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Inefficiency and Regulation of Private Liquidity*

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March 26, 2019

Abstract

We propose a simple model to study the efficiency of liquidity creation by financial intermediaries, which can take the form of either safe or risky debt. Liquidity crises arise when risky debt is defaulted on and stops providing liquidity services. Owing to a novel externality related to liquidity premia and the cost of issuing safe debt, the laissez-faire equilibrium is inefficient, characterized by an excessive supply of risky debt. However, the optimal policy requires the regulation of safe debt as well. Capital requirements targeting risky debt alone have unintended welfare-reducing consequences.

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1 Introduction

Debt performs important functions in the financial system, including liquidity services — a feature that defines the special class of debt securities labelled "safe assets." The historical evidence suggests that in normal times liquidity can be provided not only by riskless but also by risky debt, which might lose some or all of its liquidity value in times of crises (Gorton, 2017). This in fact happened during the 2007-2009 financial meltdown. As a consequence, important measures have been taken to regulate the issuance of risky debt by financial intermediaries.

The contribution of this paper is to present a liquidity-crisis model in which debt securities with differing risk and liquidity characteristics coexist. Intermediaries can issue not only safe and liquid debt but also risky debt, which provides liquidity in normal times and becomes illiquid in crisis times. The framework can help to answer important questions: Is private liquidity creation efficient? And if not, how should it be regulated? Should regulation seek to curb the supply of the safest or of the riskiest liquid assets?

These questions cannot be fully addressed by the current "safe assets" literature, which typically considers models where only risk-free securities provide liquidity services. Building on the seminal work of Gorton and Pennacchi (1990), several papers follow this approach, such as Caballero and Farhi (2017), Diamond (2016), Greenwood, Hanson, and Stein (2015), Li (2017), Magill, Quinzii, and Rochet (2016), and Stein (2012).

Our first finding is that the creation of both safe and risky liquidity by private intermediaries is inefficient. Because of a novel externality, each intermediary does not internalize the effects that its liquidity creation has on the borrowing costs of the intermediaries issuing safe assets. The externality operates through liquidity premia and thus differs from the classical fire-sale externality widely analyzed in the literature (Bianchi, 2011; Dávila and Korinek, 2017; Greenwood, Hanson, and Stein, 2015; Lorenzoni, 2008; Stein, 2012).

We then ask how private intermediaries should be regulated. One immediate implication of our first finding is that the issuance of risky liquidity should be reduced to limit the negative externality. In some cases, it is optimal to ban the production of risky liquidity entirely — a policy that, in our model, eliminates financial crises.

The key result, however, is that reducing or eliminating risky liquidity alone is not enough to achieve efficiency and could even backfire and reduce welfare. A policy designed exclusively to reduce or eliminate risky liquidity creates a void in the liquidity market. Such a void will be filled, at least in part, by newly created safe assets. Yet, it is presumably more expensive to create safe than risky liquidity, as intermediaries must devote some resources to preventing default (e.g., equity issuance costs, monitoring costs, and managerial, legal, and other costs). Thus, an over-expansion of safe liquidity generates excessive social costs. As a result, optimal regulation will involve not only a restriction on risky liquidity but also some rules on safe liquidity.

An important application of our results relates to capital requirements. A tight requirement that eliminates risky liquidity entirely is always welfare-reducing. In some cases, however, this policy is nonetheless optimal if combined with a tax on safe intermediaries to avert the unintended consequences of too great an expansion of costly safe liquidity.

In the model, the production of safe debt is costlier than risky debt. Issuers of safe debt must avoid default, which requires them to raise an adequate amount of costly equity. In an alternative formulation, intermediaries avoid default by screening and monitoring the projects they invest in — also a costly process. In any case, safe debt must command a liquidity premium sufficient to cover the equity or monitoring costs.

The externality behind the inefficiency of the unregulated equilibrium operates through liquidity premia. When issuing debt, the intermediary (whether safe or risky) does not internalize the effects of its choices on the costs sustained by other intermediaries, in particular safe ones. As each one supplies more liquidity, liquidity premia fall and so borrowing costs increase. To avoid default, safe intermediaries must offset this increase with more equity or more intense monitoring, increasing the social resources allocated to safe liquidity creation.

By taxing all intermediaries, a regulator can enable the safe ones to economize on the socially-costly resources allocated to safe liquidity creation. The tax should differ depending on whether the liquidity is safe or risky. If the tax on risky intermediaries is very high, the regulation is akin to a ban on risky liquidity. Crucially, however, even in this case the intermediaries issuing safe debt should be taxed in order to avoid excessive expansion of safe liquidity. The tax restrains the supply of safe liquidity and thus keeps the liquidity premium high, allowing safe intermediaries to borrow cheaply and thus be solvent even with little or no equity.[1] Taxes should be imposed on intermediaries' profits (i.e., ex-post) and they should be time-varying and procyclical

[1] In some cases, the optimal tax generates only a modest decrease in intermediaries' borrowing costs, and safe intermediaries need to raise equity to avoid default. It then becomes optimal to let some produce risky debt, permitting some liquidity to be supplied without too much costly equity. Nonetheless, the amount of risky debt is less than in the unregulated equilibrium.

(i.e., high in normal and low in bad times).

Our results are independent of the cost sustained by intermediaries to produce safe liquidity. Our two versions of the model — with costly equity and with costly monitoring — both deliver the same results. Crucially, there is a one-to-one mapping between the two models, in the sense that their mathematical structures are identical. That is, they are described by the same equations and so have the same unregulated equilibrium and the same optimal regulatory policies. Besides equity and monitoring costs, there might be other costs for producing safe liquidity (managerial and legal expenses, say, or the costs of credible disclosure of information about the safety of their debt). Taking these into account is likely to reinforce our results.

The model that we use builds on a related work of ours (Benigno and Robatto, 2017), but with some major differences. First of all, the focus of that work is on the optimal supply of public versus private liquidity and the need for government intervention in liquidity crises. Here, instead, we abstract from public liquidity and government intervention to focus on private liquidity only. Second, given its different focus, the model of Benigno and Robatto (2017) posits a simpler modeling of intermediaries' balance sheet, with no externality. A novelty of the present paper is its richer yet still simple modeling of financial intermediation based on well-established, important stylized facts or theoretical results (i.e., equity and equity issuance costs, monitoring, and a more general risk choice by intermediaries) in order to study the resulting pecuniary externality.

With respect to the literature on capital requirements, two closely related papers are Plantin (2014) and Kyle (2018). They present theoretical models in which tight capital requirements on commercial banks create a vacuum that is filled by shadow banks.[2] Our results complement this view. In our model too capital requirements generate a substitution, but from risky to safe, costlier liquidity production.

The paper is structured as follows. Section 2 presents the model and Section 3 studies the laissez-faire equilibrium. Section 4 discusses the planner's problem and hence the inefficiency of laissez-faire. Section 5 discusses how regulation can implement the planner's solution and analyzes capital requirements. Section 6 discusses the implications of monitoring costs rather than equity costs, and Section 7 concludes.

[2] A number of other papers also emphasize how the response to financial regulation can undermine the objective of the interventions. For instance, Gofman (2017) studies how stability in financial networks can hinder efficiency, and Malherbe (2014) shows that policies to offset fire-sale externalities can generate multiple equilibria and adverse selection.

2 Model

Time is discrete, with two periods indexed by $t = 0, 1$. At $t = 1$, an aggregate shock occurs; the economy can be in one of two states: high (i.e., h), with probability $1-\pi$, or low (i.e., l), with probability π. We interpret the low state as a recession because aggregate productivity is lower. Depending on the risk characteristics of the liquid assets supplied in equilibrium, the low state may also comprise a liquidity crunch, in which a recession is associated with a financial crisis.

We follow a large literature on the role of liquidity in facilitating households' transactions and consumption, which includes Diamond and Dybvig (1983), Gorton and Pennacchi (1990), and Lucas and Stokey (1987). This approach allows us to keep the model and analysis simple and tractable, although our results could be reformulated in more general frameworks, such as a liquidity-in-the-utility function, akin to Nagel (2016) and Stein (2012), which would capture a more general role for liquidity, beyond transaction facilitation. We could also assume that liquidity is used by firms to finance production pre-sales, as in Bigio (2015), Holmström and Tirole (2011), and Jermann and Quadrini (2012). In our view, however, our approach is the simplest and most transparent way to deliver our results. These possible alternatives would complicate the analysis without altering the main messages of the paper.

In the baseline model, intermediaries that want to issue risk-free liquidity must raise enough equity to avoid default, and equity is costly. Section 6 presents an alternative formulation in which intermediaries avoid default by screening and monitoring the projects they invest in and then shows that there is a one-to-one mapping between the two models. That is, they are described by the same equations, so all the results are identical.

In what follows, we first set out the households' problem, then that of financial intermediaries.

2.1 Households

Households are endowed with $\bar{Y}_0$ units of goods at $t = 0$, and $\bar{Y}_h$ and $\bar{Y}_l$ in the high and low states of $t = 1$, respectively. Their preferences are

$$X_0 + (1-\pi)\left[\log C_h + X_h\right] + \pi\left[\log C_l + X_l\right] \tag{1}$$

where X_0 is consumption at $t = 0$, C_h and X_h are consumption at $t = 1$ in the high state, and C_l and X_l consumption at $t = 1$ in the low state.

The formulation of the utility function builds on the cash-credit model of Lucas and Stokey (1987). In our framework, C_h and C_l denote consumption that is subject to a liquidity constraint, i.e., must be purchased using liquid assets (akin to the cash good in Lucas and Stokey, 1987), while X_h and X_l denote consumption that can be purchased using credit. Alternatively, one can think of consumption of C_h and C_l as occurring at the beginning of $t = 1$ (and subject to a liquidity constraint), and consumption of X_h and X_l at the end of $t = 1$ (and not subject to any liquidity constraint). In addition, as in Lucas and Stokey (1987), each household consists of a shopper and a seller, who act separately at the beginning of $t = 1$. That is, the shopper purchases C_h or C_l subject to a liquidity constraint, while the seller sells the endowment $\bar{Y}_h$ or $\bar{Y}_l$ to other households.[3]

To simplify the exposition, we discuss first the liquidity constraint, which applies at time $t = 1$, and then the household's budget constraints. A financial friction restricts which securities can be used to buy consumption C_h and C_l. In line with the historical evidence discussed in Gorton (2017), our first assumption is that only debt securities provide liquidity. We limit the analysis to private liquidity issued by financial intermediaries, abstracting from government-supplied liquidity, given that our aim is to study the possible inefficiency of private liquidity creation. In any event, at the cost of somewhat more complicated exposition, the results are readily generalized to a context in which some government liquidity is also available. Our second assumption on liquidity is that in each state of nature a debt security is liquid only if it is not defaulted on in that state. Such an assumption can be justified by the existence of some time requirement to complete the bankruptcy procedure. In practice, this kind of delay might preclude the use of the securities in certain trades that require liquid assets with a value known to all parties. The payoff on all securities, including the partially-defaulted, eventually accrues to the owner and can thus be used at a later time — in our model, to purchase consumption X_h and X_l.

Given these assumptions, only two types of debt securities can provide liquidity services: (i) safe securities, modeled as zero-coupon bonds with a unitary face value, which are never defaulted on; and (ii) risky securities, also zero-coupon bonds with unitary face value, which are fully repaid only in one state of nature and defaulted on (at least partially) in the other. The assumptions made above about the liquidity of debt imply that safe securities — denoted as S — always provide liquidity services,

[3] An alternative approach is to use a model in which households purchase C_h and C_l from a different set of agents. But this would entail additional notation, so we follow the shopper-seller structure of Lucas and Stokey (1987).

whereas risky securities provide liquidity only in the state in which they are fully repaid. In principle, there can be many types of risky securities, differing in their default rate. But as is shown later, intermediaries have an incentive to create only one type of risky security, denoted as D. Securities D are fully repaid in state h and partially defaulted on in state l at a rate χ_l, which is endogenously determined (i.e., their payoff is $1-\chi_l$ in the low state).

