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JEL Classification: E32, E44, G21, G28

Keywords: Banking, Bank Collateral, liquidity, capital misallocation, macroprudential policy

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Bank Assets, Liquidity and Credit Cycles*

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*The views expressed in this paper do not necessarily represent the views of the Banque Centrale du Luxembourg or the Eurosystem.

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1 Introduction

Lending relationships are typically plagued by information asymmetries that play a key role in shaping macroeconomic outcomes. In this respect, banks are of central importance, as they are involved in at least two layers of financial contracting, through their intermediation activity between savers and borrowers. This paper focuses on the macroeconomic implications of banks' portfolio decisions over different assets in the presence of chained financial frictions, intended as the combination of collateralized borrowing by both banks and entrepreneurs. To this end, we devise a tractable model that integrates limited enforceability of loan contracts—as popularized by Kiyotaki and Moore (1997) (KM, hereafter)—with an analogous friction characterizing the financial relationship between depositors and bankers. As in recent contributions on banking and the macroeconomy (see, *inter alia*, Gertler and Kiyotaki, 2015), deposits are secured by a fraction of bankers' assets that are pledged as collateral. The main departure from these studies consists of envisaging different degrees of liquidity—and, thus, pledgeability—for different types of bank assets. In doing so, our key contribution consists of detailing the mechanisms through which heterogeneous collateral assets shape bankers' incentives to intermediate funds, and ultimately affect the amplitude of credit cycles.

Financial institutions resort to collateralized debt to raise funds, providing assets as a guarantee in case of default on their debt obligations. This is the case for non-traditional banking activities—with sale and repurchase agreements (repos) employed as the main source of funding—as well as for commercial banks, where securitized-banking often supplements more traditional intermediation activities. In fact, banks employ financial collateral both for currency management purposes and, more recently, as part of non-standard monetary policy frameworks.[1] A vast literature has focused on quantifying the dynamic multiplier emerging from the limited enforceability of debt contracts in economies à la KM.[2] While most of the contributions in this tradition have emphasized the role of borrowers' collateral for the amplification of macroeconomic shocks, bank collateral has generally been overlooked. We seek to fill this gap.

In our model the ability of bankers to intermediate funds between savers and borrowers rests on the composition of different assets they are able to pledge as collateral. Along with extending bank loans, bankers may also invest in an infinitely-lived productive asset, 'capital', whose main purpose is to serve as a buffer against which the intermediary is trusted to be able to meet its financial obligations. As such, capital held by bankers provides a source of insider equity, which is necessary to overcome the agency problem in the model. In turn, deposits are bounded from above by bankers' holdings of both types of collateral assets. However, due to the vertically integrated structure of

[1] The set of assets that central banks accept from commercial banks generally includes government bonds and other debt instruments issued by the public sector and international/supranational institutions. In some cases, also securities issued by the private sector can be accepted, such as covered bank bonds, uncovered bank bonds, asset-backed securities or corporate bonds.

[2] See Kocherlakota (2000), Krishnamurthy (2003) and Cordoba and Ripoll (2004), *inter alia.*

our credit economy savers anticipate that, in case of bankers' default, liquidating their financial claims (i.e., bank loans) is subordinated to borrowers being solvent on their debt obligations. This friction, which has not been formerly investigated, affects savers' perceived liquidity of bankers' financial assets beyond that of capital,[3] inducing a transaction cost depositors have to bear in order to liquidate bank loans. If the latter are regarded as relatively illiquid, savers will be less prone to accept them as collateral.

A key feature of the model is that combining limited enforceability of deposit and loan contracts reduces the interest rate on loans below the one that would prevail in a standard economy where only loans are secured by collateral assets. This allows borrowers to extend their capital holdings, contributing to increase total production in the steady state and alleviating capital misallocation as it emerges in economies à la KM, where borrowers hold too little capital in equilibrium, due to constrained borrowing. This property has a key implication for equilibrium dynamics: as the propagation of technology shifts crucially rests on the distribution of real assets between lenders and borrowers, envisaging financially constrained intermediaries into an otherwise standard KM economy produces a 'banking attenuator' that is neither linked to the procyclicality of the external finance premium (Goodfriend and McCallum, 2007), nor to monopolistic competition and interest rate-setting rigidities in financial intermediation (Gerali *et al.*, 2010).

The main distinction between different types of bank collateral lies in the way they affect bankers' incentives to intermediate funds. Both assets have the potential to relax bankers' financial constraint. However, while increasing real assets exacerbates capital misallocation and reduces lending through a negative externality on borrowers' demand for credit, increasing bankers' holdings of financial assets compresses the spread between the loan and the deposit rate, thus attenuating capital misallocation. This feature of the model has key implications for equilibrium dynamics under different degrees of collateralization of bank loans. A relatively scarce liquidity of bankers' financial assets amplifies the response of gross output to productivity shocks. As in KM, a positive technology shift reallocates capital from the lenders to the borrowers. On one hand, this allows borrowers to expand their borrowing capacity. On the other hand, a decline in bankers' real assets is typically counteracted by an expansion in bank loans: as the latter are perceived to be increasingly illiquid, the compensation effect is gradually muted, so that bankers need to cut their capital investment further to meet borrowers' higher demand for credit. In turn, the response of total production—which increases in borrowers' real assets, *ceteris paribus*—is amplified, relative to situations in which deposit contracts involve relatively low transaction costs in case of bankers' default.

The model produces a countercyclical 'flight to quality' in bankers' optimal asset allocation (see, e.g., Lang and Nakamura, 1995): during expansions (contractions), bankers increase (decrease) their holdings of the relatively illiquid asset—bank loans—while decreasing (increasing)

[3] In the remainder we will refer to *financial assets* and *bank loans* interchangeably, while *capital goods* will also be referred to as *real assets*.

their capital holdings, which do not bear any risk of default. As a result, 'too much' borrowing capacity is allocated during boom states and 'too little' in bad states, inducing a procyclical bank leverage and generating excessive fluctuations in credit, output and asset prices. From a normative viewpoint, we study to which extent a hypothetical banking regulator may intervene to smooth the amplitude of these fluctuations, by impairing the endogenous propagation mechanism that hinges on capital misallocation. Along with removing or reducing systemic risks with a view to protecting and enhancing the resilience of the financial system, the Bank of England has recently indicated how macroprudential policy making should facilitate the supply of finance for productive investment, thus making capital allocation more efficient (Bank of England, 2016). With respect to our model, we show that a constant capital-to-asset ratio attenuates the transmission of technology shifts, although the gap between borrowers' and bankers' marginal product of capital cannot entirely be closed. By contrast, the regulator may successfully attenuate the economy's response to productivity shocks, devising a state-dependent capital buffer that induces a countercyclical bank leverage and stabilizes fluctuations in borrowers' collateral, even without resolving the distortion in capital allocation. This is accomplished by adjusting the capital-to-asset ratio in response to changes in bank lending: when the rule features enough responsiveness, movements in the value of borrowers' collateral assets are counteracted by similar-sized changes in the loan rate, so that the traditional amplification mechanism embodied by borrowers' collateral constraint is neutralized.

Related literature This paper is strictly related to a growing literature that introduces financial intermediation into well established quantitative macroeconomic frameworks, so as to account for a number of distinctive features of the last financial crisis (see, *inter alia*, Gertler and Kiyotaki, 2010). To name a few, Gertler and Karadi (2011), have devised a model that emphasizes the role of collateral that banks post to their lenders, as well as their net worth, for the transmission of unconventional monetary measures.[4] Gertler *et al.* (2012) extend the baseline insights of this framework, allowing intermediaries to issue outside equity—thus making risk exposure an endogenous choice of the banking sector—while Gertler and Kiyotaki (2015) devise a model of banking that allows for liquidity mismatch and bank runs. More recently, Hirakata *et al.* (2017) have introduced chained financial contracts into a dynamic general equilibrium models à la Bernanke *et al.* (1999).[5] The common trait of these contributions and many others in this tradition is to look at different sources of funding of financial intermediaries—thus emphasizing the composition of the right-hand side of banks' balance sheet—while typically considering only one type of asset—bank loans. We deviate from this approach and focus on the role of limited enforceability of deposit contracts in a setting where banks may invest in different assets,[6] whose

[4]See also subsequent contributions in this modeling tradition, such as Rannenberg (2012) and Garcia-Cicco *et al.* (2015), as well as earlier contributions that have stressed the role of bank capital for the transmission of a variety of shocks, such as Aikman and Paustian (2006) and Meh and Moran (2010).

[5]Unlike the present framework—which is based on costly enforcement of both deposit and loan contracts—Hirakata *et al.* (2017) consider costly state verification problems applying to both intermediaries and entrepreneurs.

[6]In this respect, our framework is closer to Chen (2001), who stresses the importance of moral hazard behavior

distinctive trait is to bear different degrees of liquidity depending on whether they are involved in more than one layer of financial relationships.

The paper is also part of a rapidly developing banking literature on the role of macroprudential policy-making. Some recent examples include Martinez-Miera and Suarez (2012), Angeloni and Faia (2013), Harris *et. al* (2015), Clerc *et. al* (2015), Begenau (2015) and Elenev *et al.* (2017). These contributions rely on medium- to large-scale dynamic general equilibrium models. While an obvious advantage of this modeling approach is to allow for a variety of shocks, transmission channels and alternative policy settings, our framework allows for a neater interpretation of the interplay between bank capital requirements, capital misallocation and the amplitude of credit cycles. In this respect, our framework is more closely related to Gersbach and Rochet (2016), who show that complete markets do not sufficiently stabilize credit-driven fluctuations, thus providing a clear rationale for macroprudential-policy intervention.

This paper contributes to the existing literature along two main routes. First, one obvious advantage of our model over the existing contributions is analytical tractability, especially if we compare our setting to the existing tradition of medium- to large-scale general equilibrium models featuring a banking sector. In this respect, we show that i) limited enforceability of deposits and loan contracts generate opposite effects on capital misallocation, and ii) a higher pledgeability of bank holdings of financial assets reduces the amplification of aggregate macroeconomic shocks. Second, a defining feature of our normative analysis is to study to which extent a regulator may promote a more efficient allocation of productive capital by 'leaning against' capital misallocation. In this respect, we emphasize that examining capital misallocation, in line with the seminal work of KM, is paramount to understand the role of credit frictions in a model with financial intermediation that is affected by a (double) moral hazard problem, as in Holmstrom and Tirole (1997) and Chen (2001). In particular, we show that shock amplification is a direct function of the misallocation of capital, and that macroprudential policies are effective only in so far as they can ameliorate distortions in the allocation of capital.

Structure The paper is organized as follows: Section 2 presents the framework; Section 3 discusses the steady-state equilibrium; Section 4 focuses on equilibrium dynamics in the neighborhood of the steady state and the amplification of shocks to productivity in connection with the degree of financial collateralization; Section 5 examines the role of macroprudential policy-making in reducing capital misallocation to smooth macroeconomic fluctuations; Section 6 concludes.

2 Model

The economy is populated by three types of infinitely-lived, unit-sized, agents: savers, borrowers and bankers. There are two layers of financial relationships: savers make deposits to the bankers,

of both bankers and entrepeneurs in a quantitative model à la Holmstrom and Tirole (1997). However, in this framework there is no role for liquidity assessment of different types of bank assets.

who act as financial intermediaries and extend credit to the borrowers. Two goods are traded in this economy: a durable asset, 'capital', and a non-durable consumption good. Capital, which is held by both bankers and borrowers, does not depreciate and is fixed in total supply to one. All agents have linear preferences defined over non-durable consumption.[7,8] The remainder of this section provides further details on the characteristics of the actors populating the economy and their decision rules.[9]

2.1 Savers

Savers are the most patient agents in the economy. In each period, they are endowed with an exogenous, non-produced income. We assume that savers are neither capable of monitoring the activity of the borrowers, nor of enforcing direct financial contracts with them. As a result, they make deposits at the financial intermediaries. The linearity of their preferences implies that savers are indifferent between consumption and deposits in equilibrium, so that gross interest rate on savings (deposits), R^S, equals their rate of time preference, $1/\beta^S$. Savers' budget constraint reads as:

$$c_t^S + b_t^S = \varepsilon^S + R^S b_{t-1}^S, \tag{1}$$

where c_t^S denotes the consumption of non-durables, b_t^S is the amount of savings and ε^S is a fixed endowment.[10]

2.2 Borrowers

Borrowers' ability to attract external funding is bounded by the limited enforceability of debt contracts. In line with Hart and Moore (1994) we assume that, should borrowers default, bankers acquire the right to liquidate the stock of capital, k_t^B. Based on the predicted outcomes of the renegotiation, borrowers are subject to an enforcement constraint. Neither bankers nor borrowers

[7] The model will be solved by log-linearizing it around its long-run equilibrium. In this respect, some assumptions will be introduced to pin down the steady state, and rule out degenerate allocations of consumption and assets across the three types of agents.

[8] In this respect, it is important to recall that a number of studies have shown that collateral constraints can act as a powerful amplification and propagation mechanism of exogenous shocks under simplifying assumptions on preferences and production technologies. Kocherlakota (2000) and Cordoba and Ripoll (2004) are some noteworthy examples, in this respect. Nevertheless, it is worth pointing out how modelling collateral constraints has become the dominant approach when introducing financial frictions into otherwise standard DSGE models, and how their combination with asymmetrc information problems affecting the banking sector typically enhances the amplification of a variety of shocks, even under more standard assumptions on preferences and production technologies.

