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Abstract

A monopoly seller advises buyers about which of two goods best fits their needs but may be tempted to steer buyers towards the higher margin good. For the seller to collect information about a buyer's needs and provide truthful advice, the profits from selling both goods must lie within an implementability cone. In the optimal regulation, pricing distortions and information-collection incentives are controlled separately by price regulation and fixed rewards respectively. This no longer holds when the seller has private information about costs as both problems interact. We study the extent to which competition and the threat by buyers to switch sellers can substitute for regulation.

JEL Classification: D82, I11, L13, L15, L51, G24

Keywords: Mis-Selling, Expertise, regulation, asymmetric information

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Controlling Sellers Who Provide Advice: Regulation and Competition*

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1. Introduction

Motivation. In many instances, customers rely on sellers for advice on the goods or services they purchase: pharmacists advise clients on which non-subscription drugs to use, and sell them those drugs; retailers for high-tech products often also educate their customers; private and corporate bankers advise clients on investment opportunities, which they then provide for a fee. Such situations are naturally prone to conflicts of interest, which result in the underprovision of advice. Indeed, sellers may under-invest in assessing their clients' actual needs. This tendency may be exacerbated by the sellers' temptation to distort their advice towards higher margin products, which further reduces their incentives to assess the clients' needs.

Important issues arise. What drives sellers' incentives to provide informed and unbiased expert advice? What is the impact of sellers' market power? How should activities in which sellers also provide expert advice be regulated? Does competition stimulate or instead hinder the provision of unbiased advice and to what extent can it offer an alternative to regulation? These questions are relevant for public and antitrust policy but remain largely unexplored.

This paper's objective is thus twofold. First, we take a normative perspective and study the optimal regulation of sellers who also provide expert advice. Second, we study the extent to which competition can be a substitute to regulation.

Main Elements of the Model. A buyer seeks to purchase one of two goods, A and B, from a seller. The buyer's needs can be of one of two types, A or B, and the buyer needs the seller's expert advice to make a purchase decision. The buyer enjoys a surplus only from the good fitting his needs. Buyer and seller have the same prior about the buyer's needs but remain ignorant of the buyer's exact needs.

To this, we add two elements. First, the seller can, at a private cost, observe a noisy signal of the buyer's needs. If he does, which we assume to be socially optimal, he is in a position to offer valuable advice to make a match more likely and thus increase surplus. However, because information collection is costly and unobservable, whether the seller advises the buyer depends on his incentives, *i.e.*, there is moral hazard.

Second, we assume that one of the goods, say good A, may or may not have a lower production cost/higher margin, and that only the seller knows whether it does. Given this information asymmetry, a seller with a low cost for good A may be tempted to push it to enjoy higher profits, which reduces his incentive to collect information in the first place.

UNREGULATED MONOPOLY. We start with the case of an unregulated monopolist seller. We show that the seller's incentives to provide advice depend on whether profits for both goods are similar enough: they must lie within an *implementatibilty cone* which we characterize. Our assumption that providing advice is socially optimal means that the social surpluses for both goods lie within the cone. However, a monopolist capturing only a fraction of the social surplus may favor good A for its higher margin *a priori*. That is, monopoly profits may lie outside the implementability cone. Two allocative distortions arise: prices exceed marginal costs and advice quality is too low.

REGULATION. We thus study the extent to which regulation can curb both distortions. In doing so, we adopt a normative perspective, assuming away practical implementation issues, notably those related to data collection. A regulator may both regulate prices to curb the seller's market power and redistribute part of the surplus so obtained to the seller to preserve his incentives to collect information.

When the seller's costs are publicly known, prices and fixed rewards are used to control pricing distortions and information gathering separately. Setting prices equal to marginal costs maximizes welfare but also means that the seller cannot recoup the cost of information gathering through higher revenues. Fixed rewards are thus used to reward the seller for accurate advice. Assuming that the seller is protected by limited liability, an optimal regulation cannot rely on punishments following incorrect advice. The cheapest way to solve the moral hazard problem is then to set symmetric rewards for both goods in case of correct advice so that the seller's profits lay at the extremal point of the implementability cone. Yet, advice has lower social value than under complete information because the rewards needed for incentive purposes also imply a *liability rent* for the seller.

This regulation is no longer feasible if the seller has private information on its costs as the implied information rent biases him towards pushing good A. Regulation must thus compensate a low-cost seller for that rent. The combination of adverse selection and moral hazard leads to pricing distortions and thus inefficiency. To make mimicking a high-cost seller unattractive, the optimal regulation combines two tools. First, it sets good

A's price above marginal cost if the seller reports a high cost. This depresses demand, thereby discouraging a low-cost seller from reporting a high cost. The positive price-cost margin implies that, unlike under complete information, a high-cost seller makes a profit, and fees diminish so that a high-cost seller's profits remain at the extremal point of the *implementability cone.* Second, the optimal regulation induces a low-cost seller to reveal information with higher fees whereas prices remain equal to marginal costs. Thus, the low-cost seller's profits now lay within the cone and no longer at its extremal point. Implementation costs are higher and asymmetric information makes gathering information even less socially valuable due to the combination of *liability* and *information rents.*

Competition and Buyer-Seller Dynamics. Such regulation frameworks may be difficult to implement as they are information-intensive: they require data on the advice the seller offers in each interaction with each buyer, including its accuracy given the buyer's needs. Transaction costs, the dispersion of information among buyers, and regulators' limited capabilities may make such level of control impossible. Instead, buyers have an advantage over the regulator in assessing the accuracy of the advice they received. However, they have more limited tools than the regulator for controlling the seller. Thus we study the extent to which buyers themselves can achieve a more decentralized control of the seller's incentives.

We show that buyers can approximate the rewards of the optimal regulation by making the probability of dropping a seller for a rival dependent on whether his advice proved accurate. Such retrospective rules help control moral hazard and adverse selection. They are akin to, but imperfect substitutes for, the optimal regulation's rewards. These rules control sellers' incentives for information gathering but not their market power.

When the seller's cost is common knowledge, the optimal rule is to switch sellers with positive probability if a low-cost seller's recommendation of good A proves incorrect. Switching implies zero profit in the continuation for a seller who made a wrong recommendation. It is thus akin to the absence of rewards following a wrong advice in the optimal regulation. This brings the seller's intertemporal profits inside the implementability cone. When the seller has private information on his costs, buyers also use this threat as a screening device and switch more often with high-cost sellers than in the pure moral hazard scenario to induce information revelation from low-cost sellers.

Hence, a regulator may favor a decentralized, indirect regulation via buyers' behavior over a more costly centralized, information-intensive regulation. In that case, regulatory intervention can take several forms such as lowering entry barriers to offer buyers alternatives to switch to, incentivizing buyers to rate sellers, penalizing contracts tying buyers to sellers and more generally lowering switching costs.

Paper Organization. Section 2 reviews the related literature. Section 3 presents the model. Section 4 characterizes the *implementability cone*, and studies the unregulated monopolist case. Section 5 studies the optimal regulation. Section 6 studies buyer-seller dynamics. Section 7 presents applications to the pharmaceutical drugs market and patient-doctor relations, and that for financial advice. All proofs are in the Online Appendix.

2. Related Literature

The disciplining role of the threat of switching is a familiar argument in economics (see, *e.g.*, Ferejohn (1986) on the role of retrospective voting for disciplining politicians). At a broad level, in a tradition à la Tiebout, competition allows buyers to *vote with their feet*. At a less general level, our paper builds on several branches of the literature.

Credence Goods. A large literature starting with Nelson (1970) and Darby and Karni (1973) studies situations in which sellers know more than buyers about product quality or buyers' needs. (Dulleck and Kerschbamer (2006) survey the theory.) In Pitchik and Schotter (1987) and Fong (2005), information being free, the incentive problem central to our analysis is absent. Emons (1997, 2001) studies how a monopolist can convey credibly and price information when information-gathering effort is verifiable. Wolinsky (1993) and Board (2009) consider competitive environments differing in the kind of information provided. Unlike our paper, they do not analyze the seller's incentives to acquire information. Bouckaert and Degryse (2000) and Emons (2000) study competition between experts and non-experts while Pesendorfer and Wolinsky (2003) and Dulleck and Kerschbamer (2009) analyze similar asymmetric competition when seller's effort is non-verifiable. Alger and Salanié (2006) also study the role of competition in this moral hazard environment. We dot not study these issues and focus, instead, on the potential mismatch between recommendations and customers' preferences. We share with this literature the assumption that consumers must rely on the expert's advice to make purchase decisions but we also shed light on other less studied issues. For instance, the expert's incentives to collect information about consumers' needs and the seller's private information on production costs are both understudied. That the optimal regulation with pure moral hazard lies at the extremal point of the implementation cone is reminiscent of Emons' (1997) equal-markup condition developed in a credence goods context. Efficiency is restored when the expert is indifferent between recommending repair and diagnosis of a failed product. Yet, Emons (1997) considers neither the expert's incentives to collect information nor the seller's private information on production costs. In a closer paper, Chen, Li and Zhang (2018) characterize the optimal liability rules for disciplining a seller in a model which, like ours, entails unobservable information gathering and private information on costs.

Some papers have studied dynamic relationships in such contexts. For pure credence goods, Fong, Liu and Meng (2018) show how an expert seller can build trust vis-à-vis consumers when consumers can monitor whether his past recommendations were correct or not. This disciplinary role of switching is different from the one we highlight, where switching incentivizes sellers to offer informed advice. Frankel and Schwarz (2014) consider repeated expert-buyer interactions, but assume long-lived experts and short-lived buyers who never observe the true state of the world. In a search model, Galenianos and Gavazza (2017) study how repeated expert-buyer interactions allow to solve a moral hazard problem. This disciplinary role of dynamic relationships has also been studied in reputation models (e.g., Board and Meyer-ter-Vehn, 2013). Nevertheless, Schneider (2012) provides evidence that reputation may have limited incentive power for expert sellers.

Incentives for Mis-Selling. Inderst and Ottaviani (2009, 2012) study incentives to collect information in a market context but focus on the agency problem arising when selling is delegated to an agent. Inderst and Ottaviani (2012) consider the choice of a contract between a seller and the agent who can recommend alternative products to buyers. Inderst and Ottaviani (2009) stress a multitask problem: the agent must both find new clients and advise them on the product's suitability. This leads to mis-selling. How much mis-selling the seller tolerates depends on his ability to control the agent via commissions contingent on client satisfaction or to commit to *ex post* penalties for mis-selling. Our analysis differs in several ways. First, we do not model agency problems between sellers and sales agents but instead focus on agency problems between sellers and buyers or regulators. Second, we allow for incentive contracts contingent on the seller's information on the buyer's needs and show that truthful advice derives from the seller's incentives to gather information. By contrast, Inderst and Ottaviani (2009, 2012) restrict the contract space to non-contingent contracts and so have to assume that information is revealed in a subsequent cheap talk stage. Third, in our setup, the seller has private information about margins. This is a further source of rent. It implies that the seller is biased even in regulated environments.

Inderst and Ottaviani (2013) focus on refund or cancellation policies when buyers vary in sophistication. The cancellation policy aligns the seller and buyers' interests, provided buyers are rational enough to understand how it affects the seller's incentives. We confirm that the buyers' sophistication matters for disciplining sellers. Indeed, we find that rational consumers adopting retrospective rules to terminate relationships with sellers can approach the optimal regulation.[1]

Several recent papers study issues relevant to the finance industry, notably the provision of nonverifiable information to customers. Bolton *et al.* (2007) show that competition among specialized financial intermediaries leads to credible information disclosure. Garicano and Santos (2004) study efficiency in matching clients with agents in a context with private information about a client's value and moral hazard in effort provision. Although they view trade as being mediated by trust and address different issues, Gennaioli *et al.* (2015) argue, as we do, that financial advice, like medical advice, maybe self-serving.

Delegated Expertise. To the extent that the seller's information-gathering choice and his signal are unobservable, our paper builds on the literature on delegated expertise initiated by Lambert (1986) and Demski and Sappington (1987) and developed by Gromb and Martimort (2007), Malcomson (2009), Chade and Kovrijnykh (2016), Grassi and Ma (2016) and Zambrano (2017) among others. A key departure from this literature is that we embed the expertise relationship into a market context to link incentives to offer advice with the distribution of profits the market structure implies.

[1] In a model with competing intermediaries, Murooka (2015) studies how commissions received by those intermediaries affect their incentives to educate customers who misperceive the value of the products.

3. The Model

Preferences and Demand Information. A risk-neutral seller offers two goods, A or B, at unit prices p_A and p_B. A risk-neutral buyer considers purchasing one unit of one of the goods. The buyer has needs denoted by $\theta \in \{A, B\}$. His valuation v for a unit of good $i \in \{A, B\}$ depends on whether the good matches his needs, *i.e.*, whether $i = \theta$: if it does (resp. does not), v is drawn from a distribution function F (resp. G) with positive density function f (resp. g) defined over the entire positive real line. We assume that F dominates G in the sense of first-order stochastic dominance. Thus, the buyer's valuation v tends to be higher for a good that matches its needs.

Neither the seller nor the buyer knows the buyer's needs θ. Both needs are equally likely. The seller can, however, collect information on the buyer's needs and advise him on which good to purchase. Specifically, at a private cost $\psi > 0$, the seller observes a signal $\sigma \in \{A, B\}$ about the buyer's needs with precision ε defined as

$$\varepsilon \equiv \Pr(\sigma = A \mid \theta = A) = \Pr(\sigma = B \mid \theta = B) \in (1/2, 1) .$$

We assume that the seller's information-collection decision and the signal's realization are unobservable. This creates the potential for moral hazard.

The timing and information structure we consider are as follows. First, the buyer's needs θ realize but remain unknown to the buyer and the seller. Second, the seller sets prices for both goods and may or may not collect information on the buyer's needs. He then recommends a good to the buyer. We assume there is no purchase without recommendation or, equivalently, that the buyer needs the seller's advice to make a purchase decision. Third, the buyer learns his valuation v for this good and, given its price, decides whether to buy it.

Remark 1. The buyer's prior on his needs plays no role in our analysis. For most of the paper (except Section 6 where we study a dynamic buyer-seller relationship), the buyer could wrongly believe he has a large number of potential needs, each corresponding to a specific good, and most of those being irrelevant. The buyer could remain unaware of his exact needs.[2] What matters for our analysis is that the buyer is unable to draw

[2] See, among others, Li, Peitz and Zhao (2014, 2016), Eliaz and Spiegler (2011) for industrial organization models of unawareness.

posterior beliefs about his needs, before a purchase, from the knowledge of his own valuation. Otherwise, he could insist on consuming a good that was not recommended. If so, the buyer would be an expert, which is at odds with our premise that the seller is an indispensable provider of information.

Remark 2. Because the buyer is unable to draw any inference before a purchase, his only decision is whether to buy the recommended good, which does not depend on priors. In particular, $G(0)$ can be arbitrarily close to 1, so that if the recommended good does not match the buyer's needs, the buyer's expected demand and thus the seller's expected profit are arbitrarily close to zero. To streamline the analysis and simplify calculations, we assume from now on that the distribution G has such a point mass at zero.[3] Note that, in a static model, whether the buyer learns if the recommendation was correct after buying the good plays no role.

Cost and Information. Finally, we assume that the two goods have different marginal costs. While good B's cost is $c_B = c$, good A's cost c_A can be either $\overline{c}_A = c$ or $\underline{c}_A = c - \Delta c$, with $\Delta c > 0$. Moreover, the seller knows the value of c_A but the buyer only has a prior $\nu \equiv \Pr(c_A = \underline{c}_A)$. The cost and information differences may stem from the goods' different nature. For instance, good A may be less common or more specific than good B. The costs may be production costs, opportunity costs of shelf or storage space, *etc.*

Notations. A buyer with valuation v derives a net surplus $v - p_i$ from consuming good i. Hence, a buyer with needs $\theta = i$ advised to purchase good i has expected surplus

$$S(p_i) = \int_{p_i}^{+\infty} (v - p_i) dF(v) = \int_{p_i}^{+\infty} (1 - F(v)) dv.$$

The (expected) demand induced by the random valuation is

$$D(p_i) = -S'(p_i) = 1 - F(p_i).$$

$S(\cdot)$ being non-increasing and convex, $D(\cdot)$ is non-increasing. The overall surplus when a buyer with needs $\theta = i$ is advised to purchase good i with cost c_i at price p_i is

$$W(c_i, p_i) = \int_{p_i}^{+\infty} (1 - F(v)) dv + (p_i - c_i)(1 - F(p_i)) = \int_{p_i}^{+\infty} (v - c_i) dF(v).$$

[3]Hence the buyer infers from observing a positive valuation that the seller's advice was accurate but observing $v = 0$ is not enough to infer that the seller's advice was inaccurate.

