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JEL Classification: D74, H41, L13

Keywords: free riding, preemption, dynamic conflict, inter-group conflict, incomplete information, waiting

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Preemption contests between groups

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1 Introduction

A first-come, first-served rule can generate preemption incentives. We study such incentives when members of a group can preempt other groups on behalf of their own group. In such a framework group members' free-riding incentives inside the own group conflict with preemption motives that apply in the inter-group competition. Each player prefers that his or her group preempts the other groups, but that not he, but another member of his group is the preempting player who bears the costs of the preemptive action.

As an illustrative example, consider two industries, both of which can press a policy to implement trade policy regulations in their favour. The CEOs of the companies of these two industries may take action on behalf of their own group: for example, when they talk to influential politicians, they can inform them and instill in them the idea that such protection policies are necessary for their industry.[1] Since access to decision makers is a scarce resource and the mental readiness and receptiveness of politicians are limited, volunteering on behalf of one's own industry also has a cost: the CEO cannot use this opportunity to lobby for a more firm specific interest. In the absence of competition from other industries, a CEO may prefer to pitch for his or her own company interests and hope that other CEO colleagues will stand up for the interests of their industry.[2] This describes a waiting game as analysed in Bliss and Nalebuff (1984) and Fudenberg and Tirole (1986). The incentives change if there is competition between industries for trade policy favors. Policy requests from different industries may directly contradict or preclude each other. But even if these requests do not interfere with each other directly, they can still be in resource rivalry to each other. For instance, politics cannot implement any number of trade protection measures without discrediting the country internationally, and the US Secretary of Commerce may also be limited in the number of bills he or she can press for in the government. Furthermore, it may be easier to implement measures during the "honeymoon" period of a newly elected politician, who therefore has to prioritize possible policies.[3] Finally, the legislators' resources may also be limited more generally.[4] If such regulatory capacity constraints exist,

[1] Examples of forums where CEOs can meet politicians include U.S. President Donald Trump's *Manufacturing Council* and the *Strategy & Policy Forum*, both of which he founded early in 2017. Other instances occur when company leaders are members of the entourage of prime ministers or governors on important state visits and travel with them in their government jets. For example, participants during the Washington state trade mission to Mexico in May 2017 "...include(d) the governor, representatives from the state departments of Commerce and Agriculture, the Seattle Metropolitan Chamber of Commerce, the Trade Development Alliance of Greater Seattle, local business leaders and other government representatives." ("Agriculture and industry among top priorities for upcoming Washington trade delegation to Mexico," 2017.)

[2] While these influence attempts are rarely made public, a very interesting example is that of "the memo...written by Robert E. Murray, a longtime Trump supporter who donated $300,000 to the president's inauguration. In it, Mr. Murray, the head of Murray Energy, presented Mr. Trump with a wish list of environmental rollbacks just weeks after the inauguration." (Friedman, 2018.) This wish list, dated March 2017 and *organized in order of priority*, starts with the elimination of the "Clean Power Plan," the overturning of the "endangerment finding for greenhouse gases," the elimination of tax credits for solar and wind energy producers, and the withdrawal from the "Paris accords." It is clear that the priorities identified by Mr. Murray benefit the entire fossil fuel industry, rather than just the coal plants owned by Murray Energy. These priorities were rapidly acted upon by the Trump administration. Only two years later did president Trump try to affect policy choices to benefit a specific firm owned by Murray Energy. (Wade and Natter, 2019.)

[3] The honeymoon effect is well-studied in political science, see e.g. Buchler and Dominguez (2005) and references therein.

[4] Legislatorial effort might also be subject to opportunity costs such as the formation of costly voting coalitions. Glazer and McMillan (1990, 1992), building theories on project choice by legislators, allude to the opportunity cost of the legislator, such as

the incentive to wait for one's own CEO colleagues to become active faces the incentive to preempt colleagues from the other industry.

The problem here combines incentives of a waiting game with incentives as in a preemption game: as in the trade-protection illustration, the waiting game emerges inside each group, whereas the preemption game applies in the relationship between the industry groups. Our analysis shows how the two motives affect the individual player's decision making and addresses questions about how the timing of action depends on the intensity of competition between groups (number of groups), the general size of the groups overall, and the size asymmetries between groups. Preemption games tilt the timing of actions such that players act earlier. Waiting games tilt the timing of actions such that players tend to delay action. With incomplete information, preemption games and waiting games tend to assign the timing of action in equilibrium to the low-cost players. As there are now multiple, partially countervailing incentives, we also ask if this general insight remains valid.

Preemption incentives and free-riding incentives are often collective action problems that cause socially inefficient outcomes. It is not surprising if, with the exception of informal and non-repeated contexts,[5] institutions emerge that overcome the intra-group collective action problem by making groups act as unitary players. Many of the empirical frameworks that come to mind therefore assume that the intra-group free-riding problems are taken care of, with examples ranging from entry problems[6], R&D competition[7] or de-facto-standard setting in industries with network effects[8] in industrial organization.

While the de-facto incentives to institutionalize groups and coordinate their behavior internally may limit the number of examples in which the free-riding incentives and the pre-emption incentives are directly at work and face each other, this does not make the study of the game with free-riding inside purely non-cooperative groups less relevant. The study of the non-cooperative equilibrium is a benchmark that establishes the implicit or explicit default threat point or status–quo for collective bargaining that may take place inside a group. As is well-known from bargaining theory (see, e.g., the discussion in Binmore, Rubinstein and Wolinsky 1986), the payoff characteristics of the non-cooperative equilibrium are relevant for the cooperative solution that might be reached, for the sustainability of such cooperation in infinitely repeated games, and for how the group members coordinate their actions and split the surplus from cooperation among themselves.

forgone opportunities for constituency services. Johnson and and Libecap (2003) emphasize the transaction cost of legislation.

[5]Possible examples outside industrial organization can be found in the military context when individuals volunteer for suicide missions for their own troops, or when informal groups benefit from one of the members assuming special responsibility in a leadership position.

[6]See, e.g., Argenziano and Schmidt-Dengler (2014), Boyarchenko and Levendorskii (2014), Bloch, Fabrizi, and Lippert (2015), Nishide and Yagi (2016), and Ruiz-Aliseda (2016) Mason and Weeds (2010), and Fudenberg, Gilbert, Stiglitz, and Tirole (1983).

[7]See, e.g., Tirole (1983), Hopenhayn and Squintani (2011, 2016), and Nishihara (2018).

[8]Strong network externalities, due to the establishment of de-facto standards (Katz and Shapiro 1985, Farrell and Saloner 1986) might generate free-rider problems with a similar flavour. Preemptive timing by an early investor might be costly for this investor, but help setting a de-facto standard and might be beneficial for a whole subset of firms who all prefer this standard compared to an alternate standard.

The applications mentioned allude to some of the related literature. From a methodological point of view, our contribution brings together the problem of preemption and the problem of contributing to a group public good. Starting with Olson and Zeckhauser (1966), many aspects of the problem of contributing to a public good have been studied. Much of this literature assumes contributors' efforts add linearly to determine the aggregate group effort. Hirshleifer (1983) introduced alternatives to this technology—particularly the "best-shot" technology according to which only the largest contribution of a member of the group matters. Barbieri and Malueg (2014) use this group-contribution technology. The question of who volunteers has led to the study of the "volunteer's game" (Diekmann 1985). In this, a group of players gains and enjoys a public good if at least one of its members makes a contribution, where the contribution has a pre-defined cost for the contributor, and all players choose simultaneously whether to contribute. A key issue in this volunteer's dilemma is coordination. Lack of coordination may lead to inefficiency: multiple (or no) players may expend effort. All efforts but the largest are wasted in the best-shot framework, and up to $n - 1$ contributions are wasted in the volunteer's game.[9] If players can choose when to contribute, then "delay" may serve as a natural coordination device (Bliss and Nalebuff, 1984, and Fudenberg and Tirole, 1986). Players with a high contribution cost or with a low stake may be inclined to wait. This can lead to provision by only the player with the lowest cost, but with some delay.

The preemption game between groups adds an element of competition to this dynamic volunteer's game. It can be seen as a contest between multiple groups, where the members of each group individually decide on their contributions to the group's contest effort. Contest theory has studied group contests under a variety of assumptions about how groups' efforts determine the winner group, how group members' efforts contribute to the aggregate group effort, and what players know about each other, inside the group and across groups. A first approach considered additivity of contributions.[10] In an important departure, Fu, Lu, and Pan (2015) allow for an allocation of tasks among the competing teams. Also more recently Hirshleifer's best-shot effort-aggregation technology has been applied in inter-group contests. This includes work by Barbieri, Malueg, and Topolyan (2014), Chowdhury, Lee, and Sheremeta (2013), and Barbieri and Malueg (2016).[11]

The preemption game between groups relates to this literature on group contests. It describes a group contest with heterogenous players, incomplete information, and with a dynamic timing structure of the contribution decision. This richer dynamic structure plays a coordinating role. Unlike in the waiting games by Bliss and Nalebuff (1984) or Fudenberg and Tirole (1986), however, the timing decision is shaped by two

[9] Many aspects of this game have been studied. See, e.g., Diekmann (1993) and He, Wang, and Li (2014) for the role of players' cost asymmetry, and Archetti (2009) and Peña and Noeldeke (2016) for considerations of group size. For a contribution focusing on timing, see Bergstrom (2017).

[10] Seminal contributions in this field are Katz, Nitzan, and Rosenberg (1990) and Esteban and Ray (2001).

[11] This literature also considered asymmetric conflict, by which one group aggregates effort according to a best-shot technology and the other group aggregates according to a weakest-link technology (Clark and Konrad, 2007, Chowdhury and Topolyan, 2016).

factors: the preemption threat by players of competing groups and the free-riding incentives. Except for the case of bunching when several players volunteer immediately, for symmetric teams the only player who expends effort is the one with the lowest cost.[12]

We proceed in several steps. Section 2 provides the main analysis. First, we outline the key building blocks of the formal model. Then we study the equilibrium and its properties, including welfare, in the parameter range in which the equilibrium is interior. Section 3 considers asymmetry between the competing groups and strategies, focusing on group size, strength of the externalities within groups, and the possibility that agents may be differentially informed about teammates and competitors. Section 4 discusses the results and concludes. All proofs are in Appendix 1.

2 The formal framework

We first describe the formal framework that combines the problem of preemption between groups with the problems of free-riding and coordination within each group. Then we turn to the characterization of equilibrium and study its properties.

2.1 Players, actions and payoffs

We define N as the set of all players i and $\{N_1, N_2, ..., N_K\}$ a partition of these players into K groups of identical size with n players in each group. A representative player is denoted by i. This player is further characterized by his cost of effort c_i. All players' cost parameters are drawn independently from the same atomless cumulative distribution function F. We assume F is continuous and differentiable on its support $[\underline{c}, \bar{c}] \subset (0, \infty)$. We denote the density of F by f and assume it to be strictly positive on $(\underline{c}, \bar{c})$. Each player i knows the value of his own c_i and knows the distribution from which all players' valuations are drawn, but not the values of other players' realized costs, neither for members of his own group nor for members of the other groups.[13]

Player i's action is denoted by T_i and is chosen from the interval $[0, \infty]$. The action is the time until which player i waits to provide the public good to his own group ("grabs"), given that none of the other $Kn - 1$ players grabbed prior to T_i. All players choose their T_i independently and simultaneously, based on the information of their own cost, the distribution F, and the rules of the game. Players cannot observe the actual choices of grabbing times T_i, but as time goes on, they observe whether another player has grabbed.

[12] More recently a literature has addressed this problem and looked at outcomes in which several group members have to take action, where some of the actions are more expensive than others, and the less expensive tasks may be awarded first. In this framework the players have countervailing incentives. They prefer not to take any of the costly actions, but conditional on taking up one of the costly tasks, they prefer to assume a task with a lower cost (see Bonatti and Hörner, 2011).

[13] See the discussion in Section 3 for a possible way to relax this assumption.

If a player has not observed any of the other players grabbing prior to time T_i, then player i takes action at this point and the game ends.

