DISCUSSION PAPER SERIES

DP13843

"WHEN OLSON MEETS DAHL": FROM INEFFICIENT GROUPS FORMATION TO INEFFICIENT POLICY-MAKING

David Martimort

INDUSTRIAL ORGANIZATION AND PUBLIC ECONOMICS

ISSN 0265-8003

"WHEN OLSON MEETS DAHL": FROM INEFFICIENT GROUPS FORMATION TO INEFFICIENT POLICY-MAKING

David Martimort

Discussion Paper DP13843
Published 06 July 2019
Submitted 11 June 2019

Centre for Economic Policy Research
33 Great Sutton Street, London EC1V 0DX, UK
Tel: +44 (0)20 7183 8801
www.cepr.org

"WHEN OLSON MEETS DAHL": FROM INEFFICIENT GROUPS FORMATION TO INEFFICIENT POLICY-MAKING

Abstract

Two conflicting interest groups buy favors from a policy-maker. Influence is modeled as a common agency game with lobbyists proposing monetary contributions contingent on decisions. When the preferences of the group members are common knowledge, groups form efficiently and lobbying competition perfectly aggregates preferences. When those preferences are instead private information, free riding in collective action arises within groups. Free riding implies that the influence of a group is weakened and that lobbying competition imperfectly aggregates preferences. By softening lobbying competition, private information might also increase groups' payoffs and hurt the policy-maker. Importantly, the magnitudes of informational frictions within each group are jointly determined at equilibrium. We draw from these findings a number of implications for the organization of interest groups.

JEL Classification: N/A

Keywords: N/A

David Martimort - david.martimort@psemail.eu
Paris School of Economics and CEPR

Working paper

"*WHEN OLSON MEETS DAHL*": FROM INEFFICIENT GROUPS FORMATION TO INEFFICIENT POLICY-MAKING

June 11, 2019

Perrin Lefebvre and David Martimort

Abstract. Two conflicting interest groups buy favors from a policy-maker. Influence is modeled as a common agency game with lobbyists proposing monetary contributions contingent on decisions. When the preferences of the group members are common knowledge, groups form efficiently and lobbying competition perfectly aggregates preferences. When those preferences are instead private information, free riding in collective action arises within groups. Free riding implies that the influence of a group is weakened and that lobbying competition imperfectly aggregates preferences. By softening lobbying competition, private information might also increase groups' payoffs and hurt the policy-maker. Importantly, the magnitudes of informational frictions within each group are jointly determined at equilibrium. We draw from these findings a number of implications for the organization of interest groups.

Keywords. Lobbying, Collective Action, Free Riding, Asymmetric Information, Common Agency, Mechanism Design.

JEL codes. D72; D82.

[1]We thank Ingela Alger, Roland Benabou, Morten Bennedsen, Stefan Berhinger, Francis Bloch, Kim Sau Chung, Julien Combe, Sidartha Gorton, Denis Gromb, Louise Guillouet, Humberto Moreira, Wilfried Sand-Zantman, Aggey Semenov, Stéphane Straub, Nicolas Treich and especially Michel Le Breton for useful suggestions and comments on this project. The comments of the Editor Sean Gailmard and two referees were instrumental in improving this paper. We also thank participants to the *2014 LAMES* held in Sao Paulo, the *2016 ECODEC Conference* at CREST-Paris, the *2016 Revue Economique Conference* at MSE-Paris, the *2017 NGO Conference* at Paris School of Economics, the *2018 Mechanism and Institution Design Conference* in Durham, and the *2018 SIEP Conference* in Padova, for comments. The usual disclaimer applies.

[a]Paris School of Economics,lefebvre.perrin@gmail.com.

[b]Paris School of Economics-EHESS,david.martimort@parisschoolofeconomics.eu.

1. INTRODUCTION

MOTIVATION. The formation of interest groups, their competition in the political arena and, more generally, their influence on policy-making are key concerns for students of modern democracies. The role of lobbying as a vehicle for the representation of diverse interests, and its impact on the democratic process, although unanimously recognized, has nevertheless raised conflicting views among both political scientists and economists.

Following Dahl (1961)'s seminal work, the so called *pluralistic approach* to politics views competition as a healthy way to aggregate preferences across diverse interest groups. Taking an optimistic stance, this view of politics argues that policy-making reaches the right balance between the various interests who have a say in the decision. In the economics literature, this approach, which certainly found its modern roots in the earlier works of Peltzman (1976) and Becker (1983), is nowadays best exemplified by the so-called *common agency model* of lobbying competition proposed earlier by Bernheim and Whinston (1986) and then pursued by Grossman and Helpman (1994) and others over a broad range of settings.[1] Within this realm, interest groups influence a policy-maker through monetary payments whose levels depend on the policy-maker's decision. Importantly, there always exist equilibria in which groups offer *truthful schedules* that perfectly reflect their preferences over alternatives. When utility is transferable, the policy-maker ends up choosing an efficient policy that maximizes the sum of his own payoff and those of all active interest groups.[2] An immediate corollary is that free riding both within and across groups does not arise at truthful equilibria; a somewhat unpalatable conclusion.[3]

In sharp contrast, Olson (1965) has viewed free riding as hindering the representation of interests. Strongly organized groups buy favors while unorganized rivals are unable to exert any influence. The major thrust of *The Logic of Collective Action* is that intra-group free riding might prevent groups from promoting their interests. Free riding being more of a curse as the size of the group increases, large latent groups might be dominated by small and better organized groups; the so called *Olson Paradox*.

While the pluralistic approach starts from the presumption that groups formation is costless, the Olsonian view fails to recognize that inefficiencies in collective action are to a large extent endogenous. Indeed, the stakes for forming as an active group depend on whether other groups might also influence decision-making or not. This paper makes progress in reconciling those two views by offering an integrated framework. To address the free riding problem in collective action, we introduce asymmetric information on the preferences of group members. Individuals shade their willingnesses to pay for a change in policy to reduce their own contribution while still benefitting from their own group's action. In our framework, the stakes for groups formation are endogenously determined as a result of the competition between lobbying groups in influencing the policy-maker. The endogenous benefits of groups formation determine whether there are enough gains from collectively acting to cover the informational cost that is needed to induce information

[1] Noticeable contributions include Aidt (1996), Dixit, Grossman and Helpman (1997), Rama and Tabellini (1998), Besley and Coate (2001) and Yu (2005).

[2] Throughout the paper, an efficient allocation is defined as being on the Pareto frontier of the set of payoffs for the policy-maker and the interest groups. This definition thus implicitly assumes that this policy-maker represents other non-organized interests in the polity.

[3] Another albeit less immediate corollary is that games of voluntary contributions (Bergstrom et al., 1986) entail inefficient free riding by arbitrarily restricting strategies to be non-contingent fixed payments.

revelation within a group. In this context, we ask whether the political process still efficiently aggregates preferences not only *within* but also *across* groups.

In this endeavor, we rely on two important bodies of theoretical work; namely *mechanism design* and *common agency*. When taken separately, those models have been extensively used to understand respectively groups formation and lobbying competition. Yet, those two paradigms have evolved independently and have not offered the more comprehensive framework needed in political economy environments. Before being active, groups must solve their own collective action problem. Asymmetric information on individual preferences comes with transaction costs. Coasean bargaining among group members might not be efficient. Analyzing such transaction costs calls for importing tools from the mechanism design literature. Within each group, incentive mechanisms are designed to ensure that members reveal their preferences. At the last stage of the game, interest groups compete for the policy-maker's influence; a standard common agency game.

Model and Main results. Two interest groups with conflicting preferences over a policy decision buy favors from a policy-maker. Within each group, individuals have private information on their preferences. To be active, a group appoints a lobbyist so as to exert pressure on the policy-maker. The group must also determine how its members share their overall lobbying effort.

Aggregating preferences across conflicting groups. We demonstrate that groups with conflicting interests are ready to compensate the policy-maker for the exact impact that their influence has on the rest of society. Contributions, endogenously determined at the equilibrium of the common agency game, are thus *Vickrey-Clarke-Groves* (thereafter VCG) payments.[4] One important property of such payments is that interest groups have no incentives to manipulate their preferences for strategic purposes. The sole source of distortions, if any, may thus come from solving the intra-group free riding problem.

Aggregating preferences within groups. Had preferences been common knowledge, the free riding problem could be solved by having each individual contribute up to his willingness to pay for the change in policy that the group's action induces. Preferences would be perfectly aggregated within each group. This efficient solution to the free riding problem together with the fact that the common agency stage of the game has a truthful equilibrium implies that policy-making is necessarily efficient. The basic take-away of this complete information scenario is that, if Olson is wrong and free riding in collective action does not matter within groups, then Dahl is also right and lobbying competition perfectly aggregates preferences across groups.

Asymmetric information radically changes the picture. Group members may now free ride by shading their willingness to pay for a change in policy. As a result, information can only be revealed if all those individuals garner some *information rent*. A group now forms when the gains from influencing the policy-maker cover the overall information rent left to its members: A key *incentive-feasibility condition*. This condition has far reaching consequences. It links the size of contributions needed to influence the policy-maker with the difficulty to learn information from group members. To illustrate, inducing a significant policy shift calls for a greater contribution by the group but raising individual contributions to do so might also exacerbate internal incentives to free ride.

[4] Green and Laffont (1977).

Inefficient groups formation. To limit information rents, two sorts of distortions are required. First, free riding is less of a concern when the group's overall contribution is reduced. The policy-maker is now more inclined towards the *status quo* policy he chooses under the sole influence of competing interests. Asymmetric information within groups thus softens lobbying competition. It also hurts the policy-maker who can no longer extract as much by playing one group against the other. Asymmetric information offers a new justification for the well documented low levels of monetary contributions in politics.[5]

Second, a group now expresses moderate preferences for modifying decision-making. Following insights from the mechanism design literature (Myerson, 1981) willingnesses to pay for a policy change are indeed replaced by *virtual willingnesses to pay* which are of a lower magnitude. Aggregating *virtual willingnesses* within a group is akin to adopting moderate preferences. When free riding is too much of a concern, a group might even fail to get organized at all. Under those circumstances, *Olson's Paradox* finds strong informational foundations: An interest group is no longer active if its own internal informational problems are too costly to solve.

The role of the distribution of types. When the support of the distribution of preference parameters contains a type who has no stake in policy-making, inefficiencies are pervasive and (with an additional technical condition) might not even depend on the competing group's strategy. This result is best exemplified for large groups. Incentives to free ride then culminate. Individuals would like to pretend having no stake so as to contribute nothing while still enjoying the benefits of any collective action. Overall, zero contribution can be collected and large groups lose any influence. *A contrario*, even large groups might be active when the lowest possible type has a stake. This is not smallness *per se* that facilitates group formation but the existence of a minimal stake for each individual.

Joint determination of informational frictions. Both the *Chicago School* (Stigler, 1971; Posner, 1974) and the more recent *New Regulatory Economics* (Laffont and Tirole, 1991) have taken as granted inefficiencies in collective action and analyzed their impact on policy-making. The political science literature has taken a less extreme view and argued that those frictions depend on the political environment (Gray and Lowery, 1996). To illustrate, when studying the formation of coalitions among already active lobbyists, Holyoke (2009) shows that lobbyists with slightly divergent interests will be more likely to get organized into a coalition when they face a tougher competition. Our results echo those findings. Inefficiencies in group formation are completely captured by the shadow costs of the incentive-feasibility conditions that pertain to those groups. Since the net benefits of collective action for a given group depend on the *status quo* policy that would have been chosen under the sole influence of its rival, these shadow costs are (in a sense to be properly defined) best responses to each other. Informational frictions in collective action are thus *jointly* determined at equilibrium. In sharp contrast with the complete information scenario, inefficiencies within groups now percolate as inefficiencies in the lobbying game. Under asymmetric information, Olson's view is incomplete and free riding also depends on how competing groups solve their own organizational problem. Yet, Dahl is also wrong and lobbying competition cannot efficiently aggregate preferences.

Towards an I.O. perspective on the formation of interest groups. That informational frictions in the formation of groups are jointly determined at equilibrium suggests that groups

[5] Ansolabehere et al. (2003).

might also want to use various commitment devices to (indirectly) increase frictions for their competitors and foster their own influence; an idea which is clearly reminiscent of a huge I.O. literature on business strategies (Fudenberg and Tirole, 1985; Bulow, Geanakoplos and Klemperer, 1985). In this respect, we analyze under which circumstances *strong coalitions*, those which are able to credibly share information, may worsen the free riding problems faced by competing groups, and so doing limit entry in the political arena. More generally, we also unveil how entry costs have not only an impact at the extensive margin, banning entry by some rival groups, but also at the intensive margin. Competing groups, even if they enter, now adopt more moderate stances since the need to cover entry costs raises individual contributions and thus exacerbates their own free riding problem.

Literature Review. This paper blends together two bodies of literature. The first one is the by now familiar common agency model of lobbying competition. The second body, which is used to describe intra-group agreements, relies instead on a mechanism design approach. We now discuss in more details how the paper borrows from those existing models but also how it significantly departs from existing works.

Common agency. Following Bernheim and Whinston (1986) and Grossman and Helpman (1994), the bulk of the common agency literature has taken as given the set of interest groups that are supposed to be active. Given that groups offer truthful schedules that perfectly reflect their preferences over alternatives, free riding does not arise either across or within groups. Henceforth, the theory in its most basic form cannot truly distinguish whether contributions are offered by individuals or by groups. Both scenarios would lead to the same policy although the distributions of payoffs might differ. There is no reason to form an interest group and influence by individuals alone suffices. On top, payoff distributions play no role in the analysis, a particularly unpalatable conclusion since the main purpose of political economy models should be to understand how these distributions affect policy-making. Our approach avoids this hurdle. In our analysis, the distribution of payoffs that comes out of the lobbying competition stage also determines informational frictions within groups and thus how groups express their influence on policy.

To reconcile common agency models with Olson's perspective and depict more realistic environments, various ingredients have been appended. Following Mitra (1999, 2002), a first line or research, mostly developed in specific contexts (in trade policy or for the organization of juridictions) has viewed entry into the lobbying game as a strategic decision (Krishna and Mitra, 2005; Brou and Ruta, 2006; Laussel, 2006; Leaver and Makris, 2006; Martimort and Semenov, 2007; Bombardini, 2008; Redoano, 2010). Free riding might occur when a player opts out while others contribute. Those models reach the unpalatable conclusion that, whether other groups enter or not the political arena, the marginal influence of an active group only reflects its own preferences over alternatives. Our model provides a richer set of results. The marginal influence of a group depends both on the magnitude of the entry cost and the identity of other active groups.

Even when entry is costless and information is complete, Dixit and Olson (2000) and Furusawa and Konishi (2011), have shown how free riding on the decision to belong to a group might occur. The *Coase Theorem* might not always apply when participation is strategic. Although we discuss how free riding in participation would modify our findings, the main reason for inefficiency hereafter is that individuals are privately informed on their willingnesses to pay and free ride by shading those willingnesses. This bounds any feasible agreement ruling a coalition to respect incentive compatibility. Finally, Felli and Merlo (2006) have supposed that the decision-maker can commit to interact with some groups; possibly leading to inefficient group representation.

A second line of research has instead studied repeated versions of the common agency game. Equilibrium contributions must thus be self-enforcing as in the single lobbying group models of Pecorino (2001), Damania and Fredriksson (2000, 2003), Damania, Fredriksson and Osang (2004) and Magee (2002). Free riding comes from the fact that payments no longer represent marginal contributions and some individuals may prefer not to pay their own (equal) share of the overall contribution of the group. A possible motivation for an equal-sharing rule is the impossibility to target individuals according to their respective willingness to pay. This argument calls for putting information at the forefront of the analysis as we do thereafter.

A recent trend of the common agency literature has viewed asymmetric information on the policy-maker's preferences as the vehicle for contracting frictions. That uninformed interest groups have to give up informational rent to influence a privately informed policy-maker might render lobbying competition inefficient (Le Breton and Salanié, 2003; Martimort and Semenov, 2008; Martimort and Stole, 2015). Each interest group is willing to extract information, thereby creating a contractual externality across groups. When a group's preferences significantly depart from those of the decision-maker, agency costs might be so large that this group is inactive. In contrast, we consider that asymmetric information is no longer on the supply side of the market for influence but rather on its demand side, i.e., on the preferences of individuals. Those earlier papers are also silent on the problem of group formation that is at the core of our analysis. They do not distinguish between contributions made by individuals and contributions made by groups. Although free riding across groups arises, free riding within groups is not a concern.

Moving away from the common agency paradigm, Esteban and Ray (2001) model political contests in the tradition of Tullock (1980). Contrary to us, groups are homogenous and the cost of lobbying is exogenous. Properties of this cost then determine whether the *Olson's Paradox* holds or not. Building informational foundations for the cost of lobbying as we do thereafter shows the proper scope of this paradox.

Mechanism design. Private information on willingnesses to pay for a change in policy is a key ingredient of our explicit modeling of the free riding problem. So doing, our analysis echoes the earlier mechanism design literature for public good provision (Laffont and Maskin, 1982; Mailath and Postlewaite, 1990; Ledyard and Palfrey, 1999; Hellwig, 2003). With private information, a conflict between incentives, budget balance and participation might preclude efficiency. This conflict is best expressed through an incentive-feasibility condition that aggregates those constraints. A major departure from the pubic good literature is that, in our analysis, stakes become endogenous: The costs and benefits of group formation are now derived from equilibrium behavior in the ensuing lobbying game. This endogenity suggests that even intuitive results on the possible existence of free riding in large groups might have to be qualified. Beyond, second-best distortions are captured by the shadow cost of the incentive-feasibility constraint that pertains to each group. In our context, these shadow costs are all simultaneously determined at equilibrium; adding a significant degree of complexity with respect to the traditional public good literature that takes a mechanism in isolation from any other outside consideration.

Incentives for intra-group free riding have also been analyzed in moral hazard settings where individual contributions to lobbying are non-verifiable. Lohmann (1998) considers a model where individuals voluntarily contribute to monitor a policy-maker. Free riding leads to political decisions that favor smallest groups. Anesi (2009) argues that, when the decision to join a lobbying group is endogenous, moral hazard favors participation in lobbying activities. Our model reaches somewhat different conclusions. Indeed, if participation to an interest group is costly for an individual, the incentive-feasibility condition that pertains to that group is hardened. Free riding is exacerbated. Siqueira (2001) shows how, by banding together as a group, individuals (principals) internalize a multi-principals contractual externality similar to that in Le Breton and Salanié (2003), Martimort and Semenov (2008) and Martimort and Stole (2015). Contrary to Olson's view, large groups may be better organized.

Organization of the paper. Section 2 describes the model. Section 3 analyzes group formation under complete information. Section 4 presents a simple incentive-feasibility condition that summarizes all difficulties that a group may face to overcome free riding. Section 5 provides conditions ensuring that groups form efficiently even under asymmetric information. In contrast, Section 6 investigates conditions for free riding in large groups. Section 7 tackles the more complex scenario of groups with finite sizes. Section 8 shows how inefficiencies and informational frictions are jointly determined across groups. Section 9 demonstrates how various commitment devices may strategically affect frictions faced by other groups and provide a competitive edge in the lobbying game. Section 10 assesses the welfare consequences of asymmetric information. Section 11 discusses the robustness of our findings to alternative scenarios. Proofs are relegated to an online Appendix.

2. The Model

2.1. *Interest Groups*

Preferences. Individuals are divided into two groups.[6] Group l (for $l \in \{1, 2\}$) has size $N_l \geq 1$. Agent i (for $i \in \{1, ...N_l\}$) in group l has quasi-linear preferences over a policy x chosen by a policy-maker (thereafter PM) from the interval $\mathcal{X} = [-x_{max}, x_{max}]$ (x_{max} being large enough) and the monetary contribution t_l^i that he pays to influence PM:

$$\frac{\alpha_l^i}{N_l} u_l(x) - t_l^i.$$

The parameter α_l^i captures the intensity of agent i's preferences while $u_l(x)$ stems for a payoff function which is specific to group l. Members of a given group rank all policies similarly although the intensity of preferences vary across individuals. Individual preferences are scaled up by the size of the group N_l to normalize the group's overall influence.[7] For tractability, each function u_l is supposed to be linear in x although we often keep a more general expression to show how our results would apply more broadly.To model conflicting interests, we posit:

$$u_1(x) = -u_2(x) = -x \quad \forall x \in \mathcal{X}.^{8}$$

[6]Our analysis has a broad scope and individuals may themselves be interpreted as interest groups or organizations. In U.S. legislative politics, coalitions of interest groups abound in various fields going from education (the *Committee for Education Funding* which involves more than one hundred organizations) to transportation (the *American Bus Association* or the *Air Transport Association*).

[7]Size has thus no role *per se*. In the spirit of McLean and Postlewaite (2002), this is the relative measure of his information that matters to determine the impact of an agent on his group.

To illustrate, the decision x may be the price of a regulated good or services in the spirit of Peltzman (1976); group 2 being then composed of producers while group 1 represents consumers. This decision could also be the level of an import tariff for some intermediate good as in Gawande, Krishna and Olarreaga (2012). Group 2 might then stand for domestic producers. This group asks for protection from foreign competitors and thus lobby for an import tariff. Group 1 is made of final domestic users who instead advocate for low protection to reduce their expenditures.

PM represents other (unorganized) groups in society or a median voter who might have more neutral stances. PM's quasi-linear utility function is also defined over the decision x and the overall monetary payments received z as:

$$u_0(x) + z.$$

The function u_0 is twice continuously differentiable, strictly concave, and symmetric around 0. Let denote $\varphi = u_0'^{-1}$ with $\varphi(0) = 0$ and $\varphi' < 0$ (which follows from $u_0'' < 0$). Some of our results below depend on the curvature of φ.

Notice that all parties are risk-neutral and have the same marginal utility of income. This simplifying assumption allows us to depart from insurance and redistributive considerations throughout the whole analysis.

Running example. We will sometimes rely on a quadratic specification of PM's preferences; a familiar workhorse of the Political Science literature:

$$u_0(x) = -\frac{\beta_0}{2}x^2 \Rightarrow \varphi(y) = -\frac{y}{\beta_0}. \tag{2.1}$$

The parameter β_0 might here represent the relative weight that PM gives to social welfare in his own objective relative to contributions as in Grossman and Helpman (1994).[9] ■

Information. Individual i in group l has private information on his own preference parameter α_l^i. For each group, these values are drawn from a common knowledge group-specific (atomless) distribution function F_l whose support is $\Omega_l = [\underline{\alpha}_l, \overline{\alpha}_l]$. Let f_l be the corresponding (positive) density function. In the sequel, the non-negative lower bound $\underline{\alpha}_l$ sometimes plays an important role. A specific case is obtained when individuals with no specific preferences for the policy might belong to the group, i.e., $\underline{\alpha}_l = 0$.[10] The mean value of the preference parameter for group l is $\alpha_l^e = \mathbb{E}_{\alpha_l^i}(\alpha_l^i)$.[11] We denote by $\boldsymbol{\alpha}_l = (\alpha_l^i)_{i\in\{1,\dots N_l\}}$ any arbitrary vector of preference parameters for group l and by $\alpha_l^*(\boldsymbol{\alpha}_l) = \frac{1}{N_l}\sum_{i=1}^{N_l} \alpha_l^i$ its

[8] Asymmetry in the strength of the groups could also be easily introduced. Suppose for instance that the two groups would like to push policies in opposite directions although with different intensities. Formally, say $u_1(x) = -kx$ while $u_2(x) = x$ with $k \neq 1$. Up to a change of variables and a modification of the support of the types distribution, our formalism would again apply.

[9] This model has received some empirical support (Goldberg and Maggi 1999; Gawande and Bandyopadhyay).

[10] The fact that some individuals might have no specific preferences for the policy under scrutiny is in particular justified when the group is a long-term venture banding agents on several related issues. A given individual may have found a positive value in belonging to that group in the past and still belong to that group nowadays even though he has no strict preferences for the current decision at stake.

[11] The expectation operator with respect to x is denoted as $\mathbb{E}_x(\cdot)$.

sample mean. Let Φ_{N_l} (resp. ϕ_{N_l}) be the cumulative distribution (resp. density) of this sample mean. Finally, we use the standard notation $\boldsymbol{\alpha}_l = (\alpha_l^i, \boldsymbol{\alpha}_l^{-i})$ when needed.

