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MIND THE GAP! STYLIZED DYNAMIC FACTS AND STRUCTURAL MODELS

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Abstract

We study what happens to identified shocks and to dynamic responses when the data generating process features q disturbances but less than q variables are used in the empirical model. Identified shocks are mongrels: they are linear combinations of current and past values of all structural disturbances and do not necessarily combine disturbances of the same type. Sound restrictions may be insufficient to obtain structural dynamics. The theory used to interpret the data and the disturbances it features determine whether an empirical model is too small. An example shows the magnitude of the distortions and the steps needed to reduce them. We revisit the evidence regarding the transmission of house price and of uncertainty shocks.

JEL Classification: C31, E27, E32

Keywords: Deformation, state variables, dynamic responses, Structural models, house price shocks, Uncertainty shocks

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Mind the gap! Stylized dynamic facts and structural models.

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August 21, 2019

Abstract

We study what happens to identified shocks and to dynamic responses when the data generating process features q disturbances but less than q variables are used in the empirical model. Identified shocks are mongrels: they are linear combinations of current and past values of all structural disturbances and do not necessarily combine disturbances of the same type. Sound restrictions may be insufficient to obtain structural dynamics. The theory used to interpret the data and the disturbances it features determine whether an empirical model is too small. An example shows the magnitude of the distortions and the steps needed to reduce them. We revisit the evidence regarding the transmission of house price and of uncertainty shocks.

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1 Introduction

It is common in macroeconomics to collect stylized facts about the dynamic transmission of certain structural shocks using (small scale) vector autoregressive (VAR) models and then build (larger scale) dynamic stochastic general equilibrium (DSGE) models to explain the patterns found in the data (see e.g. Galí [1999]; Iacoviello [2005], Basu and Bundick [2017] among many others).

Several authors, including Ravenna [2007], Fernández-Villaverde, Rubio-Ramírez, Sargent, and Watson [2007], Giacomini [2013], have emphasized that the matching exercise is imperfect as the linear solution of a DSGE model has a vector autoregressive-moving average (VARMA) format. To reduce the mismatch, the VAR should feature a large number of lags; but even a generous lag length may be insufficient in relevant cases. When long lags can not be used due to short data, the *invertibility* problem is typically taken care by i) simulating data from the linear decision rules of the same length as the actual data, ii) running the same VAR on both actual and simulated data, and iii) comparing the dynamics of the endogenous variables in the two systems after shocks are conventionally identified (see Chari, Kehoe, and McGrattan [2005]).

This paper studies a different mismatch problem, largely disregarded in the literature, which could be more important than invertibility for deciding which theory is consistent with the data. We call it *deformation*. It occurs when the data process generating (DGP) features q disturbances, but less than q variables enter in the empirical model. Will the innovations of such an empirical system provide information about "classes" of disturbances? Will they give information about a particular disturbance? In general, the answer is negative.

Deformation distortions make identified shocks mongrels with little economic interpretation for two reasons. The recoverable shocks are unlikely to combine structural disturbances of the same type, making it difficult to relate, say, an identified technology shocks to TFP or other supply disturbances present in a structural model. Furthermore, identification of individual disturbances require stringent conditions, which limit the type of disturbances one can analyze in such a system. Perhaps more importantly, recoverable shocks are, in general, linear combinations of *current and past* structural disturbances. Thus, unless the empirical model is carefully selected, identified shocks will display stronger propagation than the corresponding disturbances in the DGP.

The first problem (we named it cross sectional deformation) emerges when the DGP is such that several structural disturbances contemporaneously affect the variables entering the empirical model. The second problem (we named it time deformation) instead occurs whenever the empirical model is specified without paying sufficient attention to the theory used to explain the data and is exacerbated when the small scale empirical model i) does not respect the relationship between the endogenous variables and the states or ii) alters the law of motion of the states. Cross sectional deformation makes sound theoretical restrictions insufficient to obtain meaningful structural disturbances. Time deformation alters the information flow of the structural disturbances.

After an illustrative example to enhance intuition in section 2, section 3 derives formal results assuming a linear state space model for the DGP. We provide sufficient conditions for the identification of "classes" of disturbances and of a particular disturbance. Although we focus attention on general equilibrium models, deformation problems have identical implications in partial equilibrium settings, since the linear solution of such models also has a state space representation. In section 4 we use a standard New Keynesian model to show how to match a larger scale DGP to a small scale empirical model; the problems occurring when the empirical model is too small; and how time deformation can be reduced by more explicitly linking the theory and the empirical model. The reader should take away three main points from this section. First, if a SVAR is too small, identified shocks may not be interpretable. Second, when the DGP features more disturbances than the empirical model, the theory should be reduced to the same observables present in the empirical model prior to the computation of the decision rules. Third, the theory used to interpret the data and the disturbances it features must guide the choice of observables and the dimension of the empirical model. Thus, when deformation is present, dynamic facts can not be theory-free.

Section 5 provides suggestions to users who want to avoid the deformation trap. In section 6 we take a version of Iacoviello [2005]'s model with seven disturbances (the four originally used plus disturbances to the borrowing constraints and the wealth constraint of households) and use the same four variable VAR originally employed to construct responses to house price shocks. We show that the recovered house price shocks heavily reflect monetary policy and borrowing constraint disturbances, rather than the preference disturbances Iacoviello emphasizes, making the match with the data is weaker than previously thought. Section 7 extends the analysis to DGPs displaying higher order terms (such as those generated by higher order perturbed solutions of equilibrium models), shows that the results we derived hold unchanged, that deformation biases are likely to be more severe, and use Basu and Bundick [2017]'s model to highlight them.

Our analysis abstracts from invertibility issues (recently studied in, e.g. Beaudry, Feve, Guay, and Portier [2016], Forni, Gambetti, and Sala [2016], Plagborg Moller [2018], Pagan and Robinson [2018], Chahrour and Jurado [2018]) due to anticipatory disturbances or news. Both deformation and invertibility produce time deformation problems. However, invertibility does not create cross sectional deformation. Thus, the interpretation problems we consider are distinct, and matter even when invertibility is not an issue. Also, while one reason for choosing an empirical model with a limited number of variables is that certain theoretical quantities may be latent, cross sectional deformation is relevant even when all theoretical quantities are observables but short samples or identification convenience make applied researchers prefer small scale empirical models.

The current literature is silent about the issue we analyze. Apart from Canova and Hamidi Sahneh [2018], who analyze the effects of cross sectional deformation on the properties of Granger causality tests, and Miranda Agrippino and Ricco [2019], who examine the conditions for shock identification in SVAR-IV settings under partial identificability, we are aware only of early work by Blanchard and Quah [1989], Hansen and Sargent [1991], Marcet [1991], Lutkepohl [1984], Braun and Mittnik [1991] and Faust and Leeper [1988] discussing similar issues but in different settings. Some of results we present have similar flavor to Wolf [2018]. However, they are due by deformation distortions rather than insufficient identification restrictions.

2 Some intuition

To gain intuition into the problem, consider a simple consumption-saving model with TFP (Z_t), price of investment (V_t), and preferences (B_t) disturbances [1]. The representative agent maximizes:

$$\max_{C_t} \sum_{t=1}^{\infty} \beta^t B_t U(C_t) \tag{1}$$

subject to the constraints

$$\begin{aligned} C_t + I_t &= Z_t K_t^{\alpha} \quad (2) \\ K_{t+1} &= (1-\delta)K_t + V_t I_t \quad (3) \end{aligned}$$

We assume that $0 < \alpha < 1, 0 < \beta < 1$ and that (Z_t, V_t, B_t) are iid with unitary means and standard deviation $\sigma_i, i = Z, V, B$. When $U(C_t) = \log C_t$ and $\delta = 1$, the solution is

$$\begin{aligned} \log K_{t+1} &= \log(\alpha\beta) + \alpha \log K_t + \log V_t + \log Z_t \quad (4) \\ \log C_t &= \log(1-\alpha\beta) + \alpha \log K_t + \log B_t + \log Z_t \quad (5) \\ \log Y_t &= \alpha \log K_t + \log Z_t \quad (6) \end{aligned}$$

The model has three endogenous variables and three disturbances (two supply (Z_t, V_t) and one demand B_t). When the empirical model contains the three variables, the disturbances are recursively identifiable from the innovations in $y_t = \log Y_t, c_t = \log C_t, k_{t+1} = \log K_{t+1}$. Note that the three variable system is invertible ($\alpha < 1$).

Suppose that a researcher employs an empirical model with two observables. Would she be able to identify a demand and a supply disturbance? Would she be able to trace out the dynamics induced by the preference disturbance? The answer depends on the variables used.

The empirical model corresponding to the solution of the (k_{t+1}, c_t) system is obtained integrating out y_t, a control, from the problem:

$$\begin{aligned} k_{t+1} &= \log(\alpha\beta) + \alpha k_t + u_{1t} \quad (7) \\ c_t &= \log(1-\alpha\beta) + \alpha k_t + u_{2t} \quad (8) \end{aligned}$$

[1] We are grateful to Thomas Drechsel for suggesting a version of this example.

where $u_{1t} = \log V_t + \log Z_t$, $u_{2t} = \log B_t + \log Z_t$. In this system recursivity is lost. Furthermore, u_{2t}'s mixes demand and supply disturbances. Thus, theoretically motivated restrictions will fail to identify either "classes" or a particular disturbance and the dynamics they induce.

The empirical model corresponding to the solution of the (y_t, c_t) system is obtained integrating out k_t, a state, from the problem:

$$c_t = b_c + \alpha c_{t-1} + u_{1t} \tag{9}$$

$$y_t = b_y + \alpha y_{t-1} + u_{2t} \tag{10}$$

where $u_{1t} = \log Z_t + \log B_t - \alpha \log B_{t-1} + \alpha \log V_{t-1}$, $u_{2t} = \log Z_t + \alpha \log V_{t-1}$, and b_c, b_y are constant. This system omits k_t, but introduces two new states c_{t-1}, y_{t-1}. The innovation u_{1t} mixes demand and supply disturbances but now with different timing. Thus, this system displays cross sectional and time deformations. The (recursive) cross correlation between u_{jt} and current and lagged values of any of the structural disturbances does not go to one even when the number of lags goes to infinity. Thus, the recoverability condition of Chahrour and Jurado [2018] fails, and theoretical motivated restrictions will not identify any of the structural disturbances.

Is there a two variable system which allows the identification of a supply and a demand disturbance? If the two great ratios, $(\log k_{t+1} - \log y_t)$ and $(\log c_t - \log y_t)$ are used as observables, one can recover $\log V_t, \log B_t$. This happens because each disturbance affects the decision rule of only one variable.

In sum, when there is a dimensionality mismatch between the empirical model and the disturbances of the DGP, the variables entering the empirical system determine the informational content of the reduced form innovations; and eliminating states is, in general, more problematic than eliminating controls. Moreover, strict conditions are needed to recover either a "class" or a particular disturbance. The next section formalizes this intuition.

3 Analytical results

We assume that the DGP is of the form:

$$x_t = A(\theta)x_{t-1} + B(\theta)e_t \tag{11}$$

$$y_t = C(\theta)x_{t-1} + D(\theta)e_t \tag{12}$$

where x_t is a $k \times 1$ vector of endogenous and exogenous states, y_t is a $m \times 1$ vector of endogenous controls, $e_t \sim (0, \Sigma(\theta))$ is a $q \times 1$ vector of disturbances, $\Sigma(\theta)$ a diagonal matrix and θ a vector of structural parameters; $A(\theta)$ is $k \times k$, $B(\theta)$ is $k \times q$, $C(\theta)$ is $m \times k$, $D(\theta)$ is $m \times q$. For convenience, we let the eigenvalues of $A(\theta)$ to be all less than one in absolute value. Thus, if there are disturbances with permanent effects, (11)-(12) represent a properly scaled version of the process generating the data. Predictable disturbances or news about future disturbances are not considered to leave non-invertibility issues aside. While (11)-(12) is general, in our applications it is produced by the (log-)linear solution of the optimality conditions of a structural macroeconomic model.

In general, $m \geq q$ and some of the endogenous variables may be latent. Hence, the variables entering the empirical model are $z_t = S[x_t, y_t]'$, where S is a selection matrix. Fernández-Villaverde et al. [2007] assume $S = [0, I]$ (which implies that $m = q$), while Ravenna [2007] and Pagan and Robinson [2018] assume that either S=I (so that $m + k = q$), or $S = [0, I]$. In general, S is chosen so that the dimension of the empirical model matches the number of structural disturbances.

The reduced form (innovation representation) corresponding to (11)-(12) is

$$x_t = A(\theta)x_{t-1} + K_x(\theta)u_t \tag{13}$$

$$y_t = C(\theta)x_{t-1} + K_y(\theta)u_t \tag{14}$$

where $u_t = z_t - E_t[z_t|\Omega_{t-1}]$ is a $q \times 1$ vector of innovations, Ω_{t-1} includes (at least) lags of z_t, $K_x(\theta)$ and $K_y(\theta)$ are steady state Kalman gain matrices, and for those x_t and y_t belonging to z_t, $K_i(\theta)$ has a row with zeros except in one position.

Given (13)-(14), identification of the structural disturbances requires the mapping from u_t into e_t. When the empirical model is a VAR, Sims and Zha [2006],Plagborg Moller [2018] developed sufficient conditions to obtain e_t from current and past z_t; Chahrour and Jurado [2018] discuss sufficient conditions to recover e_t from current, past and future z_t. In the current setup, when $S = I$, one needs to invert $\begin{pmatrix} B(\theta) \\ D(\theta) \end{pmatrix} e_t = u_t$; when $S = [0, I]$, one needs to invert $D(\theta)e_t = u_t$. In both cases, standard order and rank conditions apply, see Rubio Ramirez, Waggoner, and Zha [2010].

In the identification exercise two assumptions are implicitly made. First, there is no misspecification in (11)-(12), at least, as far as sources of disturbances are concerned, so that $dim(z_t) = dim(e_t)$. If disturbances are left out, the identification exercises becomes problematic, even when excluded disturbances are orthogonal to included ones, and included disturbances account for a large portion of the variability of z_t. Second, when $z_t = y_t$, Ω_{t-1} it is typically specified to include long lags of z_t to take care of omitted states. When disturbances are left out, having a rich Ω_{t-1} is generally insufficient to make the identification problem well behaved.

In our analysis $dim(z_t) < dim(e_t)$, i.e. we focus on the situation when a small scale empirical system, say, a two variable VAR is used but the DGP features more than two disturbances. A researcher who wants to interpret the dynamics of the small scale empirical system may employ a theoretical model that is less complex than the DGP and may specify only enough disturbances to match the number of empirical variables. We show that the dynamics produced by such model are not relevant for the comparison and omitted disturbances play a crucial role. Let $z_{it} \equiv S_i[x_t, y_t]'$ where S_i is a $q_i \times q$, and $dim(z_{it}) = q_i < dim(e_t) = q, \forall i$. We will consider three S_i matrices.

