DISCUSSION PAPER SERIES

DP13758

AFFORDABLE HOUSING AND CITY WELFARE

Stijn Van Nieuwerburgh, Jack Favilukis and Pierre Mabille

FINANCIAL ECONOMICS, MACROECONOMICS AND GROWTH AND PUBLIC ECONOMICS

AFFORDABLE HOUSING AND CITY WELFARE

Stijn Van Nieuwerburgh, Jack Favilukis and Pierre Mabille

This Discussion Paper is issued under the auspices of the Centre's research programme in **FINANCIAL ECONOMICS, MACROECONOMICS AND GROWTH AND PUBLIC ECONOMICS**. Any opinions expressed here are those of the author(s) and not those of the Centre for Economic Policy Research. Research disseminated by CEPR may include views on policy, but the Centre itself takes no institutional policy positions.

The Centre for Economic Policy Research was established in 1983 as an educational charity, to promote independent analysis and public discussion of open economies and the relations among them. It is pluralist and non-partisan, bringing economic research to bear on the analysis of medium- and long-run policy questions.

These Discussion Papers often represent preliminary or incomplete work, circulated to encourage discussion and comment. Citation and use of such a paper should take account of its provisional character.

AFFORDABLE HOUSING AND CITY WELFARE

Abstract

Housing affordability is the main policy challenge for many large cities in the world. Zoning changes, rent control, housing vouchers, and tax credits are the main levers employed by policy makers. But how effective are they at combatting the affordability crisis? We build a new framework to evaluate the effect of these policies on the well-being of its citizens. It endogenizes house prices, rents, construction, labor supply, output, income and wealth inequality, as well as the location decisions of households. Its main novel features are risk, risk aversion, and incomplete risk-sharing. We calibrate the model to the New York MSA, incorporating current zoning and affordable housing policies. Housing affordability policies carry substantial insurance value but cause misallocation in labor and housing markets. Housing affordability policies that enhance access to this insurance especially for the neediest households create large net welfare gains.

Stijn Van Nieuwerburgh - svnieuwe@gsb.columbia.edu
Columbia Business School and CEPR

Jack Favilukis - jack.favilukis@sauder.ubc.ca
University of British Columbia

Pierre Mabille - pgm284@stern.nyu.edu
New York University

Affordable Housing and City Welfare [*]

Jack Favilukis[†]   Pierre Mabille[‡]   Stijn Van Nieuwerburgh[§]

May 24, 2019

Abstract

Housing affordability is the main policy challenge for many large cities in the world. Zoning changes, rent control, housing vouchers, and tax credits are the main levers employed by policy makers. But how effective are they at combatting the affordability crisis? We build a new framework to evaluate the effect of these policies on the well-being of its citizens. It endogenizes house prices, rents, construction, labor supply, output, income and wealth inequality, as well as the location decisions of households. Its main novel features are risk, risk aversion, and incomplete risk-sharing. We calibrate the model to the New York MSA, incorporating current zoning and affordable housing policies. Housing affordability policies carry substantial insurance value but cause misallocation in labor and housing markets. Housing affordability policies that enhance access to this insurance especially for the neediest households create large net welfare gains.

[†]Department of Finance, Sauder School of Business, University of British Columbia, 2053 Main Mall, Vancouver, BC, V6T 1Z2, Canada; jack.favilukis@sauder.ubc.ca; https://sites.google.com/site/jackfavilukis.

[‡]Department of Economics, Stern School of Business, New York University, 44 W. 4th Street, New York, NY 10012, USA; pmabille@stern.nyu.edu.

[§]Department of Finance, Graduate School of Business, Columbia University, Uris Hall 809, 3022 Broadway, New York, NY 10027, USA; NBER; CEPR; svnieuwe@gsb.columbia.edu; https://www0.gsb.columbia.edu/faculty/svannieuwerburgh.

1   Introduction

The increasing appeal of major urban centers has brought on an unprecedented housing affordability crisis. Ever more urban households are burdened by rents or mortgage payments that take up a large fraction of their paycheck and/or by long commutes. The share of cost-burdened renters in the United States has risen from 23.8% in the 1960s to 47.5% in 2016. Over this period, median home value rose 112%, far outpacing the 50% increase in the median owner income (Joint Center for Housing Studies of Harvard University, 2018). Hsieh and Moretti (2019) argue that our most productive cities are smaller than they should be because of lack of affordable housing options, underscoring the importance of the issue.

Policy makers are under increasing pressure to improve affordability.[1] They employ policy tools ranging from rent control, inclusionary zoning, land use restrictions, housing vouchers, to tax credits for developers. While there is much work, both empirical and theoretical, on housing affordability, what is missing is a general equilibrium model that quantifies the impact of such policies on prices and quantities of owned and rented housing, the spatial distribution of housing and households, commuting patterns, incentives to work, income and wealth inequality within and across neighborhoods, output, and ultimately on individual and city-wide welfare. This paper provides such a model. We carefully calibrate the model and use it as a laboratory to conduct a number of housing policy experiments.

Our overarching finding is that expansion of housing affordability policies can be welfare improving. In an incomplete markets model like ours with risk and risk aversion, affordability policies play a quantitatively important role as an insurance device, especially benefitting low income households. Those insurance benefits trade off against the housing and labor market distortions that usually accompany such policies. Our results highlight the importance of general equilibrium effects, which often reverse partial equilibrium logic, and of how the affordability policies are financed. In the spirit of Diamond and Saez (2011), we aim to evaluate policy reforms that are extensions of existing policies, limited in complexity, and potentially politically feasible, rather than characterizing first-best policy.

---

[1]Fifteen cities in California have rent control. A November 2018 California state ballot initiative proposed to overturn the 1995 Costa-Hawkins Act, clearing the way for more rent control. Oregon is the first U.S. state to impose statewide rent control. The New York State legislature is discussing expansion of rent control laws up for renewal in June 2019. So is Massachusetts. Bill de Blasio, the mayor of New York City, was elected on a platform to preserve or add 200,000 affordable housing units. Affordable housing is a key policy issue in large cities throughout the world.

We model a metropolitan area that consists of two zones, the central business district (zone 1) and the rest of the metropolitan area (zone 2). Working-age households who live in zone 2 commute to zone 1 for work. Commuting entails both an opportunity cost of time and a financial cost. Zones have different sizes, captured by limits on the maximum amount of housing that can be built. Finally, zones provide different levels of amenities. The spatial aspect of the model is important since affordable housing policies interfere with the optimal spatial allocation of labor and housing.

The city is populated by overlapping generations of risk averse households who face idiosyncratic labor productivity risk and mortality risk. They make dynamic decisions on location, non-housing and housing consumption, labor supply, tenure status (own or rent), savings in bonds, primary housing, investment property, and mortgage debt. Since households cannot perfectly hedge labor income and longevity risk, markets are incomplete. This incompleteness opens up the possibility for housing affordability policies to provide insurance. Progressive tax-and-transfer and social security systems capture important insurance mechanisms aside from affordable housing policies. The model generates a rich cross-sectional distribution over age, labor income, tenure status, housing wealth, and financial wealth. This richness is paramount to understanding both the distributional and aggregate implications of housing affordability policies.

On the firm side, the city produces tradable goods and residential housing in each zone, subject to decreasing returns to scale. As a zone approaches its maximum buildable housing limit, construction becomes increasingly expensive, and the housing supply elasticity falls. Wages, house prices, and market rents are determined in the city's equilibrium.

We calibrate the model to the New York metropolitan area, designating Manhattan as the urban core, or zone 1, and the rest of the metropolitan area (MSA) as zone 2. Our calibration targets key features of the data, including the relative size of Manhattan versus the rest of the MSA, the income distribution in the New York MSA, observed commuting times and costs, the housing supply elasticity, current zoning laws, the current size and scope of the affordable housing system, and the current federal, state, and local tax-and-transfer system. The baseline model generates realistic income, wealth, and home ownership patterns over the life-cycle for various percentiles of the income distribution. It matches both income and wealth inequality. The model also matches house price and rent levels for the MSA. It generates a large wedge between the prices and rents in the two zones.

We model rent control (RC) as mandatory inclusionary housing, a policy that requires developers to set aside a fraction of rental housing to low-income households at below

market rates and is allocated by lottery. Such policies are common in large U.S. cities like New York. We believe that the insights from our RC experiments apply more broadly to other government-provided or regulated housing units that rent at below-market rates. RC provides insurance in the model. We define *access to insurance* as the likelihood that a household in the bottom half of the income distribution that experiences a negative productivity shock gains access to an affordable housing unit. We define the *stability of insurance* as the likelihood that a household in the bottom half of the income distribution already in an affordable unit can remain there. The *value of insurance* depends on the size of the affordable unit, how deeply the rent is discounted, and on household risk aversion.

The RC system creates several distortions which trade off with the insurance benefits. Households who win the model's affordable housing lottery and wish to accept the win must satisfy an income qualification as well as a maximum unit size requirement upon first entry. By sheer luck, low-productivity households may end up in RC units in the city center, crowding out high-productivity households with a higher opportunity cost of time from commuting. This is a first source of misallocation. Second, the income qualification requirement leads to distortions in labor supply. We choose the income qualification parameter to match the observed position of RC tenants in the city-wide income distribution. A third source of misallocation is that a household that would otherwise live in zone 1 accepts an affordable housing unit in zone 2 or vice versa. Fourth, the maximum size constraint generates misallocation of housing. Households may choose RC units that are too small or too large because of the below-market rent. Fifth, once a household is in an affordable unit, it needs not requalify (with high probability). Subsequent positive productivity shocks or deterministic age-related increases in income cause further misallocation since households often choose to remain in their subsidized units despite a growing mismatch. RC tenancy in the model matches data on the fraction of RC residents who live in their unit for more than twenty years. When less needy households occupy scarce RC units, needy households have a lower chance of gaining access. Sixth, and maybe most prominently, the affordable housing mandate affects the supply of housing. Landlords earn lower average rents, lowering the price of rental property, and weakening the incentives for residential development. This in turn leads to higher rents and prices for market units.

In the first set of experiments, we study policies aimed at reducing the misallocation caused by RC. The first policy counter-factual tightens the income qualification requirement imposed upon first entry. By more explicitly targeting low-income households, the policy improves *access to insurance* for those who need it most. A second policy change is aimed at reducing the misallocation that occurs after first entry. It abolishes priority

3

for incumbents and forces RC tenants to re-apply and re-qualify for RC each period. Re-qualification replaces less needy insiders with needier outsiders. However, by creating more churn in the RC system, it lowers the *stability of insurance*, which hurts risk averse households. Combining tighter income requirements with re-qualification addresses misallocation both at first entry and thereafter. This combination experiment results in a large welfare gain of 3.59% in consumption equivalent units per average New Yorker. The gain arises from a large increase in access to insurance and a smaller reduction in the stability of that insurance. Policies that improve the targeting of affordable housing benefit young, low-productivity, low-income, and low-wealth households the most, thereby reducing income and wealth inequality. Because they do not expand the footprint of the RC system, such policies reduce rather than magnify the labor and housing market distortions. Because they result in a larger population in the urban core, they reduce the deadweight losses incurred by commuting.

A second set of policy experiments changes the scope of the affordable housing mandate. A 50% increase in the share of square footage that developers must set aside for affordable units increases welfare by 0.66%. With more affordable housing units, access to insurance increases while the stability of insurance is unaffected. The benefits this brings to lower-income households outweigh the costs that arise from weaker incentives to construct housing, higher rents in market units, and from more spatial misallocation of labor and housing. We explore both smaller and larger fractions of affordable housing units and find that welfare increases monotonically with the scope of the affordable housing mandate.

In contrast to received wisdom, increasing the affordable housing mandate symmetrically in both zones does not lead to a decline in the overall quantity of housing in the MSA. The housing stock declines in the urban core, consistent with partial equilibrium logic and empirical evidence from local estimates (Autor, Palmer, and Pathak, 2014; Diamond, McQuade, and Qian, 2019). But, in spatial equilibrium, the population and the demand for housing in zone 2 rise, where housing is cheaper and supply more elastic. The expansion of the housing stock in zone 2 more than offsets the fall in the housing stock in zone 1.

Another interesting finding, common across several experiments, is that standard housing affordability metrics, such as the average rent-to-income ratio and the fraction of rent-burdened households, do not capture the improved availability of affordable housing. The increase in the rent-to-income ratio in zone 1 reflects not only the higher rents resulting from a smaller housing stock in zone 1 but also the lower average income of the zone 1 population after endogenous relocation in response to the policy.

The third main policy experiment increases the maximum amount of residential building in zone 1, for example through a relaxation of land use or height restrictions. The "upzoning" policy we study increases the equilibrium population share of zone 1 by 9.28% and the housing stock by nearly as much. It generates a welfare gain of 0.37%. The policy creates benefits for all age, productivity, income, and wealth groups, at least in the long-run (when comparing steady states). In other words, it involves less redistribution compared to the previous policies. Rents fall, which benefits both market and RC renters. The affordability improvements are widespread but modest. The surprisingly small aggregate welfare gain reflects the modest improvement in the fate of low-income, high marginal-utility households.

The last main policy experiment studies an expansion of the housing voucher system. Vouchers are subsidies provided through the tax and transfer system to low income households to be used for housing expenditures. An expansion of the voucher system is strongly welfare increasing (1.04%). The policy completes markets, in that households' marginal utility growth of housing and non-housing consumption becomes less volatile. Because the vouchers redistribute wealth from low- to high-marginal utility households, they reduce inequality. There are substantial costs from this policy since we assume, consistent with reality, that a voucher expansion must be financed via a more progressive tax system. Since labor income taxation is distortionary, high-and middle-productivity households reduce labor supply, and the city suffers a decline in output. The voucher expansion, whose direct effects are independent of location, triggers an interesting spatial response. Since the housing stock in the city center falls, due to a tax-induced reduction in housing demand, some high- and middle-productivity households move out of the city center. In equilibrium, the vouchers do not allow poor households to "move to opportunity" (zone 1) but rather seem to "remove from opportunity" some high-productivity households who end up farther from their jobs after the policy change, a sign of stronger labor misallocation.

A tax credit policy of the same magnitude as the voucher expansion, discussed in the appendix, suffers from similar tax-induced distortions on labor supply and housing demand, but without producing large gains for high-marginal utility households. It is only marginally welfare increasing (0.02%). The voucher and tax credit experiments underscore the importance of modeling how housing affordability policies are paid for, an aspect that has received little consideration in the literature.

The appendix discusses several more policy experiments. The most interesting result is for a policy that relocates all affordable housing units in zone 1 to zone 2. Despite allowing more high-productivity households with a high time cost of commuting to live closer

to work, the policy also causes higher financial costs of commuting for more low-income households. The net welfare effect is essentially zero. In contrast, when the relocation is accompanied by free transportation for RC tenants, there is a substantial welfare gain.

Related Literature   Our work is at the intersection of the macro-finance and urban economics literatures. On the one hand, a large literature in finance solves partial-equilibrium models of portfolio choice between housing (extensive and intensive margin), financial assets, and mortgages. Examples are Campbell and Cocco (2003), Cocco (2005), Yao and Zhang (2004), and Berger, Guerrieri, Lorenzoni, and Vavra (2017). Davis and Van Nieuwerburgh (2015) summarize this literature. Recent work in macro-finance has solved such models in general equilibrium, adding aggregate risk, endogenizing house prices and sometimes also interest rates. E.g., Landvoigt, Piazzesi, and Schneider (2015), Favilukis, Ludvigson, and Van Nieuwerburgh (2017), Guren and McQuade (2019), and Kaplan, Mitman, and Violante (2019). Imrohoroglu, Matoba, and Tuzel (2016) study the effect of the 1978 passage of Proposition 13 which lowered property taxes in California. Like the former literature, our model features a life-cycle and a rich portfolio choice problem and captures key quantitative features of observed wealth accumulation and home ownership over the life-cycle. Like the latter literature, house prices, rents, and wages are determined in equilibrium. We abstract from aggregate risk which is not central to the question at hand. Our contribution to the macro-finance literature is to add a spatial dimension to the model by introducing commuting costs, differing housing supply elasticities, and local amenities.

On the other hand, a voluminous literature in urban economics studies the spatial location of households in urban areas. Brueckner (1987) summarizes the Muth-Mills monocentric city model. This literature studies the trade-off between the commuting costs and housing expenditures. Rappaport (2014) introduces leisure as a source of utility and argues that the monocentric model remains empirically relevant. Rosen (1979) and Roback (1982) introduce spatial equilibrium. Recent work on spatial sorting includes Van Nieuwerburgh and Weill (2010), Behrens, Duranton, and Robert-Nicoud (2014) and Eeckhout, Pinheiro, and Schmidheiny (2014). Guerrieri, Hartley, and Hurst (2013) study house price dynamics in a city and focus on neighborhood consumption externalities, in part based on empirical evidence in Rossi-Hansberg, Sarte, and Owens (2010). Couture, Gaubert, Handbury, and Hurst (2018) uses a similar device to explain the return of rich households to the urban core over the past decades, reversing an earlier wave of suburban flight. Our model also features such luxury amenities in the city center. Urban models tend to be static, households tend to be risk neutral or have quasi-linear preferences, and

landlords are absentee (outside the model). The lack of risk, investment demand for housing by local residents, and wealth effects makes it hard to connect these spatial models to the macro-finance literature. There is no insurance role of affordability policies.

Hizmo (2015) and Ortalo-Magné and Prat (2016) bridge some of the gap between these two literatures by studying a problem where households are exposed to local labor income risk, make a once-and-for-all location choice, and then make an optimal financial portfolio choice. Their models are complementary to ours in that they solve a richer portfolio choice problem in closed-form, but don't have preferences that admit wealth effects nor allow for consumption and location choice each period. Studying the welfare effects of housing affordability policies requires incorporating wealth effects and considering landlords that are inside the city.

Because it is a heterogeneous-agent, incomplete-markets model, agents choices and equilibrium prices depend on the entire wealth distribution. Because of the spatial dimension, households' location is an additional state variable that needs to be kept track of. We use state-of-the-art methods to solve the model. We extend the approach of Favilukis et al. (2017), which itself extends Gomes and Michaelides (2008) and Krusell and Smith (1998) before that. The solution approach can accommodate aggregate risk, though we abstract from it in this model.

The resulting model is a new laboratory that can be used to study how place-based policies affect the spatial distribution of people, labor supply, house prices, output, and inequality. Favilukis and Van Nieuwerburgh (2018) use a related framework to study the effect of out-of-town investors on residential property prices.

Finally, our model connects to a growing empirical literature that studies the effect of rent control and zoning policies on rents, house prices, and housing supply. Autor et al. (2014); Autor, Palmer, and Pathak (2017) find that the elimination of the rent control mandate on prices in Cambridge increased the value of decontrolled units and neighboring properties in the following decade, by allowing constrained owners to raise rents and increasing the amenity value of those neighborhoods through housing market externalities. The price increase spurred new construction, increasing the rental stock. Diamond et al. (2019) show that the expansion of the RC mandate in San Francisco led to a reduction in the supply of available housing, by decreasing owners' incentives to rent below market prices, paradoxically contributing to rising rents and the gentrification of the area. While beneficial to tenants in RC, it resulted in an aggregate welfare loss. We also find a lower housing stock and higher rents from a RC expansion, but an aggregate welfare gain for the entire MSA in spatial equilibrium. Davis, Gregory, Hartley, and Tan (2017) study the effect of housing vouchers on location choice and of location choice on children's school-

ing outcomes in a rich model of the Los Angeles housing market. In a related model of neighborhood choice, Davis, Gregory, and Hartley (2018) study Low Income Housing Tax Credits and their effect on demographic composition, rent, and children's adult earnings. Diamond and McQuade (2019) find that LIHTC buildings in high- (low-)income neighborhoods have negative (positive) effects on neighboring property prices.[2]

The rest of the paper is organized as follows. Section 2 sets up the model. Section 3 describes the calibration to the New York metropolitan area. Section 4 discusses the benchmark model's implications for quantities and prices, the distribution of households, and housing affordability. Section 5 studies the main counter-factual policy experiments. Section 6 concludes. Several appendices provide model derivations (A), additional detail on the data (B), additional calibration details and output from the benchmark model (C-F), six additional policy experiments (G), robustness analysis that investigates the sensitivity of our policy results to changes in the key parameters (H), and transitional dynamics (I).

2 Model

The model consists of two geographies, the "urban core" and the "periphery", whose union forms the "metropolitan area" or "city." The urban core, which we refer to as zone 1, is the central business district where all employment takes place. The periphery, or zone 2, contains the outer boroughs of the city as well as the suburban areas that belong to the metropolitan area. While clearly an abstraction of the more complex production and commuting patterns in large cities, the monocentric city assumption captures the essence of commuting patterns (Rappaport, 2014) and is the simplest way to introduce a spatial aspect in the model. The two zones are allowed to differ in size. The city has a fixed population.[3]

---

[2]Earlier work by Baum-Snow and Marion (2009) focuses on the effects of LIHTCs on low income neighborhoods and Freedman and Owens (2011) specifically focuses on crime. Luque, Ikromov, and Noseworthy (2019) summarize various financing methods for low-income housing development.

[3]Future work could study interactions between affordability policies and migration in a multi-city model. Such a model would need to take a stance on a reservation utility of moving and on the moving costs across MSAs. These reservation utilities would naturally differ by age, productivity, and wealth, leading to a proliferation of free parameters. The lack of guidance from the literature would pose a substantial challenge to calibration. Furthermore, the empirical evidence for the New York metropolitan area, discussed in Appendix B.6, suggests that our zero net migration assumption fits the data well. Finally, welfare would be affected by the outside option. These three considerations motivate the closed-city model assumption.

2.1 Households

Preferences The economy consists of overlapping generations of risk averse households. There is a continuum of households of a given age a. Each household maximizes a utility function u over consumption goods c, housing h, and labor supply n. Utility depends on location ℓ and age a, allowing the model to capture commuting time and amenity differences across locations.

The period utility function is a CES aggregator of c and h and leisure l:

$$U(c_t, h_t, n_t, \ell_t, a) = \frac{\left[\chi_t^{\ell,a} \mathcal{C}(c_t, h_t, l_t) \right]^{1-\gamma}}{1-\gamma}, \tag{1}$$

$$\mathcal{C}(c_t, h_t, l_t) = \left[(1-\alpha_n)\left((1-\alpha_h)c_t^\epsilon + \alpha_h h_t^\epsilon\right)^{\frac{\eta}{\epsilon}} + \alpha_n l_t^\eta \right]^{\frac{1}{\eta}}$$

$$h_t \geq \underline{h} \tag{2}$$

$$n_t^a = \begin{cases} 1 - \phi_T^\ell - l_t \geq \underline{n} & \text{if } a < 65 \\ 0 & \text{if } a \geq 65 \end{cases} \tag{3}$$

$$\chi_t^{\ell,a} = \begin{cases} \chi^1 & \text{if } \ell = 1 \text{ and } c_t < \underline{c} \\ \chi^1 \chi^W & \text{if } \ell = 1 \text{ and } a < 65 \text{ and } c_t \geq \underline{c} \\ \chi^1 \chi^R & \text{if } \ell = 1 \text{ and } a \geq 65 \text{ and } c_t \geq \underline{c} \\ 1 & \text{if } \ell = 2 \end{cases} \tag{4}$$

The coefficient of relative risk aversion is γ. The parameter ϵ controls the intra-temporal elasticity of substitution between housing and non-housing consumption.

Equation (2) imposes a minimum house size requirement ($\underline{h}$), capturing the notion that a minimum amount of shelter is necessary for a household. The city's building code often contains such minimum size restrictions.

Total non-sleeping time in equation (3) is normalized to 1 and allocated to work (n_t), leisure (l_t), and commuting time ϕ_T^ℓ. We normalize commuting time for zone 1 residents to zero: $\phi_T^2 > \phi_T^1 = 0$. Since we will match income data that exclude the unemployed, we impose a minimum constraint on the number of hours worked ($\underline{n}$) for working-age households. This restriction will also help us match the correlation between income and wealth. There is an exogenous retirement age of 65. Retirees supply no labor.

The age- and location-specific taste-shifter $\chi^{\ell,a}(c_t)$ is normalized to one for all zone 2 residents. The shifter χ^1 captures the amenity value of zone 1 relative to zone 2, including for example relative school quality. The shifter χ^W (χ^R) increases the utility for working-age (retired) households that live in zone 1 and consume above a threshold $\underline{c}$. This creates a complementarity between living in zone 1 and high consumption levels.

9

This modeling device captures that luxury amenities such as high-end entertainment, restaurants, museums, or art galleries are concentrated in the urban core. Assuming that the benefit from such luxury amenities only accrues above a certain consumption threshold, provides an extra pull for rich households to live in the city center beyond the pull provided by the opportunity cost of commuting. Guerrieri et al. (2013) achieve a similar outcome through a neighborhood consumption externality. A special case of the model arises for $\chi^1 = \chi^R = \chi^W = 1$; location choice is solely determined by commuting costs. Another special case is $\underline{c} = 0$, which gives the same amenity value of the city center χ^1 to all households, regardless of consumption level. We solve and discuss these special cases in Appendices H.2 and H.3.

There are two types of households in terms of the time discount factor. One group of households have a high degree of patience β^H while the rest have a low degree of patience β^L. This preference heterogeneity helps the model match observed patterns of wealth inequality and wealth accumulation over the life cycle. A special case of the model with $\beta^H = \beta^L$ is discussed in Appendix H.4.

Endowments   A household's labor income y_t^{lab} depends on the number of hours worked n, the wage per hour worked W, a deterministic component G^a which captures the hump-shaped pattern in average labor income over the life-cycle, and an idiosyncratic labor productivity z, which is stochastic and persistent.

After retirement, households earn a retirement income which is the product of an aggregate component $\overline{\Psi}$ and an idiosyncratic component $\psi^{a,z}$. The idiosyncratic component has cross-sectional mean of one, and is determined by productivity during the last year of work. Labor income is taxed linearly at rate τ^{SS} to finance retirement income. All other taxes and transfers are captured by the function $T(\cdot)$ which maps total pre-tax income into a net tax (negative if transfer). Net tax revenue goes to finance a public good which does not enter in household utility.

Households face mortality risk which depends on age, p^a. Although there is no intentional bequest motive, households who die leave accidental bequests. We assume that the number of agents who die with positive wealth leave a bequest to the same number of agents alive of ages 21 to 65. These recipient agents are randomly chosen, with one restriction. Patient agents (β^H) only leave bequests to other patient agents and impatient agents (β^L) only leave bequests to other impatient agents. One interpretation is that attitudes towards saving are passed on from parents to children. Conditional on receiving a bequest, the size of the bequest $\widehat{b}_{t+1}$ is a draw from the relevant distribution, which differs for β^H and β^L types. Because housing wealth is part of the bequest, the size of the

bequest is stochastic. Agents know the distribution of bequests, conditional on β type. This structure captures several features of real-world bequests: many households receive no bequest, bequests typically arrive later in life and at different points in time for different households, households anticipate bequests to some degree, and there is substantial heterogeneity among bequest sizes for those who receive a bequest.

Affordable Housing We model mandatory inclusionary zoning which requires developers to set aside a fraction η^ℓ of rental housing units in zone ℓ for low-income households at below-market rents.[4] The rent per square foot is a fraction $\kappa_1 < 1$ of the free-market rent.[5] Every household in the model enters the affordable housing lottery every period. A household that wins the lottery in a zone can choose to turn down the affordable unit, and rent or own in the location of its choice on the free market.[6] If the household accepts the RC lottery win, it must abide by two conditions: (i) its income must be below a cutoff, expressed as a fraction κ_2 of area median income (AMI), when it first moves into the unit – not in subsequent periods – and (ii), the rent paid on the unit must be below a fraction κ_3 of AMI. The latter condition effectively restricts the maximum size of the affordable unit. These conditions capture typical rent regulation and affordable housing specifications. We refer to this system as the RC system and to the housing as RC units.

Households that lived in an affordable housing unit in a given zone in the previous period have an exogenously set, high probability of winning the lottery in the current period, $p^{RC,exog}$.[7] This parameter value determines the persistence of the RC system. For households that were not previously in RC, the probability of winning the lottery for each zone is endogenously determined to equate the residual demand (once accounting for persistent RC renters) and the supply of RC units in each zone. Households form

---

[4]Examples of incentives provided for the development of affordable housing in NYC are (i) the 80/20 new construction housing program, a state program that gives low-cost financing to developers who set aside at least 20% of the units in a property for lower-income families; (ii) the 421a program, which gives tax breaks (up to 25 years) for the development of under-utilized or vacant sites often conditional on providing at least 20% affordable units (i.e., used in conjunction with the 80-20 program), (iii) the Federal Low Income Housing Tax Credits (LITCH) program, which gives tax credits to developers directly linked to the number of low-income households served, and (iv) Mandatory Inclusionary Housing, a New York City program that lets developers build bigger buildings and gives them tax breaks if they reserve some of the units for (permanently) affordable housing.

[5]Since our model is stationary, this is equivalent to assuming that RC rents grow at the same rate as private market rents. We think this is a reasonable approximation to capture a mixture of units that have a nominal rent cap and units that are rent stabilized in that rents grow at a low and stable rate. We discuss how we map our model to the NYC affordable housing data in the calibration section below.

[6]There is a single lottery for all affordable housing units. A certain lottery number range gives access to affordable housing housing in zone 1, while a second range gives access to housing in zone 2. Households with lottery numbers outside these ranges lose the housing lottery.

[7]For these households, the probability of winning the RC lottery in the other zone is set to zero.

beliefs about this probability. This belief must be consistent with rational expectations, and is updated as part of the equilibrium determination. The affordable housing mandate distorts labor supply, location choice, housing demand, and housing supply, as discussed in the introduction.

