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Massimiliano Marcellino, Andrea Carriero and Francesco Corsello

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Centre for Economic Policy Research
33 Great Sutton Street, London EC1V 0DX, UK
Tel: +44 (0)20 7183 8801
www.cepr.org

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Abstract

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JEL Classification: E31, F62, C32, E37, C53

Keywords: inflation, volatility, Global factors, large datasets, Multivariate Autoregressive Index models, Reduced Rank Regressions, Forecasting

Massimiliano Marcellino - massimiliano.marcellino@unibocconi.it
Bocconi University and CEPR

Andrea Carriero - a.carriero@qmul.ac.uk
Queen Mary College, London

Francesco Corsello - francesco.corsello@bancaditalia.it
Banca d'Italia

The Global Component of Inflation Volatility*

Andrea Carriero[†1], Francesco Corsello[‡2], and Massimiliano Marcellino[§3]

[1]Queen Mary, University of London
[2]Bank of Italy
[3]Bocconi University, IGIER and CEPR

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Abstract

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†a.carriero@qmul.ac.uk
‡francesco.corsello@bancaditalia.it
§massimiliano.marcellino@unibocconi.it

1 Introduction

Global developments play an important role in the determination of inflation rates. Papers such as Borio & Filardo (2007) and Ciccarelli & Mojon (2010) find that a substantial amount of variation in a large set of national inflation rates can be explained by global factors. Quoting Borio & Filardo (2007): "*...proxies for global economic slack add considerable explanatory power to traditional benchmark inflation rate equations, even allowing for the influence of traditional indicators of external influences on domestic inflation, such as import and Oil prices. Moreover, the role of such global factors has been growing over time, especially since the 1990s. And in a number of cases, global factors appear to have supplanted the role of domestic measures of economic slack.*" This evidence has been recently challenged by Lodge & Mikolajun (2016), whose results suggest that the relevance of global factors for forecasting domestic inflation is related to their ability to capture slow-moving trends, like those emphasized by Stock & Watson (2007) in their decomposition of US inflation into trend and cyclical components. Other empirical contributions, as Bianchi & Civelli (2015) and Auer, Borio & Filardo (2017), show that financial openness and Global Value Chains are positively related to the effects of global slack on inflation. We do not take an a priori stance on this point, but we will use an econometric model where the relative contribution of global and country-specific factors as drivers of inflation developments is estimated and can vary over time and across countries.

Another point stressed by Stock & Watson (2007), which however dates back to at least Engle (1982), is the importance of allowing for conditional time-varying volatility when modelling inflation. While Engle introduced the ARCH specification as a model for inflation volatility, Stock & Watson (2007) used stochastic volatility, which is indeed more common in macroeconomics applications and more flexible since it permits to have different shocks as drivers of the level and volatility of an economic variable. Stock and Watson found that the introduction of SV improves the out of sample forecasting power of their model for US inflation, and it is preferable also to alternative methods to allow for heterosckedasticity, such as rolling estimation or Markov switching models. Inflation volatility is also relevant for policy making as, for example, in periods of high volatility it is more difficult to understand whether inflation movements are temporary or persistent.

Volatility needs to be modeled properly in multi-country studies on inflation determinants. In particular, it seems important to understand whether and to what extent the cross-country commonality among inflation levels is also present among inflation volatilities. Furthermore, recent macro-financial literature has considered stochastic volatility as a basis to construct measures of macro and financial uncertainty (see Jurado, Ludvigson

& Ng, 2015, and Carriero, Clark & Marcellino, 2017). From this perspective, it may be important for a policymaker to disentangle whether inflation uncertainty originates locally or globally.

Mumtaz & Surico (2008) investigate co-movements in an unbalanced panel of inflation rates from the 1970s to early 2000s for 11 countries, using a large dynamic factor model that incorporates time-varying coefficients and stochastic volatility in the unobservable factors' law of motion. Mumtaz & Musso (2018) extend the analysis of Mumtaz & Surico (2008) on a large set of financial and macroeconomic variables, to disentangle the contributions of global, region-specific and country-specific uncertainty. They find that "*the volatility of inflation, interest rates and stock prices seems to be driven primarily by the global common uncertainty component in most countries, although to a varying extent over time*".

We have collected inflation rates for 20 OECD countries, over the period 1960Q1-2016Q4. Figure 1 reports the time series of CPI inflation rates for each country, together with their first principal component (PC). A visual inspection reveals a non-trivial degree of commonality at low-medium frequencies, as pointed out by Lodge & Mikolajun (2016). The first PC explains about 70% of the variability of all inflation rates. However, the figure also highlights some country-specific movements in inflation rates, and changes in the volatility of inflation, which seems overall smaller in the final part of the sample. To provide descriptive evidence on commonality also in inflation volatility, we have estimated autoregressive models with stochastic volatility (AR-SV) for each inflation rate, and in Figure 2 we report the estimated volatilities together with their first principal component, which explains almost 60% of their time variation.

Based on this empirical evidence, this paper introduces a new approach to model simultaneously commonality in the level and in the volatilities of a cross section of macro-economic time series, also allowing for idiosyncratic level and volatility components. The keyword here is "simultaneously", which emphasizes that the common factor can explain both changes in the levels and volatilities of the variables.

The approach builds on the multivariate index (MAI) model of Reinsel (1983), and its Bayesian implementation in Carriero, Kapetanios & Marcellino (2016). A MAI model is a vector autoregression (VAR) with a particular reduced rank structure imposed on the coefficient matrices, such that each variable is driven by the lags of a limited number of linear combinations of all variables (so called Indexes), which can be considered as observable common factors. Stochastic volatility (SV) was introduced in the MAI model by Carriero, Corsello & Marcellino (2018), while Cubadda & Guardabascio (2017) allowed for the possibility of autoregressive (AR) terms to capture idiosyncratic components. We combine all these features into the MAI-AR-SV model, obtain an analytical representation for the indexes (observable factors) law of motion, derive a decomposition of the SV into common and idiosyncratic terms, and develop a novel Bayesian MCMC estimation algorithm.

Importantly, the approach presented here hinges on a reduced rank Vector Autoregression rather than on a factor model. This sets our approach apart from other contributions in the literature. Contributions such as Stock & Watson (1989), Forni, Hallin, Lippi & Reichlin (2000), Mumtaz & Surico (2008) and Mumtaz & Musso (2018) rely on a factor model in which the factor has time varying volatility. Hence, in all these contributions the common volatility factor is merely the volatility of the common factor. Since the factor enters the conditional mean of the process, the volatility of the factor can explain the volatility of the conditional mean of the data. However, neither the factor nor its volatility can explain the conditional variance of the shocks. Instead in the approach presented here the common factor is common to both the conditional mean and the conditional variance of the model. Our methodology is also substantially different from Delle Monache, Petrella & Venditti (2016), who extend the model of Stock & Watson (2007) to a multi-country inflation setting for the euro area, since their model feature a common permanent component with its own changing volatility estimated in a non-Bayesian setting where time variation is driven by likelihood scores.

We work with a single index model where the index (a linear combination of all the national inflation rates) represents the global factor that drives both levels and volatilities of all national inflation rates. Inflation levels and volatilities also have an idiosyncratic, country-specific, component, whose relative importance with respect to the global component is time-varying and empirically determined.

We find that the single common factor in the MAI-SV model explains on average about 70% of the variability of all inflation rates. Moreover, there is also substantial commonality in the inflation volatilities, increased in the last two decades. The average (across countries) share of stochastic volatility explained by the global component spans from 20% to 65% throughout the sample.

While various sources can be behind the global inflation factor, it turns out that since the early '90s its level and volatility are strongly correlated with those of Chinese PPI and Oil inflation. Measures of global slack seem not to have additional explanatory power, and

US monetary policy shocks are basically uncorrelated with shocks to the global inflation factor. Hence, supply seems to matter more than demand to explain the global component of inflation and its volatility.

We also find that the global inflation factor is highly persistent, and this persistence is transmitted to the global component of the national inflation rates, in line with Ciccarelli & Mojon (2010). Level components explained by the common factor show a larger degree of persistence than idiosyncratic components.

We then repeat the same analysis on a panel of non-Food and non-Energy inflation rates for the same set of OECD countries, using data available for the period 1979Q1-2016Q4, we find a smaller but non-negligible degree of commonality. The global core inflation factor explains roughly 25% of the variability of core CPI inflation levels and the average (across countries) share of stochastic volatility explained by the global component spans from 10% to 20% throughout the sample, without displaying sizable variation over time as was the case for headline inflation rates. The remaining substantial national component of core inflation level and volatility leaves scope for national monetary policies.

The evidence provided in this paper also contributes to the long standing debate on globalisation, inflation and monetary policy. Rogoff (2003) and Rogoff (2006) discuss how various structural elements accompanying the globalisation since the early 1990s may have lowered the global long term equilibrium of inflation rates, fostering the strong global comovement of CPIs and somehow diminishing the role of domestic slack and monetary policy in determining national inflation. On this respect, and in line with our empirical results, Wang & Wen (2007) show that, within a New Keynesian DSGE model with nominal rigidities or in a sticky information monetary model, country-specific monetary shocks are not able to explain the strong correlation between inflation of several advanced economies present in the data.

However, as highlighted also in the recent speech by Carney (2017), core inflation seems to be less affected by global dynamics, already when looking at simple pairwise correlation. Our work and methodology allow to measure separately the degree of cross-country commonality in first and second moments of both headline and core inflation rates, providing precious information to monetary policy makers pursuing their inflation mandate in an increasingly global context.

Finally, point and density forecast evaluations show that the MAI-AR-SV model has a very good out of sample performance for inflation rates, when compared with a set of multivariate and univariate competitors, and the SV specification is particularly relevant for the proper calibration of density forecasts. These results hold for both all items inflation and core inflation rates, and provide further empirical support for our proposed model.

The paper is structured as follows. Section 2 introduces the econometric model and the volatility decomposition. Section 3 discusses the choice of prior distributions and the Markov Chain Monte Carlo estimation methodology to draw from the posterior distribution (with additional details in the Appendix). Section 4 presents the data and the empirical results on the commonality in inflation rate levels and volatilities, and on their drivers. Section 5 assesses the point and density forecasting performance of the MAI-AR-SV inflation model. Section 6 concludes. The Appendix provides additional details and empirical results.

2 The econometric model

2.1 The MAI-AR-SV model

We assume that the model for the n-dimensional zero mean process[1] y_t containing the inflation rates of interest is:

$$y_t = \sum_{\ell=1}^{q} \Gamma_\ell \cdot y_{t-\ell} + \sum_{\ell=1}^{p} A_\ell \cdot B_0 \cdot y_{t-\ell} + u_t, \tag{1}$$

where $A_\ell,\ \ell = 1, \ldots, p$ are $n \times r$ matrices, B_0 is an $r \times n$ matrix, and $\Gamma_\ell,\ \ell = 1, \ldots, q$ are n-dimensional diagonal matrices with diagonal elements $\gamma_{1,\ell}, \gamma_{2,\ell}, \ldots, \gamma_{n,\ell}$.

In this model, each of the n variables in y_t is driven by its own lags, capturing country-specific features of inflation, with associated coefficients Γ_ℓ, by the lags of r common observable factors ($B_0\ y_{t-\ell}$, the "indexes"), capturing global features of inflation, with associated loading matrices A_ℓ, and by error terms, u_t, whose properties are described below. With respect to an unrestricted Vector Autoregression, the model above leads to a substantial reduction in the number of parameters.[2]

The product $A_\ell \cdot B_0$ is an $n \times n$ matrix with reduced rank (r), for each $\ell \in \{1, \ldots, p\}$. As it happens also in factor models and in cointegrated VARs, it is the case that one can rotate these matrices arbitrarily, e.g. $A_\ell \cdot B_0 = A_\ell Q' \cdot Q B_0$, where Q is an orthogonal

[1] A non-zero mean can be easily allowed by inserting an intercept in the model.
[2] In our empirical application, we have $p = q = 4$, $r = 1$ and $n = 20$, so that there are 180 parameters in the MAI-AR-SV while there would be 1600 parameters in an unrestricted VAR.

matrix, and hence proper identification restrictions are needed to pin down only one of these possible rotations. Identification can be straightforwardly achieved for example as in Reinsel (1983) by assuming that the first r rows and columns of B_0 form an identity matrix, that is $B_0 = \begin{bmatrix} I_r & \widetilde{B}_0 \end{bmatrix}$. We will follow a similar approach, see Section 3.1.1 for details.

In expression (1), the error term u_t is assumed to be uncorrelated over time, with multivariate Gaussian distribution $u_t \sim N(0, \Omega_t)$, where Ω_t is a time-varying variance-covariance matrix. Following Cogley & Sargent (2005) and Primiceri (2005), we operate a triangular reduction on the full matrix Ω_t, so that the errors u_t can be written as $u_t = G^{-1}\Sigma_t\varepsilon_t$, where ε_t is i.i.d. with multivariate Gaussian distribution $\varepsilon_t \sim N(0, I_n)$, G is a triangular matrix containing reduced form covariances[3], and $\{\Sigma_t\}_{t=1}^T$ is the history of diagonal matrices containing the stochastic volatilities. This implies the following factorization for the variance covariance matrix Ω_t:

$$\Omega_t = G^{-1}\Sigma_t\Sigma_t \left(G^{-1}\right)' \tag{2}$$

$$G = \begin{bmatrix} 1 & 0 & \dots & \dots & 0 \\ g_1 & 1 & \ddots & \ddots & \vdots \\ g_2 & g_3 & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ g_{m-n+2} & g_{m-n+3} & \dots & g_m & 1 \end{bmatrix}, \qquad \Sigma_t = \begin{bmatrix} \sigma_{1,t} & 0 & \dots & \dots & 0 \\ 0 & \sigma_{2,t} & \ddots & \ddots & \vdots \\ 0 & 0 & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & \dots & 0 & \sigma_{n,t} \end{bmatrix}. \tag{3}$$

The elements in the matrix G are further collected in the vector $g = (g_1, g_2, ..., g_m)'$ which has dimension $m = n\,(n-1)/2$. Following Primiceri (2005), the law of motion for the time-varying (TV) standard deviations, collected in the vector $\sigma_t = \begin{bmatrix} \sigma_{1,t}, \dots, \sigma_{n,t} \end{bmatrix}'$, is defined in logarithms as:

$$\log \sigma_t = \log \sigma_{t-1} + \nu_{\sigma,t}, \qquad \nu_{\sigma,t} \overset{iid}{\sim} \mathcal{MN}\left(\, \mathbf{0},\ Q_\sigma \,\right). \tag{4}$$

We will refer to the model in (1) with the volatility specification in (2), (3) and (4) as the MAI-AR-SV model.

[3]The matrix G can be also made time-varying, but at the cost of a substantial increase in computational complexity when the number of variables is large.

2.2 An alternative representation of the MAI-AR-SV model

Let us define the observable factors driving all variables as

$$F_t \equiv B_0 \cdot Y_t, \tag{5}$$

and note that the following decomposition holds:[4]

$$I_n = \Omega_t B_0' \Xi_t^{-1} B_0 + B_{0\perp}' \Xi_{\perp,t}^{-1} B_{0\perp} \Omega_t^{-1}, \tag{6}$$

where $B_{0\perp}$ is the $(n-r) \times n$ orthogonal matrix of B_0 such that $B_0 B_{0\perp}' = \mathbf{0}_{r\times(n-r)}$, $\Xi_t = B_0 \Omega_t B_0'$ and $\Xi_{\perp,t} = B_{0\perp}\Omega_t^{-1} B_{0\perp}'$. Let us also define

$$G_t = B_{0\perp}\Omega_t^{-1} y_t, \tag{7}$$

where G_t are $n-r$ variables that can be interpreted as idiosyncratic components, because, as we will see later on, they are driven by shocks uncorrelated with those driving the common factors F_t.

Using (5)-(7), we can now write the MAI-AR-SV model in (1)-(2) as

$$y_t = \sum_{\ell=1}^{q} \Gamma_\ell [\Omega_t B_0' \Xi_t^{-1} B_0 + B_{0\perp}' \Xi_{\perp,t}^{-1} B_{0\perp} \Omega_t^{-1}]\ y_{t-\ell} + \sum_{\ell=1}^{p} A_\ell \cdot B_0\ y_{t-\ell} + u_t,$$

or

$$y_t = \sum_{\ell=1}^{q} \Gamma_\ell B_{0\perp}' \Xi_{\perp,t}^{-1} G_{t-\ell} + \sum_{\ell=1}^{\max(p,q)} (\Gamma_\ell \Omega_t B_0' \Xi_t^{-1} + A_\ell) F_{t-\ell} + u_t. \tag{8}$$

Next, we derive the model for the factors F_t implied by the MAI-AR-SV model. Starting from (8) and multiplying both sides of it either by B_0 or by $B_{0\perp}\Omega_t^{-1}$, we obtain:

$$F_t = \sum_{\ell=1}^{q} B_0 \Gamma_\ell B_{0\perp}' \Xi_{\perp,t}^{-1} G_{\ t-\ell} + \sum_{\ell=1}^{\max(p,q)} B_0 (\Gamma_\ell \Omega_t B_0' \Xi_t^{-1} + A_\ell) F_{t-\ell} + \omega_t, \tag{9}$$

$$G_t = \sum_{\ell=1}^{q} B_{0\perp}\Omega_t^{-1} \Gamma_\ell B_{0\perp}' \Xi_{\perp,t}^{-1} G_{t-\ell} + \sum_{\ell=1}^{\max(p,q)} B_{0\perp}\Omega_t^{-1} (\Gamma_\ell \Omega_t B_0' \Xi_t^{-1} + A_\ell) F_{\ t-\ell} + \psi_t,$$

where

$$\begin{bmatrix} \omega_t \\ \psi_t \end{bmatrix} = \begin{bmatrix} B_0 u_t \\ B_{0\perp}\Omega_t^{-1} u_t \end{bmatrix} \overset{i}{\sim} \mathcal{MN}\left(\mathbf{0}, \begin{bmatrix} \Xi_t & 0 \\ 0 & \Xi_{\perp,t} \end{bmatrix}\right), \tag{10}$$

[4]See Carriero et al. (2016) and the references therein for details.

since

$$E(\omega_t \psi_t^{'}) = E(B_0 u_t u_t^{'} \ \Omega_t^{-1} B_{0\perp}^{'}) = B_0 \Omega_t \Omega_t^{-1} B_{0\perp}^{'} = 0.$$

Hence, the r observable factors F_t and the $n - r$ idiosyncratic components G_t jointly evolve as a VAR, with block uncorrelated errors.

The model in (8)-(9) is similar to a factor augmented VAR (FAVAR) model, as for example in Bernanke et al. (2005), or Stock & Watson (2005) who also allow for variable-specific AR terms. However, it differs from a FAVAR in three important ways. First, it also features stochastic volatility both in the common (ω_t) and in the idiosyncratic (ψ_t) shocks, which is particularly relevant for modelling inflation, as we will see. Second, in the FAVAR model the factors are unobservable, while they are observable in the MAI case, which simplifies model estimation and interpretation of the results. Third, in general unobserved factors should be modeled with a VARMA rather than a VAR model, as emphasized by Dufour & Stevanović (2013), while in our case we can analytically derive the VAR model followed by the observable factors F_t (jointly with the variables G_t).

2.3 Decomposing the volatilities

We decompose the stochastic volatility of the MAI-AR-SV errors u_t into two orthogonal sets of components, one of them driven by the volatility of the common shocks ω_t, the other by that of the idiosyncratic shocks, ψ_t.

Using again the decomposition in (6), we get:

$$u_t = \Omega_t B_0' \Xi_t^{-1} \omega_t + B_{0\perp}' \left(B_{0\perp} \Omega_t^{-1} B_{0\perp}' \right)^{-1} \psi_t,$$

with $\Xi_t = B_0 \Omega_t B_0^{'}$. Hence, due to the orthogonality of ω_t and ψ_t, we can then decompose the total error volatility into the volatility of the common component and that of the idiosyncratic component:

$$\Omega_t = \Omega_t^{com} + \Omega_t^{idio},$$

where

$$\Omega_t^{com} = \Omega_t B_0' \Xi_t^{-1} B_0 \Omega_t, \qquad \Omega_t^{idio} = B_{0\perp}' \Xi_{\perp,t}^{-1} B_{0\perp}.$$

2.4 Decomposing the levels and computing IRFs

It is interesting to decompose the inflation rates in y_t into their common and idiosyncratic components, where the common component is driven by the common shocks ω_t and the idiosyncratic component by the idiosyncratic shocks ψ_t. The decomposition can be also used to compute impulse response functions (IRFs) to common shocks.

We cannot directly use the model in (8), as both F_t and G_t are driven by both ω_t and ψ_t. However, we can use the projection approach, proposed for example by Jordà (2005) for IRF computation, to obtain the decomposition:

$$y_t = B_1(L)\omega_t + B_2(L)\psi_t,$$

where ω_t and ψ_t are defined in (10). The common and idiosyncratic components are orthogonal at all leads and lags, due to temporal independence and orthogonality of ω_t and ψ_t. Therefore, empirically, we can obtain the common component as the fitted value in a regression of y_t on contemporaneous and lagged values of the (estimated) common shocks ω_t (and the idiosyncratic component as y_t minus the estimated common component), while the IRFs to common shocks are computed from the elements of $B_1(L)$[5].

3 Estimation

The model is estimated using Markov Chain Monte Carlo (MCMC) techniques. In this Section we discuss, in turn, the priors on the model coefficients and the MCMC algorithm we use to obtain draws from the posterior distributions.

3.1 Specification of the prior distributions

The prior distributions are constructed in various steps, which generally require the use of a training sample $\{-T^*, \ldots, -1, 0\}$.

3.1.1 Prior on B_0

In order to identify the model, we need to restrict at least r^2 elements in the $r \times n$ matrix B_0. Given that in our case $r = 1$, then $B_0 = \begin{bmatrix} b_{0,1} & \ldots & b_{0,n} \end{bmatrix}$ is a row vector of weights, and as identifying restrictions we simply normalize to 1 the first weight $b_{0,1}$.

[5] In our empirical application on inflation, we have a single factor ($r = 1$), which explains on average more than 70% of the cross-country inflation variability. In this case, ω_t is a scalar, which further simplifies the computation of the common component of inflation rates, and their impulse response functions to global shocks.

Prior knowledge for the unrestricted elements of B_0 is elicited with a Normal distribution. To set the prior moments for the elements of $B_0 = \begin{bmatrix} b_{0,1} & \dots & b_{0,n} \end{bmatrix}$, we compute the largest eigenvalue score S_t from the principal component analysis on the set of variables, and then run the following n univariate regressions:

$$\forall k \in \{1, \dots, n\}, \quad S_t = b_{0,k} \cdot y_{t,k} + u_{k,t}, \quad u_{k,t} \overset{iid}{\sim} \mathcal{N}\left(0, \sigma_k^2\right)$$

The OLS estimates of these regressions, and their standard errors, are used to calibrate mean and variances of the prior distributions. For identification, $b_{0,1}$ is set to 1, and the remaining elements of B_0 are divided by $\widehat{b}_{0,1}$.

3.1.2 Prior on A

Defining $A \equiv \begin{bmatrix} A_1 & \dots & A_p \end{bmatrix}$, the prior on $a = vec\left(A'\right)$ is multivariate Normal, centered on $\mathbf{0}$, and with diagonal variance V_a resembling a Minnesota prior. In particular, it holds that

$$V_a = Diag\left(\begin{bmatrix} \widehat{\sigma}^2_{y,1} \\ \vdots \\ \widehat{\sigma}^2_{y,n} \end{bmatrix}\right) \otimes \begin{bmatrix} \Psi_1 & 0 & \dots & 0 \\ 0 & \Psi_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & \Psi_p \end{bmatrix}, \forall \ell \quad \Psi_\ell = \frac{\lambda_a}{\ell^d} \cdot Diag\left(\begin{bmatrix} \widehat{\sigma}^{-2}_{F,1} \\ \vdots \\ \widehat{\sigma}^{-2}_{F,r} \end{bmatrix}\right),$$

where $\widehat{\sigma}^2_{y,j}$ and $\widehat{\sigma}^2_{F,s}$ are the residual variances of a univariate AR(1) for, respectively, each variable j and each factor s (computed using the prior mean of B_0). λ_a is a tightness parameter, and d is a decay parameter. We chose standard calibration borrowing from the VAR literature, i.e. $\lambda_a = 0.2$ and $d = 2$.

3.1.3 Prior on Γ_ℓ

The prior distribution on the AR coefficients in the matrices Γ_ℓ, collected in the column vector γ (see the Appendix for details), is a multivariate Normal distribution. In the spirit of a Minnesota prior, we choose an a priori unitary mean for the first lag of each variable whose dynamics resemble a random walk, and a zero mean for the higher lags. Regarding the a priori covariance matrix, we assume no correlation across coefficients of different lags and variables, and we set a prior structure for the variances which resembles the Minnesota prior, using the tightness and decay parameters:

$$\bar{\gamma} = \begin{bmatrix} \bar{\gamma}_1 \\ \bar{\gamma}_2 \\ \vdots \\ \bar{\gamma}_q \end{bmatrix} = \begin{bmatrix} \mathbf{1}_{n\times 1} \\ \mathbf{0}_{n\times 1} \\ \vdots \\ \mathbf{0}_{n\times 1} \end{bmatrix}, \qquad V_\gamma = \lambda_\gamma \cdot Diag\left(\begin{bmatrix} 1^{-d} \\ 2^{-d} \\ \vdots \\ q^{-d} \end{bmatrix}\right) \otimes I_n.$$

Considering the smaller number of elements in γ in our empirical application, we chose a less tight prior calibration than in the case of A, i.e. $d = 1$ and $\lambda_\gamma = 0.1$.

3.1.4 Prior on Ω_t

The prior for the elements of G in expression (2), the matrix containing the stable covariances among the MAI errors, is a multivariate Normal distribution centered at zero, with large diagonal covariance matrix.

The priors to produce inference on the elements of σ_t in (4) are set as follows. The prior for σ_0 is a multivariate Normal, centered at $\begin{bmatrix} \widehat{\sigma}^2_{y,1} & \widehat{\sigma}^2_{y,2} & \dots & \widehat{\sigma}^2_{y,n} \end{bmatrix}'$ estimated as standard deviations of univariate AR(1) residuals on each observable, with identity covariance matrix, as in Primiceri (2005). The prior distribution for the innovation covariance matrix Q_σ is calibrated as in Primiceri (2005).

3.2 Gibbs Sampler

This subsection describes each step of the Gibbs Sampler (GS) used to simulate from the joint posterior distribution of both parameters $\theta = \{\gamma, A, B_0, G, Q_\sigma\}$ and unobservable states $\{\sigma_t\}_{t=1}^T$ of the MAI-AR-SV model. Moreover, for volatility estimation the Omori et al. (2007) procedure requires drawing the indexes of Normal components of a mixture approximating a $\log\chi^2_1$ distribution labeled as $\{S_t\}_{t=1}^T$. This MCMC estimation approach is needed as the joint posterior distribution cannot be analytically determined. The algorithm innovates with respect to Carriero et al. (2018) since it introduces the step to draw the AR coefficients, and it builds upon Carriero et al. (2016) to draw the reduced rank structure of the VAR coefficients $(A \cdot B_0)$, and on Primiceri (2005) and Del Negro & Primiceri (2015) to draw the time-varying volatilities.

