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Abstract

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JEL Classification: C13, C51, E30

Keywords: Bayesian estimation, Monte-Carlo methods, Maximum Likelihood Estimation, DSGE model, Closed economy, Open economy

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Identification Versus Misspecification in New Keynesian Monetary Policy Models*

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November 2018

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1. Introduction

In this paper we study two key challenges in developing empirical New-Keynesian DSGE models for policy analysis: identification and misspecification. Following the seminal papers by Christiano, Eichenbaum and Evans (2005), and Smets and Wouters (2003), estimated dynamic stochastic general equilibrium (DSGE) models have become one of the most important tools available to policy-makers at central banks. However, when the quantitative implications of the models are of interest, it is vital that the inputs to the models, i.e. their parameter values, have empirical credibility. This has stimulated a very active research effort aimed at the estimation and empirical evaluation of DSGE models.

Most papers in the empirical DSGE literature have used Bayesian estimation techniques. The choice of this approach can partly be explained by compelling arguments for why Bayesian methods are appropriate when thinking about possibly misspecified macroeconomic models, see e.g. Sims (2007, 2008). But it is also conceivable that Bayesian methods have been applied to remedy identification problems. If some of the parameters in the model are not identified by the data, the prior distribution provide curvature and thus enables estimation of the model. The dominant narrative, see e.g. Blanchard (2016a, 2016b), is that lack of identification is the main reason for the adoption of Bayesian methods. Our paper provides a different perspective by arguing that model design and problems with model misspecification may still be a first-order issue for many monetary DSGE models, and that a key benefit of moving from a classical to a Bayesian framework is to mitigate the consequences of such problems.

In line with the dominant narrative, Canova and Sala (2009) and Iskrev (2008, 2010a), suggest that there is often insufficient information in the data to learn about the structural parameters in DSGE models, thus casting doubt on the empirical results and policy inference drawn from estimated DSGE models. Canova and Sala (2009) focus on limited information methods (i.e. the minimum distance estimator used by e.g. Christiano, Eichenbaum and Evans, 2005) and argue that problems with observational equivalence between parameterizations are widespread in DSGE models and that Bayesian methods may hide underlying identification problems. Following Canova and Sala (2009), we define a DSGE model as suffering from observational equivalence if different parameterizations of the model are indistinguishable with respect to the likelihood for a given set of observable variables. Another, arguably more relevant, case in practice is a situation where the DSGE model is only weakly identified, i.e. where the likelihood function has a unique but only weak curvature for (some or all of) the parameters that the econometrician tries to estimate. In the former case, the ML estimator will be inconsistent, whereas in the latter case, the ML estimator will be consistent but a very large sample may be required to learn about (some of) the parameters of the DSGE model.[1] Iskrev (2008, 2010a), on the other hand, considers full information methods and develops a test to check for identification prior to estimation. Iskrev's analysis of the Smets and Wouters (2007) model also points toward local indistinguishability between some of the parameters, given that the econometrician attempts to estimate a large set of parameters. In conclusion, this leads to a widely held belief that maximum likelihood estimation may not be feasible because many parameters are poorly identified, see for example the recent discussion in Blanchard (2016a,b) and the references therein.

In contrast, we show in this paper that identification should not be assumed to be weak

[1] So with Canova and Sala's (2009) definition of weak identification, sample size matters for identification strength. This definition is in analogy with, but not identical to, the weak instrument problem studied in the GMM literature as the weak instrument problem is not necessarily a small sample problem, see Stock et al. (2002). The definition by Canova and Sala is convenient for us as they study identification in DSGE models.

for all models. Models can have similar core features and yet differ in terms of the degree of identification. Identification should therefore be assessed on a case-by-case basis, by applying recently developed pre-tests of identification. Such tests are *fast* and effective in highlighting major identification issues *before* models are actually taken to the data.

To begin with, we use Iskrev's (2010a,b) tests of identification on a prototype DSGE model to show that identification problems are limited to a few parameters if a suitable set of observed variables is matched. Iskrev's tests are particularly useful because they allow us to exploit the entire prior parameter space.[2] Next, we complement Iskrev's identification tests by studying the small sample properties of the classical maximum likelihood (ML) estimator in a Monte Carlo simulation exercise. To generate artificial samples, we use a log-linearized version of a typical New Keynesian open-economy DSGE model, parameterized by the Bayesian posterior median estimate in Adolfson et al. (2008a). We look at small sample distributions for individual parameters and show that the ML estimator is unbiased for nearly all parameters, and as the sample size increases, the small sample bias that occurs for a few parameters disappears. We also show that the mean square error is small (relative to e.g. the prior uncertainty) for many parameters. In contrast to Canova and Sala (2009), we therefore conclude that, in our case, many of the structural parameters are well identified and taking the model to the data will be informative about their values.

We then estimate closed and open economy models on actual Swedish (open economy) and U.S. (closed economy) data with classical ML estimation techniques and compare the results with the Bayesian estimation results. For both countries, we obtain an outsized increase in the log likelihood at the ML estimate which we interpret as informativeness in the data. A key finding in the ML estimation is that the Calvo parameters in the price-setting equations are driven to implausibly high values. We argue that this is due to misspecified price-setting behavior in two complementary ways. First, we apply a formal test of misspecification by using the DSGE-VAR methodology of Del Negro and Schorfheide (2004, 2009). This tool measures how well the DSGE model mimics the data, and we obtain a large improvement in marginal likelihood when relaxing the cross-equation restrictions in the DSGE. It is hence clear that both the closed and open economy models are misspecified, but the test does not tell us in which dimensions the DSGEs need to be altered. Consequently, we complement our direct evidence with an informal test, where misspecification is introduced in the DSGE through an incorrectly specified Phillips curve, and study how ML estimation on simulated data is affected by this. The results provide support for problems in the price-setting bloc.

A possible limitation of our analysis is that it is restricted to two baseline models, the workhorse New Keynesian open-economy DSGE model in Adolfson et al. (2008a) and a closed economy formulation of the same model (Adolfson, 2008b). This feature may limit our ability to draw any conclusions about identification and misspecification in New Keynesian DSGE models more generally. There are, however, three distinct reasons why we think our analysis may be of importance to a wider range of studies.

First, the closed and open economy models we study have well-documented good empirical

[2] We make use of the tests in Iskrev (2010a,b) since they have been applied to the workhorse model of Smets and Wouters (2007), with which our models share many features. Other recent papers which have developed useful tools to study identification in DSGE models include Andrle (2010) and Komunjer and Ng (2011), who build on the same asymptotic theory as Iskrev (2010a); Müller (2013), who discusses uncertainty estimates robust to misspecification; Consolo, Favero and Paccagnini (2009), Guerron-Quintana, Inoue and Kilian (2013), Koop, Pesaran and Smith (2011), Qu (2014) and Andrews and Mikusheva (2015). The latter study encompasses most of the recent literature on the subject and also proposes an informal test for weak identification. In Appendix C, to complement Iskrev (2010a,b), we discuss and show that the results of such an informal test applied to our models do not alter the general outcomes.

properties (see e.g. Adolfson et al. 2008a,b).[3] Second, the parameterization and structure of the closed economy formulation of the model is literally a multi-shock variant of the Christiano, Eichenbaum and Evans (2005) model which is in turn the backbone of other key empirical models in the DSGE literature (see e.g. Smets and Wouters 2003 and 2007, and Coenen et al., 2012). To show this, impulse response functions to a monetary policy shock in six different models of the Euro Area and the US (see Wieland et al., 2012), including ours, are reported in Figure 1. As can be seen, the transmission mechanism in our model is almost identical to the other closed and open economy benchmark models. Third, several central banks use very similar models, e.g., the Federal Reserve Board's SIGMA model (Erceg et al., 2006), the European Central Bank's New Area Wide model (Christoffel et al., 2008), the International Monetary Fund's GEM model (Pesenti, 2008), and the European Commission's QUEST model (Ratto et al., 2009). See Coenen et al. (2012) for a review of these models.

The remainder of the paper is organized as follows. In the next section, we describe the closed and open economy DSGE models that we use as the data-generating processes. We perform a detailed identification analysis in Section 3: first the identification tests developed in Iskrev (2010a,b) followed by the Monte Carlo simulation exercise. In Section 4 we address misspecification issues. We estimate the DSGE model on actual Swedish and U.S. data with ML techniques and offer a comparison with the Bayesian estimation results. We subsequently apply formal and informal tests of misspecification to the model and assess the economic implications of the differences between the Bayesian posterior and ML estimate. Finally, we provide some concluding remarks in Section 5.

2. The DGP - variations of a standard New Keynesian DSGE model

As data-generating processes (DGP) we use two variations of a standard New Keynesian DSGE model. The open economy formulation of the model is identical to the model estimated in Adolfson et al. (2008a). It shares its basic closed economy features with many recent new Keynesian models, including the benchmark models of Christiano, Eichenbaum and Evans (2005), Altig, Christiano, Eichenbaum and Lindé (2011), and Smets and Wouters (2003). This section provides an overview of the open economy model. At the end of the section, we show how to recast it as a standard closed economy model.

2.1. The model

The model economy includes four different categories of operating firms. These are domestic goods firms, importing consumption, importing investment, and exporting firms, respectively. Within each category, there is a continuum of firms that each produces a differentiated good and set prices. The domestic goods firms produce their goods using capital and labor inputs, and sell them to a retailer who transforms the intermediate products into a homogenous final

[3] With the exception of the uncovered interest rate parity condition, this model is essentially identical to the model originally developed by Adolfson et al. (2007). Sims (2007) points out that this is the first estimated fully-fledged DSGE model that is in full operational use as the core model in the policy process at an inflation targeting central bank (Sveriges Riksbank). Also, many models in the open-economy literature are similar in spirit (see e.g. Cristadoro et al., 2008, Justiniano and Preston, 2010, Rabanal and Tuesta, 2010 and Smets and Wouters, 2002).

good according to the CES function

$$Y_t = \left[\int_0^1 (Y_{i,t})^{\frac{1}{\lambda_t^d}} \, di \right]^{\lambda_t^d}, 1 \leq \lambda_t^d < \infty, \tag{1}$$

where λ_t^d is a stochastic process that determines the time-varying flexible-price markup in the domestic goods market.

The production function for intermediate good i is given by

$$Y_{i,t} = z_t^{1-\alpha} \epsilon_t K_{i,t}^{\alpha} H_{i,t}^{1-\alpha} - z_t \phi, \tag{2}$$

where z_t is a unit-root technology shock capturing world productivity, ϵ_t is a domestic covariance stationary technology shock, $K_{i,t}$ the capital stock and $H_{i,t}$ denotes homogeneous labor hired by the i^{th} firm. A fixed cost $z_t\phi$ is included in the production function. We set this parameter so that profits are zero in steady state, following Christiano et al. (2005).

We allow for working capital by assuming that the intermediate firms' wage bill has to be financed in advance through loans from a financial intermediary. Cost minimization then yields the following nominal marginal cost for intermediate firm i:

$$mc_t^d = \frac{1}{(1-\alpha)^{1-\alpha}} \frac{1}{\alpha^{\alpha}} (R_t^k)^{\alpha} (W_t R_{t-1})^{1-\alpha} \frac{1}{(z_t)^{1-\alpha}} \frac{1}{\epsilon_t}, \tag{3}$$

where R_t^k is the gross nominal rental rate per unit of capital, R_{t-1} the gross nominal (economy wide) interest rate, and W_t the nominal wage rate per unit of aggregate, homogeneous, labor $H_{i,t}$.

Each of the domestic goods firms is subject to price stickiness through an indexation variant of the Calvo (1983) model. Since we have a time-varying inflation target in the model, we allow for partial indexation not only to the current inflation target, but also to last period's inflation rate in order to allow for a lagged pricing term in the Phillips curve. Each intermediate firm faces in any period a probability $(1 - \xi_d)$ that it can reoptimize its price.[4] In each period, the price is set so that the firms maximize a future stream of marginal utility discounted period-profits, taking into account that they might not be able to optimally change their price in each future period.

Log-linearization of the first-order condition of the profit maximization problem yields the following log-linearized Phillips curve:

$$\begin{aligned} \left(\widehat{\pi}_t^d - \widehat{\bar{\pi}}_t^c\right) &= \frac{\beta}{1+\kappa_d\beta}\left(\mathrm{E}_t\widehat{\pi}_{t+1}^d - \rho_{\bar{\pi}}\widehat{\bar{\pi}}_t^c\right) + \frac{\kappa_d}{1+\kappa_d\beta}\left(\widehat{\pi}_{t-1}^d - \widehat{\bar{\pi}}_t^c\right) \\ &\quad - \frac{\kappa_d\beta\,(1-\rho_{\bar{\pi}})}{1+\kappa_d\beta}\widehat{\bar{\pi}}_t^c + \frac{(1-\xi_d)(1-\beta\xi_d)}{\xi_d\,(1+\kappa_d\beta)}\left(\widehat{mc}_t^d + \widehat{\lambda}_t^d\right), \end{aligned} \tag{4}$$

where $\hat{\pi}_t^d$ denotes inflation in the domestic sector (a hat denotes percent deviation from steady state, i.e., $\hat{X}_t = dX_t/X \approx \ln X_t - \ln X$) and $\widehat{\bar{\pi}}_t^c$ the time-varying inflation target of the central bank. β is the discount factor and $\rho_{\bar{\pi}}$ the persistence coefficient in the AR(1)-process for $\widehat{\bar{\pi}}_t^c$.

We now turn to the import and export sectors. There is a continuum of importing consumption and investment firms that each buys a homogenous good at price P_t^* in the world market,

[4] For the firms that are not allowed to reoptimize their price P_{t+1}^d, we adopt the indexation scheme $P_{t+1}^d = \left(\pi_t^d\right)^{\kappa_d} \left(\bar{\pi}_{t+1}^c\right)^{1-\kappa_d} P_t^d$ where κ_d is an indexation parameter.

and converts it into a differentiated good through a brand-naming technology. The exporting firms buy the (homogenous) domestic final good at price P_t^d and turn this into a differentiated export good through the same type of brand naming. The nominal marginal costs of the importing and exporting firms are thus $S_t P_t^*$ and P_t^d / S_t, respectively, where S_t is the nominal exchange rate (domestic currency per unit of foreign currency). The differentiated import and export goods are subsequently aggregated by an import consumption, import investment and export packer, respectively, so that the final import consumption, import investment, and export good is each a CES composite according to the following:

$$C_t^m = \left[\int_0^1 \left(C_{i,t}^m \right)^{\frac{1}{\lambda_t^{mc}}} di \right]^{\lambda_t^{mc}}, \qquad I_t^m = \left[\int_0^1 \left(I_{i,t}^m \right)^{\frac{1}{\lambda_t^{mi}}} di \right]^{\lambda_t^{mi}}, \qquad X_t = \left[\int_0^1 \left(X_{i,t} \right)^{\frac{1}{\lambda_t^{x}}} di \right]^{\lambda_t^{x}}, \tag{5}$$

where $1 \leq \lambda_t^j < \infty$ for $j = \{mc, mi, x\}$ is the time-varying flexible-price markup in the import consumption (mc), import investment (mi) and export (x) sector. By assumption, the continuum of consumption and investment is invoiced by importers in the domestic currency and by exporters in the foreign currency. To allow for short-run incomplete exchange rate pass-through to import and export prices, we introduce nominal rigidities in the local currency price, modeled through the same type of Calvo setup as above. The price-setting problems of the importing and exporting firms are completely analogous to those of the domestic firms, and in total there are thus four specific Phillips curve relations determining inflation in the domestic, import consumption, import investment and export sectors.

There is a continuum of j households, whose preferences are given by

$$\mathrm{E}_0^j \sum_{t=0}^{\infty} \beta^t \left[\zeta_t^c \ln \left(C_{j,t} - b C_{j,t-1} \right) - \zeta_t^h A_L \frac{\left(h_{j,t} \right)^{1+\sigma_L}}{1+\sigma_L} + A_q \frac{\left(\frac{Q_{j,t}}{z_t P_t^d} \right)^{1-\sigma_q}}{1-\sigma_q} \right], \tag{6}$$

where $C_{j,t}$, $h_{j,t}$ and $Q_{j,t} / \left(z_t P_t^d \right)$ denote the j^{th} household's levels of aggregate consumption, labor supply and stationarized real cash holdings, respectively. Consumption is subject to habit formation through $b C_{j,t-1}$. ζ_t^c and ζ_t^h are persistent preference shocks to consumption and labor supply, respectively. Aggregate consumption is assumed to be given by the following CES function:

$$C_t = \left[(1-\omega_c)^{1/\eta_c} \left(C_t^d \right)^{(\eta_c - 1)/\eta_c} + \omega_c^{1/\eta_c} \left(C_t^m \right)^{(\eta_c - 1)/\eta_c} \right]^{\eta_c/(\eta_c - 1)}, \tag{7}$$

where C_t^d and C_t^m are consumption of the domestic and imported good (provided by the domestic and importing consumption firms, respectively). ω_c is the share of imports in consumption, and η_c is the elasticity of substitution across consumption goods.

The households invest in a basket of domestic and imported investment goods (I_t) to form the capital stock (K_t), and decide how much capital to rent to the domestic firms given costs of adjusting the investment rate. The capital accumulation equation is given by

$$K_{t+1} = (1-\delta) K_t + \Upsilon_t \left(1 - \tilde{S} \left(I_t / I_{t-1} \right) \right) I_t, \tag{8}$$

where $\tilde{S} \left(I_t / I_{t-1} \right)$ determines the investment adjustment costs through the estimated parameter $\tilde{S}''$, and Υ_t is a stationary investment-specific technology shock. Total investment is assumed to be given by a CES of aggregate domestic and imported investment goods (I_t^d and I_t^m, respectively) according to

$$I_t = \left[(1-\omega_i)^{1/\eta_i} \left(I_t^d \right)^{(\eta_i - 1)/\eta_i} + \omega_i^{1/\eta_i} \left(I_t^m \right)^{(\eta_i - 1)/\eta_i} \right]^{\eta_i/(\eta_i - 1)}, \tag{9}$$

where ω_i is the share of imports in investment, and η_i is the elasticity of substitution across investment goods.

Following Erceg, Henderson and Levin (2000), each household is a monopoly supplier of a differentiated labor service which implies that they can set their own wage. After having set their wage, households supply the firms' demand for labor at the going wage rate. Each household sells its labor to a firm which transforms household labor into a homogenous good that is demanded by each of the domestic goods-producing firms. Wage stickiness is introduced through the Calvo (1983) setup, with partial indexation to last period's CPI inflation rate, the current inflation target and technology growth. Household j reoptimizes its nominal wage rate $W_{j,t}^{new}$ according to the following

$$\begin{array}{c} \max\limits_{W_{j,t}^{new}} \quad \mathrm{E}_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \left[-\zeta_{t+s}^h A_L \frac{(h_{j,t+s})^{1+\sigma_L}}{1+\sigma_L} + \right. \\ \left. \upsilon_{t+s} \frac{(1-\tau_{t+s}^y)}{(1+\tau_{t+s}^w)} \left(\left(\pi_t^c \ldots \pi_{t+s-1}^c \right)^{\kappa_w} \left(\bar{\pi}_{t+1}^c \ldots \bar{\pi}_{t+s}^c \right)^{(1-\kappa_w)} \left(\mu_{z,t+1} \ldots \mu_{z,t+s} \right) W_{j,t}^{new} \right) h_{j,t+s} \right], \end{array} \tag{10}$$

where ξ_w is the probability that a household is not allowed to reoptimize its wage, τ_t^y a labor income tax, τ_t^w a pay-roll tax (paid for simplicity by the households), and $\mu_{z,t} = z_t/z_{t-1}$ is the growth rate of the permanent technology level.[5]

The households save in domestic and foreign bonds, and the choice between domestic and foreign bond holdings balances into an arbitrage condition pinning down expected exchange rate changes (i.e., an uncovered interest rate parity condition). To ensure a well-defined steady-state in the model, we assume that there is a premium on the foreign bond holdings which depends on the aggregate net foreign asset position of the domestic households, following, e.g. Lundvik (1992), and Schmitt-Grohé and Uribe (2001). Our specification of the risk premium also includes the expected change in the exchange rate $E_t S_{t+1}/S_{t-1}$ which is based on the vast empirical evidence of a forward premium puzzle in the data (i.e., that risk premia are strongly negatively correlated with the expected depreciation of the exchange rate), see e.g. Fama (1984) and Duarte and Stockman (2005). The risk premium is given by:

$$\Phi(a_t, S_t, \tilde{\phi}_t) = \exp\left(-\tilde{\phi}_a (a_t - \bar{a}) - \tilde{\phi}_s \left(\frac{E_t S_{t+1}}{S_t} \frac{S_t}{S_{t-1}} - 1 \right) + \tilde{\phi}_t \right), \tag{11}$$

where $a_t \equiv (S_t B_t^*)/(P_t z_t)$ is the net foreign asset position, and $\tilde{\phi}_t$ is a shock to the risk premium. The UIP condition in its log-linearized form is given by:

$$\widehat{R}_t - \widehat{R}_t^* = \left(1 - \widetilde{\phi}_s \right) E_t \Delta \widehat{S}_{t+1} - \widetilde{\phi}_s \Delta \widehat{S}_t - \widetilde{\phi}_a \widehat{a}_t + \widehat{\widetilde{\phi}}_t. \tag{12}$$

By setting $\tilde{\phi}_s = 0$, we obtain the UIP condition typically used in small open economy models (see e.g. Adolfson et al., 2007).

[5] For the households that are not allowed to reoptimize, the indexation scheme is $W_{j,t+1} = (\pi_t^c)^{\kappa_w} (\bar{\pi}_{t+1}^c)^{(1-\kappa_w)} \mu_{z,t+1} W_{j,t}^{new}$, where κ_w is an indexation parameter.

Following Smets and Wouters (2003), monetary policy is approximated with a generalized Taylor-type rule:

$$\begin{aligned}\widehat{R}_t &= \rho_R \widehat{R}_{t-1} + (1-\rho_R)\left[\widehat{\bar{\pi}}^c_t + r_\pi\left(\hat{\pi}^c_{t-1} - \widehat{\bar{\pi}}^c_t\right) + r_y \hat{y}_{t-1} + r_x \hat{x}_{t-1}\right] \\ &\quad + r_{\Delta\pi}\Delta\hat{\pi}^c_t + r_{\Delta y}\Delta\hat{y}_t + \varepsilon_{R,t},\end{aligned} \tag{13}$$

where $\hat{\pi}^c_t$ denotes CPI inflation, $\hat{y}_t$ the output gap (actual minus trend output), $\hat{x}_{t-1}$ the lagged real exchange rate $\left(\hat{x}_t \equiv \hat{S}_t + \hat{P}^*_t - \hat{P}^c_t\right)$, and $\varepsilon_{R,t}$ an uncorrelated monetary policy shock.