Location and Tenure Choice Denote by $p^{RC,\ell}$ the probability of winning the affordable housing lottery and being offered a unit in zone ℓ. The household chooses whether to accept the RC option with value $V_{RC,\ell}$, or to turn it down and go to the private housing market with value V_{free}. The value function V is:

$$V = p^{RC,1} \max \left\{ V_{RC,1}, V_{free} \right\} + p^{RC,2} \max \left\{ V_{RC,2}, V_{free} \right\} + \left(1 - p^{RC,1} - p^{RC,2} \right) V_{free}.$$

A household that loses the lottery or wins it but turns it down, freely chooses in which location $\ell \in \{1, 2\}$ to live and whether to be an owner (O) or a renter (R).

$$V_{free} = \max \left\{ V_{O,1}, V_{R,1}, V_{O,2}, V_{R,2} \right\}.$$

The Bellman equations for $V_{RC,\ell}$, $V_{R,\ell}$ and $V_{O,\ell}$ are defined below.

Let S_t be the vector which includes the wage W_t, the housing price P_t^ℓ, the market rent R_t^ℓ and previous housing stock H_{t-1}^ℓ for each location ℓ. The household forms beliefs about S_t. The household's individual state variables are: its net worth at the start of the period x_t, its idiosyncratic productivity level z_t, its age a, and its RC status in the previous period d (equal to 0 if the household was not in RC, 1 if it was in RC in zone 1, and 2 if it was in RC in zone 2). The latter affects the probabilities of winning the lottery, and whether the income constraint applies in the current period conditional on choosing RC. We suppress the dependence on β-type in the problem formulation below, but note here that there is one set of Bellman equations for each β-type.

Market Renter Problem A renter household on the free rental market in location ℓ chooses non-durable consumption c_t, housing consumption h_t, and working hours n_t

to solve:

$$V_{R,\ell}(x_t, z_t, a, d_t) = \max_{c_t, h_t, n_t, b_{t+1}} U(c_t, h_t, n_t, \ell_t) + (1 - p^a)\beta \mathbb{E}_t[V(x_{t+1}, z_{t+1}, a + 1, 0)]$$

s.t.

$$c_t + R_t^\ell h_t + Q b_{t+1} + \phi_F^\ell = \left(1 - \tau^{SS}\right) y_t^{lab} + \overline{\Psi}_t \psi^z + x_t - T\left(y_t^{tot}\right),$$
$$y_t^{lab} = W_t n_t G^a z_t,$$
$$y_t^{tot} = y_t^{lab} + \left(\frac{1}{Q} - 1\right) x_t,$$
$$x_{t+1} = b_{t+1} + \widehat{b}_{t+1} \geq 0,$$
and equations (1), (2), (3), (4).

The renter's savings in the risk-free bond, $Q b_{t+1}$, are obtained from the budget constraint. Pre-tax labor income y_t^{lab} is the product of wages W per efficiency unit of labor, the number of hours n, and the productivity per hour $G^a z$. Total pre-tax income, y^{tot}, is comprised of labor income and financial income. Financial income for renters is the interest income on bonds. The function $T(\cdot)$ transforms total before-tax income into a net tax, and captures all insurance provided through the tax code. Additionally, a Social Security tax τ^{SS} is applied to labor income. Next period's financial wealth x_{t+1} consists of savings b_{t+1} plus any accidental bequests $\widehat{b}_{t+1}$. Housing demand and labor supply choices are subject to minimum constraints discussed above. In addition to a time cost, residents of zone 2 face a financial cost of commuting ϕ_F^2. As we did for the time cost, we normalize the financial cost of commuting in zone 1 to zero: $\phi_F^1 = 0$.

RC Renter Problem A renter household in the RC system in location ℓ chooses non-durable consumption c_t, housing consumption h_t, and working hours n_t to solve:

$$V_{RC,\ell}(x_t, z_t, a, d_t) = \max_{c_t, h_t, n_t, b_{t+1}} U(c_t, h_t, n_t, \ell_t) + (1 - p^a)\beta \mathbb{E}_t[V(x_{t+1}, z_{t+1}, a + 1, \ell)]$$

s.t.

$$c_t + \kappa_1 R_t^\ell h_t + Q b_{t+1} + \phi_{F,t}^\ell = \left(1 - \tau^{SS}\right) y_t^{lab} + \overline{\Psi}_t \psi^z + x_t - T\left(y_t^{tot}\right),$$
$$x_{t+1} = b_{t+1} + \widehat{b}_{t+1} \geq 0,$$
$$y_t^{lab} \leq \kappa_2 \overline{Y}_t \text{ if } d_t = 0,$$
$$h_t \leq \frac{\kappa_3 \overline{Y}_t}{\kappa_1 R_t^\ell},$$
and equations (1), (2), (3), (4).

The per square foot rent of a RC unit is a fraction κ_1 of the market rent R_t^ℓ. For households who were not previously in the RC system ($d_t = 0$) to qualify for RC, labor income must not exceed a fraction κ_2 of area median income (AMI), $\overline{Y}_t = \text{Median}[y_t^{lab,i}]$, the median

across all residents in the metro area. The last inequality says that expenditures on rent $(\kappa_1 R_t^\ell h_t)$ must not exceed a fraction κ_3 of AMI.[8] We impose the same minimum house size constraint in the RC system. We note that a household in RC in the current period carries over her priority status for RC into the next period; the value function next period has RC flag $d_{t+1} = \ell$.

Owner's Problem   An owner in location ℓ chooses non-durable consumption c_t, housing consumption h_t, working hours n_t, and investment property $\widehat{h}_t$ to solve:

$$V_{O,\ell}(x_t, z_t, a, d_t) = \max_{c_t, h_t, \widehat{h}_t, n_t, b_{t+1}} U(c_t, h_t, n_t, \ell_t, a) + (1 - p^a)\beta E_t[V(x_{t+1}, z_{t+1}, a+1, 0)]$$

s.t.

$$c_t + P_t^\ell h_t + Q b_{t+1} + \kappa_4^\ell P_t^\ell \widehat{h}_t + \phi_{F,t}^\ell = (1 - \tau^{SS}) y_t^{lab} + \overline{\Psi}_t \psi^z + x_t + \kappa_4^\ell R_t^\ell \widehat{h}_t - T\left(y_t^{tot}\right),$$

$$x_{t+1} = b_{t+1} + \widehat{b}_{t+1} + P_{t+1}^\ell h_t(1 - \delta^\ell - \tau^{P,\ell}) + \kappa_4^\ell P_{t+1}^\ell \widehat{h}_t(1 - \delta^\ell - \tau^{P,\ell}),$$

$$-Q_t b_{t+1} \leq P_t^\ell \left(\theta_{res} h_t + \theta_{inv}\kappa_4^\ell \widehat{h}_t\right) - \kappa_4^\ell R_t^\ell \widehat{h}_t - (y_t^{tot} - c_t),$$

$$\widehat{h}_t \geq 0,$$

$$\kappa_4^\ell = 1 - \eta^\ell + \eta^\ell \kappa_1,$$

and equations (1), (2), (3), (4).

Local home owners are the landlords to the local renters. This is a departure from the typical assumption of absentee landlords.[9] Our landlords are risk-averse households inside the model. For simplicity, we assume that renters cannot buy investment property and that owners can only buy investment property in the zone of their primary residence. Landlords earn rental income $\kappa_4^\ell R_t^\ell \widehat{h}_t$ on their investment units $\widehat{h}_t$. Per the affordable housing mandate, investment property is a bundle of η^ℓ square feet of RC units and $1 - \eta^\ell$ square feet of free-market units. The effective rent earned per square foot of investment property is $\kappa_4^\ell R_t^\ell$. Since the average rent is a multiple $\kappa_4 \leq 1$ of the market rent, the average price of rental property must be the same multiple of the market price, $\kappa_4^\ell P_t^\ell$. Because prices and rents scale by the same constant, the return on investing in rental property is

---

[8]In the implementation, we assume that the income and size qualification cutoffs for RC are constants. We then compute what fractions κ_2 and κ_3 of AMI they represent. This allows us to sidestep the issue that the AMI may change with RC policies.

[9]The majority of rentals in the urban core are multi-family units owned by corporations. According to 2015 Real Capital Analytics data, 81% of the Manhattan multifamily housing stock is owned by owner-operator-developers which tend to be overwhelmingly local. Non-financial firms, some of which are also local, own 3%. The remaining 16% is owned by financial firms, private equity funds or publicly listed REITs. Since at least some of the investors in private equity funds or publicly listed REITs which hold New York multifamily apartments are locals, the local ownership share is even higher than 81%. The majority of rentals in the rest of the metro area are single-family rentals. About 99% of those are owned by small, local owners.

the same as that on owner-occupied housing. As a result, landlords are not directly affected by RC regulation. However, the lower average price for rental property ($\kappa_4 < 1$) has important effects on housing supply/development, as discussed below.

The physical rate of depreciation for housing units is δ^ℓ. The term $P^\ell h \delta^\ell$ is a financial costs, i.e., a maintenance cost. As shown in equation (10) below, the physical depreciation can be offset by residential investment undertaken by the construction sector.[10]

Property taxes on the housing owned in period t are paid in year $t+1$; the tax rate is $\tau^{P,\ell}$. Property tax revenue finances local government spending which does not confer utility to the households.[11]

Housing serves as a collateral asset for debt. For simplicity, mortgages are negative short-term safe assets. In practice, mortgage rates are higher than bond rates but mortgage interest is also tax deducible. We assume these two effects cancel out. Households can borrow a fraction θ_{res} of the market value of their primary residence and a potentially different fraction θ_{inv} against investment property. The empirically relevant case is $\theta_{res} \geq \theta_{inv}$. We exclude current-period rental income and savings from the pledgable collateral. In light of the fact that one period is four years in the calibration, we do not want to include four years worth of (future) rental income and savings for fear of making the borrowing constraint too loose.[12]

Appendix A shows that, for renters, the choices of h_t and n_t are analytic functions of c_t. Therefore, the renter's problem can be rewritten with just two choices: consumption c_t and location ℓ. For owners, the choices of h_t and n_t are analytic functions of c_t and $\widehat{h}_t$. Therefore, the owner's problem can be rewritten with just three choices: consumption c_t, investment property size $\widehat{h}_t$, and location ℓ.

2.2 Firms

Goods Producers There are a large number n_f of identical, competitive firms located in the urban core (zone 1), all of which produce the numéraire consumption good.[13] This

---

[10]The model can accommodate a higher rate of depreciation for renter-occupied properties, possibly to reflect the higher rate of depreciation for affordable housing units. We are not aware of empirical evidence that shows that mandatory inclusionary housing results in higher depreciation of the affordable units. Traditional rent control is often associated with under-maintenance. We also note that the model captures that the RC system results in lower rents and fewer units being built.

[11]This is equivalent to a model where public goods enter in the utility function, but in a separable way from private consumption. A model where the public good enters non-separably in the utility function would require taking a stance on the elasticity of substitution between private and public consumption.

[12]This assumptions helps the model match the home ownership rate. However, the affordable housing policies would have similar effects without it.

[13]We assume that the number of firms is proportional to the number of households in the city when solving the model. With this assumption, our numerical solution is invariant to the number of households.

good is traded nationally; its price is unaffected by events in the city and normalized to 1. The firms have decreasing returns to scale and choose efficiency units of labor to maximize profit each period:

$$\Pi_{c,t} = \max_{N_{c,t}} N_{c,t}^{\rho_c} - W_t N_{c,t} \tag{5}$$

Developers and Affordable Housing Mandate   In each location ℓ there is a large number n_f of identical, competitive construction firms (developers) which produce new housing units and sell them locally. All developers are headquartered in the urban core, regardless of where their construction activity takes place.

The cost of the affordable housing mandate is born by developers. Affordable housing regulation stipulates that for every $1 - \eta^\ell$ square foot of market rental units built in zone ℓ, η^ℓ square feet of RC units must be built. Developers receive an average price per foot for rental property of $\kappa_4^\ell P_t^\ell$, while they receive a price per foot of P_t^ℓ for owner-occupied units. Given a home ownership rate in zone ℓ of ho_t^ℓ, developers receive an average price per foot $\overline{P}_t^\ell$:

$$\overline{P}_t^\ell = \left(ho_t^\ell + (1 - ho_t^\ell)\kappa_4^\ell \right) P_t^\ell. \tag{6}$$

The cost of construction of owner-occupied and rental property in a given location is the same. After completion of construction but prior to sale, some of the newly constructed housing units are designated as rental units and the remainder as ownership units. The renter-occupancy designation triggers affordable housing regulation. It results in a lower rent and price than for owner-occupied units. Developers would like to sell ownership units rather than rental units, but the home ownership rate is determined in equilibrium. Developers are price takers in the market for space, and face an average sale price of $\overline{P}_t^\ell$.

A special case of the model is the case without rent control: $\kappa_4^\ell = 1$ either because $\eta^\ell = 0$ or $\kappa_1 = 1$. In that case, $\overline{P}_t^\ell = P_t^\ell$. Without rent control, the higher sale price for housing increases incentives to develop more housing.

Zoning   Given the existing housing stock in location ℓ, H_{t-1}^ℓ, and average sale price of $\overline{P}_t^\ell$, construction firms have decreasing returns to scale and choose labor to maximize profit each period:

$$\Pi_{h,t}^\ell = \max_{N_{\ell,t}} \overline{P}_t^\ell \left(1 - \frac{H_{t-1}^\ell}{\overline{H}^\ell} \right) N_{\ell,t}^{\rho_h} - W_t N_{\ell,t} \tag{7}$$

Due to decreasing return to scale, the numerical solution would depend on the number of households otherwise.

16

The production function of housing has two nonlinearities. First, as for consumption good firms, there are decreasing returns to scale because $\rho_h < 1$.

Second, construction is limited by zoning laws. The maximal amount of square footage zoned for residential use in zone ℓ is given by $\overline{H^\ell}$. We interpret $\overline{H^\ell}$ as the total land area zoned for residential use multiplied by the maximum permitted number of floors that could be built on this land, the floor area ratio (FAR). This term captures the idea that, the more housing is already built in a zone, the more expensive it is to build additional housing. For example, additional construction may have to take the form of taller structures, buildings on less suitable terrain, or irregular infill lots. Therefore, producing twice as much housing requires more than twice as much labor. Laxer zoning policy, modeled as a larger $\overline{H^\ell}$, makes development cheaper, and all else equal, will expand the supply of housing.

When $\overline{H^\ell}$ is sufficiently high, the model's solution becomes independent of $\overline{H^\ell}$, and the supply of housing is governed solely by ρ_h. When $\overline{H^\ell}$ is sufficiently low, the housing supply elasticity depends on both $\overline{H^\ell}$ and ρ_h.[14]

Per capita profits from tradeable and construction sectors are:

$$\Pi_t = \Pi_{c,t} + \Pi_{h,t}^1 + \Pi_{h,t}^2$$

We assume that goods and construction firms are owned by equity holders outside the city. Appendix H.5 considers a model where profits are redistributed to locals.

2.3 Equilibrium

Given parameters, a competitive equilibrium is a price vector $(W_t, P_t^\ell, R_t^\ell)$ and an allocation, namely aggregate residential demand by market renters $H_t^{R,\ell}$, RC renters $H_t^{RC,\ell}$, and owners $H_t^{O,\ell}$, aggregate investment demand by owners $\widehat{H}_t^\ell$, aggregate housing supply, aggregate labor demand by goods and housing producing firms $(N_{c,t}, N_{\ell,t})$, and aggregate labor supply N_t such that households and firms optimize and markets clear.

The following conditions characterize the equilibrium. First, given wages and average prices given by (6), firms optimize their labor demand, resulting in the first-order

---

[14]In this sense, the model captures that construction firms must pay more for land when land is scarce or difficult to build on due to regulatory constraints. This scarcity is reflected in equilibrium house prices.

conditions:

$$N_{c,t} = \left(\frac{\rho_c}{W_t} \right)^{\frac{1}{1-\rho_c}} \quad \text{and} \quad N_{\ell,t} = \left(\frac{\left(1 - \frac{H^\ell_{t-1}}{H^\ell}\right) \overline{P}^\ell_t \rho_h}{W_t} \right)^{\frac{1}{1-\rho_h}}. \tag{8}$$

Second, labor demand equals labor supply:

$$n_f \left(N_{c,t} + \sum_\ell N_{\ell,t} \right) = N_t. \tag{9}$$

Third, the housing market clears in each location ℓ:

$$(1 - \delta^\ell) H^\ell_{t-1} + n_f \left(1 - \frac{H^\ell_{t-1}}{H^\ell} \right) N^{\rho_h}_{\ell,t} = H^{O,\ell}_t + \widehat{H}^\ell_t. \tag{10}$$

The left-hand-side is the supply of housing which consists of the non-depreciated housing stock and new residential construction. The right-hand-side is the demand for those housing units by owner-occupiers and landlords. Fourth, the supply of rental units in each location ℓ must equal the demand, from market tenants and RC tenants, respectively:

$$\widehat{H}^\ell_t (1 - \eta^\ell) = H^{R,\ell}_t, \quad \widehat{H}^\ell_t \eta^\ell = H^{RC,\ell}_t \tag{11}$$

Fifth, total pension payments equal to total Social Security taxes collected:

$$\overline{\Psi}_t N_{ret} = \tau^{SS} N_t W_t, \tag{12}$$

where N_{ret} is the total number of retirees, which is a constant, and N_t are total efficiency units of labor. Sixth, the aggregate state S_t evolves according to rational expectations. Seventh, the value of all bequests received is equal to the wealth of all agents who die. We focus on the model's steady state where all aggregate quantities and prices are constant.

2.4 Welfare Effects of Affordability Policies

We compute the welfare effect of an affordability policy using the following procedure. Denote agent i's value function under benchmark policy θ_b as $V_i(x, z, a, S; \theta_b)$. Consider an alternative policy θ_a with value function $V_i(x, z, a, S; \theta_a)$. The alternative policy implies a change in value functions, which we express in consumption equivalent units. We

compare steady state welfare for agents belonging to a group g with cardinality G:

$$\mathcal{W}_g = \left(\frac{\frac{1}{G} \sum_{i \in g} V_i(\theta_a)}{\frac{1}{G} \sum_{i \in g} V_i(\theta_b)} \right)^{\frac{1}{(1-\gamma)(1-\alpha_n)}} - 1. \tag{13}$$

We focus on the following groups: the entire population, age groups, productivity groups, income quartiles, and wealth quartiles. Since we equally weight all households, we assume a utilitarian social welfare function.[15] The groups always have the same cardinality (number of members) under the benchmark and alternative policies. Income and wealth are endogenously determined. Therefore, different policies may lead to different households in a given income or wealth quartile. We do not aggregate welfare by tenure status, except in the transition exercise discussed in Appendix I.[16]

3 Calibration

We calibrate the model to the New York MSA. Data sources are described in Appendix B. Table 1 summarizes the chosen model parameters.

Geography The New York metro consists of 25 counties located in New York (12), New Jersey (12), and Pennsylvania (1). We assume that Manhattan (New York County) represents zone 1 and the other 24 counties make up zone 2.[17] The zones differ in size, measured by the maximum buildable residential square footage permitted by existing zoning rules, $\overline{H^\ell}$. Appendix B describes detailed calculations on the relative size of Manhattan and the rest of the metro area, which imply that $\overline{H^1} = 0.0238 \times \overline{H^2}$. We then choose $\overline{H^2}$ such that the fraction of households living in zone 1 equals 10.5% of the total, the fraction observed in in the data. Since the model has no vacancy, we equate the number of

---

[15]Because of the curvature of the value function, lower-income households implicitly receive a larger weight. We have also computed our results assuming a welfare criterion that first transforms the value functions V into V^b before aggregation, where b can be chosen to over- or under-weight the poor vs. the rich, relative to the utilitarian social welfare function. These results are available upon request.

[16]Tenure status groups do not have fixed cardinality and tenure status is strongly policy-dependent. For example, a policy may increase the number of households who obtain a RC unit, but RC units may be smaller in the alternative economy. Comparing the welfare of RC households in the steady state of the benchmark to that of RC households in the steady state of the alternative economy may show a welfare reduction because of the smaller unit size. It misses the potentially much larger welfare gain for households who were not in RC in the benchmark but are in RC under the alternative policy.

[17]Alternative choices are to designate (i) New York City (five counties coinciding with the five boroughs of NYC) as zone 1 and the rest of the metro as zone 2, or (ii) Manhattan as zone 1 and the other four counties in New York City as zone 2. Both choices ignore that the dominant commuting pattern is from the rest of the metro area to Manhattan.

households in the model with the number of occupied housing units in the data.

Demographics   The model is calibrated so that one model period is equivalent to 4 years. Households enter the model at age 21, work until age 64, and retire with a pension at age 65. Survival probabilities p^a are calibrated to mortality data from the Census Bureau. People age 65 and over comprise 19.1% of the population age 21 and over in the data. In the model, we get 21.8%. The average New York metro resident above age 21 is 47.6 years old in the data and 47.4 years old in the model.

Labor Income   Recall that pre-tax labor income for household i of age a is $y_t^{lab} = W_t n_t^i G^a z_t^i$, where the household takes wages as given and chooses labor supply n_t^i. The choice of hours is subject to a minimum hours constraint, which is set to 0.5 times average hours worked. This constraint rules out a choice of a positive but very small number of hours, which we do not see in the data given the indivisibility of jobs. It also rules out unemployment since our earnings data are for the (part-time and full-time) employed. This constraint binds for only 6.15% of workers in equilibrium.

Efficiency units of labor $G^a z_t^i$ consist of a deterministic component that depends on age (G^a) and a stochastic component z^i that captures idiosyncratic income risk. The G^a function is chosen to enable the model to match the mean of labor earnings by age. We use data from ten waves of the Survey of Consumer Finances (1983-2010) to estimate G^a.

The idiosyncratic productivity process z is chosen to both match earnings inequality in the NY MSA data and to generate realistic persistence in earnings. We discretize z as a 4-state Markov chain. The four states vary by age to capture the rising variance of earnings with age. We use the SCF data to discipline the increase in variance by age. The four grid points at the average age are chosen to match the NY metro pre-tax earnings distribution from the Census Bureau. We choose annual household-level earnings cutoffs in the data of \$41,000, \$82,000, and \$164,000. This results in four earnings groups with average earnings of \$28,125, \$60,951, \$116,738, and \$309,016. Average New York MSA household earnings are \$124,091; the median is \$88,988. The four point grid for productivity z is chosen to match the average earnings in each group. This is an iterative process since labor supply is endogenous and depends on all other parameters and features of the model.

The 4×4 transition probability matrix for z^i is age-invariant, but is allowed to depend on β type. Specifically, the expected duration of the highest productivity state is higher for the more patient agents. There are five unique parameters governing transition probabilities which are pinned down by five moments in the data. The four income groups

have population shares in the data of 16.1%, 29.8%, 34.2%, and 19.9%, respectively. Since the shares sum to 1, that delivers three restrictions on the transition matrix. Matching the persistence of labor income to 0.9 delivers a fourth restriction. Finally, the dependence on β is calibrated to deliver the observed correlation between income and wealth in the SCF data. Appendix C contains the parameter values and further details.

Taxation Since our model is an incomplete markets model, housing affordability policies can act as an insurance device and help to "complete the market." Therefore, it is important to realistically calibrate the redistribution provided through the tax code. We follow Heathcote, Storesletten, and Violante (2017) and choose an income tax schedule that captures the observed progressivity of the U.S. tax code in a parsimonious way. Net taxes are given by the function $T(\cdot)$:

$$T(y^{tot}) = y^{tot} - \lambda (y^{tot})^{1-\tau}$$

The parameter τ governs the progressivity of the tax and transfer system. We set $\tau = 0.17$ to match the average income-weighted marginal tax rate of 34% for the U.S. It is close to the value of 0.18 estimated by Heathcote et al. (2017). We set λ to match federal, state, and local government spending to aggregate income in the NY metro area, equal to 15-20%.[18] This delivers $\lambda = 0.75$. Appendix D shows the resulting tax and after-tax income along the before-tax income distribution.

Retirement Income Social Security taxes and receipts are treated separately from the tax and transfer system.[19] Social Security taxes are proportional to labor earnings and set to $\tau^{SS} = 0.10$, a realistic value. Retirement income is increasing in the household's last productivity level prior to retirement, but is capped for higher income levels. We use actual Social Security rules to estimate each productivity group's pension relative to the average pension. The resulting pension income states are $\psi^z = [0.50, 1.03, 1.13, 1.13]$, where z reflects the last productivity level prior to retirement. They are multiplied by average retirement income $\overline{\Psi}$, which is endogenously determined in equation (12) to balance the social security budget. Average retirement income $\overline{\Psi}$ is $44364, which corresponds to 36% of average earnings.

---

[18]Depending on what share of NY State spending goes to the NY metro area, we get a different number in this range.

[19]Social security is not in $T(y)$ because a large part of it is not a transfer, but rather an inter-temporal savings vehicle.

Table 1: Calibration

Description	Parameter	Value	Target
Panel A: Geography and Income			
Productivity states	Z	[0.255 0.753 1.453 3.522]	Income inequality - see App. C
Transition prob.	$(p_{11}, p_{22}, p_{33}, p_{44}, p^H)$	(0.93,0.92,0.92,0.82,0.02)	Pop. shares of income groups, persistence of income - see App. C
Tax progressivity	τ	0.17	Heathcote et al. (16)
Tax burden	λ	0.75	Pension tax revenue
Average pension	$\overline{\Psi}$	0.16	Average replacement rate of 35%
Pension distr.	ψ^z	[0.50 1.03 1.13 1.13]	U.S. pension rules
Available space	$(\overline{H^1}, \overline{H^2})$	(0.05,2.11)	Max. residential buildable area, pop. share Manhattan
Time-Commuting cost	ϕ_T^2	0.037	Average commuting time NY
Financial-Commuting cost	ϕ_F^2	0.01	Average commuting cost NY
Panel B: Preferences			
Risk aversion	γ	5	Standard value
Leisure weight	α_n	0.63	Average NY work week
Housing consumption weight	α_h	0.41	Avg. rent/avg. income of 23%
CES parameter consumption/housing	ϵ	-0.50	Elasticity of substitution consumption/housing of 2/3
Minimum hours worked	$\underline{n}$	0.15	Min. hours worked of 15% of available time
Time Preference (4yr)	(β^H, β^L)	(1.281,0.985)	Average wealth/income 5.69 and wealth Gini 0.80 in SCF
Cons. externality threshold	$\underline{c}$	0.45	House-price/income ratio of zones in NY data
Extra utility zone 1 workers	χ^W	1.004	Income ratio of zones in NY data
Extra utility zone 1 retirees	χ^R	1.038	Fraction of retirees ratio of zones in NY data
Extra utility zone 1 all	χ^1	1.080	Rent per sqft ratio of zones in NY data
Panel C: Finance, Housing, Construction			
Bond Price (4yr)	Q	0.89	Price/rent ratio in NYC area
Maximum residential LTV	θ_{res}	0.80	Median mortgage downpayment
Maximum investment LTV	θ_{inv}	0.80	Mortgage underwriting standards
Property tax (4yr)	$(\tau^{P,1}, \tau^{P,2})$	(0.029,0.053)	Property tax rate in Manhattan and NY metro area
Depreciation rate	(δ^1, δ^2)	(0.058,0.096)	Res. depreciation BEA
Minimum housing size	$\underline{h}$	0.25	Legal minimum housing size
Panel D: Production			
Return to scale consumption sector	ρ_c	0.66	Labor income share of 2/3
Return to scale housing sector	ρ_h	0.66	Housing supply elasticity for NY
Panel E: Affordable Housing			
Fraction rent control	(η^1, η^2)	(24.46%,15.97%)	Fraction of households in RC of 13% and 4.7%
Probability to win RC lottery when in RC	$p^{RC,exog}$	0.76	Fraction of RC households in same unit for $\geq$ 20 years
Rental discount	κ_1	50%	Observed rental discount
Income threshold for RC	κ_2	0.40	Income distribution of RC tenants
Rental share threshold for RC	κ_3	0.35	Standard program value

Commuting Cost   We choose the time cost to match the time spent commuting for the average New York metro area resident. This time cost is the average of all commutes, including those within Manhattan. Since the model normalizes the commuting time within zone 1 to zero, we target the additional commuting time of zone 2 residents. The additional commuting time amounts to 25 minutes per trip for 10 commuting trips per week.[20] The 4.2 hours represent 3.7% of the 112 hours of weekly non-sleeping time. Hence, we set $\phi_T^2 = 0.037$.

The financial cost of commuting ϕ_F^2 is set to 1.8% of average labor earnings, or $2281 per household per year. This is a reasonable estimate for the commuting cost in excess of the commuting cost within Manhattan, which is normalized to zero.[21]

We assume that retirees have time and financial commuting costs that are 1/3 of those of workers. This captures that retirees make fewer trips, travel at off-peak hours, and receive transportation discounts.

Preferences   The functional form for the utility function is given in equation (1). We set risk aversion $\gamma = 5$, a standard value in the macro-finance literature. Since risk aversion governs the value of insurance against risk, it is a key parameter. In Appendix H.1, we redo the analysis for a risk aversion value of 2. We assume Cobb-Douglas preferences between consumption (of non-durable goods and housing services) and leisure.

The observed average workweek for New York metro residents is 42.8 hours or 38.2% of available non-sleeping time. Since there are 1.64 workers on average per household, household time spent working is 38.2%× 1.64/2=31.3%. We set α_n to match household time spent working. The model generates 30.9% of time worked.

Our model generates an (endogenous) average Frisch elasticity of 0.65, with 25th and 75th percentiles of 0.56 and 0.76. This is in line with estimates based on the intensive margin of labor supply in micro data. This is an important object because misallocation coming from workers' persistent location and labor supply decisions partly depend on how sensitive labor supply is to wage changes.

We set α_h in order to match the ratio of average market rent to metro-wide average

---

[20]The 25 minute additional commute results from a 15 minute commute within Manhattan and a 40 minute commute from zone 2 to zone 1. With 10.5% of the population living in Manhattan, the average commuting time is 37.4 minutes per trip or 6.2 hours a week. This is exactly the observed average for the New York metro from Census data.

