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THE ARMISTICE OF THE SEXES:
GENDER COMPLEMENTARITIES IN THE
PRODUCTION FUNCTION
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# THE ARMISTICE OF THE SEXES: GENDER COMPLEMENTARITIES IN THE PRODUCTION FUNCTION 

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# THE ARMISTICE OF THE SEXES: GENDER COMPLEMENTARITIES IN THE PRODUCTION FUNCTION 


#### Abstract

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JEL Classification: O47, J24

Keywords: Female Labor Force Participation, aggregate production function, elasticity of substitution, growth

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# The Armistice of the Sexes: <br> Gender Complementarities in the Production Function 

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June 2019


#### Abstract

Macroeconomic models have largely ignored the importance of gender diversity by assuming that male and female workers are perfectly substitutable in the aggregate production function. Whether this assumption is valid is an empirical question that this paper aims to answer by estimating the elasticity of substitution (ES) between the two types of labor. We apply linear and non-linear techniques to cross-country data at the aggregate level, to cross-country data at the sectoral level, and to firm-level data for the manufacturing sector in China. We find that women and men are far from being perfect substitutes: the ES is below 1 for the aggregate sample, between 1-2 for the sectoral sample, and between 2-3 at firm-level. We discuss why the ES may vary at different levels of aggregation and conclude on the implications of these results for growth accounting and for the gains from gender equality.


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[^0]"The models that researchers build to understand the economy tend to be blind to race and gender, as if macroeconomic policies typically affect blacks the same as whites and women the same as men. Increasingly, that's looking like the wrong way to go about it. The models need to change."

Narayana Kocherlakota, July, 2017

## 1 Introduction

Female labor force participation (FLFP) remains stubbornly below male labor force participation (MLFP). There are deep cultural factors that explain the low participation of women in the workforce, in particular related to the historical role of agricultural work in the division of labor. Some progress was made in the last half century, owing in particular to technological improvements at home, to changing norms on gender roles and to contraception (Giuliano (2014)). Nonetheless, in 2014, FLFP - at 54 percent for the median OECD country - was well below MLFP ( 68 percent), and even lower for the median middle-income country at 49 percent, compared to 75 percent for MLFP.

Although the literature has started to account for gender differences in its macroeconomic models (e.g. Borella et al. (2017)), it is often assumed, including in models designed to assess the impact of gender discrimination, that raising FLFP would increase output mainly through the direct effect of adding workers to the labor force. In this paper, we argue that such mechanical exercises do not recognize that, because of social norms affecting the way men and women and raised and interact in the workplace, women bring different skills and ideas that are of important value to the economy. Specifically, production functions which impose that labor enters only as the sum of female and male workers, and thus assume perfect substitutability, may not only embody a significant departure from reality, but if used to assess the impact of gender diversity, may lead to misleading estimates of the benefits, for both men and women, of increasing FLFP.

That male and female workers bring different skills to the workplace has been given prominence recently by Lagarde (2014), for example, who emphasizes the differences in management and negotiating strategies between men and women. Microeconomic analyses support this view of the workplace. In particular, it has been recognized that different generations, cultural backgrounds, and genders contribute to better management, including risk management, as well as to higher productivity and improved bottom lines for firms (Terjesen et al. (2009)).

To tackle the limitations of the existing literature, this paper makes three main contributions. First, we clarify analytically the importance of gender diversity in cases where the elasticity of substitution is finite, and argue that -with the relevant range presumably one of imperfect but partial substitutability - it is important to model production with women and men entering as separate
arguments. In such a case, for those sectors or countries where female workers are in short supply and men are in excess supply, the effect of raising FLFP will be larger than the effect of raising MLFP by the same amount, as long as women's productivity is not substantially lower than male productivity. We show that the positive effect will be stronger the more scarce are women in the labor force (and the larger the relative surplus of men), and the lower is the elasticity of substitution.

Second, this paper applies linear and non-linear techniques to estimate the elasticity of substitution between men and women, using cross-country data at the aggregate level, cross-country data at the sectoral level, and firm-level data for manufacturing firms in China. To the best of our knowledge, this paper represents the first attempt at estimating this elasticity of substitution using production data, an exercise that permits us to cover a large panel of advanced economies and to compare the elasticity of substitution for different levels of aggregation. The model for the production function relates output to the stock of capital, male employment, and female employment, in a constant elasticity of substitution (CES) specification. We find that in most specifications the elasticity of substitution between male and female labor is below 1 for the aggregate sample, between 1-2 for the sectoral sample, and around 2-3 at the firm-level, suggesting that men and women are indeed imperfect substitutes.

Third, we discuss why the ES may vary at different levels of aggregation and we use the estimated production function to interpret past increases in total factor productivity (TFP) and to compute the potential effects of future increases in FLFP on growth and TFP. For the median OECD country, where the share of women in total employment workers is 0.43 , a modified growth accounting exercise suggests that raising female employment to match male employment would increase GDP by 8-14 percent (as with other growth accounting exercises, the calculations take the production function as given and do not attempt to explain why women are in short supply and men in excess supply). A significant contribution of this increase would come from the effect of imperfect substitutability on labor productivity, which would be between 1 and 6 percent of GDP under the range of our elasticity of substitution estimates. Looking also at the dramatic increases in FLFP over the past 50 years, our results suggest that the common interpretation of TFP growth as being driven by technology, innovation, etc., may under-estimate the contribution of FLFP to TFP growth.

The finding that the gains are higher the lower the elasticity of substitution runs counter to the argument in Baqaee and Farhi (2017) that gains from removing distortions are higher the more substitutable the inputs. This is because our calculations implicitly assume that men are initially in excess supply, so that increasing FLFP not only increases the labor force but it also improves the productivity of men, by correcting the gender imbalance. In addition, we assess the effect of achieving a given allocation for female and male employment (gender equality in employment rates), as opposed to one where a given distortion, taken as a primitive, is removed, as is done

Baqaee and Farhi (2017) -we explain the difference between these two exercises in more detail in Section 7. We think of this experiment as a relevant one because policymaking often works by setting quantitative targets in the first place. Policies then can be implemented to achieve the desired target. ${ }^{1}$

Our findings indicate that incorporating gender diversity in macroeconomic models may be quite important for policy analyses of tax reform, flexible work arrangements, childcare support, and anti-discrimination regulations, and even more generally for assessing the effects of other macroeconomic policies. Some recent literature has recognized this, but models have been calibrated using either arbitrary values of the elasticity (typically between 1 and 2, e.g. in Fontana and Wood (2000); Agénor (2017)) or using elasticities of substitution estimated on US microeconomic wage data from the 1950s (e.g. Acemoglu et al. (2004)).

The rest of the paper is organized as follows: Section 2 places the paper's contribution in the context of the existing literature. Section 3 presents a basic model to clarify the concept of gender diversity used in the empirical analysis. Section 4 discusses the empirical strategy based on linear and non-linear cross-country, cross-industry, and firm-level regressions, while Section 5 looks at the data and some stylized facts. Section 6 reports our baseline results from the linear and non-linear estimation, and Section 7 examines the implications for TFP and GDP growth. Section 8 concludes.

## 2 A Brief Literature Review

## Female labor force participation

Although economists traditionally have related low FLFP to economic incentives or institutional constraints, several recent studies have highlighted the cultural origins of low female labor force participation (see also the survey in Giuliano (2015)). Following on Boserup (1970)'s hypothesis that gender roles originate from the division of labor in agricultural societies, Alesina et al. (2013) showed that FLFP remains lower in those regions where the plough, a heavy tool which requires upper body strength and was mostly used by men, was the main instrument for cultivation. The persistence of cultural factors across generations has also been documented. Fernandez and Fogli (2009) showed, using data on second-generation immigrants, that US-born women whose families originated from countries with low FLFP were less likely to work than similar women whose parents' countries of origin had high FLFP. In addition, learning about new possibilities can be slow. Fogli and Veldkamp (2011) develop a model where women have cultural priors on the balance between

[^1]work and family, but learn about the consequences of maternal employment on child rearing using their neighbors' experiences. The model predictions are consistent with the spatial correlation and the dynamic evolution of FLFP observed in the data. The literature has also discussed the main drivers of the recent increases (and possible leveling-off) in FLFP, highlighting the importance of improvements in maternal health (Albanesi and Olivetti (2016)), new technologies in the household (Greenwood et al. (2005)), changing gender norms (Fernández et al. (2004), Fortin (2015)), and the adoption of the pill (Goldin and Katz (2002)).

Reducing gender inequality, whether obtained from changes in legal situations, cultural values, or economic incentives, should boost growth directly (with higher labor supply) and indirectly (by improving factor efficiency). Gender equality has been shown to increase the human capital of women, to increase female labor force participation, to improve access to finance for female entrepreneurs, to improve money management in households and corporates, etc. An exhaustive literature survey is outside the scope of this paper, but the reader is referred to the World Bank (2012)'s World Development Report for the development and microeconomic literature, and to Seguino (2013) for the macroeconomic literature.

