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JEL Classification: O11, O14, L16, E21

Keywords: structural change, Multi-sector growth model, non-homothetic preferences, Relative price effects, Non-monotonic Engel curves, aggregation

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We study structural change in historical consumption expenditure of the United States, the United Kingdom, Canada and Australia over more than a century. To identify preference parameters from aggregate data, we characterize the most general class of preferences in a time-additive setting that admits aggregation of the intertemporal saving decision. We parametrize and estimate such intertemporally aggregable (IA) preferences and discuss their properties in a dynamic general equilibrium framework with sustained growth. Our preferences class is considerably more flexible than the Gorman form or PIGL/PIGLOG, giving rise to a good fit of the non-monotonic pattern of structural change.

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1 Introduction

As countries develop, the consumption expenditure and value-added share of the agricultural sector tends to decline steadily, the share of manufacturing first increases and then decreases, and eventually services become the dominant sector. Qualitatively, this is a robust pattern across time and space. In this paper, we make three contributions to the structural change literature: (i) we document these robust patterns of structural transformation in the United States (USA), the United Kindom (GBR), Canada (CAN), and Australia (AUS) with new consumption expenditure data covering over a century; (ii) we analyze structural change in a multi-sector growth model and characterize the most general class of preferences for which aggregate expenditure and saving are independent of inequality — a property that we call *intertemporal aggregation*. Intertemporal aggregation is more general than Gorman aggregation, as inequality can still affect the intratemporal demand across sectors; (iii) we provide a quantitative analysis, where this demand structure allows us to consistently estimate the preference parameters from aggregate sectoral expenditure data. Our preference specfication is more flexible than the preferences frequently used in the literature, e.g., the generalized Stone-Geary or the Price-Independent Generalized Linearity (PIGL) preferences, and we show that this additional flexibility is required to fit the historical data.

Although the pattern of sectoral reallocation is well documented in other data, the empirical literature on structural change has come to different conclusions on whether stable preferences are consistent with this pattern. Herrendorf, Rogerson, and Valentinyi (2013) find that the sectoral demands derived from the standard non-homothetic generalized Stone-Geary preference specification are consistent with the USA's structural change in the postwar era. In contrast, Buera and Kaboski (2009) show, for a historical sample starting in 1870, that the same specification struggles to fit the data for the USA. However, as constructing the consumption component of value added requires input-output tables, which are not available for the prewar period, the results in Buera and Kaboski (2009) are not directly comparable to the ones in Herrendorf, Rogerson, and Valentinyi (2013). In this paper we derive results comparable for the post- and the pre-war period by focusing instead on the structural change within final consumption expenditure, where sectors are categorized according to final consumption expenditure instead of consumption in value added.

As a first step, we construct the final consumption expenditure shares for the USA, GBR, CAN, and AUS from 1900 to 2014. Three strong and robust regularities emerge from the historical data across all four countries: (i) a continued decline of the expenditure share for agriculture, (ii) a hump-shape of the manufacturing share that peaks in the middle of the last century, and (iii) an accelerated rise of the service sector. The same qualitative pattern has been documented also for value-added and employment shares (see Herrendorf, Rogerson, and Valentinyi, 2014, for a comprehensive survey and also Buera and Kaboski, 2012), but most studies of structural change that quantify demand forces restrict the analysis to the post-war period. However, in the post-war period, for example, regularity (ii) above cannot be studied as manufacturing is already declining.

To quantify the demand-side forces of structural change, it is necessary to parameterize preferences. To fit the non-monotonic pattern of consumption expenditure shares described above, we need flexible income effects that require non-homothetic preferences outside the restrictive Gorman form. However, such preferences do not allow for exact linear aggregation, and in general one needs microdata to consistently estimate the parameters.[1] This is a challenge, since no comprehensive cross-sectional expenditure data are available for the historical period that we study. We therefore propose a new class of preferences that allows us to consistently estimate parameters from aggregate sectoral data with minimal distributional information. In our multi-sector growth framework with time additivity, the household problem can be split into two decisions: (i) the optimal savings decision (the intertemporal problem); and (ii) how to spend the given expenditure level on different sectors (the intratemporal problem). Our proposed class restricts preferences such that aggregate saving and expenditure are independent of inequality — we call this property intertemporal aggregation and characterize the full class of such preferences. The preferences in our class imply that the marginal utility of any individual relative to the individual with the average expenditure level remains constant at any point in time. As a con-

[1] A quantitatively valid framework is crucial to assess the welfare effects of structural change. For example, an aggregate productivity slowdown from the Baumol (1967) cost disease could be reinforced or dampened by income effects. With non-homothetic preferences and household heterogeneity, the distributional effects of such trends are non-trivial and potentially important.

sequence, the impact of inequality on the aggregate sectoral consumption demand is reduced to a simple scalar. This property allows us to estimate all parameters from aggregate data, up to one constant that can be identified from information on the expenditure distribution at one point in time. Despite this tractable aggregation property, the intratemporal problem allows for differences in the marginal propensity to consume from specific sectors across households, i.e., inequality matters for the intratemporal expenditure structure.

The resulting class of intertemporally aggregable (IA) preferences is parsimonious and flexible. For example, the flexibility allows a specific good to be a necessity for low income levels and a luxury for high levels (at given prices). Banks, Blundell, and Lewbel (1997) have shown that this is an essential feature to fit microeconomic expenditure data. We show that our IA class directly nests the frequently used generalized Stone-Geary and the PIGL preferences (see Muellbauer, 1975, 1976) as special cases, but it allows for additional flexibility when fitting the non-monotonic pattern of structural change. Despite its flexibility, we also demonstrate the IA specification's consistency with a standard multi-sector growth model as put forward by Herrendorf et al. (2014), i.e., it supports an asymptotic balanced growth path.

In the quantitative analysis, we estimate different parameterizations of our IA preferences for both the historical and the post-war sample. By restricting parameters, we can directly compare the fit of our preferred IA specification to the one of the generalized Stone-Geary or the PIGL specification. We find in the historical data, which includes the pre-war period, that the generalized Stone-Geary specification struggles to fit the sectoral expenditure shares of agriculture, manufacturing, and services for the USA, GBR and CAN. This finding is in line with the conclusion in Buera and Kaboski (2009), which is, however, based on consumption in value added.[2] In contrast to generalized Stone-Geary preferences, for which income effects converge rapidly to zero as countries get richer, PIGL as well as IA preferences permit sustained income effects. This allows the latter specifications to fit the continued decline in agriculture much better towards the end of each sample period. Furthermore, IA preferences also have the necessary flexibility to fit the non-monotonic

[2] As is common in the literature, Buera and Kaboski (2009) consider a subsistence level of manufacturing consumption equal to zero. In our historical sample, we find that the fit of the generalized Stone-Geary preferences improves when this is relaxed.

pattern in manufacturing and the acceleration in services observed in the historical data. When we pool the historical data for all four countries, we also find that the PIGL and IA preference specifications fit the historical data best. However, once we allow for an additional parameter that separately governs the subsistence level of manufacturing consumption, which is commonly restricted to zero in the literature, the generalized Stone-Geary specification yields a good fit of the historical panel data. This finding is reminiscent of Herrendorf et al. (2013), who show that the generalized Stone-Geary specification captures the structural transformation of the USA in the post-war data well.

The remainder of the paper is organized as follows. Section 2 places our study in the related literature. Section 3 describes the historical panel data and establishes the empirical regularities. In Section 4 we present the general theoretical framework, and in Section 5 we characterize the class of IA preferences. Section 6 presents the specific parameterization of preferences and Section 7 contains the structural estimation and discusses the main empirical results. Section 8 concludes. All proofs and the continuation of the main estimation tables are in Appendix A. Additional figures, tables, and a detailed description of the historical data are delegated to Online Appendixes B and C.[3]

2 Related literature

Our paper relates to several recent developments in the macroeconomic literature on structural change. The papers closest to ours are Buera and Kaboski (2009), Herrendorf et al. (2013, 2014), Boppart (2014), and Comin et al. (2015).

As in the quantitative analysis in Herrendorf et al. (2013), we focus on the structural transformation that a specific demand system generates for given sectoral prices and nominal per-capita expenditure. Herrendorf et al. (2013) find that the generalized Stone-Geary specification fits the post-war data in the USA well. We go beyond their analysis by considering a larger dataset that also spans the pre-war period for the USA and covers in addition GBR, CAN, and AUS. Herrendorf et al. (2013) categorize the broad sectors agriculture, manufacturing, and services from

[3]The Online Appendix is available from the authors' websites or can be requested by email.

two natural perspectives: in terms of final consumption expenditures and in terms of consumption in value added. The latter requires input-output tables to separate consumption and investment, and they therefore focus on the post-war period when these tables are available. The main focus of their paper is to reconcile the differences in the estimated preference parameters that result from the two categorizations. Instead, we concentrate on the final consumption expenditure perspective that does not require data on the input-output structure. Thus, we can study a much longer time period that contains, for example, the hump-shape in manufacturing. When considering historical final consumption expenditure data from 1900 onwards, our conclusions differ in important ways from the post-war sample: the generalized Stone-Geary specification struggles to fit the historical expenditure shares for the majority of countries, including the USA, and income effects are no longer the single main force behind structural transformation in final consumption expenditure. In contrast to their results for the post-war sample, we find that it is important to also estimate a subsistence level in manufacturing consumption to fit the historical data well.

Our empirical results for the long samples resemble the findings in Buera and Kaboski (2009), who use decennial value-added data from 1870 onwards to show that the generalized Stone-Geary specification cannot fit the most salient patterns of structural change. However, our data allow us to focus on the final output that is domestically consumed, while Buera and Kaboski (2009) run the non-homothetic specification over total output, which also includes exports. Moreover, since input-output tables are not available for the historical data, they have to construct a proxy for sectoral consumption value-added by deducting all investment from manufacturing. We not only confirm their negative result for the generalized Stone-Geary specification using historical final consumption expenditure data, we also address the gap in the literature by proposing a more flexible preference specification that can fit the historical expenditure shares.

As in Boppart (2014) and Comin et al. (2015), we use a preference specification that allows for both sustained income and relative price effects in a standard multisector growth framework as presented in Herrendorf et al. (2014).[4] A theo-

[4]Kongsamut et al. (2001), Ngai and Pissarides (2007), Foellmi and Zweimüller (2008), shut down either the relative price or the income effect to be consistent with an exact balanced growth

retical drawback of the Stone-Geary specification is that the existence of an exact balanced growth path depends on knife-edge parameterizations that imply mutually exclusive income effects (Kongsamut et al., 2001) or relative price effects (Ngai and Pissarides, 2007), respectively. Boppart (2014) overcomes this limitation and proposes a PIGL preference specification where structural transformation occurs along an exact balanced growth path. While Boppart (2014) considers an economy with two broad sectors for goods and services that decrease and increase monotonically, respectively, we are splitting up the goods sector into agriculture and manufacturing as is common in the structural change literature that emphasizes the hump-shape in manufacturing. Our IA class of preferences nests PIGL and allows for additional flexibility in terms of income effects, which is important to fit the historical data that includes the pre-war period. Moreover, we provide empirical evidence for the importance of relative price and income effects for the entire 20th century and for four different countries, while Boppart (2014) focuses on the post-war period in the USA.[5]

Comin et al. (2015) apply the non-homothetic CES preference as specified by Hanoch (1975) in a multi-sector growth model and argue that such a preference specification provides a good fit to sectoral employment shares in a panel of countries during the post-war era. These non-homothetic CES preferences, however, are not part of the IA class of preferences. Hence, a theoretical drawback of the non-homothetic CES preferences is that preference parameters cannot be identified without aggregation bias from aggregate sectoral expenditure shares. The IA property of preferences is central to our application of historical structural change, where microeconomic consumer expenditure surveys do not exist on a regular basis.

Our paper is also related to the microeconomic literature on demand system estimation. The PIGL class of preferences has been introduced by Muellbauer (1975, 1976) and yields expenditure shares that are quasi-linear in the nominal expenditure level raised to some power (in the PIGLOG case it is a quasi-linear function in the

path. Like Comin et al. (2015), we consider specifications consistent with an asymptotic balanced growth path, while Boppart (2014) establishes structural change along an exact balanced growth path.

[5]Leon-Ledesma and Moro (2017) use the PIGL preferences of Boppart (2014) to analyze the U.S. post-war period. Eckert and Peters (2018) apply PIGL preferences to study structural change between the agricultural and the non-agricultural sector in a spatial equilibrium model.

logarithm of the nominal expenditure level). Banks, Blundell, and Lewbel (1997) established the QAID system that results from the quadratic generalization of the Almost Ideal Demand (AID) system, which is itself a special case of PIGLOG. Like our IA preferences, the QAID specification allows for a non-monotonic relationship between expenditure shares of different sectors and the nominal expenditure level. However, in contrast to IA preferences, QAID is not compatible with sustained growth and the demand system cannot be aggregated in a tractable manner. In a microeconomic analysis with cross-sectional data on individuals' expenditures this is not a concern, but in macroeconomic time-series data this is an important caveat.

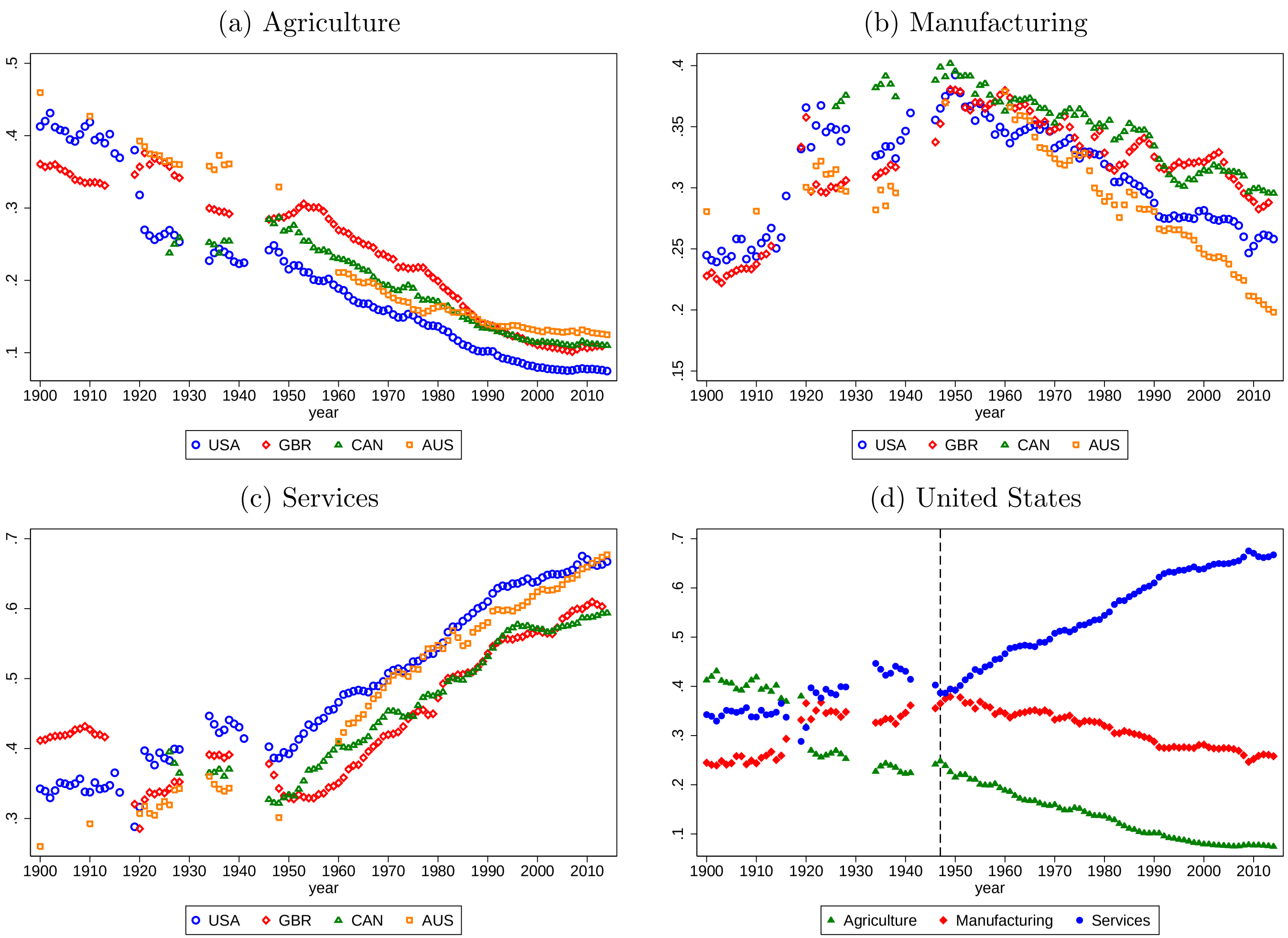
3 Historical data on structural change

We use nominal final consumption expenditure and price data for individual consumption categories in the USA, GBR, CAN, and AUS. We aggregate the consumption categories to the three broad sectors agriculture, manufacturing, and services. Roughly speaking, agriculture consists of food and beverages purchased for off-premise consumption. Manufacturing captures durable goods, clothing and footwear, gasoline and other energy goods, and other nondurable goods. Services consist of private services and for robustness we later also include government consumption. The categorization follows Herrendorf et al. (2013) and is standard in the structural change literature.[6] For each broad sector, we can then calculate its expenditure as a share of total nominal consumption expenditure.