To clear the final goods market, the foreign bond market, and the loan market for working capital, the following three constraints must hold in equilibrium:

$$C^d_t + I^d_t + G_t + C^x_t + I^x_t \leq z_t^{1-\alpha}\epsilon_t K^\alpha_t H^{1-\alpha}_t - z_t\phi, \tag{14}$$

$$S_t B^*_{t+1} = S_t P^x_t\left(C^x_t + I^x_t\right) - S_t P^*_t\left(C^m_t + I^m_t\right) + R^*_{t-1}\Phi(a_{t-1}, \widetilde{\phi}_{t-1}) S_t B^*_t, \tag{15}$$

$$\nu W_t H_t = \mu_t M_t - Q_t, \tag{16}$$

where G_t is government expenditures, C^x_t and I^x_t are the foreign demand for export goods, and $\mu_t = M_{t+1}/M_t$ is the monetary injection by the central bank. When defining the demand for export goods, we introduce a stationary asymmetric (or foreign) technology shock $\tilde{z}^*_t = z^*_t/z_t$, where z^*_t is the permanent technology level abroad, to allow for temporary differences in permanent technological progress domestically and abroad.

The log-linearized shock processes are given by the univariate representation

$$\hat{\varsigma}_t = \rho_\varsigma \hat{\varsigma}_{t-1} + \varepsilon_{\varsigma,t}, \qquad \varepsilon_{\varsigma,t} \overset{iid}{\sim} N\left(0, \sigma^2_\varsigma\right)$$

where

$$\varsigma^{open}_t = \{\mu_{z,t}, \epsilon_t, \lambda^j_t, \zeta^c_t, \zeta^h_t, \Upsilon_t, \tilde{\phi}_t, \varepsilon_{R,t}, \bar{\pi}^c_t, \tilde{z}^*_t\} \text{ and } j = \{d, mc, mi, x\}. \tag{17}$$

The government spends resources on consuming part of the domestic good, and collects taxes from the households. The resulting fiscal surplus/deficit plus the seigniorage are assumed to be transferred back to the households in a lump sum fashion. Consequently, there is no government debt. The fiscal policy variables - taxes on capital income, labor income, consumption, and the pay-roll, together with (HP-detrended) government expenditures - are assumed to follow an identified VAR model with two lags.

To simplify the analysis, we adopt the assumption that the foreign prices, output (HP-detrended) and interest rate are exogenously given by an identified VAR model with four lags. Both the foreign and the fiscal VAR models are being estimated, using uninformative priors, prior to estimating the other structural parameters in the DSGE model.[6]

To re-cast the model in its closed economy form, we set import shares ω_c and ω_i in eqs. (7) and (9) to nil. Together with a balanced trade assumption, this implies that the evolution of

[6] The scaled level of foreign output $\hat{y}^*_t$ enters the stationarized log-linear representation of the DSGE model (via in the aggregate resource constraint, eq. (14) as total export demand equals $C^x_t + I^x_t = \left(\frac{P^x_t}{P^*_t}\right)^{-\eta_f} Y^*_t$). In order to avoid joint estimation of the parameters in the foreign VAR and the deep parameters in the model, we use the HP-filter to compute $\hat{y}^*_t$ as $\ln Y^*_t - \overline{\ln Y^*_t}$ where $\overline{\ln Y^*_t}$ is the HP-trend of log trade-weighted foreign output with the smoothing coefficient set to 1600. Then the filtered series $\hat{y}^*_t$ is used to estimate the VAR. However, $\hat{y}^*_t$ is treated as an unobserved variable when the DSGE is subsequently estimated, we only observe overall foreign output growth $\Delta\ln Y^*_t$ as discussed in Section 2.2. In the working paper version of Adolfson et al. (2008a), we discuss that the baseline estimation results hold up (but are much more time-consuming to generate) if the foreign VAR is estimated jointly, presumably since $\overline{\ln Y^*_t}$ has largely similar statistical properties to the estimated stochastic unit-root shock.

all foreign variables **is** irrelevant for the dynamics in the domestic economy. For the shocks, we keep the eight disturbances

$$\varsigma_t^{close} = \left\{\mu_{z,t}, \epsilon_t, \lambda_t^d, \zeta_t^c, \zeta_t^h, \Upsilon_t, \varepsilon_{R,t}, \bar{\pi}_t^c\right\} \tag{18}$$

in (17) and allow for exogenous variations in the fiscal variables.

To compute the equilibrium decision rules, we proceed as follows. First, we stationarize all quantities determined in period t by scaling with the unit root technology shock z_t. Then, we log-linearize the model around the constant steady state and calculate a numerical (reduced form) solution with the AIM algorithm developed by Anderson and Moore (1985).

2.2. Parameterization of the model

To parameterize the model, we use the Bayesian posterior median estimates from matching quarterly Swedish data $1980Q1 - 2004Q4$, on the following observed 15 variables; the GDP deflator, the real wage, consumption, investment, the real exchange rate, the short-run interest rate, hours worked per capita (total hours divided by working age population), GDP, exports, imports, the consumer price index (CPI), the investment deflator, foreign output, foreign inflation and the foreign interest rate

$$\tilde{Y}_t^{obs,open} = \begin{array}{c} [\ \pi_t^d \quad \Delta\ln(W_t/P_t) \quad \Delta\ln C_t \quad \Delta\ln I_t \quad \hat{x}_t \quad R_t \quad \hat{H}_t \quad \Delta\ln Y_t ... \\ \Delta\ln\tilde{X}_t \quad \Delta\ln\tilde{M}_t \quad \pi_t^{cpi} \quad \pi_t^{def,i} \quad \Delta\ln Y_t^* \quad \pi_t^* \quad R_t^*\ ]'. \end{array} \tag{19}$$

The data are adapted from Adolfson et al. (2008a), and the interested reader is kindly referred to this paper for further details. Among other things, we there verify by multiple MCMC-chains that the Posterior median is not bimodal. Following Christiano et al. (2005), we calibrate parameters we think are weakly identified by the set of observables we match, and estimate the other parameters. The latter pertain mostly to the nominal and real frictions in the model as well as the exogenous shock processes. Appendix A provides details on the prior distribution and calibrated values.

3. Identification analysis

In this section we show that many parameters in the two models are not particularly affected by identification issues. While the results do not allow us to draw any firmer conclusions about identification in New Keynesian DSGE models more generally, we discussed in the introduction that these results are likely relevant to a wider range of models, beyond the actual case studies. Moreover, we also show that recent pre-tests proposed in the literature are robust and adequate in highlighting possible issues. Indeed, we establish that diagnostic tests of identification developed in Iskrev (2010a,b) are in line with more computationally intensive Monte Carlo (small sample) analysis. For large scale models, only the former pre-tests would be feasible. Therefore, identification issues should and can always be assessed on a case-by-case basis using pre-tests.

3.1. Pre-estimation tests of identification

As a starting point we apply the diagnostic tests of identification developed in Iskrev (2010a,b). Let $m_T(\theta)$ denote all first- and second-order moments for the l series with observed variables ($l = 15$ and 7 in our open and closed economy variants, respectively). If the second-order moments are computed for contemporaneous and $T-1$ lags of observed data, then $m_T(\theta)$ is a

$(T-1)\,l^2 + l\,(l+3)\,/2$ sized vector with moments in the model determined by the k estimated parameters in θ. Now, if m_T is a continuously differentiable function of θ, then θ_0 is locally identifiable if the Jacobian matrix $J\,(T) = \partial m_T/\partial\theta$ has rank k at θ_0. By applying the chain rule, Iskrev (2010a) shows that the Jacobian matrix $J\,(T)$ can be conveniently expressed as

$$J\,(T) = \frac{\partial m_T}{\partial \tau}\frac{\partial \tau}{\partial \theta} \equiv J_1\,(T)\,J_2, \tag{20}$$

where τ is a vector with all reduced-form coefficients in the DSGE model. The decomposition in eq. (20) is very useful, as it implies that if the matrix $J_2 = \partial\tau/\partial\theta$ has a rank less than k at θ_0, then some of the parameters in θ are not locally identified at θ_0 regardless of which dataseries are used in the estimation as this matrix is independent of the set of observables. For this reason, Iskrev suggests the rank of both matrices J_2 and $J(T)$ are studied to learn about identification. Full column rank (k) of J_2 is necessary for identification, whereas the rank of $J(T)$ will show if θ is locally identified for the particular set of observables and contemporaneous and lagged moments under consideration.

Iskrev (2008, 2010a) emphasizes that although both $J\,(T)$ and J_2 have rank k, they can be close to rank deficient and hence poorly conditioned. Poorly conditioned $J\,(T)$ and J_2 matrices suggest that identification of (at least) some parameters is weak. A parameter θ_i can be poorly identified because it has little impact on the reduced-form coefficients of the model, i.e. $\partial\tau/\partial\theta_i \approx 0$, or because there is a high degree of substitutability between the parameter and (possibly a linear combination of) other parameters, so that $\partial\tau/\partial\theta_i \approx \Sigma_{j\neq i} a_j\,(\partial\tau/\partial\theta_j)$ where $\Sigma_{j\neq i} a_j$ is some linear combination of the other deep parameters in θ. The problem with parameter interdependence arises when different parameter combinations play a very similar role in the model, and can be assessed by computing multiple collinearity coefficients for each parameter in θ. The multiple collinearity coefficient for θ_i measures how well the effect of θ_i in the model can be mimicked by other parameters in θ, and values close to unity (or -1) imply a very strong degree of multiple collinearity, and accordingly problems with weak identification.

In addition to reporting J_2 and $J\,(T)$ multiple collinearity statistics, where $J\,(T)$ considers all moments for contemporaneous and one lag of data, we also present in Table 1a expected standard errors following Iskrev (2010b), who discusses in detail how they can be computed from the inverse of the Fisher information matrix as a function of the set of observables and the sample size (we use 100 observations) using Cramér-Rao lower bounds.[7] Iskrev suggests the expected standard errors are used as an a priori measure of identification strength.

The first column in Table 1a reports the multiple collinearity coefficients (Mco) in the model (J_2), which range from 0.169 ($\rho_{\tilde{z}^*}$) to 0.912 (r_π). For most of the parameters, there is not much intrinsic interdependence in the theoretical model, possibly with the exception of some of the parameters in the policy rule. Next, we study the multiple collinearity in the moments of the data, $J\,(T)$. First we consider the full set of 15 observed variables (see eq. 19). We see that the collinearity coefficients are generally higher for $J\,(T)$ than J_2. Especially, there seems to be a strong linear dependence between the policy parameters (ρ_r, r_π, r_x, r_y) and the other deep parameters with Mco of around 0.99.

The expected standard deviations from the Fisher Information matrix, std(θ_i), are also somewhat high for r_π, r_x, and r_y, but economically reasonable for most of the other parameters.

[7]Because the expected standard errors in Iskrev (2010a) - denoted std(θ_i) in Table 1a - are based on the Cramér-Rao inequality it follows that the true standard devations should be greater or equal to std(θ_i) for all parameters θ_i. Even so, this inequality may not be verified in practice due to the non-linearity of the mapping between θ and the likelihood function, as well as lack of convergence in the computation of the asymptotic information matrix.

For comparable parameters, they are often quite close to those reported by e.g. Christiano, Eichenbaum and Evans (2005). We also report the identification strength measured as the ratio $s^r(\theta_i) \equiv$abs$(\theta_i)/$std(θ_i) (i.e. normalizing the parameter value with its standard deviation), where a high value means that θ_i is more tightly pinned down and thus strongly identified. The identification strength seems adequate for most of our estimated parameters, perhaps with the exception of the three policy rule parameters r_π, r_x and r_y for which data seem to be less informative. In the column labeled 'prior$(\theta_i)/$std(θ_i)', we study how large the expected standard deviations are compared to the standard deviation of the prior distribution. From this column we see that estimation cannot be expected to reduce the uncertainty about r_π, r_y, ξ_w, $\tilde{S}''$, λ_d, and ζ_c compared to the *a priori* belief, since they all have a value below one. Even so, when excluding these parameters from the set of estimated parameters, we find that they have little influence on the sampling uncertainty of the other parameters.[8]

If we specifically look at the Calvo parameters, we see that the expected standard deviations are very small for the four Phillips curves but that the data seem comparably less informative about the wage-setting parameter. Compared to Iskrev (2010b), our choice of estimated parameters and observables seems to provide a somewhat better precision; for instance we find that s^r for ξ_p and ξ_w is 33.69 and 8.29, respectively, whereas Iskrev finds 3.8 and 10.8.

It thus seem like problems of weak identification are limited to some parameters that do not appear to contaminate the informativeness of the other deep parameters. Notice, however, that the results in Table 1a and the Monte Carlo evidence in the next section are contingent upon one particular parameterization of the model. To explore identification in a larger region of the parameter space, we sample $5,000$ draws from the prior distribution. In Table 1b we report the results as the share of draws for which $J(T)$ and J_2 have full column rank along with their (median) condition number, the (median) smallest singular value and (median) tolerance value. The conditioning number is the ratio between the largest and smallest singular value of $J(T)$ and J_2, which have full rank if their smallest singular value is larger than the tolerance value. Hence, a large conditioning number is a strong indication of weak identification.

For all parameter draws, $J(T)$ does have full rank, and Table 1b makes it clear that the model is locally identified in all parts of the parameter space that we explore. Problems with weak identification could, however, still be present. In fact, they seem somewhat more noticeable when moving around in the parameter space since the conditioning numbers (for the draws with full rank) increase and the smallest singular values diminish compared to the numbers under the posterior median parameterization.

The five rightmost columns in Tables 1a and 1b display the corresponding results for the closed economy variant of the model. In this formulation of the model, we follow Smets and Wouters (2007) and match the seven variables (a subset of the variables in $\tilde{Y}_t^{obs,open}$ in eq. 19)

$$\tilde{Y}_t^{obs,close} = [\ \pi_t^d \quad \Delta\ln(W_t/P_t) \quad \Delta\ln C_t \quad \Delta\ln I_t \quad R_t \quad \hat{H}_t \quad \Delta\ln Y_t]', \tag{21}$$

using a subset of the estimated and calibrated shocks as described at the end of Section 2.1.

The results for the closed economy model in Table 1b show that $J(T)$ and J_2 are always full rank and the conditioning numbers are comparable to those obtained in the open economy model, suggesting that problems with weak identification are still an issue in certain regions of the parameter space. For the individual parameters in Table 1a, we see that the Mco $J(T)$ coefficients are notably higher for the deep parameters. Even so, std(θ) and $\theta/$std(θ) are often

[8] When calibrating r_π,r_y, ξ_w, $\tilde{S}''$, λ_d, and ζ_c, the standard deviations based on the Fisher Information matrix for the other parameters are essentially unchanged. This dispels possible objections that our results on identification may be driven by the fact that we calibrate (i.e. fix) parameters which are weakly identified or not identified at all.

comparable with the exception of a few parameters ($\tilde{S}''$ and the consumption preference shock ρ_{ζ_c}) for both the posterior and the prior median.[9]

Note finally that complementing the Iskrev tests with additional ones as proposed by Andrews and Mikusheva (2015) also supports the same conclusions discussed here, see Appendix C.

3.2. Identification in small samples: A Monte Carlo exercise

In this section, we first describe in detail how the small sample ML distributions have been generated using the DSGE models, and then present the distribution results. The Monte Carlo exercise also confirms the main findings from identification pre-tests, showing their effectiveness and robustness as ex-ante identification diagnostic tools.

3.2.1. Setup

To assess the performance of Maximum Likelihood in small samples simulated with the DSGE models, the following steps are conducted for each model:

1. Solve the DSGE model using the chosen parameterization (calibrated parameters and the Bayesian posterior median of the estimated parameters).

2. Generate an artificial sample of length T by simulating the model $1,000 + T$ periods initiated from the steady state. The first $1,000$ observations are discarded as burn-ins. The innovations in the shock series were drawn randomly from the normal distribution.

3. Given the simulated data, estimate the parameters by maximizing the likelihood function for the given set of observable variables (provided by eq. (19) for the open economy model and eq. (21) for the closed economy).[10]

4. Store the resulting parameter estimates along with the likelihood information, inverse Hessian, seed number used to generate the sample, and convergence diagnostics.

5. Repeat Step 1 to 5 N times to obtain a parameter distribution that is stable. In practice, it took between $1,000$ and $1,500$ samples to obtain approximate convergence in mean and variance in the marginal parameter distributions, and therefore we decided to use $N = 1,500$.

[9] We have also done the identification tests for the seven "closed economy" variables in (21) in the open economy model when all 43 parameters in Table 1a are estimated. In this case the identification strength diminishes, especially for the parameters pertaining to the open economy. When narrowing the number of observables, the multiple collinearity coefficients for most parameters turn out to be close to unity, and the properties of $J(T)$ – inferior to those in Table 1b, implying that problems with weak identification are more marked. This exercise thus demonstrates that the econometrician needs to think hard about the structure of the model and the variables that need to be included in order to ensure identification of a given set of parameters, consistent with the conclusions in Boivin and Giannoni (2006) and Guerron-Quintana (2010).

[10] We use Chris Sims' optimizer CSMINWEL to perform the estimations, and impose the lower (b_l) and upper bounds (b_u) that are reported in the last two columns in Table A.2. The lower and upper bounds only define a theoretically admissible range of a parameter, and not necessarily a range that is considered to be consistent with existing empirical evidence. We use the following smooth mapping function $p_{\text{mod}} = b_u - \frac{b_u - b_l}{1+e^{p_{\text{opt}}}}$ between the model parameters (p_{mod}) and the parameters that we optimize over (p_{opt}). Notice that p_{mod} converges to b_u as p_{opt} approaches ∞, and that p_{mod} converges to b_l as p_{opt} approaches $-\infty$.

We consider several sample sizes. For our benchmark results in Table 2, we set $T = 100$, which is roughly equivalent to the Swedish actual data sample. In Appendix B.4 we also report distributions for $T = 400$ to examine if we have square-root sample size convergence.[11]

In order to simplify the interpretation of the results and focus on the key model parameters, we do not include measurement errors and keep the parameters of the exogenous foreign VAR (in the open economy model) and fiscal policy VAR (both closed and open economy variants) fixed at their true values, which differs from how the model was estimated on actual Swedish data using Bayesian techniques.[12]

3.2.2. Results

In Table 2, we report distribution results for the open economy (left columns) and closed economy (right columns) formulation of the model. First, we turn to the open economy variant and see that mean and median for almost every parameter are equal or close to the true value, so the ML estimator appears to be unbiased. Important exceptions are two coefficients in the policy rule; r_π and r_y, which both have mean estimates that are notably higher than their true values. However, the median for the two parameters is of the right magnitude, indicating that the parameter distributions are skewed to the right. Given the specification of the instrument rule, where ρ_R multiplies the coefficients in the policy rule (see eq. 13), it is perhaps not surprising that the distributions for these two parameters can be skewed to the right. In samples when ρ_R is driven close to unity, the values of r_π and r_y can easily end up at very high values without affecting the composite response much. The fourth column of Table 2, labeled "Std. of distribution", shows the standard deviation of the parameter distributions, i.e. the mean square errors (MSEs), and not surprisingly the standard deviations are very high for these two parameters. The MSEs are also relatively high for the investment adjustment cost parameter, $\tilde{S}''$, and the persistence coefficient for the asymmetric technology shock, $\rho_{\tilde{z}^*}$, suggesting that also these parameters are sometimes driven to very high and low values, respectively. Interestingly, the MSEs for the key parameters pertaining to the nominal rigidities in the model reveal that the marginal distributions are much tighter for the sticky price parameters (ξ_d, ξ_{mc}, ξ_{mi} and ξ_x), relative to the parameter governing nominal wage stickiness, ξ_w, indicating that the degree of price stickiness is more strongly identified relative to the degree of wage stickiness. By and large, the patterns for the parameters we estimate in the closed economy specification are very similar to those already discussed, noting that the MSE is almost twice as high for $\tilde{S}''$ but somewhat muted for r_y.

In addition to the standard deviations of the resulting marginal parameter distributions, Table 2 reports the median standard deviation of the estimates in each sample computed from the inverse Hessian matrix. As discussed in further detail in Iskrev (2008, 2010a), weak identification induces poor conditioning of the Hessian (analogous with the poor conditioning of the Jacobians reported in Table 1b), and causes standard deviations based on the inverse Hessian to underestimate the true sampling uncertainty. The standard deviations based on the inverse

[11] Throughout the paper, the results are limited to the convergent estimations only. Dropping non-convergent optimizations reflect our belief that the econometrician would not be satisfied with a non-convergent outcome, and would redo the estimation by perturbing the starting values of the optimization until convergence was found. Given that very few samples are plagued by convergence problems, as noted in Table 2, our approach is largely inconsequential.

[12] In Appendix B we check the robustness of our results regarding, *i*) the starting values in the optimizations, *ii*) adding measurement errors and re-estimating the fiscal and foreign VARs, and *iii*) imposing the true co-integrating vectors among the set of observed variables. The key results are little affected. In the appendix we also check the consistency properties by increasing each sample size to 1,600 and 6,400 observations.

Hessian are smaller than the standard deviation of the distribution for every parameter, and in particular so for r_x and r_y, indicating problems of data informativeness. It is, however, clear from the MSEs that the key parameters are still well identified. For instance, the sticky price parameters (ξ_d, ξ_{mc}, ξ_{mi} and ξ_x in the open and ξ_d in the closed economy variant) have very small MSEs which shows that the likelihood is very informative about these parameters even when $T = 100$. Hence, although a few parameters are poorly identified, our results suggest that we can learn a lot from the data about many parameters.

Comparing the Monte-Carlo simulated MSEs in Table 2 with the Cramér-Rao (CR) lower bounds computed using Iskrev's (2010b) approach (cf. the forth and eight columns in Table 2 and the third column in Table 1a), we see that the simulated MSEs are generally higher than the CR lower bounds as expected. The exceptions are $\lambda_{m,c}$, and $\sigma_{\lambda_{m,c}}$ in the open economy model for which we counterfactually obtain lower simulated MSE values; i.e. a ratio of the simulated MSE over the CR bounds that is lower than unity. All other implied ratios are above unity as they should be.[13]

However, due to the non-linear relationships between the parameters in the model, the ratios vary quite a bit and can be as high as 5 (see e.g. the policy rule parameters r_π and r_y). In our view, this demonstrates, on the one hand, the usefulness of complementing Iskrev's (2010b) approach with Monte-Carlo simulations. On the other hand, our analysis suggests that Iskrev's approach provides expected standard errors that are often a good proxy of the MC ones, which is useful because his method is fast enough to be iteratively performed at several random locations in the prior space and thereby overcomes the contingency of the Monte Carlo simulations on a particular location in the parameter space.

In Figure 1, we complement the information in the table by plotting the kernel density estimates of the marginal parameter distributions for some key parameters in the open economy model. The solid line ($T = 100$) in the figure confirms the picture in Table 2 and shows that the distributions for $\tilde{S}''$, r_π and r_y are clearly skewed to the right. Notice that the marginal distributions for r_π are reported in logs in order to improve the visibility of the results. Comparing the densities for the benchmark sample size with the results when increasing the sample size to $T = 400$ observations (dotted lines), it is clear that this set of data suffices for identification of the parameters in the notion of Rothenberg (1971): as the sample size increases, the parameter distributions start to collapse around the true values. So, conditional on the observed variables and estimated parameters, the ML estimates are consistent in the open economy model. The same result applies in the closed economy model.

4. Analysis on actual data and misspecification

Having done all the pre-tests and the ex-ante identification analysis, we now take the models to the data and elaborate on misspecification issues. We show that ML and Bayesian estimates differ notably for some key parameters, suggesting problems with model misspecification. By applying formal and informal tests of model misspecification, we provide support for this conjecture. Finally, we assess the economic implications of misspecification by studying the difference between the impulse response functions to a monetary policy shock for the ML estimate and Bayesian posterior.