Increasing FLFP brings women into the measured labor force, but it also reduces the production of unpaid home goods and services (including reproduction, which is also a public good; see e.g., Erturk and William Darity (2000)). Economic theory suggests that the overall effect on welfare depends on: (i) the extent to which FLFP was depressed by suboptimal constraints; (ii) the externalities from home production; and (iii) the externalities from work in the market economy. When focusing more narrowly on the measurable level of market economic activity, the first order effect of increasing FLFP on GDP should depend on: (i) the productivity of paid work, and (ii) the costs for economic activity of the reduction in unpaid home work (which will depend, inter alia, on the extent of provision of public services such as childcare). ${ }^{2}$

The existing literature has taken three main directions to link FLFP and growth: (i) a few studies have made use of an accounting approach, and produced estimates of the GDP gains from increases in FLFP by assuming the gains from a female entrant are proportional to the average productivity of current workers (e.g., Aguirre et al. (2012)); (ii) in several empirical papers, labor market participation has been bundled together with other indicators of gender inequality (legal rights, human capital, etc.) to showcase the importance of gender issues for long-term growth or for income inequality (e.g., Elborgh-Woytek et al. (2013)); (iii) a few papers have calibrated general equilibrium models and assessed the benefits of removing barriers to female labor force participation, see Alvarez (2019).

[^2]
## Gender diversity

Microeconomic assessments of gender diversity have benefited from the labor economics literature on labor demand functions and substitutability between different types of workers (see e.g. an early survey by Hamermesh and Grant (1979)). Freeman (1979) estimates the elasticity of substitution between male and female workers of different cohorts and concludes that "the magnitudes of those elasticities ... are sufficiently slight as to suggest that male and female workers operate in essentially separate production processes". Merrilees (1982) arrived at a similar conclusion. On the other hand Grant and Hamermesh (1981) estimated a similar model, assuming exogenous supplies of labor and capital, and found that young workers are highly substitutable with older female workers, but the effect of increasing male employment on female wages is much smaller than on male wages, i.e. the elasticity of substitution between older male and female workers is lower. Similarly, when splitting workers by gender, Costrell et al. (1986) found very little substitution between male and female workers. Using time series data, Johnson and Blakemore (1979) estimated that the elasticity of substitution is around 1.4.

The literature has also looked at different types of employment. At the lower end of the skill spectrum, Berger (1983) and Topel (1994) found that female workers substitute more closely for low skilled or younger workers than for older male workers, but confidence in these findings is vulnerable to omitted variable bias. When Juhn and Kim (1999) control for demand shocks, they obtain the more intuitive result that skilled female workers substitute for skilled male workers, and not for unskilled ones.

Acemoglu et al. (2004) addressed endogeneity concerns by using as a natural experiment the exogenous increase in female labor supply in the US triggered by male conscription during World War II. By looking at the effect on male wages relative to female wages across US states in the 1950s, Acemoglu et al. (2004) found an elasticity of substitution between male and female workers of 3, higher than what is found typically between skilled and unskilled workers (e.g Ciccone and Peri (2005)). In a similar exercise, Pellizzari et al. (2014)) used the abolition of compulsory military service in Italy in 2000 to estimate an elasticity of substitution between male and female workers of 1.0-1.4.

At the level of specific types of employment, the literature on the determinants of the wage gap is well developed (e.g. Azmat and Ferrer (2017) show the importance of aspirations on lawyers' work performance and pay), but little evidence exists on substitution effects. Gender differences in risk behavior and reaction to incentives are also well documented in lab experiments (e.g. Shurchkov (2012) and Azmat et al. (2016); see also Azmat and Petrongolo (2014) for a survey), and have been shown to affect the performance of firms, depending on the gender composition of their board. For instance, Dezsö and Ross (2012) find that for firms whose strategy is based on innovation, gender inclusiveness in the corporate board has positive effects on firm value.

## Estimating aggregate production functions

To the best of our knowledge, there is no existing assessment of the extent of substitutability between male and female workers that is based on estimation of production functions, though our work is of course related to an extensive literature trying to estimate the extent of substitutability between labor and capital. ${ }^{3}$ Our work is closely related to Duffy and Papageorgiou (2000) who in addition to linear panel regression methods, also use Non-Linear Least Squares (NLLS) to estimate the elasticity between capital and labor. A key advantage of such NLLS estimation is that it does not rely on the assumption that firms maximize profits, which is inconsistent with the evidence that firms discriminate again female workers (Altonji and Blank (1999); Jarrell and Stanley (2004)).

## 3 Model

## Production technology

We consider a neoclassical growth model with a CES production technology. Output, $Y$, is produced with a constant returns to scale technology in the factors of production, which are represented by a labor composite input $L$ and the capital stock $K$ :

$$
\begin{equation*}
Y=A\left(\delta_{\ell} L^{\rho_{1}}+\delta_{k} K^{\rho_{1}}\right)^{\frac{1}{\rho_{1}}}, \tag{1}
\end{equation*}
$$

where the elasticity of substitution between labor and capital is $\sigma_{1}=1 /\left(1-\rho_{1}\right)$ and $A$ is a technology parameter. $\delta_{\ell}$ and $\delta_{k}$ are the share parameters. The labor variable $L$ is itself a composite of female $(F)$ and male $(M)$ labor, nested in a CES function:

$$
\begin{equation*}
L=\left(\delta F^{\rho_{2}}+M^{\rho_{2}}\right)^{\frac{1}{\rho_{2}}}, \tag{2}
\end{equation*}
$$

where the elasticity of substitution between male and female workers is $\sigma_{2}=1 /\left(1-\rho_{2}\right)$, and $\delta$ is the share parameter. Substituting equation (2) into (1) gives a two-stage CES production function:

$$
\begin{equation*}
Y=A\left[\delta_{\ell}\left(\delta F^{\rho_{2}}+M^{\rho_{2}}\right)^{\frac{\rho_{1}}{\rho_{2}}}+\delta_{k} K^{\rho_{1}}\right]^{\frac{1}{\rho_{1}}} . \tag{3}
\end{equation*}
$$

The nested-CES production in equation (3) allows for four distinct cases which will prove relevant in guiding the empirical work later on:

1. $F$ and $M$ are perfect substitutes; this is the case where $\sigma_{2}=+\infty$, i.e. $\rho_{2}=1$;
2. $F$ and $M$ are combined in a Cobb-Douglas function; this is the case where $\sigma_{2}=1$, i.e. $\rho_{2}=0$;
3. $F$ and $M$ are imperfect substitutes but more substitutable than in a Cobb-Douglas function; this is the case where $1<\sigma_{2}<+\infty$, i.e. $0<\rho_{2}<1$;

[^3]4. $F$ and $M$ are more complements than what is assumed in a Cobb-Douglas function; this is the case where $\sigma_{2}<1$, i.e. $\rho_{2}<0$.

Two points are worth noting here. First, equation (3) assumes, for tractability, identical substitutability between male labor and capital and between female labor and capital, an assumption that was not rejected in the microeconomic literature. Second, we note that our estimation of $\sigma_{2}$ can be related to the existing literature on labor demand functions, i.e. on the sensitivity of factor prices to exogenous changes in factor quantities. ${ }^{4}$ One way to show this relationship is that the parameter $\sigma_{2}$ of the CES function is equal to minus the inverse of the elasticity of the relative wage $w^{f} / w^{m}$ to relative labor quantities $F / M$, given by $\frac{\partial \ln \left(w^{f} / w^{m}\right)}{\partial \ln (F / M)}=-\frac{1}{\sigma_{2}}$.

## Linearization

Expressing in lower case all variables in growth rates, the log-linearization of equation (1) yields (see Appendix 1):

$$
\begin{equation*}
y=\lambda \ell+(1-\lambda) k+a, \tag{4}
\end{equation*}
$$

where $\lambda$ is the labor share in national income. Similarly, the log-linearization of equation (2) yields:

$$
\begin{equation*}
\ell=\mu f+(1-\mu) m, \tag{5}
\end{equation*}
$$

where $\mu$ is the share of women in total labor income:

$$
\begin{equation*}
\mu=\frac{\frac{\partial L}{\partial F} F}{L}=\frac{\delta F^{\rho_{2}}}{L^{\rho_{2}}} . \tag{6}
\end{equation*}
$$

Note that if $\rho_{2}=1$ and $\delta=1$ (i.e. male and female workers are perfect substitutes and the CES weights for men and women are equal), then $\mu=\frac{F}{L}=\frac{F}{F+M}$. This is intuitive and also implies, from equation (5), that if $\rho_{2}=1$ and $\delta=1$, then $\ell=\frac{F}{M+F} f+\frac{M}{M+F} m$, i.e. growth in the labor composite is equal to growth in the sum of female and male employment.