This expression is maximized when price equals marginal cost (*i.e.*, $p_i = c_i$). Therefore, the first-best surplus in a sale of good i is

$$W^*(c_i) \equiv W(c_i, c_i).$$

The monopoly price and profit in a sale of good i are defined as

$$p^m(c_i) = c_i - \frac{D(p^m(c_i))}{D'(p^m(c_i))} \quad \text{and} \quad \pi^m(c_i) \equiv (p^m(c_i) - c_i)D(p^m(c_i)).$$

Full Information Social Optimum. As a benchmark, consider the case in which information collection is contractible and both signal σ and cost c_A are observable.

Absent information, expected surplus is (weakly) maximized by the buyer purchasing good A as its cost is (weakly) lower. In that case, expected surplus is estimated based on the prior about good A being a good match, *i.e.*, with probability 1/2. Therefore, information collection is socially optimal for a given level of cost c_A if and only if

$$\sum_{\{i,j\}=\{A,B\}} \Pr(\theta = i)\left(\Pr(\sigma = i \mid \theta = i)\, W^*(c_i) + \Pr(\sigma = j \mid \theta = i) \cdot 0\right) - \psi$$
$$\geq \Pr(\theta = A)\, W^*(c_A) + \Pr(\theta = B) \cdot 0$$

which simplifies to

$$\frac{\varepsilon}{2} W^*(c_B) - \frac{(1-\varepsilon)}{2} W^*(c_A) \geq \psi. \tag{3.1}$$

The left-hand side's first term is information's social benefit: when the buyer's need is B (which has probability 1/2), information allows a match with probability ε, which yields surplus $W^*(c_B)$. Its second term captures information's social cost: when the buyer's need is A (which has probability 1/2), information, because it is noisy, may yield a mismatch with probability $(1-\varepsilon)$, which destroys surplus $W^*(c_A)$.

Note that $W^*(\cdot)$ being non-increasing, the condition is tighter when the cost of good A is lower, *i.e.*, it is tighter for $c_A = c - \Delta c$ than for $c_A = c$. This reflects that information's social cost increases with surplus $W^*(c_A)$ foregone due to a noisy signal.

In what follows, we assume that information gathering is socially valuable even when good A's cost is low. It is then *a fortiori* socially valuable when the cost is high.

ASSUMPTION 1. *Information collection is socially optimal irrespective of good A's cost:*

$$\frac{\varepsilon}{2}W^*(c) - \frac{(1-\varepsilon)}{2}W^*(c-\Delta c) \geq \psi.$$

4. PROFITS AND INFORMATION GATHERING

In this section, we analyze the seller's incentives to collect and reveal information in different contexts. We start by characterizing the set of seller profits for goods A and B compatible with information gathering and truthful advice (Section 4.1). We then use this analysis to study the case of an unregulated monopoly (Section 4.2).

4.1. The Implementability Cone

We first determine the seller's incentive compatibility condition. The seller's profit is zero unless his advice $\hat{\sigma}$ matches the buyer's needs θ, in which case it is denoted $\pi_\theta(c_A)$.[4]

The seller collects and reveals information under two conditions. First, his expected payoff from doing so must exceed that from remaining uninformed and recommending whichever of good A or B yields more profit *a priori*.[5] This condition is written as

$$\text{(4.1)} \qquad \frac{\varepsilon}{2}\pi_A(c_A) + \frac{\varepsilon}{2}\pi_B(c_A) - \psi \geq \max\left\{\frac{\pi_A(c_A)}{2}, \frac{\pi_B(c_A)}{2}\right\} \quad \forall c_A \in \{\underline{c}_A, \overline{c}_A\}.$$

The second condition is that conditional on having acquired information, the seller must prefer reporting it truthfully, which can be written as

$$\frac{\varepsilon}{2}\pi_i(c_A) > \frac{(1-\varepsilon)}{2}\pi_j(c_A) \quad \forall \{i,j\} = \{A,B\} \quad \forall c_A \in \{\underline{c}_A, \overline{c}_A\}.$$

Note however that this condition is implied by condition (4.1), which can be rewritten as

$$\text{(4.2)} \qquad \frac{\varepsilon}{2}\pi_i(c_A) \geq \frac{(1-\varepsilon)}{2}\pi_j(c_A) + \psi \quad \forall \{i,j\} = \{A,B\} \quad \forall c_A \in \{\underline{c}_A, \overline{c}_A\}.$$

Indeed, the seller would not collect a signal if this never affected his advice. Given this, we can now describe the set of profit levels ensuring information gathering and truthful advice, which is a cone in the seller's profits space (see Figure 2).

[4]In what follows, we make the dependence of all variables on random variable c_A explicit.
[5]Randomized strategies between those two options are weakly dominated.

Lemma 1. *The set of profits inducing information gathering and truthful reporting is characterized by*

$$\Gamma = \{(\pi_A(c_A), \pi_B(c_A)) \text{ s.t. } \pi_A(c_A) = \pi^* + (1-\varepsilon)x + \varepsilon y; \pi_B(c_A) = \pi^* + \varepsilon x + (1-\varepsilon)y; x \geq 0; y \geq 0\}$$

which is a positive cone with extremal point E defined by

$$\pi_A(c_A) = \pi_B(c_A) = \pi^* = \frac{2\psi}{2\varepsilon - 1}. \tag{4.3}$$

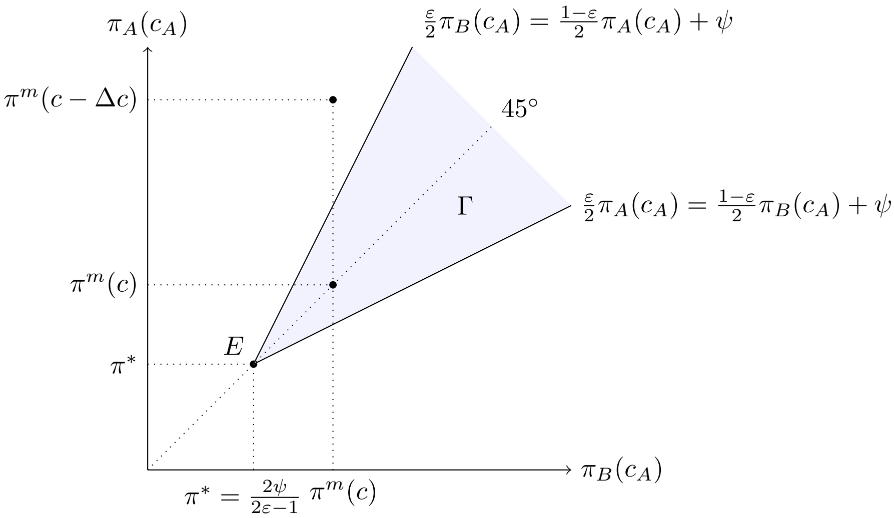

Figure 1 – The set of profits inducing information gathering and truthful advice is a cone.

4.2. Unregulated Monopolist

We now study the case of a monopoly seller charging a price p_A^m (resp. p_B^m) for good A (resp. good B) leading to a profit $\pi^m(c_A)$ (resp. $\pi^m(c_B)$) when the recommended good fits the buyer's needs. Information collection is optimal for the seller given his cost c_A if

$$\frac{\varepsilon}{2}\pi^m(c_B) - \frac{(1-\varepsilon)}{2}\pi^m(c_A) \geq \psi.$$

The condition can be understood by replacing social surplus with monopoly profits in condition (3.1). Again, it is tighter for a low-cost than for a high-cost seller because information's private cost, *i.e.*, the foregone profit $\pi^m(c_A)$ due to an inaccurate signal, decreases with cost c_A.

From now on, to focus on the relevant cases, we assume the following condition holds.

Assumption 2. *Only a high-cost seller collects information and reports it truthfully, i.e.,*

$$\frac{(2\varepsilon-1)}{2}\pi^m(c) \geq \psi \geq \frac{\varepsilon}{2}\pi^m(c) - \frac{(1-\varepsilon)}{2}\pi^m(c-\Delta c).$$

The first inequality means that a high-cost seller gathers (and reveals) information. The second one means that a low-cost seller remains uninformed and pushes good A. Assumption 2 ensures that a low-cost seller's profits $(\pi^m(c-\Delta c), \pi^m(c))$ lie outside cone Γ, while those of a high-cost seller, $(\pi^m(c), \pi^m(c))$, lie within the cone (Figure 2).[6]

Note that the seller's cost does not affect the buyer's preferences. Hence, the seller has no incentive to hide his cost which can thus be assumed common knowledge. The only issue is whether the seller's advice is informed or not.[7]

Proposition 1. *Assume the seller is an unregulated monopoly. Under Assumption 2, the unique (perfect Bayesian) equilibrium outcome is as follows.*

- *The seller charges monopoly prices for both goods: $p_A = p^m(c_A)$ and $p_B = p^m(c_B)$.*
- *A high-cost seller collects information and offers truthful advice.*
- *A low-cost seller remains uninformed and recommends good A.*

The outcome departs from social optimality in two ways: prices are above marginal costs and a low-cost seller does not offer truthful advice. This raises the issue of regulation which we analyze next.

5. Regulation

We characterize the regulation maximizing the buyer's expected surplus. Regulation aims to reduce price-cost margins to improve allocative efficiency while motivating information

[6]Assumptions 1 and 2 are compatible. Say Assumption 1 holds as an equality, *i.e.*, collecting information is socially neutral when $c_A = c - \Delta c$. Since the seller gets a fraction $k < 1$ of the overall surplus, he finds it optimal to remain uninformed and push good A. Assumption 1 also implies that collecting information has social value when $c_A = c$. Hence, k can be set close enough to 1 so that a high-cost seller opts to gather information. Thus, the condition $(2\varepsilon - 1)kW^*(c) \geq 2\psi \geq \varepsilon kW^*(c) - (1-\varepsilon)kW^*(c-\Delta c)$ is satisfied and Assumptions 1 and 2 both hold.

[7]Proposition 1 below holds even if the seller sets prices before learning his cost (Mylovanov and Tröger, 2012). This highlights the robustness of the low-cost seller's incentives to push good A *a priori*.

gathering. It relies on an incentive contract to counter the low-cost seller's bias towards pushing good A. We show that depending on the information asymmetries impeding the regulator's intervention, these goals can be reached separately (Section 5.1) or not (Section 5.2).

Unlike in the previous section, the regulator is able to set prices for the goods and rewards for the seller. We assume that the regulator can infer *ex post* the accuracy of the seller's advice and condition the seller's rewards accordingly. One interpretation is that the regulator observes numerous transactions involving the seller and can infer the demand profile from the empirical distribution of realized valuations. He can thus infer the accuracy of the seller's advice.

Contracts. From the *Revelation Principle*, we can focus on *direct*, *truthful*, *obedient* mechanisms (Myerson, 1982).[8] In direct mechanisms, the seller makes reports $\hat{c}_A$ and $\hat{\sigma}$ on cost c_A and signal σ. They specify report-contingent prices p for both goods, fixed payments T for selling each good, and fixed payments $T - R$ in case of a mismatch. A regulatory contract is thus a tuple

$$C = \{(p_{\hat{\sigma}}(\hat{c}_A), T_{\hat{\sigma}}(\hat{c}_A), R_{\hat{\sigma}}(\hat{c}_A))\}_{\hat{c}_A \in \{\underline{c}_A, \overline{c}_A\}, \hat{\sigma} \in \{A,B\}}.$$

It must induce information gathering and truthful reporting (*i.e.*, $\hat{c}_A = c_A$ and $\hat{\sigma} = \sigma$).

Timing. The game unfolds as follows. An incentive contract C that maximizes the buyer's expected surplus is designed. The seller observes cost $c_A \in \{\underline{c}_A, \overline{c}_A\}$ and makes a report $\hat{c}_A$ about c_A. The seller chooses whether to observe signal $\sigma \in \{A, B\}$ at cost ψ, and provides advice $\hat{\sigma}$. If the advice matches the buyer's needs (*i.e.*, if $\hat{\sigma} = \theta$), the buyer purchases (in expectation) $D(p_{\hat{\sigma}}) = 1 - F(p_{\hat{\sigma}})$ units of the good, the seller incurs cost $c_{\hat{\sigma}} D(p_{\hat{\sigma}})$ and receives revenue $p_{\hat{\sigma}} D(p_{\hat{\sigma}})$. Otherwise, demand and cost are zero, and the seller incurs a penalty $R_{\hat{\sigma}}$. To simplify, we assume that the seller has no gain following incorrect advice: $R_\sigma(\hat{c}_A) = T_\sigma(\hat{c}_A)$.

Remark. In the case of a contract between an upstream producer and a seller, the fixed payments T may represent fixed fees the former pays the latter, and the penalty R a pay-back transfer. In that case, assuming $T_\sigma \geq 0$ and $R_\sigma = T_\sigma$ is akin to assuming that the seller has limited liability. Here, we take this feature of the optimal contract as given

[8] As our model now combines moral hazard and adverse selection, one must deal with simultaneous deviations in actions and reports. See Laffont and Martimort (2002, Chapter 7) for a detailed analysis.

to save on notation.[9]

5.1. Regulation Under Pure Moral Hazard

Consider first the case in which information gathering and signal are unobservable but c_A is common knowledge. (This amounts to assuming $\hat{c}_A = c_A$.) The problem is thus to induce the seller to collect and report signal σ.

To build some intuition, observe that Constraint (4.1) suggests that selling either good must be rewarded and the cheapest way to do so is to make the seller indifferent between recommending good A or B based on his prior. In that case, the signal tilts the seller's decision towards truth-telling. Different price-reward combinations ensure indifference but in the least-distortionary one, prices equal marginal costs to maximize overall surplus, while fixed rewards induce information gathering and set profits at the extreme point of the cone. The seller must get some extra profit to be induced to collect information and fixed rewards are best to ensure he does.

These findings reveal an important dichotomy between pricing and information gathering incentives when costs are common knowledge. Prices determine overall surplus while rewards provide incentives for gathering information and giving truthful advice.

PROPOSITION 2. *Suppose cost c_A is common knowledge so the only incentive problem is to induce information gathering and truthful advice. The optimal regulation is as follows.*[10]

- *Both goods are priced at marginal cost:*

$$p^{mh}_\sigma(c_A) = c_\sigma, \quad \forall \sigma \in \{A, B\}. \tag{5.1}$$

- *Profits and fixed rewards are constant across goods:*

$$\pi^{mh}_\sigma(c_A) = T^{mh}_\sigma(c_A) = \pi^* = \frac{2\psi}{2\varepsilon - 1}, \quad \forall \sigma \in \{A, B\}. \tag{5.2}$$

[9] This payment structure is consistent with the *Principle of Delegated Expertise*: an optimal contract should reward experts/sellers only for recommendations confirmed by verifiable outcomes (Gromb and Martimort, 2007). Inderst and Ottaviani (2009) make a similar assumption on the payment structure.

[10] Superscript *mh* stands for *moral hazard* to stress this is the only incentive constraint considered.

- *Information gathering is induced by the regulator when:*

$$\frac{\varepsilon}{2}W^*(c) - \frac{(1-\varepsilon)}{2}W^*(c-\Delta c) \geq \psi + \frac{\psi}{2\varepsilon - 1}. \tag{5.3}$$

The seller is rewarded only for a good match. That he cannot be punished for a bad match is akin to a limited liability constraint. Hence the seller enjoys a *liability rent* $\psi/(2\varepsilon - 1)$ to gather information. Note that the lower the signal's precision, the larger the rewards and the seller's liability rent. Indeed, the noisier the mapping between information gathering and outcomes, the larger the rewards needed to induce information collection. Finally, agency costs make information gathering less valuable: condition (5.3) is tighter than condition (3.1) due to the limited liability rent. The buyer's net surplus is reduced to $W^*(c_A) - \pi^*$ for good A and $W^*(c) - \pi^*$ for good B, which may lie outside the implementability cone.

5.2. *Regulation Under Moral Hazard and Adverse Selection*

We now turn to the case of a seller having private information about the cost of good A. This information has no value absent regulation because it does not affect the buyer's utility. It has value in a regulation context, however. By manipulating cost reports to a regulator the seller can indeed steer the buyer towards the good that provides an information rent. Private information impacts incentives for information gathering.

To illustrate, we first consider the optimal contract under pure moral hazard (as in Section 5.1) and ask whether private information about cost induces advice manipulation.

Consider an uninformed low-cost seller. Based on his prior, he is tempted to report a high cost. Indeed, this does not change the fees for selling either good since condition (5.2) implies they are cost-independent, but brings the seller an extra gain $\Delta c D(c)/2$. This information rent equals the gain from selling $D(c)$ units of good A (in expectation) at a cost that is Δc below the high cost. The expectation is based on prior beliefs as the seller always recommends good A and thus remains uninformed.

The incentives to manipulate cost and their impact on information gathering can be illustrated with the implementability cone.

