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JEL Classification: E52, E58, G01, G28

Keywords: monetary and macro-prudential policy, financial crisis, zero lower bound

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House Price Dynamics, Optimal LTV Limits and the Liquidity Trap*

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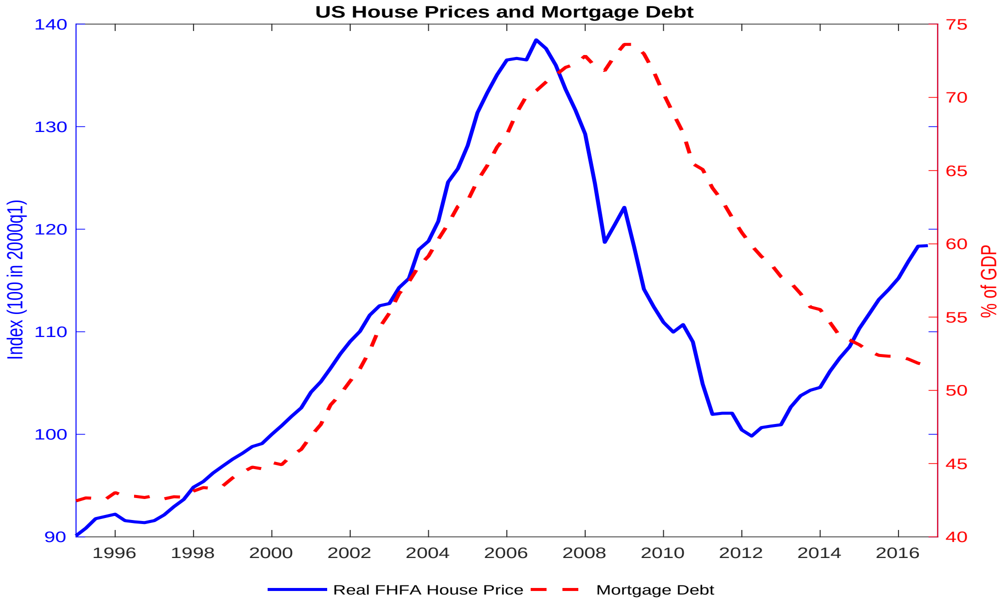

Figure 1: US real house prices (solid blue line, left scale) and mortgage debt in % of GDP (dashed red line, right scale). Data sources: FHFA (house prices), Flow of Funds (mortgage debt), and NIPA tables (GDP).

1 Introduction

A persistent boom in house prices, accompanied by a large increase in private indebtedness, planted the seeds for the financial crisis of 2008 (Figure 1). Once house prices collapsed, the turmoil in the financial sector and the deleveraging process that followed caused the worst US recession since the Great Depression.[1]

To prevent a repeat of similar episodes in the future, the recent debate has focused on enabling policy authorities to use a number of so-called 'macro-prudential' tools.[2] However, many questions still remain open regarding the interaction of these new policy tools with the existing ones, especially monetary policy, and their effect on the economy more broadly. This paper focuses on one particular macro-prudential tool—a loan-to-value (LTV) limit—and its implications for conventional monetary policy. We find that optimal macro-prudential policy is strongly countercyclical. Promptly adjusting the LTV limit can prevent an economy that suffers a large deleveraging shock from entering a liquidity trap.

Our findings contribute to a growing literature exploring the conduct of macro-prudential policy and its interaction with monetary policy. At a theoretical level, Farhi and Werning (2016)

[1] Hall (2011) provides a detailed narrative of the US Great Recession along these lines. Other countries, such as Ireland, Spain, and the UK, experienced similarly long booms before deep recessions during the same period.

[2] Institutional details on macro-prudential frameworks, both in terms of the tools available and the authorities in charge, vary greatly across countries (Akinci and Olmstead-Rumsey, 2018).

and Korinek and Simsek (2016) undertake detailed analyses of the financial market distortions that macro-prudential policy can address in the presence of aggregate demand externalities, such as nominal rigidities and the zero lower bound on the nominal interest rate, while Davila and Korinek (2018) emphasize the pecuniary externalities due to an endogenous collateral constraint.

We obtain our main results in a simple model that combines both types of externality. As standard in the New Keynesian literature, nominal rigidities arise because of staggered price setting (Calvo, 1983), so that monetary policy has real effects. The key financial friction is a borrowing constraint on relatively impatient households. Following Kiyotaki and Moore (1997), the borrowing constraint has an endogenous element because debt cannot exceed a given fraction of the value of the housing collateral. This fraction is the loan-to-value ratio, which we assume is under the control of the macro-prudential authority. The model also features a second financial friction. Borrowers obtain loans through perfectly competitive financial intermediaries (banks), which raise equity and deposits from relatively patient households. Banks would like to minimize equity, which we assume is costly to raise (as in Justiniano et al., 2017), but a capital requirement puts an upper bound on their leverage. Changes in this capital requirement generate fluctuations in the spread between borrowing and deposit rates. Together with housing demand shocks, these financial disturbances, or credit supply shocks, are the exogenous source of fluctuations in our model.

The resulting framework is rich enough to generate meaningful policy tradeoffs, but sufficiently tractable that, up to a second-order approximation, the welfare-based loss function clearly identifies how the inefficiencies in the economy map into four policy targets. Two of these loss function terms, inflation and the output gap, stem from nominal rigidities and are standard in the New Keynesian literature (e.g. Clarida et al., 1999, and Woodford, 2003). The remaining two terms are due to imperfect risk sharing among borrowers and savers. In particular, the policymaker seeks to stabilize the distribution of non-durable consumption and housing consumption between borrowers and savers—the 'consumption gap' and the 'housing gap,' respectively. The resulting welfare-based loss function is similar to those derived by Andres et al. (2013) and Benigno et al. (2016), and has elements in common with the loss function in the Cúrdia and Woodford (2016) model. While those papers only focus on optimal monetary policy, our contribution is to explore its interaction with the optimal setting of macro-prudential policy.

We use the loss function to study the optimal setting of LTV requirements, and their implications for monetary policy, taking into account the possibility that the collateral constraint and the ZLB only bind occasionally, as in Guerrieri and Iacoviello (2017).[3] Our quantitative experiments generate a prolonged boom in house prices followed by a sudden correction, cali-

[3] Rubio and Carrasco-Gallego (2014) postulate a simple rule for LTVs. Lambertini et al. (2013) study optimized simple rules for monetary policy and LTV limits in the context of boom-bust cycles generated by news shocks. Angelini et al. (2012) consider optimal monetary and macro-prudential policies (in the form of both capital requirements and LTV limits) using an ad-hoc loss function as the policymakers' objective. Our analysis also shares some similarities with De Paoli and Paustian (2017), although their model features entrepreneurs borrowing directly from households and a cost channel, giving rise to a different welfare-based loss function and a different macro-prudential policy instrument (a tax/subsidy on the cost of borrowing for entrepreneurs). None of these papers considers occasionally binding constraints.

brated to capture the salient features of the data in the US and other advanced economies pre and post-2008. The main exercise compares a baseline scenario characterized by a constant LTV ratio with a policy regime in which the macro-prudential policymaker can optimally set the LTV ratio to minimize the volatility of the consumption and housing gaps. In spite of the additional risk-sharing objectives, the optimal LTV policy prevents the zero lower bound (ZLB) on the nominal interest rate from becoming a binding constraint. LTVs decline significantly during the boom and sharply rise at the time of the crisis. Consequently, the macro-prudential policymaker avoids a large build up in private debt during the expansion such that, when house prices fall, the recessionary consequences of deleveraging are limited. As a result, the central bank can stabilize inflation and output without cutting the nominal interest rate below zero. In this sense, the optimal setting of the LTV limit is indeed prudential, at least as far as macroeconomic objectives are concerned.

Our analysis should have particular resonance given the economic conditions policymakers currently face in many advanced economies. Since the financial crisis, and in addition to a raft of unconventional monetary policy measures, interest rates have remained at very low levels almost everywhere on a persistent basis. While these policies are likely to have prevented much deeper downturns, some observers, for example Stein (2013) and Borio (2018), have expressed concerns over their impact for financial stability.[4] In some cases, the first use of the newly introduced macro-prudential instruments has been to guard against these risks. Thus, one way to summarize the current policy mix in many countries is the combination of 'loose' monetary policy and 'tight' macro-prudential policy.[5]

When the LTV tool is idle, the policy configuration in the model during the recovery is broadly consistent with the 'loose-money/tight-credit' mix observed in many economies in recent years. The policy rate remains at the ZLB for an extended period, while at the same time borrowing constraints bind tightly. Conversely, our main result shows that the optimal LTV policy is always accommodative in response to a crisis. We confirm this finding by running an experiment in which the LTV tool becomes available only after the crisis hits. In this case, a tightening of LTV requirements prolongs the period for which the monetary policy rate remains stuck at the ZLB, while an optimal loosening would tend to hasten its end.

The ability to use an LTV instrument is of particular empirical relevance because in many advanced economies mortgages are at the same time the single largest asset class on the balance sheet of banks and the single largest liability class on the balance sheet of households.[6] The

[4] Rajan (2005) and Taylor (2007) made similar a point in relation to the 'low-for-long' period before the financial crisis.

[5] For example, in explaining the decision of the Bank of England's Financial Policy Committee to limit the quantity of new lending at high loan-to-value ratios, Carney (2014) said: "The existence of macro-prudential tools allows monetary policy to focus on its primary responsibility of price stability. In other words, monetary policy does not need to be diverted to address a sector-specific risk in the housing market." Similarly, authorities in Canada have tightened macro-prudential policy several times since the financial crisis while the official policy rate has remained low (see Krznar and Morsink, 2014). The change of the policy mix in Sweden has been a subject of much controversy and debate (see, for example, Jansson, 2014, and Svensson, 2011).

[6] Cerutti et al. (2017) document how the use of borrower-based instruments, including LTV limits,

focus on LTVs in this paper complements a number of contributions that instead highlight the role of capital requirements for macro-prudential policy, such as Miles et al. (2013), Admati and Hellwig (2014), Gertler et al. (2012), Clerc et al. (2015), and Christiano and Ikeda (2016). Several papers extend the analysis to the interaction between capital requirements and monetary policy. For example, Angeloni and Faia (2013) compare alternative properties of capital requirements that mimic the Basel I, II, and III accords in a model with bank runs. Bean et al. (2010) study the optimal setting of capital requirements with ad-hoc loss functions in a simplified version of Gertler and Karadi (2011). Collard et al. (2017) and Van der Ghote (2018) derive the jointly-optimal setting of interest rates and capital requirements in environments with moral hazard frictions. Mendicino et al. (2018) evaluate the tradeoffs associated with increasing capital requirements depending on the state of the business cycle. Going forward, both LTV limits and capital requirements are likely to play a major role in the development and actual deployment of macro-prudential policy frameworks.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 introduces the loss function we use for policy analysis and discusses its interpretation vis-a-vis the New Keynesian literature on optimal monetary policy. Section 4 illustrates the optimal joint conduct of monetary and macro-prudential policy via numerical simulations. Section 5 concludes. An extensive appendix provides details on the derivations and the computations.

2 Model

The economic agents in the model are households, banks, firms, and the government. Households are heterogeneous in their degree of patience. Banks transfer funds from savers to borrowers and fund their operations with a mix of deposits and equity. Firms produce goods for consumption. The government conducts monetary and macro-prudential policy.

2.1 Households

Patient households (i.e. savers, indexed by s) have a higher discount factor than impatient households (i.e. borrowers, indexed by b). We denote with $\xi \in (0, 1)$ the mass of borrowers, and normalize the total size of the population to one. We make the standard assumptions that risk sharing is perfect within each group and that the flow of housing services is proportional to the stock of housing.

prevails in advanced economies, while emerging markets tend to rely primarily on foreign exchange related measures.

2.1.1 Preferences

A generic household $i \in (0,1)$ of type $j = \{s, b\}$ has preferences defined over consumption of goods $C_t^j(i)$, housing services $H_t^j(i)$, and hours worked $L_t^j(i)$

$$\mathbb{W}_0^j(i) \equiv \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_j^t \left[(1 - e^{-zC_t^j(i)}) + \frac{\chi_H^j e^{u_t^h}}{1-\sigma_h} H_t^j(i)^{1-\sigma_h} - \frac{\chi_L^j}{1+\varphi} L_t^j(i)^{1+\varphi} \right], \tag{1}$$

where $\beta_j \in (0,1)$ (with $\beta_b < \beta_s$) is the discount factor of type j, $z > 0$ measures the degree of absolute risk aversion, $\sigma_h \geq 0$ is the inverse elasticity of housing demand, and $\varphi \geq 0$ is the inverse Frisch elasticity of labor supply. We assume exponential preferences over consumption to facilitate aggregation and the derivation of the welfare-based loss function. The type-specific constants χ_H^j and $\chi_L^j > 0$ serve normalization purposes. Finally, preferences include an aggregate housing preference shock, u_t^h, common to all households, which follows a first-order autoregressive process with persistence $\rho_h \in [0,1)$. This *housing demand shock* is one of the two exogenous disturbances we subsequently study.

2.1.2 Savers' Budget Constraint

Patient households have a relatively high individual discount factor and can save in deposits, $D_t^s(i)$, and equity, $E_t^s(i)$, issued by financial intermediaries. Their budget constraint is

$$\begin{aligned} P_t C_t^s(i) + D_t^s(i) + E_t^s(i) + (1+\tau^h) Q_t H_t^s(i) = \\ W_t^s L_t^s(i) + R_{t-1}^d D_{t-1}^s(i) + R_{t-1}^e E_{t-1}^s(i) + Q_t H_{t-1}^s(i) + \Omega_t^s(i) - T_t^s(i) - \Gamma_t(i), \end{aligned}$$

where P_t is the consumption price index, Q_t is the nominal house price, W_t^s is the nominal wage for savers, R_{t-1}^d is the nominal return on bank deposits, and R_{t-1}^e is the nominal return on bank equity.[7] The variable $T_t^s(i)$ captures lump-sum taxes while $\Omega_t^s(i)$ denotes the savers' share of remunerated profits from intermediate goods producers. The constant τ^h is a tax/subsidy on savers' housing that we assume is set to deliver an efficient steady state in the housing market. The final term in the budget constraint is a cost associated with deviations from some preferred portfolio level of bank equity $\overline{E}_t > 0$

$$\Gamma_t(i) \equiv \frac{\Psi}{2} \left[\frac{E_t^s(i)}{\overline{E}_t} - 1 \right]^2 \overline{E}_t,$$

with $\Psi > 0$, where, for analytical convenience, we define $\overline{E}_t \equiv \tilde{\kappa}\xi D_t^b/(1-\xi)$. The introduction of this adjustment cost function follows Justiniano et al. (2017) and is a simple way to generate a liability structure in the banks' balance sheet, thus capturing the idea that in practice deposits

[7] As in Benigno et al. (2016), the introduction of type-specific wages, together with the assumption of exponential utility, simplifies aggregation, and facilitates the derivation of a welfare criterion for the economy as a whole.

are generally more liquid and easier to adjust than equity.[8]

2.1.3 Borrowers' Budget Constraint

The budget constraint for impatient households is

$$P_t C_t^b(i) - D_t^b(i) + Q_t H_t^b(i) = W_t^b L_t^b(i) - R_{t-1}^b D_{t-1}^b(i) + Q_t H_{t-1}^b(i) + \Omega_t^b(i) - T_t^b(i),$$

where $D_t^b(i)$ is the amount of borrowing at time t, $T_t^b(i)$ are lump-sum taxes, including those used to obtain an efficient allocation of consumption in the model's steady state, and $\Omega_t^b(i)$ denotes profits from ownership of intermediate goods producing firms.