Combining equation (4) with equation (5), output growth is:

$$
\begin{equation*}
y=\lambda \mu f+\lambda(1-\mu) m+(1-\lambda) k+a . \tag{7}
\end{equation*}
$$

Because the share of women's income in labor income, $\mu$, is a non-linear function of $\sigma_{2}$, we present a Taylor approximation of equation (7) to facilitate its interpretation. We define $r=1 / \sigma_{2}=1-\rho_{2} \geq 0$ as the extent of deviation from perfect substitutability. Deriving the Taylor approximation around

[^4]the point where male and female workers are perfect substitutes ${ }^{5}$ (i.e. $r \ll 1$ ), we find (see Appendix 2):
\[

$$
\begin{equation*}
\mu \approx \frac{\delta F}{\delta F+M}\left[1+\frac{M}{\delta F+M} r \ln \left(\frac{M}{F}\right)\right]=\mu^{\infty}\left[1+\left(1-\mu^{\infty}\right) r \ln \left(\frac{M}{F}\right)\right], \tag{8}
\end{equation*}
$$

\]

where $\mu^{\infty}=\delta F /(\delta F+M)$ is the women's share of labor income that would prevail if men and women were perfect substitutes. As expected, when $r>0$ (i.e. when $\sigma_{2}<\infty$ ) the women's share in labor income $\mu$ is higher than $\mu^{\infty}$ if and only if $M>F$, i.e. if female workers are in short supply, and the women's share in labor income is larger the less substitutable $M$ and $F$ are (i.e. the larger $r)$. Given $\mu$, the expression for growth is :

$$
\begin{equation*}
y \approx \lambda n+(1-\lambda) k+\underbrace{\lambda(f-m)\left(\mu-\frac{F}{N}\right)+\overbrace{a}^{\substack{\text { "true" techno- } \\ \text { logy yrowth }}},}_{\text {Solow residual (TFP growth) }} \tag{9}
\end{equation*}
$$

where $n=\frac{\Delta N}{N}=\frac{\Delta(F+M)}{F+M}=\frac{F}{N} f+\frac{M}{N} m$ is growth in the headcount of the labor force. The third term in equation (9) shows that when women's employment increases faster than men's $(f-m>0)$, the Solow residual (TFP), as estimated using growth in the capital stock $k$ and growth in total employment $n$, is growing faster than technical progress if and only if $\mu-\frac{F}{N}>0$, where $\mu$ is approximated in equation 8 .

This condition highlights the two effects of adding women to the labor force at a rate faster than men. First, women may work more or fewer hours than men, a channel captured by $\delta$. The second effect is due to imperfect substitutability between men and women, and is captured by the component $r \ln (M / F)$. Growth is thus higher the larger is $r \ln (M / F)$, i.e. the smaller the initial participation of women, and the less substitutable are men and women. The net effect on the Solow residual is a function of which channel is strongest. The first channel is 0 when there are no gender differences in work hours (i.e. $\delta=1$, in which case $\mu \geq F / N$ if $F \leq M$ ). The second channel is 0 when $M=F$ or when $\sigma_{2}=+\infty$.

We conclude this section by noting that any growth regression that seeks to estimate the effect of gender inclusiveness on growth (such as the ones surveyed in Section 2) would want to include the variable $f-m$ as an explanatory variable. Using the level of the gender gap, $F / M$, explicitly or as part of a broader gender inequality index, would miss the effects of gender diversity on growth.

## 4 Estimating the elasticity of substitution

This section uses the framework presented above to motivate the regression estimation in which the primary parameter of interest is the elasticity of substitution between women and men.

[^5]
### 4.1 Linear estimation

A standard regression that estimates GDP growth $y_{i, t}$ for a panel of countries indexed by $i$, for the period $t$, with explanatory variables including the growth rate of male and female employment is: ${ }^{6}$

$$
\begin{equation*}
y_{i, t}=\beta_{f} f_{i, t}+\beta_{m} m_{i, t}+\beta_{k} k_{i, t}+\Gamma \mathbf{Z}_{i, t}+\epsilon_{i, t} \tag{10}
\end{equation*}
$$

where $f_{i, t}$ is the growth rate of women employment, $m_{i, t}$ is the growth rate of men employment, $\beta_{f}, \beta_{m}$ are the coefficients of interest, $k_{i, t}$ is the growth rate of the stock of capital, $\beta_{k}$ is the coefficient for the capital stock $k, \mathbf{Z}_{i, t}$ is a matrix of control variables, and $\Gamma$ is the vector of coefficients corresponding to the control variables $\mathbf{Z}_{i, t} .{ }^{7}$

Identifying the coefficients by relating the theoretical expression for growth equation (7) to the growth regression (10) yields:

$$
\left\{\begin{array}{l}
\hat{\beta}_{f}=\lambda \mu  \tag{11}\\
\hat{\beta}_{m}=\lambda(1-\mu) \\
\hat{\beta}_{k}=(1-\lambda)
\end{array}\right.
$$

which implies that the women's share in labor income can be obtained from the ratio of coefficients on $f$ and $m$ :

$$
\begin{equation*}
\frac{\hat{\beta}_{f}}{\hat{\beta}_{m}}=\frac{\mu}{1-\mu} \tag{12}
\end{equation*}
$$

Since $\mu /(1-\mu)=(d F / M)^{\rho}$, where $d=\delta^{1 / \rho_{2}}$, it is possible to infer the elasticity of substitution $\sigma_{2}=1 /\left(1-\rho_{2}\right):$

$$
\begin{equation*}
\rho_{2}=\frac{\ln \left(\frac{\beta_{f}}{\beta_{m}}\right)}{\ln \left(\frac{d F}{M}\right)} \tag{13}
\end{equation*}
$$

The system (11) is however under-identified, since it is not possible to deduce the four unknowns $\left(\rho_{1}, \rho_{2}, \delta, \delta_{\ell}\right)$ from the three coefficients $\left(\hat{\beta}_{f}, \hat{\beta}_{m}, \hat{\beta}_{k}\right)$ estimated by the growth regression. One solution, presented in Appendix 3 is to derive a second order Taylor approximation of output, which leads to a quadratic function: $y=\beta+\phi_{1} f+\phi_{2} f+\phi_{3} m+\phi_{4}\left(f^{2}+m^{2}\right)$. However, the coefficient $\phi_{4}$, which is crucial for the estimation of the elasticity of substitution, was found to be very imprecisely estimated, yielding implausible values of $\sigma_{2}$. The second solution, chosen here, is to assume $d$ is fixed. ${ }^{8}$ For instance, if $\delta=1$, $\hat{\rho}_{2}=\frac{\ln \left(\hat{\beta}_{f} / \hat{\beta}_{m}\right)}{\ln (F / M)}$, i.e. $\rho_{2}$ can be interpreted as the effect of women employment growth (relative to men employment growth), in proportion to the initial values of

[^6]employment.

In the case where men and women are perfect substitutes, adding one male worker or one female worker would yield identical output, and thus the relative growth effect would be equal to the relative bases. However, if men and women are imperfect substitutes, we should find that $\rho_{2}<1$, i.e. the growth effect of adding one female worker is larger than the effect of adding one male worker when $F<M$. Finally, in the specific case of a Cobb-Douglas model with identical weights (i.e. $\delta=1$ ), we should find $\hat{\beta}_{f}=\hat{\beta}_{m}$, since $\hat{\rho}_{2}$ should equal 0 . The bottom line is that under an assumption for $d$, it is possible to estimate $\sigma_{2}$ using a linear growth regression model.

### 4.2 Non-linear estimation

The elasticity of substitution between female and male workers can also be estimated directly by using NLLS in the nested-CES aggregate production function (equation (3)). ${ }^{9}$ There are at least two advantages of NLLS over linear regression estimation. The first is that NLLS does not rely on a specific calibration of $\delta$. A second advantage is that the direct estimation does not rely on the condition that firms choose male and female labor inputs as profit maximizers, an assumption at odds with the findings of discrimination against women in the labor market. The main difficulty with NLLS is pinning down parameters in nonlinear functions due to convergence issues of the estimators used. ${ }^{10}$

Because of this challenge, we use a simpler production function in the NLLS estimations, where the elasticity of substitution between capital and the composite labor variable is equal to 1 (a CES nested in Cobb-Douglas):

$$
\begin{equation*}
Y_{i, t}=\left(A_{i} e^{\beta t}\right) K_{i, t}^{1-\alpha}\left(M_{i, t}^{\rho}+\delta F_{i, t}^{\rho}\right)^{\alpha / \rho} . \tag{14}
\end{equation*}
$$

or, in log-differences:

$$
\begin{equation*}
\Delta \ln \left(Y_{i, t}\right)=\beta+(1-\alpha) \Delta \ln \left(K_{i, t}\right)+\frac{\alpha}{\rho} \Delta \ln \left(M_{i, t}\right)+\frac{\alpha}{\rho} \Delta\left[\ln \left(1+\delta\left(\frac{F_{i, t}}{M_{i, t}}\right)^{\rho}\right)\right] . \tag{15}
\end{equation*}
$$

Equation (15) clarifies a third advantage of the NLLS method, in that it allows for variability in the ratio $F / M$, which has to be calibrated in the linear model to obtain an estimation of $\rho_{2}$ when using equation (13). Equation (14) or equation (15) can be estimated by NLLS and the asymptotic covariance matrix for the coefficients are obtained from the Jacobian matrix and mean square error of the model (Greene (2003)).