The gross benefit for each member of a group is V if a member of the group grabs first. The payoff of player i with cost c is equal to $(V-c)e^{-\rho T}$ if he is the player who grabs first and at time T, where ρ is the common discount rate. Should several players, from one or more groups, choose simultaneously, a random mechanism selects one of these players to bear the cost and his group receives the prize.[14] The payoff of player i is equal to $Ve^{-\rho T}$ if a teammate grabs first at time T, and equal to zero if a player from another group grabs first. We assume throughout that $\bar{c} \leq V$.[15] The choice of zero as the payoff of members of a non-winning group is a normalization.[16]

Before we turn to the equilibrium analysis we highlight that we consider a reduced form that encompasses other, more general frameworks, but also highlight limits of its generality.

First, the setup equivalently describes situations in which grabbing generates a common-good benefit to all members, but when the player who decides to assume the cost of grabbing receives a private extra benefit from grabbing. Such private benefits for the grabbing player may be relevant in several of the examples mentioned in the Introduction. To be more specific, consider the illustrative example on individual lobbying efforts in which the CEO uses personal access to decision makers to lobby for something that benefits the whole industry of her firm. The CEO might lobby for a policy that gives her firm a benefit $B+V$, composed of the benefit V applying to each firm in this industry plus a possible additional firm–specific benefit B. However, the lobbying decision has opportunity cost of size C. A CEO's opportunity cost is known to her, but unobserved by other CEOs and is the outcome of an independent draw from the same random distribution for all CEOs. This problem is identical to the one described above if we define $c \equiv C - B$. The assumption that c is a random variable described by a cumulative distribution function F that is continuous and differentiable on its support $[\underline{c}, \bar{c}] \subset (0, \infty)$ then perfectly corresponds with the assumption that C is described by a cumulative distribution that is obtained from F by a parallel shift of F by the parameter B and makes the two problems equivalent. Hence, our results for $B = 0$ also apply for problems with strictly positive B.

Second, our findings encompass frameworks in which the utility of grabbing and the utility of a free-riding member of the winner group are non-linear functions of the benefit of winning and the cost of grabbing. To

[14]One can compare the preemption game with a first-price auction with externalities. The player who makes the highest (earliest, in our setup) bid wins the object and pays for it, and causes positive externalities for the members of his group. The difference to this kind of auction is that all group members have waiting costs, so payoffs in the winning group are affected by the amount (how early, in our setup) of the winning bid. Furthermore, note that heterogeneity in the opportunity cost of waiting is accountable for the equilibrium outcome. We thank a reviewer for this comparison.

[15]This is a simplifying assumption so that we need not consider types $c \in (V, \bar{c}]$ for whom it is a dominant strategy never to grab. As described in footnotes 24 and 27, almost all of our results go through when $\bar{c} > V$.

[16]The analysis can easily be modified to assume that members of non-winning groups receive a non-zero loser prize and that the benefit of being a non-grabbing member of the winning group is some positive amount.

see this, normalize the utility of a member of a losing group to zero: $u(0,0) = 0$, let $u(V,0)$ be the utility of a free-riding member of the winner group and $u(V,-c)$ the utility of the grabbing player with a grabbing cost c, such that $u(V,0) > u(V,-c) > u(0,0) = 0$. Redefining quantities as

$$\hat{c} \equiv u(V,0) - u(V,-c), \text{ and } \hat{V} \equiv u(V,0),$$

the utility of grabbing is simply $\hat{V} - \hat{c}$, that of free-riding and winning is $\hat{V}$, and that of losing remains 0, while $\hat{c}$ is a random variable that has a cumulative distribution function $\hat{F}([u(V,0) - u(V,-c)])$. Structurally the resulting setup is equivalent to the one we consider, with $(\hat{c}, \hat{V}, \hat{F})$ replacing (c, V, F).

Third, we follow the assumption that is common in the waiting-game literature: grabbing time affects the payoff V and the grabbing cost c_i of players just through exponential discounting. The assumption may reasonably well apply to the CEOs' decision problems that motivate our analysis. The cost of volunteering might decline over time, due to technical progress. The cost could also increase if unsolved problems compound over time. Our assumption of time independence rests safely in the middle between these alternatives. We discuss some possible implications of time-dependent benefits or grabbing costs when we turn to a discussion of the welfare properties of preemption between groups in Section 2.3.

2.2 Properties of an interior equilibrium

The equilibrium is characterized by players' decision to grab at a time T if none of the other players grabbed earlier, with T a function of the player's own cost of grabbing. The characterization focuses on symmetric equilibria, so we describe each player's strategy by a function $T : [\underline{c}, \bar{c}] \to [0, \infty]$, where $T(\cdot)$ maps the player's own cost c to the conditional time of own grabbing, $T(c)$.

2.2.1 The equilibrium characterization

Standard incentive compatibility arguments imply that a player's optimal strategy is weakly increasing in c. We define a (symmetric) equilibrium as *interior* if the equilibrium strategy T is strictly increasing on $[\underline{c}, \bar{c}]$.[17] Given the assumptions about F there are no ties—i.e., more than one player grabbing at a given c—when T is strictly increasing. Hence, delay that does not change the probability of grabbing first is wasteful. It follows that $T(\underline{c}) = 0$ and T is continuous. Because T is nondecreasing, T is differentiable almost everywhere. Our first proposition characterizes the interior symmetric equilibrium.

[17]It will follow from Lemma 2 in Appendix 2 that an equilibrium strategy is interior if and only if it is strictly positive for all $c > \underline{c}$.

Proposition 1. *If* $\underline{c} \geq c_0 \equiv \frac{(K-1)n}{Kn-1}V$, *then the unique interior symmetric equilibrium strategy* T *satisfies*

$$T(\underline{c}) = 0 \quad \textit{and} \quad T'(c) = \frac{f(c)}{(1-F(c))}\frac{(Kn-1)}{\rho(V-c)}\left(c - \underbrace{\frac{(K-1)n}{Kn-1}V}_{c_0}\right) \quad \forall c \in (\underline{c}, \bar{c}). \tag{1}$$

The equilibrium strategy[18] in Proposition 1 follows intuitively from balancing the marginal cost of delay with its marginal benefit as follows. It is opportune to combine marginal costs and benefits into marginal changes in payoffs if the allocation is unaltered (i.e., a player is not preempted while delaying own grabbing by an instant of time) and if the allocation is altered (i.e., a player is preempted due to the choice of an additional delay). Consider a player i with cost c who plans to grab at date $T(c)$ in equilibrium. The marginal change in payoff at time $T(c)$ brought about by delaying slightly (i.e., by behaving as a player with a slightly higher cost) if the allocation is unaltered is

$$\rho\,(V-c)\,dT, \tag{2}$$

i.e., the loss in the present (net) value of the prize. If the allocation is altered, i.e., if player i is not the first to grab anymore because of the delay, then the marginal change in payoff at time $T(c)$ is $c - \frac{(K-1)n}{Kn-1}V = c - c_0$, which is the saving in the cost of grabbing minus the expected value of being preempted by another group. Now let $h(\cdot)$ be the hazard rate function for F, i.e., $h(c) \equiv f(c)/(1-F(c))$. The term $h\,(\tilde{c})\,(Kn-1)$ is the hazard rate of the minimum cost of all other agents at $\tilde{c}$: if the cdf of the minimum cost of all other agents is $G(\tilde{c}) \equiv 1 - (1-F(\tilde{c}))^{Kn-1}$, then

$$g\,(\tilde{c}) \equiv G'(\tilde{c}) = (Kn-1)\,(1-F(\tilde{c}))^{Kn-2}\,f\,(\tilde{c})\,,$$

so

$$\frac{g\,(\tilde{c})}{1-G(\tilde{c})} = \frac{(Kn-1)\,(1-F(\tilde{c}))^{Kn-2}\,f\,(\tilde{c})}{(1-F(\tilde{c}))^{Kn-1}} = (Kn-1)\frac{f(\tilde{c})}{1-F(\tilde{c})} = (Kn-1)\,h\,(\tilde{c})\,.$$

Therefore, $h\,(c)\,(Kn-1)\;dc$ is the probability that by delaying slightly beyond $T(c)$ player i is no longer the first to grab. Combining these observations, the expected marginal change in payoff at time $T(c)$ brought about by delaying slightly when the allocation is altered turns out to be

$$h\,(c)\,(Kn-1)\;dc\;(c-c_0)\,. \tag{3}$$

[18]Note that $n \geq 2$ is necessary to get any delay in equilibrium. Moreover, in the absence of possible preemption by another group ($K = 1$), equation (1) reduces to equation (4) in Bliss and Nalebuff (1984) if we take into consideration that they assume $\rho = 1$ and derive equilibrium for the game with $n+1$ agents.

Now setting (2) equal to (3) yields

$$\frac{dT}{dc} = \frac{h(c)(Kn-1)}{\rho(V-c)}(c-c_0),$$

which is equivalent to (1).

Note that, for $c > c_0$, $dT/dc > 0$: players sort such that those with a higher grabbing cost choose to grab later. Thus, the equilibrium makes an efficient selection: only the player with the lowest cost grabs. While Proposition 1 deals with a case of "sufficiently high costs," where $\underline{c} \geq c_0$, Appendix 2 deals with the possibility of "low costs," where $\underline{c} < c_0$.[19] The arguments developed there establish that the equilibrium described in Proposition 1 is the unique symmetric equilibrium if $\underline{c} \geq c_0$, without restricting attention to strictly increasing strategies. In the remainder of this section we maintain the assumption that $\underline{c} \geq c_0$ and explore the properties of the interior equilibrium.

Because $T(\underline{c}) = 0$, we have

$$T(c) = \int_{\underline{c}}^{c} T'(y)\,dy, \text{ for all } c \in [\underline{c}, \bar{c}].$$

Further, (1) implies that $\lim_{c\uparrow\bar{c}} T(c) = +\infty$. Indeed, for any $\tilde{c} \in (\underline{c}, \bar{c})$ we have

$$\begin{aligned} T(\bar{c}) - T(\tilde{c}) &= \int_{\tilde{c}}^{\bar{c}} T'(c)\,dc = \int_{\tilde{c}}^{\bar{c}} h(c)\frac{(Kn-1)(c-c_0)}{\rho(V-c)}\,dc \geq \frac{(Kn-1)(\tilde{c}-c_0)}{\rho(V-\tilde{c})}\int_{\tilde{c}}^{\bar{c}} h(c)\,dc \\ &= \frac{(Kn-1)(\tilde{c}-c_0)}{\rho(V-\tilde{c})}\left[-\lim_{c\uparrow\bar{c}} \log(1-F(c)) + \log(1-F(\tilde{c}))\right] \\ &= +\infty. \end{aligned}$$

Intuitively, equilibrium delay becomes unbounded for the highest-cost type because the rate of payoff change in (3), which captures the marginal net benefit of delay, becomes infinite at $\bar{c}$, by $\lim_{c\uparrow\bar{c}} h(c) = +\infty$, and this is true regardless of the form of F.[20]

Recalling from the proof of Proposition 1 that $U(c^*, c)$, see (16), is the payoff to a player with cost c acting as if his cost were c^*, by the envelope theorem we can write the equilibrium utility $U^E(c) \equiv U(c,c)$ as

$$U^E(c) = V - \underline{c} - \int_{\underline{c}}^{c} e^{-\rho T(y)}(1-F(y))^{Kn-1}\,dy. \tag{4}$$

We turn to the comparative-static properties of the equilibrium in the next three subsections. For this analysis it is important whether the number of group members affects the size of the benefit V each member of the winning group enjoys. If a given benefit has to be divided between the group members, then V

[19] When costs lower than c_0 are possible, those low-cost types of players will grab immediately, that is, there is partial bunching at $T = 0$ and an interior equilibrium does not exist.

