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We revisit the causes, welfare consequences, and policy implications of the dispersion in households' labor market outcomes using a model with uninsurable risk, incomplete asset markets, and home production. Allowing households to be heterogeneous in both their disutility of home work and their home production efficiency, we find that home production amplifies welfare-based differences meaning that inequality in standards of living is larger than we thought. We infer significant home production efficiency differences across households because hours working at home do not covary with consumption and wages in the cross section of households. Heterogeneity in home production efficiency is essential for inequality, as home production would not amplify inequality if differences at home only reflected heterogeneity in disutility of work.

JEL Classification: D10, D60, E21, J22

Keywords: Home Production, Labor Supply, Consumption, Inequality

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Inferring Inequality with Home Production*

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We revisit the causes, welfare consequences, and policy implications of the dispersion in households' labor market outcomes using a model with uninsurable risk, incomplete asset markets, and home production. Allowing households to be heterogeneous in both their disutility of home work and their home production efficiency, we find that home production amplifies welfare-based differences meaning that inequality in standards of living is larger than we thought. We infer significant home production efficiency differences across households because hours working at home do not covary with consumption and wages in the cross section of households. Heterogeneity in home production efficiency is essential for inequality, as home production would not amplify inequality if differences at home only reflected heterogeneity in disutility of work.

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1 Introduction

A substantial body of research examines the causes, welfare consequences, and policy implications of the pervasive dispersion across households in their labor market outcomes.[1] The literature trying to understand the dispersion in wages, hours worked, and consumption across households typically abstracts from the possibility that households can produce goods and services outside of the market sector. It is well known, however, that households spend roughly half as much time in home production activities such as child care, shopping, and cooking as in the market.

While it is understood that home production of goods and services introduces, on average, a gap between household consumption recorded in official statistics and standards of living, little is known about how household differences in home production affect inequality in standards of living. A priori there are reasons why home production could change the inferences economists draw from observing dispersion in labor market outcomes. To the extent that households are willing to substitute between market expenditures and time working at home, home production may compress welfare differences originating in the market sector. However, to the extent that differences in the home sector remain uninsurable and are large relative to the market sector, the home sector itself may emerge as a source of welfare differences across households.

We show that incorporating home production in a model with uninsurable risk and incomplete asset markets changes the inferred sources of heterogeneity across households, alters meaningfully the welfare consequences of dispersion, and leads to different policy conclusions. Surprisingly, we infer that inequality across households is larger than what one would infer without incorporating home production as long as differences at home reflect heterogeneity in both the disutility of work and production efficiency.[2] We reach this conclusion because, for households of all ages, the time input in home production does not covary negatively with consumption and wages in the cross section of households. Thus, home production does not offset differences that originate in the market sector but amplifies these differences.

We develop our findings using a general equilibrium model with home production, heterogeneous households facing idiosyncratic risk, and incomplete asset markets. In the spirit of Ghez

[1]See Heathcote, Perri, and Violante (2010) and Attanasio and Pistaferri (2016) for empirical regularities on household heterogeneity in labor market outcomes.

[2]We use the term dispersion to refer to the variation in observed outcomes (such as time allocation, consumption expenditures, and wages) or inferred sources of heterogeneity (such as permanent and transitory productivity and taste shifters). We use the term inequality to refer to the mapping from dispersion to measures which capture welfare differences across households.

and Becker (1975), households produce goods with a technology which uses expenditures and time as inputs. In the home sector, households are heterogeneous with respect to their disutility of work and their production efficiency. Home production is not tradeable and there are no assets households can purchase to directly insure against differences that originate in the home sector. In the market sector, households are also heterogeneous with respect to their disutility of work and their productivity. The structure of asset markets allows households to insure against transitory shocks in their market productivity but not against permanent productivity differences. We retain tractability and prove identification by extending the no-trade result with respect to certain assets for the one-sector model of Heathcote, Storesletten, and Violante (2014) to our model embedding multiple sectors. Therefore, we can characterize the allocations of time and consumption goods in closed form without simultaneously solving for the wealth distribution.

At the core of our approach lies an observational equivalence theorem which allows us to compare our model with home production to a nested model without home production. The observational equivalence theorem states that both models account perfectly for any given cross-sectional data on three observables: consumption expenditures, time spent working in the market, and market productivity (wages). However, the inferred sources of heterogeneity generating these data and inequality will in general differ between the two models. It is essential for our purposes that the two models are observationally equivalent so that any difference between models is exclusively driven by structural factors and not by their ability to account for cross-sectional data on labor market outcomes.

We infer heterogeneity in market productivity and disutility of market work such that the allocations generated by the standard model without home production match the cross-sectional data on the three observables. Then, we infer the sources of heterogeneity such that the allocations generated by the model with home production match the same cross-sectional data and, additionally, time spent on home production. Separating disutility of work from production efficiency at home presents a challenge for home production models because, unlike expenditures and time inputs, the market value of output in the home sector is not observable. Our solution to this identification problem is to pose that some of the cross-sectional differences in time spent working at home are driven by heterogeneity in production efficiency and that the remaining differences are driven by heterogeneity in disutility of work.

To quantify the role of home production for inequality, we use U.S. data between 1995 and

2016 on consumption expenditures, time spent on the market sector, and market productivity from the Consumption Expenditure Survey (CEX). The CEX does not contain information on time spent on home production. To overcome this problem, we use data from the American Time Use Survey (ATUS) to impute individuals' time spent on home production based on observables which are common between the two surveys. For our identification, we allow households to have different work disutility over some time activities such as cooking and cleaning because we find that these activities map closest to occupations which are intensive in manual skills. By contrast, other time activities such as child care and nursing are less intensive in manual skills and, thus, we allow households to have different production efficiencies in them.

The main result is that the U.S. economy is more unequal than we thought taking into account home production. We arrive at this conclusion using four ways to map dispersion in labor market outcomes into welfare-based measures of inequality. First, the standard deviation of equivalent variation across households is roughly 15 percent larger when we incorporate home production. Second, equalizing marginal utilities across households requires transfers with a standard deviation roughly 30 percent higher in the model with home production than in the model without home production. Third, an unborn household is willing to sacrifice 12 percent of lifetime consumption in order to eliminate heterogeneity in an environment with home production, compared to 6 percent in an environment without home production. Finally, taking into account home production, a utilitarian government would favor a more progressive tax system. For example, a household earning 200,000 dollars would face an average tax rate of 19 percent with home production, compared to 12 percent without home production. One way to understand our inequality result is in terms of the distinction between consumption and expenditures emphasized by Aguiar and Hurst (2005). We find that expenditures are less dispersed than the market value of total consumption which, in addition to expenditures, includes the market value of time spent on home production.

Our identification assumption balances two polar views on the origins of time differences at home as it attributes some of the differences to heterogeneity in home production efficiency and some to heterogeneity in the disutility of home work. We demonstrate that there is significant heterogeneity in production efficiency and that heterogeneity in production efficiency rather than in disutility of work is essential in amplifying inequality across households. If there was only heterogeneity in the disutility of work, then there would be no significant inequality gap between the model with and the model without home production. Our inference of home production efficiency comes from an intra-period optimality condition that requires households to consume more in their more efficient sector. This condition implies a log-linear relationship between production efficiency and three observables (market expenditures, time spent on home production, and market productivity). Home production efficiency is significantly dispersed across households as it cumulates the variances of these three observables which are relatively uncorrelated with each other.

The results are robust to a battery of sensitivity checks. First, our conclusions are robust to the estimated values of the elasticity of substitution across sectors, the parameter which governs the Frisch elasticity of labor supply, and the progressivity of the tax system. Second, our results apply separately within subgroups of households defined by their age, marital status, number of children, age of youngest child, the presence of a working spouse, and education levels. Third, our conclusions are robust to measures of expenditures that range from narrow (food) to broad (total spending including durables). Fourth, the inequality differences between the model with and the model without home production are robust to even large amounts of measurement error that may impact the dispersion in observables. Fifth, we examine four alternative datasets in which we do not need to impute home production time because they contain information on both expenditures and time use. We confirm our results in the Panel Study of Income Dynamics (PSID) with food expenditures, in a version of the PSID with expanded consumption categories, in a dataset from Japan, and in a dataset from the Netherlands.

There is an extensive literature which examines how non-separabilities and home production affect consumption and labor supply either over the business cycle (Benhabib, Rogerson, and Wright, 1991; Greenwood and Hercowitz, 1991; McGrattan, Rogerson, and Wright, 1997; Baxter and Jermann, 1999; Aguiar, Hurst, and Karabarbounis, 2013) or over the life cycle (Rios-Rull, 1993; Aguiar and Hurst, 2005, 2007; Dotsey, Li, and Yang, 2014). In these papers, home production provides a smoothing mechanism against differences that originate in the market sector if households are sufficiently willing to substitute expenditures with time. Our conclusions for the role of home production in understanding cross-sectional patterns differ from this literature because we find that time in home production is not negatively correlated with wages and expenditures in the cross section of households. By contrast, an assumption underlying the business cycle and life-cycle literatures is that decreases in the opportunity cost of time and in expenditures are associated with substantial increases in time spent on home production.

Even though the home production literature has emphasized shocks in the home sector in order to generate volatility in labor markets and labor wedges (Benhabib, Rogerson, and Wright, 1991; Greenwood and Hercowitz, 1991; Karabarbounis, 2014), little is known about cross-sectional differences in shocks in the home sector. We develop a methodology to infer heterogeneity in home production efficiency and disutility of work. Our results highlight the importance of these sources of heterogeneity for cross-sectional and life-cycle patterns of expenditures and time allocation.

The literature on incomplete markets has started to incorporate home production and non-separabilities into models. Kaplan (2012) argues that involuntary unemployment and non-separable preferences allow an otherwise standard model with self-insurance to account for the variation of market hours over the life cycle. Blundell, Pistaferri, and Saporta-Eksten (2016) examine consumption inequality in a model in which shocks can also be insured within the family and preferences for hours are non-separable across spouses. Blundell, Pistaferri, and Saporta-Eksten (2018) incorporate child care into a life-cycle partial equilibrium model of consumption and family labor supply. Their paper aims to understand the responsiveness of consumption and time use to transitory and permanent wage shocks and, unlike our paper, it does not quantify the extent to which home production affects inequality.

Another related literature addresses consumption inequality. Earlier work (Deaton and Paxson, 1994; Gourinchas and Parker, 2002; Storesletten, Telmer, and Yaron, 2004; Aguiar and Hurst, 2013) has examined the drivers of life-cycle consumption inequality and their welfare consequences. More recent work focuses on the increase in consumption inequality (Krueger and Perri, 2006; Blundell, Pistaferri, and Preston, 2008; Aguiar and Bils, 2015) and the decline in leisure inequality (Attanasio, Hurst, and Pistaferri, 2015) over time. Our contribution is to introduce home production data into the inequality literature and show that they change the inferences we draw about welfare. Closest to the spirit of our exercise, Jones and Klenow (2016) map differences in consumption levels and dispersion, market hours, and mortality into welfare differences across countries and find that in some cases output per capita does not track welfare closely.

Finally, our paper relates to a strand of literature which uses no-trade theorems to derive analytical solutions for a certain class of models with incomplete markets and heterogeneous agents. Constantinides and Duffie (1996) first derived a no-trade theorem in an endowment economy. Krebs (2003) extends the theorem to an environment with capital, in which households invest a constant share of wealth in physical and human capital and total income in logs follows a random walk. Most relevant for us, Heathcote, Storesletten, and Violante (2014) extend the no-trade theorem by allowing for partial insurance of wage shocks and flexible labor supply. Our contribution is to extend the theorem when households can also direct their time to multiple sectors and face heterogeneity in both their home production efficiency and disutility of work.

2 Model

We present the model and characterize its equilibrium in closed form. We then identify the sources of heterogeneity across households.

2.1 Environment

Demographics. We denote households by ι, calendar years by t, and birth years of households by j. The economy features perpetual youth demographics. Households face a constant probability of survival δ in each period. Each period a cohort of mass $1-\delta$ is born, keeping the population size constant with a mass of one.

Technologies. We denote the good purchased in the market by c_M and the various goods produced at home by c_K, where $K = 1, ..., \mathcal{K}$ indexes home produced goods. All goods are produced with labor. Hours worked in the market are h_M and hours worked in each home sector are h_K.

A household's technology in the market sector is characterized by its (pre-tax) earnings $y_t^j = z_{M,t}^j h_{M,t}^j$, where $z_{M,t}^j$ is market productivity (wage) which varies across households and over time. Aggregate production of market goods is $\int_\iota z_{M,t}^j(\iota) h_{M,t}^j(\iota) \mathrm{d}\Phi_t(\iota)$, where Φ_t is the distribution function of households. Goods and labor markets are perfectly competitive and the wage per efficiency unit of labor is one.

The government taxes labor income to finance (wasteful) public expenditures G_t of the market good. If $y_t^j = z_{M,t}^j h_{M,t}^j$ is pre-tax earnings, then $\tilde{y}_t^j = (1-\tau_0)(z_{M,t}^j)^{1-\tau_1} h_{M,t}^j$ is after-tax earnings, where τ_0 determines the level of taxes and τ_1 governs the progressivity of the tax system. When $\tau_1 = 0$ there is a flat tax rate. A higher τ_1 introduces a larger degree of progressivity into the tax system because it compresses after-tax earnings relative to pre-tax earnings.[3]

[3] Our tax schedule modifies the tax schedule considered, among others, by Guner, Kaygusuz, and Ventura (2014) and Heathcote, Storesletten, and Violante (2014) in that τ_1 is applied to market productivity z_M instead of earnings $z_M h_M$. We adopt the specification of after-tax earnings $\tilde{y} = (1-\tau_0) z_M^{1-\tau_1} h_M$ instead of $\tilde{y} = (1-\tau_0)\left(z_M h_M\right)^{1-\tau_1}$ because we can only prove the no-trade result in the home production model under the former specification. We argue that this modification does not matter for our results because market productivity z_M and hours h_M are relatively uncorrelated in the cross section of households and most of the cross-sectional variation in earnings $z_M h_M$

Households have access to $\mathcal{K}$ technologies in the home sector. Production of home goods is $c^j_{K,t} = z^j_{K,t} h^j_{K,t}$, where $z^j_{K,t}$ is home productivity which varies across households and over time. Home production is not tradeable and not storable, meaning that in every period it must be consumed.

Preferences. Households order sequences of goods and time with expected discounted utility flows $\mathbb{E}_j \sum_{t=j}^{\infty} (\beta\delta)^{t-j}\, U^j_t \left(c^j_t, h^j_{M,t}, h^j_{K,t}\right)$, where β is the discount factor and c^j_t is the consumption aggregator. Flow utility is:

$$U^j_t = \frac{(c^j_t)^{1-\gamma} - 1}{1-\gamma} - \frac{(\exp(B^j_t) h^j_{M,t} + \sum \exp(D^j_{K,t}) h^j_{K,t})^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \,. \tag{1}$$

Parameter $\gamma \geq 0$ governs the curvature of utility with respect to consumption and parameter $\eta > 0$ governs the curvature with respect to total effective hours. Hours are perfect substitutes across sectors. The disutility of work in the market sector is B^j_t and the disutility of work in each home sector is $D^j_{K,t}$. We allow B^j_t and $D^j_{K,t}$ to vary across households and over time.[4]

Consumption is given by a CES aggregator of market and home goods, with an elasticity of substitution between any goods equal to $\phi > 0$:

$$c^j_t = \left((c^j_{M,t})^{\frac{\phi-1}{\phi}} + \sum \omega^j_{K,t} (c^j_{K,t})^{\frac{\phi-1}{\phi}}\right)^{\frac{\phi}{\phi-1}} \,, \tag{2}$$

where $\omega^j_{K,t} \geq 0$ is the consumption weight of home good K relative to market consumption which varies across households and over time.[5] Our specification of preferences and technologies nests the standard model without home production when $\omega^j_{K,t} = 0$ for all K. For $\omega^j_{K,t} > 0$ our multi-sector model is a special case of the Beckerian model of home production in which expenditures and time combine to produce final utility (Becker, 1965; Ghez and Becker, 1975; Gronau, 1986).[6]

is accounted for by z_M. For this reason, our estimate of τ_1 in Section 3.2 is close to the estimates found in Guner, Kaygusuz, and Ventura (2014) and Heathcote, Storesletten, and Violante (2014).

[4]Our model features a single decision maker within each household. We model hours worked across spouses as perfect substitutes and in our quantitative results we define h_M and h_K as the sum of the respective hours worked across spouses. The perfect substitutability of hours (across sectors and spouses) is essential for the no-trade result. We can extend the model for separate disutility of work by spouse.

[5]We normalize to unity an inessential constant multiplying c_M in equation (2). In our quantitative results with $\gamma = 1$, this constant becomes an additive term in utility which does not enter equilibrium allocations and, therefore, cannot be identified from data.

[6]We use the more common formulation of the home production model as in Gronau (1986) in which time spent working in the market and at home generate disutility. As we show in Boerma and Karabarbounis (2020), this version shares many predictions with the Beckerian framework in which expenditures and home time are inputs in the production of goods which enter into utility. Further, the no-trade result can be extended in the Beckerian version of the home production model with imperfect substitutability of hours.

Home Production Efficiencies. In home production models, it is essential to distinguish between consumption weights, ω_K, which transform outputs c_K into utility and what we label production efficiencies, θ_K, which transform inputs h_K into utility. To see where production efficiencies arise in households' problem, we substitute the consumption aggregator (2) and the technologies $c^j_{K,t} = z^j_{K,t} h^j_{K,t}$ into the utility function (1) to obtain the derived utility:

$$V^j_t = \frac{\left[\left((c^j_{M,t})^{\frac{\phi-1}{\phi}} + \sum(\theta^j_{K,t} h^j_{K,t})^{\frac{\phi-1}{\phi}}\right)^{\frac{\phi}{\phi-1}}\right]^{1-\gamma} - 1}{1-\gamma} - \frac{\left(\exp(B^j_t) h^j_{M,t} + \sum \exp(D^j_{K,t}) h^j_{K,t}\right)^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}}, \quad (3)$$

where $\theta^j_{K,t} \equiv (\omega^j_{K,t})^{\frac{\phi}{\phi-1}} z^j_{K,t}$ is the production efficiency of hours in each sector, a convolution of the consumption weight and home productivity. Identifying separately ω_K from z_K is not feasible because home output c_K data are not available in common datasets. However, this lack of identification does not pose a challenge for our analyses. As we show below, all equilibrium allocations depend directly only on θ_K and not on its split between ω_K and z_K. Additionally, since equilibrium allocations and derived utility depend only on θ_K, this split does not affect any of our inequality results. In other words, even if one could separate ω_K from z_K, this split would be uninformative for equilibrium allocations and welfare analyses. For this reason, henceforth we focus our analysis on θ_K rather than ω_K and z_K.

Sources of Heterogeneity. Households are heterogeneous with respect to the work disutilities, B and D_K, and production efficiencies, z_M and θ_K. For B and z_M we impose a random walk structure which is important for obtaining the no-trade result. Under certain parametric restrictions which we discuss below, we are able to obtain the no-trade result with minimal structure on the processes governing heterogeneity in the home sector, θ_K and D_K.

Households' disutility of market work follows a random walk process:

$$B^j_t = B^j_{t-1} + \upsilon^B_t. \quad (4)$$

Households' log market productivity is the sum of a permanent component α and a more transitory component ε:

$$\log z^j_{M,t} = \alpha^j_t + \varepsilon^j_t\ . \quad (5)$$

The permanent component follows a random walk, $\alpha^j_t = \alpha^j_{t-1} + \upsilon^\alpha_t$. The more transitory component, $\varepsilon^j_t = \kappa^j_t + \upsilon^\varepsilon_t$, is the sum of a random walk component, $\kappa^j_t = \kappa^j_{t-1} + \upsilon^\kappa_t$, and an innovation

υ_t^{ε}. Finally, households are heterogeneous with respect to their production efficiency at home $\theta_{K,t}^j$ and disutility of work at home $D_{K,t}^j$. Our identification theorem below is based on cross-sectional data and does not restrict θ_K and D_K to a particular class of stochastic processes. We identify a household ι by their birth date j and a sequence $\{\theta_{K,t}^j, D_{K,t}^j, B_t^j, \alpha_t^j, \kappa_t^j, \upsilon_t^{\varepsilon}\}_{t=j}^{\infty}$.

We use υ to denote innovations and Φ_{υ_t} to denote distributions of innovations. We allow distributions of innovations to vary over time t. We assume that $\theta_{K,t}^j$ and $D_{K,t}^j$ are orthogonal to the innovations $\{\upsilon_t^B, \upsilon_t^{\alpha}, \upsilon_t^{\kappa}, \upsilon_t^{\varepsilon}\}$ and that all innovations are drawn independently from each other. The distribution of initial conditions, $\Phi_j^j(\theta_{K,j}^j, D_{K,j}^j, B_j^j, \alpha_j^j, \kappa_j^j)$, can be non-degenerate across households born in j and can vary with birth year j.

Asset Markets. It is convenient to describe the restrictions on asset markets using the definition of an island in the spirit of Heathcote, Storesletten, and Violante (2014). Islands capture insurance mechanisms available to households for smoothing more transitory shocks in the market sector. Households are partitioned into islands, with each island consisting of a continuum of households who are identical in terms of their production efficiency at home θ_K, disutilities of work D_K and B, permanent component of market productivity α, and the initial condition of κ. More formally, household $\iota = \{\theta_K^j, D_K^j, B^j, \alpha^j, \kappa^j, \upsilon^{\varepsilon}\}$ lives on island ℓ consisting of ι's with common initial state $(\theta_{K,j}^j, D_{K,j}^j, B_j^j, \alpha_j^j, \kappa_j^j)$ and sequences $\{\theta_{K,t}^j, D_{K,t}^j, B_t^j, \alpha_t^j\}_{t=j+1}^{\infty}$.

We now describe the structure of asset markets. First, households cannot trade assets contingent on $\theta_{K,t}^j$ and $D_{K,t}^j$. Second, households can trade one-period bonds $b^{\ell}(s_{t+1}^j)$ which pay one unit of market consumption contingent on $s_t^j \equiv (B_t^j, \alpha_t^j, \kappa_t^j, \upsilon_t^{\varepsilon})$ with households who live on their island ℓ. Third, households can trade economy-wide one-period bonds $x(\zeta_{t+1}^j)$ which pay one unit of market consumption contingent on $\zeta_t^j \equiv (\kappa_t^j, \upsilon_t^{\varepsilon})$ with households who live on either their island or any other island.

To preview the implications of these assumptions, differences in $(\theta_K, D_K, B, \alpha)$ across households remain uninsured by the no-trade result we will discuss below that yields $x(\zeta_{t+1}^j) = 0$ in equilibrium.[7] The more transitory component of productivity $\varepsilon_t^j = \kappa_t^j + \upsilon_t^{\varepsilon}$ becomes fully insurable because households on an island are only heterogeneous with respect to ζ_t^j and can trade state-contingent bonds $b^{\ell}(\zeta_{t+1}^j)$. As a result, the island structure generates partial insurance with respect to market productivity differences. Anticipating these results, henceforth we call α the uninsurable

[7]Households still obtain implicit insurance by substituting time across sectors. A realization of θ_K which leads to low home-produced c_K can be offset by higher purchases in the market c_M if a household desires so. Similarly, households can offset realizations of α by adjusting their time across sectors.

permanent component of market productivity and $\varepsilon = \kappa + \upsilon^{\varepsilon}$ the insurable transitory component of market productivity. We offer some examples of the type of wage shocks accommodated by the framework. Aggregate changes in wages which load differently across households, such as the skill premium, may be more difficult to insure and are captured by α. By contrast, κ may be capturing persistent shocks such as disability and υ^{ε} may be capturing transitory shocks such as unemployment which are easier to insure using asset markets, family transfers, or government transfers.[8]

Household Optimization. Households choose $\{c^j_{M,t}, h^j_{M,t}, h^j_{K,t}, b^{\ell}(s^j_{t+1}), x(\zeta^j_{t+1})\}^{\infty}_{t=j}$ to maximize the expected value of discounted flows of derived utilities in equation (3), subject to sequential budget constraints:

$$c^j_{M,t} + \int_{s^j_{t+1}} q^{\ell}_b(s^j_{t+1}) b^{\ell}(s^j_{t+1}) \mathrm{d}s^j_{t+1} + \int_{\zeta^j_{t+1}} q_x(\zeta^j_{t+1}) x(\zeta^j_{t+1}) \mathrm{d}\zeta^j_{t+1} = \tilde{y}^j_t + b^{\ell}(s^j_t) + x(\zeta^j_t)\ . \tag{6}$$

The expenditure side of the budget constraint consists of market consumption $c^j_{M,t}$, island-level state-contingent bonds $b^{\ell}(s^j_{t+1})$ at prices $q^{\ell}_b(s^j_{t+1})$, and economy-wide state-contingent bonds $x(\zeta^j_{t+1})$ at prices $q_x(\zeta^j_{t+1})$. The income side of the budget constraint consists of after-tax labor income $\tilde{y}^j_t$ and state-contingent bond payouts.