[21]In NYC, an unlimited subway pass costs around $1,400 per year per person. Rail passes from the suburbs cost around $2400 per year per person, depending on the railway station of departure. If zone 1 residents need a subway pass while zone 2 residents need a rail pass, the cost difference is about $1000 per person. With 1.64 workers per household, the cost difference is $1640 per household. The cost of commuting by car is at least as high as the cost of rail once the costs of owning, insuring, parking, and fueling the car and tolls for roads, bridges, and tunnels are factored in.

income. Income data discussed above and rental data from Zillow, detailed in Appendix B, indicate that this ratio is 23.0% for the New York MSA in 2015. The model generates 24.1%.

We set $\beta^H = 1.281$ (1.064 per year) and $\beta^L = 0.985$ (0.996 per year). A 25% share of agents has β^H, the rest has β^L. This delivers an average β of 1.06, chosen to match the average wealth-income ratio which is 5.69 in the 1998-2010 SCF data. The model generates 5.95. The dispersion in betas delivers a wealth Gini coefficient of 0.79, close to the observed wealth Gini coefficient of 0.80 for the U.S.[22] Note that because of mortality, the effective time discount rate is $(1 - p(a))\beta$.

The taste-shifter for zone 1 is parameterized as: $\chi^1 = 1.080$, $\chi^W = 1.004$, $\chi^R = 1.038$, $\underline{c} = 0.45$. Living in Manhattan gives a substantial utility boost, equivalent to a 8% higher consumption bundle. About 28% of the Manhattan population consumes above the threshold $\underline{c}$. This group derives extra utility from living in Manhattan, especially the retirees in this group ($\chi^1 \chi^R = 1.121$). We chose these four parameters to get our model to better match the following four ratios of zone 1 relative to zone 2 variables, given all other parameters: the relative fraction of retirees of 0.91, a relative household income ratio of 1.66, the relative ratio of market rents per square foot of 2.78, and the relative home ownership rate of 0.42. In the model, these ratios are 1.00, 1.69, 2.77, and 0.76, respectively.

Housing   The price for the one-period (4-year) bond is set to $Q = 0.89$ to match the average house price to rent ratio for the New York MSA, which is 17.79. The model delivers 16.94. Under the logic of the user cost model, the price-to-rent ratio depends on the interest rate, the depreciation rate, and the property tax rate.

The property tax rate in Manhattan is $\tau^{P,1} = 0.029$ or 0.73% per year, and that in zone 2 is $\tau^{P,2} = 0.053$ or 1.33% per year. These match the observed tax rates averaged over 2007-2011 according to the Brookings Institution.[23]

The housing depreciation rate in Manhattan is $\delta^1 = 0.058$ or 1.45% per year, and that in zone 2 is $\delta^2 = 0.096$ or 2.41% per year. This delivers a metro-wide average depreciation rate of 2.39% per year, equal to the average depreciation rate for privately-held residential property in the BEA Fixed Asset tables for the period 1972-2016. The annual depreciation wedge of 0.96% between zones 1 and 2 is chosen to match the relative fraction of buildings that were built before 1939.[24]

---

[22]No wealth data is available for the NY metro. We believe it is likely that wealth inequality is at least as high in the NY metro than in the rest of the U.S.

[23]The zone-2 property tax rate is computed as the weighted average across the 24 counties, weighted by the number of housing units.

[24]Data from the 5-year American Community Survey from 2017 give the distribution of housing units

We set the maximum loan-to-value ratio (LTV) for the primary residence at 80% ($\theta_{res} = 0.8$), implying a 20% down payment requirement. This is the median down-payment in the U.S. data on purchase mortgages. The LTV for investment property is also set at 80% ($\theta_{inv} = 0.8$) for simplicity.

Finally, we impose a minimum housing size of 557 square feet, or 39% of the average housing unit size of 1445 square feet. This is a realistic value for New York given the model is solved at the household level (with 1.64 members on average).

Production and Construction We assume that the return to scale $\rho_c = 0.66$. This value implies a labor share of 66% of output, consistent with the data.

For the housing sector, we also set $\rho_h = 0.66$ in order to match the housing supply elasticity, given the other parameters. The long-run housing supply elasticity in the model is derived in Appendix E. Saiz (2010) reports a housing supply elasticity for the New York metro area of 0.76. The model delivers 0.71. The housing supply elasticity is much lower in zone 1 (0.08) than in zone 2 (0.73), because in zone 1 the housing stock is much closer to $\overline{H}$ (11% from the constraint) than in zone 2 (71% from the constraint). Since the housing stock of the metro area is concentrated in zone 2, the city-wide elasticity is dominated by that in zone 2.

Affordable Housing Rent regulation plays a major role in the New York housing market, as discussed above. Direct measurement on the number of mandatory inclusionary housing units is not available. Nor is it appropriate given that there are many more affordable housing units from a range of programs. We define RC housing as all housing units that are (i) rent controlled, (ii) public housing, (iii) Mitchell Lama housing, (iv) all other government-assisted or regulated housing. We find that 13.0% of zone 1 households and 4.7% of zone 2 households live in RC units. The metro-wide average is 5.57%, the ratio of zone 1 to zone 2 is 2.77. Appendix B.5 contains a detailed description of data and definitions.[25]

We set the share of *square feet* of *rental* housing devoted to RC units, $\eta^1 = 24.46\%$ and $\eta^2 = 15.97\%$, to match the share of *households* in the *entire* population that are in RC units

---

by year built for each of the 25 counties in the New York MSA. In Manhattan, 42.8% of units are built before 1939. The housing-weighted average among the 24 counties of zone 2 is 26.6%. Assuming geometric depreciation, matching this fraction requires a 0.96% per year depreciation wedge.

[25]We chose to exclude rent stabilized units from this definition. Rents on rent stabilized units are in between those on market rentals and RC rentals. An alternative definition of RC that includes rent-stabilized units would have a higher η^ℓ and a lower rent discount ($\kappa_1 = 0.70$, or 30% below the market rent). We solve such a model as a robustness check. Below, we also study a range of policies that increase η^ℓ.

in each zone. This fraction is endogenous since housing size and ownership are choice variables.

According to the same definition and data sources, the average rent in RC units is 50% below that in all other rentals. We set the rent discount parameter $\kappa_1 = 50\%$. It follows that $\kappa_4^1 = 1 - \eta^1 + \eta^1 \kappa_1 = 0.88$ and $\kappa_4^2 = 0.92$, so that landlords earn 12% lower rents in zone 1 and 8% lower rents in zone 2 than they would in an unregulated market.

We set the income qualification threshold to a fraction $\kappa_2 = 40\%$ of AMI. We set $\kappa_3 = 35\%$ so that households in affordable housing spend no more than 35% of AMI on rent, a standard value. These two parameters affect the composition of households who live in RC housing. The lower κ_2, the larger the share of RC tenants that come from the bottom of the income distribution. Similarly, κ_3 affects who lives in RC. High-income agents in the model want to live in a house that is larger than the maximum allowed size under RC and would turn down a RC lottery win. We use data from the New York Housing and Vacancy Survey to study where in the NYC income distribution the RC tenants are located. The model matches the relative income of RC tenants for these choices of κ_2 and κ_3. Table 5 in Appendix B.5 provides more detail.

Finally, we calibrate the persistence of the RC system. We assume that households who were in RC in the previous period have a probability of 76.4% to qualify for RC in the same zone this period. The value is chosen to match the fraction of RC tenants who have lived in a RC unit for 20 years or more. That number in the data is 23.1%.[26] It is 25.2% in the model.

4 Baseline Model Results

We start by discussing the implications of the baseline model for the spatial distribution of population, housing, income, and wealth. We also discuss house prices and rents for the city as a whole and for the two zones. Then we look at the model's implications for income, wealth, and home ownership over the life-cycle.

4.1 Demographics, Income, and Wealth

Demographics The first three rows of Table 2 show that the model matches basic demographic moments. In the data, retirees represent 19.1% of Manhattan residents, and the model matches this share. On the one hand, retirees have lower time and financial costs of commuting, giving them a comparative advantage to living in zone 2. On the other

---

[26]See Table H of the NYU Furman Institutes' 2014 "Profile of Rent-Stabilized Units and Tenants in NYC."

hand, retirees tend to be wealthier making living in Manhattan financially feasible. Absent a taste for Manhattan, the commuting cost effect would drive most retirees to zone 2. A fairly strong preference for living in Manhattan is needed to offset the commuting effect ($\chi^1 = 1.080$, $\chi_R = 1.038$) and match the relative share of retirees in zone 1 to zone 2.

Table 2: New York Metro Data Targets and Model Fit

|		Data		Model	
		metro	ratio zone 1/zone 2	metro	ratio zone 1/zone 2
1	Households (thousands)	7124.9	0.12	7124.9	0.12
2	Avg. hh age, cond over 20	47.6	0.95	47.4	0.86
3	People over 65 as % over 20	19.1	0.91	21.8	1.00
4	Avg. house size (sqft)	1445	0.59	1449	0.64
5	Avg. pre-tax lab income ($)	124091	1.66	124320	1.69
6	Home ownership rate (%)	51.5	0.42	58.4	0.76
7	Median mkt price per unit ($)	510051	3.11	506420	2.34
8	Median mkt price per sqft ($)	353	5.24	344	3.57
9	Median mkt rent per unit (monthly $)	2390	1.65	2491	1.82
10	Median mkt rent per sqft (monthly $)	1.65	2.78	1.69	2.77
11	Median mkt price/median mkt rent (annual)	17.79	1.89	16.94	1.29
12	Mkt price/avg. income (annual)	3.99	1.71	4.08	1.38
13	Avg. rent/avg. income (%)	23.0	1.00	24.1	1.07
14	Avg. rent/income ratio for renters (%)	42.1	0.81	29.1	0.97
15	Rent burdened (%)	53.9	0.79	48.7	0.93
16	% Rent regulated of all housing units	5.57	2.77	5.98	2.61

Notes: Columns 2-3 report the values for the data of the variables listed in the first column. Data sources and construction are described in detail in Appendix B. Column 3 reports the ratio of the zone 1 value to the zone 2 value in the data. Column 5 reports the same ratio in the model.

Housing Units   In the data, the typical housing unit is much smaller in Manhattan than in the rest of the metro area. We back out the typical house size (in square feet) in each county as the ratio of the median house value and the median house value per square foot, using 2015 year-end values from Zillow. We obtain an average housing unit size of 897 sqft in Manhattan and 1,510 sqft in zone 2; their ratio is 0.59. In the model, households freely choose their housing size subject to a minimum house size constraint. The model generates a similar ratio of house size in zone 1 to zone 2 of 0.64.

Figure 1 shows the distribution of house sizes. The model (left panel) matches the data (right panel) quite well, even though these moments are not targeted by the calibration. The size distribution of owner-occupied housing is shifted to the right from the size distribution of renter-occupied housing units in both model and data.

Mobility   The model implies realistic moving rates from zone 1 to zone 2 and vice versa, despite the absence of moving costs. Mobility rates are not targeted by the calibration.

Figure 1: House size distribution in Model (L) and Data (R)
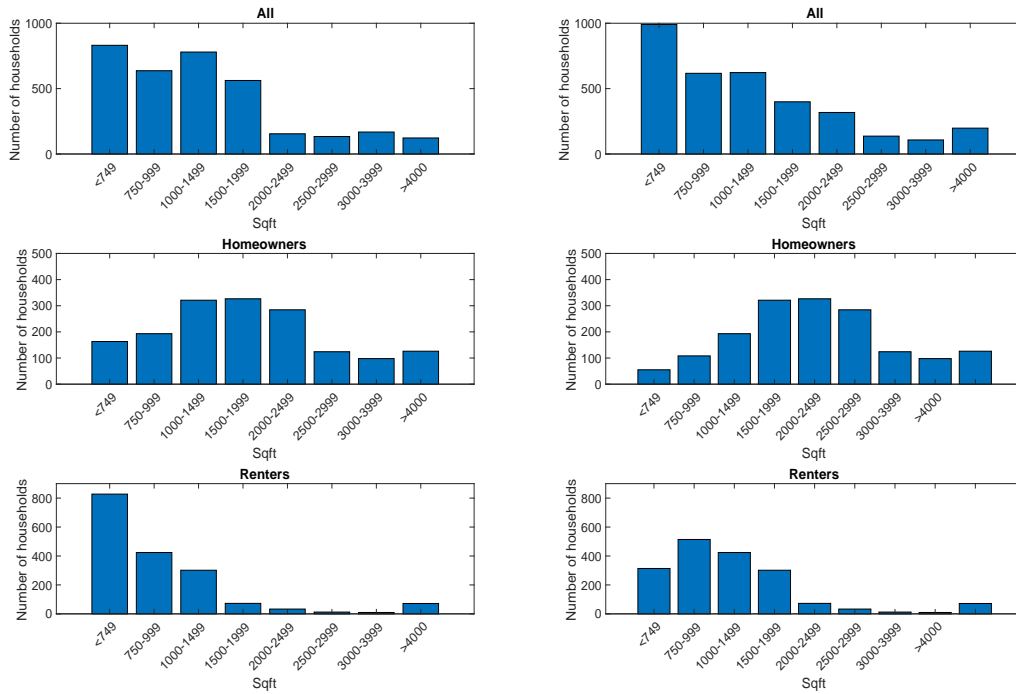
Figure 9 in Appendix F shows that mobility is highest for the young (21-25) and for middle-aged households (35-40). For these two groups, the annual mobility rate is 4% annually. The overall mobility rate across neighborhoods in the model is about 2% annually. These data are consistent with the facts for the NY MSA, where 2.1% of households move between counties within the MSA annually.[27]

Income Average income in the metro area matches the data (row 5 of Table 2) by virtue of the calibration. The ratio of average income in zone 1 to zone 2 is 1.69 in the model and 1.66 in the data.

The productivity distribution is substantially different in the two zones. Zone 1 contains workers that are on average 80% more productive than in zone 2. Productive working-age households have a high opportunity cost of time and prefer to live close to work given the time cost of commuting. Mitigating the high opportunity cost of time is the high cost of living in Manhattan. Indeed, some high-productivity workers may still be early in

---

[27]Data from the U.S. Census Bureau on annual average county-to-county migration rates for 2012-2016.

the life-cycle when earnings are lower and accumulated wealth smaller. Only 29.7% of working-age, top-productivity households live in zone 1. Since χ^W is close to 1, the luxury amenity value of living in Manhattan for working-age households is not needed to explain the observed income gap between zone 1 and zone 2. The commuting cost alone is a strong enough force.

Figure 2 plots how households of different productivity types sort across space (left panel is zone 1, right panel is zone 2) and across tenure status. The vertical axes measures the total square footage devoted to the various types of *housing* in each zone. Values reported on the top of the bars correspond to the percentage of *households* in each category. These percentages add up to 100% across the six housing categories. Colors correspond to productivity levels: increasing in shade from yellow (low, $z = 1$) to red (high, $z = 4$) for working-age households, and green for retirees. The graph shows many (wealthy) retirees in zone 1 as well as many top-productivity households. The only bottom- and lower middle-productivity households ($z = 1, 2$) that live in zone 1 are in RC housing. Low-productivity households cluster in zone 2.

Figure 2: Geographic distribution of households in the benchmark model: zone 1 (left panel) versus zone 2 (right panel).
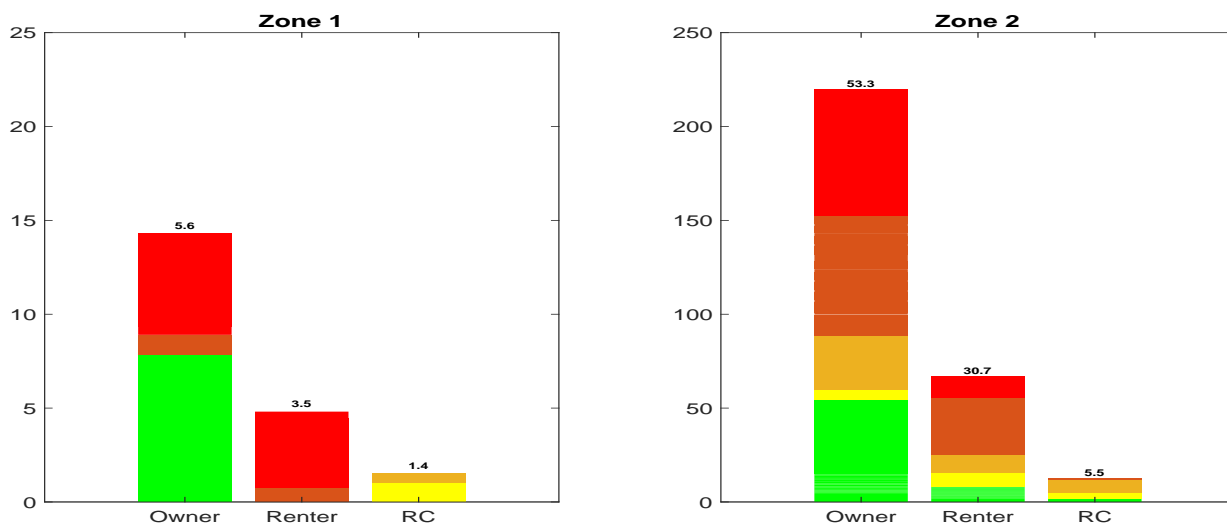
The top panel of Figure 3 shows household labor income over the life-cycle, measured as pre-tax earnings during the working phase and as social security income in retirement. We plot average income for the bottom 25% of the income distribution, for the middle of the income distribution (25-50%), and for the top 25% of the distribution, as well as the overall average income. Labor income has the familiar hump-shaped profile over the life-cycle inherited from the deterministic productivity process G^a. The model generates

a large amount of income inequality at every age. The model's earnings Gini of 0.45 matches the 0.47 value in the 2015 NY metro data.[28] Earnings inequality in the model is lower within zone 1 (Gini of 0.38) than within zone 2 (Gini of 0.45).

Wealth The model makes predictions for average wealth, the distribution of wealth across households, as well as how that wealth is spatially distributed. Average wealth to average total income (y^{tot}) in the metro area is 5.95. Wealth inequality is high, with a wealth Gini coefficient of 0.79. Both are close to the data by virtue of the calibration.

The middle panel of Figure 3 shows household wealth over the life cycle at the same *income* percentiles as in the top panel. It shows that the model endogenously generates substantial wealth accumulation for the average New York resident as well as a large amount of wealth inequality between income groups. Wealth inequality grows with age during the working phase.

4.2 Home Ownership, House Prices, and Rents

Next, we discuss the model's predictions for home ownership, house prices, and rents. The model manages to drive a large wedge between house prices, rents, and home ownership rates between zones 1 and 2 for realistic commuting costs.

Home Ownership The model generates a home ownership rate of 58.4%, fairly close to the 51.5% in the New York MSA. The bottom panel of Figure 3 plots the home ownership rate over the life-cycle. It displays a hump-shape over the life-cycle with variation across income groups. High-income households become home owners at a younger age than low-income households, achieve a higher ownership rate, and remain home owners for longer during retirement. These patterns are broadly consistent with the data.

Row 6 of Table 2 shows that the observed home ownership rate in Manhattan, at 23.1%, is far below that in the rest of the metro area, at 54.9%. The ratio of these two numbers is 0.42. The model generates a home ownership rate of 45.1% for Manhattan, which is substantially lower than the predicted 59.6% for zone 2. While its prediction for home ownership in zone 2, where 89.5% of the population lives, is close to the data, the model fails to generate the very low home ownership rate in Manhattan. The insufficiently large wedge in home ownership rates across zones is connected to the insufficiently large wedge in

---

[28]The Gini in the data is calculated by fitting a log-normal distribution to the mean and median of earnings.

Figure 3: Income, Wealth, and Home Ownership over the Life-Cycle
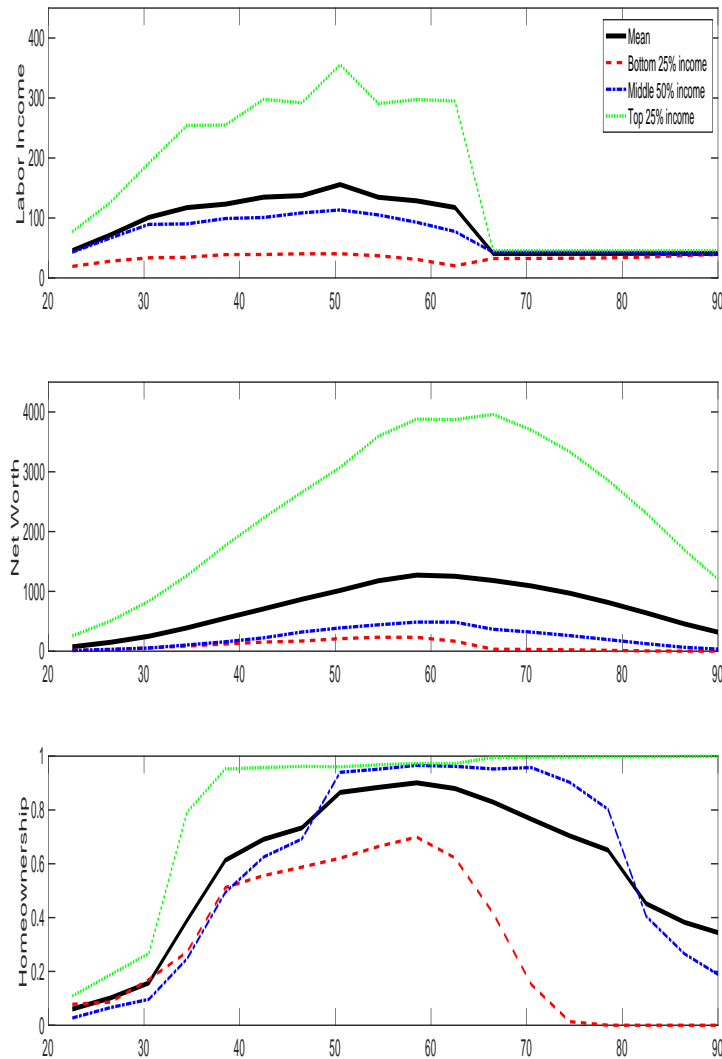
the price-rent ratios across zones. Intuitively, since owning is only somewhat more expensive than renting in zone 1 relative to zone 2, the home ownership rate in zone 1 is only somewhat lower than in zone 2. We discuss the price-rent ratio wedge further below.

Market Prices and Rents   Row 7 of Table 2 shows the median price per housing unit, row 8 the median price per square foot, row 9 the median rent per unit, and row 10 the median rent per square foot. For the data, we use the Zillow home value index (ZHVI) to measure

the median price of owner-occupied units, the Zillow median home value per square foot, and the Zillow rental index (ZRI) for the median rent per unit. These indices are available for each county in the New York metro, and we use the year-end 2015 values.[29] The ratio of the ZHVI to the ZRI in a county, is the price-rent ratio, reported in row 11 of the table.

The median house value in the NY metro area is \$510,051 in the data compared to \$506420 in the model.[30] The median is \$1.3 million in Manhattan and \$417,000 outside Manhattan in the data, a ratio of 3.11. This 3.11 house value ratio is the product of a house size ratio of 0.59 and a price per sqft ratio of 5.24. The model generates a ratio of prices per unit of 2.34, the product of a house size ratio of 0.64 and a price per sqft ratio of 3.57.

The data indicate a monthly rent on a typical market-rate unit of \$2,390 per month in the metro area. The model predicts \$2491. The ratio of rents per square foot in zone 1 to zone 2 is 2.78 in the data and matches the 2.77 in the model by virtue of the calibration. The ratio of rents per unit in zone 1 to zone 2 is 1.65 in the data, with a somewhat higher value of 1.82 obtained by the model.

The proximity to jobs and amenities is the reason why the model generates higher demand for Manhattan housing. Because of the highly inelastic housing supply in Manhattan, this translates into higher house prices and higher rents.

The model also comes close to matching the metro-wide price/rent ratio level of 17.79 (row 11). A simple user cost model would imply a steady state 4-year price-rent ratio of $(1 - Q \times (1 - \delta - \tau^P))^{-1}$. Plugging in for Q and the zone-specific property tax and depreciation parameters, the user cost formula generates price-rent ratios of 20.8 for zone 1 and 16.2 for zone 2. The price-rent ratios in the model are well approximated by the user cost formula. In the data, the price-rent ratio in Manhattan is 29.3, or 1.89 times the 15.5 value in zone 2. In the model that ratio is 1.29. In other words, the property tax and depreciation wedges generate too little spatial variation in price-to-rent ratios. Several factors outside of the model would help bridge the gap. First, houses in Manhattan may be less risky than in zone 2 which would increase the price-rent ratio wedge.[31] Second,

---

[29]Zillow excludes non-arms' length transactions and rent-controlled rentals. To aggregate across the 24 counties in zone 2, we calculate the median price as the weighted average of the median prices in each county, where the weights are the shares of housing units. Similarly, for the median rent of zone 2, we average median rents of the 24 counties using housing unit shares as weights. Zillow uses a machine-learning algorithm that ensures that the ZHVI and ZRI pertain to the same, typical, constant-quality unit, in a particular geography.

[30]To ensure consistency with the empirical procedure, we calculate the median house size in each zone including both owner- and renter-occupied units (but excluding RC units) in the model. Call these $\bar{h}^\ell$. We form the median price per unit as the product of the market price times the typical unit size $P^\ell \bar{h}^\ell$. The market rent is $R^\ell \bar{h}^\ell$. The price-rent ratio is simply $P^\ell \bar{h}^\ell / R^\ell \bar{h}^\ell = P^\ell / R^\ell$. To form metro-wide averages, we use the number housing units in each zone as weights, just like in the data.

[31]The model has no aggregate risk and risk aversion is only 5. To generate meaningful variation in hous-

32

owner-occupied housing in Manhattan may be of higher quality than in zone 2. The lower depreciation rate in zone 1 than in zone 2 may not fully capture such quality differences. Third, price-inelastic out-of-town investors may well be pushing up relative prices since they are disproportionately active in Manhattan (Favilukis and Van Nieuwerburgh, 2018). Fourth, the higher price of a Manhattan apartment may partly stem from its value as a shared/part-time rental via platforms such as AirBnB.

4.3 Housing Affordability

Price-Income and Rent-Income Row 12 of Table 2 reports the ratio of the median value of owner-occupied housing to average earnings in each zone. Average earnings are pre-tax and refer to all working-age residents in a zone, both owners and renters. The median home price to the average income is an often-used metric of housing affordability. In the NY metro data, the median owner-occupied house costs 3.99 times average income. Price-income is 6.7 in Manhattan compared to 3.6 outside Manhattan, a ratio of 1.87. The model generates a price-income ratio of 4.08 for the MSA, very close to the data. It generates a ratio across zones of 1.38. While generating a substantial spatial wedge in this housing affordability metric, the model's understatement is a direct consequence of not generating enough spatial variation in median house values, as noted above.

Row 13 reports average rent paid by market renters divided by average income of all residents in a zone; 23% in the data. This moment was the target for the housing preference parameter α^h. To get at the household-level rent burden, we compute two additional moments reported in rows 14-15 of Table 2. We use PUMS-level data from the American Community Survey for this analysis. The first statistic computes household-level rent to income ratio for renters with positive income, caps the ratios at 101%, and takes the average across households. For this calculation, income is earnings for working-age households and social security income for retirees. The observed average share of income spent on rent by renters is is 42.1% in the metro area; 35.4% in Manhattan and 43.5% in zone 2. The model generates an average rent-income ratio for renters of 29.1%, higher than the city-wide average among all residents of 23%, but lower than in the data.[32] The second statistic computes the fraction of renters with positive income whose rent is over 30% of income. These households are known as rent-burdened. In the data, 53.9%

---

ing risk premia would require, for example, changes in mortgage lending standards as in Favilukis et al. (2017). If homeowners in Manhattan are exposed less to changes in mortgage lending standards, housing in Manhattan would carry a lower risk premium. This would increase the price-rent ratio in Manhattan, and increase the wedge with the price-rent ratio in zone 2.

[32]Within the class of homothetic preferences over housing and non-housing consumption it is difficult to generate large deviations in the housing expenditure ratio without preference heterogeneity in α_h.

of households are rent-burdened; 44.1% in Manhattan and 55.9% in zone 2. The fraction of rent burdened households is 48.7% in the model, only slightly lower than in the data. Like in the data, there are fewer rent-burdened households in zone 1 than in zone 2. Taken together, the model generates a large "housing affordability crisis," with nearly half of renters spending more than 30% of their income on rent.

Affordable Housing   By virtue of the calibration, the model generates the right share of RC households in the population in each zone (row 16 of Table 2). Furthermore, the income qualification threshold κ_2 was chosen to match the observed position of RC tenants in the overall income distribution of the metro area. This ensures that the model generates the right amount of misallocation of RC in terms of income.

In Figure 4 we study the allocation of RC further and zoom in on the age dimension. It plots the fraction of households that are in RC for the bottom 25%, middle 50% and top 25% of the income distribution against age. Fractions are plotted for the model (left panel) and for data (right panel, from the New York City Housing and Vacancy Survey). In the model, some younger households in the middle of the income distribution (conditional on age) obtain RC, and even 3% of top-income young households are in RC. Their (endogenous) incomes are low enough to qualify. At later ages, those middle- and high-income households find staying in RC housing increasingly unattractive as they would like to consume more housing. This is not possible because of the constraint on RC housing size. Also middle- and high-income who were not yet in RC housing no longer satisfy the income requirement as their labor productivity grows with age. They would have to reduce labor supply too much to qualify, which is too costly. The share of RC that goes to the bottom-25% of the income and wealth distribution rises. Still, because of persistence in the RC system, a large fraction of middle-income households remains in the system for a very long time. In sum, the RC system generates a lot of misallocation.

Relative to the data, the model implies that the fraction of RC households decreases with age while it rises in the data. The elderly in RC in the data may still be the first generation of occupants and not yet reflect the steady state of the RC system, while the model describes the steady state.[33] Overall, the model captures much of the misallocation engendered by the RC system.

---

[33]RC regulations in New York have been weakened several times since 1993, resulting in a gradual loss of affordable housing units. Usually, incumbents are grand-fathered in and allowed to remain in place. The RC stock in the model is calibrated to the 2015 level. This naturally leads to more older households in RC. Also, the data may feature even stronger persistence in RC duration for incumbents than in the model, where RC tenants have a 25% chance of having to re-apply each 4-year period. Finally, retirees may prefer to age in place due to social ties (Cocco and Lopes, 2017) even if it is not in their best financial interest, a channel absent in our model.

