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Abstract

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Buyer power and mutual dependency in a model of negotiations

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We study bilateral bargaining between several buyers and sellers in a framework that allows both sides, in case of a bilateral disagreement, flexibility to adjust trade with each of their other trading partners and receive the gross benefit generated by each adjustment. A larger buyer pays a higher per-unit price when buyers' bargaining power in bilateral negotiations is sufficiently low, and a lower price otherwise. An analogous result holds for sellers. These predictions, and the implications of different technologies, are explained by the fact that size is a source of mutual dependency and not an unequivocal source of power.

1. Introduction

■ We develop a bargaining model where several buyers negotiate bilaterally with several sellers of substitute goods, which have convex costs. Transfers and quantities are determined by the ability of buyers to relocate purchases across sellers, and of sellers to relocate sales across buyers, in case of a bilateral disagreement. Specifically, the two parties in disagreement can optimally adjust their bilateral transactions with each of their other counterparties while changing the transfer to exactly reflect the changes in costs or benefits of that counterparty. Next to a foundation that builds on the axiomatic (asymmetric) Nash bargaining solution, we introduce a strategic game that generates these adjustments endogenously as reasonable inside options.

This model is applied to a question that has become increasingly important for researchers in industrial organization, as well as antitrust and business strategy practitioners, namely, whether size is always an advantage in negotiations, and more generally, the impact of market concentration on negotiated terms of trade. Consistent with most of the literature, in our model, a larger buyer obtains a lower per-unit (or average) price if there is a single seller or if adjustments are unfeasible. We explain, however, that this result does not necessarily extend to those often more realistic situations with multiple sellers and adjustments in trade. Here, our approach reveals that size

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tends to increase both buyer and seller dependency by worsening the alternative options of both sides when they seek to adjust what they trade in case of disagreement, which as explained next, can become an advantage or a disadvantage.

Disagreement of a seller with a large buyer displaces a large fraction of the potential demand and leaves a seller with few alternative buyers to replace its sales. Instead, a disagreement with a small buyer displaces a small fraction of demand and leaves the seller with many alternatives to turn to. In this sense, a seller is more dependent on a larger buyer. However, increasing marginal costs also render it relatively more costly for other sellers to accommodate a larger increase in production, and so it is equally more expensive for a large buyer to try to make up for a (off-equilibrium) shortfall when relocating its demand to other sellers of substitute goods. In this sense, a larger buyer is also more dependent on each individual seller. This second countervailing effect of size seems to have been largely overlooked by the literature.

With multiple sellers, the two conflicting effects of buyer size coexist and size becomes a source of mutual dependency, not an unequivocal source of power.[1] This demonstrates, in a stylized model that shuts down other explanations, a potential new mechanism for why size may not necessarily yield a better bargaining outcome to a firm conducting multiple bilateral bargains. In our model, the effect of size on buyer dependency dominates, and a larger buyer pays higher per-unit prices, if and only if the bilateral bargaining power of sellers is sufficiently high. So size and bilateral bargaining power are complements. Analogous results hold with respect to the size of sellers.

Mutual dependency also explains why the impact of size depends crucially on the extent to which technology allows for adjustments in case of bilateral disagreement. This suggests that practitioners and empiricists should take into account the technological specificities of the industry as a determinant of bargaining power, such as existing capacity constraints and the extent to which sellers are able to accommodate large-scale switching (e.g., by utilizing existing capacity more extensively at a reasonable cost).

By supporting the view that large purchase volumes are not per se conclusive of the existence of buyer power, in the sense of better terms of trade, our model provides conceptual guidance for policy and business strategy where buyer power has become increasingly topical.[2]

First, by changing the focus from size per se to ease of substitution and dependency, it seems to formalize concerns that have previously been expressed informally, for example, in the European Commission's guidelines on horizontal mergers and in recent sector inquiries. For instance, in the mentioned guidelines, buyer power is defined as "the bargaining strength that the buyer has vis-à-vis the seller in commercial negotiations due to its size, its commercial significance to the seller and its ability to switch to alternative suppliers." This definition highlights that, in addition to size, an assessment of buyer power needs to take into account two additional considerations: the consequences to the supplier from losing a particular buyer, and the consequences to the buyer from losing a particular supplier.

Second, the model's predictions accord well with the view taken in several influential court cases where, following inquiries with business experts, doubts were cast on the presumption that larger buyers should be able to negotiate lower per-unit prices. For example, in Hutchison/RCPM/ECT (2001) container terminal operators (sellers) argued that large carriers (buyers) had significant leverage in negotiations. Yet the European Commission argued to the contrary, stating that switching opportunities were limited for the largest carriers as "there is currently a limited number of terminal operators able to accommodate the largest vessels being used" by which "it becomes economically more difficult for the (larger) carrier to switch ports for a significant portion of its cargo." In another case, concerning toilet tissue and kitchen towels

[1] Note that, when there is a single seller, all buyers are completely dependent on that seller regardless of their size, thus, the second effect is absent, and a larger buyer then pays a lower per-unit price.

[2] Recent evidence from the UK retailing sector and the US pharmaceutical industry suggests that size alone is no guarantee to obtain discounts (see, for instance, Sorensen, 2003; Competition Commission UK, 2008; Ellison and Snyder, 2010; Grennan, 2013).

(SCA/Metsa Tissue, 2001), the European Commission observed that "buyer power can only be exercised effectively if the buyer has an adequate choice of alternative suppliers."[3] To this end, we also establish a relationship between specific changes in the Herfindahl-Hirschman-Index concentration measure (HHI) and changes in mutual dependency.

The Nash equilibria in bilateral Nash bargaining framework we use here was pioneered by Horn and Wolinsky (1998). Different assumptions on the consequences and possible adjustments in case of a bilateral disagreement underpin several prominent solutions that have been extensively adopted in both theoretical and empirical work, with, for example, quantity-forcing, two-part, and linear tariff contracts (e.g., Chipty and Snyder, 1999; Björnerstedt and Stennek, 2007; Crawford and Yurukoglu, 2012; Grennan, 2013; and Collard-Wexler, Gowrisankaran, and Lee, forthcoming). Two-part and linear tariff contracts (like ours) allow for adjustments, but these take place at a fixed per-unit price. Therefore, and of relevance to the issues of size raised here, such contracts ignore information on, and effects of, costs of production above the equilibrium level (supramarginal costs) incurred by a seller in case of disagreement, which is precisely what together with convex costs generates the novel size effects in our framework (see the first subsection of Section 6 for a detailed discussion).

In a setting with convex costs, Nash equilibria in bilateral Nash bargaining raises a related issue of credibility of the conjectured adjustments. For example, with two-part tariffs, a seller would always want to refuse adjustment requests, as a buyer would then only pay for each additional unit the marginal cost at the equilibrium quantity, which is less than the seller's true cost of production of those supramarginal units—similar issues can arise with linear tariffs. In contrast, and by construction, our approach always leads to adjustment requests that will be rationally accepted because the buyer pays the seller the true cost of the adjustment. This issue would seem even more relevant in the presence of demand uncertainty, as then adjustment requests are not only conjectured but a reality. We don't explicitly model such uncertainty in our model, but in that case, the adjustments we propose here would not only protect sellers but, under specific conditions, also lead to an ex post bilateral efficient level of trade—unlike those other above mentioned contracts.

As wholesale contracts are typically not observed, an empirical literature takes the view that the form of contracts should be part of the identifying restrictions and different models compared in terms of their overall fit with the data (e.g., Draganska, Klapper, and Villas-Boas, 2010; Bonnet et al., 2013). Here, our approach suggests a parsimonious way to complement and extend this line of work by allowing a flexibility in reacting to disagreements that should enable the recovery of additional information on costs (specifically, supramarginal costs) with seemingly manageable complexity.

A further alternative approach, referred to as "nonbinding contracts," assumes that a bilateral disagreement eventually leads to a permanent breakdown in bilateral negotiations and all parties to restart from scratch all previously successful negotiations. It has been shown that with such "global renegotiations," the solution coincides with cooperative random-order values—and in particular with the Shapley value when symmetry in bilateral bargaining power is imposed (e.g., Stole and Zwiebel, 1996; de Fontenay and Gans, 2014). Random-order values capture supramarginal costs and size may then be disadvantageous (see Gardner, 1977; Guesnerie, 1977; Segal, 2003). As we will explain in the main text, that approach leads to opposite predictions to our own, as size and bilateral bargaining power seem to be substitutes rather than complements. In contrast, our predictions echo those obtained by Postlewaite and Rosenthal (1974) for the Core, and which were the first to identify that owners of substitutable resources may lose from forming a monopoly (and therefore both their and our work hold opposite predictions to those obtained with random-order values, a fact that had also not been documented before in the cooperative games literature).

[3] The German antitrust authority in its sector inquiry into supply relationships in the grocery market has expressed *mirror* concerns about manufacturers' dependency.

To focus on a new channel that has been largely neglected, in our model, we explicitly shut down other potential sources of size effects. By assuming that buyers have the same bargaining power in bilateral negotiations, we rule out alternative explanations based on, for example, bargaining parameters that explicitly depend on size (e.g., Crawford and Yurukoglu, 2012) or that are firm specific (Grennan, 2013). In that respect, we take the prevalent view in the property rights theory of the firm, as it is understood that "it would be too easy to obtain a theory of the costs and benefits of integration if it were supposed that the bargaining process changes under integration" (Hart, 1995).

We also rule out externalities among buyers: trade is then efficient, and the aggregate gross benefit and costs are invariant in the size of buyers. This would not be the case if, for example, we allowed for negative consumption externalities that can be used to capture buyer competition downstream. With such externalities, the creation of a larger buyer leads to a direct internalization effect, which results in less quantities being traded, with an associated decrease in the gross costs of sellers and an increase in the gross surplus of all buyers. This direct internalization effect should further expand the upper and lower per-unit price bounds. However, as other buyers respond to that direct internalization by expanding their own purchases, the gross surplus of the large buyer may be significantly reduced and the effect on its price bounds reversed due to the Cournot paradox. In addition to the effects studied here, whether a large buyer pays a lower or a higher per-unit price would then also depend on the intensity of such additional effects. Iozzi and Valletti (2014), for example, analyze some of these effects in a model where each buyer negotiates a linear tariff contract with a single seller. As explored in their work, such games with externalities present additional challenges of beliefs and existence of equilibria, issues that are unrelated to the novel effects of size we wish to focus on here.[4]

Finally our results also differ from the unambiguous positive effect of buyer size that arises when a large buyer's advantage stems from a threat to access an option, that is, outside the market (such as backward integration; see Katz, 1987), or when a seller must incur fixed costs and a large buyer becomes pivotal (Raskovich, 2003). In the former case, buyer fixed costs from accessing the alternative option generate increasing returns to scale from switching (our setting is more applicable when negotiations are shaped by the, often more credible, option to relocate demand across sellers which a buyer already has a relationship with), and in the latter case, the seller fixed costs make a large buyer a complement to the other buyers (in our model, buyers remain substitutes).

The rest of the article is organized as follows. In Section 2, we describe the economy. In Section 3, we introduce and study a model of bargaining with adjustments. In Section 4, we identify conditions under which a larger buyer pays a higher or a lower per-unit price. In Section 5, we study the case of seller size. In Section 6, we discuss the effect of alternative adjustment assumptions and relate our findings to a small cooperative literature. We conclude in Section 7. The proofs, unless otherwise stated, are in the Appendix.

2. Buyers, sellers, and trades

■ There is a finite set of goods G, with $|G|$ denoting the number of goods. The cost of producing x units of goods $i \in G$ is $c(x)$, where c is a continuously differentiable, strictly increasing, and convex function, with $c(0) = 0$. Each seller I is the only producer of a subset of goods. S is the partition of G, with $|S|$ elements, such that each element $I \in S$ contains exactly the $|I|$ goods produced by seller I. Thus, $|I|$ is also the size of seller I.

There are $|N|$ symmetric consumers in a set N. The utility of a consumer $j \in N$ is $u(a_j) + t$, where t is money and a_j is a generic vector in $\mathbb{R}_+^{|G|}$, denoting the quantity of each of the goods consumed by j. The utility function u is symmetric and strictly concave, with strictly positive first-order derivatives and strictly negative cross-partial derivatives. This implies that u is submodular,

[4] On issues of existence of equilibria in bargaining with externalities, see also, for example, Collard-Wexler, Gowrisankaran, and Lee (forthcoming) and Lee and Fong (2013).

capturing substitutability and strictly decreasing benefits in the consumption of goods. Each buyer J is a union of consumers. B denotes the partition of N such that each of its $|B|$ elements contains exactly the $|J|$ consumers represented by each buyer $J \in B$.

Throughout the article, we generically refer to I as a seller and to J as a buyer. Trade between buyers and sellers is summarized by a trade matrix A with dimension $|G| \times |N|$, where each element a_{ij} is the quantity of good $i \in G$ delivered to consumer $j \in N$. As the column vector a_j represents the consumption profile of consumer j, the *gross benefit* of buyer $J \in B$ at A is represented by

$$v_J(A) = \sum_{j \in J} u(a_j).$$

The *gross cost* of seller $I \in S$ at A is

$$C_I(A) = \sum_{i \in I} c\left(\sum_{j \in N} a_{ij}\right).$$

The total surplus for a given A is

$$\Pi(A) = \sum_{J \in B} v_J(A) - \sum_{I \in S} C_I(A),$$

which is concave. The marginal cost $c'(0)$ is assumed to be sufficiently low and $c'(x)$ sufficiently large for large x such that the unique trade A^* that maximizes the economy surplus $\Pi(A)$ is strictly positive and finite. Given symmetry, convexity of costs, and concavity of utility, for each good $i \in G$ and each consumer $j \in N$, we have $a^*_{ij} = a^*$ with $c'(|N|a^*) = u_i(a^*_j)$, where a^*_j is a column vector with each element equal to a^* and u_i is the partial derivative with respect to good i. Hence, $|N|a^*$ is the total efficient production of each good.

