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Abstract

We develop a dynamic pricing model motivated by observed patterns in business-to-business (and some business-to-customer) transactions. Seller costs are perfectly correlated and evolve according to a Markov process. In every period, each buyer observes (for free) the price set by their current supplier, but not the other sellers' prices or the sellers' (common) cost level. By paying a cost s the buyer becomes "active" and benefits from (Bertrand) competition among sellers. We show that there exists a semi-separating equilibrium whereby sellers increase price immediately when costs increase and otherwise decrease price gradually. Moreover, buyers become active when prices increase but not otherwise. In sum, we deliver a theory whereby buyers become active ("search") if and only if their supplier increases price.

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Rational Buyers Search When Prices Increase

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1. Introduction

Many sellers routinely purchase inputs from regular suppliers. For example, Blinder et al. (1998) report that out of a GDP representative sample of sellers, on average 85% of sales are to regular buyers. In this business-to-business (B2B) context, buyers are faced with a dilemma: either they do "business as usual" with their current supplier (that is, pay the quoted price); or, alternatively, they try to obtain a better deal. A better deal can be gotten in several ways, including negotiating with the current supplier (Zbaracki et al., 2004) or searching for better prices from rival suppliers. For example, typically automakers such as General Motors have long-term relationships with parts suppliers, but every so often solicit bids from outside suppliers as well.Similarly, many final consumers purchase services (insurance, credit cards, wireless, cable TV, etc) through subscription plans that have a similar structure (business as usual or actively seek a better option).[1]

In this paper, we develop a dynamic model of pricing and buyer "search" which is motivated by examples like the ones above. Seller costs are perfectly correlated across sellers and evolve according to a Markov process. In every period, each buyer observes (for free) the price set by their current seller, but not the other sellers' prices or the sellers' (common) cost level. The buyer can choose to purchase at that price, or pay a cost s and benefits from (Bertrand) competition among sellers (for example, the buyer convenes a second-price auction among suppliers).[2]

Our main focus is on the interaction between cost fluctuations and buyer search behavior. When do buyers search: When prices are high or when prices are low? When prices increase or when prices decrease? Empirical and anecdotal evidence (Ho, Hogan, and Scott Morton, 2017; Paciello, Pozzi, and Trachter, 2019) suggests that buyers are more likely to search when prices are high and/or increase. However, to the best of our knowledge there is no theoretical treatment that explains such patterns as resulting from buyer rational behavior, in particular looking at how a buyer's search decision is influenced by price changes as opposed to price levels. For example, to the extent that price dispersion is lower when prices are high (a stylized fact from various markets), one would expect rational buyers to search *less* when prices increase or are high.

We begin our analysis by considering the static version of our model. Sellers (we assume there are $n \geq 2$ of them) set list prices. Buyers (we assume there is a continuum of them) decide whether to pay the list price p_i (from their assigned seller) or rather become "active" (i.e., "search"), by paying a cost s and obtaining price quotes $q_j,\ j = 1, ..., n$, from all sellers.[3] Assuming that the sellers' cost can either be high or low, we have two types of equilibria: separating and pooling.

As often happens in signaling games, we uncover a continuum of Perfect Bayesian equilibria. However, restricting to seller optimal equilibria (no alternative equilibrium is better

1. In the B2C case, there is extensive evidence of price discrimination between active and passive buyers, a practice known as "retention pricing" (Ofcom, 2010). In addition, Shelegia and Sherman (2014) find that subjects sent to shop in retail stores in Austria were 40% likely to agree on a discount when initiating a negotiation for a better price.
2. In the B2C context, a retention offer is frequently associated with a credible commitment to switch. For example, Ofcom (2010) reports that when wireless consumers request from their carrier the code required to proceed with the switch they are typically given a retention offer along with the code.
3. Our definition of "search" is different from what is normally assumed in the literature. By becoming "active", our buyers don't just learn about other sellers' prices; the actually obtain different price quotes than "passive" buyers from the same seller.

for both seller types), we are able to reduce the set of equilibria considerably. First we show that there exists a unique separating equilibrium.[4] The sellers set a price such that buyers are exactly indifferent between searching and not searching. Buyers do not search if price is low, and search with probability α if price is high, where α is the lowest value required for the seller's incentive compatibility constraint to be satisfied (a low-cost seller does not want to "masquerade" as a high-cost seller).[5]

We also show that there exists a unique pooling equilibrium: Sellers set a price p (regardless of cost) such that buyers are indifferent between searching and not searching (given the buyers' prior beliefs about sellers' cost); and buyers do not search.[6]

Next we extend the analysis to the dynamic case, that is, the case when costs evolve stochastically over time according to a Markov process with transition probabilities γ_L (switch from low to high) and γ_H (switch from high to low). If the static game admits many equilibria, in the dynamic game the scope for equilibrium multiplicity is even greater. Our refinement strategy is to restrict to equilibria with the following stationary property: whether each period's prices are separating or pooling only depends on the change in cost level with respect to the previous period. Similarly to the static model, we further restrict to seller-optimal equilibria.

We show that there exists a unique equilibrium within this set that satisfies the rockets-and-feathers property, namely the property that prices increase faster in response to a cost increase than they decrease in response to a cost decrease.[7] In this equilibrium, prices reveal cost if and only if there is an increase in cost. In other words, sellers set a high price when cost increases, and thereafter gradually decrease price along a pooling phase until the next cost increase. Buyers in turn search with strictly positive probability when prices increase and remain passive otherwise.

A number of previous papers develop models with rocket-and-feathers price dynamics (see literature review below). To the best of our knowledge, ours is the first to develop a theory of rational buyer behavior such that, in equilibrium, search is caused by price increases. In order to stress this point, in particular the difference between price level and change in price level, we use a number of existing models of search and price dynamics to generate comparable pseudo-data. We then run search regressions of the sort found in the empirical literature, where the dependent variable is given by the extent of buyer search and the set of explanatory variables includes both price level and change in price level. There are models consistent with more search when prices are high (e.g., static models) or more search when prices increase. However, ours is the only one consistent with positive coefficients on the price level and the change in price level variables, as suggested by the empirical evidence.

4. As usual, we refer to uniqueness in terms of equilibrium path; by appropriately changing off-the-equilibrium-path moves and beliefs, we can construct other equilibria which induce the same equilibrium path.
5. There also exist equilibria where buyers search with higher probability upon observing a high price, but these are not seller-optimal.
6. There also exist pooling equilibria where price is lower and buyers strictly prefer not to search, but such equilibria are not seller-optimal.
7. See Bacon (1991), Peltzman (2000), Lewis (2011). Peltzman (2000) documents, for more than 200 industries, that prices rise faster than they fall. Although much of the recent literature on rockets-and-feathers has been motivated by the dynamics of retail gasoline markets, Peltzman (2000) shows that the rockets-and-feathers feature "is found as frequently in producer goods markets as in consumer goods markets."

■ **Related literature.** Some of the literature on price dynamics with search has been motivated by the rockets-and-feathers property: Tappata (2009) develops a Varian (1980) type model of mixed-strategies and price dispersion; Yang and Ye (2008) propose a Salop and Stiglitz (1977) type bargains-and-ripoffs model; and Cabral and Fishman (2012) develop a dynamic version of a Diamond (1971) type of model. Although these models differ considerably, they all produce "rockets-and-feather" dynamics. Similarly to Diamond (1971), the equilibrium in Cabral and Fishman (2012) features no search. Search does take place along the equilibrium path in Tappata (2009) and Yang and Ye (2008). And, as we will see in Section 4, buyers are more likely to search when prices increase and/or when prices are low. The first is consistent with our model and the empirical evidence, but not the second.

To the best of our knowledge, the only paper predicting that buyers search when prices increase or are high is Lewis (2011). His "behavioral" model assumes that buyers form expectations about the price distribution based on the average price level from the previous period. In this context, when prices increase buyers expectations of the price distribution tend to be too low, causing them to search more than they otherwise would. Our paper differs from his in that we assume buyers are rational and hold correct beliefs regarding seller prices. In other words, we present a complete equilibrium narrative for the prediction that buyers search when prices increase and/or are high.

The literature on search and price dispersion extends well beyond the above papers. Relevant papers include Burdett and Judd (1983), Stahl (1989), Benabou and Gertner (1993), Janssen and Moraga-González (2004). Particularly germane are the papers on non-reservation-price buyer strategies, where costly search takes place along the equilibrium path: Dana (1994), for example, assumes that two "identical firms produce a homogeneous product and compete in price." Similarly to our setup, the firms in Dana (1994) "have the same marginal cost, which may be either high or low." Differently from us, however, there is a fraction of informed consumers who observe all of the prices. Dana (1994) shows that, if search costs are sufficiently low, then there may exist non-reservation-price equilibria where, similarly to our equilibria, buyers are indifferent between searching and not searching, and costly search takes place along the equilibrium path. Similarly, Janssen, Parakhonyak, and Parakhonyak (2017) find that "there is a region of 'high' prices that are set with positive probability such that consumers are indifferent between buying and searching."

Although these papers develop static models, one can think of the dynamic application of their results, which is a monotonic relationship between cost level and price level and with positive search intensity at high prices along the equilibrium path. That said, these papers are silent with respect to the question of buyer behavior in reaction to a price change. We return to this issue in Section 4.

Finally, considering our model allows for price discrimination between "passive" and "active" buyers, this paper also relates to the "retention offers", consumer switching, and search deterrence literatures (Cai, Deilami, and Train, 1998; Haan and Siekman, 2015; Gnutzmann, 2014; Armstrong and Zhou, 2016). For example, Gnutzmann (2014) models "retention pricing" (a lower price offered to "active" buyers) as a form of sorting heterogeneous buyers (heterogeneous with respect to their switching cost). Haan and Siekman (2015) follow a similar path, further allowing for sellers to directly poach rival sellers' buyers. As in our model, these papers consider the possibility of consumers becoming "active". However, they do not deal with the issue of price dynamics and buyer search.

■ **Roadmap.** Section 2 sets up the static version of our model. Section 3 introduces the dynamic model. In 3.1 we solve for the case when costs follow a Markov process with an absorbing state, while 3.2 considers the more general case when cost follows an ergodic stochastic Markov process. In Section 4 we compare our model to a series of alternative models of price and search dynamics. Section 5 concludes the paper.

2. Static model

In this section we consider a static model of pricing and search. This section fulfills three different goals. First, it allows us to bridge our analysis with respect to the existing literature on search in a static framework. Second, it allows us to introduce the reader to the issues of equilibrium multiplicity and equilibrium selection present in this type of models. Third, given that we propose an interpretation of search ("active" vs "passive" buyers) which differs from the standard treatment in the literature, it also allows us to introduce the reader to the particular framework we consider.

Consider an industry with $n > 2$ sellers indexed by $k \in \{1, 2, ..., n\}$ and a measure m of buyers, where each buyer is randomly assigned to a seller. Sellers produce the same product and face the same unit cost, $c \in \{c_L, c_H\}$, which is known to sellers but not the buyers. We assume that γ is the probability that $c = c_H$ (thus $1 - \gamma$ is the probability of the event $c = c_L$). Each buyer has a unit demand with choke price $\overline{u}$, which we assume is very large, so that all buyers make a purchase in equilibrium.[8]

The timing is as follows. Sellers observe the value of c and simultaneously set prices p^k, where k denotes the seller's identity. Each buyer observes the price (but not the cost) of the seller they are assigned to and chooses between two options: (a) forgo search for a better price (passive buyers); or (b) become active and search for a better deal by incurring a cost $s > 0$ (active buyers). We make the important assumptions that, by paying s, the buyer is perceived by sellers as an active buyer (i.e., a searcher); and that sellers are able to set different prices to active buyers denoted by q^k for each seller k. Once buyers make their search decisions, sellers simultaneously set prices for searchers. Finally, searchers choose a seller and all buyers make their purchase decision, at which point period payoffs are received by sellers and buyers.

A strategy for seller k consists of prices $p^k(c), q^k(c)$ as a function of cost. A strategy for a buyer attached to seller k consists of (a) a probability $\alpha(p^k)$ of becoming active (by which we mean paying cost s to get quotes q^k from all sellers $k = 1, ..., n$); and (b) a purchase decision, given the available list price (p^k for a passive buyer matched with seller k, and $\{p^k, q^1, ..., q^n\}$ for an active buyer originally attached to seller k).

We adopt the concept of Perfect Bayesian Equilibrium: a set of strategies and beliefs such that (a) each player's strategy is optimal given other players' strategies and beliefs; and (b) beliefs are Bayes-consistent with strategies. Given the game's symmetry across sellers, we further restrict attention to symmetric equilibria, that is, equilibria where all sellers follow the same strategy.

8. The assumption of unit demand is an approximation. It fits a variety of B2C examples, such as cable TV subscriptions. Moreover, in many B2B examples the particular input demanded by the buyer represents a small fraction of the overall input bill and the production function is close to Leontief (with respect to that particular part). Again, in this context, the assumption of unit demand is a good approximation.

As often is the case in games with asymmetric information and a continuum of strategies, there exists a continuum of equilibria which are supported by appropriate off-the-equilibrium-path beliefs. We therefore impose an additional restriction on equilibria, namely seller optimality.

Definition 1. *An equilibrium is* ***optimal*** *is there does not exist another equilibrium yielding both seller types a higher equilibrium payoff, with strict inequality for a least one type.*

The model is characterized by a series of parameters: seller cost $c \in \{c_H$ and $c_L\}$; buyers' belief $\beta(p)$ that $c = c_H$ (prior to their decision of whether to become active); and buyers' search cost, s.