- Case 1: $S_1 = [I, S_{12}]$. This choice generates an observable system which retains the states but integrates out part of the controls. The DGP in terms of $z_{1t} = [x_t, y_{1t}]', y_{1t} \equiv S_{12} y_t$ is:

$$x_t = A(\theta)x_{t-1} + B(\theta)e_t \tag{15}$$

$$y_{1t} = C_1(\theta)x_{t-1} + D_1(\theta)e_t \tag{16}$$

or $z_{1t} = F_1(\theta)z_{1t-1} + G_1(\theta)e_t$, where $F_1(\theta) = \begin{pmatrix} A(\theta) & 0 \\ C_1(\theta) & 0 \end{pmatrix}$ and $G_1(\theta) = \begin{pmatrix} B(\theta) \\ D_1(\theta) \end{pmatrix}$.

• Case 2: $S_2 = [S_{21}, S_{22}]$. This choice generates an observable system which integrate out part of the states and part of the controls. Let $x_t = (x_{1t}, x_{2t}), y_t = (y_{1t}, y_{2t})$, where (x_{1t}, y_{1t}) are the variables excluded from the empirical system. The DGP in terms of $z_{2t} = [x_{2t}, y_{2t}], x_{2t} \equiv S_{21}x_t, y_{2t} \equiv S_{22}y_t$, is

$$x_{2t} = A_2(\theta)x_{2t-1} + B_2(\theta)e_t + w_{1t-1} \tag{17}$$

$$y_{2t} = C_2(\theta)x_{2t-1} + D_2(\theta)e_t + w_{2t-1} \tag{18}$$

or $z_{2t} = F_2(\theta)z_{2t-1} + G_2(\theta)e_t + w_{t-1}$, where $F_2(\theta) = \begin{pmatrix} A_2(\theta) & 0 \\ C_2(\theta) & 0 \end{pmatrix}$ and $G_2(\theta) = \begin{pmatrix} B_2(\theta) \\ D_2(\theta) \end{pmatrix}$, where $w_{1t-1} = A_{21}(\theta)x_{1t-1}; w_{2t-1} = C_{21}(\theta)x_{1t-1}$. Alternatively, using (11) to separate observable and non-observable states, and integrating x_{1t} out, the DGP for z_{2t} is

$$x_{2t} = \tilde{A}_{21}(\theta)x_{2t-1} + \tilde{A}_{22}(\theta)x_{2t-2} + \tilde{B}_{20}(\theta)e_t + \tilde{B}_{21}(\theta)e_{t-1} \tag{19}$$

$$y_{2t} = \tilde{C}_{21}(\theta)x_{2t-1} + \tilde{C}_{22}(\theta)x_{2t-2} + \tilde{D}_{20}(\theta)e_t + \tilde{D}_{21}(\theta)e_{t-1} \tag{20}$$

(17)-(18) point out the misspecification present using a first order empirical model for z_{2t}. (19)-(20) shows that DGP for the observables is a VARMA(2,1).

• Case 3: $S_3 = [S_{31}, 0]$. This choice generates an empirical system which repackages the states and eliminates the controls. The DGP in terms of $z_{3t} = x_{3t} = S_{31}x_t$ is

$$x_{3t} = A_3(\theta)x_{3t-1} + B_3(\theta)e_t + w_{3t-1} \tag{21}$$

where w_{3t-1} is a function of the repackaged states. Analogously with case 2, one may write (21) as

$$z_{3t} = \bar{A}_{31}(\theta)z_{3t-1} + \bar{A}_{32}(\theta)z_{3t-2} + \bar{B}_{30}(\theta)e_t + \bar{B}_{31}e_{3t-1} \tag{22}$$

The processes for $z_{it}, i = 1, 2, 3$ are obtained substituting optimality conditions into others, prior to the computation of the decision rules. The matrices of these solutions generally differ from those obtained solving the original model and crossing out the rows corresponding to the variables absent from z_{it}, because not all the original states are necessarily used in the computation of the decision rules. Section 4 provides examples of smaller scale empirical systems which produce (15)-(16), (19)-(20), and (22) for a specific DGP.

The innovation representation of (11)-(12) when z_{it} are observables is

$$x_t = A(\theta)x_{t-1} + \hat{K}_{ix}(\theta)u_{it} \tag{23}$$

$$y_t = C(\theta)x_{t-1} + \hat{K}_{iy}(\theta)u_{it} \tag{24}$$

where $u_{it} = z_{it} - E_t[z_{it}|\Omega_{it-1}]$ is a $q_i \times 1$ vector of innovations, $\hat{K}_{ix}(\theta)$, $\hat{K}_{iy}(\theta)$ are steady state Kalman gain matrices featuring some rows with zeros except in one position.

3.1 Obtaining structural disturbances

Given that not all disturbances can be identifies, we ask whether a researcher can identify "classes" of disturbances or a particular disturbance appearing in the DGP.

The empirical system eliminates only theoretical controls We analyze the relationship between u_{1t} and e_t, when $E[z_{1t}|\Omega_{1t-1}] = \tilde{F}_1 z_{1t-1}$ and thus

$$u_{1t} = z_{1t} - \tilde{F}_1 z_{1t-1} \tag{25}$$

Proposition 1 *i) If $\tilde{F}_1 = S_1 F(\theta) \equiv F_1(\theta)$, then $u_{1t} = \lambda_1(\theta) e_t$, where $\lambda_1(\theta)$ is a $q_1 \times q$ matrix. A block diagonal $G_1(\theta)$ is sufficient to identify classes of disturbances.If $G_1(\theta)$ has at most one non-zero element in some row k, one can obtain e_{jt}, for some k and j.*
ii)If $\tilde{F}_1 \neq S_1 F(\theta)$, $u_{1t} = \lambda_1(\theta, L) e_t$, where $\lambda_1(\theta, L)$ is a $q_1 \times q$ matrix for every L and, in general, an infinite dimensional function of L.

The proof of the proposition is obtained matching (25) with (15)-(16). The first part considers the case $\tilde{F}_1 = S_1 F(\theta)$. Here the innovations u_{1t} respect the timing protocol of the structural disturbances e_t, but cross sectionally deform them because $q_1 < q$. Because $G_1(\theta)$ is rectangular, one may ask when u_{1t} carries enough information to recover some e_{jt}. It turns out that u_{k1t} compresses certain classes structural of disturbances only if $G_1(\theta)$ has a block structure. Furthermore, u_{k1t} carries information about one e_{jt} if $G_1(\theta)$ has at most one non-zero element in row k. Both restrictions are strong and unlikely to be satisfied in a large class of general equilibrium models. They require that the theory features many "conveniently" placed delay restrictions.

When $\tilde{F}_1 \neq S_1 F(\theta)$, time deformations also occurs and u_{1t} becomes a one-sided infinite moving average of the structural disturbances, $u_{1t} = \lambda_1(\theta, L) e_t \equiv (\tilde{F}_1 - S_1 F(\theta))(I - S_1 F(\theta))^{-1} G_1(\theta) e_{t-1} + G_1(\theta) e_t$. In this situation, even if $G_1(\theta)$ has at most one non-zero element in row k, the information in u_{k1t} is not enough to obtain some of the current e_{jt}.

Proposition 1 determines the properties of u_{1t}, given e_t. Thus, u_{1t} will be a mean zero process and its autocovariance function will be restricted by

$$E(u_{1t} u'_{1t-s}) \quad = \quad E(\lambda_1(\theta, L) e_t e'_{t-s} \lambda_1(\theta, L)'), \quad s \geq 0 \tag{26}$$

When e_t are iid, the variance of u_{1t} and e_t differ and the magnitude of the amplification depends on the properties of $\lambda_1(\theta, L)$. Thus, a e_{jt} disturbance with a small variance or small initial loadings $\lambda_{1j}(\theta, L = 0) \equiv \lambda_{1j0}(\theta)$ will be hard to identify from the u_{1t}. Similarly, the serial correlation properties of u_{1t} depend on the structure and magnitude of the $\lambda_1(\theta, L)$ polynomial and its row dimension. However, even when $\lambda_{1j0}(\theta) = G_1(\theta)$, cross sectional distortions may make the autocovariance function of u_{1t} insufficient to recover the autocovariance of some e_{jt}.

The states in the empirical and the theoretical models differ We analyze the relationship between $u_{it}, i = 2, 3$ and e_t when $E[z_{it}|\Omega_{it-1}] = \tilde{F}_i z_{it-1}, i = 2, 3$ so that

$$u_{it} = z_{it} - \tilde{F}_i z_{it-1} \tag{27}$$

Proposition 2 *i)* $u_{it} = \lambda_i(\theta, L)e_t$*, where* λ_i *is* $q_i \times q$ *for each* $L, \imath = 2, 3$*.*
ii) $u_{it} = \psi_i(\theta, L)u_{1t}, i = 2, 3$*.*

To prove part i), we first match (25) and (17)-(18). Then $u_{2t} = (S_2F(\theta) - \tilde{F}_2)(I - S_2F(\theta)L)^{-1}(G_2(\theta)e_{t-1} + H_2(\theta)x_{1t-2}) + G_2(\theta)e_t + H_2(\theta)x_{1t-1}$. Because x_{1t} has a VARMA(2,1) format: $M(\theta, L)x_{1t} = N(\theta, L)e_t$, where $M(\theta, L)$ is invertible, we have $u_{2t} = \lambda_2(\theta, L)e_t$, where $\lambda_2(\theta, L) = G_2(\theta) + (S_2F(\theta) - \tilde{F}_2)(I - S_2F(\theta)L)^{-1}(G_2(\theta) + H_2(\theta)M(\theta, L)^{-1}N(\theta, L)L + H_2(\theta)M(\theta, L)^{-1}N(\theta, L)L^2$. Matching (27) with (21) one similarly obtains that $u_{3t} = \lambda_3(\theta, L)e_t$. Thus, an empirical system including only the states of the DGP does not solve time deformation problems since their law of motion may be altered. Note that $S_2F(\theta) = \tilde{F}_2$, or $S_{31}A(\theta) = \tilde{F}_3$ is insufficient to avoid time distortions and that $u_{it}, i = 2, 3$ will cross sectionally and time deform the structural disturbances.

In general, $u_{it} \neq u_{1t}, i = 2, 3$ and the timing of information they contain differs even when $S_iF(\theta) = \tilde{F}_i(\theta), \forall i$. Letting $\lambda_1(\theta, L)^+$ be the generalized inverse of $\lambda_1(\theta, L)$, one can write

$$u_{it} = \lambda_i(\theta, L)\lambda_1(\theta, L)^+ u_{1t} \equiv \psi_i(\theta, L)u_{1t} \tag{28}$$

By construction $\psi_{i0}(\theta) = I$. Thus, an impulse in u_{1t} and $u_{it}, i = 2, 3$ has identical effects on the variables present in both z_{1t} and z_{it} but will last longer when z_{it} are the observables - persistence will be altered. Therefore, the dynamics induced by identified shocks in small scale empirical systems of the same dimension but featuring different variables will differ.

(27) is misspecified when states are omitted or repackaged. What happens when the innovations u_{it} are constructed using a larger information set, e.g., $u_{it} = z_{it} - \tilde{F}_i(L)z_{it-1} \quad L = 1, 2, \ldots$? Because both z_{2t} and z_{3t} are VARMA processes, standard non-invertibility and truncation issues discussed in the literature apply. In principle, $\tilde{F}_i(L)$ must be non-zero for $L \to \infty$ for time deformation biases to disappear. Still, even when $L \to \infty$, cross sectional deformations will remain.

Proposition 1 is related to the aggregation results of Faust and Leeper [1988]. Because of their DGP is a VAR, they can not analyze the consequences of omitting states or altering their law of motion. Proposition 2 has the same flavor as the result in Fernández-Villaverde et al. [2007]. The main difference is that here $u_{it}, i = 2, 3$ are reduced ranked moving averages of e_t and the reason is time deformation rather than non-invertibility.

The two propositions highlight that the variables entering in the empirical model determine the quality of the approximation of identified shocks to the structural disturbances. Eliminating theoretical controls creates innovations that cross sectionally combine the structural disturbances, but eliminating states or repackaging their law of motion may create both cross sectional and time distortions. Failure to include all the theoretical states makes the innovations computed from a finite order empirical system serially correlated. However, an empirical model with all the theoretical states may not be enough for proper inference. Section 4 discusses how careful choices may reduce time deformation when the empirical model omits or repackages some of the states.

3.2 Dynamic responses

Consider the computation of z_{it} responses to an impulse in the shocks. In the DGP they are:

$$\begin{aligned} z_{it} &= S_i \begin{pmatrix} B(\theta) \\ D(\theta) \end{pmatrix} e_t \\ z_{it+h} &= S_i \begin{pmatrix} A(\theta)^h B(\theta) \\ C(\theta)A(\theta)^{h-1}B(\theta) \end{pmatrix} e_t \qquad i = 1,2,3; h = 1,2,\ldots \end{aligned} \tag{29}$$

In the empirical system with z_{1t} as observables, they are:

$$\begin{aligned} z_{1t} &= u_{1t} \\ z_{1t+h} &= \tilde{F}_1(\theta)^h u_{1t} \end{aligned} \tag{30}$$

The impact effect differs because $u_t = G_1(\theta)e_t$ and $G_1(\theta)$ is not a square matrix. Thus, having the correct $B(\theta), D(\theta)$ matrices may be insufficient to recover some e_{jt} via $\Sigma_u = G_1(\theta)\Sigma(\theta)G_1(\theta)'$, unless $G_1(\theta)$ only has one non-zero element in the j-th row. Clearly, since $q_1 < q$, not all impact responses to e_t disturbances can be obtained. However, if $\tilde{F}_1 = \begin{pmatrix} A(\theta) \\ S_{12}C(\theta) \end{pmatrix}$ responses at longer horizons to a properly identified shock are proportional to those of the DGP. Thus, qualitatively, (30) provides a good approximation to (29), if some e_{kt} can be recovered from u_{1t}.

The responses computed in systems with $z_{it}, i = 2,3$ as observables are instead:

$$\begin{aligned} z_{it} &= u_{it} \\ z_{it+h} &= \nu_{ij} u_{it} + \tilde{F}_i(\theta)^h u_{it} \end{aligned} \tag{31}$$

Here, both the instantaneous and the dynamic responses of z_{it} will be distorted; and their pattern may have nothing to do with those produced in the DGP. We summarize the discussion in a proposition.

Proposition 3 *i) Identified impulse responses constructed in a z_{1t} system could match those of the structural model if $\tilde{F}_1(\theta) = \begin{pmatrix} A(\theta) \\ S_{12}C(\theta) \end{pmatrix}$ and $G_1(\theta)$ has at most one non-zero element in one row. ii) Even if the condition in i) holds, the dynamic responses obtained from properly identified shocks in a z_{it} system, $i = 2,3$, differ from those of the DGP.*

(30)-(31) provide an analytic approach to compute the deformation biases in impulse responses. Braun and Mittnik [1991] derived an expression of these biases when the empirical model and the DGP are both VARs.