3.1 Final consumption expenditure shares

Figure 1 illustrates three robust regularities of structural change in the USA, GBR, CAN, and AUS since the beginning of the last century: First, panel (a) shows that there has been a steady decline in the expenditure share for agricultural consumption. Historically, agriculture used to be the largest sector and then declined substantially over time. For example, in the USA, the share of agriculture in private consumption fell from 41% to only 7% during our sample period, as can be seen from

[6]The data sources and the detailed categorization of the broad sectors are described in Online Appendix C.



Notes: The figure plots the final private consumption expenditure shares for the USA, GBR, CAN, and AUS. Panels (a) to (c) plot the shares by sector, and panel (d) shows the shares for the USA. The years affected by WWI, WWII, and the Great Depression are excluded. Sources: Several, see Online Appendix C.

Figure 1: Final private consumption expenditure shares

panel (d). Second, panel (b) illustrates that the expenditure share for manufacturing consumption is hump-shaped over time. Again using the USA as an example, in 1900 the share of manufacturing was 24%, then reached its peak of 39% in 1950, and finally declined gradually to 26% by the end of the sample. Third, panel (c) shows an accelerated rise of the service sector. The share of services increased moderately between 1900 (34% in the USA) and 1950 (39%), and then increased rapidly to 67% in the second half of the sample.

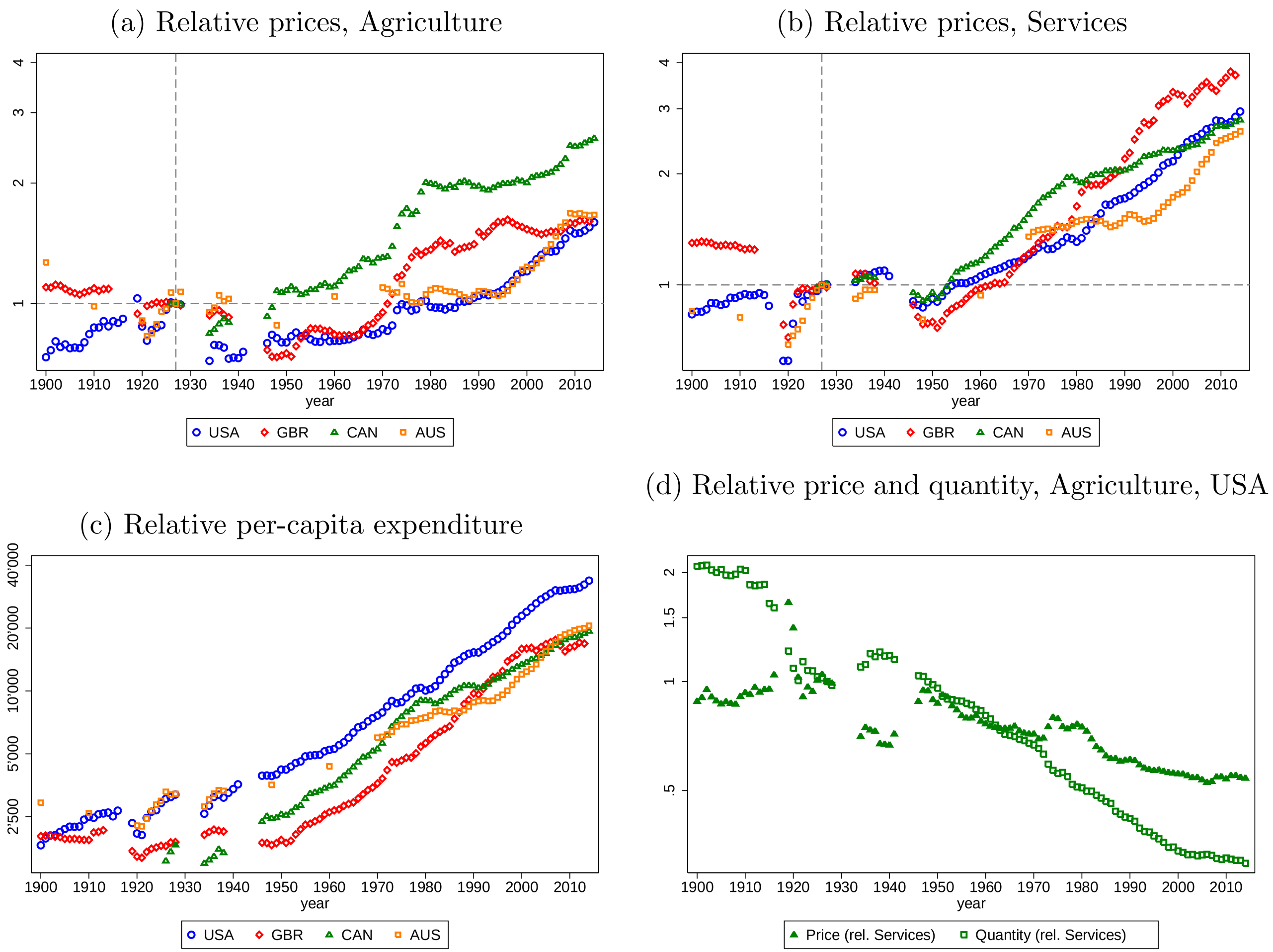
Similar regularities have been documented for other countries and complementary measures of structural changes (see for example Buera and Kaboski, 2012; Uy et al., 2013; Herrendorf et al., 2014; and Comin et al., 2015, for recent contributions). The four countries in our panel experienced similar living standards and per-capita income growth over the considered period, such that the regularities also hold up if the expenditure shares are plotted against logarithmic (log) real per-capita GDP.[7]

3.2 Relative prices and per-capita expenditure

The expenditure share of each sector is determined by the price and the real quantity demanded relative to the other sectors. In principle, structural change could be completely driven by changes in relative prices alone. However, in this section we document the development of relative prices and quantities over the last century and argue that price effects need to be complemented with sustained and flexible income effects to explain the structural transformation observed in our historical panel data.

Panels (a) and (b) of Figure 2 plot the price of the agricultural sector and the service sector relative to manufacturing on a ratio scale. All relative prices are normalized to unity in the year 1927. We see that the sectoral prices relative to manufacturing remained relatively stable in the first half of the sample and then started to increase around 1950. The price increase is more pronounced for the service sector than for agriculture, and — if substitution effects are strong enough

[7]This is illustrated in Figure B.1 in Online Appendix B, where we plot the expenditure shares over the log real per-capita income taken from the Maddison Project (2013). To test the patterns more formally, we also ran regressions of the sector shares on log GDP per capita. Following Buera and Kaboski (2012), we split the sample at the GDP per capita level that corresponds to the peak in manufacturing. The coefficients in each subsample confirm the above regularities.



Notes: The figure plots in panel (a) and (b) the prices of agriculture and services relative to manufacturing, and in panel (c) the nominal per-capita expenditure relative to the manufacturing price in the USA, GBR, CAN, and AUS. All nominal variables are based on final private consumption expenditure and expressed in PPP-adjusted 1990 international \$. In panel (a) and (b) relative prices are normalized to unity in 1927 and plotted on a ratio scale. In panel (c) relative per-capita expenditure is plotted on a ratio scale. Panel (d) shows the price and quantity of agriculture relative to services in the USA. The years affected by WWI, WWII, and the Great Depression are excluded. Sources: Several, see Online Appendix C.

Figure 2: Relative prices and private per-capita consumption expenditure

— the relative price alone could explain the late rise of the service sector documented earlier. However, for the agricultural sector both the price and the real consumption relative to services are falling over time since 1950. With homothetic preferences, such a positive relationship cannot be explained with a well-behaved substitution elasticity. Hence, in addition to relative price effects, income effects are needed to explain the historical structural change.

Why are *flexible* income effects needed? Panel (d) of Figure 2 shows that the price and quantity of agriculture relative to services fall together for more than 60 years in the USA, while in the first half of the sample, relative prices and quantities of agriculture are mostly negatively related. Since per-capita expenditure is steadily growing at the same time, this suggests that agriculture must have a substantially lower income elasticity of demand relative to the service sector in the post-war compared to the pre-war period. Hence, it is not sufficient to have income effects; they must also be flexible.

Panel (c) of Figure 2 illustrates that there has been sustained per-capita expenditure growth in all four countries (with the exception of the GBR and AUS between 1900 and 1920, where per-capita expenditure in terms of manufacturing goods is roughly constant).[8] Note that per-capita expenditure is plotted on a ratio scale; thus the slope approximates the yearly growth rate. For the USA, for example, relative per-capita expenditure has increased by more than a factor of 18 between the first observation in 1900 and the last in 2014. Thus, if income effects are important, the massive increase in per-capita expenditure will have a major role in explaining the pattern of structural change over the last century.

4 Theoretical framework

In this section, we present the general theoretical framework to consider structural change in a multi-sector growth model. On the production side, we use a similar model structure as proposed in Herrendorf, Rogerson, and Valentinyi (2014). On

[8]Note that real per-capita expenditure is unobserved in the data. Thus, in the figure, we proxy real expenditure by expressing nominal expenditure relative to the price of manufacturing. The qualitative conclusions from the figure remained unchanged if we used, for example, a Fisher-index over the sectoral prices to deflate the nominal expenditure.

the consumer side we keep preferences general and allow for heterogeneity in consumers' factor endowments. Further below in Section 5, we then discuss preference specifications consistent with our general theoretical framework.

4.1 Economic environment

We consider an infinite horizon, closed economy framework in discrete time with four production sectors. Our main focus will be on three sectors that produce the consumption goods called agriculture A, manufacturing M, and services S, but we also explicitly model a fourth sector that produces an investment good X.[9] In each sector $j \in J_+ \equiv \{A, M, S, X\}$ output $y_{j,t}$ is competitively produced according to the following Cobb-Douglas technology

$$y_{j,t} = k_{j,t}^{\alpha} \left(\gamma_j^t n_{j,t}\right)^{1-\alpha}. \tag{1}$$

Here, $k_{j,t}$ and $n_{j,t}$ denote capital and labor used in sector j and γ_j^t is a Harrod neutral technology term (where t denotes time). The technology term is normalized to one in all sectors in period $t = 0$. We have $\alpha \in (0, 1)$, and $\gamma_j \geq 1$, $\forall j$.[10] The firms in all sectors take the rental rate, $R_t = r_t + \delta$, the wage rate, w_t, and the output price, $p_{j,t}$, as given and then choose their capital and labor input to maximize profits. The capital and labor market clearing conditions are

$$\sum_{j \in J_+} k_{j,t} = k_t, \text{ and } \sum_{j \in J_+} n_{j,t} = n, \tag{2}$$

where k_t and n denote total capital and labor in the economy.

The output of agriculture, manufacturing and services is all consumed, whereas the output of sector X is invested. There is an interval of infinitely lived households

[9] The extension to more than three consumption sectors is straightforward.

[10] Furthermore, we assume that $\gamma_X > 1$ such that capital can be accumulated at a sustained positive rate. Since capital is used with a unitary elasticity and there is no technological regress in the consumption sectors, this implies that output of all sectors can grow at a steady positive rate too.

indexed by $i \in [0, N]$ with the following preferences

$$\mathcal{U}_{i,0} = \sum_{t=0}^{\infty} \beta^t v(e_{i,t}, P_t), \quad P_t \equiv (p_{A,t}, p_{M,t}, p_{S,t}). \tag{3}$$

The period utility function $v(e_{i,t}, P_t)$ is given in *indirect form*, i.e., it is defined over nominal expenditure $e_{i,t}$ and the vector P_t of prices $p_{j,t}$ of all the consumption goods $j \in J \equiv \{A, M, S\}$. We assume that the utility function $v(\cdot)$ is three times continuously differentiable in e and continuously differentiable in all prices. Furthermore, we assume a positive but diminishing marginal utility of nominal consumption expenditure, i.e., $v_e(e_{i,t}, P_t) > 0$ and $v_{ee}(e_{i,t}, P_t) < 0$. The parameter $\beta \in (0, 1)$ denotes the discount factor. We explicitly allow for household heterogeneity in factor endowments. Household i supplies inelastically $n_i \geq 0$ hours to the labor market and its initial wealth, $a_{i,0}$, is exogenously given. Since preferences are additively separable in time, the household's problem can be split up into an intertemporal and an intratemporal problem. The intertemporal problem deals with the optimal saving/spending decision where household i chooses a sequence $\{e_{i,t}, a_{i,t+1}\}_{t=0}^{\infty}$ to maximize (3) subject to a period budget constraint

$$a_{i,t+1} = a_{i,t}(1 + r_t) + w_t n_i - e_{i,t}, \tag{4}$$

and a standard no-Ponzi game condition.[11] In the intratemporal problem we can simply apply Roy's identity to find how nominal expenditure, $e_{i,t}$, is spent on the three different sectors to find the individual Marshallian demand $c_{i,j,t}$, $j \in J$ for each household i.

In the following we choose the price of the investment good as a numéraire, $p_{X,t} = 1$. Consequently, we can write the asset market clearing condition as

$$\int_0^N a_{i,t}\, di = k_t, \tag{5}$$

and the law of motion of the aggregate capital stock becomes $k_{t+1} = k_t(1 - \delta) + y_{X,t}$

[11] The no-Ponzi game condition can be expressed as $\lim_{T\to\infty} a_{i,T+1} \prod_{s=1}^{T} (1 + r_s)^{-1} \geq 0$.

where $\delta \in [0,1]$ is the depreciation rate. Furthermore, we have

$$\int_0^N n_i \, di = n, \tag{6}$$

and market clearing in the three consumption sectors implies

$$\int_0^N c_{i,j,t} \, di = y_{j,t}, \ \forall j \in J. \tag{7}$$

In macroeconomic theory it is more common to work with direct preference specifications defined over real consumption commodities instead of the indirect formulation used here. However, as we will see below, the indirect formulation allows us to characterize the optimal saving decision as simple as in a one-sector economy. This enables us to highlight the additional restrictions that the existence of a balanced growth path imposes on preferences.[12] Furthermore, for some of the preferences specifications that we will evaluate below, a closed-form utility function exists only in the indirect formulation. For these reasons, we here prefer the indirect formulation. In cases where a closed-form solution for the direct formulation exists, we will also state this direct utility function. Note, however, that in the end the main object of interest is the implied demand system, which is identical for both the direct or indirect formulation.

Although we are interested in structural change between different consumption good sectors, we nevertheless model the investment good as a separate sector.[13] In this respect, the theory is relatively general and could also accommodate investment-specific technical change.

4.2 Equilibrium definition and discussion

We define an equilibrium in this economy in the standard way.

[12]Note that we use the same generalized balanced growth definition as in Kongsamut, Rebelo, and Xie (2001) and Herrendorf et al. (2014). The word *generalized* refers to the feature that — at the sectoral level — endogenous variables are not required to grow at constant rates. Henceforth, we refer to their generalized equilibrium concept simply as balanced growth.

[13]See García-Santana et al. (2016) and Herrendorf et al. (2018) for theories of structural change induced by the investment good sector.

Definition 1. *An equilibrium is a sequence of prices and quantities that is jointly consistent with utility maximization of all households, profit maximization (and perfect competition) of all firms, as well as all the market clearing conditions* (5)–(7).

To analyze the historical consumption expenditure data presented in Section 3.1, we will in the following focus on the competitive outcome of this dynamic general equilibrium framework. As we will argue below, this is a useful benchmark model to guide our data analysis. The framework is flexible enough to allow relative prices between sectors to change. Furthermore, by allowing for capital accumulation, the model includes the intertemporal margin, which is essential to discuss consistency with sustained growth in per-capita income and expenditure. Although the framework is in some sense very standard, it seems nevertheless relevant to comment here on its generality.[14]

First, note that we focus on the decentralized market equilibrium. However, the welfare theorems apply and the decentralized market outcome is Pareto efficient (and could also be characterized as the solution to a planner's problem). Second, the imposed restrictions on the preference side, like time additivity and standard discounting, are relatively mild and standard, and we keep at this point full flexibility with respect to the period utility function. In the remainder of the paper, we will analyze the prediction of this framework under different functional form assumptions for $v(e_i, P)$. On the production side, however, the framework puts some simplifying structure; most importantly it assumes identical output elasticities of capital α across the three consumption good sectors (as well as the investment good sector). This implies that relative price changes are completely driven by differences in the productivity growth rates across sectors. It, however, precludes factor intensity differences as a source of relative price changes (à la Acemoglu and Guerrieri, 2008), and that shifts in the demand structure due to income effects have an impact on relative prices (see Caselli and Coleman, 2001). The Cobb-Douglas functional form of production could be relaxed and the output elasticity of capital could be allowed to differ between the consumption sectors and the investment sector. These generalizations would not affect the model's main predictions.