Given that there are only two cost levels, there are only two possible candidates for an equilibrium class in seller pure strategies: a separating equilibrium and a pooling equilibrium. In what follows, we show that we can pin down a unique equilibrium in each class that satisfies Definition 1.

■ **Separating equilibria.** We first show that there exists a unique optimal pooling equilibrium.

Proposition 1. *There exists a unique optimal separating equilibrium. Equilibrium seller strategies are given by*

$$\begin{aligned} p(c) &= c + s \\ q(c) &= c \end{aligned}$$

where $c \in \{c_L, c_H\}$; buyer search strategies are given by

$$\alpha(p) = \begin{cases} (c_H - c_L)/(c_H - c_L + s) & \text{if} \quad p = c_H + s \\ 1 & \text{if} \quad p > c_L + s, \;\; p \neq c_H + s \\ 0 & \text{if} \quad p \leq c_L + s \end{cases}$$

and buyer beliefs are given by

$$\beta(p) = \begin{cases} 1 & \text{if} \quad p = c_H + s \\ 0 & \text{if} \quad p \neq c_H + s \end{cases}$$

A formal proof of Proposition 1 can be found in the Appendix. We first show that no other prices can be sustained in equilibrium. Next, we examine the incentive compatibility constraint. In this case the binding constraint is that a low-cost firm does not want to "masquerade" as a high-cost firm by setting a high price (and earning a high margin). What keeps a low-cost type "honest" is precisely buyer search. This implies a lower bound on the value of α. The equilibrium corresponding to the lowest value of α must be optimal: a higher value of α implies a strictly lower payoff for a high-type seller, thus violating the optimality refinement.

One possible criticism of the equilibrium in Proposition 1 is that buyers play mixed strategies: when the seller sets $p = p_H$, buyers become active with probability $\alpha(p_H) \in (0, 1)$. However, in this context we can think of mixed-strategies as a reduced form of pure strategies with privately observed shocks. In the tradition of Harsanyi (1973), suppose that each buyer's switching cost is given by $s+\zeta$, where ζ is uniformly distributed in $[-\epsilon, \epsilon]$. Let $\Gamma(\epsilon)$ be the game that is obtained from our initial game by adding this disturbance to search costs (the original game corresponds to $\Gamma(0)$).

An equilibrium of this incomplete information game can be obtained from the equilibrium in Proposition 1 as follows. When $c = c_L$, sellers set $p = p_L - \epsilon$; when $c = c_H$ the seller sets $p = p_H - (1-2\alpha)\,\epsilon$. In this equilibrium, buyers make search decisions based on strict inequalities (almost surely), and search takes place with the probabilities indicated by Proposition 1. In particular, when $c = c_H$, buyers with search cost $s+\zeta'$ (measure zero) are indifferent between being active and being passive, where ζ' is given by

$$c_H + \big(s+\zeta'\big) = p_H - (1-2\alpha)\,\epsilon$$

An indifferent buyer then has $\zeta' = (2\alpha-1)\epsilon$. It follows that the fraction of buyers who search is given by

$$\mathbb{P}\big(\zeta < \zeta'\big) = \frac{\zeta' + \epsilon}{2\,\epsilon} = \alpha$$

where the last equality follows from the buyer's indifference condition.

■ **Pooling equilibria.** We next turn to pooling equilibria; as in the case of separating equilibria, there exists a unique optimal equilibrium within the class of pooling equilibria (proof in the Appendix).

Proposition 2. *If $s \geq (c_H - c_L)(1-\gamma)$, then there exists a unique optimal pooling equilibrium. Equilibrium strategies are given by*

$$\begin{aligned} p(c) &= \gamma\, c_H + (1-\gamma)\, c_L + s \\ q(c) &= c \end{aligned}$$

where $c \in \{c_L, c_H\}$; buyer search strategies are given by

$$\alpha(p) = \begin{cases} 0 & \text{if} \quad p = \gamma\, c_H + (1-\gamma)\, c_L + s \quad \text{or} \quad p < c_L + s \\ 1 & \text{otherwise} \end{cases}$$

and buyer beliefs are given by

$$\beta(p) = \begin{cases} \gamma & \text{if} \quad p = \gamma\, c_H + (1-\gamma)\, c_L + s \\ 0 & \text{otherwise} \end{cases}$$

The restriction that s be sufficiently high is required for the pooling price to be greater than c_H; otherwise, the high-cost firm has an incentive to deviate by setting $p \geq c_H$.

■ **Separation vs pooling.** Much of game theory is concerned with the issue of equilibrium selection in signaling games (which are prone to admit multiple equilibria). Our optimality criterion (Pareto optimality across different informed player types) allows us to select a unique separating and a unique pooling equilibrium. Can anything be said about the selection between the two types of equilibria?

One possible criterion for equilibrium selection is to consider seller expected profit (similar to what we already did when restricting to optimal equilibria within each class of equilibria).[9] In a separating equilibrium, a low-cost seller earns a profit of

$$\pi_L^S = (c_L + s) - c_L = s \tag{1}$$

whereas a high-cost seller earns

$$\pi_H^S = (1-\alpha)\, s = \left(1 - \frac{c_H - c_L}{c_H - c_L + s}\right) s = \frac{s^2}{c_H - c_L + s} \tag{2}$$

In a pooling equilibrium, a low-cost seller earns a profit of

$$\pi_L^P = \big(\gamma\, c_H + (1-\gamma)\, c_L + s\big) - c_L = s + \gamma\,(c_H - c_L) \tag{3}$$

whereas a high-cost seller earns a profit of

$$\pi_H^P = \big(\gamma\, c_H + (1-\gamma)\, c_L + s\big) - c_H = s - (1-\gamma)\,(c_H - c_L) \tag{4}$$

Comparing (1) and (3), we conclude that a pooling equilibrium gives a low-cost seller a higher profit. Comparing (2) and (4), we conclude that a high-cost seller prefers a pooling equilibrium if and only if

$$s > (c_H - c_L)\,(1-\gamma)\,/\gamma \tag{5}$$

a condition that is stricter than the condition for the existence of a pooling equilibrium. We conclude that, if

$$1 - \gamma < \frac{s}{c_H - c_L} < (1-\gamma)\,/\gamma$$

then the pooling equilibrium is better for the low cost seller but worse for the high cost seller; whereas if (5) holds then the pooling equilibrium is better for both types of seller.

In terms of the *expected*, or average, seller payoff, the pooling equilibrium (if it exists) is preferred by the seller. In fact the expected payoff of a seller in a pooling equilibrium is exactly equal to s, while in a separating equilibrium it is equal to $s\, \frac{(1-\gamma)(c_H - c_L)+s}{c_H - c_L + s} < s$. Intuitively, buyers are always indifferent between searching or not searching, thus the total amount paid is equal to the markup s above the expected price. In the pooling equilibrium with no search, this markup is entirely captured by the seller, while in a separating equilibrium some of this markup is absorbed by search activity, resulting in a lower seller profit.

From a social welfare viewpoint, the pooling equilibrium (if it exists) is clearly better. The argument is simple. First, all buyers make a purchase of exactly one unit in either equilibrium, so there is no deadweight loss. The only difference between the two equilibria is that in the separating equilibrium search takes place with strictly positive probability along the equilibrium path, whereas under pooling equilibrium there is no search, thus no search cost is wasted. We note, however, that in terms of social welfare the cards are stacked in favor of the pooling equilibrium. If demand were not L-shaped (unit demand), deadweight loss would likely be greater under a pooling equilibrium to the extent that prices do not reflect costs.

9. In this paper, all the profit calculations are normalized to be per measure one of assigned customers.

Against the seller payoff and social welfare arguments, which favor the pooling equilibrium, strong Nash equilibrium refinements favor the separating equilibrium. Specifically, the pooling equilibrium does not survive the D1 criterion (Banks and Sobel, 1987). Suppose that a seller deviates from the proposed pooling price p by increasing price by Δp. For a type i seller, this deviation is profitable if and only if the buyers' response search probability α is such that

$$\pi_i^P < (1-\alpha)\left(\pi_i^P + \Delta p\right)$$

which is equivalent to

$$\alpha < \frac{\Delta p}{\Delta p + \pi_i^P}$$

Since $\pi_L^P > \pi_H^P$, the set of values α that make the deviation profitable is strictly greater for $i = H$. Therefore, the D1 criterion implies that buyers believe that the price deviation comes from a high-cost firm. Finally, if $\Delta p < (1-\gamma)(c_H - c_L)$, then the deviation price is lower than $c_H + s$, which in turn implies that this deviation does not trigger search, which finally makes the deviation profitable for low cost type seller, thus eliminating the proposed equilibrium.

In sum, we can find arguments in favor of separation and arguments in favor of pooling. This suggests that both are potentially valid equilibria. The argument is even stronger when we consider the dynamic extension of the model, as we will do in the next section. A simple extension of the static model is to consider the "repetition" of the static equilibrium, where the underlying cost is a random draw each period. In a dynamic separating equilibrium, sellers set a high or a low price in each period as a function of their cost in that period. A dynamic pooling equilibrium has sellers set the same price in every period, independently of their cost in that period. In Section 3.2 we will see that these dynamic versions of the static model equilibria correspond to equilibria of the richer dynamic version of the model, where the underlying cost follows a Markov process. However, they fail to reflect various dynamic properties found in the data. By contrast, a combination of pooling and separation spells allows us to construct equilibria that closely match empirical stylized facts.

■ **Search and price dispersion.** Most of the search literature is based on the idea of price dispersion: to the extent that firms set different prices, a searcher expects to find a better price by searching than the price obtained without search. By contrast, in our model all sellers set the same list price p while (in some cases) "search" (i.e., active buyers) still takes place along the equilibrium path. How can there be rational search without price dispersion?

First, in our framework the incentive for becoming an active buyer is to obtain better prices from sellers, including the seller that the buyer is currently attached to: even though there are no differences in the list prices set by the various firms, active buyers are offered prices q that are lower than list prices p. To the extent that $q < p$, there is price dispersion in equilibrium, but this is the difference in prices paid by active and passive buyers, not price dispersion in the traditional sense.

Second, in the pooling equilibrium we derived buyers are uncertain about the value of cost, and thus about the value of q they will be quoted if they become active. Therefore, from the buyer's perspective there is "dispersion" of q in the probabilistic sense of the word.

3. Dynamic model

In this section we extend the model in Section 2 to the infinite-period case. Time is discrete, $t = 1, 2, ...$, and both sellers and buyers discount the future according to the factor δ. As in Section 2, seller's cost, c_t, in period t can take two different values: $c_t \in \{c_L, c_H\}$. We assume that c_0 is known to buyers and that, for $t > 0$, c_t follows a Markov process with transition matrix

$$M = \begin{bmatrix} 1-\gamma_L & \gamma_L \\ \gamma_H & 1-\gamma_H \end{bmatrix}$$

For $i \in \{L, H\}$, γ_i is the probability that cost changes when $c_t = c_i$. We assume M is common knowledge to buyers and sellers. However, as before, the value of c_t for $t > 0$ is the sellers' private information.

Let β_t denote the buyer belief that $c_t = c_H$ (measured at the *beginning* of period t), where β_t is a function of the price history up to time t (non-inclusive) and $\beta_0 = 1$ (resp. $\beta_0 = 0$) if $c_0 = c_H$ (resp. $c_0 = c_L$). A natural extension of the buyer's strategy is a probability of searching, $\alpha(p_t, \beta_t)$, as a function of the current price and of the buyer's prior belief β_t that cost is high.[10]

Regarding sellers, the time extension of the game opens the door to a rich strategy space. In order to focus our attention on a limited set of reasonable equilibria, we consider only simple equilibria. The spirit of our restriction to simple equilibria is similar to Markov equilibria: we restrict to a limited state space and assume that the nature of pricing in each period — in particular whether prices are separating or pooling — is only a function of the current state, not of other elements of price or purchase history. The optimality condition then selects a unique price set for each type.

Definition 2. *A* ***simple equilibrium*** *has the property that the nature of equilibrium prices in each period (separating, pooling) is only a function of the cost change during that period; and price levels depend only on the state variables β_t and c_t.*

In this setting, we have three possible cost-change values: $+, -, \circ$, corresponding to positive, negative or no cost change; and since there are two possible pricing strategies (separating prices and pooling prices), we have up to $2^3 = 8$ possible equilibrium configurations. However, internal consistency and the nature of cost dynamics greatly reduce the number of possible equilibrium types.

In the following subsection, we consider the case where cost dynamics are characterized by an absorbing state: either $\gamma_H = 0$ (inflationary case) or $\gamma_L = 0$ (deflationary case). The analysis of these cases is helpful as they serve as a stepping stone to the more general case (stationary cost dynamics) considered in the following subsection. The absorbing-cost analysis also helps develop economic intuition for the dynamic extension of the static game.

3.1. Absorbing cost state

In this subsection we consider a special case of our model where one of the cost states is absorbing, i.e., either $\gamma_H = 0$ or $\gamma_L = 0$. In this case the eight possible simple equilibrium

10. In what follows we denote β the belief about the cost prior to observing p_t, and $\hat{\beta}(p_t, \beta_t)$ the belief after observing p_t.

configurations of the full dynamic case are reduced to two: fully separating and fully pooling. To see this, consider, for example, the case when the high-cost state is absorbing and $c_0 = c_L$.[11] The cost change can take two values only: $\circ$ and $+$. Moreover, $+$ happens at one point in time only, when the cost increases from c_L to c_H. Suppose prices are separating at this cost-change state. It means that the cost change is revealed to the buyers and thus they can infer the underlying cost at all times. Thus both prior and post the cost change, the prices will correspond to beliefs $\beta = 0$ and $\beta = 1$ respectively, making this equilibrium fully separating. Now suppose that prices are pooling when cost-change state is $+$. Then, it must be that in state $\circ$ the seller was also charging a pooling price. Otherwise, the pooling price would identify state $+$ and buyers would optimally infer the underlying cost, making the pricing at $+$ correspond to separating equilibrium.[12]

In what follows, we consider the two equilibrium configurations separately in two cases: when c_H is an absorbing state and when c_L is an absorbing state.