[13] As noted in footnote 7, this can be due to the non-linear mapping between the deep parameters and the reduced form solution of the model, i.e. the shape of the posterior may be steeper than what is implied by the quadratic approximation underlying the estimation of the asymptotic information matrix. Moreover, the algorithm used to estimate such asymptotic information matrix in Dynare may feature slow convergence for some deep parameter, for the limit $T \to \infty$.

4.1. Maximum likelihood estimation

Our analyses in Sections 3.1 and 3.2 show that the likelihood function should be quite informative about many of the key parameters in both the closed and open economy models, under the null hypothesis that the models are correctly specified. Therefore we now attempt to obtain estimates for both variants using classical ML techniques on actual data. For the open economy variant, we use Swedish data. Here the setting in the estimation is identical to the setting that was employed in the Bayesian estimation procedure that resulted in the posterior median estimates, with the exception that the policy parameters r_π, r_y and r_x are estimated as composite coefficients in an attempt to reduce the large uncertainty bands stemming from trying to directly estimate the separate coefficients in eq. (13) as discussed in detail in Section 3.2.2.[14] In the closed economy variant, we use U.S. data for the period 1965Q1-2006Q4 for the seven variables in eq. (21) following Smets and Wouters (2007).[15] For the U.S., we also follow Smets and Wouters (2007) by assuming that the data are measured without errors. In addition, we turn off all the tax shocks. Hence, in the estimated closed economy model, we only allow for the shocks in (18) plus a government spending shock which we assume follows an AR(1)-process with persistence coefficient of 0.95 and unit innovation variance (this parameterization was determined by pre-estimation on detrended government spending series including state, federal and military expenses).[16] The priors for the "closed economy" parameters (displayed in Tables 1a and 2) that we estimate are standard and coincide with those adopted for Sweden in Table A.1.

To find the classical ML point estimates, we impose the lower and upper bounds reported in Table A.2 and perform $3,000$ optimizations with CSMINWEL by sampling starting values from the prior distribution. The ML estimates are the vector of parameters $\hat{\theta}$ out of all vectors with optimized parameters $\theta_i,\ i = 1, 2, ..., 3,000$, that returned the highest log-likelihood. To assess the uncertainty about the point estimates, i.e. how much we can learn from the log-likelihood function about the parameters, under possible misspecification, we use the standard deviations based on the sandwich form of the inverse Hessian following Newey and Windmeijer (2009) to compute 5 and 95 percentile confidence bands associated with the ML estimates.

In Table 3a, we report the classical ML estimation results for Sweden along with the Bayesian posterior median and 5 and 95 percent posterior uncertainty bands. The corresponding results for the U.S. are reported in Table 3b. From both tables, we see that the classical ML estimate moves in the same direction from the prior as the posterior median, but typically a bit further. An interesting feature of the ML estimation results is that the parameter which measures deviations from the standard UIP condition ($\widetilde{\phi}_s$) is driven almost to its upper bound of unity, suggesting the UIP does not provide a good approximation of exchange rate dynamics. Also, and in

[14] Thus, we estimate $\tilde{r}_\pi = (1-\rho_R)\, r_\pi$, $\tilde{r}_y = (1-\rho_R)\, r_y$, and $\tilde{r}_x = (1-\rho_R)\, r_x$ directly instead of r_π, r_y and r_x. Notice that the Bayesian posterior median results presented in Table 3 have been transformed to composite parameters, although the priors used in the Bayesian estimation are for the individual parameters.

[15] GDP, consumption and investment were taken from the U.S. Department of Commerce – Bureau of Economic Analysis data-bank. Real gross domestic product is expressed in billions of chained 2009 dollars. Nominal personal consumption expenditures and fixed private domestic investment are deflated with the GDP-deflator. Inflation is the first difference of the log of the implicit price deflator of GDP. Hours and wages come from the BLS (hours and hourly compensation for the non-farm business, NFB, sector for all persons). Hourly compensation is divided by the GDP price deflator in order to get the real wage variable. Hours are adjusted to take into account the limited coverage of the NFB sector compared to GDP (the index of average hours for the NFB sector is multiplied by Civilian Employment 16 years and over). All series are seasonally adjusted. The interest rate is the Federal Funds Rate. Consumption, investment, GDP, wages, and hours are expressed in 100 times log whereas inflation and the fed funds rate are annualized (400 times quarterly series).

[16] As discussed in Appendix A, we modify some of the calibrated parameters to standard values used for the U.S. to match various steady state ratios (like government spending to steady state output and the depreciation rate).

line with the results on artificial samples, the data appear to be highly informative about the sticky price parameters ξ_d, $\xi_{m,c}$, $\xi_{m,i}$ and ξ_x which are estimated to be very high. In relation to the microeconomic evidence, the median estimate of the four $\xi's$ equals 0.979 and for the U.S. we have $\xi_d = 0.989$ which implies an unrealistically high average duration between price reoptimizations of about 47 and 100 quarters, respectively. This is far too long relative to the microeconomic evidence on price-setting (see e.g. Apel, Friberg and Hallsten, 2005, for Swedish evidence, and Klenow and Malin, 2010, for U.S. evidence). The finding of a very high degree of price stickiness with classical methods is not specific to our pricing equations (see e.g. eq. 4), they are standard in the existing DSGE literature. Smets and Wouters (2003) also report a very high degree of price stickiness in their model with i.i.d. markup shocks on Euro Area data, and to reduce the degree of price stickiness on U.S. data, Smets and Wouters (2007) assume that the markup shocks follow an ARMA(1,1) (where the estimated AR term is very high) process and use the Kimball (1995) aggregator which in effect implies that firms respond less to movements in marginal costs for a given level of price stickiness.[17] However, an unappealing feature of the specification with correlated shocks in Smets and Wouters (2007) is that positive markup shocks account for a substantial part of the great inflation of the 1970s (see their Figure 4), a finding that seems hard to reconcile with Shiller's (2000) evidence that aggregate firm profits did not rise in the 1970s. Moreover, Chari, Kehoe and McGrattan (2008) have argued that the implied variance of markup shocks is implausibly high.

However, before drawing too firm conclusions about the point estimates, we need to consider the possibility that the large changes in some of the parameters (e.g. the price stickiness parameters) merely reflect small-sample uncertainty. As can be seen from Tables 3a and 3b, the 90 percent confidence interval based on the sandwich form of the inverse Hessian suggests that many parameters are relatively tightly estimated, with the exception of the investment adjustment cost $\tilde{S}''$ which has a high standard deviation of about 4 for Sweden and 1.5 for the United States. Even so, because the simulation results in Table 2 documented that the standard deviations based on the inverse Hessian are likely to underestimate the true degree of uncertainty associated with the ML estimates due to problems with weak identification, we also simulated 90 percent simulated confidence bands as follows. First, the ML point estimates and the associated inverse Hessian matrix were used to generate draws from the joint parameter distribution using the Metropolis-Hastings algorithm. The proposal distribution is taken to be the multivariate normal density centered at the previous draw with a covariance matrix proportional to the inverse Hessian. Second, all draws that could not be differentiated from the highest log-likelihood according to a standard likelihood ratio (LR-) test at the 10-percent significance level were accepted in the chain. A chain with 1,000,000 draws was simulated, and from this chain the lower and upper confidence bands were computed as the minimum and maximum values for each parameter.[18] Despite the fact that the simulated confidence bands are often notably larger than the ones based on the inverse Hessian in Tables 3a and 3b, it is clear that the log-likelihood function is very informative. For example, the Bayesian posterior median lies outside the simulated and Hessian based 90 percent ML confidence band for all the sticky price parameters (except for ξ_x in Table 3a). Therefore, we conclude that the higher relative to the

[17] An additional assumption that would reduce the implied degree of price stickiness would be to assume that capital and/or labor is specific to the firm, see Altig et al. (2011) and Woodford (2003).

[18] As we estimate 43 (26) parameters in open (closed) economy model, $2\left[\ln L\left(\hat{\theta}\right) - \ln L\left(\theta_i\right)\right]$ follows the χ^2-distribution with 43 (26) degrees of freedom and a particular parameter draw θ_i is rejected in favor of the MLE estimate $\hat{\theta}$ associated with $\ln L_{\max}$ at the 10 percent level if the χ^2-statistic exceeds 55.23 (35.56). Notice also that the robustness of the simulated confidence bands were checked by simulating and computing the confidence bands for an additional chain of 1,000,000 draws.

Bayesian posterior of the sticky price parameters ML estimate cannot be explained by small sample properties of the ML estimator.

4.2. Misspecification

All previous findings raise the issue of why the classical ML estimate differs so markedly relative to the Bayesian posterior estimate. Although the obtained log-likelihood under ML estimation by definition is associated with the largest possible likelihood, the improvement in log-likelihood from $-2128,6$ associated with the Bayesian posterior median to $-2022,2$ in the open economy model for Sweden in Table 3a under classical estimation is prima facie evidence that the likelihood is not flat when the parameters change, which is consistent with the analysis in Sections 3.1 and 3.2. In the closed economy model for the U.S., we also observe a large improvement from -1454.4 to -1410.6, a difference that is significant under the toughest possible significance level according to a standard likelihood ratio test.

Under the presumption that the data indeed are informative, the obvious candidate explanation for why the estimates differ so much is therefore model misspecification. The very low Log Posterior (-4439.2 for Sweden and -3996.7 for the U.S.) under ML estimation indicates that the ML estimates are often far out in the prior tails, which is a sign of model misspecification. To examine this formally, we apply a test of misspecification following Del Negro and Schorfheide (2004, 2009). They develop an approach where a possibly misspecified DSGE model is compared to a VAR with a prior centered on the DSGE model, where the tightness of the prior is determined by a parameter, λ. When $\lambda = \infty$ the prior imposes the DSGE restrictions with probability one and the VAR collapses to the (VAR approximation) of the DSGE, whereas a $\lambda < \infty$ implies that the cross-restrictions of the DSGE on the VAR are relaxed. By comparing the marginal likelihood for different choices of λ, one can find the optimal λ, $\widehat{\lambda}$ henceforth, which maximizes the empirical fit (i.e. the marginal likelihood) of the DSGE-VAR. A $\widehat{\lambda}$ well below infinity thus implies that the cross-equation restrictions of the DSGE model are not compatible with the data, and the model is misspecified.

In Table 4 we show (the Laplace approximation of) the log marginal likelihoods of the VAR and VECM with 8 lags when varying the tightness in the prior. The VECM with 8 lags and $\lambda = \infty$ provides a very good approximation of the DSGE model as the log-marginal likelihoods are almost identical. The VECM is a better approximation than the VAR because the latter does not exploit the cointegration relations of the DSGE model. When the VAR/VECM spans the DSGE model in terms of the marginal likelihood, the reliability of the results increases. The table shows that $\widehat{\lambda}$ is 5.5 in the VECM for the open economy model estimated on Swedish data, implying that relaxing the cross-equation restrictions of the estimated DSGE model improves the empirical fit to the data.[19] In Adolfson et al. (2008a), we found a similar degree of misspecification when allowing for a regime shift in monetary policy, and documenting it had substantial implications for the vector autocorrelation functions in the VAR. Notice also that the log-marginal likelihood improvement in the DSGE-VAR for $\widehat{\lambda}$ is much larger than the improvement in log-likelihood between the Bayesian posterior and the MLE, suggesting that even the MLE estimates impose cross-restrictions which are not satisfied in the best-fitting VAR. Thus, our open economy model is clearly misspecified according to this test.

The right panel reports the corresponding DSGE-VAR and DSGE-VECM results for our

[19] Note that the prior is proper if $\lambda \geq \lambda_{\min} = (n(p+1)+q+1)/T$, where n is the number of endogenous variables in the VAR with p lags, q the number of exogenous variables in the VAR and T the number of actual data observations. For our VAR and VECM specifications in Table 4, $\lambda_{\min}$ is 1.79 and 1.87 for the Swedish open economy model, and 0.36 and 0.39 for the U.S. closed economy model.

U.S. closed economy model. For the United States, we find that $\hat{\lambda} = 1 - 1.25$, which is in line with Del Negro et al. (2007).[20] Although $\hat{\lambda}$ is higher for the Swedish open economy model, it does not mean this model is less misspecified than the closed economy U.S. model since the longer US sample (1965Q1-2006Q4) implies that the prior is proper for a lower λ.

A limit of the test, however, is that it is not instructive to make a structural interpretation of the DSGE parameters of the VECM model with a λ less than infinity. Therefore, we cannot directly compare our estimated Calvo coefficients for the DSGE-VAR($\hat{\lambda}$) model with the ones obtained under ML estimation in Tables 3a and 3b. To strengthen our case that the high Calvo coefficients under ML estimation indicate misspecification problems, we therefore complement our DSGE-VAR analysis with some indirect evidence. Specifically, we take a structural approach and introduce misspecification through the Phillips curve for domestic prices. We do this through the domestic price setting equation for two reasons. First, we can observe both the dependent and explanatory variables in this equation. Second, the ML estimate of ξ_d is implausibly high (including its confidence bands).

To begin with, we note that the Phillips curve (4) in both DSGEs implies a positive correlation between contemporaneous domestic inflation and marginal cost: simulating artificial data under the Bayesian posterior median estimates, estimated on Swedish data, yields a correlation of 0.10, whereas it is -0.11 using the ML estimates ($1,500$ samples with $T = 100$). On actual Swedish data, we observe a negative correlation (-0.16) between domestic inflation and marginal cost (measured as the demeaned labor share) for the inflation targeting period. Accounting for a negative correlation between inflation and marginal costs in New Keynesian models is challenging, because marginal costs are normally a positively serially correlated variable. This implies that when current marginal cost rises, the present discounted sum of marginal cost also rises, and this causes firms to raise prices. Only when the direct impact of marginal cost on inflation is sufficiently low – as is the case under ML estimation – the model can generate a negative correlation through the presence of markup and inflation targeting shocks.[21]

Moreover, when we run an OLS-regression on Swedish data for the following simple pricing equation

$$\pi_t^d = (1 - c_1)(\overline{\pi} + c_2\widehat{mc}_t) + c_1\pi_{t-1}^d + \varepsilon_t \,, \tag{22}$$

where π_t^d is domestic annualized inflation, $\overline{\pi}$ the sample mean (1.69 percent for our sample) and $\widehat{mc}_t$ the marginal cost proxied by the demeaned labor share, we find $c_1 = -0.349$ and $c_2 = -0.013$. Thus, even conditional on the lagged inflation rate, a negative partial impact (c_2) of marginal costs on domestic inflation is obtained. We now proceed by examining how a misspecified domestic Phillips curve can impact the estimation under maximum likelihood.

To do this, we simulate data from an ad hoc version of the open economy DSGE model in which we replace the theoretically derived Phillips curve in eq. (4) with the simplified empirical relation in eq (22).[22] We then estimate the original open economy DSGE model, including the Phillips curve in eq. (4), noting that it is now misspecified compared to the true data generating process, with maximum likelihood. For $1,500$ artificial samples of size $T = 100$, the mean ML estimate of ξ_d in the misspecified New Keynesian Phillips domestic curve equals 0.96, which is

[20] Del Negro et al. (2007) also obtain a $\widehat{\lambda}$ of about unity. When we compute the optimal λ for the Smets and Wouters (2007) model for our U.S. dataset, we obtain similar value ($\hat{\lambda} = 1.5$).

[21] To see this, recall that in a simplified version of eq. (4) without dynamic indexation ($\kappa_d = 0$), markup and inflation targeting shocks, we have that $\hat{\pi}_t^d = \gamma_d \Sigma_{s=0}^{\infty} \beta^s \mathrm{E}_t \widehat{mc}_{t+s}$, and under the simplifying assumption that $\widehat{mc}_t = \rho\widehat{mc}_{t-1} + \varepsilon_{mc,t}$, it follows that $\hat{\pi}_t^d = (\gamma_d / (1 - \rho\beta))\,\widehat{mc}_t$. In this case, $\mathrm{corr}(\hat{\pi}_t^d, \widehat{mc}_t) = \gamma_d / (1 - \rho\beta) > 0$. Evidently, a lower γ_d (i.e. higher ξ_d) reduces the positive correlation but cannot make it negative.

[22] It does not matter for the results if we introduce eq. (22) as a true structural equation in the model to incorporate it into the beliefs of the economic agents, or as an exogenous equation where expectations are unaffected.

closely in line with our ML estimate on actual Swedish data reported in Table 3a.

Hence, misspecifying the pricing equation enables us to retrieve the implausibly high price stickiness parameter, which in turn enables the model to match the small yet negative correlation between marginal cost and inflation in the data. We believe that this exercise, together with the DSGE-VAR analysis, constitutes straightforward evidence of misspecification problems, which manifest themselves with implausible stickiness parameters under ML estimation.

4.3. Assessing the economic implications

An important question is whether the differences between the Bayesian posterior median and ML estimate in Tables 3a and 3b **matter** when the models are used for policy analysis. It is possible that many individual parameters differ, but that the propagation of key shocks remains largely unaffected. To examine this issue, we simulate the effects of monetary policy and government spending shocks under the Bayesian posterior median and the ML estimate for the closed economy U.S. model in Table 3b. In both parameterizations of the model, we scale the policy shock so that it implies an increase in the federal funds rate with 100 basis points in the first period. For the government spending shock, we scale innovation to represent the equivalence of an increase of baseline GDP with 1 percent. This scaling of the spending shock implies that the output response in the first period measures the impact spending multiplier.

The results of these experiments are reported in Figure 3. As seen from the figure, the output effects of the monetary policy shock are notably larger under the ML estimate of the model, reflecting that a nominal shock propagates more into the real equilibrium when price and wage adjustments are estimated to be notably slower relative to the Bayesian estimation results. The inflation response is also notably smaller and more persistent under the ML estimation results compared to the Bayesian posterior. For the real government spending shock, the initial output response is similar in both models, but over time the response is notably larger in the model estimated with classical ML techniques. The inflation and price level response is also much more muted in this model compared to the Bayesian posterior model. Together, these results demonstrate that the differences between the Bayesian and ML results matter immensely for the effects of core policy actions, for the very purpose of which they are constructed.

5. Concluding remarks

In this paper we have analyzed the properties of maximum likelihood estimation in state-of-the-art closed and open economy New Keynesian DSGE models for monetary policy analysis. Our asymptotic tests of identification as well as our small-sample Monte Carlo analysis suggest that the studied DSGE models are identifiable in the notion of Rothenberg (1971): if an appropriate set of variables is used to estimate the DSGE model, the ML estimator is unbiased and we have mean square convergence. We therefore argue that the econometrician can learn a lot about many parameters in DSGE models by estimating them.

Consequently, we estimate the models with classical ML techniques on actual data for Sweden (open economy) and the U.S. (closed economy) and find a large improvement in log-likelihood: above 100 relative to the Bayesian posterior median for Sweden and about 45 for the U.S. By itself, this improvement is evidence against the notion that aggregate data are not informative. However, the ML estimates of the slope of the Phillips curves (i.e., the coefficient multiplying marginal cost) imply an extremely high degree of price stickiness which is hard to reconcile with microeconomic evidence. In addition, the low slope of the Phillips curve implies that the estimated volatility of the markup shocks is implausibly large, as pointed out by Chari, Kehoe and McGrattan (2008).

Direct and indirect evidence suggests that the DSGE models under consideration appears to suffer from misspecification, and that the misspecification problem appears to be mitigated by a parameterization that is inconsistent with the microeconomic evidence. Also Del Negro et al. (2007) find evidence of misspecification in their closed economy model on U.S. data, which indicates that our results are by no means specific to our model setup.

Our findings offer an additional interpretation why Bayesian methods may have become so popular among macroeconomists: although the likelihood function is informative about many of the parameters in the model, problems with model misspecification lead to implausible ML estimates relative to existing microeconomic evidence. In this environment with parameter and model uncertainty and misspecification, Bayesian techniques offer a way to estimate models in an internally consistent way by allowing the econometrician to examine the performance of the model in a region of the parameter space that can be deemed a priori plausible as discussed by e.g. Sims (2007, 2008).

Our analysis also indicates that, while identification should not be assumed to be weak for all models; different models can have similar core features and properties and yet differ in terms of the degree of misspecification or identification. Available identification pre-tests (e.g. Iskrev, 2010b, available in Dynare) appear to be very effective in highlighting possible identification issues and should always be used on a case-by-case basis. The severity of misspecification for a given model can be assessed using the DSGE-VAR techniques developed by Del Negro and Schorfheide (2004, 2009) (also available in Dynare). However, such issues are more difficult to address, and may affect outcomes more extensively than usually assumed.

There are some important issues that we leave for future research. First, we have followed the common practice in the literature (e.g. Christiano, Eichenbaum and Evans, 2005), and calibrated (i.e. used strict priors) some parameters we believe we have relatively good prior information about and a priori know would not be well identified by the set of observable variables we match. It would be of interest to examine in future work how the results would be affected when attempting to estimate these parameters on an extended information set.

Second, the models we consider have little to say about fiscal policy, and we think an interesting extension of our work would be to study identification in models where monetary and fiscal policy switch between active and passive regimes as in Davig and Leeper (2011) or Bianchi and Melosi (2017). Moreover, it would be of interest to examine if better integration of fiscal policy aspects mitigates the misspecification issues we have identified.

Third and finally, Rubio-Ramirez and Villaverde (2005) argue that estimations based on a non-linear (second-order) approximation are much more informative about the parameters in a real business cycle model. Following the appearance of the zero lower bound during the recent financial crisis, a rapidly growing literature has started to estimate non-linear DSGE models. Therefore, an interesting extension would be to examine the extent to which identification is enhanced in non-linear frameworks relative to the standard log-linearized representation examined in this paper.

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Table 1a: Multiple collinearity statistics and expected standard errors.