[14]For instance, it coincides with the "benchmark model" proposed by Herrendorf et al. (2014), except for our generality in terms of instantaneous preferences and household heterogeneity.

4.3 Equilibrium implications

In equilibrium, the production side of the economy can be characterized by the following lemma.

Lemma 1. *The capital-labor ratio is equalized across all sectors, i.e.,*

$$\frac{k_{j,t}}{n_{j,t}} = \frac{k_t}{n}, \ \forall t, j \in J_+. \tag{8}$$

Furthermore, the prices are given by

$$p_{j,t} = \gamma_j^{-(1-\alpha)t} \left(\frac{w_t}{1-\alpha} \right)^{1-\alpha} \left(\frac{r_t + \delta}{\alpha} \right)^{\alpha} = \left(\frac{\gamma_X}{\gamma_j} \right)^{(1-\alpha)t}, \ \forall j \in J, \tag{9}$$

where the choice of numéraire $p_{X,t} = 1 = \gamma_X^{-(1-\alpha)t} \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha} \left(\frac{r_t+\delta}{\alpha}\right)^{\alpha}$ *has been used for the second equality. The equilibrium rental rate and wage rate are given by*

$$r_t + \delta = \alpha \left(\frac{\gamma_X^t n}{k_t} \right)^{1-\alpha}, \tag{10}$$

and

$$w_t = (1-\alpha)\gamma_X^t \left(\frac{k_t}{\gamma_X^t n} \right)^{\alpha}. \tag{11}$$

Finally, under optimal production, output can be expressed as

$$y_{j,t} = \gamma_j^{(1-\alpha)t} \left(\frac{k_t}{n} \right)^{\alpha} n_{j,t}, \ \forall j \in J_+. \tag{12}$$

Proof. In Section A.1 of the Appendix. ■

Lemma 1 is a direct consequence of the assumption that the output elasticity of capital is identical across sectors, which leads to equalized capital intensities across sectors (see equation (8)). Furthermore, since the production functions then only differ in the labor-augmenting technology terms γ_j^t, the marginal rate of transformation is constant in any given point in time and solely pinned down by the technology side, as shown in equation (9). Hence, shifts in the demand structure will not affect relative prices. Finally, since the capital intensity is identical across sectors, as shown by (12), output in each sector can be written as a linear function of labor used in this sector. Note again that we could allow the capital intensity in the investment good sectors to differ from the consumption good sectors (since along the balanced growth path the saving and investment rate will be constant). However, for the consumption sectors for which the expenditure shares will vary over time, the assumption of identical capital intensities is crucial, since otherwise already the technology side of the economy would exclude the coexistence of structural change with an exact balanced growth path.[15] Consequently, the assumption of identical factor shares is a restriction that we impose for theoretical reasons, but there is empirical support for this assumption as well. First note that the specified production functions are written in terms of gross final output and data on factor intensities are only readily available at the industry value-add level. Hence, to measure the capital share α of a final consumption sector, the entire input-output structure needs to be taken into account. Valentinyi and Herrendorf (2008) provide such estimates for the USA for broadly defined sectors at the gross final output level. The results show that the shares of capital and labor vary surprisingly little and the assumption of identical factor shares across consumption sectors seems to be a reasonable approximation.[16] At the value-added level, Herrendorf et al. (2015) estimate the production functions of our three sectors for the post-war USA and find that the formulation above with Cobb-Douglas technologies with identical output elasticities of capital, but different TFP growth, captures the main technological forces behind structural change. These results are for the capital-labor split. If other production factors like land or the division into skilled and unskilled labor were considered, then we expect factor intensity differences to be more relevant.

The assumption of Harrod-neutral technical change at constant rates is also made for theoretical reasons. A non-constant rate of technical change, in particular in the investment sector, would ex ante rule out the existence of a balanced growth path. Importantly, however, we allow the rate of technical change to be sector specific but restrict it to be constant over time. This assumption implies that relative prices

[15]Note that the issue arises from the intensity differences in the factor that can be accumulated, i.e., here physical capital. In a formulation with human capital that can be accumulated, the relevant assumption would be that the intensity of human plus physical capital is identical across all the sectors with changing nominal output shares.

[16]For instance, the labor income share in the service sector in the U.S. was 0.34 in 1997, which is only marginally smaller than the 0.35 found for total consumption.

will change over time, but do so at constant rates. As our data on relative prices in Section 3.2 show, this seems to be a good first approximation of the data, where we indeed see systematic changes in relative prices in the long run. However, the data also highlight that there may be some structural break in the relative price growth around WWII. To generate such a structural break in our theoretical framework, an exogenous change in the γ_j parameters is required.

The optimal saving behavior of a household i is characterized by the following Euler equation.

Lemma 2. *Solving the intertemporal household problem gives rise to the following Euler equation*

$$\frac{v_e(e_{i,t}, P_t)}{v_e(e_{i,t+1}, P_{t+1})} = \beta(1 + r_{t+1}), \tag{13}$$

where $v_e(e_{i,t}, P_t)$ is the indirect marginal utility of nominal consumption expenditure in a given period.

Proof. In Section A.2 of the Appendix. ■

Jointly with the household budget constraint, (4), the transversality condition, as well as with the initial wealth $a_{i,0}$, this fully characterizes the household's spending and saving behavior. Aggregating all the household budget constraints and combining with (5) and (6) gives

$$k_{t+1} = k_t(1 - \delta) + k_t^\alpha \left(\gamma_X^t n\right)^{1-\alpha} - E_t, \tag{14}$$

where $E_t \equiv \int_0^N e_{i,t} di$ is aggregate nominal consumption expenditure. This allows us to characterize the dynamics of the capital stock and finally solve the model.

In the following we are interested in the long-run properties of the equilibrium path. To this aim, we next define the concept of balanced growth.

Definition 2. *A balanced growth path is an equilibrium path along which the aggregate physical capital stock k_t grows at a constant positive rate. If such a balanced growth path can be reached in finite time, then we call it an exact balanced growth path. If the balanced growth path only exists as time goes to infinity, then we call it an asymptotic balanced growth path.*

The production side of the economy is potentially in line with an (exact) balanced growth path. Hence, if a balanced growth path exists, then its dynamics are fully determined by the exogenous rate of technical change. This is summarized in the next proposition.

Proposition 1. *Along a balanced growth path, the aggregate capital stock, k_t, aggregate nominal output, $y_t = k_t^\alpha \left(\gamma_X^t n\right)^{1-\alpha}$, aggregate nominal consumption, E_t, and the nominal wage rate, w_t, all grow at constant gross rate γ_X, and the nominal interest rate, r_t, is constant.*

Proof. In Section A.3 of the Appendix. ■

Whether the economy admits a balanced growth path depends on the specified period utility function. This (as highlighted in Lemma 2) essentially boils down to the question of whether the Euler equation (13) is jointly consistent with a constant interest rate, r_{t+1}, and a constant gross nominal expenditure growth rate, $\frac{e_{i,t+1}}{e_{i,t}}$, either asymptotically or in finite time. As long as preferences are well specified, *asymptotic balanced growth* is generally fulfilled since expenditure shares have converged to some asymptotic values. Consequently, asymptotically preferences look like a Cobb-Douglas with constant expenditure shares (in some case, even as a one sector economy), which is a well-known special case that supports balanced growth in the above framework. However, some utility functions, like the AID System, would imply non-positive demand for some commodities as income and relative prices grow indefinitely. We discuss this aspect below and in Section 6.

In addition to the intertemporal Euler equation, applying Roy's identity to $v(e_{i,t}, P_t)$ gives the Marshallian demands, i.e., how a given expenditure level is optimally spent on the different sectoral output. The functional form of this demand system depends on the precise formulation of the period utility function. In the rest of the paper we focus on the demand side, where we consider different preference specifications and analyze how they perform in terms of fitting the historical expenditure data. In Section 5, we ask what restriction must be imposed such that preferences preserve that the intertemporal problem can be aggregated. We then characterize the full class of such preferences and show that it accommodates as special cases frequently used formulations, such as the homothetic CES form, the generalized Stone-Geary utility function, and the PIGL class. We then discuss parameterized forms within our broader class and show how they perform in terms of fitting the historical data.

5 A general class of preferences

We have already established in Section 3 that both changes in relative prices and per-capita income have important implications for the aggregate demand structure. Moreover, we know from microeconomic data that there is a vast amount of heterogeneity in consumers' expenditures. Thus, we aim to characterize a general class of non-homothetic preferences that allows for tractable aggregation and remains theoretically consistent with our dynamic multi-sector framework.

5.1 Restricting preferences: intertemporal aggregation

Flexible demand systems typically do not allow for Gorman aggregation and therefore, in general, the preference parameters cannot be estimated from aggregate data alone. Microeconomic data are needed for identification. However, for the historical period studied in this paper, as well as in many other relevant applications, only aggregate data (like per-capita expenditure) are available. How can we consistently retrieve preference parameters from such aggregate data without restricting the utility class too much? Our approach is to rely on the dynamic framework in Section 4, restrict preferences such that aggregation in the intertemporal dimension is preserved, and then show how this allows us to identify preference parameters from aggregate data.[17] To this aim, we next define the class of intertemporally aggregable (IA) preferences.

Definition 3. *Consider our neoclassical framework with time-additive preferences* $\mathcal{U}_{i,0} = \sum_{t=0}^{\infty} \beta^t v(e_{i,t}, P_t)$ *and intertemporal optimization such that the Euler equation* (13) *holds for each individual. We call preferences* $\mathcal{U}_{i,0}$ *intertemporally aggregable (IA) if per-capita expenditure* E_t/N *satisfies the individual Euler equation irrespec-*

[17] A classic approach would be to restrict preferences to be within the Gorman class. As will be shown below, however, this is too restrictive to fit the data.

tive of the cross-sectional expenditure distribution, i.e., we have

$$\frac{v_e(E_t/N, P_t)}{v_e(E_{t+1}/N, P_{t+1})} = \beta(1 + r_{t+1}), \quad \forall P_t, P_{t+1}, r_{t+1}. \tag{15}$$

The Euler equation (13) describes the law of motion of all individual expenditure levels $e_{i,t}$ as a function of the interest rate, the rate of time preference, and prices. Aggregating all expenditure levels up then gives the law of motion for the aggregate and the per-capita expenditure level. As stated in Definition 3, preferences are IA if this path of average per-capita expenditure itself again satisfies the Euler equation — independent of the distribution of individual expenditure levels. This property implies that the economoy admits *intertemporally* a representative agent. The path of average per-capita expenditure as well as savings and capital accumulation is independent of the cross-sectional income and wealth distribution and can be viewed as being chosen by a *representative agent* that starts out with $e_{i,0} = E_0/N$.

Although IA preferences admit a representative agent for the intertemporal consumption/saving decision, they still allow for considerable flexibility of the intratemporal income effects, which is essential to match the data. Note also that the IA class of preferences does not restrict expenditure levels $e_{i,t}$ to grow at identical rates; what the Euler equation restricts is that the *marginal utility*, $v_e(\cdot)$, grows at the same rate across individuals in a given point in time. This can be fully consistent with convergence or divergence in the distribution in expenditure levels. IA is therefore a weaker restriction than for example mean-scaling as discussed in Lewbel (1989).

In the next proposition we fully characterize the class of period utility functions that allows for intertemporal aggregation according to Definition 3. We proceed by deriving the Marshallian demand system implied by IA preferences and then show how this preference class allows for identification of preference parameters from aggregate data.

Proposition 2. *Preferences* (3) *are* <u>*intertemporally aggregable*</u> *if and only if the period utility* $v(e_i, P)$ *takes (up to multiplicative or additive constants) one of the following forms*

$$v(e_i, P) = \frac{1-\epsilon}{\epsilon}\left(\frac{e_i}{\mathbf{B}(P)} - \mathbf{A}(P)\right)^{\epsilon} - \mathbf{D}(P), \quad \epsilon \notin \{0, 1\}, \tag{16}$$

$$v(e_i, P) = -\exp\left(-\left(\frac{e_i}{\mathbf{B}(P)} - \mathbf{A}(P)\right)\right) - \mathbf{D}(P), \tag{17}$$

or

$$v(e_i, P) = \mathbf{F}(P)\log\left(\frac{e_i}{\mathbf{B}(P)} - \mathbf{A}(P)\right), \tag{18}$$

where $\mathbf{A}(P)$, $\mathbf{D}(P)$, *and* $\mathbf{F}(P)$ *are functions homogenous of degree zero in prices and* $\mathbf{B}(P)$ *is a linearly homogenous function of prices.*

Proof. In Section A.4 of the Appendix. ■

The proof of Proposition 2 starts by showing that IA requires $e_{i,t+1}$ to be linearly related to $e_{i,t}$ with the same marginal response for all individuals. The intertemporal Euler equation can then be differentiated twice, reformulated as a constant, and integrated up twice to get the above restrictions on the utility function.

Given the general restriction in Definition 3, the resulting period utility function is parsimonious, fairly flexible with three non-redundant price functions, and nests (as we will show below) some well-known cases. In the special case of one commodity, the functional form collapses to the class of the "hyperbolic absolute risk aversion" (HARA) period utility function.[18] This one commodity HARA period utility function is well known to be the most general form of the period utility such that in a time additive setting overall preferences $\mathcal{U}_0$ are in the Gorman class.[19] However, the class of Gorman preferences is clearly too rigid to fit the historical data, so it is important to note that Proposition 2 broadens this class but still preserves in our intertemporal framework an aggregation result.

Note that Proposition 2 is based on a necessity proof such that further restrictions on the functions $\mathbf{A}(P)$, $\mathbf{B}(P)$, $\mathbf{D}(P)$, and $\mathbf{F}(P)$ need to be imposed to obtain a well-defined preference specification. We discuss this issue when we fully parameterize the preference specification further below. Note also that Definition 3 implicitly assumes that the Euler equation characterizes the individual choice. Hence, for applied use further restrictions need to be imposed to ensure concavity of the household problem and interiority of its solution.

[18]It is easy to show that even for our multiple commodity case the coefficient of absolute risk aversion becomes a hyperbolic function in e_i.

[19]See Pollak (1971) for a proof of this result.

The next proposition derives the Marshallian demand system of the IA preference specification (16)–(18).

Proposition 3. *If preferences are IA, then the Marshallian demand of each commodity j is given by*
(i) [period utility functions (16) and (17)]

$$c_{i,j,t} = \mathbf{A}_j(P_t)\mathbf{B}(P_t) + \frac{\mathbf{B}_j(P_t)}{\mathbf{B}(P_t)} \cdot e_{i,t} + \frac{\mathbf{D}_j(P_t)}{v_e\left(e_{i,t}, P_t\right)}, \tag{19}$$

or, (ii) [period utility function (18)]

$$c_{i,j,t} = \mathbf{A}_j(P_t)\mathbf{B}(P_t) + \frac{\mathbf{B}_j(P_t)}{\mathbf{B}(P_t)} \cdot e_{i,t} + \mathbf{F}_j(P_t)\frac{\log\left(v_e\left(e_{i,t}, P_t\right)\frac{\mathbf{B}(P)}{\mathbf{F}(P)}\right)}{v_e\left(e_{i,t}, P_t\right)}, \tag{20}$$

where $\mathbf{A}_j(P_t)$, $\mathbf{B}_j(P_t)$, $\mathbf{D}_j(P_t)$, and $\mathbf{F}_j(P_t)$ denote derivatives of the corresponding functions with respect to $p_{j,t}$. In per-capita terms, the Marshallian demand of each commodity is given by
(i) [period utility functions (16) and (17)]

$$C_{j,t}/N = \mathbf{A}_j(P_t)\mathbf{B}(P_t) + \frac{\mathbf{B}_j(P_t)}{\mathbf{B}(P_t)} \cdot E_t/N + \frac{\mathbf{D}_j(P_t)}{v_e\left(E_t/N, P_t\right)}\kappa, \tag{21}$$

or, (ii) [period utility function (18)]

$$C_{j,t}/N = \mathbf{A}_j(P_t)\mathbf{B}(P_t) + \frac{\mathbf{B}_j(P_t)}{\mathbf{B}(P_t)} \cdot E_t/N + \mathbf{F}_j(P_t)\frac{\log\left(v_e(E_t/N, P_t)\frac{\mathbf{B}(P)}{\mathbf{F}(P)}\tilde{\kappa}\right)}{v_e\left(E_t/N, P_t\right)}, \tag{22}$$

where $\kappa = \frac{1}{N}\int_0^N \frac{v_e(E_t/N,P_t)}{v_e(e_{i,t},P_t)}di$ and $\tilde{\kappa} = \exp\left(\frac{1}{N}\int_0^N \log\left(\frac{v_e(e_{i,t},P_t)}{v_e(E_t/N,P_t)}\right)\frac{v_e(E_t/N,P_t)}{v_e(e_{i,t},P_t)}di\right)$ are aggregation factors that are both constant over time.