The steps are the following:

1. Draw a history of volatilities $\{\sigma_t\}_{t=1}^T \Big| \theta, \{S_t\}_{t=1}^T$,
2. Draw $\theta, \{S_t\}_{t=1}^T \,|\, \{\sigma_t\}_{t=1}^T$. This second step is further split as follows:

(a) Draw the elements in $\theta | \{\sigma_t\}_{t=1}^T$

i. Draw the covariance of volatilities' innovations $Q_\sigma \left| \gamma, A, B_0, G, \{\sigma_t\}_{t=1}^T \right.$

ii. Draw the AR coefficients $\gamma \left| A, B_0, G, Q_\sigma, \{\sigma_t\}_{t=1}^T \right.$,

iii. Draw the loadings $A \left| \gamma, B_0, G, Q_\sigma, \{\sigma_t\}_{t=1}^T \right.$,

iv. Draw the factor weights $B_0 \left| \gamma, A, G, Q_\sigma, \{\sigma_t\}_{t=1}^T \right.$,

v. Draw the off-diagonal elements in $G \left| \gamma, A, B_0, Q_\sigma, \{\sigma_t\}_{t=1}^T \right.$

(b) Draw a history of indexes of the mixture in $\{S_t\}_{t=1}^T | \theta, \{\sigma_t\}_{t=1}^T$

It is important to note that steps *iii* and *iv* have $y_t - \mathcal{X}_t \cdot \gamma$ as dependent variable in order to draw A and B_0, while in step *ii* we use $y_t - A \cdot Z_t$ to draw the AR coefficients, where

$$Z_t \equiv (I_p \otimes B_0) \cdot vec\left(\begin{bmatrix} y_{t-1} & \dots & y_{t-p} \end{bmatrix}\right), \quad \mathcal{X}_t = \begin{bmatrix} Diag(y_{t-1}) & \dots & Diag(y_{t-q}) \end{bmatrix}.$$

The Gibbs Sampler is described in more detail in section A of the Appendix.

4 The global component of inflation volatility

4.1 Data

Following the literature on global inflation (e.g. Ciccarelli & Mojon, 2010 and Borio & Filardo, 2007) we collected a panel of Consumer Price Indices for a set of 20 OECD countries[6], downloaded from the *OECD main economic indicators* database. The dataset includes 228 observations at quarterly frequency, covering the period from 1960-Q1 to 2016-Q4. We then constructed inflation rates as year on year changes of the indexes[7].

[6] USA, Australia, Austria, Belgium, Canada, Finland, France, Germany, Greece, Italy, Japan, Luxembourg, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, UK

[7] Ciccarelli & Mojon (2010) use Year on Year changes of CPI inflation rates for the bulk of their analysis. O'Reilly & Whelan (2005) adopt the same transformation stressing that is cited in the ECB's official inflation mandate. Lodge & Mikolajun (2016) point out that using YoY changes in CPI is preferable since this transformation produces no seasonal pattern by construction.

4.2 Common component and global shock transmission

We start by estimating a MAI-SV specification (that is with $\Gamma_\ell = \mathbf{0},\ \forall \ell$), with $p = 4$ lags and with a single global factor ($r = 1$), similar to the preferred specification of Ciccarelli & Mojon (2010). The resulting model is estimated by a simplified version of the MCMC algorithm presented in Section 3.

Figure 3 reports the inflation rates for each country along with the posterior bands and median of the estimated common global inflation factor. The model is clearly able to capture the substantial co-movement of national inflation rates.

Measuring the in-sample fit of the MAI-SV model for each country, we find that on average (across countries) the estimated common component explains roughly 73% of the variance, which is in line with the Principal Component Analysis.

Next, as the residuals of the MAI-SV model are clearly serially correlated at least over parts of the sample, we estimate a MAI-AR-SV model with $p = 4$ lags for the common part, as for the MAI-SV, and $q = 4$ lags for the country-specific AR components[8]. The in-sample fit for the MAI-SV and MAI-AR-SV models are reported in the Appendix. The fit of the MAI-AR-SV is systematically higher than that of the MAI-SV specification, reaching an average explained variance of about 94%. In particular, the MAI-AR-SV specification is able to capture both the low and the high frequency variation of each inflation series, due to the presence of both common and country-specific autoregressive components.

Notwithstanding the differences mentioned above, the estimated global factor from the MAI-SV and MAI-AR-SV models are very similar, see Figure 4. They are also very similar to the first PC of the inflation rates. The latter is used to form the prior on the B_0 coefficients in the MAI models, but the prior variance is large enough so that results are data driven rather than dictated by the prior. All such measures of common components are also comparable, though with some differences, to an OECD measure of global inflation, also reported in Figure 4. These results are in line with the findings of Ciccarelli & Mojon (2010), even though their sample stops in 2008. As reported also by Ferroni & Mojon (2016), our analysis suggests strong commonality in inflation developments across OECD continues also in the more recent period, and, actually, it has been particularly high during the last financial crisis[9].

[8]Recall that the MAI-AR-SV model is a MAI-SV model with the addition of AR components.

[9]Using a more recent sample of inflation rates (1993-2014), Ferroni & Mojon (2016) find that the fraction of national inflation rates' variance that is explained by Global Inflation remains dominant.

A simple local projection analysis shows how a shock to this global factor is transmitted across countries. Figure 5 shows the responses to a unitary impulse to ω_t, the innovation of the global factor's law of motion. We observe a similar hump shaped pattern in most countries. With few exceptions, on impact and in the first periods after the impulse, national inflation tends to increase, and then after some quarters it reverts back to the impact response or even to lower values in some countries. This evidence shows how a global shock to headline inflation rates induces a similar response over a set of advanced economies, reinforcing our understanding of the degree of commonality.

4.3 Levels decomposition and persistence

Using the level decomposition discussed in section 2.4, we are able to decompose the observed inflation series of each country into orthogonal components driven, respectively, by common and idiosyncratic shocks. Moreover, for each country we measure how much variation is explained by each component.

Figures 6 reports the level decomposition of inflation rates into common and idiosyncratic components, compared with the actual series. The common components tend to explain more than 50% of almost all countries' inflation rates, and are particularly important in large economies like the US, UK, Germany and Japan[10].

Stock & Watson (2007) discuss the persistence of US inflation, using as a measure of persistence the largest autoregressive root of the levels' process. Inference about this measure of persistence is made possible by the Stock (1991) method, which is appropriate when dealing with series displaying high levels of persistence. Stock & Watson (2007) do not find strong evidence of persistence changes in US inflation from the 1970s onwards, reporting the largest AR root of US CPI inflation comprised between 0.85 and 1.05 (as 90% confidence interval). O'Reilly & Whelan (2005) report little evidence of instability for inflation persistence in the Euro Area since the 1970s; they report rolling confidence intervals for the largest AR root of Euro Area CPI inflation that are centered around 0.9 across almost the entire sample.

In light of this literature, using the entire sample, we computed the 90% confidence intervals (CI) for the largest AR roots of all national CPI inflation series, of their common and idiosyncratic components, and of the global factor. Figure 7 compares the CI for the largest AR root of the observed series, their components and the global factor, separately for each country. The picture clearly shows how the common global components tend

[10]Note that these values are lower than those reported for the explanatory power of the common global factor F_t, as the evolution of F_t is partly explained by the idiosyncratic shocks, see (9).

to preserve the high persistence of the observed series, while the idiosyncratic country-specific components display wider confidence intervals centered on slightly smaller values. The global factor shows a very narrow CI centered on 0.99.

These results are in line with what reported by Ciccarelli & Mojon (2010), who argue that "*the global component captures the most persistent and possibly nonstationary part of inflation*". Indeed, using a different methodology, they report smaller persistence for the so called "national" components; interpreting such results, they consider the global factor as an attractor and the main driver of persistence coming from the observed data. However, for this specific exercise they use annualized quarter on quarter inflation rates, which is a transformation that tends to display a smaller degree of persistence than the year on year transformation. Performing our analysis using QoQ CPI changes, we measure a degree of persistence in line with Ciccarelli & Mojon (2010) for both global and national inflation components.

4.4 Time-varying volatility decomposition

Figure 8 reports the posterior bands of the estimated conditional inflation volatilities of all countries for the MAI-AR-SV, along with the decomposition discussed in Section 2.3. The estimated volatilities display a relevant degree of commonality. Indeed, the first principal component of the volatilities explains on average about 50% of their variation.

To better understand what is driving the volatilities, Figure 9 presents the contribution of the estimated common and idiosyncratic components in determining volatilities. It turns out that the contribution of the common component is non trivial, reaching values above 50% for some countries and time periods, especially during the last decades, in particular during the Great Recession. In the 1970s the contribution of the global component to stochastic volatility is generally smaller than in the Great Recession, standing for a smaller degree of cross-country synchronization of changes in the unexplained residuals' size.

In this multi-country context, it is complex to understand the structural drivers of the common inflation volatility component. However, for a single country this can be done. Carriero et al. (2018), focusing on the US, find that supply shocks are particularly important, with demand shocks ranked second and monetary/financial shocks third. We will see later on that the importance of supply factors is also confirmed for other countries.

Figure 10 shows the posterior bands of the global factor volatility, that is $(\Xi_t)_{t=1}^{T}$. Global inflation volatility was moderate during the 1960s, increased dramatically during the 1970s before the sharp reduction starting in the 1980s associated with the change in monetary policy to fight inflation occurred in several countries. These results are in line with the US inflation volatility estimated by Stock & Watson (2007). Global inflation volatility has remained very low until mid 2000s, reaching a new spike during the Great Recession, before turning back to the historically low values in the last three/four years of the sample. Time variation is significant and relatively large throughout the entire estimation sample.

4.5 Possible drivers of global inflation's level and volatility

While there can be many drivers of the global inflation factor, Figure 11 shows that after the 90's it is correlated with the Chinese PPI inflation rate and the Oil inflation rate, with correlations around 0.7 and 0.5 respectively. To compute Oil inflation, we use the WTI price ($/barrel), while Chinese PPI inflation is available only from the early 1990s.

To obtain more reliable results on the role of these two variables as drivers of global inflation, we have regressed the global factor on its own lags and on the simultaneous and lagged values of the Chinese PPI and Oil inflation rates, starting the sample in the early '90s due to data availability. The results indicate that the regressors are significant, with the share of explained variance increasing to more than 90% when Chinese PPI and and Oil inflation are included in the specification, compared to roughly 75% in the AR model, and less autocorrelated residuals. Adding to such regressions the contemporaneous and lagged OECD global output gap does not increase the explanatory power. These findings are also qualitatively confirmed in regressions with differenced variables and suggest that supply factors are more relevant than demand ones as drivers of global inflation dynamics[11].

To further explore some other relevant sources of global inflation fluctuations, we regressed the innovation to the common component, ω_t, on simultaneous and lagged values of two estimated exogenous innovations to the US economy: the Gertler & Karadi (2015) measure of monetary policy shocks and the Beaudry & Portier (2006) measure of TFP news shock[12]. The evidence shows that such important exogenous shocks relative to the

[11]Using quarterly data from the 1990s within an empirical Phillips-Curve estimation framework, Forbes (2018) provides evidence that four global factors (global slack, commodity prices, exchange rates and the degree of global price competition among producers) contribute to national inflation dynamics, even though quite heterogeneously across countries. Moreover, the evidence provided shows that global factors have not become as important in explaining core inflation dynamics as they have for headline CPI and that the explanatory power of different global factors is very heterogeneous across time and across countries. These findings are in line with our results.

[12]Ramey (2016) reviews and discusses several shocks driving US economic fluctuations. The Gertler & Karadi (2015) monetary policy shocks are identified at high-frequency using federal funds futures, and then aggregated at lower frequencies. The Beaudry & Portier (2006) series of news shock to productivity is identified via short and long run restrictions using data on stock prices.

US economy, which typically explain a large share of business cycle fluctuations according to Ramey (2016), have almost no explanatory power for global inflation innovations. Miranda-Agrippino & Rey (2018) also find that US monetary policy shocks explain a very small share of inflation variance.

Next, to understand which global forces may correlate with global CPI inflation volatility, we estimated stochastic volatilities from univariate AR-SV models for Oil inflation, the Chinese PPI inflation and the global output gap. A comparison of the estimated (median) volatilities is reported in Figure 12. From visual inspection, a clear co-movement between Oil and global CPI inflation volatility stands out, showing a correlation around 0.5 from the early 1970s and almost 0.8 from the early 1990s. Also the Chinese PPI inflation volatility displays a positive correlation with global CPI uncertainty: the correlation is around 0.7 from the early 1990s. The volatility of the output gap is instead much lower, except for the recessionary period.

To provide additional evidence, we have regressed the (changes in the) volatility of the global inflation factor on its lags and on the contemporaneous and lagged (changes in the) volatility of Oil, Chinese PPI and output gap, either separately or jointly. The largest explanatory power is achieved by the volatility of the Chinese PPI, with an R^2 of about 0.80 with respect to about 0.39 for the AR model. Adding the other volatilities does not help. Hence, overall, we find evidence for the volatility of Chinese PPI as a driver of the volatility of global inflation, with oil volatility as second best.

So far we have analyzed separately the drivers of level and volatility of global inflation. Joint evaluation leads to the same conclusions. In particular we have conducted a joint analysis by modelling the global inflation factor, Oil inflation, Chinese PPI inflation and global output gap with a VAR model with stochastic volatility. The estimation results confirm the relevance of lags of Oil and Chinese PPI in the equation for the global inflation factor, not of global output gap. Moreover, the first principal component of the four estimated stochastic volatilities explains about 70% of their movements, even though the stochastic volatility of the global output gap has almost zero weight.

4.6 Commonality in core inflations

In light of the correlation (both in levels and volatilities) between the global component of headline CPI inflation and Oil, it is important to detect how much core components of the CPIs remain correlated across countries. To this end, the same exercises of this section have been performed using the non-Food and non-Energy Consumer Prices Indices for the same set of countries, downloaded from the *OECD main economic indicators* database. These data are available only from the late '70s onwards.

Non-Food and non-Energy inflation tends to display a lower degree of commonality, already from a quick graphical inspection[13]. Results of the decomposition are collected in Appendix C. They indicate a smaller importance of the common component both in volatilities and in levels: the global core inflation factor explains roughly 25% of the variability of core CPI inflation levels, while the average (across countries) share of stochastic volatility explained by the global component spans from 10% to 20% throughout the sample[14]. The fact that core inflation dynamics remain mostly heterogenous across countries, leaves ample scope for domestic monetary policies.

5 Forecasting inflation with the MAI-AR-SV model

To provide further evidence on the usefulness of the MAI-AR-SV as a model for multi-country inflation, we now evaluate its out of sample properties, also in comparison with a set of standard competitors.

Using the same inflation series employed in the structural analysis, several models are recursively estimated on a forecasting window going from 1990Q1 to 2016Q4. The associated out of sample forecasts are produced for six different models and 8 horizons, from 1 to 8 quarters ahead.

The models under evaluation are the following:

- the Multivariate Autoregressive Index model with AR components and Stochastic Volatility (MAI-AR-SV)
- the Multivariate Autoregressive Index model with AR components (MAI-AR)
- the univariate Autoregressive model (AR)
- the univariate Autoregressive model with Stochastic Volatility (AR-SV)

[13]The lower degree of commonality may also stand for a low degree of synchronization between national core inflation rates. Indeed, less volatile components of the CPI may be less responsive to global changes in goods' prices, and most importantly their time response can be heterogenous across countries. This phenomenon is less evident in headline inflation rates since the most volatile components of prices are more synchronous over the world and determine a non-trivial amount of variation in goods' prices.

[14]To understand whether the different results we got for headline and core inflation rates could be due to the different samples, we have also repeated the analysis for headline inflation using the same sample as for core (i.e., from 1979 onwards). The results are qualitatively similar to what illustrated in the previous subsections. There is a decline in the fraction of variance explained by the global inflation factor to around 50%, but this value is still about two times larger than that for the core inflation rates. The decline in the explanatory power of the global inflation factor over the shorter sample can be attributed to the global relevance of oil price fluctuations in the '70s.

- the Vector Autoregressive model (VAR)
- the Vector Autoregressive model with Stochastic Volatility (VAR-SV)

All models are estimated using Bayesian techniques. AR and VAR priors are constructed using the standard Litterman (1986) a priori assumption of univariate random walk processes. The SV prior in all models is calibrated as in Primiceri (2005). The MAI prior is specified as shown in section 3. Diagnostics are then computed in terms of both point and density forecasting, following the evaluation framework of Clark & Ravazzolo (2015).

Specifically, to evaluate the accuracy in terms of point forecasting, we compute the forecasts posterior medians for all vintages, models, variables and horizons. Then, we compute the Root Mean Squared Forecast Error (RMSFE) for each model, variable and horizon, using the variation across vintages. Hence, for each variable $j \in \{1, \dots, n\}$, each horizon $h \in \{1, \dots, H\}$ and each model $m \in \{1, \dots, M\}$ we compute:

$$RMSE_{j,h}^{m} = \sqrt{\frac{1}{T^*} \sum_{t=T+1}^{T+T^*} \left(y_{j,t+h} - \widehat{y}_{j,t+h}^{m} \right)^2},$$

where $\widehat{y}_{j,t+h}^{m}$ is the median of the posterior distribution $\left(\widehat{y}_{j,t+h}^{m,i}\right)_{i=1}^{L_c}$ (L_c is the length of the discretized posterior distribution). To test for significance of the squared forecast errors differences across models, we compute the Diebold & Mariano (1995) t-tests for equality of the average loss[15].

To evaluate models in terms of density forecasting, we use two measures of accuracy: the average log-predictive score and the average Continuous Ranked Probability Score (CRPS). Even in this case, to test for significantly different performances we employ the Diebold and Mariano test, following Clark & Ravazzolo (2015).

Log Predictive Scores are obtained via non-parametric kernel smoothing density estimators. Adopting a normal kernel $\mathcal{K}_{\mathcal{N}}(\cdot)$ and following an optimal selection strategy of the bandwith parameter $\widehat{\mathcal{H}}$, we can compute for each variable, model, horizon and vintage the empirical density evaluted at the actual observation $y_{j,t+h}$, that is:

$$\widehat{f}_m \left(y_{j,t+h}, \widehat{\mathcal{H}} \right) = \frac{1}{\widehat{\mathcal{H}} \cdot L_c} \sum_{i=1}^{L_c} \mathcal{K}_{\mathcal{N}} \left(\frac{y_{j,t+h} - \widehat{y}_{j,t+h}^{m,i}}{\widehat{\mathcal{H}}} \right).$$

[15] Monte Carlo evidence in Clark & McCracken (2011) and Clark & McCracken (2015) indicates that, with nested models, the Diebold-Mariano test compared against normal critical values can be viewed as a somewhat conservative (conservative in the sense of tending to have size modestly below nominal size) test for equal accuracy in the finite sample.

Then, applying logarithms and computing the average across forecasting vintages yields the average log score for each variable, model and horizon:

$$\overline{logScore}^{m}_{j,h} = \frac{1}{T^*}\sum_{t=T+1}^{T+T^*} \log \widehat{f}_m\left(y_{j,t+h}, \widehat{\mathcal{H}}\right).$$

To compute the average CRPS, following Clark & Ravazzolo (2015), we first compute the CRPS per each variable, model, horizon and vintage, making use of the actual observations, the posterior distribution $\left(\widehat{y}^{m,i}_{j,t+h}\right)_{i=1}^{L_c}$ and a random permutation of the latter $\left(\widehat{y}^{m,i'(i)}_{j,t+h}\right)_{i=1}^{L_c}$ where $i' : \{1, \ldots, L_c\} \rightarrow \{1, \ldots, L_c\}$ is randomly drawn without replacement. Lastly, we simply compute the average across time vintages:

$$\begin{aligned} CRPS^m_{j,t+h} &= \frac{1}{L_c}\sum_{i=1}^{L_c}\left|\widehat{y}^{m,i}_{j,t+h} - y_{j,t+h}\right| - \frac{1}{2\cdot L_c}\sum_{i=1}^{L_c}\left|\widehat{y}^{m,i}_{j,t+h} - \widehat{y}^{m,i'(i)}_{j,t+h}\right|, \\ \overline{CRPS}^m_{j,h} &= \frac{1}{T^*}\sum_{t=T+1}^{T+T^*} CRPS^m_{j,t+h}. \end{aligned}$$

Figure 13 portrays the relative performance of the competing set of models against the benchmark model MAI-AR-SV, for each country and four selected horizons. Models' point forecasting performance is reported as ratio between their own Root Mean Squared Errors and the benchmark's, so that values larger than one imply that the MAI-AR-SV produces more accurate point forecasts. The MAI-AR-SV model improves significantly upon its counterparts on most variables, especially at short horizons, even though in a smaller number of cases this is reversed. The AR-SV shows competitive point forecasting performance, especially at longer horizons. The highly parametrized VARs generally achieve a lower degree of point forecasting accuracy than the benchmark[16].

Moving to density forecast evaluation, Figure 14 reports the relative average log predictive scores for the chosen set of models and horizons. Alternative models' performance is reported in terms of log-scores differences with the benchmark MAI-AR-SV, so that negative values favor the MAI-AR-SV. The benchmark model clearly improves upon its competitors: the difference is negative and significant in most cases. Eventually, Figure 15 shows the CRPS reported comparatively as a ratio, where values greater than one indicates a worse density forecasting performance with respect to the MAI-AR-SV. Results are in line with the log-scores, with the benchmark model improving significantly upon

[16]Tables 1a and 1b in the online appendix report detailed results.

its competitors[17].

To conclude, our evidence shows that the MAI-AR-SV is also a good forecasting model for inflation rates. The introduction of SV is particularly relevant to improve density forecasts. This evidence is in line with findings reported by Clark & Ravazzolo (2015) and D'Agostino et al. (2013). Moreover, the introduction of the MAI component further increases the forecasting power. Furthermore, notwithstanding the smaller number of coefficients due to the reduced rank restriction imposed by the MAI structure, the benchmark model attains a higher degree of forecasting accuracy with respect to the standard unrestricted VAR estimated using a Minnesota Prior as shrinkage device[18].

6 Conclusions

Global developments play an important role in the determination of inflation rates, and indeed earlier literature has found that a substantial amount of the variation in a large set of national inflation rates can be explained by a single global factor. This literature has typically neglected inflation (conditional) volatility, while volatility is clearly relevant both from a policy point of view and for structural analysis and forecasting.

In this paper we study the evolution of inflation rates in many countries, using a novel model that allows for commonality in both levels and volatilities, in addition to country-specific components. We find that allowing for inflation volatility is indeed important, and a large fraction of it can be attributed to a global factor that is also driving the inflation levels.

While other sources can be behind this global factor, it turns out that since the early '90s it is strongly correlated with the Chinese PPI and Oil prices. Moreover, also the

[17]More detailed forecasting results are reported in Appendix B.

[18]As the common observable factor in the estimated MAI-AR-SV model is similar to the first principal component (PC) of all inflation rates, which can be interpreted as an estimated factor in a factor model, it can be expected that the forecasting performance of the MAI-AR-SV is similar to that of a factor model. To verify empirically whether this is the case, we have also estimated a set of bivariate BVAR-SV models, each of them for one inflation rate and the PC. The equation for the inflation rate can be interpreted as an AR-PC(-SV) model, commonly used in the factor forecasting literature, complemented with the second equation that describes the evolution of the PC. While direct estimation is commonly used in the factor forecasting literature, we use an iterative scheme for comparability with the MAI-AR-SV. Note also that we cannot use a single large BVAR-SV for all the inflation rates and the PC due to collinearity. It turns out that indeed the forecasting performance of the BVAR-SVs is comparable to that of the MAI-AR-SV, even slightly better in particular in terms of RMSE. Yet, as mentioned in the introduction, the possibility of jointly modelling first and second moments with the MAI-AR-SV makes it preferable to a factor model where only commonality in the first moments is considered. In this respect, it is actually encouraging that the performance of the MAI-AR-SV is overall comparable to that of a factor model. These results are available upon request.

global factor stochastic volatility is highly correlated with that of Chinese PPI and Oil prices.

Repeating the same analysis on core inflation rates for the same set of OECD countries, the model finds a smaller but non-negligible degree of commonality. The substantial national component of core inflation level and volatility leaves ample scope for domestic monetary policies.

The MAI-AR-SV shows also very good out of sample properties, achieving a comparatively better forecasting performance for inflation when compared with a set of prominent alternative models, especially in terms of density forecasting.

Finally, as we have seen that the MAI-AR-SV model can be reparametrized as an (observable) factor augmented VAR with stochastic volatility, it has a wide range of applicability in empirical macroeconomics.

