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Abstract

A key issue in VAR analysis is how best to identify economic shocks. The paper discusses the problems that the standard methods pose and proposes a new type of shock. Named an idiosyncratic shock, it is designed to identify the component in each VAR residual associated with the corresponding VAR variable. The procedure is applied to a calibrated New Keynesian model and to a VAR based on the same variables and using US data. The resulting impulse response functions are compared with those from standard procedures.

JEL Classification: C32, E32

Keywords: VAR analysis, Macroeconomic Shocks, New Keynesian Model

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Idiosyncratic shocks: a new procedure for identifying shocks in a VAR with application to the New Keynesian model.

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Abstract

A key issue in VAR analysis is how best to identify economic shocks. The paper discusses the problems that the standard methods pose and proposes a new type of shock. Named an idiosyncratic shock, it is designed to identify the component in each VAR residual associated with the corresponding VAR variable. The procedure is applied to a calibrated New Keynesian model and to a VAR based on the same variables and using US data. The resulting impulse response functions are compared with those from standard procedures.

1 Introduction

A key problem in VAR analysis is how best to identify uncorrelated shocks from the correlated residuals and give an economic interpretation to these shocks. Standard identification procedures include a transformation of the residuals that orthogonalises the residual covariance matrix through a canonical factorisation and a Choleski decomposition in which the residuals are assumed to have a recursive structure thereby giving economic significance to the order of the variables, see for example Kilian and Lutkepohl (2017). Both a Choleski decomposition and a canonical factorisation of the residual covariance matrix deliver uncorrelated shocks. All three can be implemented automatically in packages such as EViews.

This approach assumes that each structural economic shock affects only one variable in the model. It is, however, more common in economic models for a structural shock to affect more than one variable. For example, a household's decisions on consumption, labour supply and its portfolio of financial assets (including money) are all likely to be affected by the same structural shocks. This undermines the notion that structural shocks should be constructed so that they are transmitted via a single variable.

In this paper we propose an alternative way to identify shocks from a VAR. This new procedure is based on the notion that although, in general in economic applications, the VAR residuals are correlated and structural shocks affect more than one variable simultaneously, there may be a component of an equation's residual that is specific to that variable and uncorrelated with the residuals in other equations. In other words, we seek to identify a shock that is idiosyncratic to each variable.

The idiosyncratic shocks proposed in this paper are, like the standard procedures, a linear transformation of the residuals. Although each idiosyncratic shock is uncorrelated with the residuals of the other VAR equations, unlike the Choleski and orthogonalised residuals, the idiosyncratic shocks themselves will be correlated. And unlike the Choleski decomposition, the idiosyncratic shocks do not entail any economic assumptions; they simply exploit the empirical characteristics of the residuals. Their usefulness is that they identify the component in a VAR equation's residual that is uncorrelated with other residuals without imposing any economic structure that might be incorrect.

Nonetheless, it is of interest to discover the weight given to each VAR residual in the construction of each idiosyncratic shock. We can go further than this and discover the weight given to the structural shocks in constructing idiosyncratic shocks in calibrated or estimated economic models, including DSGE models. As an illustration, we show how this may be achieved for a New Keynesian model and how these calibrated idiosyncratic shocks compare with those from an estimated VAR in the same variables.

The paper is set out as follows. In Section 2 the econometric theory required to derive the idiosyncratic shocks is explained. The section concludes with an example of the differences between the idiosyncratic and the Choleski shocks. In Section 3 the idiosyncratic shocks for a calibrated New Keynesian model are derived in order to find their economic structure. First, the VAR disturbances are obtained by solving the model; they are then expressed in terms of the structural disturbances. It follows that the idiosyncratic shocks can be constructed from either. In Section 4 a VAR based on the New Keynesian model is estimated on United States data for the period 1957-2017 and the impulse response functions for the idiosyncratic shocks, a Choleski decomposition and the residuals themselves are obtained. The transformation of the VAR residuals required to construct the idiosyncratic shocks is compared with those from the calibrated New Keynesian model. Some concluding remarks are made in Section 5.