[^7]
## 5 A brief look at the data and some facts

## Cross-country data

The models presented in equation (10) and in equation (15) are estimated on country- and sectorallevel data, using 5 -year non-overlapping growth averages to filter out business cycle fluctuations. The main data constraints are for measures of the capital stock at the sectoral level, which are in general not available for non-OECD countries, and for measures of the gender composition of the labor force, for which the quality of data for non-OECD countries is doubtful. For the macro-level analysis, the World Bank's World Development Indicators (WDI) provide female labor force participation data for most advanced economies, starting in the 1990s. This dataset is complemented by data on output and capital stocks (PPP) from the Penn World Tables (version 9.0). We also check robustness to the OECD data on female employment (Annual Labor Force Statistics, ALFS) post-1990, for consistency with the World Bank data, and to the OECD data for GDP (PPP) and the IMF data on capital stock in PPP (IMF (2015)).

For sectoral value added and capital stocks, the data are taken from the OECD STructural ANalysis Database (STAN) database. STAN includes sectors where male employment dominates (e.g. mining) as well as sectors where female employment dominates (e.g. education, health and social work). The dataset coverage is heterogeneous across countries and sectors, but sufficient to be representative of all sectors. Depending on the exact series used, the macroeconomic annual dataset comprises around 1000 annual observations, which yields around 150 observations as 5 -year non-overlapping growth rates. The sectoral dataset includes 2831 annual observations, which are used to compute 513 non-overlapping 5 -year growth rates.

A brief look at the data reveals a couple of notable facts. In the last twenty years, female labor force participation has grown in most advanced and developing economies, while at the same time male labor force participation stagnated or even declined, thus yielding a reduction in gender inequality in labor market participation (see Figure 1). This reduction has, however, not been homogeneous across countries, with several countries in the Middle East and North Africa showing limited progress.

In addition, it is interesting to note that the ratio $F / M$ exhibits substantial heterogeneity both across countries and across sectors (see Figures 2 and 3, respectively). Therefore for the purpose of linear estimation, although the ratio $F / M$ can be set at its mean ( 0.77 for the OECD, or 0.64 for middle-income countries), the large observed heterogeneity may weaken the quality of the linear regression estimates.

Figure 1: Evolution of female and male labor force participation


Source: World Bank, World Development Indicators

Figure 2: Distribution of the ratio of female to male employment (annual data)


Table 1: China's firms-level data, summary statistics

| Variable | Observations | Mean | Median | Std. Dev. | 10th pctile | 90th pctile |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| In(value added) | 2,528 | 8.4 | 8.2 | 1.5 | 6.8 | 10.2 |
| Employment | 2,528 | 273.8 | 96 | 1330.8 | 30 | 464 |
| F/M ratio | 2,528 | 1.54 | 0.58 | 5.41 | 0.13 | 3.20 |
| F/(F+M) ratio | 2,528 | 0.40 | 0.37 | 5.41 | 0.12 | 0.76 |
| Source: China |  |  |  |  |  |  |

## Firm-level data

The firm-level data for China is a random subsample of 2528 firms taken from the Annual Surveys of Industrial Production conducted by the Chinese government's National Bureau of Statistics. ${ }^{11}$ The original data, which has been used in other studies of firm-level productivity (in particular Hsieh and Klenow (2009) and Feenstra et al. (2014)) covers the period 1998-2005, but data on the gender composition of the labor force of each firm is only available for the year 2004. As a result, we can only use cross-sectional information and we cannot difference the data to control for firm level total factor productivity.

This also means that the linear estimation technique proposed in Section 4.1 is not feasible and only the non-linear least squares estimation of equation (14) on the cross-section of firms can be run. This regression is appropriate under the assumption that the firm's TFP $\left(A_{i}\right)$ is uncorrelated with the other explanatory variables.

Table 1 presents summary statistics for this dataset, and Figure 4 shows how the share of female employment in total employment is distributed, for firms smaller or larger than the median firm. The dataset includes both very small firms, with less than 5 employees, and large firms, with several thousands employees. The median firm has around 100 employees. Although women remain a minority in manufacturing employment in China, the share of female employment over total employment is, at 40 percent, above what is typically observed in other countries' manufacturing sectors. Women are a majority of workers in around 30 percent of firms. Although employment in larger firms appears to be more gender-equal than in smaller firms, the differences are not pronounced.

[^8]Figure 3: Female employment over total employment (by sector)


Source: OECD Labor force survev

Figure 4: Distribution of female share of total firm's employment, China's manufacturing sector

$\square$ Small firms (empl<median) $\quad \square$ Large firms (empl>median)

Source: China's Annual Surveys of Industrial Production, 2004

Table 2: Linear model, aggregate level data, 5 year growth rates

| Variables | (1) Fixed effects (FE) WB data | (2) FE with CRS WB data | (3) FE OECD data | (4) <br> FE OECD and alternate data |
| :---: | :---: | :---: | :---: | :---: |
| Female labor supply, WB (pc change) | $\begin{gathered} 0.649 * * * \\ (2.795) \end{gathered}$ | $\begin{gathered} 0.691^{* * *} \\ (3.022) \end{gathered}$ |  |  |
| Male labor supply, WB (pc change) | $\begin{gathered} 0.531 \\ (1.403) \end{gathered}$ | $\begin{gathered} 0.225 \\ (0.987) \end{gathered}$ |  |  |
| Capital stock, PWT (pc change) | $\begin{aligned} & 0.0712 \\ & (1.486) \end{aligned}$ | $\begin{aligned} & 0.0838^{*} \\ & (1.812) \end{aligned}$ | $\begin{aligned} & 0.111^{* *} \\ & (2.498) \end{aligned}$ |  |
| Female employment, OECD (pc change) |  |  | $\begin{gathered} 0.427^{* *} \\ (2.160) \end{gathered}$ | $\begin{aligned} & 0.328^{* *} \\ & (2.044) \end{aligned}$ |
| Male employment, OECD (pc change) |  |  | $\begin{aligned} & 0.497^{*} \\ & (1.740) \end{aligned}$ | $\begin{gathered} 0.313 \\ (1.278) \end{gathered}$ |
| Capital stock, PPP, IMF (pc change) |  |  |  | $\begin{aligned} & 0.308^{* *} \\ & (2.544) \end{aligned}$ |
| Constant | $\begin{gathered} 0.0863^{* * *} \\ (6.713) \end{gathered}$ | $\begin{gathered} 0.233^{* * *} \\ (5.793) \end{gathered}$ | $\begin{gathered} 0.101^{* * *} \\ (8.208) \end{gathered}$ | $\begin{gathered} 0.0708^{* * *} \\ (4.827) \end{gathered}$ |
| No. of observations | 140 | 140 | 158 | 172 |
| Number of countries | 35 |  | 35 | 32 |
| R-squared | 0.331 |  | 0.313 | 0.218 |
| Avg. female to male empl | 0.77 | 0.77 | 0.77 | 0.77 |
| $\sigma\left(\right.$ when $\delta^{\rho}=0.87$ ) | 0.67 | 0.26 | 1.62 | 0.90 |
| $\sigma\left(\right.$ when $\left.\delta^{\rho}=1\right)$ | 0.57 | 0.19 | 2.40 | 0.85 |
| $\sigma\left(\right.$ when $\left.\delta^{\rho}=1\right)$, bootstrap 5th percentile | 0.08 | 0.07 | 0.08 | 0.07 |
| $\sigma\left(\right.$ when $\left.\delta^{\rho}=1\right)$, bootstrap 95th percentile | 6.06 | 4.08 | 5.57 | 4.61 |