Imposing symmetry—in consumers' utilities, goods' costs, and later bilateral bargaining power—will allow us to isolate the effects of size, as then any differences in the negotiated per-unit prices can be attributed to differences in buyer or seller size. We explicitly avoid interactions between the quantities purchased by two buyers on their gross benefits, ruling out situations akin to downstream competition. We do this to focus our analysis on the less understood connection between size and mutual dependency, instead of better understood competition effects.[5]

An agreement between a pair with seller $I \in S$ and buyer $J \in B$ specifies a total transfer t_{IJ} made by the buyer to the seller in exchange for the quantities a_{ij} for each $i \in I$ and $j \in J$, summarized in a $|I| \times |J|$ bilateral trade matrix A_{IJ}. Thus, an agreement is summarized by the pair (A_{IJ}, t_{IJ}), with (0,0) denoting the "default" of no agreement. A set of agreements between all buyers and sellers is summarized by the pair (A, T), where A is a $|G| \times |N|$ trade matrix and T is a $|S| \times |B|$ matrix of transfers with a typical element t_{IJ}. For a given (A, T), seller I's *net payoff* is given by

$$\theta_I(A, T) = \sum_{J \in B} t_{IJ} - C_I(A),$$

and buyer J's *net payoff* is

$$\theta_J(A, T) = v_J(A) - \sum_{I \in S} t_{IJ}.$$

In the following section, we take a bargaining approach to determine how buyers and sellers come to these agreements.

[5] Even with a single seller and multiple competing buyers, the downstream market outcome can be similar to Cournot (see, e.g., Hart and Tirole, 1990). A buyer merger could then be unprofitable due to Cournot internalization effects (see the Introduction and Iozzi and Valletti, 2014).

3. Bargaining

■ We provide two complementary formulations, applying the axiomatic asymmetric Nash bargaining solution to all bilateral negotiations before taking a strategic approach.

□ **Bilateral Nash bargaining with adjustments.** Suppose the equilibrium outcome of bilateral negotiations between each buyer and seller pair results in trade and transfers (A, T). The payoffs of a seller I and a buyer J are then $\theta_I(A, T)$ and $\theta_J(A, T)$. Let $d_{IJ}(A, T)$ and $d_{JI}(A, T)$ denote the payoffs of I and J only if this pair fails to reach the equilibrium bilateral agreement in (A, T) , their respective disagreement points. When this proves unambiguous, we drop the notational dependence of these payoff on (A, T).

To be consistent with the asymmetric Nash solution with exponent $\rho_{IJ} \in (0, 1)$, that is, to maximize the asymmetric Nash product

$$(\theta_I - d_{IJ})^{\rho_{IJ}}(\theta_J - d_{JI})^{(1-\rho_{IJ})},$$

trade A_{IJ} must maximize the bilateral gain from agreement between I and J given by

$$g_{IJ} = \theta_I + \theta_J - d_{IJ} - d_{JI},$$

and the respective seller and buyer payoffs must satisfy

$$\theta_I = d_{IJ} + \rho_{IJ} g_{IJ} \text{ and } \theta_J = d_{JI} + (1 - \rho_{IJ}) g_{IJ}, \tag{1}$$

where the simple sharing rule for the gain g_{IJ} follows from utilities being linear in payments. As seller I receives a share ρ_{IJ} of the gain from agreement, this exponent captures the bilateral bargaining power of I vis à vis J.

We follow the literature pioneered by Horn and Wolinsky (1998), and subsequently widely adopted in both theoretical and empirical work (see Introduction), by studying equilibria of multiple connected bilateral Nash bargaining problems. As explained next, we deviate from previous work in our specification of the disagreement points, which also accounts for the novel results on size.

We have in mind situations where only the buyer J and seller I, failing to reach their equilibrium agreement, would be aware of it and will therefore react to it by proposing their other respective trading partners adjustments in their bilateral transactions. We further stipulate these adjustments would make those trading partners just indifferent between the adjustment or trading according to the equilibrium agreement. This is the case if the proposed transfer changes reflect the incremental net valuation or cost of the counterpart. This idea shares in spirit the concept of "truthful strategies," introduced by Bernheim and Whinston (1986) in an auction context, in which a player is compensated for action s instead of s' by the net benefit accruing to another player due to that change. For this reason, we call the adjustments studied in this article "truthful." This leads to the following definition:

Definition 1. "Disagreement points truthful to (A, T)" are the net payoffs of seller I and buyer J when, not trading with each other but every other buyer and seller pair with $J' \neq J$ and $I' \neq I$, still trades according to (A, T), each adjusts trade optimally with, respectively, every $J' \neq J$ and $I' \neq I$, while adjusting the respective transfers $t_{IJ'}$ and $t_{I'J}$ to exactly reflect the change in the gross benefit of J' and cost of I' .

Next, we explicitly derive these disagreement points. For a given buyer J and seller I pair, we consider the trade matrix A^{IJ} adjusted from A such that

$$A^{IJ} = \text{argmax}\, \Pi(A) \text{ s.t.} \begin{cases} a_{ij}^{IJ} = 0 & \text{if } i \in I \text{ and } j \in J \\ a_{ij}^{IJ} = a_{ij} & \text{if } i \notin I \text{ and } j \notin J. \end{cases}$$

That is, to obtain from A the trade matrix A^{IJ}, the following three steps are undertaken. First, there is no trade between I and J: $a^{IJ}_{ij} = 0$ if $i \in I$ and $j \in J$. Second, we leave trade between any other buyers and sellers unchanged as in A: $a^{IJ}_{ij} = a_{ij}$ if both $i \notin I$ and $j \notin J$. Third, we adjust bilateral trade between seller I and each buyer $J' \neq J$ and between buyer J and each seller $I' \neq I$ in a way that maximizes the bilateral surplus of each pair. These adjustments are such that total surplus is also maximized subject to the constraints imposed by the first and second steps, which given our cost and benefit assumptions yields A^{IJ} as a unique solution.

To reflect "truthfulness," both I and J must capture the gross surplus generated by the respective adjustments, so transfers must be adjusted to reflect the incremental gross costs and benefits due to the change from A to A^{IJ}: the transfer received by I' from J must be increased by $C_{I'}(A^{IJ}) - C_{I'}(A)$ and the transfer paid by J' to I must be increased by $v_{J'}(A^{IJ}) - v_{J'}(A)$. It follows:

Lemma 1. "Disagreement *points are truthful to* (A, T)" if and only if, for every $I \in S$ and $J \in B$, for seller I,

$$d_{IJ}(A, T) = \sum_{J' \in B \setminus J} (t_{IJ'} + (v_{J'}(A^{IJ}) - v_{J'}(A))) - C_I(A^{IJ}),$$

and for buyer J,

$$d_{JI}(A, T) = v_J(A^{IJ}) - \sum_{I' \in S \setminus I} (t_{I'J} + C_{I'}(A^{IJ}) - C_{I'}(A)).$$

Next, we define the equilibrium concept. Let P denote the $|S| \times |B|$ matrix of bilateral Nash bargaining exponents, with a typical element $\rho_{IJ} \in (0, 1)$. We have:

Definition 2. A "truthful bargaining outcome with bargaining power P" is a trade and transfer matrix pair (A, T) such that, for every pair with seller I and buyer J , the payoffs $\theta_I(A, T)$ and $\theta_J(A, T)$ are consistent with bilateral Nash bargaining (as in (1)), and disagreement points $d_{IJ}(A, T)$ and $d_{JI}(A, T)$ are truthful to (A, T).

We find that:

Proposition 1. There is a unique truthful bargaining outcome with bargaining power P, which is given by the pair (A^*, T), with for every $I \in S$ and $J \in B$,

$$t_{IJ} = (1 - \rho_{IJ})\varphi_{IJ} + \rho_{IJ}\varkappa_{IJ}, \tag{2}$$

where

$$\varphi_{IJ} \equiv \sum_{J' \in B \setminus J} (v_{J'}(A^{*IJ}) - v_{J'}(A^*)) - (C_I(A^{*IJ}) - C_I(A^*)),$$

and

$$\varkappa_{IJ} \equiv (v_J(A^*) - v_J(A^{*IJ})) - \sum_{I' \in S \setminus I} (C_{I'}(A^*) - C_{I'}(A^{*IJ})).$$

That the outcome must be efficient, that is, that A^* is traded, follows from each a_{ij} maximizing the joint surplus of the respective buyer and seller pair while holding all other trades constant. Then, in equilibrium, the marginal cost of every good $i \in G$ must equal the partial derivative $u_i(a_j)$ for each consumer $j \in N$, which coincides with the first-order conditions for the maximization of the surplus function $\Pi(A)$. For a given P, evaluating the payoffs and disagreement points at A^* and solving (1) simultaneously for every payoff pair, we obtain a unique solution to the transfer matrix T.

It can be checked that $\varkappa_{IJ} > \varphi_{IJ}$, and therefore $\varkappa_{IJ}$ and φ_{IJ} capture, respectively, upper and lower bounds for the transfer paid by J to I. The measure of bilateral bargaining power $\rho_{IJ} \in (0, 1)$ pins down where the payment lies in that interval. How the transfers compare across buyers and sellers as a function of size will be analyzed later.

Finally, note that the difference between φ_{IJ} and $\varkappa_{IJ}$ is due to the fact that each buyer and seller enjoys some market power, as we explain next. In an online web Appendix, we show that when we replicate our economy more and more times (i.e., by adding replicas of each buyer and seller), both φ_{IJ} and $\varkappa_{IJ}$ converge to the transfer prevailing in a perfectly competitive economy, which is $|I||J|\mu a^*$ with $\mu = c'(|N|a^*) = u_i(a_j^*)$. Such competitive benchmark is intuitive as then, in the limit, at disagreement, each side could fully replace the respective trades by turning to (infinitely) many other buyers and sellers, and for each additional unit of each good, a disagreeing buyer would pay the marginal cost $c'(|N|a^*)$ to each of the remaining sellers, and a disagreeing seller would extract the marginal utility from each alternative consumer.

□ **Strategic bargaining: "Nash followed by Rubinstein."** Our disagreement points are now obtained endogenously in a strategic game. As the time between periods shrinks to zero, equilibrium contracts converge to those of Proposition 1 with an endogenous P, reflecting the relative impatience of buyers and sellers. In the interest of space, we here only describe the game informally and relegate its formal description to the Appendix.

The game takes place in periods $\tau = 0, .., \infty$, with bilateral contract offers consisting of a trade and transfer pair (A_{IJ}, t_{IJ}). It combines elements of two classic noncooperative approaches to *bilateral* bargaining. The *first approach* is Nash's (1953) demand game, the prototype model of bilateral bargaining: two players simultaneously submit an offer on how to share a pie, and each one receives its demand if the offers are exactly the same, and nothing otherwise. The *second approach* is Rubinstein's (1982) alternating offers approach.

At $\tau = 0$, similar to a Nash (1953) demand game between each pair, every buyer $J \in B$ makes a bilateral contract offer to every seller $I \in S$, and vice versa. Thus, a total of $2|S||B|$ contract offers are made simultaneously. A bilateral contract between buyer J and seller I is reached at $\tau = 0$ if and only if their bilateral offers are the same.

In each period, $\tau \geq 1$ only pairs that have failed to reach an agreement before τ engage in a bilateral alternating offers bargaining à la Rubinstein (1982), with offers made by sellers in odd periods and by buyers in even periods. Each player is potentially involved in several such bilateral negotiations, and thus, a player on the offer side may need to decide simultaneously on a set of offers to make, whereas a player on the receiving side may need to decide which to accept and reject from a set of offers. These alternating offers subgames share a similar structure to those in Collard-Wexler, Gowrisankaran, and Lee (forthcoming).[6]

Disagreement points are derived endogenously, as described next. After period τ offers have been accepted or rejected, but before the end of the period τ, a player that has not yet reached an agreement with all its counterparties can ask those counterparties they have already reached an agreement with (at or before τ) to adjust their previously agreed trade and transfer pair—and if that counterparty rejects it, then the pair trades according to their original contract.

Each player observes only the offers and adjustments it makes and receives, and it holds *passive beliefs* on all actions it does not observe—a common assumption in the vertical contracting literature.[7] At the end of each period, $\tau \geq 1$ trade takes place according to the accepted contracts or period specific adjustments (if any). Player i with discount rate r_i maximizes its expected discounted sum of period payoffs. The time between period 1 and τ is $z(\tau - 1)$. There is *immediate agreement* if all contracts are reached at $\tau = 0$.

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[6] However, the information structure of the games is different, and their game is a network game, as payoffs are a primitive that depends only on the agreements' structure and not on the level of trade (whereas both payoffs and trade must be determined endogenously in our model).

[7] See, for example, Hart and Tirole (1990); O'Brien and Shaffer (1992); McAfee and Schwartz (1994).

Proposition 2. For z sufficiently small, the strategic "Nash followed by Rubinstein" bargaining game has a unique outcome of pure-strategy passive beliefs PBE with immediate agreement. This outcome is efficient with at $\tau = 0$ every $I \in S$ and $J \in B$ making the exact same contract offer to each other

$$(A^*_{IJ}, t^{'}_{IJ}) \text{ with } t^{'}_{IJ} = \frac{(1 - e^{-zr_J})\varkappa_{IJ} + e^{-zr_J}(1 - e^{-zr_I})\varphi_{IJ}}{1 - e^{-z(r_J + r_I)}}.$$

As $z \to 0$, the equilibrium transfers converge to

$$t_{IJ} = \frac{r_J}{r_J + r_I}\varkappa_{IJ} + \frac{r_I}{r_J + r_I}\varphi_{IJ},$$

thus coinciding with those of Proposition 1 with P, such that $\rho_{IJ} = \frac{r_J}{r_J + r_I}$.

Our most relevant departure from previous work is adding a simultaneous Nash demand stage at the outset of the game. This significantly reduces the set of subgames that may be reached following any unilateral deviation from equilibrium, and in any of those subgames, at most one player can be engaged in multiple bilateral negotiations. For this reason, our game dispenses agents, which could be of independent interest to both theory and empirical researchers—as foundations of Nash equilibria in bilateral Nash bargaining where buyers and sellers do not negotiate directly but instead through agents, have met justified criticism (see Collard-Wexler, Gowrisankaran, and Lee, forthcoming). Such initial Nash demand game naturally captures the notion that a buyer and a seller should only need to start exchanging offers in negotiations if at the outset of the game, their mutual expectations prove to be incompatible.