2 Econometric theory

2.1 The idiosyncratic shocks

Consider the VAR

$$
\begin{aligned}
x_t &= \Sigma_{s=1}^{p} A_s x_{t-s} + e_t \\
Ee_t &= 0, \quad Ee_t e_t' = \Sigma \\
e_t &= \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix}, \quad \Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix}
\end{aligned}
$$

where x_t has n de-meaned variables and e_t is partitioned so that the disturbance for the first equation is e_{1t} and the disturbances for the remaining $n-1$ equations is e_{2t}[1]. Σ is partitioned conformably. Next write e_{1t} as

$$e_{1t} = \varepsilon_{1t} + B_1 e_{2t}$$

and hence

$$\varepsilon_{1t} = \begin{bmatrix} 1 & -B_1 \end{bmatrix} e_t$$

where ε_{1t} is uncorrelated with e_{2t}. In other words, ε_{1t} is an idiosyncratic shock to x_{1t}. The problem is to determine the row vector B_1.

It follows that

$$Ee_{1t}e_{2t}' = E\varepsilon_{1t}e_{2t}' + B_1 Ee_{2t}e_{2t}'$$

or

$$\Sigma_{12} = B_1 \Sigma_{22}$$

hence

$$B_1 = \Sigma_{12}\Sigma_{22}^{-1}.$$

This can be repeated for each residual e_{it} $(i = 1, ..., n)$ to obtain a row B_i for the ith residual. These rows can be formed into an $n \times n$ matrix C where $C_{ii} = 1$ and the first row of the matrix is $C_1 = \begin{bmatrix} 1 & -B_1 \end{bmatrix}$.[2] It then follows that the n idiosyncratic shocks ε_t satisfy

$$\varepsilon_t = Ce_t.$$

Thus C is the transformation matrix of the VAR residuals that generates the idiosyncratic shocks ε_t just as the lower triangular matrix K is the transformation matrix of the residuals required to generate a Choleski decomposition giving $\xi_t = K^{-1}e_t$, where $KK' = \Sigma$ and $E\xi_t\xi_t' = I$.

[1] This derivation of the idiosyncratic shocks also applies, unchanged, to a cointegrated VAR as the focus is on the residuals which are assumed to be stationary..

[2] From a computational point of view a simple way to derive the transformations associated with the residuals of the other equations is to re-order all of the residuals so that in turn each becomes he first. This can be achieved by using a simple selection matrix. After obtaining the appropriate transformation the selection matrix can be applied again to return to the original variable ordering. An example is provided in Section 3.

The Choleski transformation and the orthonormal decomposiotion produce shocks with a unit variance. In order for the idiosyncratic shocks to have a unit variance, and hence be comparable with the shocks from these two decompositions, C must be multiplied by the inverse diagonal matrix of standard deviations of the shocks. The covariance matrix of idiosyncratic shocks is given by

$$E\varepsilon_t\varepsilon_t' = C\Sigma C' = V.$$

Hence the idiosyncratic transformation matrix then becomes $C^* = D^{-1}C$ where D is a diagonal matrix with diagonal elements $V_{ii}^{\frac{1}{2}}$ $(i = 1, ..., n)$.

The correlation matrix V implies that, in general, the idiosyncratic shocks are correlated. This is because each idiosyncratic shock is a (different) linear function of the same set of residuals. Only if the shocks have a recursive structure as in a Choleski decomposition, and hence C can be ordered to be lower triangular, or if C happens to be the eigenvalues of Σ, would the idiosyncratic shocks be mutually uncorrelated. Both a Choleski decomposition and an orthonormal decomposition of Σ deliver uncorrelated shocks. If the VAR residuals are uncorrelated (Σ is diagonal) then the three measures are identical.

The VAR may now be written

$$x_t = \Sigma_{s=1}^{p} A_s x_{t-s} + C^{*-1}\varepsilon_t$$

The impulse response functions may be obtained from the companion form of the VAR

$$z_t = Az_{t-1} + u_t$$

where

$$z_t = \begin{bmatrix} x_t \\ x_{t-1} \\ \vdots \\ x_{t-p+1} \end{bmatrix}, u_t = \begin{bmatrix} C^{*-1}\varepsilon_t \\ 0 \\ \vdots \\ 0 \end{bmatrix}, A = \begin{bmatrix} A_1 & A_2 & .. & .. & A_p \\ I & 0 & .. & .. & 0 \\ 0 & I & .. & .. & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & .. & I & 0 \end{bmatrix}$$

The one unit impulse response functions are

$$\frac{\partial x_{t+s}}{\partial \varepsilon_t} = HA^s H' C^{*-1}, \qquad H = [\begin{array}{cccc} I & 0 & .. & 0 \end{array}].$$

They are therefore a transformation of the one unit residual impulse response functions $\frac{\partial x_{t+s}}{\partial e_t} = HA^sH'$. For a Choleski decomposition K replaces C^{-1}, and for an orthonormal transformation C^{-1} is replaced by the eigenvectors.