As common in the literature (e.g Kiyotaki and Moore, 1997), we assume that a collateral constraint limits impatient households' ability to borrow. The standard interpretation of such a constraint is that lenders (in this case, the financial intermediaries) require borrowers to have a stake in a leveraged investment to prevent moral hazard behavior. To provide a somewhat realistic dynamic structure of the model, we follow the specification in Guerrieri and Iacoviello (2017) and Justiniano et al. (2015)

$$D_t^b(i) \leq \gamma_d D_{t-1}^b(i) + (1 - \gamma_d)\,\Theta_t Q_t H_t^b(i), \tag{2}$$

where $\Theta_t \in [0,1]$ represents the maximum loan-to-value (LTV) ratio available to borrowers, and $\gamma_d \in [0,1)$ is a parameter controlling the extent to which the debt limit depends on the household's debt in the previous period.

As Guerrieri and Iacoviello (2017) argue, the formulation in (2) captures, in reduced form, the idea that only a fraction of borrowers experience a change to their borrowing limit each period (which may be associated with moving or re-mortgaging). This modification of the standard collateral constraint generates more persistent movements in debt and its marginal value.[9] One important implication is that movements in debt adjust only gradually to changes in the value of the housing stock, which is consistent with the data in Figure 1.

In our policy analysis, we will assume the macro-prudential authority sets the maximum LTV that banks can extend to borrowers.

2.2 Banks

A continuum of perfectly competitive banks, indexed by $k \in [0,1]$, raise funds from savers in the form of deposits and equity (their liabilities), and make loans (their assets) to borrowers. Bank k's balance sheet identity is

$$D_t^b(k) = D_t^s(k) + E_t^s(k). \tag{3}$$

[8] Little of substance would change in the first-order accurate solution to the model that we examine if we specified bank equity as a state-contingent claim.

[9] When $\gamma_d = 0$, the collateral constraint collapses to the standard contemporaneous specification.

In addition, we assume that equity must account for at least a fraction $\tilde{\kappa}_t$ of the total amount of loans banks extend to borrowers

$$E_t^s(k) \geq \tilde{\kappa}_t D_t^b(k), \tag{4}$$

where $\tilde{\kappa}_t$ follows a first-order autoregressive process with persistence $\rho_\kappa \in [0, 1)$.

The presence of equity adjustment costs breaks down the irrelevance of the capital structure (the Modigliani-Miller theorem). Savers demand a premium for holding equity, which banks pass on to borrowers in the form of a higher interest rate. From the perspective of the bank, equity is expensive, and thus deposits are the preferential source of funding. In the absence of any constraint, banks would choose to operate with zero equity and leverage would be unbounded. Equation (4) ensures finite leverage for financial intermediaries.

In equilibrium, the capital requirement constraint is always binding because financial intermediaries seek to minimize their equity requirement. Since banks are identical, if the capital constraint of all banks were slack, one bank could marginally increase its leverage, charge a lower loan rate, and take the whole market. Therefore, competition drives the banking sector against the constraint.

Banks' profits are

$$\mathcal{P}_t(k) \equiv R_t^b D_t^b(k) - R_t^d D_t^s(k) - R_t^e E_t^s(k) = [R_t^b - (1 - \tilde{\kappa}_t) R_t^d - \tilde{\kappa}_t R_t^e] D_t^b(k),$$

where the second equality follows from substituting the balance sheet constraint (3) and the capital requirement (4) at equality. The zero-profit condition implies that the loan rate is a linear combination of the return on equity and the return on deposits

$$R_t^b = \tilde{\kappa}_t R_t^e + (1 - \tilde{\kappa}_t) R_t^d,$$

where the time-varying capital requirement represents the weight on the return on equity. An increase of $\tilde{\kappa}_t$ forces banks to delever and raises credit spreads (and vice versa). As a result, henceforth we refer to $\tilde{\kappa}_t$ as a *credit supply shock*, which is the second of the two disturbances we subsequently study.

We stress that our analysis focuses on the case in which $\tilde{\kappa}_t$ is exogenous, relying on the notion that financial institutions target a certain leverage ratio due to market forces (Adrian and Shin, 2010). An alternative interpretation would be that the macro-prudential authority sets the capital requirement on financial institutions, and thus controls $\tilde{\kappa}_t$ as a macro-prudential tool. We do not pursue this approach in this paper for two reasons. First, we believe that properly studying capital requirements, and their interaction with monetary policy, would require a more detailed specification of the financial sector. While our parsimonious model of financial intermediation does capture the connection between capital requirements and spreads, the model completely abstracts from a key variable—the accumulation of net worth—that determines banks' profitability and may be crucial to understand the effects of macro-prudential policy decisions. Second, as discussed in the introduction, several papers have already studied capital requirements, either in isolation or in connection with monetary policy. We aim to complement this literature by focusing on the implications of LTV ratios and their implications for the stabilization of inflation and output.

2.3 Production

A representative retailer combines intermediate goods according to a technology with constant elasticity of substitution $\varepsilon > 1$

$$Y_t = \left[\int_0^1 Y_t(f)^{\frac{\varepsilon-1}{\varepsilon}} df \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where $Y_t(f)$ represents the intermediate good produced by firm $f \in [0, 1]$. Expenditure minimization implies that the demand for a generic intermediate good is

$$Y_t(f) = \left[\frac{P_t(f)}{P_t} \right]^{-\varepsilon} Y_t, \tag{5}$$

where $P_t(f)$ is the price of the variety produced by firm f and the aggregate price index is

$$P_t = \left[\int_0^1 P_t(f)^{1-\varepsilon} df \right]^{\frac{1}{1-\varepsilon}}.$$

Intermediate goods producers operate in monopolistic competition, are owned by savers and borrowers according to their shares in the population, and employ labor to produce variety f according to

$$Y_t(f) = L_t(f). \tag{6}$$

To simplify aggregation, we assume $L_t(f)$ is a geometric average of borrower and saver labor, with weights reflecting the shares of the two types

$$L_t(f) \equiv [L_t^b(f)]^{\xi} [L_t^s(f)]^{1-\xi},$$

and the corresponding wage index is

$$W_t \equiv (W_t^b)^{\xi} (W_t^s)^{1-\xi}.$$

Intermediate goods producers set prices on a staggered basis. As customary, we solve their optimization problem in two steps. First, for given pricing decisions, firms minimize their costs, which implies that the nominal marginal cost MC_t is independent of each firm's characteristics. The second step of the intermediate goods producers problem is to determine their pricing decision. As in Calvo (1983), we assume firms reset their price $\tilde{P}_t(f)$ in each period with a constant probability $1 - \lambda$, taking as given the demand for their variety, while the complementary measure of firms λ keep their price unchanged. The optimal price setting decision for firms that do adjust at time t solves

$$\max_{\tilde{P}_t(f)} \mathbb{E}_t \left\{ \sum_{v=0}^{\infty} \lambda^i \mathcal{M}_{t,t+v} [(1+\tau^p)\tilde{P}_t(f) - MC_{t+v}] Y_{t+v}(f) \right\},$$

subject to (5), where τ^p is a subsidy to make steady state production efficient. Households of each type own intermediate goods producers in proportion to their shares in the population. Therefore, we assume that the discount rate for future profits is

$$\mathcal{M}_{t,t+v} \equiv (\mathcal{M}^b_{t,t+v})^{\xi} (\mathcal{M}^s_{t,t+v})^{1-\xi},$$

where the stochastic discount factor between period t and $t+v$ of type j is

$$\mathcal{M}^j_{t,t+v} = \beta_j^v e^{-z(C^j_{t+v}(i) - C^j_t(i))}.$$

2.4 Equilibrium

Because of the assumption of risk-sharing within each group, all households of a given type consume the same amount of goods and housing services and work the same number of hours. Therefore, in what follows, we drop the index i and characterize the equilibrium in terms of type-aggregates. Similarly, because all financial intermediaries make identical decisions in terms of interest rate setting, we drop also the index k and simply refer to the aggregate balance sheet of the banking sector.

For a given specification of monetary and macro-prudential policy, an imperfectly competitive equilibrium for this economy is a sequence of quantities and prices such that households and intermediate goods producers optimize subject to the relevant constraints, final good producers and banks make zero profits, and all markets clear.[10] In particular, for the goods market, total production must equal the sum of consumption of the two types plus the resources spent for portfolio adjustment costs[11]

$$Y_t = \xi C^b_t + (1-\xi) C^s_t + \Gamma_t, \tag{7}$$

where

$$\Gamma_t \equiv \int_0^{1-\xi} \Gamma_t(i) di = \frac{\Psi}{2} \left(\frac{\tilde{\kappa}_t}{\tilde{\kappa}} - 1 \right)^2 \tilde{\kappa} \xi D^b_t.$$

We assume housing is in fixed supply (i.e., land). The housing market equilibrium then requires

$$H = \xi H^b_t + (1-\xi) H^s_t, \tag{8}$$

[10] Appendix A reports the equilibrium conditions for the private sector and the details of aggregation.

[11] The resource constraint follows from combining the budget constraints of the two types (aggregated over their respective measures) with the financial intermediaries balance sheet, under the assumption that the government adjusts residually the lump-sum transfers to savers.

where H is the total available stock of housing.[12] Finally, in the credit market, total bank loans must equal total household borrowing. Thus, the aggregate balance sheet for the financial sector respects

$$\xi D_t^b = (1-\xi)(D_t^s + E_t^s),$$

where per-capita real private debt (derived from the borrowers' budget constraint) evolves according to

$$\frac{D_t^b}{P_t} = \frac{R_{t-1}^b}{\Pi_t}\frac{D_{t-1}^b}{P_{t-1}} + C_t^b - Y_t + \frac{Q_t}{P_t}(H_t^b - H_{t-1}^b) + \mathcal{T}^b,$$

and $\mathcal{T}^b \equiv T_t^b/P_t$ is a subsidy that ensures the steady state allocation is efficient.

3 Loss Function

Our ultimate objective is to study the conduct of monetary and macro-prudential policy in a boom-bust scenario for house prices and private debt similar to the one that preceded and followed the financial crisis of 2008-2009. To this end, in this section, we introduce a welfare-based loss function that we use to evaluate alternative policy configurations.

We derive the loss function by taking the average of the per-period utility functions of borrowers and savers. We weight each type according to their share in the population and assume that policymakers discount the future at rate $\beta \equiv \beta_b^{\xi}\,\beta_s^{1-\xi}$.[13] A second-order approximation of the resulting objective around a zero-inflation steady state in which the collateral constraint binds gives

$$\mathcal{L}_0 \propto \frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(y_t^2 + \lambda_\pi \pi_t^2 + \lambda_c \tilde{c}_t^2 + \lambda_h \tilde{h}_t^2 \right), \tag{9}$$

where lower-case variables denote log-deviations from the efficient steady state, $\tilde{c}_t \equiv c_t^b - c_t^s$ is the consumption gap between borrowers and savers, and $\tilde{h}_t \equiv h_t^b - h_t^s$ is the housing gap between borrowers and savers.[14]

The weights on inflation, the consumption gap, and housing gap are, respectively,

$$\lambda_\pi \equiv \frac{\varepsilon}{\gamma} \qquad \lambda_c \equiv \frac{\xi(1-\xi)\sigma(1+\sigma+\varphi)}{(1+\varphi)(\sigma+\varphi)} \qquad \lambda_h \equiv \frac{\xi(1-\xi)\sigma_h}{\sigma+\varphi},$$

where $\gamma \equiv (1-\beta\lambda)(1-\lambda)(\sigma+\varphi)/\lambda$ is the slope of the Phillips curve, and $\sigma \equiv zY$ is the product of the coefficient of absolute risk aversion and steady state output.

[12] The absolute level of this variable plays no role in the analysis.

[13] Without a single discount factor, the average lifetime utility of borrowers and savers would not admit a recursive representation. To circumvent this problem, Benigno et al. (2016) assume that the discount factor of the two types is the same in the limit ($\beta_b \to \beta_s$), and that the initial distribution of wealth determines the borrowing/lending positions. We retain the heterogeneity in the discount factors but effectively assume that the policymaker is chosen at random from the population. This choice is arbitrary but we can solve the optimal policy problem for any value of $\beta \in (0,1)$.

[14] An appropriate choice of taxes and subsidies ensures that the steady state allocation is efficient. Appendix C reports the details of the derivations.

The loss function (9) features two sets of terms. The first includes output and inflation—the standard variables that appear in the welfare-based loss function of a large class of New Keynesian models.[15] Their presence in the loss function reflects the two distortions associated with price rigidities. First, such rigidities open up a 'labor wedge,' causing the level of output to deviate from its efficient level. Second, staggered price setting implies an inefficient dispersion in prices, which is proportional to the rate of inflation.

The second set of terms in (9), comprising the consumption gap and the housing gap, arise from the heterogeneity between household types. Incomplete financial markets prevent full risk sharing of goods and housing consumption. The collateral constraint further limits the amount of borrowing that impatient households can undertake, thus creating different marginal propensities to consume between borrowers and savers. Imperfect risk sharing therefore becomes a source of welfare losses a benevolent policymaker needs to take into account in setting monetary and macro-prudential policy optimally.[16]

4 Quantitative Experiments

In this section, we calibrate the model to generate the boom-bust scenario in house prices that drives our simulations. Our main experiment corresponds to a prolonged increase in house prices followed by a sharp decline. We generate the boom-bust with a sequence of either credit supply or housing demand shocks. In each scenario, monetary policy sets the nominal interest rate. We then compare the case in which the macro-prudential authority optimally sets the LTV instrument with the baseline case in which macro-prudential policy is inactive. Throughout the experiments, we allow for the presence of occasionally binding constraints on the nominal interest rate and on the collateral requirement, and discuss their effects on the ability of policy to stabilize the economy.

4.1 Parameter Values

Table 1 reports the parameter values used for the simulation exercises. The parameters in the top-half of the table correspond to the calibration (first four) or posterior mode estimates (next two) in Guerrieri and Iacoviello (2017).[17] We focus the discussion on the remaining parameters.

We set $\beta_b = 0.985$, implying that borrowers are slightly more impatient than implied by the estimate in Guerrieri and Iacoviello (2017), but in line with the value in Justiniano et al. (2017). The relative discount factors of borrowers and lenders are crucial for the extent to which changes in house price expectations cause the borrowing constraint to become slack. Our lower value of

[15] Since productivity is constant (and normalized to one), efficient output is simply equal to its steady state value, and the efficient output gap corresponds to the deviations of output from steady state.

[16] Although equity adjustment involves resource costs, the assumption that the leverage ratio is an exogenous—albeit time-varying—constraint implies that its fluctuations are independent of policy, and thus irrelevant for ranking alternative policies in terms of welfare.

[17] The implied slope of the Phillips curve is 0.0238, almost identical to the value of 0.024 in Eggertsson and Woodford (2003).

Parameter	Description	Value
β_s	Saver discount factor	0.995
σ	Inverse elasticity of substitution (consumption)	1
φ	Inverse Frisch elasticity	1
Θ	Maximum LTV	0.9
γ_d	Debt limit inertia	0.7
λ	Probability of keeping price unchanged	0.9
β_b	Borrower discount factor	0.985
ξ	Fraction of borrowers in economy	0.57
η	Debt/GDP ratio	1.8
ψ	Elasticity of funding cost to capital ratio	0.05
σ_h	Inverse elasticity of substitution (housing)	5
$\widetilde{\kappa}$	Steady state capital requirement	0.1
ρ_h	Housing demand shock persistence	0.995
ρ_κ	Credit supply shock persistence	0.995

Table 1: Parameter values.

β_b increases the steady state value of the borrowing constraint multiplier so that larger shocks are required for the constraint to become slack.

Two parameters that are important in determining the response to housing demand shocks are the persistence of the shocks and the intertemporal substitution elasticity for housing. We assume a high level of persistence, setting $\rho_\kappa = \rho_h = 0.995$, consistent with the view that market tolerance for different levels of leverage and tastes for housing change rather infrequently. For housing preferences, we set $\sigma_h = 5$ which implies that housing demand is not very responsive to movements in real house prices. Guerrieri and Iacoviello (2017) assume $\sigma_h = 1$, but incorporate habit formation in the sub-utility function for housing. The high degree of habit formation in their estimates (0.88 at the posterior mode) implies that the short-run elasticity of housing demand to changes in the house price is much lower than unity. By setting a higher value for σ_h, we aim to replicate this qualitative behavior without complicating the model, and particularly the derivation of the welfare-based loss function.[18]

We choose the remaining parameters with reference to UK data. To pick ξ, we refer to the analysis in Cloyne et al. (2018), who study the behavior of households by tenure type. Their data imply that UK household shares are roughly 30% homeowners, 40% mortgagors, and 30% renters. Since our model does not include renters, we set $\xi = 0.57$ ($\approx 0.4/0.7$) to represent the relative population shares of mortgagors and homeowners in the data.[19] We calibrate $\eta \equiv \xi D^b/Y$

[18] Of course, our approach also reduces the long-run elasticity of housing demand to house prices, and thus is less flexible than the introduction of habits.