4. Buyer power and mutual dependency

■ Following the property rights approach to the theory of the firm, we model size with a change in the control structure (Hart and Moore, 1990). With differences in the bilateral bargaining ability parameter ρ_{IJ}, it would of course be simple to construct *trivial* theories of size: for example, there would be a natural incentive for a buyer J with a low bargaining ability to be represented by a buyer J' with a higher bargaining ability, or for two buyers with similar bargaining ability to merge if size by itself increases bilateral bargaining power. Like in the property rights literature, we want to shut down such channels. We therefore assume that $\rho_{IJ} = \rho \in (0, 1)$ for every pair, regardless of name or size—so P is equal to ρ multiplied by the $|S| \times |B|$ unit matrix. This symmetry allow us to isolate and focus on the endogenous effects of size, and identify how these can be explained by the economic fundamentals of costs and preferences. In this case, the effect of a merger also does not depend on the identity of the acquirer.

Suppose that two buyers J_1 and J_2, form a larger buyer $J_3 = J_1 \cup J_2$. This will not affect the traded quantities, which are still A^*. The transfers paid by any other buyer $J \in B\backslash\{J_1, J_2\}$ are therefore unchanged. For the larger buyer J_3, the respective per-unit price is lower than the per-unit price paid jointly by J_1 and J_2 (were they to remain independent) if and only if

$$(1 - \rho)\varphi_{IJ_3} + \rho\varkappa_{IJ_3} < (1 - \rho)(\varphi_{IJ_1} + \varphi_{IJ_2}) + \rho(\varkappa_{IJ_1} + \varkappa_{IJ_2}). \tag{3}$$

To determine when (3) holds, it is instructive to first consider separately the cases where $\rho \to 0$ and $\rho \to 1$. In either case, transfers are such that the payoffs on one side of the market are determined only by the value created by their respective adjustments. Thus, we ask how a larger purchasing volume affects the value of these alternatives for both sides.

□ Seller and buyer dependency.

Seller dependency. That each seller is more dependent on the larger buyer is captured by the following result:

Lemma 2. Suppose buyers J_1 and J_2 form a buyer $J_3 = J_1 \cup J_2$. Then,

$$\varphi_{IJ_3} < \varphi_{IJ_1} + \varphi_{IJ_2} \tag{4}$$

holds, implying that as buyers hold all the bilateral bargaining power ($\rho \to 0$ and sellers are pushed to their respective "truthful" disagreement points), the per-unit price paid by J_3 is always strictly lower than the per-unit price paid jointly by J_1 and J_2.

Lemma 2 derives from two effects. One is the "incremental cost" effect previously isolated in the literature: with convex costs, per-unit incremental costs of supplying a smaller buyer are higher than those of supplying a larger buyer (see the survey in Snyder, 2012). This effect would also be present if adjustments of trades are not possible, as we will discuss below. The other effect is that off-equilibrium, a seller has the opportunity to increase sales to all remaining buyers, but a disagreement with the larger buyer leaves the seller with fewer alternatives to sell to. As consumers have decreasing marginal utility for the seller's products, the alternative to adjust sales therefore becomes less valuable following disagreement with a larger buyer.

Both effects rely on cost convexity below the equilibrium quantities and both disappear with constant marginal costs: the former because marginal and inframarginal costs are then the same, the latter because no profitable adjustments exist to be made by a seller in disagreement, as trades with all other buyers are already such that the respective marginal utilities equal a seller's constant marginal cost.

Buyer dependency. That the larger buyer also becomes more dependent on each seller is captured by the next result:

Lemma 3. Suppose buyers J_1 and J_2 form a buyer $J_3 = J_1 \cup J_2$. *If* there is more than one seller ($|I| \neq |G|$ *for every* $I \in S$), then

$$\varkappa_{IJ_3} > \varkappa_{IJ_1} + \varkappa_{IJ_2} \tag{5}$$

holds, implying that as sellers hold all the bilateral bargaining power ($\rho \to 1$ and buyers are pushed to their respective "truthful" disagreement points) the per-unit price paid by J_3 is always higher than the per-unit price paid jointly by J_1 and J_2. If there is a single seller ($|I| = |G|$ *for some* $I \in S$), then $\varkappa_{IJ_3} = \varkappa_{IJ_1} + \varkappa_{IJ_2}$, so as $\rho_1 \to 1$, there is no effect.

Recall that, as marginal costs are increasing, the average incremental costs of temporarily increasing trade with other sellers is higher for larger quantities—as demanded by J_3. In this sense, a larger buyer becomes more dependent on each seller. This negative effect of size, which tends to increase the per-unit price, relies crucially on the possibility to adjust trades: it is absent when there is only one seller (as frequently assumed in the previous theoretical literature, see the survey in Snyder, 2012), if technology rules out such adjustments (see discussion below), or if marginal costs are constant (as average incremental costs are then invariant in size).

□ **Large buyer advantage or disadvantage?** In sum, in our model, the formation of a larger buyer increases *mutual* dependency: sellers become more dependent on that buyer but that buyer also becomes more dependent on each individual seller. Recall that the per-unit price paid by the large buyer J_3 is strictly lower than the per-unit price paid jointly by J_1 and J_2 before the merger if and only if (3) holds. Rearranging terms, this condition becomes

$$\frac{\varkappa_{IJ_3} - \left(\varkappa_{IJ_1} + \varkappa_{IJ_2}\right)}{\left(\varphi_{IJ_1} + \varphi_{IJ_2}\right) - \varphi_{IJ_3}} < \frac{1-\rho}{\rho}. \tag{6}$$

Lemmas 2 and 3 allow us to sign the left-hand side: it is strictly positive and invariant in ρ when there are multiple sellers, and exactly zero when there is a single seller. As the right-hand side is monotonically decreasing in ρ, converging to ∞ as $\rho \to 0$ and to 0 as $\rho \to 1$, we obtain the following result:

Proposition 3. Suppose buyers J_1 and J_2 form a buyer $J_3 = J_1 \cup J_2$. J_3 pays strictly lower per-unit prices than those paid jointly by J_1 and J_2 if and only if

(i) there is a single seller, that is, $|I| = |G|$ for some $I \in S$;
(i) there are multiple sellers, that is, $|I| \neq |G|$ for every $I \in S$, and ρ is sufficiently low.

This result confirms and extends the current understanding of buyer power in the literature. It confirms as the typical finding in this literature is that when there is a single seller, a larger buyer pays a lower per-unit price (see the survey in Snyder, 2012). It extends as it shows that result no longer necessarily holds when there are multiple sellers of substitute products and that have convex costs. Then, a larger buyer can still obtain lower per-unit prices, but only when buyers have a sufficient advantage in bilateral bargaining. In that case, buyers are able to push sellers close to their disagreement points, which as we saw, becomes relatively less favorable to sellers as the larger buyer controls a larger share of the total demand. However, when sellers have the advantage in bilateral bargaining, a larger buyer pays a higher per-unit price. In that case, it is the sellers who are able to push buyers close to their disagreement points, which is relatively less favorable for a larger buyer because convex costs make it relatively more costly to shift a larger volume to alternative sellers.

In our model, buyers' incentives to be large are thus higher when bargaining power arising from other channels is already high. In other words, buyers' bilateral bargaining power and size have a mutually reinforcing role in bringing prices down. Size is not a substitute for the lack of bilateral bargaining power, it complements it.

Asymmetries and concentration. As discussed in the Introduction, our analysis is partly motivated by recent antitrust interest on buyer power under negotiated prices. Here, the HHI, which is the sum of the squares of individual market shares, is probably the most important first-phase screening criterion for mergers and asset sales. The HHI has its foundation in the analysis of seller market power through withholding demand below its efficient level, which is clearly distinct from the exercise of bargaining power. Antitrust authorities acknowledge this, yet they still extended the application of the HHI to markets with bilaterally negotiated contracts. [8] We analyze next to what extent our model supports this approach.

A full merger always increase the HHI. We therefore extend our analysis to asset transfers (i.e., consumers) between two buyers, which may increase or decrease the buyer HHI, denoted by HHI_B. In light of our present interest in market concentration, for the remainder of the subsection we also consider the (limit) case where products are perfect substitutes, so that $u(a_j) = u(\sum a_{ij})$. We compare market structures that differ in the respective sizes of two buyers. Starting from a situation where $|J_1| + |J_2| = M \leq N$, we consider the effect of a subset of consumers $J_m \subset J_2$ of size $|J_m| = m > 0$ joining J_1 to form buyers $\widehat{J_1} = J_1 \cup J_m$ and $\widehat{J_2} = J_2 \backslash J_m$ (the previous case where $J_m = J_2$ is covered as a limit). Total production and consumption are not affected, and this increases HHI_B if $|J_1| + m > |J_2|$, and decreases it otherwise.[9]

Proposition 4. Take the limiting case where goods are perfect substitutes. A transfer of consumers between two buyers strictly decreases $\sum_{J \in B} \varphi_{IJ}$ and strictly increases $\sum_{J \in B} \varkappa_{IJ}$ for each $I \in S$ if and only if it strictly increases buyer concentration measure HHI_B.

[8] For instance, the US guidelines state: "To evaluate whether a merger is likely to enhance market power on the buying side of the market, the Agencies employ essentially the framework described above for evaluating whether a merger is likely to enhance market power on the selling side of the market."

[9] As $HHI_B = \sum_{J \in B} (\frac{|J|}{|N|})^2$, the change is equal to the difference of $(\frac{|J_1|+m}{|N|})^2 + (\frac{|J_2|-m}{|N|})^2$ and $(\frac{|J_1|}{|N|})^2 + (\frac{|J_2|}{|N|})^2$, which is positive if and only if $|J_1| + m > |J_2|$.

Thus, a transfer of assets between two buyers increases aggregate buyer and seller dependency if and only if it increases the buyer concentration index HHI_B. In addition, the acquisition of a buyer J_1 by another buyer J_2 leads to a larger increase in HHI_B than the acquisition of J_1 by a buyer J_3 if and only if $|J_2| > |J_3|$. It follows from the previous proposition that the acquisition that leads to the highest increase in HHI_B also results in the largest increase in aggregate buyer and seller dependency. As corollary, when buyers have a sufficiently high bilateral bargaining power (i.e., ρ is close to 0), the merger with the more pronounced increase in buyer concentration, as measured by the HHI_B, will also lead to the largest drop in sellers' profits, but increase their profits if ρ is close to 1.[10] Thus, Proposition 4 only justifies the use of HHI_B to assess negative implications of a buyer merger or transfers of assets on sellers when ρ is low. These are, however, the cases where buyer power is also likely to be a concern.

□ **Factors favoring a large buyer advantage or disadvantage.** We next explore how production and consumption technology determines which of our two isolated effects is stronger. We first consider the case where no adjustments are feasible. We then identify two specific cost functions that are convex (but not strictly convex) that, respectively, shut down the buyer and the seller dependency channel. Finally, in the context of a parametric example, we explore the role of goods' substitutability and demand elasticity.

Production side factors.

No adjustments. It is instructive to first consider the case where adjustments are unfeasible. Then, A^{IJ} is such that $a_{ij}^{IJ} = 0$ if $i \in I$ and $j \in J$, and $a_{ij}^{IJ} = a^*$ otherwise. The modified transfer lower and upper bounds are

$$\widetilde{\varphi}_{IJ} = |I|\,[c\,((|N|\,a^*) - c\,((|N| - |J|)a^*)] \tag{7}$$

and

$$\widetilde{\varkappa}_{IJ} = |J|\left[u(a_j^*) - u(a_j^{IJ})\right] \text{ with } j \in J. \tag{8}$$

With respect to buyer size we no longer have two conflicting effects, as only the well-known "incremental cost" effect remains with the respective transfers from expression (8), which ensures that condition (4) still holds, but condition (5) no longer holds for the transfers in expression (7). This implies that a larger buyer always pays a lower per-unit price, and a buyer merger is always profitable.

More generally, we could have assumed that adjustments are not certain but instead are made with independent probabilities ω_J by buyer J and ω_I by seller I (and with the complementary probability trade remains unchanged).[11] This would nest our and the no adjustments cases as limits as, respectively, $\omega_J = \omega_I = 1$ or $\omega_J = \omega_I = 0$ for all $I \in S$ and $J \in B$. Having $\omega_J \in (0, 1)$ and $\omega_I \in (0, 1)$ not only seems to add realism but it is also sufficient to generate our novel predictions on the effect of size, that is, for Propositions 1 and 3 to hold, as in those cases, the equilibrium transfers are given by

$$t_{IJ} = \rho_{IJ}\varkappa'_{IJ} + (1 - \rho_{IJ})\varphi'_{IJ}$$

with

$$\varphi'_{IJ} = w_I\varphi_{IJ} + (1 - w_I)\widetilde{\varphi}_{IJ} \quad \text{and} \quad \varkappa'_{IJ} = w_J\varkappa_{IJ} + (1 - w_J)\widetilde{\varkappa}_{IJ},$$

thus a weighted combination of the bounds with and without adjustments.

[10] When bilateral bargaining power ρ is more evenly distributed, we need to consider the impact of a change in market structure not only on seller dependency but also on buyer dependency, as we did for mergers in Proposition 3. Then, a focus on the HHI_B alone is not informative.

[11] These can also alternatively capture the fraction of the bilateral adjustment surplus accruing to those players.

Binding equilibrium capacity constraints. We next suppose that capacity is limited as follows: $c(x)$ is well behaved until some K, but $c(x) = \infty$ when $x > K$. This capacity constraint is binding at equilibrium if in addition $a^* = K/|N|$. This deprives buyers of their option to adjust trades in case of disagreement, so the transfer's upper bound is given in this case by $\widetilde{\varkappa}_{IJ}$. The adjustment option persists, however, for sellers and the transfer lower bound is like in the original case φ_{IJ}.[12] Therefore, a buyer merger reduces the lower bound (reflecting that size increases seller dependency) but it has no effect on the upper bound (reflecting that the buyer dependency channel is shut down if buyers are unable to make adjustments). It follows that, for all $\rho \in (0, 1)$, with a binding equilibrium capacity but otherwise convex inframarginal costs, buyer size is unambiguously advantageous.