4 An example

To show how to match a larger DGP to a small scale empirical model; the problems occurring when the empirical model is too small; and how to reduce time deformation problems we use a standard New Keynesian models featuring five structural disturbances: a permanent a_t and a transitory ζ_t TFP shock, a preference χ_t shock, a cost push μ_t shock and a monetary policy ε_t shock (see Canova and Ferroni [2011] for details). The optimality conditions are (conditional expectations are omitted):

$$\chi_t = \chi_{t+1} - \frac{1}{1-h} g_{t+1} + \frac{h}{1-h} g_t + r_t - \pi_{t+1} \tag{32}$$

$$\pi_t = \pi_{t+1} \beta + k_p \left(\frac{h}{1-h} g_t + (1+\sigma_n)\, n_t \right) + k_p\, (\mu_t - \chi_t) \tag{33}$$

$$o_t = \zeta_t + (1-\alpha)\, n_t \tag{34}$$

$$r_t = \rho_r r_{t-1} + (1-\rho_r)\,(\phi_y\, g_t + \phi_p\, \pi_t) + \varepsilon_t \tag{35}$$

$$g_t = a_t + o_t - o_{t-1} \tag{36}$$

(32) is the Euler equation, (33) is the Phillips curve, (34) is the production function, (35) is the Taylor rule, and(36) is the definition of output growth. o_t is output and g_t its growth rate, n_t is hours worked, π_t is the inflation rate, r_t the nominal interest rate and c_t consumption. h is the coefficient of (external) consumption habit, β the discount factor, σ_n the inverse of the Frish elasticity of labor supply, κ_p the slope of the Phillips curve, α the labor share in production, ϕ_y, ϕ_π the coefficients of the Taylor rule. The disturbances evolve as:

$$\zeta_t = \rho_z\, \zeta_{t-1} + e_{zt} \tag{37}$$

$$a_t = \rho_a\, a_{t-1} + e_{at} \tag{38}$$

$$\chi_t = \rho_\chi\, \chi_{t-1} + e_{\chi t} \tag{39}$$

$$\mu_t = \rho_\mu\, \mu_{t-1} + e_{\mu_t} \tag{40}$$

$$\epsilon_t = e_{mp_t} \tag{41}$$

where $0 < \rho_j < 1, j = z, a, \chi, \mu$. The minimal state vector is $x_{t-1} = [o_{t-1}, r_{t-1}, \zeta_{t-1}, a_{t-1}, \mu_{t-1}, \chi_{t-1}]'$, and the control vector is $y_t = [g_t, o_t, \pi_t, n_t, r_t]'$. We solve the model using a first order perturbation setting $\alpha = 0.33; \beta = 0.99; \sigma_n = 1.5; h = 0.9; k_p = 0.05; \phi_y = 0.1; \phi_p = 1.5; \rho_r = 0.8; \rho_z = 0.5; \rho_a = 0.2; \rho_\chi = 0.5; \rho_\mu = 0.0$. We obtain decision rules of the form (11)-(12), where $A(\theta)$ is 6×6, $B(\theta)$ is 6×5, $C(\theta)$ is 5×6 and $D(\theta)$ is 5×5.

We consider alternative empirical systems with 4,3,or 2 variables. In the first $z_t = (o_t, \pi_t, n_t, r_t)$; it is obtained using (36) in (32)-(35):

$$\chi_t = \chi_{t+1} - \frac{1}{1-h}(a_{t+1} + o_{t+1} - o_t) + \frac{h}{1-h}(a_t + o_t - o_{t-1}) + r_t - \pi_{t+1} \quad (42)$$

$$\pi_t = \pi_{t+1}\beta + k_p\left(\frac{h}{1-h}(a_t + o_t - o_{t-1}) + (1+\sigma_n)\, n_t\right) + k_p\,(\mu_t - \chi_t) \quad (43)$$

$$o_t = \zeta_t + (1-\alpha)\, n_t \quad (44)$$

$$r_t = \rho_r\, r_{t-1} + (1-\rho_r)\,(\phi_y\,(a_t + o_t - o_{t-1}) + \phi_p \pi_t) + \varepsilon_{mp_t} \quad (45)$$

The state vector is still $x_{t-1} = [o_{t-1}, r_{t-1}, \zeta_{t-1}, a_{t-1}, \mu_{t-1}, \chi_{t-1}]'$ and $A(\theta), B(\theta)$ are unaltered. Since a control is integrated out, this system corresponds to case 1 of section 3. By proposition 1, no time distortions is present, but the innovations cross sectionally combine the structural disturbances.

The second system uses $z_t = (o_t, \pi_t, n_t)$. It is obtained using (45) into the other equations:

$$(1+\rho_r)\chi_t - \rho_r\chi_{t-1} = \chi_{t+1} - \frac{1}{1-h}(a_{t+1} + o_{t+1} - o_t) + (\frac{h+\rho_r}{1-h} + (1-\rho_r)\phi_y)(a_t + o_t - o_{t-1})$$
$$- (\frac{h\rho_r}{1-h})(a_{t-1} + o_{t-1} - o_{t-2}) + (\rho_r + (1-\rho_r)\phi_p)\,\pi_t + e_{mp_t} - \pi_{t+1} \quad (46)$$

$$\pi_t = \pi_{t+1}\beta + k_p\left(\frac{h}{1-h}(a_t + o_t - o_{t-1}) + (1+\sigma_n)\, n_t\right) + k_p\,(\mu_t - \chi_t) \quad (47)$$

$$o_t = \zeta_t + (1-\alpha)n_t \quad (48)$$

Here an endogenous state, r_{t-1} is integrated out and this makes the Euler equation (46) a second order difference equation. Thus, we loose one state, r_{t-1}, but acquire another one, o_{t-2}. Because both states and controls are eliminated, this system corresponds to case 2 of section 3. Proposition 2 then tells us that the innovations will mix cross sectionally and over time $e_{t-s}, s \geq 0$. By proposition 3, we expect identified responses to be more distorted than in the four variables system.

Distortions emerge for two reasons. First, notice that (46) is a dynamic aggregate demand equation in output and inflation while (47)-(48) define a dynamic aggregate supply equation in the same variables and that both are instantaneously moved by TFP and preference disturbances. Thus, it will be impossible to separate these disturbances in such a system. Second, (46) depends on a_{t-1}, ζ_{t-1} and, because o_{t-2} enter the equation, also on ζ_{t-2}. Hence, the aggregate demand equation evolves more persistently in response to disturbances than in the original model.

The third system uses $z_t = (\pi_t, n_t, r_t)$. It is obtained using (44) in the other equations:

$$\chi_t = \chi_{t+1} - \frac{1}{1-h}(a_{t+1} + \zeta_{t+1} - \zeta_t + (1-\alpha)\,(n_{t+1} - n_t))$$
$$+ \frac{h}{1-h}(a_t + \zeta_t - \zeta_{t-1} + (1-\alpha)\,(n_t - n_{t-1})) + r_t - \pi_{t+1} \quad (49)$$

$$\pi_t = \pi_{t+1}\beta + k_p\left(\frac{h}{1-h}(a_t + \zeta_t - \zeta_{t-1} + (1-\alpha)\,(n_t - n_{t-1})) + (1+\sigma_n)\, n_t\right) + k_p\,(\mu_t - \chi_t) \quad (50)$$

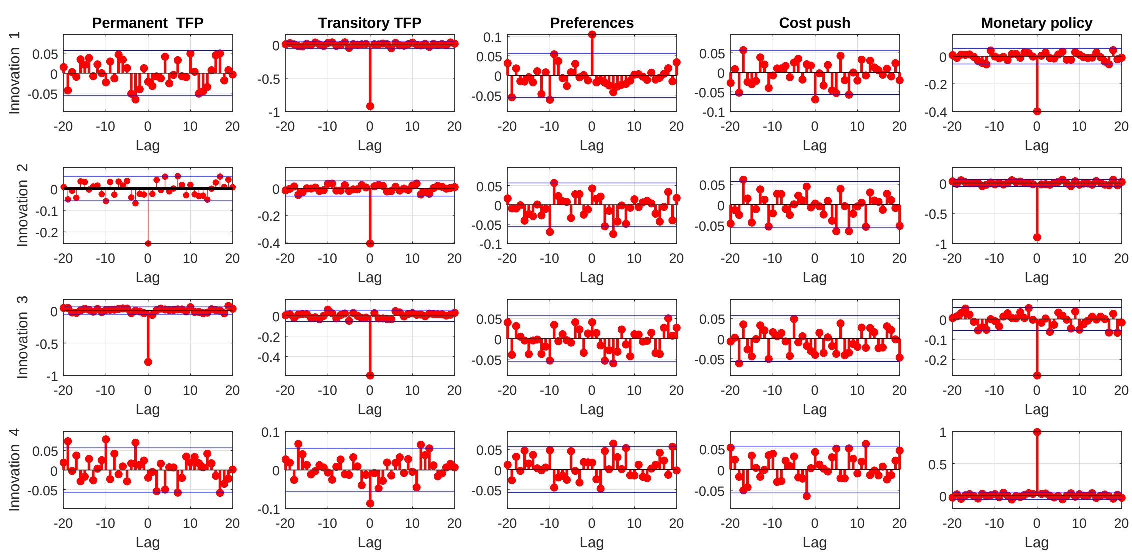
$$r_t = \rho_r\, r_{t-1} + (1-\rho_r)\,(\phi_y\,(a_t + \zeta_t - \zeta_{t-1} + (1-\alpha)\,(n_t - n_{t-1})) + \phi_p\,\pi_t) + \varepsilon_{mp_t} \quad (51)$$

In this system a state variable, o_{t-1}, is integrated out. However, the optimality conditions remain a set of first order difference equations. The reason is that n_{t-1} becomes a state variable and, given the production function, it closely proxy for o_{t-1}. Because the states are repackaged and controls omitted, biases will be present. However, because given ζ_{t-1}, n_{t-1} closely proxy for o_{t-1}, time deformations will be small. Thus, we expect the relationship between u_t and e_t and the impulse responses to be less distorted here then in the (o_t, π_t, n_t) system.

Time deformation To confirm the presence of time deformation distortions, we analytically compute the autocorrelation function of the innovations in the three systems (see figures A.1-A.3 in the appendix). The innovations of the (o_t, π_t, n_t, r_t) system are, as expected, white noise; those of the (o_t, π_t, n_t) system display serial correlation and numerous lags are significant. The innovations of the (π_t, n_t, r_t) system are, instead, serially uncorrelated.

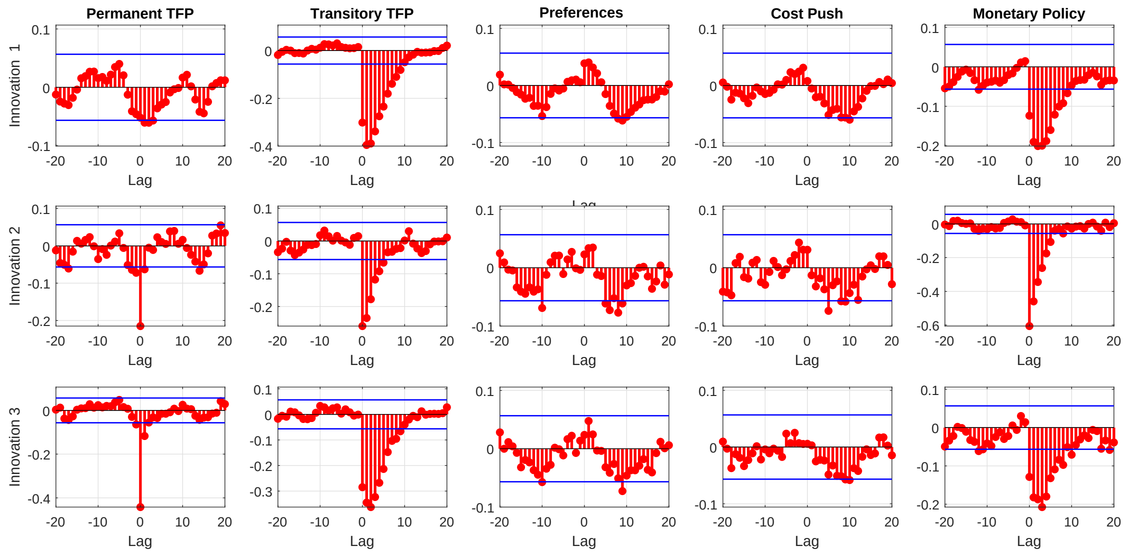
To see what causes time deformation, we analytically compute the cross correlation function between the innovations and the structural disturbances. We report the value obtained and an 95% asymptotic tunnel for the hypothesis that the cross correlation at each horizon is zero. Without time deformation, only contemporaneous correlations should be significantly different from zero. In the four variable system, u_t and e_t are only contemporaneously linked, see figure 1. This is not the case in the (o_t, π_t, n_t) system: u_t significantly correlates with several lags of the monetary policy disturbance and of the stationary TFP disturbances, see figure 2. The innovations of the (n_t, π_t, r_t) system instead show weak evidence of time aggregation, see figure 3.

Figure 1: Cross correlation function, innovations in the (o_t, π_t, n_t, r_t) system and structural shocks.



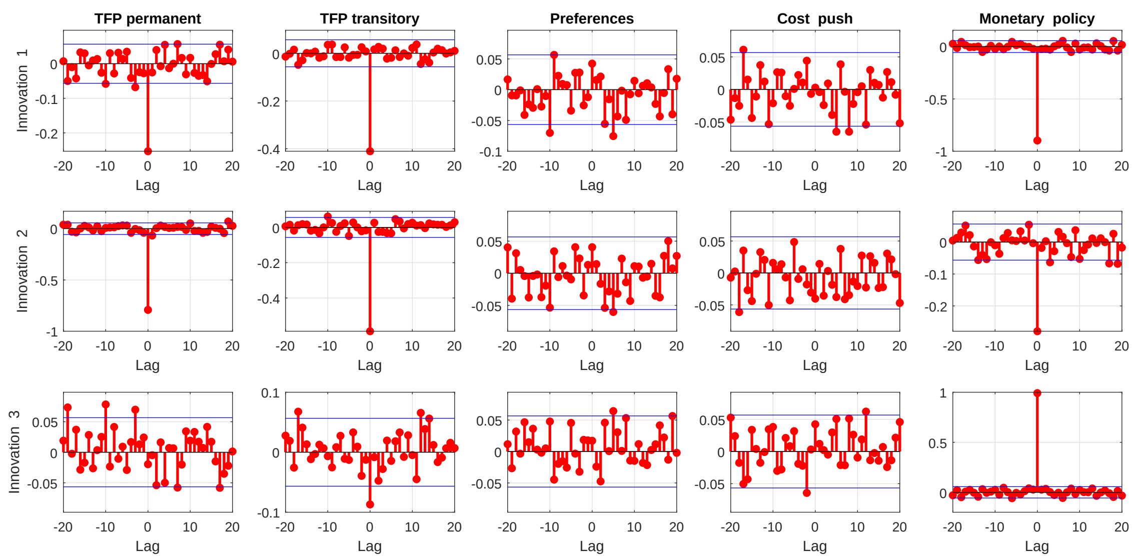
Note: Parallel line delimit the 95 % asymptotic tunnel for the hypothesis of zero cross correlations.

Figure 2: Cross correlation function, innovations in the (o_t, π_t, n_t) system and structural shocks.



Note: Parallel lines delimit 95 % asymptotic tunnel for the hypothesis of zero cross correlations.

Figure 3: Cross correlation function, innovations in the (π_t, n_t, r_t) system and structural shocks.



Note: Parallel lines delimit 95 % asymptotic tunnel for the hypothesis of zero cross correlations.

Cross sectional deformation To examine the extent of cross sectional deformation, we present the $\lambda(\theta, L)$ polynomial (for the (o_t, π_t, n_t, r_t) and (π_t, n_t, r_t) systems only $\lambda_0(\theta)$ is relevant).