Note: t-statistics in parentheses
*** $p<0.01,{ }^{* *} p<0.05,{ }^{*} p<0.1$

Table 3: Linear model, sectoral level data

| (1) | (2) | (3) | (4) |
| :---: | :---: | :---: | :---: |
| sectors 1, 4,5, 7, 9 | sectors 2, 3 and 6 | sectors 8, 10, 12 | sectors $13,14,15$ |
| Agriculture, manufacturing, electricity, gas and water, wholesale and retail trade, transport, storage and communication | Fishing, mining and quarrying, construction | Hotels and restaurants, financial intermediation, public admin. and defence | Education, health and social work, other community, social and personal services |


| Female employment (pc change) | 0.0572* | -0.00930 | 0.0880 | -0.0500 |
| :---: | :---: | :---: | :---: | :---: |
|  | [1.700] | [-0.306] | [1.590] | [-1.407] |
| Male employment (pc change) | 0.0471 | 0.113 | 0.141*** | 0.117* |
|  | [0.884] | [1.443] | [3.323] | [1.947] |
| Capital stock (pc change) | 0.243*** | 0.295** | 0.308*** | 0.216*** |
|  | [4.284] | [2.192] | [4.090] | [4.361] |
| Constant | 0.0126*** | 0.000474 | -0.00449 | 0.00682** |
|  | [3.109] | [0.0643] | [-1.009] | [2.173] |
| No. of observations | 241 | 106 | 144 | 144 |
| R-squared | 0.161 | 0.088 | 0.401 | 0.209 |
| Avg female to male empl | 0.504 | 0.138 | 1.259 | 2.477 |
| $\rho\left(\delta^{1 / \rho}=1\right)$ | -0.284 | NA | -2.042 | NA |
| $\sigma\left(\delta^{1 / p}=1\right)$ | 0.779 | NA | 0.329 | NA |

note: model controlling for sectoral fixed effects; t-statistics in brackets; ${ }^{* * *} \mathrm{p}<0.01,{ }^{* *} \mathrm{p}<0.05,{ }^{*} \mathrm{p}<0.1$

## 6 Results

We start by presenting the linear regression results using both aggregate and sectoral data, following up with the nonlinear estimation results.

### 6.1 Linear estimation results

Table 2 reports the first set of results, employing aggregated data in the linear fixed-effects model. Column (1) shows that the coefficient on growth for female employment is higher than for male employment (this latter coefficient is not even significantly different from zero). Independently of whether $\delta$ is calibrated such as to match the lower working hours of women (i.e. $d=\delta^{1 / \rho}=0.87$ ) or not, this implies a low elasticity of substitution (around 0.6), since the effect of adding female workers is stronger than the effect of adding male workers. This finding is robust to imposing constant returns to scale, i.e. $\beta_{f}+\beta_{m}+\beta_{k}=1$ (column (2)).

The dataset can be expanded by replacing the World Bank labor force data with OECD employment data, see column (3). This estimation leads to a higher elasticity of substitution, although when the IMF data are used for the capital stock and OECD data are used for GDP, the elasticity of substitution is found again to be below 1 (see column (4)).

In bootstrapping simulations it is confirmed that the point estimate of the elasticity of substitution is low, although estimation is not precise, and therefore the confidence interval includes the existing estimates based on micro data, e.g. Acemoglu et al. (2004), whose estimate is around 3, and Pellizzari et al. (2014), whose estimate is around 1.5. ${ }^{12}$ It is also worth mentioning that the results are not sensitive to reasonable changes in the parameter $\delta$.

We now turn to the sectoral data estimates, looking first at the linear model. As noted earlier, heterogeneity in the key ratio $F / M$ is an obstacle to the linear estimation strategy, the more so for sectoral data since this ratio is highly variable across sectors. One solution would be to estimate the linear model sector-by-sector, but such estimations would rely on too small a sample (around 50 observations per sector) to be reasonable. Hence, we group sectors into four broader categories according to the ratio $F / M$ shown in Figure 3. The linear estimation, shown in Table 3, replicates low estimates for the elasticity of substitution for broad sector 1 (agriculture, manufacturing, utilities, trade and transport) and broad sector 3 (hotels, financial services and administration). For the two other broad sectors (fishing, mining, construction; education, health,and other services), the OLS coefficients for female employment are negative, which is puzzling. Such a result is not consistent with a production function of the type we hypothesized, so we turn to the NLLS method.

[^9]Table 4: Non-linear model, aggregate level data, 5 year growth rates

|  | (1) <br> WB data | (2) <br> OECD data (post 1995) | (3) OECD data (whole sample) |
| :---: | :---: | :---: | :---: |
| 人 (labor share) | 0.83 | 0.84 | 0.79 |
|  | (0.05) | (0.05) | (0.04) |
| $\beta \quad$ (5-year growth in TFP) | 0.10 | 0.11 | 0.11 |
|  | (0.01) | (0.01) | (0.01) |
| $\delta$ (CES weight coef.) | 0.40 | 0.07 | 0.11 |
|  | (0.79) | (0.22) | (0.25) |
| $\rho 2(1-1 / \sigma 2)$ | -0.70 | -2.89 | -2.70 |
|  | (3.33) | (4.60) | (3.41) |
| $\sigma 2$ (elasticity of substitution) | 0.59 | 0.26 | 0.27 |
| Median bootstrapping | 0.51 | 0.26 | 0.29 |
| 1 st.dev confidence int. | [0.22-1.64] | [0.06-0.54] | [0.20-0.51] |
| No. of observations | 140 | 135 | 178 |
| Proportion of runs converged | 0.92 | 0.76 | 0.76 |

### 6.2 Nonlinear estimation results

We proceed with the non-linear least squares estimation using the aggregate data. Table 4 shows that the results for $\sigma_{2}$ are consistent with those of the linear estimation, varying between 0.2 and 0.6 . Using World Bank data (column (1)), the NLLS model estimates for the other parameters of the production function are also in the ballpark of what would be expected. TFP growth is averaging 2 percent per year $(\hat{\beta}=0.1)$. The labor share is however over-estimated, at 0.82 .

We conducted 500 bootstrapping simulations to assess the robustness of theses results and to present a confidence interval for $\sigma_{2}$. Figure 5 shows the results for the OECD data (column 3 of Table 4). The simulations show that the production function estimates are not very sensitive to the sample. The labor share and TFP growth coefficients are always close to the mean estimate, and the elasticity of substitution is most often found to be between 0.2 and 0.75 . The elasticity of substitution is below 1 in 93.5 percent of the simulations.

Moving to the sectoral estimates, Table 5 reports estimates from the non-linear least squares using the STAN OECD dataset. In column (1), where the NLLS is estimated on the whole sample, the elasticity of substitution is found to be very high, but this regression suffers from too much heterogeneity in the sectors used, and the homogeneity assumption for the CES parameters is almost certainly violated (see also the very low labor share coefficient $\alpha$ ). In particular, it is known that the labor share varies across sectors, bottoming at 0.3-0.4 for the most capital intensive sectors

Table 5: Non-linear model, sectoral level data, 5 year growth rates

|  | (1) | (2) | (3) |
| :---: | :---: | :---: | :---: |
|  | All sectors | All sectors, excl. Agr., fishing, mining, manuf. | All sectors, excl. Agr., fishing, mining, manuf., elect. and wholesale |
| $\boldsymbol{\alpha}$ (labor share) | 0.38 | 0.46 | 0.46 |
|  | (0.04) | (0.05) | (0.05) |
| $\beta$ (TFP growth) | 0.02 | 0.00 | 0.01 |
|  | (0.01) | (0.01) | (0.01) |
| $\delta$ (CES weight coef.) | 0.30 | 0.68 | 0.45 |
|  | (0.24) | (0.38) | (0.36) |
| $\rho 2(1-1 / \sigma 2)$ | 0.90 | 0.14 | 0.7 |
|  | (0.81) | (0.39) | (1.05) |
|  | 10.00 | 1.16 | 3.33 |
|  | 3.20 | 4.33 | 1.83 |
|  | [0.7-9.3] | [2.5-12.0] | [0.9-3.8] |
| No. of countries | 32 | 32 | 32 |
| No. of observations | 513 | 395 | 324 |

(mining, utilities) but exceeding 70 percent for several other sectors (hotels and restaurants; textiles); see Estrada et al. (2014). ${ }^{13}$ But the NLLS model assumes that this share, estimated by $\hat{\alpha}$, is constant across sectors. It is also possible that $\sigma_{2}$ would vary between agriculture, manufacturing, and services. Hence, we present in column (2) the NLLS estimates for the panel excluding the agriculture, fishing, mining and manufacturing sectors. The results are closer to our priors, with the labor share at 0.5 . The elasticity of substitution between male and female workers is then estimated at 1.16. The model estimation is much improved compared to earlier models, as the relatively low standard error for $\rho_{2}$ shows. Nonetheless, the 16th -84th percentile confidence interval for $\sigma_{2}$ remains very large. The estimation in column (3) excludes two more sectors that differ from traditional services (electricity and wholesale trade). The results appear to be more robust. The median bootstrapping estimate of the elasticity of substitution is 1.8.