[19] Guerrieri and Iacoviello (2017) estimate that the fraction of labor income accruing to borrowers is around 0.5. Since labor income is allocated in proportion to population share in our model, this result suggests a similar value for ξ.

with reference to the ratio of household debt to GDP. According to BIS data, this ratio averaged around 60% between 1990 and 2000.[20] Since mortgages represent around three quarters of household debt, we set $\eta = 1.8$ ($\approx 0.6 \times 0.75 \times 4$ since η is the ratio of debt to quarterly GDP). Finally, we assume the steady state capital ratio $\widetilde{\kappa}$ is 10%, close to the average reported in Meeks (2017) for UK banks over the period 1990-2008. The last parameter to pick is $\psi \equiv \Psi\widetilde{\kappa}$, which governs the elasticity of credit spreads to changes in capital requirements. In its final report to the BIS, the Macroeconomic Assessment Group (2010) estimate that a 1 percentage point rise in capital requirements would have a peak effect on annual GDP of between -0.05% and -0.35%. Conditional on the rest of the calibration, targeting the mid-range of those estimates implies a value for ψ of approximately 0.05.[21]

4.2 Simulation Methodology

Our simulation generates a prolonged rise in the real price of housing followed by a sharp fall in the absence of macro-prudential policy. Following the existing literature, we consider a boom-bust in house prices driven by either credit supply shocks (Mian and Sufi, 2011) or housing demand shocks (Adelino et al., 2016).

In practice, both supply and demand shocks are likely to have contributed to the gyrations of house prices during the first decade of the 2000s. Determining the relative importance of these two factors is beyond the scope of this paper. Instead, we study the optimal setting of monetary and macro-prudential policy in these two scenarios, and in particular the implications of actively using an LTV instrument for inflation targeting.

We use a piecewise-linear solution method to account for the possibility that (i) the zero lower bound on the short-term nominal interest rate becomes binding and/or (ii) the borrowers' collateral constraint (2) becomes slack.[22] This approach accounts for the possibility that the occasionally binding constraints may apply in future periods, but not for the risk that future shocks may cause the constraints to bind. Therefore, our method, which is similar to the OccBin toolkit developed by Guerrieri and Iacoviello (2015), ignores the skewness in the expected distribution of future outcomes (e.g. of output and inflation) arising from the possibility of being constrained in future. Appendix E.2 contains a detailed description of the approach.

[20] We use this period for our calibration of the steady state as we aim to target the average debt-to-GDP ratio that prevailed before the boom-bust episode of first decade of the 2000s.

[21] We reach this conclusion through a partial equilibrium thought experiment based on a linearized version of the borrower's Euler equation, detailed in Appendix D.7.

[22] In our model, a binding zero lower bound on the short-term nominal interest rate also implies a zero nominal rate of return to savings and a negative return to borrowing. Theoretically, a zero lower bound on saving rates arises from the existence of an unmodeled zero-interest-bearing alternative saving instrument (e.g. cash). In practice, the evidence on negative interest rates (e.g. Eisenschmidt and Smets, 2018) suggests that deposit rates feature a hard floor at zero, although anecdotally some banks may have introduced new fees on deposit accounts when official interest rates became negative.

4.3 Alternative Policy Configurations

In the two scenarios just described, we study the macroeconomic effects of a housing boom and bust under two alternative assumptions about the conduct of monetary and macro-prudential policies.

Our alternative assumptions rely on a decomposition of the welfare-based loss function which attributes the first part of (9) to the monetary policymaker and the second part to the macro-prudential policymaker

$$\mathcal{L}_0 \propto \underbrace{\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(x_t^2 + \lambda_\pi \pi_t^2 \right)}_{\equiv \mathcal{L}_0^{FIT}} + \underbrace{\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\lambda_c \tilde{c}_t^2 + \lambda_h \tilde{h}_t^2 \right)}_{\equiv \mathcal{L}_0^{MaP}} . \tag{10}$$

The loss function $\mathcal{L}_0^{FIT}$ aims to capture the objectives encoded in the 'flexible inflation targeting' mandates of many central banks during the pre-crisis period. The loss function $\mathcal{L}_0^{MaP}$ captures the 'macro-prudential' considerations arising in the model. Our decomposition relies on the observation that in the limiting case in which the share of borrowers collapses to zero ($\xi \to 0$), the model collapses to a standard New Keynesian model in which the only friction arises from price stickiness and the only policy instrument is the short-term nominal interest rate. Therefore, the existence of financial frictions in our model generates both additional terms in the loss function and the potential for additional instruments with which policy may seek to correct the associated gaps.[23]

In the baseline configuration, we assume that macro-prudential policy is idle ($\theta_t = 0,\ \forall t$). The monetary authority sets the nominal interest rate i_t to minimize $\mathcal{L}_0^{FIT}$, subject to the set of equilibrium conditions, and the additional non-negativity restrictions on the interest rate and on the multiplier associated with the collateral constraint. We interpret this arrangement as a simple characterization of the pre-crisis consensus, whereby central banks controlled the short-term nominal interest rate to pursue stabilization objectives defined in terms of inflation and aggregate real activity. As such, this configuration represents a natural benchmark against which to compare the effect of introducing macro-prudential policy.

The second configuration is a policy counterfactual that seeks to determine what would have happened had macro-prudential instruments (in particular LTV limits) been in place and actively used during the boom-bust episode. In this case, while the monetary policymaker continues to act as in the baseline, the macro-prudential policymaker sets the LTV ratio θ_t to minimize $\mathcal{L}_0^{MaP}$, subject to the equilibrium conditions and the non-negativity constraints.

From a practical standpoint, the motivation for studying these two policy configurations is the nature of central banks' remits in the past and how the introduction of new macro-prudential policy instruments may affect those remits in the future. Theoretically, we embrace the approach,

[23]In spite of this rationale, the decomposition of the loss function we employ remains arbitrary. The allocation of objectives to different policymakers can take several forms. For example, De Paoli and Paustian (2017) examine a case in which the monetary policymaker and the macro-prudential policymaker share a concern for output stabilization.

advocated in the context of monetary policy by Svensson (1999), that the policy objective is to minimize a loss function subject to a set of constraints arising from the private sector's behavior.

We assume that, within each period, the macro-prudential policymaker sets θ_t before the monetary policymaker sets the policy rate (see also De Paoli and Paustian, 2017). This 'leader-follower' assumption preserves the behavior of the monetary policymaker under the baseline policy, allowing us to trace out the effects of macro-prudential policy actions on the ability of the central bank to stabilize output and inflation.[24]

We further impose the requirement that each policy must be time-consistent. Policymakers are therefore unable to make promises about future actions in order to improve stabilization outcomes today.[25] One motivation for studying time-consistent policies is to limit the power of monetary policy at the zero lower bound. Optimal commitment policies can be very effective at mitigating the effects of the zero bound in standard New Keynesian models (see, for example, Eggertsson and Woodford, 2003), although several recent contributions question their empirical relevance (e.g. Del Negro et al., 2012). Our setting completely abstracts from these commitments and maximizes the potential scope for macro-prudential policies to improve outcomes when used alongside monetary policy.[26] The next sections present the results under the two policy configurations for each scenario.

4.4 Credit Supply Shocks

To generate a house price boom driven by credit supply shocks, we pick a sequence of innovations ϵ_t^κ to the process of the leverage constraint κ_t that reduce the spread between borrowing and deposit rates by 250 annualized basis points over five years (Justiniano et al., 2017). We then impose a large contractionary innovation that makes spreads spike up and house prices collapse. After the crisis period, the leverage constraint slowly reverts back toward its steady state value.

4.4.1 Baseline

Figure 2 shows the simulation under the baseline flexible inflation targeting policy configuration with idle macro-prudential policy. During the boom period, house prices (panel a) increase

[24] Our timing assumption is also congruent with actual institutional frameworks, whereby monetary policy decisions occur more regularly than macro-prudential policy actions. For example, in the UK, the Monetary Policy Committee meets twice as often as the Financial Policy Committee, thus typically 'inheriting' its macro-prudential policy settings.

[25] Formally, we solve for a Markov-perfect policy equilibrium. In each period, policymakers act as a Stackelberg leader with respect to private agents and future policymakers. Current policymakers take the decision rules of future policymakers as given. In equilibrium, the decisions of policymakers in the current period satisfy the decision rule followed by future policymakers. See Appendix E for technical details.

[26] Our analysis therefore contributes to an emerging literature studying monetary and macro-prudential policies under discretion. Bianchi and Mendoza (2018) argue that the nature of financial frictions generates an inherent time inconsistency problem for macro-prudential policymakers. Using a model similar to ours, Laureys and Meeks (2018) demonstrate the striking result that discretionary policies can generate better outcomes than a class of simple macro-prudential policy rules to which policymakers commit that have been widely studied in the existing literature.

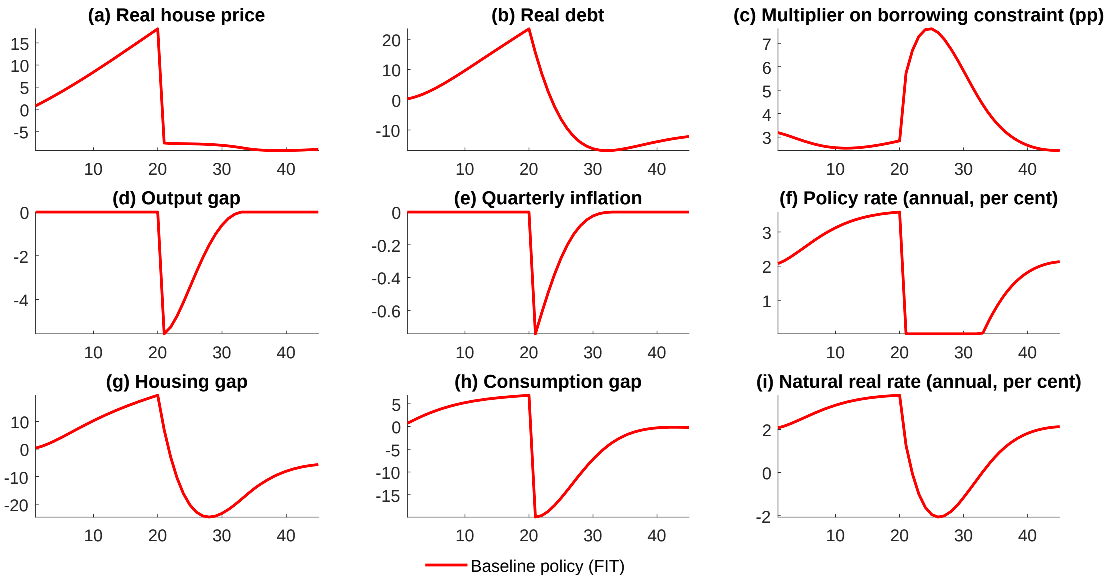

Figure 2: Credit supply scenario under 'flexible inflation targeting'.

by slightly more than 15 percentage points while debt (panel b) rises by more than 20 percent relative to its steady state value. In spite of these financial imbalances, the central bank manages to keep output and inflation on target (panels d and e, respectively) with a relatively moderate increase of the nominal interest rate by about 150 basis points (panel f).

In this phase, in effect the monetary policymaker implements a standard flexible inflation targeting criterion

$$x_t + \gamma\lambda_\pi \pi_t = 0, \tag{11}$$

which corresponds to the optimal discretionary monetary policy in the baseline New Keynesian model (Clarida et al., 1999; Woodford, 2003). Despite the additional richness of our framework, this static optimality condition remains valid because the policymaker's current decisions have no effect on the ability of future policymakers to set policy optimally.[27]

While aggregate variables are stable, the boom features notable distributional consequences. The reduction in spreads particularly benefits borrowers, who can now access credit at a cheaper price. Consequently, both the consumption gap (panel h) and especially the housing gap (panel g) widen significantly.

The crisis is the product of a sudden reversal in spreads.[28] The experiment captures the broad contours of the Great Recession. As house prices collapse, borrowers start deleveraging. The persistence in the collateral constraint slows down the process, which lasts for about two

[27] In the absence of the ZLB, the values of the endogenous state variables do not constrain the ability of future policymakers to stabilize the output gap and inflation by an appropriate choice of the nominal interest rate.

[28] The increase of more than 200 basis points is consistent with the reaction of a number of credit spreads following Lehman's bankruptcy.

years, in line with the decoupling between house prices and mortgage debt observed in the data. As in Eggertsson and Krugman (2012) and Guerrieri and Lorenzoni (2017), the deleveraging shock pushes the nominal interest rate all the way to the ZLB for a prolonged period (about three years). During this time, the economy experiences a severe recession, with output falling by almost six percentage points below trend and inflation being below target by more than two percentage points on an annualized basis. Once the downturn is over, the process of monetary policy renormalization takes until the end of the simulation horizon.

We can gain further intuition about the transmission of the credit supply shock and the optimal monetary policy response through the lens of the flexible-price—or "natural"—real interest rate. Appendix D shows that, up to a first order approximation, we can write the Euler equation for savers in terms of the output gap and the consumption gap

$$y_t - \xi \tilde{c}_t = -\sigma^{-1}(i_t - \mathbb{E}_t \pi_{t+1}) + \mathbb{E}_t(y_{t+1} - \xi \tilde{c}_{t+1}). \tag{12}$$

If prices were flexible in the current period and in the future, the Phillips curve would imply a zero output gap in every period ($y_t = 0, \ \forall t$).[29] Equation (12) then gives

$$r_t^n \equiv \sigma \xi \left(\tilde{c}_t - \mathbb{E}_t \tilde{c}_{t+1} \right), \tag{13}$$

which shows that the equilibrium real interest rate r_t^n is inversely proportional to the expected growth rate of the consumption gap.

Plugging (13) back into (12), we obtain a standard aggregate demand curve

$$y_t = \mathbb{E}_t y_{t+1} - \sigma^{-1} \left(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n \right). \tag{14}$$

By definition, if the actual real interest rate $r_t \equiv i_t - \mathbb{E}_t \pi_{t+1}$ equals the natural real interest rate r_t^n in every period, the output gap is always zero. Compared to the standard New Keynesian model, in our framework the equilibrium real interest rate is endogenous, and, in particular, through the consumption gap, is a function of the stock of debt (Eggertsson and Krugman, 2012; Benigno et al., 2016).

During the boom period, the reduction in spreads facilitates the uptake in debt by borrowers so that the consumption gap opens up. As the natural real interest rate rises, the central bank finds optimal to track its increase with the policy rate. Conversely, the collapse of the consumption gap associated with the deleveraging process pushes down the natural interest rate. The problem is that, during the recession, the natural rate turns negative. The ZLB prevents the central bank from being able to continue tracking the natural rate, and hence to fully stabilize

[29] Our definition is conditional on the actual value of the state variables, and hence differs from the standard approach (e.g. Woodford, 2003), which derives a parallel equilibrium in which prices have been, are, and will always be flexible. The notion that we use is perhaps closer to the approach often favored by policymakers in practice. For example, Ferguson (2004) defines the equilibrium real rate as "the level of the real federal funds rate that, if allowed to prevail for a couple of years, would place economic activity at its potential."