As a result, consumption of C_h and C_l at $t=1$ is subject to the liquidity constraint

$$C_h \leq S + D, \tag{2}$$

$$C_l \leq S. \tag{3}$$

The liquidity constraints (2) and (3) generalize those employed in an extensive literature on the liquidity property of safe assets (e.g., Caballero and Fahri, 2017; Diamond, 2016; Greenwood, Hanson, and Stein, 2015; Li, 2017; Magill, Quinzii, and Rochet, 2016; Stein, 2012). With this literature, we share the idea that some debt securities are valued not only for their pecuniary returns but also for their liquidity. As in the literature, safe debt, S, always provides such liquidity services. But we depart from the literature by assuming that risky debt, D, can also provide liquidity services in the contingencies where it is not defaulted on. This generalization of the liquidity constraint is in keeping with the discussion summarized in Gorton (2017). In practice, securities that provide liquidity services do not necessarily have to be risk-free. Indeed, a number of financial crises have been marked by a fall in the liquidity value of risky securities, even though such securities had the same liquidity value as safe debt before a crisis erupted. Our framework captures these facts in a simple and convenient way, suggesting implications and allowing study of policies that are hard or impossible to analyze in standard liquidity models with only risk-free assets.

The budget constraint of households at time 0 is

$$X_0 + Q^S S + Q^D D + N^S + N^D \leq \bar{Y}_0, \tag{4}$$

where Q^S and Q^D are the prices of securities S and D, and N^S and N^D denote the household's investment in the respective intermediaries' net worth.

At time $t=1$, after purchasing goods C_h or C_l subject to liquidity constraints (2) and (3), households purchase goods X_h or X_l subject to the following state-contingent budget constraints

$$X_h \leq \bar{Y}_h + (S + D - C_h) + N^S\left(1 + r_h^S\right) + N^D\left(1 + r_h^D\right) \tag{5}$$

$$X_l \leq \bar{Y}_l + (S - C_l) + D(1 - \chi_l) + N^S\left(1 + r_l^S\right) + N^D\left(1 + r_l^D\right), \tag{6}$$

where r_h^S and r_l^S are the return on equity investments N^S, in the high and low state respectively, and r_h^D and r_l^D are those on N^D. Equations (5) and (6) show that consumption of X_h and X_l can be financed by three resources: the endowment available at $t = 1$ (i.e., $\bar{Y}_h$ and $\bar{Y}_l$); the debt securities not used to purchase C_h and C_l; and the gross return on intermediaries' equity. Note that the risky securities D, which are partially defaulted on at the rate χ_l in state l and thus do not provide the liquidity needed to purchase C_l, can be used to purchase the goods X_l once the intermediaries' bankruptcy proceedings are completed, according to the recovery value $1 - \chi_l$.

Households maximize (1) subject to constraints (2)-(6). The optimal level of consumption at $t = 1$ is determined by the first-order conditions

$$\frac{1}{C_h} = 1 + \mu_h \tag{7}$$

and

$$\frac{1}{C_l} = 1 + \mu_l, \tag{8}$$

where μ_h and μ_l are the Lagrange multipliers associated with the liquidity constraints (2) and (3), respectively. Note that the first best allocation is achieved when the liquidity constraints are not binding, i.e., when $\mu_h = \mu_l = 0$. In this case, consumption is $C_h = C_l = 1$, and the marginal utilities of consuming C_h and C_l are equalized to those of consuming X_h and X_l.

At $t = 0$, the demand for debt securities S and D is determined by the respective first-order conditions, which imply

$$Q^S = (1 - \pi)(1 + \mu_h) + \pi(1 + \mu_l) \tag{9}$$

$$Q^D = (1 - \pi)(1 + \mu_h) + \pi(1 - \chi_l). \tag{10}$$

Security S always provides liquidity so the price includes the liquidity premium in both states, represented by the Lagrange multipliers μ_h and μ_l. Security D provides liquidity only in state h so its price includes the liquidity premium μ_h. Nonetheless, D may also provide a non-zero pecuniary payoff $1 - \chi_l$ in the low state, which is factored into its price.

The first-order conditions with respect to the equity holdings, N^S and N^D, specify the expected return on equity:

$$(1 - \pi)r_h^S + \pi r_l^S = 0, \tag{11}$$

$$(1 - \pi)r_h^D + \pi r_l^D = 0. \tag{12}$$

Households are willing to supply any amount of equity to intermediaries as long as the expected return is zero. This result follows from the assumption of risk neutrality in X_h and X_l, and the normalization of the discount factor between $t = 0$ and $t = 1$ to one. If we introduce a discount factor less than one, a positive return would be required but none of main results would change.

2.2 Financial intermediaries

Financial intermediaries issue debt and equity at $t = 0$ to finance investment in physical capital. As described in Section 2.1, the debt provides liquidity to households at $t = 1$.

We assume that the intermediaries' objective is to maximize rents, which are given by the payoff earned at $t = 1$ minus the repayment to debtholders and shareholders. Hence, we distinguish between the dividends that accrue to shareholders (i.e., "regular"profits) and the rents to the intermediary (i.e., "extra-profits"). In equilibrium, banking competition drives rents to zero. This approach is standard in many banking models, such as Magill, Quinzii, and Rochet (2016) and Gale and Gottardi (2017). Repayments to bondholders are bounded by a limited-liability constraint; that is, if the payoff on capital is not sufficient to repay the entire face value of outstanding debt, intermediaries default on the excess fraction.[4]

2.2.1 Intermediaries' problem

Intermediaries invest in a risky technology. Each unit of investment at $t = 0$ produces A_h in state h and A_l in state l, with $A_h > A_l > 0$ and average productivity of one; that is

$$(1 - \pi) A_h + \pi A_l = 1. \tag{13}$$

Intermediaries can finance their investments with debt or equity (i.e., net worth). Crucially, we assume that equity issuance is costly; that is, for every dollar of net worth issued, only a fraction $1 - \tau < 1$ can be used to invest. The remainder fraction $\tau > 0$ is wasted at the economy-wide level, reducing the resources available for consumption.

The banking literature has provided extensive analysis on the existence and role of equity issuance costs for banks. From a theoretical point of view, Allen and Gale (1988) are among the first to introduce a transaction costs for issuance of multiple types of securities to finance projects. Our approach broadly follows Allen and Gale

[4]We assume that intermediaries cannot abscond with assets and so are committed to repay debt and paying the dividends to equity holders, provided that they have the resources to do so.

(1988), in that intermediaries that issue both equity and debt in our model incur the transaction costs, which we model as equity issuance cost. From an empirical and applied perspective, the study and use of equity issuance costs is widespread. Calomiris and Wilson (2004) document equity issuance costs in the interwar period, that is, a relatively regulation-free environment. Corbae and D'Erasmo (2014) estimate positive equity issuance costs for banks using a structural model and use their result to conduct policy analysis. Bolton and Freixas (2000) rationalize several facts about firms' capital structure using a model in which banks face equity issuance costs. Some other papers, such as Klimenko et al. (2016), use equity issuance cost to study capital regulation. In addition, many recent macro-finance models, such as Brunnermeier and Sannikov (2014) and Gertler and Kiyotaki (2010) do not allow intermediaries to issue external equity, which is akin to imposing a very high or infinite cost of issuance. More generally, the literature shows that equity issuance costs are also important for non-financial firms, as documented by Altınkılıç and Hansen (2000) and estimated using structural models by Hennessy and Whited (2007) and Jermann and Quadrini (2012). In practice, equity issuance costs comprise such expenses as auditing, underwriting, and managing the issuance, in addition to legal and registration fees.

Going back to our model, intermediaries can in principle issue many types of debt securities with different risk characteristics (i.e., different levels of default). For simplicity, we assume that each intermediary can issue only one type of debt security, although the same type can be supplied by many different intermediaries.

Anticipating some of the results, only two types of debt security are issued in equilibrium: risk-free debt S and one type of risky debt, denoted by D. Risky debt D is issued by intermediaries with zero net worth, and its default rate is denoted by χ_l (i.e., the payoff of D in the low state is $1 - \chi_l = A_l/A_h < 1$ per dollar of debt). We solve the model by conjecturing that only S and D are issued in equilibrium, and we then verify that intermediaries do not find it profitable to issue any other debt security with a different default rate.

In our model, intermediaries' choice whether to supply S or D is endogenous, in the sense that each intermediary chooses S or D depending on which type of debt is more profitable. In equilibrium, the marginal intermediary entering the market is indifferent between S or D because competition drives intermediation rents to zero in both markets.

The budget constraint of intermediaries that issue risk-free debt S is

$$K^S = Q^S S + N^S (1-\tau), \tag{14}$$

where K^S is the capital invested by such an intermediary, and N^S is net worth. Similarly, the budget constraint of intermediaries that issue risky debt D is

$$K^D = Q^D D + N^D (1-\tau), \tag{15}$$

where K^D is the capital and N^D is net worth.

As noted, we conjecture that intermediaries issuing D choose $N^D = 0$ and hence securities D are partially defaulted on in the low state. By contrast, intermediaries issuing S raise enough net worth $N^S > 0$ to make S risk-free. Intermediaries' assets are in fact risky, so equity is required to make debt S free of default risk.

Intermediaries maximize expected rents. Rents are given by the payoff generated by capital net of repayments to debtholders and shareholders, and taking the limited liability of intermediaries into account.

For intermediaries that issue S, expected rents $\mathcal{R}^S$ are:

$$\mathcal{R}^S = (1-\pi)\left[A_h K^S - S - N^S\left(1+r_h^S\right)\right] + \pi\left[A_l K^S - S - N^S\left(1+r_l^S\right)\right], \tag{16}$$

where we have used the conjecture that these intermediaries never default. This is the case as long as the payoff on their assets in state l is sufficient to repay the face value of their debt. Formally, this condition is given by

$$A_l K^S \geq S. \tag{17}$$

Note that since productivity in the high state is $A_h > A_l$, (17) guarantees that an intermediary issuing S is solvent not only in state l, but also in state h. Using the budget constraint (14), we can rearrange (17) to obtain

$$\frac{N^S}{S} \geq \frac{1}{(1-\tau)A_l} - \frac{Q^S}{1-\tau}. \tag{18}$$

To be solvent, an intermediary issuing safe securities needs to raise sufficient equity. Since equity is costly (i.e., $\tau > 0$), this lower bound binds in equilibrium.

Equation (18) is essential to understand the pecuniary externality of our model. The mechanism works as follows. Intermediaries issuing security S are subject to the constraint (18), which depends critically on the price of safe securities Q^S. To clarify this dependence, consider the following partial-equilibrium exercise. An increase in Q^S relaxes the constraint (18), implying that intermediaries that issue safe debt need to raise less equity to be solvent. This is because each unit of S can be sold at a higher price, allowing safe intermediaries to collect more resources at $t = 0$ and thus to increase investment for any fixed level of debt S. Hence, higher Q^S allows safe intermediaries to save on the socially wasteful equity issuance cost. In general equilibrium, the price Q^S is taken as given by all agents in the economy, so no intermediary internalizes the effects of its actions on Q^S. However, a regulator or a planner that internalizes this mechanism can leverage on an increase in Q^S to reduce equity issuance, thus saving on the equity issuance cost. An increase in Q^S corresponds to a higher liquidity premium on safe assets, which is obtained if the assets used for transactions become scarcer. Indeed, limiting the supply of intermediaries' debt — i.e., the assets used for transactions — will be the channel for eliminating the pecuniary externality (see Section 4).