Adopting the parlance of the mechanism design literature (Myerson, 1981) the *virtual preference parameter* of agent i in group l is defined as:

$$h_l(\alpha_l^i) = \alpha_l^i - \frac{1 - F_l(\alpha_l^i)}{f_l(\alpha_l^i)}, \quad \forall \alpha_l^i \in \Omega_l.$$

The *monotone hazard rate property* holds, i.e., $\frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)}$ is non-increasing (Bagnoli and Bergstrom, 2005). Hence, $h_l(\alpha_l^i)$ is a non-decreasing transform of α_l^i. Following previous convention, the sample mean of virtual preference parameters is $h_l^*(\boldsymbol{\alpha}_l) = \frac{1}{N_l}\sum_{i=1}^{N_l} h_l(\alpha_l^i)$.

2.2. *Lobbying Process*

The lobbying game determines contributions and policies as endogenous objects that in turn impact on the costs and benefits of collective action. To model such feed-back, we rely on a simple two-stage model that can be viewed as a metaphor for how groups form and act in the political arena. The formation stage determines how the group's interests are aggregated and represented in the lobbying game. This first step is summarized by the appointment of a lobbyist whose objective represents that of the group, possibly modified by informational considerations. The decision of an interest group there consists in giving to this delegate some specific preferences that reflect the decision-process inside the group. The second stage determines how lobbyists compete in the lobbying arena. This step is modeled as a standard common agency game. The contributions of the groups that are needed to channel their influence on PM are determined at this stage.

Appointing Lobbyists. Following the literature on delegation in legislative bargaining,[12] the group delegates to a lobbyist the task of influencing PM. Formally, the lobbyist is endowed with the following objective:

$$\beta_l u_l(x) - T_l.$$

T_l stands for the group's overall political contribution to PM.[13] The weight β_l parametrizes the efficiency of the process of group formation. When preferences are common knowledge, we will show that each group finds a dominant strategy to choose the mean preference parameter, i.e., $\beta_l \equiv \alpha_l^*(\boldsymbol{\alpha}_l)$. When preferences are private information, the wedge between β_l and $\alpha_l^*(\boldsymbol{\alpha}_l)$ captures how informational frictions undermine collective action.

The Common Agency Game. Lobbyists compete to buy PM's favors in a standard *common agency* game (Bernheim and Whinston, 1986; Grossman and Helpman, 1994). Although, it views the political process as a black box by neglecting electoral competition and fine details of the legislative process, this framework is by now admitted by most economists as being an adequate representation of how competing groups impact

[12]Christiansen (2013), Besley and Coate (2003), Dur and Roelfsema (2005).

[13]In practice, lobbyists might have their own preferences on issues they want to influence. Our approach short-cuts any agency problem between the group and its lobbyist; the group can implement whatever lobbyist's preferences it would like.

policy-making.[14] We thus adopt the corresponding informational assumptions, timing, and equilibrium refinements.

First, the lobbyists' objectives are revealed so that lobbyists know their rival's objective.[15] Second, lobbyists offer to PM the non-negative contribution schedules $\tilde{T}_l(x)$. Such contributions are commitments to pay PM in response for the policy choice x. PM is free to accept or refuse each of those contracts.[16] Third, and because each lobbyist always has such schedule in his best-response correspondence, we restrict attention to *truthful* equilibria obtained when lobbyists use *truthful contribution schedules* of the form:[17]

$$\tilde{T}_l(x) = \max\{\beta_l u_l(x) - V_l, 0\} \quad \forall x \in \mathcal{X}. \tag{2.2}$$

Here, V_l stems for the payoff that lobbyist l can always secure for any choice x. Truthful schedules perfectly reflect the lobbyist's preferences over alternatives. Hence, the equilibrium policy maximizes the payoff of the grand-coalition made of those lobbyists together with PM. Our goal is to investigate under which circumstances the preferences of lobbyists might no longer reflect those of interest groups thanks to informational frictions.

For a vector of lobbyists preferences $\boldsymbol{\beta} = (\beta_1, \beta_2)$, let $\Delta\beta = \beta_1 - \beta_2$ be a measure of the degree of political polarization. The policy $x(\beta_1, \beta_2)$ that maximizes the payoff of the grand-coalition made of PM and the two lobbyists reflects this polarization:

$$u_0'(x(\beta_1, \beta_2)) = \Delta\beta \Leftrightarrow x(\beta_1, \beta_2) = \varphi(\Delta\beta). \tag{2.3}$$

The optimal policy is tilted towards the group whose lobbyist has the strongest preferences. Of course, the definition of $x(\beta_1, \beta_2)$ given above also applies to characterize the optimal policy taken when either one group or none is active provided that we use the convention $\beta_l = 0$ for an inactive group. For instance, $x(0, \beta_2) = \varphi(-\beta_2)$ is the decision implemented if group 1 is not organized.

2.3. *Group Formation: Mechanism Design*

We envision the formation of an interest group as a decentralized bargaining process. Through this process, individuals find a way to share their overall contribution and define an objective function for that group. Specifying *a priori* an extensive form and a particular communication protocol would be *ad hoc*. Comparing the performances of such protocols would also become a daunting task. We avoid those two obstacles by following the more normative mechanism design methodology initiated by Myerson and Sattherwaite (1982) and pursued by Mailath and Postlewaite (1990) and Laffont and Martimort (1997) in related contexts. From the Revelation Principle (Myerson, 1982), any bargaining procedure would indeed lead to an allocation rule that could as well have been proposed by an uninformed mediator that acts on behalf of that group. This mediator has no physical

[14]Possible extensions of the common agency paradigm which offer a more detailed view of the political process are found in Baron (1999, 2006) and Baron and Hirsch (2011).

[15]This assumption might *a priori* look quite strong but is in fact less demanding that it appears as it will be discussed below.

[16]The restriction to non-negative contributions is without any loss of generality. Indeed, PM being free to refuse any contract would never choose a policy corresponding to a negative payment.

[17]Insisting on *truthfulness* is akin to imposing an equilibrium refinement (Bernheim and Whinston, 1986). Inefficient equilibria may nevertheless arise without the *truthfullness* refinement (Kirshteiger and Prat, 2001). Bernheim and Whinston (1986) show that *truthful* equilibria are also coalition-proof.

existence *per se* but instead stands as a metaphor for how the bargaining process unfolds. With that perspective in mind, a mechanism for the formation for group l determines, for all possible vectors of reports $\hat{\boldsymbol{\alpha}}_l$ of its members, an objective function for the lobbyist and the individual contributions of its members, $\left(\beta_l(\hat{\boldsymbol{\alpha}}_l), (t_l^i(\hat{\boldsymbol{\alpha}}_l))_{i=1}^{N_l}\right)$.

No-Veto Constraints. An important issue is to specify what happens if an individual opposes to the mechanism that rules his group. Following the mechanism design literature,[18] we assume individual veto power. If an agent chooses not to participate to the mechanism in group l, no lobbyist is appointed and PM chooses $x(0, \beta_{-l}(\boldsymbol{\alpha}_{-l}))$ so as to reflect only group $-l$'s influence. There are two justifications for this assumption. First, coalitions may themselves be made of interest groups that band together on some specific issues; a scenario that certainly echoes practices in nowadays U.S. Legislative Politics.[19] It is then legitimate to give each group equal veto power. Second, giving individual veto power showcases an upper bound on possible informational frictions. Section 11.1 below shows that restricting veto power to key members is enough to induce free riding and inefficiency. When no player has veto power, it is well known from d'Aspremont and Gerard-Varet (1978) that the group can design efficient Bayesian incentive compatible mechanisms and behaves *as if* information was complete within the group; an unpalatable conclusion. *A contrario*, Section 11.2 also demonstrates how free riding on participation can arise if a non-ratifying agent still enjoys the influence of those who act.

To express no-veto constraints in our baseline scenario, we define the gain of acceptance for a type α_l^i individual when the preferences profile in group l is $\boldsymbol{\alpha}_l = (\alpha_l^i, \boldsymbol{\alpha}_l^{-i})$ as:

$$\frac{\alpha_l^i}{N_l}\mathbb{E}_{\boldsymbol{\alpha}_{-l}}(\Delta u_l(\beta_l(\alpha_l^i, \boldsymbol{\alpha}_l^{-i}), \beta_{-l}(\boldsymbol{\alpha}_{-l}))) - t_l^i(\alpha_l^i, \boldsymbol{\alpha}_l^{-i}) \text{ with } \Delta u_l(\beta_l, \beta_{-l}) \equiv u_l(x(\beta_l, \beta_{-l})) - u_l(x(0, \beta_{-l})).$$

Type α_l^i's expected net payoff from joining group l can thus be defined as:

$$\mathcal{U}_l^i(\alpha_l^i) = \mathbb{E}_{\boldsymbol{\alpha}_l^{-i}}\left(\frac{\alpha_l^i}{N_l}\mathbb{E}_{\boldsymbol{\alpha}_{-l}}(\Delta u_l(\beta_l(\alpha_l^i, \boldsymbol{\alpha}_l^{-i}), \beta_{-l}(\boldsymbol{\alpha}_{-l}))) - t_l^i(\alpha_l^i, \boldsymbol{\alpha}_l^{-i})\right). \tag{2.4}$$

The no-veto constraint ensures that net gains are non-negative for any agent in group l:

$$\mathcal{U}_l^i(\alpha_l^i) \geq 0, \quad \forall \alpha_l^i \in \Omega_l, \forall i. \tag{2.5}$$

Bayesian Incentive Compatibility. From the Revelation Principle (Myerson, 1982), there is no loss of generality in considering direct mechanisms such that each individual truthfully reports his type α_l^i at a Bayesian equilibrium.[20] The following incentive compatibility constraints must thus hold:

(2.6)

[18] Laffont and Maskin (1982), Mailath and Postlewaite (1990).

[19] Hula (1999).

[20] In contrast with standard mechanism design problems, our context is one of competing mechanisms with two groups, each of them relying on its own mechanism. We must thus be somewhat careful in using the Revelation Principle. Indeed, each mechanism of group formation is now a best response to the mechanism designed by the competing group; a feature that has been studied in the general model of competing hierarchies by Myerson (1982) and in more specific contexts with secret contracts by Martimort (1996). For a given mechanism $\mathcal{G}_{-l}$ that determines a deterministic allocation β_{-l} for group l, there is no loss of generality in using the Revelation Principle to characterize group l's best response in the pure-strategy equilibria of this game that will be our focus below.

$$\mathcal{U}_l^i(\alpha_l^i) = \arg\max_{\hat{\alpha}_l^i \in \Omega_l} \mathbb{E}_{\boldsymbol{\alpha}_l^{-i}} \left(\frac{\alpha_l^i}{N_l} \mathbb{E}_{\boldsymbol{\alpha}_{-l}} \left(\Delta u_l(\beta_l(\hat{\alpha}_l^i, \boldsymbol{\alpha}_l^{-i}), \beta_{-l}(\boldsymbol{\alpha}_{-l})) \right) - t_l^i(\hat{\alpha}_l^i, \boldsymbol{\alpha}_l^{-i}) \right), \quad \forall \alpha_l^i \in \Omega_l, \forall i.$$

Budget Balance. Individual contributions must cover the payment needed to influence PM.[21] Taking into account that $\boldsymbol{\alpha}_{-l}$ is a random variable at the time group l organizes, its budget constraint can be written at this stage as:

$$\sum_{i=1}^{N_l} t_l^i(\alpha_l^i, \boldsymbol{\alpha}_l^{-i}) - \mathbb{E}_{\boldsymbol{\alpha}_{-l}}(T_l(\beta_l(\boldsymbol{\alpha}_l), \beta_{-l}(\boldsymbol{\alpha}_{-l}))) \geq 0 \quad \forall (\alpha_l^i, \boldsymbol{\alpha}_l^{-i}) \in \Omega_l^{N_l}. \tag{2.7}$$[22]

A mechanism $\mathcal{G}_l$ is *incentive-feasible* if and only if it satisfies no veto (2.5), Bayesian incentive compatibility (2.6), and budget balance (2.7).

2.4. *Timing and Equilibrium Concept*

The game unfolds as follows. First, agents privately learn their preferences. Second, groups simultaneously (and secretly) propose mechanisms to their members. Third, within each group, each individual may accept or veto the proposed mechanism. If the mechanism is ratified, each agent reports his own preference parameter $\hat{\alpha}_l^i$. The lobbyist's induced preferences correspond to a weight $\beta_l(\hat{\boldsymbol{\alpha}}_l)$. Fourth, the common agency stage of the game unfolds with group l's payment to PM being $T_l(\beta_l(\hat{\boldsymbol{\alpha}}_l), \beta_{-l}(\hat{\boldsymbol{\alpha}}_{-l}))$.

The equilibrium concept is perfect Bayesian equilibrium with the addition of two refinements. First, and because mechanisms are secretly designed for each group, we impose *passive beliefs* so that members of group l still believe that group $-l$ is ruled with the equilibrium mechanism if group l were to offer an unexpected mechanism. This *"don't signal what you don't know"* refinement (Fudenberg and Tirole, 1991) is standard in the competing mechanisms literature (Martimort, 1996). Second, equilibria of the common agency stage of the game are truthful; another standard refinement.

3. Group Formation under Complete Information

Consider first the case where groups form under complete information. Our setting differs from the standard common agency model (Bernheim and Whinston, 1986; Grossman and Helpman, 1994) by the addition of an extra delegation stage where lobbysts are

[21] The payment $T_l(\beta_l, \beta_{-l})$ is a random variable. It depends on both the realization of the whole vector of valuations $\boldsymbol{\alpha}_l$ for members of group l through the impact on β_l but also on the vector of preference parameters $\boldsymbol{\alpha}_{-l}$ of members of the competing group through its impact on β_{-l}. All incentive, budget balance and no-veto constraints that apply to the mechanism for group l take into account the fact that $\boldsymbol{\alpha}_{-l}$ is viewed as being random from group l's viewpoint.

[22] The budget-balance requirement could be thought as being more demanding if those constraints had to hold for all possible realizations of group $-l$'s preferences $\boldsymbol{\alpha}_{-l}$. However, such complication would not change our results. To see why, take a mechanism with payments $t_l^i(\alpha_l^i, \boldsymbol{\alpha}_l^{-i})$ that satisfy (2.7). Construct a new set of payments $\tilde{t}_l^i(\alpha_l^i, \boldsymbol{\alpha}_l^{-i}, \boldsymbol{\alpha}_{-l})$ such that $\tilde{t}_l^i(\alpha_l^i, \boldsymbol{\alpha}_l^{-i}, \boldsymbol{\alpha}_{-l}) = t_l^i(\alpha_l^i, \boldsymbol{\alpha}_l^{-i}) - \mathbb{E}_{\boldsymbol{\alpha}_{-l}}(T_l(\beta_l(\boldsymbol{\alpha}_l), \beta_{-l}(\boldsymbol{\alpha}_{-l}))) + T_l(\beta_l(\boldsymbol{\alpha}_l), \beta_{-l}(\boldsymbol{\alpha}_{-l}))$. Those payments ensure that the budget balance constraints would hold for all possible realizations of $\boldsymbol{\alpha}_{-l}$ since

$$\sum_{i=1}^{N_l} \tilde{t}_l^i(\alpha_l^i, \boldsymbol{\alpha}_l^{-i}, \boldsymbol{\alpha}_{-l}) - T_l(\beta_l(\boldsymbol{\alpha}_l), \beta_{-l}(\boldsymbol{\alpha}_{-l}))) = \sum_{i=1}^{N_l} t_l^i(\alpha_l^i, \boldsymbol{\alpha}_l^{-i}) - \mathbb{E}_{\boldsymbol{\alpha}_{-l}}(T_l(\beta_l(\boldsymbol{\alpha}_l), \beta_{-l}(\boldsymbol{\alpha}_{-l}))) \geq 0 \quad \forall (\alpha_l^i, \boldsymbol{\alpha}_l^{-i}, \boldsymbol{\alpha}_{-l}).$$

Moreover, those payments are such that $\mathbb{E}_{\boldsymbol{\alpha}_{-l}}(\tilde{t}_l^i(\alpha_l^i, \boldsymbol{\alpha}_l^{-i}, \boldsymbol{\alpha}_{-l})) = t_l^i(\alpha_l^i, \boldsymbol{\alpha}_l^{-i})$, leaving thus Bayesian incentive compatibility and no-veto constraints unchanged.

chosen by their groups. Proposition 1 shows that this difference is immaterial, allowing us to focus on asymmetric information as the sole obstacle to efficiency.

We first focus on the common agency stage of the game. There is one such subgame for each possible realization of (β_l, β_{-l}). Following Laussel and Le Breton (2001), we know that, in the case of conflicting interest groups under scrutiny,[23] there is a unique truthful equilibrium (that may thus be indexed by the realization of the random variable (β_l, β_{-l})) in which PM gets a positive payoff as a result of the lobbyists' competition for his favors. The equilibrium payment from lobbyist l, say $T_l(\beta_l, \beta_{-l}) = \tilde{T}_l(x(\beta_l, \beta_{-l}))$, is defined as:

$$T_l(\beta_l, \beta_{-l}) = [\beta_{-l} u_{-l}(x) + u_0(x)]_{x(\beta_l,\beta_{-l})}^{x(0,\beta_{-l})}. \tag{3.1}$$
[24]

This expression is remarkable. Contributions that are endogenously determined at the truthful equilibrium of the last stage of the game are in fact VCG payments. Each group pays for the externality that inducing a change in the policy exerts both on the other group and on PM. This fact has two important implications. First, the logic of VCG mechanisms bites: There is no point in strategically choosing the lobbyst's preferences to affect subsequent lobbying competition. The lobbyist always maximizes the preferences of an individual with the mean type in the group he represents. Then, competition between interest groups reaches an efficient outcome, i.e., given that utility is transferable, it maximizes the overall payoff of the groups and PM.

Second, although the assumption that the preferences of the lobbyists are common knowledge at the common agency stage is quite convenient for expositional purposes, our results would be robust to the possibility that those preferences are kept private information. Indeed, that $T_l(\beta_l, \beta_{-l})$ are VCG payments also means that, each group would find it optimal to truthful reveal the preference parameter β_l at the last stage of the game had those parameters being kept secret.

For the time being, we obtain the following result for our benchmark scenario.

Proposition 1 Efficient group formation under complete information. *When group l forms under complete information, preferences are efficiently aggregated:*

$$\beta_l^*(\boldsymbol{\alpha}_l) = \alpha_l^*(\boldsymbol{\alpha}_l), \quad \forall \boldsymbol{\alpha}_l \in \Omega_l^{N_l}. \tag{3.2}$$

From Proposition 1, the *pluralistic approach* of politics is valid under complete information. Preferences are perfectly aggregated both within and also across groups.

4. CHARACTERIZING INCENTIVE-FEASIBILITY

We now turn to the case of asymmetric information. Our first task is to get a compact characterization of the set of incentive-feasible allocations that can be implemented within a group. We follow the mechanism design literature[25] and characterize incentive

[23] See the Appendix for details.
[24] To simplify notations, we denote $[g(x)]_{x_0}^{x_1} = g(x_1) - g(x_0)$.
[25] Laffont and Maskin (1982), Mailath and Postlewaite (1990), Ledyard and Palfrey (1999), Hellwig (2003).

compatibility conditions before aggregating (2.5), (2.6) and (2.7) into a single constraint which is both necessary and sufficient for incentive-feasibility. This incentive-feasibility constraint delineates what kind of objectives a group can pass on its lobbyist.

Bayesian Incentive compatibility. Incentive compatibility can be expressed in terms of properties of the mapping $\beta_l(\boldsymbol{\alpha}_l)$ and the individual payoffs profiles $\mathcal{U}_l^i(\alpha_l^i)$ that such mapping induces. This is the purpose of next lemma whose proof is standard.

Lemma 1 *An allocation $((\mathcal{U}_l^i(\alpha_l^i))_{1\le i\le N_l}, \beta_l(\boldsymbol{\alpha}_l))$ is Bayesian incentive compatible if and only if $\mathcal{U}_l^i(\alpha_l^i)$ is convex, and admits the integral representation:*

$$\mathcal{U}_l^i(\alpha_l^i) = \mathcal{U}_l^i(\underline{\alpha}_l) + \int_{\underline{\alpha}_l}^{\alpha_l^i} \mathbb{E}_{\boldsymbol{\alpha}_l^{-i},\boldsymbol{\alpha}_{-l}}\left(\frac{1}{N_l}\Delta u_l(\beta_l(\tilde{\alpha}_l^i, \boldsymbol{\alpha}_l^{-i}), \beta_{-l}(\boldsymbol{\alpha}_{-l}))\right) d\tilde{\alpha}_l^i. \tag{4.1}$$

Consider an individual in group l who has preferences α_l^i. To fix ideas, suppose also that $\Delta u_l(\beta_l(\alpha_l^i, \boldsymbol{\alpha}_l^{-i}), \beta_{-l}(\boldsymbol{\alpha}_{-l}))$ is non-decreasing in α_l^i for any $(\boldsymbol{\alpha}_l^{-i}, \boldsymbol{\alpha}_{-l})$.[26] By pretending to have a slightly lower valuation $\alpha_l^i - d\alpha_l^i$, type α_l^i can modify PM's decision which becomes $x(\beta_l(\alpha_l^i - d\alpha_l^i, \boldsymbol{\alpha}_l^{-i}), \beta_{-l}(\boldsymbol{\alpha}_{-l}))$. At the same time, type α_l^i also reduces his own contribution $t_l^i(\alpha_l^i - d\alpha_l^i, \boldsymbol{\alpha}_l^{-i})$ thereby letting other members of his own group pay much of what is needed to influence PM. This effect is thus at the core of the free riding problem.

More generally, type α_l^i's net gain from manipulating his own preferences is worth $\frac{1}{N_l}\Delta u_l(x(\beta_l(\alpha_l^i - d\alpha_l^i, \boldsymbol{\alpha}_l^{-i}), \beta_{-l}(\boldsymbol{\alpha}_{-l}))d\alpha_l^i \approx \frac{1}{N_l}\Delta u_l(x(\beta_l(\alpha_l^i, \boldsymbol{\alpha}_l^{-i}), \beta_{-l}(\boldsymbol{\alpha}_{-l}))d\alpha_l^i$. To reveal information, that type must thus pocket an informational rent $\mathcal{U}_l^i(\alpha_l^i) - \mathcal{U}_l^i(\alpha_l^i - d\alpha_l^i)$ which, at any point of differentiability, is approximatively worth $\dot{\mathcal{U}}_l(\alpha_l^i)d\alpha_l^i$ where $\dot{\mathcal{U}}_l^i(\alpha_l^i)$ is obtained from differentiating (4.1) as:

$$\dot{\mathcal{U}}_l^i(\alpha_l^i) = \mathbb{E}_{\boldsymbol{\alpha}_l^{-i},\boldsymbol{\alpha}_{-l}}\left(\frac{1}{N_l}\Delta u_l(\beta_l(\alpha_l^i, \boldsymbol{\alpha}_l^{-i}), \beta_{-l}(\boldsymbol{\alpha}_{-l}))\right). \tag{4.2}$$

From (4.2), $\mathcal{U}_l^i(\alpha_l^i)$ is necessarily non-decreasing. The no-veto constraint (2.5) is thus harder to satisfy for those individuals with the lowest possible type $\underline{\alpha}_l$ who are the most eager to veto. Ratification of the agreement by all types thus requires:

$$\mathcal{U}_l^i(\underline{\alpha}_l) \ge 0. \tag{4.3}$$

Incentive-feasibility condition. Equipped with the characterization of incentive compatibility (4.1) and no-veto (4.3), we now derive a feasibility condition that aggregates no veto, incentive compatibility and budget balanced constraints.

Proposition 2 Incentive-feasibility. *A mechanism $\mathcal{G}_l$ is incentive-feasible if and only if:*

1. *The virtual net gain from group l formation is non-negative:*

$$\mathbb{E}_{\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l}}\left([u_0(x) + h_l^*(\boldsymbol{\alpha}_l)u_l(x) + \beta_{-l}(\boldsymbol{\alpha}_{-l})u_{-l}(x)]_{x(0,\beta_{-l}(\boldsymbol{\alpha}_{-l}))}^{x(\beta_l(\boldsymbol{\alpha}_l),\beta_{-l}(\boldsymbol{\alpha}_{-l}))}\right) \ge 0. \tag{4.4}$$

[26] We will see below, especially in Section 5, that such *ex post* monotonicity conditions hold quite naturally under some circumstances so that the weaker interim monotonicity conditions in Item 2. of Lemma 1 is also satisfied.

2. $\mathbb{E}_{\alpha_l^{-i},\boldsymbol{\alpha}_{-l}}(\Delta u_l(\beta_l(\boldsymbol{\alpha}_l),\beta_{-l}(\boldsymbol{\alpha}_{-l})))$ *is non-decreasing in* α_l^i.