Absent private information on cost, point E corresponds to the seller's profit levels irrespective of c_A. With the proviso that prices equal marginal cost, *i.e.*, $p_\sigma(c_A) = c_A$

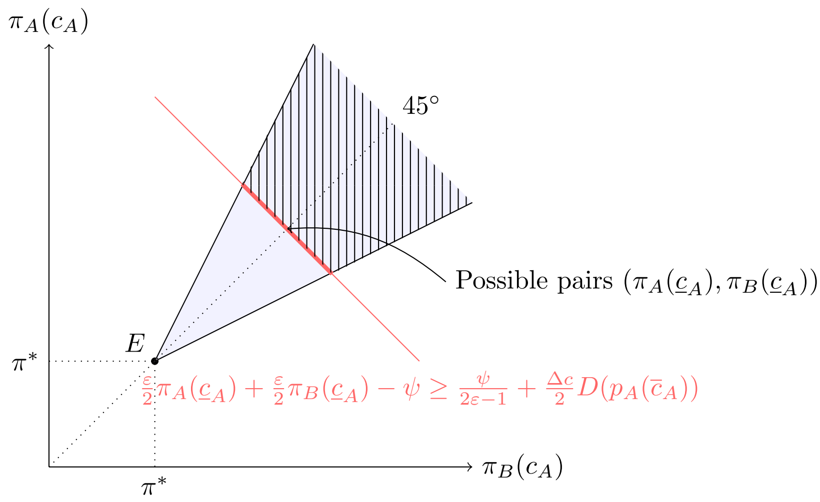

Figure 2 – Under adverse selection and moral hazard, the implementability set is a truncated cone bounded below by the constraint that ensures no cost manipulation by a low-cost seller.

for all pairs (c_A, σ), it corresponds to the optimal regulation. With private information on cost, the profit levels of a low-cost seller must also prevent him from reporting a high cost, remaining uninformed and recommending good A. When a high-cost seller is offered a contract bringing his profits to point E, the corresponding (low-cost seller's) incentive constraint is represented on Figure 2 as the downward-sloping 45-degree line, which cuts through the complete information implementability cone. Under adverse selection and moral hazard, the implementability set is thus a truncated cone bounded below by that line.

Two remarks follow. First, the optimal contract for a low-cost seller can no longer be reached at point E. Second, since the regulator minimizes the seller's expected payoff, the optimal contract should lie on the downward sloping 45-degree line defining the boundary of the implementability set under asymmetric information. To bring the optimum closer to point E, the regulator increases price $p_A(c)$ above marginal cost and depresses demand. The figure also shows that all distributions of profits $(\pi_A(\underline{c}), \pi_B(\underline{c}))$ on the 45-degree segment are possible at the optimum. Next we make this analysis more formal.

To this end, let us express the seller's information rent taking into account that information gathering and signals are non-verifiable. We define this rent as

$$\mathcal{U}(c_A) = \max_{\substack{\hat{c}_A \in \{\underline{c}_A, \overline{c}_A\} \\ x \in [0,1] \\ (y_A, y_B) \in [0,1]^2 \\ y_A + y_B = 1}} x\left(\left(\frac{\varepsilon}{2} \sum_{\sigma \in \{A,B\}} (p_\sigma(\hat{c}_A) - c_\sigma) D(p_\sigma(\hat{c}_A)) + T_\sigma(\hat{c}_A)\right) - \psi\right) + (1-x)\left(\frac{1}{2} \sum_{\sigma \in \{A,B\}} y_\sigma((p_A(\hat{c}_A) - c_\sigma) D(p_\sigma(\hat{c}_A)) + T_\sigma(\hat{c}_A))\right)$$

where x is the probability of gathering information and y_σ the probability of recommending good σ while uninformed.

We now characterize conditions for both seller types to collect information and report it truthfully, *i.e.*, $x = 1$. Inducing a high-cost seller to collect information requires that his equilibrium profits lie in the implementability cone, which can be written as

$$\mathcal{U}(\overline{c}_A) \geq \max\left\{\frac{\pi_A(\overline{c}_A)}{2}, \frac{\pi_B(\overline{c}_A)}{2}\right\}. \tag{5.4}$$

The left-hand side is the equilibrium payoff of a high-cost seller who reports his cost truthfully, gathers information and gives truthful advice. The right-hand side is the gain from remaining uninformed and making a recommendation based on prior beliefs.[11]

The key incentive problem now stems from a low-cost seller's possible "triple deviation:" he can inflate his cost, remain uninformed, and bias his advice. This deviation moves the profits of the low-cost seller out of the implementability cone. The rest of the analysis consists in determining how regulation can adjust these profits to motivate information gathering.

A low-cost seller's incentive constraint is thus

$$\mathcal{U}(\underline{c}_A) \geq \max\left\{\mathcal{U}(\overline{c}_A) + \frac{\varepsilon \Delta c}{2} D(p_A(\overline{c}_A)); \frac{\pi_B(\overline{c}_A)}{2}; \frac{\pi_A(\overline{c}_A)}{2} + \frac{\Delta c}{2} D(p_A(\overline{c}_A))\right\}. \tag{5.5}$$

The left-hand side is the equilibrium payoff of a low-cost seller who reports his cost truthfully, gathers information and gives truthful advice. The right-hand side's first term is the gain from inflating his cost, gathering information and reporting it truthfully. The second term is the gain from inflating his cost, remaining uninformed, and recommending good B. The third term is the the gain from inflating his cost, remaining uninformed,

[11]We omit a high-cost seller's option to report a low cost and check later that this constraint is slack.

and recommending good A. This strategy would be the most attractive with a contract designed only to induce information gathering.

Intuitively, making pushing good A less tempting helps incentive compatibility. Doing so requires either reducing a high-cost seller's fixed fee for selling good A (diminishing $\pi_A(\bar{c}_A)$) or increasing good A's price to lower demand (reducing $D(p_A(\bar{c}_A))$) and so reduce the information rent. However, reducing the fixed fee for selling good A can bias a high-cost seller towards good B and may in turn require increasing the liability rent of this seller type to restore information gathering incentives. This points to a trade-off between a low-cost seller's information rent and a high-cost seller's liability rent. The cheapest way of solving this trade-off is to give the high-cost seller the same liability rent as when costs are common knowledge and, at the same time, distort prices.

A higher price and lower sales for good A if the seller reports a high cost have drawbacks too. Indeed, the seller evaluates the expected gain from inflating his cost based on his prior. Because a low-cost seller expects an information rent when remaining uninformed, price distortions on good A must be large enough. Therefore, decreasing the information rent requires large distortions, which is less attractive when a low cost is more likely.

We can now characterize the optimal regulatory contract in this environment.

PROPOSITION 3. *Assume that c_A is private information and both information gathering and signals are unobservable. The optimal regulation for both types to gather information is as follows.*[12]

- *Both seller types charge prices equal to marginal cost for good B:*

$$p_B^{sb}(c_A) = c \quad \forall c_A \in \{\underline{c}_A, \bar{c}_A\}. \tag{5.6}$$

- *A low-cost seller charges a price equal to marginal cost for good A while a high-cost seller charges a price above marginal cost:*

$$p_A^{sb}(\underline{c}_A) = c - \Delta c, \tag{5.7}$$

[12]Superscript *sb* stands for *second best* to stress that all constraints are now taken into account.

$$p_A^{sb}(\bar{c}_A) = \tilde{\bar{c}}_A \textit{ where } \tilde{\bar{c}}_A = c + \tfrac{\nu}{(1-\nu)\varepsilon}\Delta c > c. \tag{5.8}$$

- *A high-cost seller's profit on each good is the same as if cost is common knowledge:*

$$\pi_A^{sb}(\bar{c}_A) = \pi_B^{sb}(\bar{c}_A) = \pi^*. \tag{5.9}$$

- *A low-cost seller's profits on both goods can be set equal to each other but greater than if cost is common knowledge:*

$$\pi_A^{sb}(\underline{c}_A) = \pi_B^{sb}(\underline{c}_A) = \pi^* + \frac{1}{2\varepsilon}\Delta c D(p_A^{sb}(\bar{c}_A)) > \pi^*. \tag{5.10}$$

An optimal regulation must afford a low-cost seller an extra rent $\Delta c D(p_A(\bar{c}_A))/2$, which shifts profits inside the cone and no longer at its extremal point E as under complete information. Many profit pairs induce information gathering by that seller type (see the red segment on Figure 2). In one of them, profits for both goods are equal, as in the pure moral hazard case. Since the cheapest way to incentivize the seller is to give him positive profits only when his advice proves correct, this information rent can be distributed over all such events so that the seller's profit following any such advice must now exceed its complete information value π^* by an amount $\Delta c D(p_A(\bar{c}_A))/4$ divided by the probability $\varepsilon/2$ that such advice is optimal.

Paying Sellers via Fees or Revenues? To reduce the low-cost seller's information rent and bring profits closer to the cone's extremal point, price distortions are needed for the high-cost seller. Indeed, increasing good A's price reduces demand and thus the low-cost seller's information rent. It is as if the high-cost seller had a *virtual cost* $\tilde{\bar{c}}_A$. Because revenues from selling good A for a high-cost seller are now positive, there is less need to pay this seller for those sales through a fee than when marginal cost pricing erodes profits as for good B

$$T_A^{sb}(\bar{c}_A) < T_B^{sb}(\bar{c}_A) = \pi^*.$$

Instead, marginal cost pricing on both goods for the low-cost seller implies no sales revenues and thus the information rent must materialize through fees

$$T_A^{sb}(\underline{c}_A) = T_B^{sb}(\underline{c}_A) = \pi^* + \frac{1}{2\varepsilon}\Delta c D(p_A^{sb}(\bar{c}_A)) > \pi^*.$$

Information Gathering. Now the cost of gathering information includes both the liability rent due to the non-verifiability of information gathering and the information rent due to private information about costs. This modifies the conditions for its optimality.

Proposition 4. *The optimal regulation requires that both a low-cost and a high-cost seller gather information when:*

$$\frac{\varepsilon}{2}W^*(c) - \frac{(1-\varepsilon)}{2}W^*(c-\Delta c) \geq \psi + \frac{\psi}{2\varepsilon - 1} + \frac{1}{2}\Delta c D(p_A^{sb}(\bar{c}_A)) \tag{5.11}$$

and

$$\frac{(2\varepsilon-1)}{2}W^*(c) \geq \psi + \frac{\psi}{2\varepsilon - 1} + \frac{\varepsilon}{2}\left(W^*(c) - W^*(\tilde{\bar{c}}_A)\right). \tag{5.12}$$

Condition (5.11) for a low-cost seller is tighter than condition (5.3) due to the information rent. While a high-cost seller gets no information rent, condition (5.12) is also tighter due to the allocative cost of replacing cost with virtual cost.

It is difficult for regulation to eliminate price distortions caused by private information while inducing information gathering. In particular, under adverse selection, a tension arises between information rent that induces price distortions and information gathering.

Besides, such a regulation may be difficult to implement in practice because it is information-intensive: the regulator must collect data on the advice the seller offers in each interaction with each buyer, including its (*ex post*) accuracy. Transaction costs, the dispersion of information among buyers, the requirement of anonymity for individual transactions, the impossibility for buyers to credibly communicate whether their needs have been met or not and regulators' limited capabilities may make such level of control impossible.[13]

Compared to a regulator, buyers have an advantage in assessing the (*ex post*) accuracy of the advice they received from the seller. However, they have more limited tools than the

[13] In Section 7, we give an example of the Japanese health system where the optimal regulation could be implemented.

regulator for controlling the seller. Thus, in the next section, we study the extent to which buyers themselves can control the seller's incentives. The benefit of such decentralized control is that buyers may use their information on the quality of advice. This allows for history-dependent strategies in which purchases with a given seller depend on the quality of his past advice. On the other hand, and because of a free-rider problem, each individual buyer may be unable to affect the pricing behavior of a given seller. In other words, even history-dependent purchase strategies might not be conditioned on posted prices, a significant difference with regulation which can control price distortions.

6. Buyer-Seller Dynamics

We now consider consider an infinitely repeated relationship and examine how a buyer can use retrospective rules to control a seller. The main insight is that the repeated interaction between a buyer and a seller may approach the outcome obtained under regulation, provided that the buyer can punish the seller by switching to a competitor when his advice proves incorrect. The threat of switching is an incentive device and asymmetric information impacts how the buyer implements switching rules.

The dynamic version of our model is as follows. We assume that several identical sellers compete to attract the buyer and the seller's cost c_A is time-invariant. The buyer's types θ_t in different periods t are i.i.d., *i.e.*, equal to A or B with equal probability. Let δ denote the discount factor, common to all players. In each period, the seller must learn the buyer's needs and incur cost ψ. To make the comparison with the case of regulation meaningful, we also assume that the buyer observes whether the seller's recommendation was correct after the purchase decision.[14]

Remark. The regulatory contracts we considered in Section 5 are purely static. In contexts with repeated interactions, this amounts to punishments for bad advice in one period arising in the same period, future trades remaining unaffected. One possible interpretation is that regulators view all transactions as anonymous. This implies that they do not keep track of the past history of advice that individual buyers may have received from a given seller and thus cannot condition sellers' future payments on such information. Buyers are typically better placed to act in response to such information

[14]That the buyer observes or not whether the recommendation was correct after the purchase decision plays no role in the static model of Section 4.

because their behavior is not constrained by any anonymity requirement.

To study the buyer's optimal strategy, there is no need to look for a Nash equilibrium between sellers. Instead we simply analyze a seller's best response to the strategies of a buyer basing future purchases on past advice quality.

The buyer can switch to a rival seller, an option ensuring him an expected surplus $\mathcal{S}_0$. Instead, $\mathcal{S}(c_A)$ denotes the value of sticking to the current seller. The rival has similar characteristics, in particular his cost is independently drawn from the same distribution, and *a priori* the relationship should give the same expected surplus up to switching costs for the buyer. We thus have $\mathcal{S}_0 = \mathbb{E}_{c_A}(\mathcal{S}(c_A)) - Z$, where Z is a switching cost. We assume that $\Delta\mathcal{S}(c_A) = \mathcal{S}(c_A) - \mathcal{S}_0 > 0$ for all c_A, *i.e.*, the buyer finds it costly to switch seller. Finally, when the buyer switches seller, the seller makes zero profit in the continuation.[15] We also assume the following condition to hold:

Assumption 3.

$$\frac{\varepsilon}{2}S(p^m(c)) + \frac{\varepsilon}{2}S(p^m(c-\Delta c)) \geq \frac{1}{2}S(p^m(c-\Delta c)).$$

This condition means that the buyer enjoys a greater expected net surplus from a static relationship with a low-cost seller if this seller, who always charges monopoly prices, provides truthful advice than if he systematically pushes good A.[16]

The problem is stationary because we assume independent draws of the buyer's needs over time. Accordingly, we describe a stationary equilibrium with constant switching probabilities. More precisely, we assume that the buyer can commit to switching seller with probability $(1-\beta_\sigma(c_A))$ (resp. $(1-\gamma_\sigma(c_A))$) when the advice following signal σ proves correct (resp. incorrect). Assuming the buyer can commit to the switching probabilities may seem extreme, but it gives its best chance to the threat of quitting as a disciplining device.

Remark. Several aspects of that commitment assumption deserve commenting. First, enforcing a switching probability is already a difficult issue, even in more standard one-shot principal-agent models. Courts may find it difficult to ascertain that the randomizing

[15] An extension of the model could allow a buyer's switching to inflict further losses on the seller, *e.g.*, the seller may incur a reputation loss *vis-à-vis* other buyers learning from their peer's experience.

[16] A similar condition always holds for a high-cost seller who is indifferent between recommending either good and always makes truthful recommendations when Assumption 2 holds.

device is as required by the mechanism.[17] However, for the informational contexts we study below, buyers find it optimal to stick to the required randomization. Second, when switching seller, the buyer incurs a switching cost. Although it helps relaxing the seller's incentive constraints, as we will show, and is optimal from an *ex ante* viewpoint, this strategy is not *ex post* optimal. Once the seller has been induced to choose an obedient and truthful behavior by the threat of switching, the buyer prefers not to switch. Thus, absent credible commitment, the stationary equilibrium would entail complex mixed strategies with the buyer randomizing between switching or not and the seller randomizing between being obedient and truthful and not in each period.[18,19] Last, note that the buyer's ability to commit to switching probabilities depends on the institutional context. A large buyer, for instance a large investor looking for advice, may build a reputation *vis-à-vis* potential sellers to switch following inaccurate advice because the history of his trades may be observed by sellers.[20]

While some of the contracting options of the regulatory context (Section 5) are impossible, some features of optimal regulation arise here too. First, the continuation payoff plays the role of the regulation's reward. Second, assuming that the buyer adopts a retrospective rule resembles the regulator's commitment power assumption.

That the buyer's problem nevertheless resembles the regulator's may be seen by defining the continuation value for the buyer's intertemporal payoff $\mathcal{S}(c_A)$ in state c_A in a

[17]See Laffont and Martimort (2002) for a discussion along these lines.