Equilibrium. Given tax parameters (τ_0, τ_1), an equilibrium consists of a sequence of allocations $\{c^j_{M,t}, h^j_{M,t}, h^j_{K,t}, b^{\ell}(s^j_{t+1}), x(\zeta^j_{t+1})\}_{\iota,t}$ and a sequence of prices $\{q^{\ell}_b(s^j_{t+1})\}_{\ell,t}$, $\{q_x(\zeta^j_{t+1})\}_t$ such that: (i) the allocations solve households' problems; (ii) asset markets clear:

$$\int_{\iota\in\ell} b^{\ell}(s^j_{t+1};\iota)\mathrm{d}\Phi_t(\iota) = 0 \quad \forall \ell, s^j_{t+1}, \quad \text{and} \quad \int_{\iota} x(\zeta^j_{t+1};\iota)\mathrm{d}\Phi_t(\iota) = 0 \quad \forall \zeta^j_{t+1}; \tag{7}$$

and (iii) the goods market clears:

$$\int_{\iota} c^j_{M,t}(\iota)\mathrm{d}\Phi_t(\iota) + G_t = \int_{\iota} z^j_{M,t}(\iota) h^j_{M,t}(\iota)\mathrm{d}\Phi_t(\iota), \tag{8}$$

where government expenditures are given by $G_t = \int_{\iota}\left[z^j_{M,t}(\iota) - (1-\tau_0) z^j_{M,t}(\iota)^{1-\tau_1}\right] h^j_{M,t}(\iota)\mathrm{d}\Phi_t(\iota)$.

2.2 Equilibrium Allocations

The model retains tractability because, under certain parametric restrictions, it features a no-trade result which allows us to solve equilibrium allocations in closed form. This section explains the

[8] We refer the reader to Heathcote, Storesletten, and Violante (2008) for a more detailed discussion of how the partial insurance framework relates to frameworks with exogenously imposed incomplete markets or to frameworks in which incompleteness arises endogenously from informational frictions or limited commitment.

logic underlying this result and Appendix A presents the proof. Our proof follows very closely the proof presented in Heathcote, Storesletten, and Violante (2014). We extend their analysis along two dimensions. First, we prove the no-trade result in an environment with multiple sectors and heterogeneity in home production efficiency and disutility of home work. Second, we allow the disutility of market work to follow a random walk instead of being a fixed effect.

We begin by guessing that the equilibrium features no trade across islands, that is $x(\zeta_{t+1}^j;\iota) = 0, \forall \iota, \zeta_{t+1}^j$. Further, we postulate that equilibrium allocations $\{c_{M,t}^j(\iota), h_{M,t}^j(\iota), h_{K,t}^j(\iota)\}$ solve a sequence of static island-level planning problems which maximize average utility within island, $\int_{\zeta_t^j} V_t^j(c_{M,t}^j(\iota), h_{M,t}^j(\iota), h_{K,t}^j(\iota);\iota)\mathrm{d}\Phi_t(\zeta_t^j)$, subject to island-level constraints equating aggregate market consumption to aggregate after-tax earnings $\int_{\zeta_t^j} c_{M,t}^j(\iota)\mathrm{d}\Phi_t(\zeta_t^j) = \int_{\zeta_t^j} \tilde{y}_t^j(\iota)\mathrm{d}\Phi_t(\zeta_t^j)$. We verify our guess by demonstrating that, at the postulated allocations, households solve their optimization problems and goods and asset markets clear.[9]

We obtain the no-trade result in two nested versions of the model. The first nested model sets consumption weights in the home sector to $\omega_K = 0$ as in Heathcote, Storesletten, and Violante (2014). The second nested model sets the curvature of utility with respect to consumption to $\gamma = 1$ for any value of $\omega_K > 0$. The home production model nests the model without home production when $\gamma = 1$, which is the case we consider below in our quantitative analysis.

To understand the no-trade result, we begin with the observation that households on each island ℓ have the same marginal utility of market consumption because they are identical in terms of $(\theta_{K,t}^j, D_{K,t}^j, B_t^j, \alpha_t^j)$ and trade in state-contingent bonds allows them to perfectly insure against $(\kappa_t^j, \upsilon_t^\varepsilon)$. Considering first the model without home production ($\omega_K = 0$), the common marginal utility of market consumption $\mu(\ell)$ at the no-trade equilibrium is:

$$\mu(\ell) = \frac{1}{c_M^\gamma} = \left(\frac{\exp\left((1+\eta)(B - \log(1-\tau_0) - (1-\tau_1)\alpha)\right)}{\int_\zeta \exp\left((1+\eta)(1-\tau_1)(\kappa + \upsilon^\varepsilon)\right)\mathrm{d}\Phi(\zeta)}\right)^{\frac{\gamma}{1+\eta\gamma}}, \tag{9}$$

where for simplicity we drop the time subscript and the cohort superscript from all variables. The no-trade result states that households do not trade state-contingent bonds across islands, $x(\zeta_{t+1}^j) = 0$. Owing to the random walk assumptions on B, α, and κ we see from equation (9) that the growth in marginal utility, μ_{t+1}/μ_t, does not depend on state variables that differentiate islands ℓ. As a result, all households value economy-wide state-contingent bonds $x(\zeta_{t+1}^j)$ identically

[9] The constraint that home production is not tradeable is implicit in the planning problem because the indirect utility $V_t^j(\iota)$ incorporates the constraint that home hours $h_{K,t}^j(\iota)$ produce only own home consumption $c_{K,t}^j(\iota)$.

Table 1: Equilibrium Allocations

Variable		No Home Production: $\omega_K = 0$	Home Production: $\omega_K > 0, \gamma = 1$
1.	c_M	$\frac{\exp\left(\frac{1+\eta}{1+\eta\gamma}(1-\tau_1)\alpha\right)}{\exp\left(\frac{1+\eta}{1+\eta\gamma}B\right)}\mathcal{C}_a^{\frac{1}{1+\eta\gamma}}$	$\frac{1}{R}\frac{\exp((1-\tau_1)\alpha)}{\exp(B)}\mathcal{C}_a^{\frac{1}{1+\eta}}$
2.	h_K		$\frac{\theta_K^{\phi-1}\left(\frac{\exp(B)/\tilde{z}_M}{\exp(D_K)}\right)^{\phi}}{R}\frac{\exp((1-\tau_1)\alpha)}{\exp(B)}\mathcal{C}_a^{\frac{1}{1+\eta}}$
3.	h_M	$\tilde{z}_M^{\eta}\frac{\exp\left(-\eta\gamma\frac{1+\eta}{1+\eta\gamma}(1-\tau_1)\alpha\right)}{\exp\left(\frac{1+\eta}{1+\eta\gamma}B\right)}\mathcal{C}_a^{-\frac{\eta\gamma}{1+\eta\gamma}}$	$\tilde{z}_M^{\eta}\frac{\exp(-\eta(1-\tau_1)\alpha)}{\exp(B)}\mathcal{C}_a^{-\frac{\eta}{1+\eta}} - \sum\frac{\exp(D_K)}{\exp(B)}h_K$

Table 1 presents the equilibrium allocation in the two models. Sources of heterogeneity $(\theta_K, D_K, B, \alpha, \varepsilon)$ and allocations (c_M, h_K, h_M) are household-specific for a household ι in period t who was born in period j. We define market productivity $z_M = \exp(\alpha+\varepsilon)$, after-tax market productivity $\tilde{z}_M = (1-\tau_0)z_M^{1-\tau_1}$, and the rate of transformation $R \equiv 1 + \sum\left(\frac{\theta_K}{\tilde{z}_M}\frac{\exp(B)}{\exp(D_K)}\right)^{\phi-1}$. Parameters γ, η, ϕ, τ_0, and τ_1 are common across all households and periods. The constant $\mathcal{C}_a \equiv \int(1-\tau_0)^{1+\eta}\exp((1+\eta)(1-\tau_1)\varepsilon)\mathrm{d}\Phi_\zeta(\zeta)$ depends on a moment of ε and, thus, is common across households within an island ℓ in any period t.

in equilibrium and there are no mutual benefits from trading $x(\zeta_{t+1}^j)$.

For the home production model with $\gamma = 1$, the marginal utility of market consumption is:

$$\mu(\ell) = \frac{1}{c_M + \tilde{z}_M\sum\frac{\exp(D_K)}{\exp(B)}h_K} = \left(\frac{\exp\left((1+\eta)(B - \log(1-\tau_0) - (1-\tau_1)\alpha)\right)}{\int_\zeta \exp\left((1+\eta)(1-\tau_1)(\kappa+\upsilon^\varepsilon)\right)\mathrm{d}\Phi(\zeta)}\right)^{\frac{1}{1+\eta}}. \tag{10}$$

The marginal utility in equation (10) has the same form as the marginal utility in equation (9) for $\gamma = 1$. Thus, marginal utility growth does not depend on the state variables that differentiate islands and the same logic explains why we obtain the no-trade result in the home production model. For this result, we note the importance of log preferences with respect to the consumption aggregator. Log preferences generate a separability between the marginal utility of market consumption and θ_K and D_K and, thus, the no-trade result holds irrespective of the value of the elasticity of substitution across sectors ϕ and further stochastic properties of θ_K and D_K.

The no-trade result allows us to derive equilibrium allocations using the sequence of planning problems without solving simultaneously for the wealth distribution.[10] We summarize the equilibrium allocations for both models in Table 1. Sources of heterogeneity $(\theta_K, D_K, B, \alpha, \varepsilon)$ and allocations (c_M, h_K, h_M) are household-specific for a household ι in period t who was born in period j. To improve the readability, we drop household ι, time t, and cohort j indices from the table.

[10] The no-trade result applies to the bonds traded across islands $x(\zeta_{t+1}^j) = 0$ and not to the within-islands bonds $b^\ell(s_{t+1}^j)$ which are traded in equilibrium. The bonds $b^\ell(s_{t+1}^j)$ are contingent on $\zeta_t^j \equiv (\kappa_t^j, \upsilon_t^\varepsilon)$ and, thus, solving for the equilibrium allocations amounts to solving a sequence of static planning problems.

Starting with the model without home production, market consumption c_M depends positively on the tax-adjusted uninsurable permanent productivity component $(1-\tau_1)\alpha$ and negatively on the disutility of market work B. By contrast, c_M does not depend on the insurable component of market productivity ε because state-contingent bonds insure against variation in ε. The final row shows that market hours h_M increase in the after-tax market productivity $\tilde{z}_M = (1-\tau_0)z_M^{1-\tau_1}$ with an elasticity η. This reflects the substitution effect on labor supply from variations in after-tax market productivity. Conditional on $\tilde{z}_M$, h_M decreases in $(1-\tau_1)\alpha$ which reflects the income effect from variations in the permanent component of market productivity. When $\gamma = 1$, substitution and income effects from variations in α cancel out and h_M depends positively only on the insurable component ε. Finally, h_M decreases in the disutility of market work B.

To understand the solutions in the home production model, households maximize their utility when the relative marginal rate of substitution between work and consumption across any two sectors, $\frac{\text{MRS}_M}{\text{MRS}_K} = \frac{\exp(B)}{\exp(D_K)} \frac{\omega_K c_M^{1/\phi}}{c_K^{1/\phi}}$, equals the ratio of after-tax productivities, $\frac{\tilde{z}_M}{z_K}$. Rearranging the optimality condition we obtain:

$$\frac{h_K}{c_M} = \theta_K^{\phi-1} \left(\frac{\exp(B)/\tilde{z}_M}{\exp(D_K)} \right)^{\phi}. \tag{11}$$

The solution for c_M in the second column of Table 1 uses equation (11) to substitute out h_K from the marginal utility in equation (10). The solution for c_M has the same form as the solution in the model without home production under $\gamma = 1$ up to the rate of transformation $R \equiv 1 + \sum \left(\frac{\theta_K}{\tilde{z}_M} \frac{\exp(B)}{\exp(D_K)} \right)^{\phi-1}$. This rate describes the incentives of households to shift hours across sectors as a function of relative efficiencies and disutilities of work.

The second row shows that home hours h_K increase in home production efficiency θ_K when $\phi > 1$, in which case the substitution effect from variations in θ_K dominates the income effect. Home hours h_K decrease in disutility of home work D_K for any value of ϕ. To understand the solution for market hours h_M in the home production model, we define effective total hours as $h_T = h_M + \sum \frac{\exp(D_K)}{\exp(B)} h_K$ and then note that the solutions for h_T coincide in the two models under $\gamma = 1$. Total effective hours h_T do not depend on α because under $\gamma = 1$ substitution and income effects from permanent changes in wages cancel out. Like in the model without home production, h_T increases in ε with a Frisch elasticity of $(1-\tau_1)\eta$.

2.3 Identification of Sources of Heterogeneity

We begin by explaining a fundamental identification challenge for home production models and then provide a solution to it. As seen in equation (11), home hours relative to market consumption h_K/c_M depend on both production efficiency θ_K and disutility of work D_K. As a result, data on h_K/c_M are informative only for a combination of θ_K and D_K. More formally, the solutions for the home production model in Table 1 reveal that we have $3+\mathcal{K}$ observed variables in the data (c_M, h_M, z_M, h_K) to inform $3+2\times\mathcal{K}$ sources of heterogeneity (α, ε, B, D_K, θ_K). The gap between observed variables and sources of heterogeneity, equal to $\mathcal{K}$, reflects the fact that both θ_K and D_K can account for h_K. This identification challenge is specific to home production models and does not arise in the standard model without home production. As evidenced in Table 1, in the latter we have 3 observed variables (c_M, h_M, z_M) to inform 3 sources of heterogeneity (α, ε, B). The challenge for home production models arises because in common datasets we observe home inputs h_K but not home outputs c_K.

Our solution to the identification challenge is to impose additional structure on θ_K and D_K. We assume that home production can be disaggregated into two sectors, N and P. In sector N, households are heterogeneous in their *non-market* production efficiency θ_N and their disutility of home work equals that in the market $D_N = B$. In sector P, households are identical in their production efficiency θ_P and heterogeneous in their *preference* for work D_P which may differ from B. These assumptions reduce the number of sources of heterogeneity to 5 (α, ε, B, D_P, θ_N) which, as we show below, can be identified from 5 observed variables (c_M, h_M, z_M, h_N, h_P).

This identification assumption balances two polar views on the origins of household differences in home work time. It allows the model to attribute some of the observed differences to heterogeneity in home production efficiency while other differences to heterogeneity in the disutility of home work. To give some concrete examples from our quantitative results below, we think of time spent on activities such as child care and nursing as belonging to h_N because these activities are relatively less intensive in manual skills and efficiency differences across households are likely to be a significant source of dispersion in hours. We think of time spent on activities such as cooking and cleaning as belonging to h_P because these activities are more intensive in manual skills and differences in disutility of work are plausibly more important than efficiency differences. Because the welfare consequences of home production depend on the origins of heterogeneity, we will also discuss below the two polar cases of all home time, $h_N + h_P$, belonging either to the sector with heterogeneity in production efficiency or to the sector with heterogeneity in disutility of work.

We now demonstrate how to infer the sources of heterogeneity, $\{\alpha, \varepsilon, B, D_P, \theta_N\}_\iota$, such that the models with and without home production both account perfectly for given cross-sectional data on consumption expenditures, hours, and wages.

Observational Equivalence Theorem. Let $\{\bar{c}_M, \bar{h}_M, \bar{z}_M, \bar{h}_N, \bar{h}_P\}_\iota$ be some cross-sectional data. Then, for any given parameters $(\eta, \phi, \tau_0, \tau_1)$:

1. There exists unique $\{\alpha, \varepsilon, B\}_\iota$ such that $\{c_M, h_M, z_M\}_\iota = \{\bar{c}_M, \bar{h}_M, \bar{z}_M\}_\iota$ under $\omega_K = 0$ for any γ.
2. There exists unique $\{\alpha, \varepsilon, B, D_P, \theta_N\}_\iota$ such that $\{c_M, h_M, z_M, h_N, h_P\}_\iota = \{\bar{c}_M, \bar{h}_M, \bar{z}_M, \bar{h}_N, \bar{h}_P\}_\iota$ under $\gamma = 1$ for any $\omega_K > 0$.

The theorem uses the fact that, in each model, the equilibrium allocations presented in Table 1 can be uniquely inverted to obtain, up to a constant, the sources of heterogeneity which generate these allocations. The formal proof is in Appendix A.5.[11]

Table 2 presents the identified sources of heterogeneity which allow the model without home production to generate the cross-sectional data $\{\bar{c}_M, \bar{h}_M, \bar{z}_M\}_\iota$ and the model with home production to generate the cross-sectional data $\{\bar{c}_M, \bar{h}_M, \bar{z}_M, \bar{h}_N, \bar{h}_P\}_\iota$. Henceforth, we drop the bar to indicate variables in the data since, by appropriate choices of the sources of heterogeneity, both models generate perfectly these data.

To understand how observables inform the sources of heterogeneity, in Table 2 we define effective total hours as:

$$h_T \equiv h_M + h_N + \left(\frac{c_M}{\theta_P h_P}\right)^{\frac{1}{\phi}} \frac{\theta_P}{\tilde{z}_M} h_P = h_M + h_N + \frac{\exp(D_P)}{\exp(B)} h_P, \tag{12}$$

and the market value of total consumption as:

$$c_T \equiv c_M + \tilde{z}_M \left(h_N + \left(\frac{c_M}{\theta_P h_P}\right)^{\frac{1}{\phi}} \frac{\theta_P}{\tilde{z}_M} h_P\right) = c_M + \tilde{z}_M \left(h_N + \frac{\exp(D_P)}{\exp(B)} h_P\right). \tag{13}$$

[11] We identify the permanent component of productivity that differs across islands and not whether permanent productivity reflects differences in α or differences in the distribution of κ across islands. Formally, we identify the sum $\alpha + \frac{1}{(1-\tau_1)(1+\eta)} \log \mathcal{C}_s$ in both models, where $\mathcal{C}_s$ is given in Table 2 and depends on the distribution of κ. The distinction between α and the distribution of κ is uninformative for allocations (for example, initial conditions, α_j^j and κ_j^j, affect similarly allocations) and, without loss in generality, we normalize $\mathcal{C}_s$ to the same constant for all islands.

Table 2: Identified Sources of Heterogeneity

No Home Production: $\omega_K = 0$		
1.	α	$\frac{1}{(1-\tau_1)(1+\eta)}\left[\log\left(\frac{c_M}{h_M}\right) + \eta(1-\tau_1)\log z_M - \log \mathcal{C}_s\right]$
2.	ε	$\log z_M - \frac{1}{(1-\tau_1)(1+\eta)}\left[\log\left(\frac{c_M}{h_M}\right) + \eta(1-\tau_1)\log z_M - \log \mathcal{C}_s\right]$
3.	B	$\frac{\eta}{1+\eta}\log(1-\tau_0) + \frac{\eta(1-\tau_1)}{1+\eta}\log z_M - \frac{\eta\gamma}{1+\eta}\log c_M - \frac{1}{1+\eta}\log h_M$
Home Production: $\omega_K > 0, \gamma = 1$		
4.	α	$\frac{1}{(1-\tau_1)(1+\eta)}\left[\log\left(\frac{c_T}{h_T}\right) + \eta\,(1-\tau_1)\log z_M - \log \mathcal{C}_s\right]$
5.	ε	$\log z_M - \frac{1}{(1-\tau_1)(1+\eta)}\left[\log\left(\frac{c_T}{h_T}\right) + \eta\,(1-\tau_1)\log z_M - \log \mathcal{C}_s\right]$
6.	B	$\frac{\eta}{1+\eta}\log(1-\tau_0) + \frac{\eta(1-\tau_1)}{1+\eta}\log z_M - \frac{\eta}{1+\eta}\log c_T - \frac{1}{1+\eta}\log h_T$
7.	D_P	$B + \frac{1}{\phi}\log\left(\frac{c_M}{h_P}\right) + \frac{\phi-1}{\phi}\log\theta_P - \log(1-\tau_0) - (1-\tau_1)\log z_M$
8.	θ_N	$(1-\tau_0)^{\frac{\phi}{\phi-1}} z_M^{\frac{(1-\tau_1)\phi}{\phi-1}} \left(\frac{h_N}{c_M}\right)^{\frac{1}{\phi-1}}$

Table 2 presents the identified sources of heterogeneity $(\alpha, \varepsilon, B, D_P, \theta_N)$ for the economy without home production (upper panel) and for the economy with home production (lower panel). The sources of heterogeneity are a function of data $(c_M, h_M, z_M, h_N, h_P)$ and parameters. We define the market value of total consumption $c_T \equiv c_M + \tilde{z}_M\left(h_N + \left(\frac{c_M}{\theta_P h_P}\right)^{\frac{1}{\phi}}\frac{\theta_P}{\tilde{z}_M}h_P\right)$ and effective total hours $h_T \equiv h_M + h_N + \left(\frac{c_M}{\theta_P h_P}\right)^{\frac{1}{\phi}}\frac{\theta_P}{\tilde{z}_M}h_P$. Parameters γ, η, ϕ, τ_0, τ_1 and the constant $\mathcal{C}_s = \int(1-\tau_0)\exp((1+\eta)(1-\tau_1)\varepsilon)\mathrm{d}\Phi_\zeta(\zeta)$ are the same across households.

The expressions first define total hours and consumption only in terms of observables and parameters. The equality uses the inferred sources of heterogeneity to express total hours and consumption in an intuitive way. Specifically, total hours h_T are the sum of hours in the three sectors, adjusted for disutility differences across sectors. The market value of total consumption c_T is the sum of market consumption, consumption in sector N valued in terms of market goods with the exchange rate $\frac{\tilde{z}_M}{z_N}$, and consumption in sector P valued in terms of market goods with the exchange rate $\frac{\tilde{z}_M}{z_P}\frac{\exp(D_P)}{\exp(B)}$.

Rows 1 to 6 show that, for $\gamma = 1$, the inferred α, ε, and B have the same functional forms between the two models. The difference is that the hours and consumption informative for the sources of heterogeneity in the home production model are h_T in equation (12) and c_T in equation (13), while in the model without home production $h_T = h_M$ and $c_T = c_M$. The inferred α depends positively on the consumption-hours ratio c_T/h_T and market productivity z_M and the inferred ε equals the difference between $\log z_M$ and α. The inferred B depends on the gap between market productivity $\log z_M$ and a combination of consumption $\log c_T$ and hours $\log h_T$.

Table 3: Numerical Example

Household	z_M	c_M	h_M	h_N	h_P	α	ε	B	D_P	θ_N	T
1	20	1,000	60			2.90	0.09	-4.00			0
2	20	600	40			2.85	0.14	-3.54			399
1	20	1,000	60	10	50	2.95	0.04	-4.74	-4.74	6.07	0
2	20	600	40	50	30	2.95	0.04	-4.74	-4.74	29.20	-765

Table 3 presents an example under parameters $\tau_0 = \tau_1 = 0$ and $\gamma = \eta = 1$. The upper panel shows inference of (α, ε, B) for two households based on data (z_M, c_M, h_M) and the model without home production. The lower panel shows inference of $(\alpha, \varepsilon, B, D_P, \theta_N)$ based on data $(z_M, c_M, h_M, h_N, h_P)$ and the model with home production under parameters $\theta_P = 20$ and $\phi = 2.35$. The last column, labeled T, shows the equivalent variation to achieve the utility level of household 1.

The additional sources of heterogeneity in the home production model, D_P and θ_N, are shown in rows 7 and 8 and are inferred by rearranging the optimality conditions (11):

$$\frac{\exp(D_P)}{\exp(B)} = \left(\frac{\theta_P}{\tilde{z}_M}\right)^{\frac{\phi-1}{\phi}} \left(\frac{c_M}{\tilde{z}_M h_P}\right)^{\frac{1}{\phi}}, \quad \text{and} \quad \frac{\theta_N}{\tilde{z}_M} = \left(\frac{\tilde{z}_M h_N}{c_M}\right)^{\frac{1}{\phi-1}}. \tag{14}$$

These expressions show how relative disutilities and efficiencies are inferred from the market value of sectoral consumptions. Holding constant relative efficiencies $\theta_P/\tilde{z}_M$, higher market expenditures relative to the market value of producing at home $c_M/\tilde{z}_M h_P$ leads to higher inferred relative disutility at home $\exp(D_P)/\exp(B)$. When home sectors are substitutes ($\phi > 1$), higher market value of producing at home relative to market expenditures $\tilde{z}_M h_N/c_M$ leads to higher inferred relative efficiency at home $\theta_N/\tilde{z}_M$.