■ **Separating equilibrium.** Consider the inflationary case, that is, $c_0 = c_L$ and $\gamma_H = 0$. In other words, cost starts at a low level, switches to a high level with probability γ_L, and once it reaches c_H it remains there forever.

Proposition 3. *Suppose that $c_0 = c_L$ and $\gamma_H = 0$. There exists a unique optimal simple separating equilibrium. Equilibrium seller strategies are given by time t prices*

$$\begin{aligned} p(c_H) &= c_H + (1-\delta)\, s \\ p(c_L) &= c_L + s\left(1 - \delta\left(1 - \alpha_{LH}\,\gamma_L\right)\right) \\ q(c_H) &= c_H - \delta\, s \\ q(c_L) &= c_L - s\,\delta\left(1 - \alpha_{LH}\,\gamma_L\right) \end{aligned}$$

where $c_t \in \{c_L, c_H\}$; buyer search strategies are given by time t search probabilities

$$\alpha(p_t, \beta_t) = \begin{cases} \alpha_H & \text{if } \; p_t = c_H + (1-\delta)s \;\; \text{and} \;\; p_t \neq p_{t-1} \\ 0 & \text{if } \; p_t = p_{t-1}; \;\; \text{or} \;\; p_t \leq c_L + s(1 - \delta\left(1 - \alpha_{LH}\,\gamma_L\right)) \\ 1 & \text{otherwise} \end{cases}$$

where

$$\alpha_{LH} \equiv \frac{c_H - c_L}{c_H - c_L + (1 - \delta\,(1 - \gamma_L))\, s}$$

and buyer beliefs are given by time t posterior beliefs

$$\hat{\beta}(p_t, \beta_t) = \begin{cases} 1 & \text{if } \; p_t = c_H + (1-\delta)s \\ 0 & \text{otherwise} \end{cases}$$

and $\beta_t = (1-\hat{\beta}_{t-1})\gamma_L + \hat{\beta}_{t-1}$ for buyers with no search activity in the previous period; $\beta_t = 1$ if a buyer was active at $t-1$ and encountered price $q(c_H)$; $\beta_t = \gamma_L$ if a buyer was active at $t-1$ and encountered a price $q(c_L)$.

11. Otherwise, if $c_0 = c_H$, then the price is constant at c_H, which is then common knowledge
12. We use terms "pooling price" and "separating price" loosely. We call a "separating price" the optimal price when $\beta_t = 0$ or $\beta_t = 1$. And a pooling price, the optimal price when beliefs are $\beta_t \in (0, 1)$.

To understand the intuition for the values of $p(c)$ and $q(c)$, suppose that $\delta = 0$. Then, for a seller, buyers have no future value and $q_H = c_H$, that is, competing for an active buyer is akin to static Bertrand competition, yielding price equal to cost (just as in the static case). Knowing this, buyers are willing to accept a price of $c_H + s$, one that exactly makes them indifferent between being active and being passive (again, just as in the static model considered in the previous section). At the opposite extreme, if $\delta = 1$ then a buyer is a valuable asset if $p_H > c_H$. Therefore, it must be that Bertrand competition for searchers implies that $p_H = c_H$.

The intuition for p_L, q_L is similar to the intuition for p_H, q_H: In the limit when $\delta = 0$, we obtain $p_L = c_L + s$ and $q_L = c_L$. In the opposite extreme, when $\delta = 1$, we get $p_L = c_L + s\,\alpha\,\gamma_L$ and $q_L = c_L - s\,(1 - \alpha\,\gamma_L)$. These latter expressions differ from their high-cost counterpart because, at state $c = c_L$, there is always a chance that the cost changes and a buyer is lost to search (which, along the equilibrium path, happens with probability $\alpha > 0$ when cost changes from c_L to c_H, which in turn happens with probability γ_L).

Next we consider the deflationary case.

Proposition 4. *Suppose that $c_0 = c_H$ and $\gamma_L = 0$. There exists a unique optimal simple separating equilibrium. Equilibrium seller strategies are given by time t prices*

$$\begin{aligned} p(c_H) &= c_H + (1-\delta)\,s + \alpha_H\,\delta s - \alpha_H\,\delta\,\gamma_H\,s \\ p(c_L) &= c_L + (1-\delta)\,s \\ q(c_H) &= c_H - \delta\,s + \alpha_H\,\delta\,s - \alpha_H\,\delta\,\gamma_H\,s \\ q(c_L) &= c_L - \delta\,s \end{aligned}$$

where $c_t \in \{c_L, c_H\}$; buyer search strategies are given by time t search probability

$$\alpha(p_t, \beta_t) = \begin{cases} \alpha_H & \text{if } \; p_t = c_H + (1-\delta)\,s + \alpha_H\,\delta\,s - \alpha_H\,\delta\,\gamma_H\,s \\ 0 & \text{otherwise} \end{cases}$$

where

$$\alpha_H \equiv \frac{\sqrt{\big((c_H - c_L) + s\,(1 - \delta(1-\gamma_H))\big)^2 + 4\,s\,\delta\,(1-\gamma_H)\,(c_H - c_L)} - \big((c_H - c_L) + s\,(1 - \delta\,(1-\gamma_H)\big)}{2\,s\,\delta\,(1-\gamma_H)}$$

and buyer beliefs are given by time t posterior beliefs

$$\hat\beta(p_t, \beta_t) = \begin{cases} 1 & \text{if } \; p_t = c_H + (1-\delta)\,s + \alpha_H\,\delta\,s - \alpha_H\,\delta\,\gamma_H\,s \\ 0 & \text{otherwise} \end{cases}$$

and $\beta_t = \hat\beta_{t-1}(1-\gamma_H)$ for buyers with no search activity in the previous period; $\beta_t = 1 - \gamma_H$ if a buyer was active at $t-1$ and encountered price $q(c_H)$; $\beta_t = 0$ if a buyer was active at $t-1$ and encountered a price $q(c_L)$.

Different from the increasing-cost case, along the equilibrium path a positive measure of buyers must be active at all times when $p_t = p(c_H)$. This follows from the seller's incentive compatibility constraint: if the seller does not lose any buyers by setting $p(c_H)$, then they have no incentive to lower price to $p(c_L)$ when $c_t = c_L$ (as a separating equilibrium requires).

We note that both separating equilibria above are simple, as the price levels follow a separating strategy in the state of no price change, and reveal the underlying cost when it changes (state "+" and "−" respectively). The equilibria are also optimal because we choose the lowest α required to satisfy the seller's incentive-compatibility constraint.

Proofs of Proposition 3 and Proposition 4 can be found in the Appendix.

Figure 1

Separating equilibria in increasing-cost and decreasing-cost cases

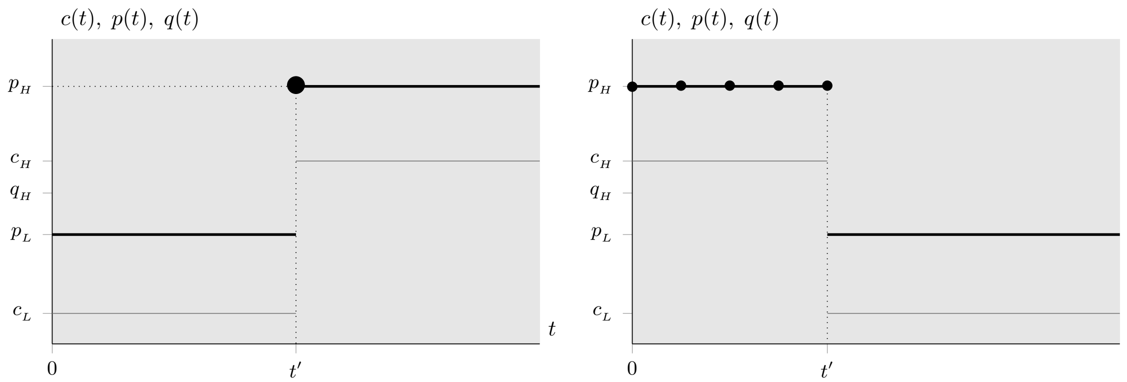


Figure 1 depicts typical equilibrium paths for the increasing cost case (left panel) and decreasing cost case (right panel). The gray lines show the evolution of cost, whereas the black lines show the evolution of the (common) list price. Finally, bullet points show the intensity of search at a given time and list price level.

In the increasing cost case, cost starts off at c_L. In each period, it increases to c_H with probability γ_L. In the particular case of Figure 1, this happens at time t'. At this point, price switches from p_L to p_H and buyers become active with probability $\alpha > 0$ (or, alternatively, a fraction α of buyers become active). Active buyers pay q_H, whereas passive buyers pay the list price p_H. Thereafter, all buyers pay p_H. Notice that, in equilibrium, buyers only become active (at most) once, namely when prices switch from p_L to p_H, which in turn happens when costs switch from c_L to c_H.

In the decreasing cost case, cost starts off at c_H. In each period, it decreases to c_L with probability γ_L. In the particular case of Figure 1, this happens at time t'. At this point, price switches from p_H to p_L. In the decreasing cost case, buyers become active with probability α every period when $p = p_H$. This differs from the increasing cost case, when buyers only become active (at most) at time t'.

Note that the intensity of search (when it happens) in this example is greater in the increasing cost case than in the decreasing cost case. However, to the extent that search takes place in more periods in the decreasing cost case, the overall total measure of active buyers may be greater in the decreasing cost case.

■ **Pooling equilibria.** We now turn to the case of pooling equilibria.

Proposition 5. *Suppose that* $s \geq (1 - \gamma_L)(c_H - c_L)$, $c_0 = c_L$ *and* $\gamma_H = 0$. *There exists a* *unique optimal simple pooling equilibrium. Equilibrium seller strategies are given by*

$$\begin{aligned}
p(0) &= c_L + s - \frac{\delta s}{1-\delta\,(1-\gamma_L)} \\
p(t) &= c_H + (1-\delta)\,s - (1-\gamma_L)^t\,(c_H - c_L - \delta\,s), \quad \forall t>0 \\
q(0,c_L) &= c_L - \frac{\delta\,s}{1-\delta\,(1-\gamma_L)} \\
q(t,c_L) &= c_L, \quad \forall t>0 \\
q(t,c_H) &= c_H - \delta\,s + \frac{\delta\,(1-\gamma_L)^{t+1}}{1-\delta\,(1-\gamma_L)}\,(c_H - c_L - \delta\,s)
\end{aligned}$$

Along the equilibrium path, non-searchers believe that $c = c_H$ with probability

$$\beta(t) = 1 - (1-\gamma_L)^t$$

If a price different from $p(t)$ is observed at time t, then $\hat{\beta}(t,p) = 0$. Buyers do not search along the equilibrium path.

Proposition 6. *Suppose that $s \geq (1-\delta)\,(c_H - c_L)/(1-\delta+\delta\,\gamma_H)$, $c_0 = c_H$ and $\gamma_L = 0$. There exists a unique optimal simple pooling equilibrium. Equilibrium seller strategies are given by*

$$\begin{aligned}
p(t) &= c_L + s + (1-\gamma_H)^t\left(c_H - c_L - s\,\delta\,\frac{1-\delta+\delta\,\gamma_H}{1-\delta}\right) \\
q(t,c_H) &= c_L + \left(1-\delta\,(1-\gamma_H)^{t+1}\right)\left(\frac{c_H - c_L}{1-\delta+\delta\,\gamma_H} + \frac{\delta\,s}{1-\delta}\right) \\
q(t,c_L) &= c_L
\end{aligned}$$

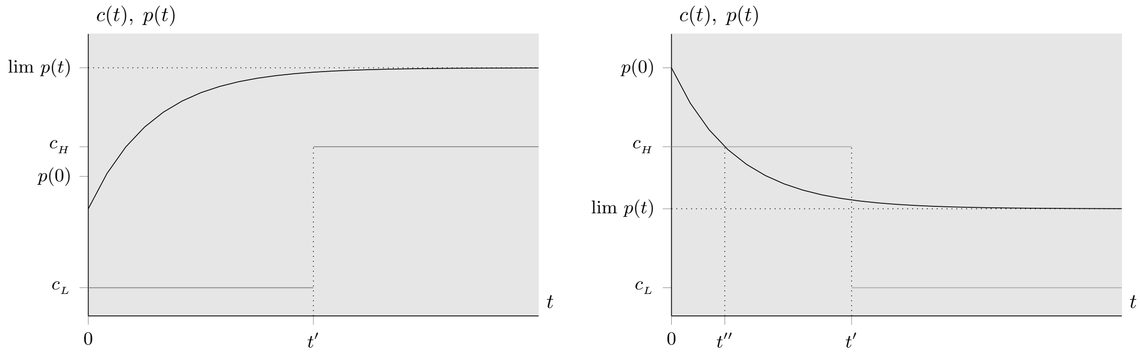
Along the equilibrium path, non-searchers believe that $c = c_H$ with probability

$$\beta(t) = (1-\gamma_H)^t$$

If a price different from $p(t)$ is observed at time t, then $\hat{\beta}(t,p) = 0$. Buyers do not search along the equilibrium path.