[18]The logic is similar to that of the audit and costly state verification literature under limited commitment. See Khalil (1997).

[19] Those equilibria are subject to the usual criticisms of mixed-strategy equilibria in more general contexts, most notably that it is hard to interpret how a player may follow such strategy. Those equilibria can be purified by expanding the game to an asymmetric information context (Harsanyi, 1973), a path that we won't follow here for simplicity.

[20]Note that the equilibria we describe below for various informational contexts all entail switching with positive probability on the equilibrium path. It implies that building a reputation for such commitment is possible since the history of trades requires visiting different sellers on path.

stationary equilibrium as the solution to the following problem:[21]

$$
\begin{aligned}
(6.1) \qquad \mathcal{S}(c_A) = \max_{\substack{(\beta_\sigma(c_A),\gamma_\sigma(c_A))_{\sigma\in\Sigma}\\ 0\leq\beta_\sigma(c_A),\gamma_\sigma(c_A)\leq 1}} \quad & \frac{\varepsilon}{2}(S(p^m(c_A)) + \delta(\mathcal{S}(c_A) - (1-\beta_A(c_A))\Delta\mathcal{S}(c_A))) \\
&+ \frac{1-\varepsilon}{2}\delta(\mathcal{S}(c_A) - (1-\gamma_A(c_A))\Delta\mathcal{S}(c_A)) \\
&+ \frac{\varepsilon}{2}(S(p^m(c)) + \delta(\mathcal{S}(c_A) - (1-\beta_B(c_A))\Delta\mathcal{S}(c_A))) \\
&+ \frac{1-\varepsilon}{2}\delta(\mathcal{S}(c_A) - (1-\gamma_B(c_A))\Delta\mathcal{S}(c_A))
\end{aligned}
$$

subject to the seller's incentive constraints which we make explicit in the next subsections.

This expression shows that switching entails a cost that the buyer may be reluctant to incur unless incentive constraints are significantly relaxed. This resembles the regulation case, where fixed payments reward the seller for good advice and punishment is not possible. Here instead, rewards for the seller are bounded by the continuation value. However, the buyer can also punish the seller for bad advice by switching to a rival. Competition affects rewards and punishments. Yet, the key issue it to find switching probabilities ensuring that the buyer's valuation of the relationship for various advice is in the implementability cone.[22] We derive those probabilities in the next subsections.

6.1. Retrospective Rules under Pure Moral Hazard

Incentive Constraints. Hereafter, the sole agency problem is to induce the seller to collect and reveal information each period. Denote by $\mathcal{U}(c_A)$ the continuation value for the seller with cost c_A on the equilibrium path. It satisfies

$$
\begin{aligned}
\mathcal{U}(c_A) = \max_{(p_A,p_B)} \frac{\varepsilon}{2}\left((p_A-c_A)D(p_A) + \delta\beta_A(c_A)\mathcal{U}(c_A)\right) + \frac{1-\varepsilon}{2}\delta\gamma_A(c_A)\mathcal{U}(c_A) \\
+ \frac{\varepsilon}{2}\left((p_B-c)D(p_B) + \delta\beta_B(c_A)\mathcal{U}(c_A)\right) + \frac{1-\varepsilon}{2}\delta\gamma_B(c_A)\mathcal{U}(c_A) - \psi.
\end{aligned}
$$

[21]Expression (6.1) is derived as follows. Given a buyer's needs, the seller's recommendation is correct with probability ε. The buyer expresses then a positive (expected) demand for the current period and obtains surplus $S(p^m(c_A))$. In the next period, the buyer stays with the seller with probability $\beta_A(c_A)$ (and earns $\mathcal{S}(c_A)$) and switches with probability $1-\beta_A(c_A)$ (and earns $\mathcal{S}_0$). When the recommendation is incorrect, there is no surplus for the current period.

[22]In our setting, the degree of competition affects the sellers' market power only through market shares and not through their prices. In alternative models, competition might also erode mark-ups. *Ceteris paribus*, the profits earned on both goods would then come closer to each other, which might facilitate their entering into the cone.

This expression makes it clear that, with history-dependent purchase strategies that only depend on the quality of a match, a buyer cannot affect the seller's prices. In a stationary equilibrium, the seller always sets monopoly prices. We thus have

$$\mathcal{U}(c_A) = \frac{\frac{\varepsilon}{2}\pi^m(c_A) + \frac{\varepsilon}{2}\pi^m(c) - \psi}{1 - \delta\left(\frac{\varepsilon}{2}(\beta_A(c_A) + \beta_B(c_A)) + \frac{1-\varepsilon}{2}(\gamma_A(c_A) + \gamma_B(c_A))\right)}. \tag{6.2}$$

Again, incentives for information gathering require preventing several possible deviations

$$\mathcal{U}(c_A) \geq \max\left\{\frac{1}{2}\pi^m(c_A) + \frac{\delta}{2}(\beta_A(c_A) + \gamma_A(c_A))\mathcal{U}(c_A); \frac{1}{2}\pi^m(c) + \frac{\delta}{2}(\beta_B(c_A) + \gamma_B(c_A))\mathcal{U}(c_A)\right\}.$$

The right-hand side stems for the seller's payoff for both goods following a one-shot deviation in which he remains uninformed and gives uninformed advice, and from then on sticks to gathering and revealing information.[23] Taken at the stationary equilibrium, the condition can be written as

$$\mathcal{U}(c_A) \geq \max\left\{\frac{\frac{1}{2}\pi^m(c_A)}{1 - \frac{\delta}{2}(\beta_A(c_A) + \gamma_A(c_A))}; \frac{\frac{1}{2}\pi^m(c)}{1 - \frac{\delta}{2}(\beta_B(c_A) + \gamma_B(c_A))}\right\}. \tag{6.3}$$

Optimal Retrospective Rules. We now analyze the buyer's retrospective rules.

Proposition 5. *Under Assumptions 2 and 3, assuming cost c_A is common knowledge and with δ sufficiently close to 1:*

- *Both seller types always gather and reveal information.*
- *The relationship with a high-cost seller is always continued:*

$$\beta_A^{mh}(\bar{c}_A) = \gamma_A^{mh}(\bar{c}_A) = \beta_B^{mh}(\bar{c}_A) = \gamma_B^{mh}(\bar{c}_A) = 1. \tag{6.4}$$

- *The relationship with a low-cost seller is always continued if he recommends good B or if he correctly recommends good A:*

$$\beta_B^{mh}(\underline{c}_A) = \gamma_B^{mh}(\underline{c}_A) = \beta_A^{mh}(\underline{c}_A) = 1, \tag{6.5}$$

[23] One-shot deviations are enough to characterize incentive compatibility in a stationary environment.

but terminated with positive probability if he wrongly recommends good A:

$$\gamma_A^{mh}(\underline{c}_A) = 1 - 2\frac{(1-\delta)K(\underline{c}_A)}{\delta\left(\frac{2\varepsilon-1}{2}\pi^m(\underline{c}_A) + \frac{\varepsilon}{2}\pi^m(c) - \psi\right)} \in [0,1) \tag{6.6}$$

where

$$K(\underline{c}_A) = \frac{1-\varepsilon}{2}\pi^m(\underline{c}_A) - \frac{\varepsilon}{2}\pi^m(c) + \psi.$$ [24]

SWITCHING AS AN INCENTIVE DEVICE. Though switching is costly, the buyer uses this threat to induce information gathering. There is no problem in continuing with a high-cost seller: he provides unbiased advice in a static relationship. The issue is with a low-cost seller who is biased towards pushing good A. The most efficient way of curbing this bias is to cut the gap between the intertemporal profit levels. This is best achieved by making continuation after a recommendation for good A less likely. The cheapest way is therefore to reduce the probability of continuation when the low-cost seller's recommendation for good A proves incorrect.

This feature of the optimal retrospective rule echoes the *Principle of Delegated Expertise*: moral hazard in information provision is best controlled by punishing the seller after erroneous advice. Of course, this threat is effective only when the future matters enough, hence the qualifier on δ.

BACK INTO THE CONE. To better understand the benefits of dynamics it is useful to return to the characterization of incentive compatible allocations through (6.2) and (6.3). We can rewrite these constraints as

$$\frac{\varepsilon}{2}\pi^m(c) - \frac{1-\varepsilon}{2}\kappa(c_A)\pi^m(c_A) \geq \psi,$$

and

$$\frac{\varepsilon}{2}\pi^m(c_A) - \frac{1-\varepsilon}{2}\kappa(c_A)\pi^m(c) \geq \psi,$$

where

$$\kappa(c_A) = \frac{1 - \frac{\delta}{1-\varepsilon}\left(\frac{\varepsilon}{2}\beta_B(c_A) + \frac{1-\varepsilon}{2}\gamma_B(c_A) + \frac{1-2\varepsilon}{2}\gamma_A(c_A)\right)}{1 - \frac{\delta}{2}(\beta_A(c_A) + \gamma_A(c_A))}.$$

The first (resp. second) constraint captures the incentives to deviate by remaining unin-

[24] Note that Assumption 2 implies $K(\underline{c}_A) > 0$.

formed and recommending good A (resp. B).

Inserting the values found in (6.5) yields $\kappa(c) = 1 > \kappa(\underline{c}_A)$. In other words, while the dynamics does not control the high-cost seller's incentives, the threat of switching is akin to lowering the stage-profit for good A, which facilitates implementation. Such a symmetry in the seller's future profits motivates information gathering.

Again, it is useful to offer a graphical representation of our findings, which is provided in Figure 3. Consider the cone defined by the following inequalities

$$\begin{cases} \frac{\varepsilon}{2}\pi_B(c_A) - \frac{1-\varepsilon}{2}\kappa(c_A)\pi_A(c_A) & \geq \psi, \\ \frac{\varepsilon}{2}\pi_A(c_A) - \frac{1-\varepsilon}{2}\kappa(c_A)\pi_B(c_A) & \geq \psi. \end{cases}$$

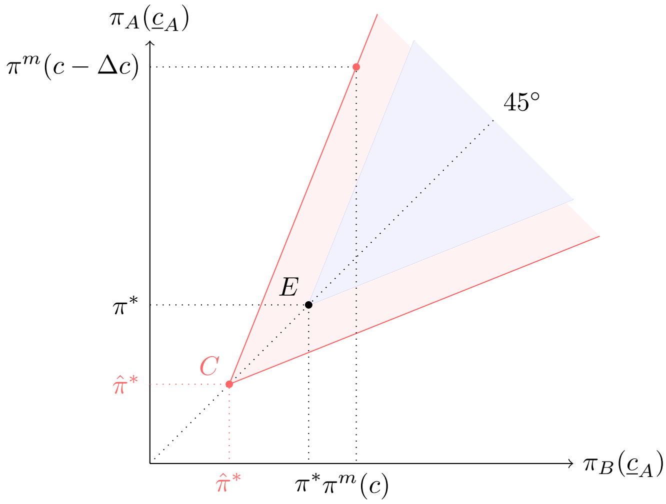

Figure 3 – Implementability cone for a low-cost seller in the buyer-seller dynamics case (in red) with respect to the one-shot case (in blue).

For a low-cost seller ($c_A = \underline{c}_A$), the new cone's extremal point C lies on the upward sloping 45-degree line but below point E since $\pi_A(c_A) = \pi_B(c_A) = \hat{\pi}^* = \frac{2\psi}{\varepsilon(1+\kappa(c_A))-1} < \pi^*$. The threat of termination acts as a punishment, not available to the regulator. Hence, the extremal point of the cone lies below the one with regulation only. Moreover, $\kappa(\underline{c}_A) < 1$ implies that lower (resp. higher) edges of the cone have now lower (resp. higher) slopes than in the regulation scenario. By switching with some probability, the buyer expands the set of implementable allocations. Switching being costly, the buyer chooses this probability so that the profit levels $(\pi^m(c), \pi^m(c - \Delta c))$ lie on the boundary of the new cone.

Information Gathering. Clearly, a high-cost seller gathers information at the optimum. If the buyer opts not to induce information gathering by a low-cost seller, he never switches seller. This leads to the following condition for inducing information gathering

$$\frac{1}{2}S(p^m(\underline{c}_A)) + \delta\mathcal{S}(\underline{c}_A) \leq \frac{\varepsilon}{2}(S(p^m(\underline{c}_A)) + S(p^m(c))) + \delta\mathcal{S}(\underline{c}_A) - \delta\Delta\mathcal{S}(\underline{c}_A)(1-\gamma_A^{mh}(\underline{c}_A)).$$

This condition can be rewritten as

$$\text{(6.7)} \qquad \frac{\varepsilon}{2}S(p^m(c)) - \frac{1-\varepsilon}{2}S(p^m(\underline{c}_A)) \geq \frac{(1-\varepsilon)(1-\delta)K(\underline{c}_A)}{\frac{2\varepsilon-1}{2}\pi^m(\underline{c}_A) + \frac{\varepsilon}{2}\pi^m(c) - \psi}\Delta\mathcal{S}(\underline{c}_A),$$

where the right-hand side is obtained after some simplifications using the definition of $\gamma_A^{mh}(\underline{c}_A)$. Assumption 3 implies that (6.7)'s left-hand side is positive. Hence, information gathering is always induced for δ close enough to 1. Importantly, condition (6.7) also shows that information gathering is facilitated when switching costs decrease. In other words, in this dynamic environment, competition facilitates information gathering.

6.2. *Retrospective Rules Under Moral Hazard and Adverse Selection*

Incentive Constraints. We now turn to the case where c_A is private information. The switching rule of Proposition 5 might no longer be optimal. A low-cost seller could mimic a high-cost seller by charging the same prices, which entails a short-run loss but ensures continuation. The switching rule must thus also prevent such deviation, *i.e.*,

$$\text{(6.8)} \qquad \mathcal{U}(\underline{c}_A) \geq \frac{1}{2}(p^m(c) - \underline{c}_A)D(p^m(c)) + \frac{\delta}{2}(\beta_A(\bar{c}_A) + \gamma_A(\bar{c}_A))\mathcal{U}(\underline{c}_A).$$

The left-hand side is the low-cost seller's equilibrium payoff from collecting and revealing information.[25] The low-cost seller may always charge the same prices as a high-cost seller and recommend good A without collecting information. By doing so, the low-cost seller enjoys a short-run profit $\pi^m(c)+\Delta c D(p^m(c))$ when selling good A. Although this is below his short-run monopoly profit $\pi^m(\underline{c}_A)$, he also secures a lower switching probability.

The switching rule must also discourage the low-cost seller from mimicking a high-cost

[25]Remember that the buyer commits to the switching rule, so that the *Revelation Principle* (Myerson, 1982) applies and all cost information is revealed in one round.

type, remaining uninformed and recommending good B

$$\mathcal{U}(\underline{c}_A) \geq \frac{1}{2}\pi^m(c) + \frac{\delta}{2}(\beta_B(\bar{c}_A) + \gamma_B(\bar{c}_A))\mathcal{U}(\underline{c}_A). \tag{6.9}$$

Finally, it must also prevent a low-cost seller from mimicking a high-cost seller but acquiring information, in which case the incentive constraint writes as

$$\begin{aligned}\mathcal{U}(\underline{c}_A) \geq & \frac{\varepsilon}{2}(p^m(c) - \underline{c}_A)D(p^m(c)) + \frac{\varepsilon}{2}\pi^m(c) - \psi \\ & + \delta\left(\frac{\varepsilon}{2}(\beta_A(\bar{c}_A) + \beta_B(\bar{c}_A)) + \frac{1-\varepsilon}{2}(\gamma_A(\bar{c}_A) + \gamma_B(\bar{c}_A))\right)\mathcal{U}(\underline{c}_A).\end{aligned} \tag{6.10}$$

Overall, the low-cost seller's incentive compatibility constraints can be summarized as follows

$$\begin{aligned}\mathcal{U}(\underline{c}_A) \geq \max\Bigg\{ & \frac{\frac{1}{2}\pi^m(c) + \frac{\Delta c}{2}D(p^m(c))}{1 - \frac{\delta}{2}(\beta_A(\bar{c}_A) + \gamma_A(\bar{c}_A))}; \frac{\frac{1}{2}\pi^m(c)}{1 - \frac{\delta}{2}(\beta_B(\bar{c}_A) + \gamma_B(\bar{c}_A))}; \\ & \frac{\varepsilon\left(\pi^m(c) + \frac{\Delta c}{2}D(p^m(c))\right) - \psi}{1 - \delta\left(\frac{\varepsilon}{2}(\beta_A(\bar{c}_A) + \beta_B(\bar{c}_A)) + \frac{1-\varepsilon}{2}(\gamma_A(\bar{c}_A) + \gamma_B(\bar{c}_A))\right)}\Bigg\}.\end{aligned} \tag{6.11}$$

PRIVATE INFORMATION MATTERS. We first show that private information on costs matters. To do so, we plug the rent profile and the switching probabilities of Proposition 5 and check whether incentive constraint (6.11) holds. First, observe that, if moral hazard is the sole concern and Assumption 2 holds, the low-cost seller's payoff satisfies

$$\mathcal{U}^{mh}(\underline{c}_A) = \frac{\frac{\varepsilon}{2}\pi^m(\underline{c}_A) + \frac{\varepsilon}{2}\pi^m(c) - \psi}{1 - \delta + \frac{\delta}{2}(1-\varepsilon)(1 - \gamma_A^{mh}(\underline{c}_A))} = \frac{\frac{1}{2}\pi^m(\underline{c}_A)}{1 - \frac{\delta}{2}(1 + \gamma_A^{mh}(\underline{c}_A))} \tag{6.12}$$

where the first equality follows from writing $\mathcal{U}^{mh}(\underline{c}_A)$ on path and the second from noticing that (6.3) is binding for a low-cost seller when Assumption 2 holds.