A numerical example in Table 3 provides insights for the mechanisms of the model and draws lessons from the observational equivalence theorem. The economy is populated by two households, there are no taxes, and preference parameters satisfy $\gamma = \eta = 1$. In the upper panel, the economist uses the model without home production to infer the sources of heterogeneity. Household 1 earns a wage $z_M = 20$, spends $c_M = 1,000$, and works $h_M = 60$. Household 2 also earns $z_M = 20$, but spends $c_M = 600$ and works $h_M = 40$. The analytical solutions in Table 2 show that households with a higher expenditures to hours ratio, c_M/h_M, or higher market productivity, z_M, have a higher uninsurable productivity component α. In Table 3 we thus infer that α is higher for household 1 than for household 2 (2.90 versus 2.85). Since both households have the same market productivity and $\alpha + \varepsilon$ add up to (log) market productivity, household 2 has a higher insurable productivity component ε than household 1. Finally, we infer that household 2 has a higher B because it spends less and works less than household 1 despite having the same market productivity.

In the lower panel the economist uses the home production model to infer the sources of heterogeneity. Now the economist also observes that the first household works $h_N = 10$ and $h_P = 50$ hours and the second household works $h_N = 50$ and $h_P = 30$ hours in the two sectors. The inferred α depends on the ratio of the market value of total consumption to total hours, c_T/h_T, rather than on the ratio of market expenditures to market hours, c_M/h_M. Since both households have the same market value of total consumption, $c_T = 2,200$, and the same total hours, $h_T = 120$, α is equal across households. Given equal market productivity, ε is also equal. Given that the two households consume and work the same, B is also equal. Equation (14) shows that D_P is also the same between the two households because they have the same value of home production in sector P relative to market expenditures, $z_M h_P/c_M$. As Table 3 shows, home production efficiency θ_N absorbs all differences in observables. We infer that θ_N is higher for household 2 because it has a higher value of production in sector N relative to market expenditures, $z_M h_N/c_M$, and the sectors are substitutes, $\phi > 1$.

We draw two lessons from this example. First, home production efficiency θ_N is dispersed across households and absorbs dispersion one would attribute to (α, ε, B) in the absence of home production. This result generalizes in our quantitative applications below in which we find that θ_N is significantly more dispersed than z_M and the dispersion in (α, ε, B) is smaller in the home production model.

The second lesson we draw is that a household's welfare ranking depends on whether the data has been generated by a model with or without home production. The last column of Table 3 shows equivalent variations T, equal to the transfers required for households to achieve a given level of utility if they re-optimize their consumption and hours choices. The reference utility level in Table 3 is the utility of household 1 and, thus, T for household 1 is always equal to zero. In the model without home production, T for household 2 equals 399. In the home production model, the two households are identical in terms of their $(\alpha, \varepsilon, B, D_P)$, but household 2 has a higher home production efficiency θ_N. Therefore, the welfare ranking changes and T becomes -765.

2.4 Discussion

Before proceeding to the quantitative results, we pause to make three comments. First, we emphasize the importance of developing an equilibrium model which expresses the arguments (c_M, h_M, h_N, h_P) of the utility function in terms of productivity and preference shifters and policy parameters. An alternative approach, followed by Krueger and Perri (2003) in their study of the welfare effects of increasing inequality in the United States and Jones and Klenow (2016) in their study of welfare and GDP differences across countries, is to plug what are endogenous variables in our framework into the utility function and conduct welfare experiments by essentially varying these variables. While our approach comes with additional complexity, it has the conceptual advantage of taking into account equilibrium responses when conducting welfare analyses with respect to changes in more primitive sources of heterogeneity and policies.

Second, we wish to highlight the merits of the Heathcote, Storesletten, and Violante (2014) framework used in our analysis compared to alternative frameworks. Standard general equilibrium models with uninsurable risk following Huggett (1993) and Aiyagari (1994) feature self-insurance via a risk-free bond. Solutions to these models are obtained computationally. While the present model also allows households to trade a risk-free bond (by setting $x(\zeta_t^j) = 1$ for all states ζ_t^j), the assumptions on asset markets, stochastic processes, and preferences allow us to derive a no-trade result and characterize equilibrium allocations in closed form. Owing to the analytical results, a major advantage of the framework is the transparency and generality of the identification.[12]

Third, our non-parametric approach to identifying the sources of heterogeneity is such that the model accounts perfectly for any given cross-sectional data on market consumption, hours, and wages. Conceptually, our approach is similar to Hsieh and Klenow (2009) who infer wedges in first-order conditions such that firm-level outcomes generated by their model match data analogs. Heathcote, Storesletten, and Violante (2014) also do not impose distributional assumptions on the sources of heterogeneity when estimating their model. A difference with Heathcote, Storesletten, and Violante (2014) is that they select moments in order to estimate parameters using the method of moments. Our approach, instead, does not require restrictions on which moments are more informative for the identification of the sources of heterogeneity.

3 Quantitative Results

We describe the data sources and the parameterization of the model. We then present the inferred sources of heterogeneity.

[12] Despite the wealth distribution not being an object of interest within this framework, a dynamic structure with non-labor income is still essential. In a framework without non-labor income, households would maximize derived utility subject to the budget constraint $c_M = z_M h_M$. Since market consumption to hours c_M/h_M equals market productivity z_M, any choice of (z_M, B, D_P, θ_N) is not sufficient to match data on $(c_M, z_M, h_M, h_N, h_P)$.

3.1 Data Sources

For the baseline analyses we use data from the Consumer Expenditure Survey (CEX) and the American Time Use Survey (ATUS). We consider married and cohabiting households with heads between 25 and 65 years old. The final sample from CEX/ATUS includes 32,993 households between 1995 and 2016. In all our results, we use sample weights provided by the surveys.[13]

Data on expenditures c_M, market productivity z_M, and market hours h_M come from CEX interview surveys collected between 1996 and 2017. Closest to the definition of Aguiar and Hurst (2013), in our baseline analyses c_M is annual non-durable consumption expenditures which include food and beverages, tobacco, personal care, apparel, utilities, household operations (including child care), public transportation, gasoline, reading material, and personal care. Expenditures exclude health and education. We adjust expenditures for household composition and size.

Our measure of income is the amount of wage and salary income before deductions earned over the past 12 months. Individual wages are defined as income divided by hours usually worked in a year, which is the product of weeks worked with usual hours worked per week. We define household market hours h_M as the sum of hours worked by spouses and market productivity z_M as the average of wages of individual members weighted by their market hours.

Data for home hours h_N and h_P come from the ATUS waves between 2003 and 2017. Randomly selected individuals from a group of households who completed their eight and final month interview for the Current Population Survey report their activities on a 24-hour time diary of the previous day. Similar to Aguiar, Hurst, and Karabarbounis (2013), total time spent on home production, $h_N + h_P$, includes housework, cooking, shopping, home and car maintenance, gardening, child care, and care for other household members.

To split total home production time between h_N and h_P, we map disaggregated time uses into occupations and then classify in h_N all the time uses mapped into occupations performing tasks with low manual content and in h_P all time uses mapped into occupations performing tasks with high manual content. The logic underlying our approach is that time activities using the same skills as occupations with high manual content are less likely to display significant heterogeneity

[13] We drop observations for households with a head who is a student, market productivity below 3 dollars per hour in 2010 dollars, working less than 20 hours per week with market productivity above 300 dollars, with expenditures in the top and bottom one percent, and with respondents who indicated working more than 92 hours in the market or at home. In the ATUS we drop respondents during weekends and in the CEX we keep only households who completed all four interviews.

in terms of production efficiency. We use the mapping from time uses to occupations together with Occupational Information Network (O*NET) task measures for various activities described in Acemoglu and Autor (2011) to create an index of manual content for each disaggregated time use.[14] We classify activities in h_N if they have a manual skill index below the median and classify activities in h_P if they have an index above the median.

The CEX does not contain information on time spent on home production. To overcome this difficulty, we impute time use data from the ATUS into the CEX. Our imputation is based on an iterative procedure in which individuals in the CEX are allocated the mean home hours h_N and h_P of matched individuals from the ATUS based on group characteristics. We begin by matching individuals based on work status, race, gender, and age. We then proceed to improve these estimates by adding a host of additional characteristics, such as family status, education, disability status, geography, hours worked, and wages, and matching individuals based on these characteristics whenever possible. We first impute home hours to individuals and, similar to market hours, then sum up these hours at the household level.

Our imputation accounts for approximately two-thirds of the variation in home hours h_N and h_P. In Appendix Table A.1 we confirm that our imputation does not introduce spurious correlations in the merged CEX/ATUS data by showing that the correlation of home hours with market hours and wages conditional on age is of similar magnitude between the ATUS sample of individuals and the merged CEX/ATUS sample of households. In Appendix Tables A.2 and A.3 we show that, conditional on age, married men, women, less educated, and more educated exhibit similar correlations between wages, market hours, and home hours in the ATUS. Further, in Appendix Tables A.4 and A.5 we show that the correlation of total home hours with market expenditures, market hours, and wages conditional on age is of similar magnitude between the CEX/ATUS and two PSID samples of households which do not require imputations since they contain information on home hours, market expenditures, market hours, and wages.

Table 4 presents summary statistics of the time allocation of married households in the CEX/ATUS sample along with the value of the manual skill index of occupations mapped to

[14] Because there are many such indices, we standardize task measures to have mean of zero and standard deviation of one and take the average across all manual tasks to create a single manual skill index. We list the mapping for time use categories displayed in Table 4. Child care is mapped to preschool teachers and child care workers; shopping is mapped to cashiers; nursing is mapped to registered nurses and nursing assistants; cooking is mapped to food preparation and serving workers; cleaning is mapped to maids and housekeeping cleaners; gardening is mapped to landscaping and groundskeeping workers; laundry is mapped to laundry and dry-cleaning workers.

Table 4: Summary Statistics of Time Allocation of Married Households

	Manual Skill Index	Hours per week		
		All Ages	Age 25-44	Age 45-65
Market hours h_M		66.1	66.8	65.5
Total home hours h_N	-0.15	21.3	25.4	17.3
Child care	-0.73	10.8	14.9	6.7
Shopping	0.08	6.4	6.5	6.3
Nursing	-0.12	1.9	1.8	2.0
All other h_N	-0.42	2.3	2.3	2.3
Total home hours h_P	0.69	16.7	16.4	17.0
Cooking	0.41	7.5	7.4	7.5
Cleaning	0.43	3.7	3.7	3.6
Gardening	1.27	2.1	1.7	2.5
Laundry	0.89	2.0	2.1	1.9
All other h_P	1.15	1.5	1.5	1.5

Table 4 presents summary statistics of the time allocation of married households in the merged CEX/ATUS sample. Hours per week are aggregated at the household level. Time allocation is split between market hours h_M, home hours h_N in the sector with production efficiency heterogeneity, and home hours h_P in the sector with disutility of work heterogeneity. Within each sector, subcategories may not add up to total hours due to rounding. The manual skill index for aggregated categories is derived as the employment-weighted manual skill indices of the subcategories that make up the aggregate. Time uses with a higher manual skill index correspond to occupations that perform tasks more intensive in manual skills.

home production activities. Beginning with market hours h_M, we note a small decline over the life cycle. The three largest time uses classified in h_N are child care, shopping, and nursing. These are activities with lower manual content (and typically higher cognitive content) than activities such as cooking, cleaning, gardening, and laundry which we classify in h_P. The allocation of time between the two types of home production is relatively balanced, but there are noticeable differences over the life cycle. As expected, child care time declines significantly in the second half of working life which generates a decline in h_N over the life cycle. By contrast, h_P increases moderately over the life cycle because gardening time increases.[15]

3.2 Parameterization

Table 5 presents parameter values for our baseline analyses. We estimate tax progressivity parameter τ_1 using data from the Annual Social and Economic Supplement of the Current Population

[15] Our life-cycle profiles are consistent with those reported in Cardia and Gomme (2018), who also embrace the view that child care has a different technology from other home production.

Table 5: Parameter Values

Parameter	$\omega_K = 0$	$\omega_K > 0$	Rationale
τ_1	0.12	0.12	$\log\left(\frac{\tilde{y}}{h_M}\right) = \mathcal{C}_\tau + (1-\tau_1)\log z_M$.
τ_0	-0.36	-0.36	Match $G/Y = 0.10$.
γ	1	1	Nesting of models.
η	0.90	0.50	Match $\beta = 0.54$ in $\log h_M = \mathcal{C}_\eta + \beta(\eta)\varepsilon$.
θ_P	—	4.64	$\theta_P = \left(\mathbb{E}\left(\frac{c_M}{\tilde{z}_M^\phi h_P}\right)^{\frac{1}{\phi}}\right)^{\frac{\phi}{1-\phi}}$.
ϕ	—	2.35	$\frac{\Delta_{65-25}\log(c_M/h_N)}{\Delta_{65-25}\log z_M} = \phi(1-\tau_1) = 2.07$.

Table 5 presents parameter values for the models without home production ($\omega_K = 0$) and with home production ($\omega_K > 0$). Parameters τ_1 and τ_0 are the progressivity and level of labor income taxes. Parameter γ governs the curvature of the utility function with respect to consumption and parameter η governs the curvature of the utility function with respect to hours. Parameter θ_P is the production efficiency in the sector with heterogeneity in disutility of work and is constant across households. Parameter ϕ is the elasticity of substitution between consumption goods.

Survey between 2005 and 2015. We use information on pre-tax personal income, tax liabilities at the federal and state level, Social Security payroll deductions, as well as usual hours and weeks worked. Our estimate of τ_1 comes from a regression of log after-tax market productivity on log market productivity before taxes. We estimate $\tau_1 = 0.12$ with a standard error below 0.01.[16] We choose $\tau_0 = -0.36$ to match an average tax rate on labor income equal to 0.10, which equals the average ratio of personal current taxes to income from the national income and product accounts.

For the home production model, we obtained the equilibrium allocations in closed form only under a curvature of the utility function with respect to consumption equal to $\gamma = 1$. We choose $\gamma = 1$ also for the model without home production, so that welfare differences across the two models do not arise from different curvatures of the utility function with respect to consumption.

Next, we estimate the parameter η for the curvature of the utility function with respect to hours. Our strategy is to choose η in each model such that a regression of log market hours $\log h_M$ on the insurable transitory component of market productivity ε yields a coefficient of 0.54. The target value of 0.54 comes from the meta analysis of estimates of the intensive margin Frisch elasticity from micro variation found in Chetty, Guren, Manoli, and Weber (2012). Consistent

[16] Our definitions of income and wage include the child care and earned income tax credits but exclude government transfers such as unemployment benefits, welfare, and food stamps because we think of fully insurable shocks ε as subsuming these transfers. Our estimated tax parameter is close to the estimate of 0.19 in Heathcote, Storesletten, and Violante (2014). Using their tax function $\log\tilde{y} = \text{constant} + (1-\tau_1)\log y$, we estimate $\tau_1 = 0.15$. We, therefore, think it is relatively inconsequential whether we apply the progressivity parameter $(1-\tau_1)$ to after-tax wages or after-tax labor income.

with the logic of Rupert, Rogerson, and Wright (2000) who argue that estimates of the Frisch elasticities are downward biased in the presence of home production, we estimate $\eta = 0.90$ in the model without home production and $\eta = 0.50$ in the model with home production.[17]

We now describe parameters specific to the home production model. To calibrate the constant level of production efficiency θ_P we use the optimality conditions (11) and take means over the population.[18] To estimate the elasticity of substitution ϕ, we again use the optimality conditions (11) to derive the regression:

$$\log\left(\frac{c_M}{h_N}\right) = \phi \log(1-\tau_0) + \phi(1-\tau_1)\log z_M - (\phi - 1)\log\theta_N. \tag{15}$$

Estimation of ϕ using data on c_M/h_N and z_M would lead to biased estimates if z_M and θ_N are correlated. For this reason, we take changes over time in equation (15) and use a synthetic panel approach to estimate ϕ based on changes in c_M/h_N and changes in z_M between the beginning and the end of the life cycle. The identifying assumption is that changes in θ_N are uncorrelated with changes in z_M between the beginning and the end of the life cycle. This assumption is consistent with the assumptions underlying the no-trade result which requires $\theta^j_{N,t+1}$ to be independent of innovations to $z^j_{M,t+1}$. Both our estimation strategy and the no-trade theorem are consistent with a correlation of production efficiency levels across sectors.

We estimate that market and home goods are substitutes with an elasticity $\phi = 2.35$. Our estimate of the elasticity of substitution is consistent with those found in the literature. For example, most estimates of Rupert, Rogerson, and Wright (1995) for couples fall between roughly 2 and 4 and Aguiar and Hurst (2007) obtain estimates of around 2.

3.3 Inferred Sources of Heterogeneity

We extract the sources of heterogeneity using CEX/ATUS data on $(c_M, h_M, z_M, h_N, h_P)$ and our parameter values into the expressions of Table 2 for each household. In Figure 1 we present the

[17] The Frisch elasticity for effective total hours h_T is $(1-\tau_1)\eta$ in both models. There are three reasons why η deviates from the targeted elasticity of 0.54. First, the progressivity of the tax system introduces the wedge $1-\tau_1$ between η and the Frisch elasticity for total hours h_T. Second, disutilities of work and home production efficiency are correlated with market wages. Third, even without such a correlation, the elasticities of market hours h_M differ between the two models because $h_M = h_T$ without home production while with home production h_M is negatively correlated to h_N and h_P. Our strategy is conservative in the sense that the inequality difference between the two models becomes larger when we set η to be equal between the two models.

[18] Rearranging this condition we obtain $\theta_P = \left(\mathbb{E}\left(\frac{c_M}{\tilde{z}_M^{\phi} h_P}\right)^{\frac{1}{\phi}} / \, \mathbb{E}\left(\frac{\exp(D_P)}{\exp(B)}\right)\right)^{\frac{\phi}{1-\phi}}$. The level of θ_P is pinned down only relative to D_P/B. We normalize $\mathbb{E}\left(\frac{\exp(D_P)}{\exp(B)}\right) = 1$ because it standardizes the mean of θ_P in symmetric way to the mean of θ_N.

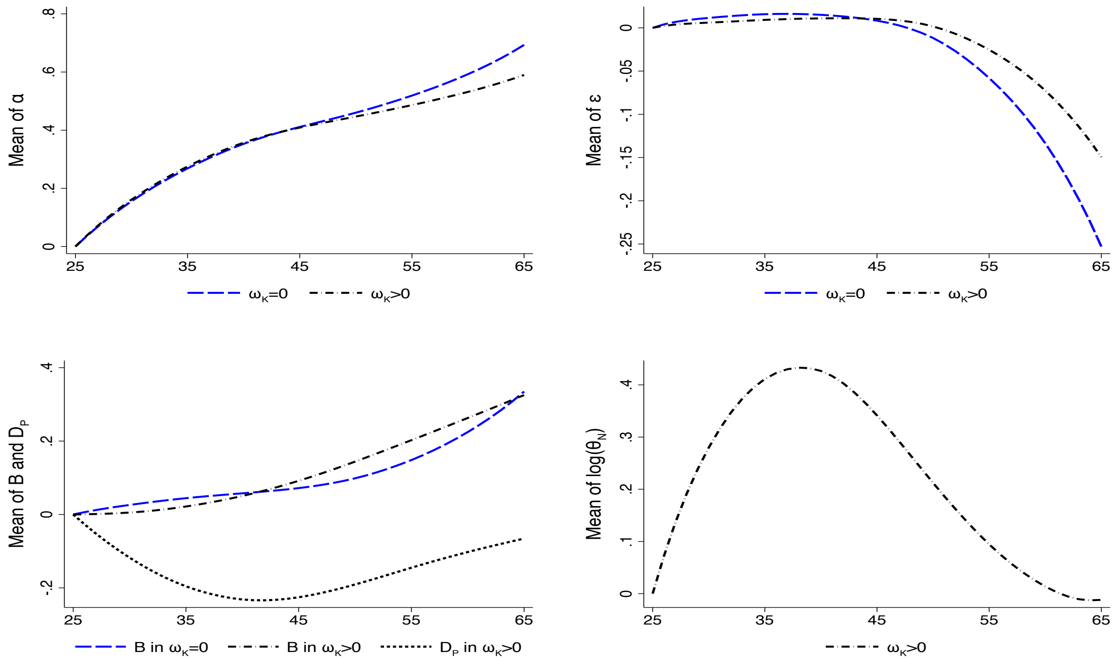


Figure 1: Means of Sources of Heterogeneity

Figure 1 plots the age means of uninsurable component of market productivity α, insurable component of market productivity ε, disutilities of work B and D_P, and home production efficiency $\log\theta_N$ for the economy with ($\omega_K > 0$, black dotted lines) and without home production ($\omega_K = 0$, blue dashed lines).

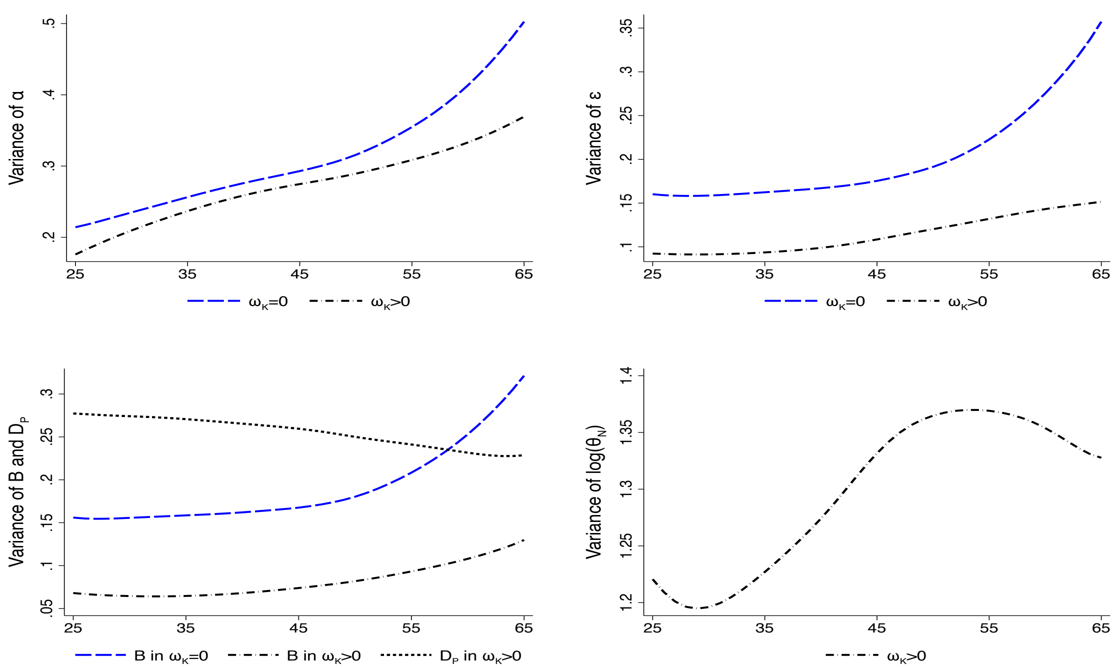


Figure 2: Variances of Sources of Heterogeneity

Figure 2 plots the age variances of uninsurable component of market productivity α, insurable component of market productivity ε, disutilities of work B and D_P, and home production efficiency $\log\theta_N$ for the economy with ($\omega_K > 0$, black dotted lines) and without home production ($\omega_K = 0$, blue dashed lines).

age profiles of the means of $(\alpha, \varepsilon, B, D_P, \log\theta_N)$. To obtain these age profiles, we regress each source of heterogeneity on age dummies, cohort dummies, and normalized year dummies as in Deaton (1997).[19] We plot the coefficients on age dummies which give the mean of each source of heterogeneity by age relative to 25. To reduce noise in the figures, we present the fitted values from locally weighted regressions of the age dummies coefficients on age.

