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UNHEDGEABLE INFLATION RISK WITHIN PENSION SCHEMES

Abstract

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JEL Classification: C61, E21, G11, G23

Keywords: Unhedgeable inflation risk, welfare loss, incomplete markets, pension contract, Valuation

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Unhedgeable Inflation Risk within Pension Schemes*

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This version: May 18, 2019

Abstract

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1 Introduction

This paper develops a framework for the valuation of pension contracts with conditional indexation in incomplete markets. Because there is no external guarantor, the value of the promises made depends on the capital adequacy of pension funds. But judging that capital adequacy becomes difficult when markets are incomplete, since a complete set of market prices is then not available and, hence, valuation will depend on preferences. We apply the framework to the Netherlands where precisely this issue plays a role in the current debate on pension reforms since one objective is to provide inflation-protected benefits. Swap contracts for Dutch consumer price index (CPI) inflation do not exist, so traditional valuation methods cannot be used to value pension contracts. While foreign indexed debt could go some way towards protecting the purchasing power of Dutch retirees, this would still leave pension fund participants exposed to real exchange rate risk. Moreover, even if it were possible to perfectly hedge consumer price inflation, workers and retirees would then still be exposed to the inflation risks associated with their own specific consumption bundle (Stewart, 2008), because it differs from the bundle consumed by the average population member: for example, the elderly consume more health care and spend more on housing than the average member of society, while the young spend more on transportation and education. This makes our analysis of much wider relevance than just for the Dutch discussions on pension reform: general inflation swaps do exist in, for example, the U.S., but not for group-specific price indices.

We estimate these valuation differences by using an intertemporal optimal consumption/portfolio allocation model and use certainty-equivalent consumption as our measure of welfare. Our results suggest that for commonly assumed degrees of risk aversion the inability to perfectly hedge general consumer-price inflation produces a lifetime welfare loss of up to 1% when expressed in terms of a decline in lifetime certainty-equivalent consumption. Furthermore we find that losses associated with unhedgeable inflation risk become much larger when there are significant differences in worker and retiree consumption bundles. The combination of the two sources of unhedgeable inflation risk produces a lifetime welfare loss of up to 6% for commonly assumed degrees of risk aversion. Hence, substantial welfare gains can be obtained by making financial markets more complete, for example by issuing index-linked bonds, or developing arrangements that allow workers and retirees to trade group-specific inflation risks.

Traditional valuation approaches are based on the construction of a replicating portfolio of traded instruments that generates cash flows identical to the arrangement to be valued. This is "risk-neutral" valuation, which is based on the no-arbitrage condition that equivalent cash flows should have the same value irrespective of the fundamental risk preferences of the investor, i.e. that risk-free arbitrage gains are precluded. However, in the absence of equivalent traded financial instruments, it is no longer possible to construct a perfect hedge, and valuation becomes dependent on the investor's risk aversion. Hence, if markets cannot be completed, regulators of collective pension funds face a dilemma. Different groups of participants have a different capacity to absorb losses from unhedgeable inflation risks and therefore have different risk appetites. In particular, in the absence of wage income and without the ability to adjust labor supply, and with less time left to adjust through consumption smoothing, elderly participants have less capacity to absorb shocks. Differences in risk appetite, and hence the regulator's dilemma, are further exacerbated if elderly participants are fundamentally more risk averse. as empirical evidence seems to suggest – see, for example, Albert and Duffy (2012); Halek and Eisenhauer (2001) and Dohmen, Falk, Golsteyn, Huffman, and Sunde (2017).

Our approach is to construct an optimal portfolio of several securities in a setting with unhedgeable inflation risk. Estimating the correlations between the returns on these assets is key for determining the hedging strategy and the price of a non-tradable indexed asset, an approach introduced by Hodges and Neuberger (1989).[1] We apply stochastic dynamic programming (SDP) to value risky cash flows as a function of the investors' risk appetite ("utility indifference pricing"). While the techniques underlying SDP go back to the 1960s, SDP has until recently remained practically unusable, because of the difficulty of addressing higher dimensionality problems, which arise when long-dated pension fund liabilities need to be valued in the presence of multiple sources of risk. Recently, however, highly-accurate numerical approximation methods have been developed that have made SDP practically usable even in higher dimensional problems. An example is the "Least Squares Monte Carlo" method (Longstaff & Schwartz, 2001). This method was developed to value American options, but is easily extended to analyse multiple sources of risks. Our paper follows the solution method applied by Koijen, Nijman, and Werker (2010), which is a variant of the "Least Squares Monte Carlo" method.

Our paper makes several contributions to the existing literature. First, we analyze the welfare losses caused by workers and retirees having different consumption bundles. This issue is studied in qualitative terms by Stewart (2008) and Munnell and Chen (2015), as inflation rates for elderly and workers have moved differently over time, for example due to higher inflation in medical costs, which affects the elderly more than working cohorts. However, to the best of our knowledge we are the first to quantify these losses using a formal welfare evaluation. Second, we analyze unhedgeable inflation risk due to market incompleteness in a more comprehensive model than the existing literature. Brennan and Xia (2002) obtain an analytical solution and De Jong (2008) applies this to a pension fund setting. Koijen et al. (2010) include an extra factor for modelling interest rate risk, to obtain quantitatively more realistic results, but they do not consider unhedgeable risk. We combine these two models and add two elements by extending the model of Koijen et al. (2010) in three ways: by including unhedgeable CPI inflation risk similar to Brennan and Xia (2002), by including unhedgeable inflation risk resulting from workers and pensioners having a different consumption bundles and by adding a third state factor. This additional factor, suggested by Driessen, Klaassen, and Melenberg (2003); Litterman and Scheinkman (1991) and Bouwman and Lord (2016), is included to produce a more accurate fit of the market interest rates and inflation rates.

The remainder of this paper is structured as follows. Section 2 presents the model. Section 3 describes the estimation approach and the data that we use for our application. In Section 4 we solve for the optimal consumption and asset allocations over an individual's life cycle. In Section 5 we calculate the welfare loss associated with unhedgeable inflation risks under different economic settings and degrees of risk aversion. Section 6 assesses the impact of consumption smoothing by analyzing the special case where an individual is subject to a mandatory saving scheme, similar to a real-world pension plan, and consumes the remainder, hence is unable to smooth consumption during the working life. Finally, Section 7 concludes the main text. Technical details and additional results are found in the appendices.

[1] Henderson (2002) applies utility maximization to obtain option prices in an incomplete market setting. This way, the optimal portfolio weights assigned to the different asset classes depend on the risk appetite of the investor.

2 An economy with a three-factor yield curve

This section constructs a three-factor model of the yield curve while allowing for unhedgeable inflation risk. Among others, Bouwman and Lord (2016); Driessen et al. (2003); Litterman and Scheinkman (1991) suggest that a three-factor model of the yield curve produces a more accurate fit of observed market rates than a two-factor model. Hence, we extend the model of Koijen et al. (2010) by adding a third state factor. However, in contrast to Koijen et al. (2010) and Bouwman and Lord (2016), but in line with the model of Brennan and Xia (2002), we also include unhedgeable inflation risk.

The state factors evolve as

$$dX_t = -KX_t dt + d\tilde{Z}_t \tag{1}$$

where $X_t = (X_{1,t}, X_{2,t}, X_{3,t})'$ is the vector containing the three state factors and $\tilde{Z}_t = (Z_{1,t}, Z_{2,t}, Z_{3,t})'$. The elements of $\tilde{Z}_t$ are independent Brownian motions. Hence, the state factors X_t are mean reverting around zero. To ensure identification of X_t and $\tilde{Z}_t$, we take K lower triangular (see Koijen et al. (2010)).

The instantaneous nominal interest risk-free rate r_t and the instantaneous *expected* inflation rate π_t are given by, respectively,

$$\begin{aligned} r_t =& \delta_{0,r} + \delta_{1,r}' X_t, \\ \pi_t =& \delta_{0,\pi} + \delta_{1,\pi}' X_t, \\ \delta_{0,r}, \delta_{0,\pi} >& 0. \end{aligned} \tag{2}$$

The nominal state price density (or pricing kernel) is given by

$$\frac{d\phi_t}{\phi_t} = -r_t dt - \Lambda_t' dZ_t, \tag{3}$$

with prices of risk

$$\begin{aligned} \Lambda_t =& \Lambda_0 + \Lambda_1 X_t, \\ \Lambda_0, \Lambda_t \in& \mathbb{R}^5 \text{ and } \Lambda_1 \in \mathbb{R}^{5\times 3}. \end{aligned} \tag{4}$$

Here, vector $Z_t = (Z_{1,t}, Z_{2,t}, \ldots, Z_{5,t}) \in \mathbb{R}^5$ expands the vector $\tilde{Z}_t \in \mathbb{R}^{3\times 1}$ with two additional Brownian motions that are independent of each other and the other elements of Z_t.

Following Duffie and Kan (1996), the price at time t of a nominal zero-coupon bond that pays out at $t+s$ is:

$$P(t, t+s) = \exp\left[A(s) + B(s)' X_t\right], \tag{5}$$

where

$$\begin{aligned} A(s) \equiv& -\int_0^s \left[B(\tau)' \tilde{\Lambda}_0 - \frac{1}{2} B(\tau)' B(\tau) + \delta_{0,r}\right] d\tau, \\ B(s) \equiv& \left(K' + \tilde{\Lambda}_1'\right)^{-1} \left[\exp\left(-\left(K' + \tilde{\Lambda}_1'\right) s\right) - I_3\right] \delta_{1,r}, \end{aligned} \tag{6}$$

where I_3 is the 3×3 identity matrix and where $\tilde{\Lambda}_t$, $\tilde{\Lambda}_0$ and $\tilde{\Lambda}_1$ are formed by the first three elements or rows of Λ_t, Λ_0 and Λ_1, respectively:

$$\begin{aligned}
\tilde{\Lambda}_t &\equiv \left[\begin{array}{ccc} \Lambda_{1,t} & \Lambda_{2,t} & \Lambda_{3,t} \end{array} \right]', \\
\tilde{\Lambda}_0 &\equiv \left[\begin{array}{ccc} (\Lambda_0)_1 & (\Lambda_0)_2 & (\Lambda_0)_3 \end{array} \right]', \\
\tilde{\Lambda}_1 &\equiv \left[\begin{array}{ccc} (\Lambda_1)_{1,1} & (\Lambda_1)_{1,2} & (\Lambda_1)_{1,3} \\ (\Lambda_1)_{2,1} & (\Lambda_1)_{2,2} & (\Lambda_1)_{2,3} \\ (\Lambda_1)_{3,1} & (\Lambda_1)_{3,2} & (\Lambda_1)_{3,3} \end{array} \right].
\end{aligned} \tag{7}$$

Hence, the yield on the nominal zero-coupon bond is given by

$$y(t, t+s) = -\frac{1}{s} \left[A(s) + B(s)' X_t \right]. \tag{8}$$

Stock prices are assumed to evolve as

$$\frac{dS_t}{S_t} = (r_t + \eta_S)\, dt + \sigma_S' dZ_t, \quad \sigma_S \in \mathbb{R}^5, \tag{9}$$

Hence, the expected stock return equals the instantaneous interest rate (r_t) plus a constant equity risk premium (η_S): $\sigma_S' \Lambda_t = \eta_S$. Hence, $\sigma_S' \Lambda_t$ is constant for all X_t, which restricts Λ_0 and Λ_1 such that

$$\sigma_S' \Lambda_0 = \eta_S \text{ and } \sigma_S' \Lambda_1 = 0. \tag{10}$$

Finally, we assume that the consumer price index (CPI) evolves as

$$\frac{d\Pi_t}{\Pi_t} = \pi_t dt + \sigma_\Pi' dZ_t + \sigma_u dZ_{u,t}, \quad \sigma_\Pi \in \mathbb{R}^5, \quad \Pi_0 = 1, \tag{11}$$

which is the sum of the process for expected inflation and two unexpected components. The first of these two components is hedgeable. The second of these components, $\sigma_u dZ_{u,t}$, is assumed to be unhedgeable. In the special case of $\sigma_u = 0$, we have a complete market setting.

3 Data Description and Estimation

3.1 Data Description

The model parameters are estimated using quarterly data on risk-free nominal interest rates, inflation rates and stock returns from January 1999 to January 2018. Hence, we have $N_T = 77$ observations for each time series. For stock returns we use the MSCI world index (in EUR). For inflation rates, we take the ex-tabacco eurostat HICP. To correct for seasonality in the inflation rates, we adjust for quarterly effects, but keep the overall average.[2] For tenors shorter than one year, we take the Euribor index for the 3-month and 6-month risk-free interest rates. We assume that counterparty risk is absent for these

[2]This is equivalent to regressing the time series on quarterly dummies and taking the residuals plus the estimated constant to keep the overall average of the deseasonalized inflation series equal to the uncorrected inflation series.

shorter tenors. For the tenors 1 to 60 years we take the European swap rates as risk-free rates. In the estimation we use only swap rates up to 30 years, as longer tenors are not available from the start of the sample period. However, when calibrating the initial state of the model for generating scenarios, we take all tenors up to 60 years.[3] Figure 1 depicts the data. While the quarterly returns on the MSCI and the Eurostat Harmonized Index of Consumer Prices (HICP) inflation rate appear to be stationary, the risk-free rate appears to be on a downward trend over the sample period under consideration, although this trend is interrupted by an increase during the eruption of the global financial crisis.

3.2 Estimation Method, because the valuation is preference dependent

We use the same optimization procedure as Draper (2014), who applies the log-likelihood estimation procedure by Goffe, Ferrier, and Rogers (1994) with similar parameter settings for the optimization. The estimation is based on a complete market setting, as Eurozone HICP inflation can be hedged using inflation swaps. This implies that we set $\sigma_u = 0$ f, because the valuation is preference dependentor the estimation. For the yield curve we use the following maturities: $\frac{1}{4}, 1, 3, 5, 10, 20$ and 30 years. We assume that the 1, 10 and 30 year yields are observed without measurement error. The reason we select these specific tenors is that we want to have sufficient dispersion across the tenors used for the estimation and to have tenors that are frequently traded. Hence, the state variables X_t are retrieved from the following 3-equation system for the yields of the tenors that are assumed to be measured with perfect precision:

$$\begin{aligned} y\left(t,t+s\right) &= -\frac{A\left(s\right)+B\left(s\right)' X_t}{s}, \text{ for all } s \in \{1,10,30\} \\ \Rightarrow X_t &= -\left(B\left(1\right), B\left(10\right), B\left(30\right)\right)'^{-1} \left(\begin{bmatrix} A\left(1\right) \\ A\left(10\right) \\ A\left(30\right) \end{bmatrix} + \begin{bmatrix} y\left(t,t+1\right) \\ 10y\left(t,t+10\right) \\ 30y\left(t,t+30\right) \end{bmatrix} \right). \end{aligned} \tag{12}$$

Then, the other yields with maturity s have measurement error $\nu_{t,s}$ given by

$$\nu_{t,s} = y\left(t,t+s\right) + \frac{A\left(s\right)+B\left(s\right)' X_t}{s} \sim N\left(0, \Sigma^s\right), \tag{13}$$

where Σ^s is the covariance matrix of the measurement errors $\nu_t = \left(\nu_{t,\frac{1}{4}}, \nu_{t,3}, \nu_{t,5}, \nu_{t,20}\right)$.

In discretized format with time steps of size h the complete model can be written (see Appendix A.1)

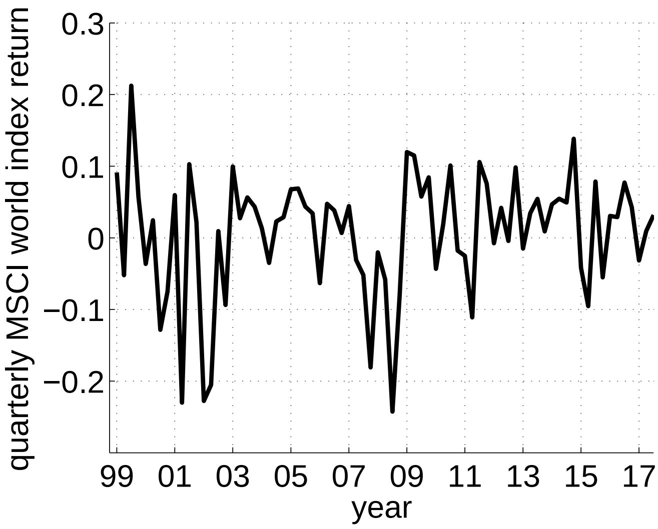
$$\begin{aligned} \Psi_t &= \hat{\alpha}^{(h)} + \hat{\Gamma}^{(h)} \Psi_{t-h} + \varepsilon_t, \quad \varepsilon_t \sim N\left(0, \hat{\Sigma}_\Psi^{(h)}\right) \\ \Psi_t &\equiv \left(X_t, \log \Pi_t, \log S_t, \log P\left(t,t+s\right)\right). \end{aligned} \tag{14}$$

Hence, the relevant measurement errors for stock returns and inflation are the corresponding elements in ε_t, i.e. $\tilde{\varepsilon}_t \equiv (\varepsilon_{4,t}, \varepsilon_{5,t})$. The log-likelihood is given by[4]

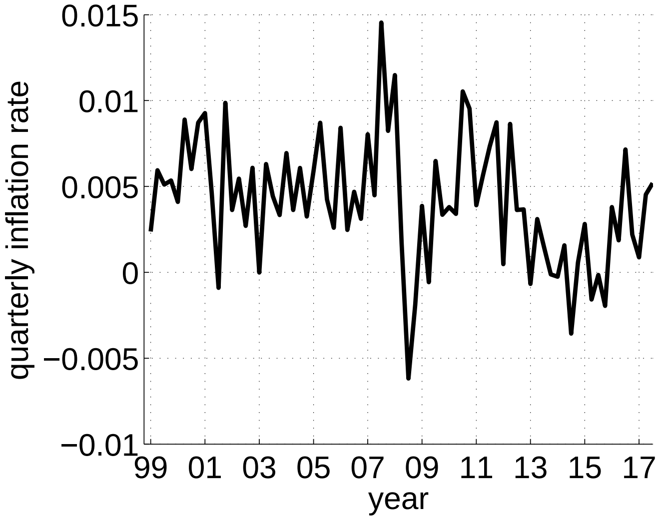
$$\ln LL = -\frac{N_T}{2} \left\{ \ln|\Sigma^s| + \frac{1}{N_T} \sum_{t=1}^{N_T} \nu_t \left(\Sigma^s\right)^{-1} \nu_t' + \ln\left|\hat{\Sigma}_\Psi\right| + \frac{1}{N_T} \sum_{t=1}^{N_T} \tilde{\varepsilon}_t \left(\hat{\Sigma}_\Psi\right)^{-1} \tilde{\varepsilon}_t' + \ln \left| \begin{bmatrix} B\left(1\right) \\ B\left(10\right) \\ B\left(30\right) \end{bmatrix} \right| \right\}. \tag{15}$$

[3] More precisely, we take maturities $\frac{1}{4}, \frac{1}{2}, 1, 2, 3, \ldots, 15, 20, 25, 30, 40, 50$ and 60 years.

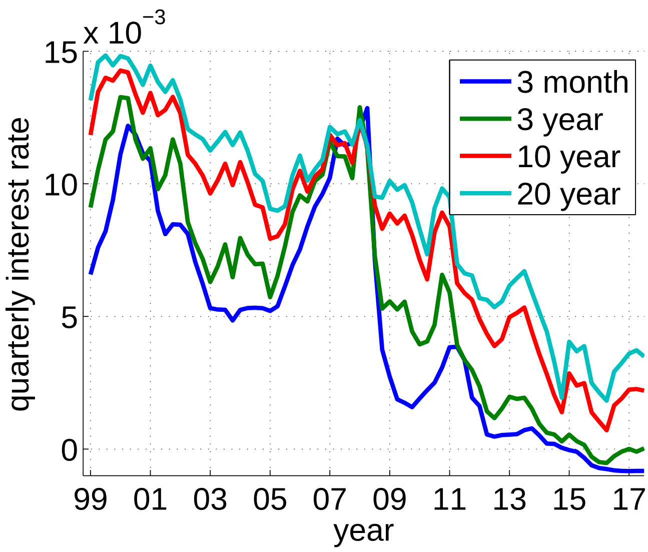
[4] The details on this log-likelihood function are provided by Duffee (2002).



(a) Quarterly MSCI world index return



(b) Quarterly returns on the eurostat HICP (ex-tobacco) after seasonality correction



(c) Quarterly risk-free interest rate (Euribor and European swap rates)

Figure 1: Quarterly data on stocks returns, inflation rates and interest rates over the period January 1999 - January 2018.