An example

An example will show clearly the differences between the idiosyncratic shocks and a Choleski decomposition. Consider the following simple example where the VAR is of first-order in two variables

$$\begin{aligned} x_t &= Ax_{t-1} + e_t \\ x_t &= [x_{1t}, x_{2t}]',\ e_t = [e_{1t}, e_{2t}]' \\ Ee_t &= 0,\ \Sigma = \begin{bmatrix} \sigma_{1^2} & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix} \end{aligned}$$

The idiosyncratic shocks are $\varepsilon_t = [\varepsilon_{1t}, \varepsilon_{2t}]'$. The residuals may therefore be written as

$$\begin{aligned} e_{1t} &= \varepsilon_{1t} + \beta_1 e_{2t} \\ e_{2t} &= \varepsilon_{2t} + \beta_2 e_{1t} \end{aligned}$$

where by construction $E\varepsilon_{1t}e_{2t} = E\varepsilon_{2t}e_{1t} = 0$. Thus

$$\varepsilon_t = \begin{bmatrix} 1 & -\beta_1 \\ -\beta_2 & 1 \end{bmatrix} e_t$$

The coefficients β_i are

$$\beta_1 = \Sigma_{12}\Sigma_{22}^{-1} = \frac{\rho\sigma_1\sigma_2}{\sigma_2^2} = \frac{\rho\sigma_1}{\sigma_2} \text{ and } \beta_2 = \frac{\rho\sigma_2}{\sigma_1}.$$

Hence, for unit idiosyncratic shocks the transformation matrix is

$$C = \begin{bmatrix} 1 & -\frac{\rho\sigma_1}{\sigma_2} \\ -\frac{\rho\sigma_2}{\sigma_1} & 1 \end{bmatrix}_t$$

This shows that, in general, the coefficients will depend on relative variances of the residuals and on their correlation. The covariance matrix of these idiosyncratic shocks is

$$\begin{aligned} E\varepsilon_t\varepsilon_t' &= \begin{bmatrix} 1 & -\beta_1 \\ -\beta_2 & 1 \end{bmatrix} \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix} \begin{bmatrix} 1 & -\beta_1 \\ -\beta_2 & 1 \end{bmatrix}' \\ &= \begin{bmatrix} 1 & -\frac{\rho\sigma_1}{\sigma_2} \\ -\frac{\rho\sigma_2}{\sigma_1} & 1 \end{bmatrix} \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix} \begin{bmatrix} 1 & -\frac{\rho\sigma_2}{\sigma_1} \\ -\frac{\rho\sigma_1}{\sigma_2} & 1 \end{bmatrix} \\ &= (1-\rho^2) \begin{bmatrix} \sigma_1^2 & -\rho\sigma_1\sigma_2 \\ -\rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix} \end{aligned}$$

It follows that the idiosyncratic shocks are uncorrelated only if the residuals are uncorrelated, in which case the idiosyncratic shocks are just the residuals.

The impulse response functions for the idiosyncratic shocks are a transformation of the one unit residual impulse response functions, namely,

$$\begin{aligned} \frac{\partial x_{t+s}}{\partial \varepsilon_t} &= \frac{\partial x_{t+s}}{\partial e_t} C^{-1} \\ &= \frac{\partial x_{t+s}}{\partial e_t} \begin{bmatrix} 1 & -\frac{\rho\sigma_1}{\sigma_2} \\ -\frac{\rho\sigma_2}{\sigma_1} & 1 \end{bmatrix}^{-1} \\ &= \frac{\partial x_{t+s}}{\partial e_t} \frac{1}{1-\rho^2} \begin{bmatrix} 1 & \frac{\rho\sigma_1}{\sigma_2} \\ \frac{\rho\sigma_2}{\sigma_1} & 1 \end{bmatrix}. \end{aligned}$$

