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# FISCAL POLICIES IN BOOMS AND BUSTS 

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#### Abstract

We introduce fiscal policies into a behavioral macroeconomic model. We show how animal spirits play an important role in the dynamics of the business cycle and of public debt. These animal spirits are able to generate different sizes of fiscal multipliers depending on the state of the economy. Depending on the interest rate regime (high or low), they affect the capacity of fiscal authorities to stabilize the economy. In the high interest rate regime the fiscal authorities face a steep trade-off between output stabilization and the stabilization of public debt, i.e. attempts to stabilize the business cycle quickly hit a limitation of debt sustainability. In the low interest rate regime, when the steady state interest rate is lower than the growth rate of the economy, the use of fiscal policy as a tool of output stabilization is made considerably stronger.


JEL Classification: N/A

Keywords: Fiscal policy, public debt sustainability, Fiscal Multiplier, low and high interest rate regimes, tradeoffs

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#### Abstract

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## 1 Introduction

An important empirical regularity is the negative correlation between the growth of the economy and government debt. This empirical regularity has been well documented (see Kumar and Woo, 2010; Panizza and Presbitero, 2013\&2014; Reinhart and Rogoff, 2010). What is less clear is the direction of the causality in this negative correlation. Does the causality go from debt to growth, i.e. increasing debt reduces the growth rate? Or does the causality run in reverse, i.e. a booming economy tends to produce a declining government debt ratio and a recession leads to an increasing government debt ratio? There is now an increasing consensus that a significant part of the negative correlation is due to this reverse causation (see Panizza and Presbitero, 2013\&2014; Schularick and Taylor, 2012). Booms and busts characterize a market economy. During booms budget deficits decline and so do government debts; during busts budget deficits increase and governments are forced to increase their debt. This dynamics arises mainly because of the automatic stabilizers in the budget.

A second empirical regularity is that the existence of booms and busts in economic activity affects the size of the fiscal multipliers. Using US data of the postwar period, Auerbach and Gorodnichenko (2012 \& 2013) find that the fiscal multipliers depend very much on the state of the economy. During recessions fiscal multipliers tend to be significantly larger than 1. For a similar conclusion see Blanchard and Leigh (2013) and Parker (2011).

The objective of this paper is to study how this boom and bust dynamics affects the effectiveness of fiscal policies (the fiscal multipliers) and how this dynamics influences the choices (the tradeoffs) the fiscal authorities face in stabilizing the business cycle.

We will perform this analysis using a behavioral macroeconomic model as developed by De Grauwe (2012). This is a model that produces booms and busts in economic activity endogenously and that, therefore, seems to be appropriate to analyze the effectiveness and the tradeoffs the authorities face when confronted by a dynamics of booms and busts.

The model takes the view that agents have cognitive limitations preventing them from having rational expectations. Instead these agents use simple rules of behaviour (heuristics). The model introduces rationality by assuming that individuals learn from
their mistakes and are willing to switch to the better performing rule. This model produces endogenous business cycles driven by "animal spirits", i.e. markets sentiments of optimism and pessimism. The model also predicts that periods of tranquility alternate with booms and busts, the timing of which, however, cannot easily be predicted. One important characteristics of this model is that the effects of policy shocks depend on the initial conditions (the state of the economy). This will be shown to be important in generating time dependent fiscal multipliers.

A final important dimension we will introduce in the analysis is the nature of the interest rate regime in which fiscal policies have to operate. As was stressed recently by Blanchard (2019) this is of great importance because it very much affects the choices policy makers have in using their fiscal tools. In Figures 1 and 2 we show the interest-rate-growth differential, i.e. the difference between the average nominal interest rate paid to service government debt and the average nominal growth rates of GDP in a number of Eurozone and non-Eurozone countries before and after the financial crisis. We find some dramatic interest-rate-growth differential reversals in the Eurozone. Some countries that experienced a negative interest-rate-growth differential regime before the crisis saw their regime turned into a positive interest-rate-growth differential which, as is well-known, makes the debt dynamics potentially unstable. It will be shown that this dramatically affects the choices the fiscal authorities of these countries face.

A comparison of Figures 1 and 2 for the post-crisis period also shows that the major industrialized countries outside the Eurozone turned into negative regimes. This is not the case in the Eurozone, where countries of the periphery experienced unfavourable positive regimes that very much affected their fiscal policy choices.

The rest of this paper is organized as follows. Section 2 presents the behavioral macroeconomic model. Section 3 discusses the basic results of the model. In section 4 we show how the fiscal multipliers depend on the state of the economy and in section 5 we perform a stability analysis. Section 6 analyzes the effectiveness of fiscal policies in two interes rate regimes, while section 7 derives and discusses the tradeoffs the authorities face under different interest regimes. Section 8 presents some empirical validation of the model. Section 9 concludes the paper.

Figure 1: Difference between Average Nominal Interest Rate and Nominal Growth GDP in Eurozone


Source: Eurostat

Figure 2: Average Nominal Interest Rate (r) and Nominal Growth GDP (y) in nonEurozone


Source: Eurostat

## 2. The Model

We extend the behavioral model of De Grauwe (2012) by adding a fiscal policy block (see also De Grauwe and Foresti, 2018). This will allow us study the role of fiscal policies in a dynamic model characterized by booms and busts produced by animal spirits.

### 2.1 Basic Equations

The model consists of an aggregate demand equation, an aggregate supply equation, and a Taylor rule. We rewrite this 3 -equation New Keynesian block by adding government spending $\left(g_{t}\right)$ as the instrument of fiscal policy in the aggregate demand equation:

$$
\begin{align*}
& y_{t}=a_{1} \tilde{E}_{t} y_{t+1}+\left(1-a_{1}\right) y_{t-1}-a_{2}\left(r_{t}-\tilde{E}_{t} \pi_{t+1}\right)+a_{3} g_{t}+\epsilon_{t}  \tag{1}\\
& \pi_{t}=b_{1} \tilde{E}_{t} \pi_{t+1}+\left(1-b_{1}\right) \pi_{t-1}+b_{2} y_{t}+\eta_{t}  \tag{2}\\
& r_{t}=\left(1-c_{3}\right)\left[c_{1}\left(\pi_{t}-\pi^{*}\right)+c_{2} y_{t}\right]+c_{3} r_{t-1}+u_{t} \tag{3}
\end{align*}
$$

Equation (1) is the aggregate demand, in which $y_{t}$ represents output, $r_{t}$ is the nominal interest rate, $\pi_{t}$ is the rate of inflation and $g_{t}$ is public expenditure. All the variables in the model should be interpreted as percentage deviations from their steady state values ${ }^{1}$. $\widetilde{E}_{t}$ is the expectations operator and the tilde above $E$ refers to the fact that expectations are not formed according to the rational expectations assumption. We will specify how these expectations are formed later in the paper.

Equation (2) represents the aggregate supply in which current inflation depends on both a forward-looking component, $\widetilde{E}_{t} \pi_{t+1}$, and lagged inflation. Inflation at time $t$ also depends on the output gap. Equation (3) represents the Taylor rule followed by the central bank, in which $\pi^{*}$ is the monetary authority's inflation target.

We have added error terms in each of the three equations. These error terms represent demand shocks $\left(\epsilon_{t}\right)$, supply shocks $\left(\eta_{t}\right)$ and monetary policy shocks $\left(\mu_{t}\right)$, respectively. These shocks are assumed to be normally distributed with zero mean and constant standard deviation.

[^0]
### 2.2 Fiscal Policy Block

Now we add to this model a fiscal policy rule and an equation representing the evolution of public debt. The fiscal policy rule specified in equation (4) assumes the following objectives of the government. First, the government wants to smooth public expenditures. This leads to inertia in public spending. This reaction is very similar to what central banks do as described in the Taylor rule. Second, the government aims at stabilizing the business cycle by reacting counter-cyclically to the changes in the output gap. Third, it has a concern of maintaining debt sustainability and therefore uses public spending to stabilize the public debt ( $\delta_{t}$ ). This leads us to the following fiscal policy rule concerning fiscal spending $g_{t}$ :

$$
\begin{equation*}
g_{t}=F_{1} g_{t-1}-F_{2} y_{t-1}-F_{3} \delta_{t}+v_{t} \tag{4}
\end{equation*}
$$

The first term in equation (4) represents the smoothing behavior: today's public spending is positively influenced by last period's public spending $g_{t-1}$ (see Muscatelli and Tirelli, 2005); $F_{2}$ measures the intensity with which the government acts countercyclically. When the output gap decreases (increases), fiscal spending increases (decreases). As it takes time for the government to get its fiscal plan approved and implemented, the government is only able to react to the past output gap $y_{t-1}$ instead of the current one. The parameter $F_{3}$ measures the government's focus on debt stabilization, i.e. when public debt increases the government reduces public spending so as to maintain debt sustainability. Fiscal policy shocks $\left(v_{t}\right)$ are introduced in equation (4) and are assumed to be normally distributed with constant standard deviation.