Proposition 2 is a fundamental step on our way to simplify the mechanism design problem of a group. Condition (4.4) indeed summarizes all difficulties that asymmetric information imposes on collective action. When computing the net benefit of forming under asymmetric information, true types are replaced by virtual valuations of a lower magnitude. Hence, the group's incentives to contribute and influence policy diminish.

For any incentive-feasible mechanism ruling group $-l$ (i.e., a mechanism which itself satisfies incentive compatibility, no veto and budget balance), group l chooses a mechanism that maximizes the sum of its members' payoffs, namely $\sum_{i=1}^{N_l}\mathbb{E}_{\alpha_l^i}(\mathcal{U}_l(\alpha_l^i))$, subject to the feasibility condition (4.4) and the monotonicity condition found in Proposition 2.

5. CONDITIONS FOR AN EFFICIENT EQUILIBRIUM UNDER ASYMMETRIC INFORMATION

We now wonder whether the outcome of the lobbying game may remain efficient even under asymmetric information. We refer to *an efficient equilibrium* as an equilibrium (if any) such that each group solves its own internal informational problem at no cost, and PM chooses an efficient decision. Suppose that such an equilibrium exists. Given that group $-l$ efficiently solves its own informational problem, group l must also do so. Lobbyists are thus given objectives that perfectly reflect the preferences of the interest groups they respectively represent, i.e., $\beta_l^*(\boldsymbol{\alpha}_l)\equiv\alpha_l^*(\boldsymbol{\alpha}_l)$, for all $\boldsymbol{\alpha}_l\in\Omega_l^{N_l}$.

PROPOSITION 3 EXISTENCE OF AN EFFICIENT EQUILIBRIUM. *An efficient equilibrium exists if and only if:*

$$\mathbb{E}_{\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l}}\left(\left[u_0(x)+\alpha_l^*(\boldsymbol{\alpha}_l)u_l(x)+\alpha_{-l}^*(\boldsymbol{\alpha}_{-l})u_{-l}(x)\right]_{x(0,\alpha_{-l}^*(\boldsymbol{\alpha}_{-l}))}^{x(\alpha_l^*(\boldsymbol{\alpha}_l),\alpha_{-l}^*(\boldsymbol{\alpha}_{-l}))}\right) \tag{5.1}$$
$$\geq\mathbb{E}_{\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l}}\left(\frac{1}{N_l}\left(\sum_{i=1}^{N_l}\frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)}\right)\Delta u_l(\alpha_l^*(\boldsymbol{\alpha}_l),\alpha_{-l}^*(\boldsymbol{\alpha}_{-l}))\right)\quad\forall l\in\{1,2\}.$$

The feasibility condition (5.1) has a simple interpretation. The l.h.s. is a measure of the welfare gain obtained from influencing PM so as to move from $x(0,\alpha_{-l}^*(\boldsymbol{\alpha}_{-l}))$, which is chosen when group l is inactive, to the efficient decision $x(\alpha_l^*(\boldsymbol{\alpha}_l),\alpha_{-l}^*(\boldsymbol{\alpha}_{-l}))$. Since $x(\alpha_l^*(\boldsymbol{\alpha}_l),\alpha_{-l}^*(\boldsymbol{\alpha}_{-l}))$ maximizes the overall payoff of groups together with PM, this gain is necessarily positive. Under complete information in group formation, this l.h.s. would be the payoff that group l could capture.

More novel is the r.h.s. of (5.1). This term stems for the overall informational cost that such change in the decision induces. Under complete information, it would disappear. Condition (5.1) is thus a fundamental equation to understand how groups solve their collective action problem. Since the l.h.s. is always non-negative, there would always exist an efficient equilibrium under complete information. We retrieve here the standard efficiency result that backs up the *pluralistic approach*. Instead, asymmetric information introduces a cost of coalition formation that may preclude such an efficient equilibrium and call for less optimistic conclusions about the efficiency of lobbying competition.

RUNNING EXAMPLE. Let us consider the case where u_0 is quadratic and suppose that types are uniformly distributed on $[\underline{\alpha}_l, \overline{\alpha}_l]$. Condition (5.1) becomes:

$$\mathbb{E}_{\alpha_l^*}\left(\left(\frac{3}{2}\alpha_l^* - \overline{\alpha}_l\right)\alpha_l^*\right) \geq 0 \quad \forall l \in \{1,2\}. \tag{5.2}$$

This condition is independent of group $-l$'s preferences; a specific feature on which we will come back soon. In this quadratic example, inefficiencies in group formation are indeed fully determined by the group's own composition and not by that of its rival. Condition (5.2) also holds when $\underline{\alpha}_l$ is close enough to $\overline{\alpha}_l$ and more specifically when $\underline{\alpha}_l \geq \frac{2}{3}\overline{\alpha}_l$. Enough homogeneity within the group (where homogeneity is defined as $\underline{\alpha}_l$ and $\overline{\alpha}_l$ being close enough) and a minimal stake ($\underline{\alpha}_l$ being necessarily positive when this latter inequality holds) altogether ensure existence of an efficient equilibrium under those circumstances. ■

6. LARGE GROUPS

Following a tradition that goes back to Bowen (1943), we now investigate how stringent Condition (5.1) is for large groups. Of course, free riding is then at its peak. We thus expect that the conditions for an efficient equilibrium might be strongly qualified.

Taking the limit $N_l \to +\infty$, the *Strong Law of Large Numbers* tells us that the sample mean of true (resp. virtual) types converges with probability one towards the mean (resp. the lowest) value of α_l^i:

(6.1)

$$\alpha_l^*(\boldsymbol{\alpha}_l) = \frac{1}{N_l}\sum_{i=1}^{N_l}\alpha_l^i \underset{N_l\to+\infty}{\overset{a.s.}{\rightarrow}} \mathbb{E}_{\alpha_l^i}(\alpha_l^i) = \alpha_l^e \quad \left(\text{resp. } h_l^*(\boldsymbol{\alpha}_l) = \frac{1}{N_l}\sum_{i=1}^{N_l} h_l(\alpha_l^i) \underset{N_l\to+\infty}{\overset{a.s.}{\rightarrow}} \mathbb{E}_{\alpha_l^i}(h(\alpha_l^i)) = \underline{\alpha}_l\right).$$

RUNNING EXAMPLE. Inserting (6.1) into (5.2), the condition that ensures existence of an efficient equilibrium in the case of a uniform distribution holds when N_l is large if $3\underline{\alpha}_l > \overline{\alpha}_l$ for all $l \in \{1,2\}$. This example suggests that efficiency is obtained if individuals have enough stakes and the distribution of types is not too disperse even for large groups. ■

Repeatedly throughout the paper, we will refer to the following assumption that requires that the lowest possible type has no stake.

ASSUMPTION 1 $\underline{\alpha}_l = 0 < \alpha_l^e \quad \forall l$.

We can now easily prove the following important result.

PROPOSITION 4 FREE RIDING IN LARGE HETEROGENOUS GROUPS. *Suppose that Assumption 1 holds. An efficient equilibrium never exists for N_l large enough.*

With a large group, inefficiencies always arise whenever the distribution of types contains individuals with no stake. To understand this property, one has to come back on the forces that lie behind such formation. On the one hand, an efficient equilibrium requires each lobbyist to behave as if he was maximizing the preferences of the average type of his group. When group l is influential, the policy has thus to move away from the decision $x(0, \alpha^*_{-l}(\boldsymbol{\alpha}_{-l}))$ that would be taken if only group $-l$ was active towards the efficient decision $x(\alpha^*_l(\boldsymbol{\alpha}_l), \alpha^*_{-l}(\boldsymbol{\alpha}_{-l}))$. The welfare gain that accrues to group l from such a move has to be compared with the overall information rent that has to be distributed internally to induce information revelation by members of the group. When group l is large, each individual only cares about minimizing his own contribution. He thus behaves as having the lowest possible valuation, namely $\underline{\alpha}_l = 0$ from Assumption 1. So overall, the group behaves as being made only of individuals with no preferences for moving the policy away from the *status quo*. The aggregate contribution of group l is thus zero, making it impossible to reach the efficient policy $x(\alpha^*_l(\boldsymbol{\alpha}_l), \alpha^*_{-l}(\boldsymbol{\alpha}_{-l}))$.

Proposition 4 has its counterpart. Efficiency is achieved when the distribution of types is not too diffuse and the lowest valuation type has *some* willingness to change the policy.

PROPOSITION 5 EFFICIENT EQUILIBRIA IN LARGE HOMOGENOUS GROUPS. *Suppose that $\alpha^e_l - \underline{\alpha}_l$ (for $l = 1, 2$) is small enough but that $\underline{\alpha}_l > 0$. An efficient equilibrium always exists when N_l (for $l = 1, 2$) is large enough.*

When α^e_l is close to $\underline{\alpha}_l$ and $\underline{\alpha}_l > 0$, the group is rather homogenous and all types are ready to pay to induce a policy shift from $x(0, \alpha^*_{-l}(\boldsymbol{\alpha}_{-l}))$ to $x(\alpha^*_l(\boldsymbol{\alpha}_l), \alpha^*_{-l}(\boldsymbol{\alpha}_{-l}))$. Aggregating over a large group, even tiny individual contributions may change the outcome. Even under asymmetric information, a large group remains influential.[27] Proposition 5 is a strong qualifier of the *Olson Paradox*. Under asymmetric information, this is not size *per se* that undermines group formation but instead the addition of size and heterogeneity.

7. GROUPS OF FINITE SIZE: ASYMMETRIC INFORMATION SOFTENS COMPETITION

Proposition 4 is reminiscent of Mailath and Postlewaite (1990) who also stressed the existing free-riding problem for large groups in a pure public good context. Beyond differences in functional forms and contexts, the main differences with our work are twofold. First, in Mailath and Postlewaite (1990), collective action is only possible when the fixed cost of producing the discrete public good has been covered. Hereafter, collective action requires to cover a variable cost; namely the cost of shifting the decision away from the sole influence of the rival group. The analysis in Hellwig (2003) shows that the nature of the cost function has major consequences on whether the free riding problem holds or not for large groups. Second, the net gains from forming are here equilibrium objects; a difference that significantly matters in the case of groups with finite sizes acting strategically. With large groups, there is indeed almost no remaining uncertainty about both aggregate preferences and aggregate contributions. Instead, for groups of finite sizes, some uncertainty remains not only about the distribution of aggregate valuations within the group, but also about the competing group's own preferences. This makes the analysis at finite sizes certainly more complex, but it also introduces some new strategic features that unveil the true determinants of inefficiencies in group formation.

[27] Hellwig (2003) criticizes Mailath and Postlewaite (1990)'s findings and argues that the free-riding problem they highlight depends a lot on the cost function for producing the public good and that efficiency may sometimes be reached with alternative assumptions. Our argument for efficiency relies instead on the properties of the distribution of types.

7.1. *Best Responses*

This Section analyzes how the true preferences of groups might no longer be reflected by the political process and how lobbying competition might in turn be modified.

Under asymmetric information, $\beta_l(\boldsymbol{\alpha}_l)$ should now satisfy the incentive-feasibility condition (4.4). When this constraint is binding, group l faces a trade-off. On the one hand, choosing $\beta_l(\boldsymbol{\alpha}_l)$ close to the efficient rule $\alpha_l^*(\boldsymbol{\alpha}_l)$ induces a significant change in policy from $x(0, \beta_{-l}(\boldsymbol{\alpha}_{-l}))$ to $x(\alpha_l^*(\boldsymbol{\alpha}_l), \beta_{-l}(\boldsymbol{\alpha}_{-l}))$. On the other hand, such a shift also requires a large contribution to PM. Individual contributions must increase which in turn exacerbates free riding. To moderate this rent-efficiency trade-off, group l chooses an objective $\beta_l^{sb}(\boldsymbol{\alpha}_l)$ which now partially reflects the group's preferences. So doing, policy changes are of a lower magnitude, contributions diminish and free riding is less of a concern.

Proposition 6 Inefficient best responses. *At a best response, group l chooses a moderate objective $\beta_l^{sb}(\boldsymbol{\alpha}_l) \leq \alpha_l^*(\boldsymbol{\alpha}_l)$:*

$$\beta_l^{sb}(\boldsymbol{\alpha}_l) = \max\left\{0, \frac{1}{N_l}\sum_{i=1}^{N_l}\alpha_l^i - \frac{\lambda_l}{1+\lambda_l}\frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)}\right\} \tag{7.1}$$

where $\lambda_l \geq 0$ is the Lagrange multiplier of the incentive-feasibility constraint (4.4).

When $\lambda_l > 0$, PM chooses a policy that no longer reflects the preferences of group l. Indeed, the overall contribution of that group is reduced to weaken intra-group free riding. PM is thus biased towards the outcome he would have chosen when selling his favors to the competing group only. Informational frictions also weaken lobbying competition. The welfare consequences of this effect are studied in Section 10.

7.2. *A Sufficient Condition Ensuring Universal Inefficiency*

We start by stating the following assumption.

Assumption 2

$$\frac{1-\Phi_{N_l}(\alpha_l^*)}{\phi_{N_l}(\alpha_l^*)} - \mathbb{E}_{\boldsymbol{\alpha}_l}\left(\frac{1}{N_l}\sum_{i=1}^{N_l}\frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)} | \frac{1}{N_l}\sum_{i=1}^{N_l}\alpha_l^i = \alpha_l^*\right) \text{ is decreasing in } \alpha_l^* \text{ for } N_l > 1.$$

Assumption 2 only depends on properties of the distribution of types.[28] It will ensure inefficiencies in group formation irrespectively of the appointment rule chosen by

[28] Suppose that types in group l are distributed according to an exponential distribution on $\mathbb{R}_+$ with density $f_l(\alpha_l^i) = \frac{1}{\mu_l} exp\left(-\frac{\alpha_l^i}{\mu_l}\right)$ and cumulative distribution function $F_l(\alpha_l^i) = 1 - exp\left(-\frac{\alpha_l^i}{\mu_l}\right)$. Assumption 1 is satisfied while the monotone hazard rate property trivially holds since $\frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)} = \mu$ and thus $\mathbb{E}_{\boldsymbol{\alpha}_l}\left(\frac{1}{N_l}\sum_{i=1}^{N_l}\frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)} | \frac{1}{N_l}\sum_{i=1}^{N_l}\alpha_l^i = \alpha_l^*\right) = \mu$. The sample mean α_l^* now follows a Gamma-distribution with density function $\phi_{N_l}(\alpha_l^*) = \frac{N_l^{N_l}}{\mu^{N_l}(N_l-1)!}(\alpha_l^*)^{N_l-1} exp\left(\frac{-N_l\alpha_l^*}{\mu}\right)$. This density function is log-concave. It follows from Bagnoli and Bergstrom (2005, Lemma 4) that $\frac{1-\Phi_{N_l}(\alpha_l^*)}{\phi_{N_l}(\alpha_l^*)}$ is thus decreasing and Assumption 2 again holds. The careful reader will have noticed that our analysis was developed in the case of distributions having a finite support, for instance a truncated exponential on a bounded interval $[0, a]$ (with a being arbitrarily large). The benefit of passing to the limit and working with an exponential distribution without truncation is to illustrate the scope of Assumption 2 with minimal technicalities.

the competing group. That inefficiencies are obtained when making assumptions on the distribution of types echoes many important contributions in the mechanism design literature (Myerson and Satterthwaite, 1983; Mailath and Postlewaite, 1990). The key aspect of Assumption 2 is that this condition is universal and, in tandem with Assumption 1, sufficient for inefficiency for any possible choice made by group $-l$.

Efficiency is not reached when the incentive-feasibility condition (4.4) fails for $\beta_l^*(\boldsymbol{\alpha}_l) = \alpha_l^*(\boldsymbol{\alpha}_l)$, i.e.

$$\mathbb{E}_{\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l}}\left(\left[u_0(x)+h_l^*(\boldsymbol{\alpha}_l)u_l(x)+\beta_{-l}(\boldsymbol{\alpha}_{-l})u_{-l}(x)\right]_{x(0,\beta_{-l}(\boldsymbol{\alpha}_{-l}))}^{x(\alpha_l^*(\boldsymbol{\alpha}_l),\beta_{-l}(\boldsymbol{\alpha}_{-l}))}\right)<0. \tag{7.2}$$

Proposition 4 already showed that Condition (7.2) holds when N_l is large enough and group $-l$ implements the efficient appointment rule, i.e., $\beta_{-l}^*(\boldsymbol{\alpha}_{-l}) = \alpha_{-l}^*(\boldsymbol{\alpha}_{-l})$. Assumptions 1 and 2 jointly ensure that this result is also true for groups of finite sizes.[29]

Proposition 7 Inefficient Group formation of finite sizes. *Suppose that Assumptions 1 and 2 both hold. Group l never forms efficiently, i.e., $\lambda_l > 0$.*

Running example (continued). When PM's utility function satisfies (2.1), the optimal policy is given by $x(\beta_1,\beta_2) = \frac{\beta_2-\beta_1}{\beta_0}$. To illustrate the input of Assumption 2, observe that Condition (7.2) becomes:[30]

$$\mathbb{E}_{\alpha_l^*}\left(\left(\frac{1-\Phi_{N_l}(\alpha_l^*)}{\phi_{N_l}(\alpha_l^*)}-\mathbb{E}_{\boldsymbol{\alpha}_l}\left(\frac{1}{N_l}\sum_{i=1}^{N_l}\frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)}\Big|\frac{1}{N_l}\sum_{i=1}^{N_l}\alpha_l^i=\alpha_l^*\right)\right)\frac{\alpha_l^*}{\beta_0}\right)<0. \tag{7.3}$$

This condition is independent of the appointment rule $\beta_{-l}(\boldsymbol{\alpha}_{-l})$ chosen by group $-l$ and it always holds thanks to Assumption 2. Indeed, from Assumption 2, the first factor in the expectation is decreasing. This term has also zero mean since $\mathbb{E}_{\alpha_l^*}\left(\frac{1-\Phi_{N_l}(\alpha_l^*)}{\phi_{N_l}(\alpha_l^*)}\right) = \mathbb{E}_{\alpha_l^i}\left(\frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)}\right) = \alpha_l^e$. The second factor $\frac{\alpha_l^*}{\beta_0}$ is increasing and thus covaries negatively with the first one. Because of negative correlation, (7.3) holds as requested since

$$\mathbb{E}_{\alpha_l^*}\left(\left(\frac{1-\Phi_{N_l}(\alpha_l^*)}{\phi_{N_l}(\alpha_l^*)}-\mathbb{E}_{\boldsymbol{\alpha}_l}\left(\frac{1}{N_l}\sum_{i=1}^{N_l}\frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)}\Big|\frac{1}{N_l}\sum_{i=1}^{N_l}\alpha_l^i=\alpha_l^*\right)\right)\frac{\alpha_l^*}{\beta_0}\right)$$
$$<\mathbb{E}_{\alpha_l^*}\left(\frac{1-\Phi_{N_l}(\alpha_l^*)}{\phi_{N_l}(\alpha_l^*)}-\mathbb{E}_{\boldsymbol{\alpha}_l}\left(\frac{1}{N_l}\sum_{i=1}^{N_l}\frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)}\Big|\frac{1}{N_l}\sum_{i=1}^{N_l}\alpha_l^i=\alpha_l^*\right)\right)\times\mathbb{E}_{\alpha_l^*}\left(\frac{\alpha_l^*}{\beta_0}\right)=0.$$

■

Proposition 7 summarizes how asymmetric information offers a drastic departure from the pluralistic view of politics. The lobbying process no longer perfectly aggregates the

[29] Similar conditions for the impossibility of implementing the first-best allocation under asymmetric information have flourished throughout the whole mechanism design literature both in public and private good contexts (Laffont and Maskin, 1982; Myerson and Sattherwaite, 1983; Cramton, Gibbons and Klemperer, 1987; Mailath and Postelwaite, 1990; Hellwig, 2003).

[30] The proof in the Appendix relies on several rounds of integration by parts.

preferences of interest groups. This failure can be extreme, with a group expressing no influence at all with positive probability. To illustrate, consider the case where all members of group l have preference parameters α_l^i such that $\alpha_l^i < \frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)}$; a possibility that arises when α_l^i is close enough to $\underline{\alpha}_l$ and $\underline{\alpha}_l < \frac{1}{f_l(\underline{\alpha}_l)}$. Under such configurations, $h_l^*(\boldsymbol{\alpha}_l) < 0$ and thus $\beta_l^{sb}(\boldsymbol{\alpha}_l) = 0$. Everything happens as if group l was inactive under asymmetric information while it would have been so under complete information.*A contrario*, even if inefficiencies still arise and the lobbyist's objectives are moderated thanks to informational frictions, some representation may always be guaranteed provided that types have enough stakes. For instance, whenever $\underline{\alpha}_l \geq \frac{1}{f_l(\underline{\alpha}_l)}$, group l always forms (i.e., $\beta_l^{sb}(\boldsymbol{\alpha}_l) > 0$) although it is almost always inefficiently so. Indeed, $\beta_l^{sb}(\boldsymbol{\alpha}_l) = \beta^*(\boldsymbol{\alpha}_l)$ only arises when $\boldsymbol{\alpha}_l = (\overline{\alpha}_l, ..., \overline{\alpha}_l)$, i.e., with zero probability.

7.3. *From Finite to Large Groups*

This section fills the gap between the finite sizes scenario and the limiting case of large groups. We are interested in asymptotic properties, making now the dependence of the optimal appointment rule $\beta_l^{sb}(\boldsymbol{\alpha}_l, N_l)$ on N_l explicit.

Proposition 8 Towards Large Groups. *Suppose that Assumptions 1 and 2 both hold. The appointment rule converges in probability towards no influence as N_l gets large:*

$$\beta_l^{sb}(\boldsymbol{\alpha}_l, N_l) \underset{N_l \to +\infty}{\overset{p}{\to}} 0. \tag{7.4}$$

Proposition 4 already showed the inexistence of an efficient equilibrium for large groups. Proposition 8 is stronger since it applies whatever group $-l$'s own choice. As size increases, the appointment rule is entirely determined by the incentive-feasibility Condition (4.4). This condition is trivially satisfied when group l exerts no influence at all. This is precisely what (7.4) shows: A large group leaves PM under the sole influence of its rival.

8. A DUAL REPRESENTATION OF EQUILIBRIA

A priori, the Lagrange multiplier λ_l that appears on the r.h.s. of (7.1) depends on the appointment rule $\beta_{-l}(\cdot)$ chosen by group $-l$. Indeed, those preferences determine how much money is paid by group l to buy the policy-maker and thus the magnitude of free riding. Since the preferences parameter $\beta_l^{sb}(\boldsymbol{\alpha}_l)$ for group l's lobbyist is fully characterized by a single non-negative parameter λ_l, it becomes quite natural to summarize an equilibrium by a pair of non-negative numbers $(\lambda_l, \lambda_{-l}) \in \overline{\mathbb{R}}_+^2$ that determine altogether appointment rules $(\beta_l^{sb}(\boldsymbol{\alpha}_l), \beta_{-l}^{sb}(\boldsymbol{\alpha}_{-l}))$ which are best responses to each other. Adopting this *dual representation*, an equilibrium amounts to a pair (λ_1, λ_2) satisfying:

$$\lambda_l = \Lambda_l^*(\lambda_{-l}) \quad \forall l \in \{1, 2\}. \tag{8.1}$$

The *"best-response mapping"* Λ_l^* defines the Lagrange multiplier characterizing group l's formation in terms of the Lagrange multiplier pertaining to group $-l$. This simple dual representation thus views an equilibrium as a pair of Lagrange multipliers,

Under complete information, each group perfectly passes its own aggregate preferences to its lobbyist; contributions to PM are always *truthful*. Whether the opposite group easily raises money to influence PM or not has no impact on group l's marginal contribution, although it might change its level. This stands in sharp contrast with the case of asymmetric information. Frictions now depend on how much money is needed to influence PM. This provides a channel by which the strength of the rival group affects group l's marginal contribution. Informational frictions in both groups are now jointly determined.

PROPOSITION 9 EXISTENCE OF EQUILIBRIA. *Suppose that Assumptions 1 and 2 both hold. There always exists a (pure-strategy) equilibrium, i.e., a pair* (λ_1, λ_2) *that solves (8.1) with* $\lambda_l > 0$ *for* $l \in \{1, 2\}$.

9. TOWARDS AN I.O. THEORY OF GROUP FORMATION

Interesting comparative statics follow from carefully looking at the the properties of best-response mappings. Slightly abusing language and using the well-known parlance of the I.O. literature, Proposition 10 shows that the game between competing groups might exhibit either *strategic complementarity* with both mappings Λ_l^* (for $l \in \{1, 2\}$) being non-decreasing or *strategic substitutability* when those mappings are instead non-increasing. Those monotonicity properties depend on fine details of PM's preferences.