Recall from Table 2 that the insurable component of market productivity α grows over the life cycle when either the ratio of consumption to hours c_T/h_T grows or when wages z_M grow. The insurable component ε falls when the increase in c_T/h_T is large relative to the increase in z_M. The upper panels of Figure 1 show that the means of α and ε grow similarly until roughly 45 between the two models and diverge after that. The slower growth of α and the smaller decline in ε in the model with home production reflect the significant decline in home hours h_N in the later part of the life cycle which implies that c_T/h_T grows by less than c_M/h_M. In the lower panels we see that both models generate a relatively similar increase in the disutility of market work B, an increase reflecting the faster growth of z_M relative to c_T and h_T over the life cycle.[20]

The home production model generates a U-shaped profile of home disutility D_P which contrasts with the increasing profile of B. To understand this difference, recall from equation (14) that $\frac{\exp(D_P)}{\exp(B)} \propto \left(\frac{c_M}{h_P}\right)^{1/\phi} \frac{1}{\tilde{z}_M}$. To rationalize the faster growth of z_M relative to c_M/h_P during the earlier stages of the life cycle, the model requires a decline in D_P relative to B. As z_M and c_M/h_P comove more closely during the later stages of the life cycle, the profile of D_P slopes upward like the profile of B.

The model with home production generates a hump-shaped profile of home efficiency θ_N. To understand this pattern, recall from equation (14) that $\theta_N = \tilde{z}_M^{\frac{\phi}{\phi-1}} \left(\frac{h_N}{c_M}\right)^{\frac{1}{\phi-1}}$. Until roughly 40, θ_N tracks market productivity z_M since $\phi > 1$. Despite z_M still rising, θ_N starts to decline after 40 and returns to its initial value by 65. This pattern is generated by the strong decline in hours h_N

[19]Results are similar when we extract the age effect in regressions which either control only for cohort dummies or only for year dummies.

[20]The flexibility in terms of initial conditions allows the model to generate arbitrary inferred life-cycle profiles of heterogeneity without violating the random walk assumptions on the sources of heterogeneity which are essential for the no-trade result. For example, the mean of α_t^j is given by $\mathbb{E}\alpha_t^j = \mathbb{E}\alpha_j^j + \sum_{s=j+1}^t \mathbb{E}\upsilon_s^\alpha$, so the difference in the mean of α_t by age is $\mathbb{E}\alpha_t^{j-1} - \mathbb{E}\alpha_t^j = [\mathbb{E}\alpha_{j-1}^{j-1} - \mathbb{E}\alpha_j^j] + \mathbb{E}\upsilon_j^\alpha$, where the term in brackets is a cohort effect and the last term is a time effect. As a result, the inferred mean of α_t by age can appear to deviate from the mean of a random walk process with an innovation which grows constantly over the life cycle due to a combination of cohort and time effects which cannot be identified separately relative to age. Similarly, the change in the inferred variance of α_t is given by $\mathrm{Var}(\alpha_t^{j-1}) - \mathrm{Var}(\alpha_t^j) = [\mathrm{Var}(\alpha_{j-1}^{j-1}) - \mathrm{Var}(\alpha_j^j)] + \mathrm{Var}(\upsilon_j^\alpha)$ and can deviate from the change in the variance of a random walk process.

after 40. As shown in Table 4, child care is the subcategory of h_N responsible for this decline.

In Figure 2 we present the age profiles of cross-sectional variances of $(\alpha, \varepsilon, B, D_P, \log\theta_N)$, which equal the variances of the residuals for each age from a regression of each source of heterogeneity on age dummies, cohort dummies, and normalized year dummies. The home production model infers significantly smaller variances of α, ε, and B than the model without home production. From the solutions in Table 2, we observe that the increasing variance of α over the life cycle is driven by the increase in the variance of the consumption-hours ratio $\log(c_T/h_T)$ and the increase in the variance of wages $\log z_M$. Because the variance of $\log(c_T/h_T)$ is lower than the variance of $\log(c_M/h_M)$, the home production model generates a lower variance of α. Given that both models match the same variance of $\log z_M$ but the home production model displays a larger covariance between α and ε than the model without home production (see Appendix Table A.6), ε turns out to be less dispersed in the home production model. The variance of B is also smaller in the home production model which reflects the smaller variance of a combination of $\log c_T$ and $\log h_T$ than a combination of $\log c_M$ and $\log h_M$.

In the lower panels we observe that the dispersion in the disutility of home work D_P exceeds the dispersion in the disutility of market work B and that home production efficiency $\log\theta_N$ is significantly more dispersed than any other source of heterogeneity. To set a benchmark for $\log\theta_N$, we note that the variance of $\log z_M$ is 0.33 in the data. What explains the almost four times as large dispersion in $\log\theta_N$? From equation (14), inferred home production efficiency is:

$$\log\theta_N = \text{constant} + \left(\frac{1}{\phi-1}\right)(\phi\log\tilde{z}_M + \log h_N - \log c_M)\,. \tag{16}$$

Our result that home production efficiency is more dispersed than market productivity reflects the fact that $\log\theta_N$ cumulates the dispersions of three observables, $\log\tilde{z}_M$, $\log h_N$, and $\log c_M$, which are relatively uncorrelated with each other.[21] When ϕ tends to zero and the goods tend to become perfect complements, we obtain $\log\theta_N = \text{constant} + \log c_M - \log h_N$. In this case the variance of $\log\theta_N$ is roughly 1.3 because the variance of $\log c_M$ is roughly 0.3, the variance of $\log h_N$ is roughly 1, and the two variables are relatively uncorrelated in the cross-section of households. When ϕ tends to infinity and the goods tend to become perfect substitutes, we obtain

[21] Recall that home production efficiency is a convolution of productivity and consumption weights, $\theta_N = \omega_N^{\frac{\phi}{\phi-1}} z_N$. As a result, its dispersion reflects dispersion in both home productivity and consumption weight as well as their covariation, $\mathrm{Var}(\log\theta_N) = \left(\frac{\phi}{\phi-1}\right)^2 \mathrm{Var}(\log\omega_N) + \mathrm{Var}(\log z_N) + 2\frac{\phi}{\phi-1}\mathrm{Cov}(\log\omega_N, \log z_N)$. Under our estimated $\phi = 2.35$, the dispersion in θ_N is roughly four times as large as the dispersion in market productivity z_M when, for example, $\mathrm{Var}(\log z_M) = \mathrm{Var}(\log z_N) = \mathrm{Var}(\log\omega_N)$ and $\mathrm{Cov}(\log\omega_K, \log z_K) = 0$.

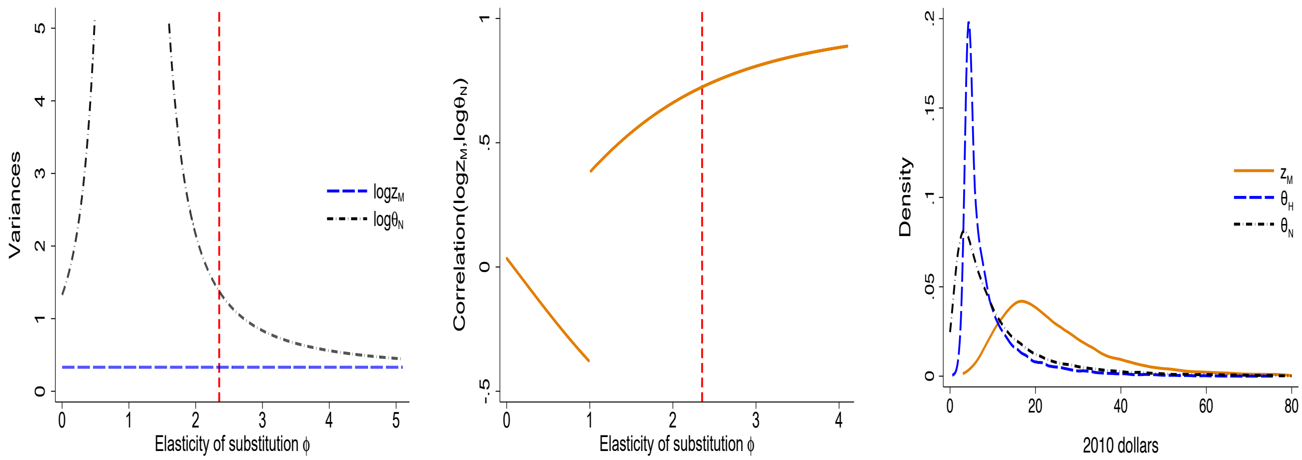


Figure 3: Production Efficiency Moments

The left panel of Figure 3 shows the variance of home production efficiency $\log\theta_N$ and market productivity $\log z_M$ and the middle panel shows the correlation between the two variables as a function of the elasticity of substitution across sectors ϕ. The dashed vertical line shows the variances and correlation at our estimated value of $\phi = 2.35$. The right panel plots estimates of the distributions of z_M, $\theta_H = \frac{h_N}{h_N+h_P}\theta_N + \frac{h_P}{h_N+h_P}\theta_P$, and θ_N at $\phi = 2.35$.

$\log\theta_N = \text{constant} + \log\tilde{z}_M$. In that case, the variance of $\log\theta_N$ converges to the variance of $\log\tilde{z}_M$. When ϕ tends to one, the variance of $\log\theta_N$ tends to infinity. To summarize, for any value of ϕ, the variance of $\log\theta_N$ exceeds the variance of $\log\tilde{z}_M$.

Figure 3 summarizes properties of production efficiencies.[22] The left panel shows the variances of $\log\theta_N$ and $\log z_M$ and the middle panel shows the correlation of the two variables as function of the elasticity of substitution across sectors ϕ. The variance of $\log\theta_N$ is larger than the variance of $\log z_M$ for any value of $\phi < 5$ in the figure.[23] The correlation between the two variables changes sign with the value of ϕ. When goods are substitutes, $\phi > 1$ as suggested by our estimation, efficiency in the home sector is positively correlated with efficiency in the market sector. If goods were complements, $\phi < 1$, the correlation would typically have been negative.

The right panel of Figure 3 plots the distributions of production efficiencies under our estimated $\phi = 2.35$. We define effective home production efficiency $\theta_H = \frac{h_N}{h_N+h_P}\theta_N + \frac{h_P}{h_N+h_P}\theta_P$. Because θ_P is a constant, θ_H is less dispersed than θ_N. The means of z_M, θ_H, and θ_N are 26.6, 10.9, and 14.3 dollars respectively. The fraction of households with efficiency exceeding 100 dollars per hour

[22] Appendix Table A.6 presents the correlation matrix of all observables and sources of heterogeneity. Appendix Figure A.1 shows estimates of the distributions of all other sources of heterogeneity.

[23] We note that the argument in the preceding paragraph referred to after-tax market productivity $\log\tilde{z}_M$ while in Figure 3 we use the more primitive pre-tax market productivity $\log z_M$. The former measure of productivity is roughly 77 percent as dispersed as the latter given our estimated tax progressivity parameter $\tau_1 = 0.12$.

equals roughly 1 percent, 0.5, and 1.2 percent respectively.

4 Inequality and Home Production

We show that home production amplifies inequality across households and that heterogeneity in production efficiency rather than disutility of work drives the increase in inequality.

4.1 Home Production Amplifies Inequality

We define inequality as a mapping from the dispersion in observed allocations and inferred sources of heterogeneity to measures capturing welfare differences across households. We acknowledge there are various such mappings and, therefore, present four inequality metrics.

4.1.1 Equivalent Variation

The equivalent variation, a broadly used metric in welfare economics, is the change in income for a household to achieve a reference level of utility. Let $\hat{\iota}$ be a reference household with a derived utility $V(\hat{c}_{M,t}, \hat{h}_{M,t}, \hat{h}_{K,t}; \hat{\iota})$, and a value function $\hat{W}_t(\hat{\iota})$. For every household ι, we compute the income transfer $T_t(\iota)$ making it indifferent between being ι and being $\hat{\iota}$ in the current period, holding constant ι's expectation over all future allocations. The equivalent variation $T_t(\iota)$ solves:

$$\hat{W}_t(\hat{\iota}; \iota) = \max_{\{c_{M,t}, h_{M,t}, h_{K,t}\}} \left\{ V(c_{M,t}, h_{M,t}, h_{K,t}; \iota) + \beta\delta\mathbb{E}_t \left[W_{t+1}(\iota')|\iota \right] \right\}, \tag{17}$$

subject to the budget constraint:

$$c_{M,t} = \tilde{y}_t + T_t(\iota) + \text{NA}_t(\iota). \tag{18}$$

In the left-hand side of equation (17) we define $\hat{W}_t(\hat{\iota}; \iota) \equiv V(\hat{c}_{M,t}, \hat{h}_{M,t}, \hat{h}_{K,t}; \hat{\iota}) + \beta\delta\mathbb{E}_t \left[W_{t+1}(\iota')|\iota \right]$ and in equation (18) we keep the net asset position $\text{NA}_t(\iota)$ constant at its value before the transfer $T_t(\iota)$ is given.

Figure 4 presents the cross-sectional dispersion in equivalent variation by age.[24] The left panel shows the standard deviation of equivalent variation, standardized by the mean value of market consumption $\int c_M(\iota) \mathrm{d}\Phi(\iota)$ which is constant across models and ages. The standard deviation is

[24] For the equivalent variation in this figure, the reference household $\hat{\iota}$ is the household with the median utility in the sample. Our results are similar when $\hat{\iota}$ is the household with the mean utility in the sample, when the identity of $\hat{\iota}$ differs by age and is the household with the median utility for each age, and when the identity of $\hat{\iota}$ differs by age and is the household with the mean utility for each age.

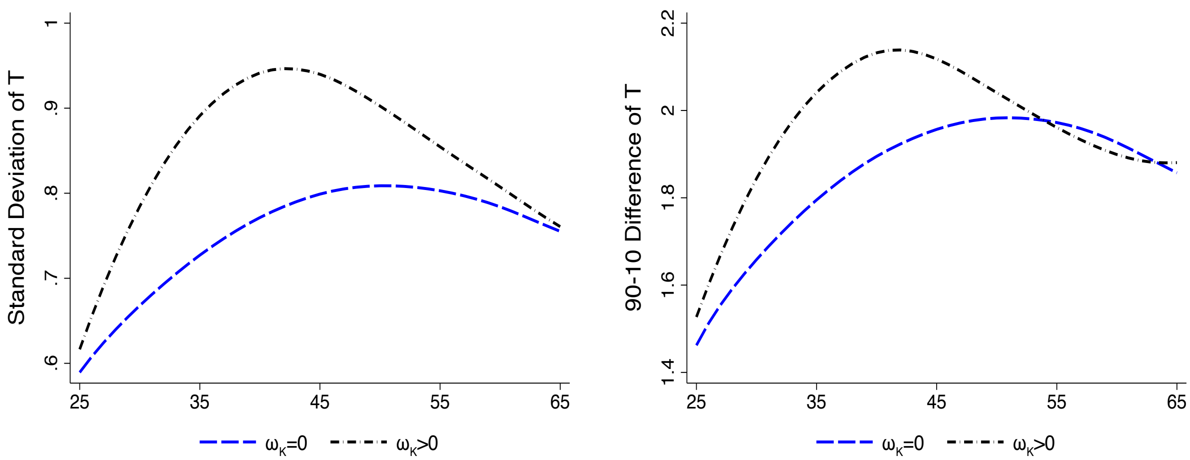


Figure 4: Dispersion in Equivalent Variation

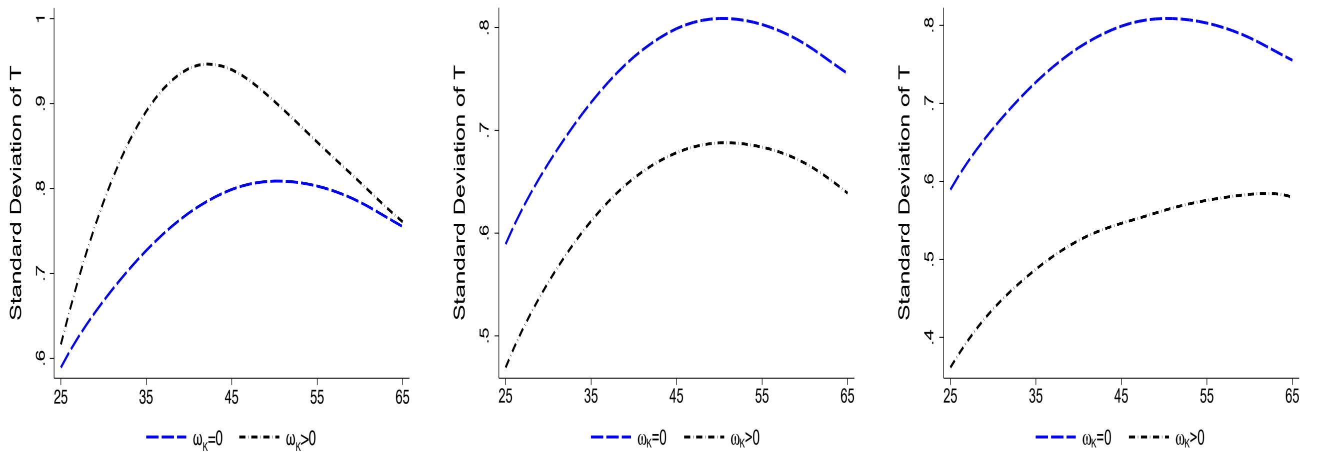
Figure 4 shows the dispersion in equivalent variation T for the model without ($\omega_K = 0$, blue dashed line) and with home production ($\omega_K > 0$, black dotted line) by age. The standard deviation of T is normalized by mean market consumption $\int c_M(\iota)\mathrm{d}\Phi(\iota)$ which is constant across models and ages.

around 0.6 in both economies at 25. By 45, the standard deviation has increased to more than 0.9 in the home production model, as opposed to below 0.8 in the model without home production. Similarly, the right panel shows different patterns until 55 between the two models using the difference between the 90th and 10th percentile in equivalent variation. Both inequality statistics tend to converge across the two models for households older than 60.

What drives our inference that inequality is higher with home production? A key feature of the data driving this inference is that home hours h_N are not negatively correlated with market consumption c_M and market productivity z_M in the cross section of households. We calculate that h_N has a correlation of 0.07 with $\log z_M$ and 0 with $\log c_M$. Thus, home production does not offset heterogeneity originating in the market sector. Instead, home production exacerbates inequality given the large dispersion in home production efficiency θ_N.[25]

To illustrate this point, in Figure 5 we repeat our analyses using a different correlation of home hours h_N with other observables in the data. The left panel repeats the age profile of the standard

[25]We focus on h_N because its low correlation with c_M and z_M is more informative than the low correlations of h_P and further discuss the role of efficiency and disutility heterogeneity in Section 4.2. Given that child care is the largest subcategory of h_N, our estimate of a weakly positive correlation between h_N and z_M is broadly consistent with the findings of Guryan, Hurst, and Kearney (2008) who document that higher educated and higher income parents tend to spend more time with their children. Appendix Tables A.1, A.2, and A.3 demonstrate that the lack of a negative correlation with wages is present both for individuals and households and is present within age, sex, and education groups. Appendix Tables A.4 and A.5 demonstrate that the correlation of home hours with both consumption and wages is broadly similar in magnitude between the CEX/ATUS sample and PSID samples in which home production time is not imputed.



(a) Baseline (b) $\text{corr}(h_N, \log z_M) = -0.8$ (c) $\text{corr}(h_N, \log c_M) = -0.8$

Figure 5: Counterfactuals of Dispersion in Equivalent Variation

Figure 5 shows the dispersion in equivalent variation T for the model without ($\omega_K = 0$, blue dashed line) and with home production ($\omega_K > 0$, black dotted line) by age. The left panel repeats the dispersion in T for our baseline model. In the other two panels we generate counterfactual data with either $\text{corr}(h_N, \log z_M) = -0.8$ or $\text{corr}(h_N, \log c_M) = -0.8$, repeat our inference of the sources of heterogeneity, and then calculate the dispersion in T.

deviation in equivalent variation $T(\iota)$ shown in the left panel of Figure 4. In the other two panels we calculate the equivalent variation $T(\iota)$ when we repeat our inference of $(\alpha, \varepsilon, B, D_P, \theta_N)$ in counterfactual data in which the correlation of home hours h_N with market productivity $\log z_M$ and market expenditures $\log c_M$ is -0.8. The figure shows that if the data featured a significantly more negative correlation between h_N and either $\log z_M$ or $\log c_M$, then we would have concluded that inequality in the model with home production is actually lower.

It is instructive to use Figure 5 to explain why our results differ from the home production literature over the business cycle (Benhabib, Rogerson, and Wright, 1991; Greenwood and Hercowitz, 1991; Karabarbounis, 2014) or over the life cycle (Rios-Rull, 1993; Aguiar and Hurst, 2005, 2007). In these literatures, home production offsets differences that originate in the market sector because decreases in the opportunity cost of time and in expenditures are associated with substantial increases in time spent on home production. Home production would also smooth welfare-based differences across households in our model had the cross-sectional data featured a negative correlation between home production time and either wages or expenditures. Our results for the role of home production in understanding cross-sectional patterns differ from previous findings because time in home production is not negatively correlated with wages and expenditures in the cross section of households.

4.1.2 Redistributive Transfers

Our second measure of inequality is the cross-sectional dispersion in redistributive transfers which equalize marginal utilities. After households choose their allocations of consumption and hours, we allow a utilitarian planner to allocate aggregate market consumption across households in order to maximize average household utility. The dispersion in these transfers captures the extent of redistribution required to maximize social welfare or, equivalently, to equalize marginal utilities of market consumption. Formally, the problem is to choose transfers $\{t(\iota)\}$ to maximize:

$$\int_{\iota} V(c_M(\iota) + t(\iota), h_M(\iota), h_N(\iota), h_P(\iota))\mathrm{d}\Phi(\iota), \tag{19}$$

subject to aggregate transfers being equal to zero $\int_{\iota} t(\iota)\mathrm{d}\Phi(\iota) = 0$.

The optimal transfers equal the gap between average and individual market value of total consumption $c_T(\iota)$:[26]

$$t(\iota) = \int_{\iota} c_T(\iota)\mathrm{d}\Phi\,(\iota) - c_T(\iota). \tag{20}$$

The dispersion in redistributive transfers $t(\iota)$ differs from the dispersion in equivalent variation $T(\iota)$ in Section 4.1.1 because it leads to an equalization of marginal utilities instead of utility levels. An advantage of using the dispersion in $t(\iota)$ as a measure of inequality is that it depends transparently only on observables and estimated parameters.

The left panel of Figure 6 shows the age profiles of the cross-sectional standard deviation in redistributive transfers $t(\iota)$ for the two models, standardized again by the mean value of market consumption $\int c_M(\iota)\mathrm{d}\Phi(\iota)$. The standard deviation of $t(\iota)$ is larger and increases more over the life cycle in the model with home production. We obtain a similar result in the right panel which shows the difference between the 90th and 10th percentile in redistributive transfers $t(\iota)$.

It is instructive to compare our findings to the those of Frazis and Stewart (2011) and Bridgman, Dugan, Lal, Osborne, and Villones (2012) who have embraced the view that home production decreases inequality. Their argument is that, since home hours do not correlate with income in the cross section of households, adding a constant value of home production across households results in a smaller dispersion in total income. Inspection of equation (13) for c_T reveals a fundamental difference in our logic. Home hours in our model are valued at their opportunity cost which varies

[26] We remind the reader that the marginal utility of market consumption under an equilibrium allocation $(c_M + t, h_M, h_N, h_P)$ equals the inverse of the market value of total consumption c_T given in equation (13).