The model is completed by a linearized version of the equation representing the evolution of debt (the government's solvency constraint). In this respect, we follow Kirsanova et al. (2007) and assume that the government buys goods and services ( $G$ ), taxes income $(\mathrm{Y})$ at a constant rate $(\tau)$, issues nominal debt $(D)$. Thus, we can write the evolution of debt in period $t$ in the following equation:

$$
\begin{equation*}
D_{t+1}=\left(1+r_{t}\right)\left(D_{t}+G_{t} P_{t}-\tau Y_{t} P_{t}\right) \tag{5}
\end{equation*}
$$

Define $d_{t}=\frac{D_{t}}{P_{t-1}}$ and $\delta_{t}=\ln d_{t}-\ln d$, (with $d$ the steady state value of $d_{t}$ ) the linearized version of this equation can be rewritten as follows (see Kirsanova et al., 2007; Kirsanova and Wren-Lewis, 2012):

$$
\begin{equation*}
\delta_{t+1}=r_{t}+(1+r s)\left(\delta_{t}-\pi_{t}+h_{1} g_{t}-h_{2} \tau y_{t}\right)+\varphi_{t} \tag{6}
\end{equation*}
$$

In this specification $r s$ is the value of the interest rate in steady state, and where all the variables $\left(\delta_{t}, r_{t}, \pi_{t}, g_{t}\right.$ and $\left.y_{t}\right)$ are expressed as deviations from their steady state values. In Appendix 1, we show the way one goes from equation (5) to (6). Finally public debt shocks $\left(\varphi_{t}\right)$ are included in equation (6) and are assumed to be normally distributed with constant standard deviation.

Equation (6) is a first order difference equation in $\delta_{t}$. We can rewrite it as follows:
$\delta_{t+1}=(1+r s) \delta_{t}+r_{t}+(1+r s)\left(h_{1} g_{t}-h_{2} \tau y_{t}\right)-(1+r s) \pi_{t}+\varphi_{t}$
In order for $\delta_{t}$ to be dynamically stable $(1+r s)<1$ (or rs $<0$ ). If this condition is not satisfied, i.e. if $\mathrm{rs} \geq 0$, the debt to GDP ratio will tend to move to infinity. This inherent instability that arises when $r s \geq 0$ can only be stopped if the fiscal authorities are willing to reduce their spending $g_{t}$ sufficiently so as to create a primary surplus on the government budget (excluding interest payments), i.e. $\left(h_{1} g_{t}-h_{2} \tau y_{t}\right)<0$, or by allowing inflation to be positive. As we assume that inflation is 0 in the steady state, we will disregards this.

We will be considering two regimes in our further discussion of this model. The first one is the regime in which $r s \geq 0$, that requires the fiscal authorities to set $g_{t}$ (in equation (4)) such that a budget surplus is created. The second regime is one in which $r s<0$. This regime does not put such a condition on $g_{t}$. As a result, as we will see, in this second regime the fiscal authorities will be able to use $g_{t}$ to pursue an objective of output stabilization. This contrasts with the first regime that, as will be shown, severely restricts the fiscal authorities to use the fiscal policy instrument to stabilize output.

It should be noted that the first regime $r s \geq 0$ is in fact a regime in which the interest rate in the steady state exceeds the growth rate of GDP in the steady state. This can be seen from the fact that in the steady state, $y_{t}=0$ and $\pi_{t}=0$. Thus, a positive interest rate means that the interest rate exceeds the nominal growth rate of the economy. Conversely, a negative interest rate in the steady state (the second regime) means that the interest rate is below the nominal growth rate of the economy.

We use the same expectations formation mechanism as in De Grauwe (2012), in which agents with cognitive limitations use simple forecasting rules (heuristics) and decide to switch between these rules depending on the relative performance of these rules in forecasting output gap and inflation (i.e. $\widetilde{E}_{t} y_{t+1}$ and $\widetilde{E}_{t} \pi_{t+1}$ ). Agents can switch between
the two rules according to a learning mechanism (See Appendix 2 for a self-contained explanation of the model expectations formation and their dynamics). This model produces waves of optimism and pessimism (animal spirits) that drive the business cycle and in turn are influenced by it. See Appendix 2 for details of the definition of animal spirits based on the learning mechanism we use. One of the questions we will ask in this paper is how fiscal policies can affect these movements and how the latter influence the sustainability of fiscal policies. We describe the solution of the model and the calibration in Appendix 3.

## 3. Animal Spirits, Output gap and Public Debt

In this section we describe the basic results from the simulation of our model for 10000 periods. In Figure 3, we set the steady state interest rate rs=-0.01. As will be remembered, this corresponds to assuming that the interest rate is lower than the nominal growth rate of the economy in the steady state. Panels A and B show the cyclical movements of the output gap $y_{t}$, animal spirits $S_{t}$ and public debt $\delta_{t}$ (all are normalized at zero). Animal spirits play a very important role. The correlation between animal spirits and output gap is very high at 0.89 .

We also find that animal spirits and public debt are negatively correlated (at -0.77 ). This is an important result. When animal spirits are positive (negative), the output gap is positive (negative). The booming (recessionary) conditions then tend to create surpluses (deficits) in the government budget and a decline (increase) in the public debt. Thus, boom (bust) conditions are associated with declining (increasing) government debt. There is a lot of empirical evidence sustaining this prediction (see Shularick and Taylor, 2012; Panizza and Presbitero, 2013 \& 2014).

In Figure 3, Panels C and D show the animal spirits and the output gap in the frequency domain. We obtain concentrations of extreme values of animal spirits when everybody becomes optimist ( $S_{t}=1$ ) or pessimist ( $S_{t}=-1$ ). This produces a boom-bust pattern in economic activity and is responsible for the non-normality found in the distribution of the output gap.

We repeat the simulations in the high interest rate regime ( $\mathrm{rs}=0.01$ ). The results are shown in Figure 4. We observe similar results compared to the low interest rate regime. This is because all the parameters (except rs) used in the simulation are the same. The
correlation of the animal spirits and the output gap is 0.89 and the correlation of the animal spirits and the government debt is -0.77 . However, a comparison of the frequency distributions of animal spirits and output gap in the two interest rate regimes reveals that the high interest rate regime produces a greater concentration of extreme values of animal spirits. This suggests that the high interest rate regime is more prone to produce more intense booms-busts in the business cycles.

We also calculated the standard deviations of the output gap in these two interest rate regimes. We find that in the high interest rate regime the output gap is more volatile than in the low interest rate regime. Standard deviations are respectively 2.10 and 1.85. The high interest rate regime also produces a higher volatility in the public debt than the low interest rate regime with standard deviations of, respectively, 22 and 30.

These results are robust as our sensitivity analysis in Figure 5 confirms. When the steady state interest rate increases, both volatility in output gap (shown in Panel A) and volatility in public debt (shown in Panel B) increase. Our explanation is the following. As the steady state interest rate increases, the public debt becomes potentially more unstable, forcing the government to use its fiscal policy to stabilize public debt. As a result, it can do less output stabilization, leading to more volatility in economic activity, except if the central bank increases its output stabilization effort. We return to this interpretation in the next few sections and we will argue that this is indeed the reason why an increasing steady state interest rate tends to make the economy less stable.

Figure 3: Output Gap and Animal Spirits (rs=-0.01)



Figure 4: Output Gap and Animal Spirits (rs=0.01)

A


C


B


D


Figure 5: Sensitivity Analysis of Volatility

A


B


## 4. Fiscal Multipliers

In this section we analyze the fiscal multipliers in our behavioural model. In order to compute these, we compute the impulse responses to a shock in government expenditures. We show these impulse responses in Appendix 4 for the two interest rate regimes. The results are very similar to each other. Here we concentrate on the shortterm fiscal multipliers. These are obtained by adding the output effects of the expenditure shock during the first 4 quarters after the shock. As the results for the two interest rate regimes are very similar we focus here on the low interest rate regime.