PROPOSITION 10 MONOTONICITY OF BEST-RESPONSE MAPPINGS. Λ_l^* *($l \in \{1, 2\}$) is everywhere non-decreasing (resp. non-increasing) if and only if* $u_0''' \geq 0$ *(resp.* $u_0''' \leq 0$*).*

The intuition for those different patterns comes from understanding how asymmetric information impacts on lobbying competition. Suppose that group $-l$ finds it more difficult to organize itself, in other words, λ_{-l} increases. The first consequence is that this group has less influence. It becomes easier for group l to shift the *status quo* towards its own preferred direction: a *policy-shifting effect*. Lower contributions from group l are needed and thus free riding is less of a curse there. On the other hand, that group $-l$ does not influence so much the decision also means that the *status quo* might already please group l. Free riding is exacerbated: a *status quo effect*.

Which effect dominates depends on the sign of u_0'''. When $u_0''' \leq 0$, the *policy-shifting effect* dominates and group l's distortions are less significant as competing interests find it more difficult to organize. *A contrario*, when $u_0''' \geq 0$, the *status quo effect* prevails. A key lesson is thus that group formation generally depends on its environment. Informational frictions within a group impacts on how easily its competitor organizes.

RUNNING EXAMPLE (CONTINUED). For quadratic preferences, $u_0''' \equiv 0$, the *status quo* and the *policy-shifting* effects cancel out. The best-response mappings Λ_l^* are flat and the Lagrange multipliers (λ_1, λ_2) are determined separately. Whether group l faces a strong opponent or not does not affect its own difficulties in solving the free riding problem. The preferences of its lobbyist are independent of the surrounding environment. ■

That informational frictions are determined altogether suggests that groups may adopt various strategies to undermine the ability of their rivals to organize themselves.

INFORMATION SHARING. Suppose that members of group $-l$ can credibly share information on their preferences. It may be so because the group is small in size and peer monitoring is possible. In some related contexts, Ostrom (1990) has argued that agents may invest resources to monitor each other and avoid free riding. Free riders could also be punished by means of group stigma, or through repeated interactions that overcome informational problems. In his analysis of coalition formation by interest groups in U.S. legislative politics, Hula (1999, p.24) first reports that *"The presence or absence of a given group is likely to be noticed in a small universe of organizations, and the potential exists for the applications of coercive sanctions by other members of the coalition to discourage free riding. At least, there must be a strong element of peer pressure."* He also points out that *"the increasing use of long-term, recurrent, and institutionalized coalitions in many policy arenas"* build *strong coalitions* of interest groups which solve the free riding problem. Finally, even when information cannot be credibly shared, private information might sometimes have a limited impact, especially when group members have limited veto power, a point that will be further discussed in Section 11.1.

Credibly sharing information within group $-l$ acts as a commitment device to fix $\lambda_{-l} = 0$. The strategic impact of such choice on the frictions faced by group l can be easily deduced from Proposition 10.

PROPOSITION 11 INFORMATION SHARING. *Let $(\lambda_l, \lambda_{-l})$ be an equilibrium of the game when both groups form under asymmetric information. Suppose that information is credibly shared within group $-l$, the new equilibrium $(\tilde{\lambda}_l, 0)$ is such that $\tilde{\lambda}_l \leq \lambda_l$ (resp. $\geq$) if $u_0''' \geq 0$ (resp. $\leq$).*

With information sharing, group $-l$ certainly finds it easier to buy PM's favors. It then becomes more difficult for group l to buy PM's favors to do so. Group l has to raise its contribution which worsens its own free riding problem. The *policy-shifting effect* exacerbates frictions within group l, while the *status quo effect* does the reverse. When the first effect dominates (i.e., $u_0''' \leq 0$), a group which has solved its own free riding problem can not only buy influence more easily but it also weakens its competitor's representation. Coalitions bound by strong ties might thus exclude rivals more easily than what less well-organized groups would do.

Illustration 1. The sugar industry is often viewed as being one of the strongest lobbying group in U.S. politics (Dixit, 1998). Between 2007 and 2014, its contribution went up to $18.5 million, according to the *Center for Responsive Politics* although sugar represents only a small fraction U.S. farm output. Because it is organized around four major producers with strong ties (the Fanjul brothers), the lobby of sugar producers has repeatedly resisted attempts by domestic sugar-users (like Pepsi Co Inc., Hershey Co., or the *National Foreign Trade Council*) but also foreign countries (like Australia) to open the market. Inefficient quotas dating back to the Great Depression have thus persisted.

Illustration 2. The strong political influence of senescent industries contrasts with the less successful lobbying exerted by growing sectors (Hillman, 1982; Brainard and Verdier, 1997; Baldwin and Robert-Nicoud, 2007). Standard arguments point out that entrants might not generate enough profits to pay for the fixed cost of lobbying while senescent industries may still be able to maintain their influence with the possible social cost of a delayed transition towards novel technologies. Our model suggests another line of arguments. Through repeated interactions over time, firms in senescent industries may have knitted tight bounds, maybe up to the point of credibly sharing information and alleviating much of the free riding problem. As these firms become more efficient at lobbying, entrants find it harder to overpass their own free riding problem and be influential.

Entry Costs. To be active, an interest group might also have to incur further organization costs beyond asymmetric information *per se*. For instance, hiring a lobbyist might require to incur search costs, to pay contingent fees, sometimes to give up extra rent if those lobbyists have to gather expert information. Collecting contributions may also rely on costly enforcement. Such entry costs harden the incentive-feasibility condition (4.4) and exacerbate frictions. To see how, let denote by K_l a fixed cost of group formation. The incentive-feasibility condition (4.4) now writes as:

$$(9.1) \qquad \mathbb{E}_{\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l}}\left(\left[u_0(x)+h_l^*(\boldsymbol{\alpha}_l)u_l(x)+\beta_{-l}(\boldsymbol{\alpha}_{-l})u_{-l}(x)\right]_{x(0,\beta_{-l}(\boldsymbol{\alpha}_{-l}))}^{x(\beta_l(\boldsymbol{\alpha}_l),\beta_{-l}(\boldsymbol{\alpha}_{-l}))}\right)\geq K_l>0.$$

Group l's contribution must increase to cover this extra fixed cost. In response, informational frictions are more pronounced. Formally, the whole best-response mapping Λ_l^* is shifted upwards, modifying accordingly the set of possible equilibria of the game.

Proposition 12 Fixed costs. *Let $(\lambda_l, \lambda_{-l})$ be an equilibrium of the game when both groups form under asymmetric information and there are no fixed costs. If group l incurs some small fixed cost of formation (i.e., K_l small enough), there exists a new equilibrium $(\tilde{\lambda}_l, \tilde{\lambda}_{-l})$ such that $\tilde{\lambda}_l \geq \lambda_l$ (resp. $\geq$) and $\tilde{\lambda}_{-l} \geq \lambda_{-l}$ (resp. $\leq$) if $u_0''' \geq 0$ (resp. $\leq$).*

If the *policy shifting effect* dominates, (i.e., $u_0''' \leq 0$), group l gets a strategic advantage when its own fixed cost of organization decreases since it also worsens free riding problem for his rival. The opposite would happen had the *status quo effect* dominated (i.e., $u_0''' \geq 0$).

Illustration 3. Following Mitra (1999, 2002), several authors have endogenized the set of active groups in complete information common agency models by assuming that groups can only form if the political benefits of doing so cover a fixed cost of groups formation (Krishna and Mitra, 2005; Brou and Ruta, 2006; Laussel, 2006; Leaver and Makris, 2006; Martimort and Semenov, 2007; Bombardini, 2008; Redoano, 2010). Fixed costs have thus only an impact at the extensive margin but no consequences on the intensive margin since all active groups always use truthful strategies. Illustrating this approach in a trade context, Bombardini (2008) provides some empirical evidence showing that, when the distribution of firm sizes has a fatter tail, more firms are able to pay for the entry cost and lobbying for protection is more effective. If firms have instead private information, the incentive-feasibility condition (9.1) directly depends on the tail of the distribution. Indeed, as the distribution puts more weight on higher types, $h^*(\boldsymbol{\alpha}_l)$ comes close to $\alpha_l^*(\boldsymbol{\alpha}_l)$ and the incentive-feasibility condition becomes easier to satisfy. Even without fixed costs, asymmetric information alone offers an alternative justification of those findings.

Coercive Stigma. Suppose now that each agent suffers from some reputation loss, or stigma, if not participating to the agreement. The term $\frac{K_l}{N}$ becomes negative and the incentive-feasibility condition (4.4) is easier to satisfy. More coercive ways of enforcing the agreement would thus shift group l's best response downwards, leading to changes in equilibria that mirror those of Proposition 12. Of course, there exists a value of those stigmas in both groups beyond which efficiency can be restored, namely

$$\mathbb{E}_{\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l}}\left(\left[u_0(x)+h_l^*(\boldsymbol{\alpha}_l)u_l(x)+\alpha_{-l}^*(\boldsymbol{\alpha}_{-l})u_{-l}(x)\right]_{x(0,\alpha_{-l}^*(\boldsymbol{\alpha}_{-l}))}^{x(\alpha_l^*(\boldsymbol{\alpha}_l),\alpha_{-l}^*(\boldsymbol{\alpha}_{-l}))}\right).$$ [31]

[31] See Makowski and Mezzetti (1994) for more general examples of conditions on budget deficit that allows efficient implementation.

10. WELFARE ANALYSIS

To understand how informational frictions within groups impact on welfare, we must remind a logic which is now familiar from Section 9. Inefficiencies in group formation have two effects on groups' payoffs. First, for a given strategy followed by his rival, each group would be better off if informational constraints *within this group* could be circumvented so as to endow its lobbyist with an objective that would perfectly reflect the group's aggregate preferences. Second, informational frictions *within the competing group* also contribute to soften competition. Each group benefits from facing a competitor with less influence. The overall impact of those competing effects on the groups' payoffs might go either way. Getting unambiguous welfare results is thus difficult in general. We thus content ourselves with pointing out a few effects that arise in specific contexts. The first one is that frictions in coalition formation softens competition between symmetric groups.

PROPOSITION 13 GROUPS' PAYOFFS. *Suppose that Assumption 1 holds, groups have the same size* $N_1 = N_2 = N$*, valuations are drawn from the same distribution on* $\Omega_1 = \Omega_2 = [0, \overline{\alpha}]$*. For* N *large enough, interest groups are ex ante better off under asymmetric information.*

Inefficiencies are pervasive in large groups when preferences are sufficiently dispersed when Assumption 1 holds. Groups have no influence on decision-making. When both groups face such a huge free riding problem, PM opts for the *status quo* which reflects only her own preferences. Under complete information, groups of equal force would compete *head-to-head* for influence, and the same decision would also be chosen.The difference is that each group would pay a lot for maintaining the *status quo* just to avoid that the other group tilts the decision in its own direction. From an *ex ante* viewpoint, the groups' expected gains remain the same under both informational scenarios. Yet, under complete information, groups waste money in competing for PM's services while they refrain from doing so under asymmetric information. Asymmetric information moderates lobbying competition and acts as a coordination device that stops wasteful lobbying.

Illustration 4. The complete information common agency model suggests that conflicting groups may use large contributions to preserve the *status quo.* This conclusion is at odds with evidence. That there is so little money used by interest groups, the so called *"Tullock Paradox"* (Tullock, 1972), is a puzzle that has indeed retained much attention.[32] Ansolabehere et al. (2003) have found explanations on the demand side of the market for influence, arguing that interest groups' political campaign contributions only account for a small fraction of the overall contributions that a legislator may gather. Focusing on the supply side, Helpman and Persson (2001) offer a model where lobbying takes place prior to a legislative bargaining stage. Competition between legislators for being part of the majority leads to small contributions. Our model offers another demand side-driven explanation. Groups reduce their contributions in response to internal free riding.

PROPOSITION 14 POLICY-MAKER'S PAYOFF. *The policy-maker's payoff is always lower under asymmetric information than under full information.*

By making lobbyists' objective less sensitive to the decision, asymmetric information

[32] See for instance Leaver and Makris (2006).

softens lobbying competition. It thus reduces the rent that PM extracts from playing one group against the other.

11. DISCUSSIONS AND EXTENSIONS

This section discusses several extensions. Section 11.1 investigates the consequences of having individuals with no veto power on group formation. Section 11.2 shows under which circumstances free riding on participation may also arise and whether it has any bite. Section 11.3 shows how our results should be modified when groups have congruent objectives. Section 11.4 highlights what sorts of *ad hoc* reduced form models could be used to model frictions in collective action problems. Finally, Section 11.5 shows how our framework can account for the possibility that interest groups have internal redistributive concerns and may follow simple, robust but suboptimal decision processes.

11.1. *Veto Power For Key Players Only*

Following the mechanism design literature, we have assumed that unanimous agreement was required to enforce a group mechanism. Beyond, we may ask what would happen if only a few players had veto power. Frictions are certainly of a lesser magnitude in that scenario. More specifically, suppose that agents indexed by $i \in \{1, ..., \hat{N}_l\}$ have no veto right while those indexed by $i \in \{\hat{N}_l + 1, ...N_l\}$ have such right. Importing an important insight due to d'Aspremont and Gerard-Varet (1979) into our specific context, we know that Bayesian incentive compatibility for agents with no veto power comes for free. Everything thus happens as if their preferences were common knowledge within the group. This immediately leads to redefine the lobbyist's preferences for that group as:

$$\beta_l^{sb}(\boldsymbol{\alpha}_l) = \max\left\{\frac{1}{N_l}\left(\sum_{i=1}^{\hat{N}_l}\alpha_l^i + \sum_{i=\hat{N}_l+1}^{N_l}\alpha_l^i - \frac{\hat{\lambda}_l}{1+\hat{\lambda}_l}\frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)}\right\}; 0\right). \tag{11.1}$$

Here, $\hat{\lambda}_l$ is the Lagrange multiplier for the new incentive-feasibility condition

$$\mathbb{E}_{\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l}}\left(\left[u_0(x) + \tilde{h}_l^*(\boldsymbol{\alpha}_l)u_l(x) + \beta_{-l}(\boldsymbol{\alpha}_{-l})u_{-l}(x)\right]_{x(0,\beta_{-l}(\boldsymbol{\alpha}_{-l}))}^{x(\beta_l(\boldsymbol{\alpha}_l),\beta_{-l}(\boldsymbol{\alpha}_{-l}))}\right) \geq 0 \tag{11.2}$$

where $\tilde{h}_l^*(\boldsymbol{\alpha}_l) = \frac{1}{N_l}\left(\sum_{i=1}^{N_l}\alpha_l^i - \sum_{i=\hat{N}_l+1}^{N_l}\frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)}\right)$.

There is nothing specific to the analysis of such environments with limited veto power and our previous results would apply *mutatis mutandis*. The only difference is the lesser inefficiency coming from the fact that free riding is less of a concern.

11.2. *Free-Riding on Participation*

Following a standard assumption of the mechanism design literature, we have posited that the whole group breaks down and thus can no longer influence PM whenever any member vetoes. Another interpretation is that the group can commit to stop lobbying if not all members agree. Such commitment is of course a strong requirement that exacerbates the conflict between incentive compatibility and participation, assuming a very weak outside option following veto. Alternatively, we could assume that the mechanism can still be somewhat enforced, and coordination somehow continues even if an agent fails to ratify.[33] In such scenario, free riding is on participation as well. A non-ratifying agent now free rides on the influence of those who remain organized and active.

There is a plethora of possible scenarios to describe such limited commitment environments. The simplest one is to assume that, had agent i vetoed the mechanism, that mechanism still enforces the allocation $\beta_l(0, \boldsymbol{\alpha}_l^{-i})$ (which is a possible offer in the menu if $\underline{\alpha}_l = 0$, an assumption that is made from now on). The mechanism thus relies on the sole contributions of ratifying agents. Free riding on participation is thus prevented when:

$$\text{(11.3)} \quad \mathcal{U}_l^i(\alpha_l^i) \geq \mathcal{U}_l^r(\alpha_l^i) = \frac{\alpha_l^i}{N_l}\mathbb{E}_{\boldsymbol{\alpha}_l^{-i},\boldsymbol{\alpha}_{-l}}(\Delta u(\beta_l(0,\boldsymbol{\alpha}_l^{-i}), \beta_{-l}(\boldsymbol{\alpha}_{-l}))).$$

A priori, this participation constraint looks more stringent one than (2.5) because the right-hand side is non-negative. Frictions in group formation might now be more significant. Nevertheless, observe that $\mathcal{U}_l^r(0) = 0$ and

$$\dot{\mathcal{U}}_l^i(\alpha_l^i) - \dot{\mathcal{U}}_l^r(\alpha_l^i) = \frac{1}{N_l}\left(\mathbb{E}_{\boldsymbol{\alpha}_l^{-i},\boldsymbol{\alpha}_{-l}}(\Delta u(\beta_l(\alpha_l^i,\boldsymbol{\alpha}_l^{-i}), \beta_{-l}(\boldsymbol{\alpha}_{-l})) - \Delta u(\beta_l(0,\boldsymbol{\alpha}_l^{-i}), \beta_{-l}(\boldsymbol{\alpha}_{-l})))\right) \geq 0$$

where the last inequality follows from the fact that $\mathbb{E}_{\boldsymbol{\alpha}_l^{-i},\boldsymbol{\alpha}_{-l}}(\Delta u(\beta_l(\alpha_l^i,\boldsymbol{\alpha}_l^{-i}), \beta_{-l}(\boldsymbol{\alpha}_{-l}))$ is non-decreasing in α_l^i. Those conditions, taken in tandem, ensure that (11.3) is only binding at $\underline{\alpha}_l = 0$ if it is so. Hence, (11.3) is no more stringent than the standard veto constraint (2.5). When $\underline{\alpha}_l = 0$, all our results are thus robust to free riding on participation.

11.3. *Congruent Groups*

Considering only two groups with conflicting preferences is consistent with casual evidence reported by Hula (1999). Interest groups with *congruent* interests tend to coalesce to push their own collective interests. Keeping two conflicting groups is thus a way to short-cut the full-fledged modeling of such coalitions.

Nevertheless, the case of two congruent groups is interesting for its own sake, and an Online Appendix briefly develops the corresponding analysis. A first important feature that distinguishes this analysis from the case of conflicting interests is that, even under complete information on preferences, *inter-group* free riding now arises. Indeed, the equilibrium contributions that are determined at the common agency stage of the game are no longer VCG payments. Congruent groups design contributions so as to leave PM indifferent between taking both contributions and his next best option which is now to refuse all contributions and choose his most preferred status quo policy free of any influence.[34] Because now contributions are no longer VCG payments, groups do not pass sincerely their aggregate preferences to their delegates. Each group shades its preferences to reduce its own share of the cost of moving PM away from the status quo. Congruent groups thus enjoy efficiency gains when merging.

Under asymmetric information, those gains should be compared with the information rents that accrue to group members. When groups remain split apart, vetoing the mechanism is not very costly for any individual. Provided that the other group forms, the decision is indeed already tilted in the right direction. Both the net benefits from forming

[33] See Martimort and Sand-Zantman (2016) for another mechanism design example of such scenario.

[34] In the parlance of Laussel and Le Breton (2001), the *"no-rent property"* holds in this setting.

and the information rent that any individual can get from doing so are rather small under this scenario. Instead, when congruent groups are merged, each individual by vetoing the mechanism induces PM to choose the status quo, a decision which is further apart. The net benefits from forming and the information rent of any individual both increase under this scenario. Whether asymmetric information is more of a blessing with split congruent groups or with a merger is thus generally difficult to assess beyond specific examples. The Online Appendix nevertheless provides an example where efficiency gains dominate even under asymmetric information. Congruent groups may then prefer to merge to solve their collective action problems. This insight justifies our earlier focus on two groups with opposite preferences throughout the paper.

Extrapolating a bit, this insight also provides a rationale for not looking at the possibility that a single individual may want to deviate from his group and directly lobby PM; a scenario that reflects an extreme case of congruence between his own objectives and those of his peers. Yet, the exact conditions under which agents with congruent preferences prefer to form coalitions rather than directly contributing on their own are left for future research.[35]

11.4. *Towards an Informational Theory of Transaction Costs*

The literature on lobbying and regulatory capture has repeatedly referred to the idea that groups incur transaction costs when targeting a policy-maker. In a nutshell, if a group's stake for changing a policy is $\alpha\Delta u$ and the group exerts some influence on PM by paying him T dollars, then its net payoff writes as

$$(11.4) \quad \alpha\Delta u - (1+\mu)T$$

where the quantity μT can be interpreted as the transaction costs of transferring those T dollars. One possible justification for such formulation is that transfers entail a deadweight loss that might reflect the imperfect enforceability of side-contracts.[36] Our analysis suggests that the group's organizational costs for collective action provide firm foundations for such transaction costs.

In Bernheim and Whinston (1986)'s model of economic influence or in Laffont and Tirole (1991)'s model of regulatory capture, such deadweight loss is group specific. Equilibrium policies then reflect the distribution of transaction costs across groups. An important albeit hidden assumption is that those frictions are independent of the group's stakes and transaction costs do not depend on the institutional environment in which this group evolves. In particular, whether there is competition against other interests or not has no impact on those costs. Even when deadweight losses are nonlinear as in Esteban and Ray (2001), a partial equilibrium perspective is adopted, fixing frictions at the outset. This is certainly a valid first step but the simple formula (11.4) above is only an approximation for the informationally-founded approach that we advocate in this paper. Policies have indeed an impact on the stakes for group formation. To showcase the possible modifications that

[35]When agents contribute directly, the game is akin to a Bayesian all-pay auctions. Examples of such games have been studied by Alboth, Lerner and Shalev (2001), Laussel and Palfrey (2003), Barbieri and Malueg (2008), and Martimort and Moreira (2010).

[36]Martimort (1999) and Faure-Grimaud, Laffont and Martimort (2003) have proposed models that endogenize those transaction costs when the relationship between a (single) interest group and the policy-maker is impacted by either asymmetric information and/or enforceability problems.

modelers should adopt when introducing transaction costs in reduced form models, we might interpret our previous results in terms of the *informational transaction costs* that characterize inefficiency in group formation. Consider an equilibrium $(\lambda_l, \lambda_{-l})$ of the game and define the *informational transaction costs* of group l's payments, say $\mu_l(\boldsymbol{\alpha}_l, \lambda_l)$, as

$$\mu_l(\boldsymbol{\alpha}_l, \lambda_l) \equiv \frac{\lambda_l}{1+\lambda_l} \frac{\frac{1}{N_l}\sum_{i=1}^{N_l} \frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)}}{\max\left\{\frac{1}{N_l}\sum_{i=1}^{N_l} \alpha_l^i - \frac{\lambda_l}{1+\lambda_l}\frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)}, 0\right\}} \tag{11.5}$$

with the convention that $\mu_l(\boldsymbol{\alpha}_l, \lambda_l) = +\infty$ when the denominator is zero.

Equipped with this formula, we may check that for all vectors $(\boldsymbol{\alpha}_l, \boldsymbol{\alpha}_{-l})$ the equilibrium weights (β_l, β_{-l}), and the subsequent decision $x(\beta_l, \beta_{-l})$ are the same as those one would have obtained by assuming that groups form under complete information, but that there is a deadweight loss of transferring money to PM that would be given by $\mu_i(\boldsymbol{\alpha}_l, \lambda_l)$. In that scenario, each group would thus maximize the following objective:

$$\alpha_l^*(\boldsymbol{\alpha}_l)\mathbb{E}_{\boldsymbol{\alpha}_{-l}}(\Delta u(\beta_l, \beta_{-l}(\boldsymbol{\alpha}_{-l}))) - (1+\mu(\boldsymbol{\alpha}_l, \lambda_l))\mathbb{E}_{\boldsymbol{\alpha}_{-l}}(T_l(\beta_l, \beta_{-l}(\boldsymbol{\alpha}_{-l}))).$$

This formula is very close to the reduced form (11.4) above. Yet, it shows that, when adopting a reduced form approach, modelers should make transaction costs dependent on various parameters whose importance has been overlooked in previous research. The profile of preferences within the group, the strength of rivals and the stakes of the decisions are all important ingredients that should appear in any *ad hoc* specification of transaction costs. In turn, such foundations might also suggest a whole set of new comparative statics to understand group behavior in politics. This could be the purpose of future research.