Table 1: Parameter estimates

Symbol	Estimated value
$\delta_{0,r}$	0.0073
$\delta_{0,\pi}$	0.0052
η_S	0.0523
$\delta_{1,r}$	$\begin{pmatrix} 0.0146, & -0.0010, & 0.0120 \end{pmatrix}$
$\delta_{1,\pi}$	$\begin{pmatrix} 0.0282, & 0.0007, & 0.0013 \end{pmatrix}$
σ_{Π}	$\begin{pmatrix} -0.0102, & 0.0012, & 0.0156, & -0.0081, & 0 \end{pmatrix}$
σ_S	$\begin{pmatrix} -0.0696, & -0.0169, & 0.1084, & -0.0276, & -0.1999 \end{pmatrix}$
K	$\begin{bmatrix} 2.3156 & 0 & 0 \\ -1.2931 & 0.1324 & 0 \\ -0.7636 & -0.3582 & 0.2946 \end{bmatrix}$
$\tilde{\Lambda}_0$	$\begin{pmatrix} 0.0716, & -0.2828, & -0.0634 \end{pmatrix}$
$\tilde{\Lambda}_1$	$\begin{bmatrix} 2.3698 & -0.0716 & 0.5015 \\ 0.7257 & -0.1132 & -0.0970 \\ 4.3813 & 0.2881 & 0.1259 \end{bmatrix}$
X_0	$\begin{pmatrix} -0.3579, & 5.8420, & -0.5069 \end{pmatrix}$

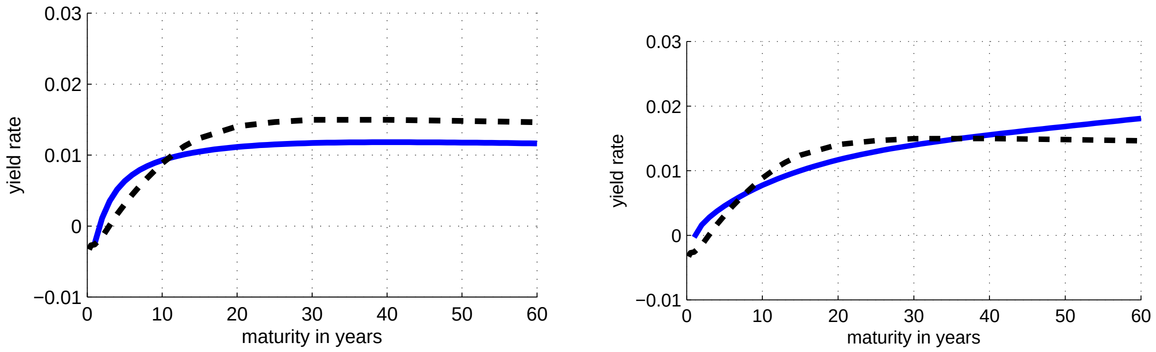
Muns (2015) shows that adding restrictions may lead to a better estimation. The most likely reason is that the estimation problem has multiple local optima (Duffee, 2002). Hence, we introduce the following restrictions. The long-run instantaneous real interest rate is non-negative, hence $\delta_{0,r} \geq \delta_{0,\pi}$. Similar to Muns (2015), we impose $\det\left(-\left(K' + \tilde{\Lambda}_1'\right)\right) < 0$ to get a finite long-term yield, $\lim_{s\to\infty} Pr\left(|y\left(t, t+s\right)| < \infty\right) = 1$, and we exclude oscillating term structures by discarding $\left(K' + \tilde{\Lambda}_1'\right)$ with complex eigenvalues. We also impose a UFR larger than the long-run instantaneous nominal interest rate. In Appendix A.2 we show that this restriction amounts to

$$\lim_{s\to\infty} E_t\left[y\left(t,t+s\right)\right] = \delta_{0,r} - \left(\left(K' + \tilde{\Lambda}_1'\right)^{-1}\delta_{1,r}\right)'\left(\tilde{\Lambda}_0 + \frac{1}{2}\left(K' + \tilde{\Lambda}_1'\right)^{-1}\delta_{1,r}\right) \geq \log\left(1 + \delta_{0,r}\right). \tag{16}$$

As in Koijen et al. (2010), we assume that the risk premium associated with unexpected inflation risk is zero, because we are unable to identify this risk premium from the available data (also see Campbell and Viceira (2002); Sangvinatsos and Wachter (2005)). Hence, we take $\left(\Lambda_0\right)_4 = \left(\Lambda_1\right)_{4,1} = \left(\Lambda_1\right)_{4,2} = \left(\Lambda_1\right)_{4,3} = 0$. Finally, similar to Koijen et al. (2010), to ensure identification, the volatility matrix $\hat{\Sigma}_\Psi$ should be triangular by imposing $\sigma_{\Pi,5} = 0$.

3.3 Estimation Results

We calibrate the initial state X_0 by minimizing for the maturities $\frac{1}{4}$ up to 60 years the sum of the squared errors between the current swap rate and the interest rates implied by the model at time zero, $y\left(0,s\right) = -\frac{A(s)+B(s)'X_0}{s}$. The initial state and the model estimates are reported in Table 1. The estimated annual equity premium equals $\eta_S = 5.23\%$ and



(a) Two states (b) Three states

Figure 2: Initial yield curves based on three and two factors. *Notes: Dashed line: swap-curve 2017 year end, solid line: fitted curve. Maturities 0-60 years.*

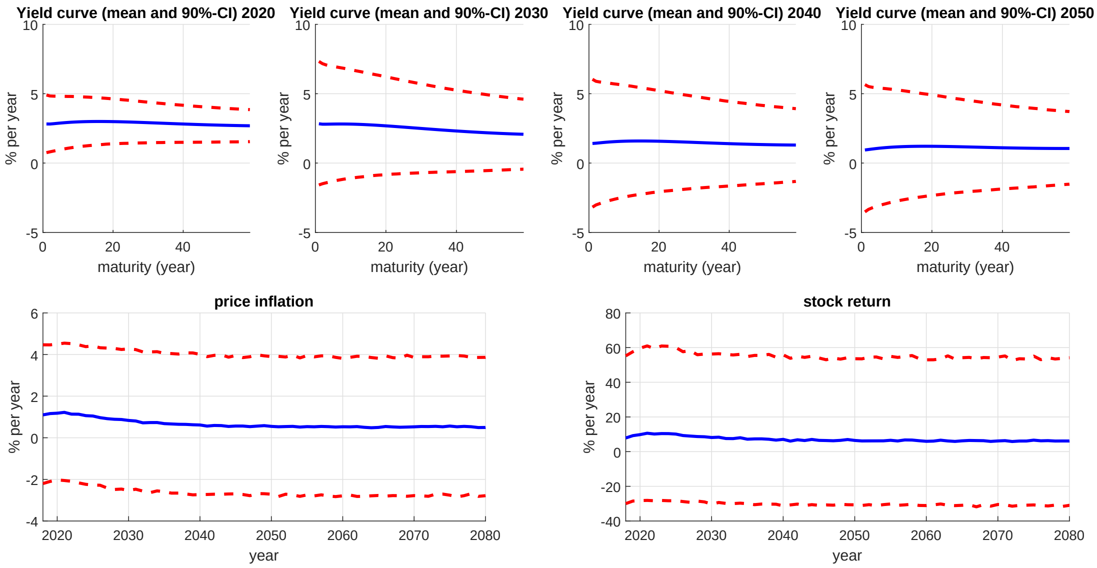


Figure 3: Mean and 90%-confidence interval of yields, inflation and stock returns

the estimated annual spot rate is $\delta_{0,r} = 0.73\%$. The estimated initial state implies the initial yield curve depicted in Figure 2. Figure 2 also shows the analogously-estimated initial yield curve based on only two factors.[5] The three-factor yield curve clearly fits the data better than the two-factor yield curve.[6]

3.4 Model simulation

Given the initial state and the parameter estimates, we can now simulate the model. Figure 3 presents the mean and 90% confidence intervals for the different economic variables based on $Q = 10^4$ simulation runs over a 60-year horizon with 2018 as the starting year. Since the interest rates in the model are normally distributed and current interest rates are at a (historically) relatively low level, the top right panel of Figure 3 indicates a substantial probability of interest rates falling below zero.

[5]In this special case we assume that the 1 and 10 year yields are measured without error.
[6]Switching from three to two factors, the sum of squared errors decreases from $14 * 10^{-5}$ to $8 * 10^{-5}$.

4 Optimal Lifetime Portfolio and Consumption with Complete Markets

To set a benchmark, we first consider a setting with complete markets. There is a representative individual who makes both a consumption-saving decision and an optimal portfolio allocation decision in each period. We assume that individuals work part of their life and are retired for the remainder of their life. First, we define the individual's preferences and the optimization problem. Second, we present the benchmark results for this case where markets are complete.

4.1 Preferences

An individual starts working at age 0, retires at age T^R and dies at age T^D. Her period-t nominal income and nominal consumption are given by Y_t and C_t, respectively. Here, t denotes both the calendar period and the age of the individual. Period utility is of the constant relative risk aversion (CRRA) type with relative risk aversion parameter γ, and is defined over real consumption. Hence, the individual's problem can be written as

$$\max_{c_t, x_t : t \in \{0,1,\ldots T^D-1\}} E_0 \left[\sum_{t=0}^{T^D-1} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma} \right], \tag{17}$$

subject to

$$\begin{aligned} w_{t+1} =& (w_t - c_t) \left(x_t' r_{t+1}^e + r_{t+1}^f \right) + y_{t+1}, \\ w_0 =& y_0, \\ c_t \leq& w_t, \\ x_{i,t} \geq& 0, \ \forall i, \\ \sum_{i=1}^{n_x} x_{i,t} \leq& 1, \end{aligned} \tag{18}$$

where β denotes the subjective discount factor, the elements $i \in \{1, \ldots, n_x\}$ of the vector x_t denote the fractions of wealth invested in the n_x different risky assets, and where we have defined the real variables:

$$c_t = \frac{C_t}{\Pi_t}, \quad w_t = \frac{W_t}{\Pi_t}, \quad y_t = \frac{Y_t}{\Pi_t}, \quad , r_t^e = \frac{R_t^e \Pi_{t-1}}{\Pi_t} \quad , r_t^f = \frac{R_t^f \Pi_{t-1}}{\Pi_t}, \tag{19}$$

where w_t is real financial wealth, y_t is the real wage, r_t^e is the real return vector on the risky assets $i \in \{1, \ldots, n_x\}$ and r_t^f is the real return on the risk-free asset. The two last restrictions, $x_{i,t} \geq 0$ and $\sum_{i=1}^{n_x} x_{i,t} \leq 1$, the no-short-selling constraints and the no-borrowing constraint, are included to make the investment problem more realistic, as life-cycle investors are typically bound by such constraints. A fraction $(1 - \sum_{i=1}^{n_x} x_{i,t})$ of financial wealth is invested in the 1-year nominal bond. We consider the real labor income process assumed by Cocco, Gomes, and Maenhout (2005):

$$y_t = \begin{cases} \exp\left(g_t + \kappa_t + \nu_t\right), & \text{for } t \in \left\{0, 1, \ldots T^R - 1\right\}, \\ 0, & \text{for } t \in \left\{T^R, T^R + 1, \ldots T^D - 1\right\}. \end{cases} \tag{20}$$

In this process for labor income, g_t represents the deterministic component of the real wage profile, where the subscript t should be interpreted as the age of the individual. The processes κ_t and ν_t represent an idiosyncratic shock and a persistent shock at time t, respectively. We consider different settings for the real wage process, which we specify later on.

4.2 The benchmark results under complete markets

Appendix B.1 describes in detail the solution method, which follows Koijen et al. (2010). This an extension of Brandt, Goyal, and Santa-Clara (2005), whose approach is inspired by the "Least Squares Monte Carlo" method (Longstaff & Schwartz, 2001). The subjective discount factor is set at $\beta = \exp(-\delta_{0,r})$. The benchmark value for the constant relative risk aversion parameter is $\gamma = 5$. We set $T^R = 40$ and $T^D = 60$, so an individual works for 40 years and is retired for $20\left(= T^D - T^R\right)$ years. The other parameter values are identical to those already assumed above.

We normalize the initial nominal wage to $Y_0 = 1$. The initial price level is also normalized to $\Pi_0 = 1$. For the benchmark setting we apply a simplified setting for real labor income, by assuming that $g_t = \kappa_t = \nu_t = 0$. Hence, the real wage equals one during the working period and is zero during retirement. From Section 5.4 onwards we extend the analysis with a stochastic wage process, by specifying more a realistic processes for g_t, κ_t and ν_t.

We start with the complete markets setup, hence $\sigma_u = 0$. We simulate an individual's life $Q = 10^4$ times and report mean outcomes and 90% symmetric confidence intervals. Figure 4 shows the optimal portfolio allocations over time for $n_x = 3$ risky assets, one stock and two index-linked bonds with maturities 5 and 30 years, and the 1 year nominal bond as risk-free asset. We select the 5 and 30 year maturities in order to have both a relatively short and relatively long maturity in the portfolio. However, the figure shows that it contains almost no 5-year debt. The reason is that the 5-year bond is relatively strongly correlated with the 30-year bond, while the latter earns the risk premium $\tilde{\Lambda}_t$. The corresponding optimal lifetime consumption, savings and wealth trajectories are shown in Figure 5. Average consumption increases with age, because the expected portfolio return exceeds the time preference rate. However, consumption also becomes more uncertain with age. Since the individual is risk averse and therefore values the worst scenarios more strongly, we see that the fifth percentile of consumption is rather stable over the lifetime. Wealth is accumulated during the working period and during retirement wealth decreases towards zero at the last period. We can also see that the investment portfolios become less risky towards the end of the individual's life. Based on the Q simulation runs, Appendix A.3 shows that "certainty-equivalent consumption" is calculated as:

$$CEC = \left[\frac{1-\beta}{Q\left(1-\beta^{T^D}\right)} \sum_{q=1}^{Q} \left(\sum_{t=0}^{T^D-1} \beta^t c_{q,t}^{1-\gamma} \right) \right]^{1/(1-\gamma)} \tag{21}$$

Certainty-equivalent consumption CEC is the certain and constant period consumption level over one's entire life that yields the same expected utility as the uncertain actual consumption stream. Using Equation (21) we find $CEC = 97.40\%$ of the annual wage during one's working life.

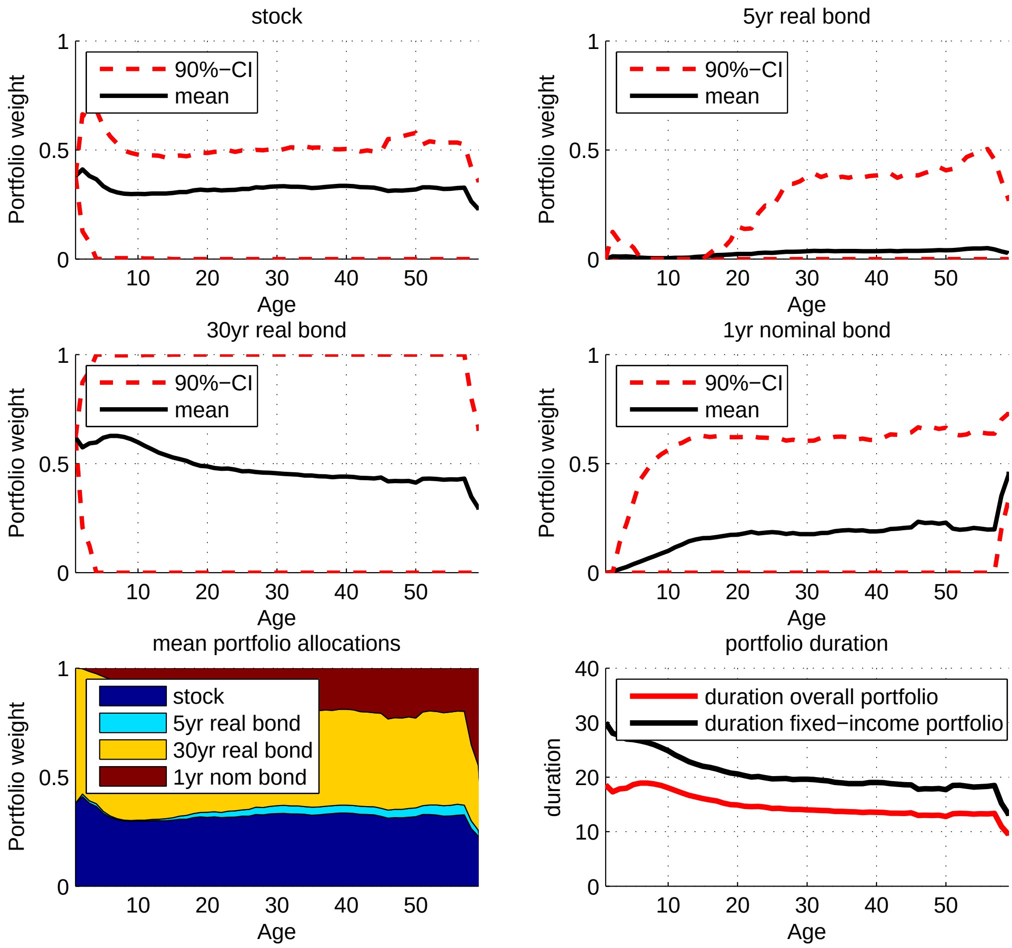


Figure 4: Optimal lifetime portfolios in the absence of unhedgeable inflation risk

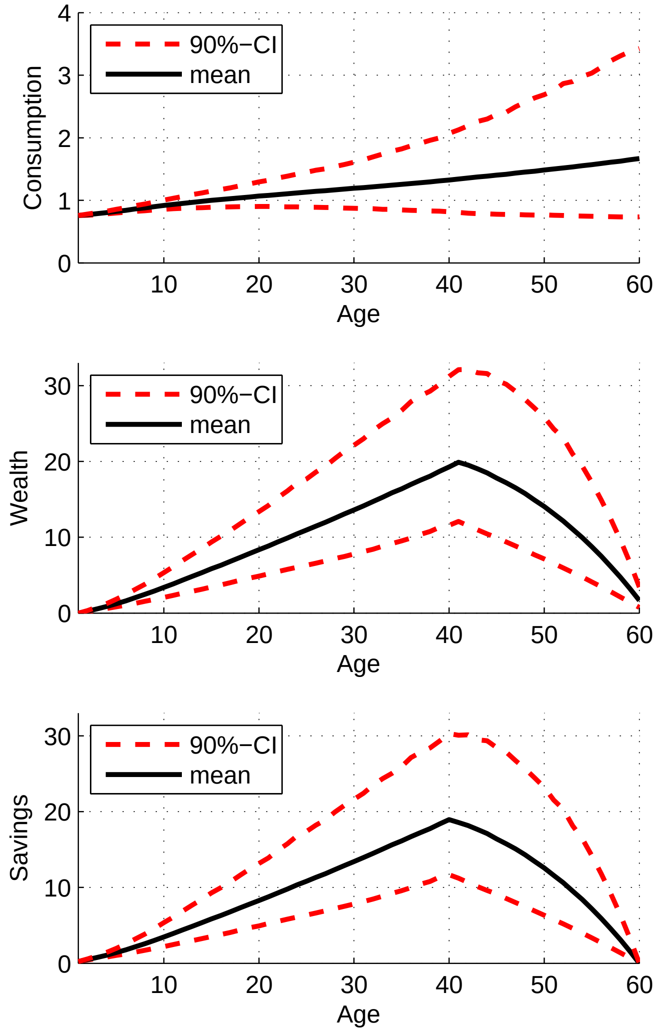


Figure 5: Optimal lifetime consumption, wealth and savings trajectories in the absence of unhedgeable inflation risk

Similar to Koijen et al. (2010), the duration of the overall portfolio in this example with only zero-coupon bonds is given by

$$Dur_{P,t} = 5x_{2,t} + 30x_{3,t} + \left(1 - \sum_{i=1}^{3} x_{i,t}\right) \tag{22}$$

using that equity has zero duration. Obviously, the fraction invested in the 30-year bond, $x_{3,t}$, has a stronger effect on the duration than the fraction invested in the 5 year bond, $x_{2,t}$. The duration of the fixed-income portfolio is in the presence of complete markets

$$Dur_{FI,t} = \frac{5x_{2,t} + 30x_{3,t} + \left(1 - \sum_{i=1}^{3} x_{i,t}\right)}{x_{2,t} + x_{3,t} + \left(1 - \sum_{i=1}^{3} x_{i,t}\right)}. \tag{23}$$

On average, the overall portfolio duration is 14.5 years, while the fixed-income portfolio duration is 20.3 years. The last panel of Figure 4 shows that the durations of the overall portfolio and the fixed-income portfolio tend to decline with age.

The benchmark asset allocation pattern is quite robust as can be seen from the impact of several parametric changes, which we discuss in detail in Appendix B.2. From these variations on the benchmark we conclude that the outcomes are qualitatively similar for a higher equity premium, a lower interest rate volatility and a portfolio with different bond maturities.

5 Incomplete markets

So far, we considered a complete market setting, as a benchmark for the main topic, the incomplete market setting. Inflation risk cannot be completely hedged in countries that do not issue inflation-linked bonds, and even if they do there are still problems if those bonds index returns on general inflation whilethe young and old participants in pension schems have significant differences in consumption patterns. Therefore in this section we first provide a quantification of the welfare loss associated with the inability to completely hedge away CPI inflation risk. The quantification is based on data for the Netherlands. Then we introduce an additional source of unhedgeable inflation risk: differences in consumption patterns of workers and retirees and quantify their welfare consequences. Finally, we extend the analysis by introducing a more realistic stochastic wage process.

5.1 Measuring the variability of unhedgeable inflation risk

We present two alternative ways to measure unhedgeable Dutch CPI inflation risk. First, since European inflation risk can be hedged using Eurozone HICP inflation swaps, we can approximate unhedgeable Dutch inflation risk as the difference between Eurostat's HICP inflation for the euro area[7] and CPI inflation of the Netherlands. The two time series are shown jointly in the left-hand panel of Figure 6, while the difference, euro-area HICP inflation minus Dutch CPI inflation, is depicted in the right-hand panel. The difference thus captures the unhedgeable component of Dutch inflation. Its standard deviation is 0.36%. A regression of this difference on its first lag shows that its coefficient, with a p-value of 0.38, is insignificant even at the 10% level.