In Section 6, we set out an alternative motivation for a constraint similar to (18).[5] There, the intermediaries that issue risk-free debt S avoid default by screening and monitoring the projects in which they invest, not by issuing equity. Screening and monitoring are costly, like equity issuance, and an increase in the price of safe assets Q^S reduces the resources devoted to them. Importantly, we identify a mapping between this baseline model with equity issuance cost and the alternative formulation with monitoring, in the sense that the two formulations are mathematically identical and thus have the same solution.

Turning now to intermediaries issuing securities D, their expected rents $\mathcal{R}^D$ are:

$$\begin{aligned}\mathcal{R}^D &= (1-\pi)\left[A_h K^D - D - N^D\left(1+r_h^D\right)\right] \\ &\quad + \pi \max\left\{A_l K^D - D - N^D\left(1+r_h^D\right), 0\right\} \\ &= (1-\pi)\left(A_h^D K^D - D\right) + \pi\left[A_l K^D - D\left(1-\chi_l\right)\right]. \qquad (19)\end{aligned}$$

The *max* operator accounts for the possibility that the limited-liability constraint might be binding in the low state, and the last line uses the conjecture that such a constraint is indeed binding. The default rate χ_l in the low state is determined such that all the resources available in state l to intermediaries issuing D are used to repay debtholders:

$$A_l K^D = (1-\chi_l)D. \qquad (20)$$

2.2.2 The optimal supply of debt securities

We begin by solving for the optimal supply of risk-free debt S. We can rewrite the intermediaries' objective function (16) using the first-order condition that deter-

[5]The two approaches can, in principle, be combined, but we keep them separate to clarify the exposition.

mines households' supply of equity, (11), the normalization in (13), and the budget constraint (14). This gives

$$\mathcal{R}^S = Q^S S - S - \tau N^S. \tag{21}$$

Intermediaries' shareholders are households, who do not discount the future and are risk-neutral, as discussed in Section 2.1; thus, they demand a zero expected return on equity. As a result, the intermediaries' rents are given by the profits earned by issuing S (i.e., the revenues $Q^S S$ from issuance minus the repayments S, given that there is no discounting between $t = 0$ and $t = 1$) minus the equity issuance cost (i.e., τN^S). Owing to this cost, intermediaries raise the minimum amount of equity needed to make their debt safe, which is determined by (18) evaluated with equality. Thus, replacing N^S in (21) using (18), we obtain

$$\mathcal{R}^S = \frac{1}{1-\tau} Q^S S - S - \frac{\tau}{1-\tau} \frac{S}{A_l}. \tag{22}$$

Intermediaries supply safe debt as long as rents $\mathcal{R}^S$ are non-negative, that is, if and only if the price of securities satisfies

$$Q^S \geq 1 + \tau \left(\frac{1}{A_l} - 1 \right).$$

Free entry ensures that in equilibrium the rents $\mathcal{R}^S$ from issuing S are zero. Therefore the supply of S will be non-negative at the price

$$Q^S = 1 + \tau \left(\frac{1}{A_l} - 1 \right). \tag{23}$$

Note that $Q^S = 1$ if there is no cost of issuing equity (i.e., if $\tau = 0$). On the contrary, with costly equity ($\tau > 0$), safe securities include a liquidity premium, which allows intermediaries to earn revenues that offset the issuance cost. In addition, since we have normalized the discount factor between $t = 0$ and $t = 1$ to one, the price of safe securities is $Q^S > 1$ if $\tau > 0$, and thus the return is negative.[6]

Next, we turn to intermediaries that issue D. Using (19), the normalization in (13), and the budget constraint (15), we obtain

$$\mathcal{R}^D = Q^D D - D \left(1 - \pi \chi_l^D \right). \tag{24}$$

Since these intermediaries issue no equity, the rents are given solely by the profits obtained by issuing debt D, i.e. revenues $Q^D D$ from issuance minus the expected

[6]If we introduce a discount factor less than one, the price Q^S can be less than one as well, so the return can be positive; such an extension would complicate the exposition without changing the main results.

repayment $D\left(1-\pi\chi_l^D\right)$.

Intermediaries issuing D are willing to supply a non-negative amount of debt as long as their rents are non-negative, that is, if

$$\begin{aligned} Q^D &\geq 1-\pi\chi_l^D \\ &= \frac{1-\pi}{1-\pi A_l} \in (1-\pi, 1), \end{aligned}$$

where the second line uses (15) and (20). Free entry drives rents to zero, and hence in equilibrium:

$$Q^D = \frac{1-\pi}{1-\pi A_l}. \tag{25}$$

That is, the price of securities D is equal to their expected payoff.

3 Laissez-faire equilibrium

We now solve the equilibrium of our model in the absence of regulation. This equilibrium entails efficient allocation of consumption in the high state and a liquidity crunch reducing consumption in the low state.

We begin by combining the expression for Q^S and Q^D, (23) and (25), obtained by solving intermediaries' supply with those obtained from households' demand, (9) and (10) respectively. We get that the Lagrange multiplier of the liquidity constraints (2) and (3) are zero in state h, $\mu_h = 0$, and positive in state l,

$$\mu_l = \frac{\tau}{\pi}\left(\frac{1}{A_l} - 1\right) > 0,$$

where the inequality follows from $A_l < 1$. Liquidity demand is completely satiated in state h but not in state l. This implies that consumption reaches the first-best level in state h but falls below that in state l:

$$C_h = 1, \tag{26}$$

$$C_l = \frac{\pi A_l}{\pi A_l + \tau(1-A_l)} < 1. \tag{27}$$

We can then determine the equilibrium quantity of safe and risky debt using the liquidity constraints (2) and (3):

$$S = \frac{\pi A_l}{\pi A_l + \tau(1-A_l)} < 1, \quad D = \frac{\tau(1-A_l)}{\pi A_l + \tau(1-A_l)}.$$

The equity cost is essential to explain the fall of consumption C_l in the low state. There, only safe securities S provide liquidity. But to offset the equity cost, safe intermediaries need to earn a liquidity premium. Hence, a shortage of liquidity — and of safe assets — in state l is a necessary condition to generate this premium, as in state h demand is satiated. The higher the equity cost, the lower the supply of safe assets, and the lower consumption in state l. With a higher equity cost, type S intermediaries supply fewer safe assets because they need a correspondingly higher liquidity premium.

The equity cost is only one necessary condition for consumption to fall in the low state. Another key factor is the risk of the technology in which intermediaries invest. If investment in physical capital is riskless (i.e., if $A_h = A_l$), then intermediaries could issue safe debt S with no need to issue equity. In this case, the equity issuance cost would be irrelevant.

The fall in consumption from state h to state l can be interpreted as a liquidity crisis. In the high state, both safe and risky debt provide liquidity services because neither is defaulted on. The economy is completely satiated with liquidity, and consumption is at the first best: $C_h = D + S = 1$. However, because in the low state only safe securities are liquid and their equilibrium quantity is $S < 1$, the equilibrium involves a reduction of consumption C_l. Although the productivity of capital drops in state l, the liquidity crisis is driven by the shortage of safe securities.

Finally, in the Appendix, we verify the conjecture made in Section 2.2.1 that intermediaries do not find it profitable to issue debt securities with default rates that differ from those of S and D.

4 The inefficiency of laissez faire

We now discuss the welfare properties of the laissez-faire equilibrium derived in the previous section. We show that the equilibrium is inefficient because of a pecuniary externality that can be corrected by regulation. We perform this analysis by characterizing the problem of a social planner who seeks to maximize households' utility subject to the same frictions as private agents, namely the equity issuance cost and the limited liquidity of risky debt.

Our ultimate objective is to reinterpret the planner as a government authority that can regulate intermediaries in such a way as to increase overall welfare. To this end, we consider a planner with "limited" powers, in the sense that it can dictate intermediaries' choices but cannot affect either households or the way securities are traded in financial markets.

The planner chooses the supply of debt and equity by intermediaries, which must comply. However, once the supply is chosen, households and intermediaries trade debt and equity at $t = 0$ as in the unregulated economy, so we continue to let Q^S and Q^D denote the price of safe and risky debt. At $t = 1$, households trade consumption goods, again as in the unregulated economy. Our planner's problem is thus akin to that of a benevolent monopolist financial intermediary that maximizes households' surplus, taking the households' demand schedule for liquid debt and equity as given.

We abstract from any power of the planner to transfer resources to intermediaries at $t = 1$ (i.e., recapitalize them). Such policies (bailouts, deposit insurance, and other government guarantees) must be implemented by a government that has a sufficiently large fiscal capacity, in particular during a crisis. The implicit assumption here is that this fiscal capacity is limited, and we accordingly focus on the case in which no such transfers are possible.[7]

4.1 The planner's problem

The planner maximizes households' utility (1) subject to three sets of constraints. We now describe these constraints in greater details; for a more formal derivation, see Appendix B.

The first set of constraints makes sure that the allocation chosen by the planner is consistent with intermediaries' budget constraint, limited liability constraint, and non-negativity constraints on assets issued (i.e., S, D, N^S, $N^D \geq 0$). In particular, the non-negativity constraints $D \geq 0$ and $N^S \geq 0$ are never binding in the laissez-faire equilibrium but, depending on parameters, they may be binding in the planner's problem.

The second set of constraints takes into account how households respond to the planner's choices. Recall that our planner has limited powers — it can choose intermediaries' supply of debt and equity at $t = 0$ but is constrained by households' demand for them (i.e., households' budget constraints at $t = 0, 1$, liquidity constraint at $t = 1$, and optimality conditions). Since the objective is to maximize households' utility, in directing intermediaries' supply of securities, the planner must take into account how households will respond.

The third and most important constraint is that securities S must be risk-free. This constraint has the same form as in the laissez-faire equilibrium, (18). Given its

[7] In a related paper (Benigno and Robatto, 2018), we study deposit insurance and government guarantees of intermediaries' debt in conjunction with more general policies relating to the supply of public liquidity. If the government's fiscal capacity is not sufficient, intermediaries' default arises in equilibrium, as we obtain here.

importance, it is worth reiterating here:

$$\frac{N^S}{S} \geq \frac{1}{(1-\tau)\,A_l} - \frac{Q^S}{1-\tau}. \tag{28}$$

There is a key difference between the intermediaries' and the planner's problem with respect to the role played by this constraint, as discussed in Section 2.2.1. Each intermediary takes prices as given, including the price of safe debt Q^S, which affects the tightness of constraint (28). The planner, however, internalizes that its choices can affect the value of Q^S, which in turn influences the minimum amount of equity required for debt S to be safe. Because of the issuance cost τ, the planner will find it optimal to tilt intermediaries' supply of debt so as to reduce equity issuance and, thus, total equity issuance costs.