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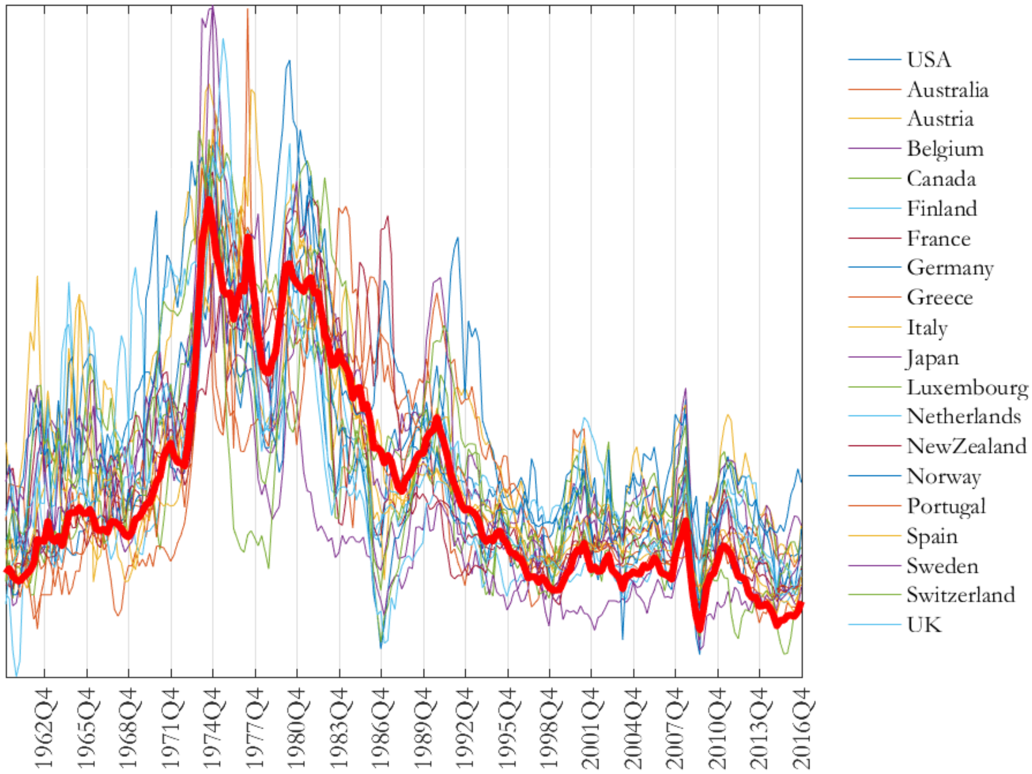


Figure 1: Inflation rates and their first principal component (thick red line)

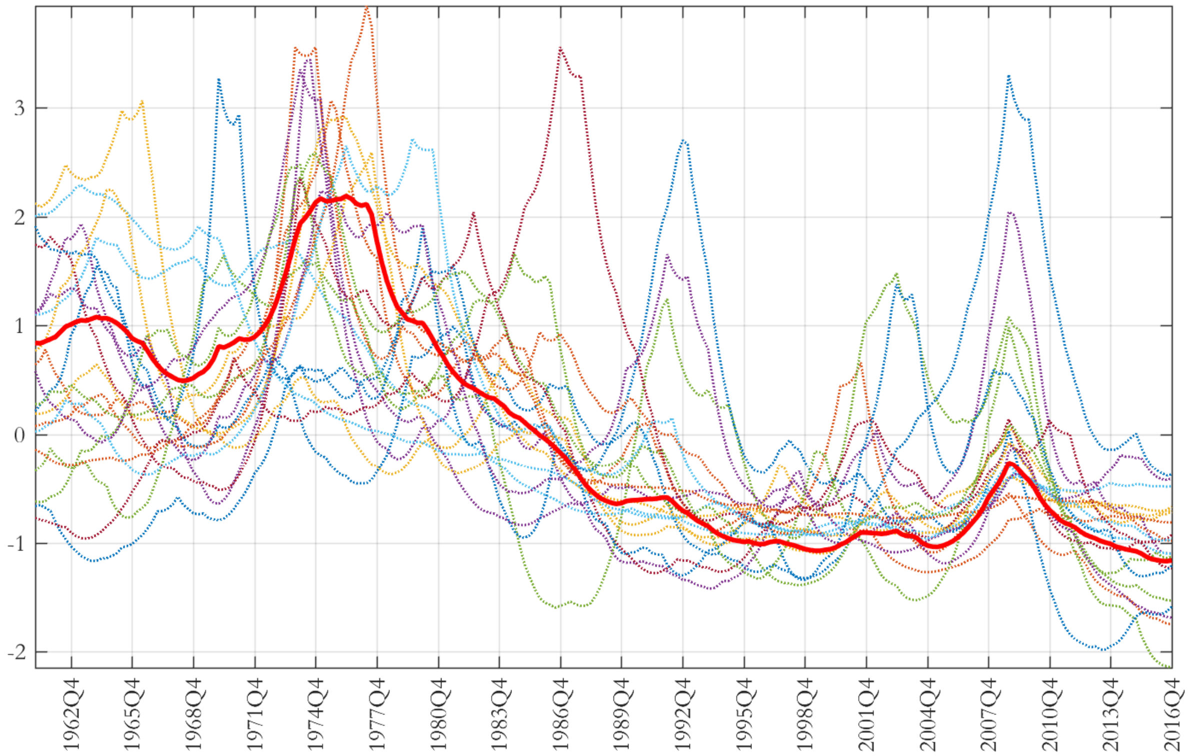


Figure 2: CPI inflation rates Stochastic Volatilities estimated from univariate AR-SV, and their first Principal Component (thick red line)

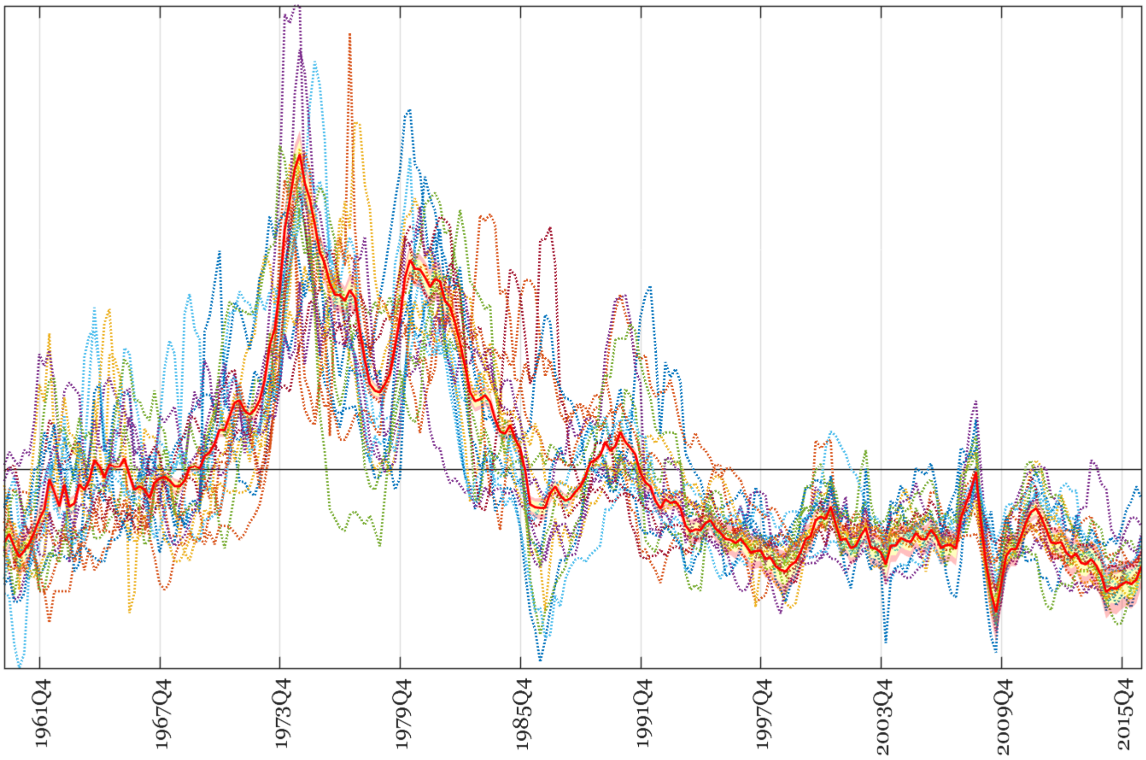


Figure 3: MAI-SV estimated common factor (with posterior bands) Vs Data

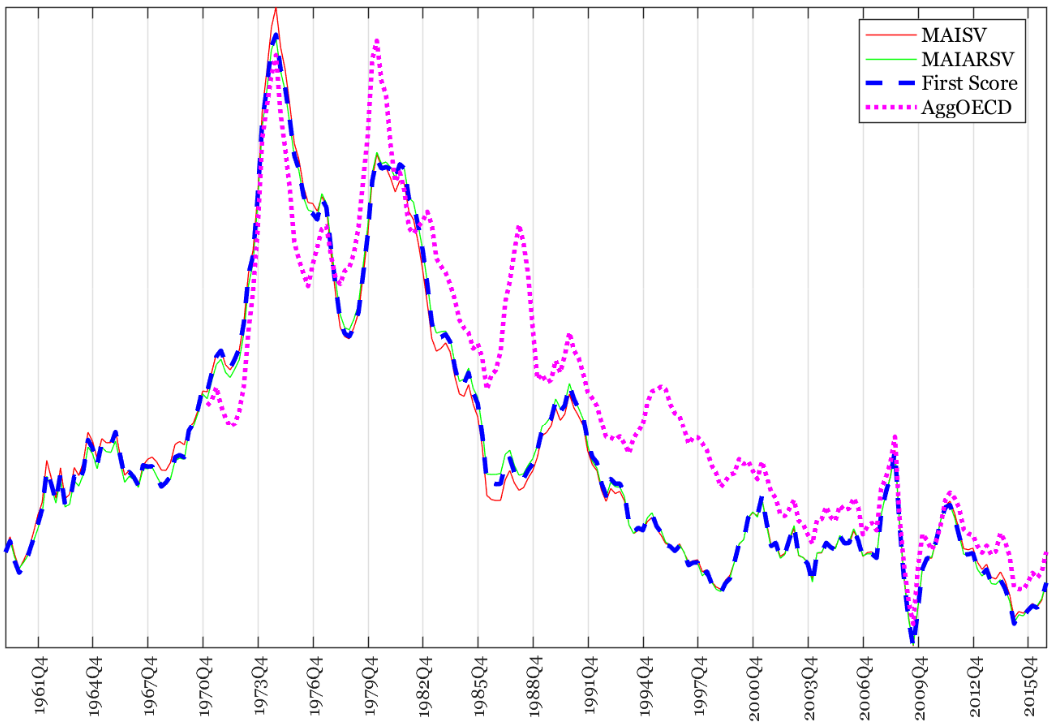


Figure 4: Comparing common factor

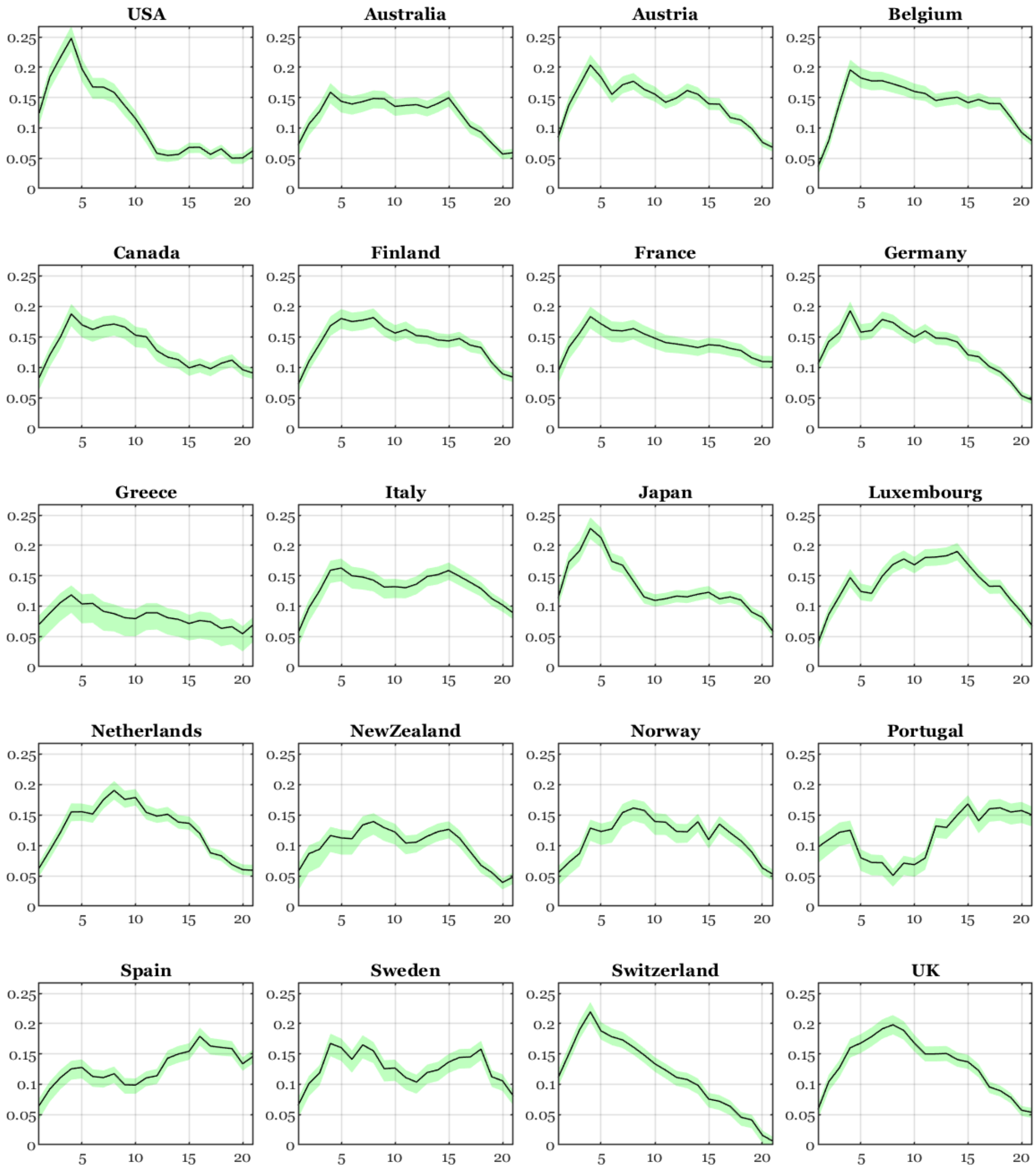


Figure 5: Responses of country inflation rates to a unitary impulse of the global inflation factor

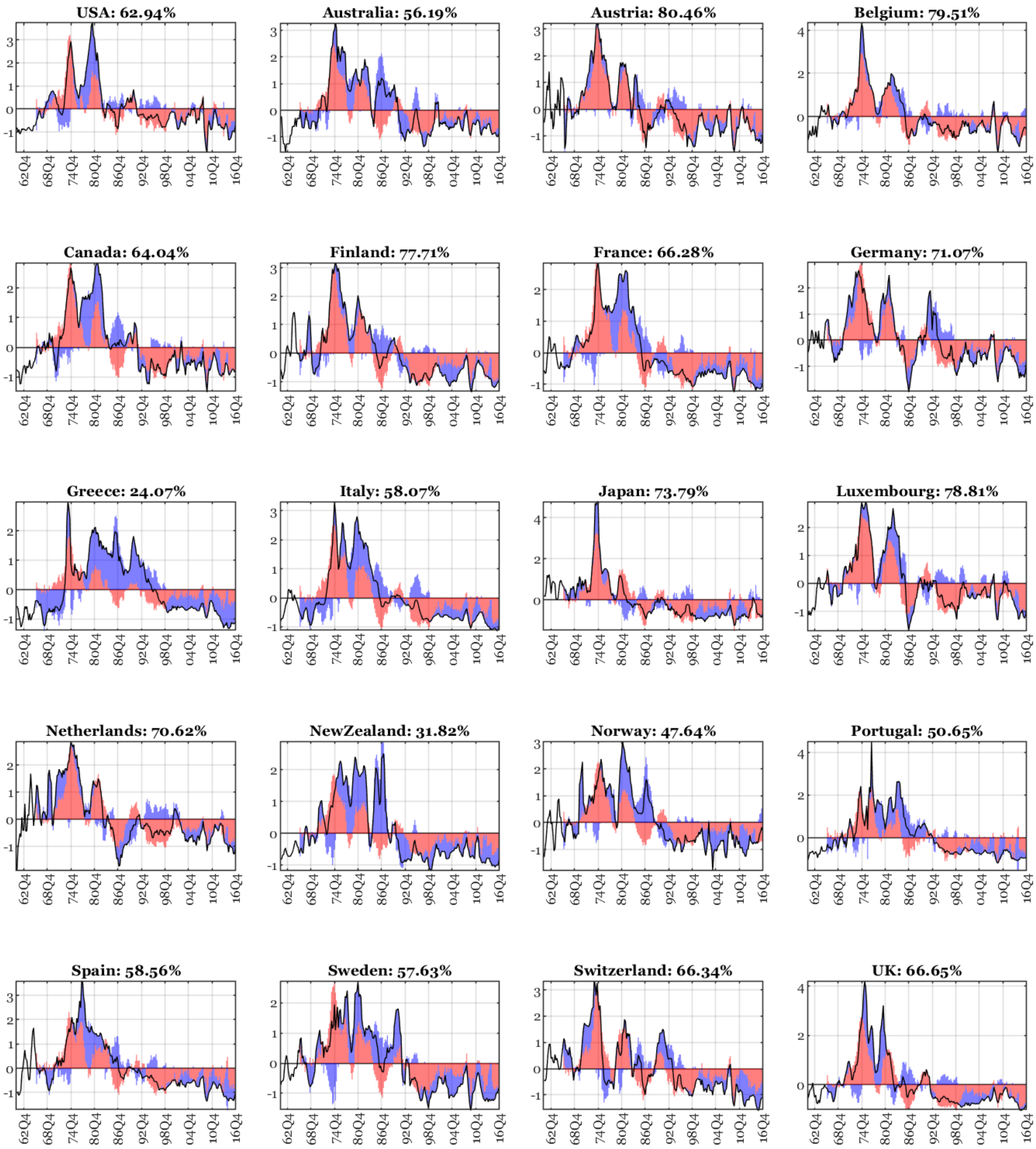


Figure 6: MAI-AR-SV, Inflation levels decomposition into common (red) and idiosyncratic (blue) components

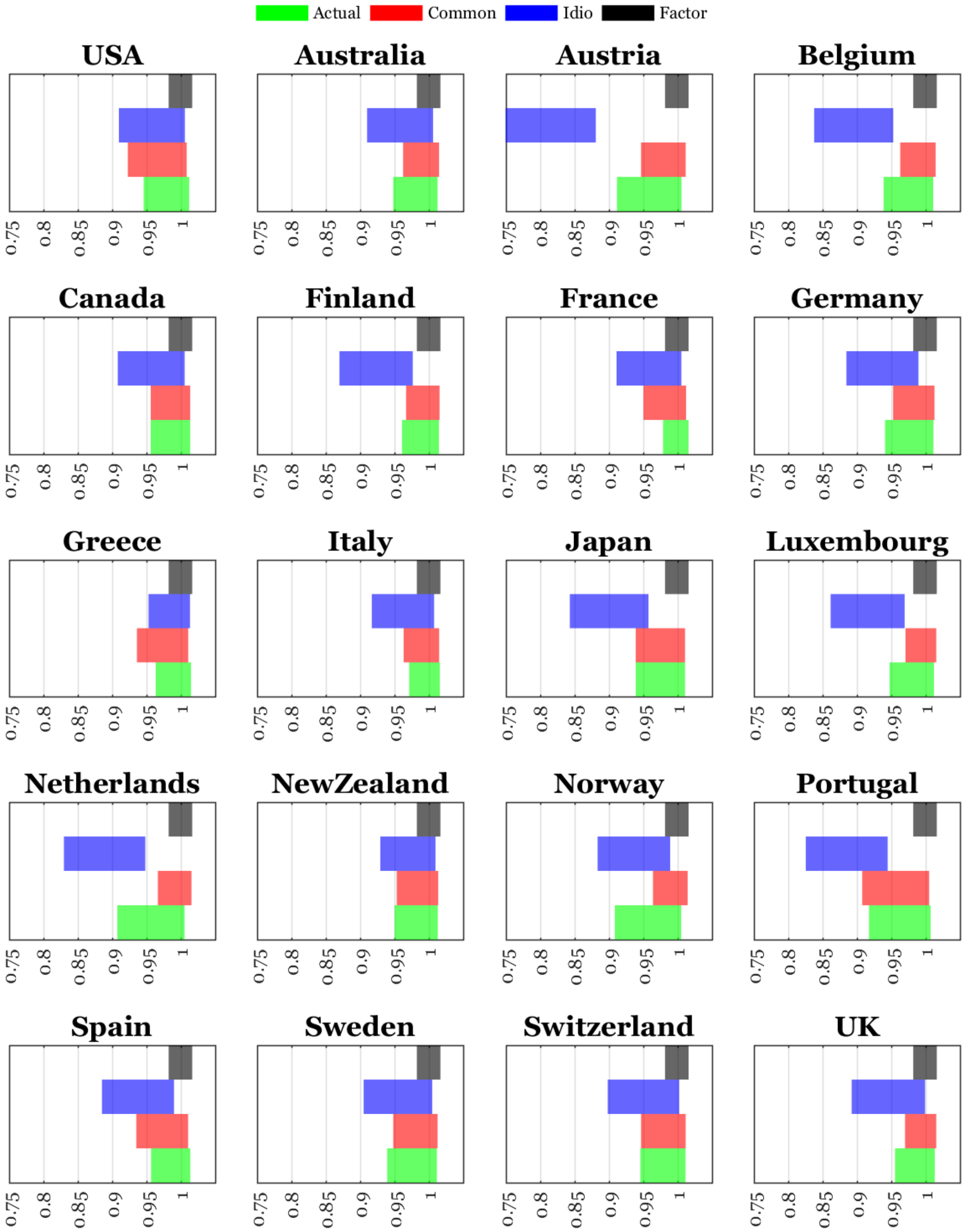


Figure 7: Largest Autoregressive Root (90% confidence intervals) CPI inflation levels, components, and global factor.

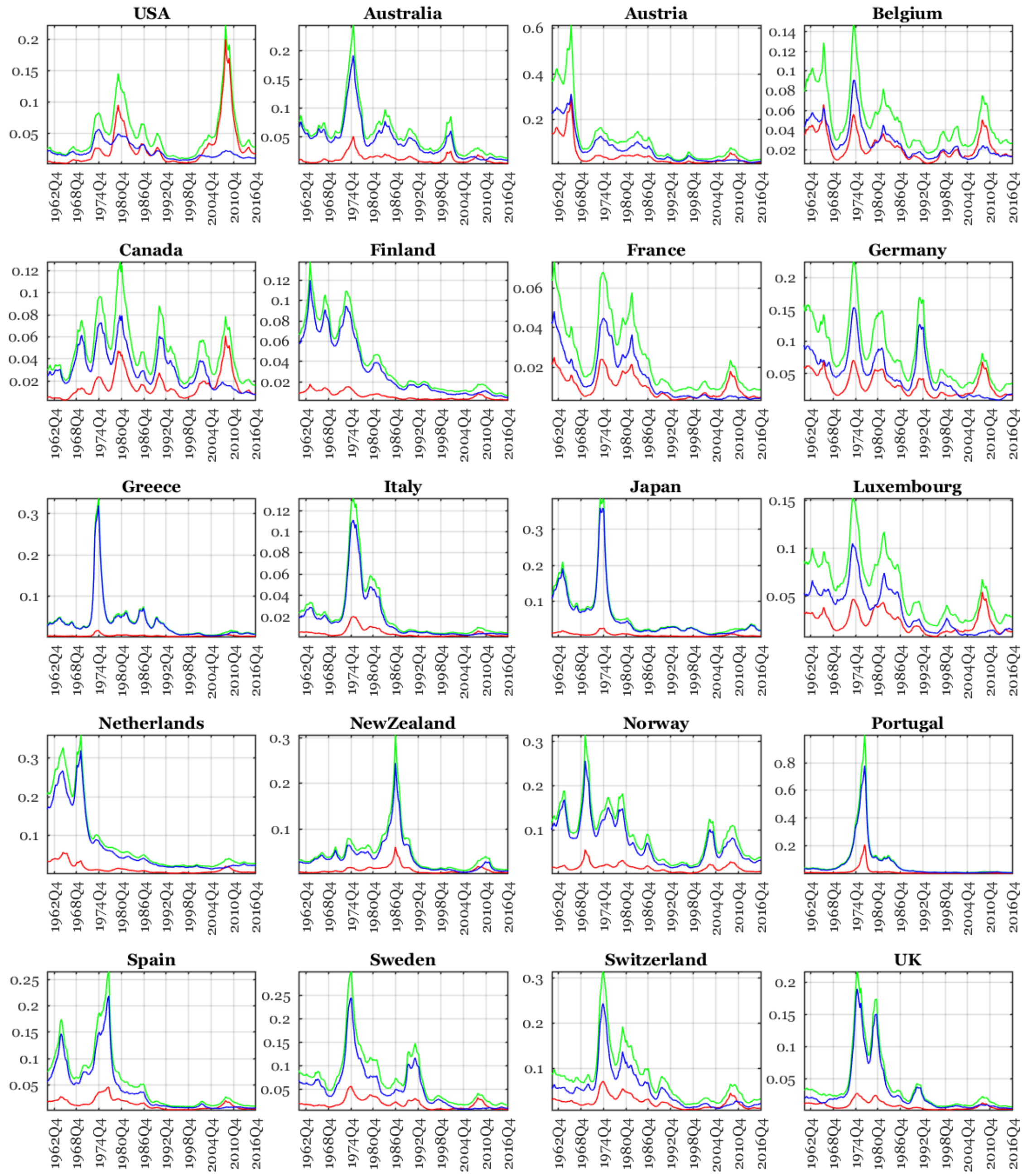


Figure 8: MAI-AR-SV, Residuals' Volatility, TV decomposition, Common (red), Idio (blue), total (green)

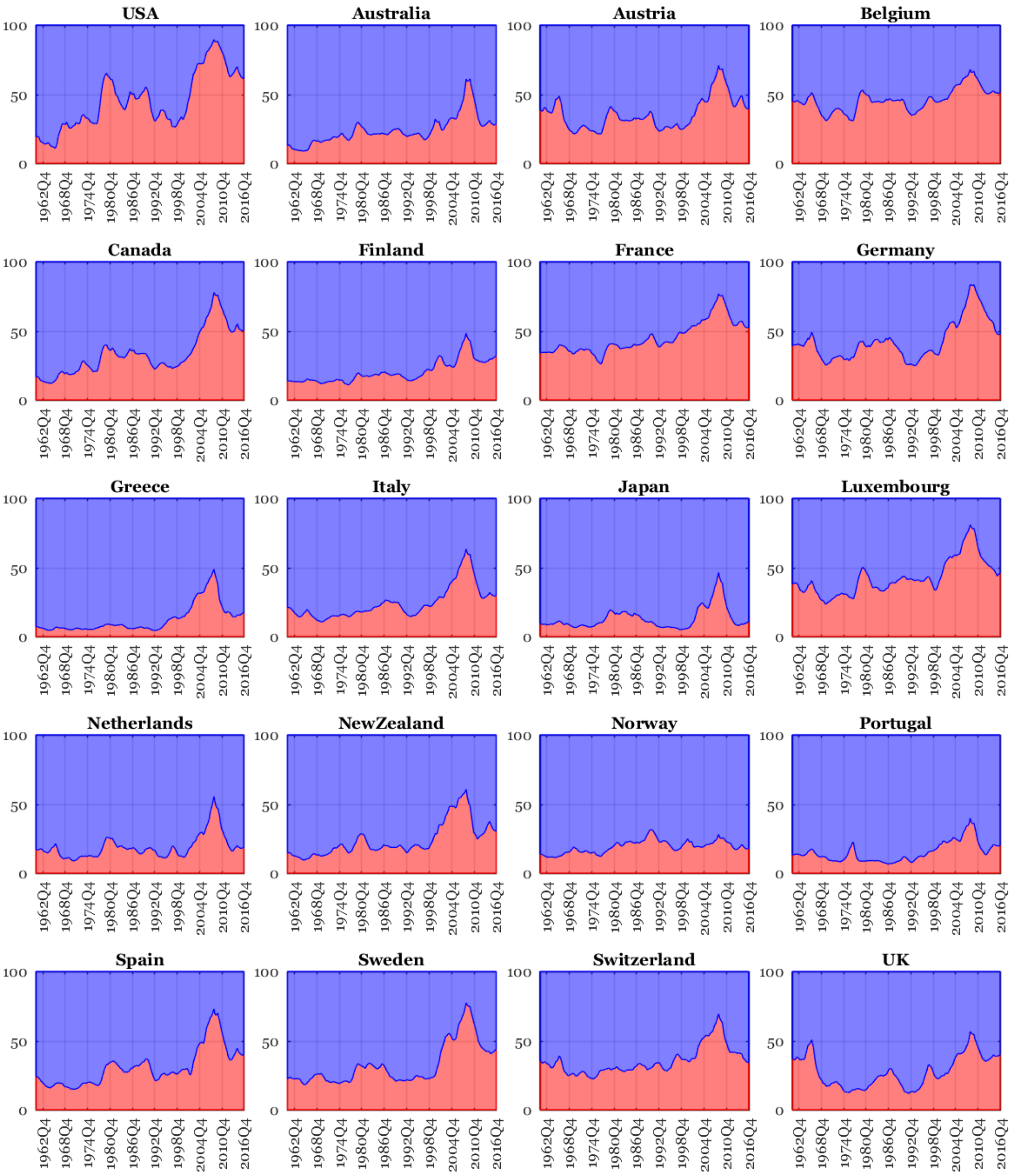


Figure 9: MAI-AR-SV, Residuals' Volatility, TV decomposition shares (%), Common (red), Idio (blue)

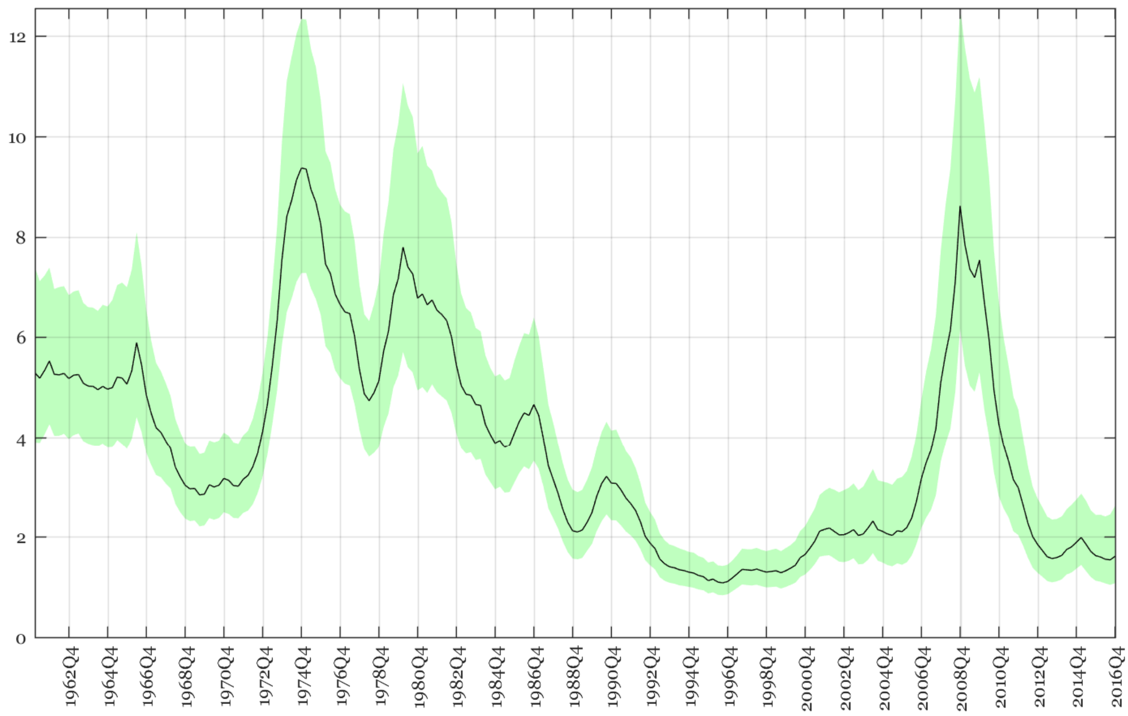


Figure 10: MAI-AR-SV, Global Factor Volatility, Posterior bands.