11.5. *Towards A More Detailed Analysis of Intra-Group Decision Processes*

Objectives. In our analysis, interest groups have utilitarian objectives. They maximize the sum of their members' expected utilities. Although this criterion of *ex ante efficiency* provides an important comparison with the complete information scenario and has thus received much attention in the mechanism design literature (Myerson and Sattherwaite, 1983; Laffont and Martimort, 1997), other selections on the Pareto frontier of incentive-feasible allocations could have been considered. In the spirit of Holmström and Myerson (1983) and Ledyard and Palfrey (1999), the whole set of interim allocations is obtained by giving type-dependant social weights to the various incarnations of the same agent. Typically, the objective functions in group l would write as

$$\sum_{i}^{N_l} \mathbb{E}_{\alpha_l^i}(\gamma_l(\alpha_l^i)\mathcal{U}_l(\alpha_l^i))$$

with the normalization $\mathbb{E}_{\alpha_l^i}(\gamma_l(\alpha_l^i)) = 1$.

Generalizing our previous approach, we would find that group l chooses a preference parameter for its lobbyist which is now given by

$$\beta_l^{sb}(\boldsymbol{\alpha}_l) = \max\left\{\frac{1}{N_l}\sum_{i=1}^{N_l} h_l(\alpha_l^i) + \frac{\Gamma_l(\alpha_l^i)}{(1+\lambda_l)f_l(\alpha_l^i)}; 0\right\} \tag{11.6}$$

where $\Gamma_l(\alpha_l^i) = \int_{\alpha_l^i}^{\overline{\alpha}_l} \gamma_l(\alpha) f_l(\alpha) d\alpha \in [0,1]$ stands for the cumulative social weight on the upper tail.[37]

Of course, this definition replicates (7.1) for a utilitarian objective. Beyond, there is a plethora of equilibria of the overall game of group formation cum lobbying. Those equilibria correspond to all possible distributions of social weights in the two groups. In sharp contrast with the complete information scenario, asymmetric information thus introduces some indeterminacy in the outcome of the political process. This indeterminacy is not due to strategic considerations; there is a unique equilibrium for a given pair of distributions of social weights. Instead, it follows from the leeway in choosing those weights, maybe in response to social norms and ethical considerations that pertain to the groups themselves.

Decision rule. In our analysis, each group chooses a point on the Pareto frontier of the set of incentive-feasible allocations. This point is implemented with a Bayesian mechanism that relies on the fact that members of the group have detailed knowledge on types distributions both within and outside the group. Alternatively, we could have insisted on more robust mechanisms.[38] The following one, suggested by a referee, is budget balanced, induces voluntary participation by all types and is implemented in dominant strategies which do not require such specific knowledge of distributions. The mechanism goes as follows. First, members of the group report their preference parameters. Second, the agent with the smallest such value is selected to serve as delegate and the contribution paid to PM is equally divided among all members of the group. Although not on the Pareto frontier, this mechanism clearly induces behavior very close to those obtained in our Bayesian scenario. To illustrate, consider the case of a large group. This group behaves like a $\underline{\alpha}_l$ agent with probability close to one. Free riding can be significant and prevents any intervention when $\underline{\alpha}_l = 0$. Beyond, asymmetric information still has a role in moderating competition between groups, in particular by reducing payments to PM and favoring moderate policies.

REFERENCES

Aidt, T. (1998), "Political Internalization of Economic Externalities and Environmental Policy," *Journal of Public Economics*, 69: 1-16.

Alboth, D., A. Lerner and J. Shalev (2001), "Profit Maximizing in Auctions of Public Goods," *Journal of Public Economic Theory*, 3: 501-525.

Anesi, V. (2009), "Moral Hazard and Free Riding in Collective Action," *Social Choice and Welfare*, 32: 197-219.

Ansolabehere, S, J. De Figueiredo and J. Snyder (2003), "Why Is There So Little Money In Politics?", *The Journal of Economic Perspectives,* 17: 105-130

d'Aspremont, C. and L.A. Gérard-Varet (1979), "Incentives and Incomplete Information," *Journal of Public Economics,* 11: 25-45.

Bagnoli, M. and T. Bergstrom (2005), "Log-Concave Probability and its Applications," *Economic Theory*, 26: 445-469.

Baldwin, R. and R. Robert-Nicoud (2007), "Entry and Asymmetric Lobbying: Why Governments Pick Losers," *Journal of the European Economic Association,* 5: 1064-1093.

[37]This condition is valid as long as the monotonicity conditions of Lemma 1 hold. A sufficient condition is that $\frac{\Gamma_l(\alpha_l^i)}{f_l(\alpha_l^i)}$ is non-decreasing. Otherwise, more complex ironing techniques must be used.

[38]See Kuzmics and Steg (2017) and the references herein.

Barbieri, S. and D. Malueg (2008), "Private Provision of a Discrete Public Good: Efficient Equilibria in the Private-Information Contribution Game," *Economic Theory,* 37: 51-80.

Baron, D. (1999), "Competitive Lobbying in a Majority-Rule Institution," Stanford Graduate Business School, Research Paper 1602.

Baron, D. (2006), "Competitive Lobbying and Supermajorities in a Majority-Rule Institution," *Scandinavian Journal of Economics,* 108: 607-642.

Baron, D. and A. Hirsch (2011), "Common Agency Lobbying Over Coalitions and Policy," *Economic Theory,* 49: 639-681.

Baumgartner, F. and B. Leech (1998), *Basic Interests: The Importance of Groups in Politics and in Political Science*, Princeton University Press.

Becker, G. (1983), "A Theory of Competition Among Pressure Groups for Political Influence," *The Quarterly Journal of Economics,* 98: 371-400.

Bergstrom, T., L. Blume and H. Varian (1986), "On the Private Provision of Public Goods," *Journal of Public Economics,* 29: 25-49.

Bernheim, B. and M. Whinston (1986), "Menu Auctions, Resource Allocation, and Economic Influence," *The Quarterly Journal of Economics*, 101: 1-31.

Besley, T. and S. Coate (2001), "Lobbying and Welfare in a Representative Democracy," *The Review of Economic Studies*, 68: 67-82.

Besley, T. and S. Coate (2003), "Centralized versus Decentralized Provision of Local Public Goods: A Political Economy Approach," *Journal of Public Economics*, 87: 2611-2637.

Bombardini, M. (2008), "Firm Heterogeneity and Lobby Participation," *Journal of International Economics,* 75: 329-348.

Bowen, H (1943), "The Interpretation of Voting in the Allocation of Economic Resource," *The Quarterly Journal of Economics*, 58: 27-48.

Brainard, S. and T. Verdier (1997), "The Political Economy of Declining Industries: Senescent Industry Collapse Revisited," *Journal of International Economics,* 42: 221-237.

Brou, D. and M. Ruta (2006), "Special Interests and the Gains from Political Integration," *Economics and Politics,* 18: 191-218.

Bulow, J., J. Geanakoplos and P. Klemperer (1985), "Multimarket Oligopoly: Strategic Substitutes and Complements," *The Journal of Political Economy,* 93: 488-511.

Christiansen, N. (2013), "Strategic Delegation in a Legislative Bargaining Model with Pork and Public Goods," *Journal of Public Economics*, 97: 217-229.

Cramton, P., R. Gibbons and P. Klemperer (1987), "Dissolving a Partnership Efficiently," *Econometrica*, 55: 615-632.

Dahl, R. (1961), *Who Governs?: Democracy and Power in an American City,* Yale University Press.

Damania, R. and P. Fredriksson (2000), "On the Formation of Industry Lobby Groups," *Journal of Economic Behavior and Organization,* 41: 315-335.

Damania, R. and P. Fredriksson (2003), "Trade Policy Reform, Endogenous Lobby Group Formation, and Environmental Policy," *Journal of Economic Behavior and Organization*, 52: 47-69.

Damania, R., P. Fredriksson and T. Osang (2004), "Collusion, Collective Action and Protection: Theory and Evidence," *Public Choice,* 121: 279-308.

Dixit, A. (1998), *The Making of Economic Policy: A Transaction-Cost Politics Perspective*, MIT Press.

Dixit, A., G. Grossman and E. Helpman (1997), "Common Agency and Coordination: General Theory and Application to Government Policy-Making," *Journal of Political*

Economy, 105: 752-769.

Dixit, A. and M. Olson (2000), "Does Voluntary Participation Undermine the Coase Theorem?" *Journal of Public Economics*, 79: 309-335.

Drazen, A., N. Limào and T. Stratmann (2007), "Political Contribution Caps and Lobby Formation: Theory and Evidence," *Journal of Public Economics,* 91: 723-754.

Dur, R. and H. Roelfsema (2005), "Why Does Centralisation Fail to Internalize Policy Externalities?" *Public Choice*, 122: 395-416.

Esteban, J. and D. Ray (2001), "Collective Action and the Group Size Paradox," *American Political Science Review,* 95: 663-672.

Faure-Grimaud, A., J.J. Laffont and D. Martimort (2003), "Collusion, Delegation and Supervision with Soft Information," *The Review of Economic Studies*, 70: 253-279.

Felli, L. and A. Merlo (2006), "Endogenous Lobbying," *Journal of the European Economic Association,* 4: 180-215.

Fudenberg, D. and J. Tirole (1984), "The Fat-Cat Effect, the Puppy-Dog Ploy, and the Lean and Hungry Look," *The American Economic Review,* 74: 361-366.

Fudenberg, D. and J. Tirole (1991), *Game Theory.* MIT Press.

Furusawa, T. and H. Konishi (2011), "Contributing or Free Riding? Voluntary Participation in a Public Good Economy," *Theoretical Economics*, 6: 219-256.

Gawande, K., and U. Bandyopadhyay (2000), "Is Protection for Sale? Evidence on the Grossman-Helpman Theory of Endogenous Protection," *The Review of Economics and Statistics,* 82: 139-152.

Gawande, K., P. Krishna and M. Olarreaga (2012), "Lobbying Competition Over Trade Policy," *International Economic Review,* 53: 115-132.

Goldberg, P. and G. Maggi (1999), "Protection for Sale: An Empirical Investigation," *The American Economic Review,* 89: 1135-1155.

Gray, V. and D. Lowery (1993), "The Diversity of State Interest Group Systems," *Political Research Quarterly*, 46: 81-97.

Green, J. and J.J. Laffont (1977), "Characterization of Satisfactory Mechanisms for the Revelation of Preferences for Public Goods," *Econometrica*, 45: 427-438.

Grossman, G. and E. Helpman (1994), "Protection for Sale," *The American Economic Review*, 84: 833-850.

Hellwig, M. (2003), "Public-Good Provision With Many Participants," *The Review of Economic Studies*, 70: 589-614.

Helpman, E. and T. Persson (2001), "Lobbying and Legislative Bargaining," *Advances in Economic Analysis and Policy*, 1: 1.

Hillman, A. (1982), "Declining Industries and Political-Support Protectionist Motives," *The American Economic Review,* 72: 1180-1187.

Holmström, B. and R. Myerson (1983), "Efficient and Durable Decision Rules with Incomplete Information," *Econometrica*, 51: 1799-1819.

Holyoke, T. (2009), "Interest Group Competition and Coalition Formation," *American Journal of Political Science*, 53: 360-375.

Hula, K. (1999), *Lobbying Together: committee Coalitions in Legislative Politics*, Georgetown University Press.

Kirchsteiger, G. and A. Prat (2001), "Inefficient Equilibria in Lobbying," *Journal of Public Economics,* 82: 349-375.

Kuzmics, C. and J. Steg (2017), "On Public Good Provision Mechanisms with Dominant Strategies and Balanced Budget," *Journal of Economic Theory,* 170: 56-69.

Laffont, J. J. and D. Martimort (1997), "Collusion under Asymmetric Information," *Econometrica*, 65: 875-911.

Laffont, J. J. and E. Maskin (1982), "The Theory of Incentives: An Overview," in *Advances in Economic Theory*, W. Hildenbrand ed., Cambridge University Press.

Laffont, J. J. and J. Tirole (1991), "The Politics of Government Decision-Making: A Theory of Regulatory Capture," *The Quarterly Journal of Economics,* 106: 1089-1127.

Laussel, D. (2006), "Special Interest Politics and Endogenous Lobby Formation," *Topics in Theoretical Economics,* 6: 1-14.

Laussel, D. and M. Le Breton (2001), "Conflict and Cooperation: The Structure of Equilibrium Payoffs in Common Agency," *Journal of Economic Theory*, 100: 93-128.

Laussel, D. and T. Palfrey (2003), "Efficient Equilibria in the Voluntary Contributions Mechanism with Private Information," *Journal of Public Economic Theory,* 5: 449-478.

Le Breton, M. and F. Salanié (2003), "Lobbying under Political Uncertainty," *Journal of Public Economics*, 87: 2589-2610.

Leaver, C. and M. Makris (2006), "Passive Industry Interests in Large Polity," *Journal of Public Economic Theory,* 8: 571-602.

Ledyard, J. and T. Palfrey (1999), "A Characterization of Interim Efficiency with Public Goods," *Econometrica*, 67: 435-448.

Lohmann, S. (1998), "An Information Rationale for the Power of Special Interests," *American Political Science Review*, 92: 809-827.

Loomis, B. (1986), "Coalitions of Interests: Building Bridges in the Balkanized State," in *Interest Group Politics*, eds. A. Cigler and B. Loomis, 258-274.

Magee, C. (2002), "Endogenous Trade Policy and Lobby Formation: An Application to the Free-Rider Problem," *Journal of International Economics,* 57: 449-471.

Mailath, G. and A. Postlewaite (1990), "Asymmetric Information Bargaining Problems With Many Agents," *The Review of Economic Studies*, 57: 351-367.

Makowski, L. and C. Mezzetti (1994), "Bayesian and Weakly Robust First-Best Mechanisms: Characterizations," *Journal of Economic Theory,* 64: 500-519.

Martimort, D. (1996), "Exclusive Dealing, Common Agency, and Multiprincipals Incentive Theory," *The RAND Journal of Economics*, 27: 1-31.

Martimort, D. (1999), "The Life Cycle of Regulatory Agencies: Dynamic Capture and Transaction Costs," *The Review of Economic Studies*, 66: 929-947.

Martimort, D. and H. Moreira (2010), "Common Agency and Public Good Provision under Asymmetric Information," *Theoretical Economics,* 5: 159-213.

Martimort, D. and A. Semenov (2007), "The Pluralistic View of Politics: Asymmetric Lobbyists, Ideological Uncertainty and Political Entry," *Economics Letters*, 97: 155-161.

Martimort, D. and A. Semenov (2008), "Ideological Uncertainty and Lobbying Competition," *Journal of Public Economics*, 92: 456-481.

Martimort, D. and L. Stole (2015), "Menu Auctions and Influence Games with Private Information," SSRN 2569703, Working Paper.

Martimort, D. and W. Sand-Zantman (2016), "A Mechanism Design Approach to Climate-Change Agreements," *Journal of European Economic Association*, 14: 669-718.

McLean, R. and A. Postlewaite (2002), "Informational Size and Incentive Compatibility," *Econometrica,* 70: 2421-2453.

Mitra, D. (1999), "Endogenous Lobby Formation and Endogenous Protection: A Long-Run Model of Trade Policy Determination," *The American Economic Review*, 89: 1116-1134.

Myerson, R. (1981), "Optimal Auction Design," *Mathematics of Operations Research,* 6:

58-73.

Myerson, R. (1982), "Optimal Coordination Mechanisms in Generalized Principal-Agent Problems," *Journal of Mathematical Economics*, 10: 67-81.

Myerson, R. and M. Satterthwaite (1983), "Efficient Mechanisms for Bilateral Trading," *Journal of Economic Theory*, 28: 265-281.

Olson, M. (1965), *The Logic of Collective Action: Public Goods and the Theory of Groups,* Harvard University Press.

Ostrom, E. (1990), *Governing the Commons: The Evolution of Institutions for Collective Action.* Cambridge University Press.

Pecorino, P. (2001), "Market Structure, Tariff Lobbying and the Free-Rider Problem," *Public Choice,* 106: 203-220.

Peltzman, S. (1976), "Toward a More General Theory of Regulation," *Journal of Law and Economics,* 19: 211-240.

Posner, R. (1974), "Theories of Economic Regulation," *The Bell Journal of Economics*, 5: 335-358.

Rama, M. and G. Tabellini (1998), "Lobbying by Capital and Labor over Trade and Labor Market Policies," *European Economic Review*, 42: 1295-1316.

Redoano, M. (2010), "Does Centralization Affect the Number and Size of Lobbies?," *Journal of Public Economic Theory,* 12: 407-435.

Siqueira, K. (2001), "Common Agency and Partial Cooperation," *Journal of Public Economic Theory*, 3: 309-339.

Stigler, G. (1971), "The Theory of Economic Regulation," *The Bell Journal of Economics,* 2: 3-21.

Tullock, G. (1972), "The Purchase of Politicians," *Western Economic Journal*, 10: 354-55.

Tullock, G. (1980), "Efficient Rent Seeking," In *Towards a Theory of the Rent-Seeking Society*, eds. J. Buchanan, R. Tollison and G. Tullock, 97-112. Texas A&M University Press.

Yu, Z. (2005), "Environmental Protection: A Theory of Direct and Indirect Competition for Political Influence," *The Review of Economic Studies*, 72: 269-286.

BIOGRAPHICAL NOTES

Perrin Lefebvre. Paris School of Economics, 48 Boulevard Jourdan, 75140 Paris, France. lefebvre.perrin@gmail.com.

David Martimort. Paris School of Economics-Ecole des Hautes Etudes en Sciences Sociales, 48 Boulevard Jourdan, 75140 Paris, France. david.martimort@psemail.eu.

ONLINE APPENDIX A: PROOFS OF THE MAIN RESULTS

Proof of Proposition 1: Laussel and Lebreton (2001) identify the properties of the lobbyists' payoffs at truthful equilibria of the common agency game with those of a cooperative game whose characteristic function is the payoff $W(\beta_1, \beta_2)$ of a coalition including a subset of lobbyists and PM. These payoffs are defined in terms of the vector of induced preferences (β_1, β_2) as:

$$W(\beta_1, \beta_2) = \max_{x \in \mathcal{X}} u_0(x) - \Delta\beta x = u_0(\varphi(\Delta\beta)) - \Delta\beta\varphi(\Delta\beta).$$

To illustrate, $W(0, \beta_2)$ stands for the payoff when group 1 fails to get organized and a similar convention applies to $W(\beta_1, 0)$ when group 2 is not organized. In our context with conflicting interest groups, this cooperative game turns out to be *sub-additive* since:

$$W(\beta_1, \beta_2) + W(0,0) \leq W(\beta_1, 0) + W(0, \beta_2) \quad \forall(\beta_1, \beta_2) \in \mathbb{R}^2_+.$$

Laussel and Le Breton (2001) then demonstrate that there is a unique truthful equilibrium with each lobbyist's payoff being his *incremental value* to the grand-coalition's surplus, namely:

$$V_l(\beta_l, \beta_{-l}) = W(\beta_1, \beta_2) - W(0, \beta_{-l}). \tag{A.1}$$

As a result, PM gets a positive share of the value of the grand-coalition:

$$W(\beta_1, 0) + W(0, \beta_2) - W(\beta_1, \beta_2) \geq W(0,0) = 0$$

(with a strict inequality whenever $\beta_l > 0$ for all l). PM pits one lobbyist against the other to extract a positive surplus. Using (2.2) and the expression of the lobbyist's payoff in (A.1) yields the expression of lobbyist l's contribution, $T_l(\beta_l, \beta_{-l}) = \tilde{T}_l(x(\beta_1, \beta_2))$, as (3.1).

Under complete information, group l's net gains from forming can thus be written as:

$$\sum_{i=1}^{N_l} \mathbb{E}_{\boldsymbol{\alpha}_l}(\mathcal{U}_l^i(\alpha_l^i)) = \mathbb{E}_{\boldsymbol{\alpha}_l, \boldsymbol{\alpha}_{-l}}\left(\alpha_l^*(\boldsymbol{\alpha}_l)\Delta u_l(\beta_l(\boldsymbol{\alpha}_l), \beta_{-l}(\boldsymbol{\alpha}_{-l})) - T_l(\beta_l(\boldsymbol{\alpha}_l), \beta_{-l}(\boldsymbol{\alpha}_{-l}))\right).$$

Using (3.1), $\sum_{i=1}^{N_l} \mathbb{E}_{\boldsymbol{\alpha}_l}(\mathcal{U}_l(\alpha_l^i))$ can be simplified as being:

$$\mathbb{E}_{\boldsymbol{\alpha}_l, \boldsymbol{\alpha}_{-l}}\left([u_0(x) + \alpha_l^*(\boldsymbol{\alpha}_l)u_l(x) + \beta_{-l}(\boldsymbol{\alpha}_{-l})u_{-l}(x)]_{x(0, \beta_{-l}(\boldsymbol{\alpha}_{-l}))}^{x(\beta_l(\boldsymbol{\alpha}_l), \beta_{-l}(\boldsymbol{\alpha}_{-l}))}\right). \tag{A.2}$$

Fix $\boldsymbol{\alpha}_l$ and consider the maximand

$$\mathbb{E}_{\boldsymbol{\alpha}_{-l}}\left([u_0(x) + \alpha_l^*(\boldsymbol{\alpha}_l)u_l(x) + \beta_{-l}(\boldsymbol{\alpha}_{-l})u_{-l}(x)]_{x(0, \beta_{-l}(\boldsymbol{\alpha}_{-l}))}^{x(\beta_l(\boldsymbol{\alpha}_l), \beta_{-l}(\boldsymbol{\alpha}_{-l}))}\right). \tag{A.3}$$

Differentiating with respect to $\beta_l(\boldsymbol{\alpha}_l)$ this expression yields:

$$\mathbb{E}_{\boldsymbol{\alpha}_{-l}}\left(\left(u_0'(x(\beta_l(\boldsymbol{\alpha}_l), \beta_{-l}(\boldsymbol{\alpha}_{-l}))) + (-1)^l(\alpha_l^*(\boldsymbol{\alpha}_l) - \beta_{-l}(\boldsymbol{\alpha}_{-l}))\right)\frac{\partial x}{\partial \beta_l}(\beta_l(\boldsymbol{\alpha}_l), \beta_{-l}(\boldsymbol{\alpha}_{-l}))\right).$$

Taking into account that $x(\beta_l(\boldsymbol{\alpha}_l), \beta_{-l}(\boldsymbol{\alpha}_{-l})) = \varphi((-1)^l(\beta_{-l}(\boldsymbol{\alpha}_{-l}) - \beta_l(\boldsymbol{\alpha}_l)))$, we obtain

$$(-1)^l(\alpha_l^*(\boldsymbol{\alpha}_l) - \beta_l(\boldsymbol{\alpha}_l))\mathbb{E}_{\boldsymbol{\alpha}_{-l}}\left(\frac{\partial x}{\partial \beta_l}(\beta_l(\boldsymbol{\alpha}_l), \beta_{-l}(\boldsymbol{\alpha}_{-l}))\right).$$

The objective function of group l (A.3) is thus quasi-concave in $\beta_l(\boldsymbol{\alpha}_l)$ and (3.2) is both necessary and sufficient for optimality. *Q.E.D.*

PROOF OF LEMMA 1: NECESSITY. Fix $\beta_{-l}(\cdot)$, and consider any incentive-feasible mechanism $\mathcal{G}_l$. To keep notations simple, we define the expected utility gains and expected payments for agent i who reports having type $\hat{\alpha}_l^i$ respectively as:

$$G_l^i(\hat{\alpha}_l^i) = \frac{1}{N_l}\mathbb{E}_{\boldsymbol{\alpha}_l^{-i},\boldsymbol{\alpha}_{-l}}\left(\Delta u_l(\beta_l(\hat{\alpha}_l^i,\boldsymbol{\alpha}_l^{-i}),\beta_{-l}(\boldsymbol{\alpha}_{-l}))\right) \text{ and } \mathcal{T}_l^i(\hat{\alpha}_l^i) = \mathbb{E}_{\boldsymbol{\alpha}_l^{-i}}\left(t_l^i(\hat{\alpha}_l^i,\boldsymbol{\alpha}_l^{-i})\right).$$

With those notations, incentive compatibility constraints (2.6) can be written as:

$$\text{(A.4)}\qquad \mathcal{U}_l^i(\alpha_l^i) = \max_{\hat{\alpha}_l^i\in\Omega_l} \alpha_l^i G_l^i(\hat{\alpha}_l^i) - \mathcal{T}_l^i(\hat{\alpha}_l^i).$$

$\mathcal{U}_l^i$ is convex as the maximum of linear functions. It thus admits a sub-differential $\partial\mathcal{U}_l^i(\alpha_l^i)$ and $G_l^i(\alpha_l^i) \in \partial\mathcal{U}_l^i(\alpha_l^i)$ at any α_l^i. Since $\mathcal{X}$ is bounded above, $\mathcal{U}_l^i$ is also Lipschitz continuous. It is thus also absolutely continuous, i.e., a.e. differentiable and admits the integral representation:

$$\text{(A.5)}\qquad \mathcal{U}_l^i(\alpha_l^i) = \mathcal{U}_l^i(\underline{\alpha}_l) + \int_{\underline{\alpha}_l}^{\alpha_l^i} G_l^i(\tilde{\alpha}_l^i)d\tilde{\alpha}_l^i.$$

This condition rewrites as (4.1). Finally, convexity of $\mathcal{U}_l^i$ amounts to G_l^i non-decreasing.