We can now state the planner's problem. In Appendix B, we show that this reduces to

$$\max_{C_h, C_l, N^S, N^D} (1-\pi)\left[\log C_h - C_h\right] + \pi\left[\log C_l - C_l\right] - \tau(N^S + N^D) \tag{29}$$

subject to two key constraints. The first is households' demand for risk-free debt, (9), evaluated at (7) and (8):

$$Q^S = \frac{1-\pi}{C_h} + \frac{\pi}{C_l}. \tag{30}$$

Constraint (30) is crucial because it reflects the planner's internalization of how its choices of C_h and C_l affect the price of safe liquidity Q^S. As observed before, this is the key difference from the laissez-faire equilibrium, where unregulated intermediaries take Q^S as given and do not internalize the effects of their own choices on it. The second constraint guarantees that securities S are risk-free and is given by (28) evaluated at $S = C_l$ according to the liquidity constraint (3):

$$\frac{N^S}{C_l} \geq \frac{1}{(1-\tau)\,A_l} - \frac{Q^S}{1-\tau}. \tag{31}$$

Finally, the planner is also subject to the non-negativity constraints $N^S \geq 0$, $N^D \geq 0$, and $C_h \geq C_l$. The latter, in particular, implies that the supply of risky liquidity satisfies $D \geq 0$; the equivalence between $C_h \geq C_l$ and $D \geq 0$ follows from (2) and (3).

After solving problem (29) for C_h and C_l, we can use the structure of the model to derive the amount of safe and risky liquidity, S and D, that is implicitly chosen by the planner. This can be done using the liquidity constraints (2) and (3).

4.1.1 The planner's solution and the inefficiency of laissez faire

We show that the planner's solution differs from the laissez-faire equilibrium. Hence, the laissez-faire equilibrium suffers an inefficiency. We characterize the planner's solution in detail in the next section.

To begin with, note that the planner chooses $N^D = 0$, as in the laissez-faire equilibrium. In fact, issuing any amount of equity that does not eliminate default is costly (because of the issuance cost) but provides no benefit (because defaulted securities do not provide liquidity in state l).

To compare the laissez-faire equilibrium with the planner's solution, we conjecture that for the planner the constraints $N^S \geq 0$ and $D \geq 0$ are not binding. Thus, we can compare the two solutions, given that under the laissez-faire equilibrium $N^S > 0$ and $D > 0$. We get that the first-order conditions with respect to the optimal levels of consumption C_h and C_t are given by

$$\pi\left(\frac{1}{C_l} - 1\right) = \frac{\tau}{1-\tau}\left[\frac{1}{A_l} - Q^S\right] + \frac{\tau}{1-\tau}C_l\left(-\frac{dQ^S}{dC_l}\right) \tag{32}$$

and

$$(1-\pi)\left(\frac{1}{C_h} - 1\right) = \frac{\tau}{1-\tau}C_l\left(-\frac{dQ^S}{dC_h}\right), \tag{33}$$

respectively. Using (30), we can compute the terms

$$\frac{dQ^S}{dC_l} = -\frac{\pi}{(C_l)^2}, \qquad \frac{dQ^S}{dC_h} = -\frac{1-\pi}{(C_h)^2},$$

which reflect the fact that the planner internalizes the possible effects of the choices of C_h and C_l on the price of safe assets Q^S, which as noted will affect equity issuance.

It is easy to verify that the laissez-faire equilibrium is inefficient, in the sense that it does not satisfy the planner's optimality conditions (32) and (33). It can also be verified that the laissez-faire equilibrium coincides with the planner's solution only if Q^S is taken as given in the planner's maximization problem; that is, if we drop constraint (30) and fix Q^S. In this case, the terms dQ^S/dC_h and dQ^S/dC_l would both be zero, and (32) and (33) would imply that C_h and C_l are the same as in the laissez faire equilibrium: $C_h = 1$, as in (26), and C_l equal to the expression in (27). Therefore, the difference between the laissez-faire equilibrium and the planner's solution is that intermediaries do not internalize the effect of their choices on Q^S.

4.1.2 Characterizing the planner's solution

The characterization of the planner's solution depends on the parameter values. In particular, we get that the planner either wants to keep some supply of risky debt (i.e., $D > 0$) or ban risky debt entirely (i.e., $D = 0$), and that it instructs intermediaries issuing safe securities either to supply a positive amount of equity (i.e., $N^S > 0$) or not to supply equity (i.e., $N^S = 0$). Note that banning risky debt is equivalent to eliminating liquidity crises (i.e., $C_h = C_l$). To cope with this rich structure, we focus most of the discussion on the planner's solution that arises in some subset of the parameter space. We motivate this focus with a set of stylized facts that enable us to pay attention to the case that is most relevant in practice. For completeness, though, we also discuss the solution that arises under other parameterization.

The result that safe debt can be issued with zero equity (i.e., $N^S = 0$) might appear extreme, but some simple extensions can reconcile the model with the thesis that in practice some equity might always be required. For instance, some equity might provide a "skin in the game" that reduces moral hazard on the part of the intermediary. To keep the analysis simple and tractable, we abstract from this and other considerations. That is, we have implicitly normalized to zero the amount of equity required to deal with frictions other than those relating to liquidity.

The central result is that the planner, under a reasonable parameterization, avoids liquidity crises by achieving equal consumption in the two states, $C_h = C_l$ (i.e., the supply of risky debt is zero, $D = 0$), and does so by instructing intermediaries to create only safe securities backed by zero equity. This result may seem somewhat surprising, but it can be explained by the planner's power to manipulate the liquidity premium. With a high premium intermediaries can borrow at very low rates and thus issue safe securities without equity. In this case, the planner avoids wasteful equity issuance costs. For completeness, we also discuss a parametrization in which the planner's solution requires $C_h > C_l$. In this case, intermediaries issue both safe and risky debt, $S, D > 0$, and the equity that backs safe securities too is positive, that is, $N^S > 0$. Even in this case, however, consumption differs from the laissez-faire equilibrium, which is thus shown to be inefficient for all parameter values.

The difficulty in characterizing the planner's solution is related to the presence of non-negativity constraints on safe intermediaries' net worth, $N^S \geq 0$, and risky debt, $D \geq 0$. While these constraints are not binding under laissez-faire, they can be binding for the planner's solution, depending on the parameterization.

To account for the possibility that the non-negativity constraints $N^S \geq 0$ and $D \geq 0$ are binding, we denote their Lagrange multipliers as $\lambda \geq 0$ and $\varphi \geq 0$. In the Appendix, we show that the general version of the first-order conditions (32) and (33) is:

$$\pi\left(\frac{1}{C_l}-1\right)=\frac{\tau-\lambda}{1-\tau}\left[\frac{1}{A_l}-Q^S\right]+\frac{\tau-\lambda}{1-\tau}C_l\left(-\frac{dQ^S}{dC_l}\right)+\varphi \tag{34}$$

and

$$(1-\pi)\left(\frac{1}{C_h}-1\right)=-\frac{\tau-\lambda}{1-\tau}C_l\left(-\frac{dQ^S}{dC_h}\right)-\varphi. \tag{35}$$

We discuss first the implied solution under our preferred parametrization, which restricts the probability of a crisis (π) to be low and the fall in productivity in the low state to be moderate (i.e., the productivity in state l, A_l, is sufficiently high). The Appendix presents more details on these parameter restrictions and the proofs of all the results.

Our focus on low π follows from the interpretation of liquidity crunches (which occur in the low state in the laissez-faire equilibrium) as financial crises, which are effectively rare events. In particular, we argue that the relevant case is when $\pi < \tau$, where τ is the equity issuance cost. The interpretation is that the probability of financial crisis is low, in relation to the social resources that must be allocated to issue safe assets. Alternatively, this restriction can be recorded to say that the cost to issue safe assets is sufficiently high, which is likely to be the case in practice.

The focus on sufficiently high A_l (i.e., sufficiently close to one, the average value of productivity) is motivated by the relatively small variations in aggregate output experienced by advanced economies. In the U.S. business cycle, downturns are ordinarily "normal recessions" (i.e., without financial crises) in which GDP variations are moderate. And even in the Great Recession, despite the financial meltdown, the overall fall in GDP was just a few percentage points. For this reason, and because in our model the realization of A_l represents an aggregate shock, A_l can be taken to be sufficiently close to A_h.

Under this parameterization, the planner's solution differs radically from the laissez-faire equilibrium in three ways: 1) consumption is equalized across states ($C_h = C_l$), so liquidity crises are avoided; 2) there is no issuance of risky debt ($D = 0$); and 3) safe debt is issued with zero equity ($N^S = 0$). The first and second features show, interestingly, that liquidity crises and the issuance of risky debt are sources of inefficiencies. The third feature shows, further, that the planner not only eliminates liquidity crises but does so without issuing equity and, thus, saves the relative issuance costs. This is possible because the planner internalizes the effect of its choice on the price of safe securities Q^S. Indeed, the planner can guarantee the safety of S not only by raising equity N^S but also by increasing the liquidity premium. It turns out that when A_l is sufficiently high, it is optimal to use only the liquidity premium, not equity issuance, to fully back safe securities in state l.

In the planner's solution, as compared with the laissez-faire equilibrium, consumption is lower in state h and, if $\pi < \tau$, higher in state l (more precisely, $C_h = C_l = A_l < 1$). The planner reduces consumption from $C_h = 1$ in the laissez-faire equilibrium to $C_h = A_l < 1$, but can improve welfare thanks to higher consumption in the low state and the saving on equity issuance costs. Indeed, because $C_h = A_l$ and A_l is relatively high, the reduction in consumption in the high state in comparison to the laissez-faire equilibrium is small.

For completeness, let us briefly discuss the solution when A_l is low. In this case, smoothing consumption across the two states would entail excessively low consumption in state h, which would be too costly. The planner's solution here shares three features with laissez-faire: 1) there is a liquidity crisis ($C_h > C_l$); 2) there is issuance of risky debt ($D > 0$); and 3) safe intermediaries raise equity to back debt S ($N^S > 0$). Despite these similarities, even in this case the laissez-faire equilibrium is inefficient. In particular, there is too much risky debt. The planner wants to achieve lower consumption $C_h < 1$ in state h, which requires less risky debt to be financed. Indeed, a marginal reduction in C_h from its laissez-faire equilibrium value ($C_h = 1$) generates a small direct welfare loss and a large indirect gain. The loss is small because unregulated consumption in the high state is at the first-best level $C_h = 1$, so moving away from it is not too costly in welfare terms. The gain arises because of a general equilibrium effect; that is, the lower level of consumption drives the liquidity premium on S up, thereby raising Q^S and allowing safe intermediaries to economize on the equity issuance cost. Issuing some risky debt, D, serves a similar purpose. It allows financing part of the consumption expenditure in state h without incurring the equity issuance cost, because intermediaries that issue D operate with zero equity ($N^D = 0$). The logic of this result is similar to that in Geanakoplos (1997, 2003), in which default is optimal to economize on scarce collateral.

Appendix D shows that under the planner's solution intermediaries make nonnegative rents. This is consistent with our assumption that the planner cannot distribute resources to intermediaries at $t = 1$, which we interpret as a restriction on bailouts.

4.2 Inspecting the pecuniary externality

We now discuss the intuition on why laissez faire is inefficient more extensively. Recall that in the laissez-faire equilibrium intermediaries issuing safe securities need to raise equity. But the higher the borrowing cost (i.e., the lower Q^S), the more equity must be raised. And recall that equity issuance is costly. The externality arises because intermediaries do not internalize the effect of their own choices on Q^S and thus on the equity issuance cost to intermediaries issuing safe securities S.

Laissez-faire entails overissuance of risky debt D up to the point of satiating liquidity demand in state h; hence, consumption C_h is at the first-best level. The high liquidity in state h reduces the liquidity premium on security S and, thus, Q^S (i.e., it increases the borrowing costs of intermediaries that issue S). As a result, safe intermediaries must raise a large amount of equity to avoid default. The social planner wants to reduce risky debt D and consumption in state h in order to lower borrowing costs for safe intermediaries (to increase Q^S). This reduces equity issuance and, thus, the associated social costs.