The solution obtained under pure moral hazard fails to satisfy the truthtelling condition when the following condition holds.

ASSUMPTION 4.

$$\mathcal{U}^{mh}(\underline{c}_A) < \max\left\{\frac{\frac{1}{2}\pi^m(c) + \frac{\Delta c}{2}D(p^m(c))}{1-\delta}; \frac{\varepsilon\left(\pi^m(c) + \frac{\Delta c}{2}D(p^m(c))\right) - \psi}{1-\delta}\right\}.$$

The right-hand side is obtained by inserting the switching probabilities of (6.4) into the right-hand side of (6.11). Assumption 4 ensures that private information on cost changes the buyer's behavior.

OPTIMAL SWITCHING RULES. We can now summarize the optimal rule's main features.

PROPOSITION 6. *Under Assumptions 2, 3 and 4, when $\underline{c}_A$ is private information of the seller and with δ sufficiently close to 1:*

- *The low- and the high-cost sellers both gather and reveal information.*
- *If the seller recommends good B, the relationship is continued:*

$$\beta_B^{sb}(\underline{c}_A) = \gamma_B^{sb}(\underline{c}_A) = \beta_B^{sb}(\bar{c}_A) = \gamma_B^{sb}(\bar{c}_A) = 1. \tag{6.13}$$

- *If the seller recommends good A, the relationship is continued if a seller correctly recommends good A and terminated with positive probability otherwise:*

$$\beta_A^{sb}(\underline{c}_A) = 1 \geq \gamma_A^{sb}(\underline{c}_A) \geq 0, \tag{6.14}$$

and

$$\beta_A^{sb}(\bar{c}_A) = 1 \geq \gamma_A^{sb}(\bar{c}_A) \geq 0. \tag{6.15}$$

- *Moreover $\gamma_A^{sb}(\underline{c}_A) = \gamma_A^{mh}(\underline{c}_A) \geq \gamma_A^{sb}(\bar{c}_A)$.*

SWITCHING AS A SCREENING DEVICE. The buyer now wants to avoid that a low-cost seller remains uninformed and recommends good A while charging the same price as a high-cost seller. The buyer should thus switch sellers with some probability following a high price for and a recommendation for good A even if this is the choice a high-cost seller who has gathered information would make. With moral hazard and adverse selection, terminating the relationship with a low-cost seller helps motivating that seller to acquire information (as in Section 6.1), and ending the relationship with a high-cost seller helps preventing a low-cost seller from mis-reporting its cost. Note that there is always more switching when dealing with an efficient seller than with an inefficient one.

COMPARISON WITH THE OPTIMAL REGULATION. Like the regulator in Section 5, the buyer is concerned with the low-cost seller's incentives to mimic a high-cost seller, charge high prices and recommend good A. Yet, the buyer has no control over prices, and fees are limited to be equilibrium continuation values. The only tool to reduce the low-cost seller's information rent is to switch sellers. Relaxing the low-cost seller's incentive constraint requires at the same time to switch sellers more often if a high price is charged for good A, and maybe less often if good A is recommended and a low price is charged for that good although such distortion is necessary in a pure moral hazard environment.

One may wonder whether a given customer could obtain more discipline from the seller by relying on a switching rule that would be contingent on the price. Suppose that a given customer uses such a switching rule. Remember that the seller charges the same price to all its customers. Assuming that there is a continuum of those, the price cannot be changed by the customers' behavior unless they would decide to act collectively. Put differently, our analysis has focused on switching rules that are contingent on the adequacy of reports only. These reports are instead targeted to each customer and the problem of collective action above no longer arises.

Indirect Regulation. Our analysis suggests that a regulator may favor a decentralized, indirect regulation via buyers' behavior over a more costly centralized information-intensive regulation. In that case, regulatory intervention can take several forms. For instance, (de)regulation may lower entry barriers so as to offer buyers more credible alternatives to which to switch. In markets where buyers learn from each other's experience and take switching decisions based on that information, regulation may internalize the information externality among buyers by incentivizing buyers to rate sellers, or penalizing contracts locking buyers to sellers.

7. Illustrations

This section illustrates our analysis with several examples where the provision of informational services is key to the retailing activity.

7.1. Health Care Sector

Drugs Markets and Pharmacists. In most countries, the pharmaceutical sector is subject to price regulation, but also to strict constraints on competition. Restrictions on drug distribution such as constraints on ownership or on the number and location of pharmacies are often justified by the fact that community pharmacists play a key role in detecting drug interactions and side-effects and facilitating suitable medicines use. Yet, critics view entry barriers as reflecting political pressure to protect pharmacists' market power, while we have seen that such market power may also undermine the provision of advice (Section 4).

Our results suggest that regulation can boost incentives for the provision of informational services and that competition can approximate this outcome if buyers use retrospective switching rules. To illustrate how our analysis sheds light on practices, France is a good example. A recent regulatory act (*Arrêté* dated of November 28/2014) allows pharmacists to collect a fee for their advising role, broadly defined as checking prescriptions, making generic substitution where needed, ensuring patients' understanding, and detecting potential drugs interactions. Since prescribed drugs are usually subject to binding price cap regulations,[26] the gains that pharmacists may derive from private information on their margins is limited. Thus, the fact that this regulation implements a constant fee across drugs is perhaps best interpreted in light of the optimal regulation that was characterized in presence of moral hazard only (Section 5.1). A constant fee is indeed a way to pay for the limited liability rent needed so that pharmacists provide careful advice.[27]

For non-prescription drugs, the situation is more complex. Because they are not subject to price regulation, pharmacists may enjoy more gains from private information on margins. This might also explain systematic biases in their recommendations. As pointed out in an Ecorys Study (2007) commissioned by the European Commission, entry barriers also induce high profit margins. The debate about the possible sources of the pharmacists' rents thus boils down to whether rents are justified by their expertise in providing advice on therapeutic choices, or whether they arise from market power and price-cost margin distortions (Philipsen and Faure, 2002). Section 4 shows that this view is still incomplete in that excessive market power may also generate mis-selling. In line with Section 6 predictions, fostering competition improve the quality of advice even though price distortions might still remain.[28]

[26]Dubois and Saethre (2016) provide evidence of those binding price constraints.

[27]It is important to notice that, in the current regulatory mode, this fee is paid on a goodwill basis in the sense that not only the pharmacists' efforts but also outcomes of his prescriptions remain unobservable.

[28]Throughout European countries but also in the U.S., non-prescription drugs also exhibit an important price dispersion. For instance, consumers study conducted in France reveals that in 2014 non-prescription drugs, which represent about 20% of total sales in pharmaceutical sector, vary from one to four while Sorensen (2000)'s empirical results reveal that non-prescription drugs are also characterized by a higher

Doctor-Patient Relationship. Unfortunately, health economics does not offer a unified view on how doctors compete and how they run their relationship with patients. Yet, three points are commonly admitted. First, doctors may exert non-contractible effort affecting health outcomes. Second, except for some special payment schemes in specific programs, health outcomes are not contractible either. Third, health outcomes may be observable by patients.[29]

Following Allard *et al.* (2009),[30] we believe that the retrospective switching rules modeled in Section 6 offer an accurate description of how doctors compete when prices/fees are not regulated.[31] The time spent by doctors with their patients allows them to establish a precise diagnostic and choose the most suitable therapy. As pointed out in McGuire (2000), this effort is not verifiable/contractible but improves health outcomes. Thus, as long as it is too costly to validate a patient's report, the implementation of the optimal regulation provided in Section 5 is not realistic in practice. It is thus more convincing to think of doctors as been disciplined by their patients' switching decisions. With that perspective in mind, Section 6 reveals that in health systems that rely on gatekeepers, it is important to guarantee low levels of switching costs to promote doctors' effort.

Instead, in some health systems, a regime similar to the optimal regulation derived in Section 6.2 could be approached. In the highly integrated Japanese health system for instance, doctors buy, prescribe and sell drugs with different mark-ups. Their profits thus directly depend on their prescription. Iizuka (2012) provides empirical evidence showing how doctors behave as imperfect agents for their patients, and that this agency relationship induces a low rate of adoption for generic drugs. In such contexts, health insurers might provide the expertise to collect all information needed to asses whether therapeutic choices have been effective, making the implementation of an optimal regulation along the lines of that described in Section 5.2 quite plausible.[32]

price dispersion in the U.S. market.

[29]The two last points are eloquently summarized in McGuire (2000) *"It may be infeasible to pay doctors on whether they are able to cure back pain because it is too costly to validate a patient's report. Nonetheless, the patient knows if his back still hurts. If the doctor is rewarded for doing a better job, because the patient is more likely to return or to recommend this doctor to friends, the doctor is encouraged to take unobserved actions to improve quality."*

[30]See also Iversen and Luras (2011) for an estimation of a dynamic model of the patient-doctor relationship in the case of Norway.

[31]A good illustration is offered by the case of France. Under the so-called *Sector II* regime, doctors can freely set their tariffs, usually above reimbursement levels of public coverage. Then, when they do not benefit from complementary health insurance coverage, patients have to pay out of their own pockets.

[32]Notice that the health insurers' role as monitors is facilitated by the fact that, for most of non-chronic diseases, the quality of a prescription is simply observed when the disease does not repeat or does not

7.2. *Financial Advising*

The global financial crisis and its aftermath have shed a crude light onto the conflicts of interest arising between financial advisers and their advisees in virtually all areas of the finance industry, from credit rating agencies to investment advisors, and from retail mortgage financing to investment banking. Some even argue conflict of interest is inherent to the intermediation nature of investment banking where the financial advisor must have a view of both sides of the market (Fox, 2010). This has led to a call for tighter regulatory oversight and, in some cases, more intense competition.

Investment Advising. Large banks are facing increasing scrutiny over their sales practices. For instance, in 2015, JPMorgan Chase agreed to pay a $307m penalty for failing to disclose to its clients that it was steering them away from investment products offered by rivals and towards a more expensive share class of proprietary mutual funds, from which it generated more profits. More generally, private bankers and other investment advisors are often accused of pushing investment strategies with higher turnover, and thus higher fees, and higher switching costs (*e.g.*, exit fees) than optimal for their clients.[33]

Our analysis points to the intricate issues involved in the regulation of such conflicts of interest. Regulators may need to deal not only with the quality of advice directly but also account for the inherent lack of transparency, and thus the high degree of information asymmetry, regarding the margins financial advisors realize on different products or strategies. One interesting aspect is the advisor's alleged ability to build up switching costs as part of the products they sell their clients. Regulatory efforts to mitigate such switching costs may also indirectly impact the quality of advice provision through two channels. First, increased competition may reduce the rent on the bank's own investment products, thereby promoting information collection. Second, lower switching costs may make it easier and more credible for clients to follow dynamic strategies of the type highlighted in Section 6, again boosting the banks' incentives to provide quality advice.

Credit Rating. Credit rating agencies were instrumental in the boom of the structured finance market in the years leading up to the financial crisis, and were accused

last over time.

[33]In a more unusual and colorful case, the Libyan Investment Authority (LIA) sued Goldman Sachs for $1.2bn to recover losses from nine "elephant trades" involving equity derivatives arranged in 2008 and which all expired worthless in 2011. The LIA alleged that Goldman exerted undue influence over its officials, who did not understand the trades, and earned about $222m from the trades. (Goldman Sachs was recently acquitted).

of having employed excessively lax credit rating standards when that market collapsed so dramatically. The structure of the credit rating industry, including its oligopolistic nature and the fact issuers, not investors, pay for ratings, was blamed by many for the excesses of the credit bubble years, leading to calls for the emergence of new agencies to offer further competition to the handful of major incumbents.

Because rating agencies are remunerated by the issuers, not the investors who rely on their ratings for their investment decisions, the industry structure is probably best captured by the buyer-seller dynamics model of Section 6 which can reflect the reputation loss a rating agency may incur (Mathis *et al.*, 2009). Agencies are arguably biased towards higher ratings as they are more likely to be accepted by issuers (Faure-Grimaud *et al.*, 2009), and generate more fees in expectation, including from repeat business. Indeed, issuers can opt not to publish a given rating, and published ratings generate ongoing fees while the issue is outstanding. Our analysis contributes to this debate on whether more stringent regulation is required. It also points to the challenges regulation might face when rating agencies have better knowledge of the margins they enjoy from different issuers, notably through consulting services.

8. Conclusion

In many instances, customers rely on sellers for advice about the goods or services they purchase from them. Such situations naturally give rise to conflicts of interest whereby sellers may steer customers towards higher margin goods or services. How to discipline expert-sellers' incentives is an issue of importance in a number of contexts.

This paper tackles this issue in a model with both moral hazard (the expert's decision to gather information is non-verifiable) and adverse selection (the expert has private information on his price-cost margins for different goods). The starting point of our analysis is the observation that information gathering incentives require that the seller's profits for both goods be similar enough. Technically, the profits must lie within an implementability cone, else the expert would have incentives to remain uninformed and recommend the highest margin good. Monopoly comes not only with the usual price distortions, it also induces under-provision of advice.

The least naive consumers should be able to use retrospective purchasing rules and buy again from a seller only if his advice proved correct. In such scenarios, consumers are *de facto* implementing (although imperfectly) an optimal regulation. Such repeated relationships might describe well market contexts where switching costs are relevant such as in the physician-patient or bank-client relationships.

One area of future research concerns the role of product market competition. A conjecture may be that some competition is desirable, to curb the sellers' market power and reduce price distortion, but excessively intense competition may lead to under-provision of advice. For sectors with some degree of competition and characterized by entry barriers, such as pharmaceutical distribution, this would allow to develop empirical tests connecting the outcome, *i.e.*, the quality of the match between the customers and the products, with competition intensity. In particular, such empirical estimations should focus on "goods maturity" to predict when more intense competition is likely to make the sellers' profits on the different goods more similar, thus motivating information gathering. The detailed analysis of specific markets may unveil new interesting features.

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Appendix

Proof of Lemma 1. The constrained set defined by (4.1) can be further developed as a pair of constraints (4.2). These constraints define a positive cone Γ in the (π_A, π_B) space with an extremal point given by (4.3) and directions given by positive vectors $(\varepsilon, 1-\varepsilon)$ and $(1-\varepsilon, \varepsilon)$. $\square$

Proof of Proposition 1. The buyer's preferences do not depend on the seller's private information c_A. Demand only depends on the price charged by the seller and not directly on the seller's cost. Thus beliefs (both on- and off-the equilibrium path) play no role in the buyer's behavior. For any specification of off-equilibrium beliefs, the best strategy for a seller with cost c_A is to charge the monopoly prices $p^m(c_A)$ for good A and $p^m(c)$ for good B, irrespective of the information he might have on the quality of the match.

Assumption 2 ensures that a high-cost seller gathers information and a low-cost seller does not. The low-cost seller always recommends good A while the high-cost seller makes a recommendation that reflects the signal he observes. This equilibrium allocation is unique and sustained by arbitrary beliefs following unexpected prices. $\square$

Proof of Proposition 2. An optimal contract maximizes the customer's expected surplus

$$\frac{\varepsilon}{2} \sum_{\sigma \in \{A,B\}} S(p_\sigma(c_A)) - T_\sigma(c_A) = \frac{\varepsilon}{2} \sum_{\sigma \in \{A,B\}} W(c_\sigma, p_\sigma(c_A)) - \pi_\sigma(c_A)$$

subject to the information gathering moral hazard constraint (4.1).

Since the implementability cone Γ expressed as (4.1) does not depend on prices, total surplus is obviously maximized with marginal cost pricing (5.1).

The maximum is obtained when $\frac{\varepsilon}{2} \sum_{\sigma \in \{A,B\}} \pi_\sigma(c_A)$ is minimized and, since Γ is a positive cone with directions $(\varepsilon, 1-\varepsilon)$ and $(1-\varepsilon, \varepsilon)$, this is achieved for the extremal point (4.3).