Proportional inframarginal costs. In a mature industry, firms may have adjusted to be able to produce up to the expected equilibrium level $x = |N|a^*$ at a constant marginal cost and marginal costs then only increase above that level, for example, as then more expensive overtime is needed. Suppose that $c(x) = cx$ holds up to $x = |N|a^*$, but $c''(x) > 0$ for $x > |N|a^*$. As marginal costs are constant below equilibrium production levels, sellers now do not find profitable adjustments and thus the lower bound is $\widetilde{\varphi}_{IJ}$. Through the channel of "seller dependency," a buyer's size cannot then be advantageous. On the other hand, as costs are still strictly convex for adjustments above the equilibrium production level, the buyer dependency channel is still present and the lower bound is like in the original case $\varkappa_{IJ}$. In this case, a buyer merger is then always unprofitable.

For the following summarizing statement, we simplify the exposition by restricting consideration to the case with more than one seller.

Proposition 5. Suppose that $|J| \neq |N|$ *and* $|I| \neq |G|$ *for every* $J \in B$ and $I \in S$. The per-unit price paid by a large buyer $J_3 = J_1 \cup J_2$ is smaller than the per-unit price paid jointly by the smaller buyers J_1 and J_2 if (i) adjustments off-equilibrium are unfeasible, or (ii) sellers have binding equilibrium capacity constraints, but (iii) that per-unit price is always larger if costs are proportional to output below equilibrium quantities and above are strictly convex.

Consumption side factors. We now turn our attention to the consumer side. Here, we wish to derive implications on the role of substitutability and hence elasticity of demand. As it turns out, for tractability we need to rely on a particular functional specification, which we set out first. Note that for the purpose of illustration, we will return to this example also in subsequent sections.

Specification of an example. Let there thus be two sellers, I_1 and I_2, producing, respectively, goods 1 and 2 at a cost $c(q) = q^2/2$. There is a unit measure of consumers with a quadratic utility,

$$U(q_1, q_2) = q_1 + q_2 - b'\left(\frac{q_1^2 + q_2^2 + 2\gamma q_1 q_2}{2}\right) + m,$$

with a sufficiently high income m and $b' > 0$. The inverse demand for good i is then

$$1 - b'(q_i + \gamma q_{-i}).$$

The parameter $\gamma \in (0, 1)$ measures the degree of product differentiation, and goods become independent as $\gamma \to 0$ and perfect substitutes as $\gamma \to 1$. Let $b' = b(1 + \gamma)^{-1}$ so that A^*, with $q_i^* = q^* = \frac{1}{b+1}$, and $U(q^*, q^*) = \frac{b+2}{(b+1)^2}$ are invariant in γ. This later ensures that, as we change γ, changes in per-unit prices are not related to changes in the equilibrium cost or consumers' equilibrium utility level. Demand becomes perfectly elastic as $b \to 0$ and perfectly inelastic as $b \to \infty$. Each buyer is a collection of consumers, buyer J_1 represents a share α_1 of the consumers

[12] We need to slightly modify the proof to take into account the corner solutions.

and buyer J_2 a share $\alpha_2 = 1 - \alpha_1$. A buyer's payoff is

$$\theta_{J_j}(A, T) = \alpha_j U\left(\frac{q_1}{\alpha_j}, \frac{q_2}{\alpha_j}\right) - t_{I_1 J_j} - t_{I_2 J_j}$$

where q_i is the total quantity purchased by the buyer of good i. When trade is efficient,

$$\theta_{J_j}(A^*, T) = \alpha_j U(q^*, q^*) - t_{I_1 J_j} - t_{I_2 J_j} \text{ and } \theta_{I_i}(A^*, T) = t_{I_i J_1} + t_{I_i J_2} - c(q^*).$$

Suppose that seller I_i and buyer J_j fail to reach an agreement. The adjusted trade matrix $A^{I_i J_j}$ from A^* has: trade between J_{-j} and I_{-i} unchanged at $(1 - \alpha_j)q_1^*$, trade between J_j and I_i set to zero, trade between J_j and I_{-i} adjusted in a bilateral optimal way to

$$\widehat{q}_j = \underset{q_j}{\text{argmax}} \left(\alpha_j U\left(0, \frac{q_j}{\alpha_j}\right) - c\left((1 - \alpha_j)q^* + q_j\right)\right) = \frac{\alpha_j(b + \alpha_j)(1 + \gamma)}{(b + 1)(\alpha_j + b + \gamma\alpha_j)},$$

and trade between I_i and J_{-j} adjusted in a bilateral optimal way to

$$\widehat{q}_i = \underset{q_i}{\text{argmax}} \left(\alpha_{-j} U\left(q^*, \frac{q_i}{\alpha_{-j}}\right) - c(q_i)\right) = \frac{\alpha_{-j}(b + \gamma + 1)}{(b + 1)(b + \alpha_{-j} + \gamma\alpha_{-j})}.$$

The truthful bargaining outcome (A^*, T) is then

$$t_{I_i J_j} = (1 - \rho_{IJ})\varphi_{I_i J_j} + \rho_{IJ}\varkappa_{I_i J_j},$$

where

$$\begin{aligned}\varphi_{I_i J_j} &= \alpha_{-j}\left(U\left(q^*, \frac{\widehat{q}_i}{\alpha_{-j}}\right) - U(q^*, q^*)\right) - (c(\widehat{q}_i) - c(q^*)) \\ &= \alpha_j \frac{2\alpha_{-j}(1 + \gamma) + b(1 + \alpha_{-j})}{2\,(b + 1)^2\left(b + \alpha_{-j}(1 + \gamma)\right)}\end{aligned}$$

and

$$\begin{aligned}\varkappa_{I_i J_j} &= \alpha_j\left(U(q^*, q^*) - U\left(0, \frac{\widehat{q}_j}{\alpha_j}\right)\right) - \left(c(q^*) - c((1 - \alpha_j)q^* + \widehat{q}_j)\right) \\ &= \alpha_j \frac{2\alpha_j(1 + \gamma) + b(2 + b(1 - \gamma) + \alpha_j)}{2\,(b + 1)^2\left(b + \alpha_j(1 + \gamma)\right)}.\end{aligned}$$

In the example, note that to have a monopolist buyer, we need to set $\alpha_j = 1$. Recall also that a buyer merger does not change the total number of units purchased. Using the explicitly derived expressions for transfers $t_{I_i J_j}$, we arrive at the next observation:

Observation. In the example, a merger of the two buyers raises the per-unit price if and only if

$$\rho > \overline{\rho} = \frac{\xi}{\xi + b^2\gamma^2} \quad \text{with} \quad \xi = (b + \gamma + 1)^2.$$

Consider first the role of substitutability. Let goods become independent, that is, $\gamma \to 0$. As expected, then $\overline{\rho} \to 1$ and a larger buyer always pays a lower per-unit price. However, $\overline{\rho} < 1$ for all $\gamma \in (0, 1)$ and $\overline{\rho}$ is decreasing in γ: thus, when goods are substitutes, a larger buyer pays a higher per-unit price if and only if buyers' bilateral bargaining power is sufficiently low, whereas within the specific example, this happens more often as goods' substitutability increases.

Consider next the role of demand elasticity. Being a monopoly buyer is also disadvantageous more often when total demand is less elastic, that is, when b increases: the critical $\overline{\rho}$ increases from $1/2$ to 1 as demands goes from perfectly inelastic with $b \to \infty$ to perfectly elastic with $b \to 0$. There are two reasons for this. First, in case of disagreement, a buyer may not only buy from the other seller but can also buy less overall. Buying less is, however, less of an option when demand is relatively inelastic. Thus, demand elasticity reduces buyer dependency from sellers, which is more beneficial to a larger buyer with a larger "displaced" quantity in case of a disagreement. Second, in case of a bilateral disagreement, a seller cannot sell more to other buyers, if either the buyer is a monopoly or demand is perfectly inelastic (because the equilibrium quantities are already all each buyer needs). Seller adjustments are valuable only when there are multiple buyers, and reducing elasticity increases buyers' bargaining power only when there are multiple buyers, but not if the buyer is a monopolist.

5. The case of seller size

■ We now turn our attention to the seemingly mirror case of seller size. In analogy to how we proceeded in the case of buyer power, let there be two sellers I_1 and I_2, and consider the effect of a merger such that $I_3 = I_1 \cup I_2$. The large seller I_3 now receives a strictly higher per-unit price from some buyer J if and only if

$$(1-\rho)\varphi_{I_3 J} + \rho\varkappa_{I_3 J} > (1-\rho)(\varphi_{I_1 J} + \varphi_{I_2 J}) + \rho(\varkappa_{I_1 J} + \varkappa_{I_2 J}).$$

Like above, we focus in turn on the effect of the merger on seller and buyer dependency. All proofs for the results in this section are relegated to the online web Appendix.

Lemma 4. Suppose sellers I_1 and I_2 form a seller $I_3 = I_1 \cup I_2$. *If* there is more than one buyer, so $|J| \neq |N|$ for all $J \in B$, then

$$\varphi_{I_3 J} < \varphi_{I_1 J} + \varphi_{I_2 J} \tag{9}$$

holds, implying that when buyers hold all the bilateral bargaining power ($\rho \to 0$), the per-unit price received by I_3 is always strictly lower than the per-unit price received jointly by I_1 and I_2. If there is a single buyer, so $|J| = |N|$ for some $J \in B$, then $\varphi_{I_3 J} = \varphi_{I_1 J} + \varphi_{I_2 J}$ (so as $\rho \to 0$, the merger has no effect).

As buyers have decreasing returns, in case of disagreement with any buyer, the seller I_3 will find it harder to sell its relatively larger number of goods to the same set of alternative buyers without depressing prices too much. Thus, a larger seller becomes more dependent on any particular buyer.

Take now the case where $\rho \to 1$. Suppose first that goods are perfect substitutes so that we can write $u(a_j) = u(\sum_{i \in G} a_{ij})$. Then, concavity of $u(\cdot)$ mirrors convexity of $c(\cdot)$, and it is intuitive that the effect mirrors that of a buyer merger. We need, however, an additional regularity assumption for this logic to hold when goods are not perfect substitutes. Putting it first informally, we need that when a consumer loses access to some good, then this reduces its utility by more when it already has access to fewer goods. Formally, consider a marginal loss when consumption of good i must be reduced, given some consumption vector a_j, that is, $-u_i(a_j)$. Take any other two goods z and y and suppose that we tilt consumption toward good z such that $da_{zj} = -da_{yj} = \Delta > 0$, if

$$\frac{d}{da_{zj}} u_i(a_j) \geq \frac{d}{da_{yj}} u_i(a_j), \tag{10}$$

then this tilt aggravates the loss associated with a reduction in the consumption of good i. We have:

Lemma 5. Suppose sellers I_1 and I_2 form a seller $I_3 = I_1 \cup I_2$ and that condition (10) holds. Then,

$$\varkappa_{I_3 J} > \varkappa_{I_1 J} + \varkappa_{I_2 J} \tag{11}$$

holds, implying that when sellers hold all the bilateral bargaining power ($\rho \to 1$), the per-unit price received by I_3 is strictly higher than that received by I_1 and I_2.

Finally, taking together the results from Lemmas 4 and 5, and with analogous construction to that on the formation of a larger buyer in Proposition 3, we have:

Proposition 6. Suppose sellers I_1 and I_2 form a seller $I_3 = I_1 \cup I_2$ and that (10) holds. The larger seller receives a strictly higher per-unit price if and only if

(i) there is only a single buyer, that is, $|J| = |N|$ for some $J \in B$,
(ii) there are multiple buyers, that is, $|J| \neq |N|$ for every $J \in B$, and ρ is sufficiently high.

A result analogous to Proposition 4 also holds for an increase in the seller concentration measure (see the online web Appendix). Concerning factors favoring a large seller advantage or disadvantage, suppose that $|J| \neq |N|$ *and* $|I| \neq |G|$ *for every* $J \in B$ and $I \in S$ and (10) holds. The per-unit price received by a large seller $I_3 = I_1 \cup I_2$ is always larger than the per-unit price jointly received by the smaller sellers I_1 and I_2 if (i) off-equilibrium adjustments are unfeasible, or (ii) if costs below equilibrium quantities are proportional to output but above are strictly convex, but (iii) this per-unit price is higher with binding equilibrium capacity constraints if and only if $\rho \geq \widetilde{\rho}$ and $\widetilde{\rho} \in (0, 1)$.

6. Alternative adjustments and solutions

■ In this section, we relate our predictions to those of three widely used Nash equilibrium in bilateral Nash bargaining models (quantity-forcing, two-part, and linear tariffs) and two of the most used cooperative solution concepts (Core and random-order values).

□ **Quantity-forcing, two-part, and linear contracts.** Like in our model, with quantity-forcing contracts, players bargain bilaterally over a quantity A_{IJ} and a transfer t_{IJ}. However, unlike in our model, no adjustments are possible in case of a bilateral disagreement and thus it corresponds to the case seen above. Importantly, only information on inframarginal utilities and costs was used there to derive the equilibrium contracts and therefore, as seen above, the per-unit price is decreasing in buyer size, and buyer mergers are always profitable. These features are shared by two-part and linear tariff models. Next, we use the parametric example introduced above to illustrate and to explain why the predictions of these two models are also different from ours.

