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JEL Classification: G33, H81, G34, G32

Keywords: Liquidation, bailout, bail-in, asset sale, agency

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Liquidation, bailout, and bail-in: Insolvency resolution mechanisms and bank lending*

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7 May 2019

Abstract

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1 Introduction

The 2008-2010 financial crisis has highlighted the need for an orderly insolvency resolution process for large financial institutions. When the crisis broke out, the FDIC only had the power to place an insured depository institution into receivership; it could not resolve failing banks or other nonbank financial companies that posed a systemic risk. The bankruptcy of Lehman sent, however, shockwaves through the financial system.

To contain the financial crisis, the US and several European authorities supported failing financial institutions with public money by providing capital injections or "bailouts" in exchange for full or partial ownership. These bailouts were met with widespread public anger as they allowed banks to "privatize profits and socialize losses".

In response to the crisis, regulators on both sides of the Atlantic devised new regulatory frameworks that attempt to minimize the use of public money to recapitalize failing banks (see Philippon and Salord (2017) for an overview). In this context the bail-in tool is probably the most important regulatory innovation.[1] Bail-in is a statutory power in the hands of resolution authorities that permits them to write down part of the bank's liabilities or to convert them into equity in order to preserve the bank as a going concern.[2],[3] Legal safeguards ensure that creditors recover no less than they would under insolvency. Culpable senior management are replaced, possibly after a transition period. Existing equityholders are wiped out (or at least heavily diluted).

The new regulatory frameworks aim to avoid future bailouts by reducing the amount banks lend and the riskiness of their loans. The subprime mortgage crisis and the

[1]Another regulatory innovation is the Orderly Liquidation Authority (OLA) established by the Dodd-Frank Act. The OLA provisions authorize the FDIC, instead of a bankruptcy court, to administer swift wind-downs of systemically important financial institutions. An OLA may involve a bail-in allowing the recapitalized bank to return to private hands under new management. We do not study OLAs separately in this paper as they are some combination of liquidation and bail-in.

[2]It is important to distinguish bail-in from contingent convertible debt obligations (CoCos). CoCos are financial instruments in which the trigger event and the conversion rate are identified in advance in the debt contract (see Flannery (2014) for a review of the literature on CoCos).

[3]In the EU, all financial contracts concluded after 1/1/2016 must have a "bail-in" clause, which means that certain obligations and liabilities may be subject to "bail-in".

phenomenon of liar's loans (see Mian and Sufi (2009)) have shown that, from bank lenders' and regulators' perspective, moral hazard is linked primarily to loan risk. According to Kashyap et al. (2008) banks' failure to offload subprime risk is a symptom of deeper governance and managerial agency problems.

Conventional wisdom says that the guarantees and safety net provided by bailouts encourage excessive lending and risk-taking, whereas liquidations and bail-ins mitigate these types of moral hazard. Is this indeed the case? How do insolvency resolution mechanisms (hereafter referred to as IRMs) affect the bank insolvency rate, the loss in default, and value at risk (VaR)? These questions remain unanswered because existing models focus on one specific IRM and do not compare the effect of different IRMs on bank policies. This paper develops a unifying, dynamic model for IRMs that addresses these questions from a microprudential perspective, i.e. we study the effects of insolvency regulation on the lending behavior and net value created by an individual bank, and not the effects on the financial system or economy as a whole. We also explore whether bailouts necessarily have to rely on public money, or whether they could be self-financed by banks.

Our paper explores how three different IRMs (i.e. liquidation, bailout and bail-in) affect a bank's payout rate, as well as the quantity and quality of loans when these three decision variables are set by risk averse inside equityholders. Insiders can invest in risky assets (loans) of which the return follows a jump diffusion process. The diffusion process reflects continuous shocks to loan returns, whereas the jumps correspond to rare, negative shocks (hereafter referred to as "crashes"). Crashes arrive according to an exogenous Poisson process, but the fraction of the bank assets that is destroyed by the crash (i.e. the crash risk exposure, a proxy for "loan quality") is a decision variable under insiders' control (e.g. through collateral requirements). Loans with a higher exposure to crash risk carry a higher expected return.

We show that insiders follow an optimal asset to net worth ratio. Banks making profits issue additional loans that are financed by debt and retained earnings. Loss making banks sell assets and use the proceeds to pay down debt. This continuous, dynamic rebalancing mechanism ensures that the target asset to net worth ratio is maintained and banks remain solvent in the absence of crashes. Following a crash, banks with low exposure remain solvent and rebalance their capital structure by selling assets. Banks with assets that are (too) highly exposed to crashes become insolvent. Whether insiders choose to put the bank at risk of insolvency depends on the reward for taking on crash risk. A high reward not only encourages banks to lend a lot but also to issue loans that become impaired in crashes (e.g. subprime mortgages). A small reward for crash risk incentivizes insiders to lend much less and to focus on high quality loans that are relatively immune to downturns. Incentives to put the bank at risk of insolvency are strongest (weakest) under the bailout (liquidation) regime. Banks are therefore most (least) prone to insolvency under the bailout (liquidation) regime, in line with conventional wisdom.

Our findings regarding loan quality turn some of the conventional wisdom on its head, however. Insiders' limited liability in the liquidation regime creates incentives to take on as much crash risk as possible by issuing low quality loans. This is a moral hazard problem from the depositors and lenders' perspective. Low quality loans give the bank a higher return in good times. However, when loans default in a crash, low recovery rates push the bank deep into insolvency. Although loan volume remains relatively low under the liquidation regime (due to the high risk-adjusted cost of borrowing), the loss in default is most severe under the liquidation regime because insiders do not care whether the bank is a little or a lot insolvent in liquidation. Moreover, payout to inside and outside equityholders is high because insiders want to milk the bank before a crash arrives and the "music stops".

With bailouts and bail-ins, on the other hand, banks remain a going concern, and managers retain "skin in the game" even in retirement.[4] Negative consequences or

[4]Following the bailout of the Royal Bank of Scotland, its CEO Sir Fred Goodwin retired in 2008

severe penalties imposed on inside equityholders when banks become insolvent mitigate insiders' incentives to take risks that crystallize in crashes. This leads to the lowest loan issuance, lowest leverage, and lowest crash risk exposure under the bail-in regime. With bailouts, the implicit government guarantee keeps the bank's cost of borrowing artificially low, causing lending activity and leverage to be highest. The payout rate to inside and outside equityholders are lowest under the bailout regime because insiders are happy to reinvest profits to stimulate long term growth.

We analyze the bank's value at risk (VaR). For short horizons VaR is primarily determined by the bank's exposure to diffusion risk and therefore by the amount of risky loans it issues. As the horizon increases, exposure to rare crash risk -and therefore loan quality- becomes relatively more important. Since the bail-in regime leads to the lowest quantity and highest quality of loans, it also coincides with the lowest VaR. The bailout and liquidation regimes lead to higher VaRs than bail-ins, but the ranking of their VaRs is not monotonic due to the interplay between diffusion and crash risk. The bailout regime generates the higher VaR for short horizons, whereas the liquidation regime generates the higher VaR for long horizons. Short term VaR measures do not convey much information about the importance of rare, adverse shocks, especially for the liquidation regime. The VaR measures complement the information contained in the credit spread of the bank debt. We show that credit spreads are highest and lowest under, respectively, the liquidation and bailout regimes.

The value created (net of recapitalizations) by an individual bank is by far highest under the bailout regime. Importantly, by distributing a fraction of all bank dividends into a bailout fund, it is possible to cover expected bailout costs without tax payers' money provided that banking (net of recapitalization costs) is a positive NPV activity. We therefore believe that "pre-funded" bailouts could be a viable way of res-

at the age of 50 with a pension entitlement of £693,000 per year. Had the government not stopped RBS from going bankrupt, Sir Fred would have received a yearly pension of £28,000 from the pension-protection fund, starting at age 65.

cuing insolvent banks.[5] Furthermore, a dividend financed bailout fund does not alter equityholders' incentives (unlike deposit guarantee schemes).

Our dynamic, continuous-time, open-horizon model captures optimal balance sheet rebalancing and recapitalizations for banks facing continuous diffusion risk and rare jump (crash) risk. To boost returns, insiders take ex ante risks that materialize ex post in states when the bank will be insolvent and managers ousted. Our model delivers tractable analytical results as well as quantitative comparative statics, allowing clear comparisons across regimes. We are unable to achieve all of this with a static two-date model.

Our paper is closely related to a growing literature on dynamic models of banks and their optimal investment and financing policies. De Nicolo, Gamba, and Lucchetta (2014) show that a resolution procedure contingent on observed levels of bank capital dominates both capital and liquidity requirements in efficiency and welfare terms. Hugonnier and Morellec (2017) develop a continuous-time dynamic model of banking and find that imposing liquidity requirements leads to lower bank losses in default at the cost of an increased likelihood of default. Combining liquidity and leverage requirements reduces both the likelihood of default and the magnitude of bank losses in default.[6] Our paper does not consider regulatory liquidity and leverage requirements but studies the role of different IRMs for loan issuance and loan quality. A comple-

[5]We only consider recapitalizations by the government or by a bailout fund. There are a variety of reasons (such as debt overhang, high uncertainty about bank asset value, and time pressure) why standard equity issues are not a realistic option for insolvent banks. Troubled banks that issued equity during the financial crisis did so after returning to solvency.

[6]While losses are usually exogenously allocated across stakeholders in the literature, a number of complementary papers endogenize the sharing rule and negotiation between the banks' claimants (e.g. Bolton and Oehmke (2018), Keister and Mitkov (2016), Colliard and Gromb (2018)). Several papers model the optimal design of insolvency resolution, bank regulation or government intervention (Gorton and Huang (2004), Kahn and Santos (2005), Acharya and Yorulmazer (2008), Philippon and Schnabl (2013), Walther and White (2016), Lucchetta, Parigi, and Rochet (2018)).

mentary working paper by Berger et al. (2018) examines theoretically and empirically how IRMs affect the initial capitalization decision, the size of the subordinated debt, the future recapitalization strategy, and the bank's net market value. Their analysis focuses on the capital structure decisions, keeping investment, payout and the riskiness of assets exogenously given. Banks do not sell assets to rebalance. Our analysis focuses on the role of IRMs for managerial risk taking and bank lending. We show that results may be significantly different depending on whether or not managers control asset risk. Earlier dynamic banking models include Merton (1977, 1978), Fries, Mella-Barral, and Perraudin (1997), Bhattacharya, Plank, Strobl, and Zechner (2002), and Décamps, Rochet and Roger (2004). These papers assume, however, the banks' asset and liability structure to be exogenously given. Unlike our model, all the above papers assume risk neutral agents. In our model high leverage results from low managerial risk aversion or high tax benefits from debt.

Our paper also relates to a recent literature that jointly models a firm's investment, payout, and borrowing decisions in a dynamic framework. Continuous-time papers in this strand include Gryglewicz (2011), Bolton, Chen, and Wang (2011), Décamps, Mariotti, et al. (2011), Décamps, Gryglewicz, et al. (2016), and Lambrecht and Myers (2017). These papers do not examine IRMs and assume that risk-neutral owners maximize market value. Lambrecht and Myers (2017) consider risk averse managers, but only consider safe debt.

There is a large literature on bank leverage, bank capital requirements and their role for bank risk taking (e.g. Gorton and Pennacchi (1990), Dewatripont and Tirole (1994), Calem and Rob (1999), Diamond and Rajan (2001), Kashyap, Rajan and Stein (2008), among others). These models, however, assume that defaulting banks cease to exists, and are usually static in nature. Several papers (e.g. Hellmann, Murdock and Stiglitz (2000)) make the point that undercapitalized or distressed banks may be more prone to moral hazard, because equityholders have "no skin in the game" and may have an incentive to "gamble for resurrection". In Davila and Walther (2017) large banks anticipate that their actions affect the government's bailout response and therefore take on more leverage than small banks. Allen et al. (2018) analyze the effect of government guarantees on the interconnection between banks' liquidity creation and likelihood of runs. For a review of the literature on bank capital regulation we refer to Santos (2001) and VanHoose (2007). Allen, Carletti and Leonello (2011) review the literature on deposit insurance and risk taking. In our model moral hazard arises before banks are undercapitalized or distressed. Insiders take risk linked to an event (in our case a crash) that will cause distress. Knowing that their game is up if a particular event arises, insiders have an incentive to ensure little or no capital is left when this event occurs.

2 Model setup

Consider a bank that invests an amount A_t in risky assets (loans) that generate an after-tax rate of return given by the following jump-diffusion process:

$$\frac{dA_t}{A_t} = [(\mu' + \kappa'\lambda f)dt + \sigma' dB_t](1-\tau) - f dy_t \quad (1)$$

$$\equiv (\mu + \kappa\lambda f)dt + \sigma dB_t - f dy_t \quad (2)$$

where B_t is a Brownian motion and y_t is a pure Poisson jump process with intensity λ and $E[dy_t] = \lambda dt$. The parameter τ is the corporate tax rate (with $0 \leq \tau < 1$). The other parameters satisfy the conditions $\mu, \lambda, \sigma > 0$ and $0 \leq f \leq 1 < \kappa$.[7]

Hence, most of the time the after-tax return follows a continuous diffusion process with drift $\mu + \lambda\kappa f$ and volatility σ, but occasionally the loan portfolio is subject to a large negative shock (i.e. crash). f captures the sensitivity of the loan portfolio's

[7] Our model does not rely on the presence of taxes. Corporate taxes help us to generate more realistic numbers for the comparative statics regarding bank leverage.

value to such a crash, i.e. f equals the fractional loss in loan portfolio's value due to an adverse shock. We allow the bank optimally to set f. In practice f can be controlled through the bank's collateral requirements (a bank with highly secured loans has a low f, whereas loans with little or no collateral generate a high f). More generally, f is determined by the quality of the loans issued. We will show that the optimal policy for f does not depend on time, and as such we can treat f to be constant in our exposition.

κ is an exogenously given parameter that determines the risk premium associated with crash risk. This premium also captures any tax deductible provisions for loan losses. The expected return is given by: $E\left[\frac{dA_t}{A_t}\right] = (\mu + \lambda f(\kappa - 1))\, dt > \mu dt$. Since $\kappa > 1$, banks are compensated with a higher expected return for issuing loans with a higher exposure to crash risk (i.e. higher f).[8]

The bank finances its investment in risky loans with equity capital (N_t) and debt (D_t), i.e. $A_t = N_t + D_t$. Banks can borrow and continuously roll over the debt. As such our model captures the short term nature of a lot of bank debt.[9] Interest on debt is a tax deductible expense:

$$dD_t = \rho_t'(1-\tau)D_t dt \equiv \rho_t D_t dt \tag{3}$$

The cost of debt equals the after-tax risk-free rate ρ if the debt is safe. A higher, risk-adjusted rate of interest (to be derived) is paid on risky bank debt.

At each instant in time inside equityholders (i.e. managers) decide how much to invest in risky loans (A_t) and how much to pay out to inside (r_t) and outside (d_t) equityholders, given the amount of equity capital (net worth, N_t) in place. Following

[8]We remain agnostic as to whether crash risk is systematic (e.g. economic downturns) or idiosyncratic (e.g. fraud or operational risk) in nature. In the latter case the risk premium could represent rents from relationship banking relative to transaction or capital market lending (see Boot and Thakor (2000)).

[9]If D_t is negative then the bank holds a net cash position. Holding a net cash position is, however, not optimal under restrictions we later impose on the model parameters.

Myers (2000), we assume that outsiders and insiders, respectively, get a fraction α and $1-\alpha$ of the firm's free cash flow and value.[10] The combined payout flow to outside and inside equityholders therefore equals $d_t + r_t = [\alpha/(1-\alpha)+1]\, r_t = [1/(1-\alpha)]\, r_t \equiv mr_t$. Payouts to outsiders are under the form of dividends, whereas payouts to insiders, hereafter referred to as "rents", may, in practice, be a combination of dividends and other managerial compensation.

We will show that, under the optimal investment and payout policies, net worth N_t follows a continuous diffusion process in the absence of crashes. A crash causes, however, a discrete fall in the bank's net worth giving rise to two possible scenarios. Under scenario 1, the bank has a strictly positive net worth position following the shock, and optimally delevers by executing *asset sales* and using the proceeds to reduce outstanding debt. Under scenario 2, the bank's equity capital is wiped out and the bank is insolvent.