[20] To see this, let $H(c) \equiv -\log(1-F(c))$ and note that $\lim_{c\uparrow\bar{c}} H(c) = +\infty$, which, by $\bar{c} < \infty$, requires $H'(\bar{c}) = +\infty$. But $H'(c) = h(c)$; hence, $\lim_{c\uparrow\bar{c}} h(c) = +\infty$.

becomes a function of n. We concentrate here on the case in which the benefit V is non-rival inside the group: winning is a pure public good for the group, such that V is invariant in n and the aggregate benefit of the winner group increases linearly in n.

2.2.2 Individual grabbing time

First we examine how individual grabbing times change with model parameters. For comparative statics we include the dependence of T and U^E on K and n.[21]

Proposition 2. *For a player of a given type c, the grabbing time $T(c; K, n)$ is (linearly) decreasing in the number K of groups and it is (linearly) increasing in the size n of the groups. Consider now a new cost distribution $\hat{F}$ obtained from F by a parallel rightward shift by $\Delta > 0$ (i.e., $\hat{F}(y + \Delta) = F(y)$), and denote with $\hat{T}$ and T the respective equilibrium strategies obtained from* (1)*. Then $\hat{T}(c+\Delta) > T(c)$ for any $c \in (\underline{c}, \bar{c})$.*

According to Proposition 2 decision makers tend to grab earlier if K, the intensity in competition, is higher, and they grab less hastily if the group has more members. If there are more groups or smaller groups, the strict order of grabbing times is maintained but the whole function $T(\cdot)$ shifts—for given behavior of other players, it turns out that each player has a higher preemption motive and a smaller free-riding incentive. The intuition can be gained from an individual player's partial optimal response to changes in n and K, as overall equilibrium effects qualitatively go in the same direction (although they may be attenuated). Delay becomes less attractive for increased K: if a player does not grab and is preempted, for higher K it is more likely that the preemptor is from a rival group and this ends up decreasing the payoff in (3). Similarly, delay becomes more attractive for increased n. For a player who is just indifferent between grabbing or waiting at some value T, it becomes more likely that another player grabs between T and $T + dT$. Moreover, it is more likely that that this other player belongs to one's own group, because $(n-1)/[(n-1) + n(K-1)]$ is increasing in n. Thus, waiting becomes strictly superior for a higher n.

The last part of Proposition 2 allows an interpretation of the consequences of a reduction in the private benefit of grabbing B described in Section 2.1. As discussed there, a decrease in the grabbing player's private gross benefit B by Δ effects a Δ-sized rightward shift of the cost distribution. Intuitively, if grabbing becomes less beneficial for everyone, the incentive to preempt decreases and agents grab later. But this does not simply result in a parallel rightward shift of the equilibrium strategy by Δ: since the overall competitive pressure has decreased, there is also an equilibrium effect that pushes each cost type to delay further.

[21] One should note well that our the comparative statics results implicitly assume changes in parameters continue to yield interior equilibria. Recall that the condition for the interior equilibrium described by Proposition 1 is $\underline{c} \geq c_0 \equiv \frac{(K-1)n}{Kn-1}V$. Because c_0 is decreasing in n, it is clear that if one starts from an interior equilibrium then increasing n continues to yield the interior equilibrium. In contrast, c_0 increases in K, with limit V. Consequently, *cet. par.*, as K becomes sufficiently large the symmetric equilibrium will not be interior.

The dynamic structure of the preemption conflict between groups can be compared to the between-groups contest problem in Barbieri and Malueg (2016) in which all players choose their all-pay contest effort in a static game with a best-shot contest technology. In contrast to Barbieri and Malueg (2016), it is interesting to note that the qualitative comparative static properties of the grabbing time in the preemption game require no assumptions regarding the shape of F (e.g., nothing is invoked about elasticity of F).[22]

2.2.3 Expected ending time

Next we calculate the expected time at which the game stops. Denote by ET^E the stopping time at which the first grabbing occurs when there are K groups of n players each and each player follows the strategy in Proposition 1. The following lemma relates ET^E to the model parameters.

Lemma 1. *The expected time at which grabbing first occurs, ET^E satisfies*

$$K\rho ET^E + (K-1) = \int_{\underline{c}}^{\bar{c}} \frac{n-1}{n} \frac{y}{V-y}\, d(1-(1-F(y))^{Kn}). \tag{5}$$

From (5) the next proposition derives comparative statics for the expected duration of the game.

Proposition 3. *The following comparative statics results hold for the expected time at which the game ends: The expected end occurs sooner (i) if the discount rate ρ is higher or if the gross benefit V for each group member is higher or (ii) if the number K of groups is higher. (iii) If $\frac{y(1-F(y))^K}{V-y}$ is decreasing (increasing) in y, then ET^E increases (decreases) in n. (iv) A first-order stochastic dominance (FOSD) increase in the costs increases ET^E. (v) A second-order stochastic dominance (SOSD) increase in the risk of the overall minimum cost increases ET^E.*

The unambiguous comparative static results (i) and (ii) in Proposition 3 are intuitively plausible: increases in ρ and V make grabbing more attractive and so do increases in K, as shown in Proposition 2. The possibility described in part (iii) that n may increase or decrease ET^E is also intuitive. An increase in n has two countervailing effects: each player with a given type c grabs later, but as the number of players increases, the probability distribution of the lowest cost type shifts. Indeed, the probability that the lowest realized cost, among those of Kn players, is higher than a given c becomes less and less as n increases, for all possible c inside the support. We identify a sufficient condition for either effect to dominate. The result in part (iv) is also intuitive. For instance, consider the rightward shift in the cost distribution analyzed in Proposition 2, which, recalling our discussion in Section 2.1, can be interpreted as a decrease in the private benefit of grabbing B. This rightward shift in costs increases T as described in Proposition 2, and therefore

[22] In general, elasticity of F, which played a crucial role in determining the direction of many comparative statics results in Barbieri and Malueg (2016), plays no role in our setup. We provide an additional instance of this fact in footnote 25.

it increases ET^E as well. Part (iv) of Proposition 3 extends these consideration to FOSD increases in costs. The results in parts (iii) and (v) are deserving of more discussion, which we provide by way of two examples in Section 2.4.[23]

The distribution of the cost of volunteering also matters for delay. Delay is, on average, larger if the *minimum* cost of volunteering is more dispersed. But, as we show in Section 2.4, delay can be shorter if the *individual* cost of volunteering is more dispersed.

2.2.4 Expected payoffs

Finally, we describe the effects parameter changes have on expected utility.

Proposition 4. *(i) The expected payoff of a player with a given cost type c is higher if the size of all groups is larger. (ii) The expected payoff is constant with respect to changes in the number of groups.*

Intuitively, the main, direct effect of an increase in n is to dilute the cost burden of grabbing among a larger number of players, while keeping each player's expected gross benefit constant at V/K in a symmetric equilibrium. As determined in Proposition 3, the expected time at which the game ends may increase, but this equilibrium effect is overwhelmed by the direct effect. The proof of the proposition shows that this holds strictly, not only *ex ante*, but also for all players, irrespective of their cost type, except for the type with the lowest possible cost realization who, in either case, grabs immediately. If there is only one group (as in Bliss and Nalebuff, 1984), this is easy to grasp: each player faces a higher probability that another player from his group grabs first, which preserves the benefit, but probabilistically shifts the cost burden of grabbing. Proposition 4 shows that this effect carries over to a multi-group framework with preemption.[24]

The payoff neutrality with respect to the number of groups is less intuitive. A larger number of groups makes it more likely that a single group is preempted. This reduces all players' expected payoffs. However, the increase in preemption pressure induces players of given types to grab earlier. This reduces wasteful delay. The proposition shows that the two effects just offset each other.

2.3 Welfare

A natural measure of welfare in the preemption game is the sum of payoffs of all players in all groups. A maximum $nV - c_{i_{\min}}$ of this sum is reached in the context of ex-ante symmetric groups of given size n if the

[23] These results extend the homologous ones in Theorems 4 and 5 of Bliss and Nalebuff (1984); in particular, their Theorem 5 characterizes only the behavior at the tails.

[24] It is now worth noting how our analysis would change if $\bar{c} > V$. For types $c \in (V, \bar{c}]$ it is a dominant strategy never to grab, i.e., to choose $T = +\infty$. For $c < V$, the differential equation in Proposition 1 continues to characterize the equilibrium strategy. Moreover, $T(c) \to \infty$ as $c \uparrow V$. Similarly, Proposition 2 continues to hold for $c < V$. However, Proposition 3 fails to hold because there is now a positive probability that no one ever grabs, implying ET^E is infinite. Finally, Proposition 4 continues to hold because $U^E(c) = U^E(V)$ for all $c \in (V, \bar{c}]$, and the earlier derivations for $c \in (\underline{c}, V]$ apply here as well.

player $i_{\min}$ who has the lowest cost $c_{i_{\min}}$ of all players takes action at $T = 0$. In comparison to this welfare benchmark, the ex-post welfare in equilibrium is $[nV - c_{i_{\min}}]\, e^{-rT(c_{i_{\min}})}$. In the symmetric case with groups of equal size it does not matter for welfare which group receives the public good. The player who has the smallest cost among all players in all groups should grab and does grabs in the equilibrium. The delay of taking action by $T(c_{i_{\min}})$ in the equilibrium constitutes the only efficiency loss. It discounts the sum of total net benefits.

Proposition 4 discusses payoffs if the number of agents in the model is allowed to change. An alternative comparison is what happens after reconstituting a given number of players into a smaller number of larger symmetric groups. By Proposition 2 we know that decreasing the number of groups (without changing team size) would increase individual grabbing times, and then increasing the number of players per group would further increase grabbing times. As individual grabbing times increase, the reorganization also implies the expected duration of the game increases. Finally, Proposition 4 implies that reorganizing the players into fewer larger groups increases expected payoffs for all types but $\underline{c}$. As discussed, these conclusions depend strongly on the question whether the winner prize is a pure public good for all members of the group, or whether a change in group size affects the benefit V which each of the members receives.

It is important to note that our setup has a time-invariant cost of grabbing. As discussed at the end of Section 2.1, there are reasons why the cost of grabbing may increase or decrease over time. We took the middle position, where these costs are constant, except for time discounting, and this is why an immediate action is always efficient here. However, the analysis is suggestive for what would happen if early action is inefficient due to a high social cost of early action. Free-riding incentives that cause a delay might then reduce the efficiency costs from hasty preemption behavior. At the same time, the fear that other groups might preempt a player's group can reduce the efficiency costs of free-riding. If there is a cost of excessive delay as well as a cost of premature action, the specific parametric situation would determine whether action is taken too early or too late from an efficiency perspective.

2.4 Numerical examples

We illustrate some of the comparative-statics results by way of numerical examples. The first illustrates a non-monotonicity result suggested by part (iii) of Proposition 3.

Example 1 (The effect of n on expected stopping time)**.** *Let $V = 2$ and assume costs are distributed according to $F(c) = 2^t \left(c - \frac{3}{2}\right)^t$, for $c \in \left[\frac{3}{2}, 2\right]$.*

First note that

$$sign\left[\frac{d}{dy}\left(\frac{y(1-F(y))^K}{V-y}\right)\right] = sign\left[V - Ky(V-y)h(y)\right]. \tag{6}$$

We begin with $K = 2$, in which case $c_0 = \frac{n}{2n-1}$. Thus, the symmetric equilibrium is interior for all $n \geq 2$. Now consider $t = 1$. Here, we see that

$$V - Ky(V - y)h(y) = 2(1 - y),$$

which is negative for the relevant range. Therefore, (6) and part (iii) of Proposition 3 imply ET^E is increasing in n; Figure 1 depicts the relationship between ρET^E and n for $n = 2, ..., 30$.