An important feature of these impulse responses is that the size of these responses depends on the initial conditions, in particular the state of animal spirits when the shock occurs (see also De Grauwe, 2012). As a result, we obtain different fiscal multipliers: the size of which will vary depending on these initial conditions. This is made clear in Figure 6. Panel A shows the frequency distribution of the short-term fiscal multipliers. We observe a wide variation of these multipliers, from 0.8 to 1.5 . We also note two peaks in the distribution, one around a multiplier of 1 and another around a multiplier between 1.2 and 1.3.

Panel B shows the origin of these two peaks. In panel B we set out the fiscal multipliers (vertical axis) against the state of animal spirits in the initial period. We find that when animal spirits are neutral (tranquil periods) the fiscal multipliers cluster around 1. When animal spirits take on extreme values, these fiscal multipliers cluster around 1.2-
1.3. Thus extreme values of animal spirits tend to amplify the effects of a fiscal expansion and lead to multipliers exceeding 1 . Note, however, that there is still a lot of noise around the non-linear relation between fiscal multipliers and animal spirits, suggesting that other initial conditions affect the size of these multipliers.

There is increasing empirical support for the view that the size of the fiscal multipliers depends on the state of the economy. As mentioned earlier, in a series of influential papers, Auerbach and Gorodnichenko (2012 \& 2013) find that the size of fiscal multiplier (spending multiplier) is state dependent in the US economy during the postwar period. In particular, they find that the multiplier exceeds 1 during recessions and tends to be smaller than 1 during periods of expansion (see also Christiano, et al., (2011)).

Figure 6: Short-term Fiscal Multipliers and Animal Spirits


## 5. Stability Analysis

In this section we perform a stability analysis of our model. We will do this in two steps. We first analyze the conditions under which the government debt is stable. We will do this by performing an impulse response analysis in which we introduce an exogenous shock to government debt and then analyze under what conditions the public debt returns to its initial equilibrium. Second we will produce a table analyzing the dynamic stability of the system for different values of the parameters $\mathrm{c}_{2}$ and $\mathrm{F}_{2}$.

### 5.1 Debt Stabilization and Fiscal Rule.

We compute the impulse response after an exogenous shock in the government debt. We do this for the two regimes about the interest rate discussed earlier, i.e. rs $<0$ and rs $>0$. We show the results in Figure 7. The left hand side panels show the impulse responses in the rs $<0$ regime; the right hand side panels show the implulse responses in the rs $>0$ regime. The contrast between the two regimes is strong. When $F_{3}=0$, i.e. the fiscal authorities do not use their fiscal instrument (expenditures) to stabilize public debt, the debt level is put on an explosive path after the shock when the interest rate exceeds the growth rate of the economy ( $\mathrm{r} s>0$ ). This is not the case in the opposite regime when the interest rate is lower than the growth rate of the economy ( $\mathrm{rs}<0$ ). Thus, in this regime the fiscal authorities do not have to use their fiscal rule to stabilize the debt. As will be shown, this frees the fiscal instrument to be used for stabilizing output. In the $r>0$ regime this will be shown to be impossible.

Figure 7 also shows the impulse responses when the fiscal authorities use their fiscal instrument to stabilize the debt. There is some critical value of $\mathrm{F}_{3}$ that will stabilize the debt in the rs> 0 regime. This turns out to be $F_{3}=0.0035$. When $F_{3}$ exceeds this number ( $F_{3}=0.005$ and $F_{3}=0.01$ ), the debt can be stabilized but note that in the $r s>0$ regime it can still take a long period for the debt to return to its initial value. As a result, it may still be the case that although the debt level is dynamically stable, it may still be unsustainable.

Figure 7: Debt Stabilization and Fiscal Rule (F3=0)



## 5.2 stability of the system

In a second step we study the dynamic stability of the system as a whole (output gap, inflation, debt) for different values of the monetary policy parameters $c_{2}$ in the Taylor rule and $F_{2}$ in the fiscal policy rule, and we ask the question of how these parameters affect the dynamic stability of the model. The results are shown in Tables 1 and 2. Table 1 shows the results when the rs $>0$ regime prevails; Table 2 when the rs $<0$ regime prevails. We observe that the area of stable outcomes is larger in the latter regime than in the former. In fact for all positive values of $\mathrm{c}_{2}$ and $\mathrm{F}_{2}$ we find stability in the $\mathrm{rs}<0$ regime. This is not the case in the rs > 0 regime where we find that fiscal and/or monetary authorities must make some minimal efforts at stabilizing the output gap to ensure stability of the system. Thus, when the interest rate is higher than the growth rate of the economy, the monetary and fiscal authorities must exert more effort (higher c2 and/or $\mathrm{F}_{2}$ ) to ensure stability of the system than when the interest rate is below the growth rate of the economy.

We also note that there is a small region of parameters that leads to chaos (indicated by " C "). This is obtained for relatively low values of c 2 and $\mathrm{F}_{2}$. Chaotic dynamics implies
cyclical movements of a variable that are aperiodic, i.e. none of the cycles repeat themselves. In addition, these cycles are produced in a deterministic way.

Table 1 Dynamic stability ( $\mathrm{rs}>0$ )

| T=10000, rs=0.01, F3=0.02 |  |  | output parameter c2 |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| output parameter F2 | 0 | 0,1 | 0,2 | 0,3 | 0,4 | 0,5 | 0,6 | 0,7 | 0,8 | 0,9 | 1 |
| 0 | U | U | C | C | S | S | S | S | S | S | S |
| 0,1 | U | C | S | S | S | S | S | S | S | S | S |
| 0,2 | C | S | S | S | S | S | S | S | S | S | S |
| 0,3 | C | S | S | S | S | S | S | S | S | S | S |
| 0,4 | C | S | S | S | S | S | S | S | S | S | S |
| 0,5 | C | S | S | S | S | S | S | S | S | S | S |
| 0,6 | C | S | S | S | S | S | S | S | S | S | S |
| 0,7 | S | S | S | S | S | S | S | S | S | S | S |
| 0,8 | S | S | S | S | S | S | S | S | S | S | S |
| 0,9 | S | S | S | S | S | S | S | S | S | S | S |
| 1 | S | S | S | S | S | S | S | S | S | S | S |

Table 2: Dynamic stability (rs < 0)

| T=10000, rs=-0.01, <br> $\begin{array}{l}\text { F3 }\end{array}$ 0.0 <br> 0.0 |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| output parameter F2 | 0 | 0,1 | 0,2 | 0,3 | 0,4 | 0,5 | 0,6 | 0,7 | 0,8 | 0,9 | 1 |
| 0 | C | C | S | S | S | S | S | S | S | S | S |
| 0,1 | C | S | S | S | S | S | S | S | S | S | S |
| 0,2 | C | S | S | S | S | S | S | S | S | S | S |
| 0,3 | S | S | S | S | S | S | S | S | S | S | S |
| 0,4 | S | S | S | S | S | S | S | S | S | S | S |
| 0,5 | S | S | S | S | S | S | S | S | S | S | S |
| 0,6 | S | S | S | S | S | S | S | S | S | S | S |
| 0,7 | S | S | S | S | S | S | S | S | S | S | S |
| 0,8 | S | S | S | S | S | S | S | S | S | S | S |
| 0,9 | S | S | S | S | S | S | S | S | S | S | S |
| 1 | S | S | S | S | S | S | S | S | S | S | S |

## 6. Fiscal Stabilization in Two Regimes

In the previous section, we contrasted the stability properties of the two interest regimes. We found that in the high interest rate regime (rs $>0$ ) the fiscal authorities have to use their fiscal tools to stabilize public debt. We suggested that this is likely to leave less space for fiscal policies to stabilize the business cycle (the output gap) as
compared to the regime of low interest rate (rs < 0). In this section we study how much space there is for fiscal policies to stabilize the business cycle.

The way we analyse this question is to vary the parameter $\mathrm{F}_{2}$ (the parameter in the fiscal rule equation (4) associated with the output gap). The higher is this parameter the more the fiscal authorities use government spending as a tool to stabilize the output gap. We then compute how successive increases in $\mathrm{F}_{2}$ affect the frequency distribution of the animal spirits. It will be remembered that the animal spirits are the driving force in producing the boom-bust behaviour in the output gap. We perform this exercise in the two interest rate regimes. We show the results in Figure 8.