Consider first the scenario where net worth remains positive following a crash. If N_t and A_t are, respectively, the bank's net worth and risky assets before the shock then net worth immediately after the shock, N_t^+, is given by:

$$N_t^+ \;=\; N_t \;-\; f\,A_t \;=\; N_t\left(1 \;-\; f\frac{A_t}{N_t}\right) \;\equiv\; N_t\;(1 \;-\; fl_t) \;\equiv\; N_t\Phi_s(l_t, f) \tag{4}$$

where $l_t \equiv \frac{A_t}{N_t} \equiv 1 + \frac{D_t}{N_t}$ is the bank's gearing ratio. Variables under the safe (i.e. solvency) regime are denoted by a subscript s. The bank's net worth remains non-negative if and only if $\Phi_s(l_t, f) \geq 0 \iff l_t \leq \frac{1}{f} \equiv \hat{l}$. Using the balance sheet identity $D_t = A_t - N_t$, and the fact that safe debt earns the risk-free rate, the process for the bank's net worth under the positive net worth condition is:

$$\begin{aligned} dN_t &= dA_t \;-\; dD_t \;-\; mr_t dt \\ &= \left[(\mu + \lambda\kappa f \;-\rho)\,A_t \;+\; \rho N_t \;-\; mq_t N_t\right]dt \;+\; \sigma A_t dB_t \;+\; \left(\Phi_s(l_t, f) - 1\right)N_t dy_t \end{aligned}$$

[10]This sharing rule, which we assume exogenously given, can be derived as the outcome of a repeated bargaining game where insiders make take-it-or-leave-it offers to outsiders subject to the threat of collective action by outsiders (see Lambrecht and Myers (2012, 2017)).

where $q_t \equiv \frac{r_t}{N_t}$ denotes insiders' payout yield. Hence:

$$\begin{aligned}\frac{dN_t}{N_t} &= \left[(\mu + \lambda\kappa f\ - \rho)\, l_t\ +\ \rho\ -\ m q_t\right] dt\ +\ \sigma l_t dB_t\ +\ \left(\Phi_s(l_t, f) - 1\right) dy_t \\ &\equiv g_s(l_t, q_t, f)\, dt\ +\ \sigma l_t dB_t\ +\ \left(\Phi_s(l_t, f) - 1\right)\ dy_t \quad \text{for}\ \ l_t \leq \frac{1}{f} \end{aligned} \tag{5}$$

Consider next the scenario where the drop in the bank's risky assets exceeds its net worth, i.e. $fA_t > N_t$ (or $l_t > \hat{l}$). Bank debt may no longer be safe and the cost of debt becomes a function of the firm's gearing ratio and loan quality, i.e. $\rho_j(l_t, f)$ where $j \in b, o, i$. Variables referring to the liquidation (i.e. bankruptcy), bailout and bail-in regimes are denoted, respectively, by the subscripts b, o and i. Similar to (5), the process for net worth is given by:

$$\begin{aligned}\frac{dN_t}{N_t} &= \left[(\mu + \lambda\kappa f\ - \rho_j(l_t, f))\, l_t\ +\ \rho_j(l_t, f)\ -\ m q_t\right] dt \\ &\qquad +\ \sigma l_t dB_t\ +\ \left(\Phi_j(l_t, f) - 1\right) dy_t \\ &\equiv g_j(l_t, q_t, f)\, dt\ +\ \sigma l_t dB_t\ +\ \left(\Phi_j(l_t, f) - 1\right)\ dy_t \ \ \text{for} \quad l_t > \frac{1}{f}\end{aligned} \tag{6}$$

A crash now wipes out the bank's net worth. The bank is recapitalized under a bailout or bail-in (i.e. $\Phi_j(l_t, f) > 0$ for $j = o, i$) but not in liquidation ($\Phi_b(l_t, f) \equiv 0$). We derive the functions for $\rho_j(l_t, f)$ and $\Phi_j(l_t, f)$ later in the paper.

2.1 Insiders' optimization problem

We assume insiders have a power utility function (and therefore constant relative risk aversion) with coefficient of risk aversion $\eta \in (0, 1)$, i.e. $U(r) = \frac{r^{1-\eta}}{1-\eta}$.[11] $\delta > 0$ is the insiders' subjective discount rate. Recall the index $j \in \{s, b, o, i\}$ for labeling the four regimes under consideration which are asset sales, liquidation, bailout and bail-in

[11] We do not explicitly consider $\eta \geq 1$ in our analysis (the special case of $\eta = 1$ corresponds to log utility). When $\eta \geq 1$, insiders' utility goes to negative infinity as the rents extracted approach zero. Insiders therefore avoid insolvency at all costs, and always adopt safe debt.

respectively. Let T_1 be the random arrival time of the first crash. The general form of insiders' (managers') optimization problem under regime j is:

$$M_j(N) = \max_{q_t, l_t, f} E\left[\int_0^{T_1} e^{-\delta t} U(q_t N_t) dt + e^{-\delta T_1} p_j M_j(\phi_j(l_{T_1-}, f) N_{T_1-}) \Big| N_0 = N\right] \qquad (7)$$

subject to $0 \leq f \leq 1$, the (in)solvency constraint $l_t \leq (>) 1/f$ when $j = s$ (b, o, i), the intertemporal budget constraint:

$$\frac{dN_t}{N_t} = [(\mu + \kappa\lambda f - \rho_j(l_t, f))l_t + \rho_j(l_t, f) - mq_t]dt + \sigma l_t dB_t \quad \text{for } t < T_1 \qquad (8)$$

and the transversality condition $\lim_{t\to\infty} E\left[e^{-\delta t} M_j(N_t)\right] = 0$.

Equation (7) implicitly defines managers' claim value M_j which consists of two components. The first component is the expected discounted utility of rents extracted up to the arrival of the first crash. The second component reflects the residual claim value to the managers after a shock has realized, and can be understood as a continuation value originating from the dynamic programming principle. As we explain next, the residual claim value depends on (1) the probability p_j of managers having a continuation claim and (2) managers' net worth recovery rate $\phi_j(l_{T_1-}, f)$ following a shock. Since a shock causes net worth to shrink and some restructuring mechanisms may further dilute managers' stake, it is the case that $0 \leq \phi_j < 1$.

When solving the optimization problem (7), the only modeling specifications required at this stage are that under each regime $\rho_j(l, f)$, $\phi_j(l, f)$ and $\Phi_j(l, f)$ are functions of the gearing ratio l and the loan quality f only. In Subsection 2.2, we describe each regime in detail and give the corresponding expressions for $\rho_j(\cdot, \cdot)$, $\phi_j(\cdot, \cdot)$, and $\Phi_j(\cdot, \cdot)$. Table 1 in Section 2.2.5 provides a summary of the definitions.

Managers solve for optimal policies and the corresponding claim values under the solvency and insolvency regimes respectively. They compare $M_s(N)$ and $M_j(N)$ (where $j = b, o$, or i is given) and ultimately adopt the policies that maximize their private value. This completes the formulation of managers' optimization problem. In what follows we solve for managers' optimal rents (q_t) policy, the optimal gearing ratio (l_t) and the optimal jump risk exposure (f) under the various restructuring mechanisms. We assume that the IRM is exogenously given. In other words, managers know ex ante whether insolvency will be resolved through liquidation, bailout or bail-in.

The proposition below gives a general characterization of the managers' optimal policies. Each restructuring mechanism is explored in detail in the subsequent sections. We define the following functions, which allow us conveniently to characterize the solutions to the managers' problem throughout the paper:

$$C(H) \equiv \left[\frac{\eta}{\lambda+\delta-(1-\eta)H}\right]^{\eta} m^{-(1-\eta)} \tag{9}$$

$$Q(H) \equiv \left(\frac{1}{mC(H)}\right)^{\frac{1}{\eta}} = \frac{\lambda+\delta-(1-\eta)H}{m\,\eta} \tag{10}$$

Proposition 1 *The optimal investment (A_t), payout to insiders (r_t), debt (D_t), crash risk exposure (f) and insiders' life-time utility (M_j) are, respectively, $A_t = l_j N_t$, $r_t = q_j N_t$, $D_t = (l_j - 1)N_t$, $f = f_j$, and $M_j(N_t) = \frac{c_j N_t^{1-\eta}}{1-\eta}$, where $c_j = C(H_j)$, $q_j = Q(H_j)$ (with $C(\cdot)$ and $Q(\cdot)$ given by* (9) *and* (10)*). l_j and f_j are the constants solving the optimization problem:*

$$\max_{l,f} G_j(l,f) \equiv \max_{l,f}\left\{[\mu+\kappa\lambda f-\rho_j(l,f)]l-\frac{\eta\sigma^2}{2}l^2+\rho_j(l,f)+\frac{\lambda p_j}{1-\eta}[\phi_j(l,f)]^{1-\eta}\right\} \tag{11}$$

and $H_j \equiv G_j(l_j, f_j)$. The subscript j takes value of either s (when it is optimal for the bank to stay solvent and the policy space in problem (11) *is restricted to $l \leq 1/f$) or $\{b, o, i\}$ (when it is optimal to put the bank at risk of insolvency and the policy space in problem* (11) *is restricted to $l > 1/f$).*

Proposition 1 states the bank's optimal investment, loan quality decision, payout and financing policy conditional on a particular restructuring mechanism j being adopted (where $j = b, o, i$ or s). The general structure of the optimal policies is the same across mechanisms. The bank's optimal loan portfolio size A_t is a constant multiple l_j of the bank's net worth N_t. We define and examine this constant l_j as well as the optimal loan quality determinant f_j in subsequent sections. Payout to insiders ("rents") and to outsiders ("dividends") are a constant fraction of net worth (i.e. $r_t = q_j N_t$ and $d_t = \alpha m q_j N_t$). Since the bank's net worth is determined by the sum of the bank's initial capital and its cumulative retained earnings, dividends are smooth and relatively insensitive to shocks in current income, except when there is a crash that reduces the bank's asset base and net worth by a discrete amount. The bank follows a constant debt to net worth ratio, which is given by $D_t/N_t = l_j - 1$. Finally, the private value of insiders' claim M_j is a concave increasing function of the bank's net worth N_t. The degree of concavity increases with insiders' coefficient of risk aversion η.

Proposition 1 does not tell us whether it is optimal to restructure through asset sales or whether it is optimal to put the bank at risk of insolvency instead. We consider this question in Section 4, but first define the various restructuring mechanisms.

2.2 Definitions of the restructuring mechanisms

We now formally define asset sales, liquidation, bailout and bail-in. A restructuring mechanism j at time T_1 is fully characterized by the following four elements: (1) the adjustment in net worth due to the restructuring, denoted by $\Phi_j(l, f)$, (2) insiders' continuation probability p_j under mechanism j, (3) insiders' net worth recovery rate, $\phi_j(l, f)$, and (4) lenders' recovery rate on the bank debt, $\Omega_j(l, f)$.

We now define each mechanism in turn. We adopt what we believe to be plausible assumptions regarding the specifications for Φ_j, ϕ_j and Ω_j. It should be clear, however, that our framework is sufficiently flexible and general to accommodate different assumptions.

2.2.1 Asset Sales

Under the asset sales regime, net worth remains positive following a crash, and the bank optimally delevers by selling some assets and using the proceeds to pay off debt. Net worth drops by a factor of $\Phi_s(l_t, f) = 1 - fl_t$ as in (4). Since the bank is always solvent, lenders incur no losses, i.e. $\Omega_s(l_t, f) = 1$, and therefore debt is safe ($\rho_s = \rho$). Managers are sure to continue, i.e. $p_s = 1$, but suffer a fractional loss on their net worth in the bank as reflected by the recovery rate $\phi_s(l_t, f) = 1 - fl_t$. The asset sale restores the bank's optimal gearing and allows managers to carry on as before, but with a reduced amount of net worth. Substitution of these expressions into (7) completes the formulation of the dynamic programming problem.

2.2.2 Liquidation

Since the bank is insolvent and not recapitalized in the liquidation regime, it follows that $\Phi_b(l_t, f) = 0$. Lenders receive the value of the assets in liquidation, $(1 - f)A_t$.[12] Inside (and outside) equityholders get nothing. Therefore, $\phi_b(l_t, f) = 0$ and $p_b = 0$. The amount of debt prior to default is $D_t = A_t - N_t = (l_t - 1)\, N_t$. Therefore, lenders' recovery rate on bank debt is $\Omega_b(l_t, f) = (1 - f)A_t/D_t = (1 - f)l_t/(l_t - 1)$. Assuming the debt is priced competitively by risk neutral lenders, the bank faces the following after-tax interest rate on its debt:

$$\rho_b(l_t, f) \;=\; \rho + \lambda(1-\tau)\,[1 - \Omega_b(l_t, f)] \;=\; \rho + \lambda(1-\tau)\left(\frac{fl_t - 1}{l_t - 1}\right) \tag{12}$$

[12] We assume there are no liquidation costs to allow for a clean comparison with bail-ins and bailouts. Introducing liquidation costs is straightforward. For example, with a proportional liquidation cost c_b the net proceeds from liquidation to bondholders are $A_t(1 - f)(1 - c_b)$.

2.2.3 Bailout

Recall that, following a crash, assets drop from A_t to $(1-f)A_t$. We assume that under the bailout regime, the government recapitalizes the entire bank which now has assets amounting to $(1-f)A_t$.[13] If the optimal asset to net worth ratio l_o is constant (which we prove below) then the bank's net worth before and after the bailout are, respectively, A_t/l_o and $(1-f)A_t/l_o$. It follows that the bank's net worth drops by a factor $1-f$, and therefore $\Phi_o(l_t, f) \equiv 1-f$ in the budget constraint (6).

In return for recapitalizing the bank the government receives equity alongside existing shareholders whose share is diluted by a factor $\xi_o(\leq 1)$.[14] The fraction ξ_o depends, for instance, on existing outside shareholders' bargaining power and how crucial it is for the government to save the bank to avoid negative externalities for the wider economy. We take ξ_o as exogenously given. Lenders' (bondholders and depositors) claims are protected, and therefore $\Omega_o(l_t, f) = 1$. This implies that debt is risk free and therefore $\rho_o(l_t, f) \equiv \rho$.[15]

We assume the government appoints new managers that replace some existing managers and dilute the stake of those managers that survive. Managers survive a bailout with some probability $p_o \in [0,1]$ and their stake in the inside equity is diluted by a factor $\xi_o(\leq 1)$, giving a net worth recovery rate of insiders upon continuation of $\phi_o(l_t, f) = (1-f)\xi_o$. The effect of dismissal and stake dilution upon the arrival of a

[13] Our results are qualitatively the same if only a fraction of the bank is recapitalized, with the remaining assets being liquidated.

[14] E.g. during the recent financial crisis the UK government bailed out the Royal Bank of Scotland and Lloyds Banking Group and acquired an equity stake of 72% and 43%, respectively.

[15] The model could be generalized by assuming that bailouts occur with a probability less than 1, and that liquidation is the alternative to a bailout. The model could also be extended to allow for the possibility that unsecured bondholders are not bailed out. The government should guarantee the claim of all depositors, though. If some deposits are unprotected then depositors may rush to get their money out of the bank after a shock because the last depositors in the queue are left with nothing.

shock can be captured by a random variable X which takes on value ξ_o with probability p_o or value 0 otherwise. In the former case insiders still have a claim after the shock, albeit a reduced one as reflected by the factor $\xi_o \leq 1$. In the latter case insiders lose their claim entirely.

Managers' optimization problem under the bailout regime can be stated as:

$$M_o(N) = \max_{q_t, l_t, f} E\left(\int_0^{T_1} e^{-\delta t} U(q_t N_t) dt + \sum_{k=1}^{\infty} \int_{T_k}^{T_{k+1}} e^{-\delta t} U(\Lambda_k q_t N_t) dt \middle| N_0 = N \right) \quad (13)$$

T_k is the random arrival time of the k^{th} crash. $\Lambda_k \equiv \prod_{n=1}^{k} X_n$ is the cumulative dismissal-adjusted dilution factor after k shocks have arrived. $X_n \sim X$ are i.i.d random variables independent of the net worth dynamics.

Due to the power form of the utility function, and X_n and N_t being independent, (13) can be expressed as:

$$M_o(N) = \max_{q_t, l_t, f} E\left(\int_0^{T_1} e^{-\delta t} U(q_t N_t) dt + \sum_{k=1}^{\infty} \int_{T_k}^{T_{k+1}} e^{-\delta t} [p_o \xi_o^{1-\eta}]^k U(q_t N_t) dt \middle| N_0 = N \right) \quad (14)$$

The second term in Equation (14) corresponds to the term $e^{-\delta T_1} p_j M_j(\phi_j(l_{T_1 -}, f) N_{T_1 -})$ in the managers' general optimization problem (7).

In summary, insolvency resolution through bailout differs from liquidation in several ways. First, the capital injection in a bailout ensures the bank's continuation rather than its liquidation. Second, following a bailout managers are able to stay in post with some probability $p_o \in [0, 1]$, albeit with a reduced net worth stake $\phi_o = (1 - f)\xi_o$. Third, the government's promise to bail out insolvent banks, guarantees lenders' claim D, and therefore debt is risk-free ($\rho_o = \rho$).