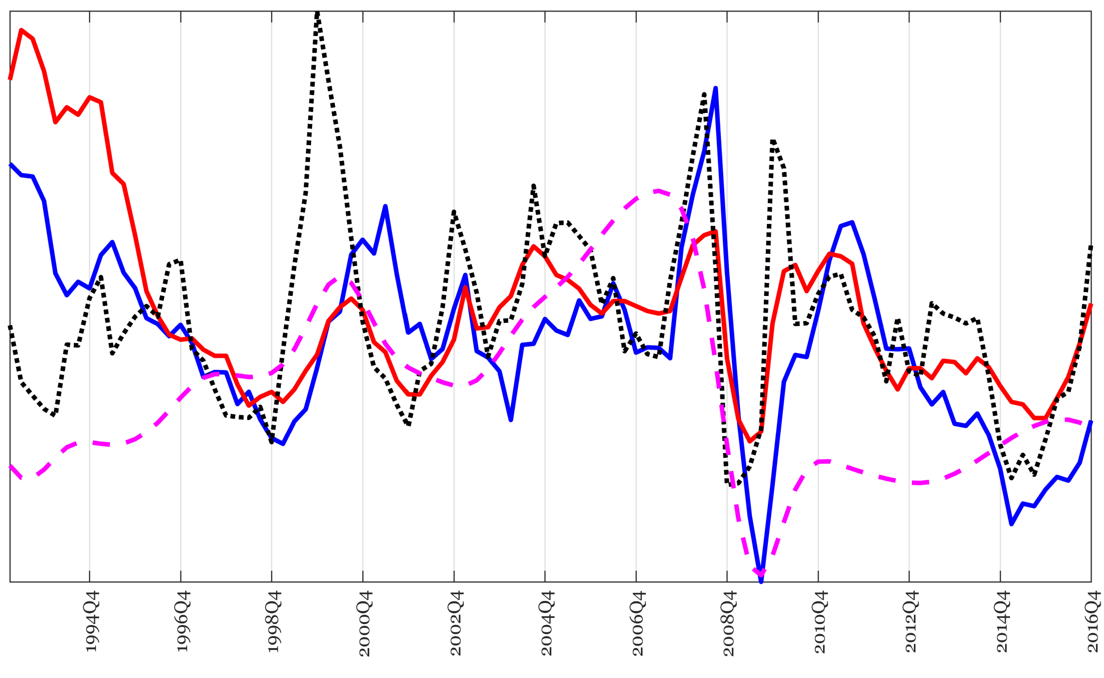


Figure 11: Comparing MAI component with Chinese PPI, Oil Inflation and Global Output Gap

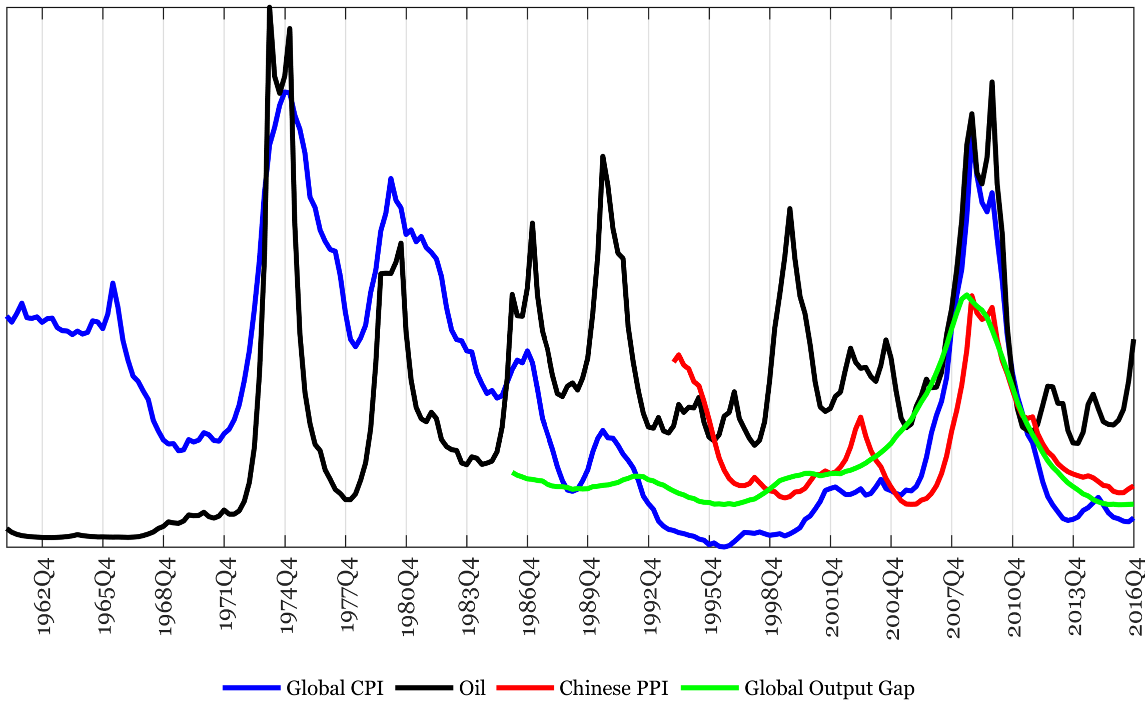


Figure 12: Median Volatilities of Global Factor (MAI-AR-SV), Oil inflation (AR-SV), Chinese PPI (AR-SV) and Global Output Gap (AR-SV)

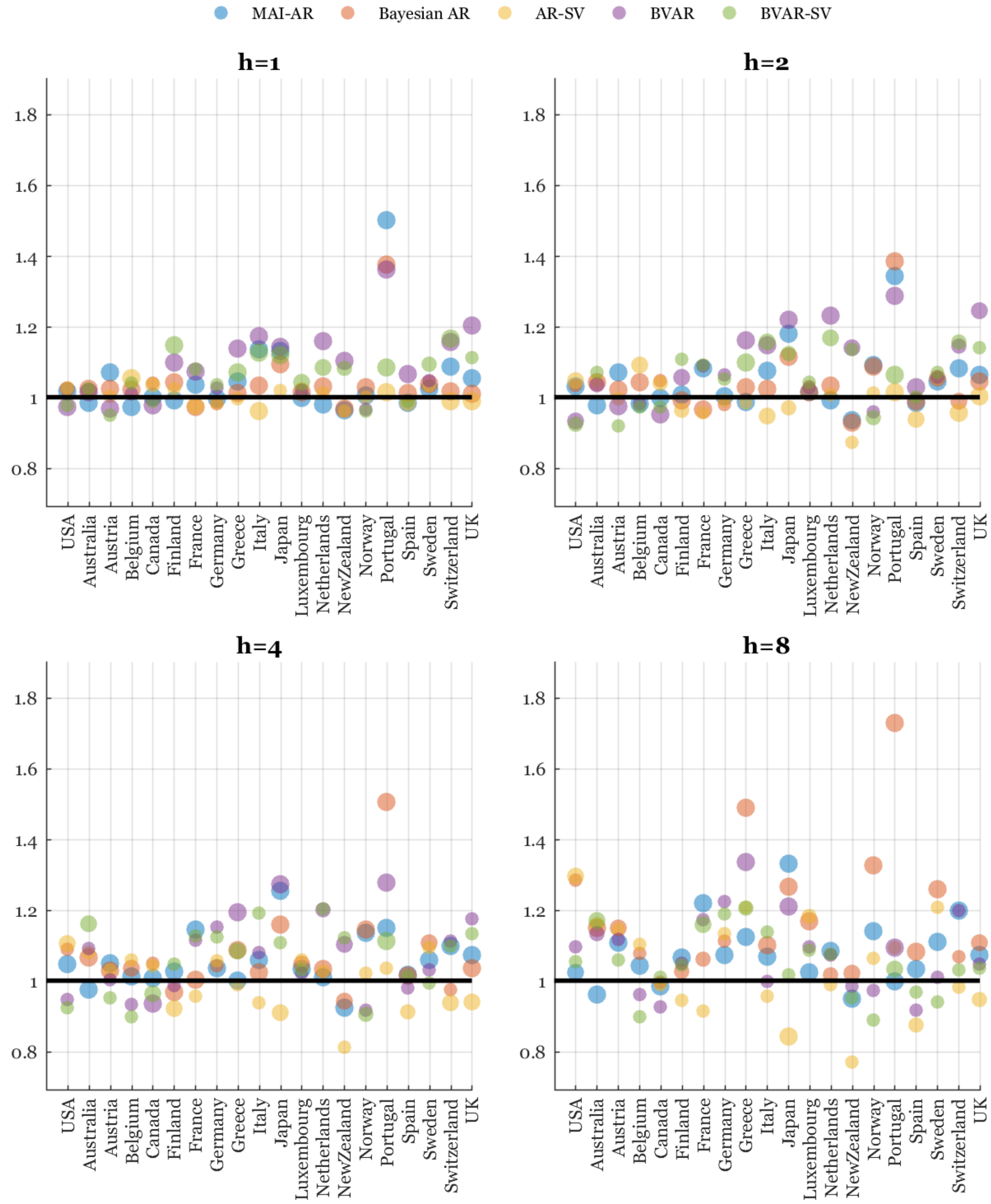
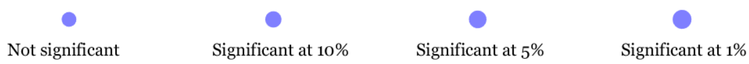


Figure 13: Relative Root Mean Squared Forecast Errors (ratios with MAI-AR-SV)

The round filled marker is larger when the difference is significant according to the Diebold Mariano *t*-statistic, see legend below.



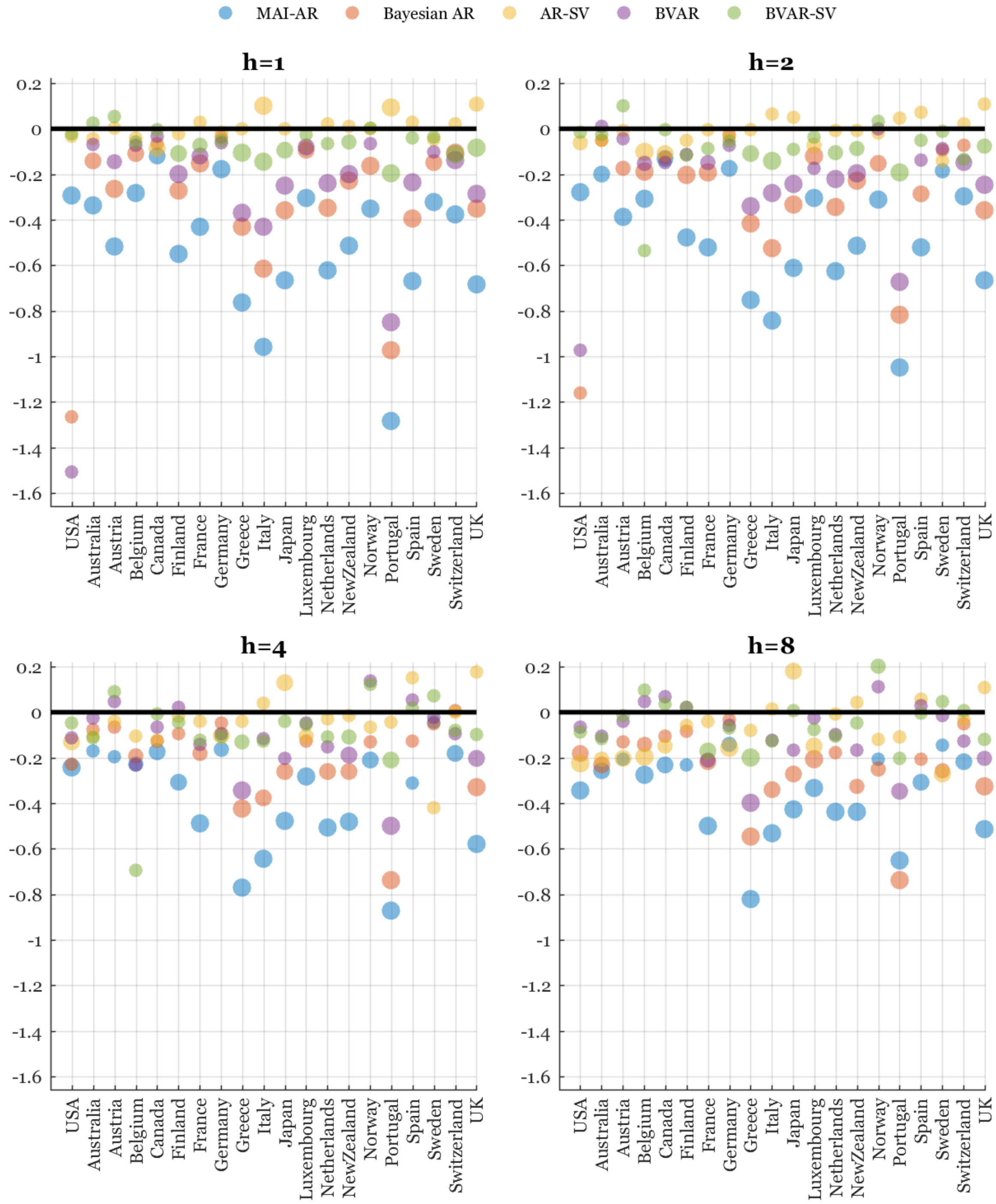
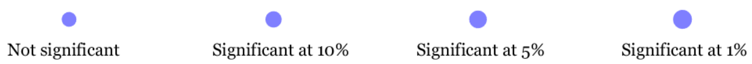


Figure 14: Relative Log Predictive Scores (differences with MAI-AR-SV)

The round filled marker is larger when the difference is significant according to the Diebold Mariano t-statistic, see legend below.



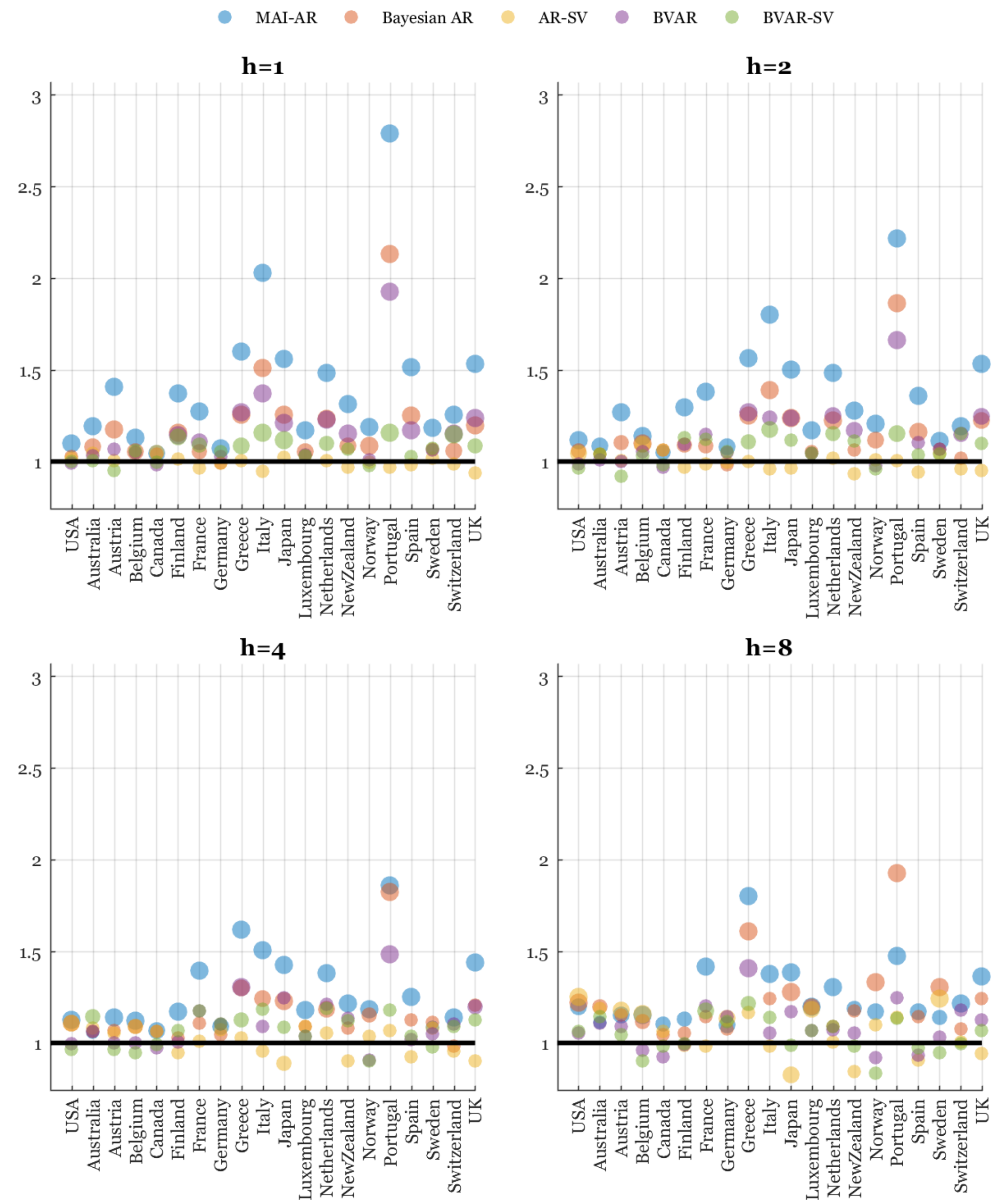
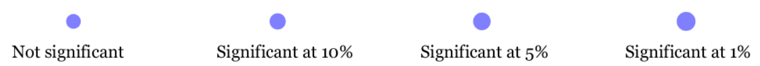


Figure 15: Relative Continuous Rank Probability Scores (ratios with MAI-AR-SV)

The round filled marker is larger when the difference is significant according to the Diebold Mariano t-statistic, see legend below.



Online Appendix for "The Global Component of Inflation Volatility"

The three sections of this online Appendix provide additional details on Gibbs Sampler algorithm for the MAI-AR-SV model (Appendix A), the forecasting evaluation tables (Appendix B), and the empirical results based on core inflation data (Appendix C).

Appendix A Detailed steps of the Gibbs Sampler for the estimation of the MAI-AR-SV model

A.1 Stacking of the model

The MAI-AR-SV model written

$$y_t = \sum_{\ell=1}^{q} \Gamma_\ell \cdot y_{t-\ell} + \sum_{\ell=1}^{p} A_\ell \cdot B_0 \cdot y_{t-\ell} + u_t, \tag{11}$$

decomposes y_t into the sum of the q autoregressive terms and the sum of the p terms depending on the r common components $B_0 \cdot y_{t-\ell}$, plus an error term. The equation in (11) can be stated in a more compact formulation.

Focusing on the first summation, each Γ_ℓ is a diagonal matrix where the main diagonal is the $n \times 1$ vector γ_ℓ:

$$\Gamma_\ell = \begin{bmatrix} \gamma_{1,\ell} & 0 & \dots & 0 \\ 0 & \gamma_{2,\ell} & \ddots & 0 \\ 0 & \ddots & \ddots & 0 \\ 0 & \dots & 0 & \gamma_{n,\ell} \end{bmatrix} = Diag \left(\begin{bmatrix} \gamma_{1,\ell} \\ \gamma_{2,\ell} \\ \vdots \\ \gamma_{n,\ell} \end{bmatrix} \right) = Diag\left(\gamma_\ell\right).$$

Recalling that $y_{t-\ell}$ is a $n \times 1$ vector of observables, the following holds:

$$\begin{aligned} \Gamma_\ell \cdot y_{t-\ell} &= Diag\left(\gamma_\ell\right) \cdot y_{t-\ell}, \\ &= Diag\left(y_{t-\ell}\right) \cdot \gamma_\ell. \end{aligned}$$

If we define $\mathcal{Y}_{t-\ell} \equiv Diag\,(y_{t-\ell})$, we can write:

$$\sum_{\ell=1}^{q} \Gamma_\ell \cdot y_{t-\ell} = \sum_{\ell=1}^{q} \mathcal{Y}_{t-\ell} \cdot \gamma_\ell = \begin{bmatrix} \mathcal{Y}_{t-1} & \mathcal{Y}_{t-2} & \dots & \mathcal{Y}_{t-q} \end{bmatrix} \cdot \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \vdots \\ \gamma_q \end{bmatrix}.$$

Defining the $n \times nq$ matrix $\mathcal{X}_t$ as $\mathcal{X}_t \equiv \begin{bmatrix} \mathcal{Y}_{t-1} & \mathcal{Y}_{t-2} & \dots & \mathcal{Y}_{t-q} \end{bmatrix}$ and the $nq \times 1$ vector γ as $\gamma \equiv \begin{bmatrix} \gamma_1' & \gamma_2' & \dots & \gamma_q' \end{bmatrix}'$, we can finally formulate the first summation more compactly as:

$$\sum_{\ell=1}^{q} \Gamma_\ell \cdot y_{t-\ell} = \mathcal{X}_t \cdot \gamma.$$

To express in compact form the second summation, we need to define the $rp \times 1$ vector of the stacked "observable" factors Z_t as

$$Z_t \equiv \begin{bmatrix} B_0 y_{t-1} \\ \vdots \\ B_0 y_{t-p} \end{bmatrix} = (I_p \otimes B_0) \cdot x_t = vec\,(B_0 \cdot x_t^\bullet)\,,$$

where $x_t^\bullet$ is the $n \times p$ matrix $x_t^\bullet = \begin{bmatrix} y_{t-1} & \dots & y_{t-p} \end{bmatrix}$ and $x_t \equiv vec\,(x_t^\bullet)$.

Stacking the $n \times r$ matrices A_ℓ in the $n \times rp$ matrix $A = \begin{bmatrix} A_1 & \dots & A_p \end{bmatrix}$, we can write the second summation as:

$$\sum_{\ell=1}^{p} A_\ell \cdot B_0 \cdot y_{t-\ell} = \begin{bmatrix} A_1 & \dots & A_p \end{bmatrix} \begin{bmatrix} B_0 y_{t-1} \\ \vdots \\ B_0 y_{t-p} \end{bmatrix} = A \cdot Z_t.$$

We can eventually write the compact formulation of the model in (11):

$$y_t = \underbrace{\mathcal{X}_t}_{n \times nq} \cdot \underbrace{\gamma}_{nq \times 1} + \underbrace{A}_{n \times rp} \cdot \underbrace{Z_t}_{rp \times 1} + u_t. \tag{12}$$

A.2 Step 1: Draw a history of volatilities $\{\sigma_t\}_{t=1}^T$

This step concerns the draw of the (unobservable) stochastic volatilities conditional on $\theta = \{\gamma, A, B_0, G, Q_\sigma\}$ and the indexes of Normal components of the mixture $\{S_t\}_{t=1}^T$. The order of these steps is in line with Del Negro & Primiceri (2015).