Cross sectional deformation is important in all systems (see table 1). With four observables, transitory TFP and monetary policy disturbances receive the largest weights in the innovation and cost push disturbance the smallest. Thus, identification of cost push disturbances is difficult, even when the correct restrictions are used; their variability has to be of an order of magnitude larger for innovations to carry information about them. In addition, while the four variable system preserves the sign of the contemporaneous responses to monetary policy disturbances (an increases interest rates and a fall in output, inflation, and hours), positive stationary TFP and negative preference disturbances will be confused, when sign restrictions are used for identification, as they both produce an instantaneous fall in (o_t, π_t, n_t, r_t).

Table 1: Entries of the $\lambda(L)$ matrix

	Structural shocks				
	a_t	ζ_t	χ_t	μ_t	ϵ_t
	Innovations in (y_t, π_t, n_t, r_t) system $\lambda_0(\theta)$				
u_{1t}	0.018	-0.722	0.087	-0.005	-0.303
u_{2t}	-0.158	-0.306	0.042	0.042	-0.716
u_{3t}	-1.464	-1.078	0.131	-0.007	-0.452
u_{4t}	-0.047	-0.086	0.014	0.012	0.778
	Innovations in (π_t, n_t, r_t) system $\lambda_0(\theta)$				
u_{1t}	-0.158	-0.306	0.042	0.042	-0.716
u_{2t}	-1.464	-1.078	0.131	-0.007	-0.452
u_{3t}	-0.047	-0.086	0.014	0.012	0.778
	Innovations in (y_t, π_t, n_t) system				
	$\lambda_0(\theta)$				
u_{1t}	-0.05	0.71	0.11	0.03	-0.29
u_{2t}	-0.19	-0.30	0.05	0.05	-0.70
u_{3t}	-1.57	-1.06	-0.17	0.05	-0.43
	$\lambda_1(\theta)$				
u_{1t}	-0.07	-0.92	0.12	0.04	-0.41
u_{2t}	-0.01	-0.28	0.03	0.01	-0.52
u_{3t}	-0.25	-1.37	0.18	0.06	-0.61
	$\lambda_2(\theta)$				
u_{1t}	-0.05	-0.90	0.11	0.04	-0.46
u_{2t}	-0.01	0.28	0.03	-0.01	-0.52
u_{3t}	-0.09	-1.35	0.16	-0.07	-0.69

In the (o_t, π_t, n_t) system, structural disturbances enter the innovations for a number of time periods ($\lambda_0(\theta), \lambda_1(\theta)$ and $\lambda_2(\theta)$ are reported for illustration). Sign restrictions can not now separate TFP, preference, and monetary policy disturbances. Positive TFP, negative preference and contractionary policy disturbances all have negative effects on (o_t, π_t, n_t).

In the (π_t, n_t, r_t) system, the sign and the magnitude of the loadings of structural disturbances are the same as in the four variable system. As compared with the (o_t, π_t, n_t, r_t) system, we loose the possibility to distinguish stationary TFP, permanent TFP and preference shocks. However, there is no change in the ability to recover monetary policy disturbances.

Cholesky factors Table 2 displays the Cholesky factors of the covariance matrix of the innovations of original model (assuming disturbances have unit variance and with the rows and columns corresponding to the variables solved out eliminated) and of the three smaller systems. While the entries of $\lambda_0(\theta)$ are such that standard zero restrictions are unlikely to identify structural disturbances, applying the same recursive restrictions to the innovations of the original and of the reduced systems makes the comparison meaningful.

Table 2: Cholesky factors

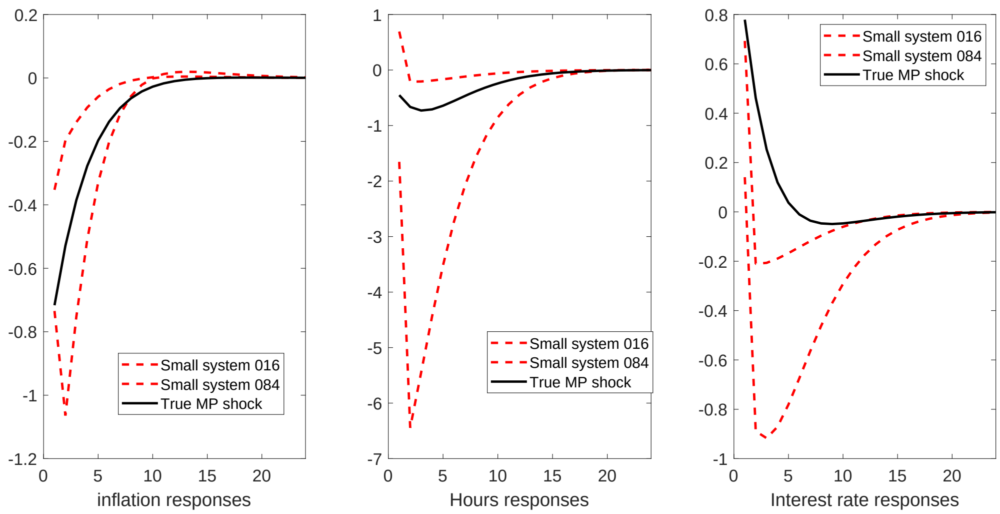
Observables	**Original system**				**Reduced system**			
(o_t, π_t, n_t, r_t)	0.75				0.78			
	0.68	0.26			0.55	0.57		
	1.06	1.14	0.95		1.14	0.44	1.14	
	-0.42	-0.13	0.16	0.07	-0.22	-0.70	0.26	0.07
(π_t, n_t, r_t)	0.26				0.79			
	1.14	0.95			1.11	1.50		
	-0.13	0.16	0.07		-0.65	0.36	0.23	
(o_t, π_t, n_t)	0.75				9.55			
	0.68	0.26			5.16	1.50		
	1.06	1.14	0.95		15.36	-0.02	1.52	

The Cholesky factor of the (o_t, π_t, n_t, r_t) system retains the signs of the Cholesky factor of the original model, but magnitudes are altered, sometimes substantially (see the (3,2) or (4,2) elements). A similar picture emerges in the (π, n_t, r_t) system. Thus, responses to orthogonal shocks in these two systems should mimic those of the original model but display magnitude distortions.

For the innovations of the (o_t, π_t, n_t) system the story is different: the signs are affected and magnitude differences are large. For example, while in the original system an orthogonal unitary shock to n_t implies a roughly similar instantaneous effect on o_t and π_t, the same shocks in the (o_t, π_t, n_t) system has a 15 times larger effect on o_t and a negative effect on π_t. As we will see, these distortions remain at longer horizons.

Impulse responses We measure dynamic deformation distortions when we identify disturbances via sign restrictions. By proposition 3, different systems will display different properties.

Figure 4: Responses to identified monetary policy shocks, (π_t, n_t, r_t) system



Note: The dashed regions report 68 % interval obtained accounting for rotation uncertainty. The solid line reports the responses in the DGP.

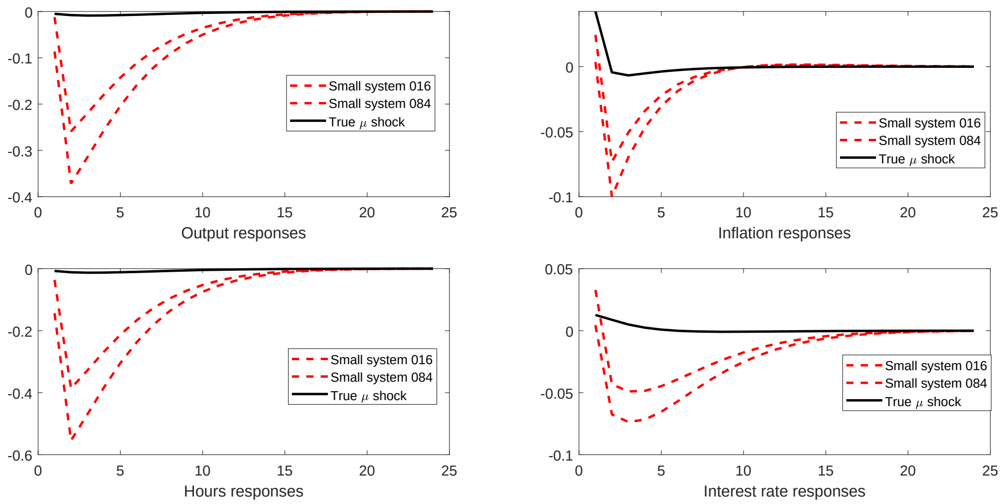
Figure 4 presents the responses to a monetary policy shock in the (π_t, n_t, r_t) systems when policy disturbances are identified assuming that an increase in r_t leads to a contemporaneous fall in the other variables. Figure A.4 in the appendix has the responses to a monetary shock in the (o_t, π_t, n_t, r_t) system. Dotted lines represent 68% credible sets across rotations satisfying the restrictions. Superimposed as continuous lines are the responses of the original 5 variable model. Even the three variable system encodes enough information to recover monetary policy disturbances. Thus, omitting output and its growth rate does not affect our ability to interpret the responses to identified monetary shocks, provided hours enter the empirical system.

The conclusion is different when cost push disturbances ae of interest. In agreement with Canova and Paustian [2011], figure 5 shows that the dynamics produced by sign-identified cost push shocks poorly approximate the dynamics induced by cost push disturbances in the original model, even when the restrictions employed are sound.

Recall that the entries of $\lambda_0(\theta)$ imply that positive stationary TFP and negative preference disturbances have the same sign implications on the four observables. Thus, imposing theoretically sound sign restrictions only identifies a linear combination of these two disturbances, a reminiscent of the masquerading effect discussed in Wolf [2018]. Figure 6, which plots responses to sign-identified preference shocks and to preferences disturbances, shows that the size of estimated impact responses is off by a large amount; and that dynamic responses are more persistent in the smaller system.

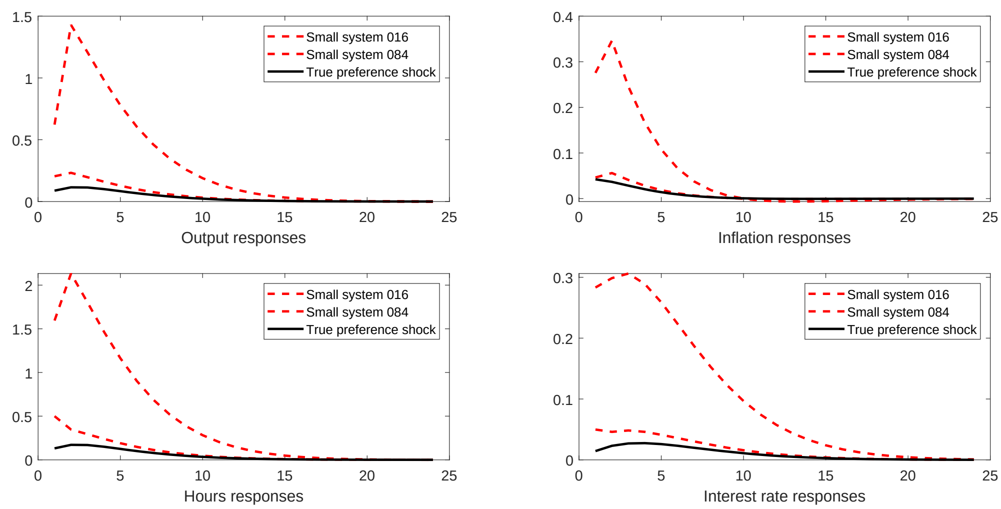
An empirical model with only the theoretical states Omission of the theoretical states or failure to proxy for them generates time deformation problems. However, an empirical system with

Figure 5: Responses to identified cost push shocks, (o_t, π_t, n_t, r_t) system



Note: The dashed regions report 68 % interval obtained accounting for rotation uncertainty. The solid line reports the responses in the DGP.

Figure 6: Responses to identified preference shocks, (o_t, π_t, n_t, r_t) system



Note: The dashed regions report 68 % interval obtained accounting for rotation uncertainty. The solid line reports the responses in the DGP.

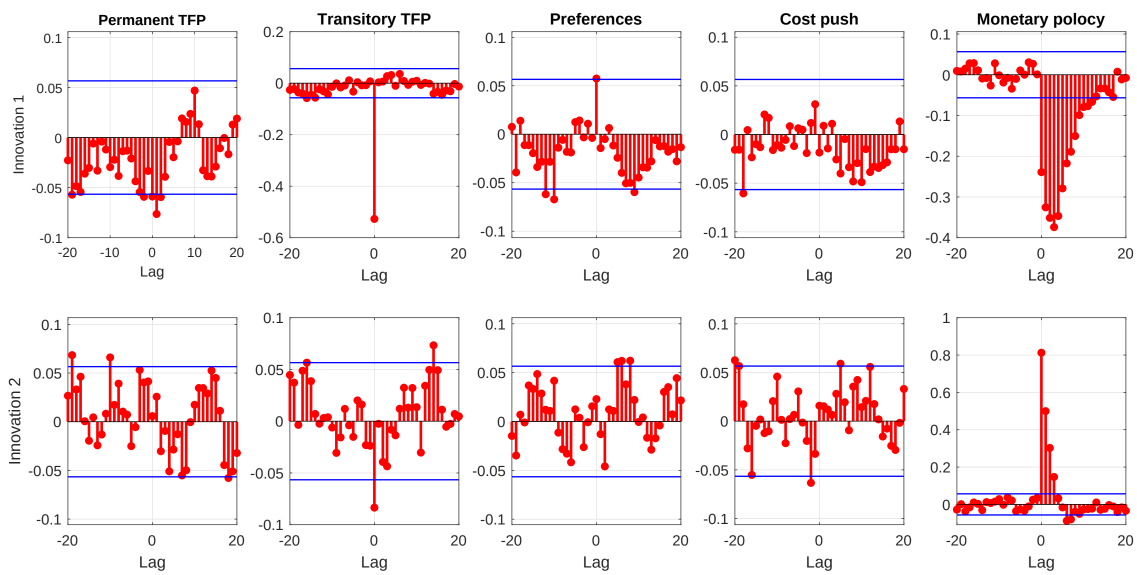
only the states (and none of the controls) will not necessarily produce interpretable identified shocks.

Starting from the (o_t, π_t, n_t, r_t) system and using the production function and the Phillips curve into the remaining two equations, the optimality conditions for $z_t = (o_t, r_t)$ are:

$$\begin{aligned}
\chi_t &= (1+\beta)\chi_{t+1} - \beta\chi_{t+2} - \frac{1}{1-h}(a_{t+1}+o_{t+1}-o_t) + \frac{\beta}{1-h}(a_{t+2}+o_{t+2}-o_{t+1}) \\
&+ (\frac{h}{1-h})(a_t+o_t-o_{t-1}) - (\frac{h\beta}{1-h})(a_{t+1}+o_{t+1}-o_t) + r_t - \beta r_{t+1} \\
&- k_p\left(\frac{h}{1-h}(a_{t+1}+o_{t+1}-o_t) + (1+\sigma_n)\frac{1}{1-\alpha}(o_{t+1}-\zeta_{t+1})\right) - k_p(\mu_{t+1}-\chi_{t+1}) \qquad (52) \\
r_t &= \beta r_{t+1} + \rho_r r_{t-1} - \beta\rho_r r_t + (1-\rho_r)\phi_y((a_t+o_t-o_{t-1}) - \beta(a_{t+1}+o_{t+1}-o_t)) \\
&+ (1-\rho_r)\phi_\pi\left(k_p\left(\frac{h}{1-h}(a_t+o_t-o_{t-1}) + (1+\sigma_n)\frac{1}{1-\alpha}(o_t-\zeta_t)\right) + k_p(\mu_t-\chi_t)\right) \\
&+ \epsilon_{mp_t} - \beta\epsilon_{mp_{t+1}} \qquad (53)
\end{aligned}$$

Here x_{t-1} is unchanged. However, inspection of (52)-(53) reveals that the optimization problem is different and, for example, o_{t+2} and r_{t+1} now appear in the equilibrium conditions. Since $(\bar{A}(\theta), \bar{B}(\theta))$ differs from the original $(A(\theta), B(\theta))$ matrices, this system will also feature timing distortions. Figure 7, which reports the cross correlation of the innovations with the five structural disturbances, indicates that u_t are serially correlated and load on a number of lags of the monetary policy disturbance.