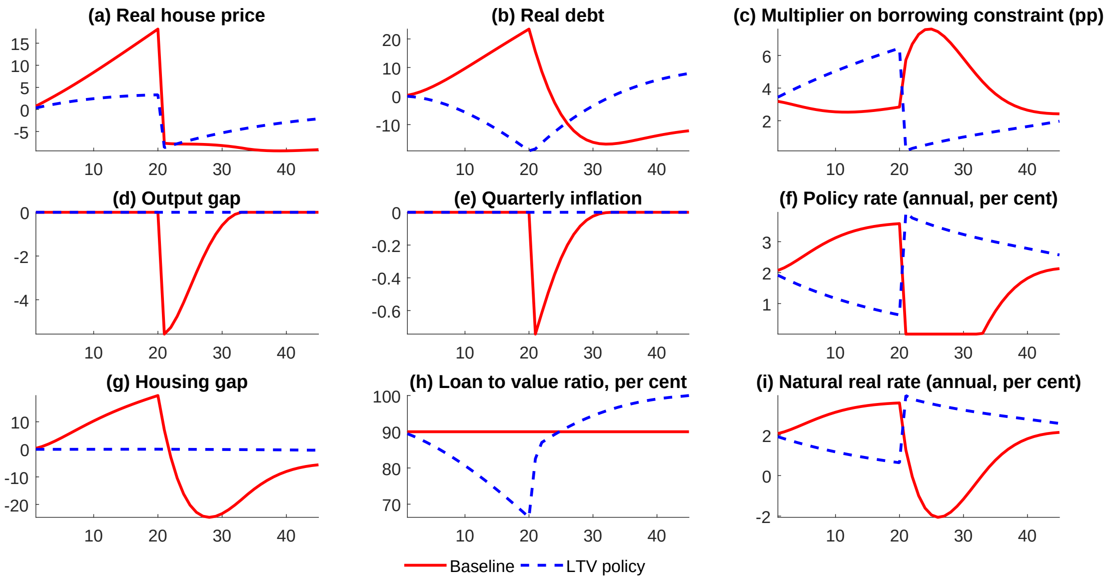

Figure 3: Credit supply scenario under 'LTV policy' (dashed blue line) and flexible inflation targeting (solid red line).

inflation and output (Eggertsson and Woodford, 2003).[30]

Lastly, we note that the collateral constraint is always binding throughout the simulation horizon, as the Lagrange multiplier (panel c) remains positive. Nevertheless, when the crisis hits, the multiplier spikes up so that the shadow value of an additional unit of debt increases significantly. As Eggertsson and Krugman (2012) and Guerrieri and Iacoviello (2017) discuss extensively, the tightening of the collateral constraint contributes to amplify the impact of the shock. The welfare-based loss (9) suggests that measuring the severity of the crisis only through the evolution of inflation and the output gap, as the flexible inflation targeting scenario does, understates the full costs of the recession.

4.4.2 LTV Policy

Next, Figure 3 compares the equilibrium outcomes in the credit supply driven boom-bust scenario under the baseline regime (solid red line) with the case in which the macro-prudential policymaker optimally sets the LTV limit θ_t to minimize the loss function $\mathcal{L}_0^{MaP}$ (dashed blue line).

The striking result is that the introduction of the LTV policy allows the economy to escape the recession. The macro-prudential policymaker aggressively tightens the LTV limit during the boom. As a consequence, house prices rise by less than 5 percentage points and debt actually declines, while its shadow value increases significantly. The macro-prudential policymaker successfully closes the housing gap. The consumption gap, however, becomes negative. The

[30]Ignoring the ZLB would imply setting a negative nominal interest rate—up to -1% in annual terms—for several quarters (see Appendix E.2.3 for details). Not surprisingly, in this case, the central bank would be able to fully stabilize output and inflation also during the deleveraging process.

inability to borrow in response to the decline in spreads actually reduces relative consumption. Since the consumption gap is proportional to the natural real interest rate, optimal monetary policy calls for a reduction of the nominal interest rate during the boom period. The optimal joint configuration, therefore, is a 'loose-money/tight-credit' policy.

When credit spreads reverse, so too does the policy stance.[31] The macro-prudential policy-maker quickly relaxes the LTV limit, which approaches 100% in the long run.[32] House prices start to recover quickly because borrowers can obtain more loans given the value of their collateral. The consumption gap becomes positive, while the housing gap remains broadly balanced. Given the expansionary boost coming from macro-prudential policy, the natural real interest rate increases. Therefore, the central bank raises the interest rate in order to avoid overheating the economy. In this phase, optimal policy features a persistent period of 'tight-money/loose-credit.'

The overall message of this experiment is that the optimal setting of the LTV limit is indeed prudential. By reducing leverage in good times, borrowers can actually use debt to cushion the negative impact of the tightening of credit spreads.

4.5 Housing Demand Shocks

The second scenario that we consider is a boom-bust in house prices driven by housing demand shocks u_t^h. The boom phase captures the idea that both borrowers and savers want to buy larger houses or upgrade their existing ones. The bust comes when, suddenly, the appetite for housing disappears.[33] For the sake of comparability, we discipline this exercise by finding a sequence of innovations to housing preference ϵ_t^h such that the dynamics of house prices under the baseline policy configuration match the ones obtained with credit supply shocks.

4.5.1 Baseline

Figure 4 compares the results in the absence of macro-prudential policy across scenarios. In all panels, the solid red lines depict variables in the credit supply scenario, while dashed blue lines represent variables in the housing demand scenario. The figure clearly highlights that the dynamics of both aggregate and distributional variables under the two scenarios are qualitatively very similar.

The main differences between the two scenarios are quantitative. The multiplier on the borrowing constraint exhibits more pronounced fluctuations with housing demand shocks. During the boom, the multiplier falls more, and monetary policy is slightly tighter than in the credit supply scenario. Conversely, at the beginning of the crisis, the multiplier increases more sharply

[31] In practice, without the boom period, the financial crisis may have not occurred and spreads may have not spiked. We nevertheless find instructive to discuss the optimal policy configuration in response to an increase in credit spreads.

[32] In the simulations, we impose that the LTV limit cannot exceed 100%, in the same way as we deal with the ZLB on the nominal interest rate and the Lagrange multiplier on the collateral constraint.

[33] A more sophisticated, but in many respects equivalent, approach to this scenario would be to introduce a non-fundamental bubble component to house prices.

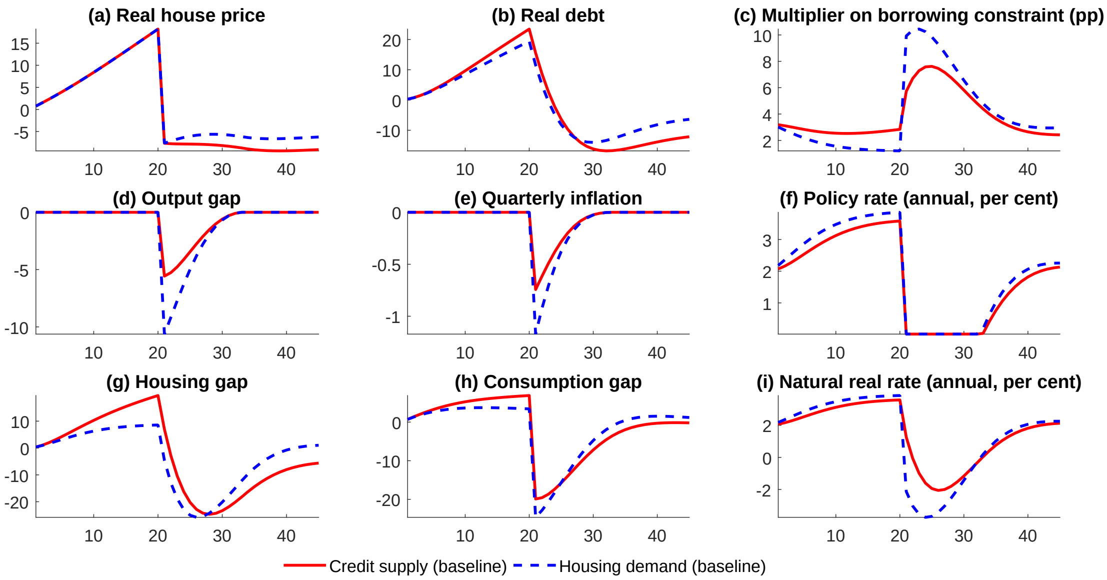

Figure 4: Housing demand scenario (dashed blue line) versus credit supply scenario (solid red line) under flexible inflation targeting.

than in the credit supply scenario. A deeper fall of the natural real interest rate, with the nominal interest rate constrained by the ZLB, roughly doubles the magnitude of the recession in terms output and inflation.

As for the distributional variables, the most notable difference is that the volatility of the risk-sharing gaps is lower in this scenario because both borrowers and savers experience the same housing demand shocks. Nevertheless, the gaps continue to move pro-cyclically because the marginal propensity to consume goods and housing services is higher for borrowers than for savers.

4.5.2 LTV Policy

Figure 5 introduces an active LTV policy (dashed blue line) over and above the baseline policy of flexible inflation targeting (solid red line) in the housing demand scenario.

As in the credit supply shock case, the introduction of macro-prudential policy is very powerful, and the economy does not enter a recession at the time of the negative housing demand shock. The macro-prudential policymaker tightens LTV requirements during the boom (though less so than in the credit supply scenario). The policy does not affect house prices directly. A tighter (i.e. more positive) multiplier on the collateral constraint compensates for the lower available LTV. The increase in debt, however, is much less pronounced, being roughly halved. While the housing gap remains stable, the consumption gap increases, although less than without macro-prudential policy. Optimal monetary policy calls for a mild increase in the nominal interest rate to stabilize both inflation and the output gap fully.

In the bust, these dynamics reverse. The aggressive loosening of LTVs at the time of the

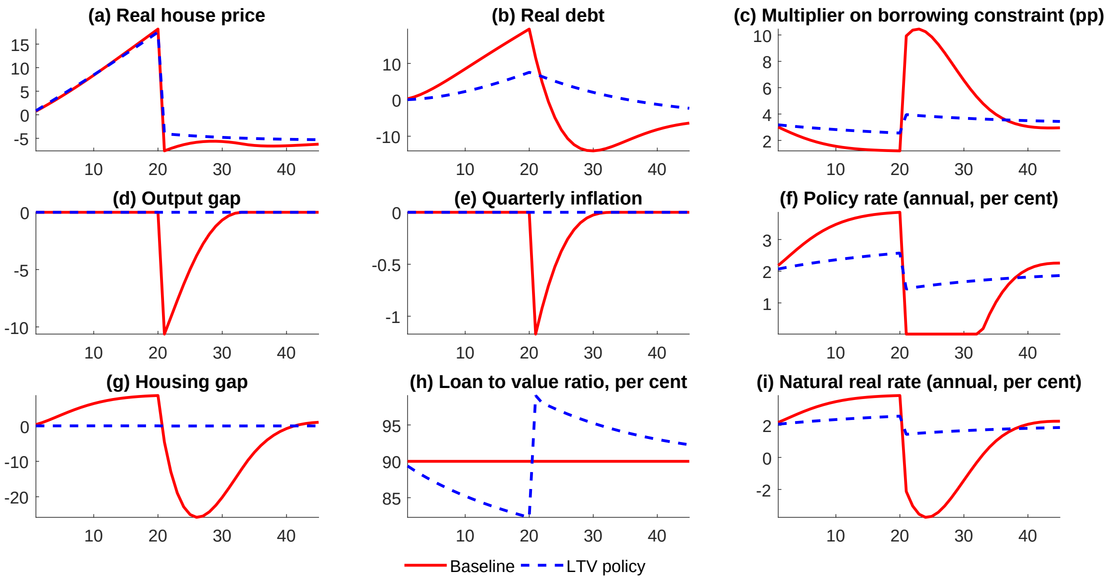

Figure 5: Housing demand scenario under LTV policy (dashed blue line) and flexible inflation targeting (solid red line).

crisis mitigates the deleveraging cycle. Not only has debt increased by less during the boom, but also its decline is extremely smooth. The housing gap remains closed and the consumption gap exhibits a small fall. Therefore, the central bank manages to keep inflation and output on target with a small, albeit persistent, cut in interest rates.

Once again, the optimal setting of the LTV limit is strongly countercyclical. While the policy barely affects house prices, the prudential nature follows from the absence of a large debt build up, which in turn avoids a protracted deleveraging period and the occurrence of a liquidity trap.

The pro-cyclicality of the nominal interest rate is the main difference in terms of implications of LTV policy for monetary policy comparing the two scenarios (Figure 3 vs. Figure 5). With credit supply shocks, the tightening of the LTV limit actually depresses the consumption gap and hence the natural real interest rate. During the boom, optimal monetary policy calls for a sequence of cuts in the nominal interest rate. Conversely, with housing demand shocks, the increase of the LTV limit only marginally affects the consumption gap. The natural real interest rate remains above its steady state value and therefore optimal monetary policy requires a sequence of interest rate increases. The complementarity or substitutability of monetary and macro-prudential policy is a theme that will carry through in the next section on exiting a liquidity trap.

4.6 Macro-Prudential Policy During the Liftoff Period

In most countries, the introduction of macro-prudential policy frameworks has occurred post-crisis, during a period of nominal interest rates at the ZLB. As economies recover, the monetary policy normalization phase (the ‘liftoff’ period) is therefore likely to be the first time we see

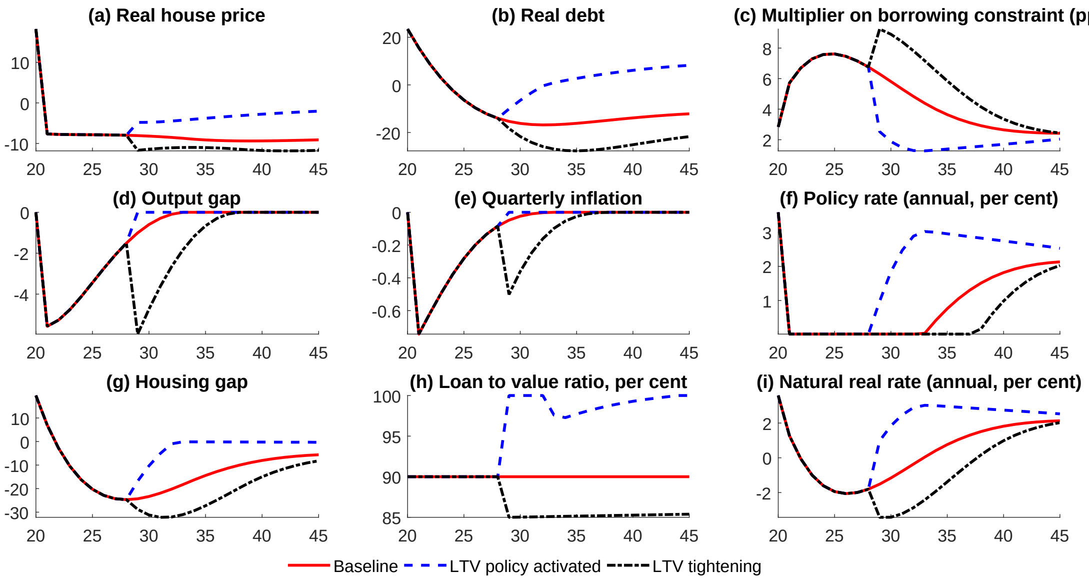

Figure 6: Exit from the credit supply scenario under baseline FIT policy (solid red), delayed introduction of LTV policy (dashed blue) and delayed LTV tightening (black dot-dashed).

macro-prudential tools in action. The effects of alternative monetary/macro-prudential policy configurations during a recovery from a recession are therefore of topical relevance. In this section, we use our model to shed light on the state-contingent nature of the interaction between the two forms of policy.