[7] See http://ec.europa.eu/eurostat/web/hicp/data/database

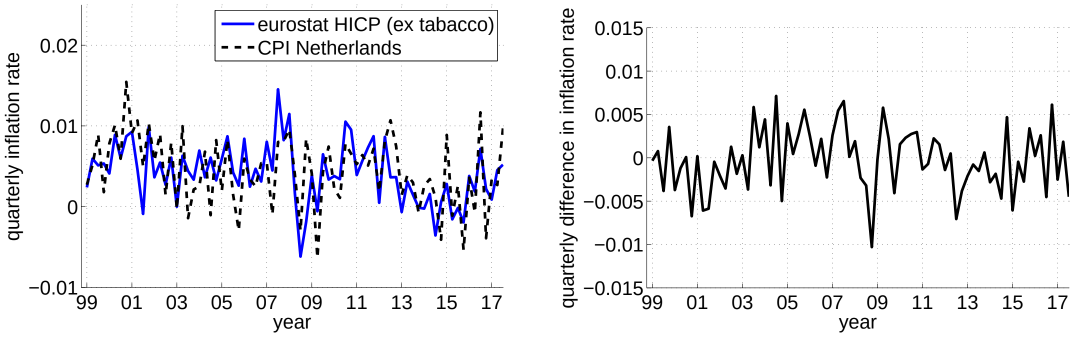


Figure 6: Quarterly inflation rates from January 1999 to January 2018 after seasonality correction. *Note: the right panel shows the difference, i.e. euro-area HICP inflation minus Dutch CPI inflation.*

The second alternative measures unhedgeable inflation risk by regressing Dutch CPI inflation on the CPI inflation rates of countries that issue index-linked bonds. Figure 7 depicts the quarterly CPI inflation rates after seasonality correction for the U.K., Germany, France, Italy, Spain, Australia, New Zealand, Denmark, Sweden, Japan, the U.S., Canada and the Netherlands.[8] The bottom right panel also shows the residual from a linear regression of Dutch CPI inflation on the CPI inflation rates of the aforementioned countries.[9] We take the regression residual as the unhedgeable component of Dutch inflation. Its standard deviation is 0.30%, very close to the standard deviation of the other alternative based on the difference between euro-area HICP inflation and Dutch CPI inflation. Again, we can reject serial correlation in the residuals.[10] We also estimate the standard deviation of the residuals by adding commodity indices, which could also be relevant for inflation hedging (Spierdijk & Umar, 2013). By including an oil price index, a gold price index and a general commodity price index, the R-squared increases to 47% and the standard deviation of residuals slightly decreases to 0.29% - see Appendix A.4 for more details.

It should be emphasized that the aforementioned estimates are likely a lower bound to the unhedgeable inflation risk. First, due to trading frictions, it may not be possible to exactly construct the desired inflation hedge and adjust its term structure to match the exposure time pattern. Moreover, we ignore the fact that index-linked bonds are illiquid financial instruments compared to nominal bonds, which would make the optimal hedge less effective on average. Possibly, to limit the cost of the hedging strategy fewer instruments than necessary may end up being used, with a higher amount of unhedged inflation risk as a consequence. Therefore, in the sequel we calibrate unhedgeable inflation risk at 0.36%, the larger of the above estimates.

[8]The data is obtained from the Federal Reserve Economic Data on May 29, 2018 from https://fred.stlouisfed.org

[9]The regression specification is $\frac{CPI_{NL,t}-CPI_{NL,t-1}}{CPI_{NL,t-1}} = \beta_0 + \sum_{i=1}^{n} \beta_i \frac{CPI_{i,t}-CPI_{i,t-1}}{CPI_{i,t-1}} + \varepsilon_t$. The R-squared is 43%.

[10]If we only consider the European countries, i.e. exclude Australia, New Zealand, Japan, the U.S. and Canada from the regression, the R-squared falls to 41% and the standard deviation of residuals remains at 0.30%. Moreover, if we redo this estimation without seasonally adjusting the CPI inflation rates, then the standard deviation of the residuals becomes 0.34%.

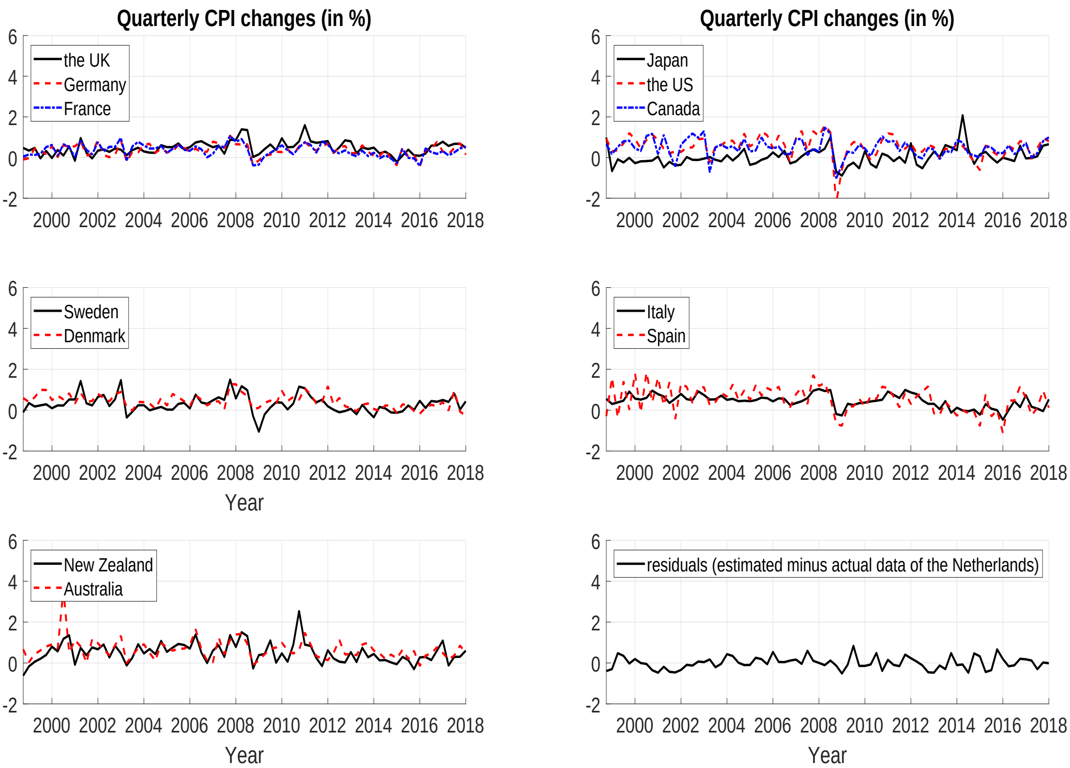


Figure 7: Quarterly CPI changes from 1999Q1 to 2018Q1 after seasonality correction
Source: Federal Reserve Economic Data, available at https://fred.stlouisfed.org

5.2 Results for unhedgeable aggregate inflation risk

This subsection studies the welfare effects of adding to our benchmark setting unhedgeable CPI inflation risk over the individual's life cycle. Based on the standard deviation of the difference between eurozone HICP inflation and Dutch CPI inflation, we set the annualized unhedgeable inflation risk parameter at $\sigma_u = \sqrt{4} * 0.36\%$. Then, Equation (11) becomes

$$\frac{d\Pi_t}{\Pi_t} = \frac{d\tilde{\Pi}_t}{\tilde{\Pi}_t} + \sigma_u dZ_{u,t}. \tag{24}$$

where $\tilde{\Pi}_t$ is the Eurozone price index and Π_t is the Dutch price index.

While unhedgeable inflation risk may seem small on an annual basis, over the long run differences in price indices potentially become large as they accumulate over time. We assume that at the start of one's life the unhedged components of the relevant price index or indices are zero. Figures 23 and 24 in Appendix B.3 show the corresponding trajectories for the optimal portfolio composition, consumption, wealth and savings, which are very similar to the corresponding trajectories in the absence of unhedgeable inflation risk. Since unhedgeable risk is orthogonal to the other risk factors, the optimal portfolio allocations over the life cycle are quite similar in the absence of unhedgeable inflation risk - see Figure 23 in Appendix B.3.[11] The differences between the trajectories with and without unhedgeable inflation risk can be explained by the fact that wealth and/or the state of the economy (in real terms) is more volatile in the setting with unhedgeable inflation.

Using Equation (21) Figure 8 depicts for both the complete and the incomplete markets setting the individual's certainty-equivalent consumption over her remaining lifetime. In the left panel we see that certainty-equivalent consumption is hump-shaped over the remaining lifetime. The reason is that there is a tradeoff between two effects. On the one hand, certainty-equivalent consumption over an individual's remaining lifetime increases as average consumption increases with age. On the other hand, the uncertainty about consumption also increases with age, which reduces welfare. These two effects are also shown in the top panel of Figure 5. The hump-shaped pattern for certainty-equivalent consumption in Figure 8 shows that the first effect dominates at young ages, while the second effect dominates later in life.

In the right panel of Figure 8, we compare the incomplete market setting with the complete market setting. Over the entire lifetime, the welfare loss is about 0.21%, while the welfare loss over the remaining lifetime becomes larger at higher ages. In particular, for a person entering retirement it equals 0.82%. For two reasons the welfare loss is smaller for a young person. First, unhedgeable inflation risk accumulates over time,

[11] We can see this immediately by noting that:

$$E_0\left[\frac{(W_T/\Pi_T)^{1-\gamma}}{1-\gamma}\right] = E_0\left[\frac{\left(W_T/\tilde{\Pi}_T\right)^{1-\gamma}}{1-\gamma}\left(\Pi_T^u\right)^{1-\gamma}\right] \Longrightarrow$$

$$\underset{x_\tau:\tau\in\{0,1,\ldots T-1\}}{\arg\max} E_0\left[\frac{(W_T/\Pi_T)^{1-\gamma}}{1-\gamma}\right] = \underset{x_\tau:\tau\in\{0,1,\ldots T-1\}}{\arg\max} E_0\left[\frac{\left(W_T/\tilde{\Pi}_T\right)^{1-\gamma}}{1-\gamma}\right]$$

where $\tilde{\Pi}_T$ denotes the hedgeable part of inflation risk and $\Pi_T^u = \tilde{\Pi}_T/\Pi_T$ denotes the unhedgeable part.

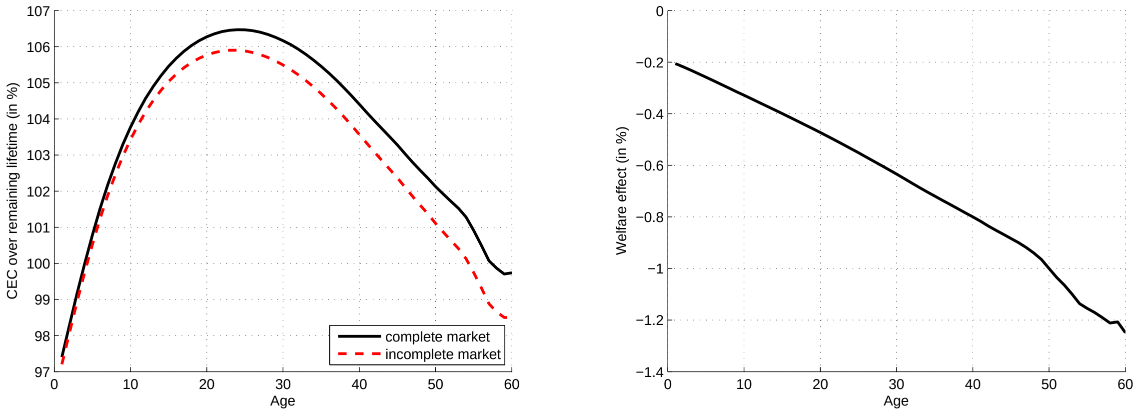


Figure 8: Certainty-equivalent consumption and welfare loss from unhedgeable consumer-price inflation risk. *Notes: the left-hand panel depicts certainty-equivalent consumption over the remaining lifetime as a function of age, while the right-hand panel depicts the welfare loss from unhedgeable inflation risk calculated as* $\left(\frac{CEC_t^{incompl} - CEC_t^{compl}}{CEC_t^{compl}} * 100\%\right)$, *where "incompl" indicates the case of incomplete markets, i.e. of unhedgeable inflation, while "compl" indicates the case of complete markets, i.e. in which all inflation risk is hedgeable.*

implying that the resulting losses materialize mostly later in life and, hence, discounting reduces their effect on the young's welfare. Second, we have assumed that the nominal wage rate follows the price index and, therefore, the individual is partially hedged against unhedgeable inflation risk during her active period. We will relax this assumption from Section 5.4 onwards.

5.3 Heterogeneous inflation risk exposures

Pensioners typically have a different consumption basket than working cohorts, so they are exposed to different inflation risk. In the U.S. there exists an experimental CPI for elderly Americans known as the CPI-E, which has been developed by the Bureau of Labor Statistics (BLS). This index attaches a higher weight to medical care and a lower weight to education and transportation. For example, Stewart (2008) shows that the CPI-E (elderly) and the CPI-W (workers) have moved differently over time. Munnell and Chen (2015) argue that if medical costs start surging, the social security index should be linked to an index designed for the elderly, e.g. the CPI-E, instead of the overall CPI. Figure 9 depicts the quarterly percentage changes in CPI-E and CPI-W obtained from the Bureau of Labor Statistics (2018). The difference between these two series is shown in the lower panel of Figure 9 and has a standard deviation of $\sigma^E = 0.30\%$.[12]

This subsection explores the welfare losses associated with worker- and elderly-specific inflation processes using the data constructed by the Bureau of Labor Statistics (2018), first when aggregate (CPI) inflation risk can be perfectly hedged and then when it cannot. Denote the price index of the elderly by Π^E and the price index of the workers by Π^W.

[12]Any conclusions based on these indices should be treated with care: there is measurement error due to the small sample sizes, while discounts for elderly are not taken into account (Munnell & Chen, 2015; Stewart, 2008).

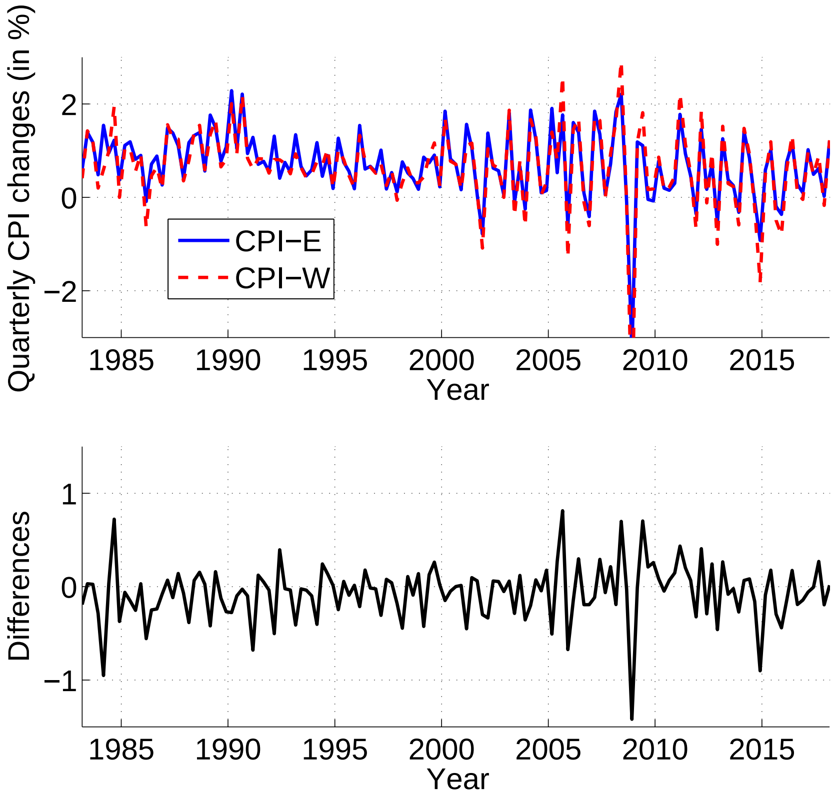


Figure 9: Quarterly percentage changes in CPI-E (elderly) and CPI-W (workers) in the United States. *Source:* Bureau of Labor Statistics (2018).

We assume that the difference in the inflation processes of the workers and the elderly is

$$\frac{d\Pi_t^W}{\Pi_t^W} - \frac{d\Pi_t^E}{\Pi_t^E} = \sigma_E dZ_{E,t}, \tag{25}$$

where $dZ_{E,t}$ is a Brownian motion independent of the other processes in the economy and σ_E is the standard deviation of the differences. Based on the empirical standard deviation of the difference, we set its annualized value at $\sigma_E = \sqrt{4} * 0.30\%$.

Furthermore, we assume that the overall inflation rate is determined by the group-specific inflation rates as follows

$$\frac{d\Pi_t}{\Pi_t} = (1-\omega)\frac{d\Pi_t^W}{\Pi_t^W} + \omega\frac{d\Pi_t^E}{\Pi_t^E}. \tag{26}$$

The parameter ω denotes the proportion of the consumer price index fluctuations determined by the price index of the elderly. By regressing the inflation rates presented in the top panel of Figure 9 on the overall quarterly CPI changes, we find the following regression fit:

$$\frac{d\Pi_t}{\Pi_t} \approx 51.2\%\frac{d\Pi_t^W}{\Pi_t^W} + 47.2\%\frac{d\Pi_t^E}{\Pi_t^E},$$

with an $R^2 = 99.3\%$. This indicates that roughly half of the CPI fluctuations is determined by the price index of the elderly. Hence, in the sequel we set $\omega = 50\%$. The elderly are a smaller group than the young, but also has accumulated relatively more wealth.

In analogy to the above definitions, we now replace the real variables defined in Equation (19) by

$$c_t = \frac{C_t}{\Pi_t^l}, \quad w_t = \frac{W_t}{\Pi_t^l}, \quad y_t = \frac{Y_t}{\Pi_t^l}, \quad ,r_t^e = \frac{R_t^e \Pi_{t-1}^l}{\Pi_t^l} \quad ,r_t^f = \frac{R_t^f \Pi_{t-1}^l}{\Pi_t^l},$$
$$\text{with } \Pi_t^l = \begin{cases} \Pi_t^W, & \text{for } t \in \left\{0, 1, \ldots T^R - 1\right\}, \\ \Pi_t^E & \text{for } t \in \left\{T^R, T^R + 1, \ldots T^D - 1\right\}. \end{cases} \tag{27}$$

Formally, the optimization problem can again be rewritten subject to the restrictions in (18).

Again, we assume that at the start of one's life the unhedged components of the relevant price index or indices are zero. Consider first the case in which there is no unhedgeable inflation risk in the overall price index, i.e. $\sigma_u = 0$, hence there is only unhedgeable inflation risk stemming from the differences in the consumption bundles of the workers and the retired. Figure 25 and Figure 26 in Appendix B.3, respectively, depict the optimal portfolio composition, consumption, savings and wealth trajectories over an individual's lifetime. Combining unhedgeable risk in aggregate inflation and in the inflation differences, i.e. $\sigma_E = \sqrt{4} * 0.30\%$ and $\sigma_u = \sqrt{4} * 0.36\%$, yields Figure 27 and Figure 28 in Appendix B.3. Again, the differences with the complete market case are small, compare these figures with the ones in Section 4.2.

Figure 10 presents the welfare effects over the individual's remaining lifetime for the various possible combinations: no unhedgeable inflation risk, one source of unhedgeable inflation risk and both sources combined. As before, welfare losses are largely concentrated during retirement. Therefore, Table 2 reports for the different cases the welfares loss from unhedgeable inflation risk at the start of the career and at the start of retirement. For CRRA parameter $\gamma = 5$, the welfare loss over lifetime from unhedgeable CPI inflation risk is about 0.21% in terms of certainty-equivalent consumption, while that from cohort-specific inflation risk is about 0.05%. If we combine the two sources of unhedgeable inflation risk, the overall lifetime welfare loss is 0.25%. The welfare loss from both sources of unhedgeable inflation risk is much larger at the start of retirement, and amounts 1.05%.

Since different groups of participants will have different degrees of risk aversion (in particular the young will have more risk appetite than the old), Table 2 reports welfare losses for different degrees of relative risk aversion. Not surprisingly, welfare losses increase with the degree of risk aversion. Since valuation in incomplete markets depends on the degree of risk aversion, we establish a range for the welfare loses associated with unhedgeable inflation risk for degrees of relative risk aversion commonly assumed in the literature. The full lifetime value of not having unhedgeable inflation risks ranges from almost zero for risk aversion levels $\gamma = 3$ to 6.07% for $\gamma = 10$. For individuals at the start of their retirement welfare losses amount to up to 7.81% percent when $\gamma = 10$ when both sources of unhedgeable inflation risks are present. This is arguably the most relevant measure, so welfare losses from unhedgeable inflation risk can be substantial. This poses a major problem for regulators having toassess the capital adequacy of a collective pension fund; since there are no market prices available for the unhedgeable component of inflation risk and the standard risk neutral approach is not applicable, the regulator will have to choose a degree of risk aversion but then it unavoidably will run into objections by groups having a diffent attitude towards risk. Choose the average of the degree of risk aversion and the young will accuse the regulator of being overly cautious

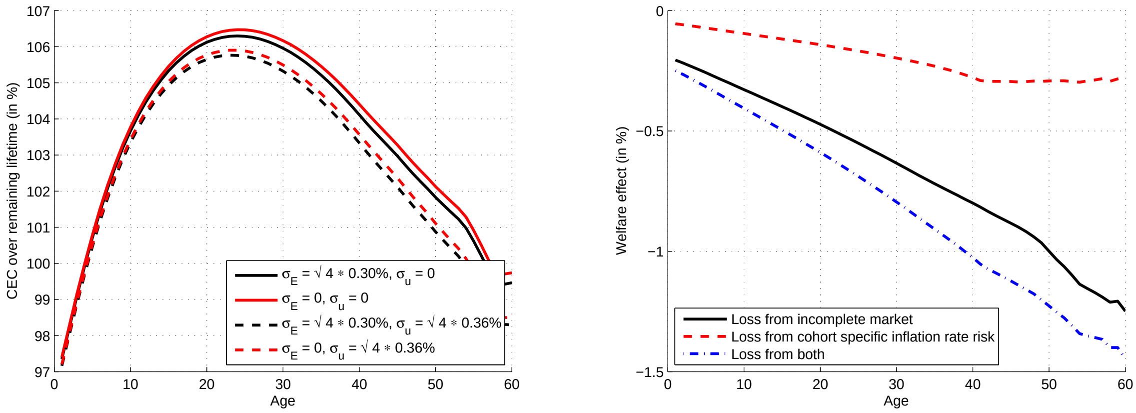


Figure 10: Certainty-equivalent consumption and welfare losses from unhedgeable consumer-price and group-specific inflation risk. *Notes: left-hand panel depicts certainty-equivalent consumption over the remaining lifetime as a function of age under different settings of unhedgeable inflation risk, while the right-hand panel depicts the corresponding welfare loss from these different sources of unhedgeable inflation risk.*

while in the optics of the old the pension fund may well be undercapitalized when the regulator gives a green light.