With two-part tariffs, players bargain bilaterally over a fixed transfer and a marginal price. Closer to our model, in case of a bilateral disagreement, adjustments are now possible but (unlike in our model) at a constant marginal price. The efficient quantities are traded in equilibrium and marginal prices are size-invariant, as for every i and j, they satisfy

$$U_i(q_i, q_{-i})|_{q_i=q_{-i}=q^*} = w_{I_i J_j} = c'(q^*) = \frac{1}{1+b} = w^*.$$

Consider next the fixed transfer $t_{I_i J_j}$. In case of disagreement, trade between J_j and I_i is zero, seller I_i still sells the equilibrium quantity $(1-\alpha_j)q^*$ to the other buyer J_{-j}, but buyer J_j increases its purchases from the other seller I_{-i} at a constant price w^* to

$$\widehat{q}_{-i} = \underset{q}{\operatorname{argmax}} \left(\alpha_j \left(\frac{q}{\alpha_j} - b\frac{\left(\frac{q}{\alpha_j}\right)^2}{2(1+\gamma)} \right) - w^* q \right) = \frac{\alpha_j(1+\gamma)}{1+b}.$$

The gains from trade between buyer J_j and seller I_i are then

$$g_{I_i J_j} = \alpha_j \left(U(q^*, q^*) - U\left(0, \frac{\widehat{q}_{-i}}{\alpha_j}\right)\right) + w^* \left(\widehat{q}_{-i} - \alpha_j q^*\right) - c(q^*) + c((1-\alpha_j)q^*).$$

A two-part tariff contract gives a seller a share ρ of the gains and satisfies

$$\theta_{I_i} = \rho g_{I_i J_j} + d_{I_i J_j} \Leftrightarrow$$

$$t_{I_i J_j} + t_{I_i J_{-j}} + w^* q^* - c(q^*) = \rho g_{I_i J_j} + (t_{I_i J_{-j}} + w^*((1-\alpha_j))q^* - c((1-\alpha_j)q^*)).$$

Importantly, note that θ_{I_i}, $g_{I_i J_j}$, and $d_{I_i J_j}$ again contain no information on costs above the equilibrium trade level. Solving, we have

$$t^*_{I_i J_j} + w^* \alpha_j q^* = (1-\rho)\varphi_{I_i J_j} + \rho \varkappa_{I_i J_j},$$

where

$$\varphi_{I_i J_j} = c(q^*) - c((1-\alpha_j)q^*) = \alpha_j \frac{(2-\alpha_j)}{2(1+b)^2}$$

and

$$\varkappa_{I_i J_j} = \alpha_j \left(U(q^*, q^*) - U\left(0, \frac{\widehat{q}_{-i}}{\alpha_j}\right)\right) + w^*(\widehat{q}_{-i} - \alpha_j q^*) = \alpha_j \frac{2 + b(1-\gamma)}{2(1+b)^2}.$$

The per-unit price is then

$$\frac{t^*_{I_i J_j} + w^* \alpha_j q^*}{\alpha_j q^*} = (1-\rho)\frac{2-\alpha_j}{2(1+b)} + \rho \frac{2 + b(1-\gamma)}{2(1+b)},$$

which is strictly decreasing in α_j. Thus, also in the case of two-part tariffs, the larger of two buyers always pays a lower per-unit price, and a buyer merger is always profitable.

Consider next the case of linear tariffs. A linear tariff maximizes the Nash product

$$(\theta_{I_i} - d_{I_i J_j})^{\rho} (\theta_{J_i} - d_{J_i I_j})^{(1-\rho)}.$$

As expected, a lower price is always beneficial to the buyer, and thus $\theta_{J_i} - d_{J_i I_j}$ is strictly decreasing in $w_{I_i J_j}$, and

$$\theta_{I_i} - d_{I_i J_j} = w_{I_i J_j} q_{I_i J_j} - (c(q_{I_i J_j} + q_{I_i J_{-j}}) - c(q_{I_i J_{-j}})),$$

is concave in $w_{I_i J_j}$ due to a standard price and quantity trade-off as the quantity traded between the pair I_i and J_j, $q_{I_i J_j}$, is strictly decreasing in $w_{I_i J_j}$. This makes it simple to obtain the upper and lower bounds of $w_{I_i J_j}$: the lower bound solves $(\theta_{I_i} - d_{I_i J_j}) = 0$ and the upper bound maximizes $(\theta_{I_i} - d_{I_i J_j})$. Importantly, note again that neither of these two problems includes information on costs above the equilibrium level.

All buyers choose their quantities optimally at the equilibrium linear tariffs, thus $q^*_{I_i J_j}$ is obtained by maximizing θ_{I_i} with respect to $q_{I_i J_j}$. Replacing this solution in the expressions above, the lower bounds are then obtained and coincide with the average incremental costs of the units traded between each pair, that is,

$$w_{I_i J_j} = \frac{c(q_{I_i J_j} + q_{I_i J_{-j}}) - c(q_{I_i J_{-j}})}{q_{I_i J_j}},$$

and, as $d_{J_i I_j}$ does not depend on $w_{I_i J_j}$, the upper bounds are obtained by maximizing the equilibrium payoff of each seller with respect to each tariff, that is, by solving

$$\frac{\partial(\theta_{J_i} - d_{J_i I_j})}{\partial w_{I_i J_j}} = \frac{\partial \theta_{J_i}}{\partial w_{I_i J_j}} = 0.$$

In the context of our example, with, respectively, two *symmetric* independent buyers and a monopoly buyer, the lower bound is $\frac{3}{4b+3}$ and $\frac{1}{2b+1}$, and the upper bound in both cases is $\frac{b-b\gamma+1}{2b-b\gamma+1}$. Thus, for all $\rho \in (0, 1)$, a monopoly buyer pays a lower per-unit price.[13]

Unlike quantity-forcing contracts, two-part and linear tariffs allow for off-equilibrium adjustments. However, in both cases, the bilateral gains from agreement, and therefore transfers and per-unit prices, still depended only on the shape of the cost curves below equilibrium production levels. The reason is that, in those cases, a buyer who does not reach an agreement with a seller is allowed to adjust purchases by paying a constant price per-unit to an alternative seller, and it is then that seller (and never the buyer) that pays the real cost of producing those units. This explains why in these cases, a larger buyer can never pay a larger per-unit price. It also has several other consequences, as discussed next.

One issue concerns the credibility of the conjectured adjustments with two-part and linear tariffs in the presence of convex costs. Indeed, there will be common knowledge that a seller receiving an adjustment request would rationally want to refuse it. In the case of two-part tariffs, for example, the seller is always made worst off by an adjustment request, as the buyer pays only the marginal cost at q^*, which is w^*, for each of the $\widehat{q}_i - \alpha_i q^*$ incremental units but, as costs are strictly convex, each of those units costs more than w^* to produce. There can also be common knowledge that in case of a bilateral disagreement, the conjectured adjustments may simply be unfeasible (e.g., due to the capacity constraints), so off-equilibrium beliefs can be inconsistent with the technology.

A second issue is that a researcher using one of those three models needs to attribute observations of higher per-unit prices paid by a larger buyer to heterogeneity in bilateral bargaining power parameters. Differences that may have been explained by sellers' higher costs of producing above equilibrium levels (or the impossibility to do so, in the case of tight capacity constraints) may then be wrongly attributed to differences in bilateral bargaining skills.

With respect to the former criticism, by construction, our approach always leads to adjustment requests that should be rationally accepted. With respect to the latter one, the approach of this article is rich enough to capture both considerations—which are all likely to play a part in real-world negotiations.

□ **Random-order values and the Core.** Another approach with Nash equilibria in bilateral Nash bargaining, known as "nonbinding contracts," assumes that (like in our model) each pair bargains bilaterally over a quantity A_{IJ} and a transfer t_{IJ} but (unlike in our model) in case of a bilateral disagreement between I and J, this pair becomes unable to agree with each other forever, and all agreements (including between third parties) are void and renegotiated. The disagreement point between I and J are therefore associated with the outcome of a similar (sub)game where trade between I and J is assumed to be zero.[14] Due to a consistency property (Hart and Mas-Colell, 1989), the bargaining outcome obtained recursively coincides with a random-order value with bilateral bargaining parameters P. Random-order values (ROV) measure the expected marginal contribution of a player to the set of players that precede it in an order of players, over the set of all possible orders.[15]

With ROV, the information on costs above the equilibrium level becomes relevant in determining the payoffs. It is then possible that size could be a disadvantage. In the cooperative game theory literature, the possibility that owners of substitutable resources may lose from forming a monopoly has been identified early on by Postlewaite and Rosenthal (1974) for the Core, by

[13] This holds for any $\alpha_i \in (0, 1)$. An implication is that a monopoly buyer always purchases a larger quantity than the two independent buyers purchase jointly.

[14] The disagreement points in that (sub)game assume that all existing agreements are again void and new agreements must then be renegotiated in a (subsub)game where trade between all thus far disagreeing parties is zero (and so on and so forth). Thus, the solution is recursive.

[15] In the Shapley value, this expectation is taken with respect to the uniform distribution over the set of all orders, and is obtained with the "nonbinding contracts" approach when bilateral gains are shared equally, that is, all ρ_{IJ} are $1/2$.

Legros (1987) for the Nucleolus, and studied by Gardner (1977), Guesnerie (1977), and Segal (2003) for random-order values. To compare (to the extent possible) those predictions to our own, we consider the simple example of Postlewaite and Rosenthal (1974), as it can be interpreted as an economy with two sellers of a homogeneous good, having each two units of capacity (with zero marginal cost), and three consumers each with unitary demand and a willingness to pay of 1 (the value for each coalition is then twice the values in their game). A merger to a buyer monopoly is equivalent to having instead a representative buyer with three units of demand, and the other two buyers becoming dummy players.

Deriving the ROV recursively for every bilateral sharing rule $\rho \in (0, 1)$, we found that with ROV, a buyer merger to monopoly is unprofitable if and only if sellers' bilateral bargaining power is sufficiently high. Thus, with ROV, or equivalently with a nonbinding contracts approach, size can be seen as a substitute to bargaining power, which contrasts with our finding that size and bargaining power are instead complements.[16]

This stands in clear disagreement not only with our predictions but also with the predictions originally obtained for the Core, where size and bargaining power are instead seen as complements. Indeed, Postlewaite and Rosenthal (1974) have shown in the context of their original example that with three independent buyers, the Core consists of a single allocation where each one gets one third of the surplus, and thus buyers appropriate jointly the full surplus. A merger to a buyer monopoly is then (weakly) disadvantageous, because it expands the set of Core payoffs by reducing the set of allowable blocking coalitions: the monopoly buyer still captures the full surplus (like before the merge) if it has all the bargaining power, but it captures less otherwise.[17]

Beyond such simple settings, as the dearth of literature attests, it becomes complex to study the effect of size in the Core.[18] For this reason, little is still known on how general the phenomenon described by Postlewaite and Rosenthal (1974) is, on which conditions are more likely to make buyer mergers unprofitable (in particular, if they do not result in a monopoly), on the relative prices paid by buyers of heterogeneous sizes, on whether size is more advantageous when buyers' bargaining power is high or low, etc. Our work extended our understanding on this phenomenon within a general industrial organization context, while capturing with relative transparency what seem to be economic forces similar to those present in the Core.

7. Conclusion

■ To study the determinants of buyer and seller power, we developed a model where buyers and sellers bargain bilaterally and can make local adjustments. A key result is that size may be an advantage or a disadvantage when trying to secure better terms of trade. This is explained by the fact that a player's size increases the dependency of its counterparties, but it also increases its own dependency on them, as switching becomes a less attractive alternative for both sides. Which effect dominates depends on the distribution of bilateral bargaining power. By considering different technologies and contracts, we made more transparent the different channels through which size advantages or disadvantages arise.

We focused on a comparative statics analysis of size, but a further topic would be the analysis of equilibrium market structures. Hart and Kurz (1983) introduced several concepts of stability for this type of analysis (see also Bloch, 2002). Assume in our setting that only players of the same type, buyers or sellers, can form groups that will bargain jointly. It follows from our results

[16] See the online web Appendix for a derivation and explanation of this result, and a more complete comparison of our results with those early results in cooperative game theory.

[17] Closer to the smooth cost functions of our work, with an *extended example* where the second unit cost is $c \in (0, 1)$, we find similar predictions: a buyer merger to monopoly strictly increases the upper bound and strictly decreases the lower bound of what buyers appropriate in type symmetric Core allocations.

[18] These cooperative solutions often prove impractical because they require "global information," a value for each possible coalition—in our setting, an "exponential" total of $(2^{|B|} - 1)(2^{|S|} - 1)$ values. Our approach only requires two disagreement values for each buyer and seller pair and the agreement payoffs, thus a "linear" total of $2|S||B| + (|S| + |B|)$ values.

that for extreme values of bilateral bargaining power, the players on the strong side would benefit from forming a monopoly. However, then the players on the other side, anticipating that they will face a monopoly, also benefit from forming a monopoly themselves (recall that size is always beneficial when there is a single counterparty). In the absence of antitrust scrutiny, a bilateral monopoly should thus emerge in equilibrium. The problem becomes significantly more complex for more balanced distributions of bargaining power, and in particular it then also depends on the way the concept of stability specifies how the members of a group will organize if a subgroup deviates. We must leave such analysis to future work.

Finally, the insights from our analysis should also be of interest outside industrial organization. For example, in the context of international trade relations, it suggests that a large manufacturing country (say, China) may not always be able to negotiate a better trade deal with the European Union or the United States than a small country (say, Vietnam). For a large country, it may be almost essential to trade with both blocks, but for a small country, it may often be sufficient to trade with just one. Also, in those cases, size should increase the mutual dependency of the contracting parties, rather than be a unequivocal source of power.

Appendix: The Appendix contains a formal description of the "Nash followed by Rubinstein game" and proofs of Propositions 2 and 4 and Lemmas 2 and 3

The "Nash followed by Rubinstein" game. The game takes place in periods $\tau = 0, .., \infty$. A bilateral contract offer in period τ from seller I to buyer J, denoted by O_τ^{IJ}, and from J to I, denoted by O_τ^{JI}, consists of a trade and transfer pair (A_{IJ}, t_{IJ}).