Figure 2 depicts typical equilibrium paths under the increasing cost case (left panel) and decreasing cost case (right panel). In each period, both a high cost and a low cost seller set the same price, a price which changes from period to period following the evolution of buyer beliefs. In the increasing cost case, price increases over time and converges asymptotically to $c_H + (1-\delta)\,s$, the price set by a high-cost seller in the separating equilibrium. In the decreasing cost case, prices decrease over time and converge asymptotically to $c_L + s$. This price is different from the separating equilibrium case, because searchers in the pooling equilibrium gain knowledge about the underlying cost. When cost is c_L, searchers are more optimistic than consumers who had never searched, and thus their search reservation price remains lower than the price charged in equilibrium, which implies that they search in every period going forward, which in turn implies zero seller profit (per searcher) in all future periods.

Figure 2
Pooling equilibria in increasing-cost and decreasing-cost cases



Note that, unlike the separating equilibrium, prices may fall below costs under a pooling equilibrium. For example, in the decreasing cost case (right panel), which corresponds to a particular realization of the stochastic process for costs, we observe that the seller prices below cost from t'' to t'. Why would a seller choose to stay in the market while pricing below cost? Given the equilibrium strategies, the alternative of setting a higher price (e.g., a price above cost) leads all buyers to search; and as we showed earlier, a searcher is worth zero, implying zero profit for any potential deviations. Therefore, in order for the putative price path to be incentive compatible, we require that the seller's value of staying in the market is positive. Although at t'' the seller makes negative period profits, its expect discounted profit is positive. Naturally, this condition implies limits on the relevant parameter values, which corresponds to the conditions in the text of Proposition 5 and Proposition 6.

■ **Discussion.** The main focus of our paper is understanding buyer behavior with respect to searching for a better deal (i.e., becoming active). How do the above equilibria inform this behavior? First, we can see that depending on how sellers behave (separating vs pooling) buyers may become active in certain periods or not become active at all. Second, we see that search is associated with high prices (decreasing cost case) or with price increases (increasing cost case). The empirical evidence seems consistent with both of these patterns. As we will see in Section 4 our model is unique in that it predicts both the effect of p and the effect of $\Delta\, p$ on buyer search.

3.2. Ergodic cost dynamics

In this section we consider the stationary case when costs follow an ergodic process: $\gamma_i > 0$ for $i \in \{L, H\}$. We continue to focus on simple equilibria. Even though, as mentioned in the previous section, there are potentially eight combinations of separating and pooling prices, internal consistency reduces the number of (simple) equilibrium configurations to four: fully separating, fully pooling, separating when price increases and otherwise pooling (rockets and feathers), separating when price decreases and otherwise pooling (bubbles and rocks). Optimality selects a unique equilibrium within each class of simple equilibria.

We prove two results: first, a result regarding the search properties of simple equilibria; and second, a result proving that there exists a unique simple equilibrium satisfying a series of properties that are consistent with the data. We begin with the central result of our paper, pertaining to the relation between price and search.

Proposition 7. *Along the equilibrium path of an optimal simple equilibrium, search only takes place when price (strictly) increases or price is at the highest observed level.*

In other words, a (strict) price increase or a price at the highest observed level is a necessary condition for search to take place. The intuition for Proposition 7 is, in a certain sense, already present in the static model of Section 2: the highest observed price corresponds to the separation price set by a high-cost seller. In order to ensure incentive compatibility, buyers must search when such a price is set; otherwise, low-cost seller would want to mimic a high-cost seller.

Similarly, in a dynamic case, buyers search when the price is at a high level. Observing a high price following either a low price (price increase) or high price (no change), some buyers choose to become active and the search intensity will be associated with the seller incentive compatibility constraint. The more attractive it is for a low-cost seller to "masquerade" as a high cost seller, the more buyers would choose to search to deter this deviation.

The proof of Proposition 7 may be found in the Appendix. The proof consists of deriving all simple equilibria and showing, by inspection, that all have the properties claimed by the proposition.

The fully dynamic model is necessary to explore many of the dynamic properties found in the data, namely asymmetries in the upward and downward movement of prices. Next we formally define the property of *rockets and feathers* — the aforementioned asymmetry in price adjustment — and show that there exists a unique simple equilibrium with such properties. As we will then see, in this equilibrium buyer search is caused by a price increase, not by the price level per se.

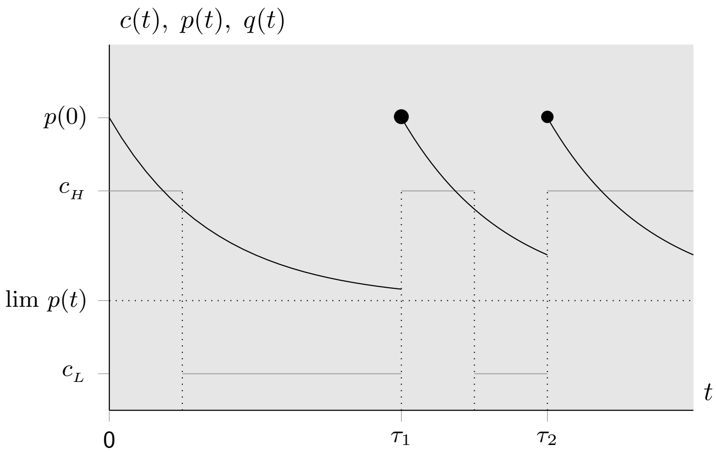
Definition 3. *An equilibrium has the* ***rockets-and-feathers*** *property if, along the equilibrium path, the average price increase is greater than the average price decrease.*

We are now ready to complete our equilibrium selection process in the stationary-cost case. The precise description of equilibrium strategies is a rather lengthy process. In fact, there exists no closed-form analytical form describing the buyer search strategies. The text of the proposition below includes the essential features of the equilibrium path, leaving the complete description of equilibrium strategies for the text of the proof itself.

Proposition 8. *There exists a unique optimal simple equilibrium featuring rockets-and-feathers price dynamics. Along the equilibrium path, sellers set $p = p_H$ when cost increases from c_L to c_H. Thereafter, sellers set a strictly declining series of prices $p(t)$ independently of cost, where t is time elapsed since the last switch from c_L to c_H. Buyers search with probability $\alpha(t)$ when price equals p_H, where $\alpha(t)$ is strictly increasing in t; and otherwise remain passive.*

Figure 3 depicts equilibrium price and search intensity along one possible cost series. In this particular case, cost switches from c_L to c_H at times τ_1 and τ_2. When that happens, sellers switch from whatever price they were setting to p_H, the highest price in the observed price distribution. Also at τ_i, buyers search with strictly positive probability. Moreover,

Figure 3
Rockets-and-feathers equilibrium



search intensity is greater the greater the price increase that takes place during that period. For example, at τ_1 price had been declining for longer than at τ_2. As a result, the price increase at time τ_1 is greater than the price increase at τ_2. This in turn implies that search intensity is higher at τ_1 than at τ_2. In Figure 3 (just as in Figure 1), this is represented by a larger bullet point at τ_1.

As can be seen from Figure 3, there are two statements one can make about search. First, search takes place if and only if the list price increases. Second, search takes place if and only if the list price is at its highest level. However, there are several reasons why the statement about price increase is more accurate than the statement about price level. First, the intensity of search is greater the greater the price increase, whereas the price level includes no information about the intensity of search other than that it's strictly positive when list price is at its highest level. Second, were we to consider the actual price paid (where searchers pay the discounted prices q), the statement about highest price is no longer true: while it is true that search takes place when the list price is at the highest value observed along the equilibrium path, the average price paid might not be the highest.

As mentioned earlier in this section, the simplicity criterion, together with optimality, reduces the set of equilibria to four types of equilibria. The additional rockets-and-feathers selection criterion reduces the set to a unique equilibrium. There are therefore three optimal simple equilibria excluded by the rockets-and-feathers selection criterion. First, similarly to what we find in the context of repeated games, the repetition of the static Nash equilibrium is a natural extensions of the Nash equilibria of the static game to the dynamic context. In particular, there exists a pure separating equilibrium in the dynamic game where prices (independent of time t) are given by

$$\begin{aligned}
p_L &= c_L + (1-\delta)\, s + \delta\, \alpha_L\, \gamma_L\, s \\
q_L &= c_L - \delta\, s + \delta\, \alpha_L\, \gamma_L\, s \\
p_H &= c_H + (1-\delta)\, s + \delta\, \alpha_H\, s - \delta\, \alpha_H\, \gamma_H\, s \\
q_H &= c_H - \delta\, s + \delta\, \alpha_H\, s - \delta\, \alpha_H\, \gamma_H\, s
\end{aligned}$$

This equilibrium satisfies Proposition 7 in the sense that buyers search when prices are high.

It is also true that buyers search when prices increase, but a high price is a sufficient of the presence and extent of search. (More on this in the next section.)

The pooling equilibrium from the static model in Section 2 can also be extended to the dynamic case.[13] In this case, prices and beliefs are given by

$$\begin{aligned} p &= \beta\, c_H + (1-\beta)\, c_L + s \\ q &= c \\ \beta &= \gamma_L / (\gamma_L + \gamma_H) \end{aligned}$$

As in the separating equilibrium, prices are independent of time t. Differently from the separating equilibrium, list prices are also independent of cost. As in the static pooling equilibrium, buyers never search. This equilibrium thus satisfies Proposition 7 in a trivial way (for no search takes place along the equilibrium path).

Finally, there is a third type of equilibrium excluded by the "rockets-and-feathers" property, one which we may refer to as "bubbles and rocks." In this equilibrium sellers separate when costs decrease, and pool otherwise. Specifically, each time cost switches from c_H to c_L sellers set a low price. Thereafter, prices increase over time with p determined by the number of periods since the last cost decrease and are independent of seller cost.

In the next section we use our model (and alternative models of search and price dynamics) as a data generating processes and run a series of regressions on the pseudo-data thus generated. As we will see, Proposition 8 implies that both price and price change are correlated with buyer search in our model.

4. Numerical simulations

Anecdotal and empirical evidence (Ho, Hogan, and Scott Morton, 2017; Paciello, Pozzi, and Trachter, 2019) suggests that buyers are more likely to search when prices increase and when prices are high. As we have seen in the previous section, our model delivers these predictions.

In order to evaluate the relative weight of price level and change in price level as determinants of buyer search behavior, we use our model as a data-generating process and run a series of regressions based on the pseudo-data thus created. In order to compare our model's (CG) relative performance with respect to alternative models of search and price dynamics, we follow the same strategy to generate alternative datasets from alternative models. In all cases, after generating equilibrium values, we normalize all of the variables, so that regression coefficients can be understood as the effect in Y of a one standard deviation in X, measured in standard deviations of Y.

One first alternative is the trivial extension of our "static" model to the dynamic game (S): as seen in the previous section, one possible equilibrium of the dynamic game consists of sellers setting a high price if cost is high and a low price if cost is low.[14] We thus generate a time series by stringing together multiple simulations of the static model. Two additional

13. For simplicity, we depart from the assumption that c_0 is known to the buyers and instead assume that buyers enter the market with ergodic beliefs about the underlying cost.

14. The static models developed in Dana (1994), Janssen, Pichler, and Weidenholzer (2011), Janssen, Parakhonyak, and Parakhonyak (2017), reviewed in Section 1, would lead to similar pseudo data.

Table 1
Comparison of various models as data generating processes
(S) Static model (Section 2); (CG) Dynamic model (Section 3.2);
(T) Tappata (2009); (YY) Yang and Ye (2008)
Dependent variable: search intensity

	S	CG	T	YY
Price	1.000***	0.293***	−0.999***	−0.996***
	(0.000)	(0.004)	(0.001)	(0.003)
Price change	−0.000	0.794***	1.284***	1.176***
	(0.000)	(0.004)	(0.001)	(0.003)
Observations	1,000	1,000	1,000	1,000
Adjusted R^2	1.000	0.988	1.000	0.993
Note:		*p<0.1; **p<0.05; ***p<0.01		

alternative data generating processes are based on Tappata's (2009) model of rockets-and-feathers dynamics (T) and Yang and Ye's (2008) model of rockets-and-feathers dynamics (YY). In all cases, we assume that

$$\begin{aligned} c_L &= 1 \\ c_H &= 6 \\ \delta &= .9 \\ s &= 3 \\ \gamma_L = \gamma_H &= .4 \end{aligned}$$

and use the various equilibria equations to create time series of $c(t), p(t), \alpha(t)$.[15]

The results are shown in Table 3.2. As can be seen, CG, T and YY are all consistent with a positive relation between price increase and search intensity. However, both T and YY imply a *negative* relation between price level and search intensity, whereas CG predicts a positive relation between the two. Both CG and S are consistent with a positive relation between price level and search intensity. However, the "static" model makes no prediction with respect to the relation between price change and search intensity. In other words, of the four models CG is the only one that is consistent with a positive relation between search intensity and both price level and change in price level.

15. Detailed notes about the simulation of the T and YY models will be provided upon request. The qualitative features of the numerical computations with the CG model are robust with respect to the choice of parameter values. Additional parameter values for simulating Tappata (2009): number of firms = 2; consumer valuation = 4; proportion of buyers with no search cost = 0.2; and $\overline{s} = 6$. Additional parameter values for simulating Yang and Ye (2008): reservation price = 4; measure of consumers = 2; capacity constraint = 4. While consumer valuation in some states is below the marginal cost of production, the seller is happy to produce at a loss in any particular period in exchange for the possibility to reap profits from the consumer in the future when the cost becomes low. Thus there is trade in every period.

To understand these results, notice first that, by construction, the static model cannot deliver dependance of search on past price realizations (since every period simulation is independent from the past). As a result, we cannot estimate a coefficient of search on Δp. At some level, this is an obvious result. That said, we decided to include this model in our comparison so as to highlight the difference between price level and changes in price level as determinants of buyer search.