[9] Appendix B reports the derivation of the first-order conditions, and a summary of the key equilibrium conditions of the model.

[10] Steady-state variables are reported without the time subscript.

are able to observe the liquidation value before the actual default, though borrowers have all the bargaining power in the liquidation process. With probability $1-\omega$ bankers expect to recover no collateral asset after a default, while with probability ω bankers expect to be able to recover $E_t q_{t+1} k_t^B$, where E_t indicates the rational expectation operator and q_{t+1} denotes the capital price at time $t+1$.

To derive the renegotiation outcome, we consider the following default scenarios:

1. *Bankers expect to recover* $E_t q_{t+1} k_t^B$. Since bankers can expropriate the whole stock of capital, borrowers have to make a payment that leaves bankers indifferent between liquidation and allowing borrowers to preserve the stock of collateral assets. This requires borrowers to make a payment at least equal to $E_t q_{t+1} k_t^B$, so that the ex-post value of defaulting from the perspective of the borrowers is:

$$R^B b_t^B - E_t q_{t+1} k_t^B, \tag{2}$$

where R^B denotes the gross loan rate and b_t^B is the loan.

2. *Bankers expect to recover no collateral.* If the liquidation value is zero, liquidation is clearly not the best option for the borrowers. Therefore, borrowers have no incentive to pay the loan back. The ex-post default value in this case is:

$$R^B b_t^B. \tag{3}$$

Therefore, enforcement requires that the expected value of non defaulting is not smaller than the expected value of defaulting, that is:

$$0 \geq \omega \left[R^B b_t^B - E_t q_{t+1} k_t^B \right] + (1-\omega) R^B b_t^B, \tag{4}$$

which reduces to

$$R^B b_t^B \leq \omega E_t q_{t+1} k_t^B. \tag{5}$$

According to (5), the maximum amount of credit borrowers may access is such that the sum of principal and interest, $R^B b_t^B$, equals a fraction of the value of borrowers' capital in period $t+1$.

Borrowers also face a flow-of-funds constraint:

$$c_t^B + R^B b_{t-1}^B + q_t (k_t^B - k_{t-1}^B) = b_t^B + y_t^B, \tag{6}$$

where c_t^B and y_t^B denote borrowers' consumption and production of perishable goods, respectively. As in KM, borrowers are assumed to combine capital and labor—the latter being supplied inelastically—through a linear production technology

$$y_t^B = \alpha_t k_{t-1}^B, \tag{7}$$

with α_t being a multiplicative productivity shifter: $\log \alpha_t = \rho \log \alpha_{t-1} + u_t$, where $\rho \in [0, 1)$ and u_t is an iid shock.

Borrowers maximize their utility under the collateral and the flow-of-funds constraints, taking R^B as given. The linearity of their preferences implies that the shadow value of borrowing amounts to $1 - \beta^B R^B$. In equilibrium, the collateral constraint holds with equality in the neighborhood of a determinate steady state:

Assumption I. $\beta^B < 1/R^B$

In light of this, borrowers' demand for capital is determined at the point where the present value of the marginal product of capital, $\beta^B E_t \alpha_{t+1}$, is equal to the opportunity cost of holding capital, $u_t^B \equiv q_t - \left[\beta^B R^B + \omega\left(1 - \beta^B R^B\right)\right]\left(R^B\right)^{-1} E_t q_{t+1}$:

$$u_t^B = \beta^B E_t \alpha_{t+1}. \tag{8}$$

2.3 Bankers

Bankers' primary activity consists of intermediating funds between savers and borrowers. However, their ability to attract savers' financial resources is bounded by the limited enforceability of the deposit contracts, given that bankers may divert assets for personal use (see also Gertler and Kiyotaki, 2015). At this stage of the analysis we abstract from the implementation of regulatory bank capital ratios to discourage bankers' moral hazard behavior, while focusing on the characteristics of the deposit contract.

We assume that, upon bankers' default, savers acquire the right to liquidate bankers' asset holdings.[11] At the time of contracting the amount of deposits, though, the liquidation value of bankers' assets is uncertain. In this respect, the enforcement problem is isomorphic to that characterizing bankers' lending relationship with the borrowers. However, due to the vertically integrated structure of the credit economy, we envisage an additional friction that limits the pledgeability of bank loans beyond that of capital. While real assets remain in the availability of the bankers for the entire duration of the deposit contract—so that savers can frictionlessly liquidate them in case

[11] There are two main considerations why this assumption is a reasonable one: first, savers have no direct use of the collateral assets; second, even if collateral assets represent an attractive investment opportunity, savers have no experience in hedging.

of bankers' default—the resources corresponding to bankers' financial claims are in the availability of the borrowers. Therefore, from the perspective of the savers the possibility to liquidate b_t^B in the event of a default of the banking sector goes beyond the capacity of the bankers to honor the deposit contract, while being subordinated to borrowers' solvency. In light of this, we assume that savers account for a transaction cost they would have to bear for seizing bank loans: $(1-\xi)\, b_t^B$, where $\xi \in [0,1]$ indexes savers' perceived liquidity of bankers' financial assets. In the extreme case, savers regard bank loans as completely illiquid and do not accept them as collateral, ξ is set to zero (i.e., financial frictions are no longer chained), while $\xi = 1$ corresponds to a situation in which savers attach no risk to their ability of liquidating financial assets in case bankers' default.

To derive the renegotiation outcome, we assume that with probability $1-\chi$ savers expect to recover no collateral, while with probability χ the expected recovery value is $E_t q_{t+1} k_t^I + \xi b_t^B$, where k_t^I denotes bankers' holdings of capital and ξb_t^B represents the amount of bank loans held as collateral, net of transaction costs. This implies the following default scenarios:

1. *Savers expect to recover* $E_t q_{t+1} k_t^I + \xi b_t^B$. Since savers expect to expropriate the stock of real and financial assets after bearing a transaction cost $(1-\xi)\, b_t^B$, bankers have to make a payment that leaves savers indifferent between liquidation and allowing borrowers to preserve the stock of collateral assets. This requires bankers to make a payment at least equal to $E_t q_{t+1} k_t^I + \xi b_t^B$, so that the ex-post value of defaulting from the perspective of the bankers is:

$$R^S b_t^S - E_t q_{t+1} k_t^I - \xi b_t^B. \tag{9}$$

2. *Savers expect to recover no collateral.* If the liquidation value is zero, liquidation is clearly not the best option for the savers. Therefore, bankers have no incentive to pay deposits back. The ex-post default value in this case is:

$$R^S b_t^S. \tag{10}$$

Enforcement requires that the expected value of not defaulting is not smaller than the expected value of defaulting, so that:

$$0 \geq \chi \left[R^S b_t^S - E_t q_{t+1} k_t^I - \xi b_t^B \right] + (1-\chi) R^S b_t^S, \tag{11}$$

which reduces to

$$R^S b_t^S \leq \chi \left(E_t q_{t+1} k_t^I + \xi b_t^B \right), \tag{12}$$

according to which deposits should be limited from above by a fraction of the discounted expected collateral value. As we shall see later, the above condition is assumed to always hold with equality. Notably, bankers' collateral constraint embodies the notion that real and financial assets have different degrees of liquidity (see also Bernanke and Gertler, 1985).[12] In fact, (12) recalls the liquidity constraint envisaged by Benigno and Nisticò (2017), where safe and pseudo-safe assets co-exist and both contribute to set the maximum amount of resources available for consumption. In their case, while the entire stock of safe assets (i.e., money) is available to finance private expenditure, only a fraction of pseudo-safe assets can be employed to cover consumption, as it displays less-than-perfect liquidity.

Some considerations are in order about the role of the capital goods held by the bankers. First, this asset mainly serves as a buffer against which the intermediary is trusted to be able to meet its financial obligations.[13] This is reminiscent of Bernanke and Gertler (1985), where the financial sector owns bank capital to provide a source of insider equity, which is necessary to overcome the agency problem in the model (from the perspective of the intermediary). In addition, k_t^I is important in that it breaks the tight link between deposits and lending—which would be otherwise embodied by a binding deposit contract—thus allowing for the possibility that a countercyclical 'flight to quality' drives the supply of credit. In the present context, such an effect would translate into bankers' allocating relatively more resources to capital investment—which, unlike bank loans, does not bear any risk of default—during adverse periods. In turn, this mechanism may open the route to the emergence of credit crunch episodes (Bernanke and Lown, 1991). Finally, as in Bernanke and Gertler (1985) we assume that capital is productive, being employed by bankers to invest in projects on their own behalf.[14] Specifically, bankers' production technology is assumed to feature the following properties:

$$y_t^I = \alpha_t G(k_{t-1}^I), \tag{13}$$

with $G' > 0$, $G'' < 0$, $G'(0) > \varrho > G'(1)$,[15] where ϱ equals the marginal product of capital for the

[12] As χ affects the collateralization of both real and financial assets, in the remainder we will refer to 'financial collateralization' as the degree of pledgeability of bank loans that is exclusively captured by their liquidity, as indexed by ξ.

[13] Broadly speaking, k_t^I may be seen as corresponding to bankers' security holdings, which are typically more liquid, as compared with bank loans.

[14] An alternative formalization of this setting is traced in Section 2.3.1.

[15] Assuming a decreasing returns to scale technology available to the borrowers would not alter our key results. As we will see in the next section, it is the relatively higher impatience of the borrowers, combined with their collateral constraint, that endows them with a suboptimal stock of capital. This point is also discussed in KM. Introducing a decreasing returns to scale technology would only hinder the analytical tractability of the model.

financial intermediary:

$$\varrho \equiv \frac{R^B \beta^B \left[R^S \left(1-\beta^I\right) - \chi \left(1-\beta^I R^S\right)\right]}{R^S \beta^I \left[R^B \left(1-\beta^B\right) - \omega \left(1-\beta^B R^B\right)\right]}, \tag{14}$$

where β^I denotes bankers' discount factor and (14) is required to ensure an internal solution in which both bankers and borrowers demand capital.[16]

Bankers' flow-of-funds constraint reads as:

$$c_t^I + b_t^B + R^S b_{t-1}^S + q_t (k_t^I - k_{t-1}^I) = b_t^S + R^B b_{t-1}^B + y_t^I, \tag{15}$$

where c_t^I denotes bankers' consumption.

Due to the linearity of their preferences, also bankers' shadow value of borrowing is constant and equal to $1 - R^S \beta^I$. In the reminder of the analysis, we make the following assumption:

Assumption II. $\beta^I < 1/R^S$

The assumption ensures that the enforcement constraint holds with equality in the neighborhood of the steady state, implying that bankers are relatively more impatient than savers.[17] In light of this, unless either χ or ξ equal zero, bankers always charge a lending rate that is lower than their rate of time preference, as extending loans allows them to relax their financial constraint:

$$R^B = \frac{R^S - \chi \xi \left(1-\beta^I R^S\right)}{\beta^I R^S}, \tag{16}$$

from which it is possible to write down the spread between the loan and the deposit rate:

$$R^B - R^S = \frac{\beta^S - \beta^I}{\beta^I \beta^S} - \chi \xi \frac{1-\beta^I R^S}{\beta^I R^S}. \tag{17}$$

The first term on the right-hand side of this equality is the spread that would prevail if bankers could not borrow off their loans (i.e., if $\chi\xi = 0$), while the second term captures how bankers' financial constraint affects their ability to intermediate funds. Increasing χ and/or ξ compresses the spread. Greater pledgeability of financial assets increases the collateral value that savers expect to recover in case of bankers' default. This relaxes the financial constraint, eases more deposits and translates into a higher credit supply, thus compressing the lending rate.

The distinction between the two types of collateral has crucial implications for bankers' incentives to intermediate funds between savers and borrowers. On one hand, while increasing k_t^I expands bankers' lending capacity, it also exerts a negative externality on borrowers' demand

[16] The role of this property will be discussed further in Section 4.1.

[17] In this respect, imposing $\beta^I R^S = 1$ reduces the model to the conventional KM economy.

for credit by decreasing their collateral. On the other hand, increasing b_t^B attenuates the debt-enforcement problem between bankers and borrowers, as implied by the reduction of the spread between the loan and the deposit rate. As it will be discussed in Section 4.2, such a distinction has key implications for equilibrium dynamics.

The Euler equation governing bankers' demand for real assets is

$$q_t = \frac{R^S \beta^I + \chi \left(1 - \beta^I R^S\right)}{R^S} E_t q_{t+1} + \beta^I E_t \left[\alpha_{t+1} G'\left(k_t^I\right)\right]. \tag{18}$$

Note that by relaxing (i) and allowing for $\beta^I R^S = 1$ (i.e., assuming that bankers are as impatient as savers), (18) reduces to lenders' Euler equation in the conventional direct-credit economy à la KM. Under these circumstances, bankers are no longer financially constrained. As we shall see in the next section, this implies both a higher loan rate and a higher user cost of capital from the perspective of the financial intermediaries, as compared with what observed when bankers face a binding collateral constraint. These properties will play a crucial role for both the long-run and the short-run behavior of the model economy.