The following experiments assume the economy enters recession at time t_0 following a house price boom and bust, during which macro-prudential policy has been inactive. We then consider two scenarios. In the first, at $\tau > t_0$, we introduce LTV requirements optimally to minimize $\mathcal{L}_{\tau}^{MaP}$. In the second scenario, we consider an ad-hoc persistent tightening of LTV requirements in period τ.[34]

While we intend our experiments to be mainly illustrative, we calibrate the size of the LTV tightening to the Canadian experience. To enhance the empirical relevance, we also assume macro-prudential policy becomes active eight quarters after the housing bust ($\tau = 8$).[35]

Figure 6 shows the credit supply scenario. The solid red lines show the dynamics of the economy under the baseline flexible inflation targeting framework discussed above. In contrast, the dashed blue lines display the responses of the model economy when the optimal LTV policy

[34] In particular, we assume the LTV requirement follows a first-order autoregressive process with persistence parameter $\rho_\theta = 0.995$.

[35] Beginning in late 2008, the Canadian financial regulation authority decreased the maximum LTV several times (Shim et al., 2013). Each time, the tightening was 5 percentage point, with no reference that the policy change would be temporary. Despite only considering one round of tightening, our calibration may nevertheless overstate the actual consequence of the regulatory change since (a) the measures applied only to mortgages with a government guarantee; and (b) some of the policy changes applied only to new or refinanced mortgages, rather than the entire stock.

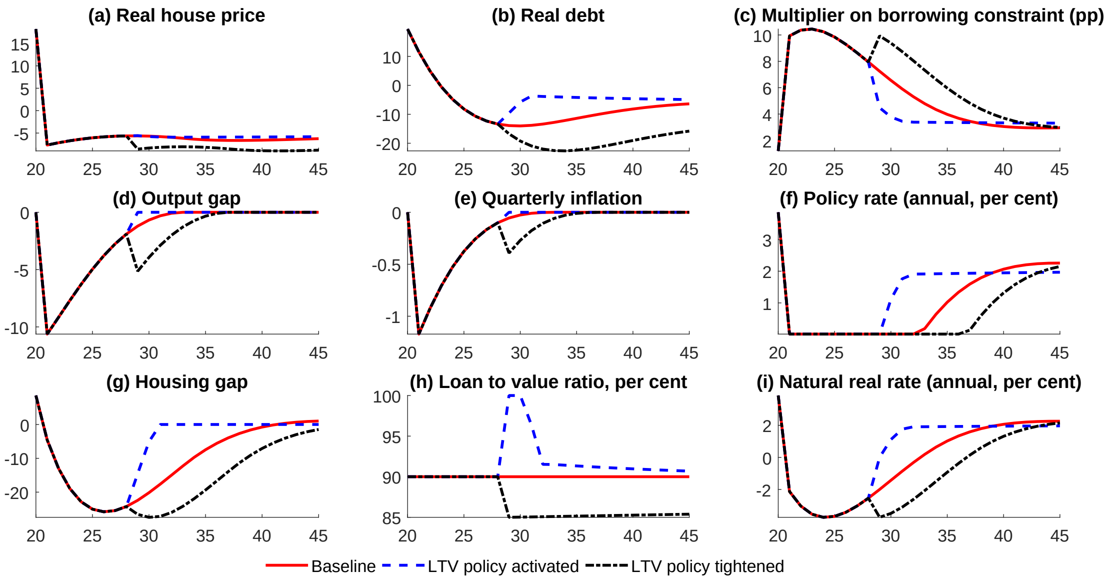

Figure 7: Exit from the housing demand shock under baseline FIT policy (solid red), delayed introduction of LTV policy (dashed blue) and delayed LTV tightening (black dot-dashed).

becomes active eight quarters after the bust. In this case, the macro-prudential authority immediately loosens the LTV requirement, which quickly hits its upper bound. The multiplier on the borrowing constraints falls sharply, and deleveraging ceases. As real house prices recover modestly, the housing gap begins to return to zero. In terms of monetary policy, the relaxation of the LTV limit means a faster renormalization of the natural real interest rate, which facilitates an early nominal interest rate liftoff. Compared to the baseline, the output gap closes more rapidly, while inflation returns to its target. Monetary and macro-prudential policy work in concert to achieve a better macroeconomic outcome.

Qualitatively, the dynamics are the opposite in the case of an LTV tightening. A lower LTV limit exacerbates the deleveraging process, and further depresses the natural real interest rate below its baseline path via the effect on the consumption gap. The distributional effects of the policy spill over onto the macro-economy, creating a double-dip recession and an additional bout of disinflation that force the monetary authority to delay the exit from the ZLB.

Figure 7 repeats the same experiment for the housing demand scenario. The qualitative features of the responses are the same. An adoption of optimal macro-prudential policy speeds up the recovery relative to the baseline case, while an ad-hoc tightening creates a second recession and prolongs the ZLB period. Relative to the credit supply scenario, the optimal LTV policy takes a somewhat different path, with an initial sharp loosening followed by a prolonged period of more modest accommodation. Equally, the recessionary effects of the LTV tightening are somewhat smaller relative to their peak compared to the credit supply scenario, partly because of the different nature of the shock. Nonetheless, the main conclusion from this exercise is robust to the underlying cause of the boom-bust scenario. The introduction of an LTV policy during a recovery requires a relaxation of the LTV limit to facilitate the exit from a liquidity trap. Conversely, a persistent tightening lengthens the duration of the ZLB period.

5 Conclusion

The presence of a strongly countercyclical LTV limit avoids a liquidity trap caused by a debt-deleveraging shock. Optimal macro-prudential policy prevents excessive accumulation of debt during a house price boom and allows for its gradual reduction if and when the boom reverses course. The central bank supports the macro-prudential authority by cutting interest rates in the case of a credit supply driven expansion. Conversely, in a demand-driven scenario, optimal monetary policy requires a series of modest hikes. Therefore, identifying the cause of house price dynamics is important for the appropriate conduct of monetary policy in the presence of active macro-prudential policy.

If macro-prudential policy tools become available only after a crisis has occurred, an aggressive relaxation of LTVs still speeds up the recovery and shortens the duration of a liquidity trap. On the other hand, a tightening of financial conditions during a recovery, possibly for reasons other than macroeconomic stabilization, may delay the liftoff of interest rates from the ZLB and even generate a new recession.

Our results demonstrate the power of active borrower-based policies to reduce the risk of future crises due to excessive private leverage arising from the housing market. To the extent that the marginal buyer in the housing market faces a binding borrowing constraint, this conclusion should be robust to the presence of additional heterogeneity in the population and to other exogenous drivers of house prices.[36]

Of course, LTV limits are only one of the many tools available to macro-prudential authorities. In this paper, we have focused on their implications for monetary policy. Going forward, an interesting aspect to address would be the extent to which the presence of multiple instruments reduces the burden of adjustment on LTVs. A second avenue to explore would be to estimate the consequences of the actual use of macro-prudential tools for the stance of monetary policy through their effects on the natural rate of interest. We leave the study of these questions for future research.

[36] An exception would be the case of unconstrained buyers dominating the market, which may be relevant for certain areas of big cities (such as, for example, London or New York) where cash purchases can drive house prices.

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Appendix

A Private Sector Optimality Conditions and Aggregation

This appendix reports the optimality conditions of the private sector (savers, borrowers, and intermediate goods producers) and the details of the aggregation.

A.1 Savers

Starting with savers, the first order condition for deposits is

$$\mathbb{E}_t \left(\mathcal{M}^s_{t+1} \frac{R^d_t}{\Pi_{t+1}} \right) = 1,$$

where $\Pi_t \equiv P_t/P_{t-1}$ is the gross inflation rate and the stochastic discount factor for savers $\mathcal{M}^s_{t+1}$ is

$$\mathcal{M}^s_{t+1} \equiv \beta_s e^{-z(C^s_{t+1}(i) - C^s_t(i))}.$$

The corresponding condition for bank equity is

$$\mathbb{E}_t \left(\mathcal{M}^s_{t+1} \frac{R^e_t}{\Pi_{t+1}} \right) = 1 + \Psi \left[\frac{E^s_t(i)}{\tilde{\kappa}\xi D^b_t/(1-\xi)} - 1 \right].$$

Combining the two Euler equation, we can obtain the no-arbitrage condition between equity and deposits

$$\mathbb{E}_t \left(\mathcal{M}^s_{t+1} \frac{R^e_t - R^d_t}{\Pi_{t+1}} \right) = \Psi \left[\frac{E^s_t(i)}{\tilde{\kappa}\xi D^b_t/(1-\xi)} - 1 \right].$$

After rearranging, the first order condition for housing services can be written as

$$(1+\tau^h)\frac{Q_t}{P_t} = \frac{\chi^s_H e^{u^h_t} H^s_t(i)^{-\sigma_h}}{z e^{-zC^s_t(i)}} + \mathbb{E}_t \left(\mathcal{M}^s_{t+1} \frac{Q_{t+1}}{P_{t+1}} \right).$$

The labor supply condition is

$$\frac{W^s_t}{P_t} = \frac{\chi^s_L L^s_t(i)^{\varphi}}{z e^{-zC^s_t(i)}}.$$

The budget constraint at equality completes the list of first order conditions for savers.

A.2 Borrowers

Moving on to borrowers, we attach a Lagrange multiplier normalized by the real marginal utility of consumption $(\tilde{\mu}_t(i) z e^{-zC^b_t(i)}/P_t)$ to the collateral constraint. The first order condition for borrowed funds is

$$\mathbb{E}_t \left(\mathcal{M}^b_{t+1} \frac{R^b_t - \gamma_d \tilde{\mu}_{t+1}(i)}{\Pi_{t+1}} \right) = 1 - \tilde{\mu}_t(i),$$

where the stochastic discount factor for borrowers $\mathcal{M}^b_{t+1}$ is

$$\mathcal{M}^b_{t+1} \equiv \beta_b e^{-z(C^b_{t+1}(i)-C^b_t(i))}.$$

The first order condition for housing demand is

$$[1-(1-\gamma_d)\Theta_t\tilde{\mu}_t(i)]\frac{Q_t}{P_t} = \frac{\chi^b_H e^{u^h_t} H^b_t(i)^{-\sigma_h}}{ze^{-zC^b_t(i)}} + \mathbb{E}_t\left(\mathcal{M}^b_{t+1}\frac{Q_{t+1}}{P_{t+1}}\right).$$

The labor supply condition is

$$\frac{W^b_t}{P_t} = \frac{\chi^b_L L^b_t(i)^{\varphi}}{ze^{-zC^s_t(i)}}.$$

The equilibrium conditions for borrower households include the complementary slackness condition

$$\tilde{\mu}_t(i)[D^b_t - \gamma_d D^b_{t-1}(i) - (1-\gamma_d)\,\Theta_t Q_t H^b_t(i)] = 0.$$

The budget constraint at equality completes the list of first order conditions for borrowers.

A.3 Firms

The text reports the optimality condition for banks, which is the result of perfect competition in the financial sector.

To derive the expression for the marginal cost, we solve the dual problem

$$\min_{L^b_t, L^s_t} \frac{W^b_t}{P_t}L^b_t(f) + \frac{W^s_t}{P_t}L^s_t(f),$$

subject to the technological constraint given by the production function. Let $MC_t(f)$ be the multiplier on the constraint (the real marginal cost). The first order conditions for the two types of labor are

$$\begin{aligned} \frac{W^b_t}{P_t} &= \xi MC_t(f) L^b_t(f))^{\xi-1} L^s_t(f)^{1-\xi} &&= \xi MC_t(f)\frac{Y_t(f)}{L^b_t(f)} \\ \frac{W^s_t}{P_t} &= (1-\xi) MC_t(f) L^b_t(f))^{\xi} L^s_t(f)^{-\xi} &&= (1-\xi) MC_t(f)\frac{Y_t(f)}{L^s_t(f)}. \end{aligned}$$

Taking the ratio between the two first order conditions above shows that at the optimum all firms choose the same proportion of labor of the two types. As a consequence, the marginal cost is independent of firm-specific characteristics ($MC_t(f) = MC_t$). Furthermore, if we take a geometric average of the two first order conditions above, with weights ξ and $1-\xi$, respectively, we obtain the expression for the marginal cost

$$MC_t = \frac{W_t/P_t}{\xi^{\xi}(1-\xi)^{1-\xi}},$$

where the expression for the aggregate wage index is reported in the text.

Intermediate goods producers set prices on a staggered basis. Their optimality condition can be summarised by a non-linear Phillips curve

$$\frac{X_{1t}}{X_{2t}} = \left(\frac{1 - \lambda \Pi_t^{\varepsilon - 1}}{1 - \lambda} \right)^{\frac{1}{-\varepsilon}},$$

where X_{1t} represents the present discounted value of real costs

$$X_{1t} = \frac{\varepsilon}{\varepsilon - 1} z e^{-zC_{t+1}} Y_t MC_t + \beta \lambda \mathbb{E}_t (\Pi_t^{\varepsilon} X_{1t+1}),$$

and X_{2t} represents the present discounted value of real revenues

$$X_{2t} = (1 + \tau^p) z e^{-zC_{t+1}} Y_t + \beta \lambda \mathbb{E}_t (\Pi_t^{\varepsilon - 1} X_{2t+1}),$$

where $\beta \equiv \beta_b^{\xi} \; \beta_s^{1-\xi}$.

A.4 Aggregation

To aggregate within types, we simply integrate over the measure of households in each group. Consumption of savers and borrowers is

$$\int_0^{1-\xi} C_t^s(i) di = (1 - \xi) C_t^s \qquad \text{and} \qquad \int_{1-\xi}^{1} C_t^b(i) di = \xi C_t^b,$$

while housing demand is

$$\int_0^{1-\xi} H_t^s(i) di = (1 - \xi) H_t^s \qquad \text{and} \qquad \int_{1-\xi}^{1} H_t^b(i) di = \xi H_t^b.$$

In the credit market, total bank loans must equal total household borrowing

$$\int_0^1 D_t^b(k) dk = \int_{1-\xi}^{1} D_t^b(i) di = \xi D_t^b.$$

Similarly, for deposits and equity holdings we have

$$\int_0^1 D_t^s(k) dk = \int_0^{1-\xi} D_t^s(i) di = (1 - \xi) D_t^s \qquad \text{and} \qquad \int_0^1 E_t^s(k) dk = \int_0^{1-\xi} E_t^s(i) di = (1 - \xi) E_t^s.$$

Using these expressions, we obtain the aggregate balance sheet for the financial sector and the economy-wide capital constraint reported in the text.

Labor market clearing requires

$$\int_0^1 L_t^s(f) df = \int_0^{1-\xi} L_t^s(i) di = (1 - \xi) L_t^s \qquad \text{and} \qquad \int_0^1 L_t^b(f) df = \int_{1-\xi}^{1} L_t^b(i) di = \xi L_t^b.$$

Aggregating production across firms yields

$$\int_0^1 Y_t(f)df = \int_0^1 L_t^b(f)^{\xi} L_t^s(f)^{1-\xi} df. \tag{15}$$

As discussed in the previous section, the ratio of hours worked of different types is independent of firm-specific characteristics. Therefore, using the labor market equilibrium conditions, we can rewrite the right-hand side of the previous expression as

$$\int_0^1 L_t^b(f)^{\xi} L_t^s(f)^{1-\xi} df = (\xi L_t^b)^{\xi}((1-\xi)L_t^s)^{1-\xi} = \xi^{\xi}(1-\xi)^{1-\xi} L_t,$$

where aggregate labor is

$$L_t \equiv (L_t^b)^{\xi}(L_t^s)^{1-\xi}. \tag{16}$$

Using the demand for firm f's product, the left-hand side of (15) can also be rewritten in terms of aggregate variables only as

$$\int_0^1 Y_t(f)df = \Delta_t Y_t,$$

where Δ_t is an index of price dispersion, defined as

$$\Delta_t \equiv \int_0^1 \left[\frac{P_t(f)}{P_t}\right]^{-\varepsilon} df.$$

Given the definition of the price index and the assumption of staggered price setting, the index of price dispersion evolves according to

$$\Delta_t = \lambda \Delta_{t-1} \Pi_t^{\varepsilon} + (1-\lambda)\left(\frac{1-\lambda \Pi_t^{\varepsilon-1}}{1-\lambda}\right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

Therefore, in the aggregate, production is described by

$$\Delta_t Y_t = \xi^{\xi}(1-\xi)^{1-\xi} L_t.$$

The last step of the aggregation is the derivation of the law of motion of debt. To obtain this equation, we start from the flow budget constraint of a generic borrower

$$P_t C_t^b(i) - D_t^b(i) + Q_t H_t^b(i) = W_t^b L_t^b(i) - R_{t-1}^b D_{t-1}^b(i) + Q_t H_{t-1}^b(i) + \Omega_t^b(i) - T_t^b(i).$$

We assume that each household $i \in [0,1]$ receives an equal share of aggregate profit

$$\Omega_t^j(i) = P_t Y_t - W_t L_t,$$

for $j = \{b, s\}$. From the first order conditions of intermediate goods producers we have

$$W_t^b L_t^b(f) = \xi MC_t Y_t(f) \qquad \text{and} \qquad W_t^s L_t^s(f) = (1-\xi) MC_t Y_t(f).$$

Integrating over firms, we obtain

$$W_t^b L_t^b = W_t^s L_t^s = W_t L_t = MC_t \Delta_t Y_t,$$

where we have used the labor market equilibrium conditions, the definition of the wage and labor indexes, and the definition of the price dispersion index.