The inefficiency of the laissez-faire equilibrium is aggravated by the fact that not even intermediaries that issues safe securities internalize the effect of their choices on the price of their securities Q^S. Thus, the planner's solution restraints the supply of safe securities as well. However, the overall effect on the supply of S is ambiguous, because there are two opposite forces. The direct effect would suggest a reduction of S and, thus, a lower consumption in state l as well. The lower borrowing costs due to the reduction of D and C_h explained above, however, lowers the level of equity required to issue safe debt S and thus, reduces the marginal cost of producing liquidity that can be used in state l (i.e., safe liquidity). This second effect suggests an increase in S and in consumption in state l. Under the reasonable parameter restrictions described in the previous section, the planner wants to increase safe liquidity S and consumption C_l.

To support the argument that the planner wants to increase the liquidity premium on safe securities S in order to economize on equity issuance, we compare the price Q^S in the planner's solution with that in the laissez-faire equilibrium. Under our preferred parameterization, which implies $C_h = C_l = A_l$ and $N^S = 0$, we can use (30) and (31) to obtain $Q^S = 1/A_l$ in the planner's solution. We can then easily compare this with the laissez-faire price: equation (23) confirms our conjecture that Q^S is higher in the planner's solution. If instead the planner's solution implies $C_h > C_l$ and $N^S > 0$, the comparison is not straightforward. Nonetheless, the same result holds as shown in Appendix C.

5 Implementing the planner's solution

This section examines the way in which a regulatory authority can implement the social planner's solution, showing this is possible via a system of taxes differentiating between safe and risky intermediaries. We also consider capital requirements that force all intermediaries to issue safe assets and, thus, the facto bans risky liquidity. However, this regulatory tool always reduces welfare, even where the planner's solution calls for a ban on risky debt. The main reason for this is that the planner's solution always requires some tax on safe intermediaries, in addition to the restrictions on the supply of risky liquidity.

5.1 Taxation to implement the planner's solution

A regulator can use a system of taxes and transfers to implement the planner's solution.[8] Taxes should be imposed on intermediaries at $t = 1$, they should be time-varying and procyclical (i.e., high in normal and low in bad times), and the proceeds should be rebated lump-sum to households.[9]

Note that this implementation does not require transferring any resources to intermediaries but rather taxing them. Thus, there is no need for bailouts or for subsidized insurance for debt holders. This is an important result, because our suggested policy can be implemented even if fiscal capacity is limited or if there are institutional, legal, or political constraints that limit the transfers to intermediaries. In addition, because the optimal policy does not require any ex-post transfer of resources, it is also likely to avoid moral hazard that heightens risk-taking, which is common under such alternative policies, such as bailouts and deposit insurance.

Let T^D and T^S denote the optimal tax per unit of debt imposed on risky and safe intermediaries at $t = 1$. The optimal tax on risky intermediaries is

$$T^D = \begin{cases} \frac{1}{C_h} - 1 & \text{in state } h \\ 0 & \text{in state } l, \end{cases} \tag{36}$$

[8] This approach is standard in models with pecuniary externality.

[9] An alternative way to eliminate externalities is to cap quantities. In our model, the cap would be imposed on the amount of debt issued by intermediaries. For a discussion of this approach in the context of fire-sales externality, see Stein (2012).

and that on safe intermediaries is

$$T^S = \begin{cases} (1-\pi)\left(\frac{1}{C_h} - 1\right) + \frac{\tau}{1-\tau}\frac{1}{C_l} & \text{in state } h, \text{ if } N^S > 0 \\ \frac{1}{A_l} - 1 & \text{in state } h, \text{ if } N^S = 0 \\ 0 & \text{in state } l. \end{cases} \tag{37}$$

Appendix D shows that these taxes imply zero rents for intermediaries under the planner's solution, and hence that they effectively implement such a solution.[10]

The optimal taxes T^D and T^S are state-contingent; this can be interpreted, in practice, as the need for time-varying regulation. That is, taxes should be positive only in the high state, nil in the low state. This is important, especially for safe intermediaries. In the low state, intermediaries that issue S use all their resources to repay their debt. If they were taxed in this state, they would have less resources to repay safe securities, and so would be obliged to issue less safe debt in the first place.[11] Therefore, a non-state-contingent tax on intermediaries would reduce liquidity by comparison with the planner's solution and thus would not implement such a solution.

5.2 The unintended consequences of capital requirements

Let us now turn to capital requirements. This policy does not implement the planner's solution, indeed, it even reduces welfare by comparison with the laissez-faire equilibrium.

The social planner's analysis shows that under certain conditions it is optimal to issue only safe liquidity and achieve a constant level of consumption across states, eliminating liquidity crises. Capital requirements are an instrument that can enforce the supply of only safe securities and so also attain constant consumption across states. But they do not replicate the planner's solution. To see this, consider the planner's problem under the parametrization that implies equal consumption across states. In that case, safe intermediaries are taxed in state h, implying that the liquidity premium on safe securities must be high enough to offset the taxes imposed on safe intermediaries. A plain capital requirement, however, just equalizes consumption without taxing safe intermediaries. Hence, the liquidity premium on safe assets is lower. Because of the link between liquidity premia on S and equity (eq. 18),

[10] If the planner's solution calls for a ban on risky liquidity (i.e., $D = 0$ and thus $C_h = C_l$), any tax T^D greater than that in (36) also implements the planner's solution; in this case, such a tax would generate negative rents for intermediaries issuing D.

[11] Formally, a tax on safe intermediaries in the low state would affect constraint (17) and thus the leverage restriction in (31).

intermediaries must raise more equity to back their safe securities, and thus incur in greater equity issuance costs. The allocation under capital requirements not only fails to implement the social planner's solution, but actually reduces welfare with respect to the laissez-faire equilibrium.

We consider a policy under which it is mandatory for all intermediaries to issue risk-free securities. This implies that risky debt is nil ($D = 0$), and all intermediaries issuing safe debt S must satisfy (17). We show below that this constraint is equivalent, in equilibrium, to a capital requirement. In particular, intermediaries' must satisfy a leverage constraint of the form

$$\frac{N^S}{S} \geq \frac{1 - A_l}{A_l}. \tag{38}$$

Note that the leverage ratio N^S/S is the same as in the laissez-faire equilibrium, so this type of capital requirement is akin to a ban on risky liquid securities. The equilibrium that results, because only safe debt is available, entails equal consumption across states, i.e. $C_h = C_l = (1 + \mu)^{-1}$, where μ is the Lagrange multiplier of the liquidity constraints.

The rents of safe security issuers are still given by (22). In equilibrium, free entry implies zero rents ($\mathcal{R}^S = 0$) and thus

$$Q^S = 1 + \tau \left(\frac{1}{A_l} - 1 \right).$$

Note that the price of safe securities is the same as under laissez-faire, but here households' demand implies $Q^S = 1 + \mu$, because consumption is equalized across states. Thus, we obtain the equilibrium level of consumption

$$C_h = C_l = \frac{A_l}{A_l + \tau (1 - A_l)}.$$

With respect to the laissez-faire equilibrium, consumption is lower in state h and higher in state l. Risky debt securities D are not supplied so liquidity is insufficient in the high state driving the liquidity premium up and consumption down. Conversely, the liquidity premium is lower in state l and consumption accordingly higher. Therefore, the overall supply of safe securities S is greater under regulation than under laissez-faire, and is given by

$$S = \frac{A_l}{A_l + \tau (1 - A_l)}. \tag{39}$$

Moreover, in equilibrium, the equity raised by intermediaries is also greater because the supply of safe securities is larger, as given by

$$N^S = \frac{1 - A_l}{A_l + \tau (1 - A_l)}. \tag{40}$$

Combining equations (39) and (40), we verify that the leverage constraint is given by (38).

We can now compare welfare under capital requirements and under the unregulated equilibrium. Welfare under laissez-faire, denoted by W^L, is

$$W^L = -\pi \ln \left[1 + \frac{\tau}{\pi}\left(\frac{1}{A_l} - 1\right)\right] - 1,$$

which follows from evaluating planner's objective function (29), at the laissez-faire equilibrium. In fact, households' welfare (29) already summarizes the resource constraints in the economy. Under capital requirements, instead, welfare is given by

$$W^C = -\ln \left[1 + \tau\left(\frac{1}{A_l} - 1\right)\right] - 1.$$

The welfare difference is:

$$\Delta^W = W^C - W^L = -\ln \left[1 + \tau\left(\frac{1}{A_l} - 1\right)\right] + \pi \ln \left[1 + \frac{\tau}{\pi}\left(\frac{1}{A_l} - 1\right)\right].$$

It is easily shown that welfare is always lower in the regulated equilibrium. First, $\Delta^W|_{\pi\to 0} < 0$ so that welfare under laissez-faire is higher when $\pi \to 0$. In fact, under laissez-faire consumption attains the first-best level in state h, and as $\pi \to 0$, state h is realized with a probability that approaches one. Second, $\Delta^W|_{\pi\to 1} = 0$, and hence welfare will be equivalent in the two equilibria. Third, $\partial\Delta^W/\partial\pi > 0$ for any parametrization. Therefore, $\Delta^W < 0$ for any $\pi \in [0, 1)$, so that capital requirements always reduce welfare with respect to the laissez-faire equilibrium.

Why are capital requirements always welfare-reducing? To clarify this, we distinguish between the case in which the planner's solution analyzed in Section 4.1.2 bans risky liquidity ($D = 0$) or just reduces it ($D > 0$).

If the planner's solution eliminates risky liquidity (set $D = 0$), capital requirements are not optimal because they do not regulate the creation of safe assets. Setting $D = 0$ reduces risky liquidity and incentivizes the issuance of safe liquidity. However, safe liquidity is costly; in particular, the safe intermediaries operating under the capital requirement do not internalize the fact that their liquidity creation lowers the liquidity premium on S, forcing all intermediaries to raise a larger amount of (costly) equity. Indeed, when enforcing $D = 0$, the planner also wants to limit the issuance of safe assets.

If the planner's solution implies some issuance of risky debt (set $D > 0$), capital requirements are not optimal for another reason as well. In the laissez-faire equilibrium, debt D is fully repaid and provides liquidity in state h, so that consumption is higher than in l. Securities D are useful because they allow the economy to economize on costly collateral — that is, on equity — since they are issued by intermediaries with zero equity. The benefit of capital requirements with respect to laissez-faire is an increase of consumption C_l in the low state. However, this requires a greater supply of risk-free debt, which must be appropriately backed by equity. In addition, consumption C_h under capital requirements is lower than in the laissez-faire equilibrium. The two costs of capital requirements — higher equity issuance and lower consumption in state h — offset the benefit of higher consumption in state l; thus the policy is not welfare-improving in this case either.

6 Alternative formulation with monitoring

In the baseline model, the social cost in the balance sheet of safe security issuers is the equity issuance cost. And although this cost is significant in practice, one might wonder about the robustness of our conclusions. This section is intended to show that the results are quite general and hold under a conceptually different mechanism.

We reformulate the model without equity (and without equity costs), but with a costly technology to improve the quality of projects. We take this technology as monitoring or screening, two activities at the core of financial intermediation (Diamond, 1986; Freixas and Rochet, 2008). In principle, we could combine the equity issuance cost with the monitoring cost in a single model, but to simplify the exposition we omit equity.

Monitoring plays the same role as equity: in making sure that the issuers of safe debt have the resources to avoid default in bad states. Indeed, we are able to map the model with monitoring onto that with equity issuance costs, in the sense that the two have the same mathematical structure. More precisely, we identify the mapping between the cost of monitoring and the equity issuance cost so that both the laissez-faire equilibrium and the planner's solution are identical in the two models.