The results we obtain are quite striking. The panels on the left hand side show the frequency distributions of animal spirits for increasing values of $\mathrm{F}_{3}$ in the high interest rate regime. In this regime the fiscal authorities have to use expenditures to stabilize the debt level. This is why $\mathrm{F}_{3}$ has to be positive. We set $\mathrm{F}_{3}=0.02$ which guarantees stability of public debt. We now observe that in this regime increasing attempts by the fiscal authorities to follow an anti-cyclical policy have no effects on the distribution of animal spirits (and therefore also on the distribution of the output gap). Thus given that in the high interest rate regime, the fiscal authorities have to use their fiscal instrument to stabilize the public debt, fiscal policies aimed at stabilizing the business cycle lose their effectiveness.

This is not the case in the low interest rate regime. In this regime the fiscal authorities can set $\mathrm{F}_{3}=0$ without endangering the stability of the public debt. As a result, successive increases in $\mathrm{F}_{2}$ have a pronounced effect on the distribution of animal spirits. The higher is $F_{2}$ the less frequent are the extreme values of animal spirits and the more these are concentrated around zero. As a result, the boom-bust nature in the business cycle is substantially reduced. Fiscal policy can be used as an effective instrument (similar to the role of a central bank) to stabilize the business cycle.

Figure 8: Frequency Distribution of Animal Spirits in Two Interest Rate Regimes
rs > 0



rs < 0




To understand the intuition of this result, consider what happens during a recession when animal spirits have turned negative. When in the high interest rate regime the fiscal authorities attempt to stabilize the output gap by increasing spending, this leads to a policy conflict: the increased spending tends to increase the debt level. Given the underlying instability of the government debt, the fiscal authorities are prevented from stabilizing the output gap because this policy destabilizes the debt level. As we have given $\mathrm{F}_{3}$ a value that prevents public debt instability, the fiscal authorities cannot use their fiscal instrument to stabilise the output gap. No such constraint exists in the low interest regime: in a recession the fiscal authorities can increase expenditure without
destabilizing the debt level and fiscal policy is an effective instrument to stabilize the business cycle.

## 7. Trade-off between Output and Debt Variability

The conventional rational expectations do not have a tradeoff between output and debt. This is well justified by the Ricardian Equivalence proposition. If agents are perfectly forward looking and are able to internalize the government's budget constraint, any attempt of the government to spend more (either by issuing debt or increasing tax) to affect the aggregate demand does not affect agents' consumption decisions, and thus it does not change aggregate demand (i.e. the fiscal multipliers should be zero). This is in contrast to what we find in our behavioural model. The results of section 4 indicate that the presence of animal spirits results in positive fiscal multipliers and sometimes the size of the multipliers can be large. The implication of our model is that government faces a tradeoff between output and debt stabilizations.

We construct the tradeoffs between output and debt variability in Figure 9. The way we do this is as follows. First, we vary the fiscal output stabilizing parameter, $\mathrm{F}_{2}$, and we compute the standard deviations of the output gap and the public debt for increasing values of $\mathrm{F}_{2}$. We do this for different interest rate regimes. The results are shown in panels A and B of Figure 9. From panel A we observe that increases in $\mathrm{F}_{2}$ lead to a monotonic decline in the volatility of output as long as the real interest rate $r s<0$. When $r s>0$ we obtain a non-linear relation, i.e. increases in $\mathrm{F}_{2}$ initially reduce the variability of output; at some point, however, the slope becomes positive, i.e. further increases in $\mathrm{F}_{2}$ have the effect of increasing the volatility of output. This non-linearity increases with the increase in rs. We will come back to this surprising result to give it an interpretation.

Panel B shows the relation between the volatility of government debt and $\mathrm{F}_{2}$. We find that for all interest rate regimes this relation is positive, i.e. increasing attempts at stabilizing output using government spending tend to increase the volatility of debt. This positive effect becomes stronger when the real interest rate rs increases.

Panel C shows the trade-offs between the volatility of output and public debt. They are obtained by combining the previous two panels. In order to understand this tradeoff it is good to start from point $A$. This shows the points of the tradeoffs where $F_{2}=0$, i.e.
there is no fiscal stabilization. As $\mathrm{F}_{2}$ increases, we move up along the tradeoffs. When rs < 0 we obtain negatively sloped tradeoffs, i.e. when fiscal authorities increase their output stabilization efforts this comes at a price of increasing the variability of the debt. When rs $=-0.03$ this negative tradeoff is relatively flat, i.e. the cost of output stabilization in terms of debt variability is small. With increasing rs, the slope of the tradeoffs increases indicating that the cost of stabilization tends to increase. When rs turns positive, we observe that at some point (when $F_{2}$ becomes large enough) the tradeoffs become positively sloped, i.e. further attempts at stabilizing output by varying government spending lead to increases in both the variability of output and debt. What is the intuition behind this surprising result?

In order to understand this result, let us analyze what happens during a recession. In that case the fiscal authorities increase spending in order to stabilize output. This leads to increases in the deficit and thus in government debt. We know, however, that when rs > 0 the debt is dynamically unstable except if the authorities keep a sufficiently positive primary balance. The anti-cyclical fiscal policy leads to a departure from this condition thereby destabilizing the debt. The latter forces the fiscal authorities to reduce spending thereby offsetting the anti-cyclical policy stance. When rs is very positive, the underlying instability of the debt is very strong. As a result, the need to reduce spending to stabilize the debt overwhelms the anti-cyclical policy stance. Fiscal policies as a whole (reflecting both $\mathrm{F}_{2}$ and $\mathrm{F}_{3}$ ) become pro-cyclical.

From the preceding, it follows that in interest rate regimes in which the interest rate exceeds the growth rate of the economy in the steady state, the use of fiscal policy to stabilize output is severely limited. The use of fiscal policy to stabilize the business cycle can quickly lead to a "loss-loss" situation in which both the government debt and the business cycle are destabilized. In a regime when the interest rate is lower than the growth rate of the economy, this problem does not arise. This is a regime that allows the fiscal authorities to follow anti-cyclical policies. However, there is always some price to pay in that these policies tend to increase the variability of the debt. Obviously when rs is very negative this cost is reduced.

Figure 9: Constructing Trade-offs


B


C


## 8. Empirical Validation

A central prediction of our behavioral model is that, by driving the booms and busts in economic activity, animal spirits are responsible for the negative correlation between the business cycle and the government debt GDP ratio. In this section we provide some empirical evidence for this prediction (see also De Grauwe, 2012; De Grauwe and Ji, 2018, for more evidence).

In table 3 we report the results of a panel fixed effect regression analysis on data of OECD countries during 2001-2017. The dependent variable is the change in the Government debt to GDP ratio. We selected the OECD Business Confidence Indicator as our measure of Animal Spirits. The results are shown in Table 3. We observe a highly significant negative relation between the changes in the government debt ratio and the Business Confidence Indicators for the sample of countries as a whole and for the subsamples of the Eurozone and the non-Eurozone countries. This confirms our prediction that that during periods of market optimism, government debt ratios tend to decline while the reverse occurs during periods of pessimism.

Table 3: Empirical Evidence of the trade-off between Animal Spirits and Public Debt

|  | (1) | (2) | (3) |
| :---: | :---: | :---: | :---: |
|  | Change in government debt to GDP ratio | Change in government debt to GDP ratio | Change in government debt to GDP ratio |
| Business | -1.9025*** | -2.1272*** | -1.2574** |
| Confidence Index | [0.2307] | [0.3018] | [0.4648] |
| Sample | Total | Eurozone | Non-Eurozone |
| Observations | 306 | 187 | 119 |
| $R^{2}$ | 0.161 | 0.186 | 0.093 |
| Note: 1. Standard errors in brackets: * $p<0.1,{ }^{* *} p<0.05$, ${ }^{* * *} p<0.01$ <br> 2. Sample includes countries annual data 2001-2017: Eurozone (Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Portugal and Spain), Non-Eurozone (Australia, Japan, Norway Sweden, Switzerland, United Kingdom and United States). <br> Source: OECD |  |  |  |

## 9. Conclusion

In this paper we extended the behavioral New Keynesian Model (De Grauwe, 2012) by adding a fiscal policy block. The model has non-linear features and generates waves of optimism and pessimism (i.e. animal spirits). These animal spirits arise because of the agents' cognitive limitations. This behavioral model has allowed us to study the effects of government spending by taking into account its interactions with animal spirits, monetary policy and public debt.

Our first important finding is that animal spirits not only are a strong force in influencing the movements in the output gap, they are also important in moving public debt. When animal spirits are positive and the economy is booming, government debt tends to decline. Conversely, when animal spirits are negative and the economy experiences a downturn, public debt increases. This negative correlation between animal spirits and public debt has a strong impact on the choices the fiscal policymakers face.