SUFFICIENCY. Suppose that the allocation $(\mathcal{U}_l^i, G_l^i)$ satisfies (4.1) with $\mathcal{U}_l^i$ convex (i.e., G_l^i non-decreasing). By convexity, $\mathcal{U}_l^i$ admits a subdifferential that contains $G_l^i(\hat{\alpha}_l^i)$ at any $\hat{\alpha}_l^i$. Incentive compatibility then follows since:

$$\alpha_l^i G_l^i(\alpha_l^i) - \mathcal{T}_l^i(\alpha_l^i) = \mathcal{U}_l^i(\alpha_l^i) \geq \mathcal{U}_l^i(\hat{\alpha}_l^i) + G_l^i(\hat{\alpha}_l^i)(\alpha_l^i - \hat{\alpha}_l^i) = \alpha_l^i G_l^i(\hat{\alpha}_l^i) - \mathcal{T}_l^i(\hat{\alpha}_l^i).$$

Q.E.D.

PROOF OF LEMMA 2: NECESSITY. Taking expectations of (2.7) with respect to $\boldsymbol{\alpha}_l$ yields:

$$\text{(A.6)}\qquad \sum_{i=1}^{N_l}\mathbb{E}_{\alpha_l^i}\left(\alpha_l^i G_l^i(\alpha_l^i) - \mathcal{U}_l^i(\alpha_l^i)\right) - \mathbb{E}_{\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l}}(T_l(\beta_l(\boldsymbol{\alpha}_l),\beta_{-l}(\boldsymbol{\alpha}_{-l}))) \geq 0.$$

Using (4.1) and integrating by parts, we obtain:

$$\mathbb{E}_{\alpha_l^i}\left(\mathcal{U}_l^i(\alpha_l^i)\right) = \mathcal{U}_l^i(\underline{\alpha}_l) + \mathbb{E}_{\alpha_l^i}\left(\frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)}G_l^i(\alpha_l^i)\right).$$

Inserting into (A.6) and simplifying yields:

$$\sum_{i=1}^{N_l}\mathbb{E}_{\alpha_l^i}\left(h_l(\alpha_l^i)G_l^i(\alpha_l^i)\right) - \mathbb{E}_{\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l}}(T_l(\beta_l(\boldsymbol{\alpha}_l),\beta_{-l}(\boldsymbol{\alpha}_{-l}))) \geq N_l\mathcal{U}_l^i(\underline{\alpha}_l) \geq 0$$

where the last inequality follows from (2.5). This condition can be written as:

$$\text{(A.7)}\qquad \mathbb{E}_{\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l}}\left(h_l^*(\boldsymbol{\alpha}_l)\Delta u_l(\beta_l(\boldsymbol{\alpha}_l),\beta_{-l}(\boldsymbol{\alpha}_{-l})) - T_l(\beta_l(\boldsymbol{\alpha}_l),\beta_{-l}(\boldsymbol{\alpha}_{-l}))\right) \geq 0.$$

Using the expression of payments from (3.1) yields (4.4).

SUFFICIENCY. Consider an allocation that satisfies (4.4), or (A.7) once (3.1) has been used, and such that G_l is non-decreasing. Define now a rent profile such that (A.5) holds and

$$\text{(A.8)}\qquad \mathcal{U}_l^i(\underline{\alpha}_l) = \mathbb{E}_{\alpha_l^i}\left(h_l(\alpha_l^i)G_l^i(\alpha_l^i)\right) - \frac{1}{N_l}\mathbb{E}_{\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l}}(T_l(\beta_l(\boldsymbol{\alpha}_l),\beta_{-l}(\boldsymbol{\alpha}_{-l}))) \geq 0.$$

From Lemma 1, such allocation is incentive compatible. From the fact that $\mathcal{U}_l^i$ is non-increasing, (A.8) also implies that (2.5) holds everywhere. Moreover, the expected payment $\mathcal{T}_l^i$ satisfies:

(A.9)
$$\mathcal{T}_l^i(\alpha_l^i) = \alpha_l^i G_l^i(\alpha_l^i) - \int_{\underline{\alpha}_l}^{\alpha_l^i} G_l^i(\tilde{\alpha}_l^i)d\tilde{\alpha}_l^i - \mathbb{E}_{\alpha_l^i}\left(h_l(\alpha_l^i)G_l^i(\alpha_l^i)\right) + \frac{1}{N_l}\mathbb{E}_{\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l}}(T_l(\beta_l(\boldsymbol{\alpha}_l),\beta_{-l}(\boldsymbol{\alpha}_{-l}))).$$

Taking expectations and integrating by parts, we get:

$$\text{(A.10)} \quad \mathbb{E}_{\alpha_l^i}(\mathcal{T}_l^i(\alpha_l^i)) = \frac{1}{N_l}\mathbb{E}_{\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l}}(T_l(\beta_l(\boldsymbol{\alpha}_l),\beta_{-l}(\boldsymbol{\alpha}_{-l}))).$$

From the expression of $\mathcal{T}_l^i(\alpha_l^i)$ in (A.9), we reconstruct the payments $t_l^i(\boldsymbol{\alpha}_l)$ as follows:

$$\text{(A.11)} \quad t_l^i(\boldsymbol{\alpha}_l) = \frac{1}{N_l}\mathbb{E}_{\boldsymbol{\alpha}_{-l}}(T_l(\beta_l(\boldsymbol{\alpha}_l),\beta_{-l}(\boldsymbol{\alpha}_{-l}))) + \varphi_l(\alpha_l^i) - \frac{1}{N_l-1}\sum_{j\neq i}\varphi_l^i(\alpha_l^j).$$

where $\varphi_l^i(\alpha_l^i) = \mathcal{T}_l^i(\alpha_l^i) - \frac{1}{N_l}\mathbb{E}_{\boldsymbol{\alpha}_l^{-i},\boldsymbol{\alpha}_{-l}}(T_l(\beta_l(\boldsymbol{\alpha}_l),\beta_{-l}(\boldsymbol{\alpha}_{-l})))$. Observe first that $\mathbb{E}_{\boldsymbol{\alpha}_l^i}(\varphi_l^i(\alpha_l^i)) = \mathbb{E}_{\boldsymbol{\alpha}_l^i}(\mathcal{T}_l^i(\alpha_l^i)) - \frac{1}{N_l}\mathbb{E}_{\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l}}(T_l(\beta_l(\boldsymbol{\alpha}_l),\beta_{-l}(\boldsymbol{\alpha}_{-l}))) = 0$ from (A.10). Second, we also have $\mathbb{E}_{\boldsymbol{\alpha}_l^{-i}}(t_l^i(\boldsymbol{\alpha}_l)) = \mathcal{T}_l^i(\alpha_l^i)$ so that Bayesian incentive compatibility and veto constraints are preserved. Finally, we can check that budget balance (2.7) is satisfied when (A.11) holds. (The payments $\mathcal{T}_l^i$ and t_l^i actually do not depend on i, so that all agents obey to the same contribution rule.) *Q.E.D.*

Proof of Proposition 3: Expressing the incentive-feasibility condition (4.4) for $\beta_l(\boldsymbol{\alpha}_l) = \alpha_l^*(\boldsymbol{\alpha}_l)$ ($l \in \{1,2\}$) and developing yields (5.1). Turning to Item 2. of Lemma 2, observe that:

$$\mathbb{E}_{\boldsymbol{\alpha}_l^{-i},\boldsymbol{\alpha}_{-l}}(\Delta u_l(\alpha_l^*(\boldsymbol{\alpha}_l),\alpha_{-l}^*(\boldsymbol{\alpha}_{-l}))) = (-1)^l\mathbb{E}_{\boldsymbol{\alpha}_l^{-i},\boldsymbol{\alpha}_{-l}}(x(\alpha_l^*(\boldsymbol{\alpha}_l),\alpha_{-l}^*(\boldsymbol{\alpha}_{-l})) - x(0,\alpha_{-l}^*(\boldsymbol{\alpha}_{-l}))).$$

Since $x(\beta_l,\beta_{-l}) = \varphi((-1)^l(\beta_{-l}-\beta_l))$ and $\varphi' < 0$, the r.h.s. above is non-decreasing in α_l^i. *Q.E.D.*

Proof of Proposition 4: Taking limits as $N_l \to +\infty$ and using (6.1), the policy chosen by PM converges *a.s.* towards:

$$\text{(A.12)} \quad x(\alpha_l^e,\alpha_{-l}^*(\boldsymbol{\alpha}_{-l})) = \arg\max_{x\in\mathcal{X}} u_0(x) + \alpha_l^e u_l(x) + \alpha_{-l}^*(\boldsymbol{\alpha}_{-l})u_{-l}(x) \quad \forall \boldsymbol{\alpha}_{-l}.$$

Taking limits as $N_l \to +\infty$, and again using (6.1), (5.1) certainly fails when:

$$\text{(A.13)} \quad \mathbb{E}_{\boldsymbol{\alpha}_{-l}}\left(\left[u_0(x) + \underline{\alpha}_l u_l(x) + \alpha_{-l}^*(\boldsymbol{\alpha}_{-l})u_{-l}(x)\right]_{x(0,\boldsymbol{\alpha}_{-l})}^{x(\alpha_l^e,\alpha_{-l}^*(\boldsymbol{\alpha}_{-l}))}\right) < 0.$$

Notice that $x(0,\alpha_{-l}^*(\boldsymbol{\alpha}_{-l})) = \arg\max_{x\in\mathcal{X}} u_0(x)+\underline{\alpha}_l u_l(x)+\alpha_{-l}^*(\boldsymbol{\alpha}_{-l})u_{-l}(x)$ when $\underline{\alpha}_l = 0$ and observe that $x(\boldsymbol{\alpha}_l^e,\alpha_{-l}^*(\boldsymbol{\alpha}_{-l})) \neq x(0,\alpha_{-l}^*(\boldsymbol{\alpha}_{-l}))$ when $\alpha_l^e > 0$. We thus have:

$$\text{(A.14)} \quad \left[u_0(x) + \underline{\alpha}_l u_l(x) + \alpha_{-l}^*(\boldsymbol{\alpha}_{-l})u_{-l}(x)\right]_{x(\alpha_l^e,\alpha_{-l}^*(\boldsymbol{\alpha}_{-l}))}^{x(0,\boldsymbol{\alpha}_{-l})} > 0 \quad \forall \boldsymbol{\alpha}_{-l}.$$

Taking expectations expectations over $\boldsymbol{\alpha}_{-l}$ yields (A.14) as requested. *Q.E.D.*

Proof of Proposition 5: To check whether an efficient equilibrium exists when N_l and N_{-l} are both large enough, suppose that group $-l$ has chosen an efficient appointment rule, i.e., $\beta_{-l}(\boldsymbol{\alpha}_{-l}) = \alpha^*_{-l}(\boldsymbol{\alpha}_{-l})$. Condition (5.1) certainly holds for N_l large enough when:

$$\text{(A.15)} \quad \left[u_0(x) + \underline{\alpha}_l u_l(x) + \alpha^*_{-l}(\boldsymbol{\alpha}_{-l})u_{-l}(x)\right]_{x(0,\alpha^*_{-l}(\boldsymbol{\alpha}_{-l}))}^{x(\alpha^e_l,\alpha^*_{-l}(\boldsymbol{\alpha}_{-l}))} > 0 \quad \forall \boldsymbol{\alpha}_{-l}.$$

By (A.12), the following strict inequality holds when $x(\alpha^e_l, \alpha^*_{-l}(\boldsymbol{\alpha}_{-l})) \neq x(0, \alpha^*_{-l}(\boldsymbol{\alpha}_{-l}))$:

$$\left[u_0(x) + \alpha^e_l u_l(x) + \alpha^*_{-l}(\boldsymbol{\alpha}_{-l})u_{-l}(x)\right]_{x(0,\alpha^*_{-l}(\boldsymbol{\alpha}_{-l}))}^{x(\alpha^e_l,\alpha^*_{-l}(\boldsymbol{\alpha}_{-l}))} > 0 \quad \forall \boldsymbol{\alpha}_{-l}.$$

Thus, (A.15) also holds if $\alpha^e_l - \underline{\alpha}_l$ is small enough and $\underline{\alpha}_l > 0$. Taking expectations over $\boldsymbol{\alpha}_{-l}$ yields (5.1). Individual contributions can then be reconstructed following the proof of sufficiency in Lemma 2 with efficient appointment rules. *Q.E.D.*

Proof of Proposition 6: The mechanism design problem for group l can be written as:

$$(\mathcal{GF}): \quad \max_{\mathcal{U}^i_l,\mathcal{G}_l} \sum_{i=1}^{N_l} \mathbb{E}_{\boldsymbol{\alpha}_l}(\mathcal{U}^i_l(\alpha^i_l)) \text{ subject to (2.4), (2.5), (2.6) and (2.7).}$$

We shall neglect the monotonicity condition, i.e., $\mathbb{E}_{\boldsymbol{\alpha}_l^{-i},\boldsymbol{\alpha}_{-l}}(\Delta u_l(\beta_l(\alpha^i_l, \boldsymbol{\alpha}^{-i}_l), \beta_{-l}(\boldsymbol{\alpha}_{-l})))$ non-decreasing. This condition will be checked *ex post* on the solution of the so relaxed problem. Taking into account (A.2) and the fact that (2.5), (2.6) and (2.7) can be aggregated into a single incentive-feasibility constraint (4.4), we rewrite $(\mathcal{GF})$ as:

$$\max_{\beta_l(\boldsymbol{\alpha}_l)\geq 0} \mathbb{E}_{\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l}}\left(u_0(x(\beta_l(\boldsymbol{\alpha}_l), \beta_{-l}(\boldsymbol{\alpha}_{-l}))) + \alpha^*_l(\boldsymbol{\alpha}_l)u_l(x(\beta_l(\boldsymbol{\alpha}_l), \beta_{-l}(\boldsymbol{\alpha}_{-l}))) + \beta_{-l}(\boldsymbol{\alpha}_{-l})u_{-l}(x(\beta_l(\boldsymbol{\alpha}_l), \beta_{-l}(\boldsymbol{\alpha}_{-l})))\right)$$

subject to (4.4). Let denote by λ_l the non-negative Lagrange multiplier for this constraint. The corresponding Lagrangean satisfies (up to terms independent of $\beta_l(\boldsymbol{\alpha}_l)$):

$$(1+\lambda_l)\mathbb{E}_{\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l}}\left(u_0(x(\beta_l(\boldsymbol{\alpha}_l), \beta_{-l}(\boldsymbol{\alpha}_{-l}))) + \tilde{\beta}_l(\boldsymbol{\alpha}_l, \lambda_l)u_l(x(\beta_l(\boldsymbol{\alpha}_l), \beta_{-l}(\boldsymbol{\alpha}_{-l}))) + \beta_{-l}(\boldsymbol{\alpha}_{-l})u_{-l}(x(\beta_l(\boldsymbol{\alpha}_l), \beta_{-l}(\boldsymbol{\alpha}_{-l})))\right)$$

where we define $\tilde{\beta}_l(\boldsymbol{\alpha}_l, \lambda_l)$ as

$$\text{(A.16)} \quad \tilde{\beta}_l(\boldsymbol{\alpha}_l, \lambda_l) = \frac{1}{N_l}\sum_{i=1}^{N_l} \alpha^i_l - \frac{\lambda_l}{1+\lambda_l}\frac{1-F_l(\alpha^i_l)}{f_l(\alpha^i_l)}.$$

For each possible realization of $\boldsymbol{\alpha}_l$ the optimality condition in $\beta_l(\boldsymbol{\alpha}_l)$ under the constraint $\beta_l(\boldsymbol{\alpha}_l) \geq 0$ writes as follows:

(A.17)

$$\mathbb{E}_{\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l}}\left(\left(u'_0(x(\beta^{sb}_l(\boldsymbol{\alpha}_l), \beta_{-l}(\boldsymbol{\alpha}_{-l}))) + (-1)^l(\tilde{\beta}_l(\boldsymbol{\alpha}_l, \lambda_l) - \beta_{-l}(\boldsymbol{\alpha}_{-l}))\right)\frac{\partial x}{\partial \beta_l}(\beta^{sb}_l(\boldsymbol{\alpha}_l), \beta_{-l}(\boldsymbol{\alpha}_{-l}))\right) \leq 0.$$

We distinguish two cases.

1. $\tilde{\beta}_l(\boldsymbol{\alpha}_l, \lambda_l) \geq 0$. Observe that, by definition, $x(\tilde{\beta}_l(\boldsymbol{\alpha}_l, \lambda_l), \beta_{-l}(\boldsymbol{\alpha}_{-l})) = \varphi((-1)^l(\beta_{-l}(\boldsymbol{\alpha}_{-l}) - \tilde{\beta}_l(\boldsymbol{\alpha}_l, \lambda_l)))$. Thus, (A.17) is satisfied as an equality by

$$\text{(A.18)} \quad \beta^{sb}_l(\boldsymbol{\alpha}_l) = \tilde{\beta}_l(\boldsymbol{\alpha}_l, \lambda_l) \geq 0.$$

2. $\tilde{\beta}_l(\boldsymbol{\alpha}_l,\lambda_l)<0$. Because $x(0,\beta_{-l}(\boldsymbol{\alpha}_{-l}))=\varphi((-1)^l\beta_{-l}(\boldsymbol{\alpha}_{-l})$, we deduce that:

(A.19)
$$u_0'(x(0,\beta_{-l}(\boldsymbol{\alpha}_{-l})))+(-1)^l(\tilde{\beta}_l(\boldsymbol{\alpha}_l,\lambda_l)-\beta_{-l}(\boldsymbol{\alpha}_{-l}))=(-1)^l\tilde{\beta}_l(\boldsymbol{\alpha}_l,\lambda_l)\begin{cases}<0 & \text{if } l=2,\\ >0 & \text{if } l=1.\end{cases}$$

Observe now that:

$$\text{(A.20)}\quad \frac{\partial x}{\partial \beta_1}(\beta_1,\beta_2)<0<\frac{\partial x}{\partial \beta_2}(\beta_1,\beta_2).$$

Putting together (A.19) with (A.20) and using (A.17) yields:

$$\text{(A.21)}\quad \beta_l^{sb}(\boldsymbol{\alpha}_l)=0.$$

Gathering (A.18) and (A.21) finally gives us (7.1).

Moreover, because of the monotonicity condition of the hazard rate, β_l^{sb} is non-decreasing in α_l^i and $\mathcal{U}_l^i$ is convex as requested by Lemma 1. *Q.E.D.*

Proof of Proposition 7: We rewrite (7.2) by means of integrals inside the expectation as:

(A.22)
$$\mathbb{E}_{\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l}}\Big(\int_0^{\alpha_l^*(\boldsymbol{\alpha}_l)}\frac{\partial x}{\partial \beta_l}(\beta,\beta_{-l}(\boldsymbol{\alpha}_{-l}))\Big(u_0'(x(\beta,\beta_{-l}(\boldsymbol{\alpha}_{-l})))+(-1)^l(\alpha_l^*(\boldsymbol{\alpha}_l)-\beta_{-l}(\boldsymbol{\alpha}_{-l}))\Big)d\beta\Big)$$
$$<(-1)^l\mathbb{E}_{\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l}}\left(\left(\frac{1}{N_l}\sum_{i=1}^{N_l}\frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)}\right)\int_0^{\alpha_l^*(\boldsymbol{\alpha}_l)}\frac{\partial x}{\partial \beta_l}(\beta,\beta_{-l}(\boldsymbol{\alpha}_{-l}))d\beta\right).$$

Using the fact that $x(\beta_l,\beta_{-l})=\varphi((-1)^l(\beta_{-l}-\beta_l))$, (A.22) becomes

$$\text{(A.23)}\quad (-1)^l\mathbb{E}_{\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l}}\left(\int_0^{\alpha_l^*(\boldsymbol{\alpha}_l)}(\alpha_l^*(\boldsymbol{\alpha}_l)-\beta)\frac{\partial x}{\partial \beta_l}(\beta,\beta_{-l}(\boldsymbol{\alpha}_{-l}))d\beta\right)$$
$$<(-1)^l\mathbb{E}_{\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l}}\left(\int_0^{\alpha_l^*(\boldsymbol{\alpha}_l)}\left(\frac{1}{N_l}\sum_{i=1}^{N_l}\frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)}\right)\frac{\partial x}{\partial \beta_l}(\beta,\beta_{-l}(\boldsymbol{\alpha}_{-l}))d\beta\right).$$

Using the *Law of Iterated Expectations*, we can rewrite:

$$\mathbb{E}_{\boldsymbol{\alpha}_l}\left(\int_0^{\alpha_l^*(\boldsymbol{\alpha}_l)}(\alpha_l^*(\boldsymbol{\alpha}_l)-\beta)\frac{\partial x}{\partial \beta_l}(\beta,\beta_{-l}(\boldsymbol{\alpha}_{-l}))d\beta\right)$$
$$=\mathbb{E}_{\alpha_l^*}\left(\mathbb{E}_{\boldsymbol{\alpha}_l}\left(\int_0^{\alpha_l^*(\boldsymbol{\alpha}_l)}(\alpha_l^*(\boldsymbol{\alpha}_l)-\beta)\frac{\partial x}{\partial \beta_l}(\beta,\beta_{-l}(\boldsymbol{\alpha}_{-l}))d\beta|\alpha_l^*(\boldsymbol{\alpha}_l)=\alpha_l^*\right)\right)$$
$$=\int_0^{\overline{\alpha}_l}\left(\int_0^{\alpha_l^*}(\alpha_l^*-\beta)\frac{\partial x}{\partial \beta_l}(\beta,\beta_{-l}(\boldsymbol{\alpha}_{-l}))d\beta\right)\phi_{N_l}(\alpha_l^*)d\alpha_l^*.$$

Integrating by parts, this last term becomes:

$$\left[-(1-\Phi_{N_l}(\alpha_l^*))\int_0^{\alpha_l^*}(\alpha_l^*-\beta)\frac{\partial x}{\partial \beta_l}(\beta,\beta_{-l}(\boldsymbol{\alpha}_{-l}))d\beta\right]_0^{\overline{\alpha}_l}$$

$$+\int_0^{\overline{\alpha}_l}(1-\Phi_{N_l}(\alpha_l^*))\left((\alpha_l^*-\alpha_l^*)\frac{\partial x}{\partial\beta_l}(\alpha_l^*,\beta_{-l}(\boldsymbol{\alpha}_{-l}))+\int_0^{\alpha_l^*}\frac{\partial x}{\partial\beta_l}(\beta,\beta_{-l}(\boldsymbol{\alpha}_{-l}))d\beta\right)d\alpha_l^*.$$

Or, simplifying

(A.24)

$$\mathbb{E}_{\boldsymbol{\alpha}_l}\left(\int_0^{\alpha_l^*(\boldsymbol{\alpha}_l)}(\alpha_l^*(\boldsymbol{\alpha}_l)-\beta)\frac{\partial x}{\partial\beta_l}(\beta,\beta_{-l}(\boldsymbol{\alpha}_{-l}))d\beta\right)=\mathbb{E}_{\alpha_l^*}\left(\frac{1-\Phi_{N_l}(\alpha_l^*)}{\phi_{N_l}(\alpha_l^*)}\int_0^{\alpha_l^*}\frac{\partial x}{\partial\beta_l}(\beta,\beta_{-l}(\boldsymbol{\alpha}_{-l}))d\beta\right).$$

Using again the *Law of Iterated Expectations,* we also get:

$$\text{(A.25)}\quad \mathbb{E}_{\boldsymbol{\alpha}_l}\left(\left(\frac{1}{N_l}\sum_{i=1}^{N_l}\frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)}\right)\int_0^{\alpha_l^*(\boldsymbol{\alpha}_l)}\frac{\partial x}{\partial\beta_l}(\beta,\beta_{-l}(\boldsymbol{\alpha}_{-l}))d\beta\right)$$

$$=\mathbb{E}_{\alpha_l^*}\left(\mathbb{E}_{\boldsymbol{\alpha}_l}\left(\left(\frac{1}{N_l}\sum_{i=1}^{N_l}\frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)}\right)\int_0^{\alpha_l^*(\boldsymbol{\alpha}_l)}\frac{\partial x}{\partial\beta_l}(\beta,\beta_{-l}(\boldsymbol{\alpha}_{-l}))d\beta|\alpha_l^*(\boldsymbol{\alpha}_l)=\alpha_l^*\right)\right)$$

$$=\mathbb{E}_{\alpha_l^*}\left(\mathbb{E}_{\boldsymbol{\alpha}_l}\left(\frac{1}{N_l}\sum_{i=1}^{N_l}\frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)}|\alpha_l^*(\boldsymbol{\alpha}_l)=\alpha_l^*\right)\int_0^{\alpha_l^*}\frac{\partial x}{\partial\beta_l}(\beta,\beta_{-l}(\boldsymbol{\alpha}_{-l}))d\beta\right).$$

Using (A.24) and (A.25) and taking expectations over $\boldsymbol{\alpha}_{-l}$, Condition (A.23) becomes:

(A.26)

$$\mathbb{E}_{\alpha_l^*,\boldsymbol{\alpha}_{-l}}\left(\left(\frac{1-\Phi_{N_l}(\alpha_l^*)}{\phi_{N_l}(\alpha_l^*)}-\mathbb{E}_{\boldsymbol{\alpha}_l}\left(\frac{1}{N_l}\sum_{i=1}^{N_l}\frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)}|\alpha_l^*(\boldsymbol{\alpha}_l)=\alpha_l^*\right)\right)\int_0^{\alpha_l^*}(-1)^l\frac{\partial x}{\partial\beta_l}(\beta,\beta_{-l}(\boldsymbol{\alpha}_{-l}))d\beta\right)<0.$$

Observe that this inequality cannot be strict for $N_l=1$ since the first bracketed term is identically null. Suppose thus that $N_l>1$. Observe then that $\int_0^{\alpha_l^*}(-1)^l\mathbb{E}_{\boldsymbol{\alpha}_{-l}}\left(\frac{\partial x}{\partial\beta_l}(\beta,\beta_{-l}(\boldsymbol{\alpha}_{-l}))\right)d\beta$ is increasing in α_l^* while $\frac{1-\Phi_{N_l}(\alpha_l^*)}{\phi_{N_l}(\alpha_l^*)}-\mathbb{E}_{\boldsymbol{\alpha}_l}\left(\frac{1}{N_l}\sum_{i=1}^{N_l}\frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)}|\alpha_l^*(\boldsymbol{\alpha}_l)=\alpha_l^*\right)$ is decreasing in α_l^* from Assumption 2. Hence, those two functions covary negatively.