The intuition for the numerical results is more complex in the Tappata (2009) and Yang and Ye (2008) models. In these models search is the result of the previously observed price due to the persistence of cost shocks: A low price in period $t-1$ signals a low cost in period t, as consumers believe cost is likely to have remained low, and thus induces more search in period t. Similarly, a high price in period $t-1$ signals a high cost in period t, as consumers believe cost is likely to have remained high, and thus induces less search in period t. Higher search then happens when price is low in period $t-1$, which in turn corresponds to no price change or a positive price change. Lower search, by contrast, happens when price is low in period $t-1$, which corresponds no price change or a negative price change. Hence the observed relation between price change and search. However, with respect to the current cost state (or price level), search is just as likely to occur in a high state as it is in a low state. Moreover, in both models cost shocks are persistent, so the HH state is more likely than the LH state. On average, this results in lower search when prices are high, which is contrary to the empirical evidence on search.

In sum, we note that our model differs significantly from Tappata (2009) and Yang and Ye (2008); and, unlike these papers, it is consistent with the empirical evidence both regarding price levels and changes in price level.[16] Moreover, the above explanation shows that the qualitative results hold for different parameter values. Finally, while various static models in the literature (and the one presented in Section 2) are consistent with a monotonic relation between price level and the extent of search, the dynamic model delivers a clear relation between change in price level and search which is not found in the dynamic projection of the static model.

5. Conclusion

Most of the economics literature on search and price dispersion is centered on final-consumer markets, markets where consumers are price takers. However, many transactions in the economy take place in a business-to-business context, a context where buyers have power to request specific price quotes (customer markets).

In this paper, we propose a model to address this type of markets. Although parsimonious, our model produces rich dynamics with different types of equilibria (pooling, separating, semi-separating). These equilibria offer an explanation for various stylized facts regarding price dynamics, including, in particular, the property that consumers seek a better deal (become “active” buyers) when they observe a price increase.

16. Admittedly, the relation between price and search was not the focus of Tappata (2009) and Yang and Ye (2008).

Appendix

Proof of Proposition 1: Consider first the price set for searchers. Since sellers simultaneously set prices that are available to all customers, Bertrand competition results in pricing at marginal cost, or $q(c) = c$ for $c \in \{c_L, c_H\}$. Searchers are therefore not profitable for sellers.

In a separating equilibrium, it must be that $\beta(p(c_L)) = 0$ and $\beta(p(c_H)) = 1$. The reservation value for search is then $c_L + s$ when $p(c_L)$ is set and $c_H + s$ when buyers observe $p(c_H)$.

We first show why $p(c_L)$ must be equal to buyer's reservation price. If the $p(c_L) > c_L + s$, then all assigned buyers search and thus unprofitable to the seller. This is suboptimal relative to charging $c_L + s$ and making profit $s > 0$ on each customer. If, on the other hand, $p(c_L) < c_L + s$, then no customers search and the sellers have a profitable deviation to charge a price that is ϵ greater than $p(c_L)$ but still below the reservation price. In order to not have a profitable deviation, it must then be that $p(c_L) = c_L + s$. This also implies that there must be no search activity at $p(c_L)$. If a positive measure of buyers search, sellers could profitably deviate by charging a price ϵ smaller than the reservation price which discourages search activity from all buyers.

In order to sustain an equilibrium with two prices, some search must be induced at the high price. If that is not the case, the seller would deviate by charging the highest price of $p(c_L)$ and $p(c_H)$, independently of the cost they face. In order to avoid such profitable deviation, the higher price must induce some search activity, whereby some customers are lost (become active and unprofitable), deterring the low cost sellers from "masquerading" as high cost sellers.

If at $p(c_H)$ all buyers search, a high cost seller receives zero profit. Such an equilibrium can be sustained only if $p(c_L) < c_H$, otherwise the high cost seller would profitably deviate and "masquerade" as a low cost seller, making a positive profit off all its buyers. Even if $p(c_L) > c_H$, we will show that such an equilibrium is not optimal as it leads to lower profits relative to the alternative equilibrium.

Consider, on the other hand, an equilibrium where the high cost seller retains some of its buyers. Then it must be that at $p(c_H)$ buyers are indifferent between staying with their assigned seller and searching. It follows that $p(c_H)$ is exactly equal to the reservation price $c_H + s$. Note that this equilibrium delivers positive profit to the high cost seller and doesn't change the profit of a low cost seller relative to the case when buyers search at $p(c_H)$. It dominates in optimality the previously discussed case where all buyers search after observing $p(c_H)$.

We now turn to α, the fraction of buyers who search at $p(c_H)$. Restrictions on α need to be consistent with incentive compatibility for each seller type. In order for the low cost type to not "masquerade" as a high cost seller, it must be that:

$$(1-\alpha)(p(c_H) - c_L) \leq p(c_L) - c_L$$

Plugging in the equilibrium prices, this condition imposes a lower bound on search activity, $\underline{\alpha} = \frac{c_H - c_L}{c_H - c_L + s}$.

It must also be that the high cost seller does not find it profitable to deviate to $p(c_L)$ and "masquerade" as a low cost buyer:

$$p(c_L) - c_H < (1-\alpha)\left(p(c_H) - c_H\right)$$

This condition imposes an upper bound on search activity, $\overline{\alpha} = \frac{c_H - c_L}{s}$. Since $\overline{\alpha} > \underline{\alpha}$, the equilibrium always exists for all $\alpha \in [\underline{\alpha}, \overline{\alpha}]$. The optimality condition selects a unique equilibrium with the lowest possible α. This is because α doesn't change the payoff for low cost sellers, but necessarily increases profits for high cost sellers, allowing them to make profit on more buyers. We have now showed that the only optimal equilibrium is one proposed in Proposition 1.

Finally, the off-equilibrium beliefs have to be such that incentive compatibility constraints are satisfied. The easiest way is to achieve this is to set all off-equilibrium beliefs to $\beta(p) = 0, \forall p \neq \{p(c_H), p(c_L)\}$. Note that in this equilibrium buyers do not have a profitable deviation by construction. ■

Proof of Proposition 2: Bertrand competition for searchers pins down $q(c) = c$ for both costs, as before. Let p be equilibrium price. Since in a pooling equilibrium price doesn't reveal any information about the underlying cost, it must be that $\beta(p) = \gamma$. This means that observing p, a buyer has a reservation price for search equal to the expected value of seach $\gamma\, c_H + (1-\gamma)\, c_L + s$. A price strictly above this reservation price will result in all buyers searching, leading to an equilibrium where sellers have zero profit. On the other hand, a price below c_H is not incentive compatible for the high cost seller as it delivers negative profit when $c = c_H$.

As long as $c_H \geq (\gamma\, c_H + (1-\gamma)\, c_L + s)$, or $s \geq (c_H - c_L)\,(1-\gamma)$, there exist pooling equilibria. Any price $p \in [c_H, \gamma c_H + (1-\gamma) c_L]$ with corresponding search activity $\alpha \in [0, 1)$ can be sustained in an equilibrium where sellers make a positive profit. Of this set, $\alpha = 0$ and $p = \gamma\, c_H + (1-\gamma)\, c_L$ is the unique optimal equilibrium: a combination of the highest possible price and the lowest possible measure of searchers.

We assume strict off-equilibrium beliefs, such that observing any price other than p leads buyers to believe that the underlying cost is low. This means that any seller deviation to a price above $c_L + s$ (the indifference price for buyers with beliefs $\beta = 0$) induces search from all buyers. Since deviating to price $c_L + s$ and below is suboptimal relative to charging the equilibrium price $p > c_L + s$, it follows that sellers do not have a profitable deviation in the proposed equilibrium. Finally, by construction buyers have no incentive to deviate. ■

Proof of Proposition 3: The equilibrium described in the model is simple, because at states $\Delta c \in \{\circ, +\}$ seller strategy is to play a separating equilibrium. We will now show that among the fully separating equilibria the one presented is the only one that satisfies optimality. We begin by showing that in such an equilibrium, there will only be two prices, and that those prices will be at the indifference point of the buyers. We then proceed to solve for those prices and show that there is only one solution. This proof is somewhat parallel to the static case, so we will be concise.

First, by the nature of separating equilibria, we know that when the cost is low (resp. high) buyer belief corresponds to $\hat{\beta} = 0$ (resp. $\hat{\beta} = 1$). The only gain from searching is then to receive an immediate discount in the current period (since the equilibrium is symmetric, the value going forward is the same whether the buyer stays with their assigned seller or switches sellers). Thus the indifference price when $c_t = c_L$ is $q(c_L) + s$, and, when $c_t = c_H$, is $q(c_H) + s$.[17] When $c = c_L$, as in the static equilibrium, it must be that the price is at the indifference level: higher, and all customers search, leaving zero profit; lower, and the

17. We assume that the q can only depend on c and β. It follows that q is constant for each cost level.

seller can deviate by charging ϵ more without inducing search. When the cost switches to c_H, sellers will charge a higher price to reveal the change. It must then induce search from some buyers, otherwise the seller would have a profitable deviation to charge this price earlier when the cost is still low. However, it must be that not all buyers are searching, otherwise, a profitable deviation might be to continue charging a low price or, if not an option, sellers make zero profit when costs are high (similar as in the static case, this is a possible equilibrium, but is not optimal, as high cost sellers can make a positive profit in the equilibrium we explore). This means that at a cost switch, buyers must be indifferent between searching and not searching, and so the price must be equal to the indifference price $q(c_H) + s$. When the cost remains high, a number of price paths are supported with punitive beliefs (where any off-equilibrium price leads buyers to believe that the underlying cost is c_L and thus any price aboce $q_L + s$ induces search): as long as it is profitable for a high cost seller, it is better than the alternative to deviating and loosing all buyers to search. Such prices are in the range of $p \in [c_H, q(c_H) + s]$. Additionally, at an indifference price, any seach activity is supported as long as some buyers remain with the seller: if $p = q(c_H) + s$, then $\alpha \in [0, 1)$ can be an equilibrium. The optimality condition selects the highest possible price with the lowest possible search probability: $p(c_H) = q(c_H) + s$ with no search except when prices jump from $p(c_L)$ to $p(c_H)$. Knowing this nature of pricing we can now solve for the value of each price.

Let v_i be seller value per assigned buyer, measured at the beginning of the period, when the cost state is i ($i \in \{L, H\}$) and the buyer does not search. When $c = c_H$, we have

$$v_H = \frac{p(c_H) - c_H}{1 - \delta} \tag{6}$$

Buyers only search (with positive probability) in the first period when cost is high. If search does not take place, then buyers do not ever search along the subsequent equilibrium path and the seller makes $p(c_H) - c_H$ margin in every period going forward.

Recall that

$$p(c_H) = q(c_H) + s \tag{7}$$

Differently from the static model, competition for searchers does not lead to pricing at cost. This is because attracting a buyer leads to profits in the current period but also in future periods. Specifically, when competing for a searcher, sellers lower prices to the point where *discounted* profit from getting a buyer is zero. This implies the zero profit condition:

$$q(c_H) - c_H + \delta\, v_H = 0 \tag{8}$$

Together, (6)–(8) imply

$$\begin{aligned} p(c_H) &= c_H + (1 - \delta)\, s \\ q(c_H) &= c_H - \delta\, s \end{aligned} \tag{9}$$

Substituting (9) for p_H in (6) we get

$$v_H = s$$

In words, given the buyer's ability to force sellers to compete head to head by paying a cost s, the value of s is also the measure of the rent that a seller earns from a non-searching buyer.

Denote α_{LH} the probability of search when price increases from $p(c_L)$ to $p(c_H)$. Then, when $c = c_L$, value per buyer is given by

$$v_L = p(c_L) - c_L + \delta \Big(\gamma_L \, (1 - \alpha_{LH}) \, v_H + (1 - \gamma_L) \, v_L \Big) \tag{10}$$

(Recall that a buyer who searches is effectively a customer lost, revenue wise; in other words, the seller is indifferent between keeping and losing a buyer who is offered price q_k.) Similarly to the case when $c = c_H$, when $c = c_L$ a buyer is indifferent between searching and not searching in a given period if and only if

$$p(c_L) = q_L + s \tag{11}$$

Moreover, when $c = c_L$ competition for searchers implies

$$q_L - c_L + \delta \Big(\gamma_L \, (1 - \alpha_{LH}) \, v_H + (1 - \gamma_L) \, v_L \Big) = 0 \tag{12}$$

Equations (10)–(12) can be solved to obtain

$$\begin{aligned} p(c_L) &= c_L + s \Big(1 - \delta \left(1 - \alpha_{LH} \, \gamma_L \right) \Big) \\ q_L &= c_L - s \, \delta \left(1 - \alpha_{LH} \, \gamma_L \right) \\ v_L &= s \end{aligned} \tag{13}$$

(The intuition for $v_L = s$ is similar to the intuition for $v_H = s$.)

Next, we consider whether there may be profitable deviations from the above strategies. First note that, by construction, buyers do not have a profitable deviation: they are always indifferent between searching and not searching. The binding constraint is therefore that firms do not want to deviate.

A deviation for a c_L type would be to "masquerade" itself as a c_H type and raise price before the cost increase. This increases markup ($p(c_H) - c_L > p(c_L) - c_L$), but also results in the loss of an α_{LH} fraction of buyers.[18] This gives a lower bound on the value of α_{LH}. Conversely, a deviation for a c_H type would be to "masquerade" itself as a c_L type and keep price at $p(c_L)$ when cost increases to c_H. By doing so, the seller retains the α_{LH} fraction of buyers who would have searched when price increases, but at a cost of a lower markup ($p(c_L) - c_H < p(c_H) - c_H$). This gives an upper bound on the value of α_{LH}.