Proof. In Section A.5 of the Appendix. ■

One indicator of the flexibility of the income effects of a demand system is its "rank". Since three additive terms that can be different functions of E_t/N show up in (21) and (22), the Marshallian demand system can have up to rank 3 (which is the maximum rank in a three-sector model as considered in this paper).[20] Our IA class of preferences also encompasses the homothetic, the quasi-homothetic, the PIGL/PIGLOG, and the quadratic demand systems as special cases. This can easily be verified from theorem 1 in Lewbel (1987). However, it is worthwhile noting that the IA specification's demand system is one rank more flexible than the one of the generalized Stone-Geary or the PIGL/PIGLOG specifications.

With $\mathbf{A}(P_t) = 0$, the demand system in (21) to (22) coincides with the demand system implied by PIGL/PIGLOG preferences as specified in Muellbauer (1975). Nevertheless, the more flexible IA preferences still preserve an attractive aggregation property. To illustrate this, the following corollary generalizes theorem 7 in Muellbauer (1975) to the class of IA preferences.

Corollary 1. *If the distribution of $v_e(e_{i,t}, P_t)/v_e(E_t/N, P_t)$ is constant over time, then IA is the most general preference specification for which, given knowledge of the distribution of $e_{i,t}$ at one point in time, there is no aggregation bias from using per-capita expenditure E_t/N as the relevant expenditure variable.*

Proof. In Section A.6 of the Appendix. ∎

The key implication of Proposition 3 and Corollary 1 is that the per-capita demand can be expressed as a function of the prices, nominal per-capita expenditure, as well as an inequality index of relative marginal utilities that is constant over time. Thus, up to the constant aggregation factors κ or $\tilde{\kappa}$, the individual demand and the market demand take an identical structure. This allows us to empirically identify all the preference parameters from aggregate data except for a scalar that multiplies the function $\mathbf{D}(P)$ in case (21), or the functions $\mathbf{A}(P)$ and $\mathbf{B}(P)$ in case (22). The terms κ and $\tilde{\kappa}$ can indeed be interpreted as aggregation factors as in Blundell, Pashardes, and Weber (1993). This result holds true although the preference class is more general than the Gorman or the PIGL/PIGLOG class. The result crucially hinges on all the households being on the intertemporal (Euler) equation and does not automatically generalize if some of the households were for instance

[20]See Lewbel (1991) for a general definition and discussion of the rank. Matching micro data typically requires a rank of (at least) three (see, e.g., Banks et al. (1997)).

credit constraint.[21] Therefore, if the goal is to retrieve preference parameters from aggregate data the IA preference class is a natural starting point. Finally, Corollary 1 shows that after having estimated preferences from (21) to (22) (or, the corresponding expenditure shares), the aggregation factor can simply be calculated with distributional data from one period.

6 Parameterizations of IA preferences

In this section we propose a simple and flexible parameterization of IA preferences that is suitable for empirical applications and remains consistent with our dynamic multi-sector general equilibrium framework. To this aim, we focus on the period utility function in (16), which implies aggregate expenditure shares, $\eta_{j,t} \equiv p_{j,t}C_{j,t}/E_t$, of the following form

$$\eta_{j,t} = \mathbf{A}_j(P_t)p_{j,t}\frac{\mathbf{B}(P_t)}{E_t/N} + \frac{\mathbf{B}_j(P_t)p_{j,t}}{\mathbf{B}(P_t)} + \kappa\mathbf{D}_j(P_t)p_{j,t}\left(\frac{E_t/N}{\mathbf{B}(P_t)} - \mathbf{A}(P_t)\right)^{1-\epsilon}\frac{\mathbf{B}(P_t)}{E_t/N}. \tag{23}$$

We focus on (16) because the power form is more flexible than the exponential and log specifications in (17) and (18), which can be derived as limit cases of the former. Moreover, we choose a parameterization that directly nests the demand systems of the generalized Stone-Geary and the PIGL preferences. In the quantitative analysis, this will allow us to directly compare the fit of different specifications in the historical data by simply restricting certain parameters.

[21] However, even in a setting with incomplete markets where the expenditure (and wealth) distribution stabilizes asymptotically the aggregation factors are guaranteed to be constant and the same aggregation results go through asymptotically.

6.1 Price Functions

We start by parametrizing the price function $\mathbf{B}(P_t)$ in (16) and (23) with the CES aggregator

$$\mathbf{B}(P_t) \equiv \begin{cases} \left(\sum_{j\in J} \omega_j p_{j,t}^{1-\sigma}\right)^{\frac{1}{1-\sigma}}, & \sigma \neq 1 \\ \prod_{j\in J} p_{j,t}^{\omega_j}, & \sigma = 1, \end{cases} \tag{24}$$

where $\sigma > 0$, $\sum_{j\in J} \omega_j = 1$, and $\omega_j \geq 0$. Thus, our parameterization nests the indirect formulation of the standard CES utility function in the special case $\mathbf{A}(P_t) = \mathbf{D}(P_t) = 0$. Next, for the price function $\mathbf{A}(P_t)$ we choose the functional form

$$\mathbf{A}(P_t) = \mathbf{B}(P_t)^{-1} \sum_{j\in J} p_{j,t}\bar{c}_j, \quad p_{j,t}(C_{j,t}/N - \bar{c}_j) \geq 0, \, \forall j \in J. \tag{25}$$

Under additional restrictions outlined in Section 6.3 below, the parameter $\bar{c}_j$ can be interpreted as the subsistence ($\bar{c}_j > 0$) or endowment ($\bar{c}_j < 0$) level of real sectoral consumption as under generalized Stone-Geary preferences.

The price function $\mathbf{D}(P_t)$ that enters (16) additively is parametrized by a quadratic form in log prices

$$\mathbf{D}(P_t) = \frac{1-\epsilon}{\kappa} \left(\sum_{j\in J} \nu_j \log(p_{j,t}) + \frac{1}{2} \sum_{j\in J} \log(p_{j,t}) \sum_{l\in J} \psi_{jl} \log(p_{l,t}) \right), \tag{26}$$

where $\sum_{j\in J} \nu_j = 0$, $\psi_{jl} = \psi_{lj}$, $\forall j, l \in J$, and $\sum_{l\in J} \psi_{jl} = 0$. We have conveniently scaled the function $\mathbf{D}(P_t)$ with the inverse of the constant inequality index κ, such that κ drops from the aggregate expenditure share (23). Moreover, the imposed parameter restrictions ensure that the expenditure shares add up to unity and that the Hessian of the expenditure function with respect to prices — the Slutsky matrix — is symmetric. Finally, we require $\epsilon \in (0, 1)$ such that the positive scalar $(1 - \epsilon)$ becomes redundant in (16).[22]

Proposition 4. *In our intertemporal framework, the period utility function in (16) with $\epsilon \in (0, 1)$ and price functions (24)–(26) supports (i) an asymptotic balanced*

[22] We also that find this is the empirically relevant range for the parameter.

growth path, (ii) non-negativity of expenditure shares as $t \to \infty$.

Proof. In Section A.7 of the Appendix. ■

Proposition 4 establishes that our parameterization of IA preferences supports an asymptotic balanced growth path in our dynamic multi-sector framework. Moreover, it yields consumption expenditure shares that remain non-negative under sustained growth of per-capita income even as time goes to infinity. The latter property is easily violated by many flexible demand systems like the AID and the QAID, for example.

We show next that this parameterization of IA preferences nests as special cases the two standard preference specifications in the literature — the PIGL and the generalized Stone-Geary.

6.2 PIGL preferences

The PIGL class of preferences defined in Muellbauer (1975, 1976) is nested in (16) when $\mathbf{A}(P_t) = 0$. The aggregate expenditure share of the PIGL is of the form

$$\eta_{j,t} = \frac{\mathbf{B}_j(P_t)p_{j,t}}{\mathbf{B}(P_t)} + \mathbf{D}_j(P_t)p_{j,t}\left(\frac{E_t/N}{\mathbf{B}(P_t)}\right)^{-\epsilon}\kappa, \tag{27}$$

where $\kappa = \frac{1}{N}\int_0^N ((E_t/N)/e_{i,t})^{\epsilon-1}\,di$. Given that $\mathbf{A}(P_t) = 0$, the flexibility of the PIGL expenditure system is reduced by one additive term compared to (23) and it only has rank 2. For the empirical application, this is a limitation relative to the more flexible IA preferences. In contrast, the expression for the constant aggregation factor κ is independent of prices, as can be seen from (27), and only the parameter ϵ is required to compute its value. Finally, in the limit case $\sigma = 1$, when the price function $\mathbf{B}(P_t)$ is of the Cobb-Douglas form outlined in (24), then the PIGL specification is even consistent with an exact balanced growth path.[23]

[23] In this case $\mathbf{B}(P_t)$ grows at a constant rate in any period; thus it is sufficient that nominal expenditure grows at a constant rate in the Euler equation (13) to support a constant interest rate even for finite t. See Boppart (2014) for an exact balanced growth consistent PIGL specification with two sectors.

6.3 Generalized Stone-Geary preferences

The generalized Stone-Geary specification is nested in (16) with price functions (24)–(25) when (i) $\mathbf{D}(P_t) = 0$, (ii) the weights of $\mathbf{B}(P_t)$ are strictly positive, $\omega_j > 0$ $\forall j \in J$, and (iii) each individual i has enough budget to cover the subsistence consumption $\bar{c}_j$ for each good,

$$p_{j,t}(c_{i,j,t} - \bar{c}_j) \geq 0, \forall j \in J. \tag{28}$$

Restriction (28) is required by the underlying generalized Stone-Geary utility function.[24] The empirical structural change literature has mostly abstracted from this regularity constraint by directly working under the representative (average) agent assumption. In that case, the regularity condition (28) concides exactly with the parameter restriction in (25), which is also the restriction that we will impose in the empirical application below.[25]

The generalized Stone-Geary specification is within the Gorman class such that market expenditure shares are functions of per-capita expenditure only:

$$\eta_{j,t} = \frac{\omega_j p_{j,t}^{1-\sigma}}{\mathbf{B}(P_t)^{1-\sigma}} + \left[p_{j,t}\bar{c}_j - \frac{\omega_j p_{j,t}^{1-\sigma}}{\mathbf{B}(P_t)^{1-\sigma}} \sum_{l \in J} p_{l,t}\bar{c}_l \right] (E_t/N)^{-1}. \tag{30}$$

Inequality does not affect the demand structure. The functional form of (30) shows that the parameter σ is an important determinant of the expenditure shares' price elasticity, i.e., when $\sigma = 1$ prices do not affect the expenditure shares other than through the terms that contain the subsistence consumption. It is also easy to see that the income elasticity of expenditure is driven by the subsistence levels $\bar{c}_j$.

[24] The indirect utility function of the generalized Stone-Geary specification follows from maximizing the utility function

$$u_{i,t} = \frac{1-\epsilon}{\epsilon} \left[\left(\sum_{j \in J} \omega_j^{\frac{1}{\sigma}} \left(c_{i,j,t} - \bar{c}_j \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \right]^{\epsilon}, \tag{29}$$

subject to the intratemporal budget restriction $\sum_{j \in J} p_{j,t} c_{i,j,t} \leq e_{i,t}$ and the regularity constraints $c_{i,j,t} - \bar{c}_j \geq 0$, $\forall j \in J$.

[25] Note that it is possible to allow for weaker restrictions on the subsistence levels. In particular, just redefine the subsistence terms in (29) as $\bar{c}_j \equiv \bar{c}_j - \bar{u}$, $\bar{u} > 0$. Then the utility function remains well-defined even if $c_{i,j,t} < \bar{c}_j$, $\forall j \in J$.

However, when in the long run per-capita consumption expenditure outgrows prices as suggested by the data and our parameterization, i.e., $\lim_{t\to+\infty} p_{j,t}/(E_t/N) = 0$, then all the terms involving the subsistence levels $\bar{c}_j$ asymptotically become irrelevant. Therefore, the generalized Stone-Geary specification mechanically implies that over time structural change must be driven more and more by relative price effects. Finally, the parameter ϵ, which determines the intertemporal elasticity of substitution, drops out of (30) and cannot be identified from the intratemporal expenditure shares.

On the theoretical side, the existing literature has emphasized that generalized Stone-Geary preferences are consistent with exact balanced growth for a narrow set of parameterizations. Kongsamut et al. (2001) consider the special case without relative price effects where $\sigma = 1$ and $\sum_{j\in J} p_{j,t}\bar{c}_j = 0$. Ngai and Pissarides (2007) on the other hand assume that there are no income effects, $\bar{c}_j = 0$ and $\epsilon \to 0$.

Finally, similar to the PIGL demand system in (27), the demand system in (30) has only rank 2. Thus, its functional flexibility remains below the maximum possible rank in a three-sector model.

6.4 Other preference specifications

We end this section by briefly discussing two other preference specifications used in the literature — the generalized PIGLOG (QAID) and the non-homothetic CES — that are not IA. These preferences also have a high flexibility, but neither of them allows for tractable aggregation. Furthermore, generalized PIGLOG is not consistent with sustained growth in per-capita expenditure.

Generalized PIGLOG (QAID) The QAID system in Banks et al. (1997) has been widely used in the microeconomic literature on consumer demand estimation. The demand system results from the generalized PIGLOG preference specification and allows the Engel curves to depend on quadratic terms of the logarithm of expenditure. Banks et al. (1997) show that this exactly aggregable system has the maximum rank of 3 and that this flexibility is necessary to fit the microeconomic expenditure data. As in our case, their preference specification allows a good to be a luxury at some income levels and a necessity at other levels even when relative prices remain constant.

While we share with QAID the feature of flexibility to fit the data, there are two important differences between our IA preference specification and their QAID system. Firstly, while they nest cases that are exactly aggregable such as the Translog model, their general specification does not allow for constant aggregation factors as discussed in Blundell et al. (1993). In contrast, our dynamic framework has a single and constant aggregation factor and allows for the identification of preference parameters from aggregate data with cross-sectional information for only one period. Second, the QAID system is not consistent with sustained growth, because the budget shares are predicted to be outside the unit interval for high expenditure levels. This is an important limitation in the context of our multi-sector growth model.

Non-homothetic CES Comin et al. (2015) propose using the non-homothetic CES preference specification from Hanoch (1975) in a multi-sector growth model to study structural change. An important feature of these preferences is that they allow for sector-specific income elasticities of demand that remain different from unity as income grows. However, non-homothetic CES preferences are not intertemporally aggregable and their parameters cannot be consistently estimated from historical macro data. Furthermore, the non-homothetic CES does not have a closed form for the Marshallian demand system and — even when the required micro data are available — the estimation of the demand system poses additional challenges.

7 Empirical application

In this section we estimate the expenditure system of the IA preferences in (16) with price functions (24)–(26) and compare its fit to the nested generalized Stone-Geary and PIGL specifications discussed above. To identify the preference parameters, we use the variation in the historical data on sectoral prices and nominal final consumption expenditure per-capita for the USA, GBR, CAN, and AUS over the period 1900 to 2014.[26] Following Herrendorf et al. (2013), we report the feasible general-

[26]Although knowing the value of the constant aggregation factor κ is not necessary for the prediction of the aggregate expenditure shares and its income elasticity, we also determine κ using

ized nonlinear least squares (FGNLS) estimator with robust standard errors.[27] As the expenditure shares for the three sectors must add up to unity, it is sufficient to estimate the parameters of the expenditure shares for only two sectors.

The main finding of our quantitative analysis is that IA preferences fit the historical data better than the less flexible generalized Stone-Geary and PIGL specifications. Particularly for the USA, GBR, and CAN, the generalized Stone-Geary specification struggles to simultaneously fit the pronounced decline of agriculture, the hump-shape of manufacturing, and the accelerated rise of services that we documented in Section 3.

7.1 Estimation of Preference Parameters

Tables 1 and 2 report the estimation results when we consider the final private consumption expenditure shares in each of the four countries individually. Similarly, Table 3 shows the parameter estimates when we pool the data for all four countries.[28] In each of the tables, the columns labeled "Stone-Geary" show the results for the specification in (30), i.e., when we impose the restriction $\mathbf{D}(P_t) = 0$ in (23). "PIGL" indicates columns with the estimation results for the specification in (27), i.e., when we impose $\mathbf{A}(P_t) = 0$ in (23). Lastly, in the remaining columns with the label "IA", both of these restrictions are relaxed and we consider our general IA parametrization. Figures 3 and 4 show the fits of the three specifications, and Sections 7.2 and 7.3 discuss the predictions of the model in more detail.