5.4 Endowment economy with stochastic real wage during working life

So far we have considered a setting in which our individual each period obtains a nominal wage perfectly linked to the price index, i.e. a constant real wage over the life cycle. This subsection turns to the case of a more realistic labor process of the individual in which the wage is not perfectly linked to the price index.

First, we estimate the deterministic part of Equation (20), which is given by a polynomial function:

$$g_t = \theta_0 + \theta_1 t + \theta_2 \frac{t^2}{10} + \theta_3 \frac{t^3}{100}. \tag{28}$$

Using data for the Netherlands on average income per age in (CBS, 2019), we obtain the following estimates

$$\hat{\theta}_0 = 2.9554, \quad \hat{\theta}_1 = 0.0964, \quad \hat{\theta}_2 = -0.0309, \quad \hat{\theta}_3 = 0.0025. \tag{29}$$

The idiosyncratic risk from Equation (20) is modelled as

$$\kappa_t \sim N\left(0, \sigma_\kappa^2\right).$$

For the parameter of the idiosyncratic risk, we take same value as in Cocco et al. (2005), which is $\sigma_\kappa^2 = 0.0738$.[13] Figure 11 shows the obtained stochastic real wage income in the case without unhedgeable risk and no persistent risk in real wage, i.e. $\sigma_u = 0$ and $\nu_t = 0$.

[13]They estimate the wage profile for three groups: no high school, high school and college. We take their middle case for our assumption of parameter σ_κ. The middle case is also the setting applied by Koijen et al. (2010).

Table 2: Welfare effect from unhedgeable inflation risk.

Parameter setting	σ_u(in %)	σ_E(in %)	Lifetime welfare effect (in %)			
			$\gamma = 3$	$\gamma = 5$	$\gamma = 7$	$\gamma = 10$
No unhedgeable inflation risk (benchmark)	0	0	-	-	-	-
Unhedgeable inflation risk overall price index	$\sqrt{4} * 0.36$	0	-0.05	-0.21	-0.40	-1.09
Cohort specific inflation risk	0	$\sqrt{4} * 0.30$	-3*10^{-3}	-0.05	-0.20	-1.05
Both sources of unhedgeable inflation risk	$\sqrt{4} * 0.36$	$\sqrt{4} * 0.30$	-0.05	-0.25	-0.60	-6.07
	σ_u(in %)	σ_E(in %)	Welfare effect during retirement (in %)			
			$\gamma = 3$	$\gamma = 5$	$\gamma = 7$	$\gamma = 10$
No unhedgeable inflation risk (benchmark)	0	0	-	-	-	-
Unhedgeable inflation risk overall price index	$\sqrt{4} * 0.36$	0	-0.52	-0.82	-0.83	-1.46
Cohort specific inflation risk	0	$\sqrt{4} * 0.30$	-0.11	-0.29	-0.53	-1.47
Both sources of unhedgeable inflation risk	$\sqrt{4} * 0.36$	$\sqrt{4} * 0.30$	-0.61	-1.05	-1.35	-7.81

Note: the welfare effect is relative to the setting without unhedgeable inflation risk, i.e. $\sigma_u = \sigma_E = 0$. *The welfare effects during retirement are evaluated at the moment of retirement.*

For the persistent shock of real labor income, we assume that ν_t represents the part of unexpected inflation which is not factored into nominal wage arrangements:

$$\nu_t = -(1-\rho)\left(\frac{\Pi_t - \Pi_{t-1}}{\Pi_{t-1}} - \pi_t\right). \tag{30}$$

If we take $\rho = 1$, then the real wage risk only consists of idiosyncratic risk κ_t, i.e. $y_t = \exp(g_t + \kappa_t)$. For $\rho < 1$, part of the unexpected inflation $\left(\frac{\Pi_t - \Pi_{t-1}}{\Pi_{t-1}} - \pi_t\right)$ is not captured in the nominal wage contracts. This introduces nominal rigidity into wages, similar to the staggered contract first formulated by Taylor (1979). This similarity is explained in more detail in Appendix A.5. The economic interpretation is that collective labour contracts are typically revised every two to four years. This way, unexpected inflation has a gradually increasing effect on the real wage, before a new nominal wage arrangement is concluded.

For the setting without unhedgeable inflation risk and with complete wage correction for unexpected inflation ($\rho = 1$), the optimal lifetime portfolios are depicted in Figure 12 and the optimal lifetime consumption, wealth and savings trajectories are shown in Figure 13. As before, the optimal allocation towards stocks is largest at the beginning of the individual's career. The reason is that human capital is relatively safe compared to equity investments and the ratio of financial wealth over total wealth, including human capital, is lower at the beginning of the individual's career. However, in a setting with an increasing expected real wage at a low age - see Figure 11 - the optimal allocation towards stocks is even larger at the beginning of the individual's career, because the ratio of financial wealth over total wealth is even lower. Concretely, the optimal allocation towards equity in the first six years of working life is on average 10%-points higher. This difference remains positive during working life, but decreases with age. During retirement the average allocation towards stocks is equal to the setting with a constant real wage.[14]

[14]These effects can be observed by comparing Figure 4 with Figure 12.

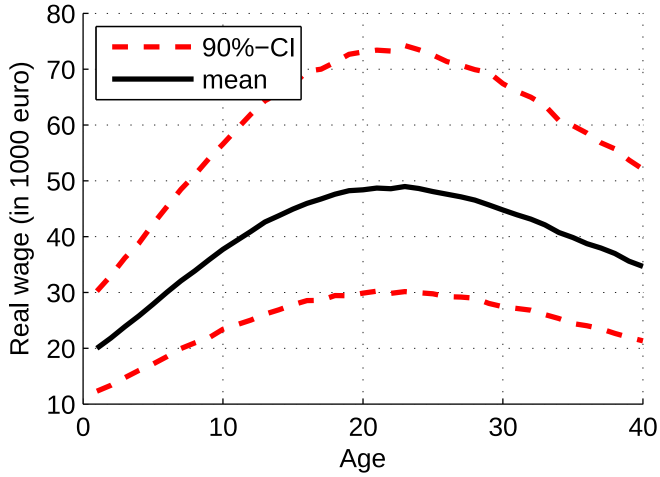


Figure 11: Stochastic real wage income

The settings with unhedgeable inflation risk are shown in Figures 29 - 34 in Appendix B.3.

In the left-hand panel of Figure 14 the welfare levels at each age over the remaining lifetime for the settings with and without the different sources of unhedgeable inflation risk are presented, while the right-hand panel shows the welfare losses relative to the case in which there is no unhedgeable inflation risk. These welfare effects are determined for constant relative risk aversion $\gamma = 5$. Compared to the setting without unhedgeable inflation risk, lifetime welfare is 0.14% to 0.20% lower, while for individuals close to the end of their life welfare over their remaining lifetime is between 0.7% and 1.6% lower. Table 3 presents the welfare effects from the different sources of unhedgeable inflation risk for different parameter values ρ. This table shows that the loss from unhedgeable inflation risk increases when a larger part of unexpected inflation risk is not incorporated in the nominal wage arrangements. However, comparing the case with $\rho = 1$ to the case with $\rho = 0$ the additional welfare loss is rather small, i.e. at most 0.02%-point over lifetime and at most 0.08%-point during retirement. Table 3 considers the case for relative risk aversion parameter $\gamma = 5$ only, while Table 4 present the results for other levels of risk aversion with $\rho = 0$ and $\rho = 1$ respectively. While, as before, the level of risk aversion has a substantial influence on the welfare losses associated with unhedgeable inflation risks, the degree of nominal wage indexation has relatively little effect (compare the results for $\rho = 1$ with the corresponding results for $\rho = 0$).

6 A Fixed Pension Contribution Rate

In this section we turn to the special case in which an indvidual does not optimally smooth consumption during the working life, but merely saves by contributing a fixed fraction of his wage to a defined-contribution (DC) retirement account. The remainder of his salary is consumed. This way, we can investigate how much of the welfare loss from unhedgeable inflation risk may be avoided by consumption smoothing. This is not just of academic interest since real-world pension plans are often of the DC type, requiring a fixed contribution rate from its participants. Moreover, optimal consumption smoothing is not always possible. For example, when necessary living expenses already

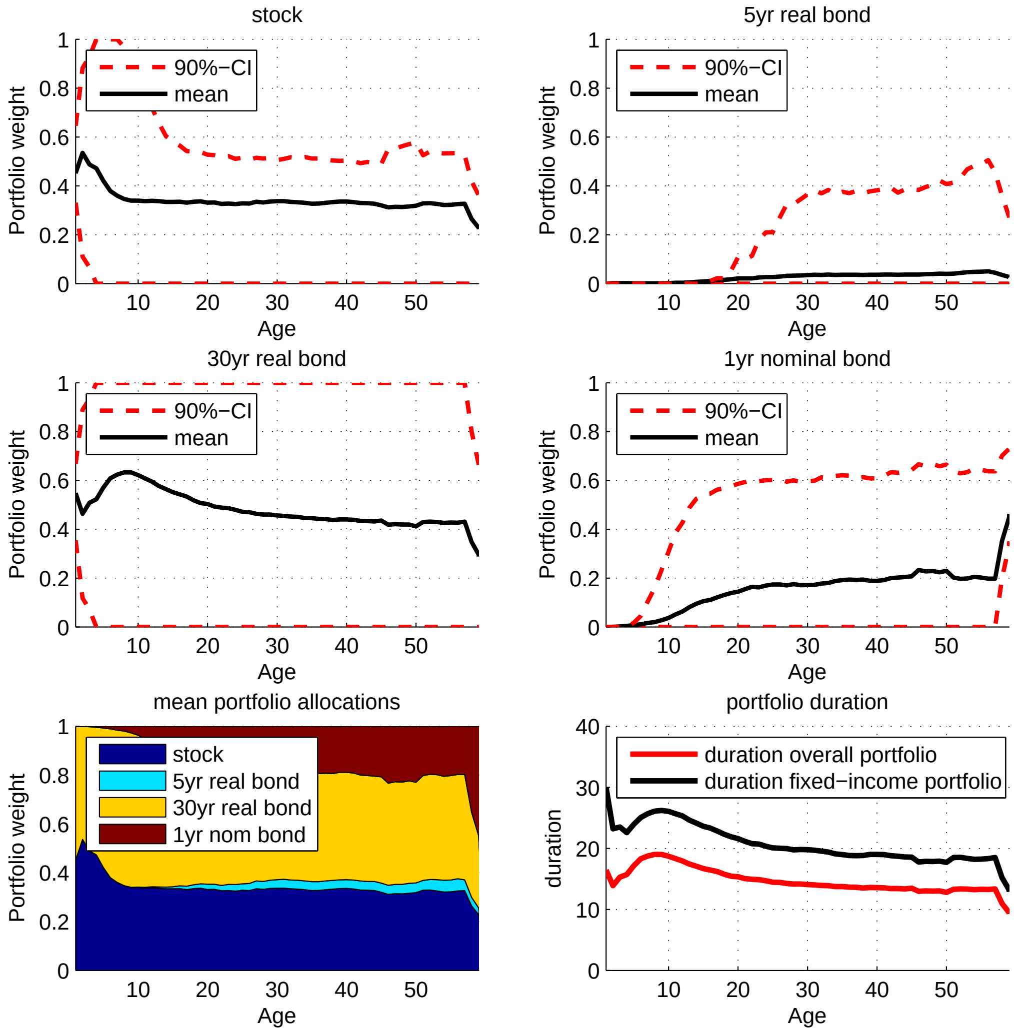


Figure 12: Optimal lifetime portfolios without unhedgeable inflation risk in a setting with stochastic labour income

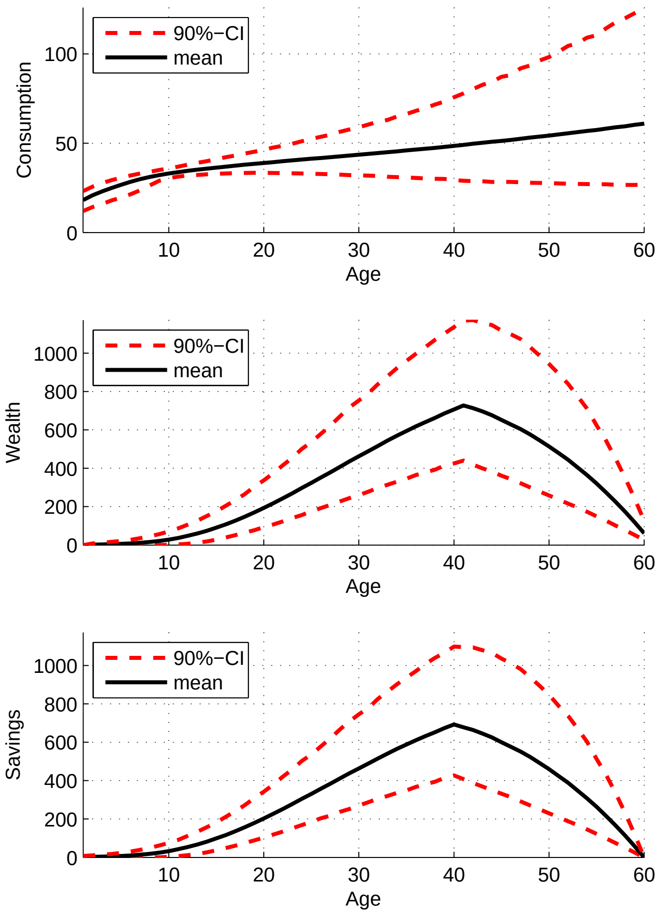


Figure 13: Optimal lifetime consumption, wealth and savings trajectories without unhedgeable inflation risk in a setting with stochastic labour income. *Note: figures are in thousands of euros.*

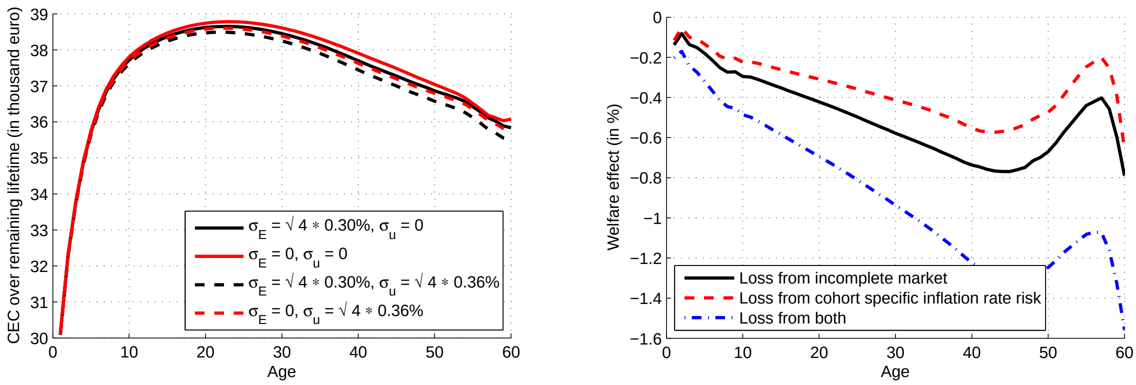


Figure 14: Certainty-equivalent consumption and welfare losses from unhedgeable inflation risk in a setting with stochastic labour income. *Notes: left-hand panel depicts certainty-equivalent consumption over the remaining lifetime as a function of age under different settings of unhedgeable inflation risk, while the right-hand panel depicts the corresponding welfare loss from these different sources of unhedgeable inflation risk. Figure is based on constant relative risk aversion* $\gamma = 5$.

equal or exceed net income, additional savings are no longer possible in the absence of borrowing possibilities. Also myopic financial behaviour of individuals may lead them to spend all disposable income. Hence, we consider it useful, certainly for the group in the lower part of the income distribution, to analyze the welfare effects in the presence of a fixed contribution rate for pension savings χ and all remaining income in each period is consumed in the same period. This is the situation we analyse in this section. So consumption during working life equals

$$c_t = (1-\chi)\, y_t. \tag{31}$$

The individual still optimizes his investment portfolio each period, but chooses his optimal consumption level only during his retirement.

In order to have a fair comparison with the earlier setting with consumption smoothing as in Section 5.4, we determine the fixed contribution rate such that the average present value of the contributions to the DC pension scheme equal the average present value of contributions under consumption smoothing:

$$\begin{aligned} \frac{1}{Q}\sum_{q=1}^{Q}\sum_{t=0}^{T^R-1}\left(y_{q,t}-c_{q,t}\right)P_q\left(t,T^R\right) =& \frac{1}{Q}\sum_{q=1}^{Q}\sum_{t=0}^{T^R-1}\chi y_{q,t}P_q\left(t,T^R\right), \\ \iff \chi =& \frac{\sum_{q=1}^{Q}\sum_{t=0}^{T^R-1}\left(y_{q,t}-c_{q,t}\right)P_q\left(t,T^R\right)}{\sum_{q=1}^{Q}\sum_{t=0}^{T^R-1}y_{q,t}P_q\left(t,T^R\right)}. \end{aligned} \tag{32}$$

where c_t is the optimal consumption level obtained in subsection 5.4. Hence, the parameter χ is calculated by taking the average across the Q simulation paths. For the risk aversion parameter $\gamma = 5$, we obtain an average contribution rate of $\chi = 7.9\%$ under the setting without unhedgeable risk ($\sigma_u = \sigma_E = 0$). Including unhedgeable risk yields the same contribution rate when rounded to one digit. The contribution rate increases with risk aversion, because a participant with larger risk aversion reduces the probability of having extremely low consumption by saving more. Specifically, for $\gamma = 3, 7$ and 10, we obtain $\chi = 4.4\%$, 9.9% and $\chi = 12.4\%$, respectively.

Table 3: Welfare effects from unhedgeable inflation risk in a setting with stochastic labour income

Parameter setting	σ_u(in %)	σ_E(in %)	Lifetime welfare effect (in %)			
			$\rho = 0$	$\rho = \frac{1}{4}$	$\rho = \frac{1}{2}$	$\rho = 1$
No unhedgeable inflation risk (benchmark)	0	0	-	-	-	-
Unhedgeable inflation risk overall price index	$\sqrt{4} * 0.36$	0	-0.16	-0.15	-0.15	-0.14
Cohort specific inflation risk	0	$\sqrt{4} * 0.30$	-0.13	-0.13	-0.12	-0.12
Both sources of unhedgeable inflation risk	$\sqrt{4} * 0.36$	$\sqrt{4} * 0.30$	-0.22	-0.22	-0.21	-0.20
	σ_u(in %)	σ_E(in %)	Welfare effect during retirement (in %)			
			$\rho = 0$	$\rho = \frac{1}{4}$	$\rho = \frac{1}{2}$	$\rho = 1$
No unhedgeable inflation risk (benchmark)	0	0	-	-	-	-
Unhedgeable inflation risk overall price index	$\sqrt{4} * 0.36$	0	-0.82	-0.80	-0.78	-0.75
Cohort specific inflation risk	0	$\sqrt{4} * 0.30$	-0.60	-0.59	-0.58	-0.57
Both sources of unhedgeable inflation risk	$\sqrt{4} * 0.36$	$\sqrt{4} * 0.30$	-1.33	-1.31	-1.29	-1.25

Note: the welfare effect is relative to the setting without unhedgeable inflation risk, i.e. $\sigma_u = \sigma_E = 0$. *The welfare effect during retirement is evaluated at the moment of retirement. Calculations are based on a constant relative risk aversion parameter* $\gamma = 5$.

We take for each value of the risk aversion parameter the corresponding fixed contribution rate as presented above. For the setting with $\gamma = 5$, without unhedgeable inflation risk and with complete nominal correction for unexpected inflation ($\rho = 1$), the optimal lifetime portfolios are depicted in Figure 15 and the optimal lifetime consumption, wealth and savings trajectories are shown in Figure 16. The kink in consumption at retirement date is the result of switching from a regime without consumption smoothing to a regime with consumption smoothing during retirement.

The left-hand panel of Figure 17 depicts welfare at each age over the remaining lifetime for the settings with and without the different sources of unhedgeable inflation risk, while the right-hand panel shows the welfare losses associated with the different sources of unhedgeable inflation risk. Compared to the setting without unhedgeable inflation risk, lifetime welfare is 0.20% to 0.52% lower, depending on which unhedgeable risks are considered. For individuals close to the end of their life, the welfare loss associated with both sources of unhedgeable inflation risk amounts to about 2.3%. Table 5 presents the welfare effects from the different sources of unhedgeable inflation risk for different risk aversion parameter values γ.[15] Comparing the numbers in Table 5 with those in the bottom half of Table 4 we see that in most cases the welfare losses from unhedgeable inflation risk are larger than under the setting with consumption smoothing. Hence, a substantial fraction of the losses associated with unhedgeable risk is eliminated through optimal consumption smoothing. The range of lifetime certainty-equivalent consumption values associated with the elimination of unhedgeable inflation risks changes from 0.03%-0.88% to 0.27%-5.96% for risk aversion ranging from $\gamma = 3$ to $\gamma = 10$. As real-life pension schemes typically apply a fixed or at least a rigid contribution rate and individuals can optimally smooth consumption only to a limited extent, the actual valuation range most

[15] The welfare effects for a contribution rate of $\chi = 10\%$ and $\chi = 20\%$ are presented in Table 6 in Appendix B.3.