2.3.1 On the Interpretation of Bank Capital

We have followed Bernanke and Gertler (1985) in that we assume the financial sector owns bank capital. Specifically, bank capital mitigates the potential excessive tendency by banks to take risks and, more generally, can serve as a cushion against solvency problems. Therefore bank capital plays a pivotal role, as it determines the amount of funds intermediated by banks. Effectively, this specification is a convenient way to introduce a source of insider capital. An alternative, yet equivalent, formulation of the model can be envisaged by introducing a second productive sector, whose firms are not affected by solvency issues. Shares (or, equivalently, corporate bonds) of firms in this sector are bought by the intermediary sector as a way of retaining (liquid) inside capital, which can be used to alleviate the agency problem in the model. Firms in this sector, whose variables are indexed by "L", produce by means of a the following technology: $y_t^L = \alpha_t G\left(k_{t-1}^L\right)$. The price of each share reflects the value of the capital used in this production sector, and the ownership of shares entails the receipt of dividends equal to the value of the production, y_t^L.[18]

This alternative formulation is entirely isomorphic to the one we have described so far, and highlights a possible interpretation of bank capital as the intermediary sector's ownership of securities. Equity and money market funds shares, as well as other debt security holdings (which include corporate bonds, mortgages and other asset-backed securities) are an important and growing share of the financial sector's balance sheet, and are often employed as collateral in interbank lending

[18] It is worth noting that, under perfect competition, the payment to bond holders would equal total revenues, y_t, in the alternative scenario where corporate bonds are purchased to provide a buffer against which the intermediary is expected to meet its financial obligations.

activities, particularly in the repo market.[19]

2.4 Market Clearing

To close the model, we need to state the market-clearing conditions. We know that the total supply of capital equals one: $k_t^I + k_t^B = 1$. As for the consumption goods market, the aggregate resource constraint reads as:

$$y_t = \epsilon^S + y_t^I + y_t^B, \tag{19}$$

where y_t denotes the total demand of consumption goods.

The aggregate demand and supply for credit are given by the two enforcement constraints (holding with equality) faced by borrowers and bankers, respectively:

$$b_t^B = \omega \frac{E_t q_{t+1} k_t^B}{R^B}, \tag{20}$$

$$b_t^B = \frac{1}{\xi\chi}\left(R^S b_t^S - \chi E_t q_{t+1} k_t^I\right), \tag{21}$$

which imply that, as savers are indifferent between any path of consumption and savings, the amount of deposits is contingent upon bankers' capitalization, so that $b_t^S = \frac{\xi\chi}{R^S}\left(\frac{\omega}{R^B}k_t^B + \frac{1}{\xi}k_t^I\right)E_t q_{t+1}$. Thus, also the market for final goods is cleared according to the Walras' Law.

2.5 Equilibrium

Market equilibrium is defined as a sequence of prices and allocations of physical capital, debt, and consumption of savers, borrowers and bankers $\{q_t, k_t^B, k_t^I, b_t^B, b_t^S, c_t^S, c_t^B, c_t^I\}$, such that: each saver chooses $\{b_t^S, c_t^S\}$ to maximize her expected discounted utility subject to the budget constraint (1); each borrower chooses $\{b_t^B, k_t^B, c_t^B\}$ to maximize the expected discounted utility subject to the production function (7), the borrowing constraint (5), and the flow-of-funds constraint (6); each banker chooses $\{b_t^B, b_t^S, k_t^I, c_t^I\}$ to maximize expected discounted utility subject to the technology embodied by (13), the collateral constraint (12), and the flow-of-funds constraint (15); and the markets for goods, capital, and debt clear. Furthermore, *Assumptions I, II* hold in equilibrium, and ensure non-degenerate steady-state consumption and asset allocations across the three types of agents.

[19] For instance, by the end of 2018 "other securities" are almost twice as large as the share of loans in the balance sheet of the US (domestic) financial sector.

3 Steady State

Financial frictions characterizing both the savers-bankers relationship and the bankers-borrowers relationship deeply affect the properties of the model. Examining their interaction in the long-run is key for understanding the propagation of technology shocks.

In the remainder we impose, without loss of generality, $G(k_{t-1}^I) = \left(k_{t-1}^I\right)^\mu$, with $\mu \in [0,1]$. Evaluating borrowers' demand for capital (8) in the non-stochastic steady state returns:

$$q = \frac{R^B \beta^B}{\left(1-\beta^B\right) R^B - \omega\left(1-\beta^B R^B\right)}. \tag{22}$$

From (18) we retrieve the marginal product of bankers' capital, as a function of its price:

$$G^{'}\left(k^I\right) = \frac{R^S\left(1-\beta^I\right) - \chi\left(1-\beta^I R^S\right)}{R^S \beta^I} q, \tag{23}$$

so that Equations (22), (23) and $k^I + k^B = 1$ pin down borrowers' and bankers' holdings of capital. In turn, these allow us to characterize the key inefficiency at work in the economy. Importantly, financial collateralization only affects the steady state of the economy through its impact on R^B, which in turn influences the capital price through borrowers' Euler, as implied by (22).[20]

[Insert Figure 1]

Figure 1 contains a sketch of the long-run equilibrium of the economy. On the horizontal axis, borrowers' demand for capital is measured from the left, while bankers' demand from the right. The sum of the two equals one. On the vertical axis we report the marginal product of capital of both borrowers and bankers. Borrowers' marginal product of capital is indicated by the line ACE^*, while bankers' marginal product is represented by the line DE^0E^*. The first-best allocation would be attained at E^0, where the product of capital owned by the bankers and the borrowers is the same, at the margin. In our economy, however, the steady-state equilibrium is at E^*, where the marginal product of capital of the borrowers ($mpk^B = 1$) exceeds that of the bankers ($mpk^I = \varrho$). That is, relative to the first best too little capital is used by the borrowers, due to their financial constraint. As discussed by KM, this type of capital misallocation implies a loss of output relative to the first-best, as indicated by the area CE^0E^*.[21] The following remark elaborates on the relationship between borrowers' and bankers' marginal product of capital:

Remark 1 *As long as $\beta^B < \beta^I$, bankers' marginal product of capital is lower than that of the borrowers.*

[20] In light of this, envisaging savers that invest in real assets in place of the bankers would alter neither the role of liquidity of bankers' financial collateral, nor the key aggregate implications of the model.

[21] The area under the solid line, ACE^*D, is the steady-state output.

In fact, imposing $G^{'}(k^I) < 1$ returns the following inequality:

$$\beta^B - \beta^I < \frac{\chi\beta^B\left(1-\beta^I R^S\right)\left(R^B-\xi\omega\right)}{R^S\left(R^B-\omega\right)}. \tag{24}$$

As we assume $\beta^B < \beta^I$, the left-hand side of the inequality is negative, while its right-hand side is positive, given that $\beta^I R^S < 1$, $R^B > \xi\omega$ and $R^B > \omega$ hold by assumption. Therefore, a distinctive feature of the equilibrium is that the marginal product of borrowers' capital is higher than that of the bankers, given that the former cannot borrow as much as they want. As a result, any shift in capital usage from the borrowers to the bankers will lead to a first-order decline in aggregate output, as it will become evident when exploring the linearized economy.

So far, the present economy is isomorphic to that put forward by KM, as the suboptimality of the steady-state equilibrium allocation ultimately rests on borrowers' financial constraint. However, it is important to note that combining limited enforceability of both deposit and loan contracts induces bankers to hold less capital and increase their marginal product—thus setting the steady-state equilibrium on a more efficient allocation—as compared with the baseline KM economy. To see this, it suffices to set $\beta^I R^S = 1$, so as to reduce the model to a direct-credit economy where savers and bankers have identical degrees of impatience. Notably, in this case the productivity gap between bankers and borrowers is higher than that obtained in the economy with financial intermediation. This is due to the lenders charging a higher loan rate and attaining a higher steady-state user cost of capital, which exacerbates the inefficiency in capital allocation. In Figure 1 this additional loss of output, relative to the first-best allocation, is captured by the trapezoid $C_{KM}CE^*E^*_{KM}$ (where E^*_{KM} indicates the steady-state equilibrium in the KM setting).

In light of this key property, the next step in the analysis consists of understanding how the collateralization of different types of bank assets impacts on capital misallocation. To this end, we define the productivity gap between borrowers and bankers as

$$mpk^B - mpk^I \equiv \Delta = 1 - \varrho. \tag{25}$$

As far as the effect of χ on the productivity gap is concerned, this is not unambiguous: first, raising χ inflates the steady-state capital price by compressing the intermediation spread, as embodied by (16); second, a higher χ increases bankers' marginal benefit of relaxing the collateral constraint by investing into an extra unit of capital: as a result, bankers have a higher incentive to accumulate capital, so that the first factor on the right-hand side of (23) decreases in χ. As it will be detailed in the next section, these competing forces tend to offset each other, so that bankers' deposit-to-value ratio has little influence on capital misallocation and the propagation of technology disturbances.

As for the pledgeability of bank loans, the following summarizes the impact of financial collateralization on the productivity gap:

Proposition 1 *Increasing the pledgeability of bank loans (ξ) reduces the gap between bankers' and* *borrowers' marginal product of capital* (Δ).

Proof. *See Appendix B.* ■

Notably, a higher degree of financial collateralization expands bankers' lending capacity and compresses the spread charged over the deposit rate. In turn, lower lending rates allow borrowers to increase their borrowing capacity through a higher collateral value, *ceteris paribus*. The combination of these effects is such that mpk^I unambiguously increases in the degree of financial collateralization, reducing the productivity gap with respect to the borrowers. This factor will play a key role in determining the size of the response of gross output to a technology shock, as it will be detailed in Section 4.1.

4 Equilibrium Dynamics

To examine equilibrium dynamics we log-linearize the key behavioral rules and constraints around the non-stochastic steady state, where the incentive compatibility constraints (5) and (12) are assumed to hold with equality.[22] This local approximation method is accurate to the extent that we limit the technology shock to be bounded in the neighborhood of the steady state, so that neither borrowers' nor bankers' default occurs as an equilibrium outcome. As for borrowers' Euler equation (8):

$$\hat{q}_t = \phi E_t \hat{q}_{t+1} + (1-\phi) E_t \hat{\alpha}_{t+1}, \tag{26}$$

where $\phi \equiv \frac{\beta^B R^B + \omega\left(1-\beta^B R^B\right)}{R^B}$. As for the bankers' Euler equation (18):

$$\hat{q}_t = \lambda E_t \hat{q}_{t+1} + (1-\lambda) E_t \hat{\alpha}_{t+1} + \frac{1-\lambda}{\eta} \hat{k}_t^B, \tag{27}$$

where $\lambda \equiv \frac{R^S \beta^I + \chi\left(1-\beta^I R^S\right)}{R^S}$ and η^{-1} is the elasticity of the bankers' marginal product of capital times the ratio of borrowers' to bankers' capital holdings in the steady state (i.e., $\eta \equiv \frac{1-k^B}{k^B(1-\mu)}$).

Once we obtain the solutions for $\hat{q}_t$ and $\hat{k}_t^B$ as linear functions of the technology shifter, we can determine closed-form expressions for the equilibrium path of other variables in the model. We first focus on (26), whose forward-iteration leads to:

$$\hat{q}_t = \gamma \hat{\alpha}_t, \tag{28}$$

where $\gamma \equiv \frac{1-\phi}{1-\phi\rho}\rho > 0$. With this expression for $\hat{q}_t$, we can resort to (27), obtaining

$$\hat{k}_t^B = \upsilon \hat{\alpha}_t, \tag{29}$$

[22] Variables in log-deviation from their steady-state level are denoted by a "^".

where $v \equiv \frac{\eta}{1-\lambda}\frac{(\lambda-\phi)(1-\rho)\rho}{1-\phi\rho} > 0$. Thus, it is possible to linearize total production in the neighborhood of the steady state, obtaining:

$$\hat{y}_t = \hat{\alpha}_t + \Delta\frac{y^B}{y}\hat{k}^B_{t-1}, \tag{30}$$

According to (30), the dynamics of gross output is shaped by $\hat{\alpha}_t$, as well as by borrowers' capital holdings at time $t-1$: the second effect captures the endogenous propagation of productivity shifts on gross output. In fact, $\hat{y}_t$ depends on the past history of shocks not only through the first-round impact of $\hat{\alpha}_t$, but also through the effect of $\hat{\alpha}_{t-1}$ on $\hat{k}^B_{t-1}$, as implied by (29). In light of this, we can rewrite (30) as

$$\hat{y}_t = \varpi\hat{\alpha}_{t-1} + u_t \tag{31}$$

where $\varpi \equiv \rho + v\Delta\frac{y^B}{y}$. According to (31), eliminating the key source of steady-state inefficiency—i.e., attaining $\Delta = 0$—implies that total output's departures from the steady state would track the path of the technology shock, so that the model would feature no endogenous propagation of productivity shifts.[23] Moreover, we need to recall that envisaging limited enforceability of both deposit and loan contracts reduces capital misallocation as it emerges in the original KM economy, thus compressing Δ with respect to the case in which $R^S\beta^I = 1$. In this respect, the model produces a 'banking attenuator' that entirely rests on the functioning of financial frictions in banking activity, as compared with analogous effects stemming from the procyclicality of the external finance premium (Goodfriend and McCallum, 2007) or monopolistic competition in the intermediation activity and staggered interest rate-setting schemes (Gerali *et al.*, 2010). Figure 2 displays the extent of the amplification effect on output in the KM model, as compared with our model, under the baseline calibration.[24] The next subsection examines the roots of this amplification under different degrees of collateralization in the banking sector.