For one standard deviation idiosyncratic shocks the transformation matrix is

$$C^* = \frac{1}{(1-\rho^2)^{\frac{1}{2}}} \begin{bmatrix} \frac{1}{\sigma_1} & -\frac{\rho\sigma_1}{\sigma_2^2} \\ -\frac{\rho\sigma_2}{\sigma_1} & \frac{1}{\sigma_2} \end{bmatrix}$$

and the impulse response functions are

$$\begin{aligned}
\frac{\partial x_{t+s}}{\partial \varepsilon_t} &= \frac{\partial x_{t+s}}{\partial e_t}\sqrt{1-\rho^2}\left[\begin{array}{cc} \frac{1}{\sigma_1} & -\frac{\rho\sigma_1}{\sigma_2^2} \\ -\frac{\rho\sigma_2}{\sigma_1^2} & \frac{1}{\sigma_2} \end{array}\right]^{-1} \\
&= \frac{\partial x_{t+s}}{\partial e_t}\frac{1}{\sqrt{1-\rho^2}}\left[\begin{array}{cc} \sigma_1 & \frac{\rho\sigma_1}{\sigma_2^2} \\ \frac{\rho\sigma_2^2}{\sigma_1^2} & \sigma_2 \end{array}\right].
\end{aligned}$$

For the Choleski decomposition the impulse response functions are $\varepsilon_t = K^{-1}e_t$ where

$$K^{-1} = \left[\begin{array}{cc} \frac{1}{\sigma_1} & 0 \\ -\frac{\rho}{\sigma_1\sqrt{1-\rho^2}} & \frac{1}{\sigma_2\sqrt{1-\rho^2}} \end{array}\right]$$

$$\begin{aligned}
\frac{\partial x_{t+s}}{\partial \varepsilon_t} &= \frac{\partial x_{t+s}}{\partial e_t}K \\
&= \frac{\partial x_{t+s}}{\partial e_t}\left[\begin{array}{cc} \sigma_1 & 0 \\ \rho\sigma_2 & \sigma_2\sqrt{1-\rho^2} \end{array}\right]
\end{aligned}$$

Hence the larger is ρ, the more significant is the order of the variables on the impulse response functions. In contrast, the order of the variables does not affect the idiosyncratic shocks.

3 Theoretical application: the New Keynesian model

To further illustrate the procedure and to provide an economic interpretation of idiosyncratic shocks, we apply it to a three equation New Keynesian model (NKM). First we consider a theoretical NKM and construct the idiosyncratic shocks from a calibrated version of the model. We then consider a numerical version based on the estimation of the VAR associated with a three equation New Keynesian model.

A standard stylised New Keynesian model where monetary policy is based on a Taylor rule is given by

$$\begin{aligned}
\pi_t &= \phi + \beta E_t\pi_{t+1} + \gamma x_t + u_{\pi t} \\
x_t &= E_t x_{t+1} - \alpha(R_t - E_t\pi_{t+1} - \theta) + u_{xt} \\
R_t &= \theta + \pi^* + \mu(\pi_t - \pi^*) + \upsilon x_t + u_{Rt}
\end{aligned}$$

where π is inflation, x is the output gap, r is the Federal Funds rate, $u_{\pi t}$, $u_{\pi t}$ and u_{Rt} are assumed to be independent, zero mean i.i.d. processes and $\phi = (1-\beta)\pi^*$. This choice of ϕ guarantees that the steady-state solution is $\pi_t = \pi^*$, $x_t = 0$ and $R_t = \pi^* + \theta$. Taylor sets $\mu = 1.5$ and $\nu = 0.5$.

It is important to note that the idiosyncratic shocks defined above cannot, in general, be interpreted as estimates of the structural disturbances. Whereas each idiosyncratic shock affects only one structural variable, the structural disturbances may affect more than one structural variable as, being endogenous, they impact on all of the variables on the right-hand side of each equation.