First consider a c_L type trying to mimic a c_H type. This will change the value function only in the periods where the cost is low. In other words, once cost switches to c_H, profit per non-searching buyer is the same regardless of whether the seller deviates to $p(c_H)$ now or when cost increases to c_H. The incentive compatibility (IC) condition is therefore given by

$$(1 - \alpha_{LH}) \left(\frac{p(c_H) - c_L}{1 - \delta \, (1 - \gamma_L)} \right) < \left(\frac{p(c_L) - c_L}{1 - \delta \, (1 - \gamma_L)} \right)$$

The left-hand side (LHS) is the discounted expected payoff until cost changes to $c = c_H$ given that the firm sets $p = p(c_H)$ now. The right-hand side (RHS) is the corresponding discounted payoff given that the firm sets a low price until cost increases. The LHS decreases

18. Note that some of these buyers may remain with the seller. However, to the extent that buyers search, their net present value is zero — just as if they actually left the seller for the rival seller.

in α_{LH} while the RHS is constant with respect to α_{LH}. Equating the two we find a lower bound on α_{LH} for this IC condition to be satisfied:

$$\alpha_{LH} \geq \underline{\alpha}_{LH} = \frac{c_H - c_L}{c_H - c_L + (1 - \delta(1 - \gamma_L))\, s}$$

Note that this quantity is less than 1. Since the price levels determined earlier do not depend on α_{LH}, optimality chooses the lowest admissible value for search probability.

We must also consider the case of a c_H type deviating to mimic a c_L type. The one-step deviation rule applies, and so the IC condition simply becomes

$$p(c_L) - c_H < (1 - \alpha_{LH})\,(p(c_H) - c_H)$$

This gives us an upper bound on α_{LH}

$$\alpha_{LH} \leq \overline{\alpha}_{LH} = \frac{c_H - c_L}{(1 - \delta(1 - \gamma_L))\, s}$$

Note that $\underline{\alpha}_{LH} \in (0,1)$ and $\underline{\alpha}_{LH} < \overline{\alpha}_{LH}$. It follows that an equilibrium exists.

Finally, the proof relies on the assumption that $p_H > p_L$. This is not necessarily true. To see this recall that p_H, p_L are closely related to profits that a seller can extract from an acquired buyer. In a dynamic case, it is not only the current profits, but future profits as well. Thus it could be that even though the current cost is c_H (high) implying low current profits from a buyer, the buyer is, for example, expected to search less in the future, resulting in higher profits going forward. The $p_H > p_L$ condition is $c_H - c_L - s\,\alpha_{LH}\,\gamma_L > 0$, or, plugging in the value for α_{LH}: $c_H - c_L + s\,(1-\delta)\,(1-\gamma_L) > 0$, which is always satisfied. ■

Proof of Proposition 4: Similarly to the absorbing up case, this equilibrium is simple, as the separating strategy is played at all times. As before, the only benefit of searching is the immediate discount, so the indifference prices associated with each cost are $q(c_L) + s$ and $q(c_H) + s$ and the beliefs must be $\hat{\beta}(p(c_L)) = 0$ and $\hat{\beta}(p(c_H)) = 1$. We will now show that the prices to assigned buyers correspond to the indifference prices in each state.

First note that, at a high cost, there must be search at all times. If that was not true, then the low cost seller would have a profitable deviation to charge the higher price. In addition, for sellers to make a positive profit, not all buyers search at the high price. It must then be that at $p(c_H)$ the buyer is indifferent between searching and not searching, so $p(c_H) = q(c_H) + s$. Once the cost decreases, the price must also be at the indifference point $p(c_L) = q(c_L) + s$: above, and it triggers search for all buyers leading to zero profit; below, and sellers could extract ϵ more profits by bringing the price up closer to the indifference point. It also must be that in equilibrium there is no search at $p(c_L)$, otherwise, a profitable deviation would be to charge an ϵ smaller price and loose a minuscule margin to discourage a measurable set of buyers from searching.

We can now find the prices associated with this equilibrium. As in the previous case, v_L, the value of having an assigned consumer, $q(c_L)$ and $p(c_L)$ solve the following system of equations:

$$\begin{aligned} v_L &= \frac{p(c_L) - c_L}{1 - \delta} \\ p(c_L) &= q(c_L) + s \\ q(c_L) - c_L + \delta\, v_L &= 0 \end{aligned}$$

Then, $q(c_L) = c_L - \delta\, s$; $p(c_L) = c_L + (1-\delta)\, s$; $v_L = s$. From here, $v_H, q(c_H), p(c_H)$ solve the following system of equations:

$$\begin{aligned} v_H = p(c_H) - c_H + \delta\,(1-\gamma_H)\,(1-\alpha_H)\, v_H + \gamma_H\, \delta\, v_L \\ p(c_H) = q(c_H) + s \\ q(c_H) - c_H + \delta\,(1-\gamma_H)\,(1-\alpha_H)\, v_H + \gamma_H\, \delta\, v_L = 0 \end{aligned}$$

Then, $q(c_H) = c_H - \delta\, s + \alpha_H\, \delta\, s - \alpha_H\, \delta\, \gamma_H\, s$; $p(c_H) = c_H + (1-\delta)\, s + \alpha_H\, \delta\, s - \alpha_H\, \delta\, \gamma_H s$; and $v_H = s$.

Note that buyers do not have a profitable deviation in this equilibrium by construction. We now turn to IC constraints of the sellers to find bounds on α_H. In order for the low cost seller to not profitably "masquerade" as a high cost seller (i.e. remain charging a high price when the cost reverts to low for one more period), it must be that

$$(1-\alpha_H)\,(p(c_H) - c_L + \delta\, v_L) \leq v_L$$

This condition imposes a lower bound on α_H:

$$\alpha_H \geq \frac{\sqrt{((c_H - c_L) + s\,(1-\delta(1-\gamma_H)))^2 + 4\, s\, \delta(1-\gamma_H)\,(c_H - c_L)} - ((c_H - c_L) + s\,(1-\delta\,(1-\gamma_H))}{2\, s\, \delta(1-\gamma_H)}$$

On the other hand, if α_H was too large, then a high cost seller would have an incentive to lower the price and masquerade as a low cost seller, so it must be that:

$$v_L - \frac{c_H - c_L}{1-\delta\,(1-\gamma_H)} \leq (1-\alpha_H)\, v_H$$

The latter condition imposes that once the firm charges a low price, it will remain charging a low price for ever, thus the value of this deviation is v_L less the difference in costs between the high and low cost until the cost actually switches to the low value. The upper bound that this condition imposes on α_H is $\alpha_H \leq \frac{c_H - c_L}{s\,(1-\delta\,(1-\gamma_H))}$. As before, optimality selects a unique equilibrium where α_H corresponds to the lower bound found here.

Note that both seller types make positive profits, so the participation constraints are not binding.

Finally, the proof relies on the condition that $p_H > p_L$, which is always true, because $p_H - p_L = c_H - c_L + \alpha_H\, \delta\, s\,(1-\gamma_H) > 0$. ■

Proof of Proposition 5: This equilibrium is simple, as the pooling strategy is played by sellers in each state "$\circ$" and "+".

In a pooling equilibrium no information is revealed, so beliefs follow the unconditional probability of cost being in a high state:

$$\beta_t = 1 - (1-\gamma_L)^{t-1}$$

The reservation price for a consumer that has never searched is:

$$\beta(t)\, u_H(t) + (1-\beta(t))\, u_L + s$$

where u_L and $u_H(t)$ is the value of searching corresponding to low and high costs.

As before, with restrictive beliefs, many pooling prices can be supported in an equilibrium. As long as firms make positive profits, they would not choose to deviate and lose all of their customers. Optimality selects the highest such price, that is, the price at which buyers are indifferent between searching and not searching. In addition, buyers in equilibrium never search, otherwise a seller would have a profitable deviation of cutting the prices by ϵ and deter searchers at an arbitrarily small cost to their profit margin.

We now find the price path that follows buyer's indifference price across time. First note that when a buyer searches after period 0, upon discovering the low cost state they become forever after more optimistic about the cost than a buyer who has never searched. This means that they continue to search in every period where a seller assigned to them charges indifference prices targeted towards non-searchers. As a result, it must be that $q(c_L, t) = c_L$ for $t \geq 1$. On the other hand, a searcher who discovers a high cost will remain for ever more pessimistic than someone who never searched and will never search again.

Define

$$u_H(t) = -q(c_H, t) - \sum_{\tau=1}^{\infty} \delta^\tau p(t+\tau)$$

the value of searching of a buyer when the cost is high. On the seller side, the zero profit condition states that

$$q(c_H, t) - c_H + \sum_{\tau=1}^{\infty} \delta^\tau (p(t+\tau) - c_H)) = 0$$

It must then be that

$$u_H(t) = -\frac{c_H}{1-\delta}$$

If search occurs in the low state, buyers continue to search every period until they find that the state is high. Together with the zero profit condition, this implies that this value is discounted prices and the expected search costs:

$$\begin{aligned} u_L &= -\frac{c_H}{1-\delta} + \frac{c_H - c_L}{1-\delta\,(1-\gamma_L)} - \delta\,\frac{s}{1-\delta\,(1-\gamma_L)} \\ &= \frac{-c_H\,\delta\,\gamma_L - (c_L + \delta\,s)\,(1-\delta)}{(1-\delta)\,(1-\delta+\delta\,\gamma_L)} \end{aligned}$$

Unless we're in $t = 0$ in which case there beliefs are the same as the non-shoppers and so there won't be any search in the future, so

$$u_L(0) = -\frac{c_H}{1-\delta} + \frac{c_H - c_L}{1-\delta\,(1-\gamma_L)}$$

Let $u(t)$ be buyer value at time t. Then

$$u(t) = -p(t) + \delta\,u(t+1)$$

Since buyers are indifferent between searching and not, we have

$$u(t) = (1-\beta(t))\,u_L + \beta(t)\,u_H - s$$

Then

$$
\begin{aligned}
p(t) &= -u(t) + \delta\, u(t+1) \\
&= c_H + s\,(1-\delta) - (1-\gamma_L)^t\,(c_H - c_L - \delta\, s)
\end{aligned}
$$

And

$$
\begin{aligned}
p(0) &= c_L + s - \delta\,\frac{s}{1-\delta\,(1-\gamma_L)} \\
q_L(0) &= c_L - \sum_{t=1}^{\infty} \delta^t (p(t) - E[c(t)]) \\
&= c_L - \frac{\delta\, s}{1-\delta\,(1-\gamma_L)}
\end{aligned}
$$

Next we compute the introductory prices. Since buyers continue to search after they have searched in the low state, we have

$$
q_L = c_L
$$

In the high state, compute from the 0 profit

$$
\begin{aligned}
q_H &= c_H - \sum_{\tau=t+1}^{\infty} \delta^{\tau-t}(p(t) - E[c(t)]) \\
&= c_H - \delta\, s + \frac{(1-\gamma_L)^{t+1}\delta}{1-\delta\,(1-\gamma_L)}(c_H - c_L - \delta\, s)
\end{aligned}
$$

To confirm participation constraint, we need to make sure that at no point a seller chooses to exit the market rather than remain charging the pooling price. This might happen when a seller's cost increases immediately, but they are still expected to charge equilibrium prices which might be below their marginal cost. Clearly, the constraint is most binding when c goes up right away. We need to specify parameter boundaries such that even in this case, a seller finds it profitable to stay:

$$
\begin{aligned}
0 &\le \sum_{t=1}^{\infty} \delta^{t-1}(p(t) - ch) \\
&= s - (1-\gamma_L)\frac{c_H - c_L - \delta\, s}{1-\delta\,(1-\gamma_L)} \\
0 &\le s - \delta\, s + \delta\,\gamma_L\, s - (1-\gamma_L)\,(c_H - c_L) + \delta\, s - \delta\,\gamma_L\, s \\
s &\ge (1-\gamma_L)\,(c_H - c_L)
\end{aligned}
$$

Finally, we check that our assumption that $p_H > p_L$ holds. The condition implies that $c_H - c_L + \alpha_H\,\delta\, s\,(1-\gamma_H) > 0$, which holds for all parameter values. ■

Proof of Proposition 6: See proof of Proposition 5 for a trivial adoption to this case in terms of equilibrium structure. In the following calculations we show how we obtained prices stated in the proposition.