Parameter		Open Economy Model					Closed Economy Model				
		Mco (J_2)	Mco ($J(T)$)	std(θ_i)	θ_i/std(θ_i)	prior(θ_i) /std(θ_i)	Mco (J_2)	Mco ($J(T)$)	std(θ_i)	θ_i/std(θ_i)	prior(θ_i) /std(θ_i)
Calvo wages	ξ_w	0.336	0.887	0.092	8.288	0.542	0.442	0.935	0.097	7.906	0.517
Calvo domestic prices	ξ_d	0.729	0.791	0.025	33.694	2.042	0.740	0.936	0.038	21.738	1.318
Calvo import cons. prices	$\xi_{m,c}$	0.578	0.888	0.015	60.869	3.382	—	—	—	—	—
Calvo import inv. prices	$\xi_{m,i}$	0.674	0.806	0.013	73.091	3.892	—	—	—	—	—
Calvo export prices	ξ_x	0.566	0.948	0.025	35.572	2.035	—	—	—	—	—
Indexation prices	κ	0.713	0.934	0.052	4.366	2.885	0.702	0.980	0.120	1.900	1.255
Indexation wages	κ_w	0.450	0.829	0.135	2.395	1.112	0.495	0.938	0.125	2.582	1.199
Investment adj. cost	$\tilde{S}''$	0.559	0.853	2.823	3.041	0.531	0.564	0.868	3.786	2.267	0.396
Habit formation	b	0.589	0.831	0.053	12.943	1.906	0.695	0.885	0.056	12.090	1.781
Markup domestic	λ_d	0.490	0.814	0.109	10.954	0.458	0.580	0.869	0.155	7.693	0.322
Subst. elasticity invest.	η_i	0.700	0.945	0.134	20.204	5.842	—	—	—	—	—
Subst. elasticity foreign	η_f	0.651	0.970	0.330	4.645	2.382	—	—	—	—	—
Markup imported cons.	$\lambda_{m,c}$	0.657	0.943	0.029	54.657	1.725	—	—	—	—	—
Markup.imported invest.	$\lambda_{m,i}$	0.725	0.922	0.021	53.809	2.373	—	—	—	—	—
Technology growth	μ_z	0.514	0.825	0.0002	4 549.07	2.263	0.432	0.857	0.001	1533.800	0.763
Risk premium	$\tilde{\phi}$	0.680	0.912	0.016	3.224	64.472	—	—	—	—	—
UIP modification	$\tilde{\phi}_s$	0.506	0.878	0.040	15.321	3.792	—	—	—	—	—
Unit root tech. persistance	ρ_{μ_z}	0.562	0.978	0.100	8.422	0.997	0.502	0.994	0.172	4.915	0.581
Stationary tech. persistance	ρ_{ε}	0.463	0.975	0.041	22.789	2.464	0.493	0.985	0.044	21.106	2.281
Invest. spec. tech. persist.	ρ_{Υ}	0.629	0.987	0.085	8.136	1.172	0.645	0.983	0.061	11.287	1.627
Risk premium persistence	$\rho_{\tilde{\phi}}$	0.654	0.999	0.079	8.644	1.264	—	—	—	—	—
Consumption pref. persist.	ρ_{ζ_c}	0.499	0.958	0.116	5.666	0.862	0.538	0.957	0.128	5.147	0.783
Labour supply persistance	ρ_{ζ_h}	0.583	0.972	0.101	2.680	0.993	0.542	0.963	0.115	2.353	0.871
Asymmetric tech. persist.	$\rho_{\tilde{z}*}$	0.169	0.759	0.067	14.437	1.498	—	—	—	—	—
Unit root tech. shock	σ_{μ_z}	0.494	0.968	0.042	3.165	23.800	0.392	0.994	0.101	1.325	9.935
Stationary tech. shock	σ_{ε}	0.333	0.967	0.048	13.886	20.787	0.318	0.976	0.050	13.437	20.112
Imp. cons. markup shock	$\sigma_{\lambda_{m,c}}$	0.615	0.834	0.128	8.774	7.792	—	—	—	—	—
Imp. invest. markup shock	$\sigma_{\lambda_{m,i}}$	0.561	0.825	0.098	11.583	10.214	—	—	—	—	—
Domestic markup shock	σ_{λ_d}	0.752	0.913	0.070	11.474	14.218	0.692	0.927	0.088	9.198	11.398
Invest. spec. tech. shock	σ_{Υ}	0.572	0.984	0.070	5.653	14.276	0.634	0.987	0.054	7.284	18.384
Risk premium shock	$\sigma_{\tilde{\phi}}$	0.627	0.998	0.159	4.974	6.272	—	—	—	—	—
Consumption pref. shock	σ_{ζ_c}	0.404	0.957	0.035	7.576	28.805	0.414	0.955	0.035	7.477	28.387
Labour supply shock	σ_{ζ_h}	0.550	0.971	0.055	7.033	18.220	0.545	0.948	0.059	6.505	16.873
Asymmetric tech. shock	$\sigma_{\tilde{z}*}$	0.178	0.710	0.044	4.298	22.862	—	—	—	—	—
Export markup shock	σ_{λ_x}	0.612	0.877	0.254	4.069	3.939	—	—	—	—	—
Monetary policy shock	σ_R	0.586	0.937	0.020	12.276	51.364	0.663	0.981	0.021	11.582	48.462
Inflation target shock	$\sigma_{\bar{\pi}^c}$	0.408	0.921	0.050	3.144	20.025	0.375	0.916	0.055	2.861	18.221
Interest rate smoothing	ρ_R	0.890	0.991	0.043	21.060	1.153	0.892	0.993	0.047	19.452	1.066
Inflation response	r_{π}	0.912	0.994	0.960	1.744	0.104	0.893	0.992	0.943	1.775	0.106
Diff. infl response	$r_{\Delta\pi}$	0.522	0.622	0.030	3.224	1.645	0.412	0.599	0.026	3.783	3.848
Real exch. rate response	r_x	0.648	0.990	0.029	0.548	1.713	—	—	—	—	—
Output response	r_y	0.797	0.990	0.110	1.139	0.455	0.755	0.988	0.106	1.180	0.471
Diff. output response	$r_{\Delta y}$	0.658	0.940	0.040	4.412	1.239	0.635	0.981	0.046	3.880	1.088

Note: The table shows Iskrev's (2008) multiple collinearity statistics (Mco) w.r.t. the reduced form coefficients in the model only, J_2, and w.r.t. the first and second moments in the data, $J(T)$, using different sets of observables. A value close to one (or -1) indicates that there is very strong degree of collinearity w.r.t. the other deep parameters. The expected standard deviation, std(θ_i), is computed using the Fisher Information Matrix at the posterior median following Iskrev (2010b). The strength of identification is calculated by θ_i/std(θ_i) and prior(θ_i)/std(θ_i) shows if uncertainty increases or decreases compared to standard deviation of the prior distribution. Dynare 4.5.5 has been used to do the calculations.

Table 1b: Rank tests and median condition numbers, smallest singular and tolerance values.

	Open Economy Model				Closed Economy Model			
	posterior median		prior sampling		posterior median		prior sampling	
	J_2	$J(T)$	J_2	$J(T)$	J_2	$J(T)$	J_2	$J(T)$
Full rank	Yes	Yes	100%	100%	Yes	Yes	100%	100%
Condition number	325.62	649.03	610.91	2.327e+6	224.435	538.697	235.559	2.33e+4
Min singular value	0.114	0.023	0.0824	7.381e-6	0.092	0.0146	0.096	4.12e-4
Tolerance value	2.677e-11	6.235e-13	2.677e-11	1.247e-12	3.09e-12	7.28e-14	3.09e-12	1.46e-13

Note: For the columns labelled "prior sampling", the rank of J_2 and $J(T)$ have been evaluated at 5,000 draws from the prior distribution in Table A:2. For the condition, smallest singular, and tolerance values we report the median from these 5,000 evaluations. The condition number measures the ratio of the largest singular number to the smallest of a matrix A. The tolerance value measures the value that would overturn the rank condition for a matrix A, i.e. tol=max(size(A)) * eps(norm(A)). Dynare 4.5.5 has been used to do the calculations.

Table 2: Small sample distribution results for different models.

Parameter		True values	Open Economy Model: Mean of distri-bution	Open Economy Model: Median of distri-bution	Open Economy Model: Std. of distri-bution	Open Economy Model: Std based on Inverse Hessian	Closed Economy Model: Mean of distri-bution	Closed Economy Model: Median of distri-bution	Closed Economy Model: Std. of distri-bution	Closed Economy Model: Std based on Inverse Hessian
Calvo wages	ξ_w	0.77	0.74	0.75	0.13	0.07	0.72	0.73	0.16	0.08
Calvo domestic prices	ξ_d	0.83	0.81	0.82	0.04	0.03	0.81	0.81	0.04	0.03
Calvo import cons. prices	$\xi_{m,c}$	0.90	0.90	0.90	0.02	0.01	—	—	—	—
Calvo import inv. prices	$\xi_{m,i}$	0.94	0.94	0.94	0.02	0.01	—	—	—	—
Calvo export prices	ξ_x	0.87	0.86	0.86	0.04	0.02	—	—	—	—
Indexation prices	κ	0.23	0.22	0.22	0.06	0.05	0.22	0.22	0.13	0.08
Indexation wages	κ_w	0.32	0.32	0.32	0.15	0.07	0,32	0.31	0.14	0.09
Investment adj. cost	$\tilde{S}''$	8.58	8.98	8.08	4.08	2.02	9.86	7.76	7.54	2.79
Habit formation	b	0.68	0.67	0.67	0.07	0.05	0.67	0.67	0.07	0.05
Markup domestic	λ_d	1.20	1.21	1.20	0.14	0.09	1.22	1.18	0.20	0.15
Subst. elasticity invest.	η_i	2.72	2.72	2.71	0.13	0.11	—	—	—	—
Subst. elasticity foreign	η_f	1.53	1.59	1.45	0.59	0.23	—	—	—	—
Markup imported cons.	$\lambda_{m,c}$	1.58	1.58	1.58	0.01	0.01	—	—	—	—
Markup.imported invest.	$\lambda_{m,i}$	1.13	1.14	1.13	0.02	0.02	—	—	—	—
Technology growth	μ_z	1.005	1.005	1.005	0.0003	0.0003	1.005	1.005	0.001	0.0006
Risk premium	$\tilde{\phi}$	0.05	0.06	0.05	0.02	0.01	—	—	—	—
UIP modification	$\tilde{\phi}_s$	0.61	0.61	0.60	0.05	0.03	—	—	—	—
Unit root tech. persistance	ρ_{μ_z}	0.85	0.80	0.83	0.14	0.06	0.72	0.80	0.24	0.08
Stationary tech. persistance	ρ_{ε}	0.93	0.89	0.90	0.08	0.03	0.88	0.90	0.08	0.04
Invest. spec. tech. persist.	ρ_{Υ}	0.69	0.65	0.67	0.13	0.06	0.67	0.68	0.10	0.07
Risk premium persistence	$\rho_{\tilde{\phi}}$	0.68	0.65	0.65	0.11	0.06	—	—	—	—
Consumption pref. persist.	ρ_{ζ_c}	0.66	0.59	0.61	0.18	0.08	0.61	0.62	0.16	0.09
Labour supply persistance	ρ_{ζ_h}	0.27	0.26	0.26	0.13	0.07	0.26	0.26	0.13	0.09
Asymmetric tech. persist.	$\rho_{\tilde{z}^*}$	0.96	0.73	0.84	0.28	0.09	—	—	—	—
Unit root tech. shock	σ_{μ_z}	0.13	0.14	0.14	0.05	0.03	0.14	0.13	0.09	0.06
Stationary tech. shock	σ_{ε}	0.67	0.66	0.65	0.06	0.05	0.65	0.65	0.06	0.05
Imp. cons. markup shock	$\sigma_{\lambda_{m,c}}$	1.13	1.13	1.12	0.11	0.10	—	—	—	—
Imp. invest. markup shock	$\sigma_{\lambda_{m,i}}$	1.13	1.14	1.13	0.11	0.10	—	—	—	—
Domestic markup shock	σ_{λ_d}	0.81	0.82	0.82	0.08	0.08	0.82	0.81	0.10	0.08
Invest. spec. tech. shock	σ_{Υ}	0.40	0.42	0.41	0.09	0.06	0.42	0.41	0.08	0.06
Risk premium shock	$\sigma_{\tilde{\phi}}$	0.79	0.82	0.80	0.21	0.12	—	—	—	—
Consumption pref. shock	σ_{ζ_c}	0.26	0.27	0.27	0.05	0.04	0.27	0.27	0.04	0.04
Labour supply shock	σ_{ζ_h}	0.39	0.39	0.39	0.06	0.04	0.39	0.39	0.07	0.05
Asymmetric tech. shock	$\sigma_{\tilde{z}^*}$	0.19	0.15	0.16	0.06	0.04	—	—	—	—
Export markup shock	σ_{λ_x}	1.03	1.13	1.09	0.41	0.21	—	—	—	—
Monetary policy shock	σ_R	0.24	0.24	0.23	0.02	0.02	0.24	0.23	0.03	0.02
Inflation target shock	$\sigma_{\bar{\pi}^c}$	0.16	0.14	0.14	0.10	0.04	0.12	0.11	0.11	0.06
Interest rate smoothing	ρ_R	0.91	0.91	0.91	0.05	0.03	0.91	0.91	0.05	0.03
Inflation response	r_π	1.67	3.80	1.59	5.08	2.70	3.80	1.52	4.94	1.24
Diff. infl response	$r_{\Delta\pi}$	0.10	0.11	0.10	0.04	0.03	0.11	0.10	0.03	0.03
Real exch. rate response	r_x	-0.02	-0.07	-0.02	0.15	0.02	—	—	—	—
Output response	r_y	0.13	0.35	0.13	0.63	0.07	0.32	0.11	0.60	0.07
Diff. output response	$r_{\Delta y}$	0.18	0.19	0.18	0.05	0.03	0.19	0.19	0.06	0.04

Note: Out of the 1,500 estimations for the sample (each 100 obs.), the results above is based on 1,452 convergent estimations (defined as estimations when the optimizer CSMINWEL terminates without an error message and when the inverse Hessian has full rank and is positive definite). Out of the 1,500 estimations for the closed economy, the results above is based on 1,448 convergent estimations. True parameter values were used as starting values in the estimations. Std based on Inverse Hessian shows the median of these estimations.

Table 3.a: Likelihood estimation results on actual data: Open economy model for Sweden.

Parameter		Bayesian Posterior Distribution			Maximum Likelihood Estimation		
		Median	5%	95%	Point estimate	5%	95%
Calvo wages	ξ_w	0.765	0.677	0.839	0.830	0.808	0.852
Calvo domestic prices	ξ_d	0.825	0.737	0.903	0.949	0.942	0.956
Calvo import cons. prices	$\xi_{m,c}$	0.900	0.870	0.926	0.989	0.963	1.015
Calvo import inv. prices	$\xi_{m,i}$	0.939	0.922	0.955	0.990	0.990	0.990
Calvo export prices	ξ_x	0.874	0.838	0.905	0.987	0.983	0.991
Indexation prices	κ_p	0.227	0.135	0.335	0.013	0.012	0.014
Indexation wages	κ_w	0.323	0.165	0.515	0.020	0.017	0.023
Investment adj. cost	$\tilde{S}''$	8.584	6.510	10.803	22.500	14.963	30.037
Habit formation	b	0.679	0.572	0.771	0.871	0.862	0.880
Markup domestic	λ_d	1.195	1.117	1.277	1.112	1.110	1.114
Subst. elasticity invest.	η_i	2.715	2.280	3.330	1.335	1.311	1.359
Subst. elasticity foreign	η_f	1.531	1.349	1.856	2.766	2.530	3.002
Markup imported cons.	$\lambda_{m,c}$	1.584	1.529	1.638	2.371	2.367	2.375
Markup.imported invest.	$\lambda_{m,i}$	1.134	1.067	1.207	2.315	2.292	2.338
Technology growth	μ_z	1.005	1.005	1.005	1.005	1.005	1.005
Risk premium	$\tilde{\phi}$	0.050	0.022	0.116	0.228	0.164	0.292
UIP modification	$\tilde{\phi}_s$	0.606	0.516	0.728	0.982	0.976	0.988
Unit root tech. shock persistence	ρ_{μ_z}	0.845	0.704	0.928	0.906	0.898	0.914
Stationary tech. shock persistence	ρ_{ε}	0.925	0.822	0.972	0.994	0.993	0.995
Invest. spec. tech shock	ρ_{Υ}	0.694	0.519	0.839	0.319	0.207	0.431
Risk premium shock persistence	$\rho_{\tilde{\phi}}$	0.684	0.503	0.855	0.416	0.395	0.437
Consumption pref.shock	ρ_{ζ_c}	0.657	0.419	0.848	0.017	-0.067	0.101
Labour supply shock persistence	ρ_{ζ_h}	0.270	0.167	0.385	0.025	0.017	0.033
Asymmetric tech. shock	$\rho_{\tilde{z}^*}$	0.964	0.947	0.978	0.933	0.929	0.937
Unit root tech. shock std. dev.	σ_z	0.133	0.098	0.184	0.064	0.053	0.075
Stationary tech. shock std. dev.	σ_{ε}	0.668	0.542	0.817	0.664	0.640	0.688
Imp. cons. markup shock std. dev.	$\sigma_{\lambda_{m,c}}$	1.126	0.956	1.349	1.285	1.187	1.383
Imp. invest. markup shock std.	$\sigma_{\lambda_{m,i}}$	1.134	0.955	1.364	1.726	1.718	1.734
Domestic markup shock std. dev.	σ_{λ}	0.807	0.684	0.960	0.823	0.820	0.826
Invest. spec. tech. shock std. dev.	σ_{Υ}	0.396	0.294	0.535	0.558	0.473	0.643
Risk premium shock std. dev.	$\sigma_{\tilde{\phi}}$	0.793	0.500	1.226	2.021	1.950	2.092
Consumption pref. shock std. dev.	σ_{ζ_c}	0.263	0.196	0.348	0.300	0.277	0.323
Labour supply shock std. dev.	σ_{ζ_h}	0.386	0.326	0.458	0.344	0.343	0.345
Asymmetric tech. shock std. dev.	$\sigma_{\tilde{z}^*}$	0.188	0.150	0.240	0.013	0.012	0.014
Export markup shock std. dev.	σ_{λ_x}	1.033	0.817	1.282	0.554	0.482	0.626
Monetary policy shock	$\sigma_{R,}$	0.239	0.207	0.279	0.219	0.218	0.220
Inflation target shock	$\sigma_{\bar{\pi}^c}$	0.157	0.089	0.247	0.263	0.244	0.282
Interest rate smoothing	ρ_R	0.913	0.882	0.938	0.957	0.956	0.958
Inflation response	$r_{\pi}(1-\rho_R)$	0.146	0.104	0.197	0.045	0.043	0.047
Diff. infl response	$r_{\Delta\pi}$	0.098	0.050	0.152	0.011	0.011	0.011
Real exch. rate response	$r_x(1-\rho_R)$	-0.001	-0.005	0.002	0.002	0.001	0.003
Output response	$r_y(1-\rho_R)$	0.011	0.005	0.018	-0.001	0.000	0.002
Diff. output response	$r_{\Delta y}$	0.178	0.118	0.241	0.060	0.024	0.096
Log Likelihood			-2128.58			-2022.16	
Log Marginal Likelihood (Laplace)			-2271.24			---	
Log Posterior			-2232.42			-4439.24	

*Note: The reported parameters for the level of inflation, real exchange and the output gap have been transformed to composite responses instead of separate responses as in Tables 1 and 2. The prior distribution used to obtain the Bayesian posterior median is provided in Table A:2. The log likelihood for the Bayesian posterior distribution is computed using the posterior median parameters. The sample period in the estimation is 1980Q1-2004Q4, where the period 1980Q1-1985Q4 is used to compute the unobserved state variables in 1985Q4 and the period 1986Q1-2004Q4 for inference. The ML estimation confidence interval is calculated as: point estimate ± 1.645*std, where std is the standard deviation according to the sandwich form of the inverse Hessian.

Table 3.b: Likelihood estimation results on actual data: Closed economy model for United States.

Parameter		Bayesian Posterior Distribution			Maximum Likelihood Estimation		
		Median	5%	95%	Point estimate	5%	95%
Calvo wages	ξ_w	0.735	0.671	0.799	0.972	0.955	0.990
Calvo domestic prices	ξ_d	0.865	0.841	0.889	0.989	0.987	0.991
Indexation prices	κ_p	0.223	0.106	0.339	0.897	0.763	1.031
Indexation wages	κ_w	0.644	0.442	0.845	0.067	0.006	0.130
Investment adj. cost	$\tilde{S}''$	6.512	4.419	8.605	2.389	0.024	4.574
Habit formation	b	0.794	0.756	0.832	0.961	0.940	0.983
Markup domestic	λ_d	1.261	1.099	1.423	1.799	1.718	1.880
Technology growth	μ_z	1.004	1.004	1.004	1.004	1.004	1.004
Unit root tech. shock persistence	ρ_{μ_z}	0.568	0.423	0.713	0.509	0.364	0.654
Stationary tech. shock persistence	ρ_ε	0.990	0.981	0.998	0.997	0.994	0.999
Invest. spec. tech shock persistence	ρ_Υ	0.612	0.518	0.706	0.363	0.149	0.577
Consumption pref.shock persistence	ρ_{ζ_c}	0.994	0.990	0.999	0.192	0.003	0.381
Labour supply shock persistence	ρ_{ζ_h}	0.428	0.302	0.553	0.329	0.086	0.573
Unit root tech. shock std. dev.	σ_z	0.388	0.274	0.501	0.490	0.384	0.596
Stationary tech. shock std. dev.	σ_ε	0.467	0.385	0.550	0.370	0.325	0.416
Domestic markup shock std. dev.	σ_λ	0.169	0.150	0.189	0.157	0.142	0.172
Invest. spec. tech. shock std. dev.	σ_Υ	0.911	0.783	1.040	1.339	1.141	1.537
Consumption pref. shock std. dev.	σ_{ζ_c}	0.150	0.112	0.188	0.173	0.144	0.203
Labour supply shock std. dev.	σ_{ζ_h}	0.207	0.164	0.251	0.199	0.124	0.274
Monetary policy shock	$\sigma_{R,}$	0.233	0.210	0.256	0.242	0.214	0.269
Inflation target shock	$\sigma_{\bar{\pi}^c}$	0.127	0.097	0.157	0.133	0.066	0.200
Interest rate smoothing	ρ_R	0.775	0.738	0.813	0.964	0.941	0.987
Inflation response	$r_\pi(1-\rho_R)$	0.407	0.330	0.484	0.311	0.188	0.435
Diff. infl response	$r_{\Delta\pi}$	0.297	0.214	0.379	0.248	0.116	0.379
Output response	$r_y(1-\rho_R)$	0.013	0.008	0.018	−0.003	−0.005	−0.001
Diff. output response	$r_{\Delta y}$	0.120	0.088	0.153	0.228	0.178	0.277
Log Likelihood			−1454.39			−1410.61	
Log Marginal Likelihood (Laplace)			−1518.85			---	
Log Posterior			−1454.27			−3996.65	

*Note: The reported parameters for the level of inflation and the output gap have been transformed to composite responses instead of separate responses as in Tables 1 and 2. The prior distribution used to obtain the Bayesian posterior median is provided in Table A:2. The log likelihood for the Bayesian posterior distribution is computed using the posterior median parameters. The sample period in the estimation is 1959Q2−2006Q4, where the period 1959Q2−1965Q4 is used to compute the unobserved state variables in 1959Q4 and the period 1966Q1−2006Q4 for inference. The ML estimation confidence interval is calculated as: point estimate ± 1.645*std, where std is the standard deviation according to the sandwich form of the inverse Hessian. The parameter μ_z has been calibrated to 1.004 (Bayesian posterior median) because it was driven to its lower bound (1.0001) in the MLE estimations, but the results are essentially unaffected (if anything, estimating it amplifies further the difference between the Bayesian and MLE log likelihood).

Table 4: Log-marginal likelihood of VAR/VECM with DSGE prior for Sweden and the U.S.