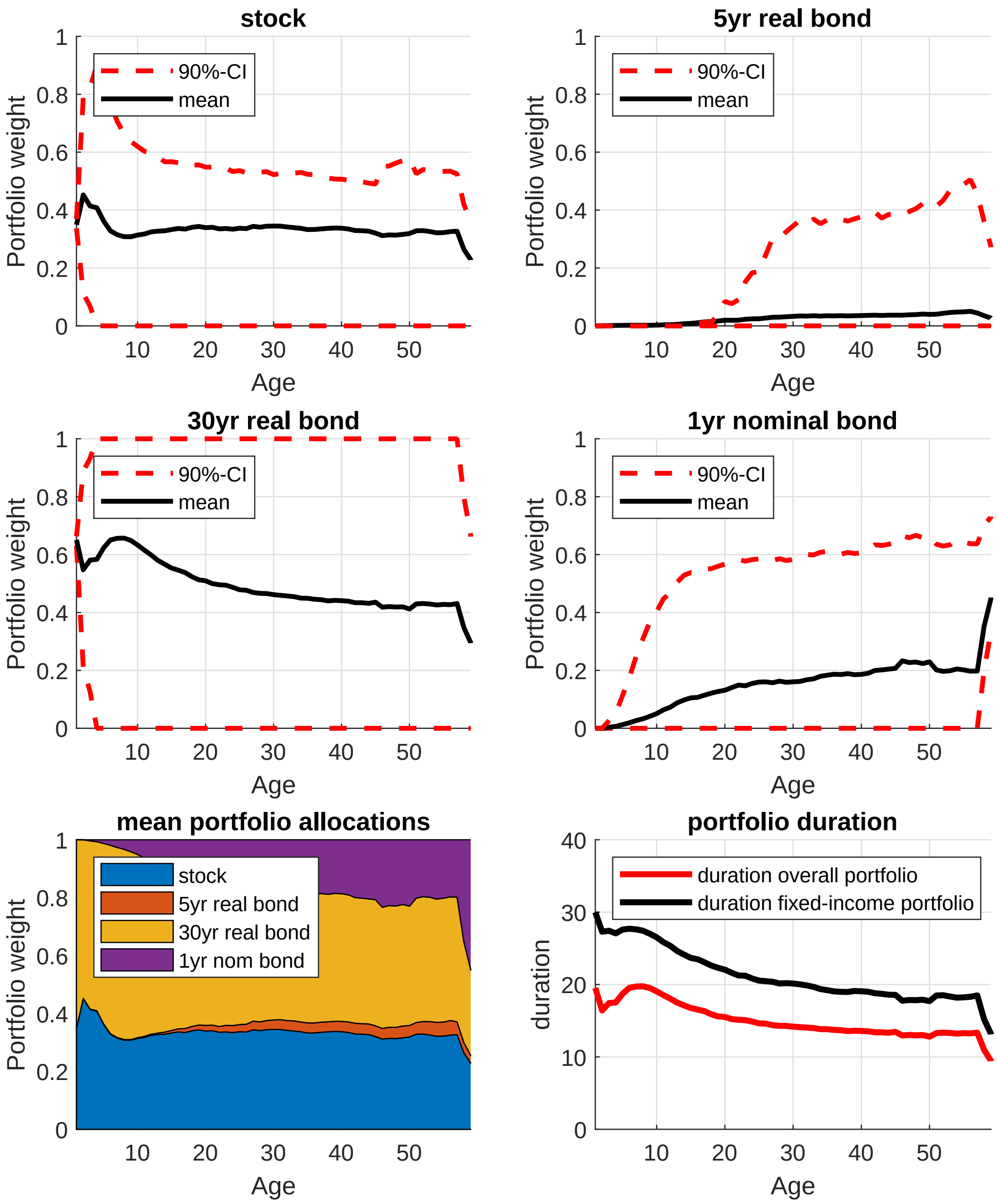


Figure 15: Optimal lifetime portfolios without unhedgeable inflation risk in a DC pension scheme. *Note: contribution rate is fixed at* $\chi = 7.9\%$

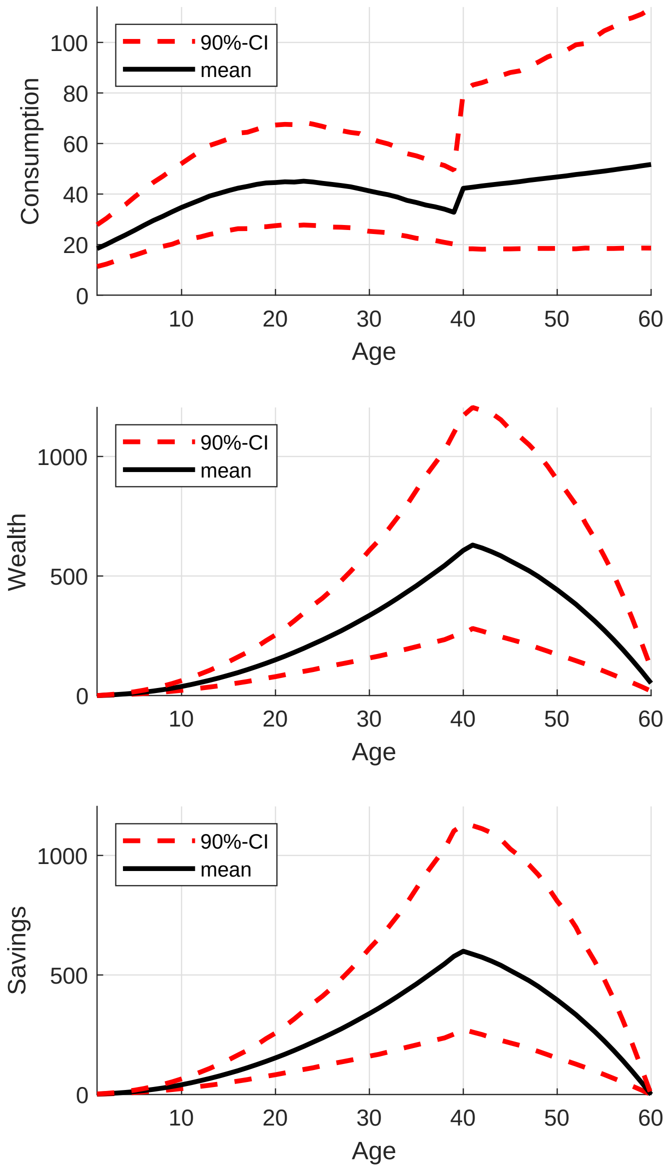


Figure 16: Optimal lifetime consumption, wealth and savings trajectories without unhedgeable inflation risk in a DC pension scheme. *Notes: savings are in thousands of euros, while the contribution rate is fixed* $\chi = 7.9\%$.

Table 4: Welfare effects from unhedgeable inflation risk in a setting with stochastic labour income

Parameter setting	$\rho = 0$		Lifetime welfare effect (in %)			
	σ_u(in %)	σ_E(in %)	$\gamma = 3$	$\gamma = 5$	$\gamma = 7$	$\gamma = 10$
No unhedgeable inflation risk (benchmark)	0	0	-	-	-	-
Unhedgeable inflation risk overall price index	$\sqrt{4} * 0.36$	0	-0.01	-0.16	-0.34	-0.52
Cohort specific inflation risk	0	$\sqrt{4} * 0.30$	-0.02	-0.13	-0.32	-0.52
Both sources of unhedgeable inflation risk	$\sqrt{4} * 0.36$	$\sqrt{4} * 0.30$	-0.03	-0.22	-0.40	-0.85
	$\rho = 0$		Welfare effect during retirement (in %)			
	σ_u(in %)	σ_E(in %)	$\gamma = 3$	$\gamma = 5$	$\gamma = 7$	$\gamma = 10$
No unhedgeable inflation risk (benchmark)	0	0	-	-	-	-
Unhedgeable inflation risk overall price index	$\sqrt{4} * 0.36$	0	-0.50	-0.80	-1.08	-3.23
Cohort specific inflation risk	0	$\sqrt{4} * 0.30$	-0.19	-0.59	-0.89	-3.00
Both sources of unhedgeable inflation risk	$\sqrt{4} * 0.36$	$\sqrt{4} * 0.30$	-0.71	-1.31	-1.73	-10.53
	$\rho = 1$		Lifetime welfare effect (in %)			
	σ_u(in %)	σ_E(in %)	$\gamma = 3$	$\gamma = 5$	$\gamma = 7$	$\gamma = 10$
No unhedgeable inflation risk (benchmark)	0	0	-	-	-	-
Unhedgeable inflation risk overall price index	$\sqrt{4} * 0.36$	0	$-1 * 10^{-3}$	-0.14	-0.31	-0.50
Cohort specific inflation risk	0	$\sqrt{4} * 0.30$	-0.02	-0.12	-0.29	-0.51
Both sources of unhedgeable inflation risk	$\sqrt{4} * 0.36$	$\sqrt{4} * 0.30$	-0.03	-0.20	-0.36	-0.88
	$\rho = 1$		Welfare effect during retirement (in %)			
	σ_u(in %)	σ_E(in %)	$\gamma = 3$	$\gamma = 5$	$\gamma = 7$	$\gamma = 10$
No unhedgeable inflation risk (benchmark)	0	0	-	-	-	-
Unhedgeable inflation risk overall price index	$\sqrt{4} * 0.36$	0	-0.46	-0.75	-0.98	-3.16
Cohort specific inflation risk	0	$\sqrt{4} * 0.30$	-0.17	-0.57	-0.83	-3.11
Both sources of unhedgeable inflation risk	$\sqrt{4} * 0.36$	$\sqrt{4} * 0.30$	-0.67	-1.25	-1.61	-11.38

Note: the welfare effects are relative to the setting without unhedgeable inflation risk, i.e. $\sigma_u = \sigma_E = 0$. *The welfare effect during retirement is evaluated at the moment of retirement.*

likely lies between these ranges.

7 Conclusion

We have developed a framework for the evaluation of welfare losses with unhedgeable inflation risk and, using calibrations based on macro and financial figures, we quantify these losses. Individuals experience welfare losses from two sources of unhedgeable inflation risk. First, CPI inflation risk cannot be fully hedged in countries that do not issue index-linked bonds. Second, the price index based on their individual consumption bundle may differ from the general consumer price index. The losses over one's remaining lifetime are rather small for young individuals, but are non-negligible for elderly individuals.

The welfare loss from not being able to hedge CPI inflation risk due to the absence of index-linked bonds may amount to up to 1% in terms of lifetime certainty-equivalent

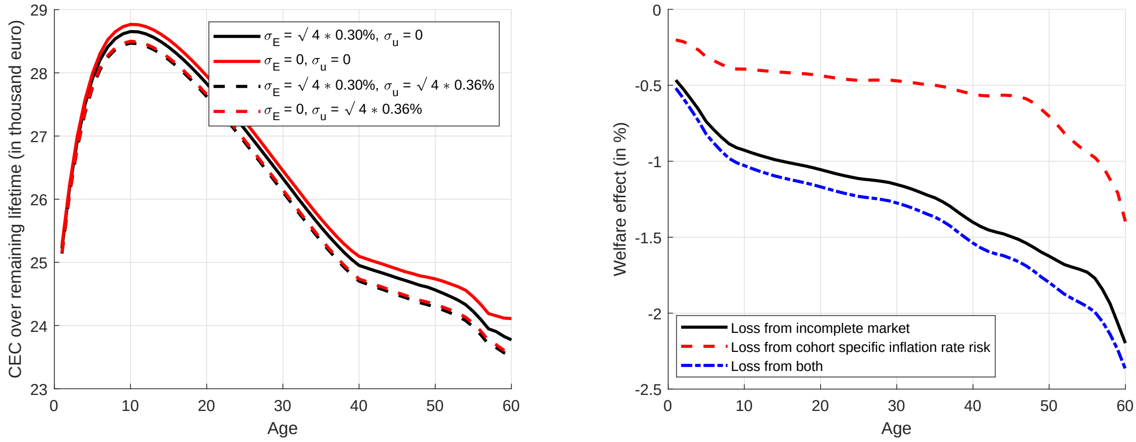


Figure 17: Certainty-equivalent consumption and welfare losses from unhedgeable inflation risk in a DC pension scheme with fixed contribution rate. *Notes: left panel depicts certainty-equivalent consumption over the remaining lifetime as a function of age under different settings of unhedgeable inflation risk, while the right panel depicts the corresponding welfare loss from these different sources of unhedgeable inflation risk. The contribution rate is fixed at* $\chi = 7.9\%$.

consumption for commonly assumed degrees of risk aversion. Moreover, the fact that workers and pensioners have their own specific consumption bundles produces a welfare loss of similar magnitude. However, if we combine the two sources of unhedgeable inflation risk, the numbers become substantially more significant: lifetime welfare loss can amount to up to 6% for commonly assumed degrees of risk aversion.

We have investigated the robustness of our baseline welfare loss quantification in various directions. In particular, welfare losses are quantitatively similar when we assume a more realistic and stochastic real wage process. Finally, the welfare losses from unhedgeable inflation risks become larger when, during their working life, individuals merely contribute a fixed percentage of their gross wage to a DC pension account, but are prevented from optimally smoothing consumption. This describes the situation of a large fraction of the population.

Different instruments may be deployed to reduce the welfare losses from unhedgeable inflation risks. The non-negligible welfare losses from unhedgeable CPI inflation risk could be reduced if governments would absorb the risk and start issuing index-linked debt. Obviously, these micro-level benefits from index-linked debt need to be weighed against its costs. For example, because of the smaller size of its market, liquidity premia associated with index-linked debt may initially be higher than with nominal debt. Also, index-linked debt may affect the stability of the public budget, although the direction into which this works is not a priori clear.[16] The welfare losses from unhedgeable risk caused by group-specific consumption packages could be reduced through appropriately designed risk-sharing arrangements in collective pension schemes in which different cohorts participate.

The substantial welfare losses that we document, in particular in the presence of different consumption patterns between the old and the young, pose a major problem for regulators having to assess the capital adequacy of a collective pension fund: different participant groups have a different capacity to absorb the losses from unhedgeable

[16]Simulations by Westerhout and Beetsma (2019) suggest that the public debt - GDP ratio is more stable under indexed debt.

Table 5: Welfare effects from unhedgeable inflation risk in a DC pension scheme with a fixed contribution rate χ

Parameter setting	σ_u(in %)	σ_E(in %)	Lifetime welfare effect (in %) $\gamma = 3$ $\chi = 4.4\%$	$\gamma = 5$ $\chi = 7.9\%$	$\gamma = 7$ $\chi = 9.9\%$	$\gamma = 10$ $\chi = 12.4\%$
No unhedgeable inflation risk (benchmark)	0	0	-	-	-	-
Unhedgeable inflation risk overall price index	$\sqrt{4} * 0.36$	0	-0.28	-0.46	-0.57	-1.57
Cohort specific inflation risk	0	$\sqrt{4} * 0.30$	-0.04	-0.20	-0.22	-0.65
Both sources of unhedgeable inflation risk	$\sqrt{4} * 0.36$	$\sqrt{4} * 0.30$	-0.27	-0.52	-0.68	-5.96
			Welfare effect during retirement (in %)			
	σ_u(in %)	σ_E(in %)	$\gamma = 3$ $\chi = 4.4\%$	$\gamma = 5$ $\chi = 7.9\%$	$\gamma = 7$ $\chi = 9.9\%$	$\gamma = 10$ $\chi = 12.4\%$
No unhedgeable inflation risk (benchmark)	0	0	-	-	-	-
Unhedgeable inflation risk overall price index	$\sqrt{4} * 0.36$	0	-0.62	-1.44	-1.96	-5.26
Cohort specific inflation risk	0	$\sqrt{4} * 0.30$	-0.09	-0.57	-0.53	-2.07
Both sources of unhedgeable inflation risk	$\sqrt{4} * 0.36$	$\sqrt{4} * 0.30$	-0.59	-1.58	-2.30	-14.85

Note: the welfare effects are relative to the setting without unhedgeable inflation risk, i.e. $\sigma_u = \sigma_E = 0$. The welfare effect during retirement is evaluated at the moment of retirement.

inflation risks. For example, the wage income of young cohorts absorbs part of the unhedgeable inflation risk, implying that they attach less value to eliminating unhedgeable risk than retirees. Moreover, since there are no market prices available for the unhedgeable component of inflation risk and the standard risk-neutral approach is not applicable in an incomplete market setting, with differences in risk aversion the regulator will have to choose a degree of risk aversion; but then the regulator unavoidably will run into objections by groups having a diffent attitude towards risk. Choose the average of the degrees of risk aversion and the young will accuse the regulator of being overly cautious, while in the optics of the old the pension fund may well be undercapitalized when the regulator gives a green light. Our analysis also highlights that the willingness to voluntarily participate in a collective pension scheme is likely to come under pressure when differences in risk preferences between groups of participants cannot be resolved through trades in outside capital markets because of market incompleteness.

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A Appendix

A.1 Discretization

In continuous time, the model can be summarized to

$$d\Psi_t = (\alpha + \Gamma'\Psi_t)\, dt + \Sigma_\Psi dZ_t, \tag{33}$$

where

$$\begin{aligned}
\Psi_t &= \begin{bmatrix} X_t \\ \log \Pi_t \\ \log S_t \\ \log P(t, t+s) \end{bmatrix} \\
\alpha &= \begin{bmatrix} 0_{3\times 1} \\ \delta_{0,\pi} - \frac{1}{2}\sigma_\Pi'\sigma_\Pi \\ \delta_{0,r} + \eta_S - \frac{1}{2}\sigma_S'\sigma_S \\ \delta_{0,r} + B(s)'\tilde{\Lambda}_0 - \frac{1}{2}B(s)'B(s) \end{bmatrix} \\
\Gamma &= \begin{bmatrix} -K & 0_{3\times 3} \\ \delta_{1,\pi}' & 0_{1\times 3} \\ \delta_{1,r}' & 0_{1\times 3} \\ \delta_{1,\pi}' + B(s)'\tilde{\Lambda}_1 & 0_{1\times 3} \end{bmatrix} \\
\Sigma_\Psi &= \begin{bmatrix} I_{3\times 3} & 0_{1\times 2} \\ \sigma_\Pi' & \\ \sigma_S' & \\ B(s)' & 0_{1\times 3} \end{bmatrix}.
\end{aligned} \tag{34}$$

Following Koijen et al. (2010), we can rewrite the model in discrete form with time steps h which is necessary for its implementation:

$$\begin{aligned}
\Psi_t &= \hat{\alpha}^{(h)} + \hat{\Gamma}^{(h)}\Psi_{t-h} + \varepsilon_t, \quad \varepsilon_t \sim N\left(0, \hat{\Sigma}_\Psi^{(h)}\right) \\
\hat{\Gamma}^{(h)} &= \exp(\Gamma h) = U \exp(Dh)\, U^{-1} \\
\hat{\alpha} &= UFU^{-1}\alpha
\end{aligned} \tag{35}$$

where D is derived from the eigenvalue decomposition, $\Gamma = UDU^{-1}$, and F is a matrix with elements:

$$F_{i,j} = \begin{cases} \frac{\exp(hD_{i,i})-1}{D_{i,i}}, & \text{for } i = j, \\ 0, & \text{for } i \neq j. \end{cases} \tag{36}$$

Finally, we have $\hat{\Sigma}_{\Psi}^{(h)} = UVU'$ where V is a matrix with elements

$$V_{i,j} = \left[U^{-1}\Sigma_{\Psi}\Sigma_{\Psi}' \left(U^{-1}\right)'\right]_{i,j} \frac{\exp\left[h\left(D_{i,i} + D_{j,j}\right)\right] - 1}{D_{i,i} + D_{j,j}}. \tag{37}$$

A.2 Derivation UFR

The derivation of the UFR is as follows

$$\begin{aligned} \lim_{s\to\infty} E_t\left[y\left(t, t+s\right)\right] =& - \lim_{s\to\infty} E_t\left[\frac{A\left(s\right)}{s}\right] - \lim_{s\to\infty} E_t\left[\frac{B\left(s\right)'}{s}\right] X_t \\ =& - \lim_{s\to\infty} \frac{A\left(s\right)}{s} \\ =& - \lim_{s\to\infty} \frac{\partial A\left(s\right)}{\partial s} \\ =& \delta_{0,r} + \lim_{s\to\infty} B\left(s\right)'\left(\tilde{\Lambda}_0 - \frac{1}{2}B\left(s\right)\right) \\ =& \delta_{0,r} - \left(\left(K' + \tilde{\Lambda}_1'\right)^{-1}\delta_{1,r}\right)'\left(\tilde{\Lambda}_0 + \frac{1}{2}\left(K' + \tilde{\Lambda}_1'\right)^{-1}\delta_{1,r}\right) \end{aligned}$$

$$\text{using } \lim_{s\to\infty} B\left(s\right) = -\left(K' + \tilde{\Lambda}_1'\right)^{-1}\delta_{1,r}. \tag{38}$$

A.3 Derivation Certainty-Equivalent Consumption

We obtain certainty-equivalent consumption from

$$\sum_{t=0}^{T^D-1} \beta^t u\left(CEC\right) = \frac{1}{Q}\sum_{q=1}^{Q}\left[\sum_{t=0}^{T^D-1} \beta^t u\left(c_{q,t}\right)\right], \tag{39}$$

where the utility function is given by $u\left(c\right) = \frac{c^{1-\gamma}}{1-\gamma}$. Then, after rearranging we obtain

$$\begin{aligned} CEC =& \left\{\left(1-\gamma\right)\frac{1-\beta}{1-\beta^{T^D}}\frac{1}{Q}\sum_{q=1}^{Q}\left[\sum_{t=0}^{T^D-1}\beta^t\frac{c_{q,t}^{1-\gamma}}{1-\gamma}\right]\right\}^{1/(1-\gamma)} \\ =& \left[\frac{1-\beta}{Q\left(1-\beta^{T^D}\right)}\sum_{q=1}^{Q}\left(\sum_{t=0}^{T^D-1}\beta^t c_{q,t}^{1-\gamma}\right)\right].^{1/(1-\gamma)} \end{aligned} \tag{40}$$

A.4 Estimating Dutch CPI using Foreign CPI and Commodity Indices

Figure 18 shows the actual Dutch inflation and two estimated Dutch inflation series. The first estimation is based on a regression of Dutch inflation on inflation of the U.K., Germany, France, Italy, Spain, Australia, New Zealand, Denmark, Sweden, Japan, the U.S.