The full results from the 500 bootstrapping for column (3) are shown in Figure 6. The mode of the distribution for $\sigma_{2}$ is 1 , and $\sigma_{2}$ was only found to be higher than 4 in 15 percent of the simulations. Nevertheless, these simulations confirm that the elasticity of substitution is likely to be higher at the sectoral level than at the aggregate level. As with any nonlinear estimator, NLLSbased estimates used in our analysis are in theory sensitive to the initial conditions used to start the procedure. It is comforting to know that the NLLS algorithm, when using both the aggregate

[^10]Figure 5: Bootstrapping of NLLS, aggregate level data


Figure 6: Bootstrapping of NLLS, sectoral level data


Figure 7: Convergence in nonlinear aggregate and sectoral models

Convergence NLLS - Aggregate dataset


Convergence NLLS - Sectoral dataset

and sectoral data, either did not converge or always converged to the same values presented in Tables 4 and 5 , respectively, as long as the algorithm's initialization values were in an economically meaningful range (see Figure 7 for detailed convergence results).

We now turn to the firm-level estimates. The results are shown in Table 6, with the different columns presenting estimations for different sub-samples: the whole sample (column 1); the sample removing outliers (column 2); the sample of firms with larger (column 3) or smaller (column 4) capital stocks than the median; and the sample of firms with a level of employment larger (column $5)$ or smaller (column 6) than the median.

The labor share is estimated to be between 0.6 and 0.75 , higher than what was found in the cross-country data. The estimate of the elasticity of substitution is nonetheless consistent with that found in sectoral data, between 2 and 3 in most estimates. The bootstrapping simulations for the entire sample, excluding outliers (column 2), are shown in Figure 8 and indicate that the mode and the median of the distribution for the elasticity of substitution are below 2 .

These baseline estimations do not correct for potential endogeneity, but not all sources of endogeneity are relevant for the different levels of disaggregation and the different techniques we use. For the macro and sectoral level estimates, an omitted variable bias is possible but the use of log-differences in the NLLS model and of fixed effects in the linear model assuages this concern. This source of bias is potentially more problematic at the firm-level, because the dataset we used is a cross-section; however, the differences in the ES were not large when comparing small firms with large firms or firms with small capital stocks with those with large capital stocks. Reverse causality is also possible but the mechanisms would be different at the firm-level (productive firms affording more gender-equality) and at the macro-level (growth driving higher female labor supply).

We address this type of endogeneity concern in the macroeconomic estimation by using an instrumental variable regression for the linear method and a GMM model for the NLLS method. The GMM-NLLS provides estimates that are close to the baseline, with an elasticity of substitution around 0.6 (Table 7). For the linear model, Equation (9) is estimated on the aggregate dataset, instrumenting $f-m$ by changes in the number of (involuntary) military conscripts, expressed in percentage of male employment, obtained from the dataset constructed by Mulligan and Shleifer (2005). This dataset, whose main source is the Military Balance annual report from the International Institute for Strategic Studies, reports the number of conscripts in the armed forces for 167 countries, every five years between 1970 and 1995. Its coverage is thus appropriate for our 5 -year non-overlapping growth rate panel of macro variables in the OECD countries.

Table 6: NLLS Estimation, China's Firms-level Data, log-level estimation

|  | (1) | (2) | (3) | (4) | (5) | (6) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | All | All: P1-99 | K $>$ Med | K<Med | L $>$ Med | L<Med |
| $\boldsymbol{\alpha}$ (labor share) | 0.75*** | 0.74*** | $0.58 * * *$ | 0.67*** | 0.72*** | 0.79*** |
|  | (0.02) | (0.02) | (0.03) | (0.03) | (0.02) | (0.02) |
| $\beta$ (constant; $\ln (\mathrm{TFP})$ ) | 2.82*** | $2.68 * * *$ | 1.97*** | 10.45 | 2.58*** | 2.87*** |
|  | (0.12) | (0.17) | (0.18) | (39.78) | (0.13) | (0.32) |
| $\delta$ (CES weight parameter) | 0.52*** | 0.49*** | 0.51*** | 0.38*** | 0.53*** | 0.58*** |
|  | (0.06) | (0.05) | (0.11) | (0.07) | (0.09) | (0.09) |
| $\rho 2(=1-1 / \sigma 2)$ | 0.67*** | 0.48*** | 0.71** | -0.03 | 0.81*** | 0.43** |
|  | (0.16) | (0.14) | (0.32) | (0.16) | $(0.24)$ | (0.19) |
| $\boldsymbol{\sigma 2}$ (elasticity of substitution) | 3.03 | 1.92 | 3.45 | 0.97 | 5.26 | 1.75 |
| Median bootstrapping | 2.89 | 1.84 | 2.04 | 1.08 | 2.58 | 1.72 |
| 1 st.dev. confidence int. | [1.91-6.06] | [1.25-2.89] | [1.12-5.71] | [0.97-2.08] | [1.52-6.22] | [1.28-2.57] |
| Obs. | 2528 | 2406 | 1203 | 1203 | 1200 | 1206 |
| R-Sq. | 0.47 | 0.38 | 0.36 | 0.08 | 0.36 | 0.05 |

[^11]Figure 8: Bootstrapping of NLLS, firm-level data


The positive relationship between military conscription and $f-m$ observed for advanced economies (see Figure 9) confirms that conscription depresses male employment (relative to female employment), and the bivariate regression of $f-m$ on military conscription yields a coefficient significant at the 10 percentile level, although the within-R-square is only 0.06 . The fixed effects IV regression coefficient on $f-m$ is then used to infer $\sigma_{2}$, which is 0.4 in the median bootstrapping estimate, with a 95 percent confidence interval of [0.1-3.7].

## 7 Interpreting the results

How do we reconcile the low estimates for the elasticity of substitution in our macroeconomic estimates with the higher elasticities found in the micro literature (and to some extent also in our sectoral and firm-level results)? One possible answer is that the elasticity of substitution between men and women is relatively high within a firm or sector, but that men and women tend to work in different sectors for which the substitution, across sectors, is low (for the production of aggregate output). McManus (1988) and Miyagiwa and Papageorgiou (2007) show indeed that the elasticity of substitution between two factors at the aggregate level is a weighted average of the elasticity of substitution across sectors and the elasticities of substitutions of the two same factors within each sector, with the weight on the elasticity of substitution across sectors being higher the more diverse the factor shares across sectors. Given the heterogeneity in FLFP across sectors documented in Figure 3, this explanation could be valid, the more so if the elasticity of substitution across sectors in the aggregate production function is low.

Table 7: IV and GMM models

|  | NLLS baseline | GMM model | IV -linear model 1/ |
| :--- | :---: | :---: | :---: |
| $\boldsymbol{\rho}$ | -0.7 | -0.82 | -1.41 |
|  | $(3.33)$ | $(1.13)$ | $(3.15)$ |
| $\boldsymbol{\sigma}$ | $\mathbf{0 . 5 9}$ | $\mathbf{0 . 5 5}$ | $\mathbf{0 . 4 1}$ |
| $\mathbf{9 5 ]}$ pctl. conf. interval |  |  |  |
| Obs |  |  | $[\mathbf{0 . 1 1 - 3 . 6 9 ]}$ |

Note: 1 / for the IV model, $\sigma$ is obtained under the assumption that $\delta^{\rho}=0.87$

Figure 9: Changes in military conscription (in percent of male employment) and $f-m$


We finally look at the implications for GDP (and for TFP) of a relatively low elasticity of substitution, by calibrating a production function and computing output for different values of $\sigma_{2}$, assuming that the gender gap in labor force participation is closed (the calculations do not require setting a horizon over which the gap is closed). The exercise is thus one where labor inputs F and M are chosen, as would be done in a growth accounting model, not one where barriers to FLFP are removed, as is often done in the literature on labor misallocation (Baqaee and Farhi (2017); Alvarez (2019)). The exercise assumes that the capital stock is fixed, and thus understates the long-run effect on GDP. The production function is calibrated using $\alpha=0.6, \delta^{1 / \rho}=0.83$, and the median value in the OECD for $F /(F+M), 0.43$.

The LHS chart in Figure 10 shows that closing the gender gap in the labor force would increase GDP by 8 to 14 percent, depending on the elasticity of substitution. When the elasticity of substitution is high (numerically, an elasticity higher than 5 is similar to the case of perfect substitutability), the increase in GDP is solely due to the larger labor force, with no effect of complementarity on labor productivity. ${ }^{14}$ For countries that start further away in terms of female labor force participation, the effect on GDP would be much stronger, around or above 20 percent of GDP (RHS chart of Figure 10 , when $F / N$ is initially 0.33 , which is the median value for Middle Income Countries). As clarified in Section 3, whether taking into account differences between women and men increases or decreases GDP depends also on the scale effect (captured by the parameter $\delta^{1 / \rho}$ ). Even if women entering the labor force have working hours lower than men, this could be compensated for by a low elasticity of substitution. In the baseline parameterization ( $\delta^{1 / \rho}=0.83$ ), when $\sigma_{2}<2$, gender complementarity is strong enough that the Solow residual is positively affected by increasing FLFP (the blue line is above the red line).