λ	Open Economy Model for Sweden DSGE-VAR 8 lags	Open Economy Model for Sweden DSGE-VECM 8 lags	Closed Economy Model for the U.S. DSGE-VAR 8 lags	Closed Economy Model for the U.S. DSGE-VECM 8 lags
λ_{min} = 0.36	—	—	—	−1 520.25
λ_{min} = 0.39	—	—	−1 577.43	—
λ_{min} = 1.79	−2 445.25	—	—	—
λ_{min} = 1.87	—	−2 469.01	—	—
0.75	—	—	−1 437.67	−1 421.84
1	—	—	−1 436.08	**−1 407.78**
1.25	—	—	**−1 421.02**	−1 408.28
1.5	—	—	−1 424.44	−1 408.95
2	−2 283.20	−2 353.32	−1 432.57	−1 409.25
3	−2 050.28	−2 071.91	−1 462.54	−1 423.30
4	−1 995.27	−2 005.99	−1 473.30	−1 429.79
5	−1 981.17	−1 987.29	−1 474.31	−1 442.96
5.5	**−1 980.23**	**−1 985.07**	−1 486.51	−1 450.06
6	−1 981.43	−1 985.36	−1 490.96	−1 452.76
10	−2 014.23	−2 017.24	−1 511.16	−1 465.30
∞	−2 244.01	−2 271.94	−1 544.49	−1 519.67
DSGE	−2 271.24	−2 271.24	−1 518.85	−1 518.85

Note: The table displays Laplace approximations of the log-marginal likelihood. λ = 1.79 and λ = 1.87 are the minimal tightnesses for the VAR and VECM for Sweden and 0.36 and 0.39 the corresponding numbers for the U.S., respectively. Bold numbers indicate the λ with the maximal log marginal likelihood. The prior is proper if $\lambda \geq \lambda_{min}=[n(p+1)+q+1]/T$, where n is the number of endogenous variables in the VAR with p lags, q the number of exogenous variables, and T the number of data points. In our Swedish application n=15, p=8, T=76 (post-training sample 86Q1-04Q4), and q=0 in the VAR whereas q=6 for the VECM. In the U.S. estimations, n=7, p=8, T=164 (post-training sample 65Q1-06Q4), and q=0 in the VAR whereas q=4 for the VECM.

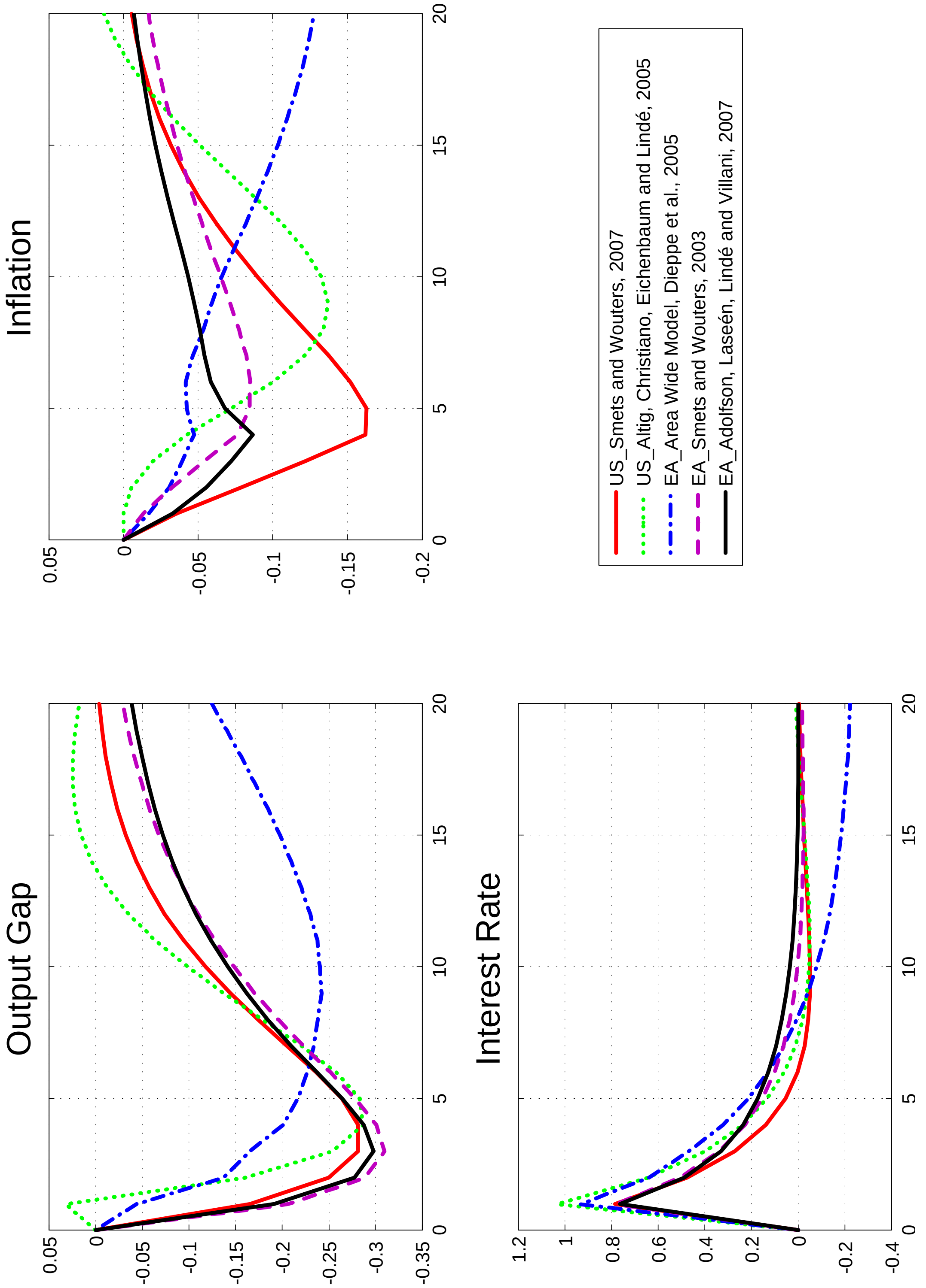

Figure 1: Impulse response functions to a monetary policy shock in different DSGE models, Smets and Wouters 2007-rule

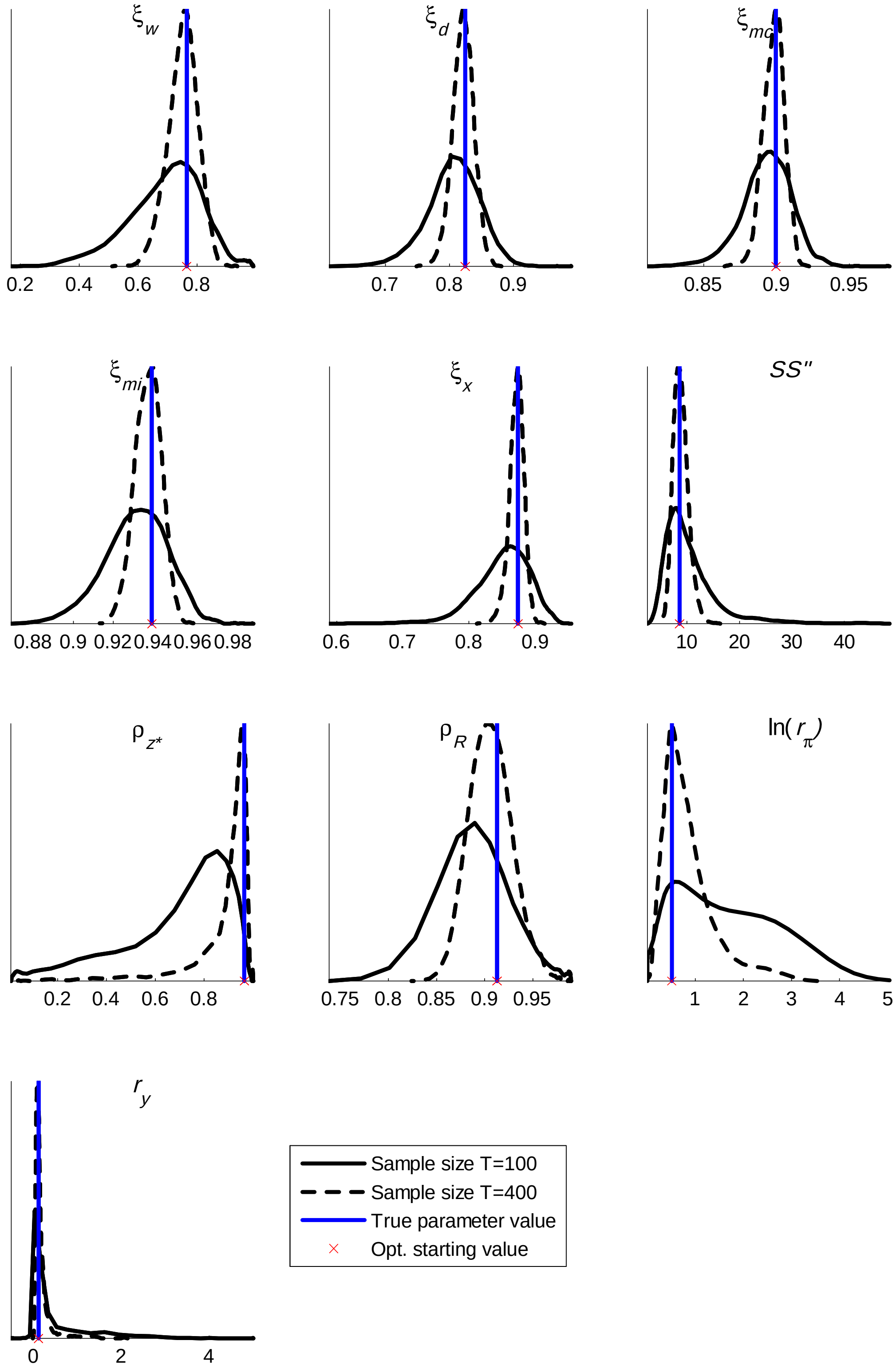

Figure 2: Kernel density estimates of the small sample distribution for the estimates of some of the model parameters. The solid line shows the parameter distribution for $T = 100$, and the dashed line shows the distribution for $T = 400$ observations. The vertical bar shows the true parameter value and the cross on the x-axis indicates the starting value in the optimizations.

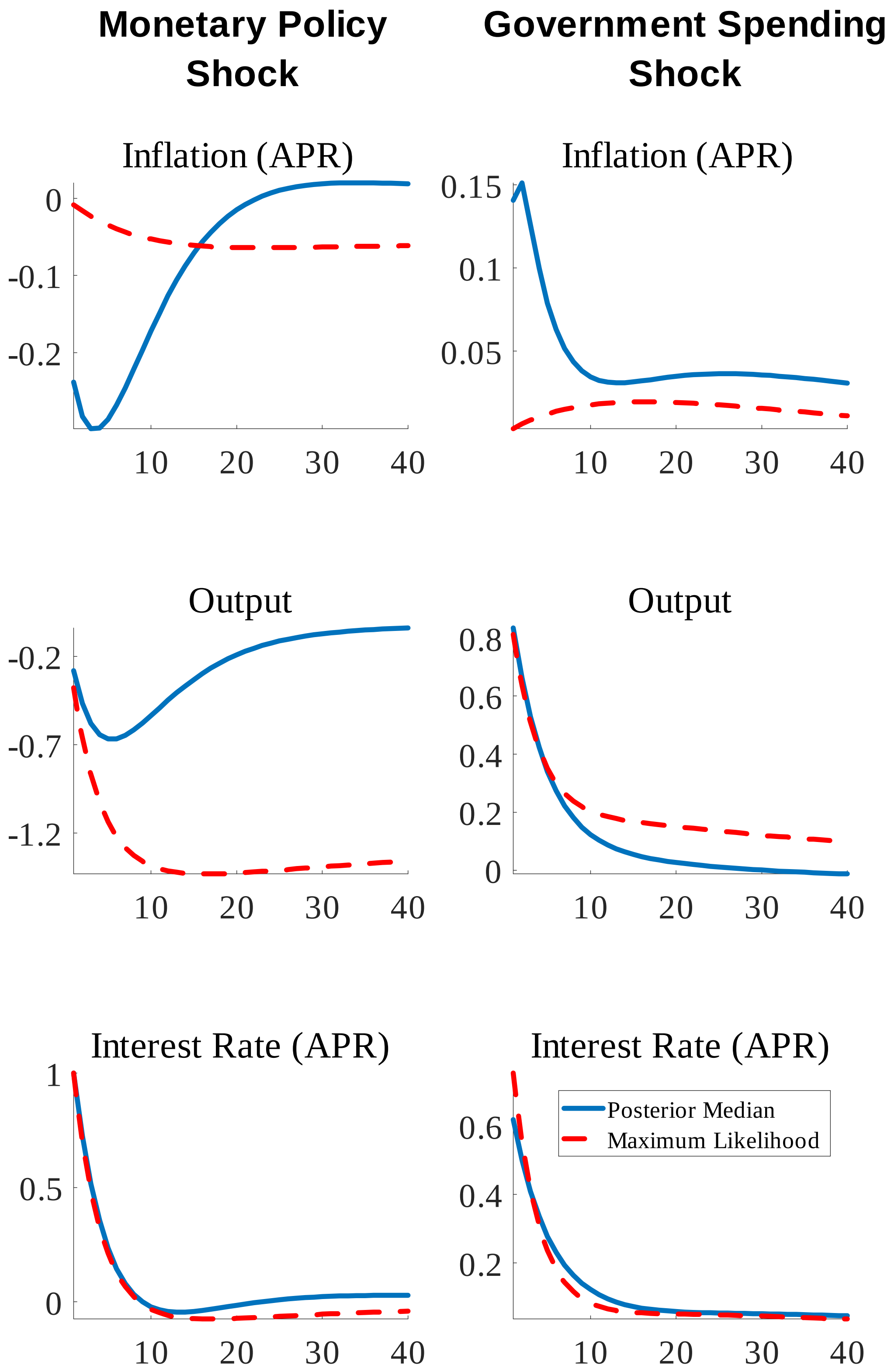

Figure 3: Comparing impulse response functions to monetary policy and government spending shocks for the Bayesian posterior median and the MLE estimate.

Appendix A. Calibrated parameters, prior and posterior distributions

The parameters we choose to calibrate are displayed in Table A.1. Following common practice in the literature (e.g. Christiano, Eichenbaum and Evans, 2005), we decided to calibrate (i.e. use strict priors) for these parameters as we have relatively good prior information about them and a priori know that they are not well identified by the set of observable variables we match. According to the results in Iskrev many of these parameters are weakly identified in his analysis of the Smets and Wouters model. But, some of them could be well identified from aggregate quantities and prices if a larger set of observed variables and first-order moments were included in the estimation. Most of these parameters are related to the steady-state values of permutations of the observables, and would be identified by inclusion of the following set of first-order moments: C/Y, $\tilde{M}/Y$, I/Y and G/Y (identifies ω_c, ω_i, δ, and g_r); π and R (identify μ and β when μ_z is estimated); $1 - WH/Y$ (identifies α); average income, pay-roll and VAT rates (identifies τ^y, τ^w, and τ^c).[23]

This said, there are a few calibrated parameters in Table A.2 $\{\eta_c, \sigma_L, \lambda_w, \rho_{\bar{\pi}}\}$ that are more problematic. The two parameters η_c and $\rho_{\bar{\pi}}$ are identified in our model (not shown), but we nevertheless decided to calibrate them to have our setup consistent with Adolfson et al. (2008a).[24] However, the average wage markup (λ_w) and the labor supply elasticity (σ_L) are better identified by micro data, see e.g. Domeij and Floden (2006), and we therefore fix also these parameters when we estimate the model on aggregate data. In principle, our model implies a distribution and history of households with different nominal wages and hours worked, and under the standard maintained assumption that the data generating process is not misspecified, the information in these distributions in conjunction with aggregate hours worked and the aggregate real wage could be used to efficiently estimate the labor supply elasticity and the steady state markup. Christiano, Eichenbaum and Evans (2005) pursue the same approach in their paper when estimating their model on U.S. data, and we therefore adapt their values for these two parameters. The fact that these parameters are weakly identified when aggregate data are used exclusively to estimate DSGE models is not an identification problem per se, it merely reflects a limitation of what can be achieved with aggregate data only. Of course, it may be the case that the DSGE model does not represent a reasonable approximation of actual wage-setting behaviour, so that estimating the first-order conditions right off the micro data would yield implausible results, but again this is a problem with misspecification and not with identification.

In comparison with other papers in the open economy literature, such as for example Justiniano and Preston (2010) and Lubik and Schorfheide (2005), we have chosen to work with a large number of variables because it facilitates identification of the parameters and shocks processes we estimate. We estimate 13 structural shocks of which 5 are assumed to be identically independently distributed and 8 follow AR(1) processes. In addition to these shocks, there are eight additional shocks provided by the exogenous and pre-estimated fiscal and foreign VARs, whose parameters are kept fixed at their posterior means throughout the estimation of the DSGE model parameters. The shocks enter in such a way that there is no stochastic singularity in the

[23] For our open economy model we consider a steady state where $\bar{a}$ is zero. Combined with our assumption that foreign and domestic inflation targets both equal $\bar{\pi}$ in the steady-state, the change in the nominal exchange rate is nil in steady state, so that $\frac{\bar{S}_{t+1}}{\bar{S}_t}\frac{\bar{S}_t}{\bar{S}_{t-1}} - 1 = 0$. We have also verified that a solution exists and is unique for our augmented specification of the risk-premium in (12) in the joint prior distribution for $\left\{\tilde{\phi}_a, \tilde{\phi}_s\right\}$.

[24] The rationale for calibrating $\rho_{\bar{\pi}}$ to 0.975 was to ensure that the inflation target is a highly persistent process. η_c was calibrated to 5 because this parameter was otherwise driven towards a very high number (around 20) due to a slightly better match of the low volatility of consumption and high volatility of imports; see Adolfson et al. (2007, 2008) for further discussion.

likelihood function.

We apply the Kalman filter to calculate the likelihood function of the observed variables, where on Swedish data the period $1980Q1$-$1985Q4$ is used to form a prior on the unobserved state variables in $1985Q4$ and the period $1986Q1$-$2004Q4$ for inference.[25] Table A.2 shows the assumptions for the prior distribution of the estimated parameters, and the resulting Bayesian posterior median estimates based on a sample of $500,000$ post burn-in draws from the posterior distribution.

When we estimate the closed economy formulation of the model on U.S. data in Section 4 we eliminate the open economy dimension from the model by setting $\omega_c = \omega_i = 0$ and the variance of all open economy shocks to nil. Reflecting somewhat different steady state ratios in the U.S. economy relative to Sweden, we also recalibrate the following parameters in Table A.1; $\beta = 0.99754$, $\alpha = 0.3$, $\mu = 0.014$, and $g_r = 0.2$. Apart from that, all the priors for the estimated parameters are identical to those in Table A.2. Another difference is that we remove the mean and variations in the tax shocks $\tau_{y,t}$, $\tau_{w,t}$ and $\tau_{c,t}$. Data for the period $1959Q1$-$1965Q4$ is used to form a prior on the unobserved state variables in $1965Q4$ and the period $1966Q1$-$2006Q4$ for inference. Following Smets and Wouters (2007), all data series are assumed to be measured without error.

[25] We include white noise measurement errors in all variables except for the short-term interest rate and the three foreign variables, since we know that those data series used are not perfectly measured and at best only approximations of the 'true' series. In particular it was hard to remove the seasonal variation in the domestic series, and there are still spikes in for example the inflation series, perhaps due to changes in the collection of the data. The variance of the white noise measurement errors is set to 0.1 percent for the real wage, consumption and output, and 0.2 percent for the other domestic variables, implying that the fundamental shocks explain about 90-95% of the variation in most of the variables measured with error.

Table A.1: Calibrated parameters for Sweden

Parameter	Description	Calibrated value
β	Households' discount factor	0.999
α	Capital share in production[a]	0.25
η_c	Substitution elasticity between C_t^d and C_t^m	5
μ	Money growth rate (quarterly rate)[a]	1.010445
σ_L	labor supply elasticity	1
δ	Depreciation rate	0.01
λ_w	Wage markup[a]	1.05
ω_i	Share of imported investment goods[a]	0.70
ω_c	Share of imported consumption goods[a]	0.40
τ^y	labor income tax rate[a]	0.30
τ^c	Consumption tax rate[a]	0.24
τ^w	Pay-roll tax rate[a]	0.30
$\rho_{\bar{\pi}}$	Inflation target persistence	0.975
g_r	Government expenditures-output ratio[a]	0.30

[a] Notice that the description of the parameter pertain to its steady state.

Table A.2: Prior and posterior distributions.

Parameter		Prior distribution type	Prior distribution mean	Prior distribution std. dev. / df	Posterior distribution median	Bounds lower	Bounds Upper
Calvo wages	ξ_w	beta	0.750	0.050	0.765	0.001	0.999
Calvo domestic prices	ξ_d	beta	0.750	0.050	0.825	0.001	0.999
Calvo import cons. prices	$\xi_{m,c}$	beta	0.750	0.050	0.900	0.001	0.999
Calvo import inv. prices	$\xi_{m,i}$	beta	0.750	0.050	0.939	0.001	0.999
Calvo export prices	ξ_x	beta	0.750	0.050	0.874	0.001	0.999
Indexation prices	κ_p	beta	0.500	0.150	0.227	0.001	0.999
Indexation wages	κ_w	beta	0.500	0.150	0.323	0.001	0.999
Investment adj. cost	$\tilde{S}''$	normal	7.694	1.500	8.584	0.1	100
Habit formation	b	beta	0.650	0.100	0.679	0.01	0.99
Markup domestic	λ_d	truncnormal	1.200	0.050	1.195	1.001	10
Subst. elasticity invest.	η_i	invgamma	1.500	4	2.715	0.01	20
Subst. elasticity foreign	η_f	invgamma	1.500	4	1.531	0.01	20
Markup imported cons.	$\lambda_{m,c}$	truncnormal	1.200	0.050	1.584	1.001	10
Markup.imported invest.	$\lambda_{m,i}$	truncnormal	1.200	0.050	1.134	1.001	10
Technology growth	μ_z	truncnormal	1.006	0.0005	1.005	1.0001	1.01
Risk premium	$\tilde{\phi}$	invgamma	0.010	2	0.050	0.0001	10
UIP modification	$\tilde{\phi}_s$	beta	0.500	0.15	0.606	0.0001	1
Unit root tech. shock persistence	ρ_{μ_z}	beta	0.850	0.100	0.845	0.0001	0.9999
Stationary tech. shock persistence	ρ_ε	beta	0.850	0.100	0.925	0.0001	0.9999
Invest. spec. tech shock persistence	ρ_Υ	beta	0.850	0.100	0.694	0.0001	0.9999
Risk premium shock persistence	$\rho_{\tilde{\phi}}$	beta	0.850	0.100	0.684	0.0001	0.9999
Consumption pref. shock persistence	ρ_{ζ_c}	beta	0.850	0.100	0.657	0.0001	0.9999
Labour supply shock persistence	ρ_{ζ_h}	beta	0.850	0.100	0.270	0.0001	0.9999
Asymmetric tech. shock persistence	$\rho_{\tilde{z}^*}$	beta	0.850	0.100	0.964	0.0001	0.9999
Unit root tech. shock std. dev.	σ_z	invgamma	0.200	2	0.133	0.001	10
Stationary tech. shock std. dev.	σ_ε	invgamma	0.700	2	0.668	0.001	10
Imp. cons. markup shock std. dev.	$\sigma_{\lambda_{m,c}}$	invgamma	1.000	2	1.126	0.001	400
Imp. invest. markup shock std. dev.	$\sigma_{\lambda_{m,i}}$	invgamma	1.000	2	1.134	0.001	400
Domestic markup shock std. dev.	σ_λ	invgamma	1.000	2	0.807	0.001	100
Invest. spec. tech. shock std. dev.	σ_Υ	invgamma	0.200	2	0.396	0.001	100
Risk premium shock std. dev.	$\sigma_{\tilde{\phi}}$	invgamma	0.050	2	0.793	0.001	10
Consumption pref. shock std. dev.	σ_{ζ_c}	invgamma	0.200	2	0.263	0.001	5
Labour supply shock std. dev.	σ_{ζ_h}	invgamma	1.000	2	0.386	0.001	15
Asymmetric tech. shock std. dev.	$\sigma_{\tilde{z}^*}$	invgamma	0.400	2	0.188	0.001	2
Export markup shock std. dev.	σ_{λ_x}	invgamma	1.000	2	1.033	0.001	20
Monetary policy shock	$\sigma_{R,}$	invgamma	0.150	2	0.239	0.001	2
Inflation target shock	$\sigma_{\bar{\pi}^c}$	invgamma	0.050	2	0.157	0.001	1.5
Interest rate smoothing	ρ_R	beta	0.800	0.050	0.913	0.001	0.999
Inflation response	r_π	truncnormal	1.700	0.100	1.674	1.01	1000
Diff. infl response	$r_{\Delta\pi}$	normal	0.300	0.050	0.098	-0.5	5
Real exch. rate response	r_x	normal	0.000	0.050	-0.016	-5	5
Output response	r_y	normal	0.125	0.050	0.125	-0.5	5
Diff. output response	$r_{\Delta y}$	normal	0.063	0.050	0.178	-0.5	5

*Note: For the inverse gamma distribution the mode and the degrees of freedom are reported. Also, for the parameters $\lambda_d, \eta_i, \eta_f, \lambda_{m,c}, \lambda_{m,i}$ and μ_z the prior distributions are truncated at 1.