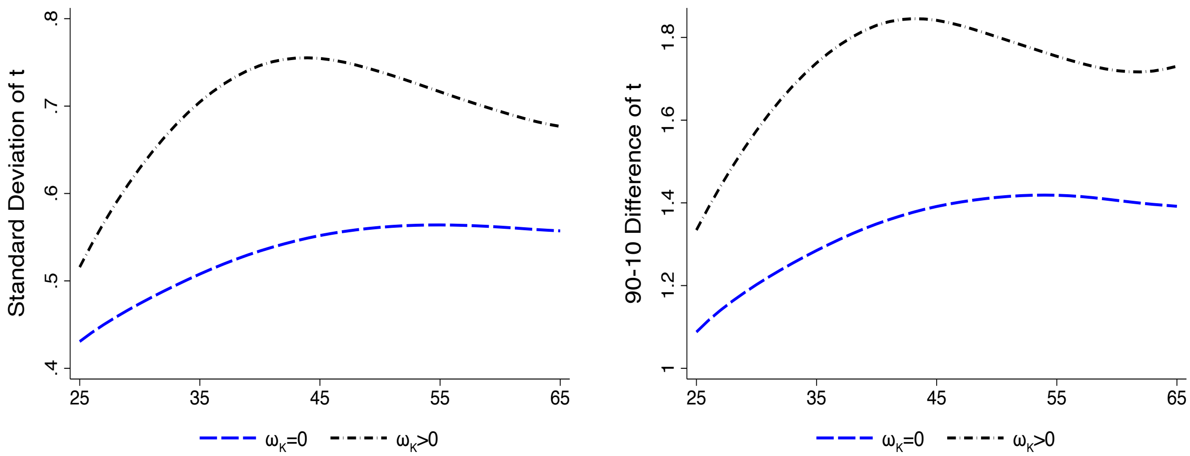


Figure 6: Dispersion in Redistributive Transfers

Figure 6 shows the dispersion in redistributive transfers t for the environment without ($\omega_K = 0$, blue dashed line) and with home production ($\omega_K > 0$, black dotted line) by age. The standard deviation of t is normalized by mean market consumption $\int c_M(\iota)\mathrm{d}\Phi(\iota)$ which is constant across models and ages.

across households. Using a constant opportunity cost does not take into account differences in the efficiency or disutility of home hours across households.[27]

4.1.3 Lifetime Welfare Cost of Heterogeneity

This section presents the lifetime welfare effects from heterogeneity across households. Our calculations contrast with our inequality metrics so far which ignore dynamic considerations. The lifetime welfare effect is the share of consumption in every period which a household is willing to sacrifice ex-ante to be indifferent between being born in the baseline environment with heterogeneity and allocations $\{c_t, h_{M,t}, h_{N,t}, h_{P,t}\}$ and a counterfactual environment in which dimensions of heterogeneity are shut down. The allocations in the counterfactual economy are denoted by $\{\hat{c}_t, \hat{h}_{M,t}, \hat{h}_{N,t}, \hat{h}_{P,t}\}$ and are generated using the equations in Table 1 after shutting down particular dimensions of heterogeneity.[28]

[27] A reasonable concern using wages to value home hours is that some households or members of the household may be at a corner solution. In practice, we are not concerned that valuing home hours at its opportunity cost biases our results for three reasons. First, in our baseline CEX/ATUS sample of married households the fraction of households with either zero market hours or zero home hours per year is less than one percent. Further, sensitivity analyses presented in Section 5 confirm our inequality results in a sample of singles and in a subsample of married households with a working spouse for which valuation at market wages is less concerning. Finally, our notion of inequality in consumption allows for a wedge between the wage and the marginal value of home hours h_P arising from disutility differences across sectors.

[28] Consistent with our definition of equilibrium in which G_t is an endogenous variable, in these counterfactuals we keep constant the tax parameters (τ_0, τ_1) because we prefer to evaluate more direct welfare effects arising from heterogeneity rather than more nuanced effects arising from changes in the tax parameters in order to satisfy the

Table 6: Lifetime Welfare Cost of Heterogeneity

	No Home Production: $\omega_K = 0$		Home Production: $\omega_K > 0$	
No dispersion in ...	λ_p	λ	λ_p	λ
z_M, θ_N, B, D_P	0.04	0.06	0.05	0.12
z_M, θ_N	0.04	0.07	0.05	0.16
θ_N, D_P	—	—	0.00	0.13
θ_N	—	—	0.00	0.13

Table 6 shows changes in aggregate productivity λ_p and welfare λ for the model without ($\omega_K = 0$) and with ($\omega_K > 0$) home production. The change is calculated as the difference between the environment described in each row and the baseline model which has all sources of heterogeneity operating at the same time. A positive number indicates a productivity or welfare gain from shutting off a particular combination of sources of heterogeneity.

The share of lifetime consumption that makes households indifferent between the actual and the counterfactual economy is given by the λ which solves:

$$\mathbb{E}_{j-1} W(\{c_t, h_{M,t}, h_{N,t}, h_{P,t}\}) = \mathbb{E}_{j-1} W(\{(1-\lambda)\hat{c}_t, \hat{h}_{M,t}, \hat{h}_{N,t}, \hat{h}_{P,t}\}) \,, \tag{21}$$

where $c_t = \left(c_{M,t}{}^{\frac{\phi-1}{\phi}} + (\theta_{N,t} h_{N,t})^{\frac{\phi-1}{\phi}} + (\theta_{P,t} h_{P,t})^{\frac{\phi-1}{\phi}} \right)^{\frac{\phi}{\phi-1}}$. When $\lambda > 0$, households prefer the counterfactual. Benabou (2002) and Floden (2001) have emphasized that total welfare effects from eliminating heterogeneity arise both from level effects when aggregate allocations change and effects capturing changes in the dispersion in allocations across households. Therefore, alongside λ, we discuss how heterogeneity influences aggregate productivity $\int_\iota z_M(\iota) h_T(\iota) \mathrm{d}\Phi(\iota) \big/ \int_\iota h_T(\iota) \mathrm{d}\Phi(\iota)$. We denote by λ_p the percent change in aggregate productivity between the counterfactual and the baseline allocation. Dispersion in market productivity z_M decreases aggregate productivity because h_T is negatively correlated with z_M in both models.

In the first row of Table 6, we shut down all sources of heterogeneity and both models collapse to a representative household economy. The welfare cost of heterogeneity λ is 12 percent in the model with home production as opposed to 6 percent in the model without home production. The difference between the two models reflects predominately the differential cost of dispersion in allocations rather than aggregate productivity changes λ_p which are relatively similar across models.[29]

government budget constraint. By contrast, when we calculate optimal taxes (τ_0, τ_1) in Section 4.1.4, we keep constant G_t to its initial equilibrium value.

[29]The welfare effects in Table 6 reflect heterogeneity both within age and over the life cycle because each counterfactual imposes a constant value of the source of heterogeneity for households of all ages. We have repeated

The larger dispersion costs of heterogeneity in the home production model reflect the costs of dispersion in the efficiency of work rather than the disutility of work. To see this, in the second row we shut down heterogeneity in efficiencies, z_M and θ_N, while we maintain heterogeneity in disutilities of work B and D_P. We find even larger welfare effects than in the first row and, thus, conclude that heterogeneity in B and D_P is not important for the welfare effects of eliminating all heterogeneity. Shutting off heterogeneity in θ_N and D_P (third row) or only in θ_N (fourth row) leads to similar welfare effects in the model with home production. This again illustrates the importance of heterogeneity in production efficiency for welfare.

4.1.4 Optimal Tax Progressivity

This section contrasts the optimal progressivity of the tax system between the model with and without home production. Relative to our previous inequality metrics, this optimal taxation exercise mixes redistribution with efficiency concerns because the optimal progressivity of the tax system increases with redistributive motives and decreases with the efficiency losses from distorting allocations. However, this exercise allows us to more directly link our inequality result to policy.

Given government expenditures G fixed at its initial equilibrium level, the government chooses tax parameters $\tau \equiv (\tau_0, \tau_1)$ to maximize utilitarian welfare:

$$\int_\iota V(c_M(\tau), h_M(\tau), h_N(\tau), h_P(\tau); \iota) \mathrm{d}\Phi(\iota) \ , \tag{22}$$

subject to the government budget constraint:

$$\int_\iota \left[z_M - (1-\tau_0) {z_M}^{1-\tau_1}\right] h_M(\tau) \mathrm{d}\Phi(\iota) = G. \tag{23}$$

In Figure 7 we plot the relationship between pre-tax labor income y and after-tax labor income $\tilde{y}$, both in thousands of 2010 dollars. The orange solid curve shows the relationship between y and $\tilde{y}$ under the parameter $\tau_1 = 0.12$ which we estimated in the data. The blue dashed and black dotted curves show this relationship under the optimal $\tau_1 = 0.06$ for the model without home production and the optimal $\tau_1 = 0.24$ for the model with home production. The relationship between y and $\tilde{y}$ is significantly more concave in the model with home production. To give an example, consider a household earning 200 thousand dollars. Under the optimal tax schedule in

these exercises by shutting down only within-age heterogeneity. Appendix Table A.7 shows similar welfare effects to those shown in Table 6 and, therefore, we conclude that the welfare effects predominately reflect the within-age component of heterogeneity.

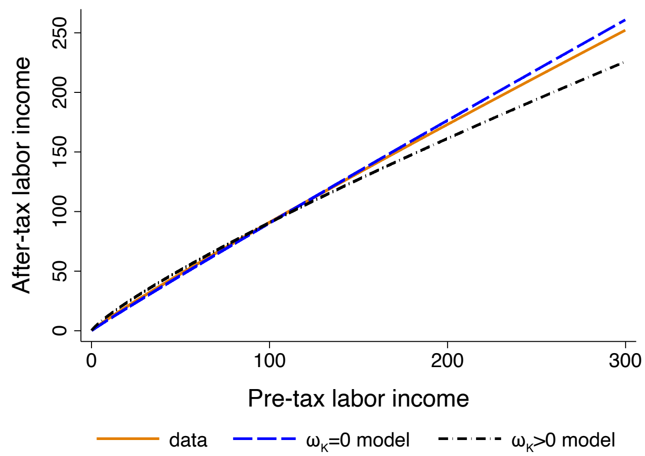


Figure 7: Optimal Tax Function

Figure 7 displays the relationship between pre-tax labor income y and after-tax labor income $\tilde{y}$ under the parameters estimates for the United States (orange solid line), under the optimal tax function for the model without home production ($\omega_K = 0$, blue dashed line), and under the optimal tax function with home production ($\omega_K > 0$, black dotted line). The progressivity parameter $\tau_1 = 0.12$ in the data, $\tau_1 = 0.06$ in the model without home production, and $\tau_1 = 0.24$ in the model with home production.

the model without home production the household faces an average tax rate of 12 percent, while in the model with home production the average tax rate increases to 19 percent.

4.2 Heterogeneity in Home Efficiency versus Disutility of Work

Using four different metrics of inequality, we have demonstrated that home production amplifies inequality across households. In our baseline model differences in the home sector reflect both heterogeneity in production efficiency θ_N and disutility of work D_P when both sectors are operating ($\omega_N > 0$ and $\omega_P > 0$). Which source of heterogeneity is quantitatively more important in elevating inferred inequality?

To quantify the importance of home production efficiency and disutility of work, we consider the two polar cases of all home time $h_N + h_P$ belonging either to the sector with heterogeneity in production efficiency or to the sector with heterogeneity in disutility of work. When we set $\omega_P = 0$, then we obtain a two-sector model in which the disutility of work B is equalized across sectors and the sectoral allocation of time depends on production efficiencies in the market z_M and at home θ_N. Instead, setting $\omega_N = 0$ yields a two-sector model in which market productivity z_M and sectoral disutility of work, B and D_P, determine the allocation.

Table 7 summarizes our results. The first column presents the four inequality metrics (averaged across all ages) in the model without home production and the last three columns present the

Table 7: The Role of Home Efficiency and Home Disutility in Amplifying Inequality

	No Home Production	Home Production		
Statistics		Efficiency	Baseline	Disutility
std(T)	0.78	1.14	0.90	0.76
std(t)	0.55	0.83	0.73	0.65
λ	0.06	0.20	0.12	0.03
τ_1	0.06	0.32	0.24	0.13

Table 7 shows the four inequality metrics for the model without home production, the home production model with only efficiency heterogeneity, the baseline home production model with both efficiency and disutility heterogeneity, and the home production model with only disutility heterogeneity. Parameters τ_0, τ_1, and ϕ are held constant to their values shown in Table 5. The estimated values for η are 0.90, 0.53, 0.50, and 0.57. The estimated value of θ_P is 4.64 for the baseline home production model and 9.74 for the model with only heterogeneity in disutility of home work.

metrics in the three versions of the home production model. In the home production model with only heterogeneity in home production efficiency, all inequality metrics are magnified relative to the baseline with heterogeneity in both efficiency and disutility. If there was only heterogeneity in disutility of home work, there would be no significant difference in inequality between the model with and the model without home production. We conclude that heterogeneity in home production efficiency rather than disutility of work is important in amplifying inequality across households.[30]

5 Sensitivity Analyses

In this section we present sensitivity analyses with respect to the parameterization of the model, subsamples of the population, and measurement error in observables. Each row in Table 8 corresponds to a different sensitivity analysis. For both models, the columns show the standard deviation in equivalent variation T, the standard deviation in transfers required to equalize marginal utilities t, the ex-ante lifetime welfare loss from shutting down all heterogeneity λ, and the degree of progressivity in the optimal tax system τ_1. In each exercise, we repeat our analysis of identifying the sources of heterogeneity $(\alpha, \varepsilon, B, D_P, \theta_N)$ and then calculate the inequality metrics. The first row of the table repeats these statistics for our baseline case.

Rows 2 to 9 vary parameters of the model. Relative to our estimated value $\tau_1 = 0.12$, changing

[30] We also consider two additional cases of interest. In the first case there is heterogeneity in home production efficiency θ_N and θ_P in both sectors and no disutility differences across sectors, $B = D_N = D_P$. We obtain nearly identical results to the $\omega_P = 0$ case. In the second case there is heterogeneity in the disutility of home work D_N and D_P in both sectors and both θ_N and θ_P are constant across households. We obtain nearly identical results to the $\omega_N = 0$ case. Appendix Table A.8 summarizes these results.

Table 8: Sensitivity Analyses of Inequality Metrics

		No Home Production: $\omega_K = 0$				Home Production: $\omega_K > 0$			
		Std(T)	Std(t)	λ	τ_1	Std(T)	Std(t)	λ	τ_1
1.	Baseline	0.78	0.55	0.06	0.06	0.90	0.73	0.12	0.24
	Parameter Values								
2.	$\tau_1 = 0.06$	0.78	0.55	0.06	0.12	0.93	0.74	0.14	0.27
3.	$\tau_1 = 0.19$	0.78	0.55	0.05	-0.04	0.88	0.72	0.10	0.20
4.	$G/Y = 0.05$	0.78	0.55	0.06	0.03	0.90	0.73	0.12	0.24
5.	$G/Y = 0.15$	0.78	0.55	0.06	0.09	0.90	0.73	0.12	0.25
6.	$\eta' = 0.8\eta$	0.75	0.55	0.05	-0.09	0.88	0.73	0.11	0.21
7.	$\eta' = 1.2\eta$	0.80	0.55	0.07	0.15	0.92	0.73	0.12	0.27
8.	$\phi = 0.5$	0.78	0.55	0.06	0.06	1.94	0.70	0.52	0.44
9.	$\phi = 20$	0.78	0.55	0.06	0.06	0.85	0.71	0.09	-0.80
	Marital, Employment, Family, and Education Groups								
10.	Singles	0.89	0.61	0.01	0.03	0.90	0.71	0.08	0.13
11.	Non-working spouse	0.80	0.55	0.10	0.22	1.34	1.07	0.21	0.33
12.	Working spouse	0.78	0.54	0.05	0.09	0.84	0.70	0.10	0.23
13.	No children	0.79	0.55	0.10	-0.06	0.81	0.67	0.18	0.13
14.	One child	0.78	0.55	0.07	0.10	0.85	0.72	0.11	0.27
15.	Two or more children	0.77	0.53	0.04	0.15	0.96	0.77	0.19	0.31
16.	Child younger than 5	0.77	0.54	0.01	0.15	1.02	0.82	0.24	0.34
17.	Less than college	0.78	0.54	0.02	-0.22	0.86	0.71	0.06	0.13
18.	College or more	0.76	0.52	0.06	-0.10	0.86	0.68	0.15	0.20
	Consumption Expenditures								
19.	Food expenditures	0.82	0.56	0.04	-0.05	0.92	0.75	0.11	0.21
20.	All expenditures	0.88	0.63	0.07	0.18	0.99	0.83	0.11	0.27
21.	Adjusted baseline	0.57	0.39	0.07	0.23	0.79	0.60	0.13	0.31
22.	Adjusted all	0.84	0.60	0.07	0.26	0.97	0.80	0.11	0.31

Table 8 presents sensitivity analyses of the four inequality metrics for the model without home production ($\omega_K = 0$) and the model with home production ($\omega_K > 0$). The first row repeats the inequality statistics in the baseline model. Each row represents a different sensitivity analysis with respect to parameters, samples, and measures of consumption expenditures.

the progressivity of the tax system to $\tau_1 = 0.06$ as in Guner, Kaygusuz, and Ventura (2014) or to $\tau_1 = 0.19$ as in Heathcote, Storesletten, and Violante (2014) does not alter significantly any result. We also obtain highly similar results when we change the target for the average labor income tax G/Y to 0.05 or 0.15. In rows 6 and 7 we vary the parameter η that governs the Frisch elasticity of labor supply by 20 percent. Raising η increases three of the inequality metrics in both models, but in all cases inequality is higher in the model with home production.

In rows 8 and 9, we vary the elasticity of substitution across goods ϕ. The Std(t) inequality metric is relatively insensitive to ϕ. When $\phi = 0.5$ and goods are complements, the Std(T), λ, and τ_1 metrics of inequality increase substantially relative to the baseline with $\phi = 2.35$. Intuitively, the complementarity between goods implies that home production amplifies differences in the market sector even more. When $\phi = 20$ and goods are almost perfect substitutes, we still find that inequality is higher with home production according to the Std(T), Std(t), and λ metrics but to a lesser extent than before. The main difference with our baseline arises in terms of the optimal progressivity which is significantly affected by the value of ϕ. Because a higher value of ϕ increases the efficiency losses from a progressive tax system, we obtain a lower τ_1 in the model with home production and $\phi = 20$ than in the model without home production.

In rows 10 to 18 of Table 8 we repeat our analyses in subsamples of households defined along their marital status, employment status of the spouse of the head, number of children, age of youngest child, and education. Repeating our analyses for different samples allows us to explore whether our inequality results reflect within group inequality or inequality across groups. Additionally, verifying our results at the subgroup level is reassuring because it allows us to control for dimensions of heterogeneity which we did not model, such as spousal employment at the extensive margin, the presence of young children, or the number of children.

Our results are remarkably stable at the subgroup level, with the home production model always generating more inequality than the model without home production according to all four metrics. Row 10 shows the sample of singles, for which the inequality gap between models is generally smaller. Rows 11 and 12 show subsamples of married households according to whether the spouse is working or not. Reassuringly for the mechanisms we have stressed, we obtain a larger inequality gap between models in the group of households with non-working spouses for which we expect home production efficiency differences to be more important. In rows 13 to 15 we differentiate according to the number of children present in the household. We obtain larger inequality gaps between models in households with more children, which highlights the importance of time spent on child care for our results. Similarly, in row 16 we find even larger inequality gaps in households with a child younger than 5. Finally, rows 17 and 18 show results for married households with a head who has not completed college and with a head who has completed college or more. Our results are similar to the baseline with the exception of the optimal progressivity τ_1 which declines substantially in the model without home production.

In rows 19 and 20 we show that our results are robust under two alternative measures of market expenditures c_M. In row 19 we use food only and in row 20 we use all expenditures including health, education, and durables. The inequality metrics and the gap between the two models are generally similar to the baseline which used nondurable consumption excluding health and education. From the four metrics, the optimal progressivity τ_1 is the most sensitive to the measure of consumption.

A concern about our results is that the dispersion in reported consumption reflects measurement error which may affect inequality differentially across the two models. We now examine the robustness of our results to measurement error in consumption expenditures. For each spending category comprising our aggregate household consumption measure, we use the elasticity of the spending category with respect to aggregate household consumption estimated by Aguiar and Bils (2015) to adjust households' spending category for measurement error. Aggregating across all spending categories produces measurement-error adjusted aggregate household consumption measures which we use to repeat our analyses.[31] We present results in row 21 for the baseline measure of nondurable consumption and in row 22 for all expenditures including health, education, and durables. We find that the model with home production still generates larger inequality than the model without home production.[32]

An alternative way to examine the sensitivity of our results to measurement error is to simulate the effects of reducing the dispersion in observables on the inequality metrics. We consider a classical measurement error model in which the reported value of variable x for household ι is:

$$\log x(\iota) = \log x^*(\iota) + m(\iota), \tag{24}$$

[31]Let $x_{M,j}(\iota)$ be reported spending of household ι in category j, $x_M(\iota) = \sum_j x_{M,j}(\iota)$ be aggregate reported consumption of household ι, and β_j be the elasticity of spending $x_{M,j}$ with respect to aggregate household consumption x_M estimated by Aguiar and Bils (2015). We allocate aggregate spending over all households in each category for a particular year, $x_{M,j} = \sum_\iota x_{M,j}(\iota)$, to households in proportion to their predicted spending in that category based on their aggregate household consumption and the spending elasticity, $x_M(\iota)^{\beta_j}$. For each household ι we obtain $c_{M,j}(\iota) = \frac{x_M(\iota)^{\beta_j}}{\sum_\iota x_M(\iota)^{\beta_j}} x_{M,j}$ and define the measurement-error adjusted aggregate household consumption as $c_M(\iota) = \sum_j c_{M,j}(\iota)$.

[32]Measurement-error adjusted consumption is less dispersed than reported consumption and, therefore, inequality according to the Std(T) and Std(t) metrics decreases in both models relative to the baseline. The measurement error adjustment lowers consumption dispersion because low-elasticity categories (such as food) account for larger fractions of aggregate spending. We calculate that the expenditure-weighted average elasticity of spending categories is 0.71 with respect to nondurable consumption and 0.95 with respect to aggregate spending. We have confirmed that our results are similar when using NIPA expenditures instead of CEX expenditures.

Table 9: Inequality Metrics and Measurement Error

		No Home Production: $\omega_K = 0$				Home Production: $\omega_K > 0$			
		Std(T)	Std(t)	λ	τ_1	Std(T)	Std(t)	λ	τ_1
1.	Baseline	0.78	0.55	0.06	0.06	0.90	0.73	0.12	0.24
	Consumption $x = c_M$								
2.	$\sigma_m^2/\text{var}(\log x) = 0.20$	0.73	0.51	0.05	0.11	0.87	0.69	0.12	0.26
3.	$\sigma_m^2/\text{var}(\log x) = 0.50$	0.62	0.41	0.04	0.15	0.80	0.60	0.12	0.27
4.	$\sigma_m^2/\text{var}(\log x) = 0.80$	0.45	0.26	0.03	0.18	0.70	0.47	0.12	0.29
	Market Hours $x = h_M$								
5.	$\sigma_m^2/\text{var}(\log x) = 0.20$	0.79	0.55	0.05	0.08	0.90	0.73	0.12	0.24
6.	$\sigma_m^2/\text{var}(\log x) = 0.50$	0.80	0.55	0.06	0.13	0.89	0.73	0.12	0.26
7.	$\sigma_m^2/\text{var}(\log x) = 0.80$	0.80	0.55	0.06	0.22	0.88	0.73	0.12	0.30
	Home Hours $x = \{h_N, h_P\}$								
8.	$\sigma_m^2/\text{var}(\log x) = 0.20$	0.78	0.55	0.06	0.06	0.92	0.74	0.12	0.24
9.	$\sigma_m^2/\text{var}(\log x) = 0.50$	0.78	0.55	0.06	0.06	0.88	0.73	0.13	0.25
10.	$\sigma_m^2/\text{var}(\log x) = 0.80$	0.78	0.55	0.06	0.06	0.78	0.70	0.14	0.25
	All Variables $x = \{c_M, h_M, h_N, h_P\}$								
11.	$\sigma_m^2/\text{var}(\log x) = 0.20$	0.74	0.51	0.05	0.11	0.89	0.70	0.12	0.25
12.	$\sigma_m^2/\text{var}(\log x) = 0.50$	0.63	0.41	0.05	0.21	0.77	0.60	0.13	0.29
13.	$\sigma_m^2/\text{var}(\log x) = 0.80$	0.46	0.26	0.05	0.30	0.52	0.40	0.14	0.33

Table 9 presents sensitivity analyses of the four inequality metrics for the model without home production ($\omega_K = 0$) and the model with home production ($\omega_K > 0$). The first row repeats the inequality statistics in the baseline model. Each row represents a different sensitivity analysis with respect to measurement error in consumption c_M, market hours h_M, home hours h_N and h_P, or all variables together. For each category, we present results when measurement error m accounts for 20, 50, and 80 percent of the variance of a particular or set of variables.

where x^* is the measurement-error adjusted value of variable x and m is a classical measurement error with variance σ_m^2.

In Table 9, rows 2 to 4 show results with measurement error in market consumption, rows 5 to 7 with measurement error in market hours, rows 8 to 10 with measurement error in home hours, and rows 11 to 13 with measurement error in all variables simultaneously. For each case we show measurement errors absorbing 20, 50, and 80 percent of the variance of the observed variable. Our process is to draw measurement error with variance σ_m^2 across households and then use simulated values x^*, which by construction display lower dispersion than reported values x, as the data input for inferring the sources of heterogeneity $(\alpha, \varepsilon, B, D_P, \theta_N)$ and measuring inequality.