Affordable housing acts as an insurance device in our incomplete markets model. We calculate the probability of getting a RC unit in the current period for a household that was not in a RC unit in the previous period and that suffered a negative productivity shock from the second to the first or from the third to the second productivity level. This probability measures *access to* the *insurance* that RC provides for middle- and low-income households who fall on hard times. If it is difficult for a low-income household to get into the RC system, then the value of that insurance is low. The access to insurance metric is 6.8% in the metro area. This breaks down into 1.6% for zone 1 and 5.2% for zone 2 RC housing. Including low-income households that already were in RC, the likelihood of getting RC housing is 14.3%. We also define the *stability* of the insurance RC offers as the probability of staying in a RC unit for a household that was in a RC unit in the previous period and that currently is in the bottom quartile of the income distribution. This probability is 72.5% in the baseline model. Risk averse households prefer a stable housing situation, i.e., a low volatility of changes in the marginal utility of housing. In a complete market, households can perfectly smooth consumption and marginal utility ratios are constant over time; their volatility is zero. Our benchmark model displays severe incompleteness with volatilities of 0.45 for both the marginal utility growth of non-housing consumption and housing consumption.[34]

Figure 4: Distribution of RC agents by age and income quartiles. Left panel: model. Right panel: data, calculated for the five New York City boroughs, from the New York City Housing and Vacancy Survey.

---
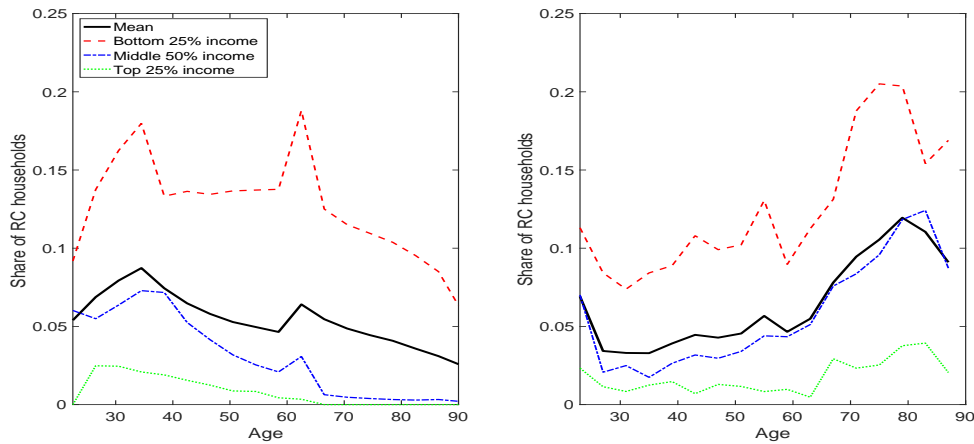
[34]The volatilities of marginal utility growth ignore the risk of being born as a low productivity household. The housing policies we study below play a role in insuring this risk.

5 Affordability Policies

Having developed a quantitatively plausible dynamic stochastic spatial equilibrium model of the New York housing market, we now turn to policy counterfactuals. We run 12 policy experiments. Six main ones are discussed in this section; the remaining six are relegated to Appendix G and summarized at the end of this section. Table 3 summarizes how key moments of the model change under the six main policy experiments. The first column reports the benchmark model, while the other columns report the percentage change in moments relative to the benchmark for each of the policy experiments. Figure 5 plots the associated welfare changes. Welfare changes are changes in value functions, expressed in consumption equivalent units; see equation (13). These are steady-state welfare comparisons. Appendix I discusses welfare along the transition path. The six policies have in common that they are all revenue neutral.

5.1 Improving the Targeting of RC

As shown above, RC in the benchmark model suffers from misallocation. The first three policies we consider aim to improve the allocation of a given amount (square footage) of affordable housing by better targeting it on the most needy households.

5.1.1 Tightening the Income Qualification Requirement

The first experiment, reported in column 1 of Table 3, lowers the income qualification threshold. Specifically, we lower κ_2 from 40% to 30% of AMI. This policy is strongly beneficial, generating an aggregate welfare gain of 1.17% (row 31). The policy is successful at allocating affordable housing units to low-income households. There is a 28.97% increase in the fraction of Q1-income households in RC (row 4), which exceeds the overall increase in the fraction of households in RC of 16.93% (row 3). Because the households in RC choose smaller apartment units (rows 6 & 7), the RC system can accommodate more households in the same square feet of affordable housing space.

The large welfare gain arises because RC becomes a better insurance device. Rows 27-30 of Table 3 provide several metrics that capture insurance provision. Row 27 reports that lowering the income qualification threshold greatly improves *access to insurance* for lower-income households who have fallen on hard times (54.35%). Row 28 reports that lowering κ_2 leaves the *stability of insurance* nearly unaffected (0.23%) from the benchmark likelihood of 72.5%. Rows 29 and 30 report the time-series standard deviation of marginal utility growth of non-housing and housing consumption, averaged across

36

Table 3: Main moments of the model under affordability policies that modify features of the RC system and the spatial allocation of housing.

		Benchm.	(1) Low κ_2	(2) Re-qualify	(3) Low κ_2 +re-qual.	(4) RC share	(5) Zoning Z1	(6) Vouchers
1	Avg(rent/inc.) among renters, Z1 (%)	28.3	6.89%	11.36%	20.18%	8.75%	0.47%	-1.12%
2	Avg(rent/inc.) among renters, Z2 (%)	29.2	-1.27%	-1.12%	-3.77%	-0.69%	0.03%	0.00%
3	Fraction of hhs in RC (%)	5.98	16.93%	30.98%	45.46%	46.17%	0.93%	-2.08%
4	Frac. in RC of those in inc. Q1 (%)	14.24	28.97%	70.88%	86.56%	43.32%	0.58%	-2.20%
5	Frac. rent-burdened (%)	48.7	-0.62%	0.73%	-5.40%	8.32%	-0.76%	-1.93%
6	Avg. size of RC unit in Z1 (sqft)	683	-9.31%	-16.66%	-18.87%	-0.29%	0.04%	-0.11%
7	Avg. size of RC unit in Z2 (sqft)	1173	-21.34%	-37.19%	-54.50%	0.15%	0.76%	0.95%
8	Avg. size of a Z1 mkt unit (sqft)	999	0.25%	0.13%	0.58%	-1.71%	-0.48%	2.95%
9	Avg. size of a Z2 mkt unit (sqft)	1521	1.32%	2.38%	3.84%	1.40%	0.40%	-1.37%
10	Frac. of population living in Z1 (%)	10.5	0.84%	2.25%	2.05%	3.96%	9.28%	-3.22%
11	Frac. of retirees living in Z1 (%)	21.5	-1.93%	-4.44%	-0.19%	-9.11%	-1.05%	9.92%
12	Housing stock in Z1	–	-0.06%	0.13%	0.08%	-0.35%	8.86%	-0.22%
13	Housing stock in Z2	–	-0.04%	-0.30%	-0.35%	0.18%	-0.73%	-0.91%
14	Rent/sqft Z1 ($)	4.21	-0.07%	-0.16%	-0.28%	1.19%	-0.70%	-0.67%
15	Rent/sqft Z2 ($)	1.52	-0.09%	-0.21%	-0.35%	1.40%	-0.86%	-0.71%
16	Price/sqft Z1 ($)	1050	-0.07%	-0.17%	-0.29%	1.18%	-0.71%	-0.66%
17	Price/sqft Z2 ($)	294	-0.09%	-0.21%	-0.35%	1.40%	-0.86%	-0.71%
18	Home ownership rate in Z1 (%)	45.1	-0.07%	-1.37%	-0.95%	-10.44%	-2.40%	6.23%
19	Home ownership rate in Z2 (%)	59.6	-0.34%	-0.77%	-2.33%	-1.72%	0.30%	0.28%
20	Avg. inc. Z1 among working-age hhs ($)	164422	-1.52%	-3.49%	-3.98%	-4.66%	-3.65%	-2.46%
21	Avg. inc. Z2 among working-age hhs ($)	100154	0.24%	0.23%	0.52%	0.53%	0.05%	-0.14%
22	Frac. of top-productivity hhs in Z1 (%)	29.7	-0.24%	0.52%	-0.04%	0.97%	-0.49%	-8.08%
23	Total hours worked in economy	–	-0.13%	-0.98%	-0.99%	-0.21%	-0.00%	-1.07%
24	Total hours worked in efficiency units	–	0.05%	-0.33%	-0.08%	-0.11%	-0.01%	-1.02%
25	Total output	–	0.02%	-0.22%	-0.06%	-0.05%	-0.03%	-0.67%
26	Total commuting time across all hhs	–	-0.14%	-0.36%	-0.25%	-0.66%	-1.17%	0.57%
27	Access to RC insurance (%)	5.6	54.35%	145.90%	185.91%	52.21%	-0.24%	-3.18%
28	Stability of RC insurance (%)	72.5	0.23%	-78.20%	-31.07%	0.19%	-0.04%	0.20%
29	Std. MU growth, nondurables	0.45	-0.14%	-0.91%	-2.28%	0.62%	-0.11%	-1.29%
30	Std. MU growth, housing	0.45	-1.03%	0.08%	-5.02%	0.58%	3.04%	-2.50%
31	**Aggregate welfare change (CEV)**	–	**1.17%**	**1.66%**	**3.59%**	**0.66%**	**0.37%**	**1.04%**

Notes: Column "Benchmark" reports values of the moments for the benchmark model. Columns "Low κ_2" to "Vouchers" report percentage changes of the moments in the policy experiments relative to the benchmark for the six main policy experiments. Rows 1-9 report housing affordability moments , rows 10-26 aggregate moments across the two zones, rows 27-30 capture insurance provision, and row 31 is the percentage aggregate welfare change, in CEV units, between the alternative and the benchmark economy. Z1 stands for zone 1 (Manhattan), Z2 for the rest of the metro area.

households. The policy lowers the volatility of the marginal utility growth of housing (-1.03%), thereby offering households more housing stability, bringing the economy closer to complete markets.

The policy also creates welfare gains from a reduction in commuting time (-0.14%, row 26). Commuting in the model wastes time, which can now be spent on leisure or work, and money, which can now be spent on consumption. The reduction in commuting reflects the larger population share of the city center (0.84%, row 10) due to the larger number of households living in smaller zone-1 RC units. Many of the new zone-1 households are of working age (the share of retirees in zone 1 falls; -1.93%, row 11). The improving

spatial allocation of labor can also be seen from the wedge between the change in the number of hours worked (-0.13%, row 23) and the change in the number of hours worked in efficiency units (0.05%, row 24). Total output increases modestly (0.02%, row 25) as does leisure.

This policy does not affect developer distortions, and has minor implications for the housing stock, rents, house prices, and home ownership rates in both zones (rows 11-19). Because there are fewer middle- and high-income households in RC, there are fewer households who are choosing sub-optimally small apartments. The average size of market units (renter- or owner-occupied) increases in both zones (rows 8 and 9). The higher housing consumption increases utility for these non-RC households.

Rent-income ratios among renters increase substantially in zone 1 (6.89%, row 1) and fall in zone 2 (-1.27%, row 2). These changes reflect the new socio-economic make-up of the two zones. There are more low-income households in zone 1 because of the policy change, so that the average income of zone 1 changes (-1.52%, row 20). The opposite is true in zone 2 (0.24%, row 21). This suggests that rent-income ratios, the most common metric of housing affordability, must be interpreted carefully as they reflect both equilibrium rents and the income of the people who have sorted into each zone in spatial equilibrium.

5.1.2 Re-qualifying for RC

A second approach to improve the targeting of RC is to force incumbent households to re-qualify periodically. In the benchmark model, RC tenants were allowed to stay in RC for another 4 years with very high probability rather than having to go through the RC lottery again. As we showed, this induces much persistence in the RC system and results in some households staying for decades. In the second policy experiment, we force every household to go through the lottery and re-qualify each period (four years). There are no more insiders or outsiders, everyone has equal probability of winning the housing lottery. By removing the preference for insiders, i.e., by setting the parameter $p^{RC,exog} = 0$, the endogenously determined probability of winning the RC lottery increases substantially.

The policy experiment results in a large average welfare gain of 1.66% per New Yorker (column 2 of Table 3). Like the previous policy, re-qualification improves access to RC insurance (145.90%, row 27). In contrast with the previous policy, it dramatically lowers the stability of this insurance (-78.20%, row 28). Housing consumption become more unstable over time (0.08%, row 30). The policy results in more households in RC (30.98%), choosing smaller units. The fraction of low-income households in RC grows much more

(70.88%), showing the improved targeting of the RC system. There is a major reduction in the misallocation of RC housing. The intuition for this result is as follows. When the income qualification threshold is fairly tight, as it is in our benchmark model ($\kappa_2 = 40\%$), there is only modest misallocation in RC housing at the time of first entry. But misallocation grows over time as incomes rise deterministically with age or stochastically due to positive productivity shocks. Making RC less persistent helps reduce the misallocation since it replaces high-income insiders with low-income outsiders. In contrast, when κ_2 is higher (e.g., 60% or more), re-qualification is much less effective since undeserving tenants are often replaced by other undeserving tenants.

The policy improves the spatial allocation of labor. The overall population of zone 1 grows (2.25%, row 10), the fraction of top-productivity, working-age households who live in zone 1 grows (0.52%, row 22), and the fraction of retirees falls (-4.44%, row 11). Labor supply falls, but much less in efficiency units. Closer inspection reveals that productivity type-2 households reduce hours the most and this reduction is concentrated in zone 1. Lower-productivity households have a strong financial incentive to live in zone 1, given that the fixed cost of commuting represents a large share of their income. If they live in zone 2, they must work harder to make up for the cost of commuting. If they live in zone 1, they reduce labor supply. This reduction in labor supply is larger in the re-qualify experiment than in the benchmark economy because (i) type-2 households now have a better chance of getting into RC housing in zone 1, (ii) rents are lower even if they do not get in, and (iii) there are more type-2 households living in zone 1 in the re-qualify experiment. In sum, the improved insurance value of RC combined with the spatial reallocation it causes, leads to a reduction in labor supply and output. The income panel of Figure 5 shows that second-income quartile households gain the most from the policy change.

5.1.3 Combining the Previous Two Policy Changes

Introducing re-qualification in an economy with tighter income qualification requirements such as those in Section 5.1.1 brings large welfare gains. Column 3 of Table 3 shows the results from combining the reduction in κ_2 and the re-qualification experiments. We find a large welfare gain of 3.59%. The gain is larger than the sum of the welfare gains from the separate experiments, suggesting amplification effects. Making the RC allocation fairer by forcing tenants to re-qualify pays large dividends when only the needy qualify in the first place. This policy produces larger welfare gains than any other policy we study. Yet, it requires no expansion of the scope of the RC program nor additional

taxes.

Figure 5 shows that households in the bottom quartile of the productivity and income distribution overwhelmingly gain from the policy while households in the top half lose. The gains are fairly uniform across age and monotonically declining in wealth.

Figure 5: Welfare effects of affordability policies by age, productivity groups, income quartiles, and net worth quartiles.
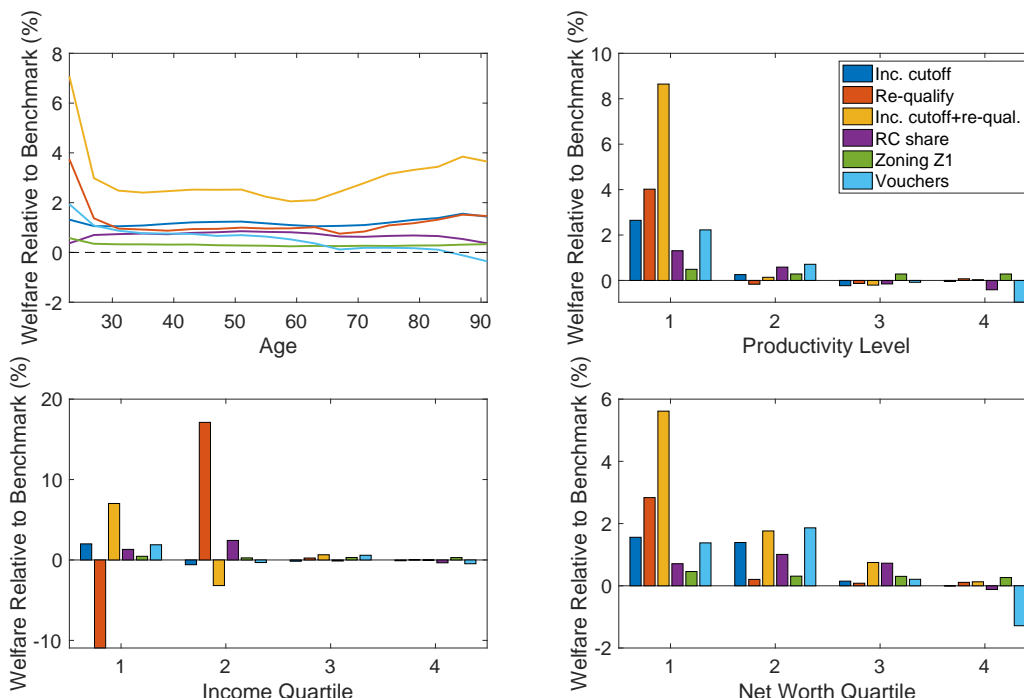
Notes: The baseline model has the following parameters: $\eta^1 = 24.46$, $\eta^2 = 15.97$, $\kappa_1 = 50\%$, $\kappa_2 = 0.40$, $\kappa_3 = 0.35$. Policy experiments, each panel: decrease the RC income cutoff by 50%, enforce re-qualification for RC every period, both, increase share of RC sqft by 50%, increase available Z1 sqft by 10%, housing vouchers. Top left panel: by age. Top right panel: by productivity level. Bottom left panel: by income quartile. Bottom right panel: by net worth quartile. The welfare changes are measured as consumption equivalent variations for an average household in each group.

5.2 Expanding the Affordable Housing Mandate

The second policy instrument we study is the scope of the affordable housing mandate. We symmetrically change η^ℓ in each zone, the fraction of rental square footage that must be set aside for affordable units. Column 4 of Table 3 increase η^ℓ by 50%. The fraction of *households* in RC increases by 46.17%. The fraction of households from the bottom 25% of the income distribution that obtains RC grows by less (43.32%), implying a slight rise in the misallocation of RC housing.

The average effect of increasing the size of the RC program is a welfare gain of 0.66% in CEV units. This modest welfare gain is the net effect of the increased insurance benefits

40

afforded by the expansion of the RC program and the increased costs of the housing and labor market distortions. Access to RC insurance increases substantially (52.21%), while the stability of the insurance is unaffected (0.19%). The increase in RC weakens developers' incentives to build since it lowers the average sale price they earn. Housing supply in zone 1 decreases (-0.35%). This is consistent with the empirical literature, which finds that increased incentives of landlords to renovate their properties and of developers to invest in new construction generate a modest housing boom in decontrolled areas (Autor et al., 2014; Diamond et al., 2019). Rents increase by 1.19% in zone 1 reflecting the increased housing scarcity. The population in zone 1 increases (3.96%) because zone 1 residents choose smaller housing units both in the market and RC rental segments.

Developer incentives are also blunted in zone 2. Nevertheless, the equilibrium housing stock in zone 2 increases by 0.18%. The reason is the increased demand for housing in zone 2. The larger housing units offset the decreased population share of zone 2. Rents increase by 1.40% in zone 2. Developers "pass through" the increased housing affordability targets into the market rent.

Increasing the affordable housing mandate city-wide triggers an increase in the city-wide housing stock because the increase in the housing stock in zone 2 exceeds the decline in zone 1. This surprising results arises from optimal relocation choices in response to the policy in spatial equilibrium.

Standard housing affordability metrics deteriorate, even though many more households are helped by cheap RC housing. Average rents increase in both zones. The rent-income ratio increases in zone 1 (8.75%). The fraction of rent-burdened renters increases by 8.32% metro-wide. These increases reflect that low- and middle-income households who do not win the RC lottery face higher rents. House prices also rise in both zones.

The policy results not only in a smaller but also in a *different* population in Manhattan. With the 46.17% growth in RC households, Manhattan becomes a more "mixed" neighborhood with lower average income (-4.66%). The reallocation of the housing stock towards affordable housing units pushes some middle- and upper-middle-income households out of the urban core. This pattern is consistent with the empirical evidence in Autor et al. (2014), who show that richer households moved into units previously occupied by poorer RC tenants after a reduction in control in Cambridge, MA.

Increasing the size of the RC program also distorts labor supply. Time spent working (-0.21%) and total efficiency units of labor (-0.11%) fall despite the reduction in time spent commuting. Expanding RC generates a drop in output (-0.05%). These are the distortions from RC emphasized by economists; we find that they are quantitatively modest, even though RC is highly persistent.

How does welfare change with the scope of RC? Figure 6 presents a sequence of experiments that vary η^ℓ. The blue line shows that aggregate welfare, expressed as a percentage change relative to the benchmark, increases with the scope of the affordable housing mandate. The insurance benefits from expanding the RC system continue to outweigh the costs until all households that want RC obtain such a unit.[35] It is worth pointing out that the experiment with a 50% increase in the amount of sqft in RC (the first point on the blue line to the right of the benchmark) already constitutes a large expansion. It implies that landlords lose $7.3 billion in forgone rent because of the RC rent discount. The voucher experiment below generates a larger welfare benefit but only costs $730 million.

Figure 6 also plots the result of the same changes in the RC system in a (recalibrated) economy featuring a lower value for risk aversion, $\gamma = 2$. The case for increasing the affordable housing mandate is qualitatively unaffected, but the welfare gains are less steeply increasing since the benefit of RC insurance is lower when risk aversion is lower. Appendix H.1 discusses the model with lower risk aversion in detail.

Figure 6: Welfare effects of varying the scope of the affordable housing mandate.
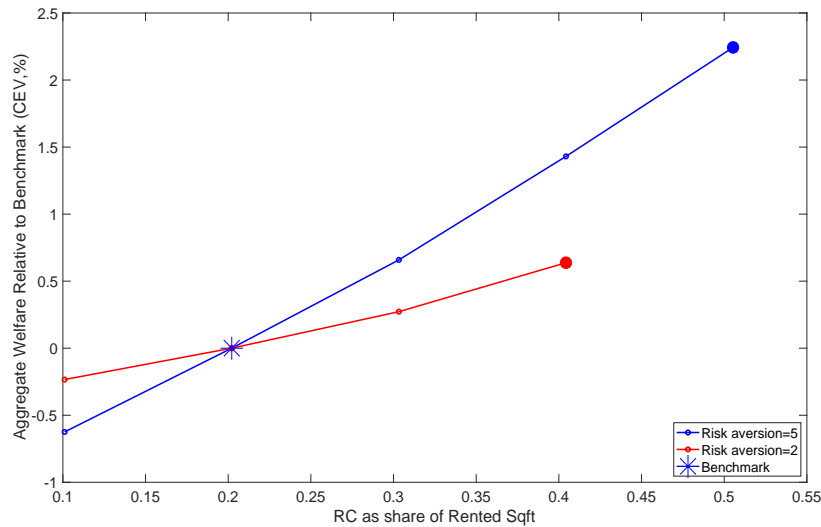
Notes: The baseline model has the following parameters: $\eta^1 = 24.46$, $\eta^2 = 15.97$, $\kappa_1 = 50\%$, $\kappa_2 = 0.40$, $\kappa_3 = 0.35$. The share of RC rented sqft is calculated as a (total sqft-)weighted average of η^1, η^2. The larger dots on the right-hand side of the graph represent the maximum RC share above which markets do not clear. The welfare changes are measured as consumption equivalent variations for an average household.

---

[35]Consistent with the result that increasing η^ℓ increases welfare, we find that an alternative calibration that includes rent-stabilized apartments in the RC sector and excludes them from the market rental sector (the opposite assumption as in the benchmark), and correspondingly adjusts the rental discount to 30% (from 50% in the benchmark) results in a 0.23% higher CEV welfare level than the benchmark model. In other words, this modeling choice makes little quantitative difference.

5.3 Relaxing Zoning Rules in the Urban Core

The fifth experiment studies a favorite policy among economists: allowing for more housing in the city center. We think of this policy as relaxing height or other land use restrictions (increasing allowable floor-area ratios), often referred to as upzoning. In the model, more construction is permitted in zone 1 when we increase $\overline{H}^1$. As in the benchmark model, for every square foot of rental housing built in zone 1, a fraction η^1 must be affordable. This policy is akin to NYC's policy of awarding developers zoning variances in exchange for affordable units. This policy has no direct fiscal outlay.[36] However, its affects all equilibrium prices and quantities indirectly.

We increase $\overline{H}^1$ by 10%. The equilibrium housing stock in Manhattan increases by 8.86%, as shown in the column 5 of Table 3. Rents and prices in zone 1 fall (-0.70% for rents). The average unit size changes by -0.40% so that the population share of Manhattan rises by 9.28%. Because of the population reallocation, average income in Manhattan decreases (-3.65%). More middle-income households can now afford Manhattan and the additional affordable housing that is associated with the new construction also brings in lower-income households. With more working-age households in zone 1, aggregate commuting time falls substantially (-1.17%). In equilibrium, most of the aggregate time saved commuting goes towards leisure. The increase in leisure boosts utility.

The housing stock in zone 2 falls (-0.73%) as developers shift their resources towards zone 1 where the population has swelled. Rents in zone 2 also fall (-0.86%) because of the weaker demand for housing in zone 2 due to the population pivot to Manhattan.

Housing affordability metrics deteriorate slightly. Average rent-to-income among renters increases by 0.47% in zone 1 and by 0.03% in zone 2. These metrics fail to capture that more households can now afford to live close to work. They mostly reflect compositional changes in the income profile of each neighborhood.

The zoning experiment leads to a small decrease in output (-0.03%). Output in the Manhattan construction sector increases substantially (8.88%), but is offset by an output loss in the much larger construction sector of zone 2 (-0.76%).

Relaxing zoning rules in zone 1 generates a modest aggregate welfare gain of 0.37%. This aggregate gain is smaller than in the previous experiments. In contrast to the preceding policies, upzoning is much less redistributive in nature. As can be seen in Fig-

---

[36]The model ignores additional costs or benefits from increased density in the city center. There could be additional utility gains from having more households live in the city center (productivity gains from agglomeration are already maximal given that everyone works in the city center), or additional utility losses from congestion. The model can be extended to incorporate such considerations. Specifically, the parameter χ^1 could be allowed to depend on the endogenous density of zone 1.

ure [5], the upzoning policy brings uniform benefits to all age, productivity, income, and wealth groups. The steady state comparison suggests a Pareto improvement. Along the transition path, some home owners are hurt by the house price declines that accompany upzoning. However, Appendix I shows that the average home owner still benefits from the policy change even along the transition path.

5.4 Vouchers

One important pillar of U.S. housing policy is the Section 8 voucher program, housing assistance provided by the federal government to low-income households. Since this policy is part of the tax and transfer system, it is already captured by the function $T(\cdot)$. We now study the effects of increasing the size of the voucher program.[37] We engineer the increase by increasing the tax progressivity parameter τ from 0.1700 to 0.1783 while changing the overall size of the tax and transfer system from λ of 0.7500 to 0.7440. Because of the increased progressivity, the experiment generates both higher tax revenues and higher transfers. The policy translates into a $738 million increase in spending on vouchers in the New York metropolitan area.[38] As in the previous experiments, there is no direct fiscal outlay associated with the voucher expansion. We impose that all households who receive an additional transfer spend at least the amount of the additional transfer on housing. This captures the institutional reality that vouchers must be used for housing expenses. The RC system remains in place with the benchmark parameter values.

The last column of Table [3] shows that aggregate welfare increases substantially when the housing voucher program is expanded (1.04%). Figure [5] shows that young, low-productivity, low-income, and low-wealth households gain substantially from the policy. The substantial reduction in the volatility of marginal utility growth (-1.29% for non-housing and -2.50% for housing) is consistent with the improved provision of insurance in society. These results are directionally consistent with the RC expansion experiments

---

[37]We increase the value of housing vouchers by an amount equal to the value of Low Income Housing Tax Credit (LIHTC) subsidies, to make the two experiments comparable quantitatively. LIHTC subsidies are designed to cover approximately 30% of developers' construction costs associated with affordable housing (see appendix G.6).

[38]Data compiled from the Housing and Urban Development department show that the housing authorities responsible for the 25 counties in the New York MSA disbursed $2.06 billion in 246,000 Section 8 vouchers in the year 2013 (latest available). This amounts to an average of $8,300 per year per voucher. The policy exercise we consider raises tax revenues from $5.434 billion to $6.215 billion, an increase of $738 million. At the cost of $8,300 per voucher, this translates into 94,100 additional vouchers. The experiment increases the size of the transfer of those who already received transfers before. The transfer increase is higher the lower is household income. It also creates new beneficiaries who now receive a transfer while they were paying a tax in the benchmark. Thus, the policy both increases the amount of the existing vouchers and disburses additional smaller vouchers.

in Section 5.2, but the redistribution in the voucher experiment is starker. This is because the vouchers are better targeted to the poor. Because RC is not as well targeted in the baseline model as it could be (as shown in Section 5.1), the RC expansion generates much smaller welfare increases dollar-for-dollar.

The large welfare gain in the voucher experiment occurs despite severe distortions. Chief among them is tax-induced labor supply distortions. In the model, as in the real world, vouchers must be paid for with distortionary labor income taxes. Because of tax progressivity, high-productivity households are disproportionately affected. Total hours worked fall sharply (-1.07% and -1.02% in efficiency units) and total output falls by a sizeable -0.67%.

The voucher expansion, which is location-neutral in its design, has interesting spatial equilibrium effects. The policy leads to a reduction in the population share of zone 1 (-3.22%) and an increase in commuting (0.57%). It also reduces the average income of zone 1 (-2.46%) and the fraction of top-productivity households who live there (-8.08%). In sum, the voucher expansion has unintended consequences for the efficiency of the labor allocation because of how it is financed. In addition, low income households who receive a larger housing voucher are not more likely to live in zone 1, close to "opportunity," in spatial equilibrium. This is consistent with the empirical evidence in Collinson and Ganong (2018) that vouchers not "move" lower-income households "to opportunity." But, surprisingly, they even crowd out higher productivity households from the city center, "removing" them "from opportunity."