Appendix B. Additional simulation results

In this appendix, we present additional simulation results for four experiments.

B.1. Robustness w.r.t. starting values

In Section 3.2.2, all the estimations were initiated from the true parameter values. This could be a clear advantage for the ML estimator in a large model. In particular, if the multidimensional likelihood surface is characterized by many local maxima, there is the possibility that the favorable results in the previous subsection was driven by the very good guesses that initialized the estimations. In this subsection we relax this assumption and instead initialize the optimizations by sampling from the prior distribution in Table 2 that were used to estimate the model on actual data. We construct a joint distribution of the parameters in the following way. First, we make $30,000$ draws from the prior distribution. Then we compute the 2.5 and 97.5th percentiles for each parameter in this distribution, and select all draws in the joint distribution that simultaneously are within the 2.5th and 97.5th percentiles. This procedure gives a distribution of starting values that can differ substantially from the true parameter values because some of the priors in Table A.2 are relatively uninformative (in particular the priors for the standard deviations of the shock processes).

In Table B.1, we report the mean, median and standard deviation of the distributions when starting out the optimizations from the prior distribution and when starting out from the true parameter values. Only results for the same samples are reported in order to be able to make an accurate comparison. The results in Table B.1 that are based on initializations with the true parameter values can also be compared to the results in Table 3 for $T = 100$, which were based on nearly all 20 additional samples. From this comparison, it is clear that the distributions are identical except for small deviations for the parameters $\tilde{S}''$ and r_π, so any conclusions drawn based on results Table B.1 are directly applicable to those in Table 2.

Comparing the marginal parameter distributions based on starting the optimizations with the true values with the ones obtained when initialized by sampling starting values from the prior distributions, it is clear from Table B.1 that they are essentially identical. Consequently, the initial guess does not seem to be of importance when assessing the performance of the ML estimator. Not surprisingly, there are some slight deviations in the distributions for the three parameters $\tilde{S}''$ and r_π and r_y, but the deviations are very small.

In Figure B.1, we confirm the conclusions in Table B.1 by comparing the distribution resulting from "true initialization" (solid black) against the distribution resulting from "prior initialization" (dashed black) along with the actual starting value distribution (dotted line). From the figure, it is clear that the prior distributions for the $1,432$ commonly convergent estimations we used are clearly off for some parameters relative to the true parameter values in line with the priors used on actual data (see Table A.2). So it is not the case that the ML estimator is able to find the optimum only because the starting values sampled from the prior are nearly identical to the true parameters. The optimizations can be initiated with parameters that are far away from the optimum and convergence can still be achieved.

To sum up, we have presented strong evidence that the performance of the ML estimator is robust even if the econometrician does not have a perfect guess of the starting value of the parameters.

B.2. Adding measurement errors and reestimating the fiscal and foreign VARs

We now examine the implications of not having measurement errors and fixing the coefficients in the VARs for the fiscal policy and foreign variables at their true values. We add measurement errors to the simulated data as described in Section 3.2.1. The measurement errors are assumed to be i.i.d. and in the estimations they are calibrated at their true values. In addition, we also reestimate the VARs for the fiscal and foreign VARs in the same way that they are estimated on actual data for each sample rather than fixing the VAR coefficients at their true values in each simulation.

A priori, we expect this alternative approach, which exactly mimics the estimation strategy on actual data, to be associated with more dispersed parameter distributions, as the added measurement errors (although calibrated at their true values) and estimated VARs induce additional uncertainty in the estimations. This prior is confirmed by the simulation results reported in Figures B.2a-c, where we see that the resulting parameter distributions are somewhat wider for some of the parameters. However, the key results are unaffected, and the ML estimator is still unbiased for almost all parameters.

B.3. Exploiting the cointegrating vectors in the simulations

One possible explanation to problems with poor identification for the degree of nominal wage stickiness is that we do not exploit the cointegrating vectors when we match the model to the data in the simulations. Instead of matching the variables in (19) where all quantities and the real wage are in quarterly growth rates, we therefore consider the following set of variables in the data instead

$$\tilde{Y}_t = \begin{matrix} [& \pi_t^d & \ln(W_t/P_t) - \ln Y_t & \ln C_t - \ln Y_t & \ln I_t - \ln Y_t & \hat{x}_t & R_t & \hat{H}_t & \Delta \ln Y_t ... \\ & & \ln \tilde{X}_t - \ln Y_t & \ln \tilde{M}_t - \ln Y_t & \pi_t^{cpi} & \pi_t^{def,i} & \ln Y_t^* - \ln Y_t & \pi_t^* & R_t^* \;]'. \end{matrix} \tag{B.1}$$

The set of variables in (B.1) imposes the true cointegrating vectors in the estimations, and by doing so it should provide more efficient estimation of the underlying parameters in the model.

However, Figures B.3a-c suggest that the efficiency gains from matching the cointegrating vectors for the quantities as opposed to the variables in first differences are note very large. In most cases the resulting parameter distributions are essentially identical. Only in a few cases the marginal parameter distributions based on the cointegrating vectors (dashed lines) are less dispersed compared to the marginal parameter distributions based on the first differenced real quantities (solid lines) in (19) .

B.4. Consistency properties of the ML estimator

Table B.2 compares the results for the sample sizes $T = 100$ as our benchmark and 400 in each simulated sample. To save space, we limit our attention to the open economy formulation of the model, but the results (not shown) are qualitatively similar in the closed economy specification estimated on the set of observables in eq. (21).

Since we have already discussed the results for the benchmark results in the main text, we immediately turn to the case of $T = 400$. When doing so, we see that the mean and median parameter estimates are getting more similar in general, and for $\tilde{S}''$ and r_π and r_y in particular. Both the mean and median are now also very similar to the true parameter values, with the exception of r_π whose mean still is too high relative to the true parameter value. In addition, it is clear that the distributions start to collapse around the true values as the standard deviations of the marginal distributions have been reduced by at least a factor of 2, consistent with square-root sample size convergence properties of the ML estimator. As can be seen from the last column, the discrepancies between the MSEs and the standard deviations based on the inverse Hessian are reduced for this larger sample size, but there is still a clear tendency that the median standard deviations computed from the inverse Hessians underestimate the true degree of uncertainty predominantly for the policy parameters.

In Table B.3, we report results for the consistency properties of the ML estimator by increasing the sample size in each of the N samples to $T = 1600$ and $T = 6400$ observations. We report results for the case when we match all 15 variables in (19) , but also when we restrict the set of observables used in the estimation to the "closed economy" variables (eq. 21 although the underlying DGP is the open economy model). As this is a very time-consuming exercise, we only report results for $N = 40$ samples for $T = 1600$ observations, and $N = 20$ samples for $T = 6400$ observations. The optimizations are initiated by the prior mode values in Table A.2.

From Table B.3, we see that the marginal parameter distributions collapse at the true parameter values as $T = 6400$, but the standard errors indicate that the rate of convergence is substantially slower for many of the parameters when only the closed economy variables are matched in the estimations. Even so, the ML estimator actually appears to be consistent also for a relatively small set of variables, although it is clearly much more efficient to work with a larger set of variables in the estimations in smaller samples. Since the results for the subset of variables are derived for the open economy formulation of the model, they imply consistency in the closed economy specification of the model.

Table B.1: Sensitivity with respect to starting values (100 observations in each sample).

Parameter		True estimates	Starting from true values: Mean of distribution	Starting from true values: Median of distribution	Starting from true values: Std. of distribution	Sampling starting values from prior distribution: Mean of distribution	Sampling starting values from prior distribution: Median of distribution	Sampling starting values from prior distribution: Std. of distribution
Calvo wages	ξ_w	0.77	0.74	0.75	0.13	0.73	0.74	0.15
Calvo domestic prices	ξ_d	0.83	0.81	0.82	0.04	0.81	0.81	0.04
Calvo import cons. prices	$\xi_{m,c}$	0.90	0.90	0.90	0.02	0.90	0.90	0.02
Calvo import inv. prices	$\xi_{m,i}$	0.94	0.94	0.94	0.02	0.94	0.94	0.02
Calvo export prices	ξ_x	0.87	0.86	0.86	0.04	0.86	0.86	0.04
Indexation prices	κ	0.23	0.22	0.22	0.06	0.22	0.22	0.06
Indexation wages	κ_w	0.32	0.32	0.32	0.15	0.32	0.32	0.15
Investment adj. cost	$\tilde{S}''$	8.58	8.99	8.08	4.10	9.05	8.11	4.32
Habit formation	b	0.68	0.67	0.67	0.07	0.67	0.67	0.07
Markup domestic	λ_d	1.20	1.21	1.20	0.14	1.21	1.20	0.14
Subst. elasticity invest.	η_i	2.72	2.72	2.71	0.13	2.72	2.71	0.13
Subst. elasticity foreign	η_f	1.53	1.59	1.45	0.59	1.59	1.45	0.60
Markup imported cons.	$\lambda_{m,c}$	1.58	1.58	1.58	0.01	1.58	1.58	0.01
Markup.imported invest.	$\lambda_{m,i}$	1.13	1.14	1.13	0.02	1.14	1.13	0.02
Technology growth	μ_z	1.01	1.01	1.01	0.00	1.01	1.01	0.00
Risk premium	$\tilde{\phi}$	0.05	0.06	0.05	0.02	0.06	0.05	0.02
UIP modification	$\tilde{\phi}_s$	0.61	0.61	0.60	0.05	0.61	0.60	0.06
Unit root tech. persistance	ρ_{μ_z}	0.85	0.80	0.83	0.14	0.81	0.84	0.14
Stationary tech. persistance	ρ_ε	0.93	0.89	0.90	0.08	0.89	0.90	0.08
Invest. spec. tech. persist.	ρ_Υ	0.69	0.65	0.67	0.13	0.65	0.67	0.13
Risk premium persistence	$\rho_{\tilde{\phi}}$	0.68	0.65	0.66	0.11	0.65	0.66	0.11
Consumption pref. persist.	ρ_{ζ_c}	0.66	0.59	0.61	0.18	0.60	0.62	0.19
Labour supply persistance	ρ_{ζ_h}	0.27	0.26	0.26	0.13	0.27	0.26	0.16
Asymmetric tech. persist.	$\rho_{\tilde{z}^*}$	0.96	0.73	0.84	0.28	0.72	0.82	0.28
Unit root tech. shock	σ_{μ_z}	0.13	0.14	0.14	0.05	0.14	0.14	0.05
Stationary tech. shock	σ_ε	0.67	0.66	0.65	0.06	0.66	0.65	0.13
Imp. cons. markup shock	$\sigma_{\lambda_{m,c}}$	1.13	1.13	1.12	0.11	1.12	1.12	0.11
Imp. invest. markup shock	$\sigma_{\lambda_{m,i}}$	1.13	1.14	1.13	0.11	1.14	1.13	0.11
Domestic markup shock	σ_{λ_d}	0.81	0.82	0.82	0.08	0.82	0.82	0.11
Invest. spec. tech. shock	σ_Υ	0.40	0.42	0.41	0.09	0.42	0.41	0.09
Risk premium shock	$\sigma_{\tilde{\phi}}$	0.79	0.82	0.80	0.21	0.82	0.80	0.21
Consumption pref. shock	σ_{ζ_c}	0.26	0.27	0.27	0.05	0.27	0.27	0.07
Labour supply shock	σ_{ζ_h}	0.39	0.39	0.39	0.06	0.39	0.39	0.07
Asymmetric tech. shock	$\sigma_{\tilde{z}^*}$	0.19	0.15	0.16	0.06	0.15	0.16	0.06
Export markup shock	σ_{λ_x}	1.03	1.13	1.08	0.41	1.14	1.09	0.42
Monetary policy shock	σ_R	0.24	0.24	0.23	0.02	0.24	0.23	0.02
Inflation target shock	$\sigma_{\bar{\pi}^c}$	0.16	0.14	0.14	0.10	0.14	0.14	0.11
Interest rate smoothing	ρ_R	0.91	0.91	0.91	0.05	0.91	0.91	0.05
Inflation response	r_π	1.67	3.77	1.59	5.06	3.79	1.56	5.18
Diff. infl response	$r_{\Delta\pi}$	0.10	0.11	0.10	0.04	0.11	0.10	0.04
Real exch. rate response	r_x	-0.02	-0.07	-0.02	0.15	-0.07	-0.02	0.16
Output response	r_y	0.13	0.35	0.13	0.62	0.35	0.12	0.65
Diff. output response	$r_{\Delta y}$	0.18	0.19	0.18	0.05	0.19	0.18	0.05

Note: Out of the 1,500 estimations, the results above are based on 1,432 commonly convergent estimations.

Table B.2: Distribution results when increasing the sample size in the open economy model.

Parameter		True values	100 observations: Mean of distribution	100 observations: Median of distribution	100 observations: Std. of distribution	100 observations: Std based on Inverse Hessian	400 observations: Mean of distribution	400 observations: Median of distribution	400 observations: Std. of distribution	400 observations: Std based on Inverse Hessian
Calvo wages	ξ_w	0.77	0.74	0.75	0.13	0.07	0.76	0.76	0.05	0.03
Calvo domestic prices	ξ_d	0.83	0.81	0.82	0.04	0.03	0.82	0.82	0.02	0.01
Calvo import cons. prices	$\xi_{m,c}$	0.90	0.90	0.90	0.02	0.01	0.90	0.90	0.01	0.01
Calvo import inv. prices	$\xi_{m,i}$	0.94	0.94	0.94	0.02	0.01	0.94	0.94	0.01	0.01
Calvo export prices	ξ_x	0.87	0.86	0.86	0.04	0.02	0.87	0.87	0.01	0.01
Indexation prices	κ	0.23	0.22	0.22	0.06	0.05	0.22	0.22	0.03	0.02
Indexation wages	κ_w	0.32	0.32	0.32	0.15	0.07	0.32	0.32	0.07	0.04
Investment adj. cost	$\tilde{S}''$	8.58	8.98	8.08	4.08	2.02	8.65	8.53	1.35	0.98
Habit formation	b	0.68	0.67	0.67	0.07	0.05	0.68	0.68	0.03	0.02
Markup domestic	λ_d	1.20	1.21	1.20	0.14	0.09	1.20	1.19	0.06	0.04
Subst. elasticity invest.	η_i	2.72	2.72	2.71	0.13	0.11	2.71	2.71	0.06	0.05
Subst. elasticity foreign	η_f	1.53	1.59	1.45	0.59	0.23	1.54	1.53	0.14	0.09
Markup imported cons.	$\lambda_{m,c}$	1.58	1.58	1.58	0.01	0.01	1.58	1.58	0.00	0.00
Markup.imported invest.	$\lambda_{m,i}$	1.13	1.14	1.13	0.02	0.02	1.13	1.13	0.01	0.01
Technology growth	μ_z	1.005	1.005	1.005	0.0003	0.00	1.005	1.005	0.0001	0.00
Risk premium	$\tilde{\phi}$	0.05	0.06	0.05	0.02	0.01	0.05	0.05	0.01	0.00
UIP modification	$\tilde{\phi}_s$	0.61	0.61	0.60	0.05	0.03	0.61	0.61	0.02	0.01
Unit root tech. persistance	ρ_{μ_z}	0.85	0.80	0.83	0.14	0.06	0.84	0.85	0.05	0.03
Stationary tech. persistance	ρ_ε	0.93	0.89	0.90	0.08	0.03	0.92	0.92	0.02	0.01
Invest. spec. tech. persist.	ρ_Υ	0.69	0.65	0.67	0.13	0.06	0.69	0.69	0.05	0.03
Risk premium persistence	$\rho_{\tilde{\phi}}$	0.68	0.65	0.65	0.11	0.06	0.68	0.68	0.04	0.03
Consumption pref. persist.	ρ_{ζ_c}	0.66	0.59	0.61	0.18	0.08	0.64	0.65	0.07	0.04
Labour supply persistance	ρ_{ζ_h}	0.27	0.26	0.26	0.13	0.07	0.27	0.27	0.06	0.04
Asymmetric tech. persist.	$\rho_{\tilde{z}*}$	0.96	0.73	0.84	0.28	0.09	0.93	0.95	0.11	0.02
Unit root tech. shock	σ_{μ_z}	0.13	0.14	0.14	0.05	0.03	0.13	0.13	0.02	0.01
Stationary tech. shock	σ_ε	0.67	0.66	0.65	0.06	0.05	0.67	0.67	0.03	0.03
Imp. cons. markup shock	$\sigma_{\lambda_{m,c}}$	1.13	1.13	1.12	0.11	0.10	1.13	1.13	0.05	0.05
Imp. invest. markup shock	$\sigma_{\lambda_{m,i}}$	1.13	1.14	1.13	0.11	0.10	1.14	1.14	0.05	0.05
Domestic markup shock	σ_{λ_d}	0.81	0.82	0.82	0.08	0.08	0.81	0.81	0.04	0.04
Invest. spec. tech. shock	σ_Υ	0.40	0.42	0.41	0.09	0.06	0.40	0.40	0.03	0.02
Risk premium shock	$\sigma_{\tilde{\phi}}$	0.79	0.82	0.80	0.21	0.12	0.80	0.80	0.08	0.06
Consumption pref. shock	σ_{ζ_c}	0.26	0.27	0.27	0.05	0.04	0.27	0.26	0.02	0.02
Labour supply shock	σ_{ζ_h}	0.39	0.39	0.39	0.06	0.04	0.38	0.38	0.03	0.02
Asymmetric tech. shock	$\sigma_{\tilde{z}*}$	0.19	0.15	0.16	0.06	0.04	0.18	0.19	0.02	0.02
Export markup shock	σ_{λ_x}	1.03	1.13	1.09	0.41	0.21	1.04	1.03	0.11	0.08
Monetary policy shock	σ_R	0.24	0.24	0.23	0.02	0.02	0.24	0.24	0.01	0.01
Inflation target shock	$\sigma_{\bar{\pi}^c}$	0.16	0.14	0.14	0.10	0.04	0.16	0.16	0.03	0.02
Interest rate smoothing	ρ_R	0.91	0.91	0.91	0.05	0.03	0.91	0.91	0.02	0.02
Inflation response	r_π	1.67	3.80	1.59	5.08	2.70	2.07	1.66	1.60	0.61
Diff. infl response	$r_{\Delta\pi}$	0.10	0.11	0.10	0.04	0.03	0.10	0.10	0.02	0.01
Real exch. rate response	r_x	-0.02	-0.07	-0.02	0.15	0.02	-0.03	-0.02	0.04	0.01
Output response	r_y	0.13	0.35	0.13	0.63	0.07	0.17	0.13	0.17	0.04
Diff. output response	$r_{\Delta y}$	0.18	0.19	0.18	0.05	0.03	0.18	0.18	0.02	0.02

Note: Out of the 1,500 estimations for the small sample (100 obs.), the results above is based on 1,452 convergent estimations (defined as estimations when the optimizer CSMINWEL terminates without an error message and when the inverse Hessian has full rank and is positive definite). Out of the 1,500 estimations for the large sample (400 obs.), the results above is based on 1,497 convergent estimations. True parameter values were used as starting values in the estimations. Std based on Inverse Hessian shows the median of these estimations.

Table B.3: Distribution results for large sample sizes, matching two sets of variables.