We find small differences relative to our baseline results. Inequality tends to decline with measurement error in consumption, but not differentially across the two models. For market hours, measurement error affects only the optimal progressivity τ_1, but we always find that progressivity is higher in the home production model. Finally, most of our results are robust to measurement error of up to 80 percent of the variance of home hours. At that level, the dispersion in equivalent variation is the same between the two models. We still obtain higher inequality with home production using the other three metrics of inequality.

6 Other Datasets and Countries

We show the similarity of the inequality results between the CEX/ATUS and three alternative datasets, the Panel Study of Income Dynamics (PSID), the Japanese Panel Survey of Consumers (JPSC), and the Longitudinal Internet Studies for the Social Sciences from the Netherlands (LISS).

6.1 Comparison between CEX/ATUS and PSID

The PSID has two advantages relative to the CEX/ATUS. It has a panel dimension and contains information on both expenditures and time spent on home production. However, we prefer using the CEX/ATUS sample for our baseline analyses for three reasons. First, the PSID survey question covers aggregated time spent on home production, which does not allow us to separate credibly home hours h_N in the sector with efficiency heterogeneity from home hours h_P in the sector with disutility heterogeneity. Second, the PSID has lower quality of time use data as compared to the time diaries from the ATUS. In particular, it is not clear if respondents include activities such as child care and shopping in their reported home hours.[33] Third, food is the only measure of consumption which is consistently covered across surveys. Later surveys cover expanded categories but the sample size is significantly smaller than the CEX/ATUS sample.

We use two versions of the PSID. In the version in which c_M includes only expenditures on food, we have 69,951 observations between 1975 and 2014 for 10,992 households. In the version in which c_M includes food, utilities, child care expenses, clothing, home insurance, telecommunication, transportation, and home repairs, we have 13,626 observations between 2004 and 2014. PSID

[33] The survey question is "About how much time do you spend on housework in an average week? I mean time spent cooking, cleaning, and doing other work around the house."

does not have information to disaggregate time spent on home production between h_N and h_P. To make the analyses as comparable as possible to CEX/ATUS, we consider three cases. The first is when all home hours belong to h_N in the sector with efficiency heterogeneity. The second case, which is more comparable to our baseline in the CEX/ATUS, is that home hours are split equally between the two sectors. The third case is when all home hours belong to h_P in the sector with disutility heterogeneity.

Table 10 reassesses our conclusions regarding inequality.[34] The first panel repeats the findings of Table 7 in the CEX/ATUS for the four inequality metrics in the model without home production, the home production model with only efficiency heterogeneity, the baseline home production model with both efficiency and disutility heterogeneity, and the home production model with only disutility heterogeneity. The second panel reports these statistics for the version of the PSID which includes an expanded set of consumption categories. The third and fourth panels report these statistics for the CEX/ATUS and PSID datasets when we restrict our measure of consumption to only food.

Our conclusions regarding inequality and the prominent role of heterogeneity in home production efficiency are stable across the four datasets. First, the baseline model with home production generates higher inequality than the model without home production. Second, in the model with only efficiency heterogeneity, all inequality metrics are magnified relative to the baseline with both efficiency and disutility heterogeneity. Third, if there was only disutility heterogeneity, there would be no significant difference in inequality between the model with and the model without home production. The only significant change in the PSID relative to the CEX/ATUS is in the optimal progressivity τ_1 which displays a smaller difference between the two models.[35]

Our results using the PSID are particularly reassuring because we do not take a stance about the classification of time uses between h_N and h_P. Therefore, the result that inequality is higher with home production does not hinge on which activities are subject to efficiency heterogeneity

[34]To isolate differences arising from samples rather than parameter values, we keep parameters fixed at their values shown in Table 5. We follow a similar strategy with the JPSC and the LISS datasets later. The exception is the constant level of production efficiency θ_P which we calibrate in each dataset to hit the same target as in the CEX/ATUS.

[35]Appendix Figures A.2 and A.3 display the age profiles of means and variances of sources of heterogeneity $(\alpha, \varepsilon, B, D_P, \log \theta_N)$ from the version of the PSID with food in the baseline case which splits home hours equally between h_N and h_P. The difference relative to the means and variances we extracted using the CEX/ATUS is that we obtain these age profiles by regressing each source of heterogeneity on age and year dummies and an individual fixed effect. Therefore, these profiles reflect the within-household evolution of the sources of heterogeneity. Despite this difference, most of age profiles in the PSID are quantitatively similar to the age profiles in the CEX/ATUS.

Table 10: Inequality and Home Production: CEX/ATUS and PSID

CEX All Statistics	No Home Production	Home Production Efficiency	Home Production Baseline	Home Production Disutility
std(T)	0.78	1.14	0.90	0.76
std(t)	0.55	0.83	0.73	0.65
λ	0.06	0.20	0.12	0.03
τ_1	0.06	0.32	0.24	0.13
PSID All Statistics	**No Home Production**	**Efficiency**	**Baseline**	**Disutility**
std(T)	0.58	0.85	0.63	0.56
std(t)	0.40	0.61	0.51	0.45
λ	0.11	0.18	0.15	0.10
τ_1	0.33	0.36	0.33	0.29
CEX Food Statistics	**No Home Production**	**Efficiency**	**Baseline**	**Disutility**
std(T)	0.82	1.15	0.92	0.80
std(t)	0.56	0.84	0.75	0.67
λ	0.04	0.20	0.11	0.02
τ_1	-0.05	0.29	0.21	0.09
PSID Food Statistics	**No Home Production**	**Efficiency**	**Baseline**	**Disutility**
std(T)	0.57	0.87	0.63	0.55
std(t)	0.40	0.62	0.51	0.45
λ	0.09	0.18	0.14	0.09
τ_1	0.28	0.33	0.29	0.24

Table 10 shows the four inequality metrics for the model without home production, the home production model with only efficiency heterogeneity, the baseline home production model with both efficiency and disutility heterogeneity, and the home production model with only disutility heterogeneity. Parameters τ_0, τ_1, and ϕ are held constant to their values shown in Table 5. For each column, the values for η are given by 0.90, 0.53, 0.50, and 0.57 (constant across panels). The estimated value of θ_P is 4.64 for the baseline home production model and 9.74 for the model with only heterogeneity in disutility of home work in the first panel; 4.01 and 6.68 in the second panel; 4.65 and 9.74 in the third panel; 3.80 and 6.31 in the fourth panel.

and which activities are subject to disutility heterogeneity. What is important for this result is that some portion of home production time is subject to heterogeneity in production efficiency.

6.2 Comparison between US, Japan, and the Netherlands

In this section, we repeat our analyses using datasets from other countries. As in the PSID, these datasets have limited information to disaggregate time spent on home production between h_N and h_P. To make the analyses comparable to CEX/ATUS and PSID, we consider the three cases of all home hours belonging to h_N in the sector with efficiency heterogeneity, of splitting home hours equally between the two sectors, and of all home hours belonging to h_P in the sector with disutility heterogeneity. We apply the same sampling restrictions as in the CEX/ATUS and focus our analyses on married households.

The first dataset is the Japanese Panel Survey of Consumers (JPSC; see, for example, Lise and Yamada, 2019). The JPSC records information for time spent on commuting, working, studying, home production and child care, leisure, and sleeping, personal care and eating. For aggregate home hours $h_N + h_P$ we use the variable for home production and child care and for market hours we use hours worked. To calculate the home and market hours for a given week, we weight the time use on workdays and days off by the number of days worked. The final dataset has 12,423 observations between 1998 and 2014. The second dataset is the Longitudinal Internet Studies for the Social Sciences from the Netherlands (LISS; see, for example, Cherchye, Demuynck, De Rock, and Vermeulen, 2017), administered by CentERdata. The dataset is based on a representative sample of Dutch households who participate in monthly surveys. We use the three waves (2009, 2010, and 2012) which contain information on time use. Home production time includes household chores, child care, and administrative chores. Market hours are measured by time spent on paid work, which includes commuting time. The final dataset has 978 observations.[36]

Table 11 summarizes our results. The upper panel repeats our findings in the CEX/ATUS and the other panels show inequality statistics in the JPSC and the LISS. Our conclusions regarding inequality and the role of production efficiency heterogeneity are stable in other countries as well. Namely, the baseline model with home production always generates higher inequality than the model without home production. All inequality statistics are magnified in the home production model with only efficiency heterogeneity, while with only disutility heterogeneity there would be

[36]For both datasets we choose a measure of consumption expenditures that is as comparable as possible to the various measures we used from the CEX and the PSID. For JPSC, consumption expenditures includes food, utilities, apparel, transport, culture and leisure, communication, trips and activities, house and land rent, durables, health, transportation, education, allowances and alimony. For LISS, consumption expenditures include food, utilities, home maintenance, transportation, daycare, child support, rent, mortgage payments, and other debt and loan payments.

Table 11: Inequality and Home Production: US, Japan, and the Netherlands

CEX/ATUS Statistics	No Home Production	Home Production Efficiency	Home Production Baseline	Home Production Disutility
std(T)	0.78	1.14	0.90	0.76
std(t)	0.55	0.83	0.73	0.65
λ	0.06	0.20	0.12	0.03
τ_1	0.06	0.32	0.24	0.13
JPSC Statistics	**No Home Production**	**Home Production Efficiency**	**Home Production Baseline**	**Home Production Disutility**
std(T)	0.65	1.00	0.77	0.67
std(t)	0.47	0.69	0.60	0.56
λ	0.04	0.11	0.07	0.02
τ_1	-0.06	0.23	0.15	0.07
LISS Statistics	**No Home Production**	**Home Production Efficiency**	**Home Production Baseline**	**Home Production Disutility**
std(T)	0.53	1.04	0.74	0.56
std(t)	0.39	0.70	0.56	0.46
λ	0.06	0.20	0.13	0.04
τ_1	0.10	0.26	0.19	0.12

Table 11 shows the four inequality metrics for the model without home production, the home production model with only efficiency heterogeneity, the baseline home production model with both efficiency and disutility heterogeneity, and the home production model with only disutility heterogeneity. Parameters τ_0, τ_1, and ϕ are held constant to their values shown in Table 5. For each column, the values for η are given by 0.90, 0.53, 0.50, and 0.57 (constant across panels). The estimated value of θ_P is 4.64 for the baseline home production model and 9.74 for the model with only heterogeneity in disutility of home work in the upper panel; 3.54 and 5.91 in the middle panel; 3.46 and 5.78 in the lower panel.

no significant difference in inequality between the models with and without home production.

7 Conclusion

The literature examining the causes, welfare consequences, and policy implications of the substantial labor market dispersion across households typically abstracts from the possibility that households can produce goods and services outside of the market sector. We revisit these issues taking into account that households spend a significant amount of their time in home production. Our model incorporates non-separable preferences between expenditures and time and heterogeneity in home production efficiency and disutility of home work into an incomplete markets model with uninsurable risk.

Separating disutility of work from production efficiency at home presents a challenge for home production models. Unlike expenditures and time inputs, the output of the home sector is not observable. We make progress on the identification of home production models by posing that some of the cross-sectional differences in time spent working at home are driven by heterogeneity in production efficiency and the remaining differences are driven by heterogeneity in the disutility of work. We discipline this split by mapping time uses to occupations and using the manual skill intensity of tasks within occupations to classify time uses into the two sectors. We envision how data on the market value of home output would improve the credibility of identifying these sources of heterogeneity but, to our knowledge, no such comprehensive data currently exist.

We reach several substantial conclusions. Allowing households to be heterogeneous in both their home production efficiency and disutility of work, we find that home production amplifies welfare-based differences across households and inequality is larger than we thought. Our result is surprising given that a priori one could expect home production to compress welfare differences originating in the market sector when households are sufficiently willing to substitute between market expenditures and time in the production of home goods. We show that home production efficiency is an important source of within-age and life-cycle differences in consumption expenditures and time allocation across households. Through the lens of the model we infer that home production does not offset differences that originate in the market sector because production efficiency differences in the home sector are significant and the time input in home production does not covary negatively with consumption and wages in the cross section of households.

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Inferring Inequality with Home Production

Online Appendix

Job Boerma and Loukas Karabarbounis

A Proofs

In this appendix, we derive the equilibrium allocations presented in Table 1 in the main text and prove the observational equivalence theorem. We proceed in four steps. First, in anticipation of the no-trade result, we solve the planner problems. Second, we postulate equilibrium allocations and prices using the solutions to the planner problems. Third, we establish that the postulated equilibrium allocations and prices indeed constitute an equilibrium as defined in Section 2 in the main text. Finally, we show how to invert the equilibrium allocations and identify the sources of heterogeneity leading to these allocations.

A.1 Preliminaries

In what follows, we define the following state vectors. The sources of heterogeneity differentiating households within each island ℓ is given by the vector ζ^j:

$$\zeta_t^j = (\kappa_t^j, \upsilon_t^\varepsilon) \in Z_t^j. \tag{A.1}$$

Households can trade bonds within each island contingent on the vector s^j:

$$s_t^j = (B_t^j, \alpha_t^j, \kappa_t^j, \upsilon_t^\varepsilon). \tag{A.2}$$

We define a household ι by a sequence of all dimensions of heterogeneity:

$$\iota = \{\theta_K^j, D_K^j, B^j, \alpha^j, \kappa^j, \upsilon^\varepsilon\}. \tag{A.3}$$

Finally, the history of all sources of heterogeneity up to period t is given by the vector:

$$\sigma_t^j = (\theta_{K,t}^j, D_{K,t}^j, B_t^j, \alpha_t^j, \kappa_t^j, \upsilon_t^\varepsilon, ..., \theta_{K,j}^j, D_{K,j}^j, B_j^j, \alpha_j^j, \kappa_j^j, \upsilon_j^\varepsilon). \tag{A.4}$$

We denote conditional probabilities by $f^{t,j}(\cdot|\cdot)$. For example, the probability that we observe σ_t^j conditional on σ_{t-1}^j is $f^{t,j}(\sigma_t^j|\sigma_{t-1}^j)$ and the probability that we observe s_t^j conditional on s_{t-1}^j is $f^{t,j}(s_t^j|s_{t-1}^j)$.

We use υ to denote innovations to processes and Φ_υ to denote the distribution of the innovation. We allow the distributions of innovations to vary over time, $\{\Phi_{\upsilon_t^\alpha}, \Phi_{\upsilon_t^B}, \Phi_{\upsilon_t^\kappa}, \Phi_{\upsilon_t^\varepsilon}, \Phi^j_{\theta_{K,t}}, \Phi^j_{D_{K,t}}\}$, and the initial distributions to vary by cohorts j, $\Phi^j_j(\theta^j_{K,j}, D^j_{K,j}, B^j_j, \alpha^j_j, \kappa^j_j)$. We assume that both $\theta^j_{K,t}$ and $D^j_{K,t}$ are orthogonal to the innovations $\{\upsilon_t^B, \upsilon_t^\alpha, \upsilon_t^\kappa, \upsilon_t^\varepsilon\}$ and that all innovations are drawn independently from each other.

A.2 Planner Problems

In every period t and in every island ℓ, the planner solves a static problem which consists of finding the allocations maximizing average utility for households on the island subject to an aggregate resource constraint. We omit j, t and ℓ from the notation for clarity.

A.2.1 No Home Production, $\omega_K = 0$

The planner chooses an allocation $\{c_M, h_M\}$ to maximize:

$$\int_Z \left[\frac{c_M^{1-\gamma} - 1}{1-\gamma} - \frac{(\exp(B) h_M)^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \right] \mathrm{d}\Phi_\zeta(\zeta) \ , \tag{A.5}$$

subject to an island resource constraint for market goods:

$$\int_Z c_M \mathrm{d}\Phi_\zeta(\zeta) = \int_Z \tilde{z}_M h_M \mathrm{d}\Phi_\zeta(\zeta) \, . \tag{A.6}$$

Denoting by $\mu(\alpha, B)$ the multiplier on the island resource constraint, the solution is characterized by the following first-order conditions (for every household ι):

$$[c_M] : \ c_M^{-\gamma} = \mu(\alpha, B), \tag{A.7}$$

$$[h_M] : \ \exp(B)^{1+\frac{1}{\eta}} h_M{}^{\frac{1}{\eta}} = \tilde{z}_M \mu(\alpha, B). \tag{A.8}$$

Equation (A.7) implies that market consumption is equal for every household ι on the island and, thus, there is full consumption insurance. Combining equations (A.6) to (A.8), we solve for market consumption and market hours for every ι:

$$c_M = \left[\frac{\int_Z \tilde{z}_M^{1+\eta} \mathrm{d}\Phi_\zeta(\zeta)}{\exp\left(\eta\left(1+\frac{1}{\eta}\right) B\right)} \right]^{\frac{\frac{1}{\eta}}{\frac{1}{\eta}+\gamma}} , \tag{A.9}$$

$$h_M = \tilde{z}_M^{\eta} \frac{\left[\int_Z \tilde{z}_M^{1+\eta} \mathrm{d}\Phi_\zeta(\zeta)\right]^{-\frac{\gamma}{\frac{1}{\eta}+\gamma}}}{\exp\left(\left(1+\frac{1}{\eta}\right) B\right)^{\frac{1}{\eta}+\gamma}} \, . \tag{A.10}$$

A.2.2 Home Production, $\omega_K > 0$

The planner chooses $\{c_M, h_M, h_K\}$ to maximize:

$$\int_Z \left[\log c - \frac{\left(\exp(B) h_M + \sum \exp(D_K) h_K \right)^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \right] \mathrm{d}\Phi_\zeta(\zeta), \tag{A.11}$$

where consumption is given by $c = \left(c_M{}^{\frac{\phi-1}{\phi}} + \sum (\theta_K h_K)^{\frac{\phi-1}{\phi}} \right)^{\frac{\phi}{\phi-1}}$ subject to the island market resource constraint (A.6).

Denoting by $\mu(\alpha, B, D_K, \theta_K)$ the multiplier on the island resource constraint, the solution to this problem is characterized by the following first-order conditions (for every household ι):

$$[c_M] : \left(c^{\frac{\phi-1}{\phi}} \right)^{-1} c_M^{-\frac{1}{\phi}} = \mu(\alpha, B, D_K, \theta_K), \tag{A.12}$$

$$[h_M] : \left(\exp(B) h_M + \sum \exp(D_K) h_K \right)^{\frac{1}{\eta}} = \tilde{z}_M \frac{\mu(\alpha, B, D_K, \theta_K)}{\exp(B)}, \tag{A.13}$$

$$[h_K] : \left(\exp(B) h_M + \sum \exp(D_K) h_K \right)^{\frac{1}{\eta}} = \theta_K^{\frac{\phi-1}{\phi}} \left(c^{\frac{\phi-1}{\phi}} \right)^{-1} \frac{h_K^{-\frac{1}{\phi}}}{\exp(D_K)} \tag{A.14}$$

Combining equations (A.12) to (A.14), we solve for the ratio of home hours to consumption:

$$\frac{c_M}{h_K} = \left(\frac{\exp(D_K)}{\exp(B)/\tilde{z}_M} \right)^{\phi} \theta_K^{1-\phi}. \tag{A.15}$$

Substituting these ratios into equations (A.12) to (A.14), we derive:

$$c_M = \frac{1}{\mu(\alpha, B, D_K, \theta_K)} \frac{1}{1 + \sum \theta_K^{\phi-1} \left(\frac{\exp(B)/\tilde{z}_M}{\exp(D_K)} \right)^{\phi-1}}, \tag{A.16}$$

$$h_K = \frac{1}{\mu(\alpha, B, D_K, \theta_K)} \frac{\theta_K^{\phi-1} \left(\frac{\exp(B)/\tilde{z}_M}{\exp(D_K)} \right)^{\phi}}{1 + \sum \theta_K^{\phi-1} \left(\frac{\exp(B)/\tilde{z}_M}{\exp(D_K)} \right)^{\phi-1}}. \tag{A.17}$$

These expressions yield solutions for $\{c_M, h_M, h_K\}$ given a multiplier $\mu(\alpha, B, D_K, \theta_K)$. The multiplier is equal to the inverse of the market value of total consumption:

$$c_M + \tilde{z}_M \sum \frac{\exp(D_K)}{\exp(B)} h_K = \frac{1}{\mu(\alpha, B, D_K, \theta_K)}. \tag{A.18}$$

The equality follows from equations (A.16) to (A.17).

Substituting equation (A.13) into equation (A.6), we obtain the solution for $\mu(\alpha, B, D_K, \theta_K)$:

$$\mu(\alpha, B, D_K, \theta_K) = \frac{\exp(B)}{\left(\int_Z \tilde{z}_M^{1+\eta} \mathrm{d}\Phi_\zeta(\zeta) \right)^{\frac{1}{1+\eta}}}. \tag{A.19}$$

The denominator is an expectation independent of ζ. Therefore, μ is independent of ζ. We also note that $\mu(\alpha, B, D_K, \theta_K)$ in the model with home production equals $\mu(\alpha, B)$ in the model without home production under $\gamma = 1$. Given this solution for $\mu(\alpha, B, D_K, \theta_K)$, we obtain the solutions:

$$c_M = \frac{\left[\int_Z \tilde{z}_M^{1+\eta} \mathrm{d}\Phi_\zeta(\zeta)\right]^{\frac{1}{1+\eta}}}{\exp(B)} \frac{1}{1 + \sum \theta_K^{\phi-1} \left(\frac{\exp(B)/\tilde{z}_M}{\exp(D_K)}\right)^{\phi-1}}, \tag{A.20}$$

$$h_K = \frac{\left[\int_Z \tilde{z}_M^{1+\eta} \mathrm{d}\Phi_\zeta(\zeta)\right]^{\frac{1}{1+\eta}}}{\exp(B)} \frac{\theta_K^{\phi-1} \left(\frac{\exp(B)/\tilde{z}_M}{\exp(D_K)}\right)^{\phi}}{1 + \sum \theta_K^{\phi-1} \left(\frac{\exp(B)/\tilde{z}_M}{\exp(D_K)}\right)^{\phi-1}}, \tag{A.21}$$

$$h_M = \tilde{z}_M^{\eta} \frac{\left[\int_Z \tilde{z}_M^{1+\eta} \mathrm{d}\Phi_\zeta(\zeta)\right]^{-\frac{1}{1+\frac{1}{\eta}}}}{\exp\big(B\big)} - \sum \frac{\exp(D_K)}{\exp(B)} h_K.$$

A.3 Postulating Equilibrium

We postulate an equilibrium in four steps.

1. We postulate that the equilibrium features no trade across islands, $x(\zeta_{t+1}^j; \iota) = 0, \forall \iota, \zeta_{t+1}^j$.

2. We postulate that the solutions $\{c_{M,t}, h_{M,t}\}$ for the model without home production and $\{c_{M,t}, h_{M,t}, h_{K,t}\}$ for the model with home production from the planner problems in Section A.2 constitute components of the equilibrium for each model.

3. We use the sequential budget constraints to postulate equilibrium holdings for the state-contingent bonds $b^\ell(s_t^j; \iota)$ which are traded within islands. For the models without home production these are given by:

$$b^\ell(s_t^j; \iota) = \mathbb{E}\left[\sum_{n=0}^{\infty} (\beta\delta)^n \frac{\mu_{t+n}(\alpha_{t+n}^j, B_{t+n}^j)}{\mu_t(\alpha_t^j, B_t^j)} \left(c_{M,t+n} - \tilde{y}_{t+n}\right)\right], \tag{A.22}$$

   where $\tilde{y} = \tilde{z}_M h_M = (1-\tau_0) z_M^{1-\tau_1} h_M$ is after-tax labor income.

   For the model with home production, state-contingent bonds $b^\ell(s_t^j; \iota)$ are given by the same expression but using the marginal utility $\mu(\alpha, B, D_K, \theta_K)$ instead of $\mu(\alpha, B)$. As shown above, the two marginal utilities are characterized by the same equation (A.19) under $\gamma = 1$.