Taken together, the baseline model and the alternative monitoring formulation reveal the key assumption of our analysis. Our results hinge on the fact that producing safe securities is costly — that is more costly than risky securities. Besides equity and monitoring, there could be other costs in practice to issue safe debt – managerial or legal costs, say, or those associated with credible disclosure of the quality of the debt. Adding these considerations is only likely to reinforce our results.

6.1 Model with monitoring

Here, we present the model and the main results, (for more details see Appendix E). The core structure is very similar to that of Section 2. The household sector is the same; the crucial difference is in the technology available to intermediaries. As mentioned, and for simplicity, we posit that intermediaries are financed only with debt (i.e., there is no equity); in principle equity could be included in the model, but this would complicate the exposition.

If an intermediary does not perform any (costly) monitoring, it can invest in technology with productivity A_h in state h and A_l in the state l, as in the baseline model. However, an intermediary that monitors its projects with intensity $M \geq 0$ (where M is chosen at $t = 0$) can reduce the volatility of the payoff, while keeping the average unchanged. Such an intermediary incurs in a cost aM per unit of capital, where $a > 0$, and the productivity of capital thus becomes

$$A_h - \frac{M}{1-\pi}, \quad A_l + \frac{M}{\pi} \tag{41}$$

in the high and low state, respectively. Let us stress again that the possibility of monitoring does not affect the average productivity; indeed, average productivity is

$$(1-\pi)\left(A_h - \frac{M}{1-\pi}\right) + \pi\left(A_l + \frac{M}{\pi}\right) = 1,$$

where the equality uses equation (13).

There is a simple way to map this monitoring model onto the baseline framework with equity issuance costs. If we set the monitoring cost parameter a to

$$a = \frac{\tau}{\pi A_l}, \tag{42}$$

both the laissez-faire equilibrium and the planner's solution of the model with monitoring are identical to those of the baseline model with costly equity. While this might seem surprising, it is in fact a natural result, because equity and monitoring play the same role, i.e. making debt safe. That is, the effects of the two forces can be understood with analogous intuitions; and hence, the mathematical structure that formalizes them is identical.

6.1.1 Financial intermediaries in monitoring model

As in the baseline model, we conjecture (and later verify) that each intermediary offers one of two types of debt security: risk-free debt S and risky debt D. Here, intermediaries that issue D do not monitor their projects (i.e., they choose $M = 0$). We proceed as in Section 2.2, positing that these features hold in equilibrium and then verifying that no deviations are profitable.

For intermediaries that issue D, one can repeat the analysis of Section 2.2, and obtain the same result. In particular, the price Q^D that implies zero rents is still given by (25).

For intermediaries that issue S, the budget constraint is

$$K^S (1 + aM) = Q^S S. \tag{43}$$

That is, the resources procured by issuing debt S at price Q^S are used for investment and to finance monitoring. Appendix E shows that the free-entry condition that drives intermediaries' rents to zero implies

$$Q^S = 1 + aM. \tag{44}$$

As in the baseline model, the price of safe assets is $Q^S > 1$ and thus includes a liquidity premium. In the baseline model, the liquidity premium compensates for the equity issuance cost, whereas here it compensates for the monitoring cost.

The safety of securities S is guaranteed if intermediaries have the resources to avoid default in the low state. In the baseline model, this requirement was formalized by equation (17), which is here replaced by

$$\left(A_l + \frac{M}{\pi}\right) K^S \geq S. \tag{45}$$

Since monitoring is costly, (45) is satisfied with equality in equilibrium.

We can now solve for the amount of monitoring M and the price of safe assets Q^S using the budget constraint (43), the zero-rent condition (44), and the no-default condition (45). We obtain that monitoring is

$$M = \pi (1 - A_l),$$

and the price of safe assets is

$$Q^S = 1 + a\pi (1 - A_l). \tag{46}$$

As noted above, we can recover the price of safe assets of the baseline model if we set

$a = \tau / (\pi A_l)$. Under this restriction, (46) is identical to (23).

Note that the constant a, which parametrizes the monitoring costs, does not affect the level of monitoring per unit of capital M. However, it does affect the price of safe assets Q^S. As a result, a rise in a results in a higher cost of securities S, which in turn reduces the resources that households invest in safe intermediaries, and thus investment in K^S. Even if M per unit of capital is not affected, total monitoring MK^S will decrease.

6.2 Laissez-faire equilibrium with monitoring

The equilibrium is identical to that of the baseline economy. To solve for it, we combine households' demand for debt securities, (9) and (10), with their supply by intermediaries, (25) and (46).

As in the baseline model, the Lagrange multiplier of the liquidity constraint is zero in state h, $\mu_h = 0$, and positive in state l,

$$\mu_l = a (1 - A_l) > 0. \tag{47}$$

We can then solve for C_h and C_l using the households' first-order conditions (7) and (8); this gives $C_h = 1$ and $C_l = 1/ (1 + a (1 - A_l)) < 1$. Finally, the amount of safe and risky debt can be determined using the liquidity constraints (2) and (3), to get:

$$S = \frac{1}{1 + a (1 - A_l)}, \quad D = \frac{a (1 - A_l)}{1 - a (1 - A_l)}.$$

As noted, all these results are identical to those for the laissez-faire equilibrium in the model with costly equity issuance if we set $a = \tau / (\pi A_l)$. Thus, as in the baseline model, the conjecture that intermediaries do not issue debt with default characteristics different from S and D can be verified.

6.3 Planner's problem and externality with monitoring

In the model with monitoring, the planner internalizes the effect of intermediaries' debt issuance on the monitoring cost incurred by safe intermediaries, which plays the same role as equity issuance costs in the baseline model. Thus, the planner here can reduce the supply of liquid securities and allow safe intermediaries (and the society as a whole) to economize on monitoring costs. We discuss here the main result of the planner's problem (for details see Appendix E).

Setting the monitoring cost a to (42), the planner's problem is the same as that in the baseline model with equity issuance cost, and so is the solution. Thus, if A_l is sufficiently large, we again get the planner's solution $C_h = C_l$ and $M = 0$. That is, even in this model with monitoring the planner can successfully eliminate liquidity crisis even if intermediaries do not monitor their projects.

As with the baseline model, we interpret the result of "zero monitoring" as a normalization implied by our simple framework. Adding other frictions might require a positive amount of monitoring to them, but the point is that the role of safe and risky assets as liquidity providers does not give rise to any additional need for monitoring over that required to deal with other financial frictions.

7 Conclusion

Liquidity can be provided not only by risk-free debt securities issued by financial intermediaries but also by risky ones. This observation, crucial to our model, stands in contrast to a large literature that focuses on models in which only safe, risk-free assets are liquid. In the absence of regulation, intermediaries internalize only a fraction of the social costs of issuing liquidity. This result derives from a novel externality that links liquidity premia to the costs of "producing" safe assets.

Optimal financial regulation can consist in a tax on all intermediaries, both safe and risky. Crucially, under reasonable parameter restrictions, the optimal policy is to ban risky liquidity altogether and couple this with a tax on safe intermediaries. Thus, capital requirements that only eliminate risky liquidity but do not tax safe intermediaries are not optimal and, indeed, reduce welfare by comparison with the unregulated laissez-faire equilibrium. Our results are complementary to those of the substantial literature on financial regulation in the context of fire sales.

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Appendix

A Laissez-faire equilibrium: checking deviations

This appendix completes the analysis of the laissez-faire equilibrium by verifying that intermediaries have no profitable deviation. In particular, Section 2.2.1 posited three conjectures that need to be verified: intermediaries issuing S hold enough equity N^S to be solvent, i.e. to satisfy (18); intermediaries issuing D hold zero equity, $(N^D = 0)$; and S and D are the only types of debt security issued in equilibrium. All these conjectures can be verified by showing that intermediaries obtain non-negative rents only if they operate with zero equity (when they issue D) or if they operate with equity such that (18) holds (when they issue S). We now show that operating with any other level of net worth leads to negative rents.

First, consider an intermediary that issues S and holds more equity than what required by (18) evaluated with equality. This deviation does not change either the payoff or the liquidity of the securities issued by such an intermediary (and, therefore, households' willingness to pay), but it does increase total equity issuance costs, implying negative rents. Thus, it is not profitable to hold more equity than is set by (18).

Next, consider an intermediary that issues risky debt and is considering deviating by issuing a strictly positive amount of equity, but not enough to make its debt risk-free. To derive the rents earned by such an intermediary, we must characterize the debt's payoff and liquidity properties. We denote as $\tilde{D}$ the debt of such an intermediary, which has payoff of one in state h and $1 - \tilde{\chi}_l$ in state l. Because the intermediary holds equity $\tilde{N} > 0$, the default rate $\tilde{\chi}_l$ will be different from the default rate χ_l of debt D issued by intermediaries with zero equity. Because $\tilde{D}$ is risky, it does not provide liquidity in state l. Thus, households' demand implies that its price equals the expected payoff, similar to D:

$$Q^{\tilde{D}} = (1 - \pi) + \pi (1 - \tilde{\chi}_l) . \tag{A.1}$$

The expression for rents is similar to (24), but it includes the costs related to equity issuance as in (21):

$$\mathcal{R}^{\tilde{D}} = Q^{\tilde{D}} \tilde{D} - \tilde{D} (1 - \pi \tilde{\chi}_l) - \tau N^{\tilde{D}} . \tag{A.2}$$

Plugging (A.1) into (A.2), we get $\mathcal{R}^{\tilde{D}} = -\tau N^{\tilde{D}} < 0$. That is, an intermediary deviating in this manner has negative rents because the equity issuance cost is not offset by any liquidity premium.

It follows that the only two securities with liquidity value supplied in equilibrium are S, with net worth equal to the lower bound in (18), and D, with zero net worth. A marginal market entrant is indifferent between these two securities.[12]

B Social planner's problem and solution

Here we provide more details about the planner's problem and first-order conditions together with sufficient conditions to characterize the solution.

B.1 Planner's problem

To derive the planner's problem, note that the planner maximizes households' utility, (1), subject to the three sets of constraints described in Section 4.1.

The first set refers to intermediaries: the budget constraints (14) and (15), the limited liability constraints, and the non-negativity constraints S, D, N^S, $N^D \geq 0$.

The second set refers to households: the budget constraints at $t = 0$, (4); the budget constraints at $t = 1$ in the high and low states, (5) and (6); and the liquidity constraints, (2) and (3). In addition, the planner takes into account households' demand schedule for debt and equity securities, which is determined by (7)-(12).

The last constraint is the one that guarantees the safety of security S. This constraint is given by (28).

We can now write the planner's problem in a compact way. After inserting the budget constraints of households and banks at $t = 0$ (i.e., (4), (14), and (15)) and those at $t = 1$ (i.e., (5) and (6)) into the utility function (1), we obtain the planner's objective function

$$\max_{C_h, C_l, N^S, N^D} (1 - \pi) [\log C_h - C_h] + \pi [\log C_l - C_l] - \tau(N^S + N^D)$$

where we have omitted the constant terms that are independent of the choice variables. The remaining constraints are: households' demand for risk-free debt (9) evaluated at (7) and (8), which is given by (30); the lower-bound requirement on equity (18) evaluated at $S = C_l$ using the liquidity constraint (3), which is given by (31); and the non-negativity constraints $N^S \geq 0$, $N^D \geq 0$, and $D \geq 0$. The latter non-negativity constraint can be rewritten $C_h \geq C_l$ using the liquidity constraints

[12]Entrants are also indifferent as regards supplying securities that are partially defaulted on in both states, but these securities do not provide liquidity and their existence does not affect any result.