[Insert Figure 2]

[23] This property echoes the role of the steady-state inefficiency for short-run dynamics in the KM model. In their setting, closing the gap between lenders' and borrowers' marginal product of capital would imply no response at all to a productivity shift. In this respect, the key difference between the two frameworks lies in that we assume an autoregressive shock, while they consider an unexpected one-off shift in technology.

[24] The discount factors are set in accordance with our assumptions about the relative degree of impatience of the three agents in the economy and are broadly in line with existing (quarterly) calibrations involving economies with heterogeneous agents: $\beta^S = 0.99$, $\beta^I = 0.98$, $\beta^B = 0.97$. We set $\rho = 0.95$, in line with the empirical evidence showing that technology shocks are generally small, but highly persistent (see, e.g., Cooley and Prescott, 1995). As for χ and ω, they are both set to 1, while $\mu = 0.4$. The response of the other variables to the technology shock is reported in Figure D1 (Appendix D).

4.1 Financial Collateral and Macroeconomic Amplification

We have now lined up the elements necessary to examine how savers' perceived liquidity of bankers' financial assets affects the amplitude of credit cycles. In this respect, there are three different channels through which ξ affects the endogenous response of total production to a technology shock:

$$\frac{\partial \varpi}{\partial \xi} = \Delta \frac{y^B}{y} \frac{\partial v}{\partial \xi} + v \frac{y^B}{y} \frac{\partial \Delta}{\partial \xi} + v \Delta \frac{\partial \left(y^B / y \right)}{\partial \xi}. \tag{32}$$

As for the first term on the right hand side of (32), *Proposition 2* details the effect induced by a marginal change in the degree of financial collateralization on the response of borrowers' capital holdings to the technology shock.

Proposition 2 *Increasing the degree of collateralization of bank loans (ξ) attenuates the impact of the technology shock on both borrowers' holdings of capital and the capital price.*

Proof. *See Appendix B.* ■

According to *Proposition 2* the sensitivity of borrowers' capital holdings to the technology shifter decreases in ξ. The intuition for this is twofold: first, increasing ξ determines a more even distribution of capital goods, as reflected by the drop in η; second, being able to pledge a higher share of financial assets reinforces the sensitivity of the capital price to the capital gain component in borrowers' Euler equation, ϕ, through the drop in the loan rate, while reducing the sensitivity to the dividend component (i.e., the shock). These effects are mutually reinforcing and ultimately exert a negative force on the overall degree of macroeconomic amplification of the system.

Turning our attention to the last two terms on the right hand side of (32), we know from *Proposition 1* that the productivity gap between borrowers and bankers shrinks as financial collateralization increases (i.e., $\partial\Delta/\partial\xi < 0$). Finally, it is immediate to prove that the last term on the right-hand side of (32) is positive, in light of greater collateralization of bank loans inducing a reallocation of capital from the bankers to the borrowers. In turn, this transfer implies both a first-order positive effect on y^B and a (milder) second-order positive impact on y, so that the overall effect on y^B/y is positive.[25]

[Insert Figure 3]

To sum up, an increase in ξ causes competing effects on ϖ. First, greater financial collateralization depresses the pass-through of $\hat{\alpha}_{t-1}$ on borrowers' capital holdings, which in turn affect total production with a lag. Second, raising ξ exerts two distinct effects on the pass-through of $\hat{k}^B_{t-1}$ on $\hat{y}_t$: on one hand, bankers' marginal product of capital increases, implying a reduction of

[25] Recall that total output is an increasing function of borrowers' capital. Therefore, the drop in y^I following a marginal increase in ξ is lower than the corresponding rise in y^B.

the productivity gap; on the other hand, borrowers' contribution to total production increases, as the reduction in the productivity gap reflects higher capital accumulation in the hands of the borrowers. The sum of these three forces potentially leads to mixed results on output amplification, as captured by the second-round effect of technology disturbances. To address this, we plot ϖ as a function of ξ and μ.[26] The aim of this exercise is to examine the direction of the overall effect exerted by financial collateralization on macroeconomic volatility, rather than quantifying an empirically plausible multiplier emerging from the interaction of bankers' and borrowers' financial constraints.[27] As it emerges from Figure 3, increasing ξ compresses ϖ, at any level of μ. By contrast, increasing the income share of capital in bankers' production technology amplifies the second-round response of output. This is because μ amplifies the productivity gap through its positive effect on η.[28] All in all, the general picture emerging from this exercise is that allowing for greater financial collateralization attenuates the overall degree of amplification of technology disturbances. The next subsection examines how this property reflects into cyclical movements in bank leverage, whose behavior is key to understanding how bankers' balance sheet affects the amplitude of credit cycles.

4.2 The Role of Leverage

To enlarge our perspective on the amplification/attenuation induced by bankers' financial collateral, we take a closer look at their balance sheet. To this end, we define bankers' equity as the difference between the value of total assets (i.e., loans and capital) and liabilities (i.e., deposits):

$$e_t^I = b_t^B + q_t k_t^I - b_t^S, \tag{33}$$

with leverage defined as the ratio between loans and equity: $lev_t^I = b_t^B / e_t^I$.

Figure 4 reports the response of selected variables to a one-standard deviation shock to technology.[29] As implied by (30), on impact output responds one-to-one with respect to the shock, regardless of the degree of financial collateralization. However, as ξ increases the second-round

[26]In Appendix C we show that different combinations of χ and ω have negligible effects on the relationship between financial collateralization on macroeconomic amplification. In fact, Figure C1 shows that varying χ has virtually no effect on the amplitude of the response to the technology shock, in light of the competing effects it has on bankers' marginal product of capital.

[27]We leave this task for future research employing larger scale dynamic general equilibrium models.

[28]It is important to emphasize that increasing μ may violate the condition $G^{'}(0) > \varrho > G^{'}(1)$, which ensures an interior solution as for how much capital bankers should hold in the neighborhood of the steady state. To see why this is the case, recall that $\mu \left(k^I\right)^{\mu-1} = \varrho$. Increasing μ inflates bankers' marginal product of capital, while leaving their user cost unaffected: Thus, as μ increases bankers are induced to hold an increasing stock of capital, so that the equality holds. An important aspect is that this effect tends to kick in earlier as ξ declines. This is because a drop in the degree of financial collateralization depresses bankers' user cost of capital. Therefore, as ξ declines and μ increases the set of steady-state allocations in which both bankers and borrowers hold capital restricts, as the condition $\varrho > G^{'}(1)$ is eventually violated and borrowers' may virtually end up with negative capital holdings.

[29]The baseline parameterization is the same as that employed in Figure 2. As for μ, we impose a rather conservative value, 0.4, which allows us to obtain a finite distribution of capital in the steady state.

response is gradually muted. To complement our analytical insight and provide further intuition on this channel, we examine the behavior of a set of variables involved in bankers' intermediation activity. In this respect, note that deposits tend to decline at low values of ξ, while increasing as bankers can pledge a higher share of their financial assets. The reason for this can be better understood by recalling the nature of the interaction between bankers' financial and real assets. The interplay takes place on two levels: on one hand, both assets have a positive effect on savers' deposits, as embodied by (12); on the other hand, it is possible to uncover a crowding out effect, as increasing bankers' real asset holdings exerts a negative force on lending by reducing borrowers' collateral.

How do these properties affect the transmission of an expansionary technology shock? Due to the capital productivity gap between borrowers and bankers, the technology shift necessarily causes a decline of bankers' real assets, thus expanding borrowers' capital and borrowing.[30] Therefore, in equilibrium deposits are influenced by two opposite forces, namely an expansion in the amount of bankers' financial assets and a contraction in their stock of real assets. In this respect, the implied allocation of bankers' assets reflects a countercyclical flight to quality pattern (see, *inter alia*, Lang and Nakamura, 1995): during expansions (contractions), bankers increase (decrease) their holdings of the inherently riskier assets—bank loans—while decreasing (increasing) their capital holdings, which do not bear any risk of default.

[Insert Figure 4]

How do these diverging forces translate in terms of bankers' ability to attract deposits and leverage? As ξ drops the impact of bank loans is gradually muted and deposits eventually track the dynamics of bankers' capital. In this context, the contraction of bankers' real asset overcomes the drop in deposits, so that lending expands in excess of bank equity, potentially leading to an increase in leverage. In fact, a procyclical leverage ratio is associated with a relevant degree of macroeconomic amplification, when bankers' financial assets are regarded as relatively illiquid. Figure 3 shows this tends to be the case for $\xi < 0.5$, under our baseline parameterization.

5 Capital Adequacy Requirements

The analysis so far has shown that limited enforceability of deposit contracts may reduce the productivity gap between borrowers and lenders, which is key to quantify the amplitude of credit cycles. In light of this, our next objective is to understand to which extent a regulator may promote a more efficient allocation of productive capital by 'leaning against' capital misallocation. In this

[30] This is a distinctive feature of lender-borrower relationships involving the collateralization of a productive asset. In fact, it is possible to show that, following a positive technology shock, the major reallocation of land from the lenders to the borrowers is only attenuated by relaxing the hypothesis of zero aggregate investment—as in the KM baseline framework—while the direction of the transfer is not inverted.

respect, the Chancellor of the Exchequer has explicitly indicated that—conditional on enhancing the resilience of the financial system—the Financial Policy Committee at the Bank of England should intend the pursual of productive capital allocation efficiency as part of its macroprudential-policy mandate:

> *"Subject to achievement of its primary objective, the Financial Policy Committee (FPC) should support the Government's economic objectives by acting in a way that, where possible, facilitates the supply of finance for productive investment provided by the UK's financial system."* (Remit and Recommendations for the Financial Policy Committee, HM Treasury, July 8, 2015).

Our economy lends itself to the analysis of this particular problem, in light of the strict connection between the capital productivity gap between borrowers and bankers and the amplitude of credit cycles.[31] To this end, we introduce two complementary tools of regulation. First, we assume deposit insurance, which ensures that savers do not suffer a loss in the event of bankers' default.[32] A direct implication of such a measure is to shift the risk of bankers' default to the government (or a hypothetical interbank deposit protection fund), so that the renegotiation of deposit contracts is redundant and bankers' financial constraint may be discarded. However, in order to mitigate bankers' moral hazard behavior, the regulator imposes an explicit capital adequacy requirement (see, e.g., Van den Heuvel, 2008). According to this regulatory constraint, equity needs to be at least a fraction θ of the loans, for bankers to be able to operate:

$$e_t^I \geq \theta b_t^B, \quad \theta \in [0,1], \tag{34}$$

where θ denotes the capital-to-asset ratio. Introducing this regulatory constraint modifies the loan rate:

$$R^B = \frac{R^S - (1-\theta)\left(1 - \beta^I R^S\right)}{\beta^I}. \tag{35}$$

Notably, (35) is isomorphic to (16), with R^B increasing in the capital-to-asset ratio. To provide an intuition for this, we combine (34) with (33), obtaining:

$$b_t^S \leq q_t k_t^I + (1-\theta) b_t^B. \tag{36}$$

[31] Notably, our model abstracts from trade-offs that impose the policy-maker to balance the incentive to improve the allocation of productive capital with an alternative financial stability objective, as in Gertler and Kiyotaki (2015), *inter alia*. However, our primary interest is to understand how far the policy-maker can go to resolve the key distortion in the credit economy, so that there is no need to introduce additional propagation mechanisms that would only hinder the analytical tractability of the model.

[32] As in Van den Heuvel (2008), deposit insurance is left unmodeled, though it is argued that it generally improves banks' ability to extend credit (see Diamond and Dybvig, 1983).

As embodied by (36), imposing a capital-to-asset ratio to bankers' intermediation activity amounts to constrain deposits from above by the current value of bankers' collateral, with the implied degree of pledgeability of bank loans being a negative function of θ. In fact, there is a direct mapping between the capital-to-asset ratio implicit in the capital requirement constraint imposed by the regulator and the degree of collateralization of bank loans as it emerges from the incentive compatibility constraint (12), which is derived in the absence of any form of deposit insurance.[33] Intuitively, a higher leverage (lower capital) ratio implies a riskier exposure of the financial intermediary. This translates into greater transaction costs savers would have to bear in order to seize bank loans in the event of bankers' default. In turn, these costs have a direct impact on degree of collateralization of bankers' financial assets that is implicit in (34).

[Insert Figure 5]

Therefore, it should come as no surprise that, absent any trade-off between enhancing capital allocation and ensuring financial stability, the optimal policy consists of setting the capital-to-asset ratio to its lower bound. Along with minimizing the fraction of bank assets that can be financed by issuing deposit liabilities, $\theta = 0$ contracts the intermediation spread, thus ensuring a more efficient allocation of capital between bankers and borrowers. Figure 5—which reports the response of the economy to a positive technology shock under this policy—confirms this view. Nevertheless, it is important to notice that even a null capital ratio is not enough to neutralize the endogenous propagation channel stemming from capital misallocation, as stated by the next proposition.