Overall, the simulations confirm that the complementarity effect is economically meaningful: for an elasticity of substitution between 0.2 and 2 , complementarity effects would contribute to an overall increase in production of 1 to 6 percent of GDP, and this effect is even stronger for countries that start with a low level of FLFP. We also note that even the marginal product of male labor (and thus men's real wage) should be increasing in FLFP when $\rho<\alpha$ (i.e., when $\sigma_{2}<2.5$, assuming $\alpha=0.6)$. Indeed, this is a case where the complementarity effect of female participation outweighs the negative effect of a higher labor supply on capital intensity. ${ }^{15}$

Our result that the gains from increasing FLFP are higher the lower the elasticity of substitution runs counter to the finding in Baqaee and Farhi (2017) that the gains from removing distortions should be higher the more substitutable the inputs. A first reason for this discrepancy is that our

[^12]Figure 10: CES production function: comparative statics

calculations implicitly assume that men are initially in excess supply, so that increasing FLFP not only increases the labor force but it also improves the productivity of men, by correcting the gender imbalance. A second reason for this discrepancy is that Baqaee and Farhi (2017) study the effect of removing a fixed wedge (in the gender literature, this would be the wedge between the MPL of women and the wage they receive) whereas we compute the gains from achieving a given outcome (gender equality in labor force participation). Intuitively, when a general equilibrium model is calibrated using observed FLFP, observed female wage data, and a given wedge in the labor market clearing condition, the marginal productivity of women is fixed. Thus, a comparative statics for $\sigma_{2}$ is accommodated by changing other parameters (for instance the CES weight coefficient $\delta$ ) such that the marginal productivity of women is unchanged. In that case, the first-order effect of changing $\sigma_{2}$ on the elasticity of growth to FLFP is null. ${ }^{16}$

On the contrary, in our exercise, the wedge in the labor market clearing condition should not be thought of as given. If the CES weight coefficient is fixed, ${ }^{17}$ then the comparative statics exercise of lowering $\sigma_{2}$ should imply that the initial barrier to FLFP is higher: intuitively, if women are very complementary to men, but are nonetheless in short supply and receive a wage lower than men, it must be that the barrier to FLFP is higher. The exercise of setting a given barrier to zero is thus substantially different from the exercise of achieving gender equality in FLFP, since the initial barrier should be understood as being a function of $\sigma_{2}$. We think that the policy experiment of achieving gender equality in labor force participation is a relevant one because policymaking

[^13]often works by setting a quantitative target in the first place. Policies then can be implemented to achieve the given objective.

## 8 Concluding remarks

The empirical growth literature has shown that gender equality is important for growth, but it has not specified exactly how FLFP contributes. Of course, adding workers to the labor force should increase GDP, and one can assess in a simple exercise how adding women to the labor force increases output (see e.g. Aguirre et al. (2012)). However, such an exercise does not do justice to the broader issue of how the diversity of skills, ideas, and management styles that gender diversity engenders may affect growth, over and above the impact of greater headcount of (female or male) workers.

In this paper we have taken seriously the question of how gender diversity -as opposed to increases in (female or male) labor supply - may affect growth, by allowing for the possibility that male and female labor are imperfect substitutes in production, a possibility that many proponents of diversity emphasize in popular discourse. The estimation of the production function, performed with linear and non-linear techniques, and using aggregate, sectoral, and firm-level data suggests that indeed the elasticity of substitution between male and female workers is low, below 1 in the aggregate data, in the range of $1-2$ in the sectoral data, and between 2-3 in the firm-level data. Although the aggregate-level elasticities of substitution are lower than what has been found in the few recent papers that looked at this issue using wage data, one possible explanation is that, while the elasticity of substitution is high within particular sectors, the share of male and female labor participation varies quite substantially across sectors and the elasticities of substitution across these sectors are low. This is consistent with our estimated elasticities of substitution that appear to decrease at higher levels of aggregation.

There are three main implications of our findings for macroeconomics. The first one is that growth accounting exercises should recognize the role that past reductions in gender inequality have had for TFP growth. Until now, TFP growth has been interpreted as originating in technology improvements, but it should also be understood that worker diversity contributes to efficiency. The second one is that we can expect general equilibrium effects of policies on gender gaps to be significant, because the elasticity of substitution is not that high. The third implication is that closing the gender gap in the labor force would lead to large increases in GDP (by 8 to 14 percent for the median advanced economy) and even to an increase in men's real wages, with complementarity effects contributing to the increase in GDP by between 1 and 6 percent of GDP, depending on the level of the elasticity of substitution. An important assumption underlying our finding that gains from gender inclusion are increasing in complementarity between women and men is that policies operate via quantitative targets for female employment.

## Appendix 1

## Log-linearization

Writing in lower case relative deviations from initial value, the equation:

$$
\begin{align*}
& X^{\rho_{1}}\left(1+\rho_{1} x\right)=A(1+a)\left[\delta_{\ell} L^{\rho_{1}}\left(1+\rho_{1} \ell\right)+\delta_{k} K^{\rho_{1}}\left(1+\rho_{1} k\right)\right]  \tag{16}\\
= & A\left[\delta_{\ell} L^{\rho_{1}}\left(1+\rho_{1} \ell\right)+\delta_{k} K^{\rho_{1}}\left(1+\rho_{1} k\right)\right]+A a\left(\delta_{\ell} L^{\rho_{1}}+\delta_{k} K^{\rho_{1}}\right), \tag{17}
\end{align*}
$$

yields, by difference from initial values (and after dividing by $\rho_{1} X^{\rho_{1}}$ ):

$$
\begin{equation*}
x=A\left[\frac{\delta_{\ell} L^{\rho_{1}}}{X^{\rho_{1}}} \ell+\frac{\delta_{k} K^{\rho_{1}}}{X^{\rho_{1}}} k\right]+\frac{A a\left(\delta_{\ell} L^{\rho_{1}}+\delta_{k} K^{\rho_{1}}\right)}{\rho_{1} X^{\rho_{1}}}=\underbrace{A \frac{\delta_{\ell} L^{\rho_{1}}}{X^{\rho_{1}}}}_{\lambda} \ell+A \frac{\delta_{k} K^{\rho_{1}}}{X^{\rho_{1}}} k+a, \tag{18}
\end{equation*}
$$

Finally, since the share of labor in income is:

$$
\begin{equation*}
\frac{\frac{\partial X}{\partial L} L}{X}=A \frac{\delta_{\ell} \rho_{1} L^{\rho_{1}}}{\rho_{1} X^{\rho_{1}}}=\lambda \tag{19}
\end{equation*}
$$

we find $x=\lambda \ell+(1-\lambda) k+a$.

Similarly, the log-linearization of $L^{\rho}$ yields $\ell=\mu f+(1-\mu) m$ where $\mu$ is the share of female labor income in total labor income.

## Appendix 2

## Taylor approximation around $\sigma_{2}=+\infty$

$$
\begin{align*}
\mu & =\frac{\delta F^{\rho_{2}}}{\delta F^{\rho_{2}}+M^{\rho_{2}}}=\frac{1}{1+\frac{1}{\delta}\left(\frac{M}{F}\right)^{1-r}}  \tag{20}\\
& \approx \frac{1}{1+\frac{1}{\delta} \frac{M}{F}-\frac{1}{\delta} \frac{M}{F} r \ln \left(\frac{M}{F}\right)}=\frac{\delta F}{(\delta F+M)\left(1-\frac{M}{M+F} r \ln \left(\frac{M}{F}\right)\right)}  \tag{21}\\
& \approx \frac{\delta F}{\delta F+M}\left(1+\frac{M}{\delta F+M} r \ln \left(\frac{M}{F}\right)\right)=\mu^{\infty}\left(1+\left(1-\mu^{\infty}\right) r \ln \left(\frac{M}{F}\right)\right), \tag{22}
\end{align*}
$$

where $\mu^{\infty}=\delta F /(\delta F+M)$ is the female labor share that would be prevail if men and women were perfect substitutes. As expected, when $r>0$ (i.e. when $\sigma_{2}<\infty$ ) the share of female labor in labor income $\mu$ is higher than $\mu^{\infty}$ if and only if $M>F$, i.e. if female workers are in short supply, and the female share in income is larger the least substitutable $M$ and $F$ (i.e. the larger $r$ ).

This translates into the growth regression:

$$
\begin{align*}
y & =\lambda \mu f+\lambda(1-\mu) m+(1-\lambda) k+a \\
& =\lambda n+\lambda\left(\mu-\frac{F}{N}\right) f+\lambda\left(1-\mu-\frac{M}{N}\right) m+(1-\lambda) k+a  \tag{23}\\
& =\lambda n+\lambda\left(\mu-\frac{F}{N}\right)(f-m)+(1-\lambda) k+a, \tag{24}
\end{align*}
$$

where $n=\frac{\Delta N}{N}=\frac{\Delta(F+M)}{F+M}=\frac{F}{N} f+\frac{M}{N} m$ is growth in the headcount of the labor force.