Aggregating the borrowers' individual budget constraints, we can then write

$$C_t^b - \frac{D_t^b}{P_t} + \frac{Q_t}{P_t} H_t^b = Y_t - \frac{R_{t-1}^b}{\Pi_t} \frac{D_{t-1}^b}{P_{t-1}} + \frac{Q_t}{P_t} H_{t-1}^b - \mathcal{T}^b, \tag{17}$$

where $\mathcal{T}^b$ is a steady state net tax/subsidy, which includes the borrowers' contribution to the firms' subsidy that make steady state output efficient, and the subsidy borrowers receive to obtain an efficient allocation. We can rewrite the last expression to capture the law of motion of debt

$$\frac{D_t^b}{P_t} = \frac{R_{t-1}^b}{\Pi_t} \frac{D_{t-1}^b}{P_{t-1}} + C_t^b - Y_t + \frac{Q_t}{P_t}(H_t^b - H_{t-1}^b) + \mathcal{T}^b.$$

B Efficient Steady State

This section first establishes the conditions under which a zero inflation ($\Pi = 1$) steady state is efficient, and then discusses how we can obtain efficiency of the steady state allocation in the decentralized equilibrium.[37]

Consider a social planner who maximises a weighted average of borrowers and savers' per-period welfare

$$\mathbb{U} \equiv \tilde{\xi} U(C^b, H^b, L^b) + (1 - \tilde{\xi}) U(C^s, H^s, L^s), \tag{18}$$

for some Pareto weights $\tilde{\xi} \in [0, 1]$, where $U(C^j, H^j, L^j)$ is the per-period utility function of type $j = \{b, s\}$. The social planner chooses allocations subject to the constraints imposed by the aggregate production function and the market clearing conditions for goods, housing, and labor. Importantly, the planner is not subject to the borrowing constraint.

In steady state, there is no price dispersion ($\Delta = 1$). We can further combine the production function with the goods and labor market constraints to yield

$$(L^b)^{\xi} (L^s)^{1-\xi} = \xi C^b + (1 - \xi) C^s.$$

Let μ_1 be the Lagrange multiplier on this constraint and μ_2 be the multiplier on the housing

[37] Without loss of generality, we normalize the price level to one so that all variables can be thought of as expressed in real terms.

resource constraint. The first-order conditions for an efficient steady state are

$$\begin{aligned}
\tilde{\xi} U_c^b &= \mu_1 \xi \\
(1-\tilde{\xi}) U_c^s &= \mu_1 (1-\xi) \\
\tilde{\xi} U_h^b &= \mu_2 \xi \\
(1-\tilde{\xi}) U_h^s &= \mu_2 (1-\xi) \\
\tilde{\xi} U_l^b &= \mu_1 \xi Y / L^b \\
(1-\tilde{\xi}) U_l^s &= \mu_1 (1-\xi) Y / L^s,
\end{aligned}$$

where U_c^j, U_h^j, and U_l^j are the marginal utilities of consumption and housing, and the marginal disutility of labor, for type j, respectively. If the Pareto weights coincide with the population weights ($\tilde{\xi} = \xi$), the marginal utility of consumption and housing are equal across types

$$U_c^b = U_c^s = \mu_1 \qquad \text{and} \qquad U_h^b = U_h^s = \mu_2,$$

and so are their levels. In addition, if the disutility of labor has a constant elasticity of substitution, as we assumed, hours supplied by borrowers and savers are proportional to each other depending on the disutility parameters χ^s and χ^b.

For a given type of household, we also obtain

$$\frac{U_c^j}{U_h^j} = \frac{\mu_1}{\mu_2}.$$

The ratio of the marginal utilities of consumption and housing for the two types are the same. The efficient steady state also implies the usual optimality conditions that equates the marginal rate of substitution between consumption and leisure to the marginal rate of transformation between labor and output

$$\frac{U_l^j}{U_c^j} = \frac{Y}{L^j}.$$

Assuming the subsidy τ^p is set as to remove the distortions from monopolistic competition in steady state ($MC = 1$), the labor market equilibrium implies

$$\left[\frac{\chi_L^b (L^b)^{\varphi}}{z e^{-zC^b}}\right]^{\xi} \left[\frac{\chi_L^s (L^s)^{\varphi}}{z e^{-zC^s}}\right]^{1-\xi} = \frac{Y}{L}.$$

Using the goods and labor market clearing conditions, and replacing output with labor from the production function, equilibrium hours solve

$$L^{\varphi} e^{zL} = \frac{z}{\left(\chi_L^b\right)^{\xi} \left(\chi_L^s\right)^{1-\xi}}.$$

We can choose the labor supply disutility parameters χ_L^j to deliver a desired target for hours worked by each group (e.g. 1/3 of the households' time endowments). Given this result, the production function pins down the equilibrium level of output. Therefore, importantly, the steady state efficient level of output and hours is independent of the distribution of wealth/debt across household types.

The next step is to find conditions under which the steady state allocation of the decentralized economy is efficient. In particular, we seek the taxes that achieve this objective. In the steady state of the decentralized economy, the savers' discount rate pins down the real rate of interest

$$R^d = \frac{1}{\beta_s}.$$

Since the ratio between equity and deposits is at its desired level, the spread between the return on equity and the return on deposits is zero, and so is the spread between loan and deposit rates

$$R^b = R^e = R^d.$$

In what follows, we drop the superscripts from returns and simply call the steady state gross real interest rate R. From the Euler equation for borrowers, we can obtain the value of the Lagrange multiplier on the collateral constraint

$$\tilde{\mu} = \frac{1 - \beta_b R}{1 - \beta_b \gamma_d},$$

which is positive as long as our initial assumption $\beta_b < 1/R = \beta_s$ is satisfied (that is, borrowers are relatively impatient).[38] With a positive multiplier, the constraint binds, and so equilibrium debt is

$$D^b = \Theta Q H^b.$$

Finally, we turn to the housing block. Starting from the law of motion of debt in steady state, we can write

$$C^b = Y - \mathcal{T}^b - (R-1)D^b.$$

In an efficient steady state, the level of consumption must be equal ($C^b = C^s$). Therefore, from the resource constraint, we have that $C^b = C^s = Y = C$. Substituting into the previous condition yields

$$\tau^b = -\frac{1-\beta_s}{\xi \beta_s}\eta,$$

where $\eta \equiv \xi D^b/Y$ is the ratio of debt to GDP and $\tau^b \equiv \mathcal{T}^b/Y$ is subsidy to borrowers (net of their contribution to the production subsidy) that equalises consumption across types.

The last element that we need to determine is the housing tax τ^h. In steady state, the

[38] Alternatively, we could write the value of the Lagrange multiplier on the borrowing constraint as $\tilde{\mu} = (\beta_s - \beta_b)/[\beta_s(1 - \beta_b\gamma_d)] > 0$, as long as $\beta_s > \beta_b$, which again corresponds to the initial assumption on the individual discount factors.

housing demand equation for borrowers is

$$[1-(1-\gamma_d)\Theta\tilde{\mu}-\beta_b]Q=\frac{\chi_H^b(H^b)^{-\sigma_h}}{ze^{-zC}},$$

while for savers we have

$$(1+\tau^h-\beta_s)Q=\frac{\chi_H^s(H^s)^{-\sigma_h}}{ze^{-zC}},$$

where we have used the equality of consumption across types. For the steady state housing allocation to be efficient, we must have that the numerator of the right-hand side of the last two expressions (the marginal utility of housing) is equal across types.[39] Therefore, the steady state housing tax must be

$$\tau^h=(\beta_s-\beta_b)\left[1-\frac{(1-\gamma_d)\Theta}{\beta_s(1-\beta_b\gamma_d)}\right],$$

where we used the expression for the steady state Lagrange multiplier $\tilde{\mu}$ derived above. Not surprisingly, the steady state tax on housing is zero in the limit $\beta_b\to\beta_s$.

B.1 Macro-Prudential Policy in the Efficient Equilibrium

This section shows that, in a flexible-price efficient equilibrium, macro-prudential policy carries distributional consequences but has no impact on the level of aggregate activity.

As derived in Appendix A, labor supply for type j's satisfies

$$W_t^j=\frac{\chi_L^j(L_t^j)^{\varphi}}{ze^{-zC_t^j}}.$$

Weighting the labor supply of each type by their respective shares, using the definition of the wage index, and equating with labor demand gives

$$\left[\frac{\chi_L^b(L_t^b)^{\varphi}}{ze^{-zC_t^b}}\right]^{\xi}\left[\frac{\chi_L^s(L_t^s)^{\varphi}}{ze^{-zC_t^s}}\right]^{1-\xi}=\frac{Y_t}{L_t}$$

Using the definition of the labor aggregator and the resource constraint, we can simplify the previous expression to

$$\frac{(\chi_L^b)^{\xi}(\chi_L^s)^{1-\xi}L_t^{1+\varphi}}{ze^{-z(Y_t-\Gamma_t)}}=Y_t,$$

where Γ_t is the portfolio adjustment cost term. In a flexible-price efficient equilibrium, the aggregate production function is simply $Y_t=L_t$. Therefore, we can express the last condition in terms of the efficient level of output Y_t^* as

$$\frac{(\chi_L^b)^{\xi}(\chi_L^s)^{1-\xi}}{ze^{-zY_t^*}}(Y_t^*)^{\varphi}=1. \tag{19}$$

[39] In addition, if the housing preference parameters are the same across households ($\chi^b=\chi^s=\chi$), then also the actual level of housing services consumed is the same ($H^b=H^s=H$).

In principle, portfolio adjustment costs associated with savers' debt-equity choice do affect output under flexible prices, but this effect is second order because Γ_t is quadratic. As a result, to a first order approximation, the efficient level of output only depends on the level of technology (which we have assumed to be constant and normalized to one) and preference parameters.

In spite of no first-order effects on aggregate supply, macro-prudential measures retain distributional consequences even in an efficient equilibrium. The macro-prudential authority will not be indifferent between different levels of LTV ratios or capital requirements. We return to this point in the next section after deriving a linear-quadratic approximation of the model that allows us to study the optimal joint conduct of monetary and macro-prudential policy. The flexible-price efficient equilibrium will be a useful starting point for our analysis.

C Derivation of the Loss Function

We define the welfare objective for the policymaker $\mathbb{W}_0$ as the present discounted value of the per-period utility of the two types, weighted by arbitrary weights $\tilde{\xi}$, and we assume the policymaker discounts the future at rate β (defined in the text)

$$\mathbb{W}_0 \equiv \mathbb{E}_0 \left(\sum_{t=0}^{\infty} \beta^t U_t \right), \tag{20}$$

where

$$U_t \equiv \tilde{\xi} U^b(C_t^b, H_t^b, L_t^b) + (1-\tilde{\xi}) U^s(C_t^s, H_t^s, L_t^s). \tag{21}$$

In order to derive a quadratic welfare objective, we take a second-order approximation of (21) around the efficient steady state in which $C^b = C^s = C = Y$, $H^b = H^s = H$, and $L^b = L^s = L$.

Ignoring terms of order three and higher, we get:

$$\begin{aligned} U_t - U \simeq\; & \tilde{\xi}[U_c^b(C_t^b - C) + \frac{1}{2}U_{cc}^b(C_t^b - C)^2] + (1-\tilde{\xi})[U_c^s(C_t^s - C) + \frac{1}{2}U_{cc}^s(C_t^s - C)^2] \\ & + \tilde{\xi}[U_h^b(H_t^b - H) + \frac{1}{2}U_{hh}^b(H_t^b - H)^2] + (1-\tilde{\xi})[U_h^s(H_t^s - H) + \frac{1}{2}U_{hh}^s(H_t^s - H)^2] \\ & + \tilde{\xi}[U_l^b(L_t^b - L) + \frac{1}{2}U_{ll}^b(L_t^b - L)^2] + (1-\tilde{\xi})[U_l^s(L_t^s - L) + \frac{1}{2}U_{ll}^s(L_t^s - L)^2], \end{aligned}$$

where U_{cc}^j, U_{hh}^j and U_{ll}^j are the second derivatives of the utility function with respect to consumption, housing, and labor, for type j, respectively.

Next, we factor out the marginal utility of consumption, housing, and the marginal disutility of labor for each group to obtain

$$
\begin{aligned}
U_t - U \simeq{}& \tilde{\xi} U_c^b [(C_t^b - C) + \frac{1}{2}\frac{U_{cc}^b}{U_c^b}(C_t^b - C)^2] + (1-\tilde{\xi}) U_c^s [(C_t^s - C) + \frac{1}{2}\frac{U_{cc}^s}{U_c^s}(C_t^s - C)^2] \\
&+ \tilde{\xi} U_h^b [(H_t^b - H) + \frac{1}{2}\frac{U_{hh}^b}{U_h^b}(H_t^b - H)^2] + (1-\tilde{\xi}) U_h^s [(H_t^s - H) + \frac{1}{2}\frac{U_{hh}^s}{U_h^s}(H_t^s - H)^2] \\
&+ \tilde{\xi} U_l^b [(L_t^b - L) + \frac{1}{2}\frac{U_{ll}^b}{U_l^b}(L_t^b - L)^2] + (1-\tilde{\xi}) U_l^s [(L_t^s - L) + \frac{1}{2}\frac{U_{ll}^s}{U_l^s}(L_t^s - L)^2].
\end{aligned}
$$

Using the first-order conditions associated with the efficient steady state, we get

$$
\begin{aligned}
U_t - U \simeq{}& \mu_1 \xi [(C_t^b - C) + \frac{1}{2}\frac{U_{cc}^b}{U_c^b}(C_t^b - C)^2] + \mu_1 (1-\xi) [(C_t^s - C) + \frac{1}{2}\frac{U_{cc}^s}{U_c^s}(C_t^s - C)^2] \\
&+ \mu_2 \xi [(H_t^b - H) + \frac{1}{2}\frac{U_{hh}^b}{U_h^b}(H_t^b - H)^2] + \mu_2 (1-\xi) [(H_t^s - H) + \frac{1}{2}\frac{U_{hh}^s}{U_h^s}(H_t^s - H)^2] \\
&- \mu_1 \xi \frac{Y}{L^b} [(L_t^b - L) + \frac{1}{2}\frac{U_{ll}^b}{U_l^b}(L_t^b - L)^2] - \mu_1 (1-\xi) \frac{Y}{L^s} [(L_t^s - L) + \frac{1}{2}\frac{U_{ll}^s}{U_l^s}(L_t^s - L)^2].
\end{aligned}
$$

Given the assumed preferences, we have

$$
\begin{aligned}
\frac{U_{cc}^j}{U_c^j} &= \frac{-z^2 e^{-zC}}{z e^{-zC}} = -z = -\frac{\sigma}{Y} \\
\frac{U_{hh}^j}{U_h^j} &= \frac{-\chi_H^j \sigma_h H^{-\sigma_h - 1}}{\chi_H^j H^{-\sigma_h}} = -\frac{\sigma_h}{H} \\
\frac{U_{ll}^j}{U_l^j} &= \frac{\chi_L^j \varphi L^{\varphi - 1}}{\chi_L^j L^{\varphi}} = \frac{\varphi}{L}
\end{aligned}
$$

After substituting for these semi-elasticities, we collect the linear terms in consumption and housing to get

$$
\begin{aligned}
U_t - U \simeq{}& \mu_1 [\xi (C_t^b - C) + (1-\xi)(C_t^s - C)] + \mu_2 [\xi (H_t^b - H) + (1-\xi)(H_t^s - H)] \\
&- \frac{1}{2}\mu_1 z \left[\xi (C_t^b - C)^2 + (1-\xi)(C_t^s - C)^2\right] \\
&- \frac{1}{2}\mu_2 \frac{\sigma_h}{H} \left[\xi (H_t^b - H)^2 + (1-\xi)(H_t^s - H)^2\right] \\
&- \mu_1 \xi \frac{Y}{L} [(L_t^b - L) + \frac{1}{2}\frac{\varphi}{L}(L_t^b - L)^2] - \mu_1 (1-\xi) \frac{Y}{L} [(L_t^s - L) + \frac{1}{2}\frac{\varphi}{L}(L_t^s - L)^2]. \qquad (22)
\end{aligned}
$$

To eliminate first-order terms, we first make use of the identity

$$
Z_t \equiv e^{\ln Z_t},
$$

for any variable Z_t. We approximate Z_t around $\ln Z$ to the second order. Let $y_t \equiv \ln Z_t$. Then, we have

$$e^{y_t} \simeq e^y + e^y(y_t - y) + \frac{1}{2}e^y(y_t - y)^2,$$

where we have ignored terms of order three and higher. Applying the transformation back, we get

$$Z_t \simeq Z + Z(\ln Z_t - \ln Z) + \frac{1}{2}(\ln Z_t - \ln Z)^2,$$

which implies

$$\frac{Z_t - Z}{Z} \simeq z_t + \frac{1}{2}z_t^2,$$

or

$$Z_t \simeq Z(1 + z_t + \frac{1}{2}z_t^2),$$

where we have defined $z_t \equiv \ln(Z_t/Z)$.