To obtain the solution the model can be written in matrix form as[3]

$$\begin{bmatrix} 1-\beta L^{-1} & -\gamma & 0 \\ -\alpha L^{-1} & 1-L^{-1} & \alpha \\ -\mu & -\nu & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \\ R_t \end{bmatrix} = \begin{bmatrix} \phi \\ \alpha\theta \\ \theta+\pi^*(1-\mu) \end{bmatrix} + \begin{bmatrix} u_{\pi t} \\ u_{xt} \\ u_{Rt} \end{bmatrix}$$

which has the general structure

$$A(L)z_t = \eta + \xi_t$$

The solution is

$$\begin{aligned} \det A(L) z_t &= adj A(L)(\eta+\xi_t) \\ &= \begin{bmatrix} 1+\alpha\nu - L^{-1} & \gamma & -\alpha\gamma \\ -\alpha(\mu - L^{-1}) & 1-\beta L^{-1} & -\alpha(1-\beta L^{-1}) \\ \mu + (\alpha\nu-\mu)L^{-1} & (\nu+\mu\gamma)+\beta\nu L^{-1} & 1+(1+\beta+\alpha\gamma)L^{-1}+\beta L^{-2} \end{bmatrix} (\eta+\xi_t) \end{aligned}$$

The determinant of $A(L)$ is

$$\det A(L) = \{[1+\alpha(\nu+\mu\gamma)]L^2 - [1+\beta(1+\alpha\upsilon)+\alpha\gamma]L+\beta\}L^{-2}.$$

If $\mu > 1$ - the Taylor rule sets $\mu = 1.5$ - monetary policy responds strongly to inflation and hence $\alpha[\nu(1-\beta)+\gamma(\mu-1)] > 0$, implying that both roots of

$$[1+\alpha(\nu+\mu\gamma)]L^2 - [1+\beta(1+\alpha\upsilon)+\alpha\gamma]L+\beta = 0$$

are either greater than or less than unity. As the product of the roots is $1 > \frac{\beta}{1+\alpha(\upsilon+\mu\gamma)} > 0$, they must both be less than unity. We denote them by $0 < \lambda_1$, $\lambda_2 < 1$. The solution is therefore

$$\begin{aligned} z_t &= \det A(L) \begin{bmatrix} 1+\alpha\nu - L^{-1} & \gamma & -\alpha\gamma \\ -\alpha(\mu - L^{-1}) & 1-\beta L^{-1} & -\alpha(1-\beta L^{-1}) \\ \mu + (\alpha\nu-\mu)L^{-1} & (\nu+\mu\gamma)-\beta\nu L^{-1} & 1+(1+\beta+\alpha\gamma)L^{-1}+\beta L^{-2} \end{bmatrix} (\eta+\xi_t) \\ \det A(L) &= [1+\alpha(\upsilon+\mu\gamma)](1-\lambda_1 L^{-1})(1-\lambda_2 L^{-1}) \end{aligned}$$

[3] Further discussion of the NKM and its solution may be found in, for example, Wickens (2012).

implying that

$$
\begin{aligned}
\begin{bmatrix} \pi_t \\ x_t \\ R_t \end{bmatrix} &= \begin{bmatrix} \pi^* \\ 0 \\ \theta + \pi^* \end{bmatrix} + \begin{bmatrix} e_{\pi t} \\ e_{xt} \\ e_{Rt} \end{bmatrix} \\
\begin{bmatrix} e_{\pi t} \\ e_{xt} \\ e_{Rt} \end{bmatrix} &= \frac{1}{1+\alpha(\upsilon+\mu\gamma)} \begin{bmatrix} 1+\alpha\nu & \gamma & -\alpha\gamma \\ -\alpha\mu & 1 & -\alpha \\ \mu & \nu+\mu\gamma & 1 \end{bmatrix} \begin{bmatrix} u_{\pi t} \\ u_{xt} \\ u_{Rt} \end{bmatrix} \\
&= D \begin{bmatrix} u_{\pi t} \\ u_{xt} \\ u_{Rt} \end{bmatrix}
\end{aligned}
$$

The solution can be interpreted as a VAR(0) in which the disturbances $e_t = (e_{\pi t}, e_{xt}, e_{Rt})'$ are linear functions of the model disturbances $u_t = (u_{\pi t}, u_{xt}, u_{Rt})'$ and hence are correlated.