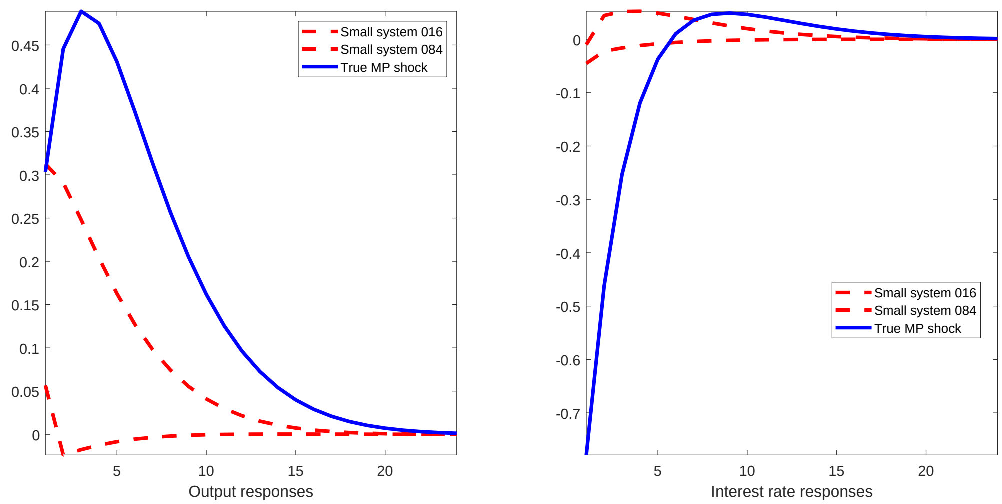
Figure 7: Cross correlation function, innovations in (o_t, r_t) system and structural shocks.



Note: Parallel lines describe the 95 % asymptotic tunnel for the hypothesis of zero cross correlations.

Cross sectional deformation is also important. With $z_t = (o_t, r_t)$, technology, monetary policy and cost push shocks are not separately sign-identifiable (they all have the same instantaneous effects on z_t). As Figure 8 shows, the monetary policy shocks identified with contemporaneous sign restrictions (r_t up and o_t down) are now a weighted average of the three underlying disturbances.

Figure 8: Responses to identified monetary policy shocks, (o_t, r_t) system.



Note: The dashed regions report 68 % interval obtained accounting for rotation uncertainty. The solid line reports the responses in the DGP.

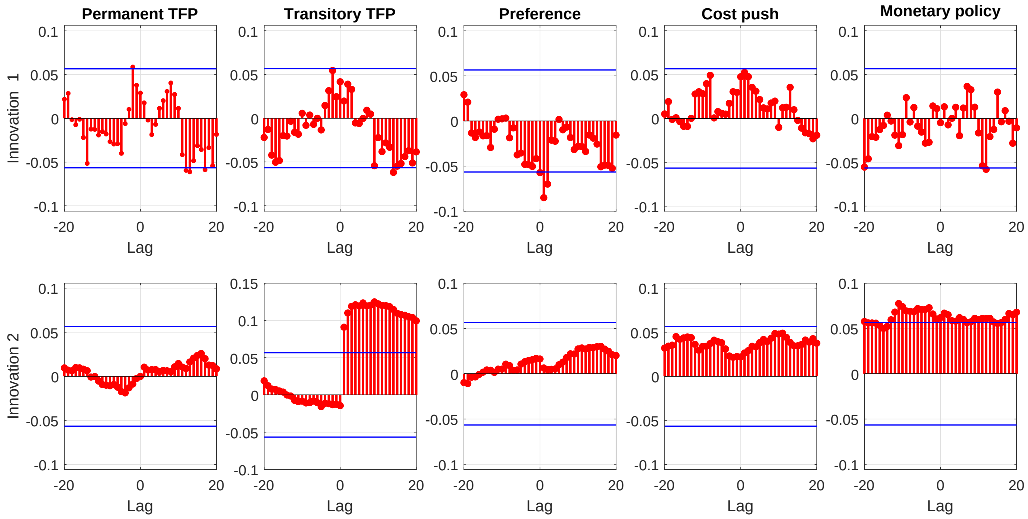
Permanent technology shocks and hours worked In the literature it is common to use an empirical model with output growth (or labor productivity) and hours to identify permanent TFP shocks. The dynamics are then compared with the dynamics permanent TFP disturbances produce in standard RBC or new Keynesian models, see e.g. Galí [1999]. While the comparison is meaningful when the DGP features only two disturbances (say, a permanent TFP and a demand shock), it may be inappropriate when the model of this section has generated the observed data. Because five disturbances are compressed into two identified shocks and the states of the original model are repackaged and their law of motion altered, there is no insurance that the identified technology shocks in the data mimic only permanent technology disturbances. To show this, we reduce the optimality conditions to contain $z_t = (g_t, n_t)$ [2]

$$\begin{aligned}(1+\rho_r)\chi_t &= \rho_r\chi_{t-1} + \chi_{t+1} + \frac{1}{1-h}g_{t+1} + (\frac{\rho_r + h}{1-h} + (1-\rho_r)\phi_y)g_t \\ &- \frac{h\rho_r}{1-h}g_{t-1} + \epsilon_{mp_t} + \kappa_p(\frac{h}{1-h}g_t + (1+\sigma_n)n_t) + \kappa_p(\mu_t - \chi_t) \qquad (54) \\ g_t &= a_t + \zeta_t + (1-\alpha)n_t - \zeta_{t-1} - (1-\alpha)n_{t-1} \qquad (55)\end{aligned}$$

Note that lagged output growth and lagged hours are now endogenous states. Figure 9, which relates the innovations of the g_t, n_t system to the structural disturbances, shows that both innovations are moving averages of the five disturbances. In particular, lags of the stationary TFP and of the monetary policy disturbances enter the second innovation; lags and leads of the permanent TFP disturbance and lags of the preference disturbance load significantly on the first innovation.

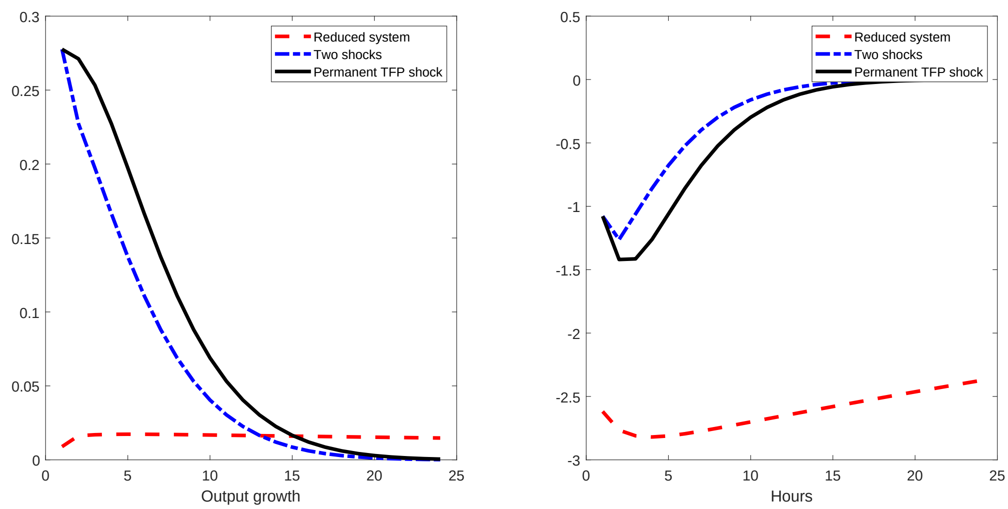
[2]To obtain these equations we assume that $\beta^{-1} = (1-\rho_r)\phi_\pi\pi + \rho_r$.

Figure 9: Cross correlation function, innovations in (g_t, n_t) system and structural shocks.



Note: Parallel lines describe the 95 % asymptotic tunnel for the hypothesis of zero cross correlations.

Figure 10: Responses to identified permanent TFP shocks, (g_t, n_t) system.



We identify a permanent TFP shock using long run restrictions. Figure 10 shows that if the DGP only has a permanent TFP and monetary policy disturbance, the responses obtained identifying a permanent shock in a VAR with $z_t = (g_t, n_t)$ capture well the original dynamics. Instead, when the model of this section has generated the data, magnitude and the persistence distortions are present. In other words, the model we consider can not be reduced to a bivariate system with output growth and hours and meaningful innovations: identified permanent technology shocks combine current and lagged values of permanent TFP as well as other stationary demand disturbances.

5 Implications for practice

Small scale empirical models are easy to estimate and to identify but problematic for interpretation and inference. Cross sectional deformation makes shock identification hard, because "classes" of disturbances need not be properly compressed into the identified shocks. This means that sound identification restrictions are generally insufficient to obtain structural disturbances. Time deformation dramatically complicates the identification process because the timing of identified shocks and structural disturbances differ. While it is tempting to associate cross sectional deformation with the elimination of theoretical controls and time deformation with the elimination of theoretical states, such an association is imperfect. Time distortions emerges also when the empirical system contains all the endogenous states. Conversely, integrating out controls may induce both biases, if the relationship between the remaining controls and the states is altered.

If we exclude Canova and Hamidi Sahneh [2018], the problems we discuss have been ignored in the recent literature. Time deformation problems have been studied in the past by Hansen and Sargent [1991], Marcet [1991], Fernández-Villaverde et al. [2007]. To the best of our knowledge, we are the first to show that their extent may depend on the dimensionality and the variables entering the empirical system and that a researcher can limit, to some extent, the magnitude of certain biases. Dimensionality reductions and non-invertibility generate similar time distortions, but a solution of the latter problem is insufficient to eliminate deformation. To be clear, assuming away all the standard pile up, cancellation and identification problems, the estimation of a VARMA model can go a long way to reduce the gap between a DGP and the empirical model due to non-invertibility. However, estimating a small scale VARMA model will not solve deformation problems if the DGP features a larger number of disturbances than the empirical model.

Our analysis has important implications for practice. If deformation problems are to be avoided, the empirical system needs to be sufficiently large. While is possible to estimate larger scale empirical models, even with relatively short datasets, their identification is still an issue. Thus, small scale empirical models are likely to be preferred by macroeconomists for some time in the future. In that case, to make the matching exercise meaningful two conditions are required. First, the size of an empirical system should be tailored to the disturbances of interest. As seen in section 4, a monetary shock can be recovered from a trivariate system with (π_t, n_t, r_t) using meaningful restrictions, but a cost push shock can not, even in four variables system. Without guidance from theory, an identified shocks may pick up the dynamics of structural disturbances which have distinct implications when a larger set of variables enter the empirical model. We have provided a way to systematically explore dimensionality reductions: in section 4 we started from a five variable structural model and analyzed whether interesting disturbances could be identified and the dynamics they produce well characterized when a smaller scale empirical system is employed. We recommend applied researchers to do the same as routine practice, prior to the estimation of the empirical system. Second, shrewdly choosing the variables entering the empirical system, can limit the magnitude of the deformation distortions. But for this to happen, empirical models can not be too small: a two variable system is likely to produce uninterpretable shocks and convoluted dynamics. In addition, one needs to be upfront about the structural model used to interpret the data. Canova and Paustian [2011] showed that shock identification is better anchored when business cycle measurement is tied up with robust identification restrictions. The connection with theory is even more important if deformation is present. If two researchers use two models with the same (New Keynesian) features but with different number or type of disturbances to interpret the data, they ought to use different empirical models to identify shocks and trace out their dynamics, even if they care about the same impulses.

While it is common to sweep deformation problems under the rug, misspecification may be pervasive. For example, Central Banks use structural models with dozens of disturbances to interpret the data and academic researchers often twist standard models in estimation so that structural parameters become exogenous disturbances (e.g an elasticity of substitution becomes a markup disturbance) to improve their fit. Unfortunately, we do not have much to say about the disturbances which are present in the DGP. The argument that one should include in the theory used to interpret the data disturbances which are important for business cycle fluctuations is, unfortunately, a catch-22 proposition, because their relevance depends on the choice and the number of disturbances one considers.

In general, the practice of comparing small scale empirical models and larger scale DSGE responses should be considerably refined. Showing that the qualitative pattern of responses to interesting impulses is similar is insufficient for a structural model to be considered successful if deformation is present. To make the gap smaller one should compare responses obtained from identified shocks in the small scale empirical system with the responses obtained in the theory, once it is reduced to the same variables as the empirical system.

It is popular to use small scale SVARs to cross off theories inconsistent with the data. For example, the responses of hours to technology shocks and of output to government spending shocks are used to limit attention to certain classes of models (see e.g. Galí [1999], Leeper, Traum, and Walker [2017]). While the qualitative features of the responses are, at times, unchanged by deformation, magnitude and persistence are generally affected. Thus, it is dangerous to exclude theories using, say, the magnitude of multipliers. To evaluate theories, one needs statistics insensitive to deformation: identified VAR shocks and the dynamics they produce do not generally fall in this class.

It is also popular to estimate the parameters of a theoretical model by matching response to certain disturbances in the data and in the theory. For the exercise to be meaningful, three conditions need to be met. First, to avoid cross sectional deformation the theory should be reduced to the same observables used in the empirical model, prior to the estimation exercise. Second, to avoid time deformation, data responses should be computed in empirical models featuring either generous lag length or carefully selected variables. Third, the disturbances of interest should be identifiable in the small scale empirical system. When any of these conditions is not satisfied, parameter estimates and dynamic responses are non-interpretable.

It has become common recently to use IV approaches to identify certain shocks and local projection techniques to compute dynamic responses (see e.g. Rossi [2019] for a survey). Would such methods reduce the deformation gap? Local projection techniques and IV estimation could be of use, but a number of conditions need to be met. Take for example case 2 of section 3, where the states are eliminated from the empirical model. The DGP for the observables is a VARMA(2,1) which, in a companion from, can be written as $W_t = QW_{t-1} + Rv_t$ where $W_t = [y_t, y_{t-1}]'$ $v_t = [e_t, e_{t-1}]'$, $Q = \begin{pmatrix} F_{21} & F_{22} \\ I & 0 \end{pmatrix}$ and $R = \begin{pmatrix} G_{20} & G_{21} \\ 0 & 0 \end{pmatrix}$. Projecting $W_{t+h},\ h = 1, 2, \dots$ on t-1 information:

$$W_{t+h} = Q^{h+1}W_{t-1} + Q^h Rv_{jt} + u_{t+h} \tag{56}$$

where v_{jt} is the disturbance of interest, $u_{t+t} = Q^h Rv_{-jt} + Q^{h-1}Rv_{t+1} + \dots + Rv_{t+h}$, and v_{-jt} are all the disturbances at t except the $j-th$ one. Because local projections do not employ the residuals of a VAR in the exercise, they are less prone to cross sectional deformation. However, for local projections to be successful in capturing $Q^h R$ the regressors of the projection equation should be W_{t-1} and v_{jt}. When v_{jt} is not observable, we need proxies that capture the effect of both e_{jt} and e_{jt-1}. If only e_{jt} is used in the projection equation, the right hand side variables are correlated with the error term, making OLS invalid. Similarly, if an IV approach is used after normalization, the instruments have to be strictly exogenous and capture only the variations in W_{jt} which are due to v_{jt}. Predetermined instruments are insufficient, unless the conditioning set of the projection equation includes an infinite number of lags of W_t.