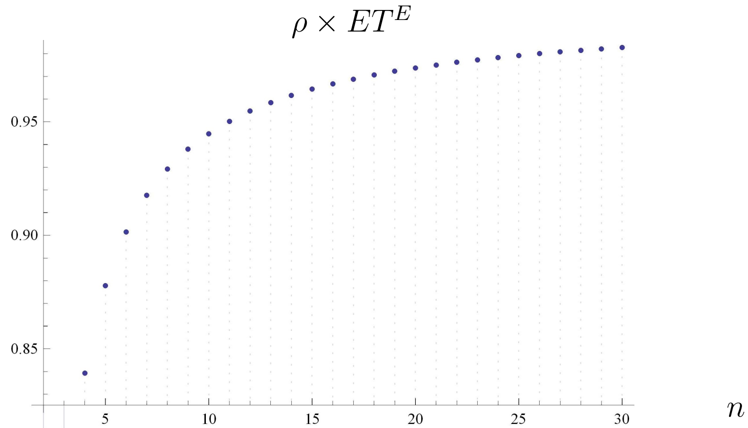

Figure 1: Effects of increasing n on ET^E: $V = K = 2$ and $F(c) = 2\left(c - \frac{3}{2}\right) \quad \forall c \in \left[\frac{3}{2}, 2\right]$.

A pattern similar to that in Figure 1 holds for any $t \leq 1$. Thus, for $t \leq 1$ we see that the free-riding effect is very strong and it overwhelms the presence of additional agents on each team, which would otherwise lead to a lower expected stopping time. In contrast, if $t = 2$, for example, we find that ET^E first increases and then decreases with n, as depicted in Figure 2. Here we see that ET^E increases for n going from 2 to 4, but further increases in team size decrease ET^E because the "order-statistic" effect of having a better distribution of the minimum cost eventually prevails. Proposition 3 implies that $\frac{y(1-F(y))^K}{V-y}$ must be increasing for at least a range of y; indeed, it is inverse U-shaped in y, which helps explain why ET^E is first increasing with n, then it turns decreasing and it stays so. As n grows, the distribution of the minimum costs is more and more concentrated towards lower values of c. Therefore, for n sufficiently large, the relevant part of $\frac{y(1-F(y))^K}{V-y}$ is increasing and the result follows as for Proposition 3. A pattern similar to that in Figure 2 is displayed by all parameterizations with $t > 1$.[25] It is interesting to note that, with more competing teams, the switch in the direction of the relationship between ET^E and n occurs later. Indeed, by (6), if K increases, then $\frac{y(1-F(y))^K}{V-y}$ becomes decreasing for a larger set of y. For example, if $t = 2$ and $K = 3$, then ET^E remains

[25]In contrast with Barbieri and Malueg (2016), elasticity of F plays no role in determining the direction of our comparative statics. Indeed, $F(c)$ is elastic for any $t > 1$, but Figure 2 shows an ambiguous effect of n on ET^E.

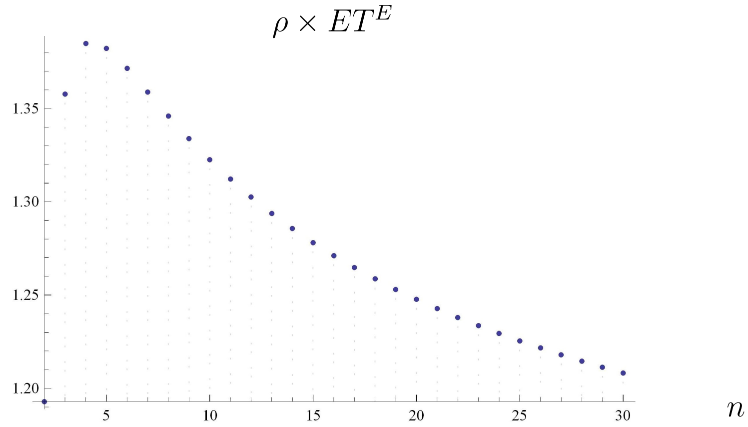

Figure 2: Effects of increasing n on ET^E: $V = K = 2$ and $F(c) = 4\left(c - \frac{3}{2}\right)^2 \quad \forall c \in \left[\frac{3}{2}, 2\right]$.

increasing in n up to $n = 7$.[26] □

We next illustrate the distributional effect described in part (v) of Proposition 3.

Example 2 (The effect of SOSD changes in the cost distribution)**.** *Fix* $K = 2$*,* $V = 3$*,* $\rho = 1$*, and* $n = 3$*, and let costs be uniformly distributed on* $[\underline{c}, \bar{c}]$.

Consider first $\underline{c} = 2$ and $\bar{c} = 3$. Then, the average overall minimum cost is approximatively 2.143 and (5) yields $ET^E \approx 0.367$. Consider now $\underline{c} = 2.1$ and $\bar{c} = 2.4$. Then, the average minimum cost remains 2.143, approximatively, and one can show that we have effected a SOSD reduction in risk of the cost distribution of the overall minimum cost. Using (5), we now have that ET^E decreases to approximatively 0.336, and the direction of the change is in accordance with part (v) of Proposition 3.

If one instead performs a mean-preserving spread of the individual cost distribution, then the result can be different. Consider for instance a uniform cost distribution with $\underline{c} = 2.1$ and $\bar{c} = 2.9$, which keeps the individual expected cost at 2.5, but is less risky than our initial uniform distribution on $[2, 3]$. Note that the average minimum cost *increases* to 2.214 for individual costs that are uniformly distributed on $[2.1, 2.9]$. And the direction of the comparative statics reverses, as ET^E increases to 0.465, approximatively. □

3 Two asymmetric groups

The analysis of preemption between symmetric groups revealed the general trade-off between waiting and grabbing, and how it depends on the size of groups and their number. In many areas in which preemption

[26] One needs to be a little careful here because for $K = 3$ and $n = 2$, $c_0 = 8/5 > 3/2$, which implies we need to analyze the corner solution. But as explained in Appendix 2, the comparative statics work out here, too. It is the case that for $K = 3$ and $n \geq 3$, we have $c_0 \leq 3/2$.

between groups emerges, groups are asymmetric; for example, they may differ in size. In a grabbing competition between groups of different size the questions arise whether a larger group is more likely to preempt, and whether its members earn larger payoffs than the members of competing groups. Formally, we consider two groups of different size and label the first group "small" and indicate the number of players in the small group with n_s. Similarly, the second group is "large" and has n_l members, with $n_l \geq n_s$. In what follows, we restrict attention to interior equilibria in which all agents within a group adopt the same strategy, but these strategies are allowed to vary between groups. We label these strategies as T_s and T_l, respectively. Our focus is to characterize equilibrium, to identify systematic differences in behavior between members of the small and large groups, and to establish which group is most likely to win.

Beyond size issues, we are also interested in payoff differences between groups and among agents. As discussed in Section 2.1, we allow the grabbing agent to have a larger gross utility than teammates who benefit from the externality. Unlike our treatment in Section 2.1, we now carry two explicit separate values for each group, rather than reinterpreting $c = C - B$. Therefore, in the small group the grabbing agent now receives V_s^g, while the other members obtain V_s^e. Similarly, in the large group we now have V_l^g and V_l^e. Our objective is to determine how the strength of the externalities affects the probability of victory and utility.

It turns out that, at a minor notation cost but with great benefit for exposition, it is possible to consider also two different cdfs of costs F_s and F_l, with densities f_s and f_l, respectively. (Similarly, one could consider two different discount factors.) Nonetheless, after characterizing equilibrium, in our comparative statics we will consider changes one at a time: first we will assume that the only difference between groups is size, and then we will consider changes in the strength of the externalities within groups.

Consider the calculus of a type-c player in the small group contemplating grabbing at $T_s(c^*)$. His expected utility is

$$\begin{aligned} U_s(c^*, c) = &\int_{c^*}^{\bar{c}} (V_s^g - c) e^{-\rho T_s(c^*)} (1 - F_l(T_l^{-1}(T_s(c^*))))^{n_l} \, d[1 - (1 - F_s(x))^{n_s - 1}] \\ &+ \int_{\underline{c}}^{c^*} V_s^e e^{-\rho T_s(x)} (1 - F_l(T_l^{-1}(T_s(x))))^{n_l} \, d[1 - (1 - F_s(x))^{n_s - 1}], \end{aligned} \tag{7}$$

which is entirely analogous to (16): the first addendum of the payoff displayed in (7) captures the possibility that this player carries his group to victory, while the second corresponds to a teammate carrying the group to victory.

We now use a construction due to Amann and Leininger (1996). Define $k(c) \equiv T_l^{-1}(T_s(c))$, so that

$k'(c) = \frac{T_s'(c)}{T_l'(k(c))}$. Then, the first-order condition $\frac{\partial U_s(c^*,c)}{\partial c^*} = 0$ at $c^* = c$ gives

$$\frac{(V_s^e - (V_s^g - c))\,(1 - F_l(k(c))(n_s - 1)f_s(c) - (V_s^g - c)(1 - F_s(c))n_l f_l(k(c))k'(c)}{(V_s^g - c)(1 - F_s(c))(1 - F_l(k(c)))\rho} = T_s'(c); \tag{8}$$

and, proceeding similarly for the large group we have that

$$\frac{(V_l^e - (V_l^g - k\,(c)))\,(1 - F_s(c))(n_l - 1)f_l(k(c))k'(c) - (V_l^g - k\,(c))(1 - F_l(k(c)))n_s f_s(c)}{(V_l^g - k\,(c))(1 - F_l(k(c)))(1 - F_s(c))\rho} = T_s'(c). \tag{9}$$

Equalizing (8) and (9) we obtain

$$\left(\frac{V_s^e(n_s - 1)}{V_s^g - c} + 1\right)\frac{f_s(c)}{1 - F_s(c)}\left[\left(\frac{V_l^e(n_l - 1)}{V_l^g - k(c)} + 1\right)\frac{f_l(k(c))}{1 - F_l(k(c))}\right]^{-1} = k'(c), \tag{10}$$

an ordinary differential equation for $k(c)$. After solving (10) with the initial condition $k(\underline{c}) = \underline{c}$, we can substitute $k(c)$ into (8) and obtain the equilibrium T_s. Since we are considering an interior equilibrium, we need to ensure that T_s is strictly increasing. Combining (9) and (10), we obtain

$$T_s'(c) = \frac{f_s(c)}{1 - F_s(c)}\frac{n_s}{\rho}\left[\frac{(V_l^e - (V_l^g - k\,(c)))\,(n_l - 1)}{V_l^e(n_l - 1) + (V_l^g - k\,(c))} \times \frac{V_s^e(n_s - 1) + (V_s^g - c)}{n_s\,(V_s^g - c)} - 1\right]; \tag{11}$$

therefore, $T_s'(c) \geq 0$ is ensured if

$$\frac{(V_l^e - (V_l^g - k\,(c)))\,(n_l - 1)}{V_l^e(n_l - 1) + (V_l^g - k\,(c))} \times \frac{V_s^e(n_s - 1) + (V_s^g - c)}{n_s\,(V_s^g - c)} \geq 1, \quad \forall c. \tag{12}$$

Since the left-hand side of (12) is strictly increasing in c by $n_s > 1$, a necessary and sufficient condition for T_s to be strictly increasing is that (12) holds at $\underline{c}$. By $k(\underline{c}) = \underline{c}$, (12) boils down to

$$\frac{(V_l^e - (V_l^g - \underline{c}))\,(n_l - 1)}{V_l^e(n_l - 1) + (V_l^g - \underline{c})} \times \frac{V_s^e(n_s - 1) + (V_s^g - \underline{c})}{n_s\,(V_s^g - \underline{c})} \geq 1. \tag{13}$$

Since $T_l(c) = T_s(k^{-1}(c))$, (13) also ensures that that $T_l(c)$ is strictly increasing. Notice that if $F_s = F_l = F$, $n_l = n_s = n$, and $V_l^e = V_s^e = V_l^g = V_s^g = V$, then $k(c) = c$ solves (10) with initial condition $k(\underline{c}) = \underline{c}$. As expected, this means $T_s = T_l = T$. Further, substituting $k(c) = c$ into (9), we obtain (1), and (13) simplifies to

$$\frac{\underline{c}(n-1)}{1}\frac{1}{n\,(V - \underline{c})} \geq 1 \Longleftrightarrow \underline{c}(2n - 1) \geq nV,$$

so we essentially recover Proposition 1 when the number of symmetric groups K equals two.