Now, we apply the approximation to the goods market resource constraint

$$Y_t = \xi C_t^b + (1-\xi)C_t^s + \Gamma_t,$$

where

$$\Gamma_t = \frac{\Psi}{2}\left(\frac{\tilde{\kappa}_t}{\tilde{\kappa}} - 1\right)^2 \tilde{\kappa}\xi D_t^b.$$

Note that in steady state Γ_t is equal to zero, and that all first and second derivatives are zero except for the second derivative with respect to $\tilde{\kappa}_t$. Therefore, up to a second order approximation, we have

$$\Gamma_t \simeq \frac{\Psi\tilde{\kappa}\xi D^b}{2}\kappa_t^2,$$

where $\kappa_t = (\tilde{\kappa}_t - \tilde{\kappa})/\tilde{\kappa}$. Consequently, up to the second order, the resource constraint becomes

$$(Y_t - Y) - \frac{\Psi\tilde{\kappa}\xi D^b}{2}\kappa_t^2 = \xi(C_t^b - C) + (1-\xi)(C_t^s - C),$$

which gives us a second order approximation of aggregate consumption in terms of aggregate output and adjustment costs

$$\xi(C_t^b - C) + (1-\xi)(C_t^s - C) = Y(y_t + \frac{1}{2}y_t^2) - \frac{\Psi\tilde{\kappa}\xi D^b}{2}\kappa_t^2. \tag{23}$$

Similarly, for the housing market resource constraint

$$H = \xi H_t^b + (1-\xi)H_t^s,$$

we have

$$0 = \xi(H_t^b - H) + (1-\xi)(H_t^s - H), \tag{24}$$

so that the second term of the first line of (22) disappears.

Going back to our approximation, we can rewrite

$$\begin{aligned}
U_t - U \simeq\ & \mu_1 Y(y_t + \frac{1}{2}y_t^2) - \frac{1}{2}\mu_1 z\left[\xi(C_t^b - C)^2 + (1-\xi)(C_t^s - C)^2\right] \\
& - \frac{1}{2}\mu_2 \frac{\sigma_h}{H}\left[\xi(H_t^b - H)^2 + (1-\xi)(H_t^s - H)^2\right] \\
& - \mu_1 \xi \frac{Y}{L}[(L_t^b - L) + \frac{1}{2}\frac{\varphi}{L}(L_t^b - L)^2] - \mu_1(1-\xi)\frac{Y}{L}[(L_t^s - L) + \frac{1}{2}\frac{\varphi}{L}(L_t^s - L)^2], \qquad (25)
\end{aligned}$$

where the equity adjustment cost term drops out because of it is independent of policy. Using the coefficients defined above, we can further rearrange the previous expression as

$$\begin{aligned}
U_t - U \simeq\ & \mu_1 Y[y_t - \xi l_t^b - (1-\xi)l_t^s] + \frac{1}{2}\mu_1 Y y_t^2 \\
& - \frac{1}{2}\mu_1 \sigma Y\left[\xi(c_t^b)^2 + (1-\xi)(c_t^s)^2\right] - \frac{1}{2}\mu_2 \sigma_h H\left[\xi(h_t^b)^2 + (1-\xi)(h_t^s)^2\right] \\
& - \frac{1}{2}\mu_1 \xi Y(1+\varphi)(l_t^b)^2 - \frac{1}{2}\mu_1(1-\xi)Y(1+\varphi)(l_t^s)^2. \qquad (26)
\end{aligned}$$

Next, we focus on eliminating the first-order terms left in the approximation. From the aggregate production function derived in section A.4 we have

$$\hat{\Delta}_t + y_t = \xi l_t^b + (1-\xi)l_t^s,$$

where $\hat{\Delta}_t \equiv \Delta_t - 1$, since there is no price dispersion in steady state. Replacing from this equation for the difference between output and the weighted average of two types' labor supply, we can write

$$\begin{aligned}
U_t - U \simeq\ & \frac{1}{2}\mu_1 Y y_t^2 - \mu_1 Y \hat{\Delta}_t \\
& - \frac{1}{2}\mu_1 \sigma Y\left[\xi(c_t^b)^2 + (1-\xi)(c_t^s)^2\right] - \frac{1}{2}\mu_2 \sigma_h H\left[\xi(h_t^b)^2 + (1-\xi)(h_t^s)^2\right] \\
& - \frac{1}{2}\mu_1 \xi Y(1+\varphi)(l_t^b)^2 - \frac{1}{2}\mu_1(1-\xi)Y(1+\varphi)(l_t^s)^2.
\end{aligned}$$

At this point, the welfare objective is fully quadratic.[40] However, we can further manipulate the approximation to obtain terms that have a more meaningful economic interpretation. To this end, we combine the terms in output, consumption, and labor

$$\begin{aligned}
U_t - U \simeq\ & -\frac{1}{2}\mu_1 Y\left\{\sigma[\xi(c_t^b)^2 + (1-\xi)(c_t^s)^2] - y_t^2 + (1+\varphi)[\xi(l_t^b)^2 + (1-\xi)(l_t^s)^2]\right\} \\
& - \frac{1}{2}\mu_2 \sigma_h H\left[\xi(h_t^b)^2 + (1-\xi)(h_t^s)^2\right] - \mu_1 Y \hat{\Delta}_t. \qquad (27)
\end{aligned}$$

Let us focus on the first line of the right-hand side of (27). Adding and subtracting $(\varphi+\sigma)y_t^2$,

[40] As we will show formally below, $\hat{\Delta}_t$ (the price dispersion index) is a term of order two.

we can write

$$\sigma[\xi(c_t^b)^2+(1-\xi)(c_t^s)^2]-y_t^2+(1+\varphi)[\xi(l_t^b)^2+(1-\xi)(l_t^s)^2]=$$
$$\sigma[\xi(c_t^b)^2+(1-\xi)(c_t^s)^2]-y_t^2+(\varphi+\sigma)y_t^2-(\varphi+\sigma)y_t^2+(1+\varphi)[\xi(l_t^b)^2+(1-\xi)(l_t^s)^2].$$

We can take σy_t^2 inside the consumption terms and $(1+\varphi)y_t^2$ inside the labor terms to write

$$\sigma[\xi(c_t^b)^2+(1-\xi)(c_t^s)^2]-y_t^2+(1+\varphi)[\xi(l_t^b)^2+(1-\xi)(l_t^s)^2]=$$
$$(\varphi+\sigma)y_t^2+\sigma[\xi(c_t^b)^2+(1-\xi)(c_t^s)^2-y_t^2]+(1+\varphi)[\xi(l_t^b)^2+(1-\xi)(l_t^s)^2-y_t^2]. \quad (28)$$

Now we work with the second term of the right-hand side of (28), which we can write as

$$\begin{aligned}\xi(c_t^b)^2+(1-\xi)(c_t^s)^2-y_t^2 &= \xi[(c_t^b)^2-y_t^2]+(1-\xi)[(c_t^s)^2-y_t^2]\\ &= \xi(c_t^b+y_t)(c_t^b-y_t)+(1-\xi)(c_t^s+y_t)(c_t^s-y_t).\end{aligned}$$

We use again the resource constraint to replace the difference between each type's consumption and output

$$\begin{aligned}\xi(c_t^b)^2+(1-\xi)(c_t^s)^2-y_t^2 &= \xi(c_t^b+y_t)(1-\xi)(c_t^b-c_t^s)-(1-\xi)(c_t^s+y_t)\xi(c_t^b-c_t^s)\\ &= \xi(1-\xi)(c_t^b-c_t^s)[(c_t^b+y_t)-(c_t^s+y_t)]\\ &= \xi(1-\xi)(c_t^b-c_t^s)^2. \qquad (29)\end{aligned}$$

Before moving on, we need to take an approximation of the labor supply conditions

$$\frac{\chi_L^b(L_t^b)^{1+\varphi}}{ze^{-zC_t^b}}=W_t^bL_t^b \qquad \text{and} \qquad \frac{\chi_L^s(L_t^s)^{1+\varphi}}{ze^{-zC_t^s}}=W_t^sL_t^s.$$

Taking a geometric average of the above two conditions, with weights reflecting the two types' shares, we get an aggregate labor supply condition of the form

$$\frac{\chi_L(L_t)^{1+\varphi}}{ze^{-z(Y_t-\Gamma_t)}}=W_tL_t.$$

Substituting on the left-hand side for aggregate employment from the aggregate production function, we can write

$$\frac{\chi_L}{ze^{-z(Y_t-\Gamma_t)}}\left[\frac{\Delta_tY_t}{\xi^\xi(1-\xi)^{1-\xi}}\right]^{1+\varphi}=W_tL_t.$$

As we proved in section A.4, $W_t^jL_t^j=W_tL_t$ for $j=\{b,s\}$. Therefore, we can write

$$\frac{\chi_L}{ze^{-z(Y_t-\Gamma_t)}}\left[\frac{\Delta_tY_t}{\xi^\xi(1-\xi)^{1-\xi}}\right]^{1+\varphi}=\frac{\chi_L^b(L_t^b)^{1+\varphi}}{ze^{-zC_t^b}}=\frac{\chi_L^s(L_t^s)^{1+\varphi}}{ze^{-zC_t^s}}.$$

Approximating these two conditions and solving for each type's labor supply, we obtain

$$\begin{aligned} l_t^b &= \hat{\Delta}_t + y_t - \frac{\sigma}{1+\varphi}(c_t^b - y_t) \\ l_t^s &= \hat{\Delta}_t + y_t - \frac{\sigma}{1+\varphi}(c_t^s - y_t). \end{aligned}$$

Using the first order approximation of the resource constraint, we can rewrite the two conditions above as

$$\begin{aligned} l_t^b &= \hat{\Delta}_t + y_t - \frac{\sigma}{1+\varphi}(1-\xi)(c_t^b - c_t^s) \\ l_t^s &= \hat{\Delta}_t + y_t + \frac{\sigma}{1+\varphi}\xi(c_t^b - c_t^s). \end{aligned}$$

Now we can move on to the third term of the right-hand side of (28) substituting out labor supply of the two types.

$$\begin{aligned} &\xi(l_t^b)^2 + (1-\xi)(l_t^s)^2 - y_t^2 = \\ &\qquad \xi\left[\hat{\Delta}_t + y_t - \frac{\sigma}{1+\varphi}(1-\xi)(c_t^b - c_t^s)\right]^2 + (1-\xi)\left[\hat{\Delta}_t + y_t + \frac{\sigma}{1+\varphi}\xi(c_t^b - c_t^s)\right]^2 - y_t^2. \end{aligned}$$

We expand the two squared terms on the right-hand side of the last equation isolating the terms in the consumption gap

$$\begin{aligned} &\xi(l_t^b)^2 + (1-\xi)(l_t^s)^2 - y_t^2 = \\ &\qquad \xi y_t^2 + \xi\left[\frac{\sigma}{1+\varphi}(1-\xi)(c_t^b - c_t^s)\right]^2 - 2\xi y_t \frac{\sigma}{1+\varphi}(1-\xi)(c_t^b - c_t^s) \\ &\qquad + (1-\xi)y_t^2 + (1-\xi)\left[\frac{\sigma}{1+\varphi}\xi(c_t^b - c_t^s)\right]^2 + 2(1-\xi)y_t\frac{\sigma}{1+\varphi}\xi(c_t^b - c_t^s) - y_t^2, \end{aligned}$$

where the term $\hat{\Delta}_t$ drops out because it is of order two, hence its square and its product with first order terms is irrelevant for welfare up to the second order.

We can now combine terms and simplify to obtain

$$\xi(l_t^b)^2 + (1-\xi)(l_t^s)^2 - y_t^2 = \xi(1-\xi)\left[\frac{\sigma}{1+\varphi}(c_t^b - c_t^s)\right]^2, \tag{30}$$

We replace (29) and (30) into (28)

$$\sigma[\xi(c_t^b)^2 + (1-\xi)(c_t^s)^2] - y_t^2 + (1+\varphi)[\xi(l_t^b)^2 + (1-\xi)(l_t^s)^2]$$
$$= (\varphi+\sigma)y_t^2 + \sigma\xi(1-\xi)(c_t^b - c_t^s)^2 + (1+\varphi)\xi(1-\xi)\left[\frac{\sigma}{1+\varphi}(c_t^b - c_t^s)\right]^2$$
$$= (\varphi+\sigma)y_t^2 + \xi(1-\xi)\sigma\left(\frac{1+\sigma+\varphi}{1+\varphi}\right)(c_t^b - c_t^s)^2.$$

Expanding the last term on the right-hand side and combining it with the first, we can write

$$\sigma[\xi(c_t^b)^2 + (1-\xi)(c_t^s)^2] - y_t^2 + (1+\varphi)[\xi(l_t^b)^2 + (1-\xi)(l_t^s)^2]$$
$$= (\sigma+\varphi)y_t^2 - 2(1+\varphi)a_t y_t + \xi(1-\xi)\sigma\left(\frac{1+\sigma+\varphi}{1+\varphi}\right)(c_t^b - c_t^s)^2.$$

We now go back to (27) and substitute the result we have just derived to obtain

$$\begin{aligned} U_t - U \simeq & -\frac{1}{2}\mu_1 Y\left[(\sigma+\varphi)y_t^2 + \xi(1-\xi)\sigma\left(\frac{1+\sigma+\varphi}{1+\varphi}\right)(c_t^b - c_t^s)^2\right] \\ & -\frac{1}{2}\mu_2\sigma_h H\left[\xi(h_t^b)^2 + (1-\xi)(h_t^s)^2\right] - \mu_1 Y\hat{\Delta}_t. \end{aligned} \tag{31}$$

Next, we work with the second line of (31). From the linear approximation of the housing market clearing condition, we have

$$\begin{aligned} h_t^b &= -(1-\xi)(h_t^b - h_t^s) \\ h_t^s &= \xi(h_t^b - h_t^s). \end{aligned}$$