2.2.4 Bail-in

In a bail-in, the claims of the creditors of the failed bank are written down and converted into equity in order to absorb the losses and recapitalize the bank. A bail-in is not negotiated (it is imposed upon the firm and its creditors by the authority responsible for resolution). The bail-in not only significantly changes the ownership structure of the firm but may coincide with restructurings (e.g. splitting up the bank) that alter the bank's investment and payout policy. Unfortunately, it is not known in advance exactly how the resolution authority will restructure the bank. This poses a real challenge for pricing bail-in bonds. We do not attempt to model the restructuring process but take its outcome as exogenously given. In particular, we assume that the optimal asset to net worth ratio after the bail-in is l^*. The corresponding market to book value is assumed to be w^*, which means that the market value of the total (inside plus outside) equity after the bail-in is given by: $F_t^+ \equiv w^*(1-f)A_t/l^*$.

As with the bailout we assume that just enough debt is converted into equity to achieve the optimal gearing ratio l^* of risky assets to net worth. To avoid a run on the bank when a crash occurs, the bank needs at least two classes of debt: secured, senior debt (e.g. deposits) that is protected and will not be bailed in, and unsecured, junior debt (e.g. long term bonds) that can be converted into equity. The amounts of senior and junior debt are denoted by D_1 and D_2, respectively, with the total amount of debt equal to $D = D_1 + D_2$.

To avoid a bank run, full protection is given to senior creditors. Therefore the amount of junior debt must (at least) equal the amount of debt that has to be bailed in. Assuming that the bank has a constant optimal gearing ratio l_i prior to the bail-in (a claim we verify below) the amount of debt prior to the shock is: $D = A - N = l_iN - N = (l_i - 1)A/l_i$. The bank's assets after the shock are $A^+ = (1-f)A$. Hence, the optimal amount of debt and net worth after the bail-in are, respectively: $D^+ = (l^* - 1)(1-f)A/l^*$ and $N^+ = (1-f)A/l^*$. To enable the bail-in we therefore require that:

$$D_1 \;=\; A\left[\frac{(l^*-1)(1-f)}{l^*}\right] \quad \text{and} \quad D_2 = D - D_1 = A\left[\frac{l_i-1}{l_i} - \frac{(l^*-1)(1-f)}{l^*}\right]$$

To recapitalize the bank, the bail-in forcibly converts unsecured debt into equity and dilutes existing equityholders. We assume that unsecured lenders and existing equityholders (inside as well as outside) receive, respectively, a fraction $1-\xi_i$ and ξ_i of the firm's equity. Secured lenders (e.g. depositors) are fully protected and earn a rate of interest ρ'. Assuming lenders are risk neutral and debt is issued competitively, the after-tax cost of debt is given by:

$$\begin{aligned} \rho_i(l_i,f) \;&=\; \frac{D_1}{D}\,\rho'(1-\tau) \;+\; \frac{D_2}{D}\left\{\rho'(1-\tau)+\lambda(1-\tau)\left[1-\frac{(1-\xi_i)w^*(1-f)A/l^*}{D_2}\right]\right\} \\ &=\; \rho \;+\; \lambda(1-\tau)\left[1-\frac{(1-f)l_i(1+h)}{l_i-1}\right] \quad \textit{where} \;\; h \;\equiv\; \frac{(1-\xi_i)w^*-1}{l^*} \end{aligned} \tag{15}$$

Or equivalently, the recovery rate on the total bank debt is:

$$\Omega_i(l_i,f) \;=\; \frac{D_1 \;+\; (1-\xi_i)w^*(1-f)A/l^*}{D} \;=\; \frac{(1-f)(1+h)l_i}{l_i-1} \tag{16}$$

Bail-ins only make economic sense if the junior debt is risky. This requires that $\rho_i(l_i,f) > \rho$, or equivalently:

$$\rho_i(l_i,f) > \rho \iff h \;<\; \frac{fl_i-1}{(1-f)l_i} \iff l_i \;>\; \frac{1}{(1+h)f-h} \equiv \hat{l}_i(f;h) \tag{17}$$

We verify later (see Proposition 4) that junior debt is indeed risky.

For bail-ins to go through, no creditor or shareholder should be worse off under the bail-in compared to what he or she would get under a hypothetical liquidation scenario (this is the so-called "No Creditor Worse off than under Liquidation" (NCWOL) test of the Bank Recovery and Resolution Directive (BRRD) in the European Union). The payoff to junior creditors in liquidation equals $(1-f)A - D_1 \;=\; (1-f)A/l^*$, whereas their payoff in a bail-in equals $(1-\xi_i)w^*(1-f)A/l^*$. Consequently, a bail-in is acceptable to junior creditors if and only if $1 \le (1-\xi_i)w^*$, or equivalently if $h \ge 0$.

The net worth process under the bail-in regime is given by:

$$\frac{dN_t}{N_t} = [(\mu + \lambda\kappa f - \rho_i(l_t, f))\, l_t \; + \rho_i(l_t, f) \; - mq_t]\, dt + \sigma l_t dB_t + \left[(1-f)\frac{A_t/N_t}{l^*} - 1\right] dy_t$$

$$\equiv g_i(l_t, q_t, f)\, dt \; + \sigma l_t dB_t \; + (\Phi_i(l_t, f) - 1)\, dy_t \;\; \text{for } l_t \; > \; 1/f \tag{18}$$

Managers' objective function under the bail-in regime becomes:

$$M_i(N) \;=\; \max_{q_t, l_t, f} E\left(\int_0^{T_1} e^{-\delta t} U(q_t N_t) dt + e^{-\delta T_1}\, p_i\, M_i((1-f)\xi_i N_{T_1 -})\Big| N_0 = N\right) \tag{19}$$

which exactly has the general form of Equation (7). Managers have a continuation claim with probability $p_i \in [0, 1]$ and their net worth stake is diluted by a factor $\xi_i (< 1)$.[16]

Note that $\phi_i(l_t, f) \equiv (1-f)\xi_i$. Therefore, $\phi_i(l_t, f)$ does not equal $\Phi_i(l_t, f) \equiv (1-f)l_i/l^*)$, i.e. managers' continuation or severance claim (implicitly defined by Equation (19)) relates to the bank's net worth after the shock, but assuming the bank's original gearing l_i is maintained. Since a bail-in typically reduces leverage ($l_t/l^* > 1$) linking managers' continuation or severance claim to the post bail-in gearing l^* would allow managers to freeride on unsecured creditors.

2.2.5 Summary of definitions

Table 1 below summarizes our assumptions regarding Φ_j, p_j, ϕ_j and Ω_j across the four restructuring mechanisms. Figure 1 illustrates the effect of each restructuring mechanism on the bank's balance sheet. The figure shows the bank's balance sheet 1) before the crash, 2) immediately after the crash, and 3) after the restructuring. Under

[16]The BRRD stipulates that management should in principle be replaced following a bail-in, unless their expertise is crucial for the restructuring. If managers are sure to be replaced then $p_i M_i$ could be interpreted as managers' severance claim consisting of pension rights and other outstanding contractual payments. In that case p_i is not a probability, but to be interpreted as the recovery rate for their claim in the firm.

liquidation all assets are sold off. Notice how also the asset sale regime leads to a significant contraction in the firm's assets. With bailouts and bail-ins the restructuring focuses on the bank's liabilities.

IRM j	$\Phi_j(l,f)$	p_j	$\phi_j(l,f)$	$\Omega_j(l,f)$
Asset sales $j=s$	$1-fl$	$p_s=1$	$1-fl$	1
Liquidation $j=b$	0	$p_b=0$	0	$\frac{(1-f)l}{l-1}$
Bailout $j=o$	$1-f$	$0\leq p_0\leq 1$	$(1-f)\xi_o$	1
Bail-in $j=i$	$\frac{(1-f)l}{l^*}$	$0\leq p_i\leq 1$	$(1-f)\xi_i$	$\frac{(1-f)(1+h)l}{l-1}$

Table 1: Summary of definitions of different IRMs. Φ_j is the net worth adjustment due to restructuring, p_j is insiders' continuation probability, ϕ_j is insiders' net worth recovery rate, Ω_j is lenders' recovery rate on bank debt and ξ_j is the dilution factor of insiders' equity stake.

3 Optimal policies without insolvency: asset sales

Under the asset sale regime banks do not become insolvent. Following a crash, banks sell off assets to pay down debt and to delever. The following proposition characterizes the bank's optimal gearing ratio (l_s) and its optimal jump risk exposure (f_s). For the bank to take on debt, it is necessary that the Merton ratio is larger than one. We therefore impose the standing assumption $(\mu-\rho)/(\sigma^2\eta)>1$ throughout the rest of the paper. Some of our results presented in the next section require a higher Merton ratio. Any additional assumptions will be explicitly stated when needed.

Proposition 2 *If the bank has to stay solvent in crashes then the optimal asset to net worth ratio, l_s, is (implicitly) given by:*

$$l_s \;=\; \frac{\mu+\kappa\lambda f_s-\rho-\frac{f_s\lambda}{(1-f_s l_s)^\eta}}{\eta\sigma^2} \quad \textit{with} \;\; l_s < \frac{1}{f_s} \tag{20}$$

The assets' optimal exposure to crashes, f_s, and corresponding investment level, l_s, are:

$$f_s \equiv \frac{\sigma^2\eta}{\mu-\rho}\left(1-\kappa^{-\frac{1}{\eta}}\right) < 1 \quad \textit{and} \quad l_s(f_s) = \frac{\mu-\rho}{\sigma^2\eta}$$

Whenever the bank's asset base drops by a factor $(1-f_s)$ from A to $(1-f_s)A$ due to a crash, the bank's net worth drops by a factor $(1-f_s l_s)$. The bank restores the optimal asset to net worth ratio, l_s, by selling an amount of assets equal to $Af_s(l_s-1)$ and using the proceeds to pay off debt.

According to the bank's optimal investment policy the amount of risky loans issued (A_t) equals a constant multiple l_s of the bank's net worth (N_t). In the absence of crashes, net worth follows a geometric Brownian motion under the optimal investment and payout policies. Therefore, absent jumps, net worth always stays positive. The optimal value for l_s under the asset sales regime is always strictly less than $1/f_s$ to ensure the bank remains solvent also when a crash occurs.[17] Nevertheless, leverage amplifies the effect of a loss in the firm's loan portfolio on the bank's net worth and asset base. Consider a levered firm with $A = 100$, $N = 20$ and suppose $l_s = 5$ and $f_s = 0.1$. A 10% loss in assets due to a crash reduces net worth by a factor $1 - f_s l_s = 0.5$ from 20 to 10. This causes the asset to net worth ratio to jump to $l = 90/10 = 9$, making the bank too risky. Managers rebalance by selling off an amount $Af_s(l_s - 1) = 40$ in loans, and using the proceeds to pay off debt, restoring the asset to net worth ratio to its optimal level $l_s = 50/10 = 5$. The example illustrates how leverage amplifies contractions in the bank's balance sheet following losses in its loan portfolio. An initial loss $f_s A$ of the bank's loans leads to a subsequent loan sale of $(l_s - 1)f_s A$. This is illustrated in panel A of Figure 1.[18]

[17] It is impossible for a levered bank to maintain solvency if f is too high. For example, when $f = 1$ the bank's asset value drops to zero in a crash, and banks with debt become insolvent. We prove in the internet appendix that there exists a critical level of jump size f above which a bank can no longer optimally take on debt if it prefers to stay solvent during a crash. See the discussion in Section 4.

[18] Asset sales happen in a frictionless manner in our model. The internet appendix shows that proportional transaction costs associated with selling assets after a jump reduce the level of the optimal

The optimal exposure to crashes increases in the premium κ associated with jump risk. A higher expected return μ and lower volatility σ associated with the diffusion risk reduce f_s. Under the optimal policies the fraction of net worth at risk in a crash equals $f_s\, l_s = 1 - \kappa^{-\frac{1}{\eta}}$. Consequently, as we approach risk neutrality (i.e. $\eta \to 0$) close to 100% of the bank's net worth is at risk if $\kappa > 1$. Risk averse managers, on the other hand, put significantly less net worth at stake. As the risk premium associated with crash risk disappears ($\kappa \to 1$) the bank's optimal exposure to crashes goes to zero ($f_s \to 0$), i.e. insiders issue loans of the highest quality.

Equation (20) is the first-order condition with respect to l_s, and allows us to analyze how the optimal investment varies as a function of f when the asset sale regime prevails. No closed-form solution for l_s exists, except for the cases $\lambda = 0$, $f = 0$, or $f = f_s$, for which we obtain the standard Merton (1969) investment policy. The multiple l_s increases with the excess return $\mu - \rho$ and decreases with volatility σ and insiders' risk aversion η. The optimal asset to net worth ratio l_s for the optimal jump exposure f_s equals exactly the Merton (1969) investment policy. Hence, the optimal level of investment declines with managers' risk aversion. Furthermore, under managers' optimal exposure f_s the optimal investment policy is independent of the frequency λ with which crashes occur. This highlights that the gearing ratio l_s can be very different depending on whether or not insiders control the bank's risk exposure.

4 Optimal policies with insolvency

The solution developed in previous section assumes that the bank must remain solvent in crashes. The optimal gearing ratio, l_s, is therefore strictly less than $\frac{1}{f_s}$. From managers' point of view it is, however, not always optimal to adopt an investment and

asset to net worth ratio. The qualitative properties associated with the bank's optimal policies remain, however, largely the same.

payout policy that guarantees the bank's solvency in crashes.

Let us assume for a moment that f is exogenously given, and consider the polar cases of highest ($f = 0$) and lowest ($f = 1$) loan quality. For $f = 0$ (crashes cause zero losses in the loan portfolio), the problem collapses to the standard Merton (1969) problem. One can verify that l_s indeed coincides with the Merton solution for $f = 0$. The solvency constraint $l_s < \hat{l} \equiv 1/f$ does not constrain the gearing ratio since $\hat{l} = +\infty$ for $f = 0$. Therefore, the bank's policy l_s is optimal for zero jump exposure. The bank remains solvent at all times given that net worth follows a geometric Brownian motion. Intuitively, it is clear that the asset sale policy remains optimal for sufficiently small f, because the solvency constraint $l_s < \hat{l} = \frac{1}{f}$ is unlikely to matter when $\hat{l}$ is large.

Consider next the polar case $f = 1$ (i.e. crashes cause a total loss in the loan portfolio). To remain solvent in crashes the bank should now adopt a negative debt level (i.e. net cash position) by setting $l_s < \hat{l} = 1$. It is intuitively clear that such a conservative investment policy is unlikely to be optimal, particularly if crashes are rare (small λ). Instead the bank may wish to adopt a more aggressive investment policy and accept the specter of insolvency in crashes.

We know that managers' desire to expose loans to jump risk is driven by the reward κ for taking on crash risk. A higher reward κ induces higher exposure f. The following proposition proves that there exists a critical reward threshold $\underline{\kappa}_j$ such that for $\kappa \leq \underline{\kappa}_j$ managers prefer to issue high quality loans to keep the bank solvent at all times, whereas for $\kappa \geq \underline{\kappa}_j$ managers prefer to issue low quality loans such that the bank may become insolvent and undergo IRM j ($j \in \{b, o, i\}$).

Proposition 3 *There exists a critical risk premium $\underline{\kappa}_j \geq 1$ for IRM j (j=b,o,i) such that managers keep the bank solvent by adopting low leverage and issuing high quality loans if the reward for taking on crash risk exposure is sufficiently low ($1 \leq \kappa < \underline{\kappa}_j$). Managers put the bank at risk of insolvency by adopting high leverage and issuing* *low quality loans if the rewards are sufficiently high (i.e. $\kappa \geq \underline{\kappa}_j$). For the safe regime ($1 \leq \kappa < \underline{\kappa}_j$) the bank optimally adopts the policies l_s and f_s as described in Proposition 2.*

We now consider three IRMs (liquidation, bailout and bail-in) and examine how they affect the bank's optimal level of investment l and jump risk exposure f. The prevailing IRM is common knowledge. In what follows we impose the following parameter restrictions for, respectively, the bailout and bail-in cases:

$$\frac{\mu - \rho}{\sigma^2 \eta} > 1 + \frac{p_o \xi_o^{1-\eta}}{\kappa} \tag{21}$$

$$\frac{\mu - \rho}{\sigma^2 \eta} > 1 + \frac{p_i \xi_i^{1-\eta}(1+h)}{\kappa - (1+h)(1-\tau)} \quad \text{and} \quad \kappa > (1+h)(1-\tau) \tag{22}$$

Conditions (21) and (22) ensure that managers' objective function ($M_j(N)$) has a unique interior maximum ((l_j, f_j)) under the bailout and bail-in regimes, respectively, when managers have a strictly positive probability of continuation and strictly positive residual equity stake after restructuring (i.e. if $p_o\xi_o, p_i\xi_i > 0$).