The idiosyncratic shocks defined above, ε_t, are related to the VAR disturbances e_t through

$$
\begin{bmatrix} \varepsilon_{\pi t} \\ \varepsilon_{xt} \\ \varepsilon_{Rt} \end{bmatrix} = C \begin{bmatrix} e_{\pi t} \\ e_{xt} \\ e_{Rt} \end{bmatrix}.
$$

They can also be related to the structural disturbances through

$$
\begin{bmatrix} \varepsilon_{\pi t} \\ \varepsilon_{xt} \\ \varepsilon_{Rt} \end{bmatrix} = CD \begin{bmatrix} u_{\pi t} \\ u_{xt} \\ u_{Rt} \end{bmatrix}.
$$

By calibrating the NKM we can obtain numerical values for the transformation matrices C and CD. The calibration is based on the following parameter values: $\alpha = 0.5$, $\gamma = 0.5$, $\nu = 0.5$, $\mu = 1.5$. In addition we require the variances of the disturbances. These are based approximately on the data used in the numerical example in the next section. Thus we use

$$
V(u) = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 12 & 0 \\ 0 & 0 & 4 \end{bmatrix}
$$

and from this construct $\Sigma = V(e)$.

This calibration gives

$$
\begin{aligned}
\begin{bmatrix} e_{\pi t} \\ e_{xt} \\ e_{Rt} \end{bmatrix} &= \frac{1}{1+\alpha(\upsilon+\mu\gamma)} \begin{bmatrix} 1+\alpha\nu & \gamma & -\alpha\gamma \\ -\alpha\mu & 1 & -\alpha \\ \mu & \nu+\mu\gamma & 1 \end{bmatrix} \begin{bmatrix} u_{\pi t} \\ u_{xt} \\ u_{Rt} \end{bmatrix} \\
&= \frac{1}{1.625} \begin{bmatrix} 1.25 & 0.5 & -0.25 \\ -0.75 & 1 & -0.5 \\ 1.5 & 1.25 & 1 \end{bmatrix} \begin{bmatrix} u_{\pi t} \\ u_{xt} \\ u_{Rt} \end{bmatrix}
\end{aligned}
$$

and

$$V(z) = V(e) = \Sigma = \begin{bmatrix} 4.828 & 0.426 & 6.533 \\ 0.426 & 6.391 & 1.988 \\ 6.533 & 1.988 & 14.485 \end{bmatrix}$$

The idiosyncratic shocks related to the VAR disturbances and to the NKM disturbances are therefore

$$\begin{aligned} \begin{bmatrix} \varepsilon_{\pi t} \\ \varepsilon_{xt} \\ \varepsilon_{Rt} \end{bmatrix} &= \begin{bmatrix} 1 & 0.077 & -0.462 \\ 0.250 & 1 & -0.250 \\ -1.333 & -0.222 & 1 \end{bmatrix} \begin{bmatrix} e_{\pi t} \\ e_{xt} \\ e_{Rt} \end{bmatrix} \\ &= \begin{bmatrix} 0.308 & 0.000 & -0.462 \\ -0.500 & 0.500 & -0.500 \\ 0.000 & 0.222 & 0.889 \end{bmatrix} \begin{bmatrix} u_{\pi t} \\ u_{xt} \\ u_{Rt} \end{bmatrix} \end{aligned}$$

Thus the dominant sources of $\varepsilon_{\pi t}$ are the structural inflation disturbance and the Taylor rule disturbance; the output disturbance does not affect $\varepsilon_{\pi t}$. A positive Taylor rule disturbance - a monetary policy tightening - causes a fall in inflation. The monetary policy shock ε_{Rt} responds to the structural output and monetary disturbances but not to the inflation disturbances. The output shock ε_{xt} falls in response to higher inflation and to monetary policy tightening.