The cone Γ has a broad appeal. Not only does it describe the seller's profit pairs that induce information gathering but, more generally, it also describes the set of payoffs that any risk-neutral agent should receive to make information gathering valuable. It thus applies to the regulator as well which allows us to find out conditions so that information gathering is socially valuable. (Similar conditions apply throughout this Appendix.) Inducing information gathering is indeed viewed as valuable by the regulator when the difference between welfare and what is left to the seller to induce to gather information, namely $(W^*(c_A) - \pi^*, W^*(c) - \pi^*)$ also belongs to Γ. Developing this expression yields (5.3) and

$$\frac{(2\varepsilon-1)}{2}W^*(c) \geq \psi + \frac{\psi}{2\varepsilon-1}. \tag{A.1}$$

It is easy to verify that (5.3) implies (A.1) since $W^*(\cdot)$ is non-increasing. Hence, if it is optimal to induce information gathering by the low-cost seller, it is also so by a high-cost seller. □

PROOF OF PROPOSITION 3 . We consider contracts that induce information gathering from both types (Proposition 3). We turn later to the characterization of the conditions that ensure it is optimal to do so (using Proposition A.1 and Proposition A.2). First, we can write the regulator's objective under asymmetric information as

$$\mathbb{E}_{c_A}\left(\frac{\varepsilon}{2}\sum_{\sigma\in\Sigma} W(c_\sigma, p_\sigma(c_A)) - \psi - \mathcal{U}(c_A)\right). \tag{A.2}$$

Second, we develop the incentive constraints (5.4) and (5.5) respectively as

$$\mathcal{U}(\bar{c}_A) = \frac{\varepsilon}{2}\pi_A(\bar{c}_A) + \frac{\varepsilon}{2}\pi_B(\bar{c}_A) - \psi \geq \frac{1}{2}\pi_A(\bar{c}_A), \tag{A.3}$$

$$\mathcal{U}(\bar{c}_A) = \frac{\varepsilon}{2}\pi_A(\bar{c}_A) + \frac{\varepsilon}{2}\pi_B(\bar{c}_A) - \psi \geq \frac{1}{2}\pi_B(\bar{c}_A), \tag{A.4}$$

and

$$\mathcal{U}(\underline{c}_A) \geq \mathcal{U}(\bar{c}_A) + \frac{\varepsilon}{2}\Delta c D(p_A(\bar{c}_A)), \tag{A.5}$$

$$\mathcal{U}(\underline{c}_A) \geq \frac{1}{2}\pi_B(\bar{c}_A), \tag{A.6}$$

$$\mathcal{U}(\underline{c}_A) \geq \frac{1}{2}\pi_A(\bar{c}_A) + \frac{\Delta c}{2}D(p_A(\bar{c}_A)). \tag{A.7}$$

Participation is ensured for both types when it is so for a high-cost seller

$$\mathcal{U}(\bar{c}_A) \geq 0. \tag{A.8}$$

An optimal contract maximizes (A.2) subject to the incentive constraints (A.3) to (A.7) and the participation constraint (A.8).[34]

BINDING CONSTRAINTS. Fixing prices, we first minimize the expected rent left to the seller

$$\mathbb{E}_{c_A}(\mathcal{U}(c_A)) = \nu\mathcal{U}(\underline{c}_A) + (1-\nu)\left(\frac{\varepsilon}{2}\pi_A(\bar{c}_A) + \frac{\varepsilon}{2}\pi_B(\bar{c}_A) - \psi\right). \tag{A.9}$$

[34] The incentive constraint of a high-cost seller and the participation constraint of a low-cost one can be shown to be satisfied.

We distinguish two cases depending on which of the constraints (A.3) to (A.7) are binding when minimizing (A.9). Of course, some of these constraints are necessarily binding.

Case 1. Constraints (A.3), (A.4) and (A.7) are binding. Consider first the case where (A.3) and (A.4) are both binding to minimize $\mathcal{U}(\bar{c}_A)$. This implies

$$\pi_A(\bar{c}_A) = \pi_B(\bar{c}_A) = \frac{2\psi}{2\varepsilon - 1} \text{ and } \mathcal{U}(\bar{c}_A) = \frac{\psi}{2\varepsilon - 1} \tag{A.10}$$

and thus (A.7) is more constraining than (A.6).

Finally, observe that the fact that (A.7) is more constraining than (A.5) amounts to

$$\frac{1}{2}\pi_A(\bar{c}_A) + \frac{1}{2}\Delta c D(p_A(\bar{c}_A)) \geq \frac{\varepsilon}{2}\pi_A(\bar{c}_A) + \frac{\varepsilon}{2}\pi_B(\bar{c}_A) - \psi + \frac{\varepsilon}{2}\Delta c D(p_A(\bar{c}_A))$$

or

$$2\psi + (1-\varepsilon)\Delta c D(p_A(\bar{c}_A)) \geq \varepsilon\pi_B(\bar{c}_A) - (1-\varepsilon)\pi_A(\bar{c}_A) = 2\psi$$

where the last equality follows from the fact that (A.3) and (A.4) are binding.

From (A.7) binding, it follows that

$$\mathcal{U}(\underline{c}_A) = \frac{\psi}{2\varepsilon - 1} + \frac{1}{2}\Delta c D(p_A(\bar{c}_A)) \tag{A.11}$$

which, altogether with (A.10), gives us the following expression of the seller's expected rent

$$\mathbb{E}_{c_A}(\mathcal{U}(c_A)) = \frac{\psi}{2\varepsilon - 1} + \frac{\nu}{2}\Delta c D(p_A(\bar{c}_A)). \tag{A.12}$$

Case 2. Constraints (A.4), (A.5) and (A.7) are binding. Consider now the case where (A.4) is binding, (A.3) is slack and the right-hand side of (A.5) is weakly greater than the right-hand side of (A.7). This latter condition writes as $\mathcal{U}(\bar{c}_A) + \frac{\varepsilon}{2}\Delta c D(p_A(\bar{c}_A)) \geq \pi_A(\bar{c}_A) + \frac{1}{2}\Delta c D(p_A(\bar{c}_A))$, or

$$\varepsilon\pi_B(\bar{c}_A) - (1-\varepsilon)\pi_A(\bar{c}_A) \geq 2\psi + (1-\varepsilon)\Delta c D(p_A(\bar{c}_A)). \tag{A.13}$$

This condition implies $\varepsilon\pi_B(\bar{c}_A) - (1-\varepsilon)\pi_A(\bar{c}_A) > 2\psi$, which ensures that (A.3) holds.

The minimization of $\mathcal{U}(\bar{c}_A)$ subject to (A.4) and (A.13) implies that both constraints are binding.

Thus, (A.5) and (A.7) are also binding. This gives the following expressions of profits

$$\pi_A(\bar{c}_A) = \frac{2\psi}{2\varepsilon-1} + \frac{(1-\varepsilon)^2}{2\varepsilon-1}\Delta c D(p_A(\bar{c}_A)), \tag{A.14}$$

$$\pi_B(\bar{c}_A) = \frac{2\psi}{2\varepsilon-1} + \frac{(1-\varepsilon)\varepsilon}{2\varepsilon-1}\Delta c D(p_A(\bar{c}_A)). \tag{A.15}$$

These conditions imply $\pi_A(\bar{c}_A) < \pi_B(\bar{c}_A)$, so that (A.3) holds. Those formula also imply

$$\mathcal{U}(\bar{c}_A) = \frac{\psi}{2\varepsilon-1} + \frac{(1-\varepsilon)\varepsilon}{2(2\varepsilon-1)}\Delta c D(p_A(\bar{c}_A)), \tag{A.16}$$

$$\mathcal{U}(\underline{c}_A) = \frac{\psi}{2\varepsilon-1} + \frac{\varepsilon^2}{2(2\varepsilon-1)}\Delta c D(p_A(\bar{c}_A)). \tag{A.17}$$

It gives us the following expression of the seller's expected rent

$$\mathbb{E}_{c_A}(\mathcal{U}(c_A)) = \frac{\psi}{2\varepsilon-1} + \frac{(\nu\varepsilon+(1-\nu)(1-\varepsilon))\varepsilon}{2(2\varepsilon-1)}\Delta c D(p_A(\bar{c}_A)). \tag{A.18}$$

The comparison of (A.12) and (A.18) shows that the optimal contract that induces information gathering from both types is found in CASE 1 (resp. CASE 2) when

$$(\nu\varepsilon+(1-\nu)(1-\varepsilon))\varepsilon \geq \text{ (resp. } \leq)\nu(2\varepsilon-1) \Leftrightarrow \nu(1-\varepsilon)+(1-\nu)\varepsilon \geq \text{ (resp. } \leq)0$$

which is always true. Thus, CASE 1 is the only relevant one.

PRICES. Accounting for the expression of the seller's rent obtained above, prices must maximize

$$\mathbb{E}_{c_A}\left(\frac{\varepsilon}{2}W(c_A, p_A(c_A)) + \frac{\varepsilon}{2}W(c, p_B(c_A)) - \psi - \mathcal{U}(c_A)\right).$$

Inserting (A.12) into the above maximand and optimizing w.r.t. to prices $p_A(c_A)$ and $p_B(c_A)$ gives us (5.7) and (5.8).

FIXED FEES. The profit levels for each good are both given by (A.10) if $\bar{c}_A$ realizes. From this and the existing distortion of $p_A^{sb}(\bar{c}_A)$ given in (5.8), we obtain

$$\frac{2\psi}{2\varepsilon-1} - \frac{\nu}{(1-\nu)\varepsilon}D(p_A^{sb}(\bar{c}_A)) = T_A^{sb}(\bar{c}_A) < T_B^{sb}(\bar{c}_A) = \pi^*.$$

Profit levels for each good (and thus fixed fees since prices are then equal to marginal costs) remain indeterminate if $\underline{c}_A$ realizes. The sum of these fees is obtained from (A.11) as

$$\frac{\varepsilon}{2}\sum_{\sigma\in\{A,B\}} T_\sigma^{sb}(\underline{c}_A) = \frac{2\varepsilon\psi}{2\varepsilon-1} + \frac{1}{2}\Delta c D(p_A^{sb}(\bar{c}_A)), \tag{A.19}$$

while information gathering in state $\underline{c}_A$ holds when

$$U^{sb}(\underline{c}_A) \geq \max\left\{\frac{T_A^{sb}(\underline{c}_A)}{2}; \frac{T_B^{sb}(\underline{c}_A)}{2}\right\},$$

which can be written as a pair of inequalities

$$\varepsilon T_A^{sb}(\underline{c}_A) - (1-\varepsilon)T_B^{sb}(\underline{c}_A) \geq 2\psi, \tag{A.20}$$

$$\varepsilon T_B^{sb}(\underline{c}_A) - (1-\varepsilon)T_A^{sb}(\underline{c}_A) \geq 2\psi. \tag{A.21}$$

It is straightforward to check that (A.19), (A.20) and (A.21) altogether define a non-empty set of fixed fees $(T_A^{sb}(\underline{c}_A), T_B^{sb}(\underline{c}_A))$ that can be used to implement the optimal contract. A particular case is to have equal fees

$$T_A^{sb}(\underline{c}_A) = T_B^{sb}(\underline{c}_A) = \frac{2\psi}{2\varepsilon-1} + \frac{1}{2\varepsilon}\Delta c D(p_A^{sb}(\bar{c}_A)) > \pi^*. \tag{A.22}$$

PROFITS. From (A.10), we get (5.9). Because prices are equal to marginal costs on each good for a low-cost seller, (A.22) imply (5.10).

PAYOFF. When information is collected only by the high-cost seller, the expected consumer surplus becomes

$$\mathcal{W}_{11} = \nu\left(\frac{\varepsilon}{2}W^*(c-\Delta c) + \frac{\varepsilon}{2}W^*(c)\right) + (1-\nu)\left(\frac{\varepsilon}{2}W(c, p_A^{sb}(\bar{c}_A)) + \frac{\varepsilon}{2}W^*(c)\right) - \psi - \frac{\psi}{2\varepsilon-1} - \frac{\nu}{2}\Delta c D(p_A^{sb}(\bar{c}_A)).$$

INFORMATION GATHERING BY ONLY ONE TYPE. Suppose that the optimal contract requests that the high-cost seller never gathers information and that, in this case, good B is always sold. This possibility allows to save on the rent of the low-cost seller since the benefits of mimicking a high-cost type then disappear. The sole incentive constraint is that inducing information gathering for the low-cost seller

$$\mathcal{U}(\underline{c}_A) = \frac{\varepsilon}{2}\pi_A(\underline{c}_A) + \frac{\varepsilon}{2}\pi_B(\underline{c}_A) - \psi \geq \max\left\{\frac{1}{2}\pi_B(\underline{c}_A); \frac{1}{2}\pi_B(\underline{c}_A)\right\} \tag{A.23}$$

while the high-cost seller's participation constraint is

$$\mathcal{U}(\bar{c}_A) = \frac{1}{2}\pi_B(\bar{c}_A) \geq 0. \tag{A.24}$$

Replicating an argument made in the case of pure moral hazard, we obtain the following result.

PROPOSITION A.1. *Suppose also that c_A is private information and that both effort in information gathering and recommendations are unobservable. The optimal contract that induces information gathering by the low-cost seller only has the following properties.*

- *A low-cost seller charges price equal to marginal cost for both goods:*

$$p_A^{sb}(\underline{c}_A) = c - \Delta c \text{ and } p_B^{sb} = c. \tag{A.25}$$

- *The high-cost seller always sells good B at marginal cost:*

$$p_B^{sb}(\bar{c}_A) = c. \tag{A.26}$$

- *Fixed fees for the low-cost seller are:*

$$T_A^{sb}(\underline{c}_A) = T_B^{sb}(\underline{c}_A) = \pi^*. \tag{A.27}$$

PROOF OF PROPOSITION A.1. An optimal contract that induces information gathering from the low-cost seller only is implemented at minimal cost when the high-cost seller always chooses good B when uninformed. It maximizes

$$\nu\left(\frac{\varepsilon}{2}W(\underline{c}_A, p_A(\underline{c}_A)) + \frac{\varepsilon}{2}W(c, p_B(\underline{c}_A)) - \psi - \mathcal{U}(\underline{c}_A)\right) + (1-\nu)\left(\frac{1}{2}W(c, p_B(\bar{c}_A)) - \mathcal{U}(\bar{c}_A)\right) \tag{A.28}$$

subject to the truthtelling (A.23) and participation (A.24) constraints. These constraints are obviously binding. Optimizing w.r.t. prices gives (A.25) and (A.26). Finding the expressions of $T_A^{sb}(\underline{c}_A)$ and $T_B^{sb}(\underline{c}_A)$ in (A.27) is easily obtained. □

When information is collected only by the low-cost seller, expected consumer surplus is

$$\mathcal{W}_{10} = \nu\left(\frac{\varepsilon}{2}W^*(c-\Delta c) + \frac{\varepsilon}{2}W^*(c) - \psi - \frac{\psi}{2\varepsilon - 1}\right) + (1-\nu)\frac{1}{2}W^*(c).$$

Suppose now that the optimal contract requests that the low-cost seller never gathers information and that good A is always sold by that type while the high-cost seller gathers information.