(2) and (3). The non-negativity constraint $S \geq 0$ can be omitted because it is never binding in equilibrium, given $C_l = S$ from (3) and the fact that households' utility is log in C_l.

We conjecture that the limited liability constraints of intermediaries are not binding in state h and then verify this conjecture after deriving the planner's solution by showing that intermediaries' rents are non-negative (see Appendix D).

B.2 Planner's solution

We can now consider the Lagrangian multiplier associated with the social planner's problem:

$$\begin{aligned}\mathcal{L} &= (1-\pi)\left[\log C_h - C_h\right] + \pi\left[\log C_l - C_l\right] - \tau N^S + \\ &\quad \mu\left[N^S - C_l\left(\frac{1}{(1-\tau)A_l} - \frac{Q^S}{1-\tau}\right)\right] + \lambda N^S + \varphi(C_h - C_l)\end{aligned}$$

in which $\mu, \lambda, \varphi \geq 0$ and Q^S is given by (30). The first-order conditions with respect to C_h and C_l are

$$(1-\pi)\left[\frac{1}{C_h} - 1\right] + \mu\frac{C_l}{1-\tau}\frac{\partial Q^S}{\partial C_h} + \varphi = 0 \tag{B.3}$$

$$\pi\left[\frac{1}{C_l} - 1\right] - \mu\left(\frac{1}{(1-\tau)A_l} - \frac{Q^S}{1-\tau}\right) + \mu\frac{C_l}{1-\tau}\frac{\partial Q^S}{\partial C_l} - \varphi = 0 \tag{B.4}$$

The first-order condition with respect to N^S is

$$\tau = \mu + \lambda$$

and the three Kuhn-Tucker conditions are

$$\begin{aligned}\mu\left[N^S - C_l\left(\frac{1}{(1-\tau)A_l^S} - \frac{Q^S}{1-\tau}\right)\right] &= 0 \\ \lambda N^S &= 0 \\ \varphi(C_h - C_l) &= 0.\end{aligned}$$

Note first that either μ or λ or both should be positive, given that $\tau > 0$. We first show that it cannot be the case that $\mu = 0$ and $\lambda > 0$. By contradiction, suppose that $\mu = 0$ and $\lambda > 0$. Then it is necessarily the case that $\varphi > 0$; otherwise the first-order conditions imply that $C_h = 1$ and $C_l = 1$, which would imply that the lower bound on net worth is positive, contradicting $\mu = 0$. However, $\varphi > 0$ and $\mu = 0$ imply that

$$(1-\pi)\left[\frac{1}{C_h} - 1\right] + \varphi = 0$$

$$\pi \left[\frac{1}{C_l} - 1\right] - \varphi = 0$$

from which it follows that $C_h > C_l$ again contradicting $\varphi > 0$. Therefore $\mu = 0$ and $\lambda > 0$ is not a possible solution. It should necessarily be the case that $\mu > 0$ with $0 < \mu \leq \tau$. Moreover, two cases are possible: $C_h > C_l$ and $C_h = C_l$. In the following Proposition, we discuss the sufficient conditions for each case to arise.

Proposition 1 *(Sufficient conditions to characterize the planner's solution)*

i. If $0 < A_l < (1-2\tau)/(1-\tau)$, then the planner's solution is $C_h > C_l$, $N^S > 0$, and $D, S > 0$, where C_h and C_l are determined by (34) and (35) evaluated at $\mu = \tau$ and $\varphi = 0$.

ii. If τ satisfies

$$0 < \tau \leq \frac{1}{2 + \pi^{-\frac{1}{2}}}$$

and

$$A_l > \max\left\{\frac{1-2\tau}{1-\tau}, \ \frac{1-2\sqrt{\pi(1-\pi)}}{(1-2\pi)^2}\right\} \tag{B.5}$$

then $N^S = 0$ and $C_h = C_l = A_l$, $D = 0$, and $S > 0$.

iii. If τ satisfies

$$\tau > \frac{1}{2 + 2\pi^{-\frac{1}{2}}}$$

and A_l is sufficiently close to one, then $N^S = 0$ and $C_h = C_l = A_l$, $D = 0$, and $S > 0$.

In Section 4.1.1, we use results (ii) and (iii) of the proposition to focus on the case in which A_l is sufficiently high. Next, we provide the proof of Proposition 1.

Proof. We begin by proving part (i) of the proposition. We conjecture that the constraints $N^S \geq 0$ and $C_h \geq C_l$ are not binding and then we verify that $N^S > 0$ and $C_h > C_l$. Under the conjecture, we have $\lambda = 0$, $\mu = \tau$ and $\varphi = 0$. We can rewrite the first-order conditions of the planner's problem (B.3) and (B.4), as

$$\frac{1}{C_h} = 1 + \frac{\tau}{1-\tau}\frac{C_l}{C_h^2}$$

$$\frac{1}{C_l} = 1 + \frac{\tau}{1-\tau}\frac{1}{C_l} + \frac{\tau}{\pi}\left(\frac{1}{(1-\tau)\,A_l} - \frac{Q^S}{1-\tau}\right).$$

We can write the latter condition as

$$\frac{1}{C_l} = 1 + \frac{1}{\pi}\frac{\tau}{1-\tau}\frac{1}{A_l} - \frac{(1-\pi)}{\pi}\frac{\tau}{1-\tau}\frac{1}{C_h}.$$

We define

$$x \equiv \frac{C_l}{C_h},$$

and

$$\beta \equiv 1 + \frac{1}{\pi}\frac{\tau}{1-\tau}\frac{1}{A_l}.$$

We can then write the above first-order conditions as

$$\frac{1}{C_h} = \frac{1}{1 - \frac{\tau}{1-\tau}x} \tag{B.6}$$

and

$$\frac{1}{C_l} = \beta - \frac{(1-\pi)}{\pi}\frac{\tau}{1-\tau}\frac{1}{C_h}$$

from which it follows that

$$\frac{1}{C_l} = \frac{\beta - \frac{\tau}{1-\tau}\beta x - \frac{(1-\pi)}{\pi}\frac{\tau}{1-\tau}}{1 - \frac{\tau}{1-\tau}x} \tag{B.7}$$

Combining (B.6) and (B.7) using the definition of x, we obtain an equation that determines x implicitly:

$$\frac{1}{x} = \beta - \frac{\tau}{1-\tau}\beta x - \frac{(1-\pi)}{\pi}\frac{\tau}{1-\tau}. \tag{B.8}$$

First, (B.8) evaluated at $A_l = (1-2\tau)/(1-\tau)$ implies that $x = 1$ is a solution. In addition, $0 < A_l < (1-2\tau)/(1-\tau)$ implies $\tau < 1/2$, and thus the right-hand side of (B.8) is decreasing in A_l for $x \in (0,1)$. Since the left-hand side of (B.8) is a hyperbole as a function of x, and the right-hand side is a downward-sloping line as a function of x, it follows that, for $A_l < (1-2\tau)/(1-\tau)$, (B.8) has a solution that satisfies $x \in (0,1)$. Since $x = C_l/C_h$, then the solution is $C_h > C_l$, hence the constraint $C_h > C_l$ is not binding, as conjectured.

To conclude the proof of part (i), we need to show that the constraint $N^S \geq 0$ is not binding. This requirement is satisfied if $Q^S A_l \leq 1$. Using (30), (B.6), and (B.7), we can rewrite Q^S as

$$Q^S = \frac{1-\pi}{1-\frac{\tau x}{1-\tau}}\frac{1-2\tau}{1-\tau} + \pi\beta.$$

Thus, we obtain

$$\begin{aligned} Q^S A_l &= \frac{1-\pi}{1-\frac{\tau x}{1-\tau}} \frac{1-2\tau}{1-\tau} A_l + \pi\beta A_l \\ &\leq \frac{1-\pi}{1-\frac{\tau x}{1-\tau}} \left(\frac{1-2\tau}{1-\tau}\right)^2 + \pi\frac{1-2\tau}{1-\tau} + \frac{\tau}{1-\tau} \\ &< (1-\pi)\left(\frac{1-2\tau}{1-\tau}\right) + \pi\frac{1-2\tau}{1-\tau} + \frac{\tau}{1-\tau} = 1 \end{aligned}$$

where the second line uses $A_l < (1-2\tau)(1-\tau)$, and the last inequality uses $x < 1$ and rearranges. Thus, the non-negativity constraint on N^S is not binding, confirming the initial conjecture.

Next, we prove part (ii). We begin by showing that the case $N^S > 0$ and $C_h > C_l$ is not a solution. Under this conjecture, we can study (B.8). Consider the right-hand side of (B.8) and define it as

$$R(A_l, x) = \beta(A_l) - \frac{\tau}{1-\tau}\beta(A_l)x - \frac{(1-\pi)}{\pi}\frac{\tau}{1-\tau}$$

which is a downward-sloping line as a function of x in which we have emphasized the dependence of β on A_l. First, note that as discussed in the proof of part (i), equation (B.8) evaluated at $A_l = (1-2\tau)/(1-\tau)$ has a solution at $x = 1$. Therefore, $R((1-2\tau)/(1-\tau), 1) = 1$. Next, we show that when $A_l = (1-2\tau)/(1-\tau)$, equation (B.8) has no solution in the interval $x \in (0, 1)$. For this to be case, it should be that the moduli of the slope of $R((1-2\tau)/(1-\tau), 1)$ is not greater than 1. This requires that

$$\frac{\tau}{1-\tau} + \frac{1}{\pi}\frac{\tau^2}{1-\tau}\frac{1}{1-2\tau} \leq 1$$

which is verified if $\tau \leq 1/(2+\pi^{-1/2})$. Note further that the intercept of $R(A_l, x)$ decreases with A_l, and that $R(A_l, 1) < 1$ if and only if $A_l > (1-2\tau)/(1-\tau)$. Therefore there is no $x \in (0, 1)$ that solves equation (B.8): hence, either $C_h = C_l$ or $N^S = 0$, or both.

Next, we can show that the case $N^S > 0$ and $C_h = C_l$ is not a solution either. Assume by contradiction that $N^S > 0$, so that $\lambda = 0$, and $C_h = C_l = C$. The first-order conditions (B.3) and (B.4) imply

$$\frac{1}{C} = 1 + \frac{\tau}{1-\tau}\frac{1}{A_l}. \tag{B.9}$$

Similarly to the proof of part (i), it should be the case that $Q^S A_l < 1$, in which

$Q^S = 1/C$ using (30) and $C_h = C_l = C$. However, using equation (B.9), we can write

$$\begin{aligned} Q^S A_l &= A_l + \frac{\tau}{1-\tau} \\ &\geq \frac{1-2\tau}{1-\tau} + \frac{\tau}{1-\tau} = 1, \end{aligned}$$

where the inequality uses the assumption $A_l \geq (1-2\tau)/(1-\tau)$, and the last equality rearranges. This is a contradiction. Thus, so far, we have shown that $N^S > 0$ is not a solution, and thus necessarily $N^S = 0$, which implies $Q^S = 1/A_l$.