We first specify the sequence of moves. Period $\tau = 0$ has a single stage with simultaneous demands by buyers and sellers. Specifically, each buyer $J \in B$ makes $|S|$ contract offers, an offer O_0^{JI} to each seller $I \in S$, and each seller $I \in S$ makes $|B|$ contract offers, an offer O_0^{IJ} to each buyer $J \in B$. Thus, a total of $2|S||B|$ contract offers are made simultaneously. Similar to a Nash (1953) demand game between each pair, a bilateral contract between a buyer J and a seller I is reached immediately at $\tau = 0$ if and only if their respective bilateral offers are equal. Thus, if $O_0^{JI} = O_0^{IJ}$, players I and J have reached an agreement at $\tau = 0$, and otherwise if $O_0^{JI} \neq O_0^{IJ}$.

Let $S_\tau^J \subseteq S$ denote the set of sellers that have a contract with buyer J at or before τ and $B_\tau^I \subseteq B$ the set of buyers that reached a contract with seller I at or before τ. If an offer (A_{IJ}, t_{IJ}) is accepted at τ, then from that period onward the pair has a contract and thus $I \in S_{\tau'}^J$ and $J \in B_{\tau'}^I$ for all $\tau' \geq \tau$. Thus, for example, if $O_0^{JI} = O_0^{IJ}$, then $I \in S_\tau^J$ and $J \in B_\tau^I$ for all $\tau \geq 0$.

Each period $\tau \geq 1$ has four stages: in stage 1, contract offers are made; in stage 2, each player chooses from the set of offers it receives which offers to accept and which offers to reject; in stage 3, temporary adjustments offers are made; in stage 4, each player chooses from the set of adjustment requests it receives which ones to accept and which ones to reject. We next specify the identity of the players making and receiving contract and adjustment offers, describing in turn odd and even periods.

In each odd period τ, at stage 1, each seller $I \in S$ makes a contract offer to each buyer $J \in B \backslash B_{\tau-1}^I$ (thus, if that set is empty, I makes no offers). In stage 2 of that odd period, each buyer J receives a set of contract offers, one from each $I \in S \backslash S_{\tau-1}^J$, and decides which offers from that set to reject and which ones to accept (thus, if that set is empty, J does nothing). If J accepts O_τ^{IJ}, then $J \in B_\tau^I$ and $I \in S_\tau^J$. In stage 3, adjustment offers are made simultaneously: each buyer J such that $S_\tau^J \neq S$ can make an adjustment request to each seller $I \in S_\tau^J$ so that at τ only they trade according to (A'_{IJ}, t'_{IJ}) rather than their previously agreed contract (A_{IJ}, t_{IJ}), and each seller I such that $B_\tau^I \neq B$ can make a similar adjustment request to each buyer $J \in B_\tau^I$. Let $\widehat{S}_\tau^J \subseteq S_\tau^J$ and $\widehat{B}_\tau^I \subseteq B_\tau^I$ denote the set of players to which, respectively, J and I make adjustment requests at τ. In stage 4, each buyer J receives an adjustment request from seller I if and only if $J \in \widehat{B}_\tau^I$ but $I \notin \widehat{S}_\tau^J$, and seller I receives an adjustment request from buyer J if and only if $I \in \widehat{S}_\tau^J$ but $J \notin \widehat{B}_\tau$, which rules out conflicting adjustment requests. Each player chooses which of the received adjustments to accept and reject: if accepted, the pair trades according to (A'_{IJ}, t'_{IJ}), if rejected, according to the original agreement (A_{IJ}, t_{IJ}).

Likewise, each even period $\tau \geq 2$ has four stages. In stage 1, each buyer $J \in B$ makes an offer to each seller $I \in S \backslash S_{\tau-1}^J$. In stage 2, seller I receives a set of contract offers, one from each $J \in B \backslash B_{\tau-1}^I$, and decides which ones to accept and which ones to reject. If I accepts, O_τ^{JI} then $I \in S_\tau^J$. Stages 3 and 4 are equal to those of odd periods.

The information and belief structure is the following. Each player observes only the offers it makes or receives. Each player also holds passive beliefs on what it does not observe, that is, offers and acceptance decisions it is not privy to (see references in the main text). Thus, a player who observes an out-of-equilibrium action at τ does not update its beliefs on the actions that have been taken by all other players and that it does not observe; rather, it still believes that all unobserved actions remain those of the conjectured equilibrium.

Payoffs are determined as follows: at the end of each period $\tau = 1, .., \infty$, trade takes place according to the accepted contracts and period specific adjustments (if any). Per-period payoffs are those presented in Section 2. The real time between period 1 and τ is $z(\tau - 1) > 0$. Buyer J and seller I have respectively finite discount rates $r_J > 0$ and $r_I > 0$. Each player maximizes its (expected) discounted sum of period payoffs.

Proof of Proposition 2. We first identify the set of subgames that need to be considered to prove that the proposed strategies form a perfect Bayesian equilibrium with immediate agreement (step 1). With this, we show that there exists no profitable unilateral deviation from the proposed equilibrium (steps 2 and 3). Next, we show that the proposed equilibrium is also the unique efficient one (step 4). Finally, we show that in the class we study, there cannot be an inefficient equilibrium (step 5).

Step 1. Given that the game starts with a Nash demand game and that in the conjectured equilibrium all offers should result in immediate agreements at $\tau = 0$, with trade and transfers in every period $\tau \geq 1$ summarized by (A^*, T'), the equilibrium payoffs are

$$\Theta_I(A^*, T') = \frac{\theta_I(A^*, T')}{r_I} \quad \text{and} \quad \Theta_J(A^*, T') = \frac{\theta_J(A^*, T')}{r_J}.$$

To check that the equilibrium strategies form a Perfect Bayesian equilibrium, we only need to consider the following set of relevant subgames: (i) those subgames that follow a unilateral deviation by a single seller I with a subset of buyers $B' \subseteq B$, but where all the other pairs with sellers $I' \neq I$ have reached their respective equilibrium agreements among themselves, and (ii) those subgames that follow a unilateral deviation by some buyer J with a subset of sellers $S' \subseteq S$ but all the other pairs with a buyer $J' \neq J$ have reached their respective equilibrium agreements among themselves. As deviations of type (ii) involve similar steps to former ones, in the interest of space, we will only report on the details for the deviations of type (i).

Step 2. Consider then subgames that follow a unilateral deviation by a single seller I. The first step is to derive the best response of some buyer J who at $\tau = 0$ receives an offer $O_0^{IJ*} \neq (A^*_{IJ}, t'_{IJ})$ from seller I. Other than seller I's deviation, both J and every seller $I' \neq I$ will still have made their equilibrium offers $O_0^{JI'*} = O_0^{I'J*} = (A^*_{I'J}, t^*_{I'J})$, and thus $S_0^J = S\backslash I$. Given that J holds passive beliefs, J also believes that every other buyer and seller will have reached their equilibrium agreements and thus that $B_\tau^I = B\backslash J$ if $S_\tau^J = S\backslash I$ (regardless of whether I has deviated with J alone or with instead with some subset of S that contains J). Thus, in stage 3 of every period τ where $S_\tau^J = S\backslash I$, J expects that only himself and I have not reached all their equilibrium agreements, and therefore both make adjustment requests to every other seller and buyer that extract the full surplus generated from those adjustments, and all such adjustments will be accepted in stage 4. Then, as long as an agreement has not yet been reached with I, J believes that its period τ payoff will be

$$v_J(A^{IJ}) - \sum_{I' \in S\backslash I} (t'_{I'J} + C_{I'}(A^{IJ}) - C_{I'}(A^*)) = d_{JI}(A^*, T'),$$

and that of seller I will be

$$\sum_{J' \in B\backslash J} \left(t'_{IJ'} + v_{J'}(A^{IJ}) - v_{J'}(A^*)\right) - C_I(A^{IJ}) = d_{IJ}(A^*, T'),$$

which coincide with disagreement points that are truthful to (A^*, T'). Buyer J also believes that if in an offer $(\widetilde{A}_{IJ}, \widetilde{t}_{IJ})$ is accepted at τ, then period τ and subsequent periods payoffs will be, respectively, for J and I,

$$\sum_{J' \in B\backslash J} t'_{IJ'} + \widetilde{t}_{IJ} - C_I(\widetilde{A}) \quad \text{and} \quad v_J(\widetilde{A}) - \sum_{I' \in S\backslash I} t'_{I'J} + \widetilde{t}_{IJ},$$

where $\widetilde{A}$ is such that $a_{ij} = \widetilde{a}_{ij}$ if $i \in I$ and $j \in J$, and $a_{ij} = a^*_{ij}$ otherwise. Therefore, J believes that an accepted offer $(\widetilde{A}_{IJ}, \widetilde{t}_{IJ})$ results in a bilateral gain,

$$g_{IJ}(\widetilde{A}_{IJ}) = \sum_{J' \in B\backslash J} t'_{IJ'} + \widetilde{t}_{IJ} - C_I(\widetilde{A}) + v_J(\widetilde{A}) - \sum_{I' \in S\backslash I} t'_{I'J} + \widetilde{t}_{IJ} - d_{JI}(A^*, T') - d_{IJ}(A^*, T')$$

$$= v_J(\widetilde{A}) - C_I(\widetilde{A}) - v_J(A^{IJ}) + \sum_{I' \in S\backslash I} (C_{I'}(A^{IJ}) - C_{I'}(A^*)) - \sum_{J' \in B\backslash J} \left(v_{J'}(A^{IJ}) - v_{J'}(A^*)\right) + C_I(A^{IJ}),$$

which is strictly positive for some $\widetilde{A}$, and is maximized with respect to $\widetilde{A}_{IJ}$ for A^*_{IJ}.

As buyer J has passive beliefs, following the unilateral deviation of seller I, the buyer J believes that it is in a complete information alternating offers bargaining game for gains of agreement $g_{IJ}(\widetilde{A}_{IJ})$ with the respective inside options $d_{JI}(A^*, T')$ and $d_{IJ}(A^*, T')$. Rubinstein (1982) showed that in such bilateral situation, the following construction also yields the unique subgame perfect equilibrium of that game. Thus, J behaves like in that equilibrium, and believes that I will behave like in that equilibrium, which is described next: both players' offers maximize the bilateral gains from trade, that is, $\widetilde{A}_{IJ} = A^*_{IJ}$, and the transfers proposed by, respectively, I in even periods and J in odd periods, t^I_{IJ} and t^J_{IJ}, satisfy

$$\frac{1}{r_I}\left(t^J_{IJ} + \sum_{J'\in B\setminus J} t'_{IJ'} - C_I(A^*)\right) = \frac{1-e^{-zr_I}}{r_I} d_{IJ}(A^*, T') + \frac{e^{-zr_I}}{r_I}\left(t^I_{IJ} + \sum_{J'\in B\setminus J} t'_{IJ'} - C_I(A^*)\right)$$

and

$$\frac{1}{r_J}\left(v_J(A^*) - t^I_{IJ} - \sum_{I'\in S\setminus I} t'_{I'J}\right) = \frac{1-e^{-zr_J}}{r_J} d_{JI}(A^*, T') + \frac{e^{-zr_J}}{r_J}\left(v_J(A^*) - t^J_{IJ} - \sum_{I'\in S\setminus I} t'_{I'J}\right),$$

which solving, gives that J expects I to offer in every odd period

$$t^I_{IJ} = \frac{(1-e^{-zr_J})\varkappa_{IJ} + e^{-zr_J}(1-e^{-zr_I})\varphi_{IJ}}{1-e^{-z(r_I+r_J)}},$$

and that J himself will make to I in every even period an offer of

$$t^J_{IJ} = \frac{e^{-zr_I}(1-e^{-zr_J})\varkappa_{IJ} + (1-e^{-zr_I})\varphi_{IJ}}{1-e^{-z(r_J+r_I)}}.$$

In every odd period τ, buyer J will also accept from seller I any contract offer that leaves it with a (weakly) higher continuation payoff than rejecting it when it expects its offer (A^*_{IJ}, t^J_{IJ}) to be accepted by I in the next period $\tau + 1$: thus, in an odd period, J accepts any offer that gives it a higher continuation payoff than any of critical offers $(\widehat{A}_{IJ}, \widehat{t}_{IJ})$ that satisfies

$$\frac{1}{r_J}\left(v_J\left(\widehat{A}^{IJ}\right) - \widehat{t}_{IJ} - \sum_{I'\in S\setminus I} t_{I'J}\right) =$$

$$\frac{1-e^{-zr_J}}{r_J} d_{JI}(A^*, T') + \frac{e^{-zr_J}}{r_J}\left(v_J(A^*) - t^J_{IJ} - \sum_{I'\in S\setminus I} t_{I'J}\right), \tag{A1}$$

with $\widehat{A}^{IJ}$ having $a_{ij} = a^*$ if $i \notin I$ and $j \notin J$. In particular, note that $(A^*_{IJ}, \widehat{t}_{IJ}) = (A^*_{IJ}, \widetilde{t}_{IJ})$.

To show that the candidate PBE is an equilibrium, we finally need to show that the proposed equilibrium strategies resist a simultaneous deviation by a single seller I with a single or with multiple buyers, and I anticipates that every buyer $J \in B$ will use the strategy outlined above in any of the possible subgames that follow such a deviation at $\tau = 0$. To do this, we further need to consider only those deviations that involve seller I making acceptable offers to a subset of buyers at $\tau = 1$, as explained next. Suppose otherwise, that I's optimal deviation is such that it results in a subgame that has some last period $\tau \geq 2$ at which some offers are accepted: if that τ is even then, it would be some buyer J making that offer (A^*_{IJ}, t^J_{IJ}), and then seller I could increase its profit by making at $\tau - 1$ an offer with the same trade but higher transfers as $t^I_{IJ} > t^J_{IJ}$; if that last period τ is odd, then the seller could increase its profit by offering the same $(\widehat{A}_{IJ}, \widehat{t}_{IJ})$ at $\tau - 2$ and avoid losses from delay; finally, as the equilibrium offers (A^*_{IJ}, t'_{IJ}) are accepted if made by seller I at $\tau = 1$ instead of $\tau = 0$, and generate the same payoffs, we can, without loss of generality, focus on deviations where seller I makes a set of acceptable offers to a subset of buyers only at $\tau = 1$, and it reaches no other agreements at either $\tau = 0$ nor $\tau \geq 2$.