In order to draw the volatilities, we start from (12) and apply the triangular reduction of the errors $u_t = G^{-1}\Sigma_t\varepsilon_t$ to transform the model in the following way:

$$y_t = \mathcal{X}_t \cdot \gamma + A \cdot Z_t + G^{-1}\Sigma_t\varepsilon_t,$$

$$\underbrace{G\left(y_t - \mathcal{X}_t \cdot \gamma - A \cdot Z_t\right)}_{\widetilde{y}_t} = \Sigma_t\varepsilon_t,$$

$$\widetilde{y}_t = Diag\left(\sigma_t\right) \cdot \varepsilon_t.$$

Moreover, using the Hadamard product operator, we can first write

$$\Sigma_t\varepsilon_t = \Sigma_t \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \vdots \\ \varepsilon_{n,t} \end{bmatrix} = \begin{bmatrix} \sigma_{1,t}\ \varepsilon_{1,t} \\ \sigma_{2,t}\ \varepsilon_{2,t} \\ \vdots \\ \sigma_{n,t}\ \varepsilon_{n,t} \end{bmatrix} = \sigma_t \odot \varepsilon_t,$$

and taking the square element by element on both sides of $\widetilde{y}_t = \Sigma_t\varepsilon_t$, we obtain:

$$\left(\widetilde{y}_t\right)^{\cdot 2} = \left(\Sigma_t\varepsilon_t\right)^{\cdot 2} = \sigma_t^{\cdot 2} \odot \varepsilon_t^{\cdot 2} \quad \iff \quad \begin{bmatrix} \widetilde{y}_{1,t}^2 \\ \widetilde{y}_{2,t}^2 \\ \vdots \\ \widetilde{y}_{n,t}^2 \end{bmatrix} = \begin{bmatrix} \sigma_{1,t}^2\ \varepsilon_{1,t}^2 \\ \sigma_{2,t}^2\ \varepsilon_{2,t}^2 \\ \vdots \\ \sigma_{n,t}^2\ \varepsilon_{n,t}^2 \end{bmatrix}.$$

We add on the left hand-side a small constant[19] $\bar{c} = 10^{-3}$ and apply the logarithm on

[19] The addition of a small constant term has numerical stability purposes, as explained in Fuller (2009) and Primiceri (2005).

both sides[20] in order to break the non-linearity, to obtain:

$$\underbrace{\begin{bmatrix} \log\ \left(\widetilde{y}_{1,t}^2 + \bar{c}\right) \\ \log\ \left(\widetilde{y}_{2,t}^2 + \bar{c}\right) \\ \vdots \\ \log\ \left(\widetilde{y}_{n,t}^2 + \bar{c}\right) \end{bmatrix}}_{\widetilde{y}_t^*} = 2 \begin{bmatrix} \log\ \sigma_{1,t} \\ \log\ \sigma_{2,t} \\ \vdots \\ \log\ \sigma_{n,t} \end{bmatrix} + \begin{bmatrix} \log\ \varepsilon_{1,t}^2 \\ \log\ \varepsilon_{2,t}^2 \\ \vdots \\ \log\ \varepsilon_{n,t}^2 \end{bmatrix},$$

$$\widetilde{y}_t^* = 2 \log \sigma_t + \log\ \left[(\varepsilon_t)^{\cdot 2}\right].$$

Since $\varepsilon_t \overset{iid}{\sim} \mathcal{MN}(\mathbf{0}, I_n)$, each term $\log \varepsilon_{j,t}^2$ has a $\log \chi_1^2$ distribution, and $\log\ \left[(\varepsilon_t)^{\cdot 2}\right]$ is a vector of independent $\log \chi_1^2$ random variables.

Then, conditioning on $\{S_t\}_{t=1}^T$, i.e. the sequence of $n \times 1$ vectors that specify the indexes of Normal components of the mixture, the vector $\log\ \left[(\varepsilon_t)^{\cdot 2}\right]$ has the following Gaussian distribution:

$$\log\ \left[(\varepsilon_t)^{\cdot 2}\right] \big| \{S_t\}_{t=1}^T \sim \mathcal{MN}\left(\underbrace{\begin{bmatrix} m_{s_{1,t}}^m \\ m_{s_{2,t}}^m \\ \vdots \\ m_{s_{n,t}}^m \end{bmatrix}}_{\varphi_t}, \underbrace{\begin{bmatrix} m_{s_{1,t}}^v & 0 & \dots & 0 \\ 0 & m_{s_{2,t}}^v & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & m_{s_{n,t}}^v \end{bmatrix}}_{\Upsilon_t} \right)$$

Hence, given the Gaussian distribution of $\log\ \left[(\varepsilon_t)^{\cdot 2}\right] \big| \{S_t\}_{t=1}^T$, we can define the needed state space form as:

$$\widetilde{y}_t^* = \varphi_t + 2 \log \sigma_t + \zeta_t, \qquad \zeta_t \overset{i}{\sim} \mathcal{MN}\left(\ \mathbf{0},\ \Upsilon_t\ \right),$$

$$\log \sigma_t = \log \sigma_{t-1} + \nu_{\sigma,t}, \qquad \nu_{\sigma,t} \overset{iid}{\sim} \mathcal{MN}\left(\mathbf{0},\ \underbrace{Q_\sigma}_{n \times n} \right).$$

At this point, the Forward Filtering Backward Sampling (FFBS) procedure, introduced by Carter & Kohn (1994), can be implemented to draw a history of volatilities $\{\sigma_t\}_{t=1}^T$. The procedure is described below. For simplicity we define $\widetilde{\sigma}_t \equiv \log \sigma_t$, since the FFBS

[20]Noticing that $\log \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \log a \\ \log b \end{bmatrix}$, and $\begin{bmatrix} a \\ b \end{bmatrix}^{\cdot 2} = \begin{bmatrix} a^2 \\ b^2 \end{bmatrix}$.

procedure is implemented on the log-volatilities.

The filter can be initialized at the following values:

$$\widetilde{\sigma}_{0|0} = \log(\bar{\sigma}), \qquad P_{\sigma,0|0} = \bar{P}_\sigma,$$

where $\log(\bar{\sigma})$ and $\bar{P}_\sigma$ are respectively the mean and the covariance matrix of the prior distribution of $\log \sigma_0$.

Recursively, for each $\left(\widetilde{\sigma}_{t-1|t-1},\ P_{\sigma,t-1|t-1}\right)$, we compute the filter:

$$\begin{aligned}
P_{\sigma,t|t-1} &= P_{\sigma,t-1|t-1} + Q_\sigma, \\
K_{\sigma,t} &= 2P_{\sigma,t|t-1}\left(4P_{\sigma,t|t-1} + \Upsilon_t\right)^{-1}, \\
\widetilde{\sigma}_{t|t} &= \widetilde{\sigma}_{t-1|t-1} + K_{\sigma,t}\left(\widetilde{y}_t^* - 2\widetilde{\sigma}_{t-1|t-1} - \varphi_t\right), \\
P_{\sigma,t|t} &= P_{\sigma,t|t-1} - 2K_{\sigma,t}P_{\sigma,t|t-1}.
\end{aligned}$$

Having an entire set of updating and prediction steps $\left(\widetilde{\sigma}_{t|t},\ P_{\sigma,t|t},\ P_{\sigma,t|t-1}\right)_{t=1}^{T}$, we start to sample backward, beginning by sampling $\widetilde{\sigma}_T$ from $\mathcal{MN}\left(\ \widetilde{\sigma}_{T|T},\ P_{\sigma,T|T}\ \right)$, and then for each $t\ \in \{T-1, T-2, \ldots, 2, 1\}$ we sample recursively each $\widetilde{\sigma}_t$ from $\mathcal{MN}\left(\ \widetilde{\sigma}_{t|t+1},\ P_{\sigma,t|t+1}\ \right)$ where:

$$\begin{aligned}
\widetilde{\sigma}_{t|t+1} &= \widetilde{\sigma}_{t|t} + P_{\sigma,t|t}P_{\sigma,t+1|t}^{-1}\left(\widetilde{\sigma}_{t+1} - \widetilde{\sigma}_{t|t}\right), \\
P_{\sigma,t|t+1} &= P_{\sigma,t|t} - P_{\sigma,t|t}P_{\sigma,t+1|t}^{-1}P_{\sigma,t|t}.
\end{aligned}$$

A.3 Step 2(a): Draw $\theta | \{\sigma_t\}_{t=1}^T$

A.3.1 Substep 2(a).i: Draw the covariance of volatilities' innovations Q_σ

Conditioning on $\{\sigma_t\}_{t=0}^T$, we can draw the covariance matrix Q_σ. Indeed, recall that:

$$\log \sigma_t = \log \sigma_{t-1} + \nu_{\sigma,t}, \qquad \nu_{\sigma,t} \overset{iid}{\sim} \mathcal{MN}\left(\mathbf{0},\ \underbrace{Q_\sigma}_{n\times n}\right).$$

But then, having a complete history of the sigmas, given the random walk law of motion, is equivalent to having a complete histories of innovations $\nu_{\sigma,t}$. Stacking the $\nu_{\sigma,t}$ across time, we get:

$$\underbrace{\nu_\sigma^*}_{n\times T} = \begin{bmatrix}\nu_{\sigma,1} & \nu_{\sigma,2} & \dots & \nu_{\sigma,T}\end{bmatrix},$$

and we can easily compute the innovations sum of squares matrix:

$$\underbrace{S_\sigma}_{n\times n} = \underbrace{\nu_\sigma^*}_{n\times T}\underbrace{\nu_\sigma^{*\prime}}_{T\times n}.$$

If the prior on the matrix Q_σ is a $n \times n$ Inverse Wishart with scale matrix $\bar{Q}_\sigma$ and degrees of freedom $\tau_{\sigma,0}$:

$$Q_\sigma \sim \mathcal{IW}_n\left(\bar{Q}_\sigma,\ \tau_{\sigma,0}\right),$$

then the posterior is conjugate and given by:

$$Q_\sigma \big|\left\{\sigma_t^i\right\}_{t=0}^T \sim \mathcal{IW}_n\left(S_\sigma + \bar{Q}_\sigma,\ \tau_{\sigma,0} + T\right).$$

A.3.2 Substep 2(a).ii: draw AR-coefficients γ

Starting from (12), and conditioning on B_0 and A, we can obtain the following linear regression model with common coefficients and variable specific regressors:

$$\begin{aligned} y_t &= \mathcal{X}_t \cdot \gamma + \ A \cdot\ Z_t + u_t, \\ y_t - A \cdot Z_t &= \mathcal{X}_t \cdot \gamma + u_t, \\ y_t^\circ &= \mathcal{X}_t \cdot \gamma + u_t, \end{aligned}$$

with

$$u_t \overset{i}{\sim} \mathcal{MN}\left(\mathbf{0},\ \underbrace{\Omega_t}_{n\times n}\right), \qquad u_t = G^{-1}\Sigma_t\varepsilon_t, \qquad \varepsilon_t \overset{iid}{\sim} \mathcal{MN}\left(\ \mathbf{0},\ I_n\ \right).$$

Considering separate equations to estimate the AR coefficients contained in γ would ignore the cross-correlations of the innovations in u_t. Since within the GS we draw directly the elements g in the matrix G and the stochastic volatilities σ_t in Σ_t, for efficiency purposes we can compute the following transformation of the equation:

$$
\begin{aligned}
y_t^{\circ} &= \mathcal{X}_t \cdot \gamma + u_t, \\
y_t^{\circ} &= \mathcal{X}_t \cdot \gamma + G^{-1}\Sigma_t \varepsilon_t, \\
\Sigma_t^{-1} G \cdot y_t^{\circ} &= \Sigma_t^{-1} G \cdot \mathcal{X}_t \cdot \gamma + \underbrace{\Sigma_t^{-1} G \cdot G^{-1} \Sigma_t}_{I_n} \cdot \varepsilon_t, \\
\Sigma_t^{-1} G \cdot y_t^{\circ} &= \Sigma_t^{-1} G \cdot \mathcal{X}_t \cdot \gamma + \varepsilon_t, \\
\widetilde{y_t^{\circ}} &= \widetilde{\mathcal{X}_t} \cdot \gamma + \varepsilon_t.
\end{aligned}
$$

We finally obtain a multivariate linear regression with homoskedastic residuals and unitary diagonal covariance matrix:

$$
\widetilde{y_t^{\circ}} = \widetilde{\mathcal{X}_t} \cdot \gamma + \varepsilon_t, \qquad \varepsilon_t \overset{iid}{\sim} \mathcal{MN}\left(\mathbf{0},\ I_n \right).
$$

The transformed model can be stacked in columns:

$$
\begin{aligned}
&\begin{bmatrix} \widetilde{y_1^{\circ}} \\ \vdots \\ \widetilde{y_T^{\circ}} \end{bmatrix} = \begin{bmatrix} \widetilde{\mathcal{X}_1} \\ \vdots \\ \widetilde{\mathcal{X}_T} \end{bmatrix} \cdot \gamma + \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_T \end{bmatrix}, \\
&\underbrace{\widetilde{Y}^{\circ}}_{nT \times 1} = \underbrace{\widetilde{\mathcal{X}}}_{nT \times nq} \cdot \underbrace{\gamma}_{nq \times 1} + \underbrace{\varepsilon^{\circ}}_{nT \times 1}, \qquad \varepsilon^{\circ} \sim \mathcal{MN}\left(\mathbf{0},\ I_T \otimes I_n \right).
\end{aligned}
$$

With the stacked version of the model, adopting the Normal semi-conjugate prior for the vector of coefficients γ:

$$
\gamma \sim \mathcal{MN}\left(\bar{\gamma},\ V_{\gamma} \right),
$$

we can eventually draw from the posterior distribution:

$$
\gamma \sim \mathcal{MN}\left(\widetilde{\gamma},\ \widetilde{V}_{\gamma} \right),
$$

where

$$
\widetilde{\gamma} = \widetilde{V}_{\gamma} \cdot \left(\widetilde{\mathcal{X}}' \cdot \widetilde{Y}^{\circ} + V_{\gamma}^{-1} \cdot \bar{\gamma} \right), \qquad \widetilde{V}_{\gamma} = \left(\widetilde{\mathcal{X}}' \cdot \widetilde{\mathcal{X}} + V_{\gamma}^{-1} \right)^{-1}.
$$

A.3.3 Substep 2(a).iii: Draw the loadings A

To draw the loadings contained in A, we restate (12) as:

$$
\begin{aligned}
y_t - \mathcal{X}_t \cdot \gamma &= A \cdot Z_t + u_t, \\
y_t^{\bullet} &= A \cdot Z_t + u_t,
\end{aligned}
$$

and stack it as:

$$
\begin{bmatrix} y_1^{\bullet\prime} \\ y_2^{\bullet\prime} \\ \vdots \\ y_T^{\bullet\prime} \end{bmatrix} = \begin{bmatrix} Z_1' \\ Z_2' \\ \vdots \\ Z_T' \end{bmatrix} A' + \begin{bmatrix} u_1' \\ u_2' \\ \vdots \\ u_T' \end{bmatrix},
$$

$$
\underbrace{y^{\bullet}}_{T \times n} = \underbrace{Z}_{T \times rp} \cdot \underbrace{A'}_{rp \times n} + u.
$$

Defining $a \equiv vec\,(A')$, and exploiting the Kronecker product's properties, this form can be vectorized and transformed in:

$$
\begin{aligned}
vec\,(y^{\bullet}) &= vec\,(Z \cdot A' \cdot I_n) + vec\,(u)\,, \\
\underbrace{Y^{\bullet}}_{nT \times 1} &= \underbrace{(I_n \otimes Z)}_{n \times nrp} \cdot \underbrace{a}_{nrp \times 1} + U,
\end{aligned}
$$

where $\underbrace{U}_{nT \times 1}$ has the following distribution:

$$
U \sim \mathcal{MN}\left(\mathbf{0},\ \underbrace{V_u}_{n \times n}\right),
$$

and

$$V_u \equiv \begin{bmatrix} \Omega_1^{(1,1)} & 0 & \cdots & 0 & \cdots & \cdots & \Omega_1^{(1,n)} & 0 & \cdots & 0 \\ 0 & \Omega_2^{(1,1)} & \ddots & \vdots & \cdots & \cdots & 0 & \Omega_2^{(1,n)} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 & \cdots & \cdots & \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \Omega_T^{(1,1)} & \cdots & \cdots & 0 & \cdots & 0 & \Omega_T^{(1,n)} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ \Omega_1^{(n,1)} & 0 & \cdots & 0 & \cdots & \cdots & \Omega_1^{(n,n)} & 0 & \cdots & 0 \\ 0 & \Omega_2^{(n,1)} & \ddots & \vdots & \cdots & \cdots & 0 & \Omega_2^{(n,n)} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 & \cdots & \cdots & \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \Omega_T^{(n,1)} & \cdots & \cdots & 0 & \cdots & 0 & \Omega_T^{(nn)} \end{bmatrix}$$

$$= \sum_{t=1}^{T} \left[\Omega_t \otimes (e_t \cdot e_t') \right].$$

To use an informative prior on a we follow the approach by Gelman et al. (2014). The strategy incorporates the prior as observations. Considering a multivariate Normal prior with the following moments:

$$a \sim \mathcal{MN}\left(\ \bar{a},\ V_a\ \right),$$

it is possible to augment the model with nrp observations that express the prior information:

$$\begin{bmatrix} Y^{\bullet} \\ \bar{a} \end{bmatrix} = \begin{bmatrix} I_n \otimes Z \\ I_{nrp} \end{bmatrix} a + \begin{bmatrix} U \\ U_a \end{bmatrix},$$

$$Y^{\diamond} = Z^{\diamond} a + U^{\diamond}, \quad U^{\diamond} \sim \mathcal{MN}\left(\ \mathbf{0}_{nT+nrp},\ V^{\diamond}\ \right),$$

$$V^{\diamond} = \begin{bmatrix} V_u & \mathbf{0}_{nT \times nrp} \\ \mathbf{0}_{nrp \times nT} & V_a \end{bmatrix}.$$

A draw for a then comes from the following posterior:

$$a \sim \mathcal{MN}\left(\ \widetilde{a},\ \left(Z^{\diamond\prime} V^{\diamond -1} Z^{\diamond}\right)^{-1}\ \right),$$

$$\widetilde{a} = \left(Z^{\diamond\prime} V^{\diamond -1} Z^{\diamond}\right)^{-1} Z^{\diamond\prime} V^{\diamond -1} Y^{\diamond}.$$

In order to decrease the computational burden of this step throughout the sampling, the strategy proposed by Carriero, Clark & Marcellino (2018) is adopted, as generalized in

Carriero, Corsello & Marcellino (2018): the triangular structure of the error is exploited, and coefficients are drawn equation by equation.

A.3.4 Substep 2(a).iv: Draw the factor weights elements in B_0

Given the restrictions and the nonlinear role of B_0, a Random Walk Metropolis step on the kernel of the posterior of each element of B_0 is implemented, nested into the GS. In order to do this, we first write the likelihood of the model. Given the model in (12):

$$y_t = \mathcal{X}_t \cdot \gamma + A \cdot Z_t + u_t \ , \qquad u_t \overset{i}{\sim} \mathcal{MN} \left(\mathbf{0}, \ \Omega_t \right),$$

using the chain rule, we can write the likelihood kernel as:

$$f \left(\left\{ y_t \right\}_{t=1}^{T} \middle| \gamma, A, (\Omega_t)_{t=1}^{T}, \ B_0 \right) \propto \left(\prod_{t=1}^{T} |\Omega_t|^{-\frac{1}{2}} \right) \exp \left\{ -\frac{1}{2} \sum_{t=1}^{T} \widehat{y}_t' \cdot \Omega_t^{-1} \cdot \widehat{y}_t \right\},$$

where

$$\widehat{y}_t \equiv y_t - A \cdot Z_t - \mathcal{X}_t \cdot \gamma.$$

Now we consider the $r^* \equiv n - r$ scalar unrestricted elements of B_0, i.e. $\left(b_{0,j}\right)_{j=1}^{r^*}$. Then, $\forall j \in \{1, \ldots, r^*\}$ we can define the set $b_{0,j-} \equiv \left(b_{0,s}\right)_{s \neq j}$.

For a given prior $f\left(b_{0,j}\right)$ on each element $b_{0,j}$, we can write the kernel of the conditional posterior of $b_{0,j}$ as:

$$f_{post} \left(b_{0,j} | \left(y_t, \Omega_t\right)_{t=1}^{T}, \ A, \ b_{0,j-} \right) \propto f \left(\left(y_t\right)_{t=1}^{T} \middle| A, \ B_0, \ \left(\Omega_t\right)_{t=1}^{T} \right) \cdot f\left(b_{0,j}\right).$$

We are now ready to design the Metropolis step, separately for each j. Given the last step B_0^{i-1}, a random walk candidate is computed as:

$$b_{0,j}^{*} = b_{0,j}^{i-1} + c_j \cdot \eta_t,$$

where c_j is a scaling factor calibrated to have an acceptance rate of approximately 30%-35% and $\eta_t \overset{iid}{\sim} \mathcal{N}\left(0, v_j\right)$, with v_j being the variance of prior $f\left(b_{0,j}\right)$. The candidate draw is accepted with probability:

$$\alpha_j = \min \left\{ 1, \frac{f_{post} \left(b_{0,j}^{*} | \left(y_t, \Omega_t^{i-1}\right)_{t=1}^{T}, \ A, \ b_{0,j-}^{i-1} \right)}{f_{post} \left(b_{0,j}^{i-1} | \left(y_t, \Omega_t^{i-1}\right)_{t=1}^{T}, \ A, \ b_{0,j-}^{i-1} \right)} \right\}.$$

When the candidate is accepted, then $b^i_{0,j-} = b^*_{0,j}$, otherwise $b^i_{0,j-} = b^{i-1}_{0,j-}$. Repeating this procedure $\forall j \in \{1, \ldots, r^*\}$, we build a draw B^i_0 from the distribution of interest.

A.3.5 Substep 2(a).v: draw the off-diagonal elements in G

To draw the off-diagonal elements, we restate the model in (12) as:

$$\begin{aligned} y_t - \mathcal{X}_t \cdot \gamma - A \cdot Z_t &= G^{-1}\Sigma_t\varepsilon_t, \\ \widehat{y}_t &= G^{-1}\Sigma_t\varepsilon_t, \\ G \cdot \widehat{y}_t &= \Sigma_t\varepsilon_t. \end{aligned}$$

Removing ones from the diagonal of G, and bringing off diagonal elements on the right hand side, produces:

$$G = I_n + G^*.$$

This can be combined in the model to obtain:

$$\begin{aligned} (I_n + G^*)\ \widehat{y}_t &= \Sigma_t\ \varepsilon_t, \\ \widehat{y}_t &= -G^*\ \widehat{y}_t + \Sigma_t\ \varepsilon_t. \end{aligned}$$

Exploiting the Kronecker product's properties, we get:

$$-\ I_n \underbrace{G^*}_{n\times n} \underbrace{\widehat{y}_t}_{n\times 1} = -\underbrace{(I_n \otimes \widehat{y}_t')}_{n\times n^2} \underbrace{vec\,(G^{*\prime})}_{n^2\times 1}$$

where $vec\,(G^{*\prime})$ has zeros in positions $[(i-1)\,n + j]^{i\in\{1,\ldots,n\}}_{j\in\{1,\ldots,n\}}$. By removing the zeros, we obtain exactly the elements below the main diagonal of G gathered in the m-dimensional vector g. Removing the corresponding columns in $-\left(I_n \otimes \widehat{y}_t'\right)$ we construct the matrix W_t, which has the following form:

$$\underbrace{W_t}_{n\times m} = -1 \cdot \begin{bmatrix} 0 & \multicolumn{11}{c}{\dotfill} & 0 \\ \widehat{y}_{1,t} & 0 & \multicolumn{10}{c}{\dotfill} & \vdots \\ 0 & \widehat{y}_{1,t} & \widehat{y}_{2,t} & 0 & \multicolumn{8}{c}{\dotfill} & \vdots \\ 0 & 0 & 0 & \widehat{y}_{1,t} & \widehat{y}_{2,t} & \widehat{y}_{3,t} & 0 & \multicolumn{5}{c}{\dotfill} & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \ddots & \ddots & \vdots & \vdots & \vdots & \vdots & 0 \\ 0 & \multicolumn{6}{c}{\dotfill} & 0 & \widehat{y}_{1,t} & \widehat{y}_{2,t} & \widehat{y}_{3,t} & \cdots & \widehat{y}_{n-1,t} \end{bmatrix}.$$

We can then rewrite the model as:

$$\widehat{y}_t = -G^* \widehat{y}_t + \Sigma_t \varepsilon_t,$$

$$\widehat{y}_t = -\left(I_n \otimes \widehat{y}_t'\right) vec\left(G^{*\prime}\right) + \Sigma_t \varepsilon_t,$$

$$\widehat{y}_t = W_t\ g + \varepsilon_t^*, \qquad \varepsilon_t^* \sim \mathcal{MN}\left(\ \mathbf{0}_{n\times 1},\ \Sigma_t^2\ \right).$$

Next, we stack the model as:

$$\begin{bmatrix} \widehat{y}_1 \\ \widehat{y}_2 \\ \vdots \\ \widehat{y}_T \end{bmatrix} = \begin{bmatrix} W_1 \\ W_2 \\ \vdots \\ W_T \end{bmatrix} g + \begin{bmatrix} \varepsilon_1^* \\ \varepsilon_2^* \\ \vdots \\ \varepsilon_T^* \end{bmatrix},$$

$$\underbrace{\widehat{y}}_{nT\times 1} = \underbrace{W}_{Tn\times m} \cdot \underbrace{g}_{m\times 1} + \varepsilon^*, \qquad \varepsilon^* \sim \mathcal{MN}\left(\ \mathbf{0}_{nT\times 1},\ \Sigma^2\ \right) m$$

where Σ is the diagonal matrix containing all the stacked stochastic volatilities vectors in the main diagonal:

$$\Sigma = Diag\left(\begin{bmatrix} \sigma_1' & \sigma_2' & \dots & \sigma_T' \end{bmatrix}'\right).$$

We can then use a similar approach as the one implemented for a, following Gelman et al. (2014). Given the prior :

$$g \sim \mathcal{MN}\left(\ \bar{g},\ V_g\ \right),$$

we augment the model with r observations that express the prior information:

$$\begin{bmatrix} \widehat{y} \\ \bar{g} \end{bmatrix} = \begin{bmatrix} W \\ I_m \end{bmatrix} g + \begin{bmatrix} \varepsilon^* \\ \varepsilon_g \end{bmatrix},$$

$$\widehat{Y}^\diamond = W^\diamond g + \varepsilon^\diamond, \quad \varepsilon^\diamond \sim \mathcal{MN}\left(\ \mathbf{0}_{nT+m},\ V_\varepsilon^\diamond\ \right),$$

$$V_\varepsilon^\diamond = \begin{bmatrix} \Sigma^2 & \mathbf{0}_{nT\times m} \\ \mathbf{0}_{m\times nT} & V_g \end{bmatrix}.$$

A draw for g is finally obtained through the following posterior:

$$g \sim \mathcal{MN}\left(\ \widetilde{g},\ \left(W^{\diamond\prime} V_\varepsilon^{\diamond -1} W^\diamond\right)^{-1}\ \right),$$

$$\widetilde{g} = \left(W^{\diamond\prime} V_\varepsilon^{\diamond -1} W^\diamond\right)^{-1} W^{\diamond\prime} V_\varepsilon^{\diamond -1} \widehat{Y}^\diamond$$

A.4 Step 2(b): Draw a history of indexes of the mixture $\{S_t\}_{t=1}^T \,|\theta, \{\sigma_t\}_{t=1}^T$

Starting from the following formulation seen in Step 1 of the algorithm

$$\widetilde{y}_t^* = 2\log\sigma_t + \log\ \left[(\varepsilon_t)^{\cdot 2}\right],$$

we can notice that, since $\varepsilon_t \overset{iid}{\sim} \mathcal{MN}(\mathbf{0}, I_n)$, each term $\log\varepsilon_{j,t}^2$ has a $\log\chi_1^2$ distribution, and $\log\ \left[(\varepsilon_t)^{\cdot 2}\right]$ is a vector of independent $\log\chi_1^2$ random variables.