## Taylor approximation around $\sigma_{2}=1$

In this case, we assume $\left|\rho_{2}\right| \ll 1$ and use the Taylor expansion $\alpha^{\rho_{2}} \approx 1+\rho_{2} \ln (\alpha)$ to $\mu$ :

$$
\begin{align*}
\mu & =\delta \frac{F^{\rho_{2}}}{\delta F^{\rho_{2}}+M^{\rho_{2}}}=\frac{1}{1+\frac{1}{\delta}\left(\frac{M}{F}\right)^{\rho_{2}}}  \tag{25}\\
& \approx \frac{1}{1+1 / \delta+\rho_{2} / \delta \ln \left(\frac{M}{F}\right)}  \tag{26}\\
& \approx \frac{1}{1+1 / \delta} \frac{1}{1+\rho_{2} /(1+\delta) \ln \left(\frac{M}{F}\right)}  \tag{27}\\
& \approx \frac{\delta}{1+\delta}\left(1-\rho_{2} /(1+\delta) \ln \left(\frac{M}{F}\right)\right) . \tag{28}
\end{align*}
$$

This shows that the share of women in labor income is $\delta /(1+\delta)$ if $\rho_{2}=0$ and it is is higher than $\delta /(1+\delta)$ if $M>F$ and the male and female labor are more complement than in a Cobb-Douglas composite (i.e. if $\rho_{2}<0$ ).

This translates into the growth regression:

$$
\begin{align*}
y & =\lambda \mu f+\lambda(1-\mu) m+(1-\lambda) k+a \\
& =\lambda n+\lambda\left(\mu-\frac{F}{N}\right)(f-m)+(1-\lambda) k+a, \tag{29}
\end{align*}
$$

where $n$ is growth in the headcount of the labor force. The second term in equation (29) shows that if $\mu>\frac{F}{N}$, growth in increasing in $f-m$.

## Appendix 3

## Second-order approximation

A second-order Taylor approximation of the production function can be derived for small changes in $L, F$ and $M$ :

$$
L^{\rho}=L_{0}^{\rho}+\rho L_{0}^{\rho-1}\left(L-L_{0}\right)+\rho(\rho-1) L_{0}^{\rho-2} \frac{\left(L-L_{0}\right)^{2}}{2}
$$

with identical formulaes applying to $F^{\rho}$ and $M^{\rho}$. Writing in lower case the relative differences from the value at $\left(L_{0}, F_{0}, M_{0}\right)$ :

$$
l+\frac{\rho-1}{2} l^{2}=\mu\left(f+\frac{\rho-1}{2} f^{2}\right)+(1-\mu)\left(m+\frac{\rho-1}{2} m^{2}\right),
$$

where $\mu=\frac{\delta F^{\rho}}{M^{\rho}}$. Note that this also implies $l^{2}=\mu^{2} f^{2}+(1-\mu)^{2} m^{2}+o\left(l^{2}\right)$. Thus:

$$
l=\mu f+(1-\mu) m+\frac{\rho-1}{2} \mu(1-\mu)\left(f^{2}+m^{2}\right)+o\left(l^{2}\right) .
$$

Finally, $y=\beta+(1-\alpha) k+\alpha l$ implies:

$$
y=\beta+(1-\alpha) k+\alpha \mu f+\alpha(1-\mu) m+\alpha \frac{\rho-1}{2} \mu(1-\mu)\left(f^{2}+m^{2}\right) .
$$

Thus, the constrained linear regression:

$$
y=\beta+\phi_{1} k+\phi_{2} f+\phi_{3} m+\phi_{4}\left(f^{2}+m^{2}\right),
$$

where $\phi_{1}=1-\phi_{2}-\phi_{3}$ allows us to identify $\alpha, \delta$ and $\rho$ :

$$
\begin{equation*}
\alpha=1-\phi_{1} ; \quad \rho=1+\frac{2\left(1-\phi_{1}\right) \phi_{4}}{\phi_{2} \phi_{3}} ; \quad \delta=\frac{\phi_{2}}{1-\phi_{1}}(M / F)^{\rho} . \tag{30}
\end{equation*}
$$

This method was applied to OECD data but the coefficient $\phi_{4}$ was very imprecisely estimated (and most often not distinguishable from 0 ), which makes the method impractical to estimate $\rho$ and thus $\sigma$.

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[^1]:    ${ }^{1}$ One example that demonstrates the relevance of our exercise is the stated objective, in many advanced economies, to achieve gender parity in corporate boards. In France, less than 15 percent of corporate board members for the top 40 listed firms were women in 2010, despite general progress with gender equality and despite governance codes having promoted gender parity in corporate boards for decades (Zenou et al. (2017)). Such slow progress led the French government to legislate in 2011 a 40 percent quota for women in large firms' corporate boards, a quota that has now been met.

[^2]:    ${ }^{2}$ The U-shaped relationship between FLFP and income (Tam (2011)) could thus be explained by the lower benefits that developing countries can expect (compared to advanced economies) when women work in the market economy (Erturk and William Darity (2000)).

[^3]:    ${ }^{3}$ Since the derivation of the CES production function by Arrow et al. (1961), a number of notable empirical attempts tried to estimate this elasticity for the U.S. manufacturing sector including Maddala (1965), Lucas (1969), Berndt (1976), and more recently, Antras (2004), and Klump et al. (2007).

[^4]:    ${ }^{4}$ The focus prior to Grant and Hamermesh (1981) was on the Allen partial elasticity of substitution, i.e. the sensitivity of quantities to prices; see Seidman (1989). We also note that the restriction to a CES specification means that in our model all inputs will be q-complements, i.e. the marginal product of each factor is increasing in the supply of the other factors.

[^5]:    ${ }^{5}$ It is also possible to linearize near the Cobb-Douglas specification (i.e. $\left|\rho_{2}\right| \ll 1$ ). See Appendix 2 for details.

[^6]:    ${ }^{6}$ The variables are expressed in growth rates, but not in per capita terms.
    ${ }^{7}$ One may want to enforce the constraint $\beta_{f}+\beta_{m}+\beta_{k}=1$ to guarantee the econometric model is consistent with the assumption of constant returns to scale, but this is without consequence for the rest of the theoretical framework.
    ${ }^{8}$ Unfortunately, assuming the coefficient $\beta_{k}$ is known and equal to the capital share in total income is not a solution. This is because the issue is with disentangling the effects of $\sigma_{2}$ and $\delta$ on the female labor share of income over total labor income.

[^7]:    ${ }^{9}$ The methodology used in our analysis follows that used in estimating aggregate production functions, e.g. Arrow et al. (1961), Bodkin and Klein (1967), Duffy and Papageorgiou (2000) and Duffy et al. (2004).
    ${ }^{10}$ For further discussion on the challenges facing nonlinear estimation, see e.g., León-Ledesma et al. (2010).

[^8]:    ${ }^{11} \mathrm{We}$ are grateful to Grace Li for providing this random sample of the original dataset.

[^9]:    ${ }^{12}$ The confidence intervals are computed using 500 bootstrapping iterations, but throwing out the results of those iterations for which $\sigma_{2}$ would not be defined.

[^10]:    ${ }^{13}$ The labor share has evolved across time, with the sharp fall in the labor share in manufacturing post-2005 being frequently documented (see e.g Karabarbounis and Neiman (2014)) Unfortunately, the relatively small size of the dataset did not allow us to split the sample by time period.

[^11]:    note: standard errors in brackets; condidenc interval for $\sigma 2$ obtained from 500 bootstrapping iterations

[^12]:    ${ }^{14}$ The blue line is below the red line when $\sigma_{2}>5$ because the red line also assumes $\delta=1$, whereas the red line assumes $\delta^{1 / \rho}=0.83$, capturing the fact that women work around 83 percent of the men work hours in the OECD.
    ${ }^{15}$ When the production function is Cobb-Douglas in capital and labor (i.e. $\rho_{1}=0$ ), the marginal product of men labor, $(\partial Y) /(\partial M)=A K^{1-\alpha}(\alpha-\rho)\left(M^{\rho}+\delta F^{\rho}\right)^{\alpha / \rho-1}$ is increasing in $F$ if $\rho<\alpha$.

[^13]:    ${ }^{16}$ Since the MPL, and thus the share of income, is what drives the first-order effect of adding workers to output, Baqaee and Farhi (2017) emphasize the need to go to the second order in order to understand the effect of substitutability on the sensitivity of output to the removal of wedges.
    ${ }^{17}$ We prefer to do this to assume gender equality in hourly productivity rather than adjust $\delta$ to match wage data as wages are affected by discrimination. Thus we set $\delta$ to 0.83 , capturing that women working hours are 17 percent lower than men's.