Our model also provides insightful results concerning fiscal multipliers. We found that the size of the fiscal multiplier depends on the state of the economy, which is consistent with recent empirical evidence. In particular, we find that when animal spirits are neutral, the fiscal multiplier tends to be small and when animal spirits take on extreme values, the fiscal multiplier is large and typically exceeds 1 . This is consistent with recent empirical findings that the fiscal multiplier is state dependent.

In order to analyze these choices we distinguished between two interest rate regimes. In the first regime, the steady state interest rate is higher than the nominal growth rate of the economy; in the second regime the steady state interest rate is below the nominal growth rate of the economy. We showed that these two regimes have strongly different stability properties that affect the governments' capacity to use the fiscal policy instrument to stabilize the business cycle.

We found that when the interest rate exceeds the growth rate of the economy (such as the condition now many periphery Eurozone countries experience since the sovereign debt crisis), the use of fiscal policy to stabilize output is severely limited. The use of fiscal policy to stabilize the business cycle can quickly lead to a "loss-loss" situation in which both the government debt and the business cycle are destabilized. In a regime
when the interest rate is lower than the growth rate of the economy, this problem does not arise. This is a regime that allows the fiscal authorities to follow anti-cyclical policies. However, there is always some price to pay in that these policies tend to increase the variability of public debt.

We conclude from this that when the economy is in the high interest rate regimes, the responsibility of stabilizing the business cycles should be borne mainly by the monetary authorities. This then puts a lot of pressure on the central bank; a pressure many central banks are reluctant to take on. In the low interest rate regime the responsibility of the stabilization of the business cycle can more easily be shared by the monetary and fiscal authorities.

## Appendix 1: From equation (5) to (6)

$$
D_{t+1}=\left(1+r_{t}\right)\left(D_{t}+G_{t} P_{t}-\tau Y_{t} P_{t}\right)
$$

Divide both sides by $P_{t}$ yields

$$
\frac{D_{t+1}}{P_{t}}=\left(1+r_{t}\right)\left(\frac{D_{t}}{P_{t-1}} \frac{P_{t-1}}{P_{t}}+G_{t}-\tau Y_{t}\right)
$$

Define $d_{t}=\frac{D_{t}}{P_{t-1}}$,

$$
\begin{gathered}
d_{t+1}=\left(1+r_{t}\right)\left(d_{t} \frac{P_{t-1}}{P_{t}}+G_{t}-\tau Y_{t}\right) \\
d_{t+1}=\left(1+r_{t}\right)\left(d_{t}\left(1-\pi_{t}\right)+G_{t}-\tau Y_{t}\right) \\
\frac{d_{t+1}}{1+r_{t}}=\left(d_{t}\left(1-\pi_{t}\right)+G_{t}-\tau Y_{t}\right)
\end{gathered}
$$

We now divide both sides by Y (the steady state value of output)

$$
\frac{\Delta_{t+1}}{1+r_{t}}=\left(\Delta_{t}\left(1-\pi_{t}\right)+\frac{G_{t}}{Y}-\tau \frac{Y_{t}}{Y}\right)
$$

where $\Delta_{t}=\frac{d_{t}}{Y}$
We can now linearize the budget constraint around its steady state values. We also use the formulas of a linear expansion around the steady state (see Uhlig,1999):

$$
\begin{gathered}
a X_{t}=a X x_{t} \\
\left(X_{t}+a\right) Z_{t}=X Z x_{t}+(X+a) Z x_{t}
\end{gathered}
$$

where $a$ is a constant term; $X$ is the steady state value of variable $X_{t}$ and $x_{t}=\ln X_{t}-$ $\ln X$, i.e. the percentage deviation of $X_{t}$ from its steady state value; $Z$ is the steady state value of variable $Z_{t}$ and $z_{t}=\ln Z_{t}-\ln Z$, i.e. the percentage deviation of $Z_{t}$ from its steady state value.

Define $\delta_{t}=\ln d_{t}-\ln d$, we obtain

$$
\frac{\Delta}{(1+r)}\left(\delta_{t+1}-\hat{r}_{t}\right)=\left(\Delta\left(\delta_{t}-\pi_{t}\right)+\frac{G}{Y} g_{t}-\tau y_{t}\right)
$$

or

$$
\delta_{t+1}=\hat{r}_{t}+(1+r)\left(\delta_{t}-\pi_{t}+h_{1} g_{t}-h_{2} y_{t}\right)
$$

where $\delta_{t}$ is the percentage deviation of real debt in period t from its steady state value, and $h_{1}=\frac{G}{Y \Delta}$ and $h_{2}=\frac{\tau}{\Delta}$.

## Appendix 2: Behavioral expectations formation

## A2.1. Introducing heuristics in forecasting output

Agents are assumed to use simple rules (heuristics) to forecast the future output $\widetilde{E}_{\mathrm{t}} y_{t+1}$. The way we proceed is as follows. We assume two types of forecasting rules. A first rule is called a "fundamentalist" one. Agents estimate the steady state value of the output gap (which is normalized at 0 ) and use this to forecast the future output gap. A second forecasting rule is an "extrapolative" one. This is a rule that does not presuppose that agents know the steady state output gap. They are agnostic about it. Instead, they extrapolate the previous observed output gap into the future. The two rules are specified as follows:

The fundamentalist rule is defined by $\widetilde{\mathrm{E}}_{\mathrm{t}}^{\mathrm{f}} \mathrm{y}_{\mathrm{t}+1}=0$

The extrapolative rule is defined by $\widetilde{\mathrm{E}}_{\mathrm{t}}^{\mathrm{e}} \mathrm{y}_{\mathrm{t}+1}=y_{t-1}$
This kind of simple heuristic has often been used in the behavioral finance literature where agents are assumed to use fundamentalist and chartist rules (see Brock and Hommes, 1997; Branch and Evans, 2006; De Grauwe and Grimaldi, 2006). It is probably the simplest possible assumption one can make about how agents who experience cognitive limitations use rules that embody limited knowledge to guide their behaviour. They only require agents to use information they understand, and do not require them to understand the whole picture. More complex rules can be used (see for example De Grauwe (2012), Hommes and Lustenhouwer (2016)).

We assume that the market forecast can be obtained as a weighted average of these two forecasts, i.e.

$$
\begin{align*}
\widetilde{\mathrm{E}}_{\mathrm{t}} y_{t+1} & =\alpha_{f, t} \widetilde{\mathrm{E}}_{\mathrm{t}}^{\mathrm{f}} \mathrm{y}_{\mathrm{t}+1}+\alpha_{e, t} \widetilde{\mathrm{E}}_{\mathrm{t}}^{\mathrm{e}} \mathrm{y}_{\mathrm{t}+1}  \tag{A3}\\
\widetilde{\mathrm{E}}_{\mathrm{t}} y_{t+1} & =\alpha_{f, t} 0+\alpha_{e, t} \mathrm{y}_{\mathrm{t}-1}  \tag{A4}\\
\alpha_{f, t}+\alpha_{e, t} & =1 \tag{A5}
\end{align*}
$$

where $\alpha_{f, t}$ and $\alpha_{e, t}$ are the probabilities that agents use a fundamentalist, respectively, an extrapolative rule.

It can be seen that when $\alpha_{f, t}=1$, i.e. the probability of all agents using the fundamentalist rule is equal to 1 , the coefficient in front of $y_{t-1}$ is $1-a_{1}$, while if $\alpha_{f, t}=0$, the probability of all agents using the extrapolative rule is equal to 1 , that coefficient is 1 . This makes clear that the source of the persistence in the output gap will be coming from the use of the extrapolative rule.

The forecasting rules (heuristics) introduced here are not derived at the micro level and then aggregated. Instead, they are imposed ex post, on the demand and supply equations. This has also been the approach in the learning literature pioneered by Evans and Honkapohja (2001). Ideally one would like to derive the heuristics from the microlevel in an environment in which agents experience cognitive problems. Our knowledge about how to model this behavior at the micro level and how to aggregate it is too sketchy, however. Psychologists and brain scientists struggle to understand how our brain processes information. There is as yet no generally accepted model we could use to model the micro-foundations of information processing in a world in which agents experience cognitive limitations. We have not tried to do so.

## A2.2. Selecting the forecasting rules in forecasting output

As indicated earlier, agents in our model are willing to learn, i.e. they continuously evaluate their forecast performance. This willingness to learn and to change one's behavior is a very fundamental definition of rational behaviour. Thus, our agents in the model are rational, not in the sense of having rational expectations. Instead our agents are rational in the sense that they learn from their mistakes. The concept of "bounded rationality" is often used to characterize this behaviour.