Since $\underline{\alpha}_l=0$ from Assumption 1 and α_l^* has mean α^e, it can be checked that:

$$\mathbb{E}_{\alpha_l^*}\left(\frac{1-\Phi_{N_l}(\alpha_l^*)}{\phi_{N_l}(\alpha_l^*)}\right)=\alpha^e-\underline{\alpha}_l=\alpha^e.$$

Using the *Law of Iterated Expectations,* we also get:

$$\mathbb{E}_{\alpha_l^*}\left(\mathbb{E}_{\boldsymbol{\alpha}_l}\left(\frac{1}{N_l}\sum_{i=1}^{N_l}\frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)}|\alpha_l^*(\boldsymbol{\alpha}_l)=\alpha_l^*\right)\right)=\mathbb{E}_{\boldsymbol{\alpha}_l}\left(\frac{1}{N_l}\sum_{i=1}^{N_l}\frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)}\right)$$

$$=\mathbb{E}_{\alpha_l^i}\left(\frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)}\right)=\alpha^e-\underline{\alpha}_l=\alpha^e.$$

Therefore, we obtain:

$$\text{(A.27)}\quad \mathbb{E}_{\alpha_l^*}\left(\frac{1-\Phi_{N_l}(\alpha_l^*)}{\phi_{N_l}(\alpha_l^*)}-\mathbb{E}_{\boldsymbol{\alpha}_l}\left(\frac{1}{N_l}\sum_{i=1}^{N_l}\frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)}|\alpha_l^*(\boldsymbol{\alpha}_l)=\alpha_l^*\right)\right)=0.$$

Because of the negative covariance pointed out above, we thus have:

$$\mathbb{E}_{\alpha_l^*}\left(\left(\frac{1-\Phi_{N_l}(\alpha_l^*)}{\phi_{N_l}(\alpha_l^*)}-\mathbb{E}_{\boldsymbol{\alpha}_l}\left(\frac{1}{N_l}\sum_{i=1}^{N_l}\frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)}|\alpha_l^*(\boldsymbol{\alpha}_l)=\alpha_l^*\right)\right)\int_0^{\alpha_l^*}(-1)^l\mathbb{E}_{\boldsymbol{\alpha}_{-l}}\left(\frac{\partial x}{\partial\beta_l}(\beta,\beta_{-l}(\boldsymbol{\alpha}_{-l}))\right)d\beta\right)<$$

$$\mathbb{E}_{\alpha_l^*}\left(\left(\frac{1-\Phi_{N_l}(\alpha_l^*)}{\phi_{N_l}(\alpha_l^*)}-\mathbb{E}_{\boldsymbol{\alpha}_l}\left(\frac{1}{N_l}\sum_{i=1}^{N_l}\frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)}|\alpha_l^*(\boldsymbol{\alpha}_l)=\alpha_l^*\right)\right)\right)\mathbb{E}_{\alpha_l^*}\left(\int_0^{\alpha_l^*}(-1)^l\mathbb{E}_{\boldsymbol{\alpha}_{-l}}\left(\frac{\partial x}{\partial \beta_l}(\beta,\beta_{-l}(\boldsymbol{\alpha}_{-l}))d\beta\right)\right)$$
$$< 0$$

where the last inequality follows from (A.27). Therefore, Condition (A.26) holds. The feasibility condition (4.4) does not hold for $\beta_l(\boldsymbol{\alpha}_l)=\alpha_l^*(\boldsymbol{\alpha}_l)$ and necessarily $\lambda_l>0$. *Q.E.D.*

Proof of Proposition 8: First, we prove a preliminary Lemma.

Lemma A.1

$$\mathbb{E}_{\boldsymbol{\alpha}_{-l}}(T_l(\beta_l,\beta_{-l}^{sb}(\boldsymbol{\alpha}_{-l})))>0 \quad \forall \beta_l>0.$$

Proof of Lemma A.1: From (3.1) and the definition of $x(\beta_l,\beta_{-l}^{sb}(\boldsymbol{\alpha}_{-l}))$, we can write:

$$T_l(\beta_l,\beta_{-l}^{sb}(\boldsymbol{\alpha}_{-l}))=\int_0^{\beta_l}(u_0'(x(\beta,\beta_{-l}^{sb}(\boldsymbol{\alpha}_{-l})))+(-1)^{-l}\beta_{-l}^{sb}(\boldsymbol{\alpha}_{-l}))\frac{\partial x}{\partial \beta_l}(\beta,\beta_{-l}^{sb}(\boldsymbol{\alpha}_{-l}))d\beta$$

and thus

$$T_l(\beta_l,\beta_{-l}^{sb}(\boldsymbol{\alpha}_{-l}))=\int_0^{\beta_l}(-1)^{l+1}\beta\frac{\partial x}{\partial \beta_l}(\beta,\beta_{-l}^{sb}(\boldsymbol{\alpha}_{-l}))d\beta. \tag{A.28}$$

Thus, $T_l(\beta_l,\beta_{-l}^{sb}(\boldsymbol{\alpha}_{-l}))>0$ if $\beta_l>0$. Therefore, $\min\limits_{\boldsymbol{\alpha}_{-l}\in[0,\overline{\alpha}]} T_l(\beta_l,\beta_{-l}^{sb}(\boldsymbol{\alpha}_{-l}))>0$ since the minimum of a continuous function on a compact set is achieved. The proof then ends by taking expectations. *Q.E.D.*

Assumption 1 implies that $\mathbb{E}_{\alpha_l^i}\left(h_l(\alpha_l^i)\right)=\underline{\alpha}_l=0$. Let us make the dependence of the sample mean on N_l explicit, namely $h_{N_l}^*(\boldsymbol{\alpha}_l)=\frac{1}{N_l}\sum_{i=1}^{N_l}h_l(\alpha_l^i)$. For any given $\hat{\lambda}_l\in\mathbb{R}_+$, we now define

$$\hat{\beta}_l(\boldsymbol{\alpha}_l,N_l,\hat{\lambda}_l)\equiv\max\left\{\frac{1}{N_l}\sum_{i=1}^{N_l}\alpha_l^i-\frac{\hat{\lambda}_l}{1+\hat{\lambda}_l}\frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)};0\right\}.$$

Since $|\Delta u_l(\hat{\beta}_l(\cdot,\cdot,\hat{\lambda}_l),\beta_{-l}^{sb})|$ is uniformly bounded in N_l on $[0,\overline{\alpha}_l]\times\mathbb{N}$, (6.1) implies that

$$h_{N_l}^*(\boldsymbol{\alpha}_l)\mathbb{E}_{\boldsymbol{\alpha}_{-l}}\left(\Delta u_l(\hat{\beta}_l(\boldsymbol{\alpha}_l,N_l,\hat{\lambda}_l),\beta_{-l}^{sb}(\boldsymbol{\alpha}_{-l}))\right)\underset{N_l\to+\infty}{\overset{a.s.}{\to}}0. \tag{A.29}$$

Since convergence almost surely implies convergence in law, we get

$$\lim_{N_l\to\infty}\mathbb{E}_{\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l}}\left(h_{N_l}^*(\boldsymbol{\alpha}_l)\Delta u_l(\hat{\beta}_l(\boldsymbol{\alpha}_l,N_l,\hat{\lambda}_l),\beta_{-l}^{sb}(\boldsymbol{\alpha}_{-l}))\right)=0. \tag{A.30}$$

In addition, we have

$$\hat{\beta}_l(\boldsymbol{\alpha}_l,N_l,\hat{\lambda}_l)\underset{N_l\to+\infty}{\overset{a.s.}{\to}}\alpha_l^e-\frac{\hat{\lambda}_l}{1+\hat{\lambda}_l}(\alpha_l^e-\underline{\alpha}_l)=\frac{1}{1+\hat{\lambda}_l}\alpha_l^e>0 \tag{A.31}$$

where the last equality follows from Assumption 1. Therefore, for all $\hat{\lambda}_l<+\infty$, Lemma A.1 implies:

$$\lim_{N_l\to\infty}\mathbb{E}_{\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l}}\left[T_l(\hat{\beta}_l(\boldsymbol{\alpha}_l,N_l,\hat{\lambda}_l),\beta_{-l}^{sb}(\boldsymbol{\alpha}_{-l}))\right]>0. \tag{A.32}$$

It follows from (A.30) and (A.32) that:

$$\lim_{N_l\to\infty} \mathbb{E}_{\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l}}\left[h^*_{N_l}(\boldsymbol{\alpha}_l)\Delta u_l(\hat{\beta}_l(\boldsymbol{\alpha}_l,N_l,\hat{\lambda}_l),\beta^{sb}_{-l}(\boldsymbol{\alpha}_{-l}))-T_l(\hat{\beta}_l(\boldsymbol{\alpha}_l,N_l,\hat{\lambda}_l),\beta^{sb}_{-l}(\boldsymbol{\alpha}_{-l}))\right]<0.$$

Therefore, for any $\hat{\lambda}_l>0$, there exists N^e such that, for $N_l\geq N^e$, the incentive-feasibility condition for group l, namely (4.4), would be violated with the appointment rule $\hat{\beta}_l(\boldsymbol{\alpha}_l,N_l,\hat{\lambda}_l)$. Hence, the Lagrange multiplier $\lambda_l(N_l)$ of the incentive-feasibility condition (4.4) must satisfy $\lambda_l(N_l)\geq\hat{\lambda}_l$ to ensure that the incentive-feasibility condition (4.4) holds as an equality for $\beta^{sb}_l(\boldsymbol{\alpha}_l,N_l)=\max\left\{\frac{1}{N_l}\left(\sum_{i=1}^{N_l}\alpha^i_l-\frac{\lambda_l(N_l)}{1+\lambda_l(N_l)}\frac{1-F_l(\alpha^i_l)}{f_l(\alpha^i_l)}\right);0\right\}$. Since $\hat{\lambda}_l$ is arbitrary, we get:

$$\text{(A.33)}\quad \lim_{N_l\to\infty}\lambda_l(N_l)=+\infty.$$

Condition (A.33) also implies the following convergence in probability for $\beta^{sb}_l(\boldsymbol{\alpha}_l,N_l)$:

$$\beta^{sb}_l(\boldsymbol{\alpha}_l,N_l)\underset{N_l\to+\infty}{\overset{p}{\to}}\max\left\{\mathbb{E}_{\alpha^i_l}\left(h_l(\alpha^i_l)\right);0\right\}=0$$

since, from Assumption 1, $\mathbb{E}_{\alpha^i_l}\left(h_l(\alpha^i_l)\right)=\underline{\alpha}_l=0$. Hence, (7.4) holds. *Q.E.D.*

Proofs of Propositions 11 and 12: Assume that $u_0'''\geq 0$. From Proposition 10, the mapping Λ^*_l is everywhere non-decreasing. Since $0<\lambda_{-l}$, we thus get $\tilde{\lambda}_l=\Lambda^*_l(0)\leq\lambda_l$. Finally, the opposite condition holds if $u_0'''\leq 0$ which ends the proof of Proposition 11. The proof of Proposition 12 is similar and omitted. *Q.E.D.*

Proof of Proposition 13: Let consider an equilibrium of the game obtained when the common size of the groups is $N_1=N_2=N$. It corresponds to the appointment rules $(\beta^{sb}_l(\cdot,N),\beta^{sb}_{-l}(\cdot,N))$ where the dependence on N is again made explicit. It is easy to check that the limiting behaviors described in Proposition 8 still apply when both groups have variable sizes:

$$\text{(A.34)}\quad \lim_{N\to\infty}\lambda_l(N)=+\infty \text{ and } \beta^{sb}_l(\boldsymbol{\alpha}_l,N)\underset{N\to+\infty}{\overset{p}{\to}}0\quad\forall l\in\{1,2\}.$$

Reminding that the ideal point of PM is 0 and that the function $x(\beta^{sb}_l(\boldsymbol{\alpha}_l,N),\beta^{sb}_{-l}(\boldsymbol{\alpha}_{-l},N))$ is continuous, (A.34) implies the following convergences in probability:

$$\Delta u_l(\beta^{sb}_l(\boldsymbol{\alpha}_l,N),\beta^{sb}_{-l}(\boldsymbol{\alpha}_{-l},N))\underset{N\to+\infty}{\overset{p}{\to}}0.$$

Inserting (A.34) into (A.28), we also deduce that

$$T_l(\beta^{sb}_l(\boldsymbol{\alpha}_l,N),\beta^{sb}_{-l}(\boldsymbol{\alpha}_{-l},N))\underset{N\to+\infty}{\overset{p}{\to}}0.$$

Defining group l's overall payoff as

$$\mathcal{U}^{sb}_l(\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l},N)=\alpha^*_N(\boldsymbol{\alpha}_l)\Delta u_l(\beta^{sb}_l(\boldsymbol{\alpha}_l,N),\beta^{sb}_{-l}(\boldsymbol{\alpha}_{-l},N))-T_l(\beta^{sb}_l(\boldsymbol{\alpha}_l,N),\beta^{sb}_{-l}(\boldsymbol{\alpha}_{-l},N)).$$

We thus have

$$\mathcal{U}^{sb}_l(\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l},N)\underset{N\to+\infty}{\overset{p}{\to}}0$$

and

$$\text{(A.35)}\quad \lim_{N\to\infty}\mathbb{E}_{\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l}}(\mathcal{U}^{sb}_l(\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l},N))=0$$

Suppose now that both groups form under complete information. When the profile of preferences is $(\boldsymbol{\alpha}_l, \boldsymbol{\alpha}_{-l})$, group l's payoff writes as:

$$\mathcal{U}_l^{fb}(\boldsymbol{\alpha}_l, \boldsymbol{\alpha}_{-l}, N) = \alpha_N^*(\boldsymbol{\alpha}_l)\Delta u_l(\alpha_N^*(\boldsymbol{\alpha}_l), \alpha_N^*(\boldsymbol{\alpha}_{-l})) - T_l(\alpha_N^*(\boldsymbol{\alpha}_l), \alpha_N^*(\boldsymbol{\alpha}_{-l}))$$

or

$$\mathcal{U}_l^{fb}(\boldsymbol{\alpha}_l, \boldsymbol{\alpha}_{-l}, N) = W(\alpha_N^*(\boldsymbol{\alpha}_l), \alpha_N^*(\boldsymbol{\alpha}_{-l})) - W(0, \alpha_N^*(\boldsymbol{\alpha}_{-l})) = \int_0^{\alpha_N^*(\boldsymbol{\alpha}_l)} (-1)^l x(\alpha, \alpha_N^*(\boldsymbol{\alpha}_l)) d\alpha.$$

Thus, we get

$$\mathcal{U}_l^{fb}(\boldsymbol{\alpha}_l, \boldsymbol{\alpha}_{-l}, N) \underset{N\to+\infty}{\overset{p}{\to}} W(\alpha^e, \alpha^e) - W(0, \alpha^e) = \int_0^{\alpha^e} (-1)^l x(\alpha, \alpha^e) d\alpha < 0$$

since $sgn(x(\alpha, \alpha^e)) = sgn((-1)^{l+1})$ when $\alpha \in [0, \alpha^e]$. Taking expectations yields:

$$\text{(A.36)} \quad \lim_{N\to\infty} \mathbb{E}_{\boldsymbol{\alpha}_l, \boldsymbol{\alpha}_{-l}} \left(\mathcal{U}_l^{fb}(\boldsymbol{\alpha}_l, \boldsymbol{\alpha}_{-l}, N) \right) < 0.$$

The result then directly follows by comparing the r.h.s. of (A.35) and (A.36). *Q.E.D.*

Proof of Proposition 14: We show that PM is always worse off under incomplete information in the *ex post* sense. The *ex ante* result will directly follow by taking expectations. The total contribution $T_0(\beta_l, \beta_{-l})$ received by PM and his payoff write respectively as:

$$T_0(\beta_l, \beta_{-l}) = [\beta_{-l}u_{-l}(x) + u_0(x)]_{x(\beta_l,\beta_{-l})}^{x(0,\beta_{-l})} + [\beta_l u_l(x) + u_0(x)]_{x(\beta_l,\beta_{-l})}^{x(\beta_l,0)}$$

and

$$\mathcal{U}_0(\beta_l, \beta_{-l}) = u_0(x(\beta_l, \beta_{-l})) + T_0(\beta_l, \beta_{-l}).$$

Differentiating with respect to β_l, we get:

$$\frac{\partial \mathcal{U}_0}{\partial \beta_l}(\beta_l, \beta_{-l}) = u_l(x(\beta_l, 0)) - u_l(x(\beta_l, \beta_{-l})) + \frac{\partial x}{\partial \beta_l}(\beta_l, 0) \left(u_0'(x(\beta_l, 0)) + \beta_l u_l'(x(\beta_l, 0)) \right)$$
$$- \frac{\partial x}{\partial \beta_l}(\beta_l, \beta_{-l}) \left(u_0'(x(\beta_l, \beta_{-l})) + \beta_l u_l'(x(\beta_l, \beta_{-l})) + \beta_{-l} u_{-l}'(x(\beta_l, \beta_{-l})) \right).$$

Using the definitions of $x(\beta_l, \beta_{-l})$ and $x(0, \beta_{-l})$, the latter expression becomes:

$$\frac{\partial \mathcal{U}_0}{\partial \beta_l}(\beta_l, \beta_{-l}) = u_l(x(\beta_l, 0)) - u_l(x(\beta_l, \beta_{-l})) = (-1)^l (x(\beta_l, 0) - x(\beta_l, \beta_{-l})).$$

It follows that:

$$\frac{\partial \mathcal{U}_0}{\partial \beta_l}(\beta_l, \beta_{-l}) > 0 \quad \forall \beta_l > 0.$$

Therefore, for all $\beta_l < \alpha_l^*$, $\beta_{-l} < \alpha_{-l}^*$, the following string of inequalities holds:

$$\mathcal{U}_0(\beta_l, \beta_{-l}) < \mathcal{U}_0(\alpha_l^*, \beta_{-l}) < \mathcal{U}_0(\alpha_l^*, \alpha_{-l}^*).$$

In particular, we may take $\beta_l = \beta_l^{sb}(\boldsymbol{\alpha}_l)$ and $\beta_{-l} = \beta_{-l}^{sb}(\boldsymbol{\alpha}_{-l})$. Taking expectations, and taking into account that those inequalities are strict on a set of positive measure, we finally obtain:

$$\mathbb{E}_{\boldsymbol{\alpha}_l, \boldsymbol{\alpha}_{-l}}(\mathcal{U}_0(\beta_l^{sb}(\boldsymbol{\alpha}_{-l}), \beta_{-l}^{sb}(\boldsymbol{\alpha}_{-l}))) < \mathbb{E}_{\boldsymbol{\alpha}_l, \boldsymbol{\alpha}_{-l}}(\mathcal{U}_0(\alpha_l^*(\boldsymbol{\alpha}_l), \beta_{-l}^{sb}(\boldsymbol{\alpha}_{-l}))) < \mathbb{E}_{\boldsymbol{\alpha}_l, \boldsymbol{\alpha}_{-l}}(\mathcal{U}_0(\alpha_l^*(\boldsymbol{\alpha}_l), \alpha_{-l}^*(\boldsymbol{\alpha}_{-l})))$$

which ends the proof. *Q.E.D.*

ONLINE APPENDIX B: PROOFS OF SECTION 9

PROOF OF PROPOSITION 9: Let us first define for any non-negative value λ, $\hat{\beta}_l(\boldsymbol{\alpha}_l, \lambda)$ as:

$$\hat{\beta}_l(\boldsymbol{\alpha}_l, \lambda) = \max\left\{\tilde{\beta}_l(\boldsymbol{\alpha}_l, \lambda); 0\right\}. \tag{C.1}$$

Observe that $\hat{\beta}_l(\boldsymbol{\alpha}_l, 0) = \alpha_l^*$ and $\hat{\beta}_l(\boldsymbol{\alpha}_l, \lambda_l) = \beta^{sb}(\boldsymbol{\alpha}_l)$ when λ_l is the equilibrium value of the Lagrange multiplier for group l. Define the function $\mathcal{W}_l$ from $\mathbb{R}_+ \times \mathbb{R}_+$ on $\mathbb{R}$ as:

$$\mathcal{W}_l(\lambda, \lambda_{-l}) = \mathbb{E}_{\boldsymbol{\alpha}_l, \boldsymbol{\alpha}_{-l}}\left(\Delta W_l(h_l^*(\boldsymbol{\alpha}_l), \hat{\beta}_l(\boldsymbol{\alpha}_l, \lambda), \hat{\beta}_{-l}(\boldsymbol{\alpha}_{-l}, \lambda_{-l}))\right) \tag{C.2}$$

where

$$\Delta W_l(\tilde{\beta}_l, \beta_l, \beta_{-l}) = \left[u_0(x) + \tilde{\beta}_l u_l(x) + \beta_{-l}u_{-l}(x)\right]_{x(0,\beta_{-l})}^{x(\beta_l,\beta_{-l})}. \tag{C.3}$$

LEMMA C.1 *The mapping $\Lambda_l^*(\lambda_{-l})$ such that $\mathcal{W}_l(\Lambda_l^*(\lambda_{-l}), \lambda_{-l}) = 0$ is defined on $\mathbb{R}_+$, single-valued and continuous.*

PROOF OF LEMMA C.1: We start with three preliminary results.