4 Numerical example

Next we consider the VAR associated with this three equation New Keynesian model. This is based on US annual data 1957-2017 for the three variables, π=INFL, x=GAP and r=FFR. The output gap x is measured as the percentage deviation of GDP from its trend as represented by third-order polynomial in time. Unlike an HP filter, this form of trend does not distort the dynamic behaviour of the gap. A VAR with 2 lags has the following estimates

	FFR	GAP	$INFL$
$FFR(-1)$	0.917	−0.673	0.200
	(5.22)	(−3.79)	(2.08)
$FFR(-2)$	−0.235	0.569	−0.246
	(1.38)	(3.31)	(−2.63)
$GAP(-1)$	0.322	1.327	0.145
	(2.38)	(9.72)	(1.96)
$GAP(-2)$	−0.343	−0.495	−0.060
	(−2.55)	(−3.63)	(−0.81)
$INFL(-1)$	0.146	0.137	0.849
	(0.52)	(0.48)	(5.46)
$INFL(-2)$	0.260	−0.084	0.081
	(0.99)	(−0.31)	(0.56)
C	0.247	0.341	0.443
	(0.62)	(0.85)	(2.02)
R^2	0.826	0.783	0.869
SE	1.567	1.583	0.861

The residual covariance matrix is

$$\Sigma = \begin{bmatrix} 2.456 & 1.413 & 0.725 \\ 1.413 & 2.507 & 0.370 \\ 0.725 & 0.370 & 0.741 \end{bmatrix}.$$

The matrix that transforms the residuals to give unit idiosyncratic shocks is

$$C = \begin{bmatrix} 1 & -0.452 & -0.753 \\ -0.602 & 1 & 0.089 \\ -0.311 & 0.027 & 1 \end{bmatrix}$$

and the transformation that gives one standard deviation idiosyncratic shocks is

$$C^{*} = \begin{bmatrix} 0.638 & -0.286 & -0.875 \\ -0.384 & 0.632 & 0.103 \\ -0.198 & 0.017 & 1.161 \end{bmatrix}$$

Hence transformation matrices to the impulse response functions for a unit shock to the residuals are

(i) one standard deviation unit shock to the residuals H

$$H = \begin{bmatrix} 1.567 & 0 & 0 \\ 0 & 1.583 & 0 \\ 0 & 0 & 0.861 \end{bmatrix}$$

(ii) the Choleski decomposition K

$$K = \begin{bmatrix} 1.567 & 0 & 0 \\ 0.902 & 1.302 & 0 \\ 0.463 & -0.036 & 0.725 \end{bmatrix}$$

(iii) for one unit idiosyncratic shock C^{-1}

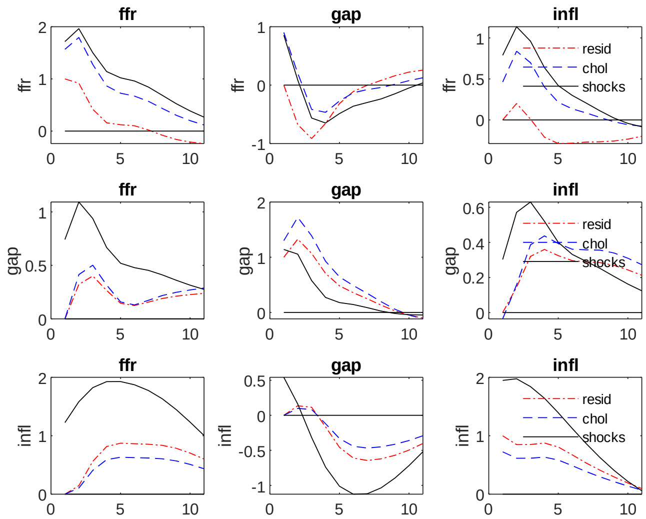
$$C^{-1} = \begin{bmatrix} 1.933 & 0.836 & 1.381 \\ 1.112 & 1.483 & 0.706 \\ 0.571 & 0.219 & 1.410 \end{bmatrix}$$

(iv) one standard deviation idiosyncratic shock C^{*-1}

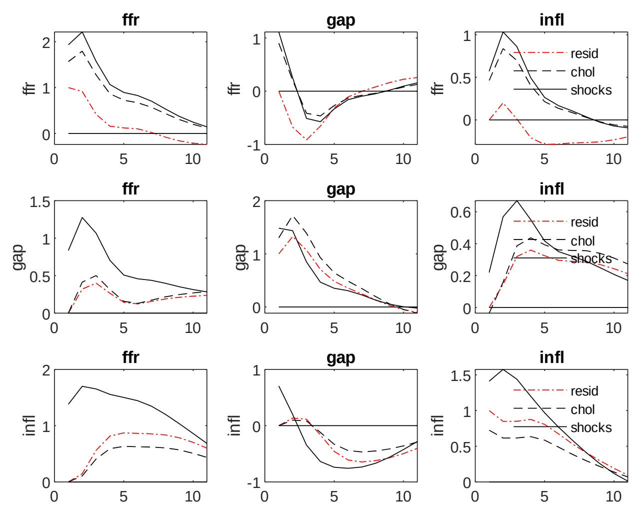
$$C^{*-1} = \begin{bmatrix} 0.787 & -0.355 & -0.593 \\ -0.355 & 0.592 & 0.052 \\ -0.593 & 0.052 & 1.903 \end{bmatrix}$$