Proposition 4 *For the insolvency regime (i.e. $\kappa \geq \underline{\kappa}_j$), the optimal investment policy (l_j) under the liquidation, bailout and bail-in regime is:*

$$l_b(f_b) = \frac{\mu - \rho}{\eta \sigma^2} + \frac{[\kappa - (1-\tau)]\, \lambda f_b}{\eta \sigma^2} \quad (liquidation) \tag{23}$$

$$l_o(f_o) = \frac{\mu - \rho}{\eta \sigma^2} + \frac{\kappa \lambda f_o}{\eta \sigma^2} \quad (bailout) \tag{24}$$

$$l_i(f_i) = \frac{\mu - \rho}{\eta \sigma^2} + \frac{[\kappa - (1-\tau)]\lambda f_i + \lambda h (1-\tau)(1-f_i)}{\sigma^2 \eta} \quad (bail-in) \tag{25}$$

If managers have a zero continuation probability or zero residual equity stake ($p_j\xi_j = 0$), then they adopt maximum crash risk exposure ($f_j = 1$). Since managers have no claim in liquidation ($p_b = 0$) they adopt maximum exposure under the liquidation regime, i.e. $f_b = 1$. If $p_o\xi_o, p_i\xi_i > 0$, then the optimal exposure level under bailout or bail-in is given *by some* $f_o, f_i \in (0,1)$ *which is the unique solution to the equation*

$$l_o(f_o) \;-\; \frac{p_o \xi_o^{1-\eta}}{\kappa(1-f_o)^\eta} = 0 \qquad (for\ bailouts) \tag{26}$$

$$l_i(f_i) \;-\; \frac{p_i \xi_i^{1-\eta}}{[\kappa-(1+h)(1-\tau)](1-f_i)^\eta} = 0 \quad (for\ bail-ins) \tag{27}$$

Junior debt is always risky under bail-ins.

The asset to net worth ratio l exceeds the Merton solution ($l = (\mu - \rho)/(\eta\sigma^2)$) under all three IRMs. Since firms adopt the Merton investment policy under the asset sale regime (see Proposition 2), it follows there is a discrete upward jump in investment and leverage when we move at $\underline{\kappa}_j$ from the asset sale regime to one of the three IRMs. This discrete increase in loan issuance coincides with a drop in loan quality. Under the liquidation regime banks even adopt the maximum possible risk exposure (f_b = 1). Given managers' limited liability and zero payoff in liquidation, they do not care whether the bank ends up insolvent a little or a lot. This creates a serious moral hazard problem from lenders' viewpoint.

Importantly, high (low) crash risk exposure is combined with high (low) leverage. In particular, if the reward for crash risk is low ($\kappa < \underline{\kappa}_j$) then we obtain an equilibrium where banks issue a low volume of high quality loans. If jump risk premiums are high ($\kappa > \underline{\kappa}_j$) banks issue high volumes of low quality loans, which is a toxic combination of high financial risk (leverage) and high business risk (exposure to crashes).

Banks do not adopt maximum crash risk exposure under the bailout and bail-in regime (i.e. $f_o, f_i < 1$) if managers retain some "skin in the game" after the restructuring (i.e. if $p_o\xi_o, p_i\xi_i > 0$) because doing so would wipe out all the firm's assets and leave no bank to be bailed out (or to be bailed-in). Only when managers are sure to lose everything ($p_o\xi_o, p_i\xi_i = 0$) do they adopt 100% exposure (i.e. $f_o = f_i = 1$) to crashes. Using Eq. (26) and Eq. (27) one can show that the optimal exposure to crashes (f_o and f_i) increases with μ, λ and κ, and decreases with ρ, σ, η and insiders' dilution adjusted probability of continuation ($p_o{\xi_o}^{1-\eta}$ and $p_i{\xi_i}^{1-\eta}$).

4.1 A comparison of the insolvency resolution mechanisms

Previously, we showed that managers choose a low (high) loan volume and a low (high) jump exposure level if the reward κ for taking on crash risk is below (above) a critical threshold $\underline{\kappa}_j$ that depends on the prevailing IRM. If κ varies across banks (due to differences in banks' operating efficiency or loan selection skills), and follows some distribution, then all banks with a κ below (above) $\underline{\kappa}_j$ are (in)solvent following a crash. As such, an IRM with a lower threshold $\underline{\kappa}_j$ leads to a higher insolvency rate in the banking industry.

In what follows we compare the critical thresholds $\underline{\kappa}_j$, the cost of debt, the bank's optimal investment and payout policies, its exposure to crashes, and managers' claim value across the three IRMs.

Proposition 5 *If the parameters are such that conditions* (21) *and* (22) *hold, then we have the following comparison results across different IRMs where* $v_j \equiv p_j \xi_j^{1-\eta}$ *denotes the dilution-adjusted continuation probability of insiders:*

i) Loan quality is highest (lowest) under the bail-in (liquidation) regime, i.e.:
$f_b \geq f_o \geq f_i$ *if either* $v_i \geq v_o$*, or* $v_o > v_i$ *and* $(1+h)(1-\tau) < \kappa < \frac{v_o(1+h)(1-\tau)}{v_o - v_i}$.

ii) The cost of bank debt is highest (lowest) under the liquidation (bailout) regime:
$\rho_b(l_b, f_b) > \rho_i(l_i, f_i) \geq \rho_o(l_o, f_o) = \rho$

iii) The asset to net worth ratio is highest (lowest) under the bailout (bail-in) regime:
$l_o(f_o) \geq l_b(f_b) \geq l_i(f_i)$

iv) Managers' claim value is highest (lowest) under the bailout (liquidation) regime:
$M_o(N) > M_i(N) > M_b(N)$ *if* $v_o \geq v_i$

v) The payout yield is highest (lowest) under the liquidation (bailout) regime:
$q_b > q_i > q_o$ *if* $v_o \geq v_i$

vi) The critical crash risk premium above which managers put the bank at risk of insolvency is highest (lowest) under the liqidation (bailout) regime, i.e.:
$\underline{\kappa}_b > \underline{\kappa}_i > \underline{\kappa}_o$ *if* $v_o \geq v_i$

i) Managers face a tradeoff when setting the optimal loan quality. On the one hand, a high crash exposure f (i.e. low loan quality) improves the risk-adjusted performance of the leveraged equity.[19] On the other hand, managers' residual claim value after a resolution is proportional to $(1-f)^{1-\eta}$ and as such a low f preserves a larger fraction of residual value. For example, the choice of $f = 1$ delivers the best risk-adjusted performance of the leveraged equity but at the same time the assets are completely wiped out in a crash leaving managers with nothing in the aftermath. The tradeoff depends on the managers' (dilution-adjusted) continuation probability v_j which acts as a weighting attached to the residual claim value.

If managers anticipate an insolvent bank is always liquidated then there is no trade-off involved. They simply expose 100% of the firm's assets to crash risk for maximum return (i.e. $f_b = 1$). Under bailout with $v_o > 0$, however, the residual claim and managers' infinite marginal utility near zero provide them with an incentive to keep some "skin" in the game. This explains why $f_o < 1 = f_b$ (and similarly why $f_i < 1$ for bail-in). Therefore, the liquidation IRM leads to the lowest loan quality.

The comparison of f_o and f_i is more subtle and crucially depends on v_o and v_i. Consider first the case $v_i \geq v_o$. Increasing f generates a higher risk adjusted performance of the equity under bailouts than bail-ins. If $v_i \geq v_o$ the managers have a stronger incentive to retain a residual claim under bail-ins than under bailouts. Hence, the trade-off that determines the optimal f unambiguously implies that $f_i \leq f_o$.

[19]The relevant criterion here is: $(\mu + \kappa\lambda f - \rho_j)l_j + \rho_j - \frac{\eta\sigma^2}{2}l_j^2$ which resembles the classical mean-variance performance measure adopted by a risk averse agent. In the cases of liquidation and bail-in, a higher f increases both the loan return and cost of debt, but the net effect on the performance measure is positive at the optimally chosen l_j.

The intuition is less clear when $v_o > v_i$. Bailouts favor a higher f than bail-ins in terms of risk-adjusted performance but a lower f in terms of their residual claim value (since $v_o > v_i$). The ranking of f_i and f_o becomes ambiguous in general, but $f_i < f_o$ still holds under an additional condition $\kappa < \frac{v_o(1+h)(1-\tau)}{(v_o - v_i)}$ which can be restated as $\frac{v_o - v_i}{v_i} < \frac{(1+h)(1-\tau)}{\kappa}$. Hence, $f_i < f_o$ as long as v_o does not exceed v_i too much.

ii) The cost of debt is highest in the liquidation regime, because (1) lenders only receive the proceeds from liquidation and (2) there are no liquidation proceeds (since $f_b = 1$). In a bail-in lenders acquire a claim on the assets of the restructured bank. Creditors are strictly better off than under the liquidation regime because (1) $h > 0$ and (2) the bailed-in bank has a positive asset base ($f_i \leq 1$). Therefore, $\rho_i(l_i, f_i) < \rho_b(l_b, f_b)$. Finally, debt is risk free in the bailout regime, and therefore $\rho_o = \rho$.

iii) Loan issuance (i.e. the asset to net worth ratio) is highest under the bailout regime because of the low cost of debt ($\rho_o = \rho$). Next, the amount of loans issued under the liquidation regime is higher than under the bail-in regime. Even though the cost of debt is highest under the liquidation regime, crash risk exposure is much higher under the liquidation regime ($f_b = 1$) than under the bail-in regime. Since investment increases in f, the higher risk exposure f under the liquidation regime dominates and causes investment to be higher under liquidation than bail-in (i.e. $l_b(f_b) \geq l_i(f_i)$).

iv) Consider next managers' claim value M_j. Managers are best (worst) off if insolvencies are resolved through bailouts (liquidation). Under the liquidation regime managers get nothing when the bank becomes insolvent. Under a bail-in, the restructuring creates extra value to creditors. This reduces the cost of debt. Furthermore, under a bail-in the bank carries on as a going concern, which creates space for a managerial severance claim. Finally, managers fare best under the bailout regime because the cost of debt is lowest, loan issuance is highest generating higher growth, and managers remain in post with some positive probability v_o.

v) Total payout equals $d_t + r_t = (1+\alpha m) q_j N_t$ and is proportional to managers' rent payout yield q_j ($\equiv r_t/N_t$). From Proposition 1 it follows that $M_j = N^{1-\eta} / \left[(1-\eta) m q_j^{\eta}\right]$. Therefore (see iv), payout is inversely related to managers' claim M_j. Consequently, payout is highest under the liquidation regime and lowest under the bailout regime. Under the former managers want to milk the firm before it is liquidated, whereas under the latter managers prefer to reinvest profits for long term growth.

vi) The ranking for $\underline{\kappa}_j$ shows that managers are most (least) likely to put the bank at risk of insolvency under the bailout (liquidation) regime. The explanation mirrors the previous argument why managers most (least) prefer the bailout (liquidation) regime. If one considers an industry of banks with different levels of κ then the critical threshold $\underline{\kappa}_j$ determines the insolvency rate in a crash. The bailout (liquidation) regime generates the highest (lowest) insolvency rate. Put differently, one could say that the bailout (liquidation) regime makes managers most (least) prone to put the bank at risk of insolvency. This is an important caveat that should be kept in mind when evaluating the results regarding value creation in Section 6.

4.2 Comparative statics

Table 2 numerically illustrates our model and shows the net debt ratio (D_j/A_j), the crash risk exposure (f_j), and insiders' payout yield (q_j) for the three IRMs. An asterix in the table indicates that the bank is safe and engages in asset sales when a crash happens.

Although our parameter values for μ', σ', ρ' and τ are standard, some model parameters values are less standard and set to reproduce the rich spectrum of possible outcomes that our model encapsulates. In particular, we choose a relatively low coefficient of relative risk aversion ($\eta = 0.65$) because for $\eta \geq 1$ banks never become insolvent and always operate within the asset sales regime. Insiders' subjective discount rate ($\delta = 0.4$) is set high enough to ensure that the transversality condition is satisfied for all parameter combinations. This condition imposes a restriction on the rate of growth of the net worth process to ensure insiders' claim value remains bounded. δ does not affect the net debt ratio (NDR), nor the bank's crash risk exposure f. A lower δ does, however, reduce insiders' payout yield q as it makes insiders more patient.[20] The base values for the crash arrival rate ($\lambda = 0.05$) and the risk premium parameter ($\kappa = 2$) imply that for $f = 0.9$, the expected before-tax return on the bank's assets is $E[dA/A] = \mu' + \lambda f(\kappa' - 1) = 0.145$. For the bail-in regime we need to make additional assumptions regarding the bank's market to book value (MBV) w^* and the gearing ratio l^* adopted by the resolution authority after the bail-in. MBVs and gearing (l) within the banking industry are empirically observed and reported. We set $w^* = 1.3$ and $l^* = 5$.[21]

The table illustrates that the safe regime occurs most (least) often in the liquidation (bailout) regime, reflecting our earlier result that $\underline{\kappa}_b > \underline{\kappa}_i > \underline{\kappa}_o$. For our parameter combinations, banks are safe under the bailout regime only if there is no crash risk ($\lambda = 0$). Under the bail-in regime, banks remain safe for zero crash risk, and for high insider risk aversion ($\eta = 0.8$). Under the liquidation regime, banks remain safe for zero crash risk, high insider risk aversion, low expected return on assets ($\mu = 0.08$) and a low crash risk premium ($\kappa = 1.8$). The absence of a safety net under the liquidation regime discourages insiders from putting the bank at risk of insolvency and mitigates risk taking, but only up to some point, as we show below.

Under the asset sale regime banks adopt a low NDR and low crash risk exposure.

[20]Reducing δ from 0.4 to 0.35 for the base parameter case reduces the payout yields in, respectively, the liquidation, bailout, and bail-in regimes from 11.1%, 8.7% and 10.8% to 9.5%, 7.2% and 9.3%.

[21]MBVs in the banking industry are relatively low. Bogdanova, Fender and Takats (2018) show that price-to-book ratios of major US banks averaged about 1.3 in 2017. Banks have been delevering since the financial crisis. For example, Cohen and Scatigna (2016) report that the ratio of capital to total risk-weighted assets for US commercial banks rose from 13.9% in 2009 to 17.4% in 2012. Capital is defined as common equity and does not include preferred shares or hybrid securities.

The NDR of safe banks ranges under the three IRMs from 43.7% to 66.2%. Crash risk exposure ranges from 7.2% to 18.7%. Payout yield ranges from 11.4% to 11.7%. A low NDR and low crash risk exposure means that the bank's equity cushion is sufficiently large to withstand a sudden drop in the value of the bank's assets.

Introducing insolvency risk causes a discrete upward shift in both the NDR and crash risk exposure, and generates a larger dispersion in the payout yield. The NDR now ranges from 77.6% to 93.2. Crash risk exposure ranges from 89.1% to 100%. Insiders' payout yield ranges from 1.9% (for $\lambda = 0.1$ under the bailout regime) to 11.7% (for $\mu' = 0.08$ under the liquidation regime). Under the bailout regime, insiders prefer to reinvest income for future growth and therefore pay out very little. This has important implication for the bank's market value (see Section 6).

Comparing the three IRMs the table confirms the results in Proposition 5. Conditional on the bank being at risk of insolvency, the NDR is highest (lowest) for bailout (bail-in). For the base case parameters the NDR equals 83.1%, 88.6% and 82.6% for respectively the liquidation, bailout, and bail-in regime. Crash risk exposure is highest (lowest) for the liquidation (bail-in) regime and equals 100%, 98.8% and 93.6% for the liquidation, bailout and bail-in regimes, respectively. Payout yield is highest (lowest) for the liquidation (bailout) regime and equals 11.1%, 8.7% and 10.8% for the liquidation, bailout and bail-in regimes, respectively. The bail-in regime reduces leverage and mitigates risk taking relative to the liquidation and bailout regime, but the effect is modest for the parameter combinations we consider. As predicted, the liquidation regime encourages maximum exposure to risk that crystalizes in bankruptcy.

The table allows us to gauge the effect of parameter changes on the control variables. Increasing the expected return on assets (μ'), the crash risk premium (κ'), or the crash arrival rate (λ) has a positive effect on the NDR and crash risk exposure, but a negative effect on insiders' payout yield. E.g., increasing μ' from 8% to 12% under the bail-in regime increases the optimal NDR from 77.6% to 85.6%, and crash risk exposure from 90.7% to 95.3%, whereas insiders' payout yield drops from 11.6% to 10%. Increasing return volatility (σ') and risk aversion (η) decreases the NDR and crash risk exposure, but increases the payout yield. For example, under the bail-in regime increasing volatility σ' from 0.18 to 0.22 reduces the NDR from 86.0% to 78.7%, and crash risk exposure from 95.5% to 91.3%, whereas insiders' payout yield increases from 10.4% to 11.2%. Finally, taxes increase the NDR but do not affect crash risk exposure nor the payout yield.