RESULT C.1

$$\frac{\partial \mathcal{W}_l}{\partial \lambda}(\lambda, \lambda_{-l}) > 0. \tag{C.4}$$

PROOF OF RESULT C.1: It can be shown that $\mathcal{W}_l(\lambda, \lambda_{-l})$ is differentiable in λ. Differentiating (C.2) with respect to λ, we find a.e.:

$$\frac{\partial \mathcal{W}_l}{\partial \lambda}(\lambda, \lambda_{-l}) = (-1)^l \mathbb{E}_{\boldsymbol{\alpha}_l, \boldsymbol{\alpha}_{-l}}\left(\left(h_l^*(\boldsymbol{\alpha}_l) - \hat{\beta}_l(\boldsymbol{\alpha}_l, \lambda)\right)\frac{\partial x}{\partial \beta_l}(\hat{\beta}_l(\boldsymbol{\alpha}_l, \lambda), \hat{\beta}_{-l}(\boldsymbol{\alpha}_{-l}, \lambda_{-l}))\frac{\partial \hat{\beta}_l}{\partial \lambda}(\boldsymbol{\alpha}_l, \lambda)\right). \tag{C.5}$$

From (C.1), we get $\hat{\beta}_l(\boldsymbol{\alpha}_l, \lambda) \geq h_l^*(\boldsymbol{\alpha}_l)$ with a strict inequality since $\lambda < +\infty$. From (C.1), we also get: $\frac{\partial \hat{\beta}_l}{\partial \lambda}(\boldsymbol{\alpha}_l, \lambda) \leq 0$ at all points of differentiability, a.e.. Gathering those facts with (A.20) and inserting into (C.5) yields (C.4). *Q.E.D.*

RESULT C.2

$$\lim_{\lambda \to +\infty} \mathcal{W}_l(\lambda, \lambda_{-l}) > 0. \tag{C.6}$$

PROOF OF RESULT C.2: Define now $\beta_l^\infty(\boldsymbol{\alpha}_l) = \max\{h_l^*(\boldsymbol{\alpha}_l); 0\}$. We can compute:

$$\lim_{\lambda \to +\infty} \mathcal{W}_l(\lambda, \lambda_{-l}) = \mathbb{E}_{\boldsymbol{\alpha}_l, \boldsymbol{\alpha}_{-l}}\left(\Delta W_l(h_l^*(\boldsymbol{\alpha}_l), \beta_l^\infty(\boldsymbol{\alpha}_l), \hat{\beta}_{-l}(\boldsymbol{\alpha}_{-l}, \lambda_{-l}))\right)$$

When $\beta_l^\infty(\boldsymbol{\alpha}_l) = h_l^*(\boldsymbol{\alpha}_l) > 0$, $x(\beta_l^\infty(\boldsymbol{\alpha}_l), \beta_{-l})$ is the unique maximizer of $u_0(x) + \beta_l^\infty(\boldsymbol{\alpha}_l)u_l(x) + \beta_{-l}u_{-l}(x)$. From which it follows that $\Delta W_l(h_l^*(\boldsymbol{\alpha}_l), \beta_l^\infty(\boldsymbol{\alpha}_l), \hat{\beta}_{-l}(\boldsymbol{\alpha}_{-l}, \lambda_{-l})) > 0$. Since $\beta_l^\infty(\boldsymbol{\alpha}_l) > 0$ on a set of positive measure, it follows that (C.6) holds. *Q.E.D.*

Result C.3 *Suppose that Assumptions 1 and 2 both hold, then we have:*

(C.7) $\mathcal{W}_l(0,\lambda_{-l}) < 0.$

Proof of Result C.3: Taking into account $\beta_l(\boldsymbol{\alpha}_l,0)=\alpha_l^*(\boldsymbol{\alpha}_l)>0$, (C.7) amounts to Condition (7.2) which holds, from Proposition 7, when Assumptions 1 and 2 are both satisfied. *Q.E.D.*

Putting together Results C.1, C.2 and C.3, there always exists a unique solution $\lambda_l>0$ to $\mathcal{W}_l(\lambda_l,\lambda_{-l})=0$. Λ_l^* is thus single-valued. The mappings Λ_l^* is also continuous on $[0,+\infty)$. Indeed, consider any converging sequence λ_{-l}^n, and denote $\overline{\lambda}_{-l}=\lim_{n\to+\infty}\lambda_{-l}^n$. We want to show that

(C.8) $$\lim_{n\to+\infty}\Lambda_l^*(\lambda_{-l}^n)=\Lambda_l^*(\overline{\lambda}_{-l}).$$

Observe first that $\Lambda_l^*(\lambda_{-l}^n)$ is bounded. Indeed, if it was not the case, there would exist a subsequence $\Lambda_l^*(\lambda_{-l}^{\varphi(n)})$, where φ is an increasing function from $\mathbb{N}$ into $\mathbb{N}$, such that $\lim_{n\to+\infty}\Lambda_l^*(\lambda_{-l}^{\varphi(n)})=+\infty$. From the fact that $\mathcal{W}_l(\Lambda_l^*(\lambda_{-l}^{\varphi(n)}),\lambda_{-l}^{\varphi(n)})=0$ for all n, it would follow that $\mathcal{W}_l(+\infty,\overline{\lambda}_{-l})=0$. This is a contradiction since $\mathcal{W}_l(+\infty,\overline{\lambda}_{-l})>0$. Second, consider any converging subsequence $\Lambda_l^*(\lambda_{-l}^{\varphi(n)})$ (the Bolzano-Weirstrass Theorem guarantees existence of such a subsequence), and define $\lambda_l^s=\lim_{n\to+\infty}\Lambda_l^*(\lambda_{-l}^{\varphi(n)})$. It is again the case that $\mathcal{W}_l(\lambda_l^s,\overline{\lambda}_{-l})=0$. This implies that $\lambda_l^s=\Lambda_l^*(\overline{\lambda}_{-l})$. We have shown that $\Lambda_l^*(\lambda_{-l}^n)$ is a bounded sequence, such that all converging subsequences have the same limit $\Lambda_l^*(\overline{\lambda}_{-l})$. Thus, $\lim_{n\to+\infty}\Lambda_l^*(\lambda_{-l}^n)$ exists and (C.8) holds. *Q.E.D.*

Since $\lim_{\lambda_{-l}\to+\infty}\frac{\lambda_{-l}}{1+\lambda_{-l}}=1$, and $\lim_{\lambda_{-l}\to+\infty}\hat{\beta}_{-l}(\boldsymbol{\alpha}_{-l},\lambda_{-l})=\max\{h^*(\boldsymbol{\alpha}_{-l});0\}$, an argument similar to that in Results C.1, C.2 and C.3 shows that $\lim_{\lambda_{-l}\to+\infty}\Lambda_l^*(\lambda_{-l})$ exists. Λ_l^* is thus bounded over $\mathbb{R}_+$. There exists $A_l>0$ such that for all $\lambda_{-l}\geq 0$, $\Lambda_l^*(\lambda_{-l})\leq A_l$. We now define the function ζ on the domain $[0,A_l]\times[0,A_{-l}]$ as $\zeta(\lambda_l,\lambda_{-l})=(\Lambda_l^*(\lambda_{-l}),\Lambda_{-l}^*(\lambda_l))$. The function ζ is continuous on a compact set and onto. From Brouwer's Fixed Point Theorem, it has a fixed point (λ_l,λ_{-l}) which gives us a dual representation of the equilibrium of the game. Moreover, Proposition 7 that holds when Assumptions 1 and 2 are satisfied implies that $\lambda_l>0$. *Q.E.D.*

Proof of Proposition 10: It can be shown that $\mathcal{W}_l(\lambda,\lambda_{-l})$ is differentiable in λ_{-l} (even though $\hat{\beta}_{-l}(\boldsymbol{\alpha}_{-l},\lambda_{-l})$ is itself just differentiable in λ_{-l} everywhere except when $\hat{\beta}_{-l}(\boldsymbol{\alpha}_{-l},\lambda_{-l})=0$). From the Theorem of Implicit Functions, we thus have:

(C.9) $$\frac{\partial\Lambda_l^*}{\partial\lambda_{-l}}(\lambda_{-l})=-\frac{\frac{\partial\mathcal{W}_l}{\partial\lambda_{-l}}(\Lambda_l^*(\lambda_{-l}),\lambda_{-l})}{\frac{\partial\mathcal{W}_l}{\partial\lambda_l}(\Lambda_l^*(\lambda_{-l}),\lambda_{-l})}$$

where the denominator is positive from Result C.2. The numerator is given by

$$\frac{\partial\mathcal{W}_l}{\partial\lambda_{-l}}(\lambda,\lambda_{-l})=\mathbb{E}_{\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l}}\left(\frac{\partial\hat{\beta}_{-l}}{\partial\lambda_{-l}}(\boldsymbol{\alpha}_{-l},\lambda_{-l})\frac{\partial\Delta W_l}{\partial\beta_{-l}}(h_l^*(\boldsymbol{\alpha}_l),\hat{\beta}_l(\boldsymbol{\alpha}_l,\lambda),\hat{\beta}_{-l}(\boldsymbol{\alpha}_{-l},\lambda_{-l})))\right).$$

Differentiating (C.3) with respect to β_{-l}, and simplifying using (2.3), we find:

(C.10)

$$\frac{\partial \Delta W_l}{\partial \beta_{-l}}(\tilde{\beta}_l, \beta_l, \beta_{-l}) = (-1)^{l+1}\left((\beta_l - \tilde{\beta}_l)\frac{\partial x}{\partial \beta_{-l}}(\beta_l, \beta_{-l}) + \tilde{\beta}_l \frac{\partial x}{\partial \beta_{-l}}(0, \beta_{-l}) + x(\beta_l, \beta_{-l}) - x(0, \beta_{-l})\right).$$

From (2.3), we also get:

$$\text{(C.11)} \quad \frac{\partial x}{\partial \beta_l}(\beta_l, \beta_{-l}) + \frac{\partial x}{\partial \beta_{-l}}(\beta_l, \beta_{-l}) = 0 \quad \forall (\beta_l, \beta_{-l}).$$

Differentiating (C.10) with respect to β_l, and using (C.11) to simplify the expression, we find:

$$\frac{\partial^2 \Delta W_l}{\partial \beta_l \partial \beta_{-l}}(\tilde{\beta}_l, \beta_l, \beta_{-l}) = (-1)^{l+1}(\beta_l - \tilde{\beta}_l)\frac{\partial^2 x}{\partial \beta_l \partial \beta_{-l}}(\beta_l, \beta_{-l}).$$

From (2.3), we know that $sgn\left(\frac{\partial^2 x}{\partial \beta_l \partial \beta_{-l}}(\beta_l, \beta_{-l})\right) = -sgn(u_0''')$. Hence, $sgn\left((-1)^{l+1}\frac{\partial^2 \Delta W_l}{\partial \beta_l \partial \beta_{-l}}(\tilde{\beta}_l, \beta_l, \beta_{-l})\right) = -sgn(u_0''')$ when $\beta_l \geq \tilde{\beta}_l$. Since $\frac{\partial \Delta W_l}{\partial \beta_{-l}}(\tilde{\beta}_l, 0, \beta_{-l}) = 0$, we finally get $sgn\left((-1)^{l+1}\frac{\partial \Delta W_l}{\partial \beta_{-l}}(\tilde{\beta}_l, \beta_l, \beta_{-l})\right) = -sgn(u_0''')$ for $\beta_l \geq \tilde{\beta}_l$. From (A.20) and the fact that $\hat{\beta}_l(\boldsymbol{\alpha}_l, \lambda_l) \geq h_l^*(\boldsymbol{\alpha}_l)$, we finally obtain:

$$\text{(C.12)} \quad \frac{\partial \mathcal{W}_l}{\partial \lambda_{-l}}(\lambda, \lambda_{-l}) \geq 0 \text{ (resp. } \geq 0) \ \Leftrightarrow u_0''' \geq 0 \text{ (resp. } \leq 0).$$

Inserting into (C.9) gives the result. *Q.E.D.*

ONLINE APPENDIX C: CONGRUENT GROUPS

Consider now the case where groups have *congruent preferences*. To mirror our previous analysis, we suppose that $u_1(x) = u_2(x) = x$ for all $x \in \mathcal{X}$. We first need to come back on the specification of payoffs in the common agency stage of the game. With congruent interest groups, the cooperative game constructed by Laussel and Le Breton (2001) turns out to be *super-additive*. Indeed, for any profile of lobbyists' preferences $(\beta_1, \beta_2) \in \mathbb{R}_+^2$, we now have:

$$W(\beta_1, \beta_2) + W(0,0) \geq W(\beta_1, 0) + W(0, \beta_2) \quad \forall (\beta_1, \beta_2) \in \mathbb{R}_+^2$$

where again $W(0,0) = 0$. Laussel and Le Breton (2001) demonstrated that the associated common agency game has the so-called *no rent property*, i.e., in all truthful continuation equilibria, the surplus of PM is always fully extracted by lobbyists. The lobbyists' payoffs lie in an interval with non-empty interior which is fully determined by the following constraints:

$$\text{(B.1)} \quad V_l(\beta_l, \beta_{-l}) \leq W(\beta_1, \beta_2) - W(0, \beta_{-l}) \quad \forall l \in \{1, 2\},$$

$$\text{(B.2)} \quad V_1(\beta_1, \beta_2) + V_2(\beta_2, \beta_1) = W(\beta_1, \beta_2).$$

The choice of the optimal appointment rules, either under complete or asymmetric information, of course depends on how the lobbyists' payoffs are precisely determined. To highlight new phenomena that might arise with congruent groups, we assume that those payoffs are given by the lobbyists' *Shapley Values* since this allocation satisfies both (B.1) and (B.2), namely:

$$\text{(B.3)} \quad V_l(\beta_l, \beta_{-l}) = \frac{1}{2}\left(W(\beta_1, \beta_2) - W(0, \beta_{-l}) + W(\beta_l, 0)\right).$$

Using (2.2), we retrieve the expression of the equilibrium payment made by lobbyist l:

$$\text{(B.4)} \quad T_l(\beta_l, \beta_{-l}) = \beta_l x(\beta_l, \beta_{-l}) - \frac{1}{2}\left(W(\beta_1, \beta_2) - W(0, \beta_{-l}) + W(\beta_l, 0)\right).$$

These payments are not VCG.[39] It is no longer a dominant strategy for each group to pass to its lobbyist the aggregate preferences of the group. Each group manipulates these preferences even under complete information. There is now inter-group free riding.

Running Example (Continued). To illustrate, consider the case of quadratic preferences. The policy chosen by PM at the last stage of the game is $x(\beta_1,\beta_2)=\frac{\beta_1+\beta_2}{\beta_0}$ while coalitional payoffs are given by $W(\beta_1,\beta_2)=\frac{(\beta_1+\beta_2)^2}{2\beta_0}$. Under complete information, each group l endows its lobbyist with an objective β_l that maximizes the net surplus of the group, taking as given its conjectures on similar choices made by group $-l$ and that preferences in that competing group remain unknown. Because mechanisms for groups formation are secret, members of group $-l$ conjecture the formation of group l even if it may be vetoed off equilibrium. Following veto, the policy chosen remains $x(0,\beta_{-l}(\boldsymbol{\alpha}_{-l}))$ where $\beta_{-l}(\boldsymbol{\alpha}_{-l})$ represents the preferences given to lobbyist $-l$. The expected gain of an individual with type α_l^i who belongs to group l is thus:

$$\mathbb{E}_{\boldsymbol{\alpha}_l^{-i}}\left(\frac{\alpha_l^i}{N_l}\mathbb{E}_{\boldsymbol{\alpha}_{-l}}\left((x(\beta_l(\boldsymbol{\alpha}_l),\beta_{-l}(\boldsymbol{\alpha}_{-l}))-x(0,\beta_{-l}(\boldsymbol{\alpha}_{-l})))-t_l^i(\boldsymbol{\alpha}_l)\right)\right).$$

Aggregating those expressions over the whole group l, $\beta_l(\boldsymbol{\alpha}_l)$ maximizes:

$$\mathbb{E}_{\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l}}\left(\alpha_l^*(\boldsymbol{\alpha}_l)(x(\beta_l(\boldsymbol{\alpha}_l),\beta_{-l}(\boldsymbol{\alpha}_{-l}))-x(0,\beta_{-l}(\boldsymbol{\alpha}_{-l})))-T_l(\beta_l(\boldsymbol{\alpha}_l),\beta_{-l}(\boldsymbol{\alpha}_{-l}))\right).$$

Using (B.4) yields the expression of group l's overall contribution:

$$T_l(\beta_l(\boldsymbol{\alpha}_l),\beta_{-l}(\boldsymbol{\alpha}_{-l}))=\beta_l(\boldsymbol{\alpha}_l)x(\beta_l(\boldsymbol{\alpha}_l),\beta_{-l}(\boldsymbol{\alpha}_{-l}))-\frac{1}{2}\left(\frac{(\beta_l(\boldsymbol{\alpha}_l)+\beta_{-l}(\boldsymbol{\alpha}_{-l}))^2}{2\beta_0}-\frac{\beta_{-l}^2(\boldsymbol{\alpha}_{-l})}{2\beta_0}+\frac{\beta_l^2(\boldsymbol{\alpha}_l)}{2\beta_0}\right)$$

or

$$T_l(\beta_l(\boldsymbol{\alpha}_l),\beta_{-l}(\boldsymbol{\alpha}_{-l}))=\frac{1}{2}\beta_l(\boldsymbol{\alpha}_l)(\beta_l(\boldsymbol{\alpha}_l)+\beta_{-l}(\boldsymbol{\alpha}_{-l})).$$

The (necessary and sufficient) first-order condition for maximization yields:

$$\beta_l^*(\boldsymbol{\alpha}_l)=\alpha_l^*(\boldsymbol{\alpha}_l)-\frac{1}{2}\mathbb{E}_{\boldsymbol{\alpha}_{-l}}(\beta_{-l}^*(\boldsymbol{\alpha}_{-l})).$$

For a symmetric equilibrium, assuming symmetric distributions, we thus obtain:

$$\text{(B.5)}\qquad \beta_l^*(\boldsymbol{\alpha}_l)=\alpha_l^*(\boldsymbol{\alpha}_l)-\frac{\alpha^e}{3}$$

where we also assume $\underline{\alpha}\geq\frac{\alpha^e}{3}\Leftrightarrow 5\underline{\alpha}>\overline{\alpha}$ to ensure that $\beta_l^*(\boldsymbol{\alpha}_l)$ remains non-negative.

From (B.5), each group chooses a lobbyist with moderate preferences. There is now inter-group free riding. The policy that ends up being chosen by PM is obviously lower than the first-best policy that would have been chosen had groups merged into a single entity so as to perfectly pass their preferences on PM :

$$x(\beta_l^*(\boldsymbol{\alpha}_l),\beta_{-l}^*(\boldsymbol{\alpha}_{-l}))=x(\alpha_l^*(\boldsymbol{\alpha}_l),\alpha_{-l}^*(\boldsymbol{\alpha}_{-l}))-\frac{2\alpha^e}{3\beta_0}<x(\alpha_l^*(\boldsymbol{\alpha}_l),\alpha_{-l}^*(\boldsymbol{\alpha}_{-l})).$$

■

[39] Even if we were to choose within the range of allocations defined by (B.1) and (B.2) another allocation than that defined with Shapley values, manipulations would still arise. It is indeed well-known that, in those contexts, there is dominant strategy mechanism that implements the first-best allocation and extracts all surplus from PM. Furosawa and Konishi (2011) find a similar result in a more specific game.

Under asymmetric information within group l, the net utility of an individual with type α_l^i when the rule $\beta_l^*(\boldsymbol{\alpha}_l)$ is still adopted within group l is thus:

(B.6)
$$\mathcal{U}_l(\alpha_l^i) = \mathbb{E}_{\boldsymbol{\alpha}_l^{-i}}\left(\frac{\alpha_l^i}{N_l}\mathbb{E}_{\boldsymbol{\alpha}_{-l}}(x(\beta_l^*(\boldsymbol{\alpha}_l), \beta_{-l}^*(\boldsymbol{\alpha}_{-l})) - x(0, \beta_{-l}^*(\boldsymbol{\alpha}_{-l}))) - t_l^i(\alpha_l^i, \boldsymbol{\alpha}_l^{-i})\right) \quad \forall \alpha_l^i \in \Omega_l.$$

We do not expect that an efficient equilibrium exists under asymmetric information. Indeed, free riding already bites across groups even under complete information. Yet, we might still be interested in determining conditions such that intra-group free riding does not add inefficiencies on top of those already brought by inter-group free riding. Proceeding as in the case of conflicting interests (Condition (5.1)), we may obtain a condition ensuring that the decision $x(\beta_l^*(\boldsymbol{\alpha}_l), \beta_{-l}^*(\boldsymbol{\alpha}_{-l}))$ remains implementable even under asymmetric information as:

(B.7)
$$\mathbb{E}_{\boldsymbol{\alpha}_l, \boldsymbol{\alpha}_{-l}}\left(\alpha_l^*(\boldsymbol{\alpha}_l)(x(\beta_l^*(\boldsymbol{\alpha}_l), \beta_{-l}^*(\boldsymbol{\alpha}_{-l})) - x(0, \beta_{-l}^*(\boldsymbol{\alpha}_{-l}))) - T(\beta_l^*(\boldsymbol{\alpha}_l), \beta_{-l}^*(\boldsymbol{\alpha}_{-l}))\right)$$
$$\geq \mathbb{E}_{\boldsymbol{\alpha}_l, \boldsymbol{\alpha}_{-l}}\left(\frac{1}{N_l}\left(\sum_{i=1}^{N_l}\frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)}\right)(x(\beta_l^*(\boldsymbol{\alpha}_l), \beta_{-l}^*(\boldsymbol{\alpha}_{-l})) - x(0, \beta_{-l}^*(\boldsymbol{\alpha}_{-l})))\right).$$

The r.h.s. above is by now familiar. It represents the expected information rent left to all members of group l. The l.h.s. is the expected net gain from group formation given the continuation equilibrium and payments.

Had groups cooperated when dealing with PM, inter-group free riding would disappear. The efficient decision $x(\alpha_l^*(\boldsymbol{\alpha}_l), \alpha_{-l}^*(\boldsymbol{\alpha}_{-l})) = \frac{1}{\beta_0}(\alpha_l^*(\boldsymbol{\alpha}_l) + \boldsymbol{\alpha}_{-l}^*(\boldsymbol{\alpha}_{-l}))$ would be implemented while PM would choose his ideal point, namely 0, when the merged group does not organize. The incentive-feasibility condition would now become:

(B.8)
$$\mathbb{E}_{\boldsymbol{\alpha}_l, \boldsymbol{\alpha}_{-l}}\left(u_0(x(\alpha_l^*(\boldsymbol{\alpha}_l), \alpha_{-l}^*(\boldsymbol{\alpha}_{-l}))) + (\alpha_l^*(\boldsymbol{\alpha}_l) + \boldsymbol{\alpha}_{-l}^*(\boldsymbol{\alpha}_{-l}))x(\alpha_l^*(\boldsymbol{\alpha}_l), \alpha_{-l}^*(\boldsymbol{\alpha}_{-l}))\right)$$
$$\geq \mathbb{E}_{\boldsymbol{\alpha}_l, \boldsymbol{\alpha}_{-l}}\left(\left(\frac{1}{N_l}\left(\sum_{i=1}^{N_l}\frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)}\right) + \frac{1}{N_{-l}}\left(\sum_{j=1}^{N_{-l}}\frac{1-F_{-l}(\alpha_j)}{f_{-l}(\alpha_j)}\right)\right)x(\alpha_l^*(\boldsymbol{\alpha}_l), \alpha_{-l}^*(\boldsymbol{\alpha}_{-l}))\right) \quad \forall l \in \{1, 2\}.$$

The comparison of the incentive-feasibility conditions (B.7) and (B.8) already highlights two important forces. On the one hand, the net gains of forming is certainly greater in the case of a merger of the two groups since the *status quo* entails no production at all while its formation induces an efficient policy. Instead, with two groups, each of them can free ride and benefit from the policy induced by the sole contribution of its rival. Overall the gains from forming for those two groups are certainly lower. On the other hand, with a merger, the *status quo* if that merged group does not form is the null policy that is chosen by PM on her own. This means that, with groups merging, the overall information rents that must be distributed are also quite large.

Running Example (Continued). For both groups, types are symmetrically and uniformly distributed on the same interval $\Theta = [\underline{\alpha}, \overline{\alpha}]$ with mean $\alpha^e = \frac{\underline{\alpha}+\overline{\alpha}}{2}$ and variance $\frac{\Delta\alpha^2}{12}$ (where also $\Delta\alpha = \overline{\alpha} - \underline{\alpha}$ and $\overline{\alpha} < 5\underline{\alpha}$ to avoid corner solutions). Groups have also the same size $N = N_1 = N_2$. Conditions (B.7) and (B.8) then amounts respectively to:
$$\frac{4}{3}(\alpha^e)^2 + \frac{9(\Delta\alpha)^2}{48N} \geq \overline{\alpha}\alpha^e \text{ and } \frac{3}{2}(\alpha^e)^2 + \frac{3(\Delta\alpha)^2}{48N} \geq \overline{\alpha}\alpha^e.$$
It can be checked that, when N is large, this second condition is easier to achieve. Inter-group free riding reduces the overall surplus and makes it more difficult to implement for each delegate the same objectives (B.5) as under complete information (even though this objective is distorted by inter-group free-riding). Merging congruent groups helps solving the collective action problem.■