Omori et al. (2007), improving upon Kim et al. (1998), show that the $\log\chi_1^2$ distribution is very well approximated by a mixture of ten Normal distributions:

$$f_{\log\chi_1^2}(x) \approx \sum_{j=1}^{10} m_j^p\ f_{\mathcal{N}}\left(x|\, m_j^m, m_j^v\right),$$

where m_j^p, m_j^m and m_j^v are contained in the following table:

j	1	2	3	4	5	6	7	8	9	10
m_j^p	0.00609	0.04775	0.13057	0.20674	0.22715	0.18842	0.12047	0.05591	0.01575	0.00115
m_j^m	1.92677	1.34744	0.73504	0.02266	−0.85173	−1.97278	−3.46788	−5.55246	−8.68384	−14.65000
m_j^v	0.11265	0.17788	0.26768	0.40611	0.62699	0.98583	1.57469	2.54498	4.16591	7.33342

Therefore, in order to have a conditionally Gaussian measurement equation, we should condition each element of the vector $\log\ \left[(\varepsilon_t)^{\cdot 2}\right]$ on the index that specifies the Normal components of the mixture. Defining the $n \times 1$ vector S_t that contains the indexes of components in period t, we can write

$$S_t \equiv \begin{bmatrix} s_{1,t} \\ \vdots \\ s_{n,t} \end{bmatrix}, \quad \text{where} \quad \left[\log\ \varepsilon_{h,t}^2 \middle|\, s_{h,t} = j\right] \sim \mathcal{N}\left(m_j^m, m_j^v\right).$$

Conditioning on a history of volatilities $(\sigma_t)_{t=1}^T$, we can restate the model as

$$\log\ \left[(\varepsilon_t)^{\cdot 2}\right] = \widetilde{y}_t^* - 2\log\sigma_t,$$

Then, the element $s_{h,t}$ that indexes the specific component from which $\log\ \varepsilon_{h,t}^2$ is drawn, has support $J = \{1, \ldots, 10\}$ and the following discrete probability distribution:

$$\forall j \in J, \qquad Pr\left[s_{h,t} = j|\, \widetilde{y}_{h,t}^*, \sigma_t\right] = \frac{m_j^p\ f_{\mathcal{N}}\left(\widetilde{y}_{h,t}^* - 2\log\sigma_{h,t}\middle|\, m_j^m, m_j^v\right)}{\sum_{\iota=1}^{10} m_\iota^p\ f_{\mathcal{N}}\left(\widetilde{y}_{h,t}^* - 2\log\sigma_{h,t}\middle|\, m_\iota^m, m_\iota^v\right)}.$$

Independent draws for all variables $h \in \{1, \dots, n\}$ at all periods $t \in \{1, \dots, T\}$ from this distribution will form the new history of indexes of mixture's components $\{S_t\}_{t=1}^{T} |\theta, \{\sigma_t\}_{t=1}^{T}$.

Appendix B Forecast Evaluation Tables

The following tables contain the Root Mean Squares Errors, Predictive Log Scores and Continous Rank Probability Scores relative to the forecast evaluation section.

Table 1a: Root Mean Squared Forecast Errors (RMSE for MAI-AR-SV, RMSE ratios in all others)

USA

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.254	1.017***	1.024*	1.026	0.972***	0.976
$h=2$	0.417	1.029***	1.032	1.045**	0.933**	0.923*
$h=3$	0.525	1.035***	1.061	1.073**	0.933	0.918
$h=4$	0.602	1.046***	1.088	1.105**	0.947	0.922
$h=5$	0.583	1.053***	1.127	1.148**	0.987	0.952
$h=6$	0.553	1.048**	1.183	1.199**	1.046	1.004
$h=7$	0.543	1.040**	1.242	1.255**	1.087	1.038
$h=8$	0.538	1.023**	1.283	1.296**	1.096	1.054

Australia

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.203	0.983***	1.022***	1.020	1.015***	1.023
$h=2$	0.309	0.976***	1.042***	1.046	1.035*	1.069
$h=3$	0.401	0.971***	1.060***	1.067	1.081*	1.137*
$h=4$	0.485	0.974***	1.065***	1.078	1.091	1.162**
$h=5$	0.518	0.969***	1.087***	1.101	1.120*	1.198**
$h=6$	0.547	0.966***	1.105***	1.119*	1.132*	1.209**
$h=7$	0.562	0.963***	1.125***	1.137*	1.134*	1.195**
$h=8$	0.576	0.961***	1.149***	1.159*	1.133*	1.169**

Austria

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.209	1.069***	1.024***	1.001	0.968***	0.948
$h=2$	0.343	1.069***	1.020***	0.995	0.974***	0.919
$h=3$	0.450	1.047***	1.022***	1.014	0.980*	0.920
$h=4$	0.535	1.049***	1.027***	1.029	1.001	0.951
$h=5$	0.575	1.062***	1.056***	1.055	1.033	0.979
$h=6$	0.595	1.078***	1.099***	1.094	1.066	1.009
$h=7$	0.618	1.094***	1.129***	1.125	1.097	1.039
$h=8$	0.637	1.107***	1.148**	1.143	1.117	1.057

Belgium

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.200	0.972***	1.020***	1.054***	1.010	1.040
$h=2$	0.331	0.980***	1.042***	1.091**	0.982	0.973
$h=3$	0.438	0.990***	1.045***	1.078*	0.959	0.927
$h=4$	0.544	1.012***	1.034**	1.058	0.932	0.897
$h=5$	0.594	1.027***	1.033*	1.057*	0.925	0.891
$h=6$	0.609	1.037***	1.039	1.066*	0.942	0.900
$h=7$	0.616	1.041***	1.053	1.078	0.955	0.902
$h=8$	0.611	1.041***	1.077	1.103	0.960	0.898

Canada

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.227	1.000***	1.040	1.036*	0.975***	0.988
$h=2$	0.363	0.997***	1.047	1.040*	0.950***	0.971
$h=3$	0.456	0.993***	1.053	1.043	0.941***	0.964*
$h=4$	0.530	1.006***	1.048	1.041	0.936***	0.963**
$h=5$	0.551	1.002***	1.017	1.011	0.929**	0.978**
$h=6$	0.560	0.989***	0.989	0.983	0.921**	0.992**
$h=7$	0.574	0.986***	0.986	0.980	0.920*	0.999
$h=8$	0.588	0.984***	0.995	0.987	0.925	1.008

Finland

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.124	0.991***	1.041***	1.022	1.098***	1.147***
$h=2$	0.233	1.006***	0.991***	0.963*	1.055**	1.108
$h=3$	0.335	1.019***	0.973***	0.931**	1.013	1.073
$h=4$	0.436	1.026***	0.966***	0.922**	0.985	1.046
$h=5$	0.513	1.042***	0.961***	0.907*	0.990	1.041
$h=6$	0.567	1.049***	0.973**	0.908	1.007	1.044
$h=7$	0.606	1.056***	0.999*	0.925	1.032	1.047
$h=8$	0.634	1.066***	1.026*	0.944	1.049	1.044

France

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.108	1.034***	0.972***	0.974**	1.072***	1.078
$h=2$	0.180	1.080***	0.964***	0.956	1.089	1.091
$h=3$	0.234	1.114***	0.991***	0.947	1.125	1.125
$h=4$	0.292	1.144***	1.003***	0.955	1.113	1.125
$h=5$	0.323	1.172***	1.016**	0.946	1.131	1.143
$h=6$	0.341	1.196***	1.036**	0.937	1.169	1.173
$h=7$	0.360	1.212***	1.050**	0.926	1.182	1.175**
$h=8$	0.382	1.22***	1.059*	0.914	1.173	1.158**

Germany

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.274	0.996***	0.982	0.984	1.026	1.035
$h=2$	0.418	1.003***	0.978	0.991	1.063	1.051
$h=3$	0.505	1.022***	1.018	1.030	1.136	1.104
$h=4$	0.594	1.035***	1.042	1.057	1.152	1.124
$h=5$	0.624	1.040***	1.063	1.078	1.186	1.171
$h=6$	0.655	1.050***	1.084	1.102	1.202	1.188
$h=7$	0.689	1.061***	1.101	1.120	1.210	1.186
$h=8$	0.717	1.072***	1.111	1.132	1.224	1.188

Greece

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.111	1.044***	1.011***	0.996	1.138***	1.069***
$h=2$	0.193	0.985***	1.027***	0.984	1.161***	1.097***
$h=3$	0.256	0.999***	1.057***	0.985	1.169***	1.082***
$h=4$	0.297	0.999***	1.087***	0.988	1.193***	1.083**
$h=5$	0.306	1.004***	1.151***	1.005	1.239***	1.110**
$h=6$	0.313	1.023***	1.239***	1.045	1.277***	1.151**
$h=7$	0.312	1.065***	1.359***	1.116	1.320***	1.190**
$h=8$	0.311	1.124***	1.490***	1.206	1.336***	1.205*

Italy

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.065	1.135***	1.032***	0.960***	1.172***	1.126***
$h=2$	0.128	1.074***	1.024***	0.946**	1.146***	1.155**
$h=3$	0.187	1.057***	1.040***	0.951*	1.116*	1.190
$h=4$	0.250	1.057***	1.023***	0.937	1.080	1.190
$h=5$	0.296	1.059***	1.030***	0.933	1.061	1.191
$h=6$	0.331	1.058***	1.054***	0.939	1.049	1.185
$h=7$	0.363	1.064***	1.071***	0.940	1.027	1.160
$h=8$	0.387	1.066***	1.101***	0.956	0.998	1.137

Statistically significant differences according to the Diebold-Mariano t-statistic are indicated by asterisks, where

*,** and ** correspond respectively to 10%,5% and 1% significance levels

Table 1b: Root Mean Squared Forecast Errors (RMSE for MAI-AR-SV, RMSE ratios in all others)

Japan

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.142	1.131***	1.092***	1.018	1.143***	1.119***
$h=2$	0.230	1.180***	1.113***	0.969*	1.218***	1.123*
$h=3$	0.313	1.230***	1.131***	0.927**	1.274***	1.134
$h=4$	0.400	1.253***	1.159***	0.908**	1.273***	1.107
$h=5$	0.464	1.289***	1.178***	0.867***	1.278***	1.085
$h=6$	0.517	1.312***	1.21***	0.851***	1.262***	1.063
$h=7$	0.563	1.322***	1.243***	0.848***	1.234***	1.033
$h=8$	0.602	1.330***	1.264***	0.842***	1.208***	1.016

Luxembourg

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.194	0.998***	1.013***	1.024	1.020	1.042**
$h=2$	0.307	1.015***	1.014***	1.030	1.027	1.042
$h=3$	0.377	1.027***	1.029***	1.042	1.038	1.048
$h=4$	0.448	1.033***	1.046**	1.059	1.021	1.029
$h=5$	0.473	1.041***	1.066**	1.083	1.027	1.033
$h=6$	0.474	1.038***	1.101**	1.118	1.046	1.046
$h=7$	0.485	1.030***	1.140**	1.155*	1.069	1.062
$h=8$	0.492	1.024***	1.168***	1.182*	1.096	1.086

Netherlands

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.156	0.980***	1.030***	1.012	1.157***	1.085**
$h=2$	0.231	0.991***	1.032***	1.007	1.231***	1.168**
$h=3$	0.283	1.000***	1.065***	1.040	1.267***	1.234*
$h=4$	0.359	1.009***	1.033***	1.025	1.200*	1.201
$h=5$	0.400	1.032***	1.025***	1.006	1.168	1.186
$h=6$	0.436	1.051***	1.033**	1.005	1.130	1.158
$h=7$	0.480	1.068***	1.019**	0.996	1.090	1.110
$h=8$	0.510	1.083***	1.015*	0.987	1.072	1.075

New Zealand

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.153	0.962***	0.966***	0.959	1.101***	1.082*
$h=2$	0.259	0.935***	0.928***	0.873	1.140**	1.134
$h=3$	0.354	0.919***	0.925**	0.813	1.133*	1.141
$h=4$	0.437	0.922***	0.942**	0.811	1.102**	1.120
$h=5$	0.491	0.921***	0.948**	0.782	1.064**	1.085**
$h=6$	0.532	0.921***	0.964**	0.765	1.036*	1.045*
$h=7$	0.561	0.931***	0.986**	0.760	1.008	0.998
$h=8$	0.575	0.947***	1.021**	0.768	0.984	0.951

Norway

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.248	1.004***	1.028***	0.999	0.965	0.961
$h=2$	0.350	1.090***	1.086***	1.012	0.957	0.940*
$h=3$	0.443	1.121***	1.121***	1.014	0.936	0.929*
$h=4$	0.538	1.134***	1.145***	1.020	0.917	0.905*
$h=5$	0.578	1.162***	1.171***	1.001	0.938	0.903
$h=6$	0.601	1.165***	1.224***	1.014	0.959	0.901
$h=7$	0.614	1.157***	1.282***	1.036	0.973	0.898
$h=8$	0.630	1.140***	1.327***	1.063	0.972	0.889

Portugal

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.079	1.500***	1.375***	1.013***	1.361***	1.085***
$h=2$	0.136	1.341***	1.384***	1.014***	1.287***	1.064***
$h=3$	0.182	1.261***	1.446***	1.023*	1.274***	1.094***
$h=4$	0.227	1.148***	1.504***	1.035	1.276***	1.112***
$h=5$	0.257	1.080***	1.580***	1.048	1.267***	1.099***
$h=6$	0.286	1.032***	1.637***	1.058	1.220***	1.076***
$h=7$	0.312	0.995***	1.690***	1.079	1.154***	1.050**
$h=8$	0.335	0.998***	1.729***	1.093	1.094***	1.032**

Spain

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.116	0.982***	1.012***	0.984**	1.065***	0.989
$h=2$	0.204	0.983***	0.987***	0.936**	1.028***	0.999
$h=3$	0.277	0.994***	1.001***	0.922**	1.006	1.011
$h=4$	0.341	1.017***	1.013***	0.911*	0.979	1.012
$h=5$	0.385	1.026***	1.027***	0.894*	0.968	1.018
$h=6$	0.421	1.026***	1.048***	0.882**	0.955	1.002
$h=7$	0.457	1.028***	1.063***	0.875*	0.934	0.977
$h=8$	0.488	1.033***	1.081***	0.874*	0.916	0.968

Sweden

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.235	1.024***	1.038***	1.033	1.047	1.093*
$h=2$	0.383	1.044***	1.053**	1.052	1.056	1.070
$h=3$	0.499	1.050***	1.089**	1.077	1.075	1.055
$h=4$	0.609	1.059***	1.108**	1.093	1.031	0.992
$h=5$	0.668	1.069***	1.126**	1.105	1.014	0.966
$h=6$	0.698	1.081***	1.170***	1.138	1.012	0.956
$h=7$	0.721	1.097***	1.215***	1.172	1.007	0.942
$h=8$	0.740	1.109***	1.257***	1.206	1.01	0.938

Switzerland

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.194	1.085***	1.017***	0.988***	1.155***	1.165***
$h=2$	0.355	1.081***	0.988**	0.956***	1.143*	1.157*
$h=3$	0.496	1.083***	0.976	0.941***	1.125	1.128
$h=4$	0.609	1.097***	0.974	0.937**	1.111	1.098
$h=5$	0.673	1.121***	0.985	0.941*	1.114	1.071
$h=6$	0.705	1.152***	1.012	0.952*	1.137	1.052
$h=7$	0.733	1.179***	1.041	0.968	1.166	1.037
$h=8$	0.762	1.198***	1.067	0.982	1.199	1.030

United Kingdom

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.114	1.054***	1.008***	0.989***	1.201***	1.112
$h=2$	0.184	1.062***	1.044***	1.003***	1.245**	1.138
$h=3$	0.245	1.069***	1.057***	0.980**	1.237	1.154
$h=4$	0.316	1.073***	1.035***	0.939**	1.173	1.132
$h=5$	0.359	1.077***	1.043***	0.921*	1.122	1.104
$h=6$	0.396	1.077***	1.056***	0.911*	1.088	1.080
$h=7$	0.433	1.074***	1.073**	0.918*	1.063	1.054
$h=8$	0.459	1.072***	1.107**	0.947*	1.047	1.035

Statistically significant differences according to the Diebold-Mariano t-statistic are indicated by asterisks, where

*,** and ** correspond respectively to 10%,5% and 1% significance levels

Table 2a: Average Log Predictive Scores (scores for MAI-AR-SV, score differences in all others)

USA

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h = 1$	0.166	-0.292^{***}	-1.263	-0.032	-1.506	-0.022
$h = 2$	-0.343	-0.277^{***}	-1.161	-0.063^{*}	-0.971	-0.016
$h = 3$	-0.600	-0.267^{***}	-0.532	-0.094^{*}	-0.410	-0.044
$h = 4$	-0.805	-0.241^{***}	-0.229	-0.130^{**}	-0.110	-0.048
$h = 5$	-0.842	-0.276^{***}	-0.180^{*}	-0.154^{**}	-0.053	-0.056
$h = 6$	-0.867	-0.308^{***}	-0.162^{**}	-0.180^{***}	-0.052	-0.073
$h = 7$	-0.892	-0.337^{***}	-0.180^{**}	-0.208^{***}	-0.067	-0.086
$h = 8$	-0.927	-0.343^{***}	-0.182^{**}	-0.223^{***}	-0.064	-0.087

Australia

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h = 1$	0.206	-0.334^{***}	-0.139^{**}	-0.044	-0.069	0.025
$h = 2$	-0.291	-0.197^{**}	-0.052	-0.050	0.012	-0.024
$h = 3$	-0.552	-0.163^{*}	-0.053	-0.083	-0.013	-0.080
$h = 4$	-0.722	-0.169	-0.075	-0.104	-0.026	-0.112
$h = 5$	-0.770	-0.215^{*}	-0.136	-0.139	-0.067	-0.139^{**}
$h = 6$	-0.809	-0.238^{**}	-0.183^{*}	-0.164^{*}	-0.091	-0.150^{*}
$h = 7$	-0.833	-0.260^{**}	-0.216^{**}	-0.182^{*}	-0.105	-0.134^{*}
$h = 8$	-0.871	-0.256^{**}	-0.233^{**}	-0.204^{*}	-0.105	-0.114

Austria

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h = 1$	0.126	-0.516^{***}	-0.265^{***}	0.004	-0.146^{*}	0.052
$h = 2$	-0.377	-0.386^{***}	-0.172^{*}	-0.007	-0.044	0.102
$h = 3$	-0.696	-0.241^{*}	-0.078	0.022	0.047	0.143
$h = 4$	-0.863	-0.195	-0.064	-0.040	0.045	0.088
$h = 5$	-0.919	-0.213^{**}	-0.102	-0.081	0.002	0.045
$h = 6$	-0.957	-0.227^{**}	-0.132	-0.128	-0.023	0.021
$h = 7$	-1.016	-0.212^{*}	-0.131	-0.170^{*}	-0.032	0.000
$h = 8$	-1.059	-0.206	-0.129	-0.204^{**}	-0.039	-0.014

Belgium

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h = 1$	0.239	-0.283^{***}	-0.110^{**}	-0.040	-0.074	-0.058
$h = 2$	-0.191	-0.307^{***}	-0.189^{***}	-0.101^{***}	-0.151^{*}	-0.535
$h = 3$	-0.453	-0.288^{**}	-0.247^{**}	-0.111^{*}	-0.376^{*}	-0.650
$h = 4$	-0.723	-0.226^{*}	-0.191^{*}	-0.106	-0.232	-0.694
$h = 5$	-0.822	-0.240^{**}	-0.151^{*}	-0.125^{*}	-0.051	0.050
$h = 6$	-0.863	-0.267^{**}	-0.154^{*}	-0.152^{**}	-0.038	0.060
$h = 7$	-0.912	-0.270^{***}	-0.140^{*}	-0.172^{***}	0.007	0.077
$h = 8$	-0.942	-0.273^{***}	-0.141^{*}	-0.195^{***}	0.046	0.096

Canada

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h = 1$	0.075	-0.119^{**}	-0.064	-0.086^{**}	-0.032	-0.003
$h = 2$	-0.352	-0.130^{**}	-0.127	-0.109^{**}	-0.147	-0.004
$h = 3$	-0.556	-0.160^{**}	-0.127	-0.137^{*}	-0.070	-0.020
$h = 4$	-0.705	-0.173^{**}	-0.126	-0.122	-0.066	-0.005
$h = 5$	-0.758	-0.193^{**}	-0.096	-0.109	0.030	0.019
$h = 6$	-0.789	-0.211^{**}	-0.086	-0.104	0.063	0.036
$h = 7$	-0.826	-0.227^{**}	-0.095	-0.124	0.069	0.047
$h = 8$	-0.862	-0.232^{**}	-0.105	-0.149^{*}	0.067	0.034

Finland

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h = 1$	0.651	-0.550^{***}	-0.271^{***}	-0.022	-0.199^{***}	-0.108^{**}
$h = 2$	0.055	-0.478^{***}	-0.204^{***}	-0.049	-0.111	-0.114
$h = 3$	-0.301	-0.402^{***}	-0.156^{*}	0.014	-0.045	-0.091
$h = 4$	-0.581	-0.307^{**}	-0.093	-0.018	0.020	-0.042
$h = 5$	-0.759	-0.259^{*}	-0.056	0.018	0.048	-0.010
$h = 6$	-0.850	-0.249^{*}	-0.068	-0.007	0.040	-0.004
$h = 7$	-0.924	-0.235^{*}	-0.074	-0.031	0.032	0.014
$h = 8$	-0.978	-0.232	-0.082	-0.056	0.020	0.022

France

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h = 1$	0.806	-0.429^{***}	-0.153^{***}	0.028	-0.119^{**}	-0.072^{*}
$h = 2$	0.340	-0.519^{***}	-0.190^{***}	-0.003	-0.149^{*}	-0.088
$h = 3$	0.078	-0.550^{***}	-0.219^{***}	-0.032	-0.172	-0.129
$h = 4$	-0.163	-0.488^{***}	-0.182^{*}	-0.040	-0.142	-0.124
$h = 5$	-0.263	-0.502^{***}	-0.207^{**}	-0.043	-0.175	-0.136
$h = 6$	-0.333	-0.514^{***}	-0.223^{**}	-0.046	-0.206^{*}	-0.163^{*}
$h = 7$	-0.410	-0.503^{***}	-0.221^{**}	-0.043	-0.211^{*}	-0.167^{**}
$h = 8$	-0.479	-0.498^{***}	-0.216^{**}	-0.039	-0.211^{*}	-0.171^{**}

Germany

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h = 1$	-0.072	-0.178^{***}	-0.035	-0.014	-0.062	-0.052
$h = 2$	-0.504	-0.172^{**}	-0.023	-0.037	-0.073	-0.055
$h = 3$	-0.706	-0.188^{**}	-0.054	-0.081	-0.127	-0.099
$h = 4$	-0.904	-0.162^{*}	-0.047	-0.103^{**}	-0.092	-0.105
$h = 5$	-0.994	-0.165^{*}	-0.039	-0.127^{**}	-0.079	-0.092
$h = 6$	-1.068	-0.162^{*}	-0.042	-0.139^{***}	-0.071	-0.091
$h = 7$	-1.140	-0.151^{*}	-0.036	-0.153^{***}	-0.064	-0.073
$h = 8$	-1.194	-0.141^{*}	-0.031	-0.155^{***}	-0.059	-0.067

Greece

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h = 1$	0.839	-0.761^{***}	-0.428^{***}	-0.002	-0.369^{***}	-0.104^{***}
$h = 2$	0.314	-0.752^{***}	-0.414^{***}	-0.003	-0.339^{***}	-0.109^{**}
$h = 3$	0.035	-0.777^{***}	-0.423^{***}	-0.022	-0.340^{***}	-0.122^{*}
$h = 4$	-0.139	-0.768^{***}	-0.424^{***}	-0.039	-0.342^{***}	-0.131^{*}
$h = 5$	-0.216	-0.793^{***}	-0.458^{***}	-0.047	-0.372^{***}	-0.145^{**}
$h = 6$	-0.269	-0.806^{***}	-0.491^{***}	-0.056	-0.391^{***}	-0.170^{**}
$h = 7$	-0.306	-0.821^{***}	-0.527^{***}	-0.067	-0.405^{***}	-0.187^{***}
$h = 8$	-0.350	-0.820^{***}	-0.545^{***}	-0.081	-0.399^{***}	-0.198^{***}

Italy

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h = 1$	1.262	-0.956^{***}	-0.616^{***}	0.101^{***}	-0.431^{***}	-0.146^{***}
$h = 2$	0.638	-0.843^{***}	-0.522^{***}	0.063	-0.283^{***}	-0.142^{***}
$h = 3$	0.254	-0.752^{***}	-0.451^{***}	0.043	-0.179	-0.139^{*}
$h = 4$	-0.026	-0.644^{***}	-0.375^{**}	0.039	-0.117	-0.125
$h = 5$	-0.190	-0.596^{***}	-0.352^{**}	0.030	-0.111	-0.127
$h = 6$	-0.298	-0.574^{***}	-0.350^{**}	0.029	-0.128	-0.131
$h = 7$	-0.394	-0.547^{***}	-0.338^{**}	0.029	-0.128	-0.126
$h = 8$	-0.468	-0.529^{***}	-0.338^{**}	0.012	-0.124	-0.126

Statistically significant differences according to the Diebold-Mariano t-statistic are indicated by asterisks, where

*,** and ** correspond respectively to 10%,5% and 1% significance levels

Table 2b: Average Log Predictive Scores (scores for MAI-AR-SV, score differences in all others)

Japan

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.505	-0.666^{***}	-0.356^{***}	0.000	-0.251^{***}	-0.094^{**}
$h=2$	0.033	-0.609^{***}	-0.331^{***}	0.051	-0.243^{***}	-0.091
$h=3$	-0.270	-0.553^{***}	-0.306^{***}	0.093^{*}	-0.241^{**}	-0.082
$h=4$	-0.537	-0.475^{***}	-0.260^{**}	0.128^{**}	-0.201	-0.040
$h=5$	-0.671	-0.474^{***}	-0.274^{**}	0.151^{**}	-0.214	-0.037
$h=6$	-0.788	-0.453^{***}	-0.267^{**}	0.167^{**}	-0.197	-0.016
$h=7$	-0.870	-0.445^{***}	-0.276^{**}	0.172^{**}	-0.190	-0.007
$h=8$	-0.946	-0.426^{***}	-0.270^{**}	0.181^{**}	-0.167	0.008