7.1.1 Generalized Stone-Geary

The first and fourth columns of Tables 1 to 3 report the result for the generalized Stone-Geary specification. The point estimate of the parameter σ, that controls the

cross-sectional consumption expenditure data for the USA in Section 7.4 below.

[27]The GNLS estimator is more efficient than the NLS estimator since it accounts for the error correlation between sectoral expenditure shares in a given year and country. The estimator of the error correlation matrix is then updated iteratively until convergence, which makes the GNLS estimator feasible. A detailed description of the estimator and robust inference is provided in Stata's documentation of the *nlsur* routine. We have implemented the FGNLS estimator in Matlab, which conveniently allows for symbolic computation of the Slutsky matrix (see Section 7.5 for details).

[28]Tables 1 to 3 report the results for the main parameters of interest. The remaining parameter estimates are shown in Tables A.1 to A.3 in Appendix A.

	USA			GBR		
	Stone-Geary	PIGL	IA	Stone-Geary	PIGL	IA
	(1)	(2)	(3)	(4)	(5)	(6)
σ	0.13	0.00	5.11	0.47	0.55	0.48
	(0.03)	(.)	(1.18)	(0.03)	(0.03)	(0.05)
$\bar{c}_A$	714.2		714.2	897.1		740.8
	(.)		(.)	(.)		(92.9)
$\bar{c}_M$	-1474.3		-1123.1	247.9		375.5
	(347.1)		(353.1)	(34.2)		(92.9)
$\bar{c}_S$	-3000.9		1288.9	952.7		1291.6
	(704.6)		(.)	(68.2)		(.)
ϵ		0.70	0.37		0.69	0.62
		(0.03)	(0.02)		(0.02)	(0.07)
ν_A		82.0	2.0		85.2	20.0
		(22)	(0.4)		(13.3)	(14.3)
ν_M		-4.3	13.0		-70.5	-6.0
		(6)	(3.1)		(8.3)	(7.7)
ν_S		-77.7	-15.0		-14.7	-14.0
		(18.9)	(3.4)		(6.6)	(12.3)
Obs	104	104	104	97	97	97
AIC	-1000	-1014	-1100	-1058	-1197	-1209
RMSE_A	0.033	0.032	0.031	0.019	0.012	0.010
RMSE_M	0.027	0.025	0.022	0.013	0.010	0.011
RMSE_S	0.017	0.017	0.015	0.022	0.017	0.014

Table 1: Estimation, Private consumption, USA and GBR

Note: Years when the USA was involved in WWI (1917–1918), WWII (1942–1945), or affected by the Great Depression (1929–1933), and years when GBR was involved in WWI (1914–1918), WWII (1939–1945), or affected by the Great Depression (1929–1933) are excluded from all estimations. All variables are based on final private consumption expenditure. AIC is the Akaike information criterion and RMSE_j is the root mean squared error for sector j. Robust standard errors are reported in parenthesis.

	CAN			AUS		
	Stone-Geary	PIGL	IA	Stone-Geary	PIGL	IA
	(1)	(2)	(3)	(4)	(5)	(6)
σ	0.66	0.00	0.00	0.15	0.00	0.00
	(0.03)	(.)	(.)	(0.1)	(0.43)	(.)
$\bar{c}_A$	721.2		352.5	947.2		947.2
	(.)		(238.9)	(.)		(.)
$\bar{c}_M$	-141.2		563.3	-2180.5		669.3
	(117.3)		(.)	(681.3)		(.)
$\bar{c}_S$	-1239.4		1088.6	-6890.7		1352.7
	(415.3)		(.)	(1636.7)		(.)
ϵ		0.32	0.55		0.72	1.00
		(0.1)	(0.04)		(0.1)	(.)
ν_A		2.6	19.4		106.7	296.5
		(1.4)	(11)		(99.9)	(16.8)
ν_M		-0.7	-0.2		-1.7	312.5
		(0.5)	(2.8)		(20.9)	(147.2)
ν_S		-1.9	-19.1		-105.1	-609.0
		(1.4)	(10)		(106)	(144.7)
Obs	77	77	77	63	63	63
AIC	-801	-839	-995	-670	-618	-677
RMSE_A	0.029	0.019	0.010	0.017	0.026	0.018
RMSE_M	0.013	0.013	0.010	0.017	0.020	0.017
RMSE_S	0.038	0.019	0.016	0.018	0.022	0.018

Table 2: Estimation, Private consumption, CAN and AUS

Note: Years when CAN and AUS were involved in WWI (1914–1918), WWII (1939–1945), or affected by the Great Depression (1929–1933) are excluded from all estimations. All variables are based on final private consumption expenditure. AIC is the Akaike information criterion and RMSE_j is the root mean squared error for sector j. Robust standard errors are reported in parenthesis.

	Stone-Geary	PIGL	IA	Stone-Geary$\vert_{\bar{c}_M=0}$
	(1)	(2)	(3)	(4)
σ	0.17	0.13	0.17	0.34
	(0.03)	(0.04)	(0.08)	(0.03)
$\bar{c}_A$	714.2		714.2	714.2
	(.)		(.)	(.)
$\bar{c}_M$	-1213.9		32.1	
	(152.3)		(180.3)	
$\bar{c}_S$	-2206.2		1088.6	75.4
	(297.2)		(.)	(76.1)
ϵ		0.73	0.55	
		(0.02)	(0.05)	
ν_A		119.6	12.1	
		(18.6)	(5.3)	
ν_M		-46.1	13.6	
		(8.3)	(3.3)	
ν_S		-73.5	-25.7	
		(14.7)	(6.9)	
Obs	341	341	341	341
AIC	-2928	-2951	-3018	-2737
RMSE_A	0.028	0.028	0.026	0.037
RMSE_M	0.029	0.029	0.027	0.030
RMSE_S	0.036	0.034	0.032	0.040

Table 3: Estimation, Private consumption, All Countries

Note: Years when the USA was involved in WWI (1917–1918), WWII (1942–1945), or affected by the Great Depression (1929–1933), and years when AUS, CAN, and GBR were involved in WWI (1914–1918), WWII (1939–1945), or affected by the Great Depression (1929–1933) are excluded from all estimations. All variables are based on final private consumption expenditure. AIC is the Akaike information criterion and RMSE_j is the root mean squared error for sector j. Robust standard errors are reported in parenthesis.

price elasticity, ranges between 0.13 and 0.66. In all samples it remains significantly smaller than 1, which shows that the specification attributes a substantial fraction of the variation in the expenditure shares to relative price effects.

The second row in the same columns of Tables 1 to 3 shows that the best fit of the generalized Stone-Geary specification to the data occurs when the estimated subsistence level of agricultural consumption, $\bar{c}_A$, is at the upper bound specified in (25) for all samples. The reason is that the wider the dispersion of per-capita expenditure in the data, the higher subsistence levels are required to generate strong enough income effects at the end of the sample where per-capita expenditures are relatively high. As a consequence, the fall in the expenditure share for agriculture predicted by the generalized Stone-Geary is generally not steep enough to fit the historical data well. This is most visible for the case of CAN, where panel (a) in Figure 3 shows that its prediction for the agricultural expenditure share deviates substantially from the actual data.

In the third and fourth row, manufacturing and services are both estimated to have an endowment level ($\bar{c}_M < 0, \bar{c}_S < 0$), except for GBR where the generalized Stone-Geary predicts a subsistence level for both. For the USA, GBR, AUS, and in the pooled sample we find that the point estimate for $\bar{c}_M$ is statistically significant and sizeable, and improves the fit of the generalized Stone-Geary specification significantly. For comparison, column (4) of Table 3 shows the estimation results in the pooled sample when $\bar{c}_M$ is restricted to zero. Relative to column (1), the Akaike Information Criterion (AIC) and the Root Mean Squared Errors (RMSE) reported at the bottom of the table increase substantially. Our estimation results also reject parameterizations of the generalized Stone-Geary specification that are consistent with an exact balanced growth path (see Section 6.3). More specifically, the subsistence/endowment levels of consumption are nonzero and the parameter σ, that controls the price elasticity, is smaller than unity.[29]

In Tables B.1 to B.3 in Online Appendix B we additionally report the main estimation results when we consider total consumption expenditure where the service

[29] The remaining estimates of the generalized Stone-Geary specification are as expected. Tables A.1 to A.3 in Appendix A show that the share parameters ω_j stay between zero and one as required by the model specification. The point estimates can generally be ordered according to $\omega_A < \omega_M < \omega_S$, which roughly reflects the order of expenditure shares at the end of the sample.

sector also includes government expenditure.[30] Our main estimation results for the generalized Stone-Geary specification remain unchanged. In particular, the parameter estimate of σ is significantly below 1 in all samples and — with the exception of AUS — the agricultural subsistence consumption is again at its upper bound.

7.1.2 PIGL

The results for the PIGL specification are reported in the second and fifth columns of Tables 1 to 3. In contrast to generalized Stone-Geary, the PIGL can also identify the parameter ϵ that also controls the intertemporal elasticity. This parameter is precisely estimated and ranges between 0.32 and 0.73. Since the estimate is significantly below unity in all samples, the PIGL specification predicts a more sustained income effect compared to the generalized Stone-Geary. The first row shows that the parameter estimate σ of the price function ranges between 0 and 0.55.[31]

The remaining parameters of the PIGL are generally well identified and the lower AIC reported at the bottom of the tables already indicates that the fit of the PIGL improves over the generalized Stone-Geary in all samples except for AUS. For instance, the RMSE of the predicted expenditure share for services in CAN drops substantially from 0.038 to 0.019. This improved fit is illustrated in panel (c) of Figure 3, which shows the prediction along with the actual data. In Table B.1 to B.3 in Online Appendix B, we show that — again with the exception of AUS — the fit of the PIGL also remains better when we estimate parameters from consumption expenditure shares that include government consumption.

[30]Due to the limited data availability of government expenditure for the USA prior to 1929 (Carter et al. (2006) reports numbers for 1902, 1913, 1922, 1927), the number of data points in the USA and the pooled sample reduces by 23 when we consider final total consumption expenditure.

[31]As discussed in Section 6.2, a value of σ below 1 corresponds to a parameterization of the PIGL that does not support an exact balanced growth path. However, even though the historical data reject both standard specifications that support an exact balanced growth path, PIGL and the generalized Stone-Geary are generally consistent with an asymptotic balanced growth path as stated in Proposition 4.

7.1.3 IA Preferences

Tables 1 to 3 report in the third and sixth columns the main results when we estimate the general specification of IA preferences in (23). The parameter ϵ is precisely estimated and ranges between 0.37 and 0.62, except for AUS, where the estimator hits the upper bound of 1. Since the parameter is significantly below 1 for USA, GBR, CAN, and the pooled sample, this confirms our earlier discussion that sustained income effects are indeed important to fit the historical data. The subsistence/endowment parameters $\bar{c}_j$ remain important to fit the data. For example, $\bar{c}_S$ is predicted to be positive in all samples and the best fit occurs when $\bar{c}_S$ is at the upper bound. Note, however, that this does not imply that services are a necessity, because the income elasticity of demand also depends on the parameters in $\mathbf{D}(P_t)$ and on the expenditure level. In fact, the flexibility of the income effects is an important feature of the IA preferences: panel (d) of Figure 5 shows, for example, that service consumption is initially predicted to be a necessity and in later periods a luxury for GBR; in panel (c) manufacturing consumption is predicted to be a luxury until the 1970s and then turns into a necessity.

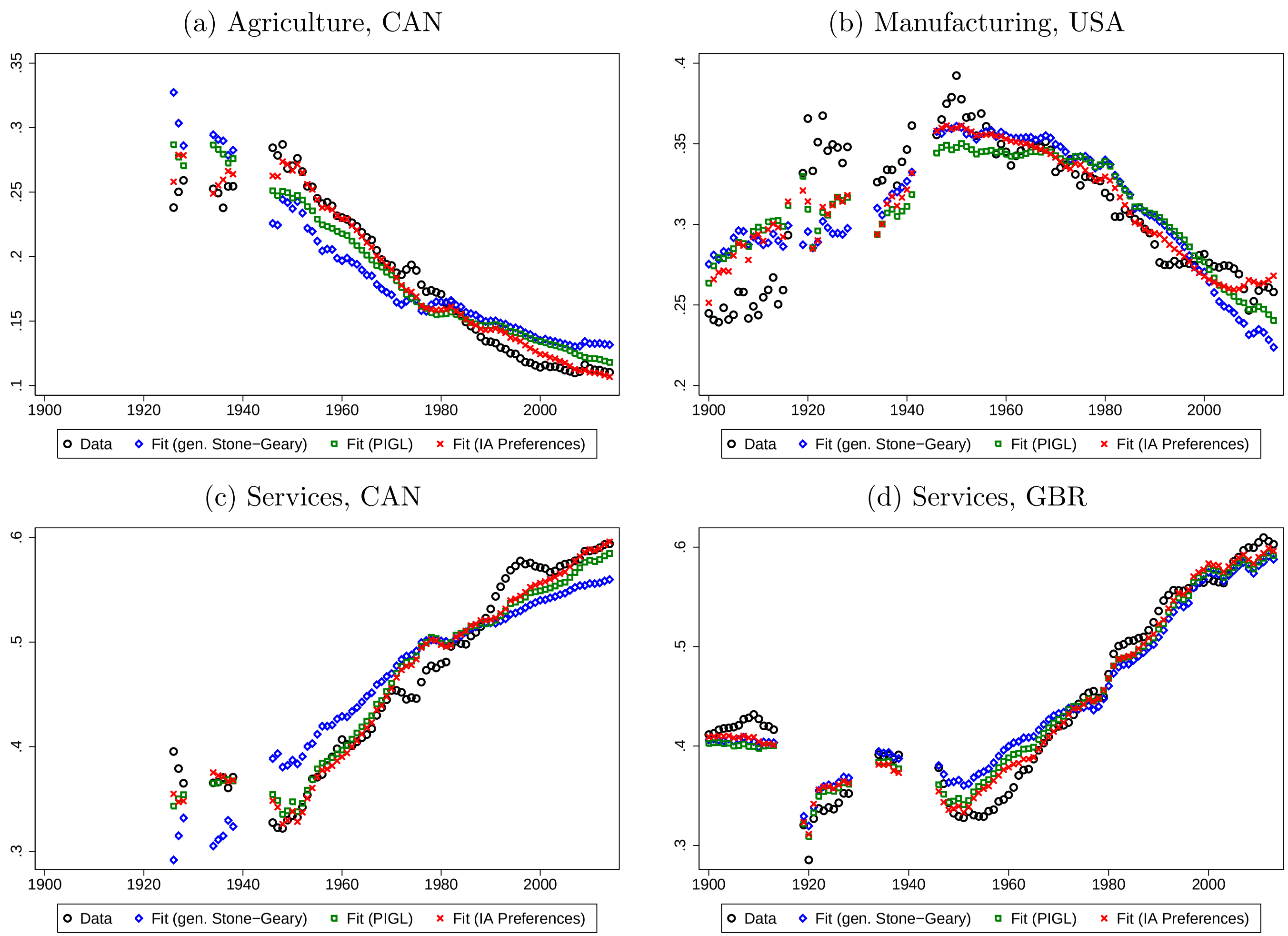
Finally, the lower AICs and RMSEs reported at the bottom of the tables suggest that the fit of the historical expenditure shares with IA improves substantially relative to the generalized Stone-Geary and the PIGL specification in several cases. For instance, in the first and third columns of Table 1 we show for the USA that the RMSE of the manufacturing sectors is 0.022 with IA, while with generalized Stone-Geary and PIGL it is 0.027 and 0.025, respectively. For GBR and CAN the drop in RMSE is the largest in the agricultural sector.