The optimal deviation of seller I at $\tau = 1$ selects a subset of buyers $B' \subseteq B$ and an acceptable offer $(\widehat{A}_{IJ}, \widehat{t}_{IJ})$ to each $J \in B'$ to maximize I's deviation payoff, which is given by

$$\frac{1}{r_I}\left[\sum_{J\in B'} \widehat{t}_{IJ} - C_I\left(\widehat{A}\right)\right],$$

with $\widehat{A}$ such that $a_{ij} = a^*$ if $i \notin I$ and $j \notin J \in B'$, and $a_{ij} = 0$ if $i \in I$ and $j \in J \notin B'$ (for notational simplicity, we have omitted the dependency of $\widehat{t}_{IJ}$ on $\widehat{A}_{IJ}$). Take condition (A1) that defines implicitly a critical $\widehat{t}_{IJ}$ that depends on $\widehat{A}$. Solving with respect to $\widehat{t}_{IJ}$, we have

$$\lim_{z\to 0} \widehat{t}_{IJ} = \frac{r_J}{r_J + r_I}\varkappa_{IJ} + \frac{r_I}{r_J + r_I}\varphi_{IJ} + \left(v_J\left(\widehat{A}\right) - v_J(A^*)\right).$$

Thus, for z arbitrarily close to zero, for a given $B' \subseteq B$, there is a set of acceptable offers $(\widehat{A}_{IJ}, \widehat{t}_{IJ})$ to each $J \in B'$ such that the deviating payoffs described above become

$$\frac{1}{r_I}\left[\sum_{J\in B'} v_J\left(\widehat{A}\right) - C_I\left(\widehat{A}\right) - \sum_{J\in B'}\left(\frac{r_J\varkappa_{IJ}}{r_J + r_I} + \frac{r_I\varphi_{IJ}}{r_J + r_I} - v_J(A^*)\right)\right]. \tag{A2}$$

Given our assumptions on u and c, the solution to

$$\max_{\widehat{A}} \left[\sum_{J\in B'} v_J\left(\widehat{A}\right) - C_I\left(\widehat{A}\right) \right] \tag{A3}$$

is *strictly* submodular with respect to buyer inclusion: the marginal contribution of each buyer J to the solution obtained for any buyer subset $B' \subseteq B$ is strictly decreasing with respect to inclusion of another buyer J' in B'. Because in each additional term in (A2),

$$\frac{r_J \varkappa_{IJ}}{r_J + r_I} + \frac{r_I \varphi_{IJ}}{r_J + r_I} - v_J(A^*),$$

there is no interaction between the term associated with J and J', if seller I prefers to deviate with $B' \cup J$ to deviating with $B'\backslash J$, then by the submodularity seen above, it also prefers to deviate with $B'' \cup J$ to deviating with $B''\backslash J$ for all $B'' \subset B'$. Thus, seller I will deviate with at most one buyer. If seller I only deviates with buyer J, then $B' = B\backslash J$ and maximizing (A3) yields for z arbitrarily close to zero a payoff

$$\frac{d_{IJ}(A^*, T')}{r_I},$$

whereas if it does not deviate, it would get instead

$$\Theta_I(A^*, T') = \frac{\theta_I(A^*, T')}{r_I}.$$

Recall from the derivation of t_{IJ}^J that, as a bilateral agreement between I and J generates a strictly positive surplus that $\Theta_I(A^*, T') > \frac{d_{IJ}(A^*,T')}{r_I}$, so any deviation with buyer J results in a strictly lower payoff. Therefore, there does not exist a weakly profitable deviation from offering the equilibrium contracts to B at $\tau = 1$, and as this yields the same payoffs to all players as the equilibrium with immediate agreement at $\tau = 0$, there is no weakly profitable deviation from the equilibrium payoff equivalent deviation that reaches all the equilibrium agreements at $\tau = 1$ (with the exception of the just mentioned payoff equivalent strategy to reach an agreement with buyers at $\tau = 1$). As no other weakly profitable deviation exists for z arbitrarily close to zero, by continuity, there is also no strictly profitable deviation for any z sufficiently close to zero.

These steps show that there is no strictly profitable deviation for a seller and, as previously mentioned, similar steps also show that there exists no profitable unilateral deviation for a buyer. This shows that the conjectured equilibrium is a PBE of the game: it is efficient and results in immediate agreement.

Step 3. That transfers T' are the unique that support a PBE that has immediate agreement with efficient trade A^* is explained next. Any PBE must resist survive unilateral deviations by every seller I with every single buyer J (in addition to other). We characterized above the best response by a buyer J to a unilateral deviation by a seller I. It follows that any equilibrium (A^*, T'') with $T'' \neq T'$ which has $t_{IJ}^{''} > t_{IJ}^I = t_{IJ}^{'}$ faces a unilateral deviation by buyer J with seller I as, conditional on making all the other proposed equilibrium offers to all sellers in $S\backslash I$ (a necessary condition for equilibrium), that buyer expects to get a higher payoff from deviating at $\tau = 0$ and reaching an agreement with I at $\tau = 1$ with a lower transfer t_{IJ}^I (the expected outcome of the unique PBE with passive beliefs of the continuation subgame). Likewise, an equilibrium that results in (A^*, T'') with $T'' \neq T'$ which has $t_{IJ}^{''} < t_{IJ}^I = t_{IJ}^{'}$ faces a unilateral deviation by seller I with seller J as, conditional on making all the other proposed equilibrium offers to all buyers in $B\backslash J$ (a necessary equilibrium condition), that seller expects to get a higher payoff from deviating at $\tau = 0$ and reaching instead an agreement with J at $\tau = 1$ with a higher transfer t_{IJ}^I (the expected outcome of the unique PBE with passive beliefs of the continuation subgame). Thus, immediate agreement at (A^*, T'') does not survive unilateral deviations, and therefore cannot be an equilibrium.

Step 4. Finally, we show that there cannot exist a PBE with immediate agreement that results in trade (A'', T'') with $A'' \neq A^*$. If $A'' \neq A^*$, then by the definition of A^*, there is some J and some I such that

$$u_i(a_j'') \neq c'\left(\sum_{j\in N} a_{ij}''\right) \quad \text{for } i \in I \quad \text{and} \quad j \in J,$$

and in the proposed equilibrium seller I and buyer J would make

$$\Theta_I = \frac{1}{r_I}\left(\sum_{J\in B} t_{IJ}'' - C_I(A'')\right) \quad \text{and} \quad \Theta_J = \frac{1}{r_J}\left(v_J(A'') - \sum_{I\in S} t_{IJ}''\right).$$

If either seller I or buyer J deviates from the proposed equilibrium at $\tau = 0$, then at $\tau = 1$, both believe that they are in a complete information bilateral bargaining game, as seen in step 2. In that game, the bilateral gains from trade are maximized for a constrained trade matrix $\widehat{A}$ such that

$$u_i(\widehat{a}_j) = c'\left(\sum_{j\in N} \widehat{a}_{ij}\right) \qquad \text{for } i \in I \quad \text{and} \quad j \in J,$$

and thus, $\widehat{A} \neq A''$. Such a deviation by either seller I or buyer J generates strictly positive gains from trade, which are shared among the two players in the issuing subgame, and with an agreement reached at $\tau = 1$. Therefore, the deviation does not create losses from delay (as trade only starts at $\tau = 1$). Therefore, such a deviation increases the payoff of I, of J, or both: so at least one of the two players has a strictly profitable deviation from the proposed equilibrium. Thus, there cannot be BPE that results in trade (A'', T'') with immediate agreement where $A'' \neq A^*$. *Q.E.D.*

Proof of Lemma 2. Given that in equilibrium the total quantity purchased by buyers J_1 and J_2 will be equal to the quantity purchased by J_3 alone, we only have to compare the respective transfer

$$\varphi_{IJ} = \sum_{J' \in B \setminus J} (v_{J'}(A^{IJ}) - v_{J'}(A^*)) - (C_I(A^{IJ}) - C_I(A^*)).$$

We will first transform this expression. Note that

$$C_I(A^*) = |I|\, c(|N|\, a^*) \quad \text{and} \quad \sum_{J' \in B \setminus J} v_{J'}(A^*) = (|N| - |J|)u(a_j^*),$$

where a_j^* is a column vector with each element equal to a^*. Recall that $u_i(a_j)$ denotes the partial derivative of u with respect to the consumption of good $i \in G$ when a_j is the consumption vector for consumer j. It is also convenient to write each a_{ij}^{IJ} element of A^{IJ} with $i \in I$ and $j \notin J$ as

$$a_{ij}^{IJ} = \frac{|N|\, a^* - |J|\, \Delta^{IJ}}{|N| - |J|},$$

where we still need to determine the (symmetric) adjustments Δ^{IJ}. From the first-order condition, we have

$$c'(|N|\, a^* - |J|\, \Delta^{IJ}) = u_i(a_j^{IJ}) \quad \text{for } i \in I \quad \text{and} \quad j \notin J, \tag{A4}$$

so that with strict convexity we have immediately $\Delta^{IJ} \in (0, a^*)$. (For future reference, note that it follows that $\Delta^{IJ} = 0$ if marginal cost is constant.) With this notation, using also that

$$C_I(A^{IJ}) = |I|\, c((|N|\, a^* - |J|\, \Delta^{IJ})$$

and that

$$\sum_{J' \in B \setminus J} v_{J'}(A^{IJ}) = (|N| - |J|)u(a_j^{IJ}),$$

we can rewrite all φ_{IJ}, and in particular, we obtain

$$\varphi_{IJ_3} = (|N| - |J_1| - |J_2|)\left(u\left(a_j^{IJ_3}\right) - u\left(a_j^*\right)\right) - |I|\,(c(|N|\, a^* - |J_3|\, \Delta^{IJ_3}) - c(|N|\, a^*)). \tag{A5}$$

To establish the assertion, we derive boundaries for the left-hand and right-hand side of expression (4). For this, let $\widehat{a}_j^{IJ_y}$ be the column vector with each element equal to a^* if $i \notin I$ and

$$\widehat{a}_{ij}^{IJ_y} = \frac{|N|\, a^* - \left|J_y\right| \Delta^{IJ_3}}{|N| - \left|J_y\right|} \quad \text{if} \quad i \in I. \tag{A6}$$

Thus, $\widehat{a}_{ij}^{IJ_y} < a_{ij}^{IJ_3}$ for $y = 1, 2$ as differentiating expression (A6) with respect to $|J_y|$, the respective sign is determined by $a^* - \Delta^{IJ_3} > 0$, which holds strictly with strictly convex costs. We then have for $y = 1, 2$ that

$$\varphi_{IJ_y} > (|N| - \left|J_y\right|)(u(\widehat{a}_j^{IJ_y}) - u(a_j^*)) - |I|\left(c\left(|N|\, a^* - \left|J_y\right| \Delta^{IJ_3}\right) - c(|N|\, a^*)\right), \tag{A7}$$

as we did not use the optimal adjustment $|J_y|\Delta^{IJ_y}$ but instead $|J_y|\Delta^{IJ_3}$. For condition (4) to hold, using (A5) and (A7), it is sufficient that the following two conditions hold:

$$\begin{aligned} &\left(c\left(|N|\, a^* - |J_3|\, \Delta^{IJ_3}\right) - c(|N|\, a^*)\right) - \left(c\left(|N|\, a^* - |J_1|\, \Delta^{IJ_3}\right)\right. \\ &\left. + c(|N|\, a^*)\right) - \left(c\left(|N|\, a^* - |J_2|\, \Delta^{IJ_3}\right) + c(|N|\, a^*)\right) \\ &> 0 \end{aligned} \tag{A8}$$

and

$$\begin{aligned} &(|N| - |J_1| - |J_2|)\left(u\left(a_j^{IJ_3}\right) - u\left(a_j^*\right)\right) - (|N| - |J_1|)\left(u\left(\widehat{a}_j^{IJ_1}\right) - u\left(a_j^*\right)\right) \\ &-(|N| - |J_2|)\left(u\left(\widehat{a}_j^{IJ_2}\right) - u\left(a_j^*\right)\right) \\ &\leq 0. \end{aligned} \tag{A9}$$

We confirm (A8) and (A9) separately. Take first condition (A8), which transforms to

$$c(|N|\, a^*) - c(|N|\, a^* - |J_1|\, \Delta^{IJ_3}) > c(|N|\, a^* - |J_2|\, \Delta^{IJ_3}) - c((|N|\, a^* - (|J_1| + |J_2|)\Delta^{IJ_3}),$$

so that this is indeed strictly satisfied when c is strictly convex. Next, take condition (A9). Observe that the left-hand side is zero if it were that $\Delta^{IJ_3} = a^*$, but $\Delta^{IJ_3} \in (0, a^*)$ and so for $y = 1, 2$, we also have that

$$\frac{|N|\, a^* - \left|J_y\right| \Delta^{IJ_3}}{|N| - \left|J_y\right|} < \frac{|N|\, a^* - |J_3|\, \Delta^{IJ_3}}{|N| - |J_3|}.$$

Differentiate the left-hand side of (A9) with respect to Δ^{IJ_3}, which by symmetry for any $i \in I$ can be simplified to

$$|I| \left[|J_1|\, u_i(\widehat{a}_j^{IJ_1}) + |J_2|\, u_i\left(\widehat{a}_j^{IJ_2}\right) - (|J_1| + |J_2|) u_i\left(a_j^{IJ_3}\right) \right]. \tag{A10}$$

As $\widehat{a}_{ij}^{IJ_y} < a_{ij}^{IJ_3}$ for $y = 1, 2$ and $i \notin I$, and the second-order derivatives are strictly negative for each $i, j \in G$, we then have that $u_i(a_j^{IJ_3}) < u_i(\widehat{a}_j^{IJ_y})$, and so that the derivative (A10) is positive. Finally, as for (strictly) convex costs $\Delta^{IJ_3} \in (0, a^*)$, the left-hand side of (A9) is increasing in that interval and equal to zero when $\Delta^{IJ_3} = a^*$, and thus, (A9) holds (strictly) when costs are (strictly) convex. *Q.E.D.*