While deriving equilibrium strategies proves analytically infeasible when groups are asymmetric, some of the comparative statics of interest can be deduced using (10) directly. The following result shows that, if values and distributions are identical, agents in the small group act more aggressively.

Proposition 5. *Consider an interior equilibrium described by* (8), (9), *and* (10) *with initial condition* $k(\underline{c}) = \underline{c}$. *If* $V_l^e = V_s^e = V_l^g = V_s^g = V$, $F_s = F_l = F$, *and* $n_l > n_s$, *then* $T_s(c) < T_l(c)$ *for any* $c \in (\underline{c}, \bar{c})$.

Intuitively, at $\underline{c}$ it must hold that $T_l(\underline{c}) = T_s(\underline{c}) = 0$. In both groups, the players with the lowest possible cost grab immediately as any such player cannot hope to be preempted by a teammate. Moreover, $T_l'(\underline{c}) > T_s'(\underline{c})$ holds. To see this, consider the incentives of a player with $c = \underline{c} + \varepsilon$ for small positive ε if he waits for some marginal unit of time dt. If the large group is, for instance, twice as large as the small group, then compared to a member of the large group, a member of the small group would face twice as high a probability of being preempted by a member of the other group. Moreover, the chances that a teammate grabs in the interval $[0, dt]$ is smaller for members of the small group; thus, waiting is less attractive for members of the small group. Similar arguments rule out any other crossings between T_s and T_l before $\bar{c}$.

Also, it is not the case that there exists a positive measure of cost types in the large group that never grab. In other words, any solution of (10) with initial condition $k(\underline{c}) = \underline{c}$ ends up satisfying $k(\bar{c}) = \bar{c}$. The apparent contradiction is resolved by noticing that $\lim_{c\uparrow\bar{c}} T_s(c) = +\infty$, just as it happens in the symmetric model.[27] We collect these observations in the following result.

Proposition 6. *Consider the interior equilibrium described by* (8), (9), *and* (10) *with initial condition* $k(\underline{c}) = \underline{c}$. *Any solution* $k(c)$ *satisfies* $k(\bar{c}) = \bar{c}$. *And the corresponding* $T_s(c)$ *derived from* (11) *with initial condition* $T_s(\underline{c}) = 0$ *has* $\lim_{c\uparrow\bar{c}} T_s(c) = +\infty$.

By Proposition 5, we see that a true trade-off emerges when we try to establish whether the small or the large group is more likely to win. Agents of a given cost type are less hesitant in the small group, but the distribution of the minimum cost in the large group is "better," assuming the only difference between groups is size. We label the probability of victory of the small group as P_s^w. By the definition of k, we have

$$P_s^w = \int_{\underline{c}}^{\bar{c}} (1 - F_l(k(c)))^{n_l} \, d[1 - (1 - F_s(c))^{n_s}]. \tag{14}$$

Table 1 describes P_s^w as we increase the size of the large group while holding the small group to two members. The calculations are performed for variations of the power distribution cdf of costs in Example 1.

We therefore see that either the large or the small group can be more likely to win.[28] It is worth noting

[27]Here too our results go through if $\bar{c} > V$. For types $c \in (V, \bar{c}]$ it is a dominant strategy never to grab, i.e., to choose $T = +\infty$. For $c < V$, (8), (9), and (10) with initial condition $k(\underline{c}) = \underline{c}$ continue to characterize equilibrium. Proposition 5 is unaffected for $c < V$. And now we have $k(V) = V$ and $T_s(c) \to \infty$ as $c \uparrow V$.

[28]In Appendix 1, Proposition 8 gives a technical condition sufficient for the group-size effect to be negative.

Table 1: *Ex ante* winning probability of the small group, $V = 2$, $n_s = 2, F_l(c) = F_s(c) = F(c)$ on $[3/2, 2]$.

$n_l \downarrow$	$F(c) = 1 - (4-2c)^5$	$F(c) = 1 - (4-2c)^2$	$F(c) = 2c - 3$	$F(c) = (2c-3)^2$	$F(c) = (2c-3)^5$
2	0.5	0.5	0.5	0.5	0.5
3	0.541360	0.535435	0.526259	0.518673	0.513964
4	0.559099	0.550223	0.536429	0.523292	0.514499
10	0.587210	0.573159	0.551014	0.522003	0.499090
20	0.595656	0.579926	0.554979	0.516549	0.483045
30	0.598380	0.582096	0.556211	0.513305	0.473895
50	0.600526	0.583802	0.557166	0.509585	0.463137
100	0.602114	0.585062	0.557862	0.505393	0.450186
1000	0.603447	0.586120	0.558421	0.497527	0.419639

that the large group can be less likely to win than the small group even if the prize is a pure public good. Thus, the size trade-off does not have a simple solution.

We now turn to the effects of asymmetries in payoffs, performing the analysis for changes in the values for the small group. However, our result applies equally well to the large group, *mutatis mutandis*. In particular, we analyze the effect of a stronger within-group externality. Two countervailing effects arise. On the one hand, agents' benefit more when a teammate grabs. On the other hand, one might expect free-riding to worsen and reduce the probability of grabbing. Our next results resolves the trade-off between these two effects.

Proposition 7. *Consider an interior equilibrium described by* (8), (9), *and* (10) *with initial condition* $k(\underline{c}) = \underline{c}$. *An increase in* V_s^e *reduces the probability of victory of the small group, but it increases the payoff of any cost type in the small group, except for* $c = \underline{c}$.

Intuitively, consider an increase of V_s^e to $V_s^e + \Delta$. Let player i in the small group be indifferent whether to grab at time T_i for given V_s^e. Suppose for a moment that the increase in V_s^e to $V_s^e + \Delta$ does not change the grabbing behavior of all other players. If the benefit of grabbing remains V_s^g but the benefit of waiting and winning increases from V_s^e to $V_s^e + \Delta$, the indifference for i no longer holds and player i prefers to wait a little longer. This partial effect is indicative for the equilibrium effect. Furthermore, an increase in V_s^e also means that the public good of winning is more valuable for each member of the small group if the member free-rides. The proposition explains that this increase in the group's payoffs overcompensates the the equilibrium effect of decreased win probability.

We conclude this section discussing the possibility that agents may be differentially informed about teammates and competitors. In many of the examples described in the Introduction, it makes sense to posit that one own's value (or the simple fact of being part of the same team) may reveal more information about teammates than competitors. It turns out that the techniques developed in this section can be used to characterize equilibrium when agents are differentially informed about teammates and competitors.

An especially tractable framework has two *ex ante* symmetric groups. Within each group, and independentely across groups, agents' values are independently drawn either from F_H or from F_L. Within each group, team members are aware of their individual values and of whether teammates' values are drawn from F_H or F_L, but they do not know which distribution is the "true" one for the other group. Therefore, groups are (potentially) asymmetric at the *interim* moment when the distribution of values becomes common knowledge among teammates. Thus, the tools developed here for asymmetric groups become relevant.

In a symmetric equilibrium, there will be one grabbing function T_H, which is optimal after observing F_H, and one grabbing function T_L, which is optimal after observing F_L. Characterization of T_H and T_L proceeds just as in earlier in this section,[29] with FOCs analogous to (8) and (9), and a differential equation for $T_L^{-1}(T_H(c))$ similar to (10).

While the resulting equations do not admit a closed-form solution, one can show that an interesting pattern emerges. Suppose that $F_H(c)$ hazard-rate "dominates" $F_L(c)$, so that costs under $F_H(c)$ tend to be higher than under $F_L(c)$. Then one can show that agents are more aggressive in grabbing if they know that within their group costs are derived from the "higher" distribution, rather than if they knew costs were drawn from the "lower" distribution. Intuitively, this occurs because knowing that costs within a group tend to be large decreases the incentive to free-ride on teammates, and therefore grabbing occurs sooner.

4 Conclusion

We have given a full characterization of symmetric equilibrium preemption between groups, when each group member can volunteer to preempt other groups on behalf of the own group. The analysis is based on the assumption that the act of preemption is costly for the member who carries it out and members have private information about their own cost of the preemption activity.

Considering symmetric groups, the equilibrium exists and is unique. Other than for a possible non-degenerate set of lowest costs types who all may act immediately, we found that players wait some time before taking action. They wait longer the higher are their own costs of acting. Hence, the preemption task is carried out by the player with the lowest cost. A player of a given cost type also waits longer the larger the group and the smaller the number of rival groups. Overall, however, an increase in group size need not cause an increase in the expected equilibrium delay: while each player type plans to wait longer in this case, the random composition of the minimum cost type improves if the group size increases, and this makes it

[29] The details of the derivation are available from the authors upon request.

more likely that a group has a member with a very low cost who would take early preemption action.

We have also analyzed differences in group size, concentrating on the case of two groups. We find that members with a given cost type choose to act earlier if they belong to the smaller group. The larger group has a "better" order statistic for the lowest-cost member, but has stronger free-riding incentives. Consequently, depending on parameter values, either group may win more often in expectation. These results are important for the role of group size as a competitive factor, showing no clear overall advantage for the small or the large group.

Appendix 1: Proofs

Proof of Proposition 1. If the strategy T is strictly increasing, then a player with value $\underline{c}$ knows he will grab first, preempting all others. Therefore, he will choose $T(\underline{c}) = 0$ because if $T(\underline{c})$ were larger than zero, then grabbing the prize an instant sooner would be profitable deviation as it would reduce wasteful delay. Similarly, given that T is strictly increasing it must be continuous, for otherwise some types just above the point of discontinuity could increase their payoffs by reducing their grabbing time to avoid wasteful delay.

Now suppose $T(\cdot)$ is continuous and strictly increasing. Consider the calculus of a single player i contemplating grabbing at $T(c^*)$. We first determine the probability that someone *else* will take action before date t. This probability depends on the minimum realized cost among the other $Kn-1$ players. The cumulative distribution function of the minimum cost among them, denoted by $c_{\min}$, is given by

$$\Pr(c_{\min} \leq x) = 1 - \Pr(\text{all other } Kn-1 \text{ costs exceed } x) = 1 - (1 - F(x))^{Kn-1},$$

for which the associated density function is

$$(Kn-1)(1 - F(x))^{Kn-2} f(x). \tag{15}$$

Moreover, if the game stops before $T(c^*)$, then the probability that i's team has won is $\frac{n-1}{Kn-1}$ because players are acting symmetrically. Following Bliss and Nalebuff (1984), we can now write the payoff to a player with cost c (a "type-c player") acting as if his cost were c^* as

$$\begin{aligned} U(c^*, c) &= (V - c)e^{-\rho T(c^*)}(1 - F(c^*))^{Kn-1} + \frac{n-1}{Kn-1} V \int_{\underline{c}}^{c^*} e^{-\rho T(x)}(Kn-1)(1 - F(x))^{Kn-2} f(x)\, dx \\ &= (V - c)e^{-\rho T(c^*)}(1 - F(c^*))^{Kn-1} + (n-1)V \int_{\underline{c}}^{c^*} e^{-\rho T(x)}(1 - F(x))^{Kn-2} f(x)\, dx. \end{aligned} \tag{16}$$

The first addendum of the payoff displayed in (16) captures the possibility that this player carries his group to victory, while the second corresponds to a teammate carrying the group to victory. The type-c player's first-order condition for choosing c^* is

$$\begin{aligned}
0 = \frac{\partial U(c^*, c)}{\partial c^*} &= -\rho T'^{*})(V-c)e^{-\rho T(c^*)}(1-F(c^*))^{Kn-1} \\
&\quad - (Kn-1)(V-c)e^{-\rho T(c^*)}(1-F(c^*))^{Kn-2} f(c^*) \\
&\qquad + (n-1)Ve^{-\rho T(c^*)}(1-F(c^*))^{Kn-2} f(c^*) \\
&= e^{-\rho T(c^*)}(1-F(c^*))^{Kn-2} \\
&\quad \times \Big\{ V(n-1)f(c^*) - (Kn-1)(V-c)f(c^*) - \rho(V-c)(1-F(c^*))T'^{*}) \Big\} \\
&= e^{-\rho T(c^*)}(1-F(c^*))^{Kn-2} \\
&\quad \times \Big\{ \big[(Kn-1)c - (K-1)nV\big] f(c^*) - \rho(V-c)(1-F(c^*))T'^{*}) \Big\}.
\end{aligned} \tag{17}$$