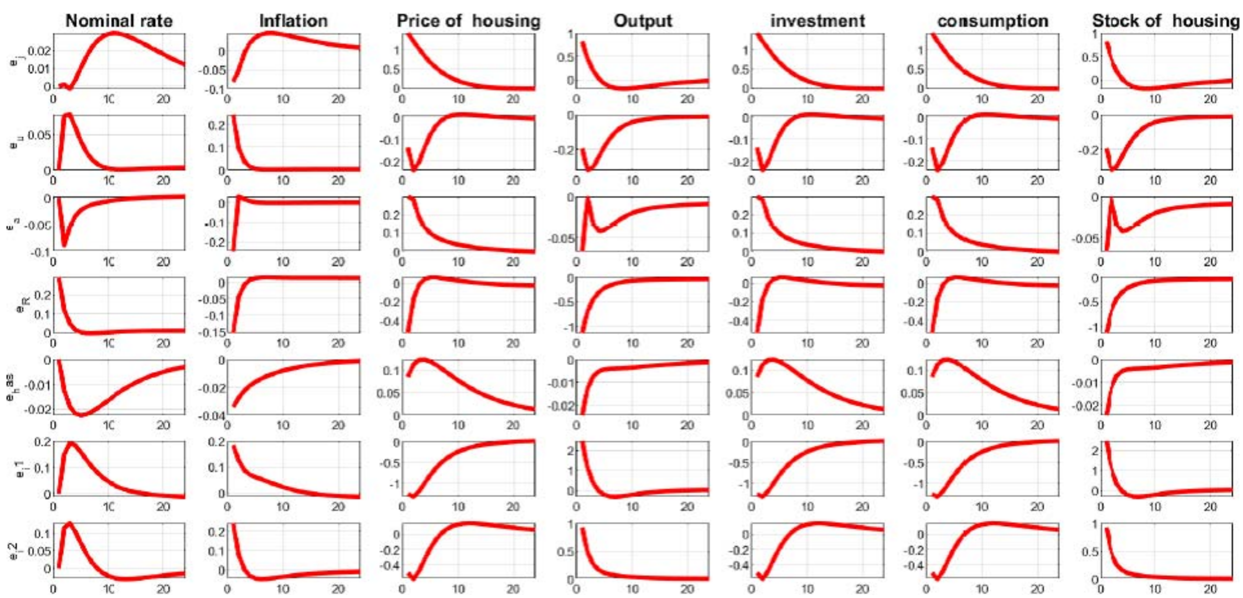
One may also be able to reduce time deformation with FAVARs, where factors are constructed using the omitted states. Still, also in this case, we need a model defining what the relevant states are. In addition, FAVARs do not necessarily eliminate cross sectional distortions: statistical principal components are unlikely to properly combine classes of structural disturbances and to make the mapping between innovations and structural disturbances better behaved.

To sum up, alternatives to SVARs could work in making the match with the theory tighter. However, to the best of our knowledge, they have not yet come into the mainstream of stylized fact production. Furthermore, they have to be appropriately rigged to deliver correct conclusions.

6 The effect of house price disturbances

The dynamics of output and inflation following house price disturbances are of primary policy importance following the 2008 financial crisis. Starting with Iacoviello [2005] many authors have tried to understand whether the responses obtained in the data can be rationalized with a structural model featuring housing, leveraged agents, and standard macroeconomic frictions. Since house price disturbances are not necessarily the major source of fluctuations in macroeconomic variables, at least in normal times, the theoretical models employed to interpret the data typically contain several other disturbances, see e.g. Rabanal [2018], Linde' [2018] for recent examples. However, apart from obvious core choices, it is not clear what one should include and, depending on the focus of the investigation, alternative disturbances may be considered. For example, for monetary policy the interaction between house price and other demand disturbances is important; for financial stability house price and leverage disturbances are at the center of attention.

Figure 11: Impulse responses theory



Iacoviello [2005] selects the minimum number of disturbances needed to map the empirical evidence into a structural model. He uses a four variable VAR to construct the dynamic responses to identified house price shocks and a model with preferences, monetary policy, technology and cost push disturbances to estimate the structural parameters and interprets the dynamics in the data using preference disturbances. Here we work with the same model but, for illustration, add disturbances to the borrowing constraints of entrepreneurs and impatient consumers and a wealth disturbance to the budget constraint of impatient consumers. These disturbances have been used in many exercises and by including them, we try to evaluate whether identified house price shocks may also be capturing the effect of other disturbances.

The optimality conditions and the law of motion of the disturbances are in the Appendix. The model features 8 endogenous states (lagged house holdings of impatient consumers and of entrepreneurs, lagged bond holdings of patients and impatient consumers, lagged capital shock, lagged output, lagged nominal interest rate, and lagged inflation) and 15 endogenous controls. The theoretical responses of the nominal rate, inflation, house prices, output, consumption, investment, the stock of housing of constrained consumers to the 7 disturbances are in figure 11. Positive preference disturbances increase all variables - the nominal rate and inflation only after a few quarters (see the first row). Qualitatively, the dynamics produced by preferences disturbances need not be confused with those of other disturbances, once we restrict attention to the four endogenous variables used by Iacoviello (nominal rate, inflation, house price and output). Nevertheless, since the seven disturbances are compressed into four innovations, cross sectional deformation is present. What do identified house price shocks capture? Preference or a combination of several disturbances? In addition, since a number of states are excluded from the VAR, time deformation is present.

We take data for real GDP, the nominal interest rate, inflation, and real house prices from the FRED data base for the period 1975:1-2018:3 and identify house price shocks using the same lag setting, the same data transformation, and the same identification scheme of Iacoviello [2005] [3].

The first row of figure 12, which plots the posterior 68% response intervals to an identified house price shock in the data and the responses to preference disturbances in the theory with four disturbances, reproduces Iacoviello's main results. After a temporary house price increase, output, inflation and the nominal interest rate persistently rise; and the same pattern is generated by preference disturbances, even though in the data, the maximum response of output is delayed by 4-5 quarters. Thus, the theory seems to do well in accounting for the data. Unfortunately, the first row of figure 12 is misleading: it displays theoretical dynamics when all states are used to calculate responses; and disregards that there are only four recoverable shocks in the VAR.

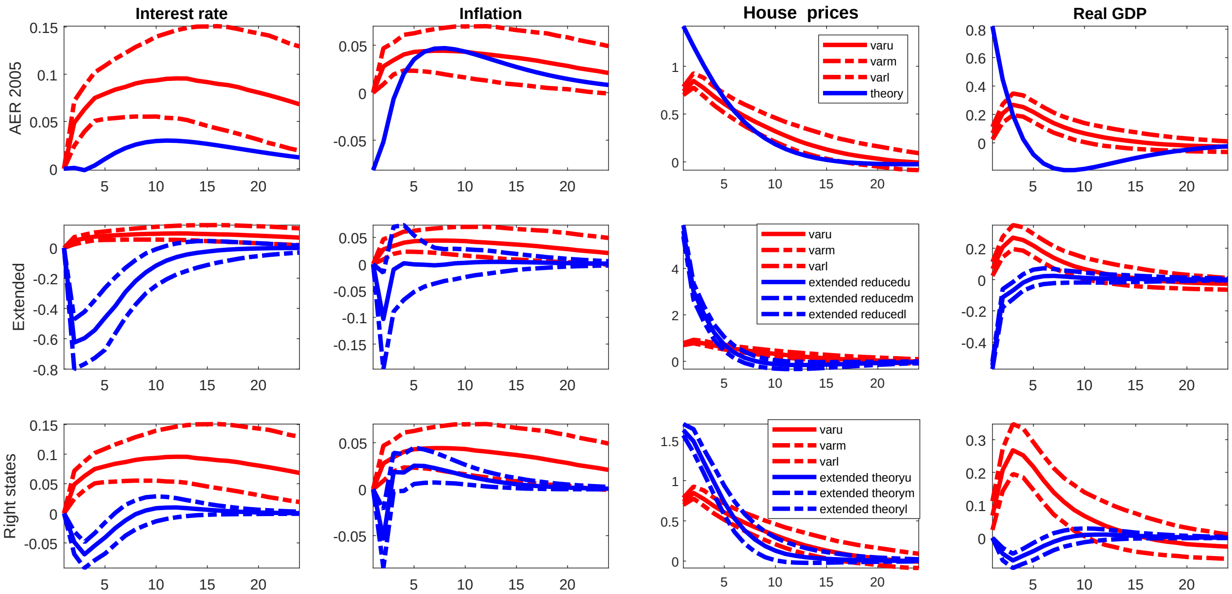
To appreciate the effects of deformation we solve equations out and reduce the first order conditions to have same four endogenous variables used in the VAR as unknown. The second row of figure 12 still plots the posterior 68% response intervals to an identified house price shock in the data but now reports the posterior 68% response intervals to an identified house price shock using data simulated from this reduced system, using the original parameterization and the same identification restrictions [4]. The sign and the persistence of the responses are now altered: output and the nominal interest rate now respond negatively; and inflation is insignificant after a few quarters. Thus, deformation matters: a four variable VAR is too small to produce identified house price shocks with the same interpretation as preference disturbances when the theory features six other disturbances.

What is it the cause of the drastic change in the dynamic responses? The third row of figure 12 evaluates the contribution of time deformation to the changes. We use the decision rules of the extended model with seven disturbances, simulate data for the four relevant endogenous variables, and identify house price shocks as in the first two rows. Because the innovations contain information about all model states, only cross sectional deformation is present.

[3] Iacoviello HP filters real GDP and house prices prior to their use in the VAR. While this choice has important implication for the timing of house price shocks and for the responses it generates, we decided to stick to this transformation since the purpose of the exercise is to show the effects of deformation, rather than those of filtering.

[4] The three new disturbances have persistence equal 0.75 and standard deviation 1.0, 1.0, 0.25, respectively. Since we normalize the impulse to unity, the magnitude of the standard deviations is irrelevant for the comparison.

Figure 12: Data and models, q_t innovations



Note: The first row reports the response to preference shocks in Iacoviello (2005) model and the 68% highest posterior interval in the data; the second row responses of the extended Iacoviello model with 7 shocks compressed to four observables and the 68% highest posterior interval in the data; the third row the responses of the extended Iacoviello model, when data is generated with the right states and the 68% highest posterior interval in the data.

The responses in rows 2 and 3 have similar sign and, quantitatively, differences are small suggesting that time deformation is relative unimportant. Truncation problems are also minor: the lag length of the estimated VAR produced by the theory does not matter for the conclusions. Hence, the responses in rows 1 and 2 differ due to cross sectional deformation; the sign of output and interest rate responses changes because seven structural disturbances are compressed into four VAR innovations. To understand what house price shocks capture, we compute the matrix of loadings of the four innovations on the seven structural disturbances. Without deformation, the q_t row should display zeros everywhere, except in the position corresponding to the preference disturbance e_j. Interestingly, there is no deformation when characterizing monetary policy disturbances: there is a one-to-one mapping between identified monetary policy shocks and theory-based monetary policy disturbances (contemporaneous cross correlation is 0.99). However, house price innovations are strongly contaminated: house price innovations heavily load on monetary policy disturbances (-1.83), on borrowing constraint disturbance of the impatient household (-1.27), and on the cost push disturbance (0.80) while the weight on the preference disturbance is small (0.05). Hence, if the model correctly represents the DGP, identified house price shocks are a mixture of monetary policy, borrowing constraints, and cost push disturbances with preference disturbances play a minor role.

Table 3: Loading of innovations in (R, π, q, Y) on structural disturbances

	e_R	e_j	e_u	e_a	e_{has}	e_{i1}	e_{i2}
R_t	1.0	0	0	0	0	0	0
π_t	-0.53	-0.003	1.43	-0.11	-0.13	0.18	0.24
q_t	-1.83	0.05	-0.80	0.13	0.33	-1.27	-0.51
y_t	-3.92	0.03	-1.14	-0.02	-0.09	2.46	0.92

The sixth row of figure 11 shows that positive borrowing constraint disturbances increase output and of the nominal rate. Thus, the negative output and interest rate responses observed in rows 2 and 3 of figure 12 are produced by the large negative loading that borrowing constraint disturbances have on identified house price shocks.

To support the conclusion, we compute the contemporaneous cross-correlation between identified house price shocks and preference disturbance in the model with four disturbances and in the extended model with seven disturbances. In the former the point estimate is 0.92 (95% confidence range across simulations (0.88, 0.96)); in the second 0.67 (95% confidence range (0.60,0.72)). Furthermore, in the latter system the contemporaneous cross-correlation between identified house price shocks and borrowing constraints disturbances is -0.63 (95% confidence range (-0.67,-0.59)).

In conclusion, if the DGP features LTV disturbances, a VAR with four endogenous variables is too small to make the comparison between preference disturbances and identified house price shocks meaningful because a number of theoretical disturbances load on identified house price shocks. To make responses to identified theoretical house price shocks look like those of preference disturbances we need a VAR with at least seven variables. If one sticks to a four variable VAR, she can compare the theory and the data only for undeformed disturbances. Monetary policy disturbances have this property; preference disturbances do not have it.

7 An extension

The process in (11)-(12) may be restrictive in certain situations. For example, when analyzing uncertainty disturbances (see e.g. Basu and Bundick [2017]), the model used for comparison is solved using higher order methods. Hence, a non-linear DGP specification is needed. This section analyzes how the conclusions of section 3 change in this case.

As Andreasan, Fernandez Villaverde, and Rubio Ramirez [2018] have shown, the pruned solution of a nonlinear state space model approximated with higher order perturbations has a linear representation of the form:

$$X_t = \mu_x(\theta) + \nu_1(\theta) X_{t-1} + \nu_2(\theta) E_t \quad (57)$$

$$Y_t = \mu_y(\theta) + \nu_3(\theta) X_t \quad (58)$$

where, for example in the case of a second order approximation, $X_t = ((x_t^f)', (x_t^s)', (x_t^f \otimes x_t^f)')'$, and x_t^f are the states of the first order system, x_t^s are the states of the second order system; $E_t = (e_t', (e_t \otimes e_t - vec(I_{n_e}))', (e_t \otimes x_{t-1}^f)'(x_{t-1}^f \otimes e_t)')'$, where e_t are the structural disturbances and I_{n_e} the identify matrix of dimension n_e; Y_t are the controls of the problem and the matrices $\mu_x(\theta), \mu_y(\theta), \nu_1(\theta), \nu_2(\theta), \nu_3(\theta)$ are given in the appendix A of Andreasan et al. [2018]. Comparing (57) – (58) and (11) – (12) one can immediately see that a higher order DGP features a larger number of states and of structural disturbances. Thus, if the empirical system is specified to be linear and features $\tilde{Z}_t = \tilde{S}[X_t, Y_t]$ as observables, where $\tilde{S} = [\tilde{S}_1, \tilde{S}_2]$, the conclusions derived in propositions 1-3 still hold. However, the reduction to $\tilde{Z}_t$ observables is potentially more damaging because the dimension of E_t is likely to be larger than the dimension of $\tilde{Z}_t$, and a larger number of states is eliminated (all those involving higher order and cross terms), making cross section and time deformations more severe.