Parameter		True estimates	1,600 observations				6,400 observations			
			All variables		Closed variables		All variables		Closed variables	
			Mean of distribution	Std. of distribution	Mean of distribution	Std. of distribution	Mean of distribution	Std. of distribution	Mean of distribution	Std. of distribution
Calvo wages	ξ_w	0.77	0.80	0.06	0.77	0.02	0.76	0.01	0.76	0.01
Calvo domestic prices	ξ_d	0.83	0.83	0.01	0.83	0.01	0.83	0.00	0.82	0.00
Calvo import cons. prices	$\xi_{m,c}$	0.90	0.90	0.01	0.90	0.03	0.90	0.00	0.90	0.02
Calvo import inv. prices	$\xi_{m,i}$	0.94	0.94	0.01	0.94	0.01	0.94	0.00	0.94	0.00
Calvo export prices	ξ_x	0.87	0.86	0.03	0.87	0.04	0.87	0.00	0.87	0.02
Indexation prices	κ	0.23	0.24	0.03	0.23	0.03	0.23	0.01	0.23	0.01
Indexation wages	κ_w	0.32	0.33	0.06	0.32	0.03	0.32	0.02	0.32	0.02
Investment adj. cost	$\tilde{S}''$	8.58	9.09	1.95	8.90	1.15	8.58	0.30	8.69	0.58
Habit formation	b	0.68	0.68	0.02	0.68	0.02	0.68	0.01	0.68	0.01
Markup domestic	λ_d	1.20	1.25	0.10	1.20	0.03	1.20	0.01	1.20	0.02
Subst. elasticity invest.	η_i	2.72	2.72	0.03	2.74	0.76	2.71	0.01	2.66	0.46
Subst. elasticity foreign	η_f	1.53	1.47	0.17	1.60	0.46	1.53	0.01	1.57	0.31
Markup imported cons.	$\lambda_{m,c}$	1.58	1.58	0.00	1.60	0.12	1.58	0.00	1.58	0.06
Markup.imported invest.	$\lambda_{m,i}$	1.13	1.13	0.01	1.16	0.10	1.13	0.00	1.14	0.04
Technology growth	μ_z	1.01	1.01	0.00	1.01	0.00	1.01	0.00	1.01	0.00
Risk premium	$\tilde{\phi}$	0.05	0.05	0.01	0.05	0.01	0.05	0.00	0.05	0.01
UIP modification	$\tilde{\phi}_s$	0.61	0.54	0.15	0.62	0.04	0.61	0.00	0.61	0.02
Unit root tech. persistance	ρ_{μ_z}	0.85	0.82	0.08	0.85	0.04	0.85	0.01	0.85	0.02
Stationary tech. persistance	ρ_{ε}	0.93	0.92	0.02	0.93	0.01	0.92	0.00	0.92	0.01
Invest. spec. tech. persist.	ρ_{Υ}	0.69	0.69	0.04	0.70	0.03	0.70	0.01	0.70	0.01
Risk premium persistence	$\rho_{\tilde{\phi}}$	0.68	0.74	0.10	0.46	0.33	0.68	0.01	0.56	0.24
Consumption pref. persist.	ρ_{ζ_c}	0.66	0.68	0.05	0.66	0.04	0.66	0.01	0.65	0.02
Labour supply persistance	ρ_{ζ_h}	0.27	0.27	0.05	0.27	0.02	0.27	0.02	0.27	0.02
Asymmetric tech. persist.	$\rho_{\tilde{z}^*}$	0.96	0.98	0.02	0.79	0.12	0.96	0.00	0.85	0.06
Unit root tech. shock	σ_{μ_z}	0.13	0.14	0.02	0.13	0.03	0.13	0.00	0.13	0.01
Stationary tech. shock	σ_{ε}	0.67	0.67	0.01	0.67	0.01	0.67	0.01	0.67	0.01
Imp. cons. markup shock	$\sigma_{\lambda_{m,c}}$	1.13	1.13	0.03	1.17	0.37	1.12	0.01	1.13	0.19
Imp. invest. markup shock	$\sigma_{\lambda_{m,i}}$	1.13	1.14	0.04	1.06	0.36	1.13	0.01	1.12	0.13
Domestic markup shock	σ_{λ_d}	0.81	0.80	0.02	0.80	0.02	0.81	0.01	0.80	0.01
Invest. spec. tech. shock	σ_{Υ}	0.40	0.40	0.04	0.39	0.02	0.39	0.01	0.39	0.01
Risk premium shock	$\sigma_{\tilde{\phi}}$	0.79	0.72	0.15	1.92	1.52	0.79	0.02	1.31	0.98
Consumption pref. shock	σ_{ζ_c}	0.26	0.26	0.01	0.26	0.01	0.26	0.01	0.26	0.01
Labour supply shock	σ_{ζ_h}	0.39	0.38	0.03	0.38	0.01	0.39	0.01	0.39	0.01
Asymmetric tech. shock	$\sigma_{\tilde{z}^*}$	0.19	0.18	0.02	0.56	0.50	0.19	0.00	0.41	0.31
Export markup shock	σ_{λ_x}	1.03	1.12	0.24	0.81	0.47	1.03	0.01	0.97	0.24
Monetary policy shock	σ_R	0.24	0.24	0.01	0.24	0.01	0.24	0.00	0.24	0.00
Inflation target shock	$\sigma_{\bar{\pi}^c}$	0.16	0.16	0.03	0.16	0.02	0.16	0.00	0.16	0.01
Interest rate smoothing	ρ_R	0.91	0.90	0.03	0.91	0.02	0.91	0.01	0.91	0.01
Inflation response	r_{π}	1.67	1.49	0.35	1.83	1.44	1.67	0.15	1.69	0.17
Diff. infl response	$r_{\Delta\pi}$	0.10	0.09	0.01	0.09	0.01	0.10	0.01	0.10	0.01
Real exch. rate response	r_x	-0.02	-0.01	0.01	-0.02	0.04	-0.02	0.00	-0.02	0.01
Output response	r_y	0.13	0.10	0.05	0.16	0.23	0.13	0.02	0.13	0.02
Diff. output response	$r_{\Delta y}$	0.18	0.17	0.01	0.17	0.01	0.18	0.01	0.18	0.01

Note: The results above are based on 35 convergent estimations with 1,600 observations in each sample, and 20 convergent estimations with 6,400 observations in each sample. The optimizations are initialized by the prior mode values in Table 2.

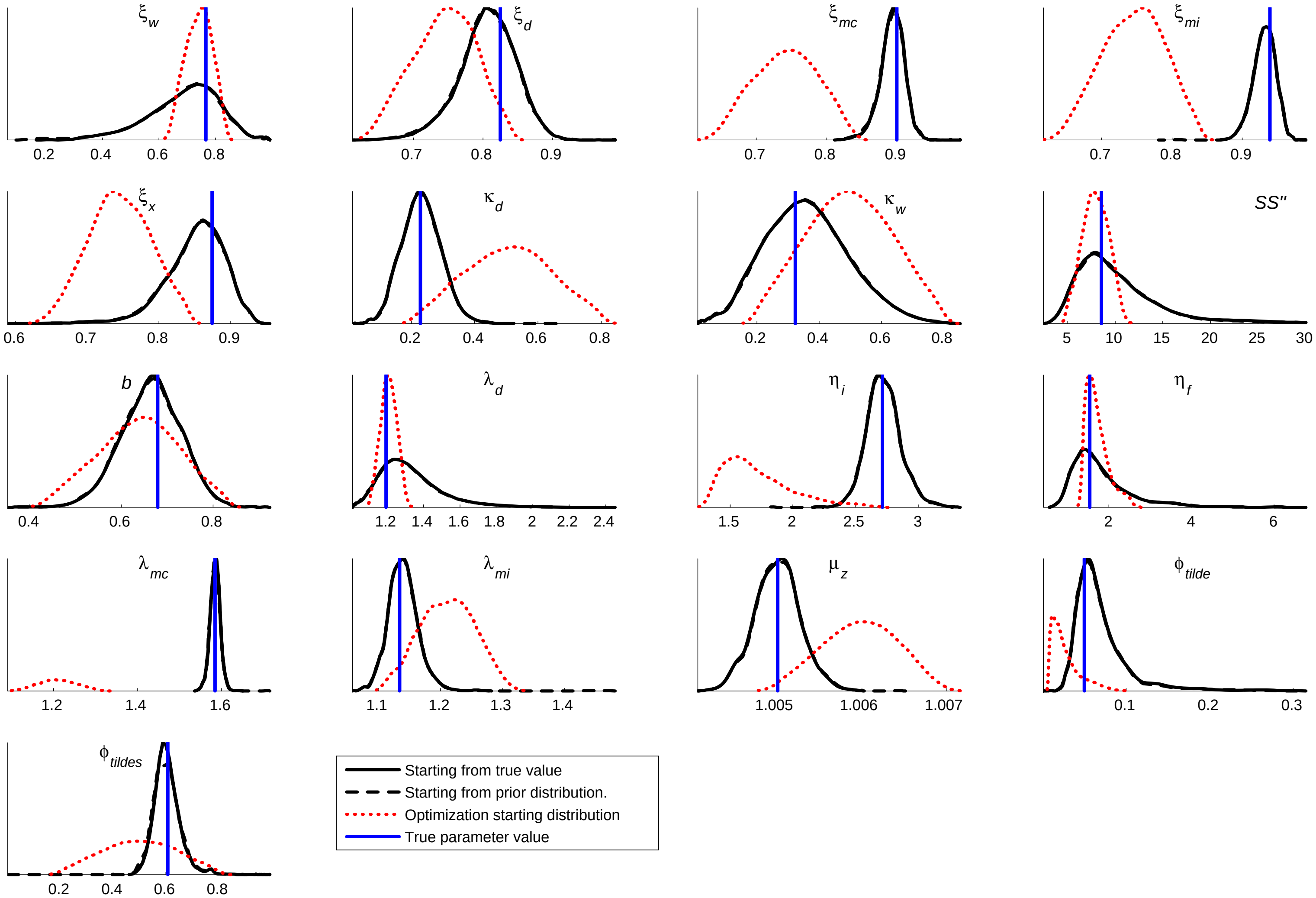

Figure B.1a: Kernel density estimates of the small sample distribution for the estimates of the deep model parameters. The solid line shows the parameter distribution when initializing the estimations with the true parameters (vertical bars), and the dashed lines show the distribution when initializing the estimations using a sample from the prior (dotted line). $T = 100$ observations in each of the N artificial samples.

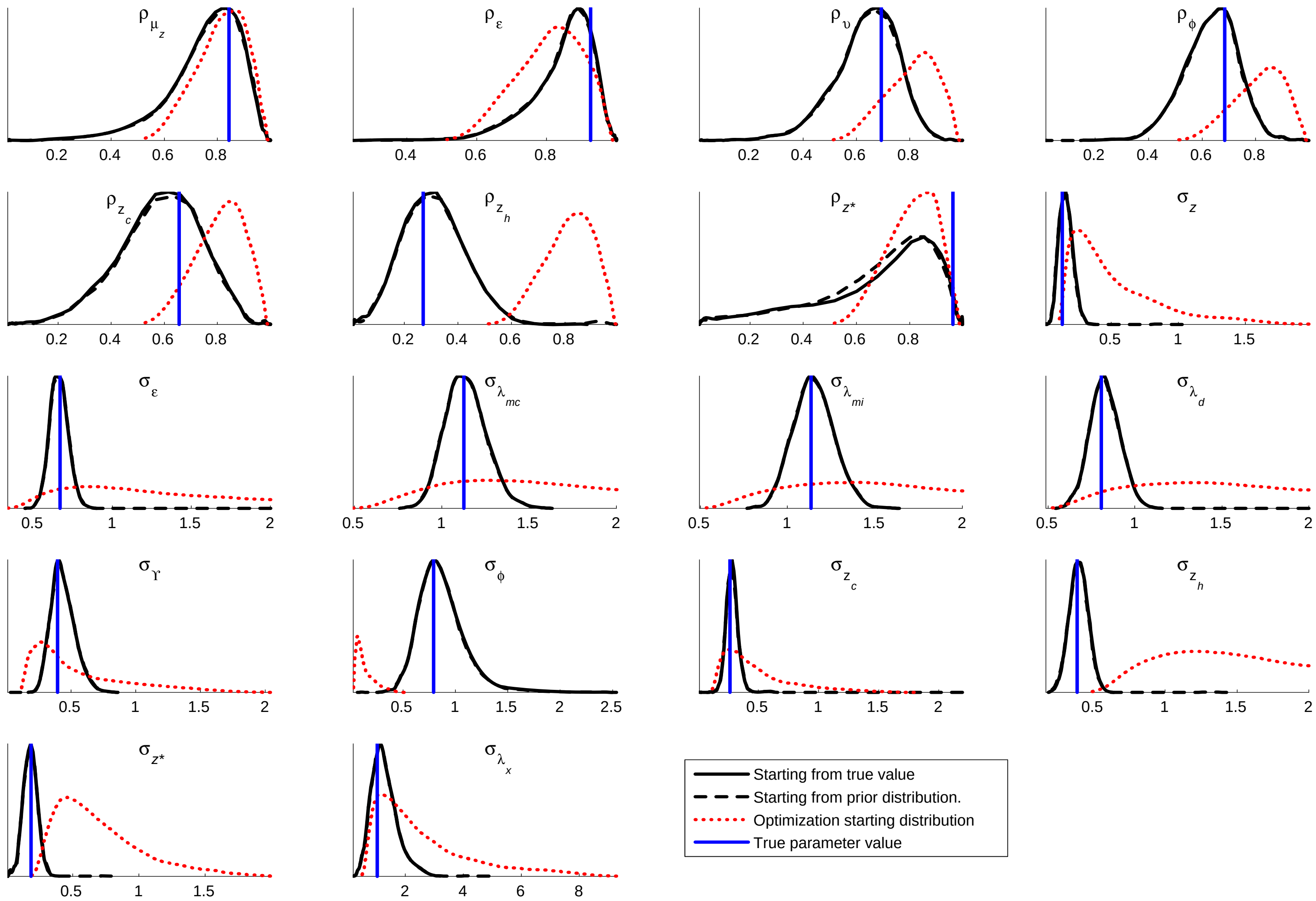

Figure B.1b: Kernel density estimates of the small sample distribution for the estimates of the shock process parameters. The solid line shows the parameter distribution when initializing the estimations with the true parameters (vertical bars), and the dashed lines show the distribution when initializing the estimations using a sample from the prior (dotted line). $T = 100$ observations in each of the N artificial samples.

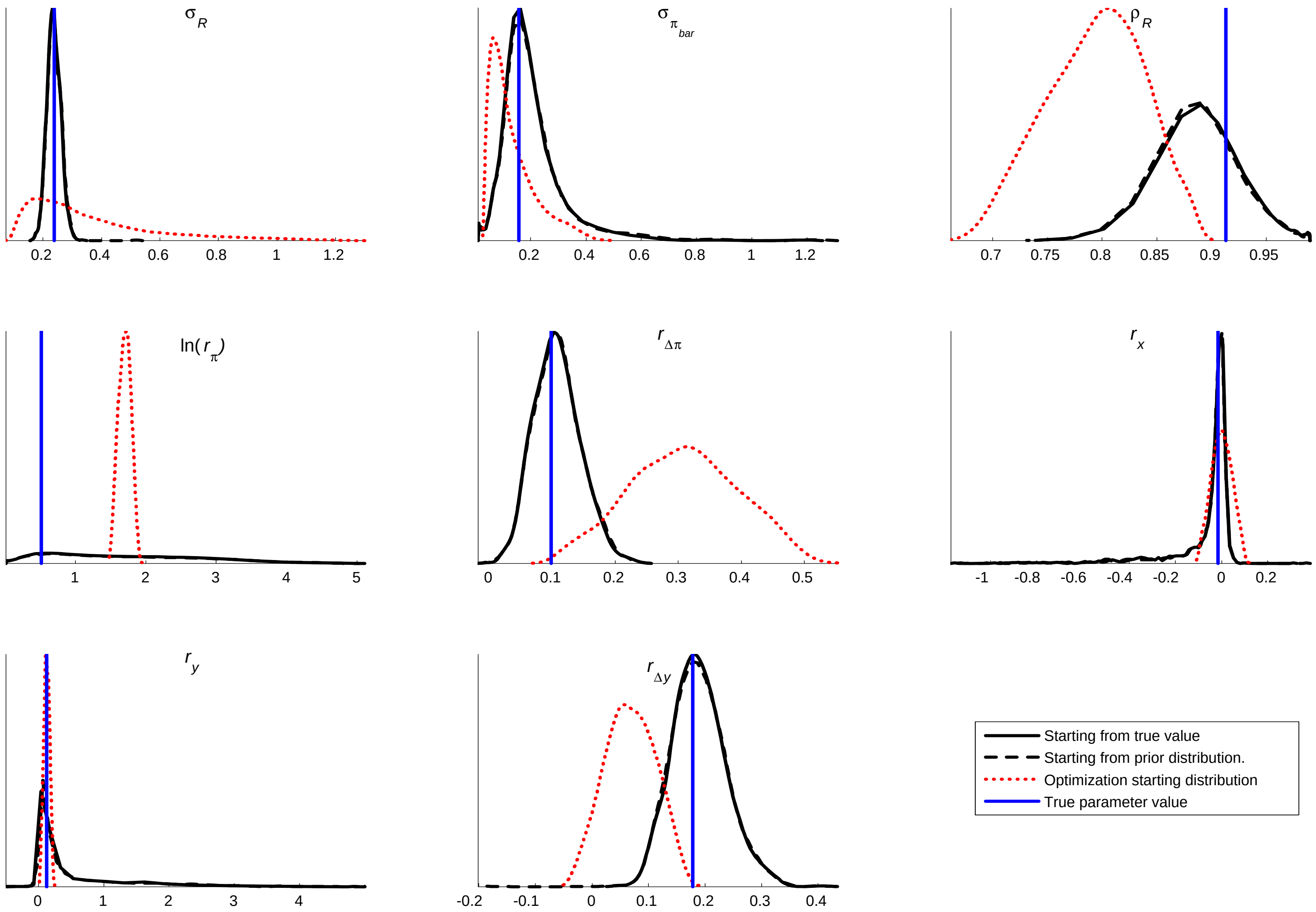

Figure B.1c: Kernel density estimates of the small sample distribution for the estimates of the policy rule parameters. The solid line shows the parameter distribution when initializing the estimations with the true parameters (vertical bars), and the dashed lines show the distribution when initializing the estimations using a sample from the prior (dotted line). $T = 100$ observations in each of the N artificial samples.

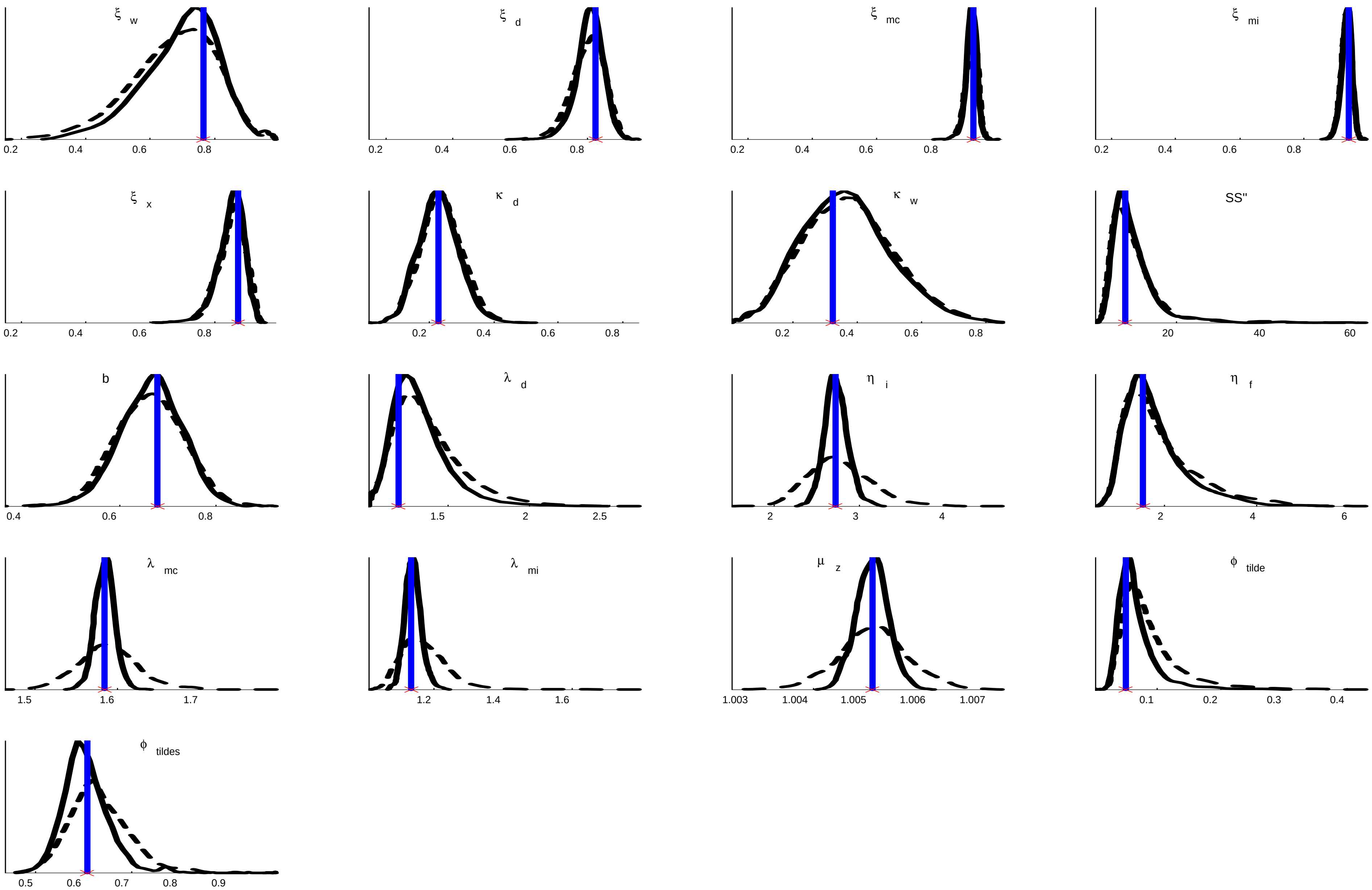

Figure B.2a: Kernel density estimates of the small sample distribution for the estimates of the deep model parameters. The solid line shows the parameter distribution when estimating the model without measurement errors and keeping the foreign and fiscal VAR models fixed, and the dashed line shows the distribution when estimating the model with measurement errors and reestimating the foreign and fiscal VAR models for each sample. The vertical bar shows true parameter value, and the cross on the x-axis denotes the starting value in the optimizations. $T = 100$ observations in each of the N artificial samples.

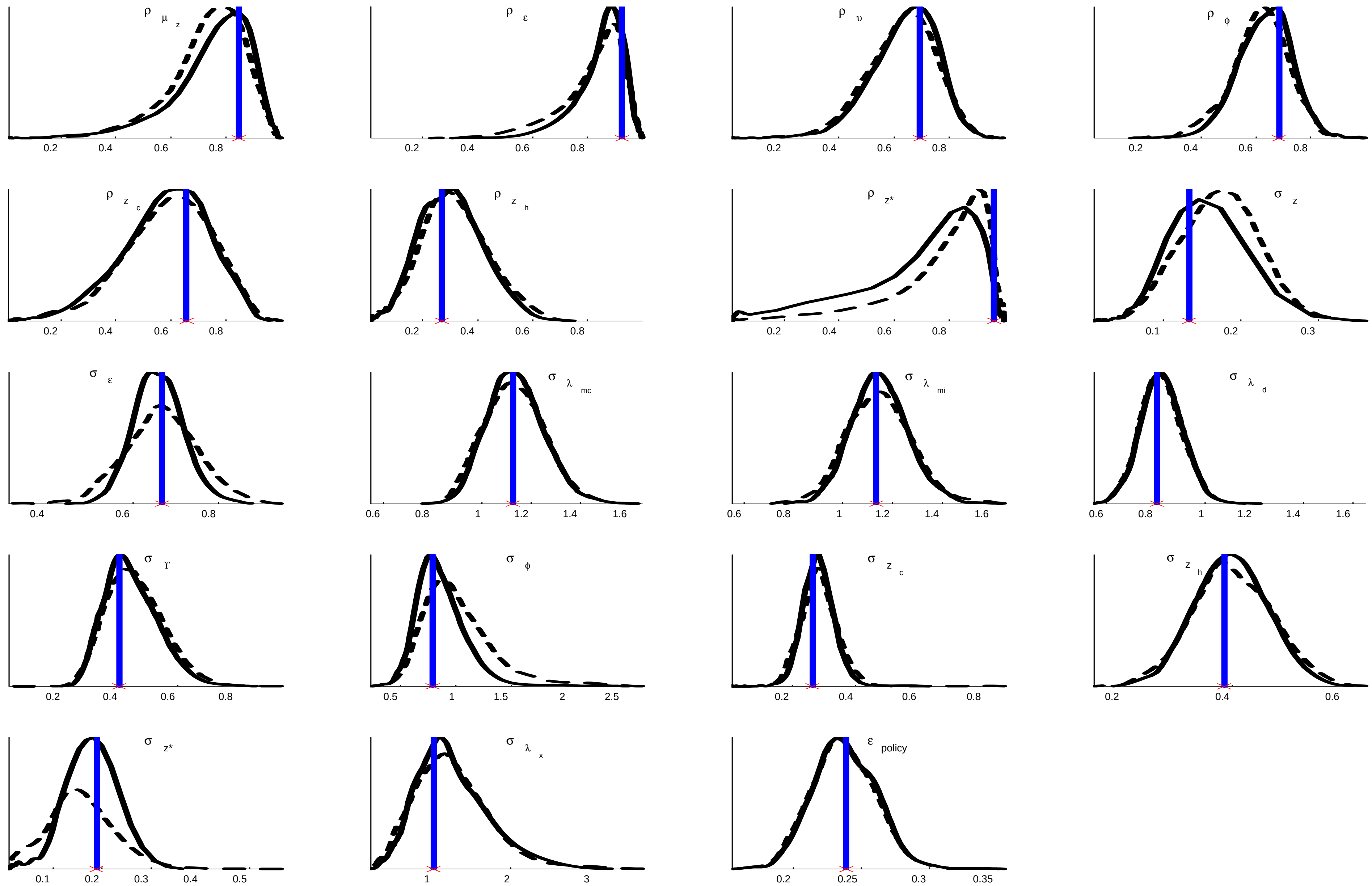

Figure B.2b: Kernel density estimates of the small sample distribution for the estimates of the shock parameters. The solid line shows the parameter distribution when estimating the model without measurement errors and keeping the foreign and fiscal VAR models fixed, and the dashed line shows the distribution when estimating the model with measurement errors and reestimating the foreign and fiscal VAR models for each sample. The vertical bar shows true parameter value, and the cross on the x-axis denotes the starting value in the optimizations. $T = 100$ observations in each of the N artificial samples.