5 Value at risk

Since its inception in the 1990s, value at risk (VaR) has become the standard by which risk is managed by financial institutions and measured by regulators today. We are not aware of any study that has examined the effect of the IRM on VaR. Our analysis focuses on the change in the asset value of the bank and looks at its VaR under different IRMs. Let $\tilde{A}_t$ be the asset value at time t. Since the bank adopts a constant leverage target under each IRM, $\tilde{A}_t = l_j N_t$ for the liquidation and bailout regimes (i.e. $j = b, o$). For the bail-in regime ($j = i$), the investment ratio changes from l_i to l^* after the bank is bailed-in at T_1, and as such $\tilde{A}_t = l^* N_t$ for $t > T_1$. Denote by $F_t^j(x) \equiv P(\tilde{A}_t/\tilde{A}_0 \leq x)$ the probability distribution function of the asset return at time t under IRM j. We define $VaR_p^j(t)$ as the percentage loss threshold over a time horizon t when the probability of breach is p, i.e:

$$P(1 - \tilde{A}_t/\tilde{A}_0 \geq VaR_p^j(t)) = p \tag{28}$$

This can be expressed in terms of the inverse of $F_t^j(x)$ as:

$$VaR_p^j(t) \;=\; 1 \;-\; (F_t^j)^{-1}(p) \tag{29}$$

Analytical expressions for $VaR_p^j(t)$ can be derived directly from equation (29) for the liquidation and bailout regime. For the bail-in regime we need to make additional assumptions regarding the bank's market to book value (MBV) w^* and the corporate policies (l^*, f^*, q^*) adopted by the resolution authority after the bail-in.

The market value of the bank's equity immediately after the bail-in is given by

$$w^* N_{T_1} \;=\; E\left[\int_{T_1}^{\infty} e^{-\rho(t-T_1)}(r_t + d_t)dt \middle| N_{T_1}\right] = (1+\alpha m)q^* E\left[\int_{T_1}^{\infty} e^{-\rho(t-T_1)} N_t dt \middle| N_{T_1}\right] \tag{30}$$

from which we can obtain:

$$w^* \;=\; \frac{(1+\alpha m)q^*}{\rho - g^* + \lambda f^* l^*} = \frac{(1+\alpha m)q^*}{mq^* - [\mu + (\kappa - 1)\lambda f^* - \rho]l^*} \tag{31}$$

Finally, we assume that the resolution authority sets the loan quality f^* to maximize the bank's market value, but subject to the constraint that the bank will not become insolvent in future, i.e. $l^* \le 1/f^*$. Since the MBV w^* is increasing in f^*, we obtain the corner solution $f^* = 1/l^*$. This choice for f^* implies that the bank's net worth will drop exactly to zero when another crash hits a bank that was previously bailed in. This means that the bank disappears after the second bail-in.[22]

The analytical expressions for $VaR_p^j(t)$ $(j = b, i, o)$ in Appendix 8.2 are fairly complicated and hard to interpret. The numerical plot for the term structure of VaR (see Figure 2) conveys a clearer picture. The parameter values used to generate Figure 2 are the same as for Table 2. As to be expected, the VaR increases with the time horizon t for all three IRMs. The VaR is determined by the bank loans' exposure to continuous diffusion risk and rare crash risk. The former affects VaR in the short run through gearing (l), whereas the latter kicks in over the longer term through jump risk exposure (f). Under the bail-in regime, banks have the lowest leverage and the lowest crash risk exposure. Consequently, VaR is lowest under the bail-in regime. Under the bailout regime, banks have higher gearing $(l_o > l_b)$, but lower crash exposure $(f_b > f_o)$

[22] One could, under different assumptions, let the bank go through an infinite number of bail-ins. E.g. we could assume that $l^* = l_i$, $q^* = q_i$, $f_i^* = f_i$, and then pin down w^* using the condition $w^* N_{T_1} = \sum_{j=1}^{\infty} E\left[\int_{T_j}^{T_{j+1}} \xi_i^{j-1} e^{-\rho(t-T_1)}(r_t + d_t)dt \middle| N_{T_1}\right]$.

than under the liquidation regime. As a result VaR is higher (lower) under the bailout regime than under the liquidation regime for short (long) horizons. Note how VaR shoots up to 100% VaR at $t = 1.026$ for the liquidation regime, because crashes result in total loss ($f_b = 1$) when a crash occurs. This critical time point is related to the lower 5-percentile ($p = 0.05$) of the exponential distribution with arrival rate $\lambda = 0.05$. This critical time point equals $(1/\lambda) * ln(1/(1-p))$, which is the mean time of jump arrival multiplied by an adjustment factor for the p% VaR significance level.[23]

Our analysis demonstrates that IRMs have important implications for the VaR of financial institutions. Furthermore, the bank's VaR depends non-trivially on the horizon being considered. Our results highlight the limitations of short horizon (e.g. one day ahead) VaR measures. Finally, our VaR measure uncovers dimensions of risk that are not captured by the credit spread on bank debt ($\rho_j - \rho$). For example, bank debt may be subject to little or no credit risk under the bailout regime, and at the same time the bank may have the highest VaR for short horizons.

6 Bank Value and IRMs

In this section we examine which IRM maximizes the bank's total market value (net of any recapitalizations). We find that the bailout regime generates the highest bank value. We show that it is feasible to set up a self-financing bailout fund in which banks make contributions during good times that cover the expected costs of bailouts. We illustrate how such a bailout fund can be implemented in our setting without altering insiders' incentives regarding loan volume and loan quality.

[23]Under the bailout regime, VaR asymptotically approaches 100% (as $t \to \infty$) but never reaches this level as the bank is being recapitalized after each crash. Under the bail-in regime VaR reaches 100% after the second crash. This is not noticeable in the figure as the critical time point that corresponds to the second crash equals 7.07 (as given by the solution to (49) in Appendix 8.2), which is in the distant future.

Under the optimal policies, the bank's total net worth has the dynamics defined in (6) (for $j = b, o, i$). Since debt is competitively priced, our analysis can be restricted to the value created for inside and outside equityholders. Recall that the total payout to both outside and inside equityholders is $d_t + r_t = (1 + \alpha m)q_j N_t$. The net market value created is the expected discounted value of the payout flow net of any capital injections. We assume that market participants are well diversified and have a subjective discount rate $\delta (> \rho)$.[24] We calculate the net market value for each IRM using risk neutral valuation and also compare the Internal Rate of Return (IRR) across mechanisms.

In liquidation all proceeds go to lenders.[25] Hence, equityholders only receive a payout up to the arrival time of the first crash. The net market value created under the liquidation regime, W_b, is therefore:

$$W_b \equiv E\left[\int_0^{T_1} e^{-\delta t}(r_t + d_t)dt\right] - N_0$$

where T_1 is the random arrival time of the first shock.

Under the bailout regime the bank operates forever, but new capital is injected after every crash. The net market value created is given by:

$$W_o \equiv E\left[\int_0^{\infty} e^{-\delta t}(r_t + d_t)dt\right] - E\left[\sum_{k=0}^{\infty} e^{-\delta T_k} N_{T_k}\right] \equiv I_o - C_o$$

where T_k is the k-th arrival time of the Poisson shock and $T_0 \equiv 0$.

Under the bail-in regime inside and outside equityholders receive a combined payout flow $r_t + d_t$ up to the first shock, after which they receive a fraction ξ_i of the bank's

[24]For example, if all market participants are mortal and subject to sudden death with Poisson arrival rate ω, then $\delta \equiv \rho + \omega$ (assuming sudden death is uncorrelated with other shocks in the economy).

[25]We also considered the scenario where a new group of investors and managers step in after liquidation to set up a new bank. The liquidation proceeds from the failed bank are taken as the new asset base. In this case, the payout flow exists indefinitely but any subsequent capital injections have to be taken into account. The net value created is higher in general, but the rate of return remains the same.

post bail-in equity. The net market value created is given by:

$$W_i \equiv E\left[\int_0^{T_1} e^{-\delta t}(r_t + d_t)dt\right] + E\left[e^{-\delta T_1}\xi_i w^* \frac{(1-f_i)A_{T_1-}}{l^*}\right] - N_0$$

where $\frac{(1-f_i)A_{T_1-}}{l^*}$ is the book value of the equity after the bail-in and w^* the corresponding market to book value ratio. Note the change in net worth following a crash:

$$\frac{(1-f_i)A_{T_1-}}{l^*} = \frac{(1-f_i)A_{T_1-}}{l_i}\frac{l_i}{l^*} = (1-f_i)\frac{l_i}{l^*}N_{T_1-} \equiv \Phi_i(l_i, f_i)N_{T_1-}. \tag{32}$$

The following proposition states the net market value created and the IRR under each IRM. The proposition holds for any constant target loan exposure level f, and thus also for the optimally chosen f_j under IRM j.

Proposition 6 *Suppose that $\delta + \lambda f - g_j > 0$,[26] and that $\kappa > \underline{\kappa}_j$ for $j = b, o, i$ then the net market value created by the bank under IRM j is given by:*

$$W_b(N_0; f) = \left(\frac{q_b(1+\alpha m)}{\delta + \lambda - g_b} - 1\right)N_0, \tag{33}$$

$$W_o(N_0; f) = I_o - C_o \equiv \frac{(1+\alpha m)q_o}{\delta + \lambda f - g_o}N_o - \frac{\delta + \lambda - g_o}{\delta + \lambda f - g_o}N_0 \tag{34}$$

$$W_i(N_0; f) = \left[\frac{(1+\alpha m)q_i + \lambda \Phi_i(l_i, f_i)\xi_i w^*}{\delta + \lambda - g_i} - 1\right]N_0 \tag{35}$$

The internal rate of return (IRR) is:

$$\delta_j(f) = \rho_j + (\mu + \kappa\lambda f - \rho_j)l_j - \lambda \quad \text{for} \quad j = b, o \tag{36}$$

$$\delta_i(f) = \rho_i + (\mu + \kappa\lambda f - \rho_i)l_i - \lambda\left[1 - \Phi_i(l_i, f)\xi_i w^*\right] \tag{37}$$

The expressions for the breakeven hurdle rate are very similar across the three mechanisms with the following components: $\rho_j + (\mu + \kappa\lambda f - \rho_j)l_j$ is the leveraged return of the loans, λ is the default intensity, and $1 - \Phi_i\xi_i w^*$ reflects the fractional loss to existing equityholders and managers in bail-in. Note that equityholders are likely to be wiped out in a bail-in and therefore $\xi_i \approx 0$. We can prove the following proposition.

[26] Recall that g_j is the growth rate of the net worth under IRM j as defined in (6) and (18).

Proposition 7 *If managers optimally choose the crash risk f_j and the parameters are such that $\xi_i = 0$, $\frac{\mu' - \rho'}{(\sigma')^2 \eta} > 2$ and $\delta > g_j - \lambda f_j$ for $j \in \{b, o, i\}$ then the bank's value creation is highest (lowest) under the bailout (bail-in) regime for both the IRR and the net market value criteria, i.e.:*

$$\delta_o(f_o) > \; \delta_b(f_b) \; > \; \delta_i(f_i)$$
$$W_o(N_0; f_o) > \; W_b(N_0; f_b) \; > \; W_i(N_0; f_i)$$

The discount rate δ used for computation of W_j is restricted to $\delta_o(f_o) > \delta > \delta_i(f_i)$.

Under managers' optimal risk exposure, bailouts and bail-ins create the highest and lowest net value, respectively. At first sight, $W_b(N; f_b) > W_i(N; f_i)$ might appear surprising considering that managers prefer bail-in to bankruptcy (i.e. $M_i(N) > M_b(N)$). This ranking is primarily driven by the fact that managers pick loans with 100% crash exposure in the liquidation regime, and the associated high return is more favorable under a risk-neutral market valuation criterion. It should be noted that we assumed there are no bankruptcy costs. Introducing bankruptcy costs could cause the IRR and net market value for liquidation to drop below the IRR and net market value for bail-in.

The highest net value is achieved under the bailout regime. The payout stream generated under the bailout regime more than compensates for the recapitalizations in crashes. Therefore, we can create a self-financing bailout fund. For instance, we can make the fund a recipient of dividends alongside outside equityholders by splitting the fraction α of free cash flows that currently accrue to outside equityholders into two components: a fraction α_1 going to outside equityholders and a fraction α_2 going to the bailout fund (with $\alpha_1 + \alpha_2 = \alpha$). Since the optimal loan volume (l_o) and loan quality (reflected by f_o) do not depend on α, the creation of the fund does not alter insiders' lending incentives. How big does α_2 have to be to meet the expected costs of recapitalizations? Using equation (34), α_2 is the solution to $\alpha_2 q_o/(1 - \alpha_1 - \alpha_2) = (\delta + \lambda - g_o)$. Solving for α_2, and using the equation (10) for q_o gives the following:

Corollary 1 *The expected costs of future bailouts can be covered by putting a fraction α_2 of total payouts into a bailout fund, where α_2 is given by:*

$$\alpha_2 \;=\; \frac{(\delta+\lambda-g_o)\eta}{\lambda+\delta-(1-\eta)H_0} \leq 1 \quad \Longleftrightarrow \quad \delta \leq \delta_o \tag{38}$$

The higher investors' discount rate δ, the larger the bank's required contribution to the bailout fund. As δ converges to δ_o, the bank's IRR, 100% of the bank's payouts go into the bailout fund (i.e. $\alpha_2(\delta_o) = 1$), leaving nothing for inside and outside equityholders. The corollary demonstrates that bailouts can be self-financing and need not rely on tax payers' money, provided that the bank generates a strictly positive NPV net of recapitalization costs. Considering that bailouts also generate the most value (compared to liquidation and bail-ins), our model suggests there is a strong case for retaining bailouts as a possible tool for resolving bank insolvencies.

7 Policy implications and conclusion

In the wake of the financial crisis a new framework for resolving bank insolvencies is being developed. Some politicians have argued that governments must commit never to bail out banks again. This may be throwing out the baby with the bath water. Leaving aside the fact that bailouts are a quick way to contain systemic risk, our model shows that, from a micro-prudential perspective, banks create the most value net of any recapitalization costs under the bailout regime. The implicit government guarantee subsidizes the cost of borrowing, which increases loan issuance and growth. On the downside, banks insolvency rates and VaR (for short horizons) are likely to be highest under the bailout regime. The exposure of bank assets to crashes can be kept low by giving insiders skin in the game in the event of a bailout. Excessive risk taking can be curbed by penalizing (rather than rewarding) managers for failure. To avoid that tax payers have to bail out banks, a fraction of total bank payouts during good times can be put in a bailout fund to cover expected bailout costs. Such a bailout fund is viable if banking (net of recapitalization costs) is a positive NPV activity before and after the bailout. Our model supports empirical evidence that bailouts can be an efficient way of resolving bank crises.[27]

Banks are least prone to insolvency under the liquidation regime. However, given that managers receive nothing in liquidation (lenders have a senior claim) they have a strong incentive to take on maximum exposure to states of the world that lead to liquidation by issuing loans of the lowest quality. Hence the liquidation regime leads to the largest loss in default, and the highest VaR for long horizons.

If the aim is to keep the amount of lending as well as the banks' exposure to crashes low then bail-ins can be a superior alternative to liquidation or bailouts. Bail-ins also lead to the lowest VaR. The price to pay is that banks grow more slowly and generate less value under the bail-in regime. Our model also highlights a number of caveats associated with bail-ins. First, banks need a sufficient amount of unsecured creditors that can be bailed in. If managers know how large the shock and resulting losses in crashes are (as is the case in our model), then they know how much unsecured, bail-inable debt is required. Problems arise if losses are larger than expected and depositors are at risk. This could trigger a bank run and cause the bail-in to unravel. Second, whether bail-ins mitigate managers' incentives to issue low quality loans, depends on managers' payoff in a bail-in. As with bailouts, it is important that managers' fortunes remain closely linked at all times to the state of the bank; managers may have to be punished in the event of heavy losses. The BRRD stipulates that management should in principle be replaced in a bail-in. If that means that managers have no liability and

[27]Dell'Ariccia, Detragiache and Rajan (2008) show that banking crises have real effects, partially through the lending channel. Looking at various international cases of banking crisis, Dewatripont (2014) finds that speedy recapitalization through a bailout is crucial to minimize the effect on the economy. He shows that in many cases the public money is eventually repaid in full, and argues that the negative impact of bailouts in terms of moral hazard and of taxpayer risk can be contained by punishing managers and shareholders that receive bailouts.

walk away scot-free then merely replacing managers could exacerbate moral hazard problems. Third, while a bail-in turns an insolvent bank into a solvent one, it does not inject any new capital (unlike bailouts). Bail-ins may therefore not resolve a bank's liquidity problems. Finally, a bail-in (or bailout for that matter) cannot turn a bank that is inherently unprofitable into one that is profitable. If no part of the bank is viable on its own, then liquidation is inevitable.