Finally, a change to one housing affordability program may affect the benefits from other programs. Increasing vouchers reduces the number of households benefitting from RC because RC tenants choose larger RC units on average (across zones 1 and 2).

5.5   Other Affordable Housing Policies

Appendix G discusses six more policies that change the various policy levers of the RC system. We start with the opposite policy of that in section 5.2: a reduction in the size of the RC system by half. We find a welfare loss (-0.62%), similar to the welfare gain from expanding the RC system by the same percentage.

Second, we lower the rent subsidy, governed by κ_1. The policy reduces welfare by a similar amount (-0.49%). It results in more households gaining access to RC but the value of the insurance provided by the RC system falls because the rent discount is smaller and RC households live in much smaller RC units.

Third, reducing the size limit on RC units, governed by κ_3, increases welfare modestly

(0.23%). The policy improves the targeting of RC housing towards the neediest households, but is not as effective as tightening the income requirement or re-qualification, discussed in Section 5.1.

Fourth, we explore the spatial dimension of the affordable housing system and conduct a policy experiment that shifts all RC households from zone 1 to zone 2. The urban core gentrifies with more high-income and top-productivity households, fewer retirees, and a higher home ownership rate. Despite the improvements in the spatial allocation of labor, aggregate welfare does not increase (-0.03%). Relocation of RC housing to zone 2 is welfare increasing once accompanied by free transportation. The aggregate welfare gain of this fifth experiment is 0.37%, a substantial increase from the -0.03% aggregate welfare without free transit. The reason is that financial transportation costs are important for low-income households and were the source of the welfare loss in the fourth experiment[39].

Finally, appendix G.6 studies a policy that subsidizes construction costs associated with affordable housing development. This tax credit policy, modeled after the federal Low Income Housing Tax Credit (LIHTC) program, increases the average price $\overline{P}^{\ell}$ developers earn, thereby stimulating new construction. The policy is sized to to have the same cost of the voucher expansion experiment. It generates a small welfare gain of 0.02%. The envisioned increase in the housing stock materializes in equilibrium, but is blunted by a reduction in housing demand. The latter arises from the distortionary taxation imposed to pay for the tax credits. Tax credits in difficult to develop areas, like New York City, create too few additional affordable housing units to make a meaningful dent in the welfare of low-income households. This experiment underscores the importance of general equilibrium effect, of targeted policies, and of how affordability programs are financed.

6   Conclusion

In a world with rising urbanization rates, the high cost of housing has surfaced as a daunting challenge. Existing affordable housing policy tools affect the supply of housing, how the housing stock is used (owned, rented, affordable), and how it is distributed in space. Households of different tenure status, age, income, and wealth are differentially affected by changes in policy. This paper develops a novel dynamic stochastic spatial equilibrium model with wealth effects and rich household heterogeneity that allows us to quantify the welfare implications of the main housing affordability policy tools.

---

[39]The 0.37% welfare gain takes into account the distortions caused by the extra tax revenue needed to pay for the free transit for RC households.

The model is calibrated to the New York metropolitan area. It matches patterns of average earnings, wealth accumulation, and home ownership over the life-cycle, delivers realistic house prices, rents, and wages, as well as large spatial differences in income and rents between the urban core and the periphery. The calibration captures the key features of New York's affordable housing system as well as restrictions on residential land use.

We use the model to evaluate changes to the affordable housing system, zoning policy, an expansion of the housing voucher system, and tax credits for the development of affordable housing. These policies have quantitatively important aggregate, distributional, and spatial implications. General equilibrium effects are sometimes at odds with partial equilibrium logic.

Consistent with conventional wisdom, increasing the housing stock in the urban core by relaxing zoning regulations is welfare improving. Contrary to conventional wisdom, increasing the generosity of the affordable housing or housing voucher systems is also welfare improving. The main reason is that housing affordability policies generate important insurance benefits which trade off against the larger housing and labor market distortions. Increasing the housing safety net for the poorest households creates welfare gains for society. How the affordability policies are financed has first-order effects on welfare gains. Finally, the insurance view of affordability points towards large advantages from better targeting of RC housing towards the neediest households.

These results underscore the need for rich models of household heterogeneity to understand both the aggregate and the distributional implications of place-based policies. In future work, we plan to use this framework to analyze investment in transportation infrastructure which lowers the time and/or financial cost of commuting. Applying this framework to study other cities with different institutional features is another useful direction. Finally, embedding the model in a multi-city framework is a challenging but fruitful extension for future work.

References

AUTOR, D. H., C. J. PALMER, AND P. A. PATHAK (2014): "Housing Market Spillovers: Evidence from the End of Rent Control in Cambridge, Massachusetts," *Journal of Political Economy*, 122, 661–717.

———— (2017): "Gentrification and the Amioneity Value of Crime Reductions: Evidence from Rent Deregulation," .

BAUM-SNOW, N. AND J. MARION (2009): "The Effects of Low Income Housing Tax Credit Developments on Neighborhoods," *Journal of Public Economics*, 93, 654–666.

BEHRENS, K., G. DURANTON, AND F. ROBERT-NICOUD (2014): "Productive Cities: Sorting, Selection, and Agglomeration," *Journal of Political Economy*, 122, 507–553.

BERGER, D., V. GUERRIERI, G. LORENZONI, AND J. VAVRA (2017): "House prices and consumer spending," *Review of Economic Studies*, 85, 1502–1542.

BRUECKNER, J. (1987): *Handbook of Regional and Urban Economics*, Amsterdam: North-Holland, chap. The Structure of Urban Equilibria: A Unified Treatment of the Muth-Mills Model.

CAMPBELL, J. Y. AND J. COCCO (2003): "Household Risk Management and Optimal Mortgage Choice," *Quarterly Journal of Economics*, 118, 1449–1494.

COCCO, J. AND P. LOPES (2017): "Reverse Mortgage Design," *Working Paper London Business School and London School of Economics*.

COCCO, J. F. (2005): "Portfolio Choice in the Presence of Housing," *Review of Financial Studies*, 18, 535–567.

COLLINSON, R. AND P. GANONG (2018): "How Do Changes in Housing Voucher Design Affect Rent and Neighborhood Quality?" *American Economic Journal: Economic Policy*, 10, 62–89.

COUTURE, V., C. GAUBERT, J. HANDBURY, AND E. HURST (2018): "Income Growth and the Distributional Effects of Urban Spatial Sorting," Working Paper University of Chicago Booth.

DAVIS, M. A., J. GREGORY, AND D. A. HARTLEY (2018): "The Long Run Effects of Low Income Housing on Neighborhood Composition," .

DAVIS, M. A., J. GREGORY, D. A. HARTLEY, AND K. T. K. TAN (2017): "Neighborhood Choices, Neighborhood Effects and Housing Vouchers," .

DAVIS, M. A. AND S. VAN NIEUWERBURGH (2015): *Handbook of Regional and Urban Economics*, North Holland, chap. Housing, Finance, and the Macroeconomy, Chapter 12.

DIAMOND, P. AND E. SAEZ (2011): "The Case for a Progressive Tax: From Basic Research to Policy Recommendations," *Journal of Economic Perspectives*, 25, 165–190.

DIAMOND, R. AND T. MCQUADE (2019): "Who Wants Afordable Housing in their Backyard? An Equilibrium Analysis of Low Income Property Development," *Journal of Political Economy*.

DIAMOND, R., T. MCQUADE, AND F. QIAN (2019): "The Effects of Rent Control Expansion on Tenants, Landlords, and Inequality: Evidence from San Francisco," .

EECKHOUT, J., R. PINHEIRO, AND K. SCHMIDHEINY (2014): "Spatial Sorting," *Journal of Political Economy*, 122, 554–620.

FAVILUKIS, J., S. C. LUDVIGSON, AND S. VAN NIEUWERBURGH (2017): "The Macroeconomic Effects of Housing Wealth, Housing Finance and Limited Risk Sharing in General Equilibrium," *Journal of Political Economy*, 125, 140–222.

FAVILUKIS, J. AND S. VAN NIEUWERBURGH (2018): "Out-of-town Investors and City Welfare," Working Paper Columbia GSB and UBC Sauder.

FREEDMAN, M. AND E. OWENS (2011): "Low-income Housing Development and Crime," *Journal of Urban Economics*, 70, 115–131.

GOMES, F. AND A. MICHAELIDES (2008): "Asset Pricing with Limited Risk Sharing and Heterogenous Agents," *Review of Financial Studies*, 21 (1), 415–449.

GUERRIERI, V., D. HARTLEY, AND E. HURST (2013): "Endogenous gentrification and housing price dynamics," *Journal of Public Economics*, 100, 45–60.

GUREN, A. AND T. MCQUADE (2019): "How Do Foreclosures Exacerbate Housing Downturns?" *Review of Economic Studies*.

HEATHCOTE, J., K. STORESLETTEN, AND G. L. VIOLANTE (2017): "Optimal Tax Progressivity: An Analytical Framework," *Quarterly Journal of Economics*, 132, 1693–1754.

HIZMO, A. (2015): "Risk in Housing Markets: An Equilibrium Approach," Working Paper New York University.

HSIEH, C.-T. AND E. MORETTI (2019): "Housing Constraints and Spatial Misallocation," *American Economic Journal: Macroeconomics*, 11.

IMROHOROGLU, A., K. MATOBA, AND S. TUZEL (2016): "Proposition 13: An Equilibrium Analysis," Working Paper University of Southern California.

JOINT CENTER FOR HOUSING STUDIES OF HARVARD UNIVERSITY (2018): "The State of the Nation's Housing 2018," .

KAPLAN, G., K. MITMAN, AND G. VIOLANTE (2019): "Consumption and House Prices in the Great Recession: Model meets Evidence," Working Paper New York University.

KRUSELL, P. AND A. A. SMITH (1998): "Income and Wealth Heterogeneity in the Macroeconomy," *Journal of Political Economy*, 106, 867–896.

LANDVOIGT, T., M. PIAZZESI, AND M. SCHNEIDER (2015): "The Housing Market(s) of San Diego," *American Economic Review*, 105, 1371–1407.

LUQUE, J., N. IKROMOV, AND W. NOSEWORTHY (2019): *Affordable Housing Development: Financial Feasibility, Tax Increment Financing, and Tax Credits*, Springer.

ORTALO-MAGNÉ, F. AND A. PRAT (2016): "Spatial Asset Pricing: A First Step," *Economica*, 83, 130–171.

RAPPAPORT, J. (2014): "Monocentric City Redux," Working Paper Federal reserve Bank of Kansas City.

ROBACK, J. (1982): "Wages, rents, and the quality of life," *The Journal of Political Economy*, 90, 1257.

ROSEN, S. (1979): "Wage-based indexes of urban quality of life," *Current issues in urban economics*, 3.

ROSSI-HANSBERG, E., P. D. SARTE, AND R. OWENS (2010): "Housing Externalities," *Journal of Political Economy*, 118, 485–535.

SAIZ, A. (2010): "The Geographic Determinants of Housing Supply," *Quarterly Journal of Economics*, 1253–1296.

STORESLETTEN, K., C. TELMER, AND A. YARON (2006): "Asset Pricing with Idiosyncratic Risk and Overlapping Generations," *Review of Economic Dynamics*, Forthcoming, working Paper, University of Oslo.

VAN NIEUWERBURGH, S. AND P.-O. WEILL (2010): "Why Has House Price Dispersion Gone Up?" *Review of Economic Studies*, 77, 1567–1606.

YAO, R. AND H. H. ZHANG (2004): "Optimal Consumption and Portfolio Choices with Risky Housing and Borrowing Constraint," *Review of Financial Studies*.

A Model Appendix

A.1 Analytical solution for housing and labor supply choices

Preferences over non-durables, housing, and leisure are represented by a CES aggregator of non-durable consumption and housing services, nested within a CES aggregator of consumption and leisure. Those are embedded within a CRRA utility function, which represents preferences over different consumption profiles over time and across states of the world. We only solve the worker's problem here. A retiree's problem is analogous, but simpler because $n_t = 0$.

The utility function is $U(c, h, n) = \frac{\mathcal{C}(c,h,n)^{1-\gamma}}{1-\gamma}$, where

$$\mathcal{C}(c,h,n) = \left[(1-\alpha_N) \left[(1-\alpha_H)c^\epsilon + \alpha_H h^\epsilon \right]^{\frac{\eta}{\epsilon}} + \alpha_N \left[1 - \Phi_T - n \right]^{\chi_0 \eta} \right]^{\frac{1}{\eta}}$$

when $|\eta|, |\epsilon| > 0$ (case (i)). χ_0 makes leisure nonlinear in hours (we set it to 1), and we impose a lower bound on hours, $n \geq n_{min}$.

When $|\eta| > 0, |\epsilon| = 0$, $u(c, h, n) = \left[(1-\alpha_N) \left[c^{1-\alpha_H} h^{\alpha_H} \right]^\eta + \alpha_N \left[1 - \Phi_T - n \right]^{\chi_0 \eta} \right]^{\frac{1}{\eta}}$ (case (ii)).

When $|\eta| = 0, |\epsilon| > 0$, $u(c, h, n) = \left[(1-\alpha_H)c^\epsilon + \alpha_H h^\epsilon \right]^{\frac{(1-\alpha_N)}{\epsilon}} \left[1 - \Phi_T - n \right]^{\chi_0 \alpha_N}$ (case (iii)).

When $|\eta| = |\epsilon| = 0$, $u(c, h, n) = \left[c^{1-\alpha_H} h^{\alpha_H} \right]^{1-\alpha_N} \left[1 - \Phi_T - n \right]^{\chi_0 \alpha_N}$ (case (iv)).

Renter   First, consider the renter's problem and let λ_t be the Lagrange multiplier on the budget constraint, ν_t be the Lagrange multiplier on the borrowing constraint, and ξ_t be the Lagrange multiplier on the non-negativity labor constraint. The numerical strategy is to choose c_t in order to maximize the household's utility, and l_t to solve the non-linear equation for labor supply. Here we will show that the other choices (h_t and b_{t+1}) can be written as analytic functions of c_t and n_t. Denote $\mathcal{C}_t = \mathcal{C}(c_t, h_t)$. We ignore the taste shifter (which is multiplicative and raised to the power $1 - \gamma$ in the equations involving $\mathcal{C}_t$), and assume $\widehat{b}_{t+1} = 0$. The budget constraint simplifies to:

$$c_t + R_t^\ell h_t + Q_t b_{t+1} + \phi_{F,t}^\ell = \overline{\Psi}_t \psi^z + \left(2 - \frac{1}{Q_t} \right) x_t + \lambda \left(W_t G^a z_t n_t + \left(\frac{1}{Q_t} - 1 \right) x_t \right)^{1-\tau} - \tau^{SS} W_t G^a z_t n_t \tag{14}$$

The first order conditions for c_t, l_t, h_t, b_{t+1} are respectively:

$$\mathcal{C}_t^{1-\gamma-\eta}(1-\alpha_n)(1-\alpha_h)((1-\alpha_h)c^\epsilon + \alpha_h h^\epsilon)^{\frac{\eta-\epsilon}{\epsilon}} c^{\epsilon-1} = \lambda_t$$
$$\mathcal{C}_t^{1-\gamma-\eta}\alpha_n(1-\phi_T-n_t)^{\eta-1} = \lambda_t W_t G^a z_t \left[\lambda(1-\tau) \left(W_t G^a z_t (1-\phi_{T,t}^\ell - l_t) + \left(\frac{1}{Q_t} - 1 \right) x_t \right)^{-\tau} - \tau^{SS} \right] + \xi_t$$
$$\mathcal{C}_t^{1-\gamma-\eta}(1-\alpha_n)\alpha_h((1-\alpha_h)c^\epsilon + \alpha_h h^\epsilon)^{\frac{\eta-\epsilon}{\epsilon}} h^{\epsilon-1} = \lambda_t R_t^\ell$$
$$\lambda_t Q_t = (1-p^a)\beta \mathbb{E}_t \left[\frac{\partial}{\partial x_{t+1}} V\left(x_{t+1}, z_{t+1}, a+1, S_{t+1} \right) \right] + \nu_t \tag{15}$$

where $\frac{\partial}{\partial x_t} V\left(x_t, z_t, a, S_t \right) = \lambda_t \left(2 - \frac{1}{Q_t} + \lambda(1-\tau)\left(\frac{1}{Q_t} - 1 \right) \left(W_t G^a z_t n_t + \left(\frac{1}{Q_t} - 1 \right) x_t \right)^{-\tau} \right)$, since the marginal value of net worth is the same in the problem of renters, RC renters and owners, and the probabilities to receive RC sum to 1.

    Case 1: $\nu_t = 0$ and $\xi_t = 0$. In this case the household is unconstrained. Residential housing is

obtained by combining the conditions for c_t and h_t: $h_t = \left(\frac{\alpha_h}{(1-\alpha_h)R_t^\ell}\right)^{\frac{1}{1-\epsilon}} c_t$.

Case (i) The nonlinear equation for hours n_t is obtained by combining the first order conditions for c_t and n_t, and substituting for h_t as a function of c_t in the CES aggregator:

$$\chi_0 \alpha_N (1 - \Phi_T - n)^{\chi_0 \eta - 1} - (1 - \alpha_N)(1 - \alpha_H) \left[(1 - \alpha_H) + \alpha_H \left[\frac{\alpha_H}{(1-\alpha_H)R} \right]^{\frac{\epsilon}{1-\epsilon}} \right]^{\frac{\eta - \epsilon}{\epsilon}} WG_a z$$
$$\left\{ \lambda(1 - \tau)(WG_a zn + rx)^{-\tau} - \tau^{SS} \right\} c^{\eta - 1} = 0 \tag{16}$$

The Jacobian is:

$$-\alpha_N \chi_0 (\chi_0 \eta - 1)(1 - \Phi_T - n)^{\chi_0 \eta - 2} + (1 - \alpha_N)(1 - \alpha_H) \left[(1 - \alpha_H) + \alpha_H \left[\frac{\alpha_H}{(1-\alpha_H)R} \right]^{\frac{\epsilon}{1-\epsilon}} \right]^{\frac{\eta - \epsilon}{\epsilon}} (WG_a z)^2$$
$$\lambda(1 - \tau)\tau(WG_a zn + rx)^{-\tau - 1} c^{\eta - 1} \tag{17}$$

Absent HSV taxes, the analytic solution for hours is:

$$n = 1 - \Phi_T - \left[\frac{(1-\alpha_N)(1-\alpha_H)}{\chi_0 \alpha_N} \left[(1 - \alpha_H) + \alpha_H \left(\frac{\alpha_H}{(1-\alpha_H)R} \right)^{\frac{\epsilon}{1-\epsilon}} \right]^{\frac{\eta - \epsilon}{\epsilon}} WG_a z(1 - \tau^{SS}) \right]^{\frac{1}{\chi_0 \eta - 1}} c^{\frac{\eta - 1}{\chi_0 \eta - 1}} \tag{18}$$

Case (ii) The nonlinear equation for hours is:

$$\chi_0 \alpha_N (1 - \Phi_T - n)^{\chi_0 \eta - 1} - (1 - \alpha_N)(1 - \alpha_H) \left(\frac{\alpha_H}{(1-\alpha_H)R} \right)^{\eta \alpha_H} WG_a z$$
$$\left\{ \lambda(1 - \tau)(WG_a zn + rx)^{-\tau} - \tau^{SS} \right\} c^{\eta - 1} = 0 \tag{19}$$

The Jacobian is:

$$-\chi_0 \alpha_N (\chi_0 \eta - 1)(1 - \Phi_T - n)^{\chi_0 \eta - 2} + (1 - \alpha_N)(1 - \alpha_H) \left(\frac{\alpha_H}{(1-\alpha_H)R} \right)^{\eta \alpha_H} (WG_a z)^2$$
$$\lambda(1 - \tau)\tau(WG_a zn + rx)^{-\tau - 1} c^{\eta - 1} \tag{20}$$

Absent HSV taxes, the analytic solution for hours is:

$$n = 1 - \Phi_T - \left[\frac{(1-\alpha_N)(1-\alpha_H)}{\chi_0 \alpha_N} \left(\frac{\alpha_H}{(1-\alpha_H)R} \right)^{\eta \alpha_H} WG_a z(1 - \tau^{SS}) \right]^{\frac{1}{\chi_0 \eta - 1}} c^{\frac{\eta - 1}{\chi_0 \eta - 1}} \tag{21}$$

Case (iii) The nonlinear equation for hours is:

$$\chi_0 \alpha_N \left[(1 - \alpha_H) + \alpha_H \left(\frac{\alpha_H}{(1-\alpha_H)R} \right)^{\frac{\epsilon}{1-\epsilon}} \right] c$$
$$- (1 - \alpha_N)(1 - \alpha_H)(1 - \Phi_T - n)WG_a z \left\{ \lambda(1 - \tau)(WG_a zn + rx)^{-\tau} - \tau^{SS} \right\} = 0 \tag{22}$$

The Jacobian is:

$$(1 - \alpha_N)(1 - \alpha_H)WG_a z \left[\left\{ \lambda(1 - \tau)(WG_a zn + rx)^{-\tau} - \tau^{SS} \right\} + \right.$$
$$\left. (1 - \Phi_T - n)\lambda(1 - \tau)\tau(WG_a zn + rx + \Pi)^{-\tau - 1} \right] \tag{23}$$

Absent HSV taxes, the analytic solution for hours is:

$$n = 1 - \Phi_T - \left[(1 - \alpha_H) + \alpha_H \left(\frac{\alpha_H}{(1-\alpha_H)R} \right)^{\frac{\epsilon}{1-\epsilon}} \right] \frac{\chi_0 \alpha_N}{(1-\alpha_N)(1-\alpha_H)WG_a z(1-\tau^{SS})} c \tag{24}$$

Case (iv) The nonlinear equation for hours is:

$$\chi_0 \alpha_N c - (1-\alpha_N)(1-\alpha_H)(1 - \Phi_T - n)WG_a z \left\{ \lambda(1-\tau)(WG_a zn + rx)^{-\tau} - \tau^{SS} \right\} = 0 \tag{25}$$

The Jacobian is:

$$(1-\alpha_N)(1-\alpha_H)WG_a z \left[\left\{ \lambda(1-\tau)(WG_a zn + rx + \Pi)^{-\tau} - \tau^{SS} \right\} + \right. \\ \left. (1 - \Phi_T - n)\lambda(1-\tau)\tau (WG_a zn + rx)^{-\tau-1} \right] \tag{26}$$

Absent HSV taxes, the analytic solution for hours is:

$$n = 1 - \Phi_T - \frac{\chi_0 \alpha_N}{(1-\alpha_N)(1-\alpha_H)WG_a z(1-\tau^{SS})} c \tag{27}$$

Given c_t, we obtain l_t (hence n_t) by numerically solving the labor supply equation. Given c_t, n_t, h_t, we obtain b_{t+1} from the budget constraint:

$$b_{t+1} = \frac{1}{Q_t} \left[\overline{\Psi}_t \psi^z - \phi^{\ell}_{F,t} + \left(2 - \frac{1}{Q_t}\right) x_t + \lambda \left(W_t G^a z_t n_t + \left(\frac{1}{Q_t} - 1\right) x_t \right)^{1-\tau} - \tau^{SS} W_t G^a z_t n_t - c_t - R^{\ell}_t h_t \right] \tag{28}$$

Case 2: $\nu_t > 0$ and $\xi_t = 0$. In this case the borrowing constraint binds and $b_{t+1} = 0$ but the labor constraint does not. The first order conditions in the first three lines of equation 15 are still correct. It is still the case that conditional on choosing a location ℓ, $h_t = \left(\frac{\alpha_h}{(1-\alpha_h)R^{\ell}_t} \right)^{\frac{1}{1-\epsilon}} c_t$ and l_t is the solution to the nonlinear equation in Case 1. Given those, c_t can be obtained from the budget constraint:

$$c_t = \frac{\overline{\Psi}_t \psi^z - \phi^{\ell}_{F,t} + \left(2 - \frac{1}{Q_t}\right) x_t + \lambda \left(W_t G^a z_t n_t + \left(\frac{1}{Q_t} - 1\right) x_t \right)^{1-\tau} - \tau^{SS} W_t G^a z_t n_t}{1 + R^{\ell}_t \left(\frac{\alpha_h}{(1-\alpha_h)R^{\ell}_t} \right)^{\frac{\epsilon}{1-\epsilon}}} \tag{29}$$

Case 3: $\nu_t = 0$ and $\xi_t > 0$. In this case the borrowing constraint does not bind, but the labor constraint does, implying $l_t = 1 - \phi^{\ell}_{T,t}$, hence $n_t = 0$. The first order conditions in the first, third, and fourth lines of equation 15 are still correct. As in Case 1, conditional on choosing a location ℓ, $h_t = \left(\frac{\alpha_h}{(1-\alpha_h)R^{\ell}_t} \right)^{\frac{1}{1-\epsilon}} c_t$. We obtain b_{t+1} from the budget constraint.

Case 4: $\nu_t > 0$ and $\xi_t > 0$. In this case both constraints bind, implying $n_t = 0$ and $b_{t+1} = 0$. The first order conditions in the first and third lines of equation 15 are still correct, so conditional on choosing a location ℓ, $h_t = \left(\frac{\alpha_h}{(1-\alpha_h)R^{\ell}_t} \right)^{\frac{1}{1-\epsilon}} c_t$. By plugging this into the budget constraint, we can explicitly solve for $c_t = \frac{\overline{\Psi}_t \psi^z - \phi^{\ell}_{F,t} + \left(2 - \frac{1}{Q_t}\right) x_t + \lambda \left(\left(\frac{1}{Q_t} - 1\right) x_t \right)^{1-\tau}}{1 + R^{\ell}_t \left(\frac{\alpha_h}{(1-\alpha_h)R^{\ell}_t} \right)^{\frac{\epsilon}{1-\epsilon}}}$.

In the case with $\lambda = 1, \tau = 0$, we simply have $c_t = \frac{\overline{\Psi}_t \psi^z - \phi^\ell_{F,t} + x_t}{1 + R^\ell_t \left(\frac{\alpha_h}{(1-\alpha_h)R^\ell_t} \right)^{\frac{\epsilon}{1-\epsilon}}}$.

RC renter   Next, the RC renter's problem is the same as the renter's problem, with additional Lagrange multipliers on the income and the rent restriction constraints, ξ^y_t and ξ^r_t, and κ_1 the RC discount multiplying R^ℓ_t wherever it appears. Note that we cannot have both $\xi_t, \xi^y_t > 0$ (we rule out $\frac{\kappa_2 \overline{Y}_t}{W_t G^a z_t} = 0$). Because households are atomistic we ignore the derivative $\frac{\partial Y_t}{\partial n_t}$.

Case 1: $\xi_t = 0$ and $\xi^y_t = 0$. There is an interior solution $n_{min} < n_t < \frac{\kappa_2 \overline{Y}_t}{W_t G^a z_t}$, which solves the same labor supply equation as the market renter (with $\kappa_1 R^\ell_t$ instead of R^ℓ_t).
If $\xi^r_t > 0$, then the residential housing choice is constrained: $h_t = \frac{\kappa_3 \overline{Y}_t}{\kappa_1 R^\ell_t}$. If $\xi^r_t = 0$, then combining the conditions for c_t and for h_t, we obtain $h_t = \left(\frac{\alpha_h}{(1-\alpha_h)\kappa_1 R^\ell_t} \right)^{\frac{1}{1-\epsilon}} c_t$.
If $\nu_t > 0$, then savings are constrained and $b_{t+1} = 0$. If $\nu_t = 0$, then given c_t, n_t, h_t, we obtain b_{t+1} from the budget constraint.

Case 2: $\xi_t > 0$ and $\xi^y_t = 0$. The leisure choice is constrained at its upper bound, and $n_t = 0$. Choices for h_t and b_{t+1} as functions of c_t are identical to Case 1.

Case 3: $\xi_t = 0$ and $\xi^y_t > 0$. The labor choice is constrained at the upper bound implied by RC, and $n_t = \frac{\kappa_2 \overline{Y}_t}{W_t G^a z_t}$. Choices for h_t and b_{t+1} as functions of c_t are identical to Case 1.

Owner   Finally, consider the owner's problem and let λ_t be the Lagrange multiplier on the budget constraint, ν_t be the Lagrange multiplier on the borrowing constraint, and ξ_t be the Lagrange multiplier on the labor constraint. The numerical strategy is to choose c_t and $\widehat{h}_t$ in order to maximize the household's utility (therefore, we ignore the multiplier on the non-negativity constraint $\widehat{h}_t \geq 0$), and n_t to solve the nonlinear equation for labor supply. Here we will show that the other choices (h_t and b_{t+1}) can be written as analytic functions of c_t and $\widehat{h}_t$. The budget constraint simplifies to:

$$c_t + P^\ell_t h_t + Q_t b_{t+1} + \kappa^\ell_4 P^\ell_t \widehat{h}_t + \phi^\ell_{F,t}$$
$$= \overline{\Psi}_t \psi^z + \left(2 - \frac{1}{Q_t} \right) x_t + \kappa^\ell_4 R^\ell_t \widehat{h}_t + \lambda \left(W_t G^a z_t n_t + \left(\frac{1}{Q_t} - 1 \right) x_t \right)^{1-\tau} - \tau^{SS} W_t G^a z_t n_t \tag{30}$$

The first-order conditions for c_t, l_t are identical to the market renter. The first-order conditions for $h_t, \widehat{h}_t, b_{t+1}$ are respectively:

$$c_t^{1-\gamma-\eta}(1-\alpha_n)\alpha_h((1-\alpha_h)c^\epsilon + \alpha_h h^\epsilon)^{\frac{\eta-\epsilon}{\epsilon}} h^{\epsilon-1} +$$
$$(1-p^a)\beta(1-\delta-\tau^P)\mathbb{E}_t \left[P^\ell_{t+1} \frac{\partial}{\partial x_{t+1}} V(x_{t+1}, z_{t+1}, a+1, S_{t+1}) \right] + \nu_t \theta_{res} P^\ell_t = \lambda_t P^\ell_t$$
$$(1-p^a)\beta(1-\delta-\tau^P)\mathbb{E}_t \left[P^\ell_{t+1} \frac{\partial}{\partial x_{t+1}} V(x_{t+1}, z_{t+1}, a+1, S_{t+1}) \right] + \nu_t \theta_{inv} P^\ell_t = \lambda_t (P_t - R_t) \tag{31}$$
$$\lambda_t Q_t = (1-p^a) \beta \mathbb{E}_t \left[\frac{\partial}{\partial x_{t+1}} V(x_{t+1}, z_{t+1}, a+1, S_{t+1}) \right] + \nu_t Q_t$$

Case 1: $\nu_t = 0$ and $\xi_t = 0$. In this case the household is unconstrained. Combining the conditions for h_t and $\widehat{h}_t$, and combining the result with the condition for c_t, we can solve analytically for $h_t = \left(\frac{\alpha_h}{(1-\alpha_h)R^\ell_t} \right)^{\frac{1}{1-\epsilon}} c_t$, as in the market renter's case.