Where $T(c) > 0$, it must be that the first-order condition holds at $c^* = c$, so (17) implies

$$T'(c) = \frac{f(c)}{1-F(c)} \times \frac{(Kn-1)c - (K-1)nV}{\rho(V-c)}. \tag{18}$$

Note that the monotonicity requirement for $T(c)$ requires that $\frac{(Kn-1)c-(K-1)nV}{\rho(V-c)}$ is positive. This defines a lower bound $c_0(K, n)$ for values of c. This lower bound is a function of K and n as stated in Proposition 1. Note further that $T(c)$ as described in (1) identifies the global best response: equations (17) and (18) imply

$$\begin{aligned}
\frac{\partial U(c^*, c)}{\partial c^*} &= e^{-\rho T(c^*)}(1-F(c^*))^{Kn-2} f(c^*) \\
&\quad \times \underbrace{\left\{ (Kn-1)c - (K-1)nV - \left(\frac{V-c}{V-c^*}\right)[(Kn-1)c^* - (K-1)nV] \right\}}_{\equiv \varphi(c^*;c)}.
\end{aligned}$$

Now observe that $\varphi(c; c) = 0$ and

$$\frac{\partial \varphi(c^*; c)}{\partial c^*} = -(V-c)\frac{(n-1)V}{(V-c^*)^2} < 0,$$

so $U(c^*, c)$ is strictly quasi-concave in c^* with a maximum at $c^* = c$. Thus, $\varphi(c; c) = 0$ yields the best response in (1). □

Proof of Proposition 2. Consider first an increase in K. We have, using (1),

$$\frac{\partial T'(c;K,n)}{\partial K} = \frac{h(c)n}{\rho(V-c)}(c-V) = -\frac{h(c)n}{\rho} < 0. \tag{19}$$

We now have

$$\begin{aligned} T(c;K,n) - T(c;K+1,n) &= \int_{K+1}^{K} \frac{\partial T(c;k,n)}{\partial k}\, dk \\ &= \int_{K+1}^{K} \left(\int_{\underline{c}}^{c} \frac{\partial T'(y;k,n)}{\partial k}\, dy \right) dk \\ &= \int_{K+1}^{K} \left(\int_{\underline{c}}^{c} -\frac{h(y)n}{\rho}\, dy \right) dk \\ &= \frac{n}{\rho} \int_{K+1}^{K} \log\left(1 - F(c)\right) dk \\ &= -\frac{n}{\rho} \log\left(1-F(c)\right) \\ &\equiv \Delta_n(c), \end{aligned}$$

where it is clear that $\Delta_n(c) > 0$ for all $c \in (\underline{c}, \bar{c})$ and $\Delta_n(c)$ is independent of K. By induction, $T(c;K+1,n) = T(c;2,n) - (K-1)\Delta_n(c)$.

Consider next an increase in n. We have

$$\begin{aligned} T(c;K,n+1) - T(c;K,n) &= \int_{\underline{c}}^{c} \frac{h(z)\left[(K(n+1)-1)z - (K-1)(n+1)V - ((Kn-1)z - (K-1)nV)\right]}{\rho(V-z)}\, dz \\ &= \int_{\underline{c}}^{c} \frac{h(z)}{\rho(V-z)} \left[Kz - (K-1)V\right]\, dz \equiv \Delta_K(c). \end{aligned}$$

Then, for $z > \underline{c}$,

$$z > \underline{c} \geq c_0 = \frac{(K-1)n}{Kn-1}V \geq \frac{K-1}{K}V,$$

implying $\Delta_K(c) > 0$ for all $c > \underline{c}$. Consequently, $T(c;n+1) > T(c;n)$ for all $c \in (\underline{c}, \bar{c}]$. That is, as the number of players per team increases, free-riding is more pervasive, with all types above $\underline{c}$ grabbing at a later dates. Further, since $\Delta_K(c)$ is independent of n, we have $T(c;K,n+1) = T(c;K,2) + (n-1)\Delta_K(c)$.

Consider now a cost distribution $\hat{F}(y+\Delta) = F(y)$ for $y \in (\underline{c}, \bar{c})$ that describes a parallel right-shift of the distribution of cost types by some Δ. An interior equilibrium now has $\hat{T}(\underline{c}+\Delta) = 0$, and it has

$$\hat{T}'(y+\Delta) = \frac{\hat{f}(y+\Delta)}{1-\hat{F}(y+\Delta)} \times \frac{(Kn-1)(y+\Delta) - (K-1)nV}{\rho(V-(y+\Delta))}$$

for $y \in (\underline{c}, \bar{c})$, which can be written equivalently as

$$
\begin{aligned}
\hat{T}'(y+\Delta) &= \frac{f(y)}{1-F(y)} \times \frac{(Kn-1)(y+\Delta)-(K-1)nV}{\rho(V-(y+\Delta))} \\
&> \frac{f(y)}{1-F(y)} \times \frac{(Kn-1)y-(K-1)nV}{\rho(V-y)} \\
&= T'(y).
\end{aligned}
$$

The inequality uses

$$
\frac{\partial \frac{(Kn-1)c-(K-1)nV}{\rho(V-c)}}{\partial c} = V\frac{n-1}{\rho\,(V-c)^2} > 0.
$$

This implies: for all corresponding types $y \in (\underline{c}, \bar{c})$ for F and $y+\Delta \in (\underline{c}+\Delta, \bar{c}+\Delta)$ for $\hat{F}$ the respective types for $\hat{F}$ grab later than for F. □

Proof of Lemma 1. The expected time at which the game stops equals

$$
\begin{aligned}
ET^E &\equiv \int_{\underline{c}}^{\bar{c}} T(c)\, d(1-(1-F(c))^{Kn}) \\
&= \int_{\underline{c}}^{\bar{c}} \left(\int_{\underline{c}}^{c} T'(y)\, dy\right) d(1-(1-F(c))^{Kn}) \\
&= \int_{\underline{c}}^{\bar{c}} T'(y) \left(\int_{y}^{\bar{c}} d(1-(1-F(c))^{Kn})\right) dy \\
&= \int_{\underline{c}}^{\bar{c}} T'(y)(1-F(y))^{Kn} dy \\
&= \int_{\underline{c}}^{\bar{c}} \frac{f(y)}{1-F(y)} \times \frac{(Kn-1)y-(K-1)nV}{\rho(V-y)}(1-F(y))^{Kn}\, dy \\
&= \int_{\underline{c}}^{\bar{c}} \frac{1}{K\rho}\left(\frac{n-1}{n}\frac{y}{V-y} - (K-1)\right) d(1-(1-F(y))^{Kn}), \qquad (20)
\end{aligned}
$$

where the third equality follows from interchanging the order of integration, and the fifth uses the strategy in (1). Equation (20) can be rewritten as

$$
K\rho ET^E + (K-1) = \int_{\underline{c}}^{\bar{c}} \frac{n-1}{n}\frac{y}{V-y}\, d(1-(1-F(y))^{Kn}).
$$

□

Proof of Proposition 3. (i) From (5) it is immediate that an increase in the discount rate ρ or in the value of the gross benefit V decrease ET^E.

(ii) The result on an increase in the number of groups follows immediately from the comparative statics of $T(\cdot)$ in Proposition 2 by which increasing K leads each type to grab sooner, which, even without there being more players would result in the expected stopping time decreasing.

(iii) Consider an increase in n. We can rewrite the right-hand side of (5) as

$$\int_{\underline{c}}^{\bar{c}} \frac{y(1-F(y))^K}{V-y}\, d\left(1-(1-F(y))^{K(n-1)}\right).$$

We see that the probability distribution in this equation is that of the minimum cost out of $K(n-1)$ independent realizations. As n increases, this probability distribution decreases in the sense of first-order stochastic dominance; therefore, if $\frac{y(1-F(y))^K}{V-y}$ is decreasing (increasing) in y, then ET^E increases (decreases) in n.

(iv) and (v) Consider equation (5), here reproduced:

$$K\rho ET^E + (K-1) = \int_{\underline{c}}^{\bar{c}} \frac{n-1}{n} \frac{y}{V-y}\, d(1-(1-F(y))^{Kn}).$$

A FOSD increase in individual cost effects a FOSD increase in the minimum cost, which is distributed with cdf $1-(1-F(y))^{Kn}$. The results follow because the function $\frac{y}{V-y}$ at the right-hand side of (5) is increasing and convex for $y \in (0, V)$. □

Proof of Proposition 4. (i) Considering (4), we see that both $e^{-\rho T(y;n)}$ and $(1-F(y))^{Kn-1}$ decrease with n, so $U^E(c)$ strictly increases for $c > \underline{c}$, and the larger c, the larger the increase.

(ii) Constancy of $U^E(c)$ with respect to changes in K follows from the fact that the integrand in (4) is constant with respect to K. To see this, for notational convenience we begin by rewriting (4) as applied to cost type c' as follows:

$$U^E(c') = V - \underline{c} - \int_{\underline{c}}^{c'} e^{-\rho T(c)}(1-F(c))^{Kn-1}\, dc.$$

Then, we take the natural log of the integrand to get

$$-\rho T(c; K) + (Kn-1)\log(1-F(c)),$$

and finally we differentiate with respect to K to get

$$-\rho \frac{\partial T(c;K)}{\partial K} + n \log(1-F(c)).$$

Now we use $\frac{\partial T(c;K)}{\partial K} = \int_{\underline{c}}^{c} \frac{\partial T'(y;K)}{\partial K}\, dy$ and substitute for $\frac{\partial T'(y;K)}{\partial K}$ from (19) to obtain

$$\begin{aligned} -\rho \int_{\underline{c}}^{c} \frac{-h\,(y)\, n}{\rho}\, dy + n \log\left(1 - F(c)\right) &= n\left(\log\left(1 - F(c)\right) + \int_{\underline{c}}^{c} h\,(c)\; dc\right) \\ &= n\left(\log\left(1 - F(c)\right) + \int_{\underline{c}}^{c} \frac{f(y)}{1 - F(y)}\, dy\right) \\ &= 0. \end{aligned}$$

Because the log of the integrand is constant with respect to K, so too is the integrand itself. □

Proof of Proposition 5. We prove $T_s(c) < T_l(c)$ by showing the equivalent relation $k(c) < c$ for all $c \in (\underline{c}, \bar{c})$. To see this, note that, at $c = \underline{c}$, (10) implies $k'(\underline{c}) < 1$, so in a sufficiently small right-neighborhood of $\underline{c}$ we have $k(c) < c$. Consider now by contradiction the smallest cost $\tilde{c}$ at which $k(\tilde{c}) = \tilde{c}$. Since $k(c)$ starts smaller than c, this requires $k'(\tilde{c}) \geq 1$. But (10) and $k(\tilde{c}) = \tilde{c}$ imply the contradiction $k'(\tilde{c}) < 1$. □

Proof of Proposition 6. Assume by contradiction that $k(\bar{c}) < \bar{c}$. Consider an arbitrary $\tilde{c} \in (\underline{c}, \bar{c})$. Using (10), we have

$$k(\bar{c}) - k(\tilde{c}) = \int_{\tilde{c}}^{\bar{c}} \frac{f_s(c)}{1 - F_s(c)} \cdot \frac{\frac{V_s^e(n_s - 1)}{V_s^g - c} + 1}{\frac{f_l(k(c))}{1 - F_l(k(c))}\left(\frac{V_l^e(n_l - 1)}{V_l^g - k(c)} + 1\right)}\, dc.$$

