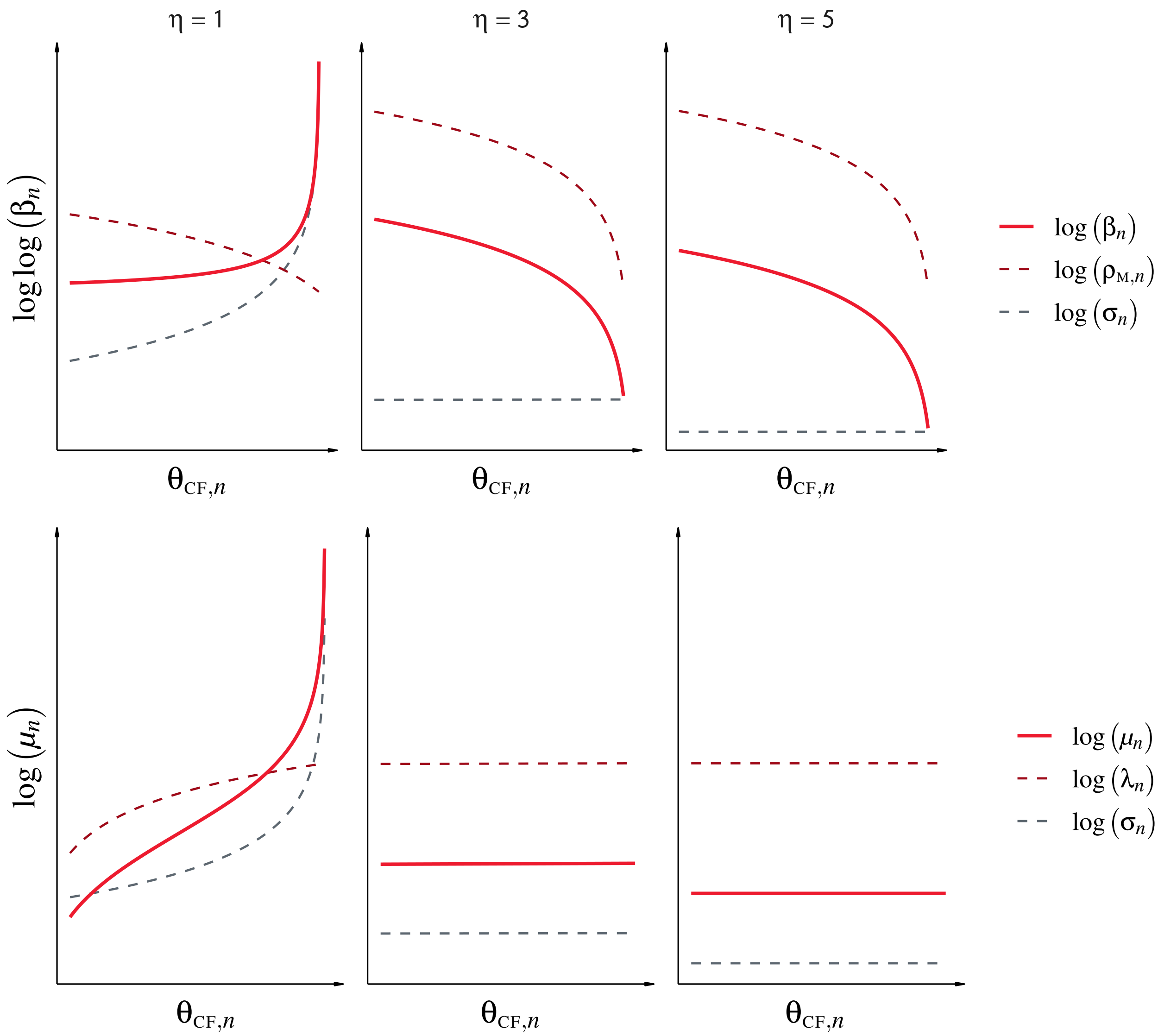


Figure 10

Relationships between Beta, Expected Return, and State Risk Exposure. This figure a firm's log beta (top row) and log expected return (bottom row) to its state risk exposure, $\theta_{\mathrm{CF},n}$. Each column corresponds to a specific value of the production elasticity coefficient η. The charts in the top row decompose log beta into log market correlation and log volatility. The charts in the bottom row decompose log expected return into log Sharpe ratio and log volatility. All y-values have been re-scaled linearly in order to fit on the same panel. The figure is based on the symmetric version of the model in which firms have equal cash-flow correlation.

	$\log(\beta_n)$		
	(1)	(2)	(3)
$\log(\rho_{M,n})$	1.000***	0.596***	
	(0.0003)	(0.013)	
$\log(\sigma_n)$	1.000***		0.294***
	(0.0004)		(0.022)
Number of Stocks	2,478	2,478	2,478
R^2	1.000	0.465	0.064

Table 1

Variance Decomposition of Beta. This table reports the estimates of the cross-sectional regressions of firms' log beta on their log market correlation and log volatility on the last day of our sample (Dec 31, 2018). Standard errors are reported in parentheses. The sample excludes 9 firms with negative correlation to the market. All variables are described in the Appendix.