Then, the nonlinear labor supply equation for n_t is the same.

Given $c_t, \widehat{h}_t, h_t, n_t$, we obtain b_{t+1} from the budget constraint:

$$b_{t+1} = \frac{1}{Q_t}\left[\overline{\Psi}_t\psi^z - \phi^\ell_{F,t} + \left(2 - \frac{1}{Q_t}\right)x_t + \kappa^\ell_4\left(R^\ell_t - P^\ell_t\right)\widehat{h}_t \right.$$
$$\left. + \lambda\left(W_t G^a z_t n_t + \left(\frac{1}{Q_t} - 1\right)x_t\right)^{1-\tau} - \tau^{SS}W_t G^a z_t n_t - c_t - P^\ell_t h_t\right] \tag{32}$$

Case 2: $\nu_t > 0$ and $\xi_t = 0$. In this case the borrowing constraint binds implying $b_{t+1} = -\frac{\theta_{res}P^\ell_t h_t + \theta_{inv}\kappa^\ell_4 P^\ell_t \widehat{h}_t}{Q_t}$, but the leisure constraint does not bind. We solve for l_t as in Case 1, as the nonlinear equation for hours is unaffected. Given $c_t, \widehat{h}_t, n_t$, we use the budget constraint to solve for h_t analytically:

$$h_t = \frac{1}{P^\ell_t(1-\theta_{res})}\left[\overline{\Psi}_t\psi^z + \left(2 - \frac{1}{Q_t}\right)x_t + \kappa^\ell_4 R^\ell_t\widehat{h}_t + \lambda\left(W_t G^z z_t n_t + \left(\frac{1}{Q_t} - 1\right)x_t\right)^{1-\tau}\right.$$
$$\left. - \phi^\ell_{F,t} - c_t - (1-\theta_{inv})\kappa^\ell_4 P^\ell_t\widehat{h}_t - \tau^{SS}W_t G^z z_t n_t\right] \tag{33}$$

In the case with $\lambda = 1, \tau = 0$, we have

$$h_t = \frac{1}{P^\ell_t(1-\theta_{res})}\left[\overline{\Psi}_t\psi^z + x_t + \left(1 - \tau^{SS}\right)W_t G^a z_t n_t + \kappa^\ell_4 R^\ell_t\widehat{h}_t - \phi^\ell_{F,t} - c_t - (1-\theta_{inv})\kappa^\ell_4 P^\ell_t\widehat{h}_t\right]$$

Case 3: $\nu_t = 0$ and $\xi_t > 0$. In this case the borrowing constraint does not bind, but the leisure constraint does, implying $n_t = 0$. Conditional on choosing a location ℓ, h_t is identical to Case 1. From the budget constraint, we deduce:

$$b_{t+1} = \frac{\overline{\Psi}_t\psi^z - \phi^\ell_{F,t} + x_t\left(2 - \frac{1}{Q_t}\right) + \kappa_4(R^\ell_t - P^\ell_t)\widehat{h}_t + \lambda\left(\left(\frac{1}{Q_t} - 1\right)x_t\right)^{1-\tau} - \left(1 + \left(\frac{\alpha_h}{(1-\alpha_h)R^\ell_t}\right)^{\frac{1}{1-\epsilon}}P^\ell_t\right)c_t}{Q_t} \tag{34}$$

Case 4: $\nu_t > 0$ and $\xi_t > 0$. In this case both constraints bind, implying $n_t = 0$ and $b_{t+1} = -\frac{\theta_{res}P^\ell_t h_t + \theta_{inv}\kappa^\ell_4 P^\ell_t\widehat{h}_t}{Q_t}$. Eliminating b_{t+1} and n_t from the budget constraint, we can solve analytically for h_t as a function of c_t and $\widehat{h}_t$ just as in case 2:

$$h_t = \frac{1}{P^\ell_t(1-\theta_{res})}\left[\overline{\Psi}_t\psi^z + \left(2 - \frac{1}{Q_t}\right)x_t + \kappa^\ell_4 R^\ell_t\widehat{h}_t + \lambda\left(\left(\frac{1}{Q_t} - 1\right)x_t\right)^{1-\tau} - \phi^\ell_{F,t} - c_t - (1-\theta_{inv})\kappa^\ell_4 P^\ell_t\widehat{h}_t\right] \tag{35}$$

In the case with $\lambda = 1, \tau = 0$, we have

$$h_t = \frac{1}{P^\ell_t(1-\theta_{res})}\left[\overline{\Psi}_t\psi^z + x_t + \kappa^\ell_4 R^\ell_t\widehat{h}_t - \phi^\ell_{F,t} - c_t - (1-\theta_{inv})\kappa^\ell_4 P^\ell_t\widehat{h}_t\right]$$

A.2 Special case which can be solved analytically

Here we use Cobb-Douglas preferences as a special case of the CES aggregator described earlier. Consider a perpetual renter who is facing a constant wage W and a constant rent R, who is not choosing location, who is not constrained, who faces no idiosyncratic shocks ($A = 1$), and whose productivity and utility are not age dependent ($G^a = 1$, $\alpha_{c,a} = \alpha_c$, and $\alpha_{h,a} = \alpha_h \ \forall a$). His problem

55

can be written as:

$$v(xs_t, a) = \max_{cs,hs,n} \frac{1}{1-\gamma} \left(cs_t^{\alpha_c} hs_t^{\alpha_h} (1-n_t)^{\alpha_n} \right)^{1-\gamma} + \beta E_t[v(xs_{t+1}, a+1)] \text{ s.t.}$$
$$xs_{t+1} = \frac{1}{Q}(xs_t + n_t W - cs_t - hs_t R) \tag{36}$$

As shown earlier, the optimal housing and labor choices satisfy: $hs_t = \frac{\alpha_h}{\alpha_c} \frac{1}{R} cs_t$ and $n_t = 1 - \frac{\alpha_n}{\alpha_c} \frac{1}{W} cs_t$. Redefining $\widehat{cs} = \frac{1}{\alpha_c} cs$ and plugging these into the maximization problem, the problem is rewritten as:

$$v(xs_t, a) = \max_{\widehat{cs}} \frac{\overline{U}}{1-\gamma} \widehat{cs}_t^{1-\gamma} + \beta E_t[v(xs_{t+1}, a+1)] \text{ s.t.}$$
$$xs_{t+1} = \frac{1}{Q}(xs_t + W - \widehat{cs}_t) \tag{37}$$

where $\overline{U} = \left(\alpha_c^{\alpha_c} \alpha_h^{\alpha_h} \alpha_n^{\alpha_n} R^{-\alpha_h} W^{-\alpha_n} \right)^{1-\gamma}$. Next we can guess and verify that the value function has the form $v(xs_t, a) = \frac{v_a}{1-\gamma} * \left(xs_t + \frac{1}{1-Q_a} W \right)^{1-\gamma}$ where v_a and Q_a are constants that depend on age a. Suppose this is true for $a+1$. Then the problem is:

$$
\begin{aligned}
v(xs_t, a) &= \max_{\widehat{cs}} \frac{\overline{U}}{1-\gamma} \widehat{cs}_t^{1-\gamma} + \frac{v_{a+1}}{1-\gamma} \beta Q^{-(1-\gamma)} (xs_t + W - \widehat{cs}_t + W \frac{Q}{1-Q_{a+1}})^{1-\gamma} \\
&= \max_{\widehat{cs}} \frac{\overline{U}}{1-\gamma} \widehat{cs}_t^{1-\gamma} + \frac{v_{a+1}}{1-\gamma} \beta Q^{-(1-\gamma)} (xs_t - \widehat{cs}_t + W \frac{1-Q_{a+1}+Q}{1-Q_{a+1}})^{1-\gamma}
\end{aligned} \tag{38}
$$

Define $X_{a+1} = v_{a+1} Q^{-(1-\gamma)} \beta$. Then the first order condition is: $\overline{U} * \widehat{cs}_t^{-\gamma} = X_{a+1} * (xs_t - \widehat{cs}_t + W \frac{1-Q_{a+1}+Q}{1-Q_{a+1}})^{-\gamma}$. Rearranging, we can solve for optimal consumption:

$$
\begin{aligned}
\widehat{cs}_t &= \frac{\left(\frac{X_{a+1}}{\overline{U}} \right)^{-1/\gamma}}{1 + \left(\frac{X_{a+1}}{\overline{U}} \right)^{-1/\gamma}} \left(xs_t + W \frac{1-Q_{a+1}+Q}{1-Q_{a+1}} \right) \\
xs_{t+1} + \frac{1}{1-Q_{a+1}} W &= \frac{1}{1 + \left(\frac{X_{a+1}}{\overline{U}} \right)^{-1/\gamma}} \left(xs_t + W \frac{1-Q_{a+1}+Q}{1-Q_{a+1}} \right)
\end{aligned} \tag{39}
$$

Plugging this back into the original problem:

$$
\begin{aligned}
v(xs_t, a) &= \left(\overline{U} \left(\frac{\left(\frac{X_{a+1}}{\overline{U}} \right)^{-1/\gamma}}{1 + \left(\frac{X_{a+1}}{\overline{U}} \right)^{-1/\gamma}} \right)^{1-\gamma} + X_{a+1} \left(\frac{1}{1 + \left(\frac{X_{a+1}}{\overline{U}} \right)^{-1/\gamma}} \right)^{1-\gamma} \right) \frac{\left(xs_t + W \frac{1-Q_{a+1}+Q}{1-Q_{a+1}} \right)^{1-\gamma}}{1-\gamma} \\
&= \overline{U} \left(1 + \left(\frac{X_{a+1}}{\overline{U}} \right)^{-1/\gamma} \right)^{-(1-\gamma)} \left(\left(\frac{X_{a+1}}{\overline{U}} \right)^{-(1-\gamma)/\gamma} + \left(\frac{X_{a+1}}{\overline{U}} \right) \right) \frac{\left(xs_t + W \frac{1-Q_{a+1}+Q}{1-Q_{a+1}} \right)^{1-\gamma}}{1-\gamma} \\
&= X_{a+1} \left(1 + \left(\frac{X_{a+1}}{\overline{U}} \right)^{-1/\gamma} \right)^{\gamma} \frac{\left(xs_t + W \frac{1-Q_{a+1}+Q}{1-Q_{a+1}} \right)^{1-\gamma}}{1-\gamma}
\end{aligned} \tag{40}
$$

This verifies the conjecture. The age dependent constants take the following form:

$$
\begin{aligned}
v_a &= X_{a+1} \left(1 + \left(\frac{X_{a+1}}{\overline{U}} \right)^{-1/\gamma} \right)^{\gamma} \\
&= \beta Q^{-(1-\gamma)} v_{a+1} \left(1 + \left(v_{a+1} \beta Q^{-(1-\gamma)} \overline{U}^{-1} \right)^{-1/\gamma} \right)^{\gamma} \\
Q_a &= \frac{Q}{1+Q-Q_{a+1}}
\end{aligned} \tag{41}
$$

Note that $Q_\infty = Q$ and $v_\infty = \overline{U}\left(1 - \beta^{\frac{1}{\gamma}} Q^{\frac{-(1-\gamma)}{\gamma}}\right)^{-\gamma}$.

A.3 Commuting costs and composition of Zone 1

From the household's FOC, we know that $\frac{\partial U}{\partial C} = \frac{\partial U}{\partial N} \times \frac{1}{w}$ where C is the numeraire, N is hours worked, and w is the wage. Suppose that moving one unit of distance towards center decreases the hourly commuting cost by ϕ_T and the financial commuting cost by ϕ_F. Also, suppose that the price is a function of distance from center $P(x)$.

First, consider time costs only ($\phi_F = 0$). The cost of decreasing the commute by d is $d \times H \times P'(x) \times \frac{\partial U}{\partial C}$, this is the amount of housing consumed H, multiplied by the price increase at the current location $P'(x) \times d$, multiplied by the marginal utility of the numeraire good. The benefit of decreasing the commute by d is $d \times \phi_T \times \frac{\partial U}{\partial N} = d \times \phi_T \times w \times \frac{\partial U}{\partial C}$, this is the marginal utility of leisure, multiplied by the extra leisure $d \times \phi_T$. Equating the cost to the benefit and rearranging: $P'(x) = \phi_T \frac{w}{H}$. The left hand side represents one's willingness to pay per square foot implying that agents with high $\frac{w}{H}$ are willing to pay a higher price. For a fixed amount of wealth, high income agents have higher $\frac{w}{H}$ because individual productivity is stationary, therefore high income agents tend to save relatively more and consume relatively less of their wealth ($\frac{w}{H}$ would be constant if individual productivity had permanent shocks). For a fixed income, high wealth agents have higher $\frac{w}{H}$ because, consistent with the Permanent Income Hypothesis, for a fixed w, high wealth agents are willing to spend more on housing.

Next, consider financial costs only ($\phi_T = 0$). The cost of decreasing the commute is the same as before $d \times H \times P'(x) \times \frac{\partial U}{\partial C}$. The benefit of decreasing the commute is $d \times \phi_F \times \frac{\partial U}{\partial C}$, this is the financial saving $d \times \phi_F$ multiplied by the marginal utility of the numeraire. Equating the cost to the benefit: $P'(x) = \phi_F \frac{1}{H}$. Low H agents are willing to pay a higher price. Agents who have low wealth or low income tend to have lower housing demand H and are willing to pay more per square foot to reduce their commute. The intuition is that the financial cost is fixed, thus agents with low housing demand are willing to pay a much higher price per square foot to 'ammortize' the benefit of not paying the fixed cost.

A.4 One-period case which can be solved analytically

There are m agents, m^c consumption producing firms, m^1 construction firms in zone 1, and m^2 construction firms in zone 2. There are two zones with sizes $m\overline{h}^1$ and $m\overline{h}^2$. Agents have initial wealth $W = 0$ and earn a wage w. They live for one period only, and there is no resale value for the housing that they buy.

Conditional on a zone, a household maximizes $U = c^{\alpha_c} h^{\alpha_h}(1 - \lambda - x)^{\alpha_n}$ subject to $c + P * h = W + w * x$ where λ is a zone specific time cost and P is a zone specific housing price ($\lambda = 0$ in zone 1). This can be rewritten as:

$$U = \max_{h,x}(W + w * x - P * h)^{\alpha_c} h^{\alpha_h}(1 - \lambda - x)^{\alpha_n} \tag{42}$$

The first order conditions imply the following solution:

$$
\begin{aligned}
c &= \alpha_c((1 - \lambda)w + W) \\
h &= \alpha_h((1 - \lambda)w + W) \\
x &= (\alpha_c + \alpha_h)(1 - \lambda) - \alpha_n \frac{W}{w} \\
U &= \left(\frac{1}{P}\right)^{\alpha_h} \left(\frac{1}{w}\right)^{\alpha_n} \alpha_c^{\alpha_c} \alpha_h^{\alpha_h} \alpha_n^{\alpha_n} ((1 - \lambda)w + W)
\end{aligned}
\tag{43}
$$

Here we used $\alpha_c + \alpha_h + \alpha_n = 1$.

Each consumption producing firm chooses hours x_c to maximize $\pi_c = x_c^{\rho_c} - wx_c$ which implies that $w = \rho_c x_c^{\rho_c - 1}$. Each construction firm in zone 1 maximizes $\pi_1 = \left(1 - \frac{H^1}{m\overline{h^1}}\right) P_1 x_1^{\rho_h} - wx_1$ which implies that $w = \left(1 - \frac{H^1}{m\overline{h^1}}\right) P_1 \rho_h x_1^{\rho_h - 1}$. Each construction firm in zone 2 maximizes $\pi_2 = \left(1 - \frac{H^2}{m\overline{h^2}}\right) P_2 x_2^{\rho_h} - wx_2$ which implies that $w = \left(1 - \frac{H^2}{m\overline{h^2}}\right) P_2 \rho_h x_2^{\rho_h - 1}$. Here H^1 and H^2 are the total amount of housing built in each zone.

Equilibrium implies that the following equations must be satisfied.

$$P_2 = P_1(1 - \lambda)^{1/\alpha_h} \tag{44}$$

Equation 44 says that for households to be indifferent between the two zones, their utility of living in each zone must be the same.

$$n_1 = \frac{H^1 P^1}{\alpha_h w} \tag{45}$$

$$n_2 = \frac{H^2 P^2}{\alpha_h w(1 - \lambda)} \tag{46}$$

$$n_1 + n_2 = m \tag{47}$$

Equations 45 and 46 say that the total number of households in each zone (N_1 and N_2) must equal to the total housing in each zone, divided by the housing size an agent in that zone would demand. The housing size comes from the solution of the agent's problem. Equation 47 says that the sum of agents living in zones 1 and 2 must equal to the total number of agents.

$$w = \rho_c x_c^{\rho_c - 1} \tag{48}$$

$$w = \left(1 - \frac{H^1}{m\overline{h^1}}\right) P_1 \rho_h x_1^{\rho_h - 1} \tag{49}$$

$$w = \left(1 - \frac{H^2}{m\overline{h^2}}\right) P_2 \rho_h x_2^{\rho_h - 1} \tag{50}$$

Equations 48, 49, and 50 relate each firm's optimal behavior to the wage.

$$H^1 = \left(1 - \frac{H^1}{m\overline{h^1}}\right) m_1 x_1^{\rho_h} \tag{51}$$

$$H^2 = \left(1 - \frac{H^2}{m\overline{h^2}}\right) m_1 x_2^{\rho_h} \tag{52}$$

Equations 51 and 52 relate each firm's output to the total output of housing in each zone. They can be rewritten as $H^1 = \frac{m\overline{h^1} m_1 x_1^{\rho_h}}{m\overline{h^1} + m_1 x_1^{\rho_h}}$ and $H^2 = \frac{m\overline{h^2} m_2 x_2^{\rho_h}}{m\overline{h^2} + m_2 x_2^{\rho_h}}$.

$$(\alpha_c + \alpha_h)(n_1 + n_2(1 - \lambda)) = m_c x_c + m_1 x_1 + m_2 x_2 \tag{53}$$

Equation 53 relates labor supply, on the left side, to labor demand, on the right side.

This is 10 equations and 10 unknowns: prices P_1, P_2; labor demand for each firm type x_1, x_2, x_c; number of households living in each zone n_1, n_2; total housing in each zone H^1, H^2; and the wage w. This can can be reduced to a single equation.

First, plug H and P into equations (49) and (50): $w = P_1 \rho_h \frac{m\overline{h^1}x_1^{\rho_h-1}}{m\overline{h^1}+m_1 x_1^{\rho_h}} = P_2 \rho_h \frac{m\overline{h^2}x_2^{\rho_h-1}}{m\overline{h^2}+m_2 x_2^{\rho_h}}$

Second, plug the wage into equations (45) and (46): $n_1 = \frac{m_1 x_1}{\alpha_h \rho_h}$ and $n_2 = \frac{m_2 x_2}{\alpha_h \rho_h (1-\lambda)}$.

Third, plug n_1 and n_2 into equation (47) to solve for x_2 in terms of x_1: $x_2 = \frac{1-\lambda}{m_2}(m\alpha_h \rho_h - m_1 x_1) = A_0 + A_1 x_1$ where $A_0 = \frac{1-\lambda}{m_2}m\alpha_h \rho_h$ and $A_1 = -m_1 \frac{1-\lambda}{m_2}$.

Fourth, plug $x_2 = A_0 + A_1 x_1$ into the equality between zone 1 and zone 2 firms' wages derived earlier and use equation (44) to get rid of prices: $\frac{m\overline{h^1}x_1^{\rho_h-1}}{m\overline{h^1}+m_1 x_1^{\rho_h}} = (1-\lambda)^{1/\alpha_h} \frac{m\overline{h^2}(A_0+A_1 x_1)^{\rho_h-1}}{m\overline{h^2}+m_2(A_0+A_1 x_1)^{\rho_h}}$. This is now one equation with one unknown and can be solved numerically.

Fifth, once we have x_1 we can immediately calculate x_2, n_1, n_2, H^1, H^2 but we still need to solve for w and P_1. We can solve for w as a function of P_1 using equation (49). We can then solve for x_c as a function of P_1 using equation (48). We can then plug this into equation (53) to solve for P_1.

B Data Appendix: New York

B.1 The New York Metro Area

U.S. Office of Management and Budget publishes the list and delineations of Metropolitan Statistical Areas (MSAs) on the Census website (https://www.census.gov/population/metro/data/metrodef.html). The current delineation is as of July 2015. New York-Newark-Jersey City, NY-NJ-PA MSA (NYC MSA) is the most populous MSA among the 382 MSAs in the nation.

NYC MSA consists of 25 counties, spanning three states around New York City. The complete list of counties with state and zone information is presented in Table 4. As previously defined, only New York County (Manhattan borough) is categorized as zone 1; the other 24 counties are categorized as zone 2. For informational purposes, the five counties of New York City are appended with parenthesized borough names used in New York City.

B.2 Population, Housing Stock, and Land Area

The main source for population, housing stock and land area is US Census Bureau `American FactFinder` (http://factfiner.census.gov). `American FactFinder` provides comprehensive survey data on a wide range of demographic and housing topics. Using the Advanced Search option on the webpage, topics such as population and housing can be queried alongside geographic filters. We select the DP02 table (selected social characteristics) for population estimates, the DP04 table (selected housing characteristics) for housing estimates, and the GCT-PH1 table (population, housing units, area and density) for land area information. Adding 25 counties separately in the geographic filter, all queried information is retrieved at the county level. We then aggregate the 24 columns as a single zone 2 column.

Since the ACS (American Community Survey) surveys are conducted regularly, the survey year must be additionally specified. We use the 2015 1-year ACS dataset as it contains the most up-to-date numbers available. For Pike County, PA, the 2015 ACS data is not available and we use the 2014 5-year ACS number instead. Given that Pike County accounts only for 0.3% of zone 2 population, the effect of using lagged numbers for Pike County is minimal.

The ratio of the land mass of zone 1 (Manhattan) to the land mass of zone 2 (the other 24 counties of the NY MSA) is 0.0028. However, that ratio is not the appropriate measure of the

Table 4: Counties in the New York MSA

County	State	Zone
New York (Manhattan)	NY	Zone 1
Bergen	NJ	Zone 2
Bronx (Bronx)	NY	Zone 2
Dutchess	NY	Zone 2
Essex	NJ	Zone 2
Hudson	NJ	Zone 2
Hunterdon	NJ	Zone 2
Kings (Brooklyn)	NY	Zone 2
Middlesex	NJ	Zone 2
Monmouth	NJ	Zone 2
Morris	NJ	Zone 2
Nassau	NY	Zone 2
Ocean	NJ	Zone 2
Orange	NY	Zone 2
Passaic	NJ	Zone 2
Pike	PA	Zone 2
Putnam	NY	Zone 2
Queens (Queens)	NY	Zone 2
Richmond (Staten Island)	NY	Zone 2
Rockland	NY	Zone 2
Somerset	NJ	Zone 2
Suffolk	NY	Zone 2
Sussex	NJ	Zone 2
Union	NJ	Zone 2
Westchester	NY	Zone 2

relative maximum availability of housing in each of the zones since Manhattan zoning allows for taller buildings, smaller lot sizes, etc.

Data on the maximum buildable residential area are graciously computed and shared by Chamna Yoon from Baruch College. He combines the maximum allowed floor area ratio (FAR) to each parcel to construct the maximum residential area for each of the five counties (boroughs) that make up New York City. Manhattan has a maximum residential area of 1,812,692,477 square feet. This is our measure for $\overline{H}^1$. The other four boroughs of NYC combine for a maximum buildable residential area of 4,870,924,726 square feet. Using the land area of each of the boroughs (expressed in square feet), we can calculate the ratio of maximum buildable residential area (sqft) to the land area (sqft). For Manhattan, this number is 2.85. For the other four boroughs of NYC it is 0.62. For Staten Island, the most suburban of the boroughs, it is 0.32. We assume that the Staten Island ratio is representative of the 20 counties in the New York MSA that lie outside NYC since these are more suburban. Applying this ratio to their land area of 222,808,633,344 square feet, this delivers a maximum buildable residential square feet for those 20 counties of 71,305,449,967 square feet. Combining that with the four NYC counties in zone 2, we get a maximum buildable residential area for zone 2 of 76,176,377,693 square feet. This is $\overline{H}^2$. The ratio $\overline{H}^1/\overline{H}^2$ is 0.0238. We argue that this ratio better reflects the relative scarcity of space in Manhattan than the corresponding land mass ratio.

B.3 Income

The main source for the income distribution data is the US Census Bureau's `American FactFinder`. From table DP03 (selected economic characteristics), we retrieve the number of individuals in each of nine income brackets, ranging from "less than $10,000" for the lowest to "$100,000 or more" for the highest bracket. The distribution suffers from top-coding problem, so we additionally estimate the conditional means for the individuals in each income bracket. For the seven income brackets except for the lowest and the highest, we simply assume the midpoint of the interval as the conditional mean. For example, for the households in $50,000 to $64,999 bracket, the conditional mean income is assumed to be $57,500. For the lowest bracket (less than $10,000), we assume the conditional mean is $7,500. Then we can calculate the conditional mean of the highest income bracket, using the average income and conditional means of the other brackets, since the reported unconditional mean is based on all data (uncensored).

Our concept of labor income is full-time earnings before taxes for year-round workers with earnings. It includes income from wages, salaries, commissions, bonuses, or tips. To convert it to household-level income, we multiply it by 1.64 household members, which is the average number of individuals per household in the New York MSA.

We aggregate the county-level income distribution into a zone 2 income distribution in two steps. First, the aggregate number of households included in each income bracket is the simple sum of county-level household numbers in the bracket. Second, we calculate the zone 2 conditional mean of the income brackets using the weighted average methods. For the lower eight income brackets, the conditional means are assumed to be constant across counties, so zone 2 conditional means are also the same. For the highest income bracket, we use the county-specific conditional mean of the highest bracket, and calculate its weighted average over the 24 counties. Using these conditional means, and the household distribution over 9 income brackets, the zone 2 average household income can be calculated.

B.4 House Prices, Rental Prices, and Home Ownership

Housing prices and rental prices data come from `Zillow` (http://www.zillow.com/research/data) indices. Zillow publishes Zillow Home Value Index (ZHVI) and Zillow Rent Index (ZRI) monthly. The main advantage of using Zillow indices compared to other indices is that it overcomes sales-composition bias by constantly estimating hypothetical market prices, controlling for hedonics such as house size. We use 2015 year-end data to be consistent with the ACS dataset. There are a few missing counties in ZHVI and ZRI. For the five counties with missing ZHVI index price, we search those counties from `Zillow` (http://www.zillow.com) website, and use the median listing prices instead. For the two counties with missing ZRI index price, we estimate the rents using the price/rent ratio of comparable counties.

Home ownership data is directly from `American FactFinder`. In table DP04 (selected housing characteristics), the *Total housing units* number is divided by *Occupied housing units* and *Vacant housing units*. *Occupied housing units* are further classified into *Owner-occupied* and *Renter-occupied* housing units, which enables us to calculate the home ownership ratio.

B.5 Rent Regulation

The main source for rent regulation data is US Census Bureau New York City Housing and Vacancy Survey (NYCHVS; http://www.census.gov/housing/nychvs). NYCHVS is conducted every three years to comply with New York state and New York City's rent regulation laws. We use

the 2014 survey data table, which is the most recent survey data. In Series IA table 14, the number of housing units under various rent-control regulations are available for each of the five NYC boroughs (corresponding to five counties). We define rent-controlled units as those units that are (i) rent controlled, (ii) public housing, (iii) Mitchell Lama housing, (iv) all other government-assisted or regulated housing.

We exclude rent-stabilized units from our definition. Rent stabilized units are restricted in terms of their annual rent increases. The vast majority of units built after 1947 that are rent stabilized are so voluntarily. They receive tax abatement in lieu of subjecting their property to rent stabilization for a defined period of time. Both rent levels and income levels of tenants in rent-stabilized units are in between those of rent-regulated and unregulated units.

We calculate the proportion of rent-controlled units (i.e., households) among all the renter-occupied units. The proportion is 16.9% for Manhattan and 13.2% for the other four NYC boroughs. We use a different data source for the other 20 counties outside of New York City. `Affordable Housing Online` (http://affordablehousingonline.com) provides various rent-related statistics at the county level. For each of the 20 counties outside NYC, we calculate the fraction of rent-regulated units by dividing *Federally Assisted Units* number by *Renter Households* number reported on each county's webpage. We then multiply these %-numbers with the renter-occupied units in ACS data set to calculate the rent-regulated units for the 20 counties. Along with the NYCHVS numbers for the four NYC boroughs, we can aggregate all the 24 counties in zone 2 to calculate the fraction of rent-regulated units. The four NYC boroughs have 1.53 million renter-occupied housing units while the rest of zone 2 has 1.30 million. The resulting fraction of rent-regulated units in zone 2 is 10.4%.