Proposition 3 *The gap between bankers' and borrowers' marginal product of capital (Δ) cannot be closed by setting the capital-to-asset ratio (θ) within the range of admissible values.*

Proof. *See Appendix A.* ■

To dig deeper on this property, Figure 6 maps the spread between the loan and the deposit rate (y-axis) and the productivity gap (x-axis), for different values of the capital-to-asset ratio (θ) and the loan-to-asset ratio applying to the borrowers (ω). As we move down along each locus, θ decreases from its upper bound to the value consistent with $\Delta = 0$. The color of a given line switches from green to blue when θ drops below its lower bound. In line with *Proposition 3*, closing the productivity gap through a capital requirement within the set of its admissible values proves to be infeasible. However, it is important to acknowledge that higher loan-to-value ratios applying to the loan contracts compress the productivity gap at any value of θ. In fact, raising ω relaxes borrowers' collateral constraint, allowing them to increase their capital holdings, so that bankers' marginal product of capital increases in equilibrium.

[Insert Figure 6]

[33] Figures D2 and D3 in Appendix D stress such isomorphism in connection with the transmission of a shock to the degree of collateralization, under (12), and one to the capital-to-asset ratio, under (36), respectively.

Figure 5 also shows that $\Delta = 0$ may only be attained at negative values of θ and $R^B - R^S$. In Figure 5—where the capital-to-asset ratio compatible with a null productivity gap is denoted by $\theta_{\Delta=0}$—the endogenous propagation of the shock is actually switched off under a negative capital-to-asset ratio, so that gross output tracks the dynamics of the productivity shifter and leverage is completely acyclical. However, according to (34), setting $\theta = \theta_{\Delta=0}$ would induce bankers to hold negative equity, for any level of credit being extended. Although we rule this out as an equilibrium outcome, it is interesting to briefly examine the underlying incentives of the bankers in such a scenario: according to (36), through a negative capital-to-asset ratio the regulator implicitly pushes for 'hyper-collateralizing' bank loans. In turn, this eventually induces bankers to set a loan rate below the interest rate on deposits, which amounts to subsidizing borrowers' capital investment so as to resolve the distortion.

5.1 A Countercyclical Capital Buffer

We now turn our attention to an alternative regulatory tool, in the attempt to reduce output fluctuations by affecting the cyclicality of bankers' balance sheet, without necessarily neutralizing the distortion stemming from capital misallocation. Recent years have witnessed an increasing interest of policymakers towards leaning against credit imbalances, pursuing macroeconomic stabilization through policy rules that set a countercyclical capital buffer. *De facto*, countercyclical capital regulation is a key block of the Basel III international regulatory framework for banks.[34] Based on the analysis of the transmission mechanism and the response of bank capital, we now examine the functioning of this type of policy tool within our framework. Thus, we allow for capital requirements to vary with the macroeconomic conditions (see, e.g., Angeloni and Faia, 2013, Nelson and Pinter, 2016 and Clerc *et al.*, 2015):

$$\frac{\theta_t}{\theta} = \left(\frac{b_t^B}{b^B}\right)^{\varphi}, \quad \varphi \geq 0, \tag{37}$$

where $\varphi = 0$ implies a constant capital-to-asset ratio, while $\varphi > 0$ induces a countercyclical capital buffer.[35]

By linearizing the time-varying counterpart of (35) in the neighborhood of the steady state we obtain:

$$\hat{R}_t^B = \psi \hat{\theta}_t, \tag{38}$$

[34] The regulatory framework evolved through three main waves. Basel I has introduced the basic capital adequacy ratio as the foundation for banking risk regulation. Basel II has reinforced it and allowed banks to use internal risk-based measure to weight the share of asset to be hold. Basel III has been brought in response to the 2007-2008 crisis, with the key innovation consisting of introducing countercyclical capital requirements, that is, imposing banks to build resilience in good times with higher capital requirements and relax them during bad times.

[35] According to the Basel III regime, capital regulation can respond to a wide range of macroeconomic indicators. Here we assume it to respond to deviations of b_t^B from its long-run equilibrium, b^B.

where $\psi = \frac{1-\beta^I R^S}{\beta^I R^B}\theta$ is positive, in light of assuming $\beta^I R^S < 1$. We then linearize (37), obtaining:

$$\hat{\theta}_t = \varphi \hat{b}_t^B. \tag{39}$$

After linearizing borrowers' financial constraint, we can substitute for $\hat{b}_t^B$ in (39) and plug the resulting expression into (38), so as to obtain:

$$\hat{R}_t^B = \frac{\psi\varphi}{1+\psi\varphi}\left(E_t\hat{q}_{t+1} + \hat{k}_t^B\right). \tag{40}$$

Thus, it is possible to establish a connection between the loan rate and borrowers' expected collateral value. Increasing the responsiveness of the capital-to-asset ratio to changes in aggregate lending amplifies this channel: raising φ implies that marginal deviations of b_t^B from its steady state transmit more promptly to the capital-to-asset ratio and, in turn, to the loan rate through the combined effect of (38) and (39). This induces a feedback effect on borrowers' capacity to attract external funding, as embodied by their collateral constraint: higher sensitivity of the loan rate to variations in aggregate lending (i.e., a steeper loan supply function) implies stronger discounting of borrowers' expected collateral. In the limit (i.e., as $\varphi \to \infty$) there is a perfect pass-through of $E_t\hat{q}_{t+1} + \hat{k}_t^B$ on $\hat{R}_t^B$. Therefore, as in the face of a technology shock both terms move in the same direction and by the same extent, borrowing does not deviate from its steady-state level and output displays no endogenous propagation.

[Insert Figure 7]

To assess the stabilization performance of the countercyclical capital buffer rule, in Figure 7 we set the steady-state capital-to-asset ratio to 8%—in line with the full weight level of Basel I and the treatment of non-rated corporate loans in Basel II and III—while varying φ over the support $[0, 1]$.[36] As expected, at $\varphi = 0$ (i.e., a capital-to-asset ratio kept at its steady-state level) we observe the strongest amplification of the output response, while the lending rate and bank leverage are both acyclical. By contrast, increasing the degree of countercyclicality of the capital buffer proves to be effective at attenuating the response of gross output to the shock, progressively compressing bank leverage. Notably, as $\varphi \to \infty$ leverage displays a strong degree of countercyclicality,[37] while lending does not deviate from its steady-state level, as conjectured above. In turn, this results in the response of gross output featuring no endogenous propagation of technology shocks, despite the regulator's policy action is not aimed at tackling capital misallocation and, therefore, the steady-state productivity gap is not closed.

[36] Alternative values of θ would only alter the quantitative implications of the exercise, while not affecting its key qualitative result.

[37] This is accomplished by setting $\varphi = 10$, meaning that, for a 1% deviation of debt from its steady-state level, the capital-to-asset ratio is adjusted from its 8% steady-state level up to 8.8%.

6 Concluding Remarks

We have devised a credit economy where bankers intermediate funds between savers and borrowers, assuming that bankers' ability to collect deposits is affected by limited enforceability: as a result, if bankers default, savers acquire the right to liquidate bankers' asset holdings. In this context, we have examined the role of bank loans as a form of collateral in deposit contracts. Due to the structure of our credit economy, which may well account for different forms of financial intermediation, savers anticipate that liquidating financial assets is conditional on borrowers being solvent on their debt obligations. This friction limits the degree of collateralization of bankers' financial assets beyond that of capital. We have demonstrated three main results: i) limited enforceability of deposit contracts counteracts the effects of limited enforceability of loan contracts, thus reducing capital misallocation as it emerges in KM; ii) greater collateralization of bankers' financial assets dampens macroeconomic fluctuations by reducing the degree of procyclicality of bank leverage; iii) while imposing a fixed capital-to-asset ratio to the bankers cannot fully neutralize capital misallocation and enhance a more efficient allocation of productive capital—thus switching off the associated endogenous propagation channel of productivity shock—a countercyclical capital adequacy requirement proves to be rather effective at smoothing credit cycles.

Our model is necessarily stylized, though it can be generalized along a number of dimensions. For instance, a realistic extension could consist of allowing bankers to issue equity (outside equity), so as to evaluate how a different debt-equity mix may affect macroeconomic amplification over expansions—when equity can be issued frictionlessly—and contractions, when equity issuance may be precluded due to tighter information frictions. This factor should counteract the role of financial assets and help obtaining a countercyclical leverage. In connection with this point, we could also allow for occasionally binding financial constraints, so as to evaluate how the policy-maker should behave across contractions—when constraints tighten—and expansions, when constraints may become non-binding. However, as this type of extensions necessarily hinder the analytical tractability of our problem, we leave them for future research projects based on large-scale models.

Figure 1: Steady-state equilibrium

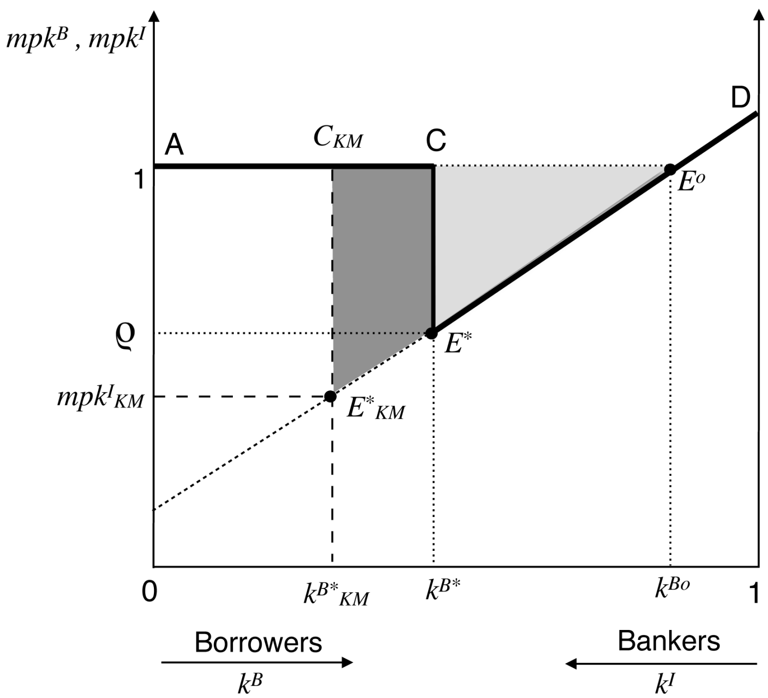
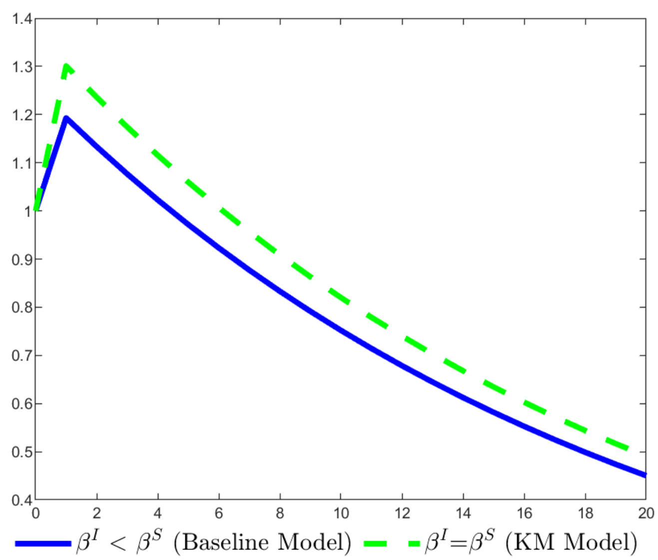
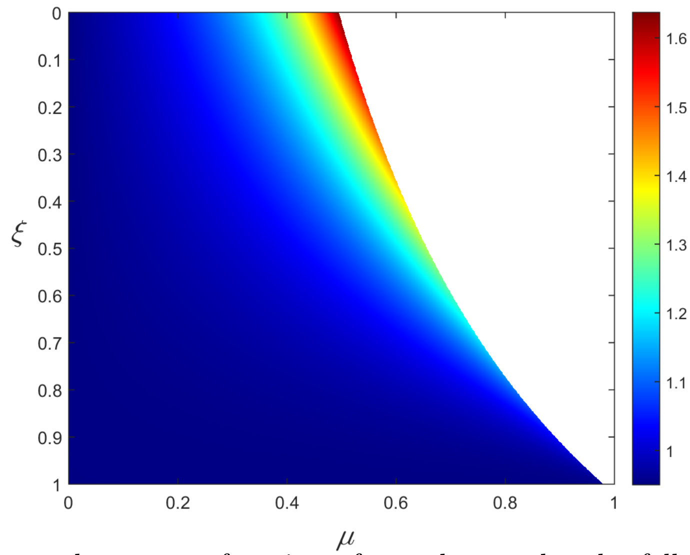