The following impulse response functions for a one standard deviation shock give the effect of each idiosyncratic shock, a Choleski shock and the VAR residuals on inflation, the output gap and the Federal Funds rate.

Impulse response functions for the residuals, a Choleski decomposition and the idiosyncratic shocks
(one standard deviation shock, rows are variable response, columns are shocks)



Impulse response functions for the residuals, a Choleski decomposition and the idiosyncratic shocks
(one unit shock, rows are variable response, columns are shocks)



The two sets of impulse response functions are similar but, as is to be expected, the shocks have different effects on the variables. In particular, the responses of inflation and the output gap to a temporary idiosyncratic shock is much larger than for the other two shocks which have no impact initially. Moreover, the effect on both is positive.

We can compare the estimated transformation matrix C with the calibrated matrix in the previous section. We re-order the variables in the estimated VAR to conform to that in the theoretical model, namely, to π, x, r. The calibrated and VAR transformation matrices C are

$$C^{CAL} = \begin{bmatrix} 1 & 0.077 & -0.462 \\ 0.250 & 1 & -0.250 \\ -1.333 & -0.222 & 1 \end{bmatrix} \quad C^{VAR} = \begin{bmatrix} 1 & 0.027 & -0.311 \\ 0.089 & 1 & -0.602 \\ -0.753 & -0.452 & 1 \end{bmatrix}$$

Although the two are different, the differences are not as large as might be expected given that the calibrated model is only a stylised theoretical model and its VAR solution is static.

5 Conclusions

It is well-known that the way shocks are defined in VAR analysis can have a large bearing on the economic interpretation of the resulting impulse response functions. In general, VAR residuals are correlated perhaps reflecting possible common shocks as found in the New Keynesian models above. Nonetheless, it is often assumed that shocks should be structural and uncorrelated. Different ways of defining such shocks have been proposed by imposing identifying restrictions on the VAR residuals. Examples are a Choleski decomposition and the shocks obtained from the eigenvectors of the residual covariance matrix. In this paper a new way of measuring shocks is proposed that does not rely on imposing identifying restrictions. The idea is to isolate from the VAR residuals for each equation a component that is uncorrelated with the residuals in the other equations. Each such shock is idiosyncratic to each endogenous variable in the VAR and is a linear combination of the residuals. Idiosyncratic shocks can also be calculated from the solution to a DSGE model and related to its disturbances. Unlike Choleski shocks, the idiosyncratic shocks are correlated with each other.

We apply the procedure to a calibrated New Keynesian DSGE model and to a VAR based on the same three variables - inflation, the output gap and the Federal Funds rate - and estimated on U.S. data for the period 1957-2017. The transformations of the residuals required to construct the idiosyncratic shocks in each case are similar. They show that monetary policy tightening and a negative disturbance to inflation both cause a negative idiosyncratic shock to inflation. A positive inflation disturbance and monetary tightening both cause a negative idiosyncratic shock to output. The idiosyncratic shock to interest rates is found to respond to the structural output and monetary disturbances but not to the inflation disturbances.

The impulse response functions obtained from the estimated VAR show the effect of the idiosyncratic shocks on the variables themselves. It is found that a positive idiosyncratic shock to the Federal Funds rate produces positive effects on inflation and the output gap. In contrast, a residual shock and a Choleski shock have little or no effect on inflation and the output gap.

We conclude that obtaining idiosyncratic shocks is a useful addition to the standard shocks included in econometric packages.

6 Reference

Lutz Kilian and Helmut Lutkepohl (2017), *Structural Vector Autoregressive Analysis,* Cambridge University Press.

Michael Wickens (2012). *Macroeconomic Theory: a Dynamic General Equilibrium approach* (2nd ed.), Princeton University Press.