Under the contradiction hypothesis, the second factor in the integrand displayed above is bounded below by a number $b > 0$. Therefore, we obtain

$$k(\bar{c}) - k(\tilde{c}) \geq b \int_{\tilde{c}}^{\bar{c}} \frac{f_s(c)}{1 - F_s(c)}\, dc = -b\left[\lim_{c \uparrow \bar{c}} \log(1 - F_s(c)) - \log(1 - F_s(\tilde{c}))\right] = +\infty,$$

and we have a contradiction to $k(\bar{c}) < \bar{c}$. Similarly, assume by contradiction that $T(\bar{c}) < \infty$. Using (11) we have

$$T_s(\bar{c}) - T_s(\tilde{c}) = \int_{\tilde{c}}^{\bar{c}} \frac{f_s(c)}{1 - F_s(c)} \frac{n_s}{\rho} \cdot \left[\frac{\left(V_l^e - \left(V_l^g - k\,(c)\right)\right)(n_l - 1)}{V_l^e(n_l - 1) + \left(V_l^g - k\,(c)\right)} \frac{V_s^e(n_s - 1) + \left(V_s^g - c\right)}{n_s\left(V_s^g - c\right)} - 1\right] dc.$$

The term in brackets in the integrand above is bounded below by a number $d > 0$. This is because, by (13), (12) holds with strict inequality away from $\underline{c}$. Therefore, with similar steps as before, we obtain

$$T_s(\bar{c}) - T_s(\tilde{c}) > d \cdot \frac{n_s}{\rho} \int_{\tilde{c}}^{\bar{c}} \frac{f_s(c)}{1 - F_s(c)}\, dc = +\infty,$$

a contradiction. □

Proof of Proposition 7. We consider two values for the externality, V_s^e and $\tilde{V}_s^e$, with $\tilde{V}_s^e > V_s^e$. The corresponding equilibrium strategies are indicated with $(T_s(c), T_l(c), k(c))$ and $\left(\tilde{T}_s(c), \tilde{T}_l(c), \tilde{k}(c)\right)$. Using (14), we prove that the probability of victory of the small group decreases by showing that $k(c) < \tilde{k}(c)$ for any $c \in (\underline{c}, \bar{c})$. To see this, note that, at $c = \underline{c}$, (10) and $k(\underline{c}) = \tilde{k}(\underline{c}) = \underline{c}$ imply $k'(\underline{c}) < \tilde{k}'(\underline{c})$, so in a right-neighborhood of $\underline{c}$ we have $k(c) < \tilde{k}(c)$. Consider now by contradiction the smallest cost z at which $k(z) = \tilde{k}(z)$. Since $k(c)$ starts smaller than $\tilde{k}(c)$, this requires $k'(z) \geq \tilde{k}'(z)$. But (10) and $k(z) = \tilde{k}(z)$ imply the contradiction $k'(\tilde{c}) < \tilde{k}'(z)$. We can now see that $T_s(c) < \tilde{T}_s(c)$ for any $c \in (\underline{c}, \bar{c})$: we know $T_s(\underline{c}) = \tilde{T}_s(\underline{c})$ and, by (11), $\tilde{T}_s'(c) > T_s'(c)$, since the term in brackets in (11) is increasing in $k(c)$ and in V_s^e. Now consider the equilibrium utility $U_s^E(c) \equiv U_s(c, c)$. Using the envelope theorem and (7), we have

$$U^E(c) = V_s^g - \underline{c} - \int_{\underline{c}}^{c} e^{-\rho T_s(y)} (1 - F(k(y)))^{n_l} \; (1 - F_s(y))^{n_s - 1} \, dy.$$

But since $k(c) < \tilde{k}(c)$ and $T_s(c) < \tilde{T}_s(c)$ for any $c \in (\underline{c}, \bar{c})$, we conclude that an increase in V_s^e increases the utility of all cost types in the small group. □

Proposition 8 (A sufficient condition for a negative group-size effect)**.** *Consider an interior equilibrium described by* (8)*,* (9)*, and* (10) *with initial condition* $k(\underline{c}) = \underline{c}$*. Let* $V_l^e = V_s^e = V_l^g = V_s^g = V$*,* $F_s = F_l = F$*, and* $n_l > n_s$*. Define the function* $\tilde{k}(c)$ *as the solution to*

$$(1 - F(\tilde{k}(c)))^{n_l} = (1 - F(c))^{n_s}. \tag{21}$$

If, $\forall c \in [\underline{c}, \bar{c}]$*, we have*

$$\tilde{k}(c) > \frac{V n_l (n_s - 1) c}{V n_s (n_l - 1) - (n_l - n_s) c}, \tag{22}$$

then $P_s^w > 0.5$.[30]

Proof of Proposition (8). Proposition (8) is stated for $V_l^e = V_s^e = V_l^g = V_s^g = V$ and $F_s = F_l = F$. Therefore, we begin by restating the differential equation for k in (10) under these assumptions. Further, to clarify the dependence on n_l, we include it, when needed, as an argument of the relevant functions. Recalling that $h(\cdot)$ denotes the hazard rate of F, we obtain

$$k'(c; n_l) = \frac{V n_s - c}{V - c} h(c) \left[\frac{V n_l - k(c; n_l)}{V - k(c; n_l)} h(k(c; n_l)) \right]^{-1}. \tag{23}$$

Note also that, if n_l were equal to n_s, then the only solution to (23) with $k(\underline{c}; n_s) = \underline{c}$ would be $k(c; n_s) = c$. Furthermore, in this case the probability of victory of the "small" group would be 0.5. The strategy of proof

[30]Consider for example $n_s = 2$, $n_l = 4$, and $F(c) = 2c - 3$ on $[3/2, 2]$. Then (21) yields $\tilde{k}(c) = 2 - \frac{\sqrt{4-2c}}{2}$, and one can check numerically that (22) is satisfied if and only if $V \geq V^* \approx 2.21634$.

is to show that $(1-F(k(c;n_l)))^{n_l} > (1-F(k(c;n_s)))^{n_s}$, for all $c \in (\underline{c}, \bar{c})$, so that (14) ends up implying $P_s^w > 0.5$.

Using (23), we obtain

$$\frac{d}{dc}(1-F(k(c;n_l)))^{n_l} = -\frac{Vn_s - c}{V-c}(1-F(k(c;n_l)))^{n_l}\left[\frac{n_l(V-k(c;n_l))}{Vn_l - k(c;n_l)}\right]; \tag{24}$$

similarly,

$$\frac{d}{dc}(1-F(k(c;n_s)))^{n_l} = -\frac{Vn_s - c}{V-c}(1-F(k(c;n_s)))^{n_s}\left[\frac{n_s(V-k(c;n_s))}{Vn_s - k(c;n_s)}\right]. \tag{25}$$

Note that, by $k(\underline{c};n_l) = \underline{c} = k(\underline{c};n_s)$, (24) yields

$$\frac{d}{dc}(1-F(k(c;n_l)))^{n_l}\bigg|_{c=\underline{c}} = -\frac{Vn_s - \underline{c}}{V-\underline{c}}\left[\frac{n_l(V-\underline{c})}{Vn_l - \underline{c}}\right];$$

while (25) gives

$$\frac{d}{dc}(1-F(k(c;n_s)))^{n_l}\bigg|_{c=\underline{c}} = -\frac{Vn_s - \underline{c}}{V-\underline{c}}\left[\frac{n_s(V-\underline{c})}{Vn_s - \underline{c}}\right].$$

Hence, by

$$\frac{n_l(V-\underline{c})}{Vn_l - \underline{c}} < \frac{n_s(V-\underline{c})}{Vn_s - \underline{c}},$$

using (24) and (25) we obtain

$$0 > \frac{d}{dc}(1-F(k(c;n_l)))^{n_l}\bigg|_{c=\underline{c}} > \frac{d}{dc}(1-F(k(c;n_s)))^{n_s}\bigg|_{c=\underline{c}}.$$

So, we see that $(1-F(k(c;n_l)))^{n_l}$ starts above $(1-F(k(c;n_s)))^{n_s}$ in a right-neighborhood of $\underline{c}$. Then, the proposition is proven if there is no intersection in $(\underline{c}, \bar{c})$ between $(1-F(k(c;n_l)))^{n_l}$ and $(1-F(k(c;n_s)))^{n_s}$.

Proceeding by contradiction, suppose there exists $\tilde{c} \in (\underline{c}, \bar{c})$ such that

$$(1-F(k(\tilde{c};n_l)))^{n_l} = (1-F(k(\tilde{c};n_s)))^{n_s}. \tag{26}$$

As $(1-F(k(c;n_l)))^{n_l}$ starts above $(1-F(k(c;n_s)))^{n_s}$ in a right-neighborhood of $\underline{c}$, we must have that for at least one $\tilde{c}$ that satisfies (26), the following also holds:

$$\frac{d}{dc}(1-F(k(c;n_l)))^{n_l}\bigg|_{c=\underline{c}} \leq \frac{d}{dc}(1-F(k(\tilde{c};n_s)))^{n_s}\bigg|_{c=\underline{c}}. \tag{27}$$

But using (24), (25), and $k(\tilde{c}; n_s) = \tilde{c}$, equation (27) requires

$$\frac{n_s(V - \tilde{c})}{Vn_s - \tilde{c}} \leq \frac{n_l(V - k(\tilde{c}; n_l))}{Vn_l - k(\tilde{c}; n_l)}. \tag{28}$$

Note also that (26), which can be equivalently restated as $(1 - F(k(\tilde{c}; n_l)))^{n_l} = (1 - F(\tilde{c}))^{n_s}$, can be concatenated to the definition of $\tilde{k}$ in (21) to give

$$(1 - F(k(\tilde{c}; n_l)))^{n_l} = (1 - F(\tilde{c}))^{n_s} = (1 - F(\tilde{k}(\tilde{c})))^{n_l},$$

which implies $k(\tilde{c}; n_l)) = \tilde{k}(\tilde{c})$. Hence, (28) implies

$$\frac{n_s(V - \tilde{c})}{Vn_s - \tilde{c}} \leq \frac{n_l(V - k(\tilde{c}; n_l))}{Vn_l - k(\tilde{c}; n_l)} = \frac{n_l(V - \tilde{k}(\tilde{c}))}{Vn_l - \tilde{k}(\tilde{c})}. \tag{29}$$

But the extremes of equation (29) are directly contradicted by the hypothesis in (22). □

Appendix 2: Equilibrium with low costs

Proposition 1 provided the symmetric equilibrium strategy under the assumption that $\underline{c} \geq c_0 \equiv \frac{(K-1)n}{Kn-1}V$. The strategy there fails to be weakly increasing if $\underline{c} < c_0$ because (1) would imply T is strictly decreasing for $c \in [\underline{c}, c_0)$. To maintain weak monotonicity of the equilibrium strategy, we now investigate the possibility that an equilibrium strategy has a "flat spot," that is, an interval over which it is constant. The following lemma shows that if a symmetric equilibrium strategy T has a flat spot, then it must occur at 0, which implies that, for some $\hat{c} \geq \underline{c}$, $T(c) = 0$ on $[\underline{c}, \hat{c}]$ and T is strictly increasing for $c > \hat{c}$. Moreover, the strategy T must be continuous.[31]

Lemma 2. *Suppose T is a symmetric equilibrium strategy. Then T is continuous and if T is constant on $[\tilde{c}_l, \tilde{c}_h]$ and $\underline{c} \leq \tilde{c}_l < \tilde{c}_h \leq \bar{c}$, then $T(c) = 0$ for all $c \in [\tilde{c}_l, \tilde{c}_h]$. Further, $\tilde{c}_h \leq c_0$.*

Proof. Suppose T is a symmetric equilibrium strategy for which types $c \in [\tilde{c}_l, \tilde{c}_h]$ grab at $\tilde{T} > 0$, i.e. a strategy with a strictly positive flat spot: $T(c) = \tilde{T}$ for all $c \in [\tilde{c}_l, \tilde{c}_h]$. We establish two facts to show this cannot be an equilibrium strategy. First, under the equilibrium conjecture, type $\tilde{c}_h$ must prefer a contribution of $\tilde{T}$ to one of $\tilde{T} + \varepsilon$. As $\varepsilon \downarrow 0$, this will imply $\tilde{c}_h \leq c_0$. Second, type $\tilde{c}_l$ must prefer a contribution of $\tilde{T}$ to one of $\tilde{T} - \varepsilon$. As $\varepsilon \downarrow 0$, this will imply $\tilde{c}_l \geq c_0$. Therefore, a flat spot at $\tilde{T} > 0$ cannot exist in equilibrium.