7.2 Predicted Nominal Expenditure Shares

In this section we present the predicted expenditure shares based on the parameter estimates with the generalized Stone-Geary, PIGL, and our IA specification in Tables 1–3. We plot the predictions in Figure 3 and 4 along with the actual shares observed in the data for the shown combinations of sectors and countries.[32]

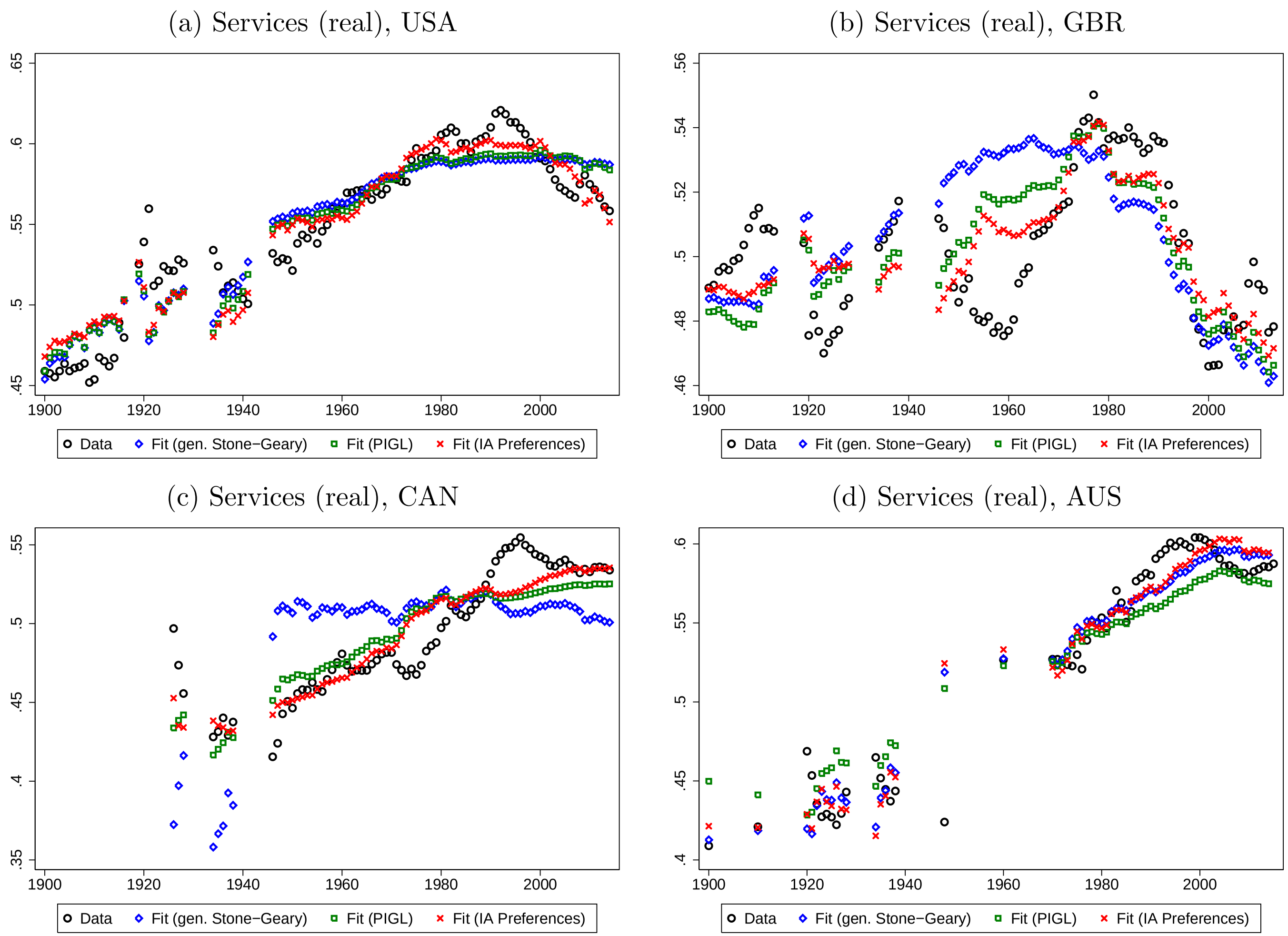
Panel (a) of Figure 3 illustrates that the generalized Stone-Geary specification

[32]The predictions for all sectors and countries can be found in Figures B.2–B.4 in Online Appendix B.



Notes: The figure plots the predicted final private nominal consumption expenditure shares based on the estimates in Tables 1–2 and A.1–A.2. The black dots show the data, the blue diamonds show the fit of the generalized Stone-Geary, the green squares show the fit of the PIGL, and the red crosses show the fit of the IA preferences.

Figure 3: Prediction final private nominal consumption expenditure shares



Notes: The figure plots the predicted final private real consumption expenditure shares by sector for each country based on the estimates in Tables 1–2 and A.1–A.2. The black dots show the data, the blue diamonds show the fit of the generalized Stone-Geary, the green squares show the fit of the PIGL, and the red crosses show the fit of the IA preferences.

Figure 4: Predicted final private real consumption expenditure shares in services

underpredicts the decline of the agricultural expenditure share in CAN over the last century. In contrast, both PIGL and IA can predict the decline well, because they generate sustained income effects, which in the case of generalized Stone-Geary vanishes quickly as income grows.[33] Panel (b) shows that PIGL underpredicts the increase in the USA's manufacturing sector until 1950, while it overpredicts the decline toward the end of the sample period. IA provides a better fit of the hump shape.[34] Panels (c) and (d) of Figure 3 show for CAN and GBR that the generalized Stone-Geary underpredicts the accelerated increase in the service sector, while IA provides a particularly good fit of the data and PIGL is relatively similar.[35]

An alternative to the nominal expenditure shares is to visualize the data and the predictions as "real shares", as for instance suggested by Herrendorf et al. (2014). To this aim, we normalize all the prices to 1 in the year 1990, calculate real quantities of agriculture, manufacturing, and services in the data as well as from the predictions, and express each sector's quantity as a share of the sum over all sectors' quantities.[36] Figure 4 shows the fit of the real expenditures shares in services for all countries in the sample.[37] Panels (a)–(c) in Figure 4 show that both generalized Stone-Geary and PIGL struggle to match the pronounced hump shape in the real quantity share of services in the USA and the fit of IA is generally better in all countries. The difference is starkest in panel (c), which shows that in CAN the real manufacturing share increased substantially in the second half of the century and then decreases again, although less than in the USA. PIGL and IA predict the strong initial increase and then flattening out of the data well, while general-

[33]The actual share of agriculture in 2014 is 11.0 percent, while the share predicted by generalized Stone-Geary is 13.2 percent. PIGL and IA predict shares of 11.8 and 10.7 percent, respectively.

[34]The prediction with PIGL is initially too high (26.4 instead of 24.4 percent), and then too low in the middle of the century (34.8 vs. 39.3) as well as towards the end of the sample (24.0 vs 25.8).

[35]The actual service share in CAN increases by 26.0 percentage points (from 33.4 percent in 1950 to 59.4 percent in 2014). PIGL predicts an increase by 23.7 percentage points, and IA is even closer with 25.8 percentage points, while generalized Stone-Geary predicts only an increase by 17.3 percentage points. In GBR, the actual share of services increases by 27.4 percentage points (from 32.9 percent in 1950 to 60.3 percent in 2013). IA matches this the best and predicts an increase by 25.7 percentage points, while generalized Stone-Geary and PIGL predict 22.2 and 24.4 percentage points, respectively.

[36]More precisely, the share of real consumption of good j is expressed as a share of the sum of real consumption across all goods, i.e. $c_j/(c_A + c_M + c_S)$, for $j = A, M, S$. The normalization of prices implies that all nominal and real shares coincide in the year 1990.

[37]For completeness, we report in Figures B.5–B.6 in Online Appendix B the analogue predictions for agriculture and manufacturing.

ized Stone-Geary yields a relative constant share in the second half of the century. Overall, the IA preference specification can, due to the more flexible income effects, generate the non-monotone pattern of structural change the most accurately. We document the role and importance of the flexible income effects in more detail in the next section and in Figure 5.

7.3 Predicted income elasticities

In all of the considered specifications, the income effects of the sectoral expenditure shares depend on the nominal per-capita expenditure and the sectoral prices, and therefore change over time. In this section we present the predicted income elasticities of the expenditure shares for the generalized Stone-Geary and IA specifications using again the point estimates of the preference parameters in Table 1 and Table 2.[38] When the income elasticity of the expenditure share is positive, the corresponding sector has a luxury character: as income increases a luxury sector absorbs a higher fraction of the budget. Sectors with a negative elasticity of the share have the character of a necessity.

Figure 5 illustrates the predicted income elasticities for the USA and GBR predicted by the generalized Stone-Geary and IA specification.[39] Panels (a) and (b) of Figure 5 show that the generalized Stone-Geary specification predicts income effects that are almost monotone and tend toward zero over time as per-capita expenditure increases. This makes it difficult for the specification to match the continued decline in the agricultural sector towards the end of the sample.

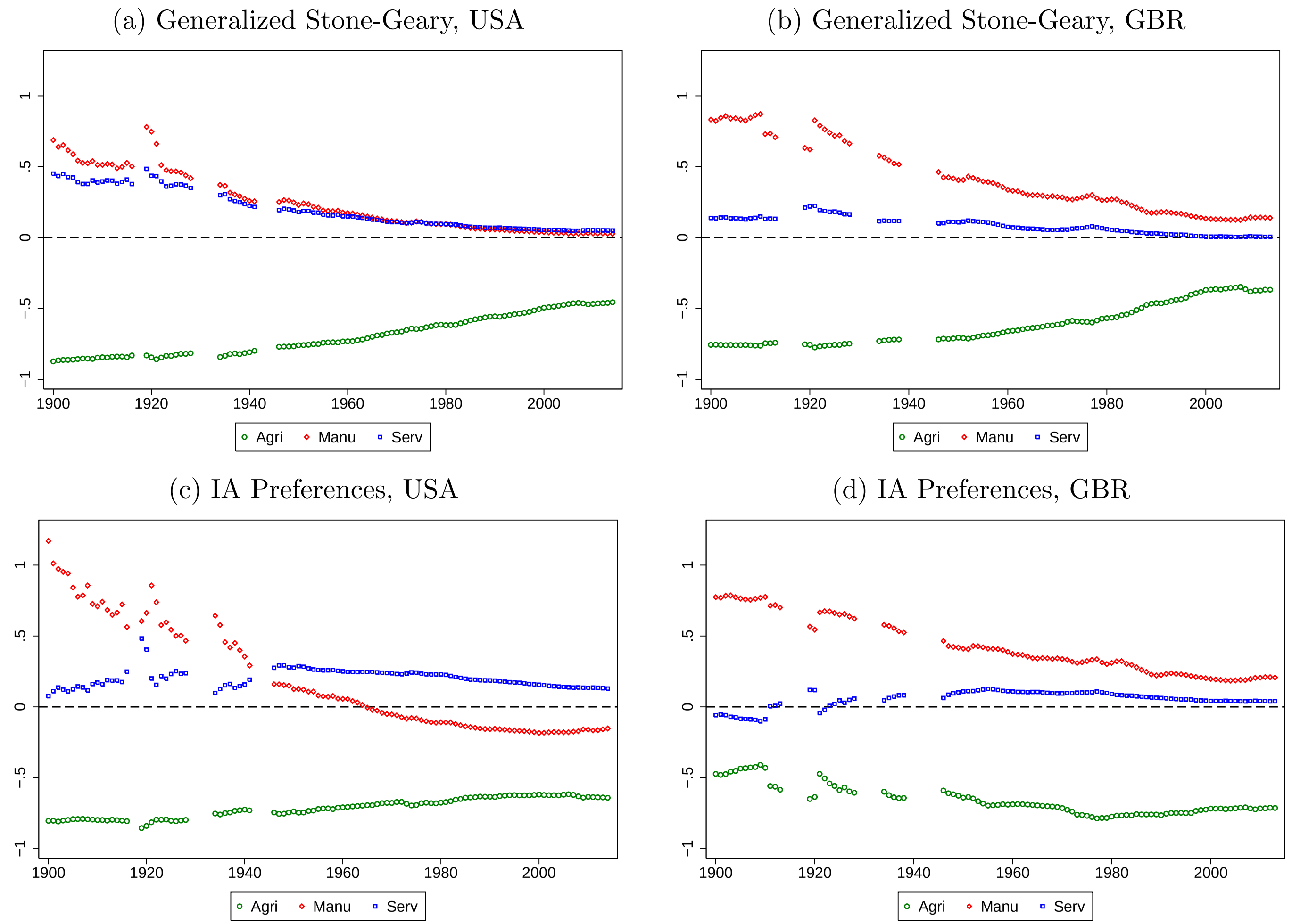
For the IA preference specification shown in panels (c) and (d) of Figure 5, the predicted income effects are more flexible and sustained. We consider first the agricultural sector. Its income elasticity is substantially below zero over the considered period, which is essential to fit the continued decline of the agricultural

[38]The income elasticity of the expenditure share for the IA specification is

$$\frac{\partial \eta_{j,t}/\eta_{j,t}}{\partial(E_t/N)/(E_t/N)} = \frac{-p_{j,t}}{\eta_{j,t}} \left[\mathbf{A}_j(P_t)\frac{\mathbf{B}(P_t)}{E_t/N} + \kappa \mathbf{D}_j(P_t)\left(\epsilon - \mathbf{A}(P_t)\frac{\mathbf{B}(P_t)}{E_t/N}\right)\left(\frac{E_t/N}{\mathbf{B}(P_t)} - \mathbf{A}(P_t)\right)^{-\epsilon}\right].$$

The elasticity for the nested generalized Stone-Geary and PIGL specifications are easily obtained with the appropriate parameter restrictions discussed in Sections 6.2–6.3.

[39]The further elasticities are shown in Figures B.7–B.8 in Appendix B.



Notes: The figure plots the predicted income elasticities of the sectoral expenditure shares for the USA and GBR based on the estimates in Tables 1 and A.1. Panel (a) shows the elasticities predicted by the generalized Stone-Geary specification for the USA, panel (b) shows the elasticities predicted by the generalized Stone-Geary specification for GBR, panel (c) shows the elasticities predicted by the IA specification for the USA, and panel (d) shows the elasticities predicted by the IA specification for GBR.

Figure 5: Predicted income elasticities of the expenditure shares in the USA and GBR

share in the USA and GBR. The manufacturing sector starts out as a clear luxury good with a high income elasticity to generate the increasing part of its hump-shaped expenditure share. The income elasticity of the manufacturing share then decreases over time and turns even negative for the USA. Thus, in the later periods of the USA sample, flexible income effects are important to fit the falling expenditure share of manufacturing. Finally, the service expenditure share's income elasticity starts out close to zero or slightly negative and is then predicted to be a luxury throughout the later sample period.

7.4 Aggregation factor

We also quantify the constant aggregation factor κ using distributional data from the U.S. Consumer Expenditure Survey between 1984 and 2014.[40] The following iterative procedure is applied to compute the constant. (i) We estimate the preference parameters for the USA (see Tables 1 and A.1) given an initial guess for the value of κ. (ii) Based on the resulting preference parameters' point estimates, the distributional data in each year, we update the value of κ according to the definition in Proposition 3. (iii) We go back to step (i) until the average value of κ over the period 1984–2014 (which we use as next round's guess) converges. For the PIGL and the IA specifications the resulting aggregation factor is 0.980 and 0.966, respectively, which is strictly below one and shows how expenditure inequality affects the aggregate expenditure structure of the economy.[41] We then use the same value of κ for the estimation of parameters in the other countries.[42]

[40] We consider data on average annual expenditures by quintiles of income before taxes from the U.S. Consumer Expenditure Survey. The data are available for the years 1984–2014 from the U.S. Bureau of Labor Statistics. This provides us with cross-sectional variation in final private consumption expenditures.

[41] For comparison, since the generalized Stone-Geary's indirect utility is of the Gorman form, its aggregation constant is trivially equal to one.

[42] Note that κ does not affect the aggregate expenditure shares in the other samples. However, when we impose the Slutsky restrictions on the parameter estimates, we check the Slutsky matrix at the individual level, which requires knowing the value of κ. Higher inequality in expenditures yields lower values of κ and tighter restrictions for the parameters of the function $\mathbf{D}(P)$. Thus, using the κ from the USA — which has a high expenditure inequality — for the other countries yields conservative estimates and predictions.

7.5 Slutsky restrictions

Working with a direct formulation of a utility function like the generalized Stone-Geary in (29) has the advantage that it is straightforward to impose parameter restrictions that ensure the consumer's maximization problem is well-behaved. Imposing the same restrictions on an indirect utility function like the PIGL and IA specification is more challenging: under the condition that the Slutsky matrix is symmetric (SM) and negative semi-definite (NSD), we can be sure that there exists a period utility function that generates the corresponding demand system, as shown for example in Hosoya (2017, Corollary 1). All point estimates reported in the tables of the main text and the appendix satisfy SM and NSD point-wise, i.e., when the individual Slutsky matrix is evaluated at the per-capita expenditure and prices observed in each sample.[43] We enforce the Slutsky restrictions by imposing prohibitive penalties for preferences parameters that yield NSD violations in the standard FGNLS estimation procedure.