Luxembourg

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.245	-0.302^{***}	-0.094^{**}	-0.075	-0.079^{**}	-0.027
$h=2$	-0.199	-0.304^{***}	-0.120^{***}	-0.072^{*}	-0.174	-0.038
$h=3$	-0.426	-0.307^{***}	-0.130^{**}	-0.070	-0.19	-0.055
$h=4$	-0.619	-0.282^{***}	-0.127	-0.105^{*}	-0.048	-0.054
$h=5$	-0.687	-0.312^{***}	-0.149^{*}	-0.117^{**}	-0.042	-0.060
$h=6$	-0.718	-0.332^{***}	-0.182^{**}	-0.133^{**}	-0.038	-0.054
$h=7$	-0.766	-0.332^{***}	-0.193^{***}	-0.145^{**}	-0.027	-0.058
$h=8$	-0.803	-0.333^{***}	-0.207^{***}	-0.148^{**}	-0.025	-0.076

Netherlands

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.388	-0.623^{***}	-0.347^{***}	0.020	-0.237^{***}	-0.064
$h=2$	-0.002	-0.625^{***}	-0.342^{***}	-0.006	-0.219^{***}	-0.106^{*}
$h=3$	-0.213	-0.604^{***}	-0.338^{***}	-0.037	-0.211^{**}	-0.130^{*}
$h=4$	-0.441	-0.508^{***}	-0.261^{**}	-0.030	-0.151	-0.108
$h=5$	-0.540	-0.500^{***}	-0.255^{**}	-0.032	-0.141	-0.111
$h=6$	-0.634	-0.479^{***}	-0.232^{**}	-0.022	-0.123	-0.109
$h=7$	-0.729	-0.451^{***}	-0.197^{*}	-0.016	-0.108	-0.102
$h=8$	-0.792	-0.438^{***}	-0.176	-0.006	-0.099	-0.105

New Zealand

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.501	-0.511^{***}	-0.227^{***}	0.009	-0.198^{***}	-0.056^{*}
$h=2$	0.008	-0.512^{***}	-0.226^{***}	-0.008	-0.195^{***}	-0.087^{*}
$h=3$	-0.249	-0.524^{***}	-0.264^{***}	-0.015	-0.214^{***}	-0.112^{*}
$h=4$	-0.442	-0.481^{***}	-0.259^{**}	-0.014	-0.189^{**}	-0.109^{*}
$h=5$	-0.540	-0.465^{***}	-0.275^{*}	0.012	-0.171^{*}	-0.083
$h=6$	-0.591	-0.463^{***}	-0.308^{*}	0.024	-0.186	-0.079
$h=7$	-0.645	-0.450^{***}	-0.319^{*}	0.037	-0.183	-0.062
$h=8$	-0.691	-0.436^{***}	-0.323^{*}	0.041	-0.167	-0.048

Norway

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.040	-0.350^{***}	-0.162^{***}	0.000	-0.064	0.003
$h=2$	-0.361	-0.311^{***}	-0.152^{**}	-0.019	0.001	0.032
$h=3$	-0.624	-0.265^{***}	-0.144	-0.047	0.068	0.058
$h=4$	-0.836	-0.210^{**}	-0.129	-0.065	0.136	0.121
$h=5$	-0.940	-0.184	-0.128	-0.052	0.152	0.165
$h=6$	-0.984	-0.188	-0.167	-0.070	0.144	0.198^{*}
$h=7$	-0.995	-0.204	-0.218	-0.095	0.115	0.201^{*}
$h=8$	-1.022	-0.204	-0.249^{*}	-0.120	0.110	0.201^{*}

Portugal

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.998	-1.281^{***}	-0.973^{***}	0.094^{***}	-0.848^{***}	-0.194^{***}
$h=2$	0.525	-1.049^{***}	-0.818^{***}	0.045	-0.673^{***}	-0.193^{***}
$h=3$	0.258	-0.949^{***}	-0.765^{***}	-0.010	-0.581^{***}	-0.210^{***}
$h=4$	0.051	-0.870^{***}	-0.737^{***}	-0.044	-0.497^{***}	-0.208^{**}
$h=5$	-0.067	-0.803^{***}	-0.738^{***}	-0.064	-0.464^{***}	-0.212^{*}
$h=6$	-0.155	-0.753^{***}	-0.744^{***}	-0.092	-0.429^{***}	-0.217^{*}
$h=7$	-0.232	-0.698^{***}	-0.750^{***}	-0.106	-0.390^{**}	-0.213
$h=8$	-0.304	-0.650^{***}	-0.738^{***}	-0.107	-0.345^{**}	-0.202

Spain

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.734	-0.670^{***}	-0.393^{***}	0.029	-0.236^{***}	-0.040
$h=2$	0.189	-0.520^{***}	-0.286^{**}	0.071	-0.136	-0.050
$h=3$	-0.196	-0.376	-0.173	0.159	-0.005	0.022
$h=4$	-0.429	-0.310	-0.127	0.152	0.052	0.019
$h=5$	-0.485	-0.361^{**}	-0.201	0.089	-0.001	-0.036
$h=6$	-0.577	-0.340^{*}	-0.205	0.094	0.002	-0.033
$h=7$	-0.653	-0.328^{**}	-0.212	0.065	0.011	-0.020
$h=8$	-0.729	-0.308^{**}	-0.204	0.058	0.027	-0.005

Sweden

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.108	-0.320^{***}	-0.146^{**}	-0.046	-0.102	-0.036
$h=2$	-0.425	-0.184^{*}	-0.092	-0.140	-0.086	-0.012
$h=3$	-0.784	-0.052	-0.025	-0.567	-0.047	0.063
$h=4$	-0.981	-0.044	-0.052	-0.420	-0.022	0.073
$h=5$	-1.081	-0.040	-0.060	-0.095	0.020	0.105
$h=6$	-1.098	-0.083	-0.134	-0.157	0.010	0.089
$h=7$	-1.103	-0.125	-0.208	-0.209	-0.006	0.071
$h=8$	-1.118	-0.145	-0.256^{*}	-0.272^{**}	-0.014	0.047

Switzerland

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.199	-0.374^{***}	-0.103^{***}	0.020	-0.135^{***}	-0.111^{***}
$h=2$	-0.362	-0.297^{***}	-0.072	0.021	-0.148^{**}	-0.134
$h=3$	-0.715	-0.215^{**}	-0.021	0.000	-0.118	-0.104
$h=4$	-0.935	-0.179^{**}	0.008	-0.001	-0.094	-0.078
$h=5$	-1.048	-0.183^{**}	0.007	-0.010	-0.076	-0.036
$h=6$	-1.102	-0.201^{**}	-0.019	-0.016	-0.088	-0.009
$h=7$	-1.153	-0.213^{***}	-0.029	-0.013	-0.102	0.009
$h=8$	-1.197	-0.215^{**}	-0.050	-0.027	-0.126	0.007

United Kingdom

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.738	-0.681^{***}	-0.351^{***}	0.106^{*}	-0.284^{***}	-0.082^{***}
$h=2$	0.227	-0.663^{***}	-0.356^{***}	0.108	-0.245^{***}	-0.074^{*}
$h=3$	-0.061	-0.645^{***}	-0.364^{***}	0.141	-0.229^{***}	-0.091^{*}
$h=4$	-0.287	-0.579^{***}	-0.330^{***}	0.176	-0.202^{**}	-0.096
$h=5$	-0.414	-0.553^{***}	-0.319^{**}	0.191	-0.187^{**}	-0.101
$h=6$	-0.500	-0.536^{***}	-0.321^{**}	0.182	-0.188^{**}	-0.110
$h=7$	-0.580	-0.520^{***}	-0.316^{**}	0.163	-0.189^{**}	-0.108
$h=8$	-0.636	-0.511^{***}	-0.324^{***}	0.107	-0.204^{*}	-0.121

Statistically significant differences according to the Diebold-Mariano t-statistic are indicated by asterisks, where

*,** and ** correspond respectively to 10%,5% and 1% significance levels

Table 3a: Average Continuous Rank Probability Scores (CRPS for MAI-AR-SV, CRPS ratios in all others)

USA

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.123	1.098***	1.028	1.02	0.993	1.000
$h=2$	0.206	1.115***	1.060*	1.042**	0.987	0.966
$h=3$	0.264	1.126***	1.097**	1.071**	0.998	0.970
$h=4$	0.315	1.126***	1.109**	1.106**	0.993	0.966
$h=5$	0.318	1.155***	1.125**	1.141***	1.006	0.983
$h=6$	0.316	1.172***	1.153***	1.185***	1.028	1.019
$h=7$	0.319	1.194***	1.195***	1.227***	1.056	1.055
$h=8$	0.327	1.197**	1.218***	1.249***	1.052	1.063

Australia

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.107	1.191***	1.081**	1.045	1.033	1.004
$h=2$	0.176	1.086**	1.041	1.040	1.008	1.039
$h=3$	0.231	1.051	1.055	1.071	1.043	1.107
$h=4$	0.277	1.056	1.063	1.081	1.063	1.141*
$h=5$	0.293	1.075	1.103	1.117	1.095	1.175**
$h=6$	0.306	1.090	1.137	1.143	1.107	1.184**
$h=7$	0.312	1.107	1.172*	1.165	1.109	1.165*
$h=8$	0.320	1.112	1.196*	1.186	1.107	1.141

Austria

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.114	1.404***	1.172***	1.006	1.065	0.951
$h=2$	0.191	1.268***	1.102*	1.002	1.001	0.918
$h=3$	0.248	1.192***	1.087	1.031	0.998	0.929
$h=4$	0.301	1.138***	1.066	1.053	1.001	0.963
$h=5$	0.327	1.142***	1.092	1.081	1.027	0.987
$h=6$	0.343	1.148***	1.124	1.121*	1.052	1.008
$h=7$	0.361	1.149***	1.142	1.158**	1.074	1.030
$h=8$	0.375	1.153**	1.152	1.179**	1.088	1.043

Belgium

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.107	1.128***	1.053**	1.047	1.050	1.060
$h=2$	0.169	1.139***	1.099***	1.096**	1.055	1.029
$h=3$	0.215	1.143***	1.117**	1.103**	1.042	0.989
$h=4$	0.274	1.122***	1.087*	1.089*	0.999	0.944
$h=5$	0.309	1.127***	1.083	1.099*	0.975	0.921
$h=6$	0.325	1.144***	1.095	1.121**	0.973	0.913
$h=7$	0.339	1.147***	1.101	1.135**	0.972	0.910
$h=8$	0.346	1.150***	1.117*	1.158***	0.959	0.900

Canada

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.126	1.044**	1.039	1.049*	0.983	0.996
$h=2$	0.196	1.048*	1.060	1.060	0.969	0.984
$h=3$	0.246	1.053*	1.073	1.073	0.976	0.993
$h=4$	0.285	1.065**	1.060	1.063	0.971	0.990
$h=5$	0.297	1.081**	1.039	1.044	0.968	1.001
$h=6$	0.305	1.087**	1.029	1.034	0.945	0.994
$h=7$	0.314	1.093**	1.034	1.048	0.931	0.983
$h=8$	0.324	1.102*	1.046	1.063	0.922	0.981

Finland

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.069	1.368***	1.155***	1.014	1.141***	1.133**
$h=2$	0.128	1.295***	1.091*	0.970	1.092	1.129
$h=3$	0.184	1.232***	1.052	0.948	1.046	1.107
$h=4$	0.241	1.172***	1.027	0.945	1.003	1.068
$h=5$	0.286	1.149**	1.011	0.934	0.987	1.044
$h=6$	0.319	1.138**	1.017	0.942	0.981	1.026
$h=7$	0.345	1.129*	1.035	0.966	0.984	1.009
$h=8$	0.365	1.13*	1.053	0.986	0.989	0.997

France

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.059	1.271***	1.053*	0.966	1.108**	1.090*
$h=2$	0.095	1.382***	1.083*	0.984	1.146	1.120
$h=3$	0.122	1.436***	1.124*	0.998	1.199	1.180
$h=4$	0.156	1.392***	1.105	1.007	1.172	1.174
$h=5$	0.174	1.407***	1.121	1.002	1.191	1.185
$h=6$	0.188	1.421***	1.137	0.995	1.214	1.201*
$h=7$	0.202	1.420***	1.140	0.989	1.215	1.193**
$h=8$	0.217	1.416***	1.144	0.980	1.200	1.175**

Germany

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.147	1.072***	0.989	0.992	1.028	1.053
$h=2$	0.225	1.079**	0.981	0.995	1.050	1.055
$h=3$	0.274	1.093**	1.027	1.042	1.102	1.097
$h=4$	0.329	1.089**	1.046	1.070	1.100	1.101
$h=5$	0.354	1.100**	1.059	1.096*	1.107	1.116
$h=6$	0.379	1.099**	1.065	1.114*	1.122	1.124
$h=7$	0.405	1.1**	1.073	1.128**	1.132	1.12
$h=8$	0.427	1.098**	1.077	1.134**	1.142	1.115

Greece

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.059	1.597***	1.252***	1.003	1.270***	1.085**
$h=2$	0.101	1.563***	1.250***	1.000	1.267***	1.107*
$h=3$	0.133	1.616***	1.286***	1.016	1.286***	1.120*
$h=4$	0.158	1.615***	1.300**	1.025	1.305***	1.124*
$h=5$	0.167	1.667***	1.366**	1.043	1.351***	1.150*
$h=6$	0.174	1.711***	1.440***	1.076	1.385***	1.184*
$h=7$	0.178	1.761***	1.533***	1.117	1.415***	1.209*
$h=8$	0.182	1.800***	1.609***	1.164	1.408***	1.215*

Italy

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.036	2.026***	1.509***	0.948	1.370***	1.158***
$h=2$	0.068	1.797***	1.388***	0.959	1.238*	1.174**
$h=3$	0.101	1.651***	1.321***	0.968	1.150	1.193
$h=4$	0.136	1.502***	1.243**	0.957	1.089	1.182
$h=5$	0.163	1.442***	1.221	0.954	1.074	1.178
$h=6$	0.183	1.408***	1.224	0.960	1.078	1.173
$h=7$	0.201	1.384***	1.225	0.964	1.07	1.155
$h=8$	0.216	1.376***	1.239	0.983	1.054	1.14

Statistically significant differences according to the Diebold-Mariano t-statistic are indicated by asterisks, where

*,** and ** correspond respectively to 10%,5% and 1% significance levels

Table 3b: Average Continuous Rank Probability Scores (CRPS for MAI-AR-SV, CRPS ratios in all others)

Japan

	MAI-AR-SV	MAI-AR	AR	AR-SV	BVAR	BVAR-SV
$h=1$	0.079	1.558***	1.253***	1.021	1.209***	1.117***
$h=2$	0.129	1.502***	1.236***	0.962	1.234**	1.117
$h=3$	0.177	1.456***	1.219***	0.911*	1.252*	1.115
$h=4$	0.226	1.423***	1.227***	0.889*	1.245	1.083
$h=5$	0.264	1.418***	1.235***	0.851**	1.247	1.053
$h=6$	0.299	1.406***	1.244***	0.835**	1.223	1.025
$h=7$	0.328	1.397***	1.266***	0.831**	1.199	1.000
$h=8$	0.353	1.384***	1.276***	0.826**	1.169	0.985

Luxembourg

	MAI-AR-SV	MAI-AR	AR	AR-SV	BVAR	BVAR-SV
$h=1$	0.103	1.171***	1.054**	1.031	1.036	1.030
$h=2$	0.164	1.168***	1.051	1.043	1.047	1.047
$h=3$	0.201	1.191***	1.076	1.061	1.060	1.059
$h=4$	0.244	1.177***	1.089	1.087	1.034	1.037
$h=5$	0.263	1.190***	1.107	1.106*	1.035	1.036
$h=6$	0.269	1.206***	1.150**	1.141**	1.038	1.034
$h=7$	0.280	1.201***	1.179**	1.171**	1.053	1.051
$h=8$	0.289	1.197***	1.196**	1.185**	1.066	1.066

Netherlands

	MAI-AR-SV	MAI-AR	AR	AR-SV	BVAR	BVAR-SV
$h=1$	0.087	1.483***	1.233***	1.006	1.230***	1.096*
$h=2$	0.130	1.484***	1.224***	1.017	1.250**	1.151*
$h=3$	0.160	1.465***	1.239***	1.053	1.271*	1.212*
$h=4$	0.199	1.379***	1.184**	1.054	1.208	1.189
$h=5$	0.224	1.361***	1.163	1.038	1.172	1.169
$h=6$	0.246	1.347***	1.152	1.029	1.133	1.147
$h=7$	0.275	1.311***	1.112	1.018	1.09	1.112
$h=8$	0.295	1.303***	1.089	1.005	1.071	1.090

New Zealand

	MAI-AR-SV	MAI-AR	AR	AR-SV	BVAR	BVAR-SV
$h=1$	0.082	1.313***	1.085**	0.969	1.152***	1.067
$h=2$	0.139	1.276***	1.061	0.933	1.170**	1.114
$h=3$	0.186	1.265***	1.080	0.904	1.179*	1.137
$h=4$	0.229	1.213***	1.081	0.899	1.135	1.120
$h=5$	0.256	1.193**	1.093	0.872	1.105	1.089
$h=6$	0.274	1.181*	1.118	0.849	1.089	1.061
$h=7$	0.289	1.180*	1.144	0.841	1.068	1.020
$h=8$	0.298	1.187*	1.174	0.844	1.052	0.983

Norway

	MAI-AR-SV	MAI-AR	AR	AR-SV	BVAR	BVAR-SV
$h=1$	0.132	1.189***	1.084***	0.996	1.007	0.979
$h=2$	0.194	1.206***	1.115**	1.010	0.978	0.959
$h=3$	0.250	1.194***	1.137**	1.018	0.940	0.935
$h=4$	0.303	1.183***	1.151*	1.036	0.908	0.903
$h=5$	0.333	1.184***	1.171*	1.024	0.913	0.881
$h=6$	0.349	1.186***	1.225**	1.045	0.919	0.863
$h=7$	0.356	1.184***	1.291**	1.071	0.926	0.848
$h=8$	0.368	1.168**	1.331***	1.098	0.919	0.836

Portugal

	MAI-AR-SV	MAI-AR	AR	AR-SV	BVAR	BVAR-SV
$h=1$	0.046	2.787***	2.130***	0.971	1.922***	1.158***
$h=2$	0.076	2.214***	1.864***	1.004	1.661***	1.150**
$h=3$	0.101	2.006***	1.821***	1.037	1.551***	1.170*
$h=4$	0.124	1.857***	1.821***	1.069	1.482***	1.179
$h=5$	0.140	1.739***	1.870***	1.095	1.447***	1.181
$h=6$	0.155	1.639***	1.902***	1.118	1.394***	1.175
$h=7$	0.170	1.547***	1.923***	1.134	1.323**	1.158
$h=8$	0.184	1.475***	1.923***	1.137	1.247	1.135

Spain

	MAI-AR-SV	MAI-AR	AR	AR-SV	BVAR	BVAR-SV
$h=1$	0.063	1.514***	1.248***	0.981	1.171***	1.027
$h=2$	0.109	1.358***	1.163***	0.941	1.102	1.036
$h=3$	0.148	1.298***	1.147**	0.932	1.069	1.041
$h=4$	0.186	1.251***	1.127	0.923	1.018	1.033
$h=5$	0.212	1.234***	1.132	0.909	1.001	1.033
$h=6$	0.236	1.208***	1.138	0.901	0.980	1.014
$h=7$	0.260	1.180**	1.137	0.900	0.951	0.985
$h=8$	0.280	1.17**	1.142	0.905	0.931	0.974

Sweden

	MAI-AR-SV	MAI-AR	AR	AR-SV	BVAR	BVAR-SV
$h=1$	0.124	1.182***	1.067	1.016	1.064	1.070
$h=2$	0.209	1.113***	1.065	1.044	1.069	1.041
$h=3$	0.275	1.093*	1.099	1.071	1.095	1.032
$h=4$	0.342	1.083	1.112	1.082	1.047	0.977
$h=5$	0.381	1.082	1.125	1.090	1.024	0.947
$h=6$	0.399	1.097	1.179*	1.131	1.023	0.941
$h=7$	0.410	1.120	1.246**	1.186*	1.026	0.936
$h=8$	0.420	1.137*	1.305***	1.240***	1.033	0.944

Switzerland

	MAI-AR-SV	MAI-AR	AR	AR-SV	BVAR	BVAR-SV
$h=1$	0.108	1.256***	1.057**	0.985	1.151***	1.145***
$h=2$	0.194	1.192***	1.017	0.960	1.149*	1.151
$h=3$	0.274	1.151***	0.996	0.954	1.133	1.128
$h=4$	0.343	1.138***	0.984	0.953	1.103	1.088
$h=5$	0.384	1.153***	0.994	0.957	1.096	1.050
$h=6$	0.406	1.175***	1.019	0.969	1.115	1.021
$h=7$	0.426	1.199***	1.047	0.987	1.144	1.001
$h=8$	0.446	1.215***	1.074	1.003	1.179	0.995

United Kingdom

	MAI-AR-SV	MAI-AR	AR	AR-SV	BVAR	BVAR-SV
$h=1$	0.063	1.533***	1.195***	0.936	1.238***	1.086*
$h=2$	0.104	1.531***	1.224**	0.952	1.243**	1.100
$h=3$	0.138	1.517***	1.245**	0.933	1.241**	1.130*
$h=4$	0.176	1.436***	1.205	0.903	1.195*	1.123
$h=5$	0.202	1.406***	1.195	0.883	1.158	1.105
$h=6$	0.222	1.382***	1.197	0.878	1.135	1.087
$h=7$	0.241	1.368***	1.214	0.898	1.126	1.074
$h=8$	0.255	1.361***	1.241	0.940	1.126	1.068

Statistically significant differences according to the Diebold-Mariano t-statistic are indicated by asterisks, where