[31] The proof is not just an adaptation of the discrete-gain vs. marginal-loss comparison familiar from standard auction theory because, by grabbing an instant sooner and breaking a tie, an agent increases discretely both his benefit and cost.

Consider first $\tilde{c}_h$. The utility of one agent in group 1 with cost realization equal to $\tilde{c}_h$ that contributes $\tilde{T}$ is

$$\frac{n-1}{Kn-1}V \times \int_{\underline{c}}^{\tilde{c}_l} e^{-\rho T(x)}\, d\left(1-\left(1-F(x)^{Kn-1}\right)\right) + \left(1-F\left(\tilde{c}_l\right)^{Kn-1}\right) e^{-\rho\tilde{T}} U_I(\tilde{c}_h), \tag{30}$$

where $U_I(c)$ represents the payoff of a type c agent if the minimum cost of all other agents is above $\tilde{c}_l$, i.e., conditional on all other agents having cost above $\tilde{c}_l$.

Now the logic behind (30) is this. The first addendum is the expected payoff if the minimum cost of all other agents is below $\tilde{c}_l$: the average present value of V multiplied by the probability that one of the other group-1 agents wins, calculated under symmetry. The second addendum is the product of the probability that the minimum cost of all other agents is above $\tilde{c}_l$, multiplied by the present value of $U_I(\tilde{c}_h)$, with $U_I(c)$ defined as

$$U_I(c) = VS_1(n,K) + (V-c)S_2(n,K),$$

where

$$S_1(n,K) = \sum_{j=1}^{n-1}\sum_{k=0}^{(K-1)n}\binom{n-1}{j}\binom{(K-1)n}{k}p^j(1-p)^{n-j-1}p^k(1-p)^{(K-1)n-k}\frac{j}{1+j+k},$$

$$S_2(n,K) = \sum_{j=0}^{n-1}\sum_{k=0}^{(K-1)n}\binom{n-1}{j}\binom{(K-1)n}{k}p^j(1-p)^{n-j-1}p^k(1-p)^{(K-1)n-k}\frac{1}{1+j+k},$$

and

$$p \equiv \frac{F(\tilde{c}_h)-F(\tilde{c}_l)}{1-F(\tilde{c}_l)}. \tag{31}$$

The payoff U_I can be understood as follows. Here j indexes other group 1 players and k indexes group 2 players. Beginning with S_2, if there are j other players in group 1 bidding $\tilde{T}$ and k players in the other $K-1$ groups bidding $\tilde{T}$ (as well as this player of interest in group 1), then the player of interest in group 1 is selected with probability $\frac{1}{1+j+k}$, in which case he earns payoff $V-c$; but (moving to S_1) if one of the other group-1 players is selected, which happens with probability $\frac{j}{1+j+k}$, then he gets the benefit V without incurring any cost. And, of course, the probability of this configuration of other players bidding $\tilde{T}$ is

$$\binom{n-1}{j}p^j(1-p)^{n-j-1}\binom{(K-1)n}{k}p^k(1-p)^{(K-1)n-k}.$$

With a similar logic, the limit for $\varepsilon \downarrow 0$ of the utility of one agent in group 1 with cost $\tilde{c}_h$ that contributes $\tilde{T}+\varepsilon$ is

$$\frac{n-1}{Kn-1}V \times \int_{\underline{c}}^{\tilde{c}_l} e^{-\rho T(x)}\, d\left(1-\left(1-F(x)^{Kn-1}\right)\right) + \left(1-F\left(\tilde{c}_l\right)^{Kn-1}\right) e^{-\rho\tilde{T}} U_R(\tilde{c}_h), \tag{32}$$

where $U_R(\tilde{c}_h)$ is this player's payoff "from the right":

$$U_R(\hat{c}) = VS_3(n,K) + (V - \hat{c})(1-p)^{Kn-1},$$

where

$$S_3(n,K) = \sum_{j=1}^{n-1} \sum_{k=0}^{(K-1)n} \binom{n-1}{j} \binom{(K-1)n}{k} p^j (1-p)^{n-j-1} p^k (1-p)^{(K-1)n-k} \frac{j}{j+k}$$

and p is again given by (31).

Using properties of a binomial distribution, one can establish

$$S_1(n,K) = \frac{n-1}{Kn-1}\left(1 - \frac{1-(1-p)^{Kn}}{pKn}\right) \tag{33}$$

$$S_2(n,K) = \frac{1-(1-p)^{Kn}}{pKn} \tag{34}$$

$$S_3(n,K) = \frac{n-1}{Kn-1}\left(1-(1-p)^{Kn-1}\right). \tag{35}$$

Since utility in (30) must be at least as large as the one in (32), we have

$$VS_1 + (V - \tilde{c}_h) S_2 = U_I(\tilde{c}_h) \geq U_R(\tilde{c}_h) = VS_3 + (V - \tilde{c}_h)(1-p)^{Kn-1}, \tag{36}$$

and the extremes of (36) imply $V(S_1 + S_2 - S_3 - (1-p)^{Kn-1}) \geq \tilde{c}_h(S_2 - (1-p)^{Kn-1})$, and therefore, substituting from (33)–(35), we have $\tilde{c}_h \leq c_0$.[32]

Moving now to $\tilde{c}_l$, the utility of one agent in group 1 with cost realization equal to $\tilde{c}_l$ that contributes $\tilde{T}$ is

$$\frac{n-1}{Kn-1} V \times \int_{\underline{c}}^{\tilde{c}_l} e^{-\rho T(x)}\, d\left(1 - \left(1 - F(x)^{Kn-1}\right)\right) + \left(1 - F(\tilde{c}_l)^{Kn-1}\right) e^{-\rho \tilde{T}} U_I(\tilde{c}_l), \tag{37}$$

while the limit for $\varepsilon \downarrow 0$ of the utility of one agent in group 1 with cost $\tilde{c}_l$ that contributes $\tilde{T} - \varepsilon$ is

$$\frac{n-1}{Kn-1} V \times \int_{\underline{c}}^{\tilde{c}_l} e^{-\rho T(x)}\, d\left(1 - \left(1 - F(x)^{Kn-1}\right)\right) + \left(1 - F(\tilde{c}_l)^{Kn-1}\right) e^{-\rho \tilde{T}} (V - \tilde{c}_l). \tag{38}$$

Since utility in (37) must be at least as large as the one in (38), we have

$$VS_1 + (V - \tilde{c}_l) S_2 = U_I(\tilde{c}_l) \geq V - \tilde{c}_l,$$

[32] We note that $S_2 - (1-p)^{Kn-1} > 0$ is equivalent to

$$1 > (1-p)^{Kn-1}(1 - p + pKn) \equiv \psi(p).$$

This latter inequality is satisfied because $\psi(0) = 1$ and $\psi'(p) < 0$. Therefore, if a flat spot exists, then $p > 0$ and $S_2 - (1-p)^{Kn-1} > 0$.

and the extremes of the above-displayed equation imply $\tilde{c}_l(1 - S_2) \geq V(1 - S_1 - S_2)$, or $\tilde{c}_l \geq c_0$. Thus, we obtain $\tilde{c}_l = \tilde{c}_h$, so a flat spot at $\tilde{T} > 0$ is impossible in equilibrium.

To see that T is continuous note that any discontinuity must be a jump discontinuity. If such a jump occurs at c', then for sufficiently small $\delta > 0$ the types in $(c', c' + \delta)$ will find it strictly profitable to decrease their grabbing times discretely to avoid wasteful delay (there is no chance of a tie since there are no flat spots at positive times).

Finally, note that the logic leading to (36) remains valid even if $\tilde{T} = 0$. Therefore, even if T is flat at zero for $c \in [\underline{c}, \tilde{c}_h]$, we obtain $\tilde{c}_h \leq c_0$. □

From Lemma 2 we now see that if an equilibrium strategy has a flat spot it must be over an interval of the form $[\underline{c}, \tilde{c}_h]$, where T takes on value 0, and $\tilde{c}_h \leq c_0$. Furthermore, for any c where T is strictly increasing, the equilibrium analysis is precisely as for an interior equilibrium, thus requiring $c \geq c_0$.

This reasoning has two implications. First, the equilibrium in Proposition 1 is unique among all symmetric ones, without focusing only on strictly increasing strategies. Second, we have the following:

Proposition 9 (Equilibrium when preemption costs may be low)**.** *If $\underline{c} < c_0$, then the unique symmetric equilibrium strategy T satisfies $T(c) = 0$ for $c \in [\underline{c}, c_0]$ and*

$$T'(c) = \frac{f(c)}{(1 - F(c))} \frac{(Kn - 1)}{\rho(V - c)} (c - c_0) \quad \forall c \in (c_0, \bar{c}). \tag{39}$$

Therefore, a flat spot can (and indeed does) appear in the symmetric equilibrium strategy if costs are "low." Once c_0 is established as the point at which T begins increasing, the comparative statics described in Section 2.2 can be seen to hold generally.

Proposition 10. *For each $c \in [\underline{c}, \bar{c}]$, the equilibrium strategy $T(c; K, n)$ is weakly decreasing in K and weakly increasing in n. Furthermore, $c_0(K, n+1) < c_0(K, n) < c_0(K+1, n)$, and if $\underline{c} < c_0(K, n) \equiv \frac{(K-1)n}{Kn-1} V$, then*

$$T(c; K, n+1) > T(c; K, n) \quad \forall c \in (c_0(K, n+1), \bar{c})$$

and

$$T(c; K, n) > T(c; K+1, n) \quad \forall c \in (c_0(K, n), \bar{c}).$$

Moreover, increasing K also decreases ET^E.

Proof. First consider the effect of increasing n. Because $c_0(K, n+1) < c_0(K, n)$, it follows that $T(c; K, n + 1) = T(c; K, n) = 0$ for $c \leq c_0(K, n)$ and $T(c; K, n+1) > T(c; K, n)$ for $c \in (c_0(K, n+1), c_0(K, n)]$. Finally,

$T(c;K,n+1) > T(c;K,n)$ for $c \in (c_0(K,n), \bar{c}]$ because $T(c_0(K,n);K,n+1) > T(c_0(K,n);K,n)$ and T' increases with n on $(c_0(K,n), \bar{c}]$. To see this latter property, note that

$$\begin{aligned}
\frac{\partial T'(c;K,\hat{n})}{\partial \hat{n}} &= \frac{h(c)K}{\rho(V-c)}\left(c - \frac{K-1}{K}V\right) \\
&> \frac{h(c)K}{\rho(V-c)}\left(c - \frac{(K-1)\hat{n}}{K\hat{n}-1}V\right) \qquad \text{(because } K \geq 2\text{)} \\
&> 0,
\end{aligned}$$

where the second inequality follows because $T(c;K,\hat{n}) > 0$ implies $c > c_0(K,\hat{n}) \equiv \frac{(K-1)\hat{n}}{K\hat{n}-1}V$.

One similarly shows that an increase in K reduces T. Analogously to the proof of the effect of increasing n, here we use the fact that increasing K increases $c_0(K,n)$ and decreases T' (see (19)). Moreover, because individual grabbing strategies decrease with an increase in K and because increasing K increases the number of players, it follows immediately that ET^E also decreases with K. □

From Proposition 10 we see that an increase in the number of teams, by reducing individual players' grabbing times, has the effect of decreasing the expected time at which the game ends. Indeed, because $c_0(K,n) \to V$ as $K \to \infty$, everyone grabs almost instantly and $ET^E \to 0$. Not surprisingly, as the effect of increasing n on the expected duration of the contest was ambiguous for interior equilibria, so too is it ambiguous for corner equilibria. Surprisingly, however, while increasing n had the effect of increasing interim payoffs at interior equilibria, examples show the effect is ambiguous for corner equilibria.

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