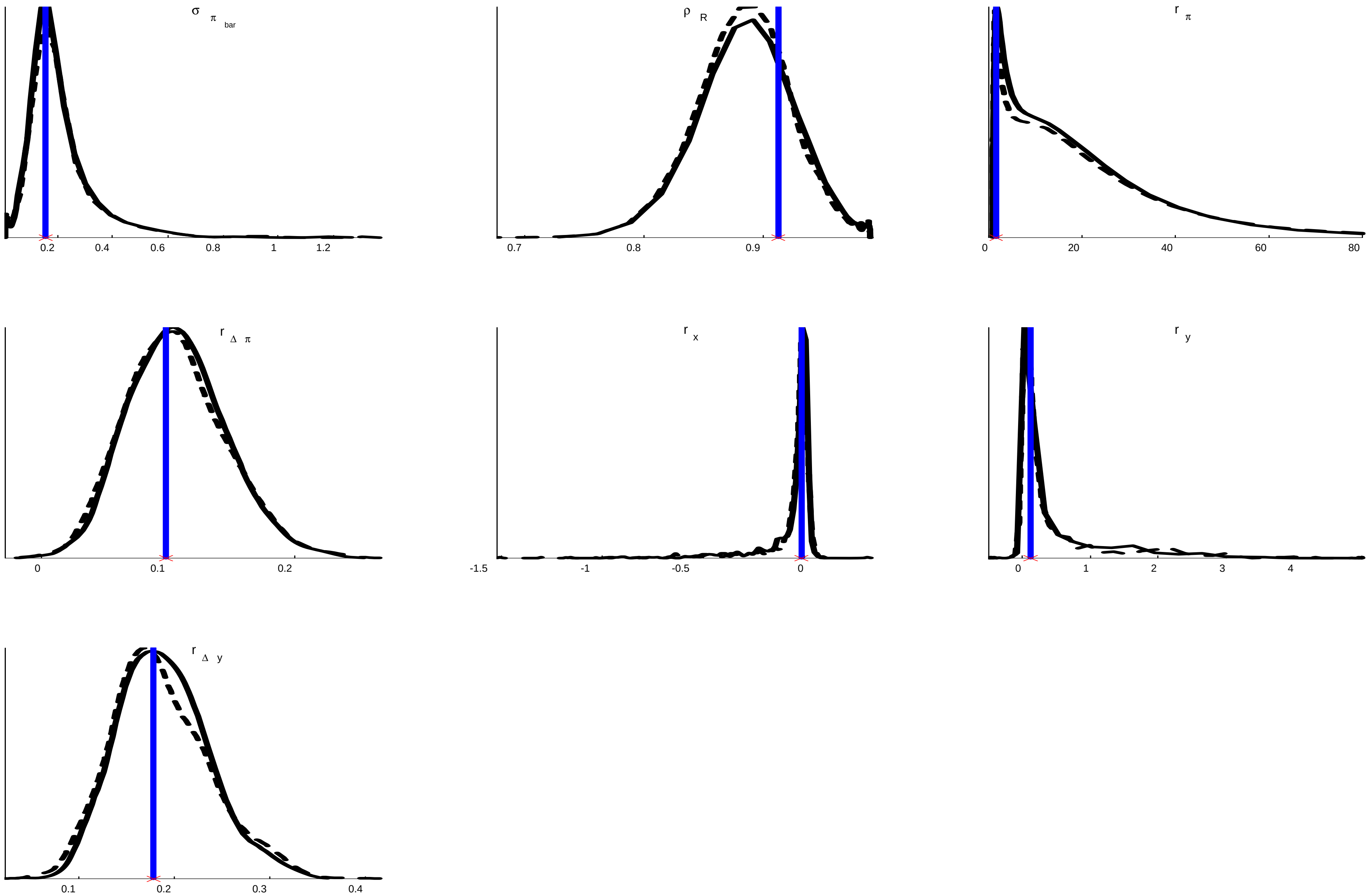

Figure B.2c: Kernel density estimates of the small sample distribution for the estimates of the policy parameters. The solid line shows the parameter distribution when estimating the model without measurement errors and keeping the foreign and fiscal VAR models fixed, and the dashed line shows the distribution when estimating the model with measurement errors and reestimating the foreign and fiscal VAR models for each sample. The vertical bar shows true parameter value, and the cross on the x-axis denotes the starting value in the optimizations. $T = 100$ observations in each of the N artificial samples.

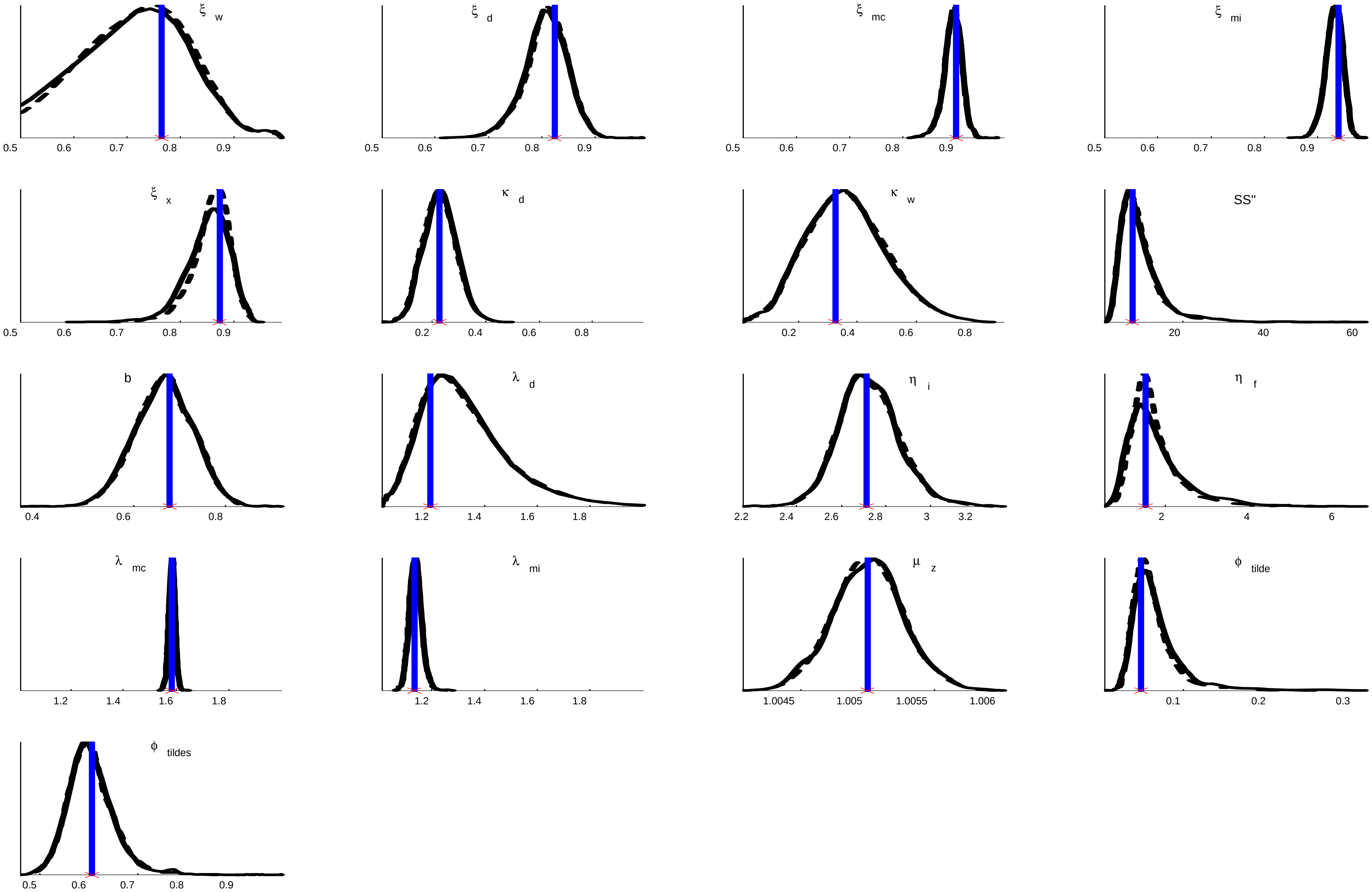

Figure B.3a: Kernel density estimates of the small sample distribution for the estimates of the deep model parameters. The solid line shows the parameter distribution when estimating the model in first differences, and the dashed line shows the distribution when estimating the model using the true cointegrating vectors. The vertical bar shows true parameter value, and the cross on the x-axis denotes the starting value in the optimizations. $T = 100$ observations in each of the N artificial samples.

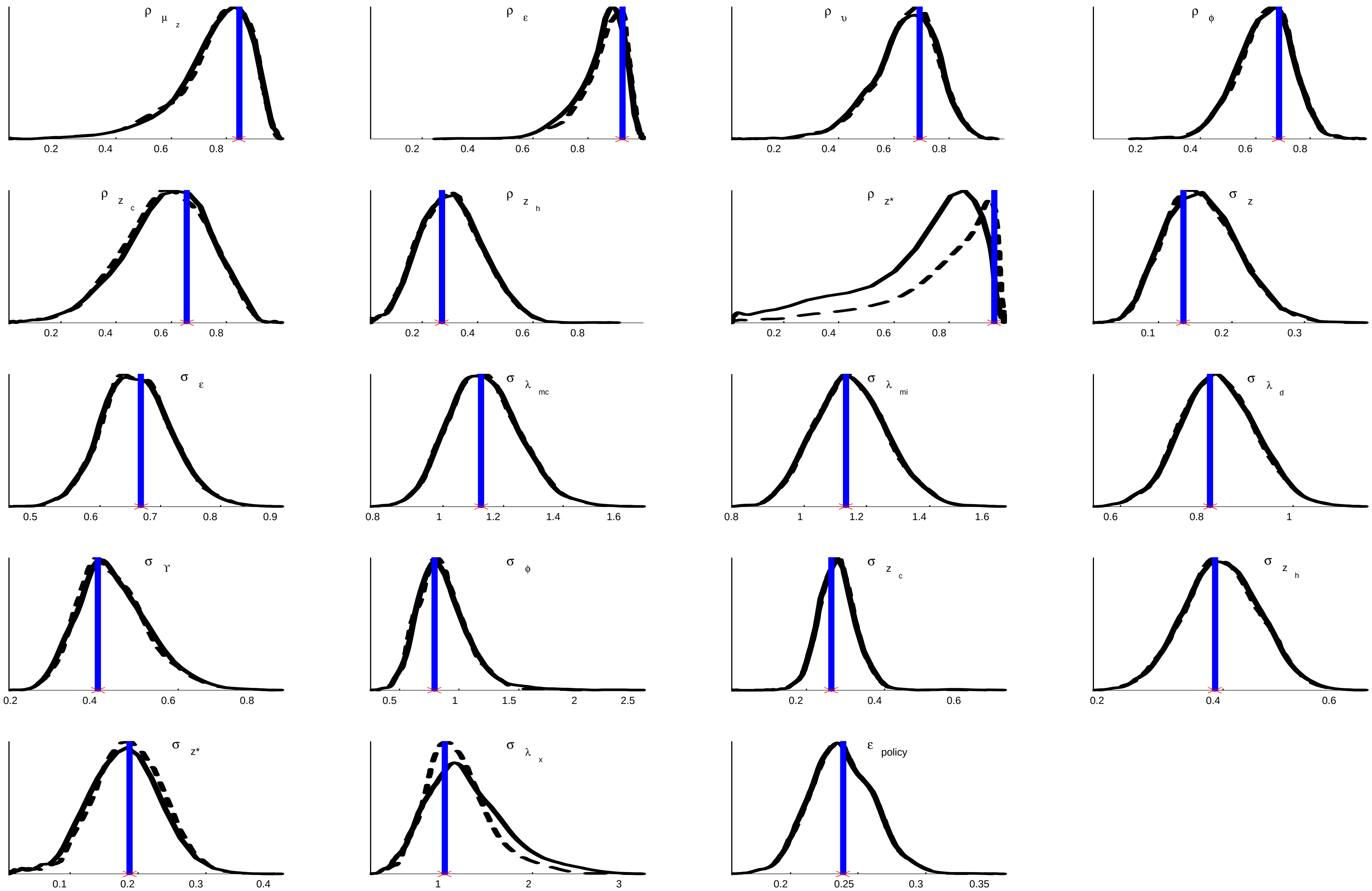

Figure B.3b: Kernel density estimates of the small sample distribution for the estimates of the shock parameters. The solid line shows the parameter distribution when estimating the model in first differences, and the dashed line shows the distribution when estimating the model using the true cointegrating vectors. The vertical bar shows true parameter value, and the cross on the x-axis denotes the starting value in the optimizations. $T = 100$ observations in each of the N artificial samples.

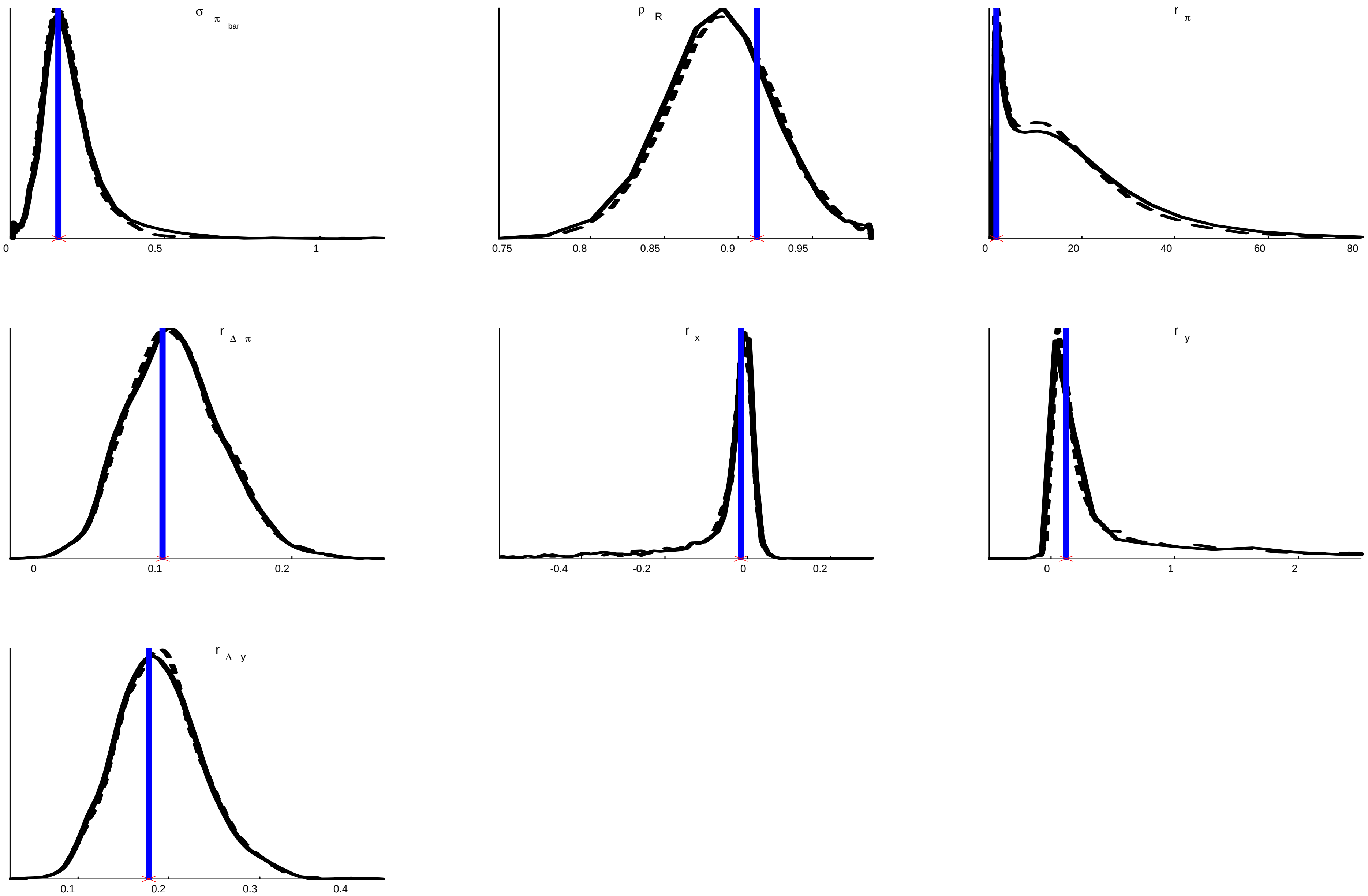

Figure B.3c: Kernel density estimates of the small sample distribution for the estimates of the monetary policy parameters. The solid line shows the parameter distribution when estimating the model in first differences, and the dashed line shows the distribution when estimating the model using the true cointegrating vectors. The vertical bar shows true parameter value, and the cross on the x-axis denotes the starting value in the optimizations. $T = 100$ observations in each of the N artificial samples.

Appendix C. Further tests for weak identification

Recent papers that have developed tools to study identification in DSGE models include Andrle (2010), Consolo, Favero and Paccagnini (2009), Guerron-Quintana, Inoue and Kilian (2013), Koop, Pesaran and Smith (2011), Komunjer and Ng (2011), Müller (2013), Qu (2014) and Andrews and Mikusheva (2015). Andrle (2010) and Komunjer and Ng (2011) are based on the same asymptotic theory as Iskrev (2010a), and hence their results would essentially reproduce ours. Müller (2013) points out that the 'sandwich' estimator is more robust in providing uncertainty of the estimated parameters in the case of misspecification. Consolo, Favero and Paccagnini (2009) use approximations of the original DSGE model (in the form of VAR models) to assess DSGE models and show that the factor-augmented version FAVAR provides a more correct tool for assessment. Our tests and the others discussed hereafter use, instead, the exact DSGE formulation for testing weak identification. Andrews and Mikusheva (2015) make a thorough synthesis of methods that are robust to weak identification, which encompasses Koop, Pesaran and Smith (2011), Guerron-Quintana, Inoue and Kilian (2013) and Qu (2014). Very shortly, this paper discusses ML inference that is robust to weak identification, based on the martingale theory and Lagrange multiplier tests. Although the paper does do not provide formal tests for weak identification, it also suggest an 'informal' test for weak identification, which analyses the positive definiteness of the Hessian of the true parameter vector θ when computed on short sample simulated data of length T. The idea is that non-positive definiteness can point to weak identification, since it is linked to the residual measure $A_T(\theta) = J_T(\theta) - I_T(\theta)$, which is the difference between two estimators of the observed information matrix based on outer product gradient ($J_T(\theta)$, i.e. the incremental observed information) and Hessian ($I_T(\theta)$, i.e. the observed information). Andrews and Mikusheva (2015) show that while the expectation $E[A_T(\theta)]$ is zero also for weak identification, in the latter case its variance may be still large even at large sample size, so that the convergence of $A_T(\theta)$ to zero will be extremely slow. Essentially the weak identification issue is linked to the relative volatility of $A_T(\theta)$ w.r.t. $J_T(\theta)$, the latter estimator being positive definite by definition. In the paper it is shown than in all the weakly identified models analysed, there is a large probability that $I_T(\theta)$ is non positive-definite. The 'informal' test therefore consists of performing several model simulation replicas of length T for a given parameter set θ and computing $I_T(\theta)$ for each simulation replica. Then, one computes the percentage of $I_T(\theta)$ matrices that is not positive-definite. The higher this percentage, the more this will be an indicator of possible weak identification. It seems useful to add here that non-positive definiteness of the Hessian $I_T(\theta)$ may also be linked to the underlying non-linearity of the mapping between θ and the reduced form solution. This non-linearity is inherent to the DSGE model formulation, and may imply deviations from classical asymptotic theory in terms of bounded uncertainty of estimates. Although Iskrev's identification strength test can be taken as a valid indicator for ranking the strength across estimated parameters and point to those more prone to weak identification, we summarize here the results of the informal test suggested by Andrews and Mikusheva (2015) to complement Iskrev (2010a). We applied this test for both the closed and the open economy versions of the model. We performed replicas of simulations of the model for length $T = 100$ periods (corresponding to the length of the actual sample size used in practical estimation) and we obtained that for about 45% of the simulation there was at least one negative eigenvalue in $I_T(\theta)$, and for about 15% of replicas there were 2 negative eigenvalues, while we never found more that two non-positive eigenvalues. We repeated this exercise for increasing simulation length: $T = 250$ and $T = 400$. The share of non-positive eigenvalues falls, for $T = 250$ ($= 400$), to about 30% (14%) for one non-positive and to 5% (1%) for two non-positive eigenvalues. We also note here that, with respect to the (simpler) example reported in Andrews and Mikusheva (2015), the share of non-positive defined Hessian's is smaller [they report almost 50% cases for $T = 200$], in spite our model is larger w.r.t. the ones discussed there. These results also show that there is indeed a convergence path, in which the share of non-positive definite matrices $I_T(\theta)$ tends to vanish at a rate $1/T^{\alpha}$ with $\alpha in (0.5, 1)$. This is suggestive that the model is not largely affected by the weak identification concerns discussed in Andrews and Mikusheva (2015), which, in contrast to the model(s) discussed here, should persist also at very large sample sizes. Applying the same methodology to the closed economy version of the model, we get 65% (6%) simulations with at least one (two) negative eigenvalue for $T = 100$. Those probabilities fall to 45% (2%) for $T = 250$ and to 22% (0%) for $T = 400$. The closed economy version of the model has therefore a larger probability of weakly identified parameters, but it features a similar (or even larger) rate of convergence. Hence, also for the less rich closed economy version of the model we can conclude that there is still a lot to learn from estimation. These results confirm what we already concluded in the core of our paper. There is indeed some degree of weak identification in the estimation of the model: there are a couple of directions in the parameter space where the likelihood shape will be quite flat. However, although a few parameters may be poorly identified according to the identification tests, the simulation results in Section 4.2 for MLE estimations with $T = 100$ suggest that we can learn a lot from the data about many parameters. This is not in contrast with Andrews and Mikusheva (2015), who also note, discussing their results, that "We can see that while the confidence intervals for many parameters are wide, in all instances they exclude some values and in most cases they cover only a small portion of the parameter space". In conclusion, applying more tests to address weak identification is indeed interesting and helps in better understanding the issue for DSGE models. However, the performance of these tests does not seem to change the main findings of our paper.