7.6 Taking stock

Overall, we conclude that the IA is the preferred preference specification for our empirical application. IA not only provides the best fit of the historical pattern of structural change over different samples and time periods, but it also remains point-wise consistent with the utility maximization paradigm such that welfare statements based on the estimated preference parameters are meaningful. Moreover, our proposed parameterization nests the standard generalized Stone-Geary and the PIGL preferences, and whether the standard specifications fit the data equally well can easily be checked by simply restricting the parameters of our preferred IA specification. The last point is particularly relevant in empirical applications with short samples or in samples that display monotone expenditure shares and relative prices. In such samples it is often sufficient to use a generalized Stone-Geary specification. However, we find that allowing all sectors to have subsistent/endowment level of consumption may be required to fit the data.

[43] Formally, the Slutsky matrix is given by the Hessian of the household's expenditure function. Since we have already imposed parameter restrictions that guarantee SM, we only need to impose restrictions that ensure the eigenvalues of the Slutsky matrix are non-positive (to check NSD).

8 Conclusion

Structural transformation is a stylized fact of modern economic development, but the existing literature has struggled to provide a theory of consumer demand that can fit the observed reallocation across sectors well and remains consistent in theoretical environments with sustained growth in per-capita expenditure. We contribute to this literature in three ways. First, we document the reallocation of consumption shares across the three broad sectors agriculture, manufacturing, and services using historical final consumption expenditure data for more than one century in the United States, the United Kingdom, Canada and Australia. The data allow us to analyze relative prices and consumption shares for a larger sample and a longer period than existing studies.

Second, we analyze the features of the data that make it difficult for existing demand theories to match the observed patterns. Generalized Stone-Geary preferences, which are often used in the structural change literature, face the challenges that (i) the estimates imply unreasonably high subsistence levels of consumption, (ii) the income effects are not sustainable in long samples with a higher variation in per-capita expenditure, and (iii) it is difficult to generate the non-monotonic sectoral expenditure shares observed in the historical data.

Our third contribution is to propose an intertemporally aggregable class of preferences that fits the historical data better, allows for tractable aggregation, and is also consistent with sustained growth environments. IA preferences are a more general form of the PIGL class which also allows for flexible and sustained income effects. We show that the flexible IA preferences provide a better fit of the historical data on consumption expenditures than existing theories when one is considering data for more than a century.

We believe that our findings are important in a number of ways. The existing literature has come to different conclusions regarding whether the generalized Stone-Geary preferences can fit the observed patterns of structural change. Our analysis of historical consumption expenditure data over more than 100 years and across four countries confirms the view that the standard preference specifications struggle to fit the long-run data well. Our findings also have important implications for the external validity of structural transformation in the development process. The observation that the generalized Stone-Geary preferences imply subsistence levels in agriculture that are binding for (not unreasonably) low income levels limits the ability to apply it to contexts with large variation in incomes. We expect that IA preferences avoid this problem and will provide a useful basis for the analysis of structural change in a wide development context. We therefore plan to consider in future work a broader sample of countries. Similarly, there is a prominent debate on the effects of deindustrialization, in particular in developing countries (see for example Rodrik, 2016). To analyze the welfare effects of such trends and of potential policies, a dynamic multi-sector general equilibrium framework and an empirically robust parameterization of preferences are essential.

Because of the lack of historical data on home production, we have focused the analysis on the structural change exclusively within market consumption expenditure. It would be interesting to extend our analysis and consider how endogenous labor supply and home production interact with the structural change in market expenditure.[44]

References

Acemoglu, D. and V. Guerrieri (2008). Capital Deepening and Nonbalanced Economic Growth. *Journal of Political Economy 116*(3), 467–498.

Banks, J., R. Blundell, and A. Lewbel (1997). Quadratic Engel Curves and Consumer Demand. *Review of Economics and Statistics 79*(4), 527–539.

Baumol, W. J. (1967). Macroeconomics of Unbalanced Growth: The Anatomy of Urban Crisis. *American Economic Review 57*(3), 415–426.

Blundell, R., P. Pashardes, and G. Weber (1993). What Do We Learn About Consumer Demand Patterns from Micro Data? *American Economic Review 83*(3), 570–597.

Boppart, T. (2014). Structural Change and the Kaldor Facts in a Growth Model

[44]See Moro et al. (2017) for such an analysis of home production in the post-war period in combination with generalized Stone-Geary preferences.

With Relative Price Effects and Non-Gorman Preferences. *Econometrica 82*(6), 2167–2196.

Buera, F. J. and J. P. Kaboski (2009). Can traditional theories of structural change fit the data? *Journal of the European Economic Association 7*(2-3), 469–477.

Buera, F. J. and J. P. Kaboski (2012). Scale and the origins of structural change. *Journal of Economic Theory 147*(2), 684–712.

Carter, S. B., S. S. Gartner, M. R. Haines, A. L. Olmstead, R. Sutch, and G. Wright (Eds.) (2006). *Historical statistics of the United States: Millennial Edition*. New York: Cambridge University Press.

Caselli, F. and W. J. Coleman (2001). The U.S. Structural Transformation and Regional Convergence: A Reinterpretation. *Journal of Political Economy 109*(3), 584–616.

Comin, D. A., D. Lashkari, and M. Mestieri (2015). Structural Change with Long-run Income and Price Effects. NBER Working Paper No. 21595.

Eckert, F. and M. Peters (2018). Spatial Structural Change. Mimeo, Yale University.

Foellmi, R. and J. Zweimüller (2008). Structural change, Engel's consumption cycles and Kaldor's facts of economic growth. *Journal of Monetary Economics 55*(7), 1317–1328.

García-Santana, M., J. Pijoan-Mas, and L. Villacorta (2016). Investment Demand and Structural Change. *CEPR Discussion Paper 11636*.

Hanoch, G. (1975). Production and Demand Models with Direct or Indirect Implicit Additivity. *Econometrica 43*(3), 395–419.

Herrendorf, B., C. Herrington, and Á. Valentinyi (2015). Sectoral Technology and Structural Transformation. *American Economic Journal: Macroeconomics 7*(4), 104–133.

Herrendorf, B., R. Rogerson, and Á. Valentinyi (2013). Two Perspectives on Preferences and Structural Transformation. *American Economic Review 103*(7), 2752–2789.

Herrendorf, B., R. Rogerson, and Á. Valentinyi (2014). Chapter 6 - Growth and Structural Transformation. In P. Aghion and S. N. Durlauf (Eds.), *Handbook of Economic Growth*, Volume 2, pp. 855–941. Elsevier.

Herrendorf, B., R. Rogerson, and Á. Valentinyi (2018). Structural Change in Investment and Consumption: A Unified Approach. Mimeo, Princeton University.

Hosoya, Y. (2017). The relationship between revealed preference and the Slutsky matrix. *Journal of Mathematical Economics 70*(C), 127–146.

Kongsamut, P., S. Rebelo, and D. Xie (2001). Beyond Balanced Growth. *Review of Economic Studies 68*, 869–882.

Leon-Ledesma, M. and A. Moro (2017). The Rise of Services and Balanced Growth in Theory and Data. Centre for Macroeconomics Discussion Paper 1714.

Lewbel, A. (1987). Characterizing Some Gorman Engel Curves. *Econometrica 55*(6), 1451–1459.

Lewbel, A. (1989). Exact Aggregation and a Representative Consumer. *The Quarterly Journal of Economics 104*(3), 621–633.

Lewbel, A. (1991). The Rank of Demand Systems: Theory and Nonparametric Estimation. *Econometrica 59*(3), 711–730.

Maddison Project (2013). Maddison Project Database version 2013.

Moro, A., S. Moslehi, and S. Tanaka (2017). Does Home Production Drive Structural Transformation? *American Economic Journal: Macroeconomics 9*(3), 116–46.

Muellbauer, J. (1975). Aggregation, Income Distribution and Consumer Demand. *Review of Economic Studies 42*(4), 525–543.

Muellbauer, J. (1976). Community Preferences and the Representative Consumer. *Econometrica 44*(5), 979–999.

Ngai, R. and C. A. Pissarides (2007). Structural Change in a Multisector Model of Growth. *American Economic Review 97*(1), 429–443.

Pollak, R. A. (1971). Additive Utility Functions and Linear Engel Curves. *Review of Economic Studies 38*(4), 401–414.

Rodrik, D. (2016). Premature deindustrialization. *Journal of Economic Growth 21*(1), 1–33.

U.S. Bureau of Labor Statistics (2018). Quintiles of Income Before Taxes: Average Annual Expenditures and Characteristics, Consumer Expenditure Survey. Extracted 9 May 2018.

Uy, T., K.-M. Yi, and J. Zhang (2013). Structural change in an open economy. *Journal of Monetary Economics 60*(6), 667–682.

Valentinyi, Á. and B. Herrendorf (2008). Measuring factor income shares at the sectoral level. *Review of Economic Dynamics 11*(4), 820 – 835.

A Proofs and additional tables

A.1 Proof of Lemma 1

Proof. In each period t, the representative firm in each sector $j \in J_+$ solves

$$\min_{k_{j,t}, n_{j,t}} k_{j,t}(r_t + \delta) + n_{j,t} w_t,$$

subject to an exogenously given output level $\bar{y}_{j,t} = k_{j,t}^{\alpha} \left(\gamma_j^t n_{j,t}\right)^{1-\alpha}$. The first-order conditions of the firms' problems are

$$\lambda_{j,t} \alpha \bar{y}_{j,t} / k_{j,t} = r_t + \delta,$$

and

$$\lambda_{j,t} (1-\alpha) \bar{y}_{j,t} / n_{j,t} = w_t,$$

where $\lambda_{j,t}$ denotes the multiplier attached to the constraint. These first-order conditions directly imply

$$\frac{k_{j,t}}{n_{j,t}} = \frac{w_t}{r_t + \delta} \cdot \frac{\alpha}{1-\alpha}, \tag{31}$$

which together with (5) and (6) implies (8). Furthermore, this allows us to write output as (12). Note that $\lambda_{j,t}$ can be interpreted as marginal cost and will be equal to the sectoral price $p_{j,t}$. Solving the first-order conditions for $\lambda_{j,t}$ and combining them with (31) gives (9). Finally, with our choice of the numéraire the first-order conditions of the investment sector imply (10) and (11) and establish the lemma. ■

A.2 Proof of Lemma 2

Proof. The Lagrangian of the household problem can be written as

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t v(e_{i,t}, P_t) + \sum_{t=0}^{\infty} \lambda_{i,t} \beta^t \left(a_{i,t}(1 + r_t) + w_t n_i - e_{i,t} - a_{i,t+1}\right). \tag{32}$$

The first-order conditions are then given by

$$v_e(e_{i,t}, P_t) = \lambda_{i,t},$$

$$\lambda_{i,t} = \lambda_{i,t+1}\beta\left(1 + r_{t+1}\right),$$

and

$$a_{i,t}(1 + r_t) + w_t n_i - e_{i,t} = a_{i,t+1}.$$

The increasing but diminishing marginal utility, i.e., $v_e(\cdot) > 0$ and $v_{ee}(\cdot) < 0$ guarantees an interior solution. Combining the first two first-order conditions then establishes the lemma. ■

A.3 Proof of Proposition 1

Proof. Positive capital growth requires positive savings and investments. Hence, along a balanced growth path we must have $k_t^{\alpha}\left(\gamma_X^t n\right)^{1-\alpha} > E_t$. Then, the resource constraint (14) implies that a constant capital growth rate requires $\frac{k_{t+1}}{k_t} = \gamma_X$. It is then straightforward to see that along this path nominal output and nominal expenditure grow at the same gross rate γ_X. Finally, (10) and (11) imply that the interest rate is constant and that the wage rate grows at gross rate γ_X as well. ■

A.4 Proof of Proposition 2

Proof. We start the proof of the proposition with a lemma.

Lemma 3. *Preferences $\mathcal{U}_{i,0}$ are intertemporally aggregable if and only if there exists a function $z : R \to R$ such that*

$$v_e(e, P) = z\left(\frac{e}{\mathcal{B}(P)} - \mathcal{A}(P)\right), \tag{33}$$

where $\mathcal{B}(P)$ and $\mathcal{A}(P)$ are functions of prices only.

Proof. The marginal utility function must be homogenous of degree minus one, i.e., $v_e(e, P) = xv_e(xe, xP)$, for any $x > 0$. Thus, (13) can be expressed as

$$v_e(e_{i,t}, P_t) = v_e(x_{t+1}e_{i,t+1}, x_{t+1}P_{t+1}), \tag{34}$$

where $x_{t+1} \equiv [\beta(1 + r_{t+1})]^{-1}$. Consider a degenerated expenditure distribution with $e_{i,t} = E_t/N$, $\forall i$ where the Euler equation tirvially holds at the averages $e_{i,t} = E_t/N$

and $e_{i,t+1} = E_{t+1}/N$. Any mean-preserving cross-sectional distribution can be generated by sequentially redistributing Δ from some individual j to another individual l. After redistribution (34) continues to hold at the average if and only if current expenditure's marginal impact on future spending is the same for both individuals, $\partial e_{j,t+1}/\partial(e_{j,t} - \Delta) = \partial e_{l,t+1}/\partial(e_{l,t} + \Delta)$ such that E_{t+1}/N remains unchanged as well. Since the function $v_e(\cdot)$ is time invariant this is satisfied if and only if $e_{i,t+1}$ is linearly related to $e_{i,t}$ in the following way:

$$\frac{e_{i,t}}{\mathcal{B}(P_t)} - \mathcal{A}(P_t) = \frac{x_{t+1}e_{i,t+1}}{\mathcal{B}(x_{t+1}P_{t+1})} - \mathcal{A}(x_{t+1}P_{t+1}). \tag{35}$$

Applying the transformation $z : R \to R$ to both sides of the above equation yields the individual Euler equation

$$z\left(\frac{e_{i,t}}{\mathcal{B}(P_t)} - \mathcal{A}(P_t)\right) = v_e(e_{i,t}, P_t) = v_e(x_{t+1}e_{i,t+1}, x_{t+1}P_{t+1}). \tag{36}$$

This establishes the lemma. ■

Based on Lemma 3, we can now prove the proposition. We have

$$v_e\left(\hat{e}_{i,t}\right) = x_{t+1}^{-1} v_e\left(\hat{e}_{i,t+1}\right), \tag{37}$$

where $\hat{e}_{i,t} \equiv \frac{e_{i,t}}{\mathcal{B}(P_t)} - \mathcal{A}(P_t)$ and $\hat{e}_{i,t+1} \equiv \frac{e_{i,t+1}}{\mathcal{B}(P_{t+1})} - \mathcal{A}(P_{t+1})$. Using (36), (37) can be expressed as

$$z\left(\hat{e}_{i,t}\right) = x_{t+1}^{-1} z\left(\hat{e}_{i,t+1}\right). \tag{38}$$

Furthermore, we know from (35) that $e_{i,t}$ is afine-linearly related to $e_{i,t+1}$ and this property is inherited by $\hat{e}_{i,t}$ and $\hat{e}_{i,t+1}$. Thus, we can write

$$\hat{e}_{i,t+1} = q_0 + q_1 \hat{e}_{i,t},$$

where the terms $q_0 \equiv \frac{\mathcal{A}(x_{t+1}P_{t+1})\mathcal{B}(x_{t+1}P_{t+1})}{x_{t+1}\mathcal{B}(P_{t+1})} - \mathcal{A}(P_{t+1})$ and $q_1 \equiv \frac{\mathcal{B}(x_{t+1}P_{t+1})}{x_{t+1}\mathcal{B}(P_{t+1})}$ are functions of prices in the two periods and x_{t+1}. Since (38) needs to hold for all $\hat{e}_{i,t}$ we can differentiate two times with respect to it and arrive at

$$z'(\hat{e}_{i,t}) = x_{t+1}^{-1} z'(\hat{e}_{i,t+1})\, q_1, \tag{39}$$

$$z''(\hat{e}_{i,t}) = x_{t+1}^{-1} z''(\hat{e}_{i,t+1})\, (q_1)^2. \tag{40}$$

We can then use equations (38)–(40) to get

$$\frac{z''(\hat{e}_{i,t})\, z(\hat{e}_{i,t})}{[z'(\hat{e}_{i,t})]^2} = \frac{z''(\hat{e}_{i,t+1})\, z(\hat{e}_{i,t+1})}{[z'(\hat{e}_{i,t+1})]^2} = Z. \tag{41}$$

Hence the second derivative with respect to $\hat{e}_{i,t}$ times the function itself divided by the first derivative squared needs to be equal to a constant (independent of prices, x_{t+1}, and the expenditure level) which we define as Z. We can drop the time index and rewrite (41) as

$$\frac{z''(\hat{e}_i)}{z'(\hat{e}_i)} = Z \frac{z'(\hat{e}_i)}{z(\hat{e}_i)}. \tag{42}$$

Hence we have

$$z'(\hat{e}_i) = \mathcal{F}\, [z(\hat{e}_i)]^Z, \tag{43}$$

where $\mathcal{F}$ is a constant. Now we have to distinguish two cases (i) $Z = 1$ and (ii) $Z \neq 1$.