The incentive constraint of a low-cost seller willing to mimic a high-cost one is

$$\mathcal{U}(\underline{c}_A) = \frac{1}{2}\pi_A(\underline{c}_A) \geq \mathcal{U}(\bar{c}_A) + \frac{\varepsilon}{2}\Delta c D(p_A(\bar{c}_A)) \tag{A.29}$$

while the high-cost seller's participation constraint remains

$$\mathcal{U}(\bar{c}_A) = \frac{\varepsilon}{2}\pi_A(\bar{c}_A) + \frac{\varepsilon}{2}\pi_B(\bar{c}_A) - \psi \geq \max\left\{\frac{1}{2}\pi_B(\underline{c}_A); \frac{1}{2}\pi_B(\underline{c}_A)\right\}. \tag{A.30}$$

PROPOSITION A.2. *Suppose also that c_A is private information and that both effort in information gathering and recommendations are unobservable. The optimal contract that induces information gathering by the high-cost seller only has the following properties.*

- *The low-cost seller only sells good A at price equal to marginal cost:*

$$p_A^{sb}(\underline{c}_A) = c - \Delta c. \tag{A.31}$$

- *The high-cost seller always sells good B at marginal cost and good A at price above marginal cost*

$$p_A^{sb}(\bar{c}_A) = c + \frac{\nu}{(1-\nu)\varepsilon}\Delta c, \ \textit{and } p_B^{sb}(\bar{c}_A) = c. \tag{A.32}$$

- *Profits for the high-cost seller are identical to those when the sole incentive problem comes from information gathering which gives the following expressions of fixed fees*

$$T_A^{sb}(\bar{c}_A) + \frac{\nu}{(1-\nu)\varepsilon}\Delta c D(p_A^{sb}(\bar{c}_A)) = T_B^{sb}(\bar{c}_A) = \pi^*. \tag{A.33}$$

The fixed fee for the low-cost seller selling good A is

$$T_A^{sb}(\underline{c}_A) = \frac{\varepsilon}{2}\Delta c D(p_A^{sb}(\bar{c}_A)). \tag{A.34}$$

PROOF OF PROPOSITION A.2. An optimal contract that induces information gathering from the high-cost seller only is implemented at minimal cost when it maximizes

$$\nu\left(\frac{1}{2}W(\underline{c}_A, p_A(\underline{c}_A)) - \mathcal{U}(\underline{c}_A)\right) + (1-\nu)\left(\frac{\varepsilon}{2}W(c, p_A(\bar{c}_A)) + \frac{\varepsilon}{2}W(c, p_B(\bar{c}_A)) - \psi - \mathcal{U}(\bar{c}_A)\right) \tag{A.35}$$

subject to the truthtelling (A.29) and participation (A.30) constraints. Those constraints are obviously binding. Optimizing w.r.t. prices gives (A.25) and (A.26). The proof for finding the expressions of $T_A^{sb}(\bar{c}_A)$ and $T_B^{sb}(\bar{c}_A)$ in (A.33) and (A.34) is then similar to that of Proposition A.1 although it takes into account that $p_A^{sb}(\bar{c}_A) > c$ so that profits net of fees on good A are positive. Condition (A.33) follows immediately from (A.29) binding and (A.31). □

When information is collected only by the low-cost seller, expected consumer surplus is

$$\mathcal{W}_{01} = \nu\frac{1}{2}W^*(c-\Delta c) + (1-\nu)\left(\frac{\varepsilon}{2}W(c, p_A^{sb}(\bar{c}_A)) + \frac{\varepsilon}{2}W^*(c) - \psi - \frac{\psi}{2\varepsilon - 1}\right).$$

OPTIMALITY OF INFORMATION GATHERING. Information gathering by both types is optimal when

$$\mathcal{W}_{11} \geq \max\{\mathcal{W}_{10}; \mathcal{W}_{01}\}.$$

Simplifying yields conditions (5.11) and

$$\frac{\varepsilon}{2}W(c, p_A^{sb}(\bar{c}_A)) - \frac{(1-\varepsilon)}{2}W^*(c) \geq \psi + \frac{\psi}{2\varepsilon - 1} + \frac{1}{2}\frac{\nu}{1-\nu}\Delta c D(p_A^{sb}(\bar{c}_A)).$$

Manipulating the left-hand side yields (5.12). □

PROOF OF PROPOSITION 5. To ease notations, let us define $K(c_A) = \frac{1-\varepsilon}{2}\pi^m(c_A) - \frac{\varepsilon}{2}\pi^m(c) + \psi$. Assumption 2 can be rewritten as

$$K(\bar{c}_A) < 0 < K(\underline{c}_A). \tag{A.36}$$

On top, observe that the following condition, that will be encountered in the analysis below,

$$\frac{\varepsilon}{2}\pi^m(\underline{c}_A) > \frac{2(1-\delta)}{\delta}K(\underline{c}_A) \tag{A.37}$$

holds when δ is close enough to 1.

SIMPLIFYING THE SET OF INCENTIVE COMPATIBLE CONSTRAINTS. We may rewrite (6.3) as a pair of constraints

$$\mathcal{U}(c_A) \geq \frac{\frac{1}{2}\pi^m(c)}{1 - \frac{\delta}{2}(\beta_B(c_A) + \gamma_B(c_A))}, \tag{A.38}$$

$$\mathcal{U}(c_A) \geq \frac{\frac{1}{2}\pi^m(c_A)}{1 - \frac{\delta}{2}(\beta_A(c_A) + \gamma_A(c_A))}. \tag{A.39}$$

We will neglect (A.38) and check that it is satisfied *ex post* once we have derived the solution to the so-called relaxed problem.

Together (6.2) and (A.39) imply that we may express the moral hazard incentive constraint when the seller has cost c_A in a more compact form as

$$\frac{\frac{\varepsilon}{2}(\pi^m(c_A)+\pi^m(c))-\psi}{1-\delta\left(\frac{\varepsilon}{2}(\beta_A(c_A)+\beta_B(c_A))+\frac{1-\varepsilon}{2}(\gamma_A(c_A)+\gamma_B(c_A))\right)}\geq\frac{\frac{1}{2}\pi^m(c_A)}{1-\frac{\delta}{2}(\beta_A(c_A)+\gamma_A(c_A))}.$$

After manipulations, we obtain

$$\text{(A.40)}\quad \delta\Bigg(\frac{\pi^m(c_A)}{2}\left(\frac{\varepsilon}{2}(\beta_A(c_A)+\beta_B(c_A))+\frac{1-\varepsilon}{2}(\gamma_A(c_A)+\gamma_B(c_A))\right)$$
$$-\left(\frac{\varepsilon}{2}(\pi^m(c_A)+\pi^m(c))-\psi\right)\frac{(\beta_A(c_A)+\gamma_A(c_A))}{2}\Bigg)\geq K(c_A).$$

Reciprocally, for a vector of probabilities of keeping the relationship $(\beta_A(c_A),\beta_B(c_A),\gamma_A(c_A),\gamma_B(c_B))$ that satisfies (A.40), we may recover the seller's continuation value $\mathcal{U}(c_A)$ from (6.2). Hence, (A.40) is both necessary and sufficient for characterizing the feasible set for problem (A.41).

Optimal probabilities of continuing the relationship. Isolating current period and continuation, we may rewrite the maximand in (6.1) as

$$\text{(A.41)}\quad (1-\delta)\mathcal{S}(c_A)=\max_{\substack{(\beta_\sigma,\gamma_\sigma)_{\sigma\in\Sigma}\\ 0\leq\beta_\sigma\leq1\\ 0\leq\gamma_\sigma\leq1}}\frac{\varepsilon}{2}(S(p^m(c_A))+S(p^m(c)))-\delta\Bigg(\frac{\varepsilon}{2}(1-\beta_A(c_A)+1-\beta_B(c_A))$$
$$+\frac{1-\varepsilon}{2}(1-\gamma_A(c_A)+1-\gamma_B(c_A))\Bigg)\Delta\mathcal{S}(c_A).$$

We look for probabilities of continuation that maximize the above expressions subject to the constraints (6.2) and (6.3). Up to some constants, the corresponding Lagrangean writes as

$$\delta\left(\frac{\varepsilon}{2}(\beta_A(c_A)+\beta_B(c_A))+\frac{1-\varepsilon}{2}(\gamma_A(c_A)+\gamma_B(c_A))\right)\Delta\mathcal{S}(c_A)$$
$$+\lambda\Bigg(\delta\Bigg(\frac{\pi^m(c_A)}{2}\left(\frac{\varepsilon}{2}(\beta_A(c_A)+\beta_B(c_A))+\frac{1-\varepsilon}{2}(\gamma_A(c_A)+\gamma_B(c_A))\right)$$
$$-\left(\frac{\varepsilon}{2}(\pi^m(c_A)+\pi^m(c))-\psi\right)\frac{(\beta_A(c_A)+\gamma_A(c_A))}{2}\Bigg)-K(c_A)\Bigg)$$

where λ is the non-negative multiplier of (A.40). This Lagrangean is linear in the probabilities $(\beta_\sigma(c_A),\gamma_\sigma(c_A))$. Whether the optimal probabilities are zero, one or interior depends on the sign of the coefficients of each of this linear expression. We now express these coefficients as

$$\text{(A.42)} \qquad \text{for } \beta_A(c_A) : \frac{\delta}{2}\left(\varepsilon\left(\Delta\mathcal{S}(c_A)+\lambda\frac{\pi^m(c_A)}{2}\right)-\lambda\left(\frac{\varepsilon}{2}(\pi^m(c_A)+\pi^m(c))-\psi\right)\right);$$

$$\text{(A.43)} \qquad \text{for } \beta_B(c_A) : \frac{\delta}{2}\varepsilon\left(\Delta\mathcal{S}(c_A)+\lambda\frac{\pi^m(c_A)}{2}\right);$$

$$\text{(A.44)} \qquad \text{for } \gamma_A(c_A) : \frac{\delta}{2}\left((1-\varepsilon)\left(\Delta\mathcal{S}(c_A)+\lambda\frac{\pi^m(c_A)}{2}\right)-\lambda\left(\frac{\varepsilon}{2}(\pi^m(c_A)+\pi^m(c))-\psi\right)\right);$$

$$\text{(A.45)} \qquad \text{for } \gamma_B(c_A) : \frac{\delta}{2}(1-\varepsilon)\left(\Delta\mathcal{S}(c_A)+\lambda\frac{\pi^m(c_A)}{2}\right).$$

Several facts follow immediately.

Fact 1. Suppose that (A.40) is slack so that moral hazard is not an issue. Making $\lambda = 0$ in the expressions (A.42) to (A.45) shows that all coefficients are positive so that all probabilities would be set to one. Inserting those values into (A.40) leads to $\delta K(c_A) > K(c_A)$; a contradiction when $c_A = \underline{c}_A$ since then $K(\underline{c}_A) > 0$ and a condition that holds when $c_A = c$ since then $K(c) < 0$ from (A.36). We deduce from this that, necessarily, (A.40) is binding if $c_A = \underline{c}_A$ and thus $\lambda > 0$ in that case. Instead, (A.40) is slack if $c_A = c$, thus $\lambda = 0$ in that case and the solution is obtained as in (6.4).

Fact 2. Turning now to the case $c_A = \underline{c}_A$, we first observe that the expressions (A.43) and (A.45) are necessarily positive which implies (6.5).

Fact 3. Comparing the expressions in (A.42) and (A.44) and noting that $\varepsilon > 1/2$, we have

$$\frac{\delta}{2}\left(\varepsilon\left(\Delta\mathcal{S}(\underline{c}_A)+\lambda\frac{\pi^m(\underline{c}_A)}{2}\right)-\lambda\left(\frac{\varepsilon}{2}(\pi^m(\underline{c}_A)+\pi^m(c))-\psi\right)\right)$$
$$> \frac{\delta}{2}\left((1-\varepsilon)\left(\Delta\mathcal{S}(\underline{c}_A)+\lambda\frac{\pi^m(\underline{c}_A)}{2}\right)-\lambda\left(\frac{\varepsilon}{2}(\pi^m(\underline{c}_A)+\pi^m(c))-\psi\right)\right).$$

Two cases must thus *a priori* be studied.

CASE 1. $\lambda > 0$ is such that the coefficient in (A.42) is zero while that in (A.44) is negative. In that case, we should have

$$\gamma_A^1(\underline{c}_A) = 0 \leq \beta_A^1(\underline{c}_A) \leq 1 \text{ and } \lambda = \frac{\varepsilon\Delta\mathcal{S}(\underline{c}_A)}{\frac{\varepsilon}{2}\pi^m(c)-\psi} > 0$$

where the denominator is positive since $\frac{\varepsilon}{2}\pi^m(c)-\psi > -K(c) > 0$.

CASE 2. $\lambda > 0$ is such that the coefficient in (A.42) is positive while that in (A.44) is zero. In that case, we should have

$$0 \leq \gamma_A^{mh}(\underline{c}_A) \leq \beta_A^{mh}(\underline{c}_A) = 1 \text{ and } \lambda = \frac{(1-\varepsilon)\Delta\mathcal{S}(\underline{c}_A)}{\frac{2\varepsilon-1}{2}\pi^m(\underline{c}_A)+\frac{\varepsilon}{2}\pi^m(c)-\psi} > 0$$

where the denominator is again positive since $\frac{2\varepsilon-1}{2}\pi^m(\underline{c}_A)+\frac{\varepsilon}{2}\pi^m(c)-\psi>\frac{\varepsilon}{2}\pi^m(c)-\psi>-K(c)>0$.

The comparison of Case 1 and Case 2 is straightforward. The customer's expected payoff is always greater in Case 2 since $\varepsilon\beta_A^1(\underline{c}_A)\le(1-\varepsilon)\gamma_A^{mh}(\underline{c}_A)+\varepsilon$.

Inserting the expressions of $\beta_B^{mh}(\underline{c}_A)$ $\gamma_B^{mh}(\underline{c}_A)$ and $\beta_A^{mh}(\underline{c}_A)$ obtained from (6.5), we obtain that $\gamma_A^{mh}(\underline{c}_A)$, when interior, must solve

$$\delta\left(\frac{\pi^m(\underline{c}_A)}{2}\left(\frac{1-\varepsilon}{2}(\gamma_A^{mh}(\underline{c}_A)-1)+1\right)-\left(\frac{\varepsilon}{2}(\pi^m(\underline{c}_A)+\pi^m(c))-\psi\right)\frac{(1+\gamma_A^{mh}(\underline{c}_A))}{2}\right)=K(\underline{c}_A)$$

which gives

$$\gamma_A^{mh}(\underline{c}_A)=1-2\frac{(1-\delta)K(\underline{c}_A)}{\delta\left(\frac{2\varepsilon-1}{2}\pi^m(\underline{c}_A)+\frac{\varepsilon}{2}\pi^m(c)-\psi\right)}\in[0,1). \tag{A.46}$$

Checking the omitted constraint (A.38). This condition clearly holds in the case $c_A=\bar{c}_A$. For $c_A=\underline{c}_A$, we can rewrite the seller's value as in (6.12) and we must thus check that

$$\frac{\frac{1}{2}\pi^m(\underline{c}_A)}{1-\frac{\delta}{2}(1+\gamma_A^{mh}(\underline{c}_A))}\ge\frac{\frac{1}{2}\pi^m(c)}{1-\delta}. \tag{A.47}$$

Using the expression for $\gamma_A^{mh}(\underline{c}_A)$ found in (A.46), we find

$$1-\frac{\delta}{2}(1+\gamma_A^{mh}(\underline{c}_A))=(1-\delta)\frac{\frac{1-\varepsilon}{2}\pi^m(\underline{c}_A)}{\frac{2\varepsilon-1}{2}\pi^m(\underline{c}_A)+\frac{\varepsilon}{2}\pi^m(c)-\psi}.$$

Inserting into (A.47) amounts to checking that $\frac{2\varepsilon-1}{2}(\pi^m(\underline{c}_A)+\pi^m(c))-\psi>0$, which obviously holds since Assumption 2 is satisfied.

□

Proof of Proposition 6. The proof follows similar steps to that of Proposition 5.

Simplifying the set of incentive compatible constraints. First, we observe that (6.2) and (A.39) are still true for type $\bar{c}_A$. The moral hazard constraint for that type writes again as

$$\delta\left(\frac{\pi^m(c)}{2}\left(\frac{\varepsilon}{2}(\beta_A(\bar{c}_A)+\beta_B(\bar{c}_A))+\frac{1-\varepsilon}{2}(\gamma_A(\bar{c}_A)+\gamma_B(\bar{c}_A))\right)\right.$$
$$\left.-(\varepsilon\pi^m(c)-\psi)\frac{(\beta_A(\bar{c}_A)+\gamma_A(\bar{c}_A))}{2}\right)\ge K(\bar{c}_A). \tag{A.48}$$

Following again (6.2) and (A.39), the moral hazard constraint for type $\underline{c}_A$ who reports truthfully his cost parameter must also be written as

$$\text{(A.49)}\quad \delta\Bigg(\frac{\pi^m(\underline{c}_A)}{2}\left(\frac{\varepsilon}{2}(\beta_A(\underline{c}_A)+\beta_B(\underline{c}_A))+\frac{1-\varepsilon}{2}(\gamma_A(\underline{c}_A)+\gamma_B(\underline{c}_A))\right) - \left(\frac{\varepsilon}{2}(\pi^m(\underline{c}_A)+\pi^m(c))-\psi\right)\frac{(\beta_A(\underline{c}_A)+\gamma_A(\underline{c}_A))}{2}\Bigg) \geq K(\underline{c}_A).$$

We then assume that (6.8) is a more stringent constraint than (6.9) and (6.10) and we check *ex post* this assertion once we have derived the optimal contract. Using also (6.2) to express the value $\mathcal{U}(\underline{c}_A)$ on the equilibrium path, incentive compatibility becomes

$$\mathcal{U}(\underline{c}_A) = \frac{\frac{\varepsilon}{2}(\pi^m(\underline{c}_A)+\pi^m(c))-\psi}{1-\delta\left(\frac{\varepsilon}{2}(\beta_A(\underline{c}_A)+\beta_B(\underline{c}_A))+\frac{1-\varepsilon}{2}(\gamma_A(\underline{c}_A)+\gamma_B(\underline{c}_A))\right)} \geq \frac{\frac{1}{2}\pi^m(c)+\frac{\Delta c}{2}D(p^m(c))}{1-\frac{\delta}{2}(\beta_A(\bar{c}_A)+\gamma_A(\bar{c}_A))}.$$

Or, after manipulations,

$$\text{(A.50)}\quad \left(1-\frac{\delta}{2}(\beta_A(\bar{c}_A)+\gamma_A(\bar{c}_A))\right)\left(\frac{\varepsilon}{2}(\pi^m(\underline{c}_A)+\pi^m(c))-\psi\right) \geq \left(1-\delta\left(\frac{\varepsilon}{2}(\beta_A(\underline{c}_A)+\beta_B(\underline{c}_A))+\frac{1-\varepsilon}{2}(\gamma_A(\underline{c}_A)+\gamma_B(\underline{c}_A))\right)\right)\left(\frac{1}{2}\pi^m(c)+\frac{\Delta c}{2}D(p^m(c))\right).$$

Optimal probabilities of continuing the relationship. We can thus write the customer's problem under asymmetric information as

$$(1-\delta)\mathbb{E}_{c_A}(\mathcal{S}(c_A)) = \max_{\substack{(\beta_\sigma(c_A),\gamma_\sigma(c_A))_{\sigma\in\Sigma}\\ 0\leq\beta_\sigma(c_A)\leq 1\\ 0\leq\gamma_\sigma(c_A)\leq 1}} \mathbb{E}_{c_A}\Bigg(\frac{\varepsilon}{2}(S(p^m(c_A))+S(p^m(c))) - \delta\left(\frac{\varepsilon}{2}(1-\beta_A(c_A)+1-\beta_B(c_A))+\frac{1-\varepsilon}{2}(1-\gamma_A(c_A)+1-\gamma_B(c_A))\right)\Delta\mathcal{S}(c_A)\Bigg)$$

subject to (A.48), (A.49) and (A.50).