Proof of Lemma 3. As the total quantity purchased by J_1 and J_2 is equal to the quantity purchased by J_3 alone, we only have to compare the respective transfers

$$\varkappa_{IJ} = (v_J(A^*) - v_J(A^{IJ})) - \sum_{I' \in S \setminus I} (C_{I'}(A^*) - C_{I'}(A^{IJ})),$$

as used in condition (5). As $v_J(A^*) = |J| u(a_j^*)$, these elements cancel in (5) and this expression can therefore be rewritten as

$$\begin{aligned} & v_J(A^{IJ_3}) - \sum_{I' \in S \setminus I} C_{I'}(A^{IJ_3}) + \sum_{I' \in S \setminus I} C_{I'}(A^*) \\ & \leq v_J(A^{IJ_1}) - \sum_{I' \in S \setminus I} C_{I'}(A^{IJ_1}) + v_J(A^{IJ_2}) - \sum_{I' \in S \setminus I} C_{I'}(A^{IJ_2}) + 2 \sum_{I' \in S \setminus I} C_{I'}(A^*). \end{aligned} \tag{A11}$$

We proceed as in the proof of Lemma 2 by first deriving properties of the optimal reallocation of purchases. Let Λ^{IJ} denote the (symmetric) increase in the consumption by consumer $j \in J$ of each good i that is not produced by I, so that for these elements $a_{ij}^{IJ} = a^* + \Lambda^{IJ}/|J|$, and note that $c'(|N|a^* + \Lambda^{IJ}) = u_i(a_j^{IJ})$ for $j \in J$ and $i \notin I$. Note that $\Lambda^{IJ} > 0$ as the cross-partial derivatives of $u(\cdot)$ are strictly negative. Using that

$$v_J(A^{IJ}) - \sum_{I' \in S \setminus I} C_{I'}(A^{IJ}) = |J|\, u(a_j^{IJ}) - (|G| - |I|) c(|N|\, a^* + \Lambda^{IJ}) \tag{A12}$$

and

$$\sum_{I' \in S \setminus I} C_{I'}(A^*) = (|G| - |I|) c(|N|\, a^*),$$

we can further rewrite condition (A11) as follows:

$$\begin{aligned} & |J_3|\, u(a_j^{IJ_3}) - (|G| - |I|) c(|N|\, a^* + \Lambda^{IJ_3}) - (|G| - |I|) c(|N|\, a^*) \\ & \leq |J_2|\, u(a_j^{IJ_2}) - (|G| - |I|) c(|N|\, a^* + \Lambda^{IJ_2}) + |J_1|\, u(a_j^{IJ_1}) - (|G| - |I|) c(|N|\, a^* + \Lambda^{IJ_1}). \end{aligned} \tag{A13}$$

If there is a single seller, that is, $|I| = |G|$, then Λ^{IJ} is zero and therefore both sides of (A13) are equal. Suppose for the remainder that $|I| \neq |G|$. We proceed by deriving boundaries. Inequality (A13) will then be satisfied if a more stringent inequality is satisfied, which is obtained by replacing in the inequality above the optimizers Λ^{IJ_2} and Λ^{IJ_1}, respectively, by $\Lambda^{IJ_3} \frac{|J_2|}{|J_3|}$ and $\Lambda^{IJ_3} \frac{|J_1|}{|J_3|}$. However, then the respective column vectors used as arguments in $u(\cdot)$ in (A13) are all the same, namely, equal to $a_j^{IJ_3}$ (i.e., each element equal to $a^* + \Lambda^{IJ_3}/|J_3|$ for each good that is not produced by I and 0 for each good produced by I). The respective utilities in (A13) then cancel out and after dividing by $(|G| - |I|)$, we are left with the condition

$$c\left(|N|\, a^* + \Lambda^{IJ_3}\right) - c\left(|N|\, a^* + \Lambda^{IJ_3} \frac{|J_1|}{|J_3|}\right) - c\left(|N|\, a^* + \Lambda^{IJ_3} \frac{|J_2|}{|J_3|}\right) + c\left(|N|\, a^*\right) \geq 0.$$

This is finally equivalent to

$$c(x + y) - c(x + \lambda y) - c(x + (1 - \lambda) y) + c(x) \geq 0,$$

where $x = |N| a^*$, $\lambda = \frac{|J_1|}{|J_3|}$, and $y = \Lambda^{IJ_3}$. We can then further rewrite this as

$$\int_0^{(1-\lambda)y} \left[c'(x + \lambda y + s) - c'(x + s)\right] ds \geq 0,$$

which holds strictly by strict convexity of $c(\cdot)$. *Q.E.D.*

Proof of Proposition 4. There are three possible cases when m assets are transferred by J_2 to J_1. Case 1: $J_1 > J_2$ and thus $J_1 + m > J_2$. In this case, HHI_B also increases and the final sizes of the largest and the smallest are, respectively, larger and smaller than the initial sizes of the two buyers. Case 2: $J_1 < J_2$ and $J_1 + m > J_2$. In this case, HHI_B also increases and we have again that the final size of the largest and the smallest of the two buyers are, respectively, larger and smaller than the initial sizes of the two buyers, that is, $\widehat{J}_1$ is larger than the initial size of J_2 by $J_1 + m - J_2$ assets and $\widehat{J}_2$ is smaller than the initial size of J_1 by $J_1 + m - J_2$ assets. Thus, its effect is similar to those of a Case 1 transfer of $J_1 + m - J_2$ assets from the smaller J_1 to the larger J_2. Case 3: $J_1 < J_2$ and $J_1 + m < J_2$. In this case, HHI_B decreases and the final size of the largest and the smallest of the two buyers are, respectively, smaller and larger than the initial sizes of the two buyers. Thus, its effect has the inverse sign of a Case 1 transfer of m assets from a smaller buyer $J_1 + m$ to a larger buyer J_2. We conclude that studying Case 1 transfers allows us to sign the effects of any transfer of m assets between two buyers, and the proposition follows if (as we show next) this has the effect of, respectively, decreasing φ_{IJ} and increasing $\varkappa_{IJ}$.

As goods are perfect substitutes, we can simplify expressions as follows. Considering a disagreement between I and J, we generically denote by $\widehat{a}$ the (adjusted) consumption of each good of seller I by each consumer of buyer $J' \neq J$, so that

$$\varphi_{IJ} = \max_{\widehat{a}} \left[(|N| - |J|)\, u\left((|G| - |I|)\, a^* + |I|\, \widehat{a}\right) - |I|\, c\left((|N| - |J|)\, \widehat{a}\right)\right] \tag{A14}$$
$$- \left[(|N| - |J|)\, u\left(|G|\, a^*\right) - |I|\, c\left(|N|\, a^*\right)\right],$$

and thus $\widehat{a}$ solves

$$u'\left((|G| - |I|)\, a^* + |I|\, \widehat{a}\right) - c'\left((|N| - |J|)\, \widehat{a}\right) = 0.$$

As described in the main text, we consider an increase in $|J_1|$ and a corresponding decrease in $|J_2|$. Though $|J|$ takes on only integer values, note that expression (A14) is defined also generally for real-valued $|J|$. Denote the respective real-valued expression by a function $\widetilde{\varphi}_I(x = |J|)$ and let x_1 denote the size of buyer J_1, x_2 the size of buyer J_2, and $x_2 = M - x_1$. From the application of the Envelope Theorem, we have

$$\frac{d(\widetilde{\varphi}_I(x_1) + \widetilde{\varphi}_I(x_2))}{dx_1} = \left[u\left((|G| - |I|)\, a^* + |I|\, \widehat{a}_2\right) - u\left((|G| - |I|)\, a^* + |I|\, \widehat{a}_1\right)\right] \tag{A15}$$
$$- |I| \left[\widehat{a}_2 c'\left((|N| - x_2)\, \widehat{a}_2\right) - \widehat{a}_1 c'\left((|N| - x_1)\, \widehat{a}_1\right)\right],$$

where $\widehat{a}_y$ (for $y = 1, 2$) solves

$$u'\left((|G| - |I|)\, a^* + |I|\, \widehat{a}_y\right) - c'\left(\left(|N| - x_y\right) \widehat{a}_y\right) = 0, \tag{A16}$$

so that from strict convexity of $c(\cdot)$ and strict concavity of $u(\cdot)$, we also have $\widehat{a}_1 > \widehat{a}_2$ as long as $x_1 > x_2$. Substituting (A16) into (A15), this expression is strictly negative if

$$u\left((|G| - |I|)\, a^* + |I|\, \widehat{a}_1\right) - u\left((|G| - |I|)\, a^* + |I|\, \widehat{a}_2\right) \tag{A17}$$
$$> |I|\, \widehat{a}_1 u'\left((|G| - |I|)\, a^* + |I|\, \widehat{a}_1\right) - |I|\, \widehat{a}_2 u'\left((|G| - |I|)\, a^* + |I|\, \widehat{a}_2\right).$$

Using strict concavity of $u(\cdot)$ and $\widehat{a}_1 > \widehat{a}_2$, (A17) surely holds if, on the right-hand side, we replace $u'((|G| - |I|)a^* + |I|\widehat{a}_2)$ by $u'((|G| - |I|)a^* + |I|\widehat{a}_1)$, which yields the following sufficient requirement:

$$\frac{u\left((|G| - |I|)\, a^* + |I|\, \widehat{a}_1\right) - u\left((|G| - |I|)\, a^* + |I|\, \widehat{a}_2\right)}{|I|\, (\widehat{a}_1 - \widehat{a}_2)} > u'\left((|G| - |I|)\, a^* + |I|\, \widehat{a}_1\right).$$

This holds from strict concavity of $u(\cdot)$ as then for $\beta > \alpha$, we have that

$$\frac{u(\beta) - u(\alpha)}{(\beta - \alpha)} > u'(\beta).$$

The assertion in the proposition concerning seller dependency then follows as, first, by leaving $|J_1| + |J_2| = M$ constant, all other φ_{IJ} for $J \in B \backslash \{J_1, J_2\}$ are not affected, and, second, we can express

$$(\varphi_{I\widehat{J}_1} + \varphi_{I\widehat{J}_2}) - (\varphi_{IJ_1} + \varphi_{IJ_2}) = \int_{|J_1|}^{|J_1|+m} \frac{d(\widetilde{\varphi}_I(x_1) + \widetilde{\varphi}_I(M - x_1))}{dx_1} dx_1 < 0.$$

We consider now the case of buyer dependency. Considering a disagreement between I and J, when goods are perfect substitutes, we had

$$\varkappa_{IJ} = |J|\, u(|G|\, a^*) - (|G| - |I|)c(|N|\, a^*)$$
$$- \max_{\widehat{a}} \left[|J|\, u((|G| - |I|)\, \widehat{a}) - (|G| - |I|)c((|N| - |J|)\, a^* + |J|\, \widehat{a})\right].$$

Again, we consider an increase in $|J_1|$ and a corresponding decrease in $|J_2|$. Though $|J|$ takes on only integer values, note that the expression above is defined also generally for real-valued $|J|$. Denote the respective real-valued expression by a function $\widetilde{\varkappa}_I(x = |J|)$. We have from application of the Envelope Theorem that

$$\begin{aligned}\frac{d(\widetilde{\varkappa}_I(x_1) + \widetilde{\varkappa}_I(x_2))}{dx_1} &= -u((|G| - |I|)\widehat{a}_1) + u((|G| - |I|)\widehat{a}_2)\\ &\quad +(\widehat{a}_1 - a^*)(|G| - |I|)c'((|N| - x_1)\,a^*\\ &\quad +x_1\widehat{a}_1) - (\widehat{a}_2 - a^*)(|G| - |I|)c'((|N| - x_2)\,a^* + x_2\widehat{a}_2),\end{aligned}$$

where $\widehat{a}_y$ (for $y = 1, 2$) solves

$$u'((|G| - |I|)\widehat{a}_y) - c'\left(\left(|N| - x_y\right)a^* + x_y\widehat{a}_y\right) = 0. \tag{A18}$$

From strict convexity of $c(\cdot)$ and strict concavity of $u(\cdot)$, we have $\widehat{a}_1 < \widehat{a}_2$ as $x_1 > x_2$. Substitute (A18) into $\frac{d(\widetilde{\varkappa}_I(x_1)+\widetilde{\varkappa}_I(x_2))}{dx_1}$ and the expression becomes

$$\begin{aligned}&-u((|G| - |I|)\widehat{a}_1) + u((|G| - |I|)\widehat{a}_2) + (\widehat{a}_1 - a^*)(|G| - |I|)u'((|G| - |I|)\widehat{a}_1)\\ &-(\widehat{a}_2 - a^*)(|G| - |I|)u'((|G| - |I|)\widehat{a}_2),\end{aligned}$$

which is decreasing in a^*, as $u'((|G| - |I|)\widehat{a}_1) > u'((|G| - |I|)\widehat{a}_2)$ when $x_1 > x_2$, and thus it is made smaller if we replace a^* by $\widehat{a}_1$. It will therefore be positive if

$$\frac{u((|G| - |I|)\widehat{a}_2) - u((|G| - |I|)\widehat{a}_1)}{(|G| - |I|)(\widehat{a}_2 - \widehat{a}_1)} > u'((|G| - |I|)\widehat{a}_2).$$

This holds again due to the strict concavity of $u(\cdot)$. The assertion concerning buyer dependency then follows as, first, by leaving $|J_1| + |J_2| = M$ constant, all other $\varkappa_{IJ}$ for $J \in B\backslash\{J_1, J_2\}$ are not affected, and, second, we can express

$$(\varkappa_{I\widetilde{J}_1} + \varkappa_{I\widetilde{J}_2}) - (\varkappa_{IJ_1} + \varkappa_{IJ_2}) = \int_{|J_1|}^{|J_1|+m} \frac{d(\widetilde{\varkappa}_I(x_1) + \widetilde{\varkappa}_I(M - x_1))}{dx_1} dx_1 > 0.$$

Q.E.D.

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