Figure 2: Output response to a technology shock: comparison with KM



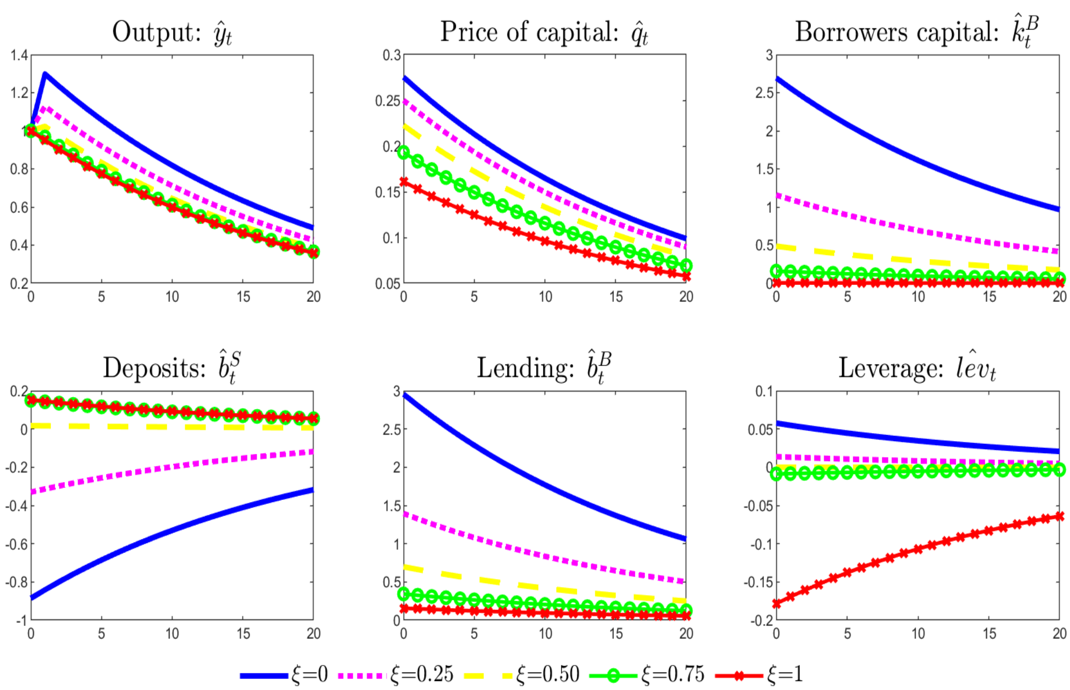
Notes. Figure 2 graphs the response of $\hat{y}_t$ to a one-standard-deviation shock to technology, under the following parameterization: $\beta^S = 0.99$, $\beta^I = 0.98$, $\beta^B = 0.97$, $\rho = 0.95$, $\chi = \omega = 1$, $\mu = 0.4$. We consider two situations: the KM case, where $1 - \beta^I R^S = 0$ (green-dashed line), and the baseline model (blue-continuos line).

Figure 3: Business cycle amplification



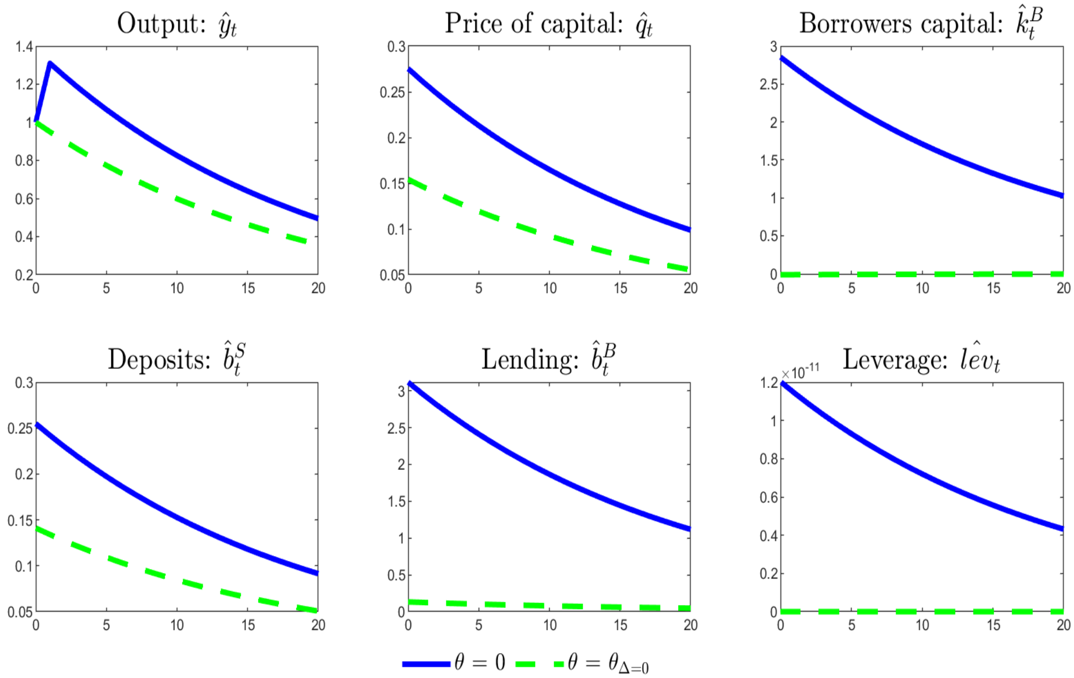
Notes. Figure 3 graphs ϖ as a function of ξ and μ, under the following parameterization: $\beta^S = 0.99$, $\beta^I = 0.98$, $\beta^B = 0.97$, $\rho = 0.95$, $\chi = \omega = 1$. The white area denotes inadmissible equilibria where bankers' capital holdings are virtually negative.

Figure 4: Impulse responses to a positive technology shock



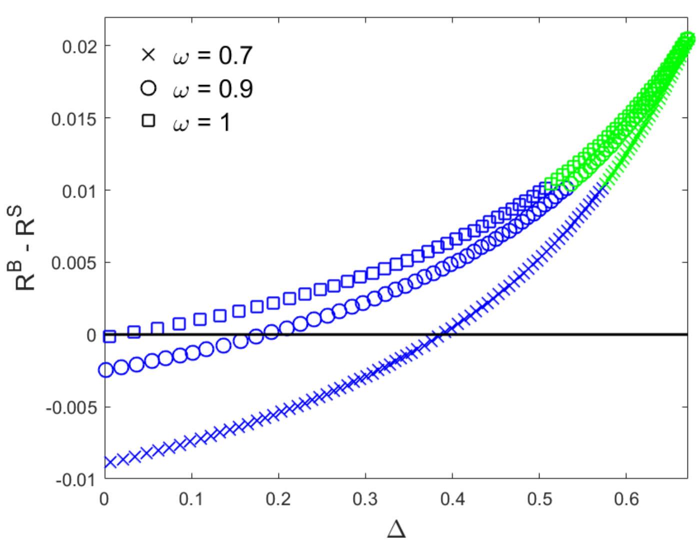
Notes. Responses of selected variables to a one-standard-deviation shock to technology, under the following parameterization: $\beta^S = 0.99$, $\beta^I = 0.98$, $\beta^B = 0.97$, $\rho = 0.95$, $\chi = \omega = 1$, $\mu = 0.4$.

Figure 5: Impulse responses under $\theta = 0$ and $\theta = \theta_{\Delta=0}$



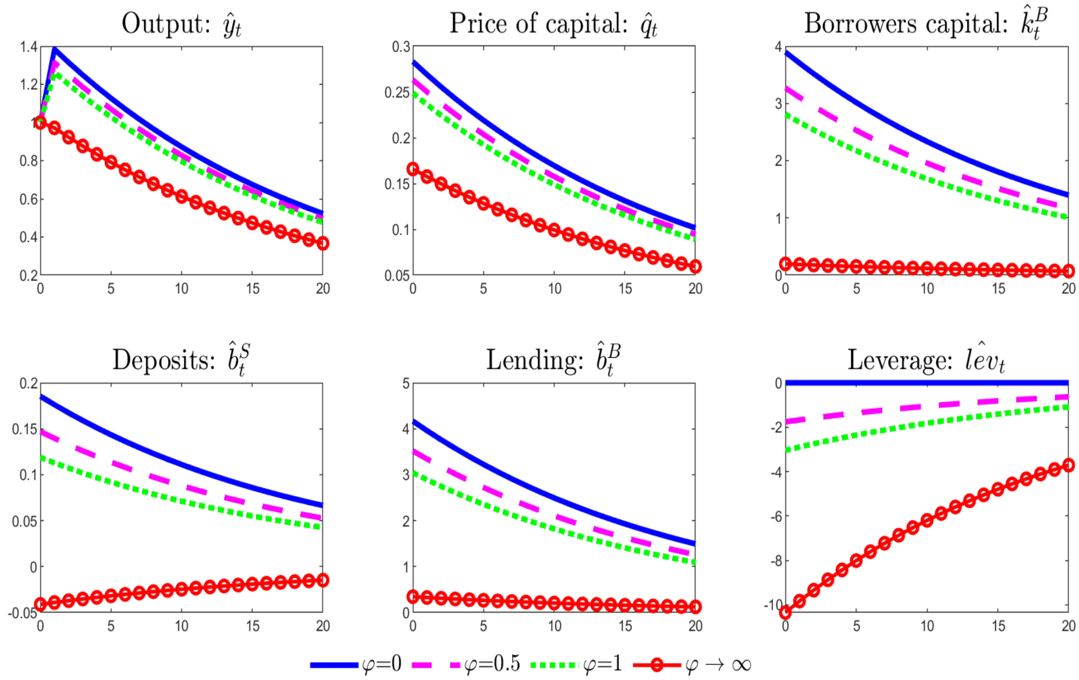
Notes. Responses of selected variables to a one-standard-deviation shock to technology, under the following parameterization: $\beta^S = 0.99$, $\beta^I = 0.98$, $\beta^B = 0.97$, $\rho = 0.95$, $\chi = \omega = 1$, $\mu = 0.4$.

Figure 6: Interaction between the capital-to-asset and the loan-to-value ratios



Notes. Figure 6 maps $R^B - R^S$ (y-axis) and Δ (x-axis) for different values of θ and ω. As we move down along each line, θ decreases from its upper bound to the value consistent with $\Delta = 0$. The color of a given locus switches from green to blue when θ drops below zero. The rest of the parameters are set as follows: $\beta^S = 0.99$, $\beta^I = 0.98$, $\beta^B = 0.97$, $\rho = 0.95$, $\mu = 0.4$.

Figure 7: Impulse responses under different φs



Notes. Responses of selected variables to a one-standard-deviation shock to technology, under the following parameterization: $\beta^S = 0.99$, $\beta^I = 0.98$, $\beta^B = 0.97$, $\rho = 0.95$, $\chi = \omega = 1$, $\mu = 0.4$, $\theta = 0.08$.

References

[1] Angeloni, I. and E. Faia, 2013, Capital Regulation and Monetary Policy with Fragile Banks, Journal of Monetary Economics, 60:311–324.

[2] Aikman, D. and M. Paustian, 2006, Bank Capital, Asset Prices and Monetary Policy, Bank of England Working Papers, No. 305.

[3] Bank of England, 2016, Understanding and Measuring Finance for Productive Investment, Discussion Paper.

[4] Begenau, J., 2015, Capital Requirements, Risk Choice, and Liquidity Provision in a Business Cycle Model, mimeo, Harvard Business School.

[5] Benigno, P. and S. Nisticò, 2017, Safe Assets, Liquidity and Monetary Policy, American Economic Journal: Macroeconomics, 9:182–227.

[6] Bernanke, B.S. and M. Gertler, 1985, Banking in General Equilibrium, NBER Working Papers 1647, National Bureau of Economic Research, Inc.

[7] Bernanke, B. S., M. Gertler and S. Gilchrist, 1999, The Financial Accelerator in a Quantitative Business Cycle Framework, in Handbook of Macroeconomics, J. B. Taylor and M. Woodford (eds.), Vol. 1, chapter 21:1341–1393.

[8] Bernanke, B.S. and C. S. Lown, 1991, Credit Crunch, Brookings Papers on Economic Activity, 22:205–248.

[9] Chen, N.-K., 2001, Bank Net Worth, Asset Prices and Economic Activity, Journal of Monetary Economics, Elsevier, 48:415–436.

[10] Clerc, L., A. Derviz, C. Mendicino, S. Moyen, K. Nikolov, L. Stracca, J. Suarez and A. P. Vardoulakis, 2015, Capital Regulation in a Macroeconomic Model with Three Layers of Default, International Journal of Central Banking, 11:9–63.

[11] Cooley, T. F. and E. C. Prescott, 1995, Economic Growth and Business Cycles. Princeton University Press.

[12] Cordoba, J. C. and M. Ripoll, 2004, Credit Cycles Redux, International Economic Review, 45:1011–1046.

[13] Diamond, D. W. and P. H. Dybvig, 1983, Bank Runs, Deposit Insurance, and Liquidity, Journal of Political Economy, 91:401–419.

[14] Elenev. V., T. Landovoigt and S. Van Nieuwerburgh, 2017, A Macroeconomic Model with Financially Constrained Producers and Intermediaries, SSRN Library.

[15] Garcia-Cicco, J., M. Kirchner and S. Justel, 2015, Domestic Financial Frictions and the Transmission of Foreign Shocks in Chile, in Global Liquidity, Spillovers to Emerging Markets and Policy Responses, edited by Raddatz, Saravia and Ventura, Central Bank of Chile.

[16] Gerali, A., S. Neri, L. Sessa and F.M. Signoretti, 2010, Credit and Banking in a DSGE Model of the Euro Area, Journal of Money, Credit and Banking, 42:107–141.

[17] Gersbach, H. and J.-C. Rochet, 2016, Capital Regulation and Credit Fluctuations, forthcoming, Journal of Monetary Economics.