The first step in the analysis then consists in defining a criterion of success. This will be the forecast performance (utility) of a particular rule. We define the utility of using the fundamentalist and extrapolative rules as follows ${ }^{2}$ :

[^1]\[

$$
\begin{align*}
U_{f, t} & =-\sum_{\mathrm{k}=0}^{\infty} \omega_{\mathrm{k}}\left[\mathrm{y}_{\mathrm{t}-\mathrm{k}-1}-\widetilde{\mathrm{E}}_{\mathrm{f}, \mathrm{t}-\mathrm{k}-2} \mathrm{y}_{\mathrm{t}-\mathrm{k}-1}\right]^{2}  \tag{A6}\\
U_{e, t} & =-\sum_{\mathrm{k}=0}^{\infty} \omega_{\mathrm{k}}\left[\mathrm{y}_{\mathrm{t}-\mathrm{k}-1}-\widetilde{\mathrm{E}}_{\mathrm{e}, \mathrm{t}-\mathrm{k}-2} \mathrm{y}_{\mathrm{t}-\mathrm{k}-1}\right]^{2} \tag{A7}
\end{align*}
$$
\]

where $U_{f, t}$ and $U_{e, t}$ are the utilities of the fundamentalist and extrapolating rules, respectively. These are defined as the negative of the mean squared forecasting errors (MSFEs) of the forecasting rules; $\omega_{k}$ are geometrically declining weights. We make these weights declining because we assume that agents tend to forget. Put differently, they give a lower weight to errors made far in the past as compared to errors made recently. The degree of forgetting turns out to play a major role in our model. This was analyzed in De Grauwe (2012).

The next step consists in evaluating these utilities. We apply discrete choice theory (see Anderson et al., 1992; Brock and Hommes, 1997) in specifying the procedure agents follow in this evaluation process. If agents were purely rational they would just compare $U_{f, t}$ and $U_{e, t}$ in (A6) and (A7) and choose the rule that produces the highest value. Thus, under pure rationality, agents would choose the fundamentalist rule if $U_{f, t}>$ $U_{e, t}$, and vice versa. However, psychologists have stressed that when we have to choose among alternatives we are also influenced by our state of mind (see Kahneman, 2002). The latter is to a large extent unpredictable. It can be influenced by many things (the weather, recent emotional experiences, etc). One way to formalize this is that the utilities of the two alternatives have a deterministic component (these are $U_{f, t}$ and $U_{e, t}$ in (A6) and (A7)) and a random component $\varepsilon_{f, t}$ and $\varepsilon_{e, t}$ The probability of choosing the fundamentalist rule is then given by

$$
\begin{equation*}
\alpha_{f, t}=P\left[\left(U_{f, t}+\varepsilon_{f, t}\right)>\left(U_{e, t}+\varepsilon_{e, t}\right)\right] \tag{A8}
\end{equation*}
$$

In words, this means that the probability of selecting the fundamentalist rule is equal to the probability that the stochastic utility associated with using the fundamentalist rule exceeds the stochastic utility of using an extrapolative rule. In order to derive a more precise expression one has to specify the distribution of the random variables $\varepsilon_{f, t}$ and $\varepsilon_{e, t .}$. It is customary in the discrete choice literature to assume that these random variables are logistically distributed (see Anderson et al., 1992 p.35). One then obtains the following expressions for the probability of choosing the fundamentalist rule:

$$
\begin{equation*}
\alpha_{f, t}=\frac{\exp \left(\gamma U_{f, t}\right)}{\exp \left(\gamma U_{f, t}\right)+\exp \left(\gamma U_{e, t}\right)} \tag{A9}
\end{equation*}
$$

Similarly the probability that an agent will use the extrapolative forecasting rule is given by:

$$
\begin{equation*}
\alpha_{e, t}=\frac{\exp \left(\gamma U_{e, t}\right)}{\exp \left(\gamma U_{f, t}\right)+\exp \left(\gamma U_{e, t}\right)}=1-\alpha_{f, t} \tag{A10}
\end{equation*}
$$

Equation (A9) says that as the past forecast performance (utility) of the fundamentalist rule improves relative to that of the extrapolative rule, agents are more likely to select the fundamentalist rule for their forecasts of the output gap. Equation (A10) has a similar interpretation. The parameter $\gamma$ measures the "intensity of choice". It is related to the variance of the random components. Defining $\varepsilon_{t}=\varepsilon_{f, t}-\varepsilon_{e, t}$ we can write (see Anderson, et al., 1992):

$$
\gamma=\frac{1}{\sqrt{\operatorname{var}\left(\varepsilon_{t}\right)}} .
$$

When $\operatorname{var}\left(\varepsilon_{t}\right)$ goes to infinity, $\gamma$ approaches 0 . In that case agents decide to be fundamentalist or extrapolator by tossing a coin and the probability to be fundamentalist (or extrapolator) is exactly 0.5 . When $\gamma=\infty$ the variance of the random components is zero (utility is then fully deterministic) and the probability of using a fundamentalist rule is either 1 or 0 . The parameter $\gamma$ can also be interpreted as expressing a willingness to learn from past performance. When $\gamma=0$ this willingness is zero; it increases with the size of $\gamma$.

As argued earlier, the selection mechanism used should be interpreted as a learning mechanism based on "trial and error". When observing that the rule they use performs less well than the alternative rule, agents are willing to switch to the more performing rule. Put differently, agents avoid making systematic mistakes by constantly being willing to learn from past mistakes and to change their behaviour. This also ensures that the market forecasts are unbiased.

## A2.3. Heuristics and selection mechanism in forecasting inflation

Agents also have to forecast inflation $\tilde{E}_{t} \pi_{t+1}$. A similar simple heuristics is used as in the case of output gap forecasting, with one rule that could be called a fundamentalist rule
and the other an extrapolative rule (see Brazier et al., 2008 for a similar setup). We assume an institutional set-up in which the central bank announces an explicit inflation target. The fundamentalist rule then is based on this announced inflation target, i.e. agents using this rule have confidence in its credibility and use it to forecast inflation. Agents who do not trust the announced inflation target use the extrapolative rule, which consists in extrapolating inflation from the past into the future.

The fundamentalist rule will be called an "inflation targeting" rule. It consists in using the central bank's inflation target to forecast future inflation, i.e.

$$
\begin{equation*}
\widetilde{\mathrm{E}}_{t}^{t a r} \pi_{t+1}=\pi^{*} \tag{A11}
\end{equation*}
$$

where the inflation target is $\pi^{*}$
The "extrapolators" are defined by

$$
\begin{equation*}
\widetilde{\mathrm{E}}_{t}^{\text {ext }} \pi_{t+1}=\pi_{t-1} \tag{A12}
\end{equation*}
$$

The market forecast is a weighted average of these two forecasts, i.e.

$$
\begin{array}{ll} 
& \widetilde{\mathrm{E}}_{t} \pi_{t+1}=\beta_{t a r, t} * 0+\beta_{\text {ext }, t} * \pi_{t-1} \\
\text { and } & \beta_{t a r, t}+\beta_{\text {ext }, t}=1 \tag{A14}
\end{array}
$$

The same selection mechanism is used as in the case of output forecasting to determine the probabilities of agents trusting the inflation target and those who do not trust it and revert to extrapolation of past inflation, i.e.

$$
\begin{align*}
& \beta_{t a r, t}=\frac{\exp \left(\gamma U_{t a r, t}\right)}{\exp \left(\gamma U_{t a r, t}\right)+\exp \left(\gamma U_{e x t, t}\right)}  \tag{A15}\\
& \beta_{e x t, t}=\frac{\exp \left(\gamma U_{e x t, t}\right)}{\exp \left(\gamma U_{t a r, t}\right)+\exp \left(\gamma U_{e x t, t}\right)} \tag{A16}
\end{align*}
$$

where $U_{\text {tar, },}$ and $U_{\text {ext }, t}$ are the forecast performances (utilities) associated with the use of the fundamentalist and extrapolative rules in equation (A17) and (A18). These are defined in the same way as in (A6) and (A7), i.e. they are the negatives of the weighted averages of past squared forecast errors of using fundamentalist (inflation targeting) and extrapolative rules, respectively.

$$
\begin{align*}
U_{t a r, t} & =-\sum_{k=0}^{\infty} \omega_{k}\left[\pi_{t-k-1}-\tilde{E}_{f, t-k-2} \pi_{t-k-1}\right]^{2}  \tag{A17}\\
U_{\text {ext }, t} & =-\sum_{k=0}^{\infty} \omega_{k}\left[\pi_{t-k-1}-\widetilde{E}_{e, t-k-2} \pi_{t-k-1}\right]^{2} \tag{A18}
\end{align*}
$$

This inflation forecasting heuristics can be interpreted as a procedure of agents to find out how credible the central bank's inflation targeting is. If this is very credible, using the announced inflation target will produce good forecasts and as a result, the probability that agents will rely on the inflation target will be high. If on the other hand the inflation target does not produce good forecasts (compared to a simple extrapolation rule), the probability that agents will use it will be small.