From the NYCHVS, we also calculate the percentage difference in average rent in New York City between our definition of regulated rentals and the others (unregulated plus rent-stabilized). That percentage difference is 49.9%. We apply the same percentage difference to all of the MSA in our model.

To calibrate the income qualification parameter κ_2, we collect data on the income of tenants in RC from the New York Housing and Vacancy Survey. For each percentile of the income distribution of the RC tenants, we calculate what percentile of the overall income distribution in New York City it represents. This is reported in the second column of Table 5 for RC income percentiles listed in the first column. For example, a household at the 25th percentile of the income distribution among RC households is at the 7.7th percentile of the overall income distribution. Another household at the median of the RC income distribution is at the 11th percentile in the overall income distribution. The last three columns show the same statistics computed within the model, for three different values of the RC income qualification threshold κ_2. The value of $\kappa_2 = 40\%$ provides the best fit to the data. The higher value of $\kappa_2 = 60\%$ has RC tenants that are too rich compared to the data. For example, the median RC tenant sits at the 39th percentile of the overall income distribution, while in the data the median RC tenant only sits at the 19th percentile. This overstates the misallocation in the RC system, compared to the data. The lower value of $\kappa_2 = 30\%$ has RC tenants that are too poor compared to the data. It understates the amount of misallocation. In sum, the model with $\kappa_2 = 40\%$ matches well where RC tenants sit in the overall income distribution.

B.6 Migration

We use county-to-county migration data for 2006-2010 and 2010-2014 from the 5-year American Community Survey for the 25 counties in the New York metropolitan area. For each county and

Table 5: Income Distribution of RC Tenants

RC inc pctile	data	$\kappa_2 = 30\%$	$\kappa_2 = 40\%$	$\kappa_2 = 60\%$
RCp1	0.4	0.3	0.4	0.5
RCp5	1.9	1.0	1.4	3.4
RCp10	3.9	3.0	3.6	8.6
RCp15	5.2	3.9	8.7	9.1
RCp25	7.7	8.8	8.7	15.7
RCp33	11.0	8.8	11.2	26.8
RCp50	19.3	11.2	18.3	39.4
RCp66	30.2	14.8	27.4	39.8
RCp75	38.4	27.1	51.9	52.0
RCp85	51.5	51.7	59.9	61.2
RCp90	60.4	55.6	63.4	70.6
RCp95	74	60.9	73.8	79.7
RCp99	94.7	80.2	89.8	93.5

survey wave, we compute net migration rates (inflow minus outflow divided by population). When one person enters the New York labor market and another one leaves, the model is unchanged, so net migration is the relevant concept for the model. We aggregate net migration for the 24 counties in zone 2. The net migration rate over the 5-year period between 2010-2014 for the entire MSA is -0.15%, or -0.03% per year. First, this net migration rate is minuscule: only about 30,000 people moved in over a 5-year period on a MSA population of 20 million. Of course, this masks much larger gross flows: about 824,000 came into the MSA and 854,000 left. Second, Manhattan (zone 1) saw a net inflow of 30,000 people coming from outside the MSA while the rest of the MSA (zone 2) saw a net outflow to the rest of the country/world of 60,000. Third, comparing the net migration in the 2010-2014 period to that in the 2006-2010 period, we find that the net migration rate rose, from -73,000 to -30,000. The net migration rate rises from -0.38% in 2006-2010 to -0.15% in the 2010-2014 period. In other words, not only are the metro-wide net migration rates tiny, they also have little cross-sectional and time-series variation.

C   Earnings Calibration

Before-tax earnings for household i of age a is given by:

$$y_{i,a}^{lab} = W_t n_t^i G^a z^i$$

where G^a is a function of age and z^i is the idiosyncratic component of productivity. Since endogenous labor supply decisions depend on all other parameters and state variables of the model, exactly matching earnings in model and data is a non-trivial task.

We determine G^a as follows. For each wave of the Survey of Consumer Finance (SCF, every 3 years form 1983-2010), we compute average earnings in each 4-year age bucket (above age 21), and divide it by the average income of all households (above age 21). This gives us an average relative income at each age. We then average this relative age-income across all 10 SCF waves.

We also use SCF data to determine how the dispersion of income changes with age. We choose four grid points for income, corresponding to fixed percentiles (0-25, 25-75, 75-90, 90-100). To

compute the idiosyncratic income $z_{a,i}$ of each group $i \in \{1, 2, 3, 4\}$ at a particular age a, relative to the average income of all households of that age we do the following:

Step 1: For each positive-earnings household, we compute which earnings group it belongs to among the households of the same age.

Step 2: For each 4-year age bucket, we compute average earnings of all earners in a group.

Step 3: We normalize each group's income by the average income in each age group, to get each group's relative income.

Step 4: Steps 1-3 above are done separately for each wave of the SCF. We compute an equal-weighted average across all 10 waves to get an average relative income for each age and income group. This gives us four 11x1 vectors $z_{a,i}$ since there are 11 4-year age groups between ages 21 (entry into job market) and 65 (retirement). Note that the average z across all households of a particular age group is always one: $E[z_{a,j}|a] = 1$.

Step 5: We regress each vector, on a linear trend to get a linearly fitted value for each group's relative income at each age. The reason we perform Step 5, rather stopping at Step 4 is that the relative income at age 4 exhibits some small non-monotonicities that are likely caused by statistical noise (sampling and measurement error). Step 5 smoothes this out.

We set the productivity states to $z^i \in Z = [0.255, 0.753, 1.453, 3.522]$ to match the observed mean NY household income levels, scaled by the NY metro area average, in the income groups below \$41,000, between \$41,000 and \$82,000, between \$82,000 and \$164,000, and above \$164,000. Those bins respectively correspond to bins for individual earnings below \$25,000, between \$25,000 and \$50,000, between \$50,000 and \$100,000, and above \$100,000, adjusted for the number of working adults in the average New York household (1.64). The NY income data is top-coded. For each county in the NY metro area, we observe the number of individuals whose earnings exceed \$100,000. Because we also observe average earnings (without top-coding), we can infer the average income of those in the top coded group.

The transition probability matrix for z is $\mathbb{P}$ for β^L agents. We impose the following restrictions:

$$
\mathbb{P} = \begin{bmatrix} p_{11} & 1 - p_{11} & 0 & 0 \\ (1 - p_{22})/2 & p_{22} & (1 - p_{22})/2 & 0 \\ 0 & (1 - p_{33})/2 & p_{33} & (1 - p_{33})/2 \\ 0 & 0 & 1 - p_{44} & p_{44} \end{bmatrix}
$$

For β^H types, the transition probability matrix is the same, except for the last two entries which are $1 - p_{44} - p^H$ and $p_{44} + p^H$, where $p^H < 1 - p_{44}$. We pin down the five parameters

$$(p_{11}, p_{22}, p_{33}, p_{44}, p^H) = (0.93, 0.92, 0.92, 0.82, .02)$$

to match the following five moments. We match the population shares in each of the four income groups defined above: 16.1%, 29.8%, 34.2%, and 19.9%, respectively (taken from the individual earnings data). Given that population shares sum to one, that delivers three moments. We match the persistence of individual labor income to a value of 0.9, based on evidence form the PSID in Storesletten, Telmer, and Yaron (2006). Finally, we choose p^H to match the fraction of high-wealth households in the top 10% of the income distribution.

Table 6 summarizes the results we obtain in the model. Average earnings are reported annually. Earnings autocorrelation and volatility are reported for 4 years.

Table 6: Labor Earnings Calibration

	Group 1	Group 2	Group 3	Group 4	All
Avg earnings	25440	66685	123460	343643	
(Data)	(28125)	(60951)	(116738)	(309016)	
Pop shares	21.4%	28.3%	33.3%	17.1%	
(Data)	(16.1%)	(29.8%)	(34.2%)	(19.9%)	
Earnings autocorr.					0.77
Earnings vol.					0.125
Corr. (income,wealth)					0.23

Notes: Averages by bins in the data are obtained by multiplying average labor earnings ($124,091 annually) by the ratio of average earnings in each bin to the overall average (0.23, 0.50, 0.96, 2.42). Average household earnings and population shares in the data are denoted in parentheses and obtained from Census Bureau data for the year 2016.

D Progressive Taxation

Following Heathcote et al. (2017), households with income $y^{tot} < y_0^{tot} = \lambda^{\frac{1}{\tau}}$ receive transfers $T(y^{tot}) < 0$, and those with $y^{tot} \geq y_0^{tot}$ pay taxes $T(y^{tot}) \geq 0$. We set $\tau = 0.17$ and $\lambda = 0.75$, as discussed in the calibration section. As a result of our calibration, 35% of households are subsidized by the progressive tax system, and 29% receive a subsidy after subtracting Social Security taxes. Figure 7 describes the progressive taxation system. At low total income values, some households receive a subsidy, which progressively decreases. At higher incomes, taxes increase faster than income. This is reflected in households' after-tax income, shown in Figure 8.

Figure 7: Progressive Taxes
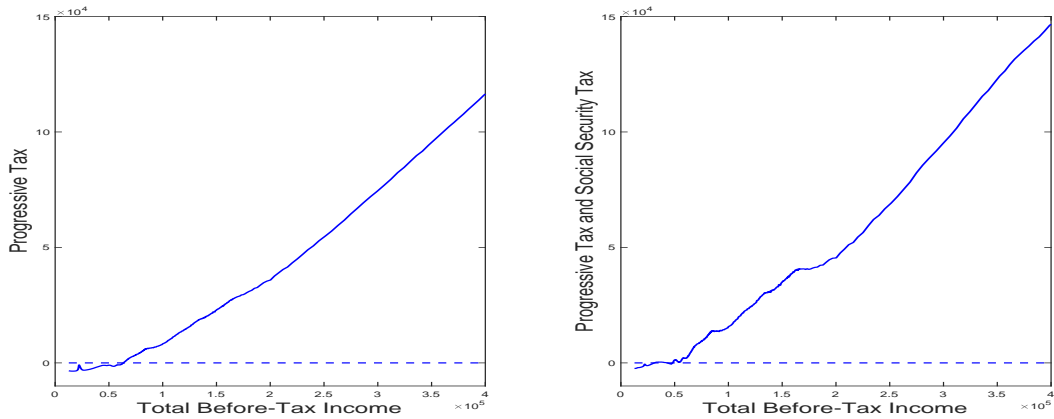
Notes: Horizontal axis: total income (in dollars, annual), measured as the sum of labor earnings, pensions, and financial income. Vertical axis: taxes minus transfers excluding Social Security taxes (in dollars, annual; left panel), total taxes minus transfers including Social Security taxes (in dollars, annual; left panel). The dashed line plots the zero-tax case.

Figure 8: After-tax Total Income
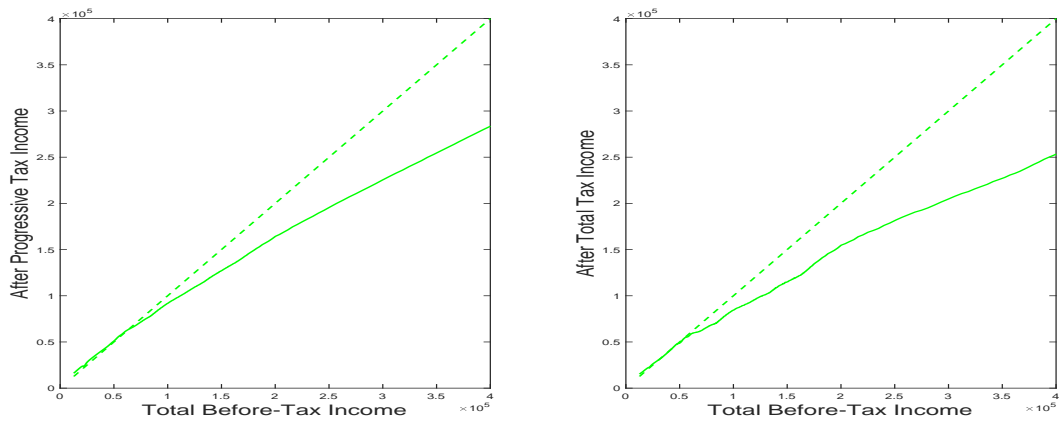
Notes: Horizontal axis: total income (in dollars, annual). Vertical axis: post-tax income excluding Social Security taxes (in dollars, annual; left panel), post-tax income including Social Security taxes (in dollars, annual; left panel). The dashed line is the 45 degree line.

66

E Housing Supply Elasticity Calibration

We compute the long-run housing supply elasticity. It measures what happens to the housing quantity and housing investment in response to a 1% permanent increase in house prices. Define housing investment for a given zone, dropping the location superscript since the treatment is parallel for both zones, as:

$$Y_t^h = \left(1 - \frac{H_{t-1}}{\overline{H}}\right) N_t^{\rho_h}.$$

Note that $H_{t+1} = (1 - \delta)H_t + Y_t^h$, so that in steady state, $Y^h = \delta H$. Rewriting the steady state housing investment equation in terms of equilibrium quantities using (8) delivers:

$$H = \frac{1}{\delta}\left(1 - \frac{H}{\overline{\overline{H}}}\right)^{\frac{1}{1-\rho_h}} \rho_h^{\frac{\rho_h}{1-\rho_h}} \overline{P}^{\frac{\rho_h}{1-\rho_h}} W^{\frac{-\rho_h}{1-\rho_h}}$$

Rewrite in logs, using lowercase letters to denote logs:

$$h = -\log(\delta) + \frac{1}{1-\rho_h}\log(1 - \exp(h - \overline{h})) + \frac{\rho_h}{1-\rho_h}\overline{p} - \frac{\rho_h}{1-\rho_h}w$$

Rearrange and substitute for $\overline{p}$ in terms of the market price $\overline{p} = \log(ho + (1 - ho)\kappa_4) + p$:

$$p = \frac{1-\rho_h}{\rho_h}h - \frac{1}{\rho_h}\log(1 - \exp(h - \overline{h})) + k$$

where

$$k \equiv \frac{1-\rho_h}{\rho_h}\log(\delta) + w - \log(ho + (1 - ho)\kappa_4)$$

Now take the partial derivative of p w.r.t. h:

$$\frac{\partial p}{\partial h} = \frac{1-\rho_h}{\rho_h} + \frac{1}{\rho_h}\frac{\exp(h - \overline{h})}{1 - \exp(h - \overline{h})} + \frac{\partial k}{\partial h}$$

Invert this expression delivers the housing supply elasticity:

$$\frac{\partial h}{\partial p} = \frac{\rho_h}{1 - \rho_h + \frac{\exp(h-\overline{h})}{1-\exp(h-\overline{h})} + \left[\rho_h\frac{\partial w}{\partial h} - \rho_h\frac{(1-\kappa_4)}{ho+(1-ho)\kappa_4}\frac{\partial ho}{\partial h}\right]} \tag{54}$$

If (i) the elasticity of wages to housing supply is small ($\frac{\partial w}{\partial h} \approx 0$) and either the RC distortions are small ($\kappa_4 \approx 1$) or the home ownership rate is inelastic to the housing supply ($\frac{\partial ho}{\partial h} \approx 0$), or (ii) if the two terms in square brackets are positive but approximately cancel each other out, then the last two terms are small. In that case, the housing supply elasticity simplifies to:

$$\frac{\partial h}{\partial p} \approx \frac{\rho_h}{1 - \rho_h + \frac{\exp(h-\overline{h})}{1-\exp(h-\overline{h})}}$$

Since, in equilibrium, $Y^h = \delta H$, $\partial y^h/\partial p = \partial h/\partial p$.

Note that $h - \overline{h}$ measures how far the housing stock is from the constraint, in percentage terms.

As H approaches $\overline{H}$, the term $\frac{\exp(h-\overline{h})}{1-\exp(h-\overline{h})}$ approaches $+\infty$ and the elasticity approaches zero. This is approximately the case in zone 1 for our calibration. If H is far below $\overline{H}$, that term is close to zero and the housing supply elasticity is close to $\frac{\rho_h}{1-\rho_h}$. That is approximately the case for zone 2 in our calibration. Since zone 2 is by far the largest component of the New York metro housing stock, zone 2 dominates the overall housing supply elasticity we calibrate to.

In the calibration, we use equation (54) to measure the housing supply elasticity and set $\frac{\partial w}{\partial h} = 0.25$ based on evidence from Favilukis and Van Nieuwerburgh (2018), who study a model with aggregate shocks to housing demand driven by out-of-town home buyers.

F Mobility Rates

Figure 9: Moving Rates
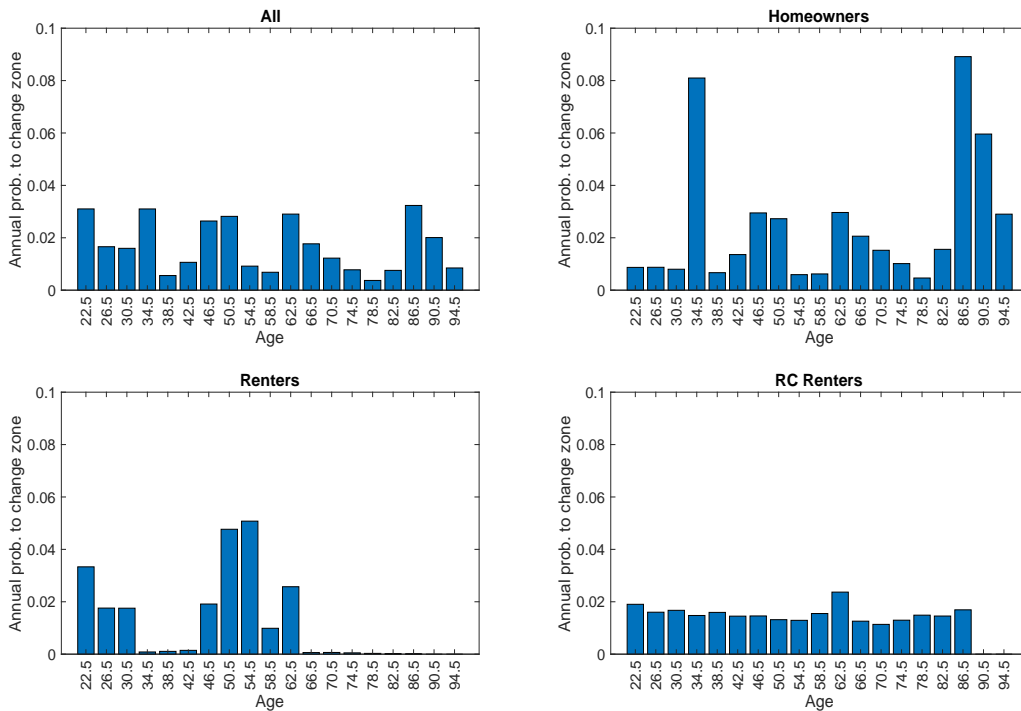
Notes: Mobility rates by age are measured as the annual probability to move across zones.

G   Additional Affordability Policies

In this appendix, we study six additional housing affordability experiments. The first five of them change features of the RC program, generally in the direction of making the RC program less generous. We (i) reduce the size of the system (governed by η^ℓ), (ii) reduce the rent subsidy (governed by κ_1), (iii) lower the size of the RC units (governed by κ_3), (iv) move all RC housing in zone 1 to zone 2, and (v) move all RC housing from zone 2 to zone 1 but provide for free transportation to RC residents (in zone 2). Experiment (vi) studies an expansion of the Low Income Housing Tax Credit program. Table 7 and Figure 10 summarize the results.

G.1   Reducing the Amount of RC Housing

This policy is the opposite of the one discussed in Section 5.2. We reduce η^ℓ in each zone by 50%. This allows us to study the symmetry of the welfare results. Given the curvature of the value function at low wealth levels, poor households are more adversely affected by a discrete reduction in the RC system than they are helped by a same-sized expansion. However, a reduction in the scope of the RC system also reduces distortions. On balance, a 50% reduction in the scope of the RC program leads to a -0.62% welfare loss, fairly similar in absolute value to the 0.66% gain from a 50% expansion.

The 50% reduction in the square footage of RC leads to a -76.78% drop in the fraction of households in RC due to endogenous changes in the size of RC units. The fraction of low-income households in RC falls only slightly less. Thus, access to RC insurance deteriorates substantially (-75.06%). Lower rents benefit market renters in both zones. A slight improvement in output (0.04%) and a reduction in commuting times (-0.04%) offset, but cannot make up for the concentrated losses on the poor. Housing affordability metrics improve, underscoring that maximizing welfare and improving housing affordability metrics can be conflicting objectives.

G.2   Reducing the Rent Subsidy for RC Housing

In the second additional experiment, labeled "RC discount," we reduce the rental discount that RC households enjoy. Specifically, we lower the rent discount from 50% to 25%; the parameter κ_1 goes from 0.5 to 0.75. The total square footage that goes to RC housing in each zone (η^ℓ) remains at the benchmark values. Surprisingly, the reduced generosity of the RC system leads to a 23.15% increase in the fraction of households in RC. The reason is that RC households tend to choose a much smaller average RC apartment size (-23.06% on average across zones). Thus, this policy has similar effects as a policy that has more "micro units" in the affordable housing system.[40] RC housing becomes less efficiently allocated; the share of bottom-quartile households that ends up in RC increases by 20.00%, less than the overall increase. Not surprisingly, those in RC under the new policy are worse off than those in RC under the benchmark policy since they are paying higher rents (due to the smaller discount) and they live in much smaller units. On the other hand, there are now more households benefiting from a lower rent level. Access to RC insurance improves substantially. On net, the reduced generosity of the RC insurance results in a non-trivial welfare loss (-0.49%). In sum, the main source of the welfare loss is the reduced *value* of the insurance provided by the RC system –RC rents are not as low as before– which disproportionately hurts

---

[40]Such a policy is advocated by several policy institutes, for instance the NYU Furman Center (2018). New York City historically discouraged the development of small apartment units due to a variety of rules and regulations.

low-income households. While access to insurance improves, the extent of insurance provided deteriorates.

Despite the reduced distortions on housing construction, the aggregate housing stock ends up falling (due to the decrease in zone 2's demand), illustrating the pitfalls of partial equilibrium logic. Output falls modestly as hours worked fall.

Table 7: Additional housing affordability policies

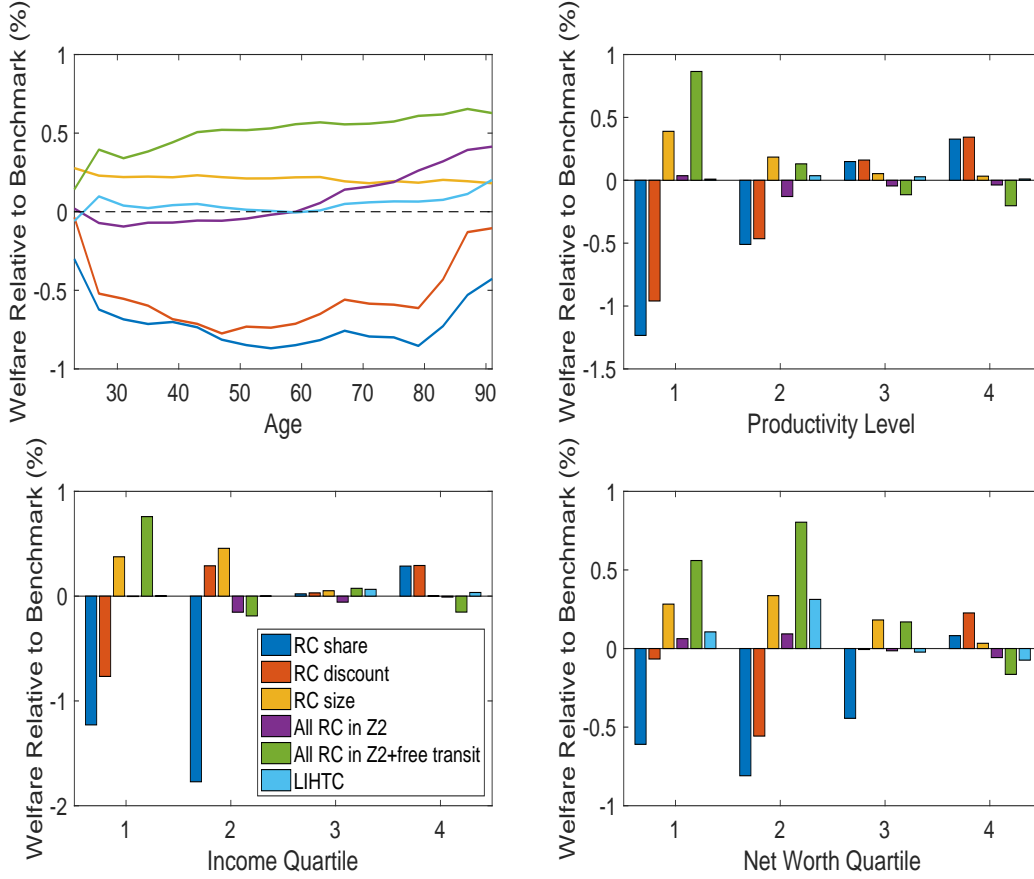|		Benchmark	RC share	RC discount	RC size	RC in Z2	RC in Z2 +free transit	LIHTC
1	Avg(rent/income) renters, Z1 (%)	28.3	-8.24%	9.67%	0.19%	-16.34%	-17.65%	-0.90%
2	Avg(rent/income) renters, Z2 (%)	29.2	0.76%	1.67%	-0.71%	0.68%	1.36%	-0.23%
3	Frac. RC (%)	5.98	-76.78%	23.15%	8.36%	–	–	0.55%
4	Frac. RC of those in inc. Q1 (%)	14.24	-75.32%	20.00%	7.83%	0.62%	-0.30%	-0.03%
5	Frac. rent-burdened (%)	48.7	-9.61%	-5.26%	-0.69%	0.22%	-1.73%	-2.05%
6	Avg. size RC unit in Z1 (sqft)	683	-1.01%	-13.37%	-13.02%	–	–	-0.02%
7	Avg. size RC unit in Z2 (sqft)	1173	1.65%	-25.76%	-7.15%	0.26%	2.19%	0.47%
8	Avg. size Z1 mkt unit (sqft)	999	-2.06%	-1.15%	0.20%	-2.63%	-1.79%	-0.03%
9	Avg. size Z2 mkt unit (sqft)	1521	-0.76%	1.81%	0.49%	-0.25%	-0.54%	0.25%
10	Frac. pop. Z1 (%)	10.5	-0.12%	3.63%	1.49%	-1.09%	-2.04%	0.35%
11	Frac. retirees Z1 (%)	21.5	-2.79%	-1.28%	-0.33%	0.21%	1.93%	-0.93%
12	Housing stock Z1	–	0.18%	0.56%	-0.01%	0.58%	0.50%	0.24%
13	Housing stock Z2	–	-0.08%	-0.20%	-0.10%	-0.47%	-0.52%	0.22%
14	Rent/sqft Z1 ($)	4.21	-0.93%	-0.90%	-0.11%	0.10%	0.07%	-0.40%
15	Rent/sqft Z2 ($)	1.52	-1.17%	-1.20%	-0.14%	0.17%	0.15%	-0.51%
16	Price/sqft Z1 ($)	1050	-0.94%	-0.90%	-0.11%	0.10%	0.07%	-0.41%
17	Price/sqft Z2 ($)	294	-1.18%	-1.20%	-0.14%	0.17%	0.15%	-0.52%
18	Home ownership Z1 (%)	45.1	5.27%	-3.51%	-1.35%	5.11%	6.73%	-0.38%
19	Home ownership Z2 (%)	59.6	1.96%	-1.68%	-0.20%	-0.55%	-0.66%	-0.20%
20	Avg. income Z1 ($)	164422	2.27%	-2.37%	-1.45%	5.01%	5.87%	0.11%
21	Avg. income Z2 ($)	100154	-0.33%	0.16%	0.11%	-0.88%	-1.04%	-0.02%
22	Frac. high prod. Z1 (%)	29.7	-1.65%	-2.65%	-0.31%	0.74%	-5.00%	-5.61%
23	Total hours worked	–	0.22%	-0.12%	-0.09%	-0.07%	-0.31%	0.00%
24	Total hours worked (efficiency)	–	0.08%	-0.04%	-0.06%	-0.05%	-0.11%	-0.01%
25	Total output	–	0.04%	-0.03%	-0.04%	-0.06%	-0.10%	-0.00%
26	Total commuting time	–	-0.04%	-0.47%	-0.18%	0.13%	0.28%	-0.06%
27	Access to RC insurance (%)	5.6	-75.06%	19.02%	10.19%	-1.83%	-3.47%	0.13%
28	Stability of RC insurance (%)	72.5	-0.63%	-2.88%	0.05%	0.44%	0.69%	-0.02%
29	Std. MU growth, nondurables	0.45	-0.51%	-0.61%	0.07%	-0.06%	0.33%	0.30%
30	Std. MU growth, housing	0.45	0.99%	-0.38%	-0.15%	6.16%	5.96%	0.28%
31	**Aggregate welfare change**	–	**-0.62%**	**-0.49%**	**0.23%**	**-0.03%**	**0.37%**	**0.02%**

Notes: Column "Benchmark" reports values of the moments for the benchmark model. Columns "RC share" to "LIHTC" report percentage changes of the moments in the policy experiments relative to the benchmark. Rows 1-8 report housing affordability moments , rows 9-24 aggregate moments across the two zones, and rows 25-26 welfare moments. Z1 stands for zone 1 (Manhattan), Z2 for the rest of the metro area. Row 20 reports what fraction of working age top-productivity households live in zone 1.

G.3 Reducing the Maximum Size of RC Housing

The third additional experiment changes how "deeply affordable" RC units must be, governed by the parameter κ_3. In the benchmark, affordable housing units have a rent expenditure cap of 35% of AMI on rent, or about $2400 per month. Here we change this cap to 20% of AMI or about $1400 per month. This effectively tightens the constraint on the maximum size of a RC unit.