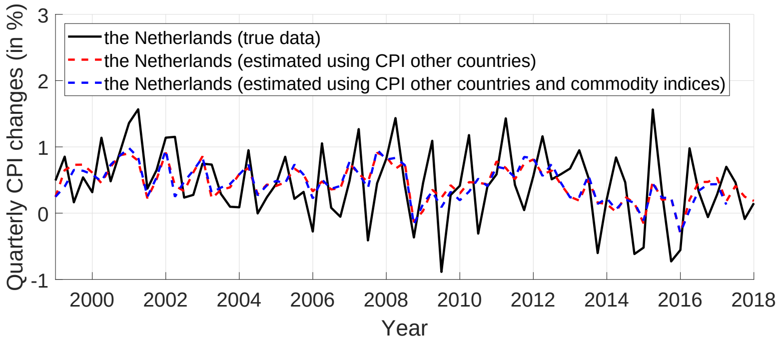

Figure 18: Quarterly Dutch inflation rates from January 1999 to January 2018 after seasonality correction

and Canada. The second also includes the following commodity indices as explanatory variables: (i) Brent Crude Oil, (ii) General Crude Oil, (iii) Gold index, and (iv) General Commodity Price Index.[17] From the first estimation we obtain an R-squared of 43% and a standard deviation of the residuals of 0.30%. From the second estimation we obtain an R-squared of 47% and a standard deviation of residuals of 0.29%.

A.5 Staggered Contract

Under the staggered contract by Taylor (1979) the nominal wage process is

$$\begin{aligned} Y_t &= \lambda Y_{t-1} + (1-\lambda)\, m_t, \\ m_t &= m_{t-1} + \varepsilon_{m,t}, \\ \varepsilon_{m,t} &\sim N\left(0, \sigma_m^2\right), \end{aligned} \tag{41}$$

where m_t denotes the stochastic money supply. Hence, if $\lambda = 1$, there is perfect nominal rigidity, while for $\lambda < 1$, the nominal wages adjust to their new state. In our setting, we replace this stochastic money supply by unexpected inflation $\left(\frac{\Pi_t - \Pi_{t-1}}{\Pi_{t-1}} - \pi_t\right)$ and we model the wage rate in real terms. Hence, inspired by the staggered nominal wage contracting approach by Taylor (1979) we define

$$\nu_t = -\lambda \left(\frac{\Pi_t - \Pi_{t-1}}{\Pi_{t-1}} - \pi_t \right). \tag{42}$$

By replacing λ by $(1-\rho)$, we arrive at the expression defined in Equation (30). This means that for $\lambda = (1-\rho) = 0$, Equation (20) becomes

$$y_t = \exp\left(g_t + \kappa_t\right). \tag{43}$$

This way, the real wage is (exponential of) the sum of a deterministic wage profile and idiosyncratic shocks only. If we take $\lambda = (1-\rho) = 1$, then Equation (20) becomes

$$y_t = \exp\left(g_t + \kappa_t\right) \exp\left[- \left(\frac{\Pi_t - \Pi_{t-1}}{\Pi_{t-1}} - \pi_t \right) \right], \tag{44}$$

hence unexpected inflation erodes the real wage rate.

[17] The data is obtained from IndexMundi on October 19, 2018 from https://www.indexmundi.com/commodities

B Appendix

B.1 Solution Method Optimal Portfolio and Consumption Problem

We follow the solution method as described by Koijen et al. (2010).

We define $J_t(w_t, f_t)$ as the value function. At time $t = T^D - 1$, we have $c_t^* = w_t$, as there is no bequest motive. In other words, the individual-optimally consumes all wealth in the last period, so

$$J_{T^D-1}(w_{T^D-1}, f_{T^D-1}) = u(w_{T^D-1}). \tag{45}$$

The Bellman equation for periods $t = 0, \dots, T^D - 2$ is

$$\begin{aligned} J_t(w_t, f_t) &= \max_{c_t, x_t \in \Upsilon} u(c_t) + \beta E_t\left[J_{t+1}(w_{t+1}, f_{t+1})\right], \\ \text{with } \Upsilon(w_t) &= \left\{(c, x) : c \leq w_t, x \geq 0, \sum x \leq 1\right\}. \end{aligned} \tag{46}$$

Following Carroll (2006), we define the amount of savings $s_t \equiv w_t - c_t$, i.e. whatever is not consumed out of beginning of period wealth. We set up a grid for s with size n_S and elements $j = 1, \dots, n_S$. We also set up a grid with H portfolios, i.e. $x_1, \dots, x_H \in \Upsilon$.

Step 1: set up grids For the portfolio grid, we take all possible combinations with step size 20%.[18] For savings we take a non-linear grid which is finer on the lower values than on the higher values, in order to be more accurate at the relevant values for determining welfare and to be more accurate for the majority of scenarios. First, we make a linear grid $\hat{s} = \left\{0, \frac{\log(100)}{n_s - 1}, \frac{2\log(100)}{n_s - 1}, \dots, \frac{(n_s - 1)\log(100)}{n_s - 1}\right\}$. Then, we define element j of the grid for savings as $\exp(\hat{s}_j) - 1$. Hence, the largest grid value is 100 and the lowest value is 0. We take $n_s = 40$ grid points. The step size of the grid points strictly increases, where the first step size of the grid is 0.125, while the last step size is 11.27.[19]

Step 2: last period At time $t = T^D - 1$, we have $c_t^* = w_t$ as there is no bequest motive. In other words, the individual optimally consumes all wealth in the last period. The value function is

$$J_{T^D-1}(w_{T^D-1}, f_{T^D-1}) = u(w_{T^D-1}). \tag{47}$$

As there are no savings in the last period, there is no portfolio to optimize.

Step 3a: first to last period At time $t = T^D - 2$, the problem given $s_{T^D-2} = s_j$ is

$$\max_{c_{T^D-2}, x_{T^D-2} \in \Upsilon} u(c_{T^D-2}) + \beta E_{T^D-2}\left[u(w_{T^D-1}) \mid f_{T^D-2}, s_{T^D-2} = s_j\right] \tag{48}$$

[18] We also calculated the result from Section 4.2 with a finer grid, by adjusting the step size to 12.5%. The obtained results are identical, but computation time increases substantially.

[19] We also calculated the result from Section 4.2 with a finer grid, by increasing the number of grid points for savings to $n_s = 50$. The obtained results are identical, but computation time heavily increases. The difference in lifetime CEC is negligible (less than 0.005%).

Step 3b: optimize asset allocation The FOC's for the asset allocation reads

$$E_{T^D-2}\left[r^e_{T^D-1}w^{-\gamma}_{T^D-1}|f_{T^D-2},s_{T^D-2}=s_j\right]+\xi-\zeta\iota=0_{3\times 0}, \tag{49}$$

where ξ and ζ are the Kuhn-Tucker multipliers corresponding to $x\geq 0$ and $\sum x\leq 1$, respectively. We then approximate for each asset i and each grid portfolio x_h the following:

$$E_{T^D-2}\left[r^e_{i,T^D-1}w^{-\gamma}_{T^D-1}|f_{T^D-2},s_{T^D-2},x_h\right]\approx\beta'_{i,x,s}\left(1,X_1,X_2,X_3\right). \tag{50}$$

Since $\hat{\beta}_{i,x,s}$ is a function of the portfolio weights x we can parameterize these coefficients as a function of x as follows:[20]

$$\hat{\beta}_{i,x,s_j}\approx g\left(x\right)\alpha_{s_j,i}. \tag{51}$$

Then we should solve the following

$$\begin{aligned} g\left(x^*\right)\hat{\alpha}_{s_j,i}\left(1,X_1,X_2,X_3\right)+\xi_i-\zeta&=0\\ \xi_i x_i=0,\forall i,\ \text{ and }\zeta\left(x'\iota-1\right)&=0\\ \xi\geq 0\text{ and }\zeta&\geq 0. \end{aligned} \tag{52}$$

We take a linear function of $g\left(\cdot\right)$ as follows

$$\begin{aligned} \left(1,x^*_1,x^*_2,x^*_3\right)'\hat{\alpha}_{s_j,i}\left(1,X_1,X_2,X_3\right)+\xi_i-\zeta&=0\\ \xi_i x_i=0,\forall i,\ \text{ and }\zeta\left(x'\iota-1\right)&=0\\ \xi\geq 0\text{ and }\zeta&\geq 0. \end{aligned} \tag{53}$$

This can be rewritten into the following form

$$\begin{aligned} 0_{3\times 1}&=\Xi_0+\Xi_1x^*+\xi-\zeta\iota\\ \Xi_0&=\left(\hat{\alpha}_{s_j,1,1},\hat{\alpha}_{s_j,2,1},\hat{\alpha}_{s_j,3,1}\right)'\left(1,X_1,X_2,X_3\right)\\ \Xi_{1,i}&=\left(\hat{\alpha}_{s_j,i,2},\hat{\alpha}_{s_j,i,3},\hat{\alpha}_{s_j,i,4}\right)'\left(1,X_1,X_2,X_3\right)\\ \Xi_1&=\left[\begin{array}{ccc}\Xi_{1,1}&\Xi_{1,2}&\Xi_{1,3}\end{array}\right]'. \end{aligned} \tag{54}$$

We can solve this numerically, to obtain the optimal portfolio x^*.

Step 3c: optimize consumption Using the FOC and the optimal asset allocation $x^*_{T^D-2}$, the optimal consumption is then given by

$$c^*_{T^D-2}\left(f_{T^D-2},s_j\right)=\left\{\beta E_{T^D-2}\left[\left(x^*_{T^D-2}r^e_{T^D-1}+r^f_{T^D-1}\right)w^{*-\gamma}_{T^D-1}|f_{T^D-2},s_{T^D-2}=s_j\right]\right\}^{-1/\gamma} \tag{55}$$

where $w^*_{T^D-1}$ corresponds to the optimal portfolio strategy obtained one step before. This conditional expectation is approximated in a similar way as we approximated the asset allocation in steps 2 and 3. However, in order to ensure strictly positive consumption in the approximation, we take the logarithm of Equation (55):

$$\log\left(\left\{\beta E_{T^D-2}\left[\left(x_{T^D-2}r^e_{T^D-1}+r^f_{T^D-1}\right)w^{*-\gamma}_{T^D-1}|f_{T^D-2},s_{T^D-2}=s_j\right]\right\}^{-1/\gamma}\right)\approx\beta^{c\prime}_{i,x,s}\left(1,X_1,X_2,X_3\right).$$

[20] A similar approach is performed by Diris, Palm, and Schotman (2014).

Again, since $\hat{\beta^c}_{i,x,s}$ is a function of the portfolio weights x we can parameterize these coefficients as a function of x:

$$\hat{\beta}^c_{i,x,s_j} \approx (1, x_1, x_2, x_3)' \, \alpha^c_{s_j}. \tag{56}$$

Then, we obtain the optimal consumption, using $x^*_{T^D-2}$, from

$$c^*_{T^D-2}\left(f_{T^D-2}, s_j\right) = \exp\left[\left(1, x^*_1, x^*_2, x^*_3\right)' \hat{\alpha}^c_{s_j}\left(1, X_1, X_2, X_3\right)\right].$$

Step 3d: endogenous grid solution Determine the endogenous grid by

$$w_{T^D-2}\left(f_{T^D-2}, s_j\right) = c^*_{T^D-2}\left(f_{T^D-2}, s_j\right) + s_j. \tag{57}$$

Step 4a: other periods At time t, the problem given $s_t = s_j$ is

$$J_t\left(w_t, f_t\right) = \max_{c_t, x_t \in \Upsilon} u\left(c_t\right) + \beta E_t\left[J_{t+1}\left(w_{t+1}, f_{t+1}\right) | f_t, s_t = s_j\right] \tag{58}$$

The FOC's are

$$\begin{aligned} E_t\left[r^e_{t+1} J'_{t+1}\left(w_{t+1}, f_{t+1}\right)\right] + \xi - \zeta\iota &= 0, \\ \left\{\beta E_t\left[\left(x^{*\prime}_t r^e_{t+1} + r^f_{t+1}\right) J'_{t+1}\left(w_{t+1}, f_{t+1}\right)\right]\right\}^{-1/\gamma} &= c^*_t. \end{aligned} \tag{59}$$

The Kuhn-Tucker multipliers are non-negative and satisfy

$$\xi_i x_i = 0, \forall i, \text{ and } \zeta\left(x'\iota - 1\right) = 0. \tag{60}$$

The envelope theorem implies that

$$J'_{t+1}\left(w_{t+1}, f_{t+1}\right) = u'\left(c^*_{t+1}\right). \tag{61}$$

This we can use to rewrite the FOC's, yielding

$$\begin{aligned} E_t\left[r^e_{t+1} c^{*-\gamma}_{t+1}\right] + \xi - \zeta\iota &= 0, \\ \left\{\beta E_t\left[\left(x^{*\prime}_t r^e_{t+1} + r^f_{t+1}\right) c^{*-\gamma}_{t+1}\right]\right\}^{-1/\gamma} &= c^*_t. \end{aligned} \tag{62}$$

Step 4b: optimize asset allocation Again, we approximate for each asset i and each grid portfolio x_h the following:

$$E_t\left[r^e_{t+1} c^{*-\gamma}_{t+1} | f_t, s_t, x_h\right] \approx \beta'_{i,x,s}\left(1, X_1, X_2, X_3\right). \tag{63}$$

Similar to step 3, we can parameterize the coefficients $\hat{\beta}_{i,x,s}$ as a function of x as follows:

$$\hat{\beta}_{i,x,s_j} \approx (1, x_1, x_2, x_3)' \, \alpha_{s_j,i}. \tag{64}$$

Then we should solve

$$\begin{aligned} \left(1, x^*_1, x^*_2, x^*_3\right)' \hat{\alpha}_{s_j,i}\left(1, X_1, X_2, X_3\right) + \xi_i - \zeta = 0 \\ \xi_i x_i = 0, \forall i, \text{ and } \zeta\left(x'\iota - 1\right) = 0 \\ \xi \geq 0 \text{ and } \zeta \geq 0. \end{aligned} \tag{65}$$

Again we use the rewritten form

$$\begin{aligned}
0_{3\times 1} =& \Xi_0 + \Xi_1 x^* + \xi - \zeta\iota \\
\Xi_0 =& \left(\hat{\alpha}_{s_j,1,1}, \hat{\alpha}_{s_j,2,1}, \hat{\alpha}_{s_j,3,1}\right)' (1, X_1, X_2, X_3) \\
\Xi_{1,i} =& \left(\hat{\alpha}_{s_j,i,2}, \hat{\alpha}_{s_j,i,3}, \hat{\alpha}_{s_j,i,4}\right)' (1, X_1, X_2, X_3) \\
\Xi_1 =& \left[\begin{array}{ccc} \Xi_{1,1} & \Xi_{1,2} & \Xi_{1,3} \end{array}\right]', \qquad (66)
\end{aligned}$$

which can be solved numerically, to obtain the optimal portfolio x^*.

Step 4c: optimize consumption Using the FOC and the optimal asset allocation x_t^*, the optimal consumption is then given by

$$\log\left(c_t^*\right) = \log\left(\left\{\beta E_t\left[\left(x_t^{*\prime} r_{t+1}^e + r_{t+1}^f\right) c_{t+1}^{*-\gamma}\right]\right\}^{-1/\gamma}\right). \qquad (67)$$

where c_{t+1}^* corresponds to the optimal consumption obtained one step before. This conditional expectation is approximated as:

$$\log\left(\left\{\beta E_t\left[\left(x_t^{*\prime} r_{t+1}^e + r_{t+1}^f\right) c_{t+1}^{*-\gamma}\right]\right\}^{-1/\gamma}\right) \approx \beta_{i,x,s}^{c\prime}\left(1, X_1, X_2, X_3\right).$$

Again, since $\hat{\beta}^c{}_{i,x,s}$ is a function of the portfolio weights x we can parameterize these coefficients as a function of x:

$$\hat{\beta}_{i,x,s_j}^c \approx \left(1, x_1, x_2, x_3\right)' \alpha_{s_j}^c. \qquad (68)$$

Then, we obtain the optimal consumption, using x_t^*, from

$$c_t^*\left(f_t, s_j\right) = \exp\left[\left(1, x_1^*, x_2^*, x_3^*\right)' \hat{\alpha}_{s_j}^c \left(1, X_1, X_2, X_3\right)\right].$$

Step 4d: endogenous grid solution Determine the endogenous grid by

$$w_t\left(f_t, s_j\right) = c_t^*\left(f_t, s_j\right) + s_j. \qquad (69)$$

Step 5: solving recursively Repeat steps 4a to 4d up to $t = 0$.

B.2 Variations on the benchmark

The benchmark asset allocation pattern is quite robust as can be seen from the impact of the following parametric changes. First, we simulate the model with a higher equity premium (η_S), resulting into a higher optimal allocation in stocks, but optimal portfolio and consumption trajectories over the life cycle that are qualitatively similar to the benchmark trajectories in Section 4.2. Second, simulating the model with a lower interest rate volatility ($\delta_{1,r}$), results in a lower optimal allocation to stocks due to a better risk-return trade-off for bonds. Third, replacing the 30-year bond by a 10-year bond produces a slightly higher stock exposure and a slightly higher allocation towards 5-year bonds within the fixed-income portfolio. The portfolio share allocated to 5-year debt rises substantially when we replace the 30-year bond by a 3-year bond. Figures 19 to 22 in Appendix B.3 show the effects of these variations in the asset menu.

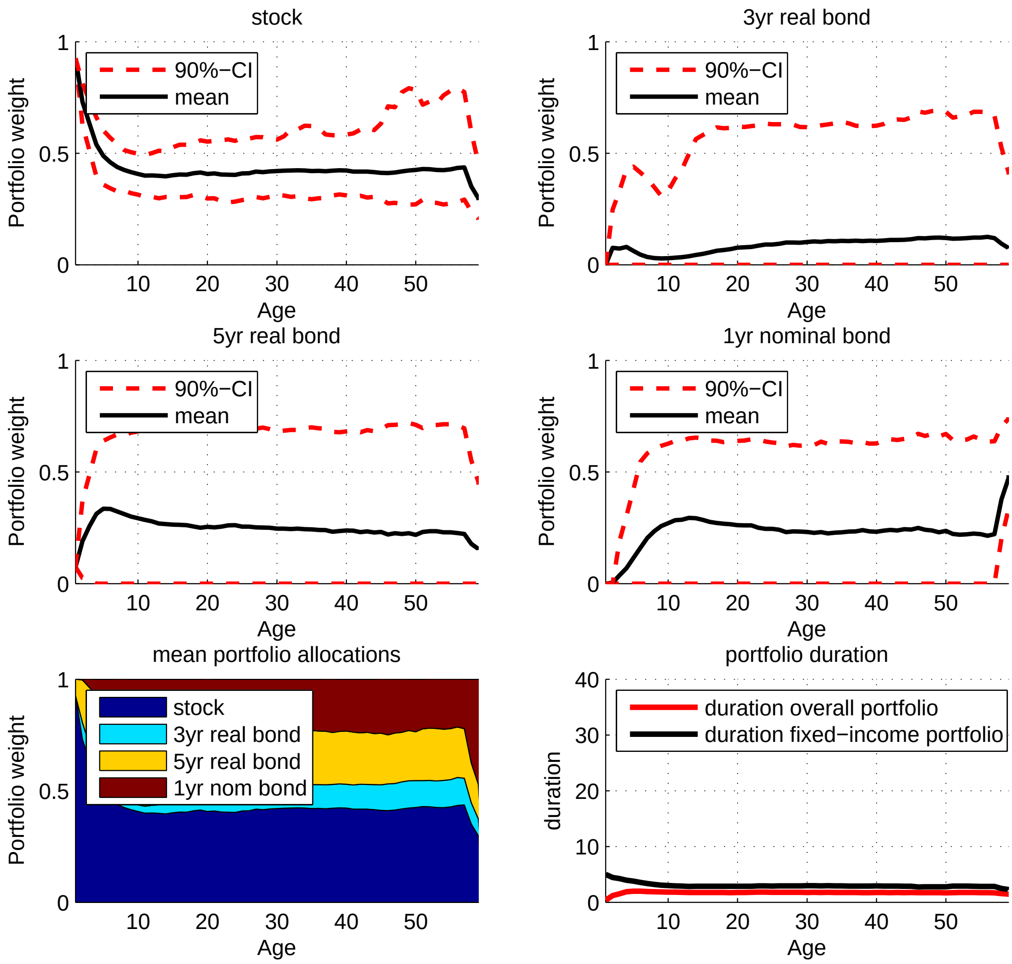

Figure 19: Optimal lifetime portfolios without unhedgeable risk and with alternative bond portfolio (with a 3 year and 5 year index-linked bond)

B.3 Additional Figures

The Figures 19 up to 34 and Table 6 are provided in this appendix in order to present a more comprehensive and complete overview of the results.