Finally, it should be mentioned that the two prediction rules for the output gap and inflation are made independently. This is a strong assumption. What we model is the use of different forecasting rules. The selection criterion is exclusively based on the forecasting performances of these rules. Agents in our model do not have a psychological predisposition to become fundamentalists or extrapolators. However, it is possible that despite the assumption of independence, the realized choices generated from our model are actually correlated due to the interactions of the different variables in the model.

## A2.4. Defining animal spirits

The forecasts made by extrapolators and fundamentalists play an important role in the model. In order to highlight this role we define an index of market sentiments, which we call "animal spirits", and which reflects how optimistic or pessimistic these forecasts are.

The definition of animal spirits is as follows:

$$
S_{t}= \begin{cases}\alpha_{e, t}-\alpha_{f, t} & \text { if } y_{t-1}>0  \tag{A19}\\ -\alpha_{e, t}+\alpha_{f, t} & \text { if } y_{t-1}<0\end{cases}
$$

where $S_{t}$ is the index of animal spirits. This can change between -1 and +1 . There are two possibilities:

- When $y_{t-1}>0$, extrapolators forecast a positive output gap. The fraction of agents who make such a positive forecasts is $\alpha_{e, t}$. Fundamentalists, however, then make a
pessimistic forecast since they expect the positive output gap to decline towards the equilibrium value of 0 . The fraction of agents who make such a forecast is $\alpha_{f, t}$. We subtract this fraction of pessimistic forecasts from the fraction $\alpha_{e, t}$ who make a positive forecast. When these two fractions are equal to each other (both are then 0.5 ) market sentiments (animal spirits) are neutral, i.e. optimists and pessimists cancel out and $S_{t}=0$. When the fraction of optimists $\alpha_{e, t}$ exceeds the fraction of pessimists $\alpha_{f, t}, \quad S_{t}$ becomes positive. As we will see, the model allows for the possibility that $\alpha_{e, t}$ moves to 1 . In that case there are only optimists and $\mathrm{S}_{t}=1$.
- When $y_{t-1}<0$, extrapolators forecast a negative output gap. The fraction of agents who make such a negative forecasts is $\alpha_{e, t}$. We give this fraction a negative sign. Fundamentalists, however, then make an optimistic forecast since they expect the negative output gap to increase towards the equilibrium value of 0 . The fraction of agents who make such a forecast is $\alpha_{f, t}$. We give this fraction of optimistic forecasts a positive sign. When these two fractions are equal to each other (both are then 0.5 ) market sentiments (animal spirits) are neutral, i.e. optimists and pessimists cancel out and $S_{t}=0$. When the fraction of pessimists $\alpha_{e, t}$ exceeds the fraction of optimists $\alpha_{f, t} S_{t}$ becomes negative. The fraction of pessimists, $\alpha_{e, t}$, can move to 1 . In that case there are only pessimists and $S_{t}=-1$.

We can rewrite (A19) as follows:

$$
S_{t}=\left\{\begin{array}{lc}
\alpha_{e, t}-\left(1-\alpha_{e, t}\right)=2 \alpha_{e, t}-1 & \text { if } y_{t-1}>0  \tag{A20}\\
-\alpha_{e, t}+\left(1-\alpha_{e, t}\right)=-2 \alpha_{e, t}+1 & \text { if } y_{t-1}<0
\end{array}\right.
$$

## APPENDIX 3: Solving the Model

The solution of the model is found by using the system of equations (1)-(4) and (6). It can be rewritten in matrix notation to yield:

$$
\left[\begin{array}{ccccc}
1 & 0 & a_{2} & -a_{3} & 0 \\
-b_{2} & 1 & 0 & 0 & 0 \\
-c_{2} & -c_{1} & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & F_{3} \\
0 & 0 & 0 & 0 & 1
\end{array}\right]\left[\begin{array}{c}
y_{t} \\
r_{t} \\
r_{t} \\
g_{t} \\
\delta_{t}
\end{array}\right]=\left[\begin{array}{ccccc}
a_{1} & a_{2} & 0 & 0 & 0 \\
0 & b_{1} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{array}\right]\left[\begin{array}{c}
\tilde{E}_{t} y_{t+1} \\
\tilde{E}_{t} \pi_{t+1} \\
\tilde{E}_{t} r_{t+1} \\
\tilde{E}_{t} g_{t+1} \\
\tilde{E}_{t} \delta_{t+1}
\end{array}\right]+\left[\begin{array}{r}
0 \\
0 \\
-c_{1} \\
0 \\
0
\end{array}\right] \pi^{*}+
$$

$$
\left[\begin{array}{ccccc}
1-a_{1} & 0 & 0 & 0 & 0 \\
0 & 1-b_{1} & 0 & 0 & 0 \\
0 & 0 & c_{3} & 0 & 0 \\
-F_{2} & 0 & 0 & F_{1} & 0 \\
-h_{2} \tau(1+r) & -(1+r) & 1 & h_{1}(1+r) & 1+r
\end{array}\right]\left[\begin{array}{c}
y_{t-1} \\
\pi_{t-1} \\
r_{t-1} \\
g_{t-1} \\
\delta_{t-1}
\end{array}\right]+\left[\begin{array}{c}
\epsilon_{t} \\
\eta_{t} \\
\mu_{t} \\
v_{t} \\
\varphi_{t}
\end{array}\right]
$$

In compact form, this system can be written as $A Z_{t}=B \tilde{E}_{t} Z_{t+1}+C \pi^{*}+D Z_{t-1}+\Sigma_{t}$.
Then, under the condition that $A$ is non-singular, a solution for $Z_{t}$ can be obtained as $Z_{t}=A^{-1}\left(B \tilde{E}_{t} Z_{t+1}+C \pi^{*}+D Z_{t-1}+\Sigma_{t}\right)$.

The model has non-linear features, making it difficult to arrive at analytical solutions. Thus, we will use numerical methods to analyze its dynamics. To this aim, we have to calibrate the model.

Table A1 presents the values used in the calibration exercise, together with references to articles where single parameters are calibrated like in our study. Note that the five shocks (demand, supply, interest rate, public expenditure and public debt shocks) are independently and identically distributed, with zero mean and constant standard deviation equal to 0.5.

Table A1: Calibrated Parameters of the Model

| $\mathrm{a}_{1}=0.5$ | Coefficient of expected output in output equation ${ }^{\text {8,* }}$ |
| :---: | :---: |
| $\mathrm{a}_{2}=0.2$ | Interest elasticity of output demand ${ }^{\text {8,* }}$ |
| $\mathrm{a}_{3}=0.25$ | Coefficient of public expenditure in output equation ${ }^{\times, 3}$ |
| $\mathrm{b}_{1}=0.5$ | Coefficient of expected inflation in inflation equation ${ }^{\text {, }}$ * |
| $\mathrm{b}_{2}=0.05$ | Coefficient of output in inflation equation ${ }^{\text {8, }}$, |
| $\mathrm{c}_{1}=1.5$ | Coefficient of inflation in Taylor rule ${ }^{\text {8,* }}$ |
| $\mathrm{c}_{2}=0.5$ | Coefficient of output in Taylor rule ${ }^{\text {V,* }}$ |
| $\mathrm{C}_{3}=0.5$ | Interest smoothing parameter in Taylor rule ${ }^{\text {Y }}$ |
| $\pi^{*}=0$ | Central bank's inflation target* |
| $\mathrm{F}_{1}=0.6$ | Public expenditure smoothing in fiscal rule ${ }^{¢}$ |
| $\mathrm{F}_{2}=0.2$ | Coefficient of output in fiscal rule ${ }^{\mathrm{H}}$ |
| $\mathrm{F}_{3}=0.03$ | Public debt parameter in fiscal rule ${ }^{\text {H }}$ |
| $\mathrm{r}=0.01$ | Steady state interest rate ${ }^{3}$; we also experiment with r=-0,01 |
| $\mathrm{h}_{1}=0.4$ | Coefficient of public expenditure in debt equation ${ }^{\dagger}$ |
| $\mathrm{h}_{2}=1.6$ | Coefficient of output in debt equation ${ }^{\dagger}$ |
| $\tau=0.3$ | Income tax rate ${ }^{\ddagger}$ |
| $\gamma=2$ | Intensity of choice parameter ${ }^{\text {8 }}$ |
| $\rho=0.5$ | Measures the speed of declining weights in mean squares errors (memory parameter) ${ }^{\mathrm{X}}$ |
| Note: See, among others, ${ }^{\text { }}$ De Grauwe (2012), ${ }^{\text {H }}$ Kirsanova et al. (2005), ${ }^{〔}$ Muscatelli and Tirelli (2005), ${ }^{3}$ Kirsanova and Wren-Lewis (2012) and Galí and Monacelli (2005), ${ }^{\dagger}$ Kirsanova et al. (2012), $\ddagger$ Ferrero (2009), ${ }^{*}$ De Grauwe and Ji (2018). |  |