To highlight the effects of deformation when the DGP features higher order terms, we take the model of Basu and Bundick [2017], which features a disturbance to the volatility of a preference, and two first moment disturbances: to the level of technology and to the level of preferences. The model is solved with a third order perturbation so that $E_t = [E_{1t}', E_{2t}']'$ where

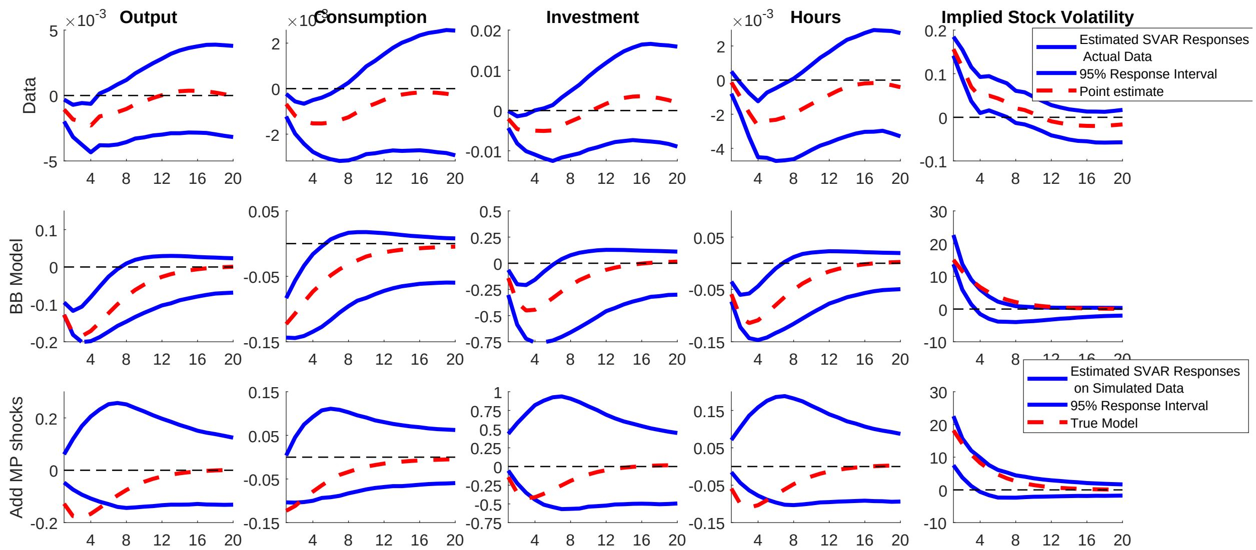
$$\begin{aligned} E_{1t} &= (e_t', (e_t \otimes e_t - vec(I_{n_e}))', (e_t \otimes x_{t-1}^f)'(x_{t-1}^f \otimes e_t)'(e_t \otimes x_{t-1}^s)')' \quad (59) \\ E_{2t} &= ((e_t \otimes x_{t-1}^f \otimes x_{t-1}^f)'(x_{t-1}^f \otimes x_{t-1}^f \otimes e_t)'(x_{t-1}^f \otimes e_t \otimes x_{t-1}^s)'(x_{t-1}^f \otimes e_t \otimes e_t)'(e_t \otimes x_{t-1}^f \otimes e_t)' \\ &\quad (e_t \otimes e_t \otimes x_{t-1}^f)'((e_t \otimes e_t \otimes e_t) - E(e_t \otimes e_t \otimes e_t)))' \quad (60) \end{aligned}$$

Since e_t is a 3×1 vector, and x_t^f a 9×1 vector including lagged values of consumption, capital, hours, output, the nominal rate, of expected utility and of the three disturbances, X_t is a 432×1 vector and E_t is a 1112×1 vector. They use an eight variables VAR to trace out the effects of uncertainty shocks, which are identified with a Cholesky decomposition having the VXO index ordered first. The VAR variables include four of the endogenous states (output, consumption, hours and nominal rate), a proxy for the capital state (investment), two controls (inflation, and a volatility measure) and money supply variable, which is not present in the model.

The first row of figure 13 presents the point estimate and the 95% response intervals of output, consumption, investment, hours and VXO to an uncertainty shock in the data. The second row has the responses to an uncertainty shocks in Basu and Bundick [2017]'s original setup and parameterization: the dashed line reports theoretical responses, and the solid lines the estimated 95% SVAR response intervals in simulated data, identifying the uncertainty shock as in the first row. The match between the theory and the data appears to be good. Furthermore, the theoretical responses and the SVAR responses to an uncertainty shock in the simulated data are similar

Two features of the authors' specification are, however, questionable. Despite the fact that the nominal interest rate is used in the VAR of the actual data, the model has little to say about interest rates because it posits a deterministic Taylor rule with no persistence (see equation (7),

Figure 13: Data and Models, VXO innovations



Note: The solid lines in the first row report 95% response intervals and the dashed line the point estimate using the actual data; the solid lines in the second and third row report the 95% response interval in the simulated data and the dashed line the conditional response in the theory.

page 945). Second, it is not obvious why changes in uncertainty of the economy are only demand driven. In principle, second moment shocks to technology could generate similar dynamics in real aggregate variables via a precautionary saving channel. Thus, the DGP potentially features more disturbances than those used in the model and the restrictions used to identify uncertainty shocks may be insufficient, making deformation important. For illustration, we add a monetary policy disturbance to the model, keeping the structure and the parameterization unchanged. As row 3 of figure 13 shows the theoretical response and the estimated response intervals differ significantly. Moreover, VAR responses in the data and in the theory do not line up.

Rows 2 and 3 differ because monetary policy and uncertainty disturbances get mixed up - they both increase the nominal rate and make all other variables fall. While the theoretical responses are constructed conditional on the monetary policy disturbances being zero, in the VAR with simulated data, the monetary policy disturbance can be both positive and negative. Hence, the sign of the responses of output, consumption, investment and hours to uncertainty shocks could be both positive or negative depending on the relative importance of uncertainty and monetary disturbances and the sign of monetary policy disturbance at each t. Because VAR responses are insignificant, identified uncertainty shocks pick up positive uncertainty and negative monetary policy disturbances [5].

To support the argument, we compute the contemporaneous cross-correlation between identified

[5]When the class of models suggested by Arouba, Boccola, and Schorfeide [2017] is used in place of a linear VAR, some of the deformation problems discussed here are eased.

volatility shocks with volatility disturbance in the original model and in the extended model with monetary policy disturbances. In the former, the point estimate is 0.77 (95% confidence range across simulations (0.68, 0.86)); in the second it is 0.62 (95% confidence range (0.50,0.74)). Furthermore, in the latter system, the contemporaneous cross-correlation between identified volatility shocks and monetary policy disturbances is -0.46 (95% confidence range (-0.50,-0.41)).

8 Conclusions

It is common in macroeconomics to collect stylized facts about the transmission of certain structural shocks using small scale SVAR models and then build DSGE models to interpret the dynamics found in the data. However, the DGP may feature more shocks than a SVAR. This paper argues that this dimensionality gap may create important inferential distortions.

Cross sectional deformation emerges when several structural disturbances contemporaneously affect the variables of the empirical model. Time deformation occurs whenever the empirical model is specified without paying sufficient attention to the theory used to explain the data. Cross sectional distortions may make sound theoretical restrictions insufficient to obtain meaningful disturbances. Time distortions make identified shocks distributed lags of the structural disturbances.

We use a standard New Keynesian model to show the problems that occur when the empirical model is too small, and describe how to reduce time distortions explicitly linking the theory and the empirical model. We argue that the theory used to interpret the data and the disturbances of interest must guide both the choice of observables and the minimal dimension of the empirical model. Thus, the empirical model used to derive dynamic facts is not theory-free, when deformation matters.

We provide suggestions on how to avoid the deformation trap when one insists in matching the dynamics produced by identified shocks in small scale empirical models and in a potentially larger scale DSGE. We revisit Iacoviello [2005]'s evidence about the transmission of house price shocks when the DGP includes LTV disturbances. We extend the analysis to consider particular non-linear DGPs and revisit Basu and Bundick [2017]'s evidence about uncertainty shocks when a monetary policy disturbance is added to the DGP. In both cases, the gap between the theory and the data may be larger than previously thought.

Because there is no guidance in choosing the number and the type of disturbances present in a DGP and because the interaction of different sets of disturbances may be crucial to understand the data, it should be clear that the problems we study in this paper are pervasive in applied macroeconomics. Furthermore, since small scale VAR models will remain the basic empirical tool to examine shock transmission for a while, researchers ought to be aware of the problems they face in practice and of ways to minimize the impact of deformation on the results they present.

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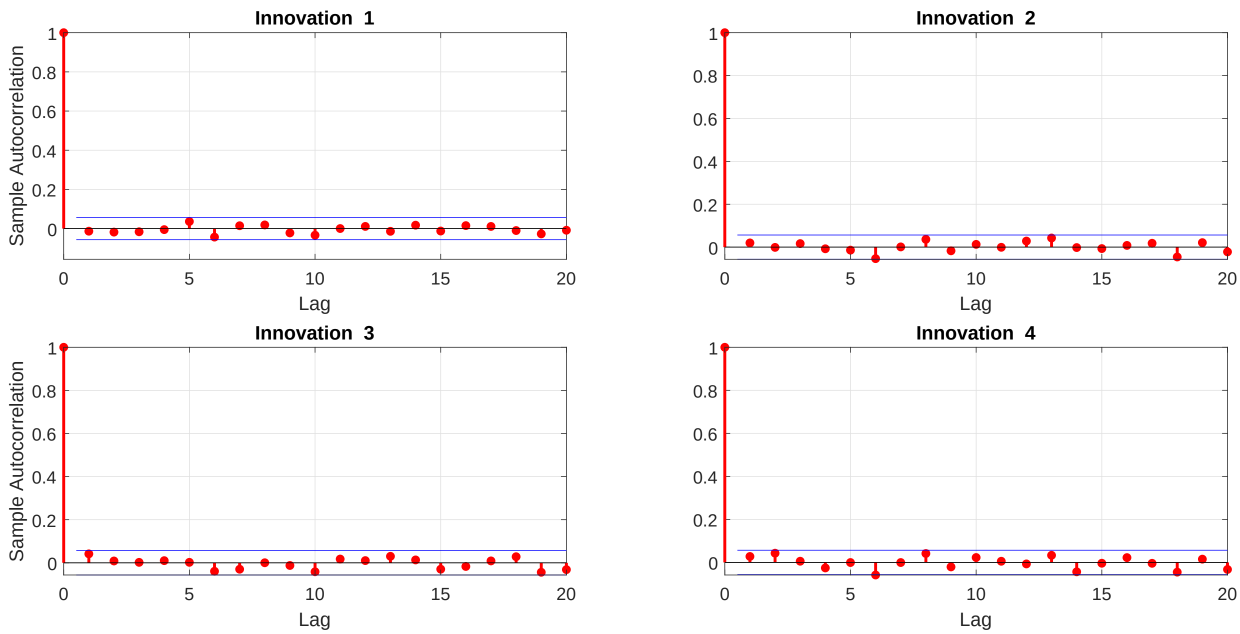
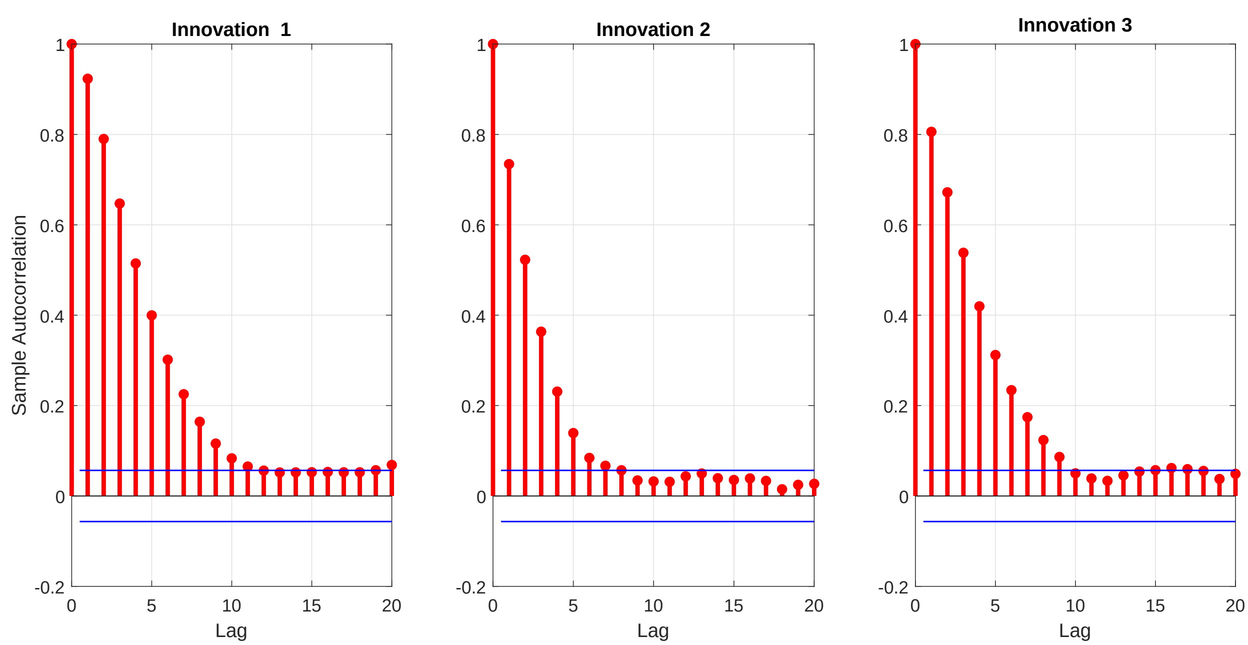


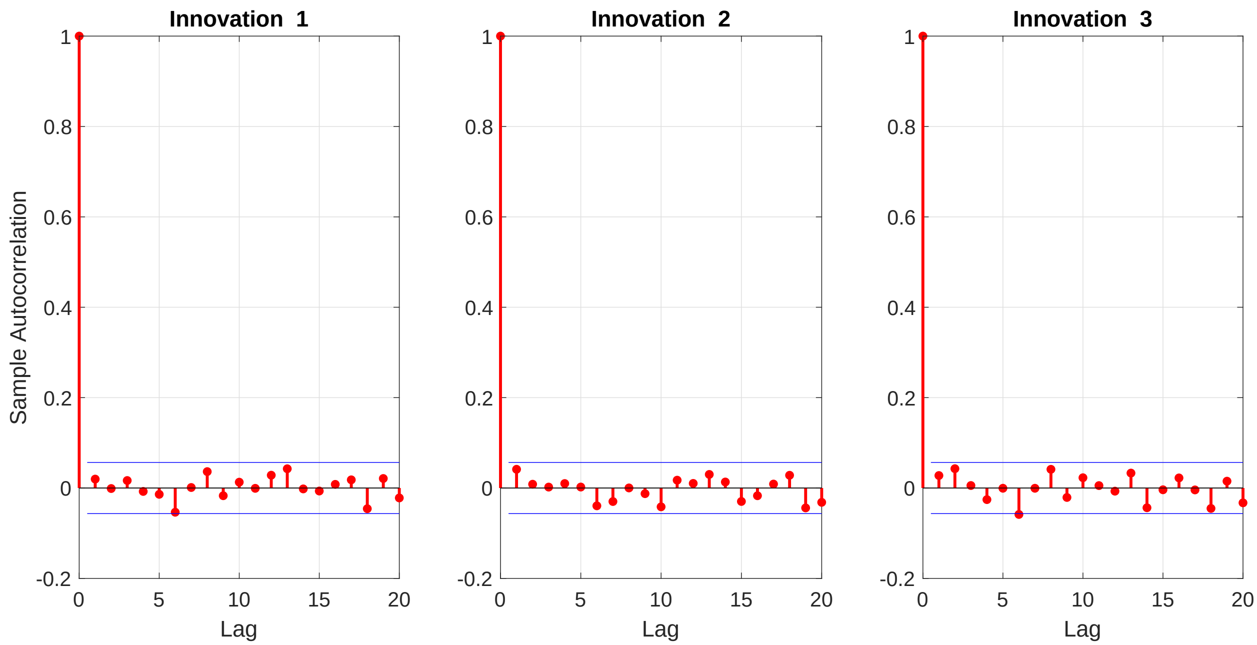
Note: Parallel lines describe 95 % asymptotic tunnel for the hypothesis of zero autocorrelations.