As a preliminary remark, it is worth noticing that (A.48) already holds even when the relationship is not continued under any circumstances. Since the customer's payoff diminishes when the relationship is terminated, this means that (A.48) is always slack at the optimum. We thus denote by λ and μ the non-negative multipliers of the remaining constraints (A.49) and (A.50) respectively and we form the corresponding Lagrangean as

$$
\begin{aligned}
&\mathbb{E}_{c_A}\Bigg(\delta\left(\frac{\varepsilon}{2}(\beta_A(c_A)+\beta_B(c_A))+\frac{1-\varepsilon}{2}(\gamma_A(c_A)+\gamma_B(c_A))\right)\Delta\mathcal{S}(c_A)\Bigg)\\
&+\lambda\Bigg(\delta\Bigg(\frac{\pi^m(\underline{c}_A)}{2}\left(\frac{\varepsilon}{2}(\beta_A(\underline{c}_A)+\beta_B(\underline{c}_A))+\frac{1-\varepsilon}{2}(\gamma_A(\underline{c}_A)+\gamma_B(\underline{c}_A))\right)\\
&\qquad-\left(\frac{\varepsilon}{2}(\pi^m(\underline{c}_A)+\pi^m(c))-\psi\right)\frac{(\beta_A(\underline{c}_A)+\gamma_A(\underline{c}_A))}{2}\Bigg)-K(\underline{c}_A)\Bigg)\\
&+\mu\Bigg(\left(1-\frac{\delta}{2}(\beta_A(\bar{c}_A)+\gamma_A(\bar{c}_A))\right)\left(\frac{\varepsilon}{2}(\pi^m(\underline{c}_A)+\pi^m(c))-\psi\right)\\
&\qquad-\left(1-\delta\left(\frac{\varepsilon}{2}(\beta_A(\underline{c}_A)+\beta_B(\underline{c}_A))+\frac{1-\varepsilon}{2}(\gamma_A(\underline{c}_A)+\gamma_B(\underline{c}_A))\right)\right)\left(\frac{1}{2}\pi^m(c)+\frac{\Delta c}{2}D(p^m(c))\right)\Bigg).
\end{aligned}
$$

This Lagrangean is again linear in the probabilities of continuing the relationship, and we obtain the following expressions of the various coefficients

$$
\begin{aligned}
\text{(A.51)}\quad \text{for } \beta_A(\underline{c}_A):\ &\frac{\delta}{2}\Bigg((\varepsilon\left(\nu\Delta\mathcal{S}(\underline{c}_A)+\lambda\frac{\pi^m(\underline{c}_A)}{2}+\mu\left(\frac{1}{2}\pi^m(c)+\frac{\Delta c}{2}D(p^m(c))\right)\right)\\
&\qquad-\lambda\left(\frac{\varepsilon}{2}(\pi^m(\underline{c}_A)+\pi^m(c))-\psi\right)\Bigg);
\end{aligned}
$$

$$
\begin{aligned}
\text{(A.52)}\quad \text{for } \gamma_A(\underline{c}_A):\ &\frac{\delta}{2}\Bigg((1-\varepsilon)\left(\nu\Delta\mathcal{S}(\underline{c}_A)+\lambda\frac{\pi^m(\underline{c}_A)}{2}\right)+\mu\left(\frac{1}{2}\pi^m(c)+\frac{\Delta c}{2}D(p^m(c))\right)\\
&\qquad-\lambda\left(\frac{\varepsilon}{2}(\pi^m(\underline{c}_A)+\pi^m(c))-\psi\right)\Bigg);
\end{aligned}
$$

$$
\text{(A.53)}\qquad \text{for } \beta_B(\underline{c}_A):\ \frac{\delta}{2}\varepsilon\left(\nu\Delta\mathcal{S}(\underline{c}_A)+\lambda\frac{\pi^m(\underline{c}_A)}{2}+\mu\left(\frac{1}{2}\pi^m(c)+\frac{\Delta c}{2}D(p^m(c))\right)\right);
$$

$$
\text{(A.54)}\qquad \text{for } \beta_A(\bar{c}_A):\ \frac{\delta}{2}\left(\varepsilon(1-\nu)\Delta\mathcal{S}(\bar{c}_A)-\mu\left(\frac{\varepsilon}{2}(\pi^m(\underline{c}_A)+\pi^m(c))-\psi\right)\right);
$$

$$
\text{(A.55)}\qquad \text{for } \gamma_B(\underline{c}_A):\ \frac{\delta}{2}(1-\varepsilon)\left(\nu\Delta\mathcal{S}(\underline{c}_A)+\lambda\frac{\pi^m(\underline{c}_A)}{2}+\mu\left(\frac{1}{2}\pi^m(c)+\frac{\Delta c}{2}D(p^m(c))\right)\right);
$$

$$
\text{(A.56)}\qquad \text{for } \beta_B(\bar{c}_A):\ \frac{\delta}{2}\varepsilon(1-\nu)\Delta\mathcal{S}(\bar{c}_A);
$$

$$
\text{(A.57)}\qquad \text{for } \gamma_A(\bar{c}_A):\ \frac{\delta}{2}\left((1-\varepsilon)(1-\nu)\Delta\mathcal{S}(\bar{c}_A)-\mu\left(\frac{\varepsilon}{2}(\pi^m(\underline{c}_A)+\pi^m(c))-\psi\right)\right);
$$

$$
\text{(A.58)}\qquad \text{for } \gamma_B(\bar{c}_A):\ \frac{\delta}{2}(1-\varepsilon)(1-\nu)\Delta\mathcal{S}(\bar{c}_A).
$$

Several facts immediately follow.

Fact 1. Coefficients in (A.53), (A.55), (A.56) and (A.58) are positive. Thus, we have necessarily (6.13).

Fact 2. Suppose that (A.49) is slack so that moral hazard is not an issue for type $\underline{c}_A$. Making $\lambda = 0$ in the expressions (A.51) to (A.55) shows that all coefficients are positive so that all probabilities would be set to one. Inserting those values into (A.49) leads to $\delta K(\underline{c}_A) > K(\underline{c}_A)$; a contradiction. We then deduce that (A.49) is necessarily binding and thus $\lambda > 0$.

Fact 3. Comparing the expressions in (A.51) and (A.52) and noting that $\varepsilon > 1/2$, we have

$$\frac{\delta}{2}\left(\varepsilon\left(\nu\Delta\mathcal{S}(\underline{c}_A) + \lambda\frac{\pi^m(\underline{c}_A)}{2}\right) + \mu\left(\frac{1}{2}\pi^m(c) + \frac{\Delta c}{2}D(p^m(c))\right) - \lambda\left(\frac{\varepsilon}{2}(\pi^m(\underline{c}_A) + \pi^m(c)) - \psi\right)\right)$$

$$> \frac{\delta}{2}\left((1-\varepsilon)\left(\nu\Delta\mathcal{S}(\underline{c}_A) + \lambda\frac{\pi^m(\underline{c}_A)}{2}\right) + \mu\left(\frac{1}{2}\pi^m(c) + \frac{\Delta c}{2}D(p^m(c))\right) - \lambda\left(\frac{\varepsilon}{2}(\pi^m(\underline{c}_A) + \pi^m(c)) - \psi\right)\right).$$

From there, we could proceed as in the proof of Proposition 5, and recognize that the customer's expected surplus is maximized when the coefficient in (A.54) is positive while that in (A.57) is zero. It follows that the optimal probabilities of continuing satisfy

$$0 \leq \gamma_A^{sb}(\underline{c}_A) \leq \beta_A^{sb}(\underline{c}_A) = 1$$

i.e., (6.14), while the multiplier of (A.49) is

$$\lambda = \frac{(1-\varepsilon)(1-\nu)\Delta\mathcal{S}(\underline{c}_A) + \mu\left(\frac{1}{2}\pi^m(c) + \frac{\Delta c}{2}D(p^m(c))\right)}{\frac{2\varepsilon-1}{2}\pi^m(\underline{c}_A) + \frac{\varepsilon}{2}\pi^m(c) - \psi} > 0.$$

Since λ is positive, the moral hazard constraint is active so $\gamma_A^{sb}(\underline{c}_A) = \gamma_A^{mh}(\underline{c}_A)$.

Fact 4. Suppose that $\mu = 0$ at the solution. Then, it should be that $\gamma_A^{sb}(\bar{c}_A) \leq \beta_A^{sb}(\bar{c}_A) = 1$ and $\gamma_A^{sb}(\underline{c}_A) = \gamma_A^{mh}(\underline{c}_A)$. Inserting those values into (A.50), this constraint would be slack when

$$(1-\delta)\left(\frac{\varepsilon}{2}(\pi^m(\underline{c}_A) + \pi^m(c)) - \psi\right) \geq \left(1 - \delta + \delta\frac{1-\varepsilon}{2}(1 - \gamma_A^{mh}(\underline{c}_A))\right)\left(\frac{1}{2}\pi^m(c) + \frac{\Delta c}{2}D(p^m(c))\right).$$

But this condition is violated by the first condition that follows from Assumption 4 which ensures that in fact $\mu > 0$.

Two cases must thus *a priori* be studied.

CASE 1. $\mu > 0$ is such that the coefficient in (A.54) is zero while that in (A.57) is negative. In that case, we must have

$$\gamma_A^1(\bar{c}_A) = 0 \leq \beta_A^1(\bar{c}_A) \leq 1 \text{ and } \mu = \frac{\varepsilon(1-\nu)\Delta\mathcal{S}(\bar{c}_A)}{\frac{\varepsilon}{2}(\pi^m(\underline{c}_A) + \pi^m(c)) - \psi}.$$

Case 2. $\mu > 0$ is such that the coefficient in (A.54) is positive while that in (A.57) is zero. In that case, we should have (6.15) since

$$\mu = \frac{(1-\varepsilon)(1-\nu)\Delta\mathcal{S}(\bar{c}_A)}{\frac{\varepsilon}{2}(\pi^m(\underline{c}_A)+\pi^m(c))-\psi} \text{ and } 0 \leq \gamma_A^{sb}(\bar{c}_A) \leq \beta_A^{sb}(\bar{c}_A) = 1.$$

Because $\varepsilon > 1/2$, the coefficient in (A.54) is always greater than that in (A.57). The comparison of Case 1 and Case 2 is straightforward. The customer's expected payoff is always greater in Case 2 since $\varepsilon\beta_A^1(\bar{c}_A) \leq (1-\varepsilon)\gamma_A^{sb}(\bar{c}_A)+\varepsilon$.

Fact 5. Inserting the expressions of $\beta_B^{sb}(\bar{c}_A)$ $\gamma_B^{sb}(\underline{c}_A)$ and $\beta_A^{sb}(\underline{c}_A)$ obtained respectively from (6.13) and (6.15), we obtain that $\gamma_A^{sb}(\bar{c}_A)$, when interior, must solve

$$\text{(A.59)} \quad \left(1-\frac{\delta}{2}(1+\gamma_A^{sb}(\bar{c}_A))\right)\left(\frac{\varepsilon}{2}(\pi^m(\underline{c}_A)+\pi^m(c))-\psi\right) = \left(1-\delta+\delta\frac{1-\varepsilon}{2}(1-\gamma_A^{mh}(\underline{c}_A))\right)\left(\frac{1}{2}\pi^m(c)+\frac{\Delta c}{2}D(p^m(c))\right).$$

Existence of such $\gamma_A^{sb}(\bar{c}_A) < 1$ follows from Assumption 4. The second item in (6.14) follows.

Checking the omitted constraints (6.9) and (6.10). We need to check that

$$\mathcal{U}(\underline{c}_A) = \frac{\frac{\varepsilon}{2}(\pi^m(\underline{c}_A)+\pi^m(c))-\psi}{1-\delta+\delta\frac{1-\varepsilon}{2}(1-\gamma_A^{mh}(\underline{c}_A))} = \frac{\frac{1}{2}\pi^m(\underline{c}_A)}{1-\frac{\delta}{2}(1+\gamma_A^{mh}(\underline{c}_A))}$$
$$\geq \max\left\{\frac{\frac{1}{2}\pi^m(c)+\frac{\Delta c}{2}D(p^m(c))}{1-\frac{\delta}{2}(1+\gamma_A^{sb}(\bar{c}_A))};\frac{\frac{1}{2}\pi^m(c)}{1-\delta};\frac{\varepsilon\left(\pi^m(c)+\frac{\Delta c}{2}D(p^m(c))\right)-\psi}{1-\delta+\delta\frac{1-\varepsilon}{2}(1-\gamma_A^{sb}(\bar{c}_A))}\right\}.$$

By (A.59) the left-hand side is equal to the first term in the maximand on the right-hand side. The second and third terms in this maximand are dominated by the first one when

$$\text{(A.60)} \quad \frac{\frac{\varepsilon}{2}(\pi^m(\underline{c}_A)+\pi^m(c))-\psi}{1-\delta+\delta\frac{1-\varepsilon}{2}(1-\gamma_A^{mh}(\underline{c}_A))} \geq \max\left\{\frac{\frac{1}{2}\pi^m(c)}{1-\delta};\frac{\varepsilon\left(\pi^m(c)+\frac{\Delta c}{2}D(p^m(c))\right)-\psi}{1-\delta+\delta\frac{1-\varepsilon}{2}(1-\gamma_A^{sb}(\bar{c}_A))}\right\}$$

When δ goes to 1, (6.6) shows that $\gamma_A^{mh}(\underline{c}_A)$ goes also to 1 from below. Then, the first inequality in (A.60) follows from the fact that, in the limit, we have

$$\frac{\frac{\varepsilon}{2}(\pi^m(\underline{c}_A)+\pi^m(c))-\psi}{1-\delta+\delta\frac{1-\varepsilon}{2}(1-\gamma_A^{mh}(\underline{c}_A))} > \frac{\frac{1}{2}\pi^m(c)}{1-\delta}$$

which itself follows from $\frac{\varepsilon}{2}\pi^m(\underline{c}_A)+\frac{\varepsilon}{2}\pi^m(c)-\psi \geq \frac{1}{2}\pi^m(c)$, which is implied by Assumption 2. Last, tedious but straightforward manipulations show that, when $\gamma_A^{sb}(\bar{c}_A)$ is interior, we have

$$\gamma_A^{mh}(\underline{c}_A)-\gamma_A^{sb}(\bar{c}_A) = \frac{2D(p^m(c))(1-\delta)\Delta c\,\varepsilon}{\delta\left((2\varepsilon-1)\pi^m(\underline{c}_A)+\varepsilon\pi^m(c)-2\psi\right)} > 0.$$

When δ goes to 1, (A.59) shows that $\gamma_A^{sb}(\bar{c}_A)$ goes also to 1 from below. Then, the second inequality in (A.60) follows from when, again in the limit, we have

$$\frac{\varepsilon}{2}(\pi^m(\underline{c}_A) + \pi^m(c)) - \psi \geq \varepsilon \left(\pi^m(c) + \frac{\Delta c}{2} D(p^m(c))\right) - \psi$$

which itself follows from $\pi^m(\underline{c}_A) > \pi^m(c) + \Delta c D(p^m(c))$. Those arguments show that, when δ is close enough to 1, the omitted constraints (6.9) and (6.10) are satisfied.

Conditions for inducing information gathering. When δ is close enough to 1, inducing partial information gathering is costly because the customer incurs the cost of switching, while continuing the relationship with the optimal probabilities $\gamma_A^{sb}(\bar{c}_A)$ and $\gamma_A^{mh}(\underline{c}_A)$ found above to incentivize the seller approximates the full information outcomes since these probabilities are very close to 1. Hence, Assumption 3 again ensures that inducing information gathering by both types is optimal. □