Having established that $N^S = 0$, we need to show that $C_h > C_l$ is not a solution. Assume by contradiction that $C_h > C_l$ and thus $\varphi = 0$. The first-order conditions (B.3) and (B.4) become

$$\frac{1}{C_h} - 1 = \frac{\tau - \lambda}{1-\tau} \frac{C_l}{C_h^2} \tag{B.10}$$

$$\frac{1}{C_l} - 1 = \frac{\tau - \lambda}{1-\tau} \frac{1}{C_l} \tag{B.11}$$

where we have used $\mu = \tau - \lambda$, and $N^S = 0$ implies $Q^S = 1/A_l$. In addition, $N^S = 0$ implies $Q^S = 1/A_l$ and, together with (30), we obtain

$$\frac{1-\pi}{C_h} + \frac{\pi}{C_l} = A_l. \tag{B.12}$$

Equations (B.10), (B.11), and (B.12) form a system of three equations in three unknowns: C_h, C_l, and λ, which can be solved analytically. One solution has $C_h = C_l = A_l$ and thus $D = 0$. The other two solutions imply

$$D = C_h - C_l = A_l(1-2\pi) \pm \sqrt{(1-A_l)^2 - 4\pi \left(A_l\right)^2 (1-\pi)}.$$

A necessary condition for a solution with $C_h > C_l$ is that the term inside the square root is positive:

$$(1-A_l)^2 - 4\pi \left(A_l\right)^2 (1-\pi) > 0. \tag{B.13}$$

We argue that (B.13) does not hold if $A_l > \hat{A}$, where $\hat{A}$ solves

$$\left(1-\hat{A}\right)^2 - 4\pi \hat{A}^2 (1-\pi) = 0 \tag{B.14}$$

Indeed, the left-hand side of (B.13) is strictly decreasing in A_l for $A_l \in (0,1)$, is strictly positive as A_l approaches zero, and strictly negative as A_l approaches 1. The value of $\hat{A} \in (0,1)$ that solves (B.14) is given by

$$\hat{A} = \frac{1 - 2\sqrt{\pi(1-\pi)}}{(1-2\pi)^2}. \tag{B.15}$$

By assumption, $A_l > \hat{A}$, and thus $C_h > C_l$ cannot be a solution. As a result, the only possible solution is $N^S = 0$ and $C_h = C_l = A_l$.

Finally, we prove part (iii). First, show that the case $C_h > C_l$ and $N^S > 0$ is not a solution. To do so, rearrange (B.8) to express it as a quadratic equation in x. Assuming $A_l \to 1$, this can be written as

$$\tau \left(1 + \frac{1}{\pi} \frac{\tau}{1-\tau}\right) x^2 - x + (1-\tau) = 0,$$

which has no real solution if the determinant is negative, that is, if

$$1 - 4\tau(1-\tau)\left(1 + \frac{1}{\pi}\frac{\tau}{1-\tau}\right) < 0$$

$$1 - 4\tau(1-\tau) - 4\tau^2 \frac{1}{\pi} < 0.$$

The determinant is equal to zero at $\tau = \frac{\sqrt{\pi}}{2+2\sqrt{\pi}}$ and downward-sloping in τ. Thus, the determinant is negative for all $\tau > \frac{1}{2+2\pi^{-1/2}}$.

We can then establish that $N^S = 0$ as in the proof of part (ii). To rule out the case $N^S = 0$ and $C_h > C_l$, we also proceed as in the proof of part (ii), obtaining the expression in (B.13). The assumption that A_l is sufficiently close to 1 implies that the left-hand side of (B.13) is negative; hence, $N^S = 0$ and $C_h > C_l$ cannot be a solution. Thus, the solution must be $N^S = 0$ and $C_h = C_l = A_l$. ■

C Price of safe debt, Q^S, in planner's solution versus laissez-faire equilibrium

This appendix formalizes the result that Q^S is higher in the social planning solution when such a solution is characterized by $C_h > C_l$ and $N^S > 0$.

Proposition 2 *If $C_h > C_l$ and $N^S > 0$, the price Q^S is higher in the social planning solution than in the laissez-faire equilibrium.*

Proof. We conjecture that Q^S can be represented by the following expression

$$Q^S = (1-\pi) + \pi \left[1 + \frac{\tau}{1-\tau}\frac{1}{\pi}\left(\frac{1}{A_l} - Q^S\right)\right] + \Lambda$$

for some Λ to be defined, where Λ differs depending on whether we are considering the laissez-faire equilibrium or the planner's solution. Rearrange the previous expression to obtain

$$Q^S = 1 - \tau + \frac{\tau}{A_l} + \Lambda\,(1-\tau)$$

and thus $\Lambda = 0$ in the laissez-faire equilibrium. If instead we set

$$\Lambda = \frac{\tau}{1-\tau}\left(\frac{(1-\pi)\,C_l}{C_h^2} + \frac{\pi}{C_l}\right)$$

we recover the price Q^S that arises in the planner's solution. Thus, we can rearrange the price in the planner's solution as:

$$\begin{aligned} Q^S &= 1-\tau+\frac{\tau}{A_l}+\Lambda\,(1-\tau) \\ &= 1-\tau+\frac{\tau}{A_l}+\tau\left(\frac{(1-\pi)\,C_l}{C_h^2}+\frac{\pi}{C_l}\right) \\ &> 1-\tau+\frac{\tau}{A_l^S} \end{aligned}$$

completing the proof, because the last line corresponds to the price Q^S in the laissez-faire equilibrium. ■

D Intermediaries' rents in the planner's solution

Here we verify that intermediaries earn non-negative rents under the planner's solution and that the taxes T^D and T^S described in Section 5.1 drive the rents to zero.

For intermediaries that issue risky debt D, equation (19) implies that rents are given by

$$\begin{aligned} \mathcal{R}^D &= (1-\pi)\left[A_h K^D - D\right] + \pi\left[A_l K^D - D\,(1-\chi_l)\right] \\ &= \left[K^D - D\,(1-\pi\chi_l)\right] \\ &= (1-\pi)\,D\left(\frac{1}{C_h}-1\right) > 0 \end{aligned} \tag{D.16}$$

where the second line uses (13) and the third line uses the intermediaries' budget constraint (15), households' demand for risky debt (10), and the first-order condition for consumption (7). The inequality follows from the fact that in the planner's solution $C_h < 1$. Thus, intermediaries that issues D earn positive rents.

For intermediaries that issue risk-free debt S, we distinguish between two cases: (*i*) $N^S > 0$ and (*ii*) $N^S = 0$. Consider first the case $N^S > 0$; we use (21) to obtain that

$$
\begin{aligned}
\mathcal{R}^S &= Q^S S - S - \tau N^S \\
&= S\left[\left(Q^S - 1\right) - \frac{\tau}{1-\tau}\left(\frac{1}{A_l^S} - Q^S\right)\right] \\
&= S\left[\left(Q^S - 1\right) - \pi\left(\frac{1}{C_l} - 1\right) + \frac{\tau}{1-\tau}\frac{1}{C_l}\right] \\
&= S\left[(1-\pi)\left(\frac{1}{C_h} - 1\right) + \frac{\tau}{1-\tau}\frac{1}{C_l}\right] > 0 \qquad \text{(D.17)}
\end{aligned}
$$

where the second line uses (18), the third line uses the planner's first-order condition (34) evaluated at $\mu = \tau$ (which holds in this case, as shown in Appendix B), and the fourth line uses (30) and rearranges. The inequality uses the fact that the planner's optimal choice of C_h is $C_h < 1$. Next, consider the second case, in which $N^S = 0$. Equation (21), the result $Q^S = 1/A_l$ derived using (30) and (31), and $N^S = 0$ imply

$$\mathcal{R}^S = S\left(\frac{1}{A_l} - 1\right) > 0 \qquad \text{(D.18)}$$

where the inequality follows from the assumption $A_l < 1$.

Finally, we show that the rents earned by intermediaries under taxes T^D and T^S described in Section 5.1 are zero. The rents of risky intermediaries are

$$\mathcal{R}^D = (1-\pi)\left[A_h K^D - D - D T_h^D\right] + \pi\left[A_l K^D - D\left(1-\chi_l\right)\right] \qquad \text{(D.19)}$$

and those of safe intermediaries are

$$\mathcal{R} = (1-\pi)\left[A_h K^S - S - S T_h^S\right] + \pi\left[A_l K^S - S\right], \qquad \text{(D.20)}$$

where T_h^D and T_h^S denote the taxes in the high state, and we have already accounted for the fact that taxes in the low state are zero. Plugging (36) and (37) into (D.19) and (D.20), and rearranging using the same steps described above for (D.16)-(D.18), we obtain that rents are indeed zero: $\mathcal{R}^D = 0$ and $\mathcal{R}^S = 0$.

E Model with monitoring: additional details

In this appendix, we provide additional details on solving the model with monitoring presented in Section 6.

E.1 Rents of S-intermediaries in the laissez-faire equilibrium

The rents earned by an intermediary that issues S are

$$\begin{aligned}\mathcal{R}^S &= (1-\pi)\left[\left(A_h - \frac{M}{1-\pi}\right)K^S - S\right] + \pi\left[\left(A_l + \frac{M}{\pi}\right)K^S - S\right] \\ &= S\left(\frac{Q^S}{1+aM} - 1\right),\end{aligned}$$

where the second line uses (13) and the budget constraint (43). Thus, the price Q^S that guarantees zero rents is $Q^S = 1 + aM$.

E.2 Planner's problem

We show that the planner's problem in the model with monitoring is the same as in the baseline model with equity issuance if we impose the parameter restriction on a stated in (42).

We begin by deriving the planner's problem in the model with monitoring. Following the same steps as in Section 4.1, we have

$$\max_{C_h, C_l, K^S, M} (1-\pi)\left[\log C_h - C_h\right] + \pi\left[\log C_l - C_l\right] - K^S a M \tag{E.21}$$

subject to the budget constraint of S-intermediaries, (43); the non-default condition of S-intermediaries, (45); households' demand for risk-free debt, which is the same as in the baseline model and thus given by (30); and the non-negativity constraints $M \geq 0$ and $C_h \geq C_l$.

To compare the planner's problems in the two models, we proceed in two steps. First, we show that the objective function is the same, and then we show that the non-negativity constraint on monitoring, $M \geq 0$, is equivalent to the non-negativity constraint on equity issuance, $N^S \geq 0$.

To compare the objective functions, we solve for M using (43) and (45):

$$M = \pi A_l \frac{\frac{1}{A_l} - Q^S}{Q^S - a\pi}. \tag{E.22}$$

Then we plug (E.22) into the planner's objective function, and we also use the budget constraint of S-intermediaries, (43), households' demand for risk-free debt, (30), the liquidity constraint (3), and the restriction on a in (42). We obtain:

$$\max_{C_h, C_l} (1-\pi)\left[\log C_h - C_h\right] + \pi\left[\log C_l - C_l\right] - C_l \frac{\tau\left(\frac{1}{A_l} - Q^S\right)}{1-\tau}. \tag{E.23}$$

This is indeed the same objective function as that obtained by plugging constraint (31) into the planner's objective function (29) in the baseline model with equity issuance cost.

As a final step, we show that the constraint $M \geq 0$ in the model with monitoring is equivalent to $N^S \geq 0$ in the baseline model. (E.22) implies that the constraint $M \geq 0$ is equivalent to $Q^S \leq 1/A^S$ if we impose the parameter restriction on a stated by (42). To see this, note that the denominator of (E.22) is positive under (42). Indeed, in the laissez-faire equilibrium Q^S is given by (46), and thus

$$Q^S - a\pi = 1 + a\pi\left(1 - A_l\right) = 1 - \tau > 0, \tag{E.24}$$

where the last equality uses (42). In addition, the planner wants to raise the price of securities S above their laissez-faire equilibrium value, as discussed in Section 4.2; thus, $Q^S - a\pi$ is positive even under the planner's solution. We then note that the condition $Q^S \leq 1/A_l$ is also necessary and sufficient to satisfy the constraint $N^S \geq 0$ in the baseline model (see (31)). Thus, there is a one-to-one mapping between the constraints $M \geq 0$ and $N^S \geq 0$ under the restriction in (42).