As shown in the column "RC housing size" of Table 7, the policy change indeed lowers the average RC unit size, by -13.02% in zone 1 and -7.15% in zone 2. Because the total amount of

Figure 10: Welfare effects of additional housing affordability policies

square footage is unaffected by the policy change, the fraction of households in RC increases by nearly the same amount (8.36% on average across zones). Tighter RC size requirements (lower κ_3) are less effective at improving the targeting of the RC system than tighter income requirements (lower κ_2). The fraction of low-income households in RC grows at 7.83%, a slower rate than the aggregate. This suggests that the RC size constraint has only limited bite, given the income limit already in place.

The policy generates a moderate welfare gain of 0.23%. Access to RC insurance expands, commuting times are reduced, while output only falls marginally. Figure 10 shows that the welfare gains are declining in productivity and income, and uniformly positive among age groups. These results are similar to those for the policies we studied in Section 5.1, which also aimed at improving targeting. But clearly, reducing the house size is a less potent way of addressing the inefficiencies in the RC system.

G.4 Spatial Allocation of RC Housing

In the fourth additional experiment, we explore the spatial dimension of our model by studying the effects of a policy that shifts all RC *units* from Manhattan to zone 2. That is, η^1 is now set to

zero and η^2 is increased to make up for the loss of RC housing in zone 1 until the overall number of *households* that enjoy RC is the same as in the benchmark model. As a result of this relocation, the average RC unit becomes 10.47% larger. This is a composition effect because RC units in zone 1 were much smaller than in zone 2 in the benchmark model. Thus, this policy requires a modest expansion in the square feet of RC housing.

The removal of the affordable housing mandate in Manhattan spurs on housing development. The housing stock in zone 1 increases (0.58%). The space formerly occupied by RC units in zone 1 is absorbed by market renters and owners. With the departure of RC tenants, the population of Manhattan becomes more affluent. Average income in zone 1 is 5.01% higher and the average dwelling size across all units increases (1.67%). Reflecting the more affluent population, the home ownership rate increases 5.11%. Zone 1 sees a small increase in rents (0.10%) reflecting the greater equilibrium housing demand. Because of higher average income and despite slightly higher rents, the rent-income ratio among renters in zone 1 falls substantially (-16.34%). Clearly, this does not mean that housing became more affordable in zone 1. In fact, there are no more affordable housing units in zone 1 left and the market rent went up.

The expansion of RC in zone 2 reduces developer incentives to build in zone 2. The housing stock decreases (-0.47%) along with average unit size (-0.60%), while rents increase (0.17%). Average rent-to-income increases by 0.68% in zone 2. As measured by the fraction of rent-burdened households metro-wide (+0.22%), and rent-to-income ratios, overall housing affordability deteriorates with this spatial reallocation of RC. With fewer households in zone 1 (-1.09%) and more retirees, total commuting time increases (0.13%). The policy attains its presumed goal, which is to attract more top-productivity households to zone 1. But the gain is small (0.74%).

The policy generates a small welfare loss (-0.03%). The main reasons for the loss are, first, smaller housing units in both zones, as zone 1 and zone 2 market rate rental units become smaller and more expensive. Second, the probability of first-time access for low-income households with a negative income shock drops (-1.83%). Incumbents are now more reluctant to voluntarily give up their RC apartment given that it is larger than the average RC unit in the benchmark model. Most importantly, low income households in RC now must pay for commuting expenses which affects their budgets disproportionally. Figure 10 summarizes the welfare effects. Young households lose while older households gain. The welfare losses would be larger still if households had an explicit preference for socio-economic diversity in every neighborhood.

G.5   Spatial Allocation of RC Housing with Free Transit

Fifth, we consider a policy that combines the relocation of RC housing from zone 2 to zone 1 with a policy that provides free transportation to all RC tenants. This policy is reported in the fifth column of Table 7. To finance the free transportation, taxes are increased on everyone. This is accomplished by changing the parameter λ in the tax and transfer function by the right amount. The total cost of providing this free transportation for all RC residents is \$974 million; this is the amount of additional taxes raised. We find that relocation of RC housing to zone 2 is welfare increasing once accompanied by free transportation; the aggregate welfare gain is 0.37%, a substantial increase from the -0.03% aggregate welfare without free transit. The reason is that financial transportation costs are meaningful for low-income households and were the source of the welfare loss in the previous experiment. For comparison, a policy that provides free transport but leaves the spatial distribution of RC housing unchanged generates a welfare gain of 0.29%. In sum, conditional on providing free transportation, moving RC housing from zone 2 to zone 1 is good for welfare.

G.6 Low-Income Housing Tax Credits

The final policy experiment studies a common policy tool in affordable housing: tax credits for developers. Tax credits directly incentivize the development of affordable housing units by giving developers subsidies to offset the cost of affordable housing construction.

G.6.1 Institutional Background

Developers who receive tax credits for affordable housing development can sell them to other profitable firms; they fetch prices above 90 cents on the dollar. In other words, they are (nearly) equivalent to cash subsidies.

The main program, the federal Low Income Housing Tax Credit (LIHTC) subsidizes 30% of the construction cost associated with affordable housing units. This is known as the 4% program. A 4% subsidy of construction costs is given over a 10-year period, and is worth 30% in present-value terms. There is a second program, the 9% subsidy for 10 years, which is worth 70% in present-value terms, which is aimed at more deeply affordable housing units. We focus on the 4% program. Total spending on LIHTC is $9 billion annually nationwide; about $50 million in the New York MSA. Additional programs, like Tax Incremental Financing, are ran by municipal governments. We focus on LIHTC.

LIHTCs are often used to subsidize mixed market rate-affordable housing projects. In practice, it is up to the states to decide how to spend their federal LIHTC allocations depending on which areas and which points of the income distribution they want to target. In places like Manhattan with high costs of land and high construction costs, the only way to break even on an affordable housing development through tax credits, given the rules of the LIHTC, is to build a mixed property with market-rate and affordable units. Such areas are known as *Difficult to Develop Areas*. They are our focus. Davis et al. (2017) study instead low-income tracts with a poverty rate above 25%, the so-called the QRT areas.

G.6.2 Modeling Tax Credits

In our model, developers in a given zone earn a price per sqft built equal to the market price times a discount; recall equation (6), repeated here for convenience:

$$\overline{P}_t^\ell = \left(ho_t^\ell + (1 - ho_t^\ell)\kappa_4^\ell \right) P_t^\ell.$$

The discount depends on the fraction of units that are owned (ho_t^ℓ) and the rent discount due to RC housing κ_4^ℓ. We now assume that developers receive a subsidy to help offset the rent discount due to RC:

$$\kappa_4^\ell = 1 - \eta^\ell + \eta^\ell \kappa_1 (1 + LIHTC).$$

In the benchmark model, the parameter $LIHTC = 0$. In the LIHTC experiment, we choose $LIHTC$ such that the total value of LIHTC subsidies, aggregated first across all firms within each zone and then across zones, is equal to 30% of the construction costs associated with the construction of RC housing. We compute the construction costs of RC housing as follows. Since the only input is labor, we take the total wage bill in each zone (aggregating across firms) and multiply it by the share of RC sqft to compute the construction costs associated with RC housing in that zone; then we sum across zones and multiply by 30%. That gives us the total value of LIHTC subsidies. Matching the construction costs of RC housing requires a value for the parameter $LIHTC$ of 0.39 or 39%. We assume it is identical across zones. When accounting for ownership rates and shares

of RC sqft in each zone, the tax credits increases the average price $\overline{P}_t^{\ell}$ earned by developers in zone 1 by 18.41% in zone 1 and 13.99% in zone 2. We then change the value of λ in the tax-and-transfer function to generate enough additional tax revenue to exactly pay for the aggregate tax credit outlay. The policy is budget neutral, like the previous experiments. The size of the program, the extra tax dollars raised (and spent), is calibrated to be the same as for the voucher program discussed in Section 5.4, and equals \$738mi. Hence, these two programs are directly comparable. Developers continue to build market rate and affordable units in proportions $1 - \eta^{\ell}$ and η^{ℓ}.

G.6.3 Results

The LIHTC policy has only very modest aggregate welfare benefits (0.02%). Most households gain across age, productivity, and income groups; while the wealthy suffer minor welfare losses. The policy succeeds in its intended purpose, to increase the equilibrium housing stock (0.24% in zone 1 and 0.22% in zone 2). While developer incentives to build are strengthened, equilibrium housing demand becomes weaker because of the higher tax burden to pay for the tax credits. The increase in supply and fall in demand for housing results in lower rents and house prices. The reduction in house prices is consistent with Diamond and McQuade (2019), who find negative spillovers on house prices in higher-income neighborhoods from LIHTC properties, since Manhattan and the average county in zone 2 of the New York metro area represent fairly wealthy areas.[41] Higher taxes reduce labor supply, but a negative wealth effect from lower house prices counters the effect, leaving hours and output are nearly unaffected. The policy only leads to a modest improvement in housing affordability (small decreases in the rent-income ratios and fraction of rent-burdened households). Poor, high-marginal utility households gain only modestly. The volatility of marginal utility growth increases, suggesting a deterioration in risk sharing.

One reason for the modest success is that the LIHTC policy in Difficult to Develop Areas increases developers' incentives to build in general, rather than directly targeting low-income housing. Thus, the benefits are diffuse and offset by the negative effects from distortionary taxation.

---

[41]They find the opposite effect in low-income neighborhoods, and attribute the positive spillovers to neighborhood revitalization and crime reductions which attract a different population. We are holding the amenity value of a neighborhood constant.

H Robustness

We study how our policy results change with key model parameters. For each parameterization, we first do a comparative statics exercise describing how the equilibrium differs from the equilibrium in our benchmark calibration. Then, we conduct a difference-in-difference exercise, describing the responses of welfare, prices, and quantities to the main policy experiments, and contrasting them with the responses in the main text. For reasons of space, Table 8 reports a subset of the moments, where the row numbers refer to the full table (see Table 3). The complete set of moments for each of the exercises are available upon request.

H.1 Lower Risk Aversion

In the benchmark calibration, the coefficient of relative risk aversion is $\gamma = 5$. We study an alternative economy where $\gamma = 2$ and compare it to the benchmark economy. For this case (only), we recalibrate the other parameters to match the model's targets, in particular the average level of the time discount factor β, to match average net worth/income, and its dispersion, to match the Gini coefficient for net worth. This calibration matches the targets about equally well as the benchmark calibration. We then compute the main experiments within this new economy.

There are a few important differences in the benchmark model with $\gamma = 2$ compared to the benchmark model with $\gamma = 5$. The home ownership rate in zone 1 is more than 10 percentage points higher than in the $\gamma = 5$ case, while that of zone 2 is mostly unchanged. As a result, the ownership rate wedge between zones decreases, a counterfactual implication which does not occur for $\gamma = 5$. This is because average income of zone 1 and the share of top-productivity people living in zone 1 are noticeably higher when $\gamma = 2$. Most importantly, a lower value for γ lowers the *value of insurance* provided by affordable housing programs. The volatilities of marginal utility growth of non-housing (0.26 vs. 0.45) and housing consumption (0.26 vs. 0.45) fall substantially, as a result of the reduced curvature of the utility function.

Turning to the economy's response to the policy experiments, we generally find qualitatively consistent but quantitatively smaller welfare changes. The combination policy of lowering the income qualification threshold and abolishing incumbent priority still generates the largest welfare gains. Expanding the scope of the affordable housing mandate and upzoning are about equally welfare improving. The voucher experiment changes the most, from strongly welfare increasing to mildly welfare decreasing. Starting off from a lower level of incompleteness and lower marginal utility for low-productivity households, improving risk sharing opportunities is not as valuable as in the $\gamma = 5$ world. However, the distortions engendered by the RC system and the taxes to pay for the voucher expansion remain as large.

H.2 No Taste Shifter for Zone 1 Living

We solve a special case of the model where $\chi^1 = \chi^W = \chi^R = 1$, so that there is no extra utility from living in zone 1. All other parameters are the same as in the benchmark economy. We do not recalibrate the model. Relative to our benchmark calibration, more households choose to live in zone 1. This is because the cost of housing in zone 1 is about 40% lower as in the benchmark calibration. Zone 1 residents tend to be younger, less productive, and poorer. Average income in zone 1 is now substantially *lower* than in zone 2, with the magnitudes essentially reversed from the benchmark calibration. There are a lot fewer retirees in zone 1 because luxury amenities were key to attract them in the benchmark calibration. They do not work, and are no longer willing

to spend more on housing to live in zone 1 when $\chi^1 = \chi^R = 1$. The higher population share of Manhattan is due to general equilibrium effects: zone 1 is less desirable, so housing becomes cheaper, attracting more households. Because financial commuting costs represent a larger share of poor households' expenditures, more of those households choose to live in zone 1 to avoid those costs. As a result, the spatial misallocation of labor increases, as nearly all of the top-productivity, working-age households live in zone 2 and commute to zone 1 for work. Without extra amenities in zone 1, the location choice solely depends on purchasing power considerations; based on comparing house prices, rents, commuting costs, and income. Thus, prices and rents induce more low-productivity households to live in zone 1, and more high-productivity households to live in zone 2. Because of its lower-income and lower-wealth residents, zone 1 now has a home ownership rate that is 10% points lower than in our baseline calibration. Income inequality within zone 1 is dramatically lower; the earnings Gini is 0.17 compared to 0.46 in the baseline calibration. This exercise underscores the importance of the amenity value of zone 1 in the baseline model.

Next, we turn to the policy experiments. In the first three experiments, which improve the targeting of RC, the gains are even larger than before. The population growth of zone 1 is larger and so is the reduction in commuting time. Even more low-income households can live in zone 1 and save the commuting cost. Access to insurance increases substantially. In the fourth experiment, increasing the scope of RC has about the same welfare gain as in the model with amenity shifters. The population of zone 1 now falls in response to the policy, while it rose in the baseline calibration, but in both cases the population of zone 1 becomes poorer in response to the policy. The upzoning policy in zone 1 naturally creates a smaller welfare gain in the absence of preferences for living in zone 1 since upzoning not only creates better access to jobs for more households but also enjoyment of city-center amenities for more households. Ignoring the amenity value of housing in the city center leads to weaker welfare results. Finally, the voucher experiment creates much smaller welfare gains than in the baseline calibration. The reason is that there is less (housing) inequality, and therefore less benefit from redistribution.

H.3 No Luxury Taste Shifter for Manhattan

We now keep the overall taste shifter for zone 1, $\chi^1 > 1$, at its benchmark value, and set $\chi^R = \chi^W = 1$, so that there is no extra taste shifter for zone 1 for wealthy households ($c_t > \underline{c}$). In the benchmark calibration, the luxury taste shifter for retirees $\chi^R = 1.038$ was substantially above 1 while that for workers $\chi^W = 1.004$ was not.

The main change compared to the original calibration is that there are very few retirees who choose to live in zone 1 now. Because they do not work and face lower time and financial commuting costs, they do not value the time and financial savings of living close to work. The model features a larger overall fraction of households live in zone 1 in equilibrium, like in Section H.2, and dwellings that are even smaller in Manhattan relative to zone 2. Manhattan now contains more young households and also significantly more working-age top-productivity households. Put differently, the luxury taste shifter in the main text acts as a barrier to entry in the Manhattan housing market for young, high-productivity households. Once the barrier removed, the spatial misallocation of labor is lower than in the baseline calibration. Average income in Manhattan is slightly lower than in the baseline calibration and so is income inequality between the two zones. However, the reversal in the relative incomes of zone 1 and zone 2 we saw for the case with no taste shifters is now gone. This shows that $\chi^1 > 1$ is crucial for generating higher average income in zone 1, and that luxury taste shifters (esp. χ^R) mostly affect the fraction of wealthy retirees in the city center. Price and rent per unit fall with the reduction in unit size, but the price and rent

per square foot as well as their ratio across zones are not affected. We also observe lower mobility rates after initial location choices have been made because more young high-productivity people already live in zone 1 rather than having to move there later once they have accumulated sufficient wealth.

Turning to the difference-in-difference results in Panel D of Table 8, we find that the first three experiments that improve the targeting of RC have nearly the same effect as in the benchmark model. Increasing the share of RC sqft by 50% also has nearly the same positive welfare effect as in the benchmark calibration. One difference is that the population of zone 1 now falls rather than rises because the size of a zone 1 market units becomes larger rather than smaller. This occurs because the policy change triggers a smaller reduction in the average income of zone 1 residents. There were already more low-income residents in Manhattan to start with in the model without luxury taste shifters. The effects of upzoning in the urban core remain positive and of similar magnitude. The absence of wealthy and/or old households who live in zone 1 for preference reasons permits an increase of top-productivity households who move to zone 1 after the upzoning. The voucher experiment has nearly unchanged welfare effects from those that start from the baseline calibration.

H.4 No Discount Factor Heterogeneity

We now impose that all agents have the same discount factor, $\beta^H = \beta^L$, and recalibrate the average β to keep the same net worth to income ratio as in the benchmark model. As a result, wealth inequality is much lower (the Gini coefficient for net worth decreases from 0.80 to 0.60 in the metro area), and so is inequality *between* zones (the ratio of zone 1 to zone 2 net worth decreases from about 1.16 to 0.68). With less wealth inequality, the investment demand for housing from the richest households falls. As a result, the price-to-rent ratios fall in both zones, and home ownership rates are significantly higher than in the baseline calibration. The model has a slightly larger share of households living in zone 1. The fraction of retirees is smaller, because there are fewer high-wealth retirees for whom the luxury amenity value of living in the city center kicks in. But there are also substantially fewer working-age, top-productivity households living in zone 1. Thus, the spatial allocation of labor is worse than in the model with beta heterogeneity. The equal-beta economy features somewhat better risk sharing than in our main model. With less wealth inequality, there are fewer low-wealth, high-MU households with volatile MU growth. Finally, the fraction of households in RC is much lower than in the baseline; access to RC insurance is much lower. This occurs because RC tenants choose larger units, again reflecting the fact that there are fewer low-wealth households in this economy.

The equal-β economy's reaction to the various experiments is qualitatively similar to that of the heterogeneous-β economy. Quantitatively, the results are slightly weakened. Increasing the generosity of affordable housing programs is less valuable in societies with lower wealth inequality.

H.5 Profit Redistribution

In the benchmark model, all profits from tradeable and construction firms left the city because we assumed that their shareholders were outside the city. In reality, some goods firms and construction firms may be locally owned. As a last robustness check, we assume that 50% of profits are redistributed to local households. We assume that this profit redistribution occurs uniformly across households and lump sum. This generates conservative estimates for the effects on wealth

inequality, as the empirical distribution of profit shares is likely to be right-skewed. All other parameters are kept at the benchmark level.

We find that zone 1 is smaller than in the benchmark and contains many more retirees. Wealth inequality between zones increase. Essentially, there are more high-net-worth households in Manhattan, living in much larger apartments than before. House prices are much higher in both zones but especially in zone 1. Zone 1 is less affordable as measured by price-income and rent-income ratios, and there are substantially more rent-burdened households in this economy. More households crowd into much smaller RC units. The profit redistribution economy features less market incompleteness, so that risk sharing is better than in our main model. The volatility of marginal utility growth decreases because the profit income provides an extra buffer for households, making it easier to smooth consumption.

The economy's reaction to the various experiments is qualitatively similar, except for the vouchers experiment. Quantitatively, the results are similar to those in the equal-beta economy which also featured less wealth inequality. The largest difference is for the voucher experiment, where welfare increases by much less than in the baseline calibration and in the equal-beta calibration. As for the other redistributive experiments, a larger voucher program has less value in an economy where poor households have a lower marginal utility of consumption because they are richer. Additionally, the distortions from progressive taxation of the voucher expansion are larger. Because the redistributed profits increases the tax base, the progressive tax system now hurts middle-income households more. Some households switch from receiving a transfer to paying a tax. Because those taxes make households poorer, they end up demanding less housing in equilibrium.

I   Transitional Dynamics

Figure 11 plots the results from a transition experiment. For each policy change discussed in the main text, we solve the following model. We start the model off in the benchmark economy B. With some small probability (set to 1%), the economy transitions to the alternative economy (A) which has undergone the policy change. With the same small probability it transitions back from A to B at some future date. Agents in the model take into account the possibility of the policy change; it is reflected in all their actions and in their value function. After having solved for policy and value functions, we simulate the economy for 5,000 periods and 2,000 households. We select all transitions where the economy switches from B to A. We then plot the change in value function $V(A)/V(B)$, expressed as a CEV. We average this change across all households in a group and across all switches we observe in the simulation. The first four panels plot the change in value function by age, productivity, income, and net worth. The identity of the households, who enters in which group, is determined based on the value in the baseline economy B in the last period before the switch. For example, we follow a group with the same households in the bottom of the income distribution *in the baseline economy*. In the bottom left panel, we sort households into market renters, owners, and RC renters, based on their status just prior to the policy change. We note that it is possible that some policy may change the tenure status of a given household. If one wants to understand how upzoning affects home owners, one needs to hold the identity fixed of a group of households who were homeowners just prior to the policy change. The bottom right panel plots house prices in zone 1 and zone 2, and rents in zone 1 and 2. Like value functions, house prices are forward looking and their change in the first period of the transition reflects the expected discounted present value of what happens in future periods.

We note that upzoning reduces house prices. This affects homeowners; the more primary and

78

investment housing they own, the larger the hit their net worth takes. The average owner still gains from upzoning (by a small amount), even after taking into account the house price. But some owners lose. Also, not every income group gains. We conclude that the zoning experiment is not quite a Pareto improvement.

The second interesting take-away from the transitional graph is that households who are in RC in the benchmark economy suffer from a policy that tightens income requirements with or without re-qualification. This is the classic insider-outsider problem. The same is true for an expansion of RC. Even though these policies increase access to RC insurance on average, the incumbents may lose their access after the policy change.

Figure 11: Welfare in a Model with Transitions.
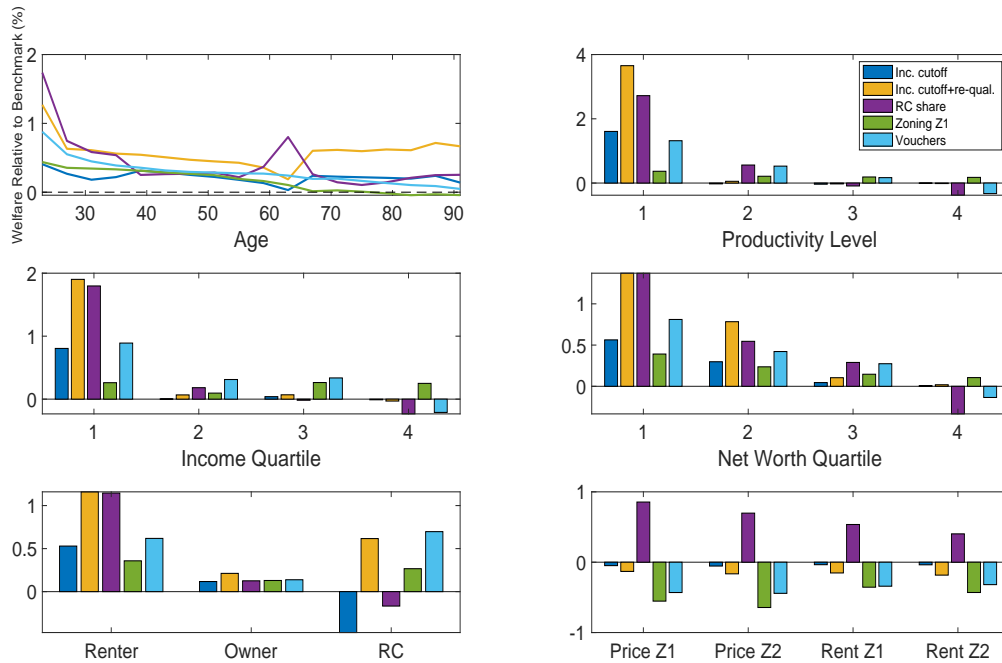
Notes: The baseline model has the following parameters: $\eta^1 = 24.46$, $\eta^2 = 15.97$, $\kappa_1 = 50\%$, $\kappa_2 = 0.40$, $\kappa_3 = 0.35$. Policy experiments, each panel: decrease the RC income cutoff by 50%, enforce re-qualification for RC every period, both, increase share of RC sqft by 50%, increase available Z1 sqft by 10%, housing vouchers. Top left panel: by age. Top right panel: by productivity level. Middle left panel: by income quartile. Middle right panel: by net worth quartile. Bottom left panel: by tenure status. Bottom right panel: prices and rents in each zone. The welfare changes are measured as consumption equivalent variations for an average household in each group.

Table 8: Policy Experiments under Alternative Parametrizations

	Bench.	Low κ_2	Re-qual	Low κ_2 +re-qual.	RC share	Zoning	Voucher
				Panel A: Baseline Calibration			
31 **Aggregate welfare change (CEV)**	–	**1.17%**	**1.66%**	**3.59%**	**0.66%**	**0.37%**	**1.04%**
				Panel B: Lower Risk Aversion			
10 Frac. of population living in Z1 (%)	10.6	1.64%	3.56%	3.92%	4.99%	6.16%	-0.66%
25 Total output	–	0.02%	-0.22%	-0.05%	-0.04%	-0.02%	-0.67%
26 Total commuting time across all hhs	–	-0.27%	-0.58%	-0.56%	-0.86%	-0.57%	0.22%
27 Access to RC insurance (%)	5.5	54.01%	132.90%	179.29%	52.82%	0.50%	-3.49%
28 Stability of RC insurance (%)	70.6	0.59%	-89.51%	-37.68%	0.73%	0.44%	0.64%
29 Std. MU growth, nondurables	0.26	0.75%	1.08%	0.29%	0.83%	-0.10%	-1.81%
30 Std. MU growth, housing	0.26	-0.17%	7.47%	-1.31%	2.93%	0.30%	-0.82%
31 **Aggregate welfare change (CEV)**	–	**0.28%**	**0.13%**	**0.71%**	**0.27%**	**0.26%**	**-0.05%**
				Panel C: No Taste Shifter for Manhattan			
10 Frac. of population living in Z1 (%)	14.4	2.20%	3.37%	6.71%	-2.86%	7.52%	-1.77%
25 Total output	–	0.01%	-0.23%	-0.09%	-0.06%	0.00%	-0.80%
26 Total commuting time across all hhss	–	-0.39%	-0.72%	-1.15%	0.75%	-1.59%	0.30%
27 Access to RC insurance (%)	5.6	53.00%	146.86%	191.51%	50.11%	-1.45%	3.90%
28 Stability of RC insurance (%)	73.1	0.26%	-77.54%	-25.93%	0.14%	-0.05%	-0.39%
29 Std. MU growth, nondurables	0.45	-0.52%	-1.33%	-3.85%	0.54%	-0.06%	0.33%
30 Std. MU growth, housing	0.41	-0.49%	0.93%	-4.27%	1.98%	0.03%	0.86%
31 **Aggregate welfare change (CEV)**	–	**1.41%**	**2.12%**	**4.78%**	**0.71%**	**0.24%**	**0.24%**
				Panel D: No Luxury Taste Shifter for Manhattan			
10 Frac. of population living in Z1 (%)	14.4	-0.15%	2.30%	2.47%	-1.30%	9.93%	-0.18%
25 Total output	–	0.08%	-0.21%	-0.06%	-0.08%	0.02%	-0.69%
26 Total commuting time across all hhs	–	-0.09%	-0.52%	-0.46%	0.37%	-2.12%	-0.03%
27 Access to RC insurance (%)	5.6	56.64%	145.07%	185.21%	54.53%	0.02%	-5.08%
28 Stability of RC insurance (%)	72.5	0.22%	-79.20%	-31.87%	0.03	0.12	0.01
29 Std. MU growth, nondurables	0.45	0.43%	-0.69%	-1.81%	0.76%	-0.13%	-1.33%
30 Std. MU growth, housing	0.49	-0.10%	0.84%	-2.43%	0.30%	0.36%	-1.00%
31 **Aggregate welfare change (CEV)**	–	**1.13%**	**1.55%**	**3.48%**	**0.72%**	**0.34%**	**1.04%**
				Panel E: No Discount Factor Heterogeneity			
10 Frac. of population living in Z1 (%)	12.6	1.07%	1.78%	2.20%	1.34%	8.30%	3.57%
25 Total output	–	0.02%	-0.06%	-0.01%	-0.03%	0.00%	-0.51%
26 Total commuting time across all hhs	–	-0.17%	-0.36%	-0.37%	-0.20%	-1.37%	-0.72%
27 Access to RC insurance (%)	3.2	37.82%	140.22%	168.89%	47.31%	1.50%	0.53%
28 Stability of RC insurance (%)	72.5	0.75%	-146.48%	-108.24%	0.84%	0.33%	0.54%
29 Std. MU growth, nondurables	0.40	-0.07%	-0.17%	-0.99%	0.41%	-0.20%	-1.63%
30 Std. MU growth, housing	0.42	-0.31%	1.52%	0.92%	0.29%	2.86%	4.50%
31 **Aggregate welfare change (CEV)**	–	**0.52%**	**0.74%**	**1.21%**	**0.44%**	**0.36%**	**0.82%**
				Panel F: Profit Redistribution			
10 Frac. of population living in Z1 (%)	6.8	1.12%	1.71%	2.13%	4.18%	8.92%	-1.72%
25 Total output	–	0.09%	-0.21%	0.10%	-0.05%	0.01%	-0.65%
26 Total commuting time across all hhs	–	-0.08%	-0.17%	-0.16%	-0.38%	-0.62%	0.00%
27 Access to RC insurance (%)	8.4	78.53%	133.56%	176.89%	52.55%	1.75%	-0.73%
28 Stability of RC insurance (%)	73.2	1.43%	-50.38%	-12.76%	0.65%	0.01%	0.30%
29 Std. MU growth, nondurables	0.32	-0.92%	-2.44%	-4.48%	0.19%	0.10%	-1.03%
30 Std. MU growth, housing	0.39	-0.25%	10.31%	-0.55%	2.91%	0.56%	-0.49%
31 **Aggregate welfare change (CEV)**	–	**0.97%**	**0.46%**	**1.69%**	**0.24%**	**0.39%**	**0.19%**

Notes: Panel A reports the aggregate welfare changes for the six main policy experiments under the benchmark calibration. They are taken from Table 3. Panels B-F study the same policy experiments under different calibrations of the model, as described in Appendix H