Define values of search. If buyers search in a low state, they search forever after. Combined with a zero profit condition of the seller, we get that the value of search is equal to the discounted costs save the search cost:

$$u_L = -\frac{c_L}{1-\delta} - \frac{\delta s}{1-\delta}$$

If a buyer searches in a high cost state, they will never search again, so u_H is simply equal to the discounted costs (due to the zero profit condition):

$$u_H = -\frac{c_L}{1-\delta} + \frac{c_L - c_H}{1-\delta\,(1-\gamma_H)}$$

Let $u(t)$ be buyer's value function at time t.[19] Then

$$u(t) = -p(t) + \delta\, u(t+1)$$

Since buyers are indifferent between searching and not, we have

$$u(t) = (1-\beta(t))\, u_L + \beta(t)\, u_H - s$$

Then

$$\begin{aligned} p(t) &= -u(t) + \delta\, u(t+1) \\ &= c_L + s + (1-\gamma_H)^t \left(c_H - c_L - \frac{\delta\, s\,(1-\delta+\delta\gamma_H)}{1-\delta} \right) \end{aligned}$$

The introductory prices are as follows

$$\begin{aligned} q_L &= c_L \\ q_H &= c_H - \sum_{\tau=t+1}^{\infty} \delta^{\tau-t}(p(t) - E[c(t)]) \\ &= c_L + \left(1-\delta\,(1-\gamma_H)^{t+1}\right) \left(\frac{c_H - c_L}{1-\delta+\delta\,\gamma_H} + \frac{\delta\, s}{1-\delta} \right) \end{aligned}$$

As in the previous case, an existence condition is needed to make sure that the participation constraint is satisfied for sellers in all possible states of the world. This condition is most binding if cost remains high but the price has converged to it's lowest possible value.

$$0 < \sum_{t=0}^{\infty} \delta^t (p_{lim} - (1-\gamma_H)^t c_H - (1-(1-\gamma_H)^t) c_L)$$

19. As before, we normalize the buyer valuation of the good to 0. This doesn't change the results because a purchase is always made in the equilibrium.

or simply

$$\frac{(c_H - c_L)(1-\delta)}{1-\delta\,(1-\gamma_H)} < s$$

which concludes the proof. ■

Proof of Proposition 7: Table 2 considers all possible combination of pooling (P) and separation (S) along an equilibrium path. Even though there are potentially 8 possible cases, only four of these are consistent:

- Always separating, which we refer to as pure separation equilibrium
- Always pooling, which we refer to as pure pooling equilibirum
- Separating at $+$, pooling at $-, \circ$, which we refer to as rockets-and-feathers equilibrium
- Separating at $-$, pooling at $+, \circ$, which we refer to as bubbles-and-rocks equilibrium

Table 2
Possible equilibrium configurations

$+$	$-$	$\circ$	Equilibrium
S	S	S	Pure separation
S	S	P	(impossible)
S	P	S	(impossible)
S	P	P	Rockets and feathers
P	S	S	(impossible)
P	S	P	Bubbles and rocks
P	P	S	(impossible)
P	P	P	Pure pooling

We strike down the "impossible" equilibria where a pooling price can not take place, because the underlying cost state is perfectly revealed to buyers through separating strategies in other states. For example, if sellers always reveal prices going up and always reveal them going down, it must be that any other price means prices haven't changed, meaning that beliefs are $\beta = 0$ or $\beta = 1$ leading to pooling price in this case equal to separating price. Thus any situation where two out of three cost changes produce separating prices is impossible (or coincide with all separating case). Similarly, if the prices reveal costs with no change in underlying cost, then any different price signals a cost change, leading buyers to perfectly discover the cost at all times.

We now show that there is a unique simple optimal equilibrium in each of the four classes of equilibria. We assume for simplicity that buyers begin in period 0 with an ergodic belief about the underlying cost. In essence, the following proofs rely on the fact that a simple equilibrium type implies a certain set of beliefs from the buyers after observing prices ($\hat{\beta}(p,t) \in \{0,1\}$, for separating prices, and $\hat{\beta}(p,t{+}1) = \beta(t{+}1) = (1{-}\gamma_H)\,\beta(t){+}\gamma_L\,(1{-}\beta(t))$, for pooling prices). With beliefs in hand, optimality selects the highest prices that a seller can charge without inducing search, or a price at which some buyers search, but not all - both coinciding with the indifference price for buyers. The indifference price of buyers is related to the discount price they would get from searching. The discount price, in turn, satisfies a zero profit condition for the sellers via Bertrand competition. Together, this set of equations delivers a unique set of prices. We then proceed with finding a unique search strategy by buyers to satisfy incentive compatibility of sellers in each cost type. Finally, we derive a condition of existence from a participation constraints by both seller types.

■ **Separating Equilibrium.** First, beliefs in this equilibrium are as follows: $\beta(t) = 1$, when $c = c_H$ and $\beta(t) = 0$, when $c = c_L$. This means that the reservation prices are $q_L + s$ and $q_H + s$, where q_L and q_H are the introductory prices in each state. These reservation prices will correspond to the actual prices in each state.[20]

Next, the search strategies must be such that sellers do not have an incentive to deviate. Further, out of a set of feasible search probabilities, optimality will select the lowest among them. As in the absorbing case, it must be that there is some search when the prices are at the highest level and there has to be no search when prices are low. Suppose that p_H induces α_H fraction of buyers to search.

We are now ready to solve for optimal pricing strategies from the following set of equations:

$$
\begin{aligned}
p_L &= q_L + s \\
p_H &= q_H + s \\
0 &= q_L - c_L + \delta\,(1-\gamma_L)\,v_L + \delta\,\gamma_L\,(1-\alpha_H)\,v_H \\
0 &= q_H - c_H + \delta\,(1-\gamma_H)\,(1-\alpha_H)\,v_H + \delta\,\gamma_H\,v_L \\
v_L &= p_L - c_L + \delta\,(1-\gamma_L)\,v_L + \delta\,\gamma_L\,(1-\alpha_H)\,v_H \\
v_H &= q_H - c_H + \delta\,(1-\gamma_H)\,(1-\alpha_H)\,v_H + \delta\,\gamma_H\,v_L
\end{aligned}
$$

Here, v_L and v_H are seller's values of having an assigned customer when the costs are equal to c_L and c_H respectively. The first two equations correspond to buyer's indifference conditions at each price. The third and fourth are the zero profit condition for the sellers. The last two define seller's value at each state. The solution to this system is:

$$
\begin{aligned}
q_L &= c_L - \delta\,s + \alpha_H\,\delta\,\gamma_L\,s \\
q_H &= c_H - \delta\,s + \alpha_H\,\delta\,s - \alpha_H\,\delta\,\gamma_H\,s \\
p_L &= c_L + (1-\delta)\,s + \alpha_H\,\delta\,\gamma_L\,s \\
p_H &= c_H + (1-\delta)\,s + \alpha_H\,\delta\,s - \alpha_H\,\delta\,\gamma_H\,s \\
v_L &= s \\
v_H &= s
\end{aligned}
$$

Given these values, we can solve for α_H that satisfies the incentive compatibility constraint. The lower bound on α_H is imposed by the IC constraint of the low cost seller: α_H needs to be high enough so that at $c = c_L$ the seller does not find it profitable to "masquerade" as a high cost seller for one period. On the other hand, if α_H is too high, then a high cost seller

20. We will not expand on this point as the intuition is the same as in the static set up. For the prices where no search happens, the optimality selects the highest possible price with no search, that is the indifference price. For prices where search is present it must be that either every buyer searches (0 profit for sellers) or that only some buyers search (then buyers must be indifferent between searching and not), and optimality again selects the latter.

might find it profitable to "masquerade" as a low cost seller in order to not loose many buyers. This imposes an upper bound on α_H. The two constraints are:

$$(1-\alpha_H)(v_L - p_L + p_H) \leq v_L$$
$$v_H - p_H + p_L \leq (1-\alpha_H)\, v_H$$

An equilibrium exists for $\alpha_H \in (\underline{\alpha}, \overline{\alpha})$, where

$$\begin{aligned}
\underline{\alpha} = & \frac{\left(c_H - c_L + \delta\,\gamma_H\, s + \delta\,\gamma_L\, s - \delta\, s + s\right)}{2\,\delta\, s(\gamma_H + \gamma_L - 1)} \\
& - \frac{\sqrt{(c_H - c_L)^2 - 2\,c_H\,\delta\, s(\gamma_H + \gamma_L) + 2\,c_H s\,(1+\delta) + 2\,c_L\,\delta\, s\,(\gamma_H + \gamma_L) - 2\,c_L\, s\,(1+\delta)}}{2v\delta\, s\,(\gamma_H + \gamma_L - 1)} \\
& + \frac{\sqrt{(\delta\,\gamma_H\, s + \delta\,\gamma_L\, s)^2 - 2\,\delta^2\,\gamma_H\, s^2 - 2\,\delta^2\,\gamma_L\, s^2 + \delta^2\, s^2 + 2\,\delta\,\gamma_H\, s^2 + 2\,\delta\,\gamma_L\, s^2 - 2\,\delta\, s^2 + s^2}}{2\,\delta\, s\,(\gamma_H + \gamma_L - 1)} \\
\overline{\alpha} = & \frac{c_H - c_L}{s\,(\delta\,\gamma_H + \delta\,\gamma_L - \delta + 1)}
\end{aligned}$$

Optimality selects the equilibrium corresponding to the lower bound on α_H.

We note that the participation constraint is always satisfied as the equilibrium prices are above the marginal cost for both types of sellers.

Finally, the construction of the equilibrium relies on the assumption that $p_H > p_L$. This is true as long as $c_H - c_L + \alpha_H\,\delta\, s\,(1-\gamma_H - \gamma_L) > 0$. This condition might not be binding, depending on values of α_H, and is always satisfied if states are persistent ($\gamma_H < 0.5, \gamma_L < 0.5$).

■ **Pooling Equilibrium.** First, beliefs in this equilibrium are constant and equal to the ergodic probability of $c = c_H$, that is, $\beta = \frac{\gamma_L}{\gamma_L + \gamma_H}$.

Next, we turn to prices that are identified by the following set of equations:

$$\begin{aligned}
u_L &= -q_L - \sum_{\tau=t+1}^{\infty} \delta^{\tau-t}\widehat{p}(\tau) - \frac{\delta s}{1-\delta\,(1-\gamma_L)} \\
u_H &= -q_H - \sum_{\tau=t+1}^{\infty} \delta^{\tau-t}(p(t) - E_t[c(t)]) \\
0 &= q_L - c_L + \sum_{\tau=t+1}^{\infty} \delta^{\tau-t}\widehat{p}(\tau) - E_t c(\tau) \\
0 &= q_H - c_H + \sum_{\tau=t+1}^{\infty} \delta^{\tau-t} p(\tau) - E_t c(\tau) \\
u &= \beta\, u_H + (1-\beta)\, u_L - s \\
u &= -p + \delta\, u
\end{aligned}$$

In the first and third equations, $\widehat{p}$ is the price that a buyer who discovers a low cost state will pay going forward (either the accepted p, or the q_L or q_H if they choose to search). The first pair of equations defines the value of searching for a buyer in each state. Note that if a buyer identifies a low cost upon searching, they continue to search in future periods until they discover a high cost. The second pair of equations describes the zero profit condition for sellers in each cost state when setting an introductory price. Finally, the last two equations identify buyer's indifference condition between searching or not. We know that the optimality will select the price that corresponds to this value. Solutions to these equations identify prices:

$$\begin{aligned} q_L &= c_L \\ p &= \frac{(c_H\,\gamma_L + c_L\,\gamma_H)(\delta\,\gamma_L - \delta + 1) - \delta^2\,\gamma_L\,(\gamma_H\,s - \gamma_L\,s + s)}{\delta\gamma_H\,\gamma_L - \delta\,\gamma_H + \delta\,\gamma_L^2 - \delta\,\gamma_L + \gamma_H + \gamma_L} \\ &+ \frac{\delta\,\gamma_L\,(\gamma_H\,s + \gamma_L\,s - 2\,s) + s\,(\gamma_H + \gamma_L - \delta\,\gamma_H)}{\delta\,\gamma_H\,\gamma_L - \delta\,\gamma_H + \delta\,\gamma_L^2 - \delta\,\gamma_L + \gamma_H + \gamma_L} \\ q_H &= \frac{c_H(1 - \delta\,(1 - \gamma_L)) + \delta\,\gamma_H\,c_L}{(1 - \delta\,(1 - \gamma_H))(1 - \delta\,(1 - \gamma_L)) - \delta^2\,\gamma_L\,\gamma_H} - \frac{\delta}{1 - \delta}\,p \end{aligned}$$

Finally, the participation constraint imposes a parameter constraint for which a seller with a high cost finds it profitable to charge a pooling price. This price might be below their marginal cost, but the discounted sum of future profits needs to be positive.

$$\frac{p}{1-\delta} - \frac{c_H\,(1 - \delta\,(1 - \gamma_L)) + \delta\,\gamma_H\,c_L}{(1 - \delta\,(1 - \gamma_H))(1 - \delta\,(1 - \gamma_L)) - \delta^2\,\gamma_L\,\gamma_H} \geq 0$$

■ **Rockets and Feathers Equilibrium.** In this equilibrium beliefs are defined as follows:

$$\beta(\tau) = (1 - \gamma_H)^\tau$$

where τ is the time since last cost increase. When cost switches from c_L to c_H (and the price signals the switch), buyers correctly believe that cost is high, that is $\beta(0) = 1$. In subsequent periods and until the next price increase buyers believe that cost has not decreased and increased again, that is, buyers know that the cost is high only if it has remained high since last increase.