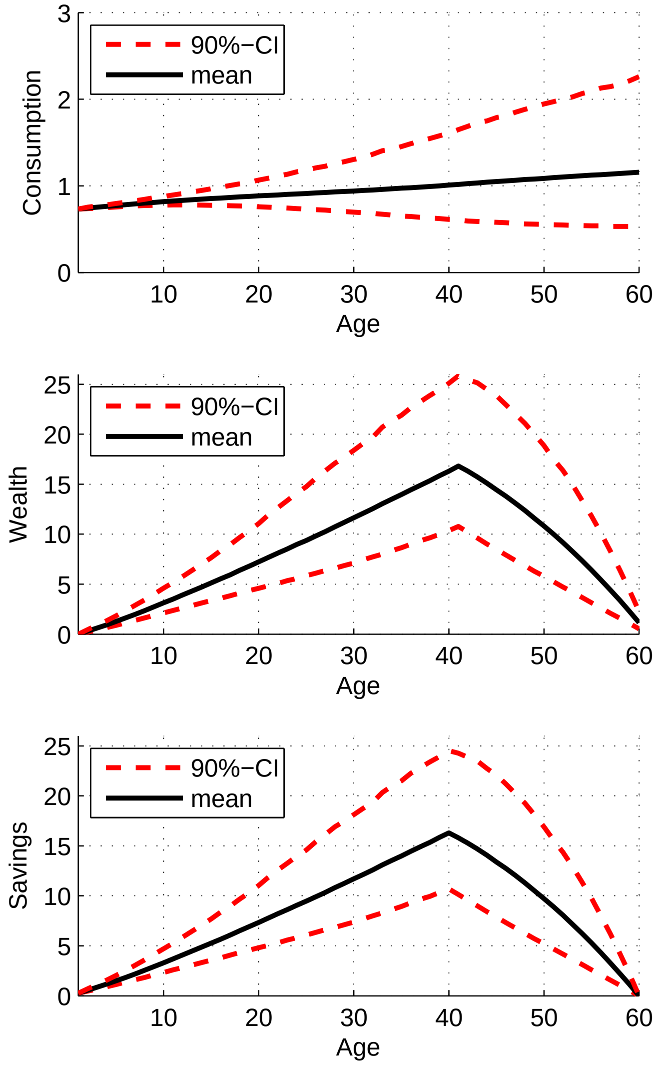

Figure 20: Optimal lifetime consumption, wealth and savings trajectories without unhedgeable risk and with alternative bond portfolio (with a 3 year and 5 year index-linked bond)

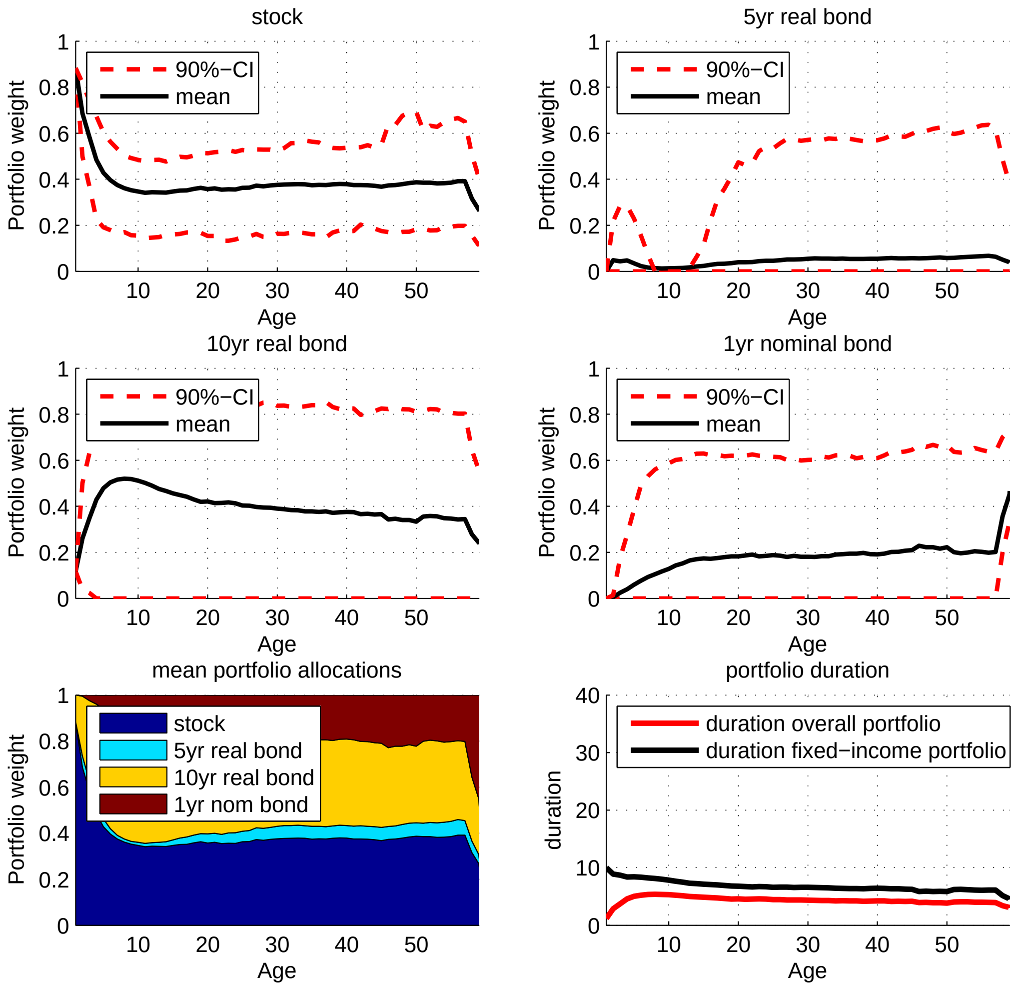

Figure 21: Optimal lifetime portfolios without unhedgeable risk and with alternative bond portfolio (with a 5 year and 10 year index-linked bond)

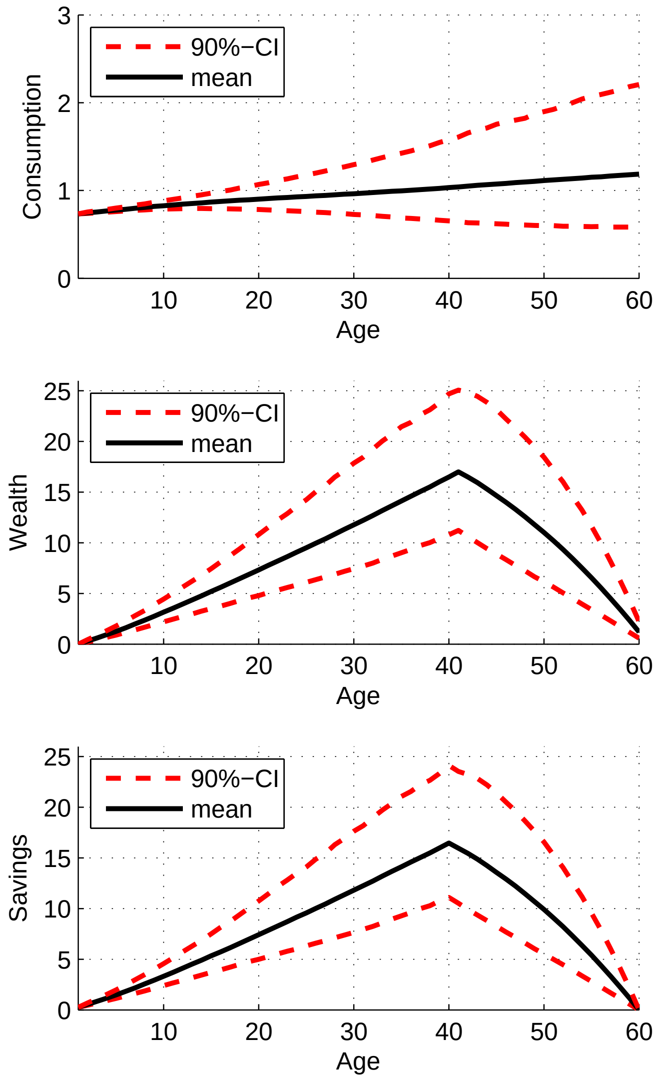

Figure 22: Optimal lifetime consumption, wealth and savings trajectories without unhedgeable risk and with alternative bond portfolio (with a 5 year and 10 year index-linked bond)

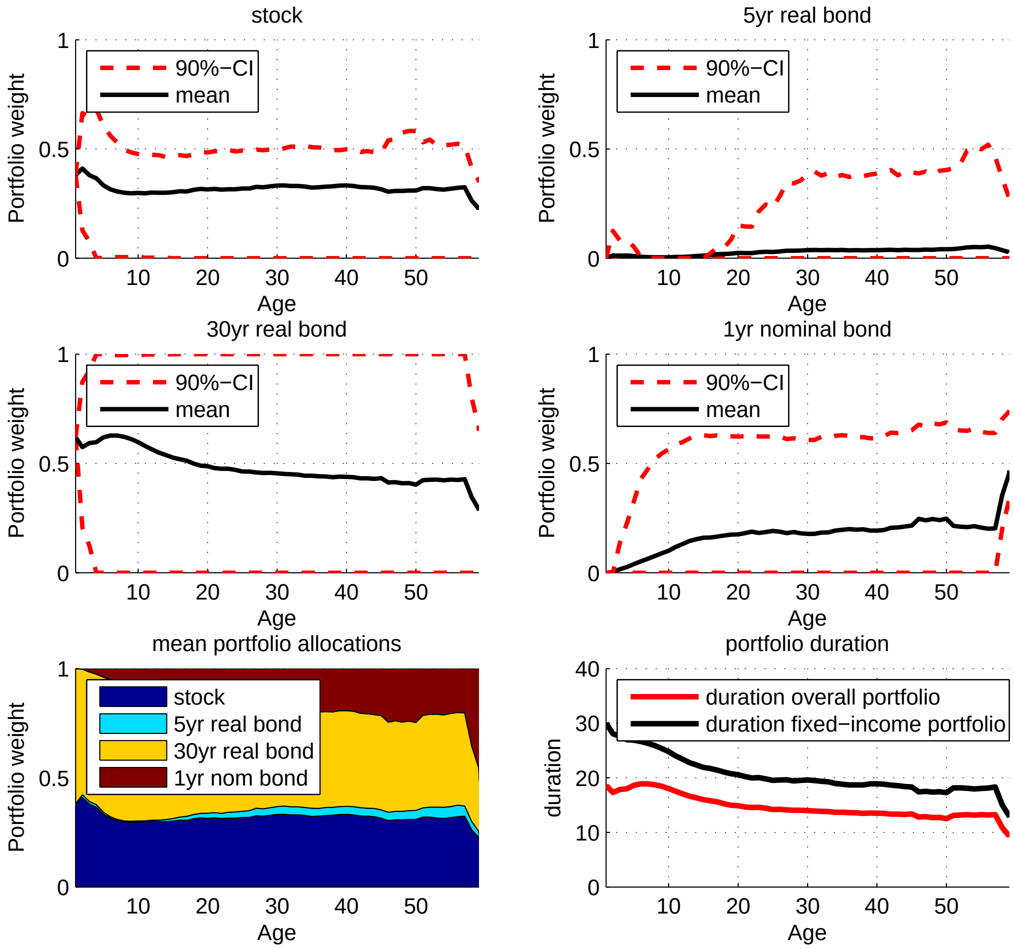

Figure 23: Optimal lifetime portfolios with unhedgeable inflation risk with $\sigma_u = \sqrt{4} * 0.36\%$ and $\sigma_E = 0$

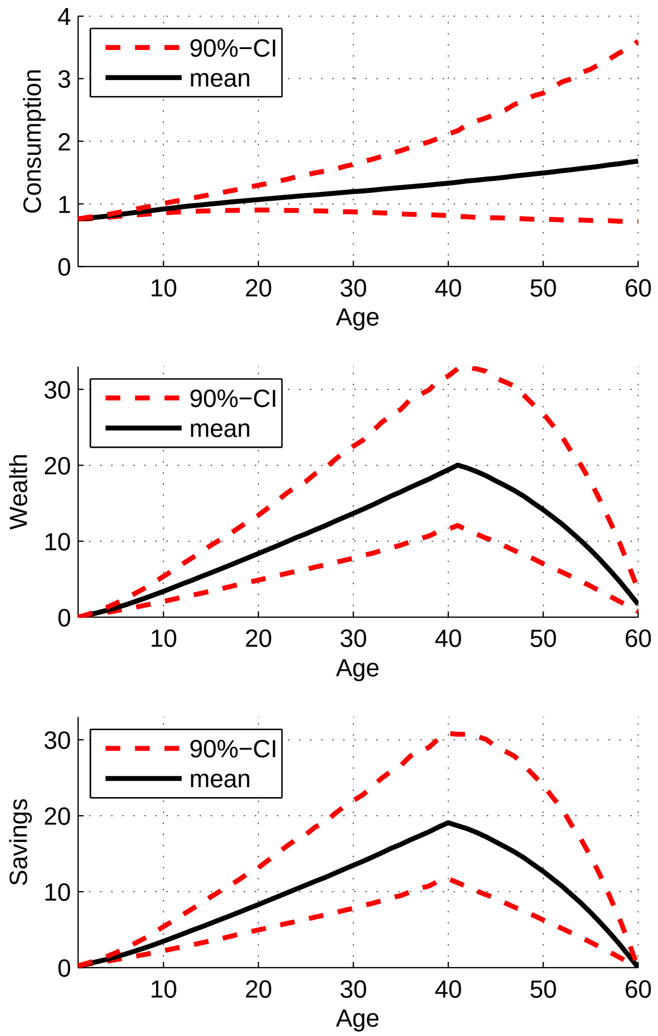

Figure 24: Optimal lifetime consumption, wealth and savings trajectories with unhedgeable inflation risk with $\sigma_u = \sqrt{4} * 0.36\%$ and $\sigma_E = 0$

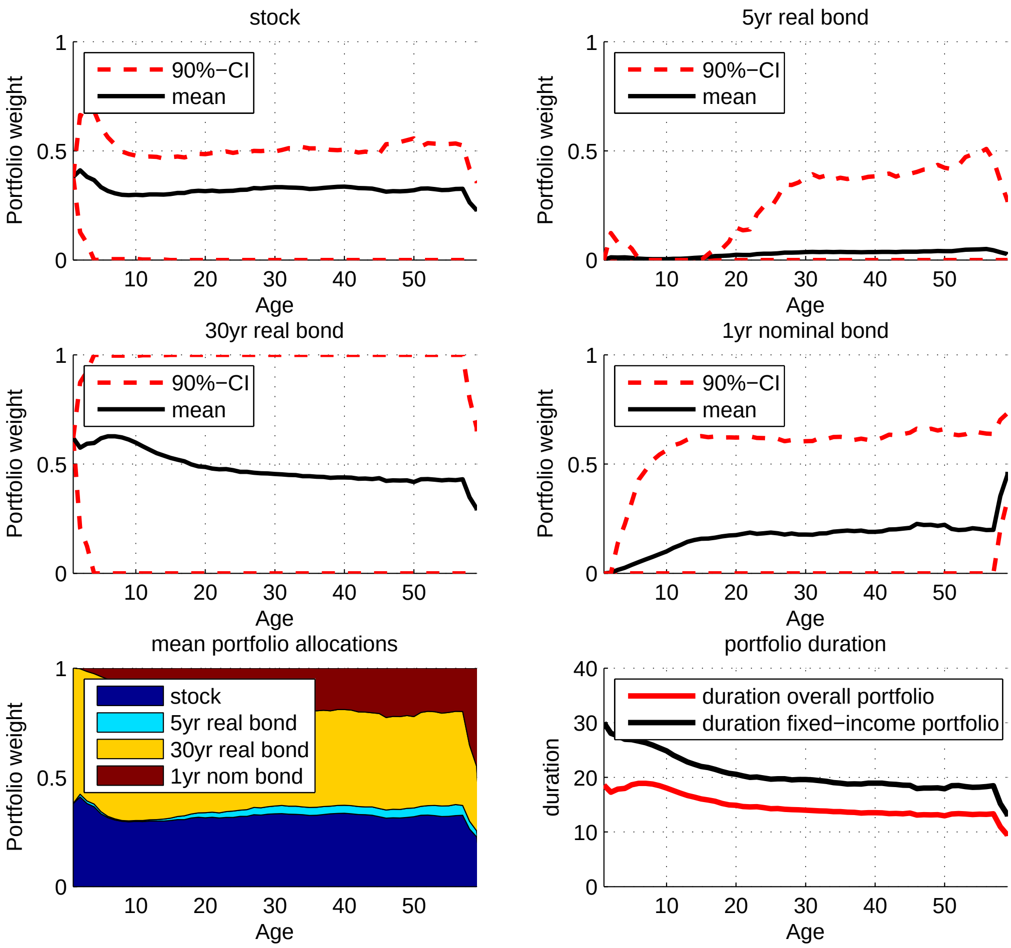

Figure 25: Optimal lifetime portfolios with unhedgeable inflation risk stemming from the differences in consumption between workers and retired. *Note: we assume* $\sigma_E = \sqrt{4} * 0.30\%$ *and* $\sigma_u = 0$.

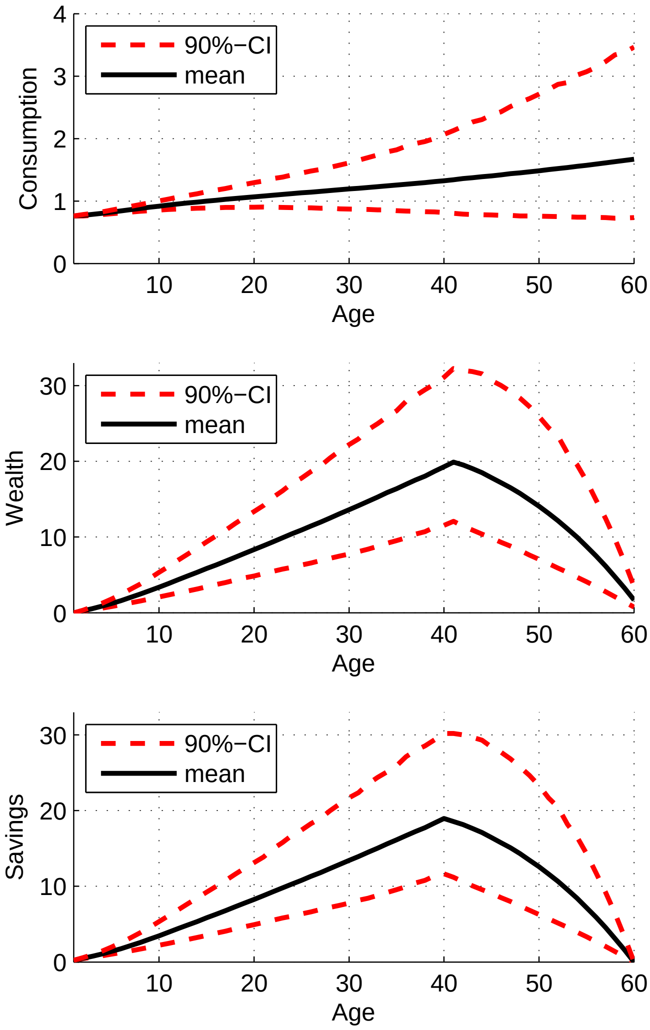

Figure 26: Optimal lifetime consumption, wealth and savings trajectories with unhedgeable inflation risk stemming from the differences in consumption between workers and retired. *Note: we assume* $\sigma_E = \sqrt{4} * 0.30\%$ *and* $\sigma_u = 0$.

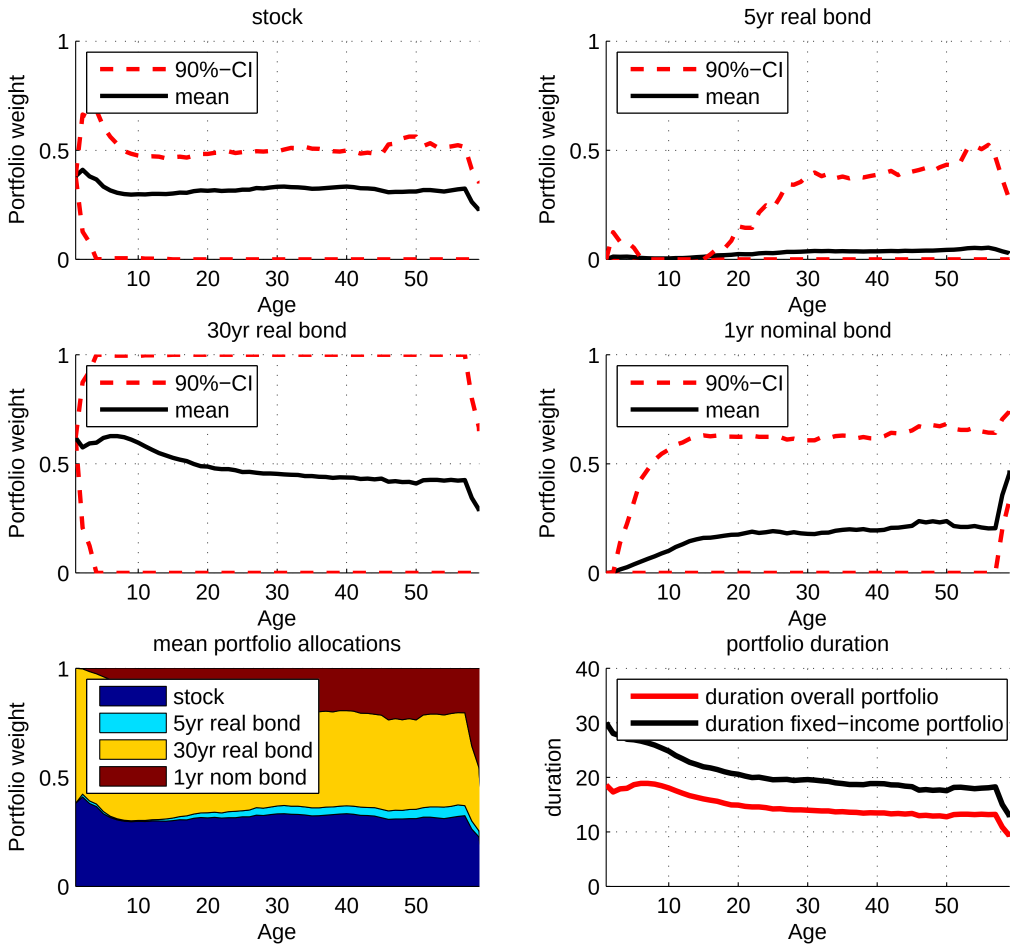

Figure 27: Optimal lifetime portfolios with unhedgeable inflation risk stemming from the differences in consumption between young and old with $\sigma_E = \sqrt{4} * 0.30\%$ and also unhedgeable overall inflation risk with $\sigma_u = \sqrt{4} * 0.36\%$