*,** and ** correspond respectively to 10%,5% and 1% significance levels

Appendix C Additional Figures

C.1 Headline Inflation

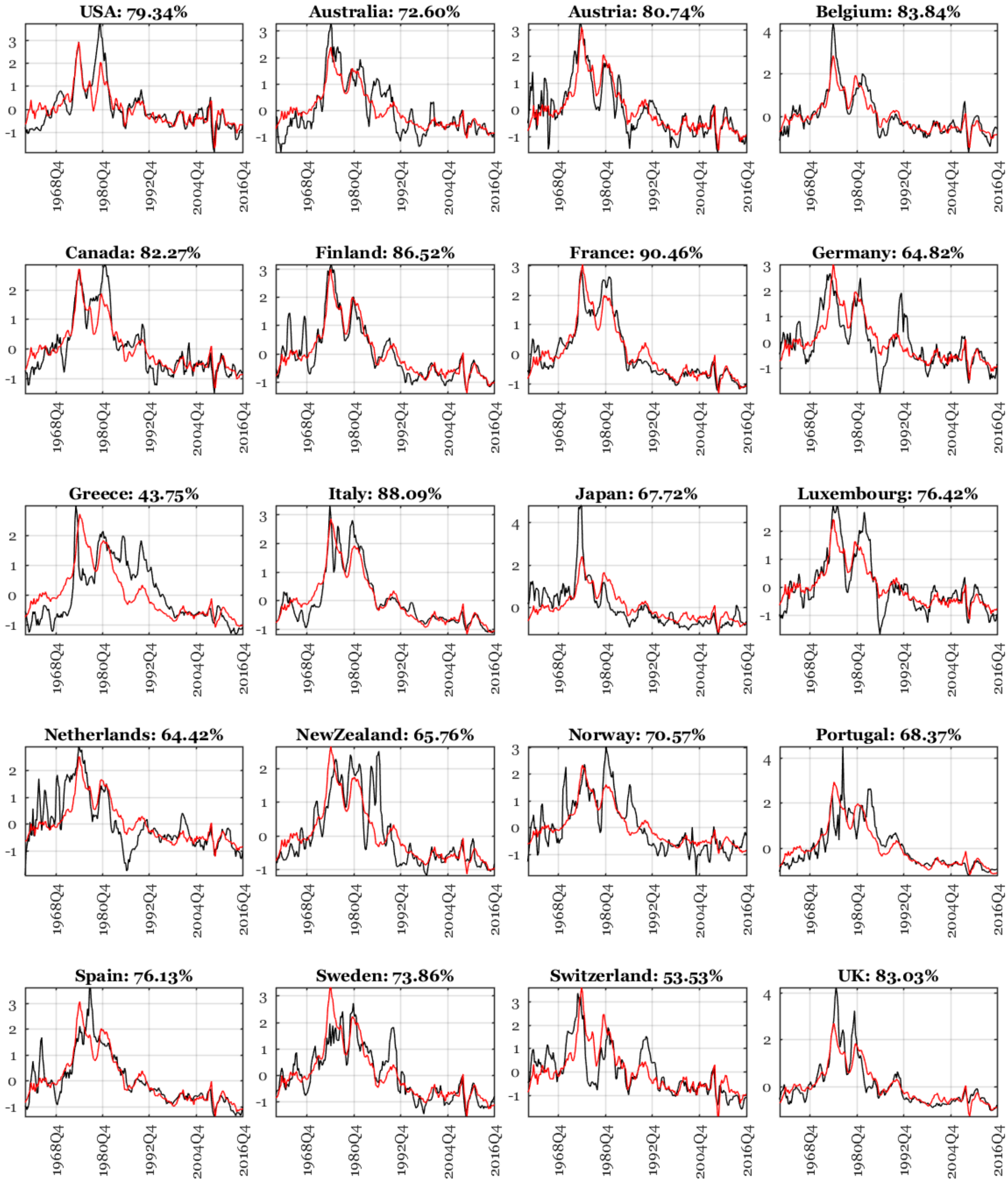


Figure 16: MAI-SV in-sample fit

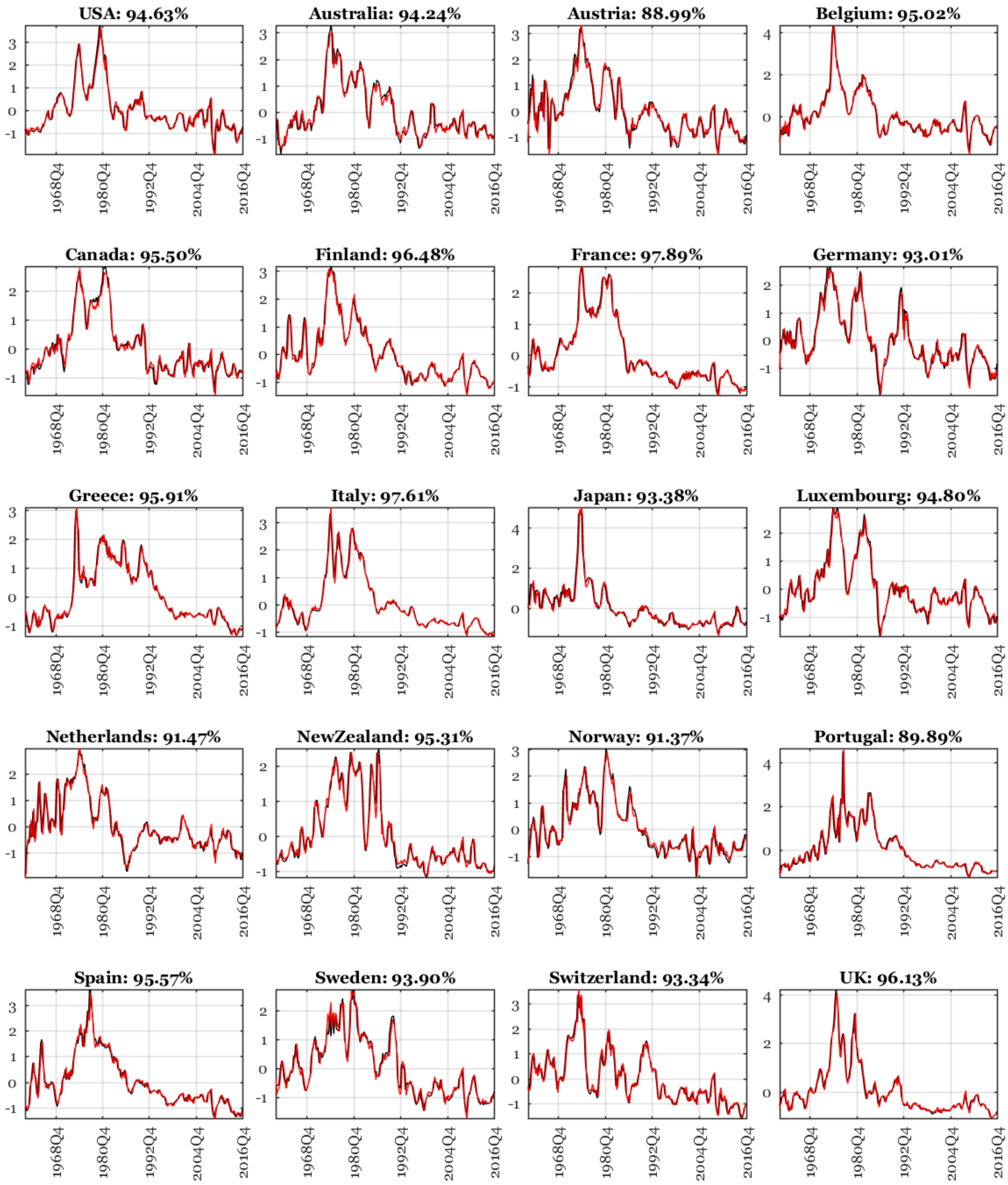


Figure 17: MAI-AR-SV in-sample fit

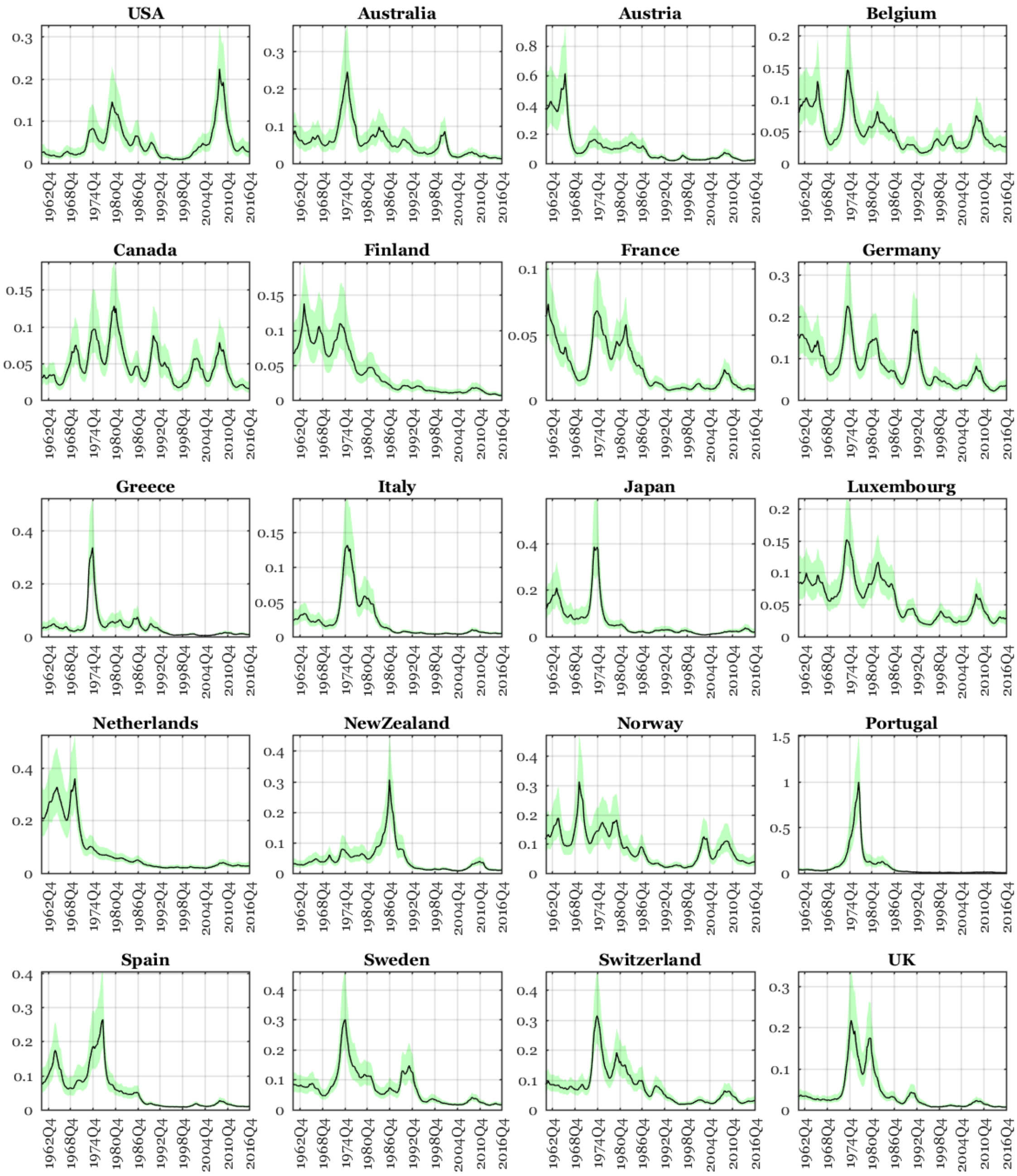


Figure 18: MAI-AR-SV, Residuals' Volatility, posterior bands

C.2 Core Inflation, Data and Decompositions

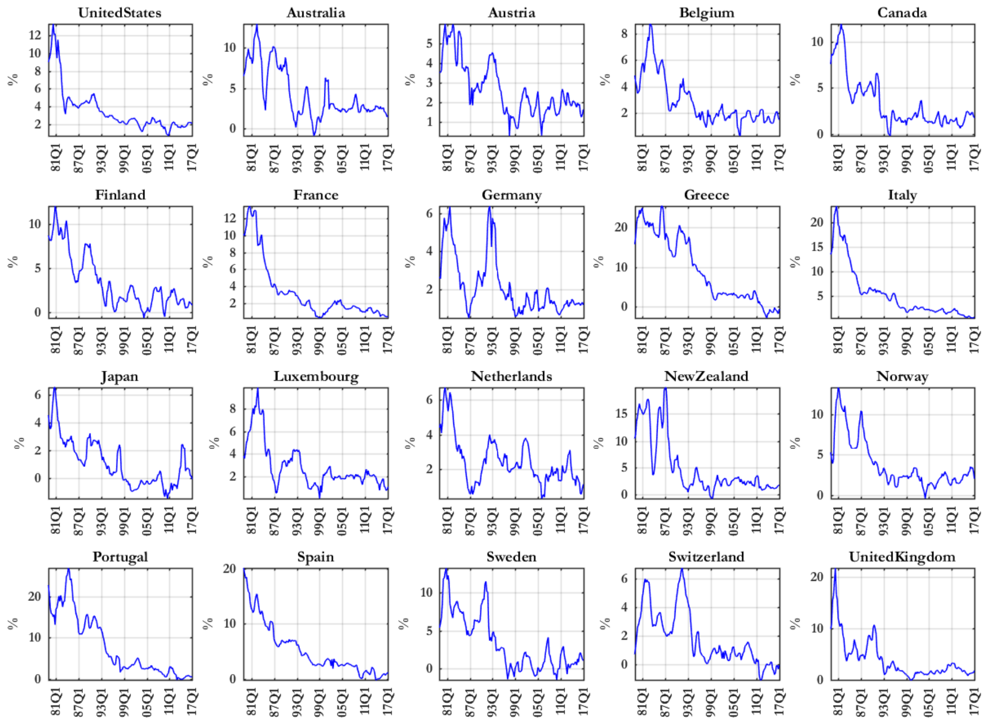


Figure 19: Non-Food & non-Energy inflation rates (year on year growth rates in quarterly CPIs)

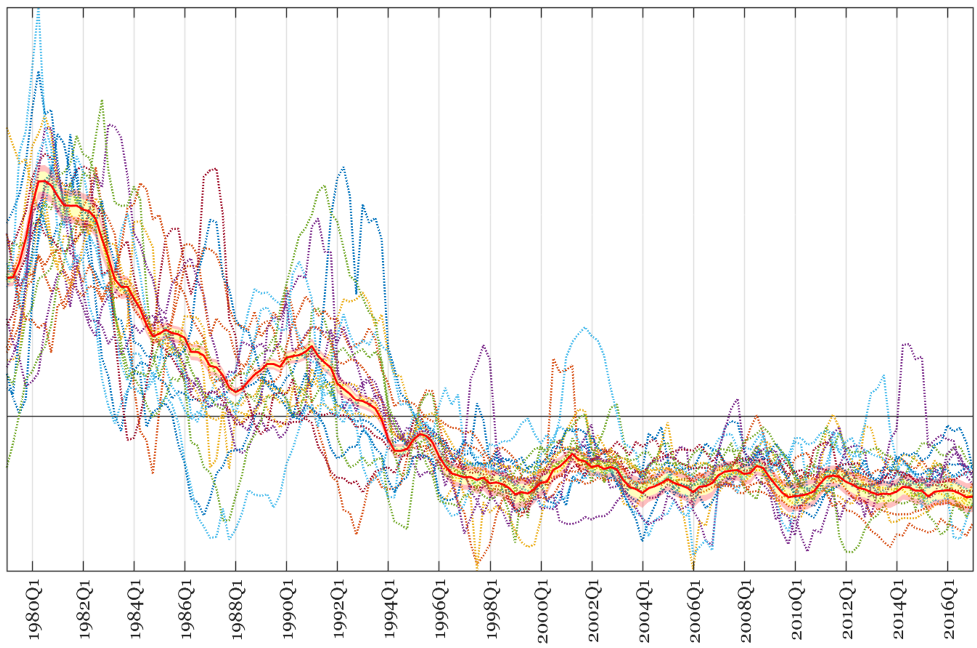


Figure 20: MAI-AR-SV estimated common factor (with posterior bands) Vs Data. Core inflation

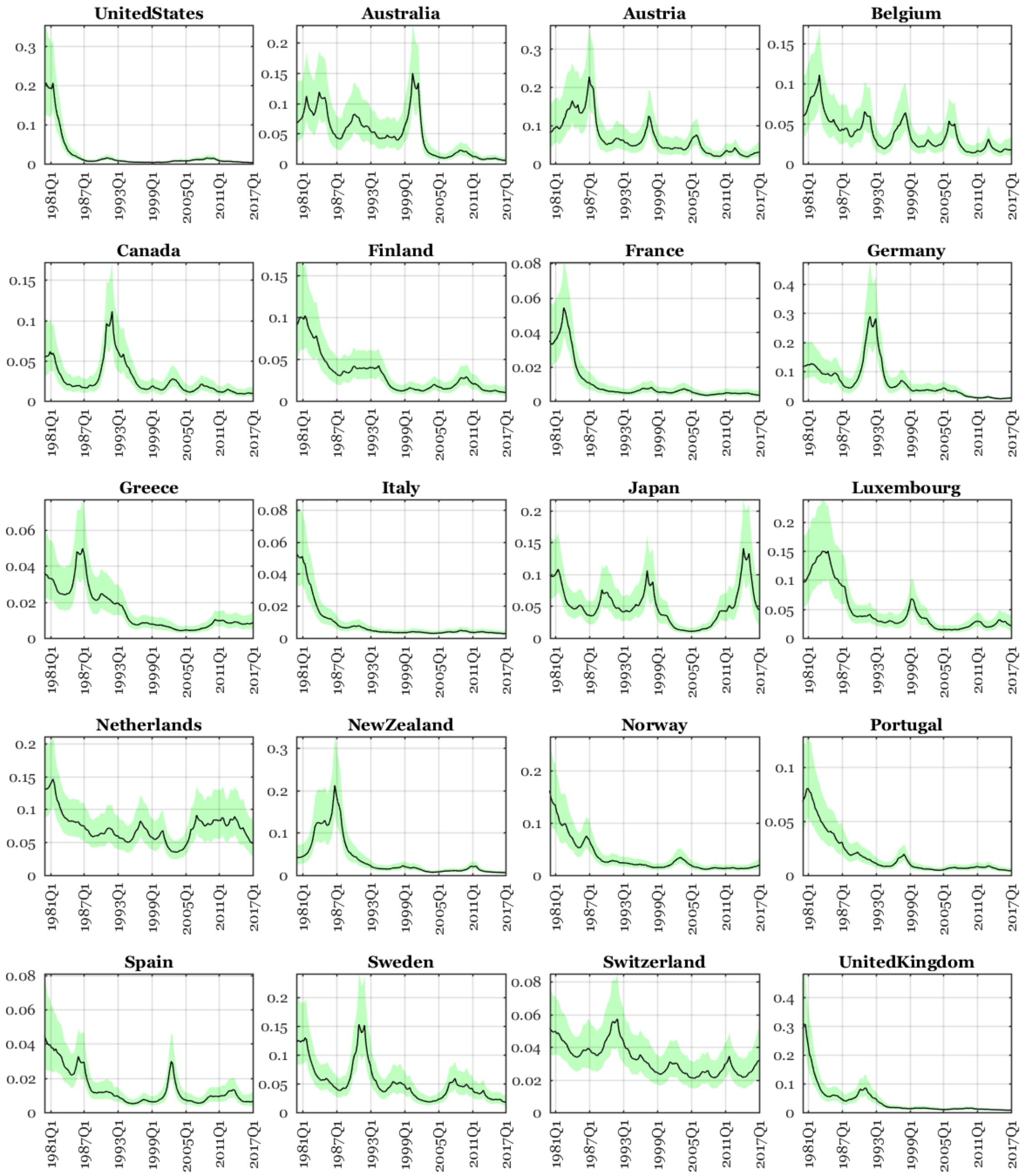


Figure 21: MAI-AR-SV, Residuals' Volatility, posterior bands. Core inflation

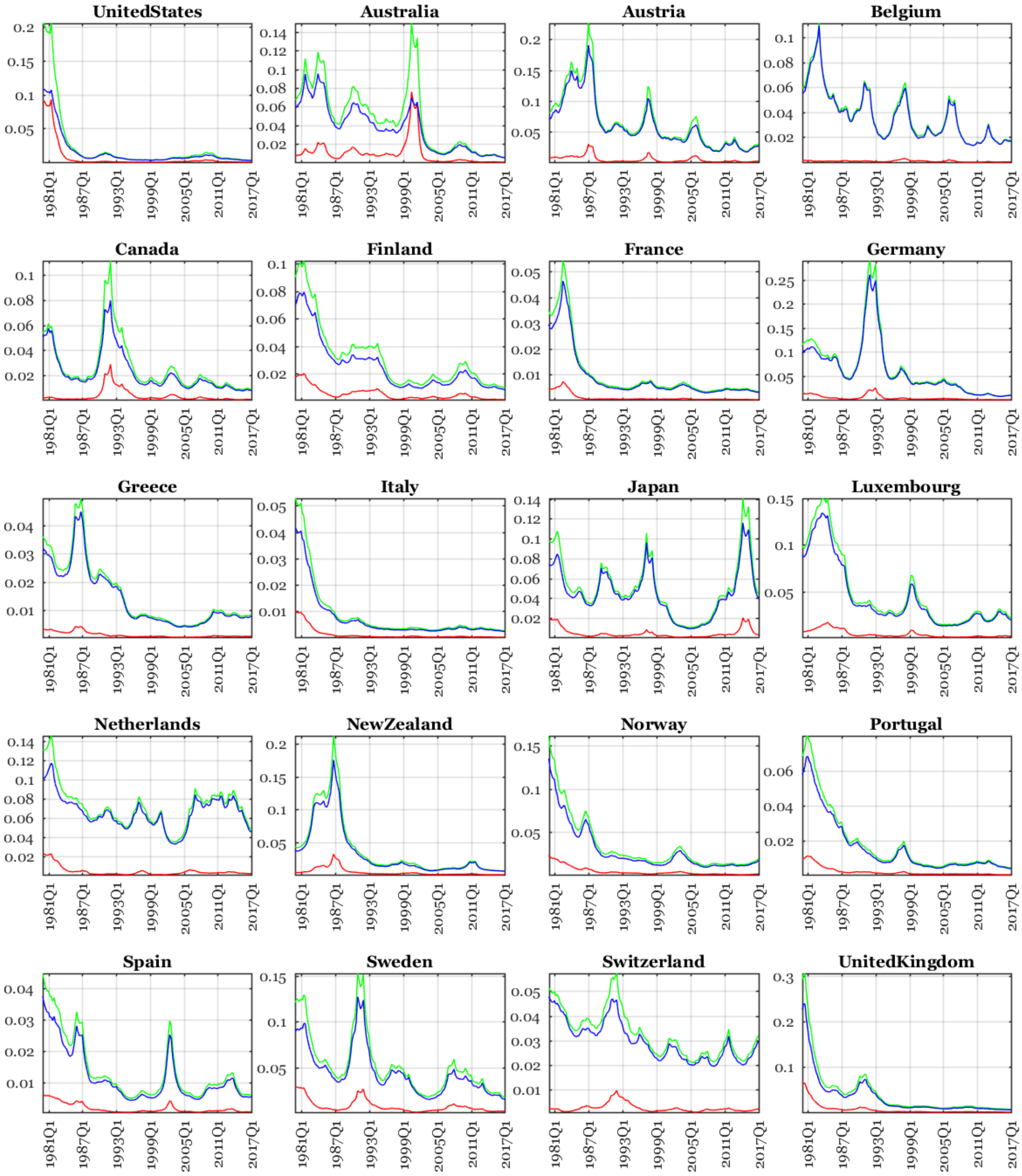


Figure 22: MAI-AR-SV, Residuals' Volatility, TV decomposition, Common (red), Idio (green), total (blue). Core inflation

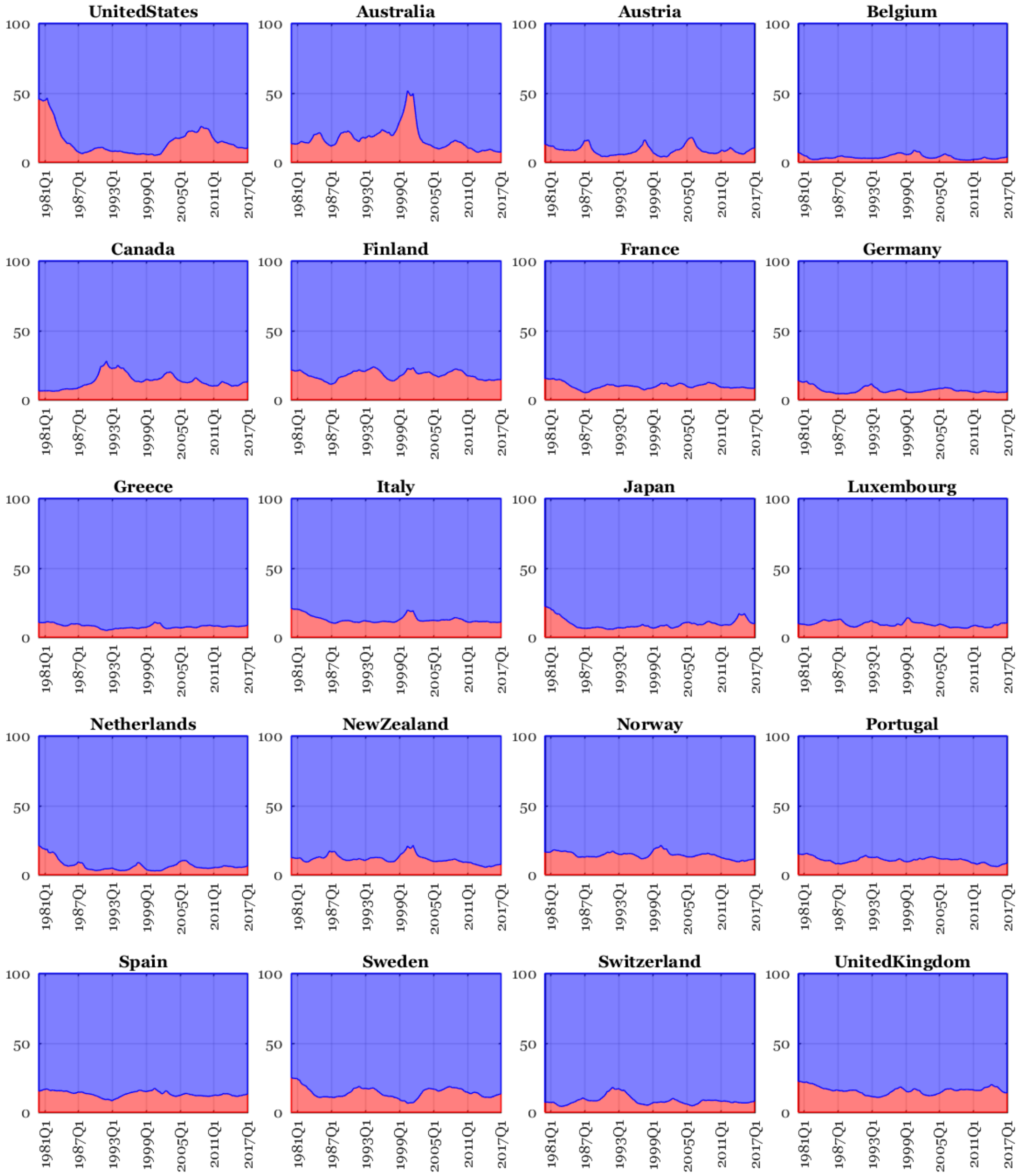


Figure 23: MAI-AR-SV, Residuals' Volatility, TV decomposition shares (%), Common (red), Idio (blue). Core inflation

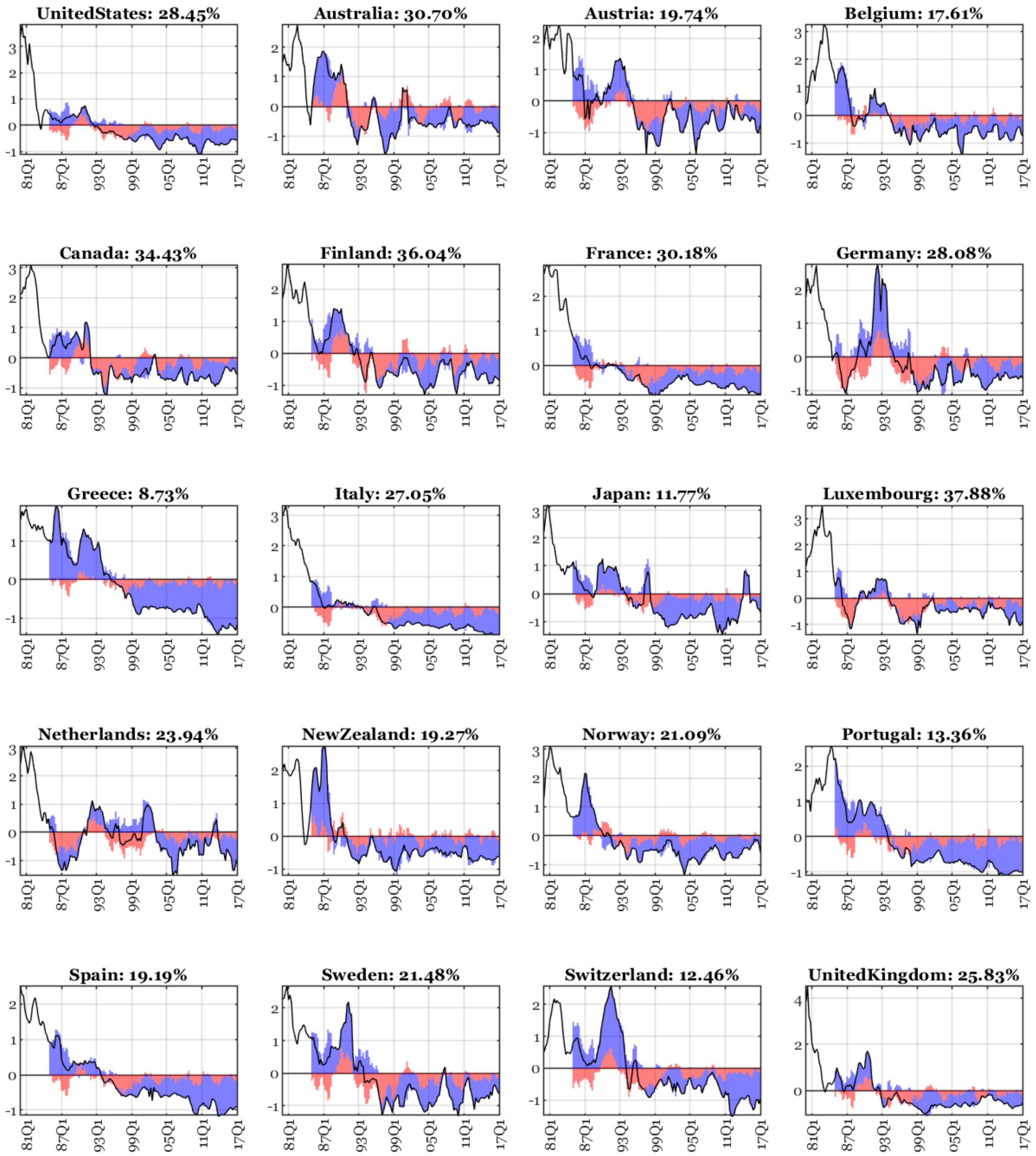


Figure 24: MAI-AR-SV, Core inflation levels decomposition into common (red) and idiosyncratic components