Figure A.1: Autocorrelation function, innovations in (o_t, π_t, n_t, r_t) system.



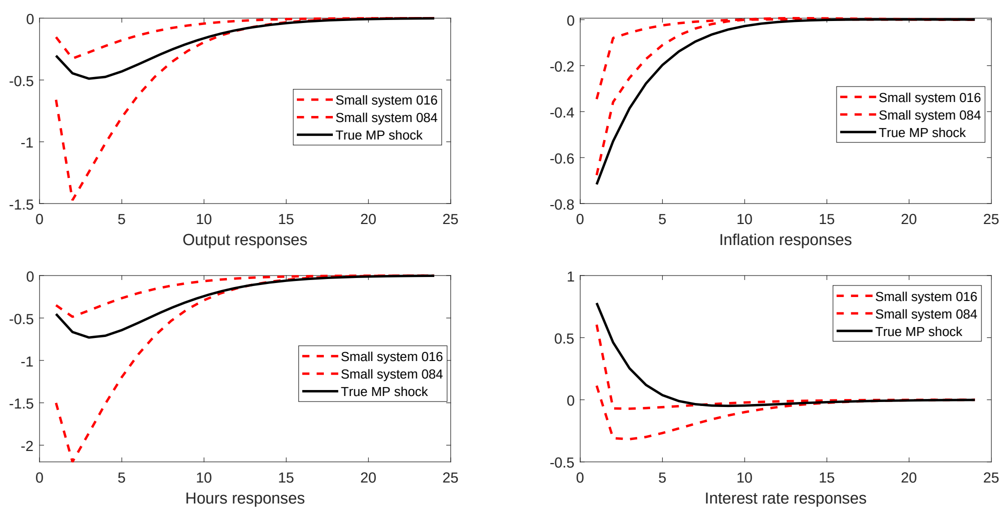
Note: Parallel lines describe 95 % asymptotic tunnel for the hypothesis of zero autocorrelations.

Figure A.2: Autocorrelation function, innovations in (o_t, π_t, n_t) system.



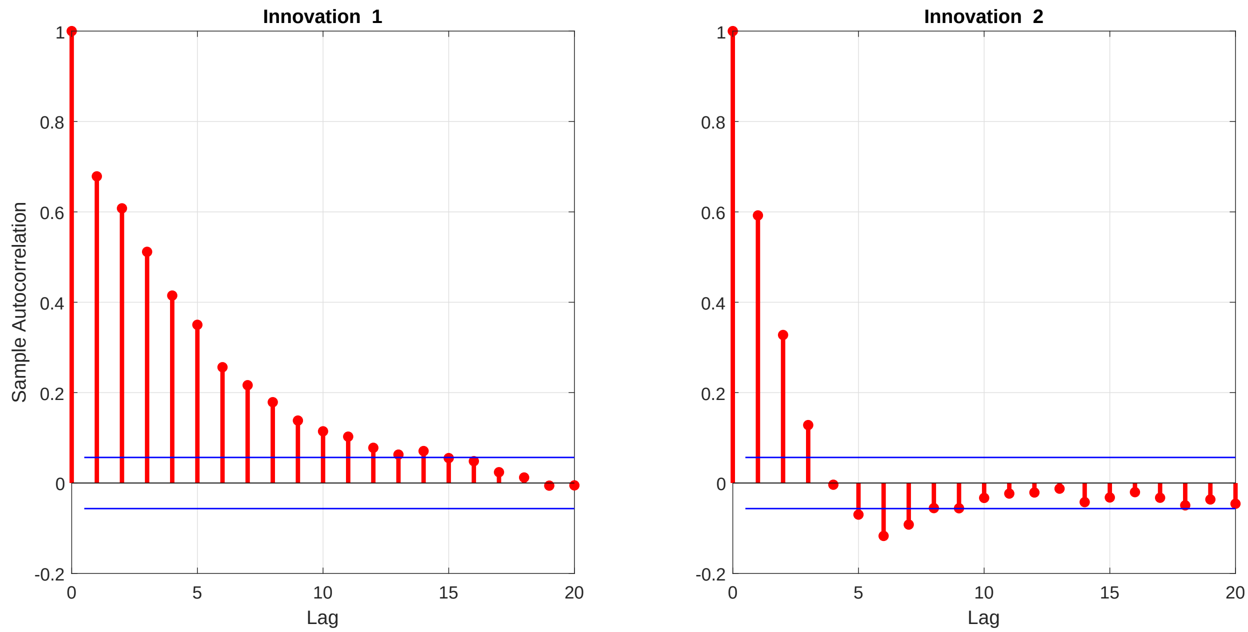
Note: Parallel lines delimit the 95 % asymptotic tunnel for the hypothesis of zero autocorrelations.

Figure A.3: Autocorrelation function, innovations in (π_t, n_t, r_t) system.



Note: The dashed regions report 68 % interval obtained accounting for rotation uncertainty. The solid line reports the responses in the DGP.

Figure A.4:Responses to monetary policy shocks, (y_t, π_t, n_t, r_t) system.



Note: Parallel lines describe the 95 % asymptotic tunnel for the hypothesis of zero autocorrelations.

Figure A.5: Autocorrelation function, innovations in (o_t, r_t) system.

The (linearized) equations of Iacoviello [2005]'s model

$$rr_t = r_t - pi_{t+1} \tag{61}$$

$$\begin{aligned}
y_t &= c_y c_t + (1 - c_y - cii_y - i_y)ci_t + cii_y cii_t + i_y i_t && (62)\\
ci_t &= ci_{t+1} - rr_t && (63)\\
i_t - k_{t-1} &= \gamma(i_{t+1} - k_t) + \frac{(1-\gamma(1-\delta)}{\psi}(y_{t+1} - x_{t+1} - k_t) + (\frac{1}{\psi}(c_t - c_{t+1}) && (64)
\end{aligned}$$

$$\begin{aligned}
q_t &= \gamma_E q_{t+1} + (1-\gamma_E)(y_{t+1} - x_{t+1} - h_t) - m\beta\ rr_t - i_{1,t} - (1 - m\beta)(c_{t+1} - c_t)\\
&- \phi_E(h_t - h_{t-1} - \gamma(h_{t+1} - h_t)) && (65)\\
q_t &= \gamma_H q_{t+1} + (1-\gamma_H)(j_t - hii_t) - mii\beta\ rr - i_{2,t} + (1 - mii\beta)(cii_t - \omega cii_{t+1})\\
&- \phi_H(hii_t - hii_{t-1} - \beta_{ii}(hii_{t+1} - hii_t) && (66)\\
q_t &= \beta q_{t+1} + (1-\beta)j_t + \iota h_t + \iota_{ii} hii_t + ci_t - betaci_{t+1} + \frac{phi_H}{hi}(h(h_t - h_{t-1})\\
&+ hii(hii_t - hii_{t-1}) - \beta h(h_{t+1} - h_t) - \beta hii(hii_{t+1} - hii_t)) && (67)
\end{aligned}$$

$$\begin{aligned}
b_t &= q_{t+1} + h_t - rr_t + i_{1,t} && (68)\\
bii_t &= q_{t+1} + hii_t - rr_t + i_{2,t} && (69)
\end{aligned}$$

$$\begin{aligned}
y_t &= \frac{\eta}{\eta - (1-\nu-\mu)}(a_t + \nu h_{t-1} + \mu k_{t-1}) - \frac{1-\nu-\mu}{\eta - (1-\nu-\mu)}(x_t + \alpha ci_t + (1-\alpha)cii_t) && (70)\\
\pi_t &= \beta\pi_{t+1} - \kappa x_t + u_t && (71)
\end{aligned}$$

$$\begin{aligned}
k_t &= \delta i_t + (1-\delta)k_{t-1} && (72)\\
b_y b_t &= c_y c_t + qh_y(h_t - h_{t-1}) + i_y i_t + \frac{b_y}{\beta}(r_{t-1} + b_{t-1} - \pi_t) - (1 - si - sii)(y_t - x_t) && (73)\\
bii_y bii_t &= cii_y cii_t + qhii_y(hii_t - hii_{t-1}) + \frac{bii_y}{\beta}(bii_{t-1} + r_{t-1} - \pi_t) - sii(y_t - x_t) + w_t && (74)
\end{aligned}$$

$$\begin{aligned}
r_t &= (1-\rho_R)(1+\rho_\pi)\pi_{t-1} + \rho_y(1-\rho_R)y_{t-1} + \rho_R r_{t-1} + e_R && (75)\\
j_t &= \rho_j j_{t-1} + e_j && (76)\\
u_t &= \rho_u u_{t-1} + e_u && (77)\\
a_t &= \rho_a a_{t-1} + e_a && (78)\\
i_{1,t} &= \rho_1 i_{1,t-1} + e_b c1 && (79)\\
i_{2,t} &= \rho_2 i_{2,t-1} + e_b c2 && (80)\\
w_t &= \rho_w w_{t-1} + e_h as && (81)\\
tc_t &= c_y c_t + (1 - c_y - cii_y - i_y)ci_t + cii_y cii_t && (82)\\
th_t &= h_t + hii_t && (83)
\end{aligned}$$

The equations of extended Basu and Bundick [2017]'s model

$$y_t + fixedcost_t = productionconstant * (z_t * n)t)^(1 - \alpha) * (u_t * k_{t-1}))^(\alpha) \quad (84)$$

$$c_t + leverageratio * k_t / rr_t = w_t * n_t + de_t + leverageratio * k_{t-1} \quad (85)$$

$$w_t = ((1-\eta)/\eta) * c_t/(1-n_t) \quad (86)$$

$$vf = (utilityconstant * a_t * (c_t^(\eta) * (1-n_t)^(1-\eta))^(1-\sigma)/thetavf) + \beta * expvf1sigma_t^(1/thetavf))^(thetavf/(1-\sigma)) \quad (87)$$

$$expvf1sigma_t = vf_{t+1}^(1-\sigma) \quad (88)$$

$$w_t * n_t = (1-\alpha) * (y_t + fixedcost)/mu_t \quad (89)$$

$$rrk_t * u_t * k_{t-1} = \alpha * (y_t + fixedcost)/mu_t \quad (90)$$

$$q_t * deltauprime_t * u_t * k_{t-1} = \alpha * (y_t + fixedcost)/mu_t \quad (91)$$

$$k_t = ((1 - deltau_t) - (\phi_k/2) * (inv_t/k_{t-1} - delta_0)^(2)) * k_{t-1} + inv_t \quad (92)$$

$$deltau_t = delta_0 + delta_1 * (u_t - 1) + (\delta_2/2) * (u_t - 1)^2 \quad (93)$$

$$deltauprime_t = delta_1 + delta_2 * (u_t - 1) \quad (94)$$

$$sdf_t = \beta * (a_t/a_{t-1}) * ((c_t^(\eta) * (1-n_t)^(1-\eta))/(c_{t-1}^(\eta) * (1-n_{t-1})^(1-\eta)))^(\frac{1-\sigma}{thetavf}) * (c_{t-1}/c_t) * (vf_t^(1-\sigma)/expvf1sigma_{t-1})^(1-1/thetavf) \quad (95)$$

$$1 = rr_t * sdf_{t+1} \quad (96)$$

$$1 = r_t * sdf_{t+1} * (pie_{t+1})^(- 1) \quad (97)$$

$$1 = sdf_{t+1} * (u_{t+1} * rrk_{t+1} + q_{t+1} * ((1 - deltau_{t+1}) - (\phi_k/2) * (inv_{t+1}/k_t - \delta_0)^2 + \phi_k * (inv_{t+1}/k_t - \delta_0) * (inv_{t+1}/k_t)))/q_t \quad (98)$$

$$1 = sdf_{t+1} * (de_{t+1} + pe_{t+1})/pe_t \quad (99)$$

$$\log r_t = (1-\rho_r) * (\log(rss) + \rho_{pie} * \log(pie_t/piess) + \rho_y * \log(y_t/y_{t-1})) + \rho_r * \log(r_{t-1}) + e \quad (100)$$

$$de_t = y_t - w_t * n_t - inv_t - (\phi_p/2) * (pie_t/piess - 1)^2 * y_t - leverageratio * (k_{t-1} - k_t/rr_t) \quad (101)$$

$$q_t^{-1} = 1 - \phi_k * (inv_t/k_{t-1} - \delta_0) \quad (102)$$

$$\phi_p * (pie_t/piess - 1) * (pie_t/piess) = (1 - thetamu) + (thetamu)/mu_t + sdf_{t+1} * \phi_p * (pie_{t+1}/piess - 1) * (y_{t+1}/y_t) * (pie_{t+1}/piess) \quad (103)$$

$$profit_t = (\mu_t - 1) * y_t - fixedcost \quad (104)$$

$$expre_t = (de_{t+1} + pe_{t+1})/pe_t \quad (105)$$

$$expre2_t = (de_{t+1} + pe_{t+1})^2/pe_t^2 \quad (106)$$

$$varexpre_t = expre2_t - expre_t^2 \quad (107)$$

$$a_t = (1-\rho_a) * ass + \rho_a * a_{t-1} + vola_{t-1} * ea_t \quad (108)$$

$$vola_t = (1-\rho_{vola}) * volass + \rho_{vola} * vola_{t-1} + volvola * evola_t \quad (109)$$

$$z_t = (1-\rho_z) * zss + \rho_z * z_{t-1} + volz * ez_t \quad (110)$$

$$e_t = (1-\rho_e) * ess + \rho_b * e_{t-1} + voless * ee_t \quad (111)$$