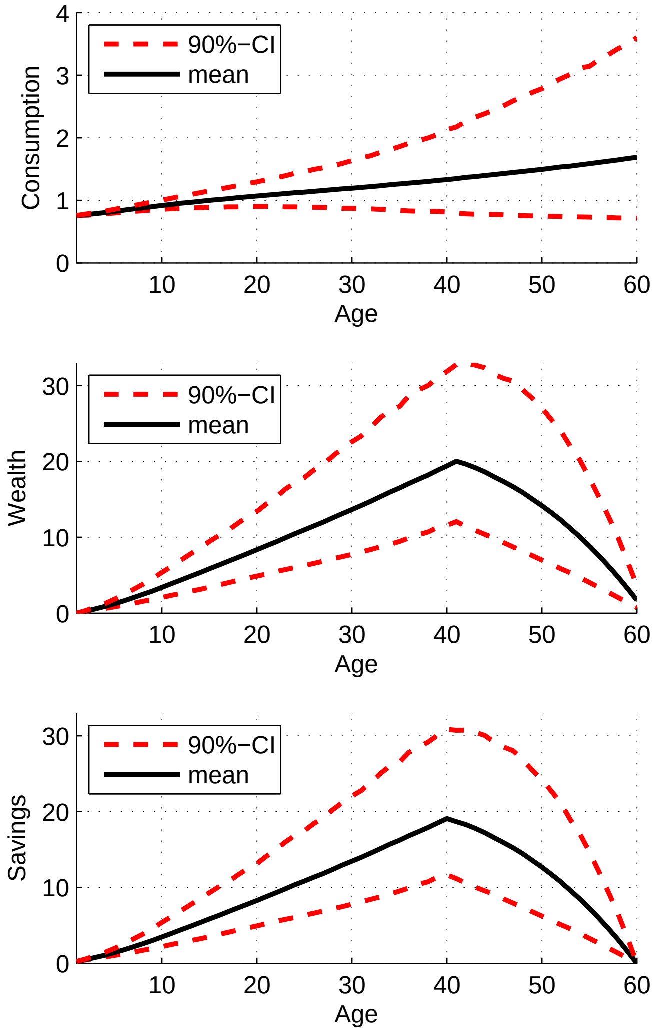

Figure 28: Optimal lifetime consumption, wealth and savings trajectories with unhedgeable inflation risk stemming from the differences in consumption between young and old with $\sigma_E = \sqrt{4} * 0.30\%$ and also unhedgeable overall inflation risk with $\sigma_u = \sqrt{4} * 0.36\%$

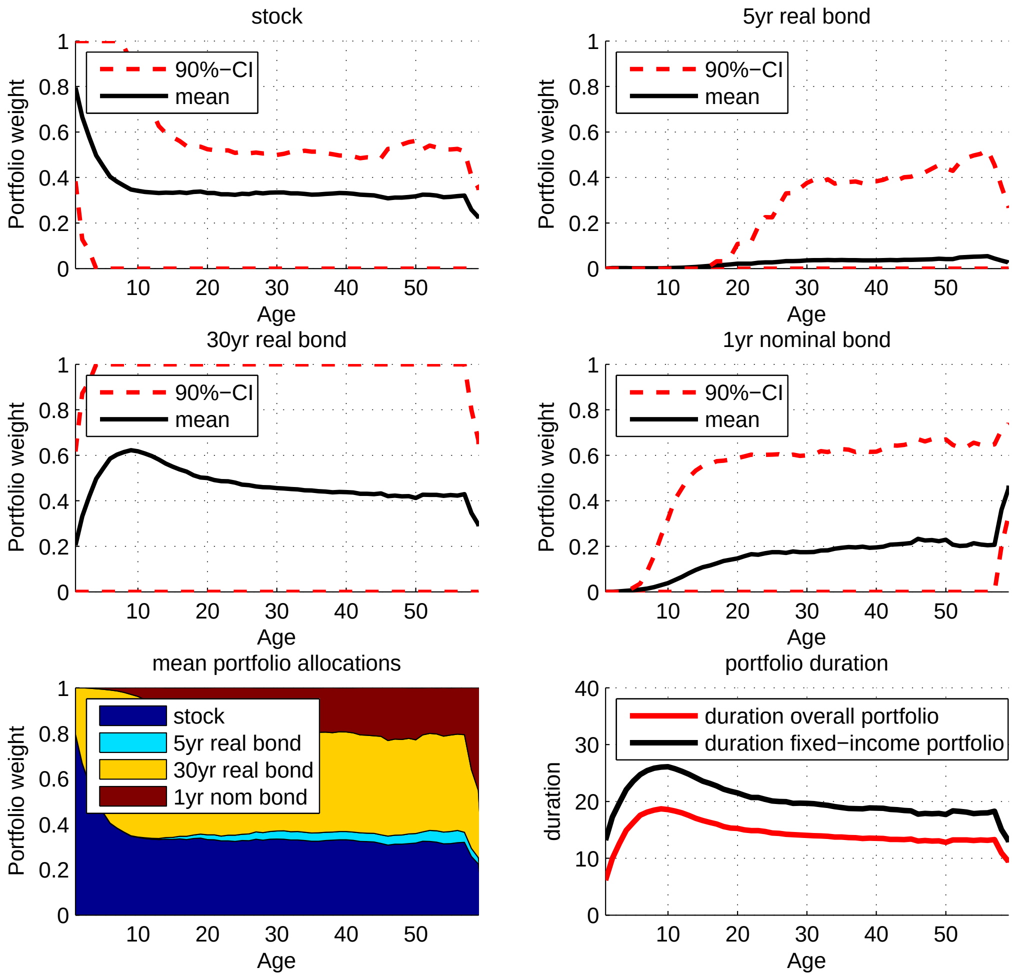

Figure 29: Optimal lifetime portfolios in the setting with stochastic labour income without unhedgeable inflation risk stemming from the differences in consumption between young and old ($\sigma_E = 0$), but with unhedgeable overall inflation risk with $\sigma_u = \sqrt{4} * 0.36\%$

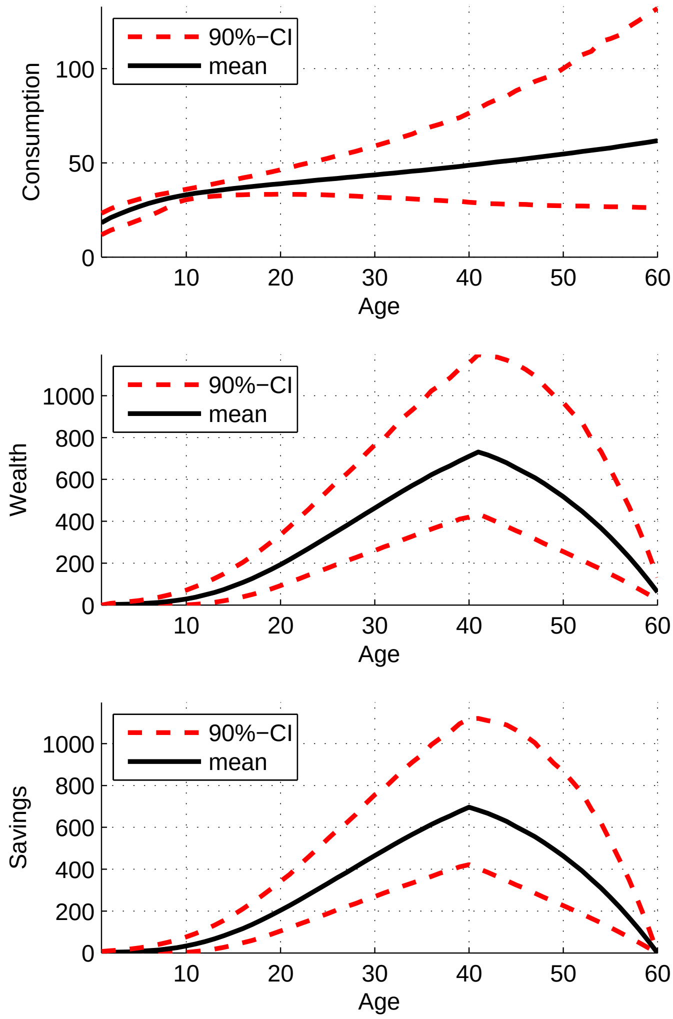

Figure 30: Optimal lifetime consumption, wealth and savings trajectories in the setting with stochastic labour income without unhedgeable inflation risk stemming from the differences in consumption between young and old ($\sigma_E = 0$), but with unhedgeable overall inflation risk with $\sigma_u = \sqrt{4} * 0.36\%$

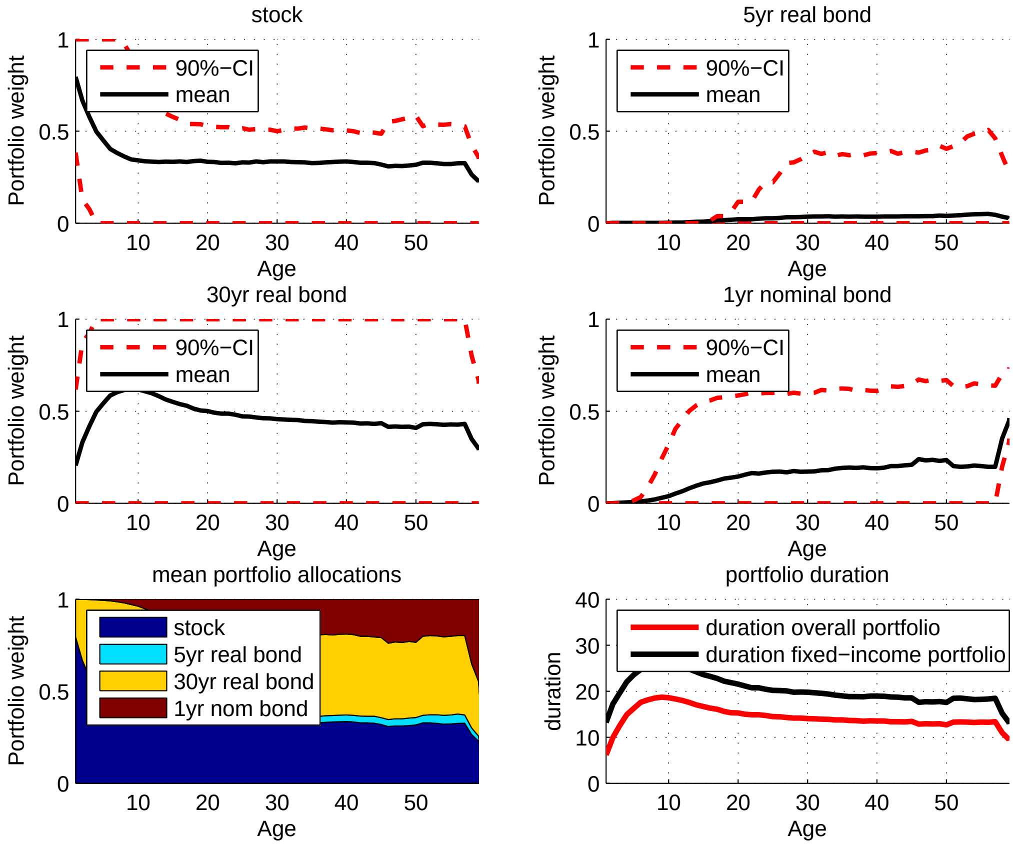

Figure 31: Optimal lifetime portfolios in the setting with stochastic labour income with unhedgeable inflation risk stemming from the differences in consumption between young and old with $\sigma_E = \sqrt{4} * 0.30\%$, but without unhedgeable overall inflation risk ($\sigma_u = 0$)

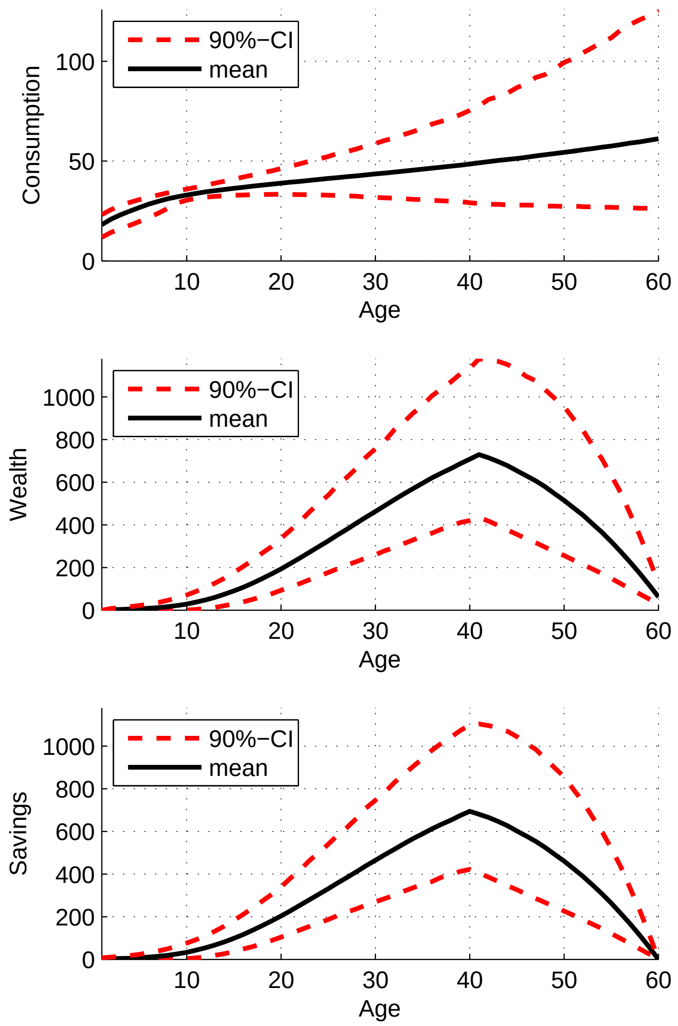

Figure 32: Optimal lifetime consumption, wealth and savings trajectories in the setting with stochastic labour income with unhedgeable inflation risk stemming from the differences in consumption between young and old with $\sigma_E = \sqrt{4} * 0.30\%$, but without unhedgeable overall inflation risk ($\sigma_u = 0$)

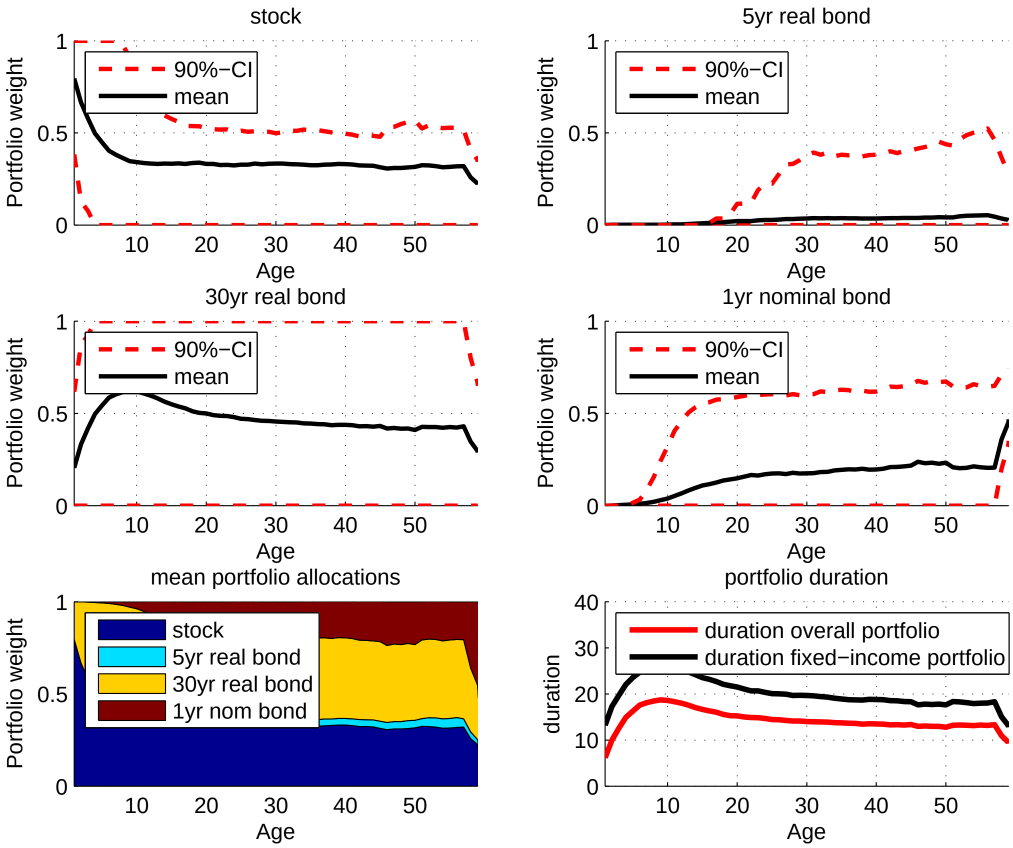

Figure 33: Optimal lifetime portfolios in the setting with stochastic labour income with unhedgeable inflation risk stemming from the differences in consumption between young and old with $\sigma_E = \sqrt{4} * 0.30\%$ and also unhedgeable overall inflation risk with $\sigma_u = \sqrt{4} * 0.36\%$

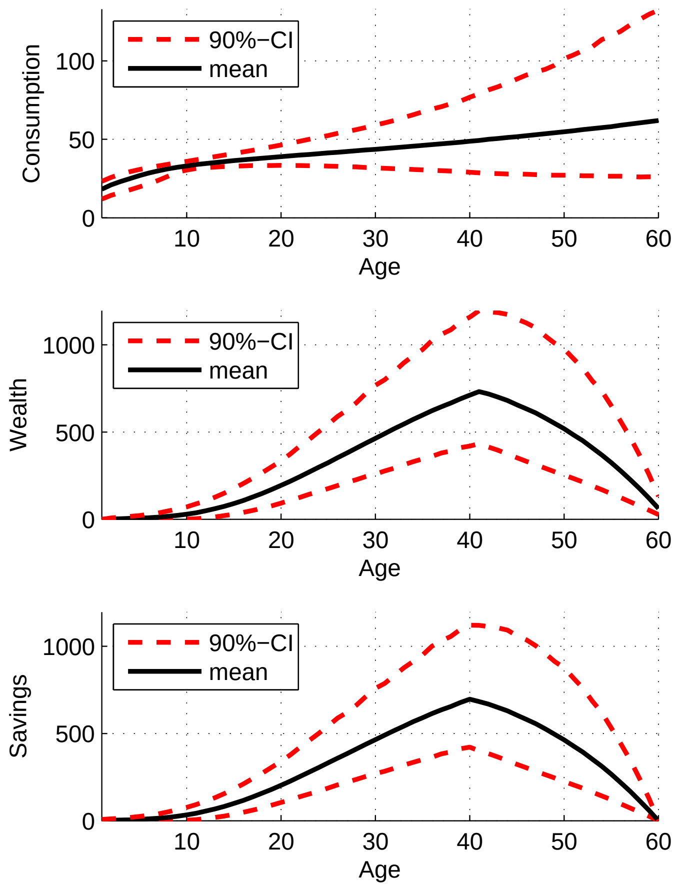

Figure 34: Optimal lifetime consumption, wealth and savings trajectories in the setting with stochastic labour income with unhedgeable inflation risk stemming from the differences in consumption between young and old with $\sigma_E = \sqrt{4} * 0.30\%$ and also unhedgeable overall inflation risk with $\sigma_u = \sqrt{4} * 0.36\%$

Table 6: Welfare effect from unhedgeable inflation risk in a DC pension scheme with fixed contribution rate

Parameter setting (fixed contribution rate $\chi = 10\%$)	σ_u(in %)	σ_E(in %)	Lifetime welfare effect (in %)			
			$\gamma = 3$	$\gamma = 5$	$\gamma = 7$	$\gamma = 10$
No unhedgeable inflation risk (benchmark)	0	0	-	-	-	-
Unhedgeable inflation risk overall price index	$\sqrt{4} * 0.36$	0	-0.08	-0.24	-0.55	-3.87
Cohort specific inflation risk	0	$\sqrt{4} * 0.30$	-0.01	-0.11	-0.21	-1.06
Both sources of unhedgeable inflation risk	$\sqrt{4} * 0.36$	$\sqrt{4} * 0.30$	-0.08	-0.27	-0.65	-10.71
	σ_u(in %)	σ_E(in %)	Welfare effect during retirement (in %)			
			$\gamma = 3$	$\gamma = 5$	$\gamma = 7$	$\gamma = 10$
No unhedgeable inflation risk (benchmark)	0	0	-	-	-	-
Unhedgeable inflation risk overall price index	$\sqrt{4} * 0.36$	0	-0.62	-1.41	-1.95	-5.62
Cohort specific inflation risk	0	$\sqrt{4} * 0.30$	-0.02	-0.51	-0.52	-1.57
Both sources of unhedgeable inflation risk	$\sqrt{4} * 0.36$	$\sqrt{4} * 0.30$	-0.58	-1.55	-2.30	-13.94
Parameter setting (fixed contribution rate $\chi = 20\%$)	σ_u(in %)	σ_E(in %)	Lifetime welfare effect (in %)			
			$\gamma = 3$	$\gamma = 5$	$\gamma = 7$	$\gamma = 10$
No unhedgeable inflation risk (benchmark)	0	0	-	-	-	-
Unhedgeable inflation risk overall price index	$\sqrt{4} * 0.36$	0	$-5 * 10^{-3}$	-0.03	-0.10	-0.21
Cohort specific inflation risk	0	$\sqrt{4} * 0.30$	0.02	-0.03	-0.11	-0.22
Both sources of unhedgeable inflation risk	$\sqrt{4} * 0.36$	$\sqrt{4} * 0.30$	-0.01	-0.04	-0.12	-0.37
	σ_u(in %)	σ_E(in %)	Welfare effect during retirement (in %)			
			$\gamma = 3$	$\gamma = 5$	$\gamma = 7$	$\gamma = 10$
No unhedgeable inflation risk (benchmark)	0	0	-	-	-	-
Unhedgeable inflation risk overall price index	$\sqrt{4} * 0.36$	0	-0.64	-1.27	-1.48	-4.63
Cohort specific inflation risk	0	$\sqrt{4} * 0.30$	0.06	-0.31	-0.26	-2.89
Both sources of unhedgeable inflation risk	$\sqrt{4} * 0.36$	$\sqrt{4} * 0.30$	-0.59	-1.41	-1.80	-16.29

Note: the welfare effect is relative to the setting without unhedgeable inflation risk, i.e. $\sigma_u = \sigma_E = 0$. *Welfare effect during retirement is evaluated at the start of retirement.*