[18] Gertler, M. and P. Karadi, 2011, A Model of Unconventional Monetary Policy, Journal of Monetary Economics, 58:17–34.

[19] Gertler, M. and N. Kiyotaki, 2010, Financial Intermediation and Credit Policy in Business Cycle Analysis, Handbook of Monetary Economics, in: Benjamin M. Friedman & Michael Woodford (ed.), 3:547–599.

[20] Gertler, M. and N. Kiyotaki, 2015, Banking, Liquidity, and Bank Runs in an Infinite Horizon Economy, American Economic Review, 105:2011–2043.

[21] Gertler, M., N. Kiyotaki and A. Queralto, 2012, Financial Crises, Bank Risk Exposure and Government Financial Policy, Journal of Monetary Economics, 59:17–34.

[22] Goodfriend, M. and B.T. McCallum, 2007, Banking and Interest Rates in Monetary Policy Analysis: a Quantitative Exploration, Journal of Monetary Economics, 54:1480–1507.

[23] Harris, M., C. C. Opp and M. M. Opp, 2014, Macroprudential Bank Capital Regulation in a Competitive Financial System, mimeo, University of California, Berkeley (Haas).

[24] Hart, O. and J. Moore, 1994, A Theory of Debt Based on the Inalienability of Human Capital, Quarterly Journal of Economics 109:841–879.

[25] Hirakata, N., N. Sudo and K. Ueda, 2009, Chained Credit Contracts and Financial Accelerators, forthcoming, Economic Inquiry.

[26] HM Treasury, 2015, Remit and Recommendations for the Financial Policy Committee, 8 July. Available at http://www.bankofengland.co.uk/financialstability/Documents/fpc/letters

[27] Holmstrom, B. and J. Tirole, 1997, Financial Intermediation, Loanable Funds and the Real Sector, Quarterly Journal of Economics 112:663–691.

[28] Kiyotaki, N. and J. Moore, 1997, Credit Cycles, Journal of Political Economy, 105:211–248.

[29] Kocherlakota, M. R., 2001, Risky Collateral and Deposit Insurance, The B.E. Journal of Macroeconomics, 1:1–20.

[30] Krishnamurthy, A., 2003, Collateral Constraints and the Amplification Mechanism, Journal of Economic Theory, 111:277–292.

[31] Lang, W. and L. Nakamura, 1995, Flight to Quality in Banking and Economic Activity, Journal of Monetary Economics, 36:145–164.

[32] Martinez-Miera, D. and J. Suarez, 2012, A Macroeconomic Model of Endogenous Systemic Risk Taking, CEPR Discussion Papers, No. 9134.

[33] Meh, C. A. and K. Moran, 2010, The Role of Bank Capital in the Propagation of Shocks, Journal of Economic Dynamics and Control, 34:555–576..

[34] Nelson, B. D. and G. Pinter, 2016, Macroprudential Capital Regulation in General Equilibrium, mimeo, Bank of England.

[35] Rannenberg, A., 2012, Asymmetric Information in Credit Markets, Bank Leverage Cycles and Macroeconomic Dynamics, European Central Bank, Working Paper Series, No. 1487.

[36] Van den Heuvel, S. J., 2008, The Welfare Cost of Bank Capital Requirements, Journal of Monetary Economics, 55:298–320.

Appendix A. Proofs

Proof of Proposition 1

As borrowers' marginal product of capital equals one in the steady state, we restrict our analysis to the impact of ξ on mpk^I:

$$\frac{\partial mpk^I}{\partial \xi} = \frac{\partial mpk^I}{\partial R^B}\frac{\partial R^B}{\partial \xi}. \tag{1}$$

As for the partial derivative of bankers' marginal product of capital with respect to the loan rate:

$$\frac{\partial mpk^I}{\partial R^B} = -\frac{\varkappa \omega \beta^B}{\kappa^2 R^S \beta^I}, \tag{2}$$

where $\kappa \equiv R^F\left(1-\beta^F\right) - \omega\left(1-\beta^F R^F\right) > 0$ and $\varkappa \equiv R^S\left(1-\beta^I\right) - \chi\left(1-\beta^I R^S\right) > 0$, so that $\partial mpk^I/\partial R^B < 0$.

As for $\partial R^B/\partial \xi < 0$, this is negative, in light of assuming $\beta^I R^S < 1$:

$$\frac{\partial R^B}{\partial \xi} = -\frac{\chi\left(1-\beta^I R^S\right)}{\beta^I R^S}. \tag{3}$$

Thus, both factors on the right-hand side of (1) are negative and, since $\partial\Delta/\partial\xi = -\partial mpk^I/\partial\xi$, increasing ξ inevitably reduces the productivity gap.■

Proof of Proposition 2

We first prove that increasing ξ attenuates the impact of the technology shock on borrowers' capital-holdings. According to Equation (35) in the main text, v quantifies the pass-through of $\hat{\alpha}_t$ on $\hat{k}_t^B$. In turn, the marginal impact of ξ on v can be computed as:

$$\frac{\partial v}{\partial \xi} = \frac{(\lambda-\phi)(1-\rho)\rho}{(1-\lambda)(1-\phi\rho)}\frac{\partial \eta}{\partial \xi} + \frac{(\lambda\rho-1)(1-\rho)\rho}{(1-\phi\rho)^2}\frac{\partial \phi}{\partial \xi}, \tag{4}$$

where:

$$\frac{\partial \eta}{\partial \xi} = \frac{\partial \eta}{\partial k^B}\frac{\partial k^B}{\partial R^B}\frac{\partial R^B}{\partial \xi} \quad \text{and} \quad \frac{\partial \phi}{\partial \xi} = \frac{\partial \phi}{\partial R^B}\frac{\partial R^B}{\partial \xi}. \tag{5}$$

Focusing on the second term on the right-hand side of (4), we can show this is negative, as: (i) $\frac{(\lambda\rho-1)(1-\rho)\rho}{(1-\phi\rho)^2} < 0$, given that $\lambda\rho < 1$; (ii) $\partial\phi/\partial R^B = -\omega/\left(R^B\right)^2 < 0$; (iii) $\partial R^B/\partial\xi < 0$, as implied by (3).

As for the first term on the right-hand side of (4): $\frac{(\lambda-\phi)(1-\rho)\rho}{(1-\lambda)(1-\phi\rho)} > 0$. Furthermore:

$$\frac{\partial \eta}{\partial k^B} = -\frac{1}{(1-\mu)\left(k^B\right)^2} < 0$$

and

$$\frac{\partial k^B}{\partial R^B} = \frac{\omega}{\kappa R^B(\mu-1)}\left(\frac{1}{\mu}\frac{R^B\beta^B\varkappa}{R^S\beta^I\kappa}\right)^{\frac{1}{\mu-1}} < 0, \tag{6}$$

where $\kappa \equiv R^B\left(1-\beta^B\right) - \omega\left(1-\beta^B R^B\right)$ and $\varkappa \equiv R^S\left(1-\beta^I\right) - \chi\left(1-\beta^I R^S\right)$. As $\partial R^B/\partial\xi < 0$, also the first term on the right-hand side of (4) is negative. Therefore, ν is a negative function of ξ.

As for the impact of technology shocks on the capital price:

$$\frac{\partial \gamma}{\partial \xi} = \frac{\partial \gamma}{\partial \phi}\frac{\partial \phi}{\partial \xi}. \tag{7}$$

As for $\partial \gamma / \partial \phi$:

$$\frac{\partial \gamma}{\partial \phi} = -\frac{1-\rho}{\left(1-\phi\rho\right)^2}\rho < 0, \tag{8}$$

while we already know that $\partial \phi / \partial \xi > 0$. Therefore, the overall effect of ξ on γ is negative.■

Proof of Proposition 3

We know that $G^{'}(k^I)$ is a decreasing function of θ. Thus, we aim to prove that the gap between bankers' and borrowers' marginal product of capital is greater than zero at $\theta = 0$. To this end, we combine the capital Euler equations of bankers and borrowers, obtaining:

$$G^{'}(k^I)\Big|_{\theta=0} = \frac{R^B \beta^B \left(R^S - 1\right)}{\left(1-\beta^B\right) R^B - \omega\left(1-\beta^B R^B\right)}.$$

We then impose $G^{'}(k^I)\Big|_{\theta=0} < 1$ to obtain

$$R^B > \frac{\omega}{1-\beta^B\left(R^S-\omega\right)}.$$

As $R^B\big|_{\theta=0} = \frac{R^S\left(1+\beta^I\right)-1}{\beta^I}$, all we need to prove is that

$$\frac{R^S\left(1+\beta^I\right)-1}{\beta^I} > \frac{\omega}{1-\beta^B\left(R^S-\omega\right)},$$

which can be manipulated to obtain

$$\left(1-\beta^B R^S\right)\left[\beta^I\left(R^S-\omega\right)+R^S-1\right]+\left(R^S-1\right)\beta^B\omega > 0.$$

As $\beta^B R^S < 1$, it is immediate to verify that both terms on the left-hand side of the last inequality are positive.

■

Appendix B. Equilibrium conditions

Derivation of key equilibrium conditions

Borrowers maximize their utility under the collateral and the flow-of-funds constraints, taking R^B as given. The corresponding Lagrangian reads as:

$$\begin{aligned} \mathcal{L}_t^B &= E_0 \sum_{t=0}^{\infty}\left(\beta^B\right)^t \Big\{c_t^B - \vartheta_t^B\left[c_t^B + R^B b_{t-1}^B + q_t(k_t^B - k_{t-1}^B) - b_t^B - \alpha_t k_{t-1}^B\right] \\ &\quad -\upsilon_t\left(b_t^B - \omega\frac{q_{t+1}k_t^B}{R^B}\right)\Big\}, \end{aligned} \tag{9}$$

where ϑ_t^B and υ_t are the multipliers associated with borrowers' budget and collateral constraint, respectively. The first-order conditions are:

$$\frac{\partial \mathcal{L}_t^B}{\partial b_t^B} = 0 \Rightarrow -\beta^B R^B E_t \vartheta_{t+1}^B + \vartheta_t^B - \upsilon_t = 0; \tag{10}$$

$$\frac{\partial \mathcal{L}_t^B}{\partial k_t^B} = 0 \Rightarrow -\vartheta_t^B q_t + \beta^B E_t \left[\vartheta_{t+1}^B q_{t+1}\right] + \beta^B E_t \left[\vartheta_{t+1}^B \alpha_{t+1}\right] + \omega \upsilon_t E_t \left[\frac{q_{t+1}}{R^B}\right] = 0. \tag{11}$$

Condition (10) implies that a marginal decrease in borrowing today expands next period's utility and relaxes the current period's borrowing constraint. As for (11), acquiring an additional unit of capital today allows to expand future consumption not only through the conventional capital gain and dividend channels, but also through the feedback effect of the expected collateral value on the price of capital. As we consider linear preferences (i.e., $\vartheta_t^B = \vartheta^B = 1$), (10) implies $\upsilon_t = \upsilon = 1 - \beta^B R^B$.[1] Thus, the collateral constraint binds in the neighborhood of the steady state as long as $R^B < 1/\beta^B$, which is imposed throughout the rest of the analysis. Finally, (11) can be rewritten as

$$q_t = \frac{\beta^B R^B + \omega\left(1 - \beta^B R^B\right)}{R^B} E_t q_{t+1} + \beta^B E_t \alpha_{t+1}. \tag{12}$$

The Lagrangian for bankers' optimization reads, instead, as

$$\begin{aligned} \mathcal{L}_t^I = {} & E_0 \sum_{t=0}^{\infty} \left(\beta^I\right)^t \{c_t^I - \vartheta_t^I [c_t^I + R^S b_{t-1}^S + b_t^B + q_t (k_t^I - k_{t-1}^I) \\ & -b_t^S - R^B b_{t-1}^B - \alpha_t G(k_{t-1}^I)] - \delta_t \left(b_t^S - \chi \frac{q_{t+1}}{R^S} k_t^I - \chi \xi \frac{b_t^B}{R^S}\right)\}, \end{aligned} \tag{13}$$

where ϑ_t^I and δ_t are the multipliers associated with bankers' budget constraint and enforcement constraint, respectively. The first-order conditions are:

$$\frac{\partial \mathcal{L}_t^I}{\partial b_t^S} = 0 \Rightarrow -R^S \beta^I E_t \vartheta_{t+1}^I + \vartheta_t^I - \delta_t = 0; \tag{14}$$

$$\frac{\partial \mathcal{L}_t^I}{\partial b_t^B} = 0 \Rightarrow R^B \beta^I E_t \vartheta_{t+1}^I - \vartheta_t^I + \frac{1}{R^S} \chi \xi \delta_t = 0; \tag{15}$$

$$\frac{\partial \mathcal{L}_t^I}{\partial k_t^I} = 0 \Rightarrow -\vartheta_t^I q_t + \beta^I E_t \left[\vartheta_{t+1}^I q_{t+1}\right] + \beta^I E_t \left[\vartheta_{t+1}^I \alpha_{t+1} G'(k_t^I)\right] + \delta_t \chi \frac{E_t [q_{t+1}]}{R^S} = 0. \tag{16}$$