While bail-ins have some advantages over liquidation and bailouts, they also have potential weaknesses that we overlooked. Gleeson (2012) and Avgouleas, Goodhart and Schoenmaker (2013) note that the restructurings required in a bail-in (such as breaking up the bank) may be hard to achieve for multinational banks in the absence of an internationally agreed system. Furthermore, bail-ins could increase systemic risk if bail-in debt is owned by other banks. This suggests that future research on IRMs should attempt to study their effect on the global banking sector and wider economy.

8 Appendix

8.1 Proofs of the main results

Proof of Proposition 1. The Hamilton-Jacobi-Bellman (HJB) equation associated with equation (7) has the following general form:

$$\begin{aligned}\delta M_j(N_t) = \max_{q_t,l_t,f}\Bigg\{ & u(q_tN_t) \; - mq_tN_t\frac{\partial M_j(N_t)}{\partial N_t} + [\mu + \kappa\lambda f - \rho_j(l_t,f)]l_tN_t\frac{\partial M_j(N_t)}{\partial N_t} \\ & + \frac{1}{2}\sigma^2 l_t^2 N_t^2 \frac{\partial^2 M_j(N_t)}{\partial N_t^2} + \rho_j(l_t,f)N_t\frac{\partial M_j(N_t)}{\partial N_t} \\ & + \lambda\left[p_j M_j(\phi_j(l_t,f)N_t) - M_j(N_t)\right]\Bigg\} \qquad (39)\end{aligned}$$

Conjecturing the value function in form of $M_j(N) = \frac{C_j N^{1-\eta}}{1-\eta}$, the HJB equation can be written as:

$$\frac{\lambda+\delta}{1-\eta} = \max_{q>0,l,f}\left\{\frac{q^{1-\eta}}{C_j(1-\eta)} - mq + [\mu+\kappa\lambda f - \rho_j(l,f)]l - \frac{\sigma^2\eta}{2}l^2 \right.$$
$$\left. + \rho_j(l,f) + \frac{\lambda p_j}{1-\eta}[\phi_j(l,f)]^{1-\eta}\right\} \tag{40}$$

The right-hand-side of (40) decouples into:

$$\max_{q>0}\left\{\frac{q^{1-\eta}}{C_j(1-\eta)} - mq\right\} + \max_{l,f}\left\{[\mu+\kappa\lambda f - \rho_j(l,f)]l - \frac{\sigma^2\eta}{2}l^2 + \rho_j(l,f) + \frac{\lambda p_j}{1-\eta}[\phi_j(l,f)]^{1-\eta}\right\}$$

In every regime, the optimal q is given by a simple first order condition leading to $q^* = (mC_j)^{-1/\eta}$. Meanwhile, the feasible domain of (l,f) depends on whether we are in the solvency or insolvency regime. For $j = s$ the constraint is $l \le 1/f$ where for $j = b,o,i$ we have $l > 1/f$ instead. The optimal (l,f) can then be obtained by maximizing the following investment objective function on the relevant regime:

$$\max_{l,f} G_j(l,f) \equiv \max_{l,f}\left\{[\mu+\kappa\lambda f - \rho_j(l,f)]l - \frac{\sigma^2\eta}{2}l^2 + \rho_j(l,f) + \frac{\lambda p_j}{1-\eta}[\phi_j(l,f)]^{1-\eta}\right\}$$

Denote the optimizers by l_j and f_j, and the optimized investment function by $H_j \equiv G_j(l_j,f_j)$. The unknown claim value multiplier C_j can be solved by putting $q = q^*$, $l = l_j$ and $f = f_j$ in (40) which gives $\frac{\eta}{1-\eta}m^{1-\frac{1}{\eta}}C_j^{-\frac{1}{\eta}} + H_j - \frac{\lambda+\delta}{1-\eta} = 0$ and in turn $C_j = \left[\frac{\eta}{\lambda+\delta-(1-\eta)H_j}\right]^\eta m^{\eta-1}$. C_j is well-defined for as long as $H_j < \frac{\lambda+\delta}{1-\eta}$.[28] C_j is increasing in H_j since we work under $\eta < 1$. Thus to compare the managers' claim value under different regimes, it is sufficient to compare the H_j's. ■

Proof of Proposition 2. In the asset sales regime, $\rho_s = \rho$, $\phi_s(l,f) = 1 - fl$ and $p_s = 1$. Then the investment objective function is:

$$G_s(l,f) \equiv (\mu+\kappa\lambda f - \rho)l - \frac{\sigma^2\eta}{2}l^2 + \rho + \frac{\lambda}{1-\eta}(1-fl)^{1-\eta}$$

and our goal is to find the pair (l,f) satisfying $l \le 1/f$ and $0 \le f \le 1$ which maximizes $G_s(l,f)$.

[28]Indeed, $H_j < \frac{\lambda+\delta}{1-\eta}$ is the necessary and sufficient condition for the managers' claim to have finite value under regime j.

The solution strategy is the following sequential optimization approach which we will also adopt for the other regimes. In the first stage, we consider f as a given constant and we find l satisfying $l \leq 1/f$ which maximizes $G_s(l, f)$. Denote the maximizer by $l_s(f)$ which depends on the value of the fixed f. Then the second stage optimization involves finding $0 \leq f \leq 1$ which maximizes $G_s(l_s(f), f)$. Suppose the maximizer is f_s. Then the pair of optimizers to the original problem is given by $(l_s(f_s), f_s)$.

In the first stage problem, direct differentiation gives $\frac{\partial}{\partial l} G_s(l, f) = \mu + \kappa\lambda f - \rho - \sigma^2\eta l - \frac{f\lambda}{(1-fl)^\eta}$ and $\frac{\partial^2}{\partial l^2} G_s(l) = -\sigma^2\eta - f^2\lambda\eta(1-fl)^{-\eta-1} < 0$. Note that $\frac{\partial}{\partial l} G_s(l, f) \to -\infty$ as $l \to \frac{1}{f}$, and since $\kappa > 1$ we have $\frac{\partial}{\partial l} G_s(l, f)\Big|_{l=0} = \mu + (\kappa - 1)\lambda f - \rho > \mu - \rho > 0$. The first order condition:

$$\mu + \kappa\lambda f - \rho - \sigma^2\eta l - \frac{f\lambda}{(1 - fl)^\eta} = 0 \tag{41}$$

has exactly one root given by some $l_s(f) \in (0, 1/f)$ which is the maximizer of $G_s(l, f)$ on $l \leq 1/f$ under a fixed f.

Write $H_s(f) \equiv G_s(l_s(f), f)$. In the second stage problem we want to find $0 \leq f \leq 1$ maximizing $H_s(f)$. Since $l_s(f)$ satisfies the first order condition $\frac{\partial G_s}{\partial l}\Big|_{l=l_s(f)} = 0$, we have:

$$\begin{aligned} H_s'(f) = \frac{\partial G_s}{\partial l}\Big|_{l=l_s(f)} \times \frac{dl_s(f)}{df} + \frac{\partial G_s}{\partial f}\Big|_{l=l_s(f)} &= \lambda l_s(f) \left(\kappa - \frac{1}{(1 - f l_s(f))^\eta}\right) \\ &= \frac{l_s(f)}{f} \left(-\mu + \rho + \sigma^2\eta l_s(f)\right) \end{aligned}$$

where the last equality is due to (41). The first order condition $H_s'(f) = 0$ gives $l_s(f) = \frac{\mu-\rho}{\sigma^2\eta}$,[29] and the associated f is obtained from $\kappa - \frac{1}{(1-fl_s(f))^\eta} = 0$ leading to a candidate solution $f = f_s \equiv \frac{\sigma^2\eta}{\mu-\rho}\left(1 - \kappa^{-\frac{1}{\eta}}\right)$. Note that $l_s(f_s)f_s = 1 - \kappa^{-\frac{1}{\eta}} < 1$. The condition $\frac{\mu-\rho}{\sigma^2\eta} > 1$ ensures $f_s < 1$.

We have shown that $H_s'(f) = 0$ has a unique root at some $0 < f_s < 1$. It remains to check this candidate solution f_s indeed corresponds to a maximum of $H_s(f)$. By

[29] It is easy to check that $l_s(f) = 0$, the alternative solution of the first order condition, will lead to a candidate optimizer $f = -\frac{\mu-\rho}{\lambda(\kappa-1)} < 0$ which is not feasible.

considering (41), it can be easily verified that $l_s(0) = \frac{\mu-\rho}{\sigma^2\eta} > 0$ and $l_s(1) < 1$. Hence $H_s'(0) = (\kappa-1)\lambda l_s(0) > 0$ and $H_s'(1) = l_s(1)\left(-\mu+\rho+\sigma^2\eta l_s(1)\right) < 0$. Then we must have $H_s'(f) \geq 0$ for $0 \leq f \leq f_s$ and $H_s'(f) \leq 0$ for $f_s \leq f \leq 1$. We conclude a maximum is attained at f_s. ■

Proof of Proposition 3 and 4 (Complemented by the internet appendix). In each of the following subsections, we will first prove for each IRM the form of the optimal l_j and f_j (i.e. Proposition 4), and then verify the existence of $\underline{\kappa}_j$ above which managers will put the bank at risk of insolvency (i.e. Proposition 3).

(i) Liquidation regime

In the liquidation regime, $\rho_b(l,f) = \rho + \lambda(1-\tau)\left[\frac{fl-1}{l-1}\right]$, $\phi_b = 0$ and $p_b = 0$. The investment objective function is then:

$$\begin{aligned} G_b(l,f) &\equiv (\mu + \kappa\lambda f - \rho_b(l,f))l - \frac{\sigma^2\eta}{2}l^2 + \rho_b(l,f) \\ &= (\mu + (\kappa - 1 + \tau)\lambda f - \rho)l - \frac{\sigma^2\eta}{2}l^2 + \rho + \lambda(1-\tau) \end{aligned}$$

We first find the maximizer of the above function over $l > \frac{1}{f}$ under a fixed f. There are two possibilities. If $1/f < l_b(f) \equiv \frac{\mu+(\kappa-1+\tau)\lambda f-\rho}{\sigma^2\eta}$, then since $l = l_b(f)$ solves the first order condition:

$$\frac{\partial}{\partial l}G_b(l,f) = \mu + (\kappa-1+\tau)\lambda f - \rho - \sigma^2\eta l = 0 \tag{42}$$

and G_b is concave in l, it must attain the maximum at $l = l_b(f)$ on $l > 1/f$. Otherwise if $1/f \geq l_b(f)$, then G_b is strictly decreasing in l on $l > 1/f$ and the maximum is attained at $1/f$. If we define $\hat{f}_b \in (0,1)$ as the unique solution to the equation $l_b(f) = 1/f$ or equivalently:

$$\zeta_b(f) \equiv \frac{\mu + (\kappa-1+\tau)\lambda f - \rho}{\sigma^2\eta} - \frac{1}{f} = 0$$

then the condition $1/f < (\geq) l_b(f)$ is equivalent to $f > (\leq) \hat{f}_b$. The optimized value function is hence given by:

$$H_b(f) \equiv \begin{cases} G_b\left(\frac{1}{f}, f\right) = \frac{(\mu+(\kappa-1+\tau)\lambda f-\rho)}{f} - \frac{\sigma^2\eta}{2f^2} + \rho + \lambda(1-\tau), & f \le \hat{f}_b \\ G_b\left(l_b(f), f\right) = \frac{(\mu+(\kappa-1+\tau)\lambda f-\rho)^2}{2\sigma^2\eta} + \rho + \lambda(1-\tau), & f > \hat{f}_b \end{cases}$$

In the second stage of the optimization problem we differentiate $H_b(f)$ on $f \le \hat{f}_b$ and $f > \hat{f}_b$ respectively. On $f > \hat{f}_b$, $H_b'(f) = \lambda(\kappa - 1 + \tau)\frac{\mu+(\kappa-1+\tau)\lambda f-\rho}{\sigma^2\eta} > 0$. On $f \le \hat{f}_b$:

$$\begin{aligned} H_b'(f) = \frac{d}{df}G_b\left(\frac{1}{f}; f\right) &= \frac{\partial}{\partial l}G_b(l, f)\Big|_{l=1/f} \times \frac{d}{df}\left(\frac{1}{f}\right) + \frac{\partial}{\partial f}G_b(l, f)\Big|_{l=1/f} \\ &= -\frac{\partial}{\partial l}G_b(l, f)\Big|_{l=1/f} \times \frac{1}{f^2} + \lambda(\kappa - 1 + \tau)\frac{1}{f} \ge 0 \end{aligned}$$

since $G_b(l, f)$ is decreasing for all $l \ge 1/f$ when $f \le \hat{f}_b$ and hence $\frac{\partial}{\partial l}G_b(l, f)\Big|_{l=1/f} \le 0$. In both cases, H_b is increasing in f such that it is maximized at $f = f_b \equiv 1$. The corresponding investment level is $l_b(f_b) = \frac{\mu+(\kappa-1+\tau)\lambda-\rho}{\sigma^2\eta}$.

To show the existence of $\underline{\kappa}_b$ above (below) which managers will prefer a risky (safe) bank and engage in liquidation (asset sales) in a crash, view $H_b = G_b(l_b(f_b), f_b)$ and $H_s = G_s(l_s(f_s), f_s)$ as functions of κ and let $J_b(\kappa) = H_b - H_s = H_b(\kappa) - H_s(\kappa)$. The general strategy of the proof, which we will also adopt for the other regimes, is to show that the function J_b is increasing in κ and thus there exists critical $\underline{\kappa}_b \ge 1$ such that $H_b \ge (<) H_s$ when $\kappa \ge \underline{\kappa}_b$ $(1 \le \kappa < \underline{\kappa}_b)$.

Since f_s and $l_s(f_s)$ are available in closed-form from Proposition 2, we can compute:

$$H_s = G_s(l_s(f_s), f_s) = \frac{(\mu - \rho)^2}{2\sigma^2\eta} + \kappa\lambda + \rho + \frac{\lambda\eta}{1-\eta}\kappa^{-\frac{1-\eta}{\eta}}$$

and then we obtain $\frac{dH_s}{d\kappa} = \lambda - \lambda\kappa^{-\frac{1}{\eta}}$. On the other hand:

$$\frac{dH_b}{d\kappa} = \frac{d}{d\kappa}\left(\frac{(\mu + (\kappa - 1 + \tau)\lambda - \rho)^2}{2\sigma^2\eta} + \rho + \lambda(1 - \tau)\right) = \frac{\mu + (\kappa - 1 + \tau)\lambda - \rho}{\sigma^2\eta}\lambda = l_b\lambda$$

such that $J_b'(\kappa) = \lambda\kappa^{-\frac{1}{\eta}} + \lambda(l_b - 1) > 0$. Hence $J_b(\kappa)$ is strictly increasing and there exists $\underline{\kappa}_b \ge 1$ such that $J_b(\kappa) < (\ge) 0$ for $1 \le \kappa < \underline{\kappa}_b$ $(\kappa \ge \underline{\kappa}_b)$.[30]

[30]Strictly speaking, to rule out the case of $\underline{\kappa}_b = \infty$ we should also verify that $J_b(\infty) > 0$. This

(ii) Bailout regime

In the bailout regime we have $\rho_o = \rho$, $\phi_o(l, f) = (1-f)\xi_o$ and $p_o \in [0, 1]$. If we define $v_o \equiv p_o \xi_o^{1-\eta}$ as the dilution-adjusted continuation probability of insiders (the same notation we have used in Proposition 5), the investment objective function is:

$$G_o(l, f) \equiv (\mu + \kappa\lambda f - \rho)l + \frac{\sigma^2\eta}{2}l^2 + \rho + \frac{\lambda v_o}{1-\eta}(1-f)^{1-\eta}$$

Similar to the analysis of the liquidation regime, we can define $\hat{f}_o \in (0, 1)$ as the unique solution to the equation:

$$\zeta_o(f) \equiv \frac{\mu + \kappa\lambda f - \rho}{\sigma^2\eta} - \frac{1}{f} = 0$$

Then under a fixed f the maximizer of $G_o(l, f)$ on $l > 1/f$ is given by $l = l_o(f) \equiv \frac{\mu+\kappa\lambda f-\rho}{\sigma^2\eta}$ when $f > \hat{f}_o$, or $l = 1/f$ when $f \leq \hat{f}_o$. Substituting the maximizer into $G_o(l, f)$ gives optimized value function under a fixed f as:

$$H_o(f) \equiv \begin{cases} G_o(1/f, f) = \frac{\mu+\kappa\lambda f-\rho}{f} - \frac{\sigma^2\eta}{2f^2} + \rho + \frac{\lambda v_o}{1-\eta}(1-f)^{1-\eta}, & 0 \leq f \leq \hat{f}_o \\ G_o(l_o(f), f) = \frac{(\mu+\kappa\lambda f-\rho)^2}{2\sigma^2\eta} + \rho + \frac{\lambda v_o}{1-\eta}(1-f)^{1-\eta}, & \hat{f}_o < f \leq 1 \end{cases}$$

If $v_o = 0$, the optimization problem then resembles the one in the liquidation regime and it is straightforward to verify that $H_o(f)$ is increasing such that the maximum is attained at $f_o \equiv 1$. We only outline the strategy of the proof here for the case of $v_o > 0$ and defer the technical details to the internet appendix. The main complication here originates from the piecewise definition of $H_o(f)$ on $f \leq \hat{f}_o$ and $f > \hat{f}_o$ respectively leading to two different first order conditions. Under condition (21) on the Merton ratio $\frac{\mu-\rho}{\sigma^2\eta} > 1 + \frac{v_o}{\kappa}$, we can show that $H_o(f)$ is indeed monotonically increasing on $f \leq \hat{f}_o$ and attains a global maximum on $f > \hat{f}_o$. Hence $f_o \in (\hat{f}_o, 1)$ is given by the first order condition derived over the second regime of $\hat{f}_o < f \leq 1$:

$$H_o'(f) = \frac{\kappa\lambda(\mu + \kappa\lambda f - \rho)}{\sigma^2\eta} - \frac{v_o\lambda}{(1-f)^\eta} \equiv \lambda\kappa\Theta_o(f) = 0 \tag{43}$$

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result is not hard to be established, and can be done by making use of the analytical expression of H_s and observing that H_b has a quadratic growth for large κ. Note that it is possible to have $\underline{\kappa}_b = 1$ and in this case $H_b \geq H_s$ for all $\kappa \geq 1$.

and the corresponding investment level is $l_o(f_o) = \frac{\mu+\kappa\lambda f_o-\rho}{\sigma^2\eta}$.