Case $Z = 1$: The solution to (43) is

$$z(\hat{e}_i) = \mathcal{G} \exp(\mathcal{F} \hat{e}_i), \tag{44}$$

where $\mathcal{G} > 0$ is some positive constant to ensure positive marginal utility. Hence, Lemma 3 requires that

$$v_e(e_i, P) = \mathcal{G} \exp\left(\mathcal{F} \left(\frac{e_i}{\mathcal{B}(P)} - \mathcal{A}(P) \right) \right). \tag{45}$$

We then integrate with respect to e_i to yield the indirect utility function

$$v(e_i, P) = \frac{\mathcal{G}\mathcal{B}(P)}{\mathcal{F}} \exp\left(\mathcal{F} \left(\frac{e_i}{\mathcal{B}(P)} - \mathcal{A}(P) \right) \right) + \mathcal{D}(P), \tag{46}$$

where $\mathcal{D}(P)$ is a new arbitrary function of prices. Since the strict concavity of (46) in e_i requires that $\mathcal{B}(P)/\mathcal{F} < 0$, a straightforward redefinition of the price functions yields the exponential form of the period utility function in (17).

Case $Z \neq 1$: In this case the solution to (43) is

$$z(\hat{e}_i) = v_e(\hat{e}_i) = [(1-Z)\mathcal{F}\hat{e}_i + \mathcal{G}]^{1/(1-Z)}\,, \tag{47}$$

where $\mathcal{F}$ and $\mathcal{G}$ are constants and $(1-Z)\mathcal{F}\hat{e}_i + \mathcal{G} > 0$. When $Z \neq 2$, integration with respect to e_i yields the indirect utility function

$$v(e_i, P) = \frac{\mathcal{B}(P)}{\mathcal{F}(2-Z)}\,[(1-Z)\mathcal{F}\hat{e} + \mathcal{G}]^{\frac{2-Z}{1-Z}} + \mathcal{D}(P), \tag{48}$$

where $\mathcal{D}(P)$ is a new arbitrary function of prices. Defining $\epsilon \equiv \frac{2-Z}{1-Z}$ in (48), then gives

$$v(e_i, P) = -\frac{\mathcal{B}(P)}{\mathcal{F}}\frac{1-\epsilon}{\epsilon}\left(\frac{1}{1-\epsilon}(-\mathcal{F})\hat{e}_i + \mathcal{G}\right)^{\epsilon} + \mathcal{D}(P). \tag{49}$$

Since $v_{ee}(e_i, P) < 0$ requires that $-\mathcal{B}(P)/\mathcal{F} > 0$, we can redefine the price functions in (49) in an obvious way to yield (16).

Similarly, when $Z = 2$, we can rewrite (47) as

$$z(\hat{e}_i) = v_e(\hat{e}_i) = [-\mathcal{F}\hat{e}_i + \mathcal{G}]^{-1}\,, \tag{50}$$

where $\mathcal{F}$ and $\mathcal{G}$ are constants and $-\mathcal{F}\hat{e}_i + \mathcal{G} > 0$. Integration with respect to e_i yields the indirect utility function

$$v(e_i, P) = -\frac{\mathcal{B}(P)}{\mathcal{F}}\log\left[-\mathcal{F}\hat{e}_i + \mathcal{G}\right] + \mathcal{D}(P), \tag{51}$$

where $\mathcal{D}(P)$ is a new function of prices. Since we could add an arbitrary constant to (51), we can assume without loss of generality that $\mathcal{D}(P) = \log(\widetilde{\mathcal{D}}(P)) > 0$. Redefining the price functions, (51) can then be expressed as (18).

Finally, the homogeneity restrictions on the price functions are required to ensure the zero homogeneity of the indirect utility functions in prices and nominal expenditure. This concludes the proof of the proposition. ■

A.5 Proof of Proposition 3

Proof. The Marshallian demand (19) to (20) follows immediately from applying Roy's identity to (16)–(18). Equations (21)–(22) are derived by substituting

$$v_e(e_{i,t}, P_t) = v_e(E_t/N, P_t)\frac{v_e(e_{i,t}, P_t)}{v_e(E_t/N, P_t)}$$

in (19)–(20), aggregating over all indviduals, and rearranging terms. Finally, the aggregation factors κ and $\tilde{\kappa}$ are constant because IA preferences imply that both $v_e(e_{i,t}, P_t)$ and $v_e(E_t/N, P_t)$ grow with the same gross rate $\beta(1 + r_{t+1})$ over time for all individuals i. ■

A.6 Proof of Corollary 1

Proof. Since $v_e(e_{i,t}, P_t)$ satisfies the individual Euler equation, the distribution of relative marginal utilities $v_e(e_{i,t}, P_t)/v_e(E_t/N, P_t)$ is constant if and only if preferences are IA. With aggregate data on per-capita expenditure and sectoral prices only, (21) allows to identify all parameters of the IA preferences up to the scale of the function $\mathbf{D}(P)$ and in (22) all parameters are identified up to a common scalar for $\mathbf{A}(P)$ and $\mathbf{B}(P)$. Furthermore, when distributional data for $e_{i,t}$ is available at some point in the data period, then the unknown scales of $\mathbf{D}(P)$ or $\mathbf{A}(P)$ and $\mathbf{B}(P)$, respectively, can easily be separated from the corresponding aggregation factors, κ and $\tilde{\kappa}$, which only depend on parameters that can be identified with aggregate data alone. ■

A.7 Proof of Proposition 4

Proof. We start the proof by showing part (i) of the proposition. Let $E_t/N \equiv e_t$. Along a balanced growth path (BGP) nominal expenditure grows at rate $\gamma_X > 1$, which is strictly greater than any price's growth rate $(\gamma_X/\gamma_j)^{1-\alpha}$. Thus, along a BGP

$$\lim_{t\to\infty} p_{j,t}/e_t = 0, \ \forall j \in J. \tag{52}$$

Consequently, since $\mathbf{A}(P_t)\left[e_t/\mathbf{B}(P_t)\right]^{-1} = \sum_{j\in J}(p_{j,t}/e_t)\bar{c}_j$, (52) implies that along a BGP

$$\lim_{t\to\infty} \mathbf{A}(P_t)\left[e_t/\mathbf{B}(P_t)\right]^{-1} = 0. \tag{53}$$

Next, the price function $\mathbf{B}(P_t)$ grows at the rate

$$\gamma_{\mathbf{B},t} = \left(\sum_{j\in J} \frac{w_j p_{j,t}^{1-\sigma}}{\sum_{l\in J}\omega_l p_{l,t}^{1-\sigma}} \left(\frac{\gamma_X}{\gamma_j}\right)^{(1-\alpha)(1-\sigma)}\right)^{1/(1-\sigma)}.$$

This growth rate is constant for finite t in the special cases $\sigma = 1$ or $\gamma_j = \gamma_l\,\forall j,l \in J$. In all other cases, the growth rate only approaches a constant in the limit with $\lim_{t\to\infty}\gamma_{\mathcal{B},t} = \max_{j\in J}(\gamma_X/\gamma_j)^{1-\alpha}$ if $\sigma < 1$ or $\lim_{t\to\infty}\gamma_{\mathcal{B},t} = \min_{j\in J}(\gamma_X/\gamma_j)^{1-\alpha}$ if $\sigma > 1$. We define this constant growth rate as $\lim_{t\to\infty}\gamma_{\mathbf{B},t} \equiv \gamma_{\mathbf{B}}$. The Euler equation can be expressed as

$$\left(\frac{1-\mathbf{A}(P_t)\left[e_t/\mathbf{B}(P_t)\right]^{-1}}{1-\mathbf{A}(P_{t+1})\left[e_{t+1}/\mathbf{B}(P_{t+1})\right]^{-1}}(e_t/e_{t+1})\gamma_{\mathbf{B},t}\right)^{\epsilon-1}\left(\frac{\mathbf{B}(P_t)}{\mathbf{B}(P_{t+1})}\right)^{-1} = \beta(1+r_{t+1}).$$

Using (53), it is easy to see that along an asymptotic BGP the left-hand side of the Euler equation approaches the constant $(\gamma_{\mathbf{B}}/\gamma_X)^{\epsilon-1}\gamma_{\mathbf{B}}$ and supports a constant interest rate on the right-hand side. In summary, we have shown that the period utility function in (16) with price functions (25)-(26) supports an asymptotic balanced growth path.

Next we prove part (ii) of the proposition. Consider the generic form of the expenditure shares in (23). Given the parameterization, the second term can be expressed as a share $\omega_j p_{j,t}^{1-\sigma}/\left(\sum_{l\in J}\omega_l p_{l,t}^{1-\sigma}\right)$ which is bounded between zero and one. Given (25), the first term can be expressed as

$$\mathbf{A}_j(P_t)p_{j,t}\left(\frac{e_t}{\mathbf{B}(P_t)}\right)^{-1} = \frac{p_{j,t}\bar{c}_j}{e_t} - \frac{\omega_j p_{j,t}^{1-\sigma}}{\sum_{l\in J}\omega_l p_{l,t}^{1-\sigma}}\mathbf{A}(P_t)\left(\frac{e_t}{\mathbf{B}(P_t)}\right)^{-1}.$$

Using (52) and (53) it is easy to see that $\lim_{t\to\infty}\mathbf{A}_j(P_t)p_{j,t}\left[e_t/\mathbf{B}(P_t)\right]^{-1} = 0$. Finally, the third term can be written as

$$\frac{\mathbf{D}_j(P_t)p_{j,t}}{v_e(e_t,P_t)\mathbf{B}(P_t)}\left(\frac{e_t}{\mathbf{B}(P_t)}\right)^{-1}\kappa=\left(\nu_j+\sum_{l\in J}\psi_{jl}\log(p_{l,t})\right)\frac{\left(1-\mathbf{A}(P_t)\left[e_t/\mathbf{B}(P_t)\right]^{-1}\right)^{1-\epsilon}}{\left[e_t/\mathbf{B}(P_t)\right]^{\epsilon}}.$$

Since $\gamma_{\mathbf{B}}$ is a weighted average of the goods prices' growth rates, it must be true that asymptotically the ratio $e_t/\mathbf{B}(P_t)$ is growing at the constant rate $1+\gamma_X-\gamma_{\mathbf{B}}>1$. Moreover, because $\epsilon>0$ the ratio $[e_t/\mathbf{B}(P_t)]^{\epsilon}$ grows an order of magnitude faster than $\sum_{l\in J}\psi_{jl}\log(p_{l,t})$. Using (53), we can therefore conclude that

$$\lim_{t\to\infty}\frac{\mathbf{D}_j(P_t)p_{j,t}}{v_e(e_t,P_t)\mathbf{B}(P_t)}\left(\frac{e_t}{\mathbf{B}(P_t)}\right)^{-1}\kappa=0.$$

In summary, we have shown that $\lim_{t\to\infty}\eta_{j,t}=\omega_j p_{j,t}^{1-\sigma}/\left(\sum_{l\in J}\omega_l p_{l,t}^{1-\sigma}\right)\in[0,1]$. This concludes the proof of Proposition 4. ■

A.8 Additional tables (continuation of Tables 1–3)

This section contains the continuation of Tables 1–3 in Section 7.1. We report the remaining parameter estimates of the generalized Stone-Geary, PIGL, and IA specifications for all samples in Tables A.1–A.3 below.

	USA			GBR		
	Stone-Geary	PIGL	IA	Stone-Geary	PIGL	IA
	(1)	(2)	(3)	(4)	(5)	(6)
ω_A	0.047	0.000	0.000	0.086	0.000	0.000
	(0.003)	(.)	(.)	(0.004)	(.)	(.)
ω_M	0.322	0.312	0.008	0.390	0.457	0.421
	(0.003)	(0.007)	(0.006)	(0.003)	(0.009)	(0.024)
ω_S	0.632	0.688	0.992	0.525	0.543	0.579
	(0.004)	(0.007)	(0.006)	(0.005)	(0.009)	(0.024)
ψ_{AA}		43.0	-1.1		36.5	-15.8
		(21.9)	(1.1)		(6)	(6.7)
ψ_{MM}		-72.0	6.1		21.6	-2.7
		(16.7)	(2.3)		(4.3)	(2.7)
ψ_{SS}		-23.2	7.5		18.4	-6.5
		(8)	(2.3)		(3.8)	(5.7)
$\psi_{AM} = \psi_{MA}$		2.9	1.3		-19.9	6.0
		(10.9)	(0.6)		(4.3)	(1.8)
$\psi_{AS} = \psi_{SA}$		-45.9	-0.2		-16.7	9.8
		(13.4)	(0.5)		(2.2)	(5.3)
$\psi_{MS} = \psi_{SM}$		69.1	-7.4		-1.7	-3.3
		(18.5)	(2.3)		(4.2)	(2.7)
Obs	104	104	104	97	97	97
AIC	-1000	-1014	-1100	-1058	-1197	-1209

Table A.1: Estimation remaining parameters, Private consumption, USA and GBR

Note: Years when the USA was involved in WWI (1917-1918), WWII (1942-1945), or affected by the Great Depression (1929-1933), and years when GBR was involved in WWI (1914-1918), WWII (1939-1945), or affected by the Great Depression (1929-1933) are excluded from all estimations. All variables are based on final private consumption expenditure. Robust standard errors are reported in parenthesis.

	CAN			AUS		
	Stone-Geary	PIGL	IA	Stone-Geary	PIGL	IA
	(1)	(2)	(3)	(4)	(5)	(6)
ω_A	0.077	0.004	0.000	0.020	0.000	0.000
	(0.005)	(0.085)	(.)	(0.003)	(.)	(.)
ω_M	0.325	0.374	0.349	0.276	0.279	0.260
	(0.006)	(0.042)	(0.017)	(0.027)	(0.033)	(0.03)
ω_S	0.598	0.622	0.651	0.704	0.721	0.740
	(0.01)	(0.069)	(0.017)	(0.027)	(0.033)	(0.03)
ψ_{AA}		0.0	2.3		97.4	247.4
		(1.6)	(4)		(86.8)	(151.6)
ψ_{MM}		-1.9	-16.8		-27.6	-413.4
		(1.7)	(7.5)		(42.9)	(181.1)
ψ_{SS}		-0.3	-3.0		0.5	-566.9
		(0.3)	(2.1)		(38)	(217.2)
$\psi_{AM} = \psi_{MA}$		0.8	5.7		-34.6	-200.4
		(1.4)	(1.8)		(43.2)	(113.5)
$\psi_{AS} = \psi_{SA}$		-0.8	-8.0		-62.8	-47.0
		(0.7)	(5.5)		(44.1)	(61.9)
$\psi_{MS} = \psi_{SM}$		1.1	11.1		62.2	613.9
		(0.8)	(5.9)		(48.6)	(231.7)
Obs	77	77	77	63	63	63
AIC	-801	-839	-995	-670	-618	-677

Table A.2: Estimation remaining parameters, Private consumption, CAN and AUS

Note: Years when AUS and CAN were involved in WWI (1914-1918), WWII (1939-1945), or affected by the Great Depression (1929-1933) are excluded from all estimations. All variables are based on final private consumption expenditure. Robust standard errors are reported in parenthesis.

	Stone-Geary	PIGL	IA	Stone-Geary$\vert_{\bar{c}_M=0}$
	(1)	(2)	(3)	(4)
ω_A	0.057	0.000	0.000	0.095
	(0.002)	(.)	(.)	(0.003)
ω_M	0.340	0.365	0.274	0.330
	(0.004)	(0.009)	(0.03)	(0.003)
ω_S	0.603	0.635	0.726	0.575
	(0.005)	(0.009)	(0.03)	(0.005)
ψ_{AA}		72.2	6.9	
		(14.6)	(4.2)	
ψ_{MM}		-14.7	-3.2	
		(6.4)	(4)	
ψ_{SS}		-24.9	-2.7	
		(8.2)	(4.5)	
$\psi_{AM} = \psi_{MA}$		-41.2	-3.1	
		(9)	(2.4)	
$\psi_{AS} = \psi_{SA}$		-31.0	-3.7	
		(6.2)	(2)	
$\psi_{MS} = \psi_{SM}$		55.9	6.4	
		(11.1)	(5.9)	
Obs	341	341	341	341
AIC	-2928	-2951	-3018	-2737

Table A.3: Estimation remaining parameters, Private consumption, All Countries

Note: Years when the USA was involved in WWI (1917-1918), WWII (1942-1945), or affected by the Great Depression (1929-1933), and years when AUS, CAN, and GBR were involved in WWI (1914-1918), WWII (1939-1945), or affected by the Great Depression (1929-1933) are excluded from all estimations. All variables are based on final private consumption expenditure. Robust standard errors are reported in parenthesis.