4. We use the intertemporal marginal rates of substitution implied by the planner solutions to postulate asset prices for $b^\ell(s_{t+1}^j; \iota)$ and $x(\zeta_{t+1}^j; \iota)$. For the model without home production,

we obtain:

$$q_b^\ell(s_{t+1}^j) = \beta\delta \exp\left(\gamma\frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma}\upsilon_{t+1}^B\right)\exp\left(-\left(1-\tau_1\right)\gamma\frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma}\upsilon_{t+1}^\alpha\right)$$

$$\times\left[\frac{\int\exp\left(A\upsilon_{t+1}^\kappa\right)\mathrm{d}\Phi_{\upsilon_{t+1}^\kappa}(\upsilon_{t+1}^\kappa)\int\exp\left(A\upsilon_{t+1}^\varepsilon\right)\mathrm{d}\Phi_{\upsilon_{t+1}^\varepsilon}(\upsilon_{t+1}^\varepsilon)}{\int\exp\left(A\upsilon_t^\varepsilon\right)\mathrm{d}\Phi_{\upsilon_t^\varepsilon}(\upsilon_t^\varepsilon)}\right]^{-\frac{\frac{\gamma}{\eta}}{\frac{1}{\eta}+\gamma}} f^{t+1,j}(s_{t+1}^j|s_t^j), \quad \text{(A.23)}$$

$$q_x(Z_{t+1}) = \beta\delta\int\exp\left(\gamma\frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma}\upsilon_{t+1}^B\right)\mathrm{d}\Phi_{\upsilon_{t+1}^B}(\upsilon_{t+1}^B)\int\exp\left(-\left(1-\tau_1\right)\gamma\frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma}\upsilon_{t+1}^\alpha\right)\mathrm{d}\Phi_{\upsilon_{t+1}^\alpha}(\upsilon_{t+1}^\alpha)$$

$$\times\left[\frac{\int\exp\left(A\upsilon_{t+1}^\kappa\right)\mathrm{d}\Phi_{\upsilon_{t+1}^\kappa}(\upsilon_{t+1}^\kappa)\int\exp\left(A\upsilon_{t+1}^\varepsilon\right)\mathrm{d}\Phi_{\upsilon_{t+1}^\varepsilon}(\upsilon_{t+1}^\varepsilon)}{\int\exp\left(A\upsilon_t^\varepsilon\right)\mathrm{d}\Phi_{\upsilon_t^\varepsilon}(\upsilon_t^\varepsilon)}\right]^{-\frac{\frac{\gamma}{\eta}}{\frac{1}{\eta}+\gamma}}\mathbb{P}\left(\left(\upsilon_{t+1}^\kappa,\upsilon_{t+1}^\varepsilon\right)\in Z_{t+1}\right), \quad \text{(A.24)}$$

where $A \equiv (1+\eta)(1-\tau_1)$. For the model with home production, we obtain the same expressions under $\gamma = 1$.

A.4 Verifying the Equilibrium Allocations and Prices

We verify that the equilibrium postulated in Section A.3 constitutes an equilibrium by showing that the postulated allocations solve the households' problem and that all markets clear.

A.4.1 Household Problem

The problem for a household ι born in period j is described in the main text. We denote the Lagrange multiplier on the household's budget constraint by $\tilde{\mu}_t$. We drop ι from the notation for simplicity.

No Home Production, $\omega_K = 0$. The optimality conditions are:

$$(\beta\delta)^{t-j}\, c_{M,t}^{-\gamma} f^{t,j}(\sigma_t^j|\sigma_j) = \tilde{\mu}_t, \quad \text{(A.25)}$$

$$(\beta\delta)^{t-j}\exp(B_t)^{1+\frac{1}{\eta}}\left(h_{M,t}\right)^{\frac{1}{\eta}} f^{t,j}(\sigma_t^j|\sigma_j) = \tilde{z}_{M,t}^j\tilde{\mu}_t, \quad \text{(A.26)}$$

$$q_b^\ell(s_{t+1}^j) = \frac{\tilde{\mu}_{t+1}}{\tilde{\mu}_t}, \quad \text{(A.27)}$$

$$q_x(Z_{t+1}) = \int\frac{\tilde{\mu}_{t+1}}{\tilde{\mu}_t}\mathrm{d}\upsilon_{t+1}^B\mathrm{d}\upsilon_{t+1}^\alpha. \quad \text{(A.28)}$$

Comparing the planner solutions to the household solutions we verify that they coincide for market consumption and hours when the multipliers are related by:

$$\tilde{\mu}_t = (\beta\delta)^{t-j}\, f^{t,j}(\sigma_t^j|\sigma_j)\mu(\alpha_t^j, B_t^j). \quad \text{(A.29)}$$

Then, the Euler equations become:

$$q_b^\ell(s_{t+1}^j) = \beta\delta \frac{\mu(\alpha_{t+1}^j, B_{t+1}^j)}{\mu(\alpha_t^j, B_t^j)} f^{t+1,j}(\sigma_{t+1}^j | \sigma_t^j), \tag{A.30}$$

$$q_x(Z_{t+1}) = \beta\delta \int \frac{\mu(\alpha_{t+1}^j, B_{t+1}^j)}{\mu(\alpha_t^j, B_t^j)} f^{t+1,j}(\sigma_{t+1}^j | \sigma_t^j) \mathrm{d}\upsilon_{t+1}^B \mathrm{d}\upsilon_{t+1}^\alpha. \tag{A.31}$$

Home Production, $\omega_K > 0$. Total hours, taking into account the respective disutility, are $\tilde{h} = \exp(B)(h_M) + \sum \exp(D_K)(h_K)$. Using again the correspondence between the planner and the household first-order conditions to relate the multipliers $\tilde{\mu}_t$ and $\mu(\alpha_t^j, B_t^j, D_{K,t}^j, \theta_{K,t}^j)$, we write the optimality conditions as:

$$\frac{\tilde{z}_{M,t}}{\exp(B_t)} \left(c^{\frac{\phi-1}{\phi}}\right)^{-1} c_{M,t}^{-\frac{1}{\phi}} = \tilde{h}_t^{\frac{1}{\eta}}, \tag{A.32}$$

$$\frac{\theta_{K,t}^{\frac{\phi-1}{\phi}}}{\exp(D_{K,t})} \left(c^{\frac{\phi-1}{\phi}}\right)^{-1} h_{K,t}^{-\frac{1}{\phi}} = \tilde{h}_t^{\frac{1}{\eta}}, \tag{A.33}$$

$$q_b^\ell(s_{t+1}^j) = \beta\delta \int \frac{\mu(\alpha_{t+1}^j, B_{t+1}^j, D_{K,t+1}^j, \theta_{K,t+1}^j)}{\mu(\alpha_t^j, B_t^j, D_{K,t}^j, \theta_{K,t}^j)} f^{t+1,j}(\sigma_{t+1}^j | \sigma_t^j) \mathrm{d}\theta_{K,t+1}^j \mathrm{d}D_{K,t+1}^j, \tag{A.34}$$

$$q_x(Z_{t+1}) = \beta\delta \int \frac{\mu(\alpha_{t+1}^j, B_{t+1}^j, D_{K,t+1}^j, \theta_{K,t+1}^j)}{\mu(\alpha_t^j, B_t^j, D_{K,t}^j, \theta_{K,t}^j)} f^{t+1,j}(\sigma_{t+1}^j | \sigma_t^j) \mathrm{d}\upsilon_{t+1}^B \mathrm{d}\upsilon_{t+1}^\alpha \mathrm{d}\theta_{K,t+1}^j \mathrm{d}D_{K,t+1}^j. \tag{A.35}$$

A.4.2 Euler Equations

We next verify that the Euler equations are satisfied at the postulated allocations and prices.

No Home Production, $\omega_K = 0$. Using the marginal utility of market consumption of the planner problem $\mu(\alpha_t^j, B_t^j)$, we write the Euler equation for the state-contingent bonds $b^\ell(s_{t+1}^j)$ at the postulated equilibrium as:

$$\begin{aligned} q_b^\ell(s_{t+1}^j) &= \beta\delta \frac{\mu(\alpha_{t+1}^j, B_{t+1}^j)}{\mu(\alpha_t^j, B_t^j)} f^{t+1,j}(\sigma_{t+1}^j | \sigma_t^j) \qquad \text{(A.36)} \\ &= \beta\delta \frac{\exp\left(\gamma \frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma} B_{t+1}^j\right) \left[\int \left(\tilde{z}_{M,t+1}^j\right)^{1+\eta} \mathrm{d}\Phi_{\zeta_{t+1}^j}(\zeta_{t+1}^j)\right]^{-\frac{\frac{\gamma}{\eta}}{\frac{1}{\eta}+\gamma}}}{\exp\left(\gamma \frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma} B_t^j\right) \left[\int \left(\tilde{z}_{M,t}^j\right)^{1+\eta} \mathrm{d}\Phi_{\zeta_t^j}(\zeta_t^j)\right]^{-\frac{\frac{\gamma}{\eta}}{\frac{1}{\eta}+\gamma}}} f^{t+1,j}(\sigma_{t+1}^j | \sigma_t^j), \end{aligned}$$

where the second line follows from equations (A.7) and (A.9). Using that B_t^j follows a random walk-process with innovation υ_t^B we rewrite $q_b^\ell(s_{t+1}^j)$ as:

$$q_b^\ell(s_{t+1}^j) = \beta\delta \exp\left(\gamma \frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma}\upsilon_{t+1}^B\right) \frac{\left[\int \left(\tilde{z}_{M,t+1}^j\right)^{1+\eta} \mathrm{d}\Phi_{\zeta_{t+1}^j}(\zeta_{t+1}^j)\right]^{-\frac{\frac{\gamma}{\eta}}{\frac{1}{\eta}+\gamma}}}{\left[\int \left(\tilde{z}_{M,t}^j\right)^{1+\eta} \mathrm{d}\Phi_{\zeta_t^j}(\zeta_t^j)\right]^{-\frac{\frac{\gamma}{\eta}}{\frac{1}{\eta}+\gamma}}} f^{t+1,j}(s_{t+1}^j|s_t^j). \tag{A.37}$$

To simplify the fraction in $q_b^\ell(s_{t+1}^j)$ we use that:

$$\tilde{z}_{M,t+1}^j = (1-\tau_0)\exp\left((1-\tau_1)\left(\alpha_t^j + \upsilon_{t+1}^\alpha + \kappa_t^j + \upsilon_{t+1}^\kappa + \upsilon_{t+1}^\varepsilon\right)\right).$$

The expectation over the random variables in the numerator is given by:

$$\begin{aligned}&\int \exp\left(A\left(\kappa_t^j + \upsilon_{t+1}^\kappa + \upsilon_{t+1}^\varepsilon\right)\right)\mathrm{d}\Phi_{\zeta_{t+1}^j}(\zeta_{t+1}^j)\\ =&\int \exp(A\kappa_t^j)\mathrm{d}\Phi_{\kappa_t^j}(\kappa_t^j)\int \exp\left(A\upsilon_{t+1}^\kappa\right)\mathrm{d}\Phi_{\upsilon_{t+1}^\kappa}(\upsilon_{t+1}^\kappa)\int \exp\left(A\upsilon_{t+1}^\varepsilon\right)\mathrm{d}\Phi_{\upsilon_{t+1}^\varepsilon}(\upsilon_{t+1}^\varepsilon)\ ,\end{aligned} \tag{A.38}$$

where the final equality follows from the assumption that the innovations are drawn independently. Similarly, the expectation over the random variables in the denominator equals:

$$\int \exp(A\kappa_t^j)\mathrm{d}\Phi_{\kappa^j,t}(\kappa_t^j)\int \exp\left(A\upsilon_t^\varepsilon\right)\mathrm{d}\Phi_{\upsilon_t^\varepsilon}(\upsilon_t^\varepsilon). \tag{A.39}$$

As a result, the price $q_b^\ell(s_{t+1}^j)$ is:

$$\begin{aligned}q_b^\ell(s_{t+1}^j) =& \beta\delta\exp\left(\gamma\frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma}\upsilon_{t+1}^B\right)\exp\left(-(1-\tau_1)\gamma\frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma}\upsilon_{t+1}^\alpha\right)\\ &\times\left[\frac{\int\exp\left(A\upsilon_{t+1}^\kappa\right)\mathrm{d}\Phi_{\upsilon_{t+1}^\kappa}(\upsilon_{t+1}^\kappa)\int\exp\left(A\upsilon_{t+1}^\varepsilon\right)\mathrm{d}\Phi_{\upsilon_{t+1}^\varepsilon}(\upsilon_{t+1}^\varepsilon)}{\int\exp\left(A\upsilon_t^\varepsilon\right)\mathrm{d}\Phi_{\upsilon_t^\varepsilon}(\upsilon_t^\varepsilon)}\right]^{-\frac{\frac{\gamma}{\eta}}{\frac{1}{\eta}+\gamma}} f^{t+1,j}(s_{t+1}^j|s_t^j),\end{aligned} \tag{A.40}$$

where $f^{t+1,j}(s_{t+1}^j|s_t^j) = f(\upsilon_{t+1}^B)f(\upsilon_{t+1}^\alpha)f(\upsilon_{t+1}^\kappa)f(\upsilon_{t+1}^\varepsilon)$. This confirms our guess in equation (A.23). The key observation is that the distributions for next-period innovations are independent of the current period state and, therefore, the term in square brackets is independent of the state vector which differentiates islands ℓ. As a result, all islands ℓ have the same state-contingent bond prices, $q_b^\ell(s_{t+1}^j) = Q_b\left(\upsilon_{t+1}^B, \upsilon_{t+1}^\alpha\right)$.

We next calculate the state-contingent bond price for a set of states $\mathcal{V}_{t+1} \subseteq \mathbb{V}_{t+1}$:

$$\begin{aligned}q_b^\ell(\mathcal{V}_{t+1}) =& \beta\delta\int_{\mathcal{V}^B}\exp\left(\gamma\frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma}\upsilon_{t+1}^B\right)\mathrm{d}\Phi_{\upsilon_{t+1}^B}\left(\upsilon_{t+1}^B\right)\int_{\mathcal{V}^\alpha}\exp\left(-(1-\tau_1)\gamma\frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma}\upsilon_{t+1}^\alpha\right)\mathrm{d}\Phi_{\upsilon_{t+1}^\alpha}\left(\upsilon_{t+1}^\alpha\right)\\ &\times\left[\frac{\int\exp\left(A\upsilon_{t+1}^\kappa\right)\mathrm{d}\Phi_{\upsilon_{t+1}^\kappa}(\upsilon_{t+1}^\kappa)\int\exp\left(A\upsilon_{t+1}^\varepsilon\right)\mathrm{d}\Phi_{\upsilon_{t+1}^\varepsilon}(\upsilon_{t+1}^\varepsilon)}{\int\exp\left(A\upsilon_t^\varepsilon\right)\mathrm{d}\Phi_{\upsilon_t^\varepsilon}(\upsilon_t^\varepsilon)}\right]^{-\frac{\frac{\gamma}{\eta}}{\frac{1}{\eta}+\gamma}}.\end{aligned} \tag{A.41}$$

Similarly, all islands face the same price $q_b^\ell(\mathcal{V}_{t+1}) = Q_b(\mathcal{V}_{t+1})$.

Finally, we calculate the price for a claim which does not depend on the realization of $(\upsilon_{t+1}^B, \upsilon_{t+1}^\alpha)$:

$$q_b^\ell(\mathbb{V}_{t+1}) = \beta\delta \int_{\mathbb{V}^B} \exp\left(\gamma \frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma} \upsilon_{t+1}^B\right) \mathrm{d}\Phi_{\upsilon_{t+1}^B}\left(\upsilon_{t+1}^B\right) \int_{\mathbb{V}^\alpha} \exp\left(-\left(1-\tau_1\right)\gamma \frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma} \upsilon_{t+1}^\alpha\right) \mathrm{d}\Phi_{\upsilon_{t+1}^\alpha}\left(\upsilon_{t+1}^\alpha\right)$$
$$\times \left[\frac{\int \exp\left(A\upsilon_{t+1}^\kappa\right) \mathrm{d}\Phi_{\upsilon_{t+1}^\kappa}(\upsilon_{t+1}^\kappa) \int \exp\left(A\upsilon_{t+1}^\varepsilon\right) \mathrm{d}\Phi_{\upsilon_{t+1}^\varepsilon}(\upsilon_{t+1}^\varepsilon)}{\int \exp\left(A\upsilon_t^\varepsilon\right) \mathrm{d}\Phi_{\upsilon_t^\varepsilon}(\upsilon_t^\varepsilon)}\right]^{-\frac{\frac{\gamma}{\eta}}{\frac{1}{\eta}+\gamma}}. \tag{A.42}$$

All islands face the same price $q_b^\ell(\mathbb{V}_{t+1}) = Q_b(\mathbb{V}_{t+1})$.

By no arbitrage, the prices of bonds x and b which are contingent on the same set of states must be equalized. Therefore, the price of a claim traded across islands for some set Z_{t+1} is equalized across islands at the no-trade equilibrium and given by:

$$q_x(Z_{t+1}) = \mathbb{P}\left((\upsilon_{t+1}^\kappa, \upsilon_{t+1}^\varepsilon) \in Z_{t+1}\right) Q_b(\mathbb{V}_{t+1}), \tag{A.43}$$

where $\mathbb{P}\left((\upsilon_{t+1}^\kappa, \upsilon_{t+1}^\varepsilon) \in Z_{t+1}\right)$ is the probability of $(\upsilon_{t+1}^\kappa, \upsilon_{t+1}^\varepsilon)$ being a member of Z_{t+1}. The expression for $q_x(Z_{t+1})$ confirms our guess in equation (A.24)

Home Production, $\omega_K > 0$. For the model with home production, we use the solution for the marginal utility of market consumption in the planner problem $\mu(\alpha_t^j, B_t^j, D_{K,t}^j, \theta_{K,t}^j)$ to write the Euler equation for the state-contingent bonds $b^\ell(s_{t+1}^j)$ at the postulated equilibrium as:

$$q_b^\ell(s_{t+1}^j) = \beta\delta \int \frac{\mu(\alpha_{t+1}^j, B_{t+1}^j, D_{K,t+1}^j, \theta_{K,t+1}^j)}{\mu(\alpha_t^j, B_t^j, D_{K,t}^j, \theta_{K,t}^j)} f^{t+1,j}(\sigma_{t+1}^j|\sigma_t^j) \mathrm{d}\theta_{K,t+1}^j \mathrm{d}D_{K,t+1}^j \tag{A.44}$$
$$= \beta\delta \int \frac{\exp\left(B_{t+1}^j\right)\left[\int \left(\tilde{z}_{M,t+1}^j\right)^{1+\eta} \mathrm{d}\Phi_{\zeta_{t+1}^j}(\zeta_{t+1}^j)\right]^{-\frac{1}{1+\eta}}}{\exp\left(B_t^j\right)\left[\int \left(\tilde{z}_{M,t}^j\right)^{1+\eta} \mathrm{d}\Phi_{\zeta_t^j}(\zeta_t^j)\right]^{-\frac{1}{1+\eta}}} f^{t+1,j}(\sigma_{t+1}^j|\sigma_t^j) \mathrm{d}\theta_{K,t+1}^j \mathrm{d}D_{K,t+1}^j.$$

where the second equality follows from equation (A.19). Using equations (A.38) and (A.39), and the fact that $\theta_{K,t+1}^j$ and $D_{K,t+1}^j$ are orthogonal to the innovations, the price $q_b^\ell(s_{t+1}^j)$ simplifies to:

$$q_b^\ell(s_{t+1}^j) = \beta\delta \exp\left(\upsilon_{t+1}^B - (1-\tau_1)\,\upsilon_{t+1}^\alpha\right)$$
$$\times \left[\frac{\int \exp\left(A\upsilon_{t+1}^\kappa\right) \mathrm{d}\Phi_{\upsilon_{t+1}^\kappa}(\upsilon_{t+1}^\kappa) \int \exp\left(A\upsilon_{t+1}^\varepsilon\right) \mathrm{d}\Phi_{\upsilon_{t+1}^\varepsilon}(\upsilon_{t+1}^\varepsilon)}{\int \exp\left(A\upsilon_t^\varepsilon\right) \mathrm{d}\Phi_{\upsilon_t^\varepsilon}(\upsilon_t^\varepsilon)}\right]^{-\frac{1}{1+\eta}} f^{t+1,j}(s_{t+1}^j|s_t^j). \tag{A.45}$$

The price $q_b^\ell(s_{t+1}^j)$ is identical to equation (A.40) for the model without home production under $\gamma = 1$. The remainder of the argument is identical to the argument for the model without home production.

A.4.3 Household's Budget Constraint

We now verify our guess for the state-contingent bond positions $b_t^\ell(s_t^j)$ and confirm that the household budget constraint holds at the postulated equilibrium allocations. The proof to this claim is identical for both models. We define the deficit term by $d_t \equiv c_{M,t} - \tilde{y}_t$. Using the expression for the price $q_b^\ell(s_{t+1}^j)$ in equation (A.30), the budget constraint at the no-trade equilibrium is given by:

$$b_t^\ell(s_t^j) = d_t + \beta\delta \int \frac{\mu(\alpha_{t+1}^j, B_{t+1}^j, D_{K,t+1}^j, \theta_{K,t+1}^j)}{\mu(\alpha_t^j, B_t^j, D_{K,t}^j, \theta_{K,t}^j)} b_{t+1}^\ell(s_{t+1}^j) f^{t+1}(\sigma_{t+1}^j|\sigma_t^j)\mathrm{d}s_{t+1}^j \mathrm{d}\theta_{K,t+1}^j \mathrm{d}D_{K,t+1}^j.$$

By substituting forward using equation (A.30), we confirm the guess for $b_t^\ell(s_t^j)$ in equation (A.22) and show that the household budget constraint holds at the postulated equilibrium allocations.

A.4.4 Goods Market Clearing

Aggregating the resource constraints in every island, we obtain that the allocations solving the planner problems satisfy the aggregate goods market clearing condition:

$$\int_\iota c_{M,t}\mathrm{d}\Phi(\iota) + G_t = \int_\iota z_{M,t} h_{M,t} \mathrm{d}\Phi(\iota). \tag{A.46}$$

A.4.5 Asset Market Clearing

We now confirm that asset markets clear. The asset market clearing conditions $\int_\iota x(\zeta_t^j;\iota)\mathrm{d}\Phi(\iota) = 0$ hold trivially in a no-trade equilibrium with $x(\zeta_t^j;\iota) = 0$. Next, we confirm that asset markets within each island ℓ also clear, that is $\int_{\iota\in\ell} b^\ell(s_t^j;\iota)\mathrm{d}\Phi(\iota) = 0$, $\forall \ell, s_t^j$.

Omitting the household index ι for simplicity, we substitute the postulated state-contingent bond holdings in equation (A.22) into the asset market clearing conditions:

$$\begin{aligned}
\int b^\ell(s_t^j)\mathrm{d}\Phi(\iota) &= \int \mathbb{E}\left[\sum_{n=0}^{\infty} (\beta\delta)^n \frac{\mu(\alpha_{t+n}^j, B_{t+n}^j, D_{K,t+n}^j, \theta_{K,t+n}^j)}{\mu(\alpha_t^j, B_t^j, D_{K,t}^j, \theta_{K,t}^j)} d_{t+n}\right] \mathrm{d}\Phi(\iota) \\
&= \sum_{n=0}^{\infty} (\beta\delta)^n \int \frac{\mu(\alpha_{t+n}^j, B_{t+n}^j, D_{K,t+n}^j, \theta_{K,t+n}^j)}{\mu(\alpha_t^j, B_t^j, D_{K,t}^j, \theta_{K,t}^j)} d_{t+n} f(\sigma_{t+n}^j|\sigma_{t-1}^j)\mathrm{d}\sigma_{t+n}^j \mathrm{d}\Phi(\iota).
\end{aligned}$$

For simplicity we omit conditioning on σ_{t-1}^j and write the density function as $f(\sigma_{t+n}^j|\sigma_{t-1}^j) = f(\{\upsilon_{t+n}^B\})f(\{\upsilon_{t+n}^\alpha\})f(\{\upsilon_{t+n}^\kappa\})f(\{\upsilon_{t+n}^\varepsilon\})f(\{\theta_{K,t+n}\})f(\{D_{K,t+n}\})$. Further, the expression for the growth in marginal utility is identical between the two models and equals $\mathcal{Q}\left(\upsilon_{t+n}^B, \upsilon_{t+n}^\alpha\right) \equiv$

$\frac{\mu(\alpha^j_{t+n},B^j_{t+n},D^j_{K,t+n},\theta^j_{K,t+n})}{\mu(\alpha^j_t,B^j_t,D^j_{K,t},\theta^j_{K,t})} = \frac{\mu(\alpha^j_{t+n},B^j_{t+n})}{\mu(\alpha^j_t,B^j_t)}$. Hence, we write aggregate state-contingent bond holdings $\int b^\ell(s^j_t)\mathrm{d}\Phi(\iota)$ as:

$$\sum_{n=0}^{\infty}(\beta\delta)^n \int\int \mathcal{Q}\left(\upsilon^B_{t+n},\upsilon^\alpha_{t+n}\right) d_{t+n} f(\{\upsilon^B_{t+n}\})f(\{\upsilon^\alpha_{t+n}\})f(\{\upsilon^\kappa_{t+n}\})f(\{\upsilon^\varepsilon_{t+n}\})f(\{\theta_{K,t+n}\})\dots$$
$$\dots\ f(\{D_{K,t+n}\})\mathrm{d}\{\upsilon^B_{t+n}\}\mathrm{d}\{\upsilon^\alpha_{t+n}\}\mathrm{d}\{\upsilon^\kappa_{t+n}\}\mathrm{d}\{\upsilon^\varepsilon_{t+n}\}\mathrm{d}\{\theta^j_{K,t+n}\}\mathrm{d}\{D^j_{K,t+n}\}\mathrm{d}\Phi(\iota)$$
$$=\sum_{n=0}^{\infty}(\beta\delta)^n \int\int d_{t+n} f(\{\upsilon^\kappa_{t+n}\})f(\{\upsilon^\varepsilon_{t+n}\})\mathrm{d}\{\upsilon^\kappa_{t+n}\}\mathrm{d}\{\upsilon^\varepsilon_{t+n}\}\mathrm{d}\Phi(\iota)$$
$$\times\mathcal{Q}\left(\upsilon^B_{t+n},\upsilon^\alpha_{t+n}\right) f(\{\upsilon^B_{t+n}\})f(\{\upsilon^\alpha_{t+n}\})f(\{\theta^j_{K,t+n}\})f(\{D^j_{K,t+n}\})\mathrm{d}\{\upsilon^B_{t+n}\}\mathrm{d}\{\upsilon^\alpha_{t+n}\}\mathrm{d}\{\theta^j_{K,t+n}\}\mathrm{d}\{D^j_{K,t+n}\}.$$

Recalling that the deficit terms equal $d_t = c_{M,t} - \tilde{y}_t$, the state-contingent bond market clearing condition holds because the first term is zero by the island-level resource constraint.