As we assume linear preferences, $\vartheta_t^I = \vartheta^I = 1$. Therefore, conditions (14) and (15) imply that the financial constraint holds with equality in the neighborhood of the steady state (i.e., $\delta_t = \delta > 0$) as long as (i) $R^S \beta^I < 1$ and (ii) $R^B \beta^I < 1$. By combining (14) and (15) we obtain

$$R^B = \frac{R^S - \chi \xi \left(1 - \beta^I R^S\right)}{\beta^I R^S}, \tag{17}$$

Finally, from (16) we can retrieve the Euler equation governing bankers' investment in real assets:

$$q_t = \frac{R^S \beta^I + \chi\left(1 - \beta^I R^S\right)}{R^S} E_t q_{t+1} + \beta^I E_t \left[\alpha_{t+1} G'(k_t^I)\right]. \tag{18}$$

Summary of the model

We have 12 endogenous variables: $\{q_t\}_{t=0}^{\infty}$, $\{c_t^S, b_t^S\}_{t=0}^{\infty}$, $\{c_t^B, b_t^B, k_t^B, y_t^B\}_{t=0}^{\infty}$, $\{c_t^I, b_t^I, b_t^S, k_t^I, y_t^I\}_{t=0}^{\infty}$, along with the aggregate productivity shifter: $\{\alpha_t\}_{t=0}^{\infty}$. The general equilibrium is characterized by

[1] Steady-state variables are reported without the time subscript.

the following equations:

- Market clearing (goods, credit and capital market, respectively):

$$\epsilon^S + y_t^B + y_t^I = \underbrace{c_t^S + c_t^B + c_t^I}_{y_t}, \tag{19}$$

$$b_t^S = \frac{\chi\xi}{R^S}\left(\omega\frac{k_t^B}{R^B} + \frac{1}{\xi}k_t^I\right)E_t q_{t+1}, \tag{20}$$

$$k_t^B + k_t^I = 1. \tag{21}$$

- Production technologies:

$$y_t^B = \alpha_t k_{t-1}^B, \tag{22}$$

$$y_t^I = \alpha_t G\left(k_{t-1}^I\right), \tag{23}$$

where

$$\log\alpha_t = \rho\log\alpha_{t-1} + u_t. \tag{24}$$

- Credit demand and supply:

$$b_t^B = \omega\frac{\mathbb{E}_t q_{t+1}}{R^B}, \tag{25}$$

$$b_t^B = \frac{1}{\chi\xi}\left(R^S b_t^S - \chi\mathbb{E}_t q_{t+1}k_t^I\right). \tag{26}$$

- Capital demand schedules:

$$q_t - \frac{\beta^I R^S + \chi\left(1-\beta^I R^S\right)}{R^S}\mathbb{E}_t q_{t+1} = \beta^I\mathbb{E}_t\alpha_{t+1}G'\left(k_t^I\right), \tag{27}$$

$$q_t - \left[\beta^B + \omega\left(\frac{1}{R^B} - \beta^B\right)\right]\mathbb{E}_t q_{t+1} = \beta^B\mathbb{E}_t\alpha_{t+1}. \tag{28}$$

- Budget constraints:

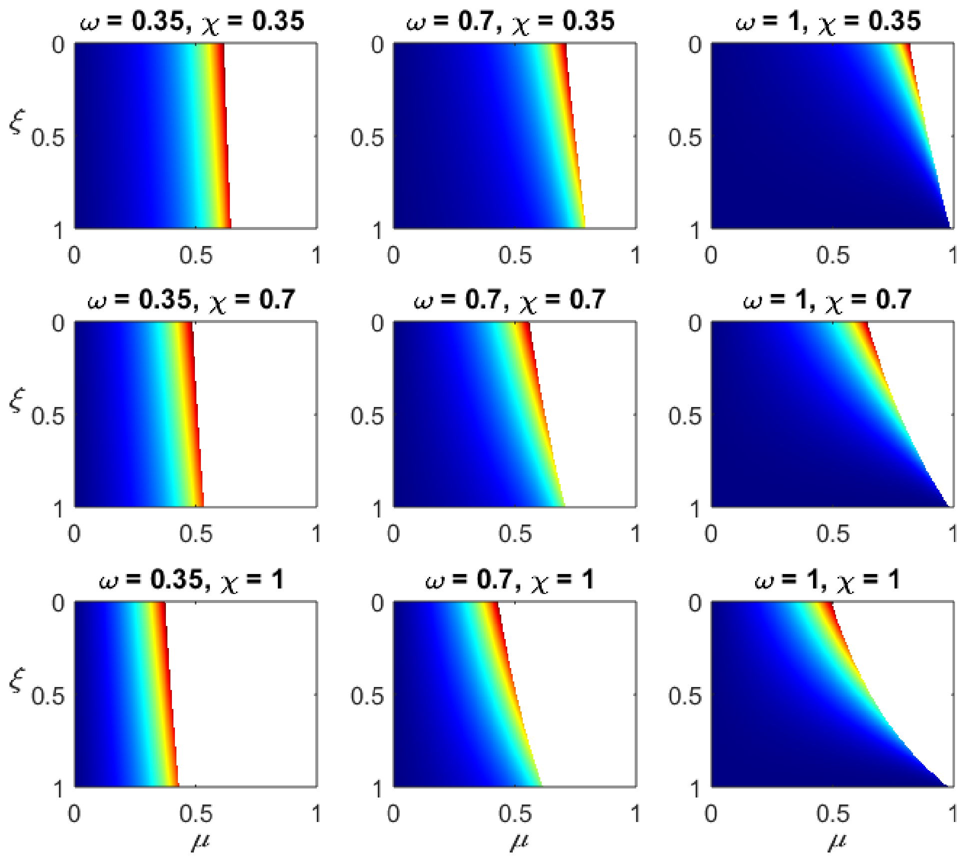
$$c_t^S + b_t^S = \epsilon^S + R^S b_{t-1}^S, \tag{29}$$

$$c_t^B + q_t k_t^B = y_t^B + q_t k_{t-1}^B + b_t^B - R^B b_{t-1}^B, \tag{30}$$

$$c_t^I + b_t^B + R^S b_{t-1}^S + q_t \left(k_t^I - k_{t-1}^I\right) = y_t^I + b_t^S + R^B b_{t-1}^B. \tag{31}$$

Appendix C. Robustness exercises

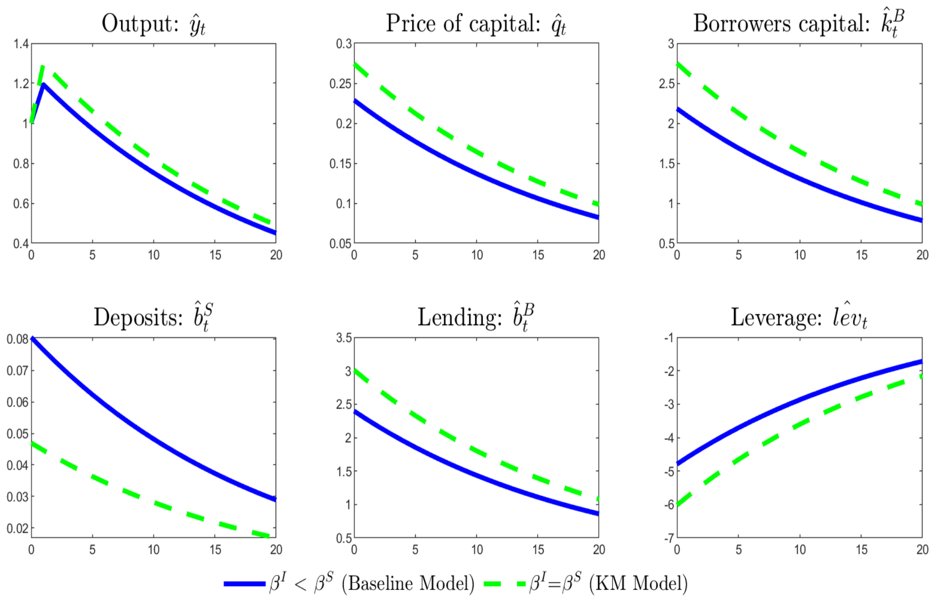
Figure C.1 Business cycle amplification.



Notes. Figure C.1 graphs ϖ as a function of ξ (y-axis) and μ (x-axis), and for different values of χ and ω, under the following parameterization: $\beta^S = 0.99$, $\beta^I = 0.98$, $\beta^B = 0.97$, $\rho = 0.95$. The white area denotes inadmissible equilibria where bankers' capital-holdings are virtually negative.

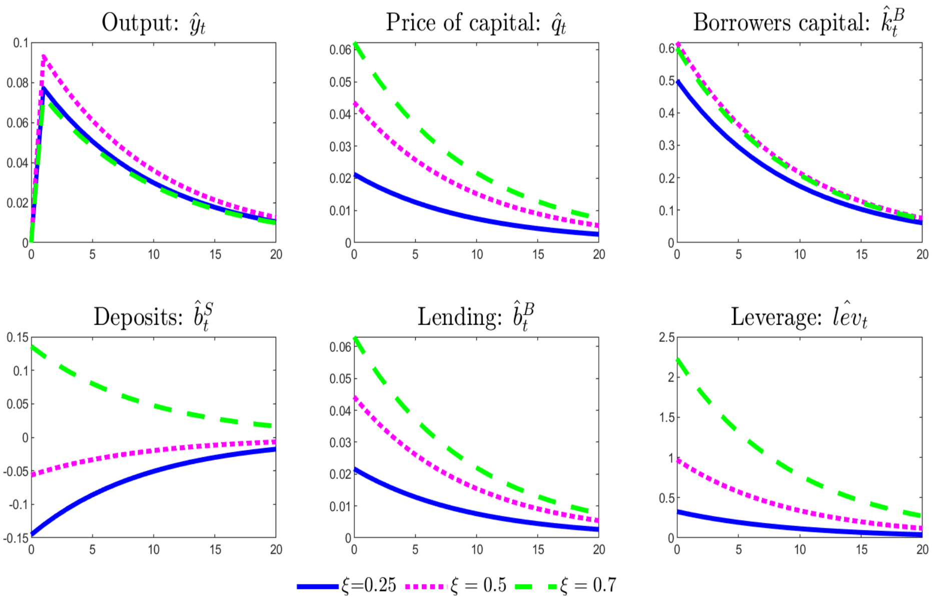
Appendix D. Additional figures

Figure D1: Comparison with KM under a technology shock.



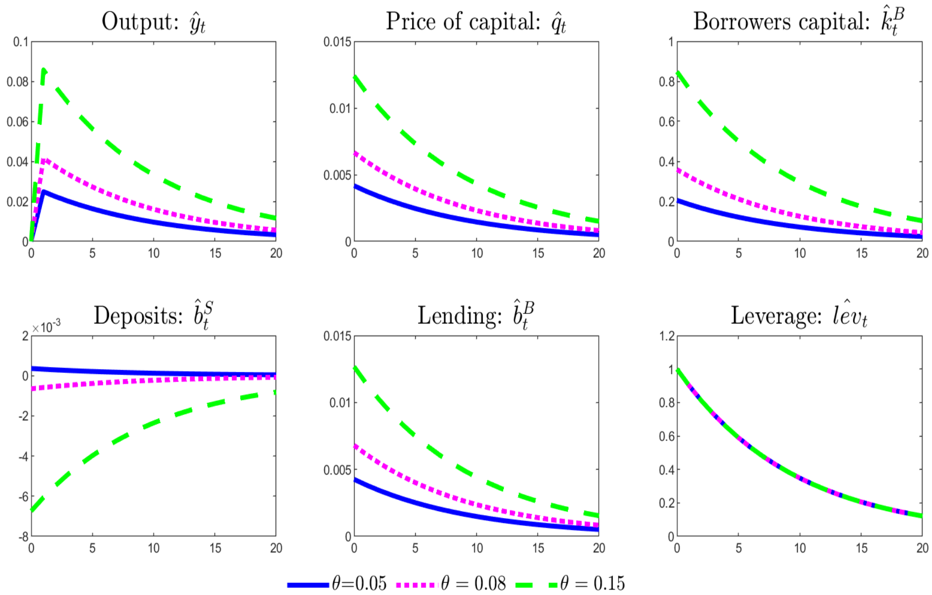
Notes. Figure D1 graphs the response to a one-standard-deviation shock to technology, under the following parameterization: $\beta^S = 0.99$, $\beta^I = 0.98$, $\beta^B = 0.97$, $\rho = 0.95$, $\chi = \omega = 1$, $\mu = 0.4$. We consider two situations: the KM case, where $1 - \beta^I R^S = 0$ (green-dashed line), and the baseline model (blue-continuos line).

Figure D2: Responses to a financial shock.



Notes. Figure D2 graphs the responses of selected variables to a one-standard-deviation shock to the degree of collateralization, ξ, under the following parameterization: $\beta^S = 0.99$, $\beta^I = 0.98$, $\beta^B = 0.97$, $\rho = 0.95$, $\chi = \omega = 1$, $\mu = 0.4$.

Figure D3: Responses to a shock to the capital-to-asset ratio.



Notes. Figure D3 graphs the responses of selected variables to a (negative) one-standard-deviation shock to capital-to-asset ratio, θ, under the following parameterization: $\beta^S = 0.99$, $\beta^I = 0.98$, $\beta^B = 0.97$, $\rho = 0.95$, $\omega = 1$, $\mu = 0.4$.