Finally, similar to the proof of the liquidation regime, the existence of $\underline{\kappa}_o$ can be verified by showing that $J_o(\kappa) \equiv H_o(\kappa) - H_s(\kappa)$ is increasing. We give the proof for the case of $v_o > 0$ as an illustration. The case of $v_o = 0$ is easier since $f_o = 1$ which leads to an analytical expression of H_o.

Note that:

$$H_o = G_o(l_o, f_o) = G_o(l_o(f_o(\kappa); \kappa), f_o(\kappa); \kappa)$$

which depends on κ explicitly via the definition of G_o as well as implicitly via $f_o = f_o(\kappa)$ and $l_o(f_o) = l_o(f_o(\kappa); \kappa)$. But since f_o and l_o satisfy the first order conditions $\frac{\partial G_o}{\partial l}\Big|_{l=l_o, f=f_o} = \frac{\partial G_o}{\partial f}\Big|_{l=l_o, f=f_o} = 0$ when $v_o > 0$, envelope theorem leads to $\frac{dH_o}{d\kappa} = \frac{\partial G_o}{\partial \kappa}\Big|_{l=l_o, f=f_o} = \lambda f_o l_o$. Then:

$$J_o'(\kappa) = H_o'(\kappa) - H_s'(\kappa) = \lambda\kappa^{-\frac{1}{\eta}} + \lambda(f_o l_o - 1) > 0$$

as $f_o l_o > 1$ on the insolvency regime. Hence $J_o(\kappa)$ is strictly increasing.

(iii) Bail-in regime

Under bail-in, ρ_i is given by (15), $\phi_i = (1-f)\xi_i$ and $p_i \in [0, 1]$. If we define $v_i \equiv p_i \xi_i^{1-\eta}$, the investment objective function is thus:

$$\begin{aligned} G_i(l, f) &= [\mu + \kappa\lambda f - \rho_i(l, f)]l - \frac{\sigma^2\eta}{2}l^2 + \rho_i(l, f) + \frac{\lambda v_i}{1-\eta}(1-f)^{1-\eta} \\ &= \{\mu + [\kappa - (1-\tau)(1+h)]\lambda f - \rho + \lambda h(1-\tau)\}l - \frac{\sigma^2\eta}{2}l^2 + \rho + \lambda(1-\tau) \\ &\quad + \frac{\lambda v_i}{1-\eta}(1-f)^{1-\eta} \qquad (44) \end{aligned}$$

As before, we first solve for the l maximizing $G_i(l, f)$ over $l > 1/f$ under a fixed f which can be derived using the exact same argument as in the bailout case. In particular, the optimizer is given by:

$$l = \begin{cases} \frac{1}{f}, & f \le \hat{f}_i \\ l_i(f) \equiv \frac{\mu+[\kappa-(1-\tau)(1+h)]\lambda f+\lambda h(1-\tau)-\rho}{\sigma^2\eta}, & f > \hat{f}_i \end{cases}$$

where $\hat{f}_i \in (0,1)$ is the solution to the equation:

$$\zeta_i(f) \equiv \frac{\mu+[\kappa-(1-\tau)(1+h)]\lambda f+\lambda h(1-\tau)-\rho}{\sigma^2\eta} - \frac{1}{f} = 0$$

In the second stage problem, we are solving for f which maximizes $H_i(f)$ where:

$$H_i(f) \equiv$$

$$\begin{cases} G_i(1/f, f) = \frac{\mu+[\kappa-(1+h)(1-\tau)]\lambda f-\rho+\lambda h(1-\tau)}{f} - \frac{\sigma^2\eta}{2f^2} + \rho + \lambda(1-\tau) + \frac{\lambda v_i}{1-\eta}(1-f)^{1-\eta}, & f \le \hat{f}_i \\ G_i(l_i(f), f) = \frac{\{\mu+[\kappa-(1+h)(1-\tau)]\lambda f-\rho+\lambda h(1-\tau)\}^2}{2\sigma^2\eta} + \rho + \lambda(1-\tau) + \frac{\lambda v_i}{1-\eta}(1-f)^{1-\eta}, & f > \hat{f}_i \end{cases}$$

When $v_i = 0$, it is easy to verify that $H_i(f)$ is increasing under the condition (22) that $\kappa > (1+h)(1-\tau)$ such that the maximizer is given by $f = f_i \equiv 1$, and then $l_i = l_i(f_i) = \frac{\mu+(\kappa-1+\tau)\lambda-\rho}{\sigma^2\eta} > 1 = \frac{1}{(1+h)f_i-h} = \hat{l}_i(f_i;h)$ such that $\rho_i(l_i, f_i) > \rho$.

Suppose $v_i > 0$. Define $\hat{f}_{i,h} \in (\frac{h}{1+h}, 1)$ as the solution to the equation:

$$\zeta_{i,h}(f) \equiv \frac{\mu+[\kappa-(1-\tau)(1+h)]\lambda f+\lambda h(1-\tau)-\rho}{\sigma^2\eta} - \frac{1}{(1+h)f-h} = 0$$

It can be easily seen that $\hat{f}_{i,h} > \hat{f}_i$ from their constructions. Under condition (22) that $\frac{\mu-\rho}{\sigma^2\eta} > 1+\frac{v_i(1+h)}{\kappa-(1+h)(1-\tau)}$, we show in the internet appendix that H_i is increasing on $f \le \hat{f}_i$ and attains an interior maximum at $f = f_i$ on $f > \hat{f}_i$ where f_i is given by the solution to the first order condition:

$$\Theta_i(f) \equiv \frac{\mu+[\kappa-(1-\tau)(1+h)]\lambda f-\rho}{\sigma^2\eta} - \frac{v_i}{[\kappa-(1-\tau)(1+h)](1-f)^\eta} = 0 \qquad (45)$$

Moreover, the condition on $\frac{\mu-\rho}{\sigma^2\eta}$ indeed also implies $f_i > \hat{f}_{i,h}$. Hence $0 < \zeta_{i,h}(f_i) = l_i(f_i) - \hat{l}_i(f_i;h)$ and as such $\rho_i(l_i, f_i) > \rho$.

The existence of $\underline{\kappa}_i$ can be proven using the same method as in the case of bailout.

■

Proof of Proposition 5. i) Since $f_b = 1$ we must have $f_b \geq f_o$ and $f_b \geq f_i$. Further recall again that under conditions (21) and (22), f_o and f_i are the unique roots of the equations in (43) and (45):

$$\Theta_o(f) \equiv \frac{\mu + \kappa\lambda f - \rho}{\sigma^2\eta} - \frac{v_o}{\kappa(1-f)^\eta} = 0$$
$$\Theta_i(f) \equiv \frac{\mu + [\kappa - (1+h)(1-\tau)]\lambda f - \rho + \lambda h(1-\tau)}{\sigma^2\eta} - \frac{v_i}{[\kappa - (1+h)(1-\tau)](1-f)^\eta} = 0$$

respectively. Note that:

$$\begin{aligned}\mu + [\kappa - (1+h)(1-\tau)]\lambda f - \rho + \lambda h(1-\tau) &= \mu + \kappa\lambda f - \rho - \lambda(1-\tau)[(1+h)f - h] \\ &< \mu + \kappa\lambda f - \rho\end{aligned}$$

on $f > \frac{h}{1+h}$. Moreover, $\frac{v_o}{\kappa} < \frac{v_i}{\kappa-(1+h)(1-\tau)}$ provided that either $v_i \geq v_o$, or $v_o > v_i$ and $(1+h)(1-\tau) < \kappa < \frac{v_o(1+h)(1-\tau)}{v_o - v_i}$. Then $\Theta_o(f) > \Theta_i(f)$ and hence $f_i < f_o$ since again each root f_j is given by a down-crossing of $\Theta_j(f) = 0$ $(j = o, i)$.

ii) We have shown in the bail-in regime that $\rho_i(l_i, f_i) > \rho$. Moreover, it can be easily verified from construction of ρ_i that $\rho_i(l, f) \leq \rho + \lambda(1-\tau)$ for any $l > 1/f$. Then the result follows since $\rho_o = \rho$ and $\rho_b(l_b, f_b) = \rho_b(l_b, 1) = \rho + \lambda(1-\tau)$.

iii) On the one hand, we have:

$$\begin{aligned}l_i &= \frac{\mu + [\kappa - (1+h)(1-\tau)]\lambda f_i - \rho + \lambda h(1-\tau)}{\sigma^2\eta} \\ &\leq \frac{\mu + [\kappa - (1+h)(1-\tau)]\lambda \times 1 - \rho + \lambda h(1-\tau)}{\sigma^2\eta} = \frac{\mu + (\kappa - 1 + \tau)\lambda - \rho}{\sigma^2\eta} = l_b\end{aligned}$$

On the other hand, we want to show $l_o \geq l_b$ which is:

$$\frac{\mu + \kappa\lambda f_o - \rho}{\sigma^2\eta} \geq \frac{\mu + (\kappa - 1 + \tau)\lambda f_b - \rho}{\sigma^2\eta} = \frac{\mu + (\kappa - 1 + \tau)\lambda - \rho}{\sigma^2\eta}$$

or equivalently $f_o \geq \frac{\kappa-1+\tau}{\kappa}$. We make use of the function $\Theta_o(f)$ in (43) where f_o is defined as the solution to $\Theta_o(f) = 0$. Check that

$$\Theta_o\left(\frac{\kappa-1+\tau}{\kappa}\right) = \frac{\mu+(\kappa-1+\tau)\lambda-\rho}{\sigma^2\eta} - \upsilon_o\kappa^{\eta-1}(1-\tau)^{-\eta}$$
$$\geq \frac{\mu-\rho}{\sigma^2\eta} - \frac{\upsilon_o}{\kappa}\left(\frac{\kappa}{1-\tau}\right)^{\eta} = \frac{1}{1-\tau}\left(\frac{\mu'-\rho'}{\sigma'^2\eta} - \upsilon_o(\kappa')^{\eta-1}\right) \geq 0 = \Theta_o(f_o)$$

since $\frac{\mu'-\rho'}{\sigma'^2\eta} \geq 1$ (recall each parameter with a prime symbol is its pre-tax value). The result follows as f_o is a down-crossing of $\Theta_o(f) = 0$.

iv) For $j = b, o, i$, $G_j(l, f) = [\mu + \kappa\lambda f - \rho_j(l, f)]l - \frac{\sigma^2}{2}\eta l^2 + \rho_j(l, f) + \frac{\lambda\upsilon_j}{1-\eta}(1-f)^{1-\eta}$ with $\upsilon_b = 0$ and $0 \leq \upsilon_o, \upsilon_i \leq 1$. On $l \geq \hat{l}_i(f; h) \geq \hat{l} = 1/f \geq 1$, it is not hard to verify that $\rho_o(l, f) \leq \rho_i(l, f) \leq \rho_b(l, f)$ and in turn $G_o(l, f) \geq G_i(l, f) \geq G_b(l, f)$ for as long as $\upsilon_o \geq \upsilon_i$. This translates into the ordering of $G_j(l_j, f_j)$, in turn C_j and finally $M_j(N)$.

v) and vi) These follow immediately from the ranking of $M_j(N; f_j)$ for $j = b, o, i$. ■

Proof of Proposition 6 (Complemented by the internet appendix). The results under each IRM can be established somewhat similarly. We provide the proof of the bailout case as an illustration. On the one hand:

$$I_o = E\left[\int_0^\infty e^{-\delta t}(r_t + d_t)dt\right] = (1+\alpha m)q_o \int_0^\infty e^{-\delta t}E(N_t)dt$$
$$= (1+\alpha m)q_o N_0 \int_0^\infty \exp\left[-(\delta - g_o + \lambda f)t\right]dt = \frac{(1+\alpha m)q_o}{\delta + \lambda f - g_o}N_o$$

On the other hand:

$$E[e^{-\delta T_k}N_{T_k}] = E[e^{-\delta T_{k-1}}e^{-\delta(T_k - T_{k-1})}N_{T_k}] = E\left[E_{T_{k-1}}\left[e^{-\delta T_{k-1}}e^{-\delta(T_k-T_{k-1})}N_{T_k}\right]\right]$$
$$= E\left[e^{-\delta T_{k-1}}N_{T_{k-1}}E_{T_{k-1}}\left[e^{-\delta(T_k-T_{k-1})}\frac{N_{T_k}}{N_{T_{k-1}}}\right]\right]$$

where we have used the law of iterated expectation. But conditioning on the information up to time T_{k-1}, $e^{-\delta(T_k-T_{k-1})}\frac{N_{T_k}}{N_{T_{k-1}}} \overset{d}{=} e^{-\delta T}\frac{N_T}{N_0}$ where T is an $Exp(\lambda)$ random variable due to the stationary properties of the underling Brownian motion and the Poisson process. Hence using an identity proven in the internet appendix:

$$E_{T_{k-1}}\left[e^{-\delta(T_k-T_{k-1})}\frac{N_{T_k}}{N_{T_{k-1}}}\right]=E\left[e^{-\delta T}\frac{N_T}{N_0}\right]=\frac{(1-f)\lambda}{\delta+\lambda-g_o}\equiv\theta$$

and $E[e^{-\delta T_k}N_{T_k}]=\theta E[e^{-\delta T_{k-1}}N_{T_{k-1}}]$. Then we can deduce iteratively that $E[e^{-\delta T_k}N_{T_k}]=\theta^k N_0$. Finally:

$$C_o=E\left[\sum_{k=0}^{\infty}e^{-\delta T_k}N_{T_k}\right]=N_0\sum_{k=0}^{\infty}\theta^k=\frac{1}{1-\theta}N_0=\frac{\delta+\lambda-g_o}{\delta+\lambda f-g_o}N_0$$

as $\theta=1-\frac{\delta+\lambda f-g_o}{\delta+\lambda-g_o}<1$, and we obtain $W_o=\left[\frac{(1+\alpha m)q_o-(\delta+\lambda f-g_o)}{\delta+\lambda f-g_o}\right]N_0$.

The IRR is given by the value of δ leading to $W_o=0$. The result can be obtained after substituting g_o by its analytical formula and $m=\frac{1}{1-\alpha}$. ■

Proof of Proposition 7. We first verify the ranking of net market value in terms of the IRR's δ_j (we will simply write $\delta_j(f_j)$ as δ_j for brevity). The ranking $\delta_i<\delta_b$ can be established easily using the fact that $f_i<f_b=1$. We are going to establish that $\delta_o>\delta_b$ which is equivalent to:

$$\begin{aligned}
&\frac{(\mu+\kappa\lambda f_o-\rho)^2}{\sigma^2\eta}+\rho>\frac{(\mu+(\kappa-1+\tau)\lambda-\rho)^2}{\sigma^2\eta}+\rho+\lambda(1-\tau)\\
\Longleftrightarrow\ &(\mu+\kappa\lambda f_o-\rho)^2-(\mu+(\kappa-1+\tau)\lambda-\rho)^2>\lambda\eta\sigma^2(1-\tau)\\
\Longleftrightarrow\ &\left(2\frac{\mu-\rho}{\sigma^2\eta}+\frac{\kappa\lambda f_o+(\kappa-1+\tau)\lambda}{\sigma^2\eta}\right)\left(f_o-\frac{\kappa-1+\tau}{\kappa}\right)>\frac{1-\tau}{\kappa}\\
\Longleftarrow\ &2\left(f_o-\frac{\kappa-1+\tau}{\kappa}\right)>\frac{1-\tau}{\kappa}
\end{aligned}$$

It is hence sufficient to show $f_o>1-\frac{1-\tau}{2\kappa}$. This can be done using the same argument in the proof of part iii) of Proposition 5 under a stronger condition that $\frac{\mu'-\rho'}{(\sigma')^2\eta}>2$.