A.5 Observational Equivalence Theorem

We derive the identified sources of heterogeneity presented in Table 2. We invert the equilibrium allocations in Table 1 and solve for the sources of heterogeneity leading to these allocations. The identification is unique up to constants because $\mathcal{C}_s$ appearing in the equations of Table 2 depends on the ε's.

A.5.1 No Home Production, $\omega_K = 0$

Given cross-sectional data $\{c_{M,t}, h_{M,t}, z_{M,t}\}_\iota$ and parameters $\gamma, \eta, \tau_0, \tau_1$, we show that there exists a unique $\{\alpha_t, \varepsilon_t, B_t\}_\iota$ such that the equilibrium allocations generated by the model are equal to the data for every household ι. We divide the solution for c_M with the solution for h_M to obtain:

$$\frac{c_{M,t}}{h_{M,t}} = (1-\tau_0)\, z_{M,t}^{-\eta(1-\tau_1)} \exp((1-\tau_1)(1+\eta)\alpha_t) \int_{\zeta_t} \exp((1-\tau_1)(1+\eta)\varepsilon_t)\mathrm{d}\Phi_{\zeta^j_t}(\zeta^j_t)\ . \tag{A.47}$$

Since the left-hand side is a positive constant and the right-hand is increasing in α_t, the value of α_t is determined uniquely for every household ι from this equation. Since $\log z_{M,t} = \alpha_t + \varepsilon_t$, ε_t is also uniquely determined. Finally, we can use the solution for $c_{M,t}$ or $h_{M,t}$ in Table 1 to solve for B_t.

A.5.2 Home Production, $\omega_K > 0$

Given cross-sectional data $\{c_{M,t}, h_{M,t}, z_{M,t}, h_{N,t}, h_{P,t}\}_\iota$ and parameters $\phi, \gamma, \eta, \tau_0, \tau_1$, we show that there exists a unique $\{\alpha_t, \varepsilon_t, B_t, \theta_{N,t}, D_{P,t}\}_\iota$ such that the equilibrium allocations generated by the model are equal to the data for every household ι.

Dividing the solution for h_N with the solution for c_M we obtain θ_N from the following equation:

$$\frac{h_{N,t}}{c_{M,t}} = \theta_{N,t}^{\phi-1}\tilde{z}_{M,t}^{-\phi}\,. \tag{A.48}$$

Next, we divide the solutions for h_P with the solution for h_N, we solve for the ratio of disutilities $\exp(D_P)/\exp(B)$:

$$\frac{h_{P,t}}{h_{N,t}} = \left(\frac{\theta_{P,t}}{\theta_{N,t}}\right)^{\phi-1}\left(\frac{\exp(B_t)}{\exp(D_{P,t})}\right)^{\phi}. \tag{A.49}$$

Next, we divide the solution for h_T with the solution for c_M and use equation (A.48) to obtain:

$$\begin{aligned}\frac{h_{M,t}+h_{N,t}+\frac{\exp(D_{P,t})}{\exp(B_t)}h_{P,t}}{c_{M,t}} &= \frac{z_{M,t}^{\eta(1-\tau_1)}}{1-\tau_0}\frac{\exp(-(1+\eta)(1-\tau_1)\alpha_t)}{\int_{Z_t}\exp((1+\eta)(1-\tau_1)\varepsilon_t)\mathrm{d}\Phi_{\zeta^j,t}(\zeta_t^j)}\\ &\times\left[1+\left(\frac{\theta_{N,t}}{\tilde{z}_{M,t}}\right)^{\phi-1}+\left(\frac{\exp\left(B_t\right)/\tilde{z}_{M,t}}{\exp\left(D_{P,t}\right)/\theta_{P,t}}\right)^{\phi-1}\right]\end{aligned} \tag{A.50}$$

Since the left-hand side is a positive constant and the right-hand is increasing in α_t, the value of α_t is determined uniquely for every household ι from this equation. Since $\log z_{M,t} = \alpha_t + \varepsilon_t$, the ε_t is also uniquely determined. Next, we can identify B using the first-order conditions with respect to market consumption and equations (A.18), (A.48) and (A.49) to obtain:

$$\exp\big((1+\eta)\,B_t\big) = \frac{\left(\frac{\bar{c}_{M,t}}{\tilde{z}_{M,t}}+h_{N,t}+\left(\frac{\bar{c}_{M,t}}{\bar{h}_{P,t}}\right)^{\frac{1}{\phi}}\theta_{P,t}{}^{\frac{\phi-1}{\phi}}\frac{h_{P,t}}{\tilde{z}_{M,t}}\right)^{-\eta}}{\bar{h}_{M,t}+h_{N,t}+\left(\frac{\bar{c}_{M,t}}{\bar{h}_{P,t}}\right)^{\frac{1}{\phi}}\theta_{P,t}{}^{\frac{\phi-1}{\phi}}\frac{h_{P,t}}{\tilde{z}_{M,t}}}\,. \tag{A.51}$$

Finally, once we know B, we can solve for D_P from equation (A.49).

B Additional Results

In this appendix we present summary statistics from various datasets and additional results and sensitivity analyses.

- Table A.1 shows summary statistics of wages and hours for married individuals in the ATUS and for married households in the CEX in which we have imputed home hours. The ATUS sample excludes respondents during weekends and, so, market hours are noticeably higher.
- Tables A.2 and A.3 show summary statistics of wages and hours for married individuals in the ATUS by sex and education.

- Tables A.4 and A.5 present summary statistics of wages, hours, and expenditures in the CEX and PSID samples.
- Table A.6 presents the correlation matrix of observables and sources of heterogeneity in the two models.
- Figure A.1 presents distributions of the sources of heterogeneity in the two models.
- Table A.7 presents the welfare effects of eliminating heterogeneity within age groups.
- Table A.8 compares the four inequality metrics in 6 versions of the home production model.

  1. One sector model with heterogeneity only in home production efficiency θ_N.
  2. Two sector model with heterogeneity in home production efficiency θ_N and disutility of work D_P (the baseline case).
  3. One sector model with heterogeneity only in home disutility of work D_P.
  4. Two sector model with heterogeneity in home production efficiencies θ_N and θ_P.
  5. Two sector model with reversal of classification of home hours relative to baseline (efficiency θ_P and disutility D_N).
  6. Two sector model with heterogeneity in home disutilities of work D_N and D_P.

  The first three cases repeat the cases shown in Table 7 in the main text. The second panel of Table A.8 shows the three alternative cases.

- Figures A.2 and A.3 present the life-cycle means and variances of the sources of heterogeneity in the version of the PSID with food expenditures. We obtain these age profiles by regressing each inferred source of heterogeneity on age and year dummies and an individual fixed effect. Therefore, these age profiles reflect the within-household evolution of the sources of heterogeneity.

Table A.1: ATUS (Raw) versus CEX (Imputed) Samples

	ATUS Married Individuals			CEX Married Households		
Age	All	25-44	45-65	All	25-44	45-65
Mean h_M	42.1	41.9	42.2	66.1	66.8	65.5
Mean h_N	12.5	14.6	10.5	21.3	25.4	17.3
Mean h_P	10.6	10.7	10.5	16.7	16.4	17.0
$\text{corr}(z_M, h_M)$	0.06	0.03	0.08	-0.15	-0.14	-0.14
$\text{corr}(z_M, h_N)$	0.01	0.04	-0.01	0.10	0.16	0.12
$\text{corr}(z_M, h_P)$	-0.08	-0.06	-0.09	0.02	0.00	0.03
$\text{corr}(h_M, h_N)$	-0.44	-0.46	-0.42	-0.25	-0.36	-0.23
$\text{corr}(h_M, h_P)$	-0.45	-0.44	-0.46	-0.42	-0.42	-0.41
$\text{corr}(h_N, h_P)$	0.10	0.14	0.08	0.15	0.20	0.17

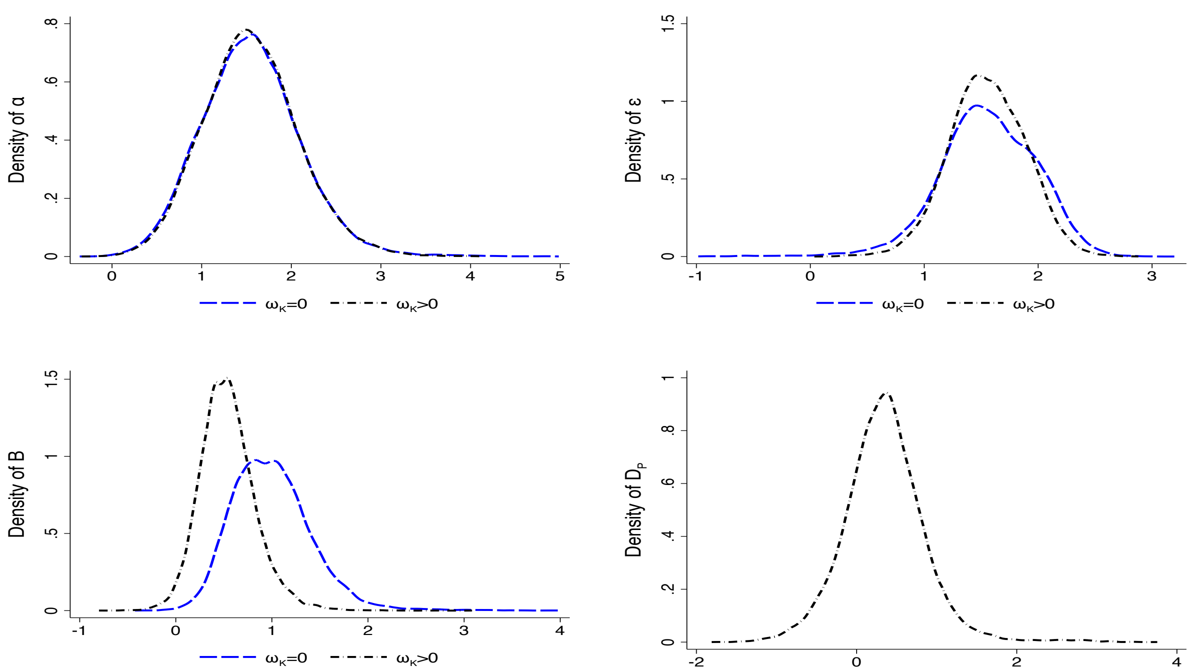

Figure A.1: Distributions of Sources of Heterogeneity

Table A.2: Correlations in ATUS Married by Sex

	ATUS All			ATUS Men			ATUS Women		
Age	All	25-44	45-65	All	25-44	45-65	All	25-44	45-65
$\text{corr}(z_M, h_M)$	0.06	0.03	0.08	0.02	0.00	0.04	0.04	0.02	0.06
$\text{corr}(z_M, h_N)$	0.01	0.04	-0.01	0.03	0.07	0.01	0.03	0.05	0.01
$\text{corr}(z_M, h_P)$	-0.08	-0.06	-0.09	-0.02	0.00	-0.04	-0.08	-0.08	-0.09
$\text{corr}(h_M, h_N)$	-0.44	-0.46	-0.42	-0.40	-0.41	-0.39	-0.44	-0.47	-0.43
$\text{corr}(h_M, h_P)$	-0.45	-0.44	-0.46	-0.39	-0.38	-0.41	-0.46	-0.44	-0.47
$\text{corr}(h_N, h_P)$	0.10	0.14	0.08	0.06	0.09	0.05	0.07	0.10	0.07

Table A.3: Correlations in ATUS Married by Education

	ATUS All			ATUS Less than College			ATUS College or More		
Age	All	25-44	45-65	All	25-44	45-65	All	25-44	45-65
$\text{corr}(z_M, h_M)$	0.06	0.03	0.08	0.05	0.04	0.03	0.05	0.02	0.07
$\text{corr}(z_M, h_N)$	0.01	0.04	-0.01	-0.01	0.01	-0.01	-0.02	0.02	-0.04
$\text{corr}(z_M, h_P)$	-0.08	-0.06	-0.09	-0.05	-0.03	-0.06	-0.07	-0.06	-0.09
$\text{corr}(h_M, h_N)$	-0.44	-0.46	-0.42	-0.42	-0.44	-0.41	-0.47	-0.50	-0.45
$\text{corr}(h_M, h_P)$	-0.45	-0.44	-0.46	-0.45	-0.43	-0.46	-0.45	-0.45	-0.45
$\text{corr}(h_N, h_P)$	0.10	0.14	0.08	0.08	0.12	0.06	0.14	0.17	0.13

Table A.4: CEX/ATUS (1995-2016) versus PSID (1975-2014) Moments

	CEX/ATUS			PSID		
Age	All	25-44	45-65	All	25-44	45-65
Mean h_M	66.1	66.8	65.5	67.8	65.3	70.3
Mean $h_N + h_P$	38.0	41.8	34.3	25.9	27.1	24.7
corr(z_M, h_M)	-0.15	-0.14	-0.14	-0.15	-0.15	-0.14
corr$(z_M, h_N + h_P)$	0.09	0.12	0.10	0.00	0.02	-0.02
corr(z_M, c_M^{food})	0.22	0.21	0.22	0.28	0.29	0.27
corr$(h_M, h_N + h_P)$	-0.42	-0.49	-0.42	-0.24	-0.28	-0.20
corr(h_M, c_M^{food})	0.10	0.09	0.12	0.06	0.06	0.08
corr$(h_N + h_P, c_M^{\text{food}})$	-0.03	-0.01	-0.02	0.01	0.03	-0.01

Table A.5: CEX/ATUS (1995-2016) versus PSID (2004-2014) Moments

	CEX/ATUS			PSID		
Age	All	25-44	45-65	All	25-44	45-65
Mean h_M	66.1	66.8	65.5	64.8	67.6	62.0
Mean $h_N + h_P$	38.0	41.8	34.3	24.3	24.1	24.6
corr(z_M, h_M)	-0.15	-0.14	-0.14	-0.09	-0.15	-0.06
corr$(z_M, h_N + h_P)$	0.09	0.12	0.10	-0.01	0.03	-0.03
corr(z_M, c_M^{nd})	0.25	0.25	0.25	0.26	0.29	0.25
corr$(h_M, h_N + h_P)$	-0.42	-0.49	-0.42	-0.23	-0.27	-0.20
corr(h_M, c_M^{nd})	0.14	0.16	0.13	0.20	0.21	0.20
corr$(h_N + h_P, c_M^{\text{nd}})$	-0.05	-0.04	-0.03	-0.03	-0.03	-0.03

Table A.6: Within-Age Correlations

$\omega_K = 0$	$\log z_M$	$\log c_M$	$\log h_M$	$\log h_N$	$\log h_P$	α	ε	B	D_P	$\log \theta_N$
$\log z_M$	1.00	0.29	-0.07	—	—	0.70	0.42	0.42	—	—
$\log c_M$		1.00	0.13	—	—	0.69	-0.50	-0.55	—	—
$\log h_M$			1.00	—	—	-0.46	0.50	-0.71	—	—
$\log h_N$				—	—	—	—	—	—	—
$\log h_P$					—	—	—	—	—	—
α						1.00	-0.35	0.23	—	—
ε							1.00	0.26	—	—
B								1.00	—	—
D_P									—	—
$\log \theta_N$										—

$\omega_K > 0$	$\log z_M$	$\log c_M$	$\log h_M$	$\log h_N$	$\log h_P$	α	ε	B	D_P	$\log \theta_N$
$\log z_M$	1.00	0.29	-0.07	0.07	-0.02	0.82	0.42	0.45	-0.58	0.69
$\log c_M$		1.00	0.13	0.00	-0.06	0.66	-0.54	-0.43	-0.01	-0.14
$\log h_M$			1.00	-0.17	-0.30	-0.32	0.38	-0.48	0.06	-0.20
$\log h_N$				1.00	0.18	0.13	-0.08	-0.29	-0.36	0.68
$\log h_P$					1.00	0.08	-0.15	-0.03	-0.70	0.12
α						1.00	-0.18	0.23	-0.41	0.46
ε							1.00	0.40	-0.34	0.46
B								1.00	-0.06	0.31
D_P									1.00	-0.66
$\log \theta_N$										1.00

Table A.7: Within-Age Heterogeneity and Lifetime Consumption Equivalence

No within-age dispersion in ...	$\omega_K = 0$ model	$\omega_K > 0$ model
z_M, θ_N, B, D_P	0.07	0.14
z_M, θ_N	0.07	0.16
θ_N, D_P	—	0.11
θ_N	—	0.12

Table A.8: The Role of Home Efficiency and Home Disutility in Amplifying Inequality

Statistics	No Home Production	Home Production		
		Efficiency θ_N	Baseline (θ_N, D_P)	Disutility D_P
std(T)	0.78	1.14	0.90	0.76
std(t)	0.55	0.83	0.73	0.65
λ	0.06	0.20	0.12	0.03
τ_1	0.06	0.32	0.24	0.13
Statistics		Efficiencies (θ_N, θ_P)	Reversed (θ_P, D_N)	Disutilities (D_N, D_P)
std(T)	0.78	1.13	0.82	0.73
std(t)	0.55	0.83	0.68	0.63
λ	0.06	0.19	0.12	0.02
τ_1	0.06	0.31	0.21	0.09

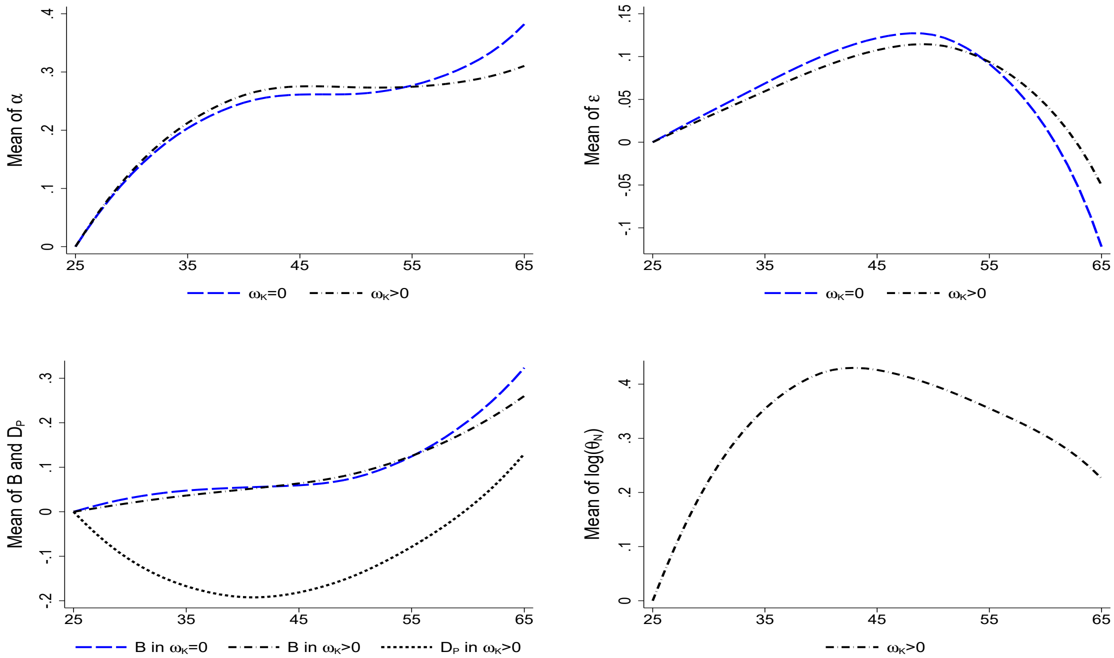

Figure A.2: Means of Sources of Heterogeneity (PSID Food)

Figure A.2 plots the age means of uninsurable component of market productivity α, insurable component of market productivity ε, disutilities of work B and D_P, and home production efficiency $\log \theta_N$ for the economy with ($\omega_K > 0$, black dotted lines) and without home production ($\omega_K = 0$, blue dashed lines).

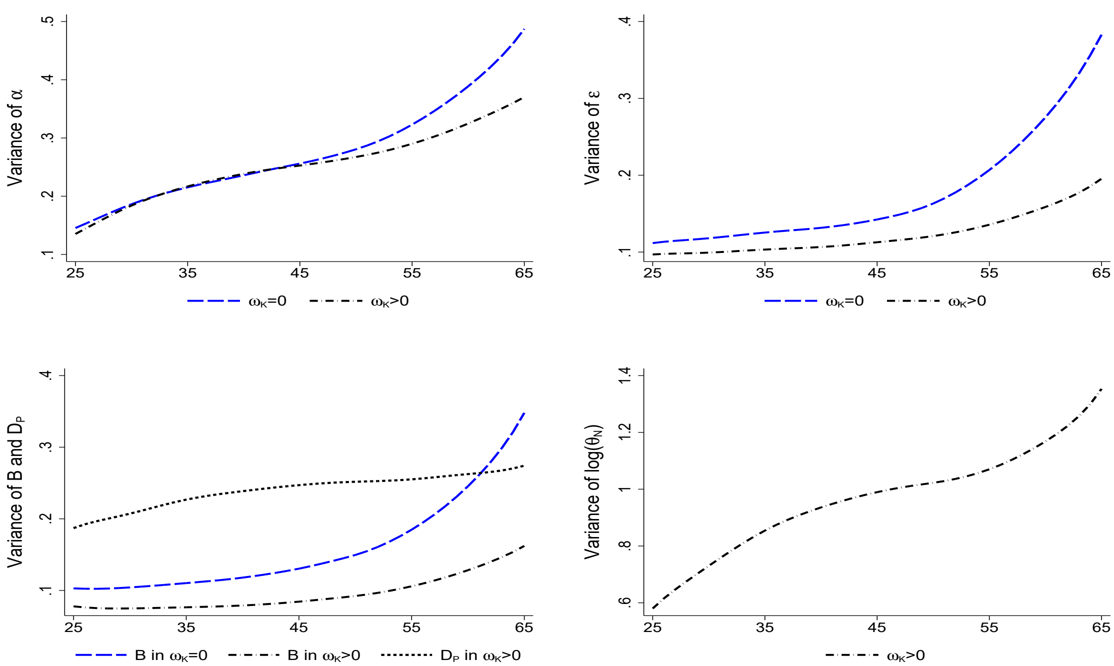

Figure A.3: Variances of Sources of Heterogeneity (PSID Food)

Figure A.3 plots the age variances of uninsurable component of market productivity α, insurable component of market productivity ε, disutilities of work B and D_P, and home production efficiency $\log \theta_N$ for the economy with ($\omega_K > 0$, black dotted lines) and without home production ($\omega_K = 0$, blue dashed lines).