As before, the prices reflect the indifference condition of an assigned buyer. Let u_L and u_H be the values of searching corresponding to low and high cost. Then,

$$\begin{aligned} &\beta(\tau)\,u_H + \big(1 - \beta(\tau)\big)\,u_L - s = \\ &\qquad = -p(\tau) + \delta\left(\mathbb{E}_\tau\left[\beta(\tau+1)\right]\,u_H + \big(1 - \mathbb{E}_\tau\left[\beta(\tau+1)\right]\big)\,u_L - s\right) \end{aligned}$$

The left-hand side is the expected price from searching in the current period; the right-hand side is the expected price from not searching in the current period and searching in the next period. As in the previous pooling equilibrium, we know that u_L is equal to the negative discounted future costs save the search cost for as long as the costs are expected to stay low. Whereas u_H is simply the negative of discount costs going forward from c_H.

$$u_L = -c_L - \sum_{\tau=t+1}^{\infty} \delta^{\tau-t} E_t[c(t)] - \frac{\delta\, s}{1-\delta\,(1-\gamma_L)}$$

$$u_H = -c_H - \sum_{\tau=t+1}^{\infty} \delta^{\tau-t} E_t[c(t)]$$

Solving with respect to p,

$$p(t) = (1-\delta)\,(s-u_L) - (\beta_t - \delta\, E_t\beta_{t+1})(u_H - u_L) \tag{14}$$

where

$$\begin{aligned}
u_H &= -\frac{c_H\,(1-\delta) + c_L\,\delta\,\gamma_H + c_H\,\delta\,\gamma_L}{(1-\delta)\,(1-\delta\,(1-\gamma_H-\gamma_L))} \\
u_L &= -\frac{c_L\,(1-\delta) + c_L\,\delta\,\gamma_H + c_H\,\delta\,\gamma_L}{(1-\delta)\,(1-\delta\,(1-\gamma_H-\gamma_L))} - \frac{s\,\delta}{1-\delta(1-\gamma_L)} \\
\beta_t &= (1-\gamma_H)^t \\
E_t\beta_{t+1} &= (1-\beta_t)\,\gamma_L + (1-(1-\beta_t)\,\gamma_L)\,\beta_t\,(1-\gamma_H)
\end{aligned}$$

We note here that $\lim_{t\to\infty} p(t) = c_L + s$. Let $v_H(\tau)$ (resp. $v_L(\tau)$) be seller value per unit mass of buyers when the cost state is high (resp. low) and τ periods have elapsed since the last cost shift. Note that $v_L(0)$ is never realized in the equilibrium, since $t=0$ is defined as a period when $c = c_H$; however, this value will be important for computing incentive compatibility conditions. In equilibrium, these value functions satisfy

$$v_H(\tau) = p(\tau) - c_H + \delta\big((1-\gamma_H)\,v_H(\tau+1) + \gamma_H\,v_L(\tau+1)\big) \tag{15}$$

$$v_L(\tau) = p(\tau) - c_L + \delta\Big((1-\gamma_L)\,v_L(\tau+1) + \gamma_L\,\big(1-\alpha\,(\tau+1)\big)\;v_H(0)\Big) \tag{16}$$

Specifically, suppose that τ periods have elapsed since the last cost increase and suppose that cost is still at c_H. In the current period, the seller sets $p(\tau)$ and pays a cost c_H. Assuming the seller has a unit mass of customers, its current profit is given by $p(\tau) - c_H$. Beginning in the next period, two things may happen: cost remains high, yielding a continuation payoff $v_H(\tau+1)$; or cost drops to c_L, yielding a continuation payoff $v_L(\tau+1)$.

Now suppose that the current cost is c_L. By a similar argument as in the preceding paragraph, current profit is $p(\tau) - c_L$. Beginning in the next period, two things may happen: cost remains low, yielding a continuation payoff $v_L(\tau+1)$; or cost increases to c_H. The latter outcome leads to a fraction $\alpha(\tau+1)$ buyers search (which implies zero value for the seller) and a continuation value $v_H(0)$ for remaining buyers.

Next we consider seller's incentive compatibility (IC) constraints. First, a seller may want to increase price when cost is low. This implies the following IC constraint:

$$v_L(\tau) \geq \big(1-\alpha(\tau)\big)\;v_L(0) \tag{17}$$

Second, a seller may want to increase price when cost is (still) high. This implies the following IC constraint:

$$v_H(\tau) \geq \big(1 - \alpha(\tau)\big)\ v_H(0) \tag{18}$$

Finally, in a period when cost shifts from c_L to c_H a seller might deviate by *not* increasing price, and instead continuing with the value $p(\tau)$ corresponding to no cost increase This implies the following IC constraint:

$$v_H(\tau) \leq \big(1 - \alpha(\tau)\big)\ v_H(0) \tag{19}$$

Equation (18) implies (17). Moreover, (18) and (19) uniquely pin down the value of $\alpha(\tau)$:

$$\alpha(\tau) = 1 - \frac{v_H(\tau)}{v_H(0)} \tag{20}$$

We cannot find a closed-form solution to the value functions. However, we can derive them numerically and recursively. Since $p(\tau) \to \overline{p}$, so $v_i(\tau) \to \overline{v}$. These v_i limits can be solved from

$$\begin{aligned}\overline{v}_H &= p(T) - c_H + \delta\big((1-\gamma_H)\,\overline{v}_H + \gamma_H\,\overline{v}_L\big)\\ \overline{v}_L &= p(T) - c_L + \delta\big((1-\gamma_L)\,\overline{v}_L + \gamma_L\,\overline{v}_H\big)\end{aligned}$$

The values of $p(\tau)$ and $v_i(\tau)$ can be approximated arbitrarily close by considering a sufficiently large terminal T, assuming $v_i(T+1) = \overline{v}_i$, and solving recursively[21].

Next, the participation constraint imposes that at all times it is profitable for sellers to be in the market. The worst outcome for sellers is when they are charging the lowest possible price while the cost is still high. We know their value then is $\overline{v}_H$, which from the above equation is equal to:

$$\overline{v}_H = \frac{p(T) - c_H + \delta\,\gamma_H\,\frac{p(T)-c_L}{1-\delta\,(1-\gamma_L)}}{1 - \delta\,(1-\gamma_H) - \delta^2\,\gamma_H\,\gamma_L 1 - \delta\,(1-\gamma_L)}$$

where $p(T)$ is the limit of equation 14 and is equal to $p(T) = c_L + s$. Plugging in $p(T)$ and simplifying leads to:

$$\overline{v}_H = \frac{s}{1-\delta} - \frac{(c_H - c_L)(1-\delta\,(1-\gamma_L))}{(1-\delta)(1-\delta\,(1-\gamma_H-\gamma_L))} \tag{21}$$

$$\tag{22}$$

A sufficient condition for the participation constraint is then $\overline{v}_H > 0$, or

$$s > \frac{(c_H - c_L)(1-\delta(1-\gamma_L))}{1-\delta(1-\gamma_H-\gamma_L)}$$

Finally, we examine the assumption that the highest charged price is indeed $p(0)$, when the cost jumps from c_L to c_H. This condition is met if $u_L > u_H$, or if

$$\frac{c_H - c_L}{1-\delta\,(1-\gamma_L-\gamma_H)} - \frac{s\,\delta}{1-\delta\,(1-\gamma_L)} > 0$$

21. Here we plugged in the condition for α, such that $\overline{v}_H = (1-\alpha_T)v_H(0)$

implying that

$$s < \frac{(c_H - c_L)(1-\delta\,(1-\gamma_L))}{\delta(1-\delta\,(1-\gamma_L-\gamma_H))}$$

To summarize, the equilibrium exists as long as

$$s \in \left[\frac{(c_H - c_L)(1-\delta(1-\gamma_L))}{1-\delta\,(1-\gamma_L-\gamma_H)}, \frac{(c_H - c_L)(1-\delta\,(1-\gamma_L))}{\delta(1-\delta\,(1-\gamma_L-\gamma_H))}\right]$$

■ **Bubbles and Rocks Equilibrium.** In this equilibrium beliefs are defined as follows:

$$\beta(\tau) = 1 - (1-\gamma_L)^\tau$$

where τ is the time since last cost decrease. When cost switches from c_H to c_L (and the price signals the change), buyers correctly believe that the cost is low, that is $\beta(0) = 0$. In subsequent periods and until the next price decrease buyers believe that costs have not increased and decreased again, that is, buyers know that the cost is low only if it has remained low since the last decrease. Note that in order for sellers to want to lower the price when the cost decreases, it must be that charging a high price is associated with search. Then it must be that there is search everywhere but the lowest level of price.

As before, the prices reflect an indifference condition of an assigned buyer. Let u_L and u_H be the values of search corresponding to low and high cost. Then, as before

$$\begin{aligned}\beta(\tau)\,u_H + \big(1-\beta(\tau)\big)\,u_L - s = \\ = -p(\tau) + \delta\left(\mathbb{E}_\tau\left[\beta(\tau+1)\right]\,u_H + \big(1-\mathbb{E}_\tau\left[\beta(\tau+1)\right]\big)\,u_L - s\right)\end{aligned}$$

The left-hand side is the expected price from searching in the current period; the right-hand side is the expected price from not searching in the current period and searching in the next period. As in the previous pooling equilibrium, we know that u_L is equal to the negative discounted future costs save the search costs for as long as the costs are expected to stay low. Whereas u_H is simply the negative of discounted costs going forward from c_H.

$$\begin{aligned}u_L &= -c_L - \sum_{\tau=t+1}^{\infty} \delta^{\tau-t} E_t[c(t)] - \frac{\delta\,s}{1-\delta\,(1-\gamma_L)} \\ u_H &= -c_H - \sum_{\tau=t+1}^{\infty} \delta^{\tau-t} E_t[c(t)]\end{aligned}$$

Solving for p,

$$p(\tau) = \delta\left(\mathbb{E}_\tau\left[\beta(\tau+1)\right]\,u_H + \big(1-\mathbb{E}_\tau\left[\beta(\tau+1)\right]\big)\,u_L - s\right) - \beta(\tau)\,u_H - \big(1-\beta(\tau)\big)\,u_L + s$$

Let $v_H(\tau)$ (resp. $v_L(\tau)$) be seller's value per unit mass of buyers when the cost state is high (resp. low) and τ periods have elapsed since the last cost shift. (Note that $v_H(0)$ is never realized, since $t = 0$ is defined as a period when $c = c_L$; however, this value will be important for computing incentive compatibility conditions.) In equilibrium, these value functions satisfy

$$v_H(\tau) = p(\tau) - c_H + \delta\Big((1-\gamma_H)\left(1-\alpha(\tau+1)\right)\, v_H(\tau+1) + \gamma_H\, v_L(0)\Big) \tag{23}$$

$$v_L(\tau) = p(\tau) - c_L + \delta\Big((1-\gamma_L)\left(1-\alpha(\tau+1)\right)\, v_L(\tau+1) + \gamma_L\,\left(1-\alpha(\tau+1)\right)\, v_H(\tau+1)\Big) \tag{24}$$

Specifically, suppose that τ periods have elapsed since the last cost decrease and suppose that cost is still at c_H. In the current period, the seller sets $p(\tau)$ and has cost c_H. Assuming the seller has a unit mass of customers, its current profit is given by $p(\tau)-c_H$. Beginning next period, two things may happen: cost remains high, yielding a continuation payoff $v_H(\tau+1)$ for the remaining $1-\alpha(\tau+1)$ customers; or cost drops to c_L, yielding a continuation payoff $v_L(0)$.

Now suppose that current cost is c_L. Current profit is $p(\tau)-c_L$. Beginning next period, two things may happen: cost remains low, yielding a continuation payoff $v_L(\tau+1)$ for the remaining $1-\alpha(\tau+1)$ customers; or cost increases to c_H for the remaining $1-\alpha(\tau+1)$ customers.

Next we consider the seller incentive compatibility (IC) constraints. First, a seller may drop the price when cost is high. This implies the following IC constraint:

$$\left(1-\alpha(\tau)\right)\, v_H(\tau) \geq v_L(0) \tag{25}$$

Second, in a period when the cost has not shifted to the high value yet, the seller might drop price to its lowest level again in order to not loose customers. This implies

$$v_L(\tau)\left(1-\alpha(\tau)\right) \geq v_L(0) \tag{26}$$

Finally, a seller may keep the price high when cost switches to c_L. This implies the following IC constraint:

$$v_L(0) \geq \left(1-\alpha(\tau)\right)\, v_L(\tau) \tag{27}$$

(26) implies (25). Moreover, (26) and (27) uniquely pin down the value of $\alpha(\tau)$:

$$\alpha(\tau) = 1 - \frac{v_L(\tau)}{v_L(0)} \tag{28}$$

As in the rockets and feathers, we cannot find a closed-form solution to the value functions. However, we can derive them numerically and recursively, since $p(\tau) \to \overline{p}$ and $v_i(\tau) \to \overline{v}$. These v_i limits can be solved from

$$\overline{v}_H = p(T) - c_H + \delta\Big((1-\gamma_H)\left(1-\alpha(T)\right)\, \overline{v}_H + \gamma_H\,\left(1-\alpha(T)\right)\, \overline{v}_L\Big)$$
$$\overline{v}_L = p(T) - c_L + \delta\Big((1-\gamma_L)\left(1-\alpha(T)\right)\, \overline{v}_L + \gamma_L\,\left(1-\alpha(T)\right)\, \overline{v}_H\Big)$$

The values of $p(\tau)$ and $v_i(\tau)$ can be approximated arbitrarily close by considering a sufficiently large terminal T, assuming $v_i(T+1) = \overline{v}_i$, and solving recursively.[22]

22. Here we assumed that $(1-\alpha_T)\,\overline{v}_L = v_L(0)$

Next, the participation constraint imposes that at all times it is profitable for sellers to be in the market. The worst outcome for sellers is when they are charging the lowest possible price while the cost switches to high. We know their value then is $v_H(1)$. We then need to numerically verify that this does not happen.

Finally, we examine the assumption that the lowest charged price is indeed $p(0)$, when the cost jumps from c_H to c_L. This condition is met if $u_L > u_H$, or

$$\frac{c_H - c_L}{1 - \delta(1 - \gamma_L - \gamma_H)} - \frac{s\delta}{1 - \delta(1 - \gamma_L)} > 0$$

which implies

$$s < \frac{(c_H - c_L)(1 - \delta(1 - \gamma_L))}{\delta(1 - \delta(1 - \gamma_L - \gamma_H))}$$

Finally, note that, in all four equilibria derived in this proof, search takes place either when the prices are at their highest level, or following a price increase. ■

Proof of Proposition 8: See the proof of Proposition 7 for the full derivation of the rockets and feathers equilibrium. ■

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