The net market value associated with an IRM under arbitrary discount rate δ can be expressed in terms of its IRR. For bailout versus liquidation, note that

$$W_o=\frac{\delta_o-\delta}{\delta-\lambda(1-f_o)-\delta_o+mq_o}>\frac{\delta_o-\delta}{\delta-\delta_o+mq_o}>\frac{\delta_b-\delta}{\delta-\delta_b+mq_b}=W_b$$

for $\delta < \delta_o(f_o)$ since $\delta_o > \delta_b$ and $q_o < q_b$ (recall part v) of Proposition 5).

For liquidation versus bail-in, it is clear that $W_b > W_i$ on $\delta_i < \delta < \delta_b$ since on this range we have $W_b > 0 > W_i$. It remains to show that $W_b > W_i$ on $\delta > \delta_b$. Expressing the net market value W_j in terms of δ_j, it is required to show that

$$\begin{aligned}
W_b > W_i \iff & \frac{\delta_b - \delta}{\delta - \delta_b + mq_b} > \frac{\delta_i - \delta}{\delta - \delta_i + mq_i} \\
\iff & (\delta_b - \delta)(\delta - \delta_i + mq_i) > (\delta_i - \delta)(\delta - \delta_b + mq_b) \quad (\delta > \delta_j - mq_j \text{ by assumption}) \\
\iff & \delta(q_b - q_i) > \delta_i q_b - \delta_b q_i \\
\iff & \delta > \frac{\delta_i q_b - \delta_b q_i}{q_b - q_i} \quad (\text{since } q_b > q_i) \\
\Longleftarrow & \delta > \frac{\delta_b q_b - \delta_b q_i}{q_b - q_i} = \delta_b \quad (\text{since } \delta_b > \delta_i)
\end{aligned}$$

The result immediately follows. ■

8.2 Derivations of VaR under each IRM

In this section we derive the probability distribution function of the bank's asset return under each IRM, which in turn allows us to compute the VaR by (29).

Consider first the regimes of liquidation and bailout ($j = b, o$) where bank's net worth evolves as (6) with $\Phi_j = 1 - f_j$ such that:

$$N_t = N_0(1 - f_j)^{Y_t} \exp\left[\left(g_j - \frac{\sigma^2 l_j^2}{2}\right) t + \sigma l_j B_t\right]$$

Then:

$$\begin{aligned}
F_t^j(x) &= P(\tilde{A}_t/\tilde{A}_0 \le x) = P(N_t/N_0 \le x) = \sum_{k=0}^{\infty} P(Y_t = k) P(N_t/N_0 \le x | Y_t = k) \\
&= \sum_{k=0}^{\infty} P(Y_t = k) P\left\{(1 - f_j)^k \exp\left[\left(g_j - \frac{\sigma^2 l_j^2}{2}\right) t + \sigma l_j B_t\right] \le x\right\} \\
&= \sum_{k=0}^{\infty} \frac{e^{-\lambda t}(\lambda t)^k}{k!} Z\left[\frac{\ln \frac{x}{(1-f_j)^k} - \left(g_j - \frac{\sigma^2 l_j^2}{2}\right) t}{\sigma l_j \sqrt{t}}\right] \qquad (46)
\end{aligned}$$

where $Z(\cdot)$ is the cumulative distribution function of a standard normal random variable. For the liquidation regime, $f_b = 1$ and hence:

$$
\begin{aligned}
F_t^b(x) &= e^{-\lambda t} Z\left[\frac{\ln x - \left(g_b - \frac{\sigma^2 l_b^2}{2}\right) t}{\sigma l_b \sqrt{t}}\right] + \sum_{k=1}^{\infty} \frac{e^{-\lambda t}(\lambda t)^k}{k!} \\
&= e^{-\lambda t} Z\left[\frac{\ln x - \left(g_b - \frac{\sigma^2 l_b^2}{2}\right) t}{\sigma l_b \sqrt{t}}\right] + 1 - e^{-\lambda t}
\end{aligned}
$$

and we can easily invert the function to obtain:

$$
VaR_p^b(t) = \begin{cases} 1 - (F_t^b)^{-1}(p) & p > 1 - e^{-\lambda t} \iff t < \frac{1}{\lambda} \ln \frac{1}{1-p} \\ 100\% & p \leq 1 - e^{-\lambda t} \iff t \geq \frac{1}{\lambda} \ln \frac{1}{1-p} \end{cases}
$$

For the bailout regime with $f_o < 1$, the main difficulty here is that (46) is an infinite sum, which makes the inversion of $F_t^o(x)$ non-trivial. However, for numerical implementation purposes it is sufficient to approximate (46) by only considering the first M terms of the summation. See the internet appendix.

Now we proceed to consider the bail-in regime $j = i$. Recall the discussion in Section 5 that the corporate policies before and after bail-in are given by (l_i, q_i, f_i) and (l^*, q^*, f^*) respectively, and that the bank will be sold down entirely on the first shock after the bail-in. The net worth dynamics are therefore:

$$
N_t = \begin{cases} N_0 \exp\left[\left(g_i - \frac{\sigma^2 l_i^2}{2}\right) t + \sigma l_i B_t\right] & t < T_1 \\ N_{T_1^-} \Phi_i \exp\left[\left(g^* - \frac{\sigma^2 (l^*)^2}{2}\right)(t - T_1) + \sigma l^* B_{t-T_1}\right] & T_1 \leq t < T_2 \\ 0 & t \geq T_2 \end{cases}
$$

where Φ_i is the change in net worth during bail-in as in (32). Then:

$$
\begin{aligned}
F_t^i(x) &= P(\tilde{A}_t/\tilde{A}_0 \leq x) \\
&= P(t < T_1) P(\tilde{A}_t/\tilde{A}_0 \leq x | t < T_1) + P(T_1 \leq t < T_2) P(\tilde{A}_t/\tilde{A}_0 \leq x | T_1 \leq t < T_2) \\
&\quad + P(t \geq T_2) P(\tilde{A}_t/\tilde{A}_0 \leq x | t \geq T_2) \qquad (47)
\end{aligned}
$$

Each term in (47) can be computed where we defer the technical details to the internet appendix. We can eventually deduce:

$$F_t^i(x) = e^{-\lambda t} Z\left[\frac{\ln x - \left(g_i - \frac{\sigma^2 l_i^2}{2}\right)t}{\sigma l_i \sqrt{t}}\right] + \lambda e^{-\lambda t}\int_0^t \vartheta(x,t;u)du + 1 - e^{-\lambda t} - \lambda t e^{-\lambda t} \tag{48}$$

where

$$\vartheta(x,t;u) \equiv Z\left[\frac{\ln \frac{x}{1-f_i} - \left(g_i - \frac{\sigma^2 l_i^2}{2}\right)u - \left(g^* - \frac{\sigma^2 (l^*)^2}{2}\right)(t-u)}{\sigma\sqrt{l_i^2 u + (l^*)^2(t-u)}}\right]$$

with $Z(\cdot)$ again being the cumulative normal distribution function. Note that $F_t^i(0) = 1 - e^{-\lambda t} - \lambda t e^{-\lambda t}$, which corresponds to $P(N_t \geq 2) = P(t \geq T_2)$. I.e., the probability that the second shock has arrived by time t and that the net worth of the bank has gone to zero as a result of the entire sell down of the bank during the second shock. The value at risk is then given by:

$$VaR_p^i(t) = \begin{cases} 1 - (F_t^i)^{-1}(p) & p > 1 - e^{-\lambda t} - \lambda t e^{-\lambda t} \iff t < \bar{t}_i \\ 100\% & p \leq 1 - e^{-\lambda t} - \lambda t e^{-\lambda t} \iff t \geq \bar{t}_i \end{cases}$$

where $\bar{t}_i$ is the solution to the equation:

$$p = 1 - e^{-\lambda t} - \lambda t e^{-\lambda t} \tag{49}$$

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		D_b/A_b	D_o/A_o	D_i/A_i	f_b	f_o	f_i	q_b	q_o	q_i
Baseline		83.10	88.64	82.57	100.00	98.79	93.65	11.08	8.70	10.85
μ'	0.08	43.67*	86.85	77.61	18.71*	98.48	90.68	11.74*	9.85	11.56
	0.12	85.92	90.00	85.64	100.00	99.00	95.29	10.16	7.38	9.96
σ'	0.18	86.31	90.82	86.01	100.00	99.13	95.47	10.59	7.62	10.39
	0.22	79.55	86.22	78.66	100.00	98.37	91.34	11.43	9.49	11.18
κ'	1.8	66.20*	87.80	80.31	7.25*	98.41	89.06	11.43*	9.28	11.20
	2.2	84.64	89.37	84.30	100.00	99.05	95.96	10.64	8.07	10.45
λ	0	66.20*	66.20*	66.20*	11.23*	11.23*	11.23*	11.44*	11.44*	11.44*
	0.1	88.73	93.21	88.48	100.00	99.45	96.65	9.67	1.88	9.31
τ	0.25	80.50	86.89	79.88	100.00	98.79	93.65	10.97	8.64	10.74
	0.45	85.70	90.39	85.25	100.00	98.79	93.65	11.18	8.75	10.96
η	0.5	87.00	91.32	86.90	100.00	99.81	98.43	11.70	6.07	11.63
	0.8	58.40	85.79	58.40	11.63	96.33	11.63	9.63	9.00	9.63

Table 2:

Comparative statics.

Optimal corporate policies under different model parameters. Base parameters used are $\mu' = 0.1$, $\sigma' = 0.2$, $\rho' = 0.05$, $\kappa' = 2$, $\tau = 0.35$, $\lambda = 0.05$, $\eta = 0.65$, $\delta = 0.4$, $\alpha = 0.8$, $v_o \equiv p_0\xi_o^{1-\eta} = 0.65$, $v_i \equiv p_i\xi_i^{1-\eta} = 0.6$, $l^* = 5$, $w^* = 1.3$ and $\xi_i = 0.1$. Numerical results are all expressed in terms of percentage. An asterisk $*$ indicates that the bank is safe and engages in asset sales when a crash arrives.

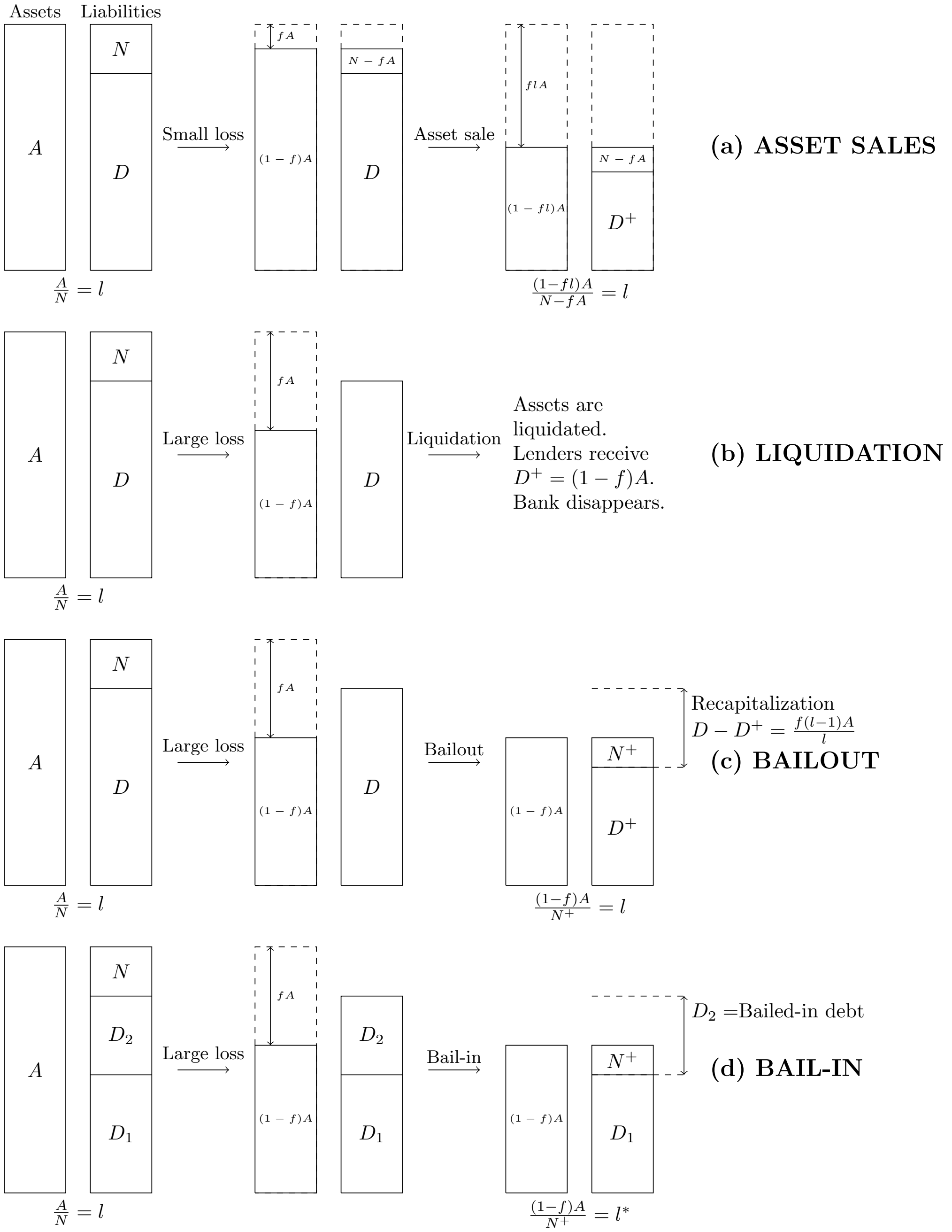

Figure 1:

Balance sheet illustrations under each restructuring mechanism.

Panel A shows that after a small loss in assets the bank rebalances its capital structure by selling assets and using the proceeds to pay off debt. After a large loss in assets, the bank becomes insolvent. Insolvency is resolved through liquidation (panel B), bailout (panel C) or bail-in (panel D).

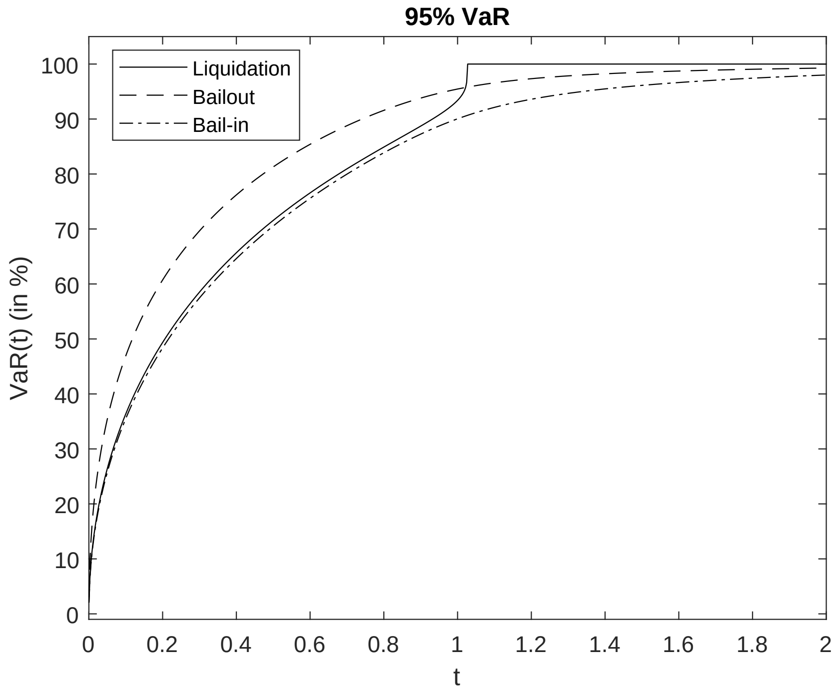

Figure 2:

95% VaR under each restructuring mechanism.

The figure plots the 95% VaR under the liquidation, bailout and bail-in regime as a function of the time horizon t. The parameter values used to generate the plot are the same as for Table 2, i.e.: $\mu' = 0.1$, $\sigma' = 0.2$, $\rho' = 0.05$, $\kappa' = 2$, $\tau = 0.35$, $\lambda = 0.05$, $\eta = 0.65$, $\delta = 0.4$, $\alpha = 0.8$, $v_o \equiv p_0 \xi_o^{1-\eta} = 0.65$, $v_i \equiv p_i \xi_i^{1-\eta} = 0.6$, $l^* = 5$, $w^* = 1.3$ and $\xi_i = 0.1$.