## APPENDIX 4: Impulse Response Functions (IRFs)t

$$
r=0.01
$$

$$
r=-0.01
$$










## References

Anderson, S., de Palma, A. and Thisse, J.F. (1992), "Discrete Choice Theory of Product Differentiation", MIT Press, Cambridge, Mass.

Auerbach, A. and Gorodnichenko, Y. (2012), "Measuring the Output Responses to Fiscal Policy." American Economic Journal: Economic Policy, 4(2), pp. 1-27.

Auerbach, A. and Gorodnichenko, Y. (2013), "Fiscal Multipliers in Recession and Expansion." In Fiscal Policy after the Financial Crisis, edited by Alberto Alesina and Francesco Giavazzi, pp. 63-98. Chicago: Univ. Chicago Press.

Blanchard, O., and Leigh, D., (2013), Growth Forecast Errors and Fiscal Multipliers, IMF Working Papers, No. 13/1, Washington, D.C.

Blanchard, O., (2019), Public Debt and Low Interest Rates, AEA Presidential Lecture, January, https://www.aeaweb.org/aea/2019conference/program/pdf/14020_paper_etZgf bDr.pdf

Branch, W.A. (2004), "The Theory of Rationally Heterogeneous Expectations: Evidence from Survey Data on Inflation Expectations", Economic Journal, 114(497), pp. 592621.

Branch, W., and Evans, G., (2006), Intrinsic heterogeneity in expectation formation, Journal of Economic theory, 127, 264-95.

Brock, W. and Hommes, C. (1997), "A Rational Route to Randomness", Econometrica, Econometric Society, 65(5), pp. 1059-96, September.

Brock W. and Hommes C. (1998), "Heterogeneous Beliefs and Routes to Chaos in a Simple Asset Pricing Model", Journal of Economic Dynamics and Control, 22(8-9), pp. 1235-74.

Caprioli, F. (2015), "Optimal Fiscal Policy under Learning", Journal of Economic Dynamics and Control, Elsevier, 58(C), pp. 101-124.

Çebi, C. (2012), "The Interaction between Monetary and Fiscal Policies in Turkey: An Estimated New Keynesian DSGE Model", Economic Modelling, Elsevier, 29(4), pp. 1258-67.

Christiano, L., Eichenbaum, M. and Rebelo, S. (2011), "When Is the Government Spending Multiplier Large?", Journal of Political Economy, 119(1), pp. 78-121.

De Grauwe, P. (2012), "Lectures on Behavioral Macroeconomics", Princeton University Press.

De Grauwe, P. and Ji, Y., (2018), Inflation Targets and the Zero Lower Bound in a Behavioural Macroeconomic Model, Economica, Volume 86.

De Grauwe, P. and Foresti, P., (2018), Animal Spirits and Fiscal Policy, CEPR Discussion Paper, DP13376.

De Grauwe, P., and Grimaldi, M., (2006), The Exchange Rate in a Behavioural Finance Framework, Princeton University Press.

Evans, G., and Honkapohja, S., 2001, Learning and Expectations in Macroeconomics, Princeton University Press, 421pp.

Ferrero, A. (2009), "Fiscal and Monetary Rules for a Currency Union", Journal of International Economics, Elsevier, 77, pp. 1-10.

Fragetta, M. and Kirsanova, T. (2010), "Strategic Monetary and Fiscal Policy Interactions: An Empirical Investigation", European Economic Review, Elsevier, 54(7), pp. 855-79, October.

Gabaix, X. (2016), "A Behavioral New Keynesian Model", CEPR Discussion Papers 11729, C.E.P.R. Discussion Papers.

Galí, J. and Monacelli, T. (2005), "Optimal Monetary and Fiscal Policy in a Currency Union", Journal of International Economics, Elsevier, 76(1), pp. 116-32, September.

Hommes, C., and Lustenhouwer, J., (2016), Managing Heterogeneous and Unanchored Expectations: A Monetary Policy Analysis, Working Paper, Tinbergen Institute, Rotterdam.

Kahneman, D., 2002, Maps of Bounded Rationality: A Perspective on Intuitive Judgment and Choice, Nobel Prize Lecture, December 8, Stockholm (also published in American Economic Review, 2003).

Kirsanova, T., Satchi, M., Vines, D. and Wren-Lewis, S. (2007). "Optimal Fiscal Policy Rules in a Monetary Union," Journal of Money, Credit and Banking, Blackwell Publishing, 39(7), pp. 1759-84, October.

Kirsanova, T., Stehn, S.J. and Vines, D. (2005), "The Interactions between Fiscal Policy and Monetary Policy", Oxford Review of Economic Policy, 21(4), Fiscal Policy (Winter), pp. 532-64.

Kirsanova, T. and Wren-Lewis, S. (2012). "Optimal Fiscal Feedback on Debt in an Economy with Nominal Rigidities", Economic Journal, Royal Economic Society, 122(559), pp. 238-64.

Kumar, M. S and Woo, S. (2010), "Public Debt and Growth", IMF Working Papers, No. 10/174, International Monetary Fund.

Muscatelli V.A. and Tirelli, P. (2005), "Analyzing the Interaction of Monetary and Fiscal Policy: Does Fiscal Policy Play a Valuable Role in Stabilisation?", CESifo Economic Studies, CESifo, 51(4), pp. 549-85.
Panizza, U. and Presbitero, A.F., 2013. Public debt and economic growth in advanced economies: A survey. Swiss Journal of Economics and Statistics, 149(2), pp.175-204.
Panizza, U. and Presbitero, A. F. (2014). "Public debt and economic growth: Is there a causal effect?," Journal of Macroeconomics, Elsevier, vol. 41(C), pages 21-41.
Parker, J.A. (2011), "On Measuring the Effects of Fiscal Policy in Recessions", Journal of Economic Literature, 49(3), pp. 703-718.

Reinhart, C. M. and Rogoff, K.S. (2010), "Growth in a Time of Debt." American Economic Review, 100(2), pp. 573-78.

Schularick, Moritz, and Alan M Taylor (2012), "Credit Booms Gone Bust: Monetary Policy, Leverage Cycles, and Financial Crises, 1870-2008", The American Economic Review 102(2): 1029-61.


[^0]:    ${ }^{1}$ See the Appendix 1 for further insights on the definition of variables.

[^1]:    ${ }^{2}$ (A6) and (A7) can be derived from the following equation:

    $$
    U_{t}=\rho U_{t-1}+(1-\rho)\left[y_{t-1}-E_{t-2} y_{t-1}\right]^{2} \quad\left(\mathrm{~A}^{\prime}\right)
    $$

    where $\rho$ can be interpreted as a memory parameter. When $\rho=0$ only the last period's forecast error is remembered; when $\rho=1$ all past periods get the same weight and agents have infinite memory. We will generally assume that $0<\rho<1$. Using ( $\mathrm{A}^{\prime}$ ) we can write

    $$
    U_{t-1}=\rho U_{t-2}+(1-\rho)\left[y_{t-2}-E_{t-3} y_{t-2}\right]^{2}\left(\mathrm{~A} 6^{\prime \prime}\right)
    $$

    Substituting (A6") into (A $6^{\prime}$ ) and repeating such substitutions ad infinitum yields the expression (A6) where

    $$
    \omega_{k}=(1-\rho) \rho^{k}
    $$