Therefore, we have

$$\begin{aligned} \xi(h_t^b)^2 + (1-\xi)(h_t^s)^2 &= \xi(1-\xi)^2(h_t^b - h_t^s)^2 + (1-\xi)\xi^2(h_t^b - h_t^s)^2 \\ &= \xi(1-\xi)(h_t^b - h_t^s)^2. \end{aligned}$$

Notice also that the coefficient multiplying the housing term is $\mu_2 H$. Using the conditions for the efficient steady state, we can rewrite

$$\mu_2 H = \mu_1 Y\frac{\mu_2 H}{\mu_1 Y} = \mu_1 Y\frac{U_h^j}{U_c^j}\frac{H}{Y}.$$

We can choose the housing utility parameters χ_H^j so that

$$\frac{U_h^j}{U_c^j}\frac{H}{Y} = 1.$$

Substituting back into the welfare approximation, we arrive at

$$U_t - U \simeq -\frac{1}{2}\mu_1 Y \left[(\sigma+\varphi)y_t^2 + \xi(1-\xi)\sigma\left(\frac{1+\sigma+\varphi}{1+\varphi}\right)(c_t^b - c_t^s)^2 + \xi(1-\xi)\sigma_h(h_t^b - h_t^s)^2\right] - \mu_1 Y\hat{\Delta}_t. \tag{32}$$

Lastly, we take a second order approximation of the price dispersion index, which yields

$$\hat{\Delta}_t = \lambda\hat{\Delta}_{t-1} + \frac{1}{2}\frac{\lambda\varepsilon}{1-\lambda}\pi_t^2.$$

Solving the previous difference equation backward, we have

$$\hat{\Delta}_t = \lambda\hat{\Delta}_{-1} + \frac{1}{2}\frac{\lambda\varepsilon}{1-\lambda}\sum_{j=0}^{t}\lambda^{t-j}\pi_j^2,$$

for some initial level of price dispersion $\hat{\Delta}_{-1}$. We are interested in the present discounted value of the previous expression, that is

$$\sum_{t=0}^{\infty}\beta^t\hat{\Delta}_t = \frac{1}{2}\frac{\lambda\varepsilon}{1-\lambda}\sum_{t=0}^{\infty}\beta^t\sum_{j=0}^{t}\lambda^{t-j}\pi_j^2,$$

where we have dropped the initial level of price dispersion as it is independent of policy. Let us now focus on the double sum on the right-hand side of the last expression, which we can expand to obtain

$$\begin{aligned}
\sum_{t=0}^{\infty}\beta^t\sum_{j=0}^{t}\lambda^{t-j}\pi_j^2 &= \pi_0^2 + \beta(\lambda\pi_0^2 + \pi_1^2) + \beta^2(\lambda^2\pi_0^2 + \lambda\pi_1^2 + \pi_2^2) + ... \\
&= \sum_{j=0}^{\infty}(\beta\lambda)^j\pi_0^2 + \beta\sum_{j=0}^{\infty}(\beta\lambda)^j\pi_1^2 + \beta^2\sum_{j=0}^{\infty}(\beta\lambda)^j\pi_2^2 + ... \\
&= \pi_0^2\sum_{j=0}^{\infty}(\beta\lambda)^j + \beta\pi_1^2\sum_{j=0}^{\infty}(\beta\lambda)^j + \beta^2\pi_2^2\sum_{j=0}^{\infty}(\beta\lambda)^j + ... \\
&= \sum_{t=0}^{\infty}\beta^t\pi_t^2\sum_{j=0}^{\infty}(\beta\lambda)^j = \frac{1}{1-\beta\lambda}\sum_{t=0}^{\infty}\beta^t\pi_t^2
\end{aligned}$$

Therefore, we can write

$$\sum_{t=0}^{\infty}\beta^t\hat{\Delta}_t = \frac{1}{2}\frac{\lambda\varepsilon}{(1-\lambda)(1-\beta\lambda)}\sum_{t=0}^{\infty}\beta^t\pi_t^2.$$

Substituting back into the approximation of utility, we obtain

$$U_t - U \simeq -\frac{1}{2}\mu_1 Y \left[(\sigma+\varphi)y_t^2 + \xi(1-\xi)\sigma\left(\frac{1+\sigma+\varphi}{1+\varphi}\right)(c_t^b - c_t^s)^2 + \xi(1-\xi)\sigma_h(h_t^b - h_t^s)^2 \right.$$
$$\left. + \frac{\lambda\varepsilon}{(1-\lambda)(1-\beta\lambda)}\pi_t^2\right]. \tag{33}$$

Therefore, up to the second order and ignoring terms independent of policy, we can rewrite the welfare objective as

$$\mathbb{W}_0 \simeq -\frac{\Omega}{2}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left(y_t^2 + \lambda_\pi\pi_t^2 + \lambda_c\tilde{c}_t^2 + \lambda_h\tilde{h}_t^2\right) \tag{34}$$

where $\tilde{c}_t \equiv c_t^b - c_t^s$ and $\tilde{h}_t \equiv h_t^b - h_t^s$ are the consumption and housing gaps, respectively. The composite parameters in the loss function are

$$\begin{aligned}
\Omega &\equiv \mu_1 Y(\sigma+\varphi) \\
\lambda_\pi &\equiv \frac{\lambda\varepsilon}{(1-\lambda)(1-\beta\lambda)(\sigma+\varphi)} \\
\lambda_c &\equiv \frac{\xi(1-\xi)\sigma(1+\sigma+\varphi)}{(1+\varphi)(\sigma+\varphi)} \\
\lambda_h &\equiv \frac{\xi(1-\xi)\sigma_h}{\sigma+\varphi}.
\end{aligned}$$

Observe that the higher is σ, the greater the weight on the output and consumption gap terms, though the weight on the consumption gap grows quadratically in σ, whereas the weight on the output gap is linear in σ. The higher is φ, the greater the weight on the aggregate output gap, and the smaller the weight on the consumption gap.

To give a rough idea of magnitudes, take $\sigma = \varphi = 1$ as a baseline case. Then, the relative weight on the consumption gap is $\lambda_c = 3\xi(1-\xi)/4$. Since $\xi(1-\xi)$ reaches a maximum of $1/4$, the maximum relative weight on the consumption gap is $3/16$. In general, the policymaker will attribute more weight to the volatility of aggregate output than to the volatility of relative consumption. Nevertheless, for a given policy, the latter may be large, thus becoming a significant source of welfare costs. The ability to use multiple policy instruments to deal with different tradeoffs should mitigate these costs.

D Linearized Constraints

In this section, we derive a first-order approximation of the equilibrium conditions that constitute the constraints for the optimal policy problem in our linear-quadratic setting. Unless otherwise stated, lower-case variables denote log-deviations from steady state, that is, for a generic variable Z_t with steady state value Z, $Z_t \equiv \ln(Z_t/Z)$.

D.1 Savers

The Euler equation for savers is

$$c_t^s = -\sigma^{-1}(i_t - \mathbb{E}_t \pi_{t+1}) + \mathbb{E}_t c_{t+1}^s, \tag{35}$$

where $i_t \equiv \ln R_t^d$ is the net nominal interest rate on deposits and $\sigma \equiv zY$ is the inverse of the elasticity of intertemporal substitution. A similar condition applies to equity investment, taking into account portfolio adjustment costs. Up to the first order, no arbitrage therefore implies

$$i_t^e = i_t + \frac{(1-\xi)\Psi}{\xi}\kappa_t. \tag{36}$$

The labor supply condition for savers is

$$w_t^s = \varphi l_t^s + \sigma c_t^s,$$

where w_t^s is the log-deviation of the savers' real wage.

Demand for housing by savers is

$$q_t = \frac{1+\tau^h-\beta_s}{1+\tau^h}(u_t^h + \sigma c_t^s - \sigma_h h_t^s) + \frac{\beta_s}{1+\tau^h}\mathbb{E}_t[q_{t+1} - \sigma(c_{t+1}^s - c_t^s)], \tag{37}$$

where q_t is the log-deviation of the real price of housing from its steady state value and the housing demand shock follows

$$u_t^h = \rho_h u_{t-1}^h + \epsilon_t^h,$$

with $\rho_h \in (0,1)$ and $\epsilon_t^h \sim \mathcal{N}(0, \varsigma_h^2)$.

D.2 Borrowers

The Euler equation for borrowers takes into account the effect of the collateral constraint

$$c_t^b = -\sigma^{-1}\left[\frac{1-(1-\beta_b\gamma_d)\tilde{\mu}}{1-\tilde{\mu}} i_t^b - \mathbb{E}_t\pi_{t+1} + \frac{\tilde{\mu}}{1-\tilde{\mu}}(\mu_t - \beta_b\gamma_d\mathbb{E}_t\mu_{t+1})\right] + \mathbb{E}_t c_{t+1}^b, \tag{38}$$

where $i_t^b \equiv \ln R_t^b$ is the net nominal interest rate faced by borrowers. Note that, everything else equal, a tightening of the collateral constraint ($\mu_t > 0$) tends to raise the cost of borrowing for impatient households, while an expected rise ($\mathbb{E}_t\mu_{t+1} > 0$) tends to lower it.

Labor supply for borrowers is

$$w_t^b = \varphi l_t^b + \sigma c_t^b,$$

where w_t^b is the log-deviation of the borrowers' real wage.

Borrowers' demand for housing is

$$q_t = \frac{(1-\gamma_d)\tilde{\mu}\Theta}{1-(1-\gamma_d)\tilde{\mu}\Theta}(\mu_t + \theta_t) + \frac{1-\beta_b-(1-\gamma_d)\tilde{\mu}\Theta}{1-(1-\gamma_d)\tilde{\mu}\Theta}(u_t^h + \sigma c_t^b - \sigma_h h_t^b)$$
$$+ \frac{\beta_b}{1-(1-\gamma_d)\tilde{\mu}\Theta}\mathbb{E}_t[q_{t+1} - \sigma(c_{t+1}^b - c_t^b)], \quad (39)$$

where θ_t, which can be either a shock or a macro-prudential policy instrument, is the log-deviation of the collateral constraint parameter (the LTV ratio) from its steady state value.

The linearized borrowing constraint at equality is

$$d_t^b \leq \gamma_d(d_{t-1}^b - \pi_t) + (1-\gamma_d)(\theta_t + q_t + h_t^b), \quad (40)$$

where d_t^b denotes the log-deviation of the real quantity of debt from its steady state value.

Finally, from the borrowers' budget constraint, we can derive the law of motion for debt as

$$d_t^b = \frac{1}{\beta_s}(i_{t-1}^b + d_{t-1}^b - \pi_t) + \frac{1}{\Theta}(h_t^b - h_{t-1}^b) + \frac{\xi}{\eta}(c_t^b - y_t), \quad (41)$$

where we have used the fact that in steady state gross returns equal the inverse of the savers' discount factor.

D.3 Banks

Banks price loans as a weighted average between the return on equity and the deposit rate

$$i_t^b = \tilde{\kappa} i_t^e + (1-\tilde{\kappa}) i_t.$$

Using the no arbitrage condition between return on equity and on deposits from the savers' problem (36), we obtain an expression for the spread of the the loan rate on the deposit rate

$$i_t^b = i_t + \frac{1-\xi}{\xi}\psi\kappa_t, \quad (42)$$

where $\psi \equiv \Psi\tilde{\kappa}$ is the semi-elasticity of the borrowing rate to capital requirements and κ_t is the log-deviation of the capital requirement from its steady state value, which follows

$$\kappa_t = \rho_\kappa \kappa_{t-1} + \epsilon_t^\kappa,$$

with $\rho_\kappa \in (0,1)$ and $\epsilon_t^\kappa \sim \mathcal{N}(0, \varsigma_\kappa^2)$.

D.4 Production

Up to a linear approximation, the production function is

$$y_t = l_t.$$

The labor demand equation is simply

$$w_t = mc_t,$$

where mc_t is the first order approximation of real marginal cost. The wage bill must be equal across types

$$w_t^s + l_t^s = w_t^b + l_t^b,$$

where the wage index is

$$w_t = \xi w_t^b + (1-\xi)w_t^s$$

and labor market clearing requires

$$l_t = \xi l_t^b + (1-\xi)l_t^s.$$

Finally, the Phillips Curve is

$$\pi_t = \frac{(1-\lambda)(1-\beta\lambda)}{\lambda}mc_t + \beta\mathbb{E}_t\pi_{t+1}.$$

D.5 Market Clearing

Goods market clearing entails

$$y_t = \xi c_t^b + (1-\xi)c_t^s, \tag{43}$$

while housing market clearing requires

$$\xi h_t^b + (1-\xi)h_t^s = 0. \tag{44}$$

The market clearing conditions complete the description of the linearized model.

D.6 Gaps and Aggregate Variables

In what follows, we combine the equilibrium relations to obtain a parsimonious set of constraints for the optimal policy problem.

On the supply side, we can rewrite the Phillips curve in terms of the efficient output gap, which is equal to the deviations of output from steady state since we have assumed a constant level of productivity (see equation 19).[41]

[41] Since our model does not have markup shocks, the flexible-price level of output is efficient.

Weighting the labor supply equations by population shares and using the equality between the real wage and marginal costs, we can write an aggregate labor market equilibrium equation

$$\varphi l_t + \sigma c_t = mc_t.$$

Using the resource constraint and the production function, we can then rewrite

$$mc_t = (\sigma + \varphi) y_t.$$

Finally, replacing into the Phillips curve gives

$$\pi_t = \gamma y_t + \beta \mathbb{E}_t \pi_{t+1}, \tag{45}$$

where $\gamma \equiv (\sigma + \varphi)(1 - \lambda)(1 - \beta\lambda)/\lambda$.

On the demand side, we start from the savers' Euler equation (35) and replace savers' consumption from the resource constraint (43) to obtain

$$y_t - \xi \tilde{c}_t = -\sigma^{-1}(i_t - \mathbb{E}_t \pi_{t+1}) + \mathbb{E}_t(y_{t+1} - \xi \tilde{c}_{t+1}). \tag{46}$$

The second condition that characterises aggregate demand comes from substituting housing demand for borrowers from the market clearing condition (44) into the borrowing constraint equality (40)

$$d_t^b \leq \gamma_d (d_{t-1}^b - \pi_t) + (1 - \gamma_d)[\theta_t + q_t + (1 - \xi)\tilde{h}_t]. \tag{47}$$

Finally, we can replace the goods and housing market resource constraints, and the banking condition (42) into the borrowers' budget constraint (41) to obtain an equation for the law of motion of debt

$$d_t^b = \frac{1}{\beta_s}\left(i_{t-1} + \frac{1-\xi}{\xi}\psi\kappa_{t-1} + d_{t-1}^b - \pi_t\right) + \frac{1-\xi}{\Theta}(\tilde{h}_t - \tilde{h}_{t-1}) + \frac{\xi(1-\xi)}{\eta}\tilde{c}_t. \tag{48}$$

The housing market (housing gap and house prices) is characterized by the housing demand equations of savers and borrowers. We can rewrite the former in terms of aggregate variables as

$$q_t = \frac{1 + \tau^h - \beta_s}{1 + \tau^h}[u_t^h + \sigma_h \xi(\tilde{h}_t - \phi \tilde{h}_{t-1})] + \sigma(y_t - \xi \tilde{c}_t) + \frac{\beta_s}{1 + \tau^h}\mathbb{E}_t[q_{t+1} - \sigma(y_{t+1} - \xi \tilde{c}_{t+1})]. \tag{49}$$

For borrowers, we have

$$\begin{aligned} q_t = \frac{(1-\gamma_d)\tilde{\mu}\Theta}{1-(1-\gamma_d)\tilde{\mu}\Theta}(\mu_t + \theta_t) + \frac{1-\beta_b-(1-\gamma_d)\tilde{\mu}\Theta}{1-(1-\gamma_d)\tilde{\mu}\Theta}[u_t^h - \sigma_h(1-\xi)(\tilde{h}_t - \phi\tilde{h}_{t-1})] \\ + \sigma[y_t + (1-\xi)\tilde{c}_t] + \frac{\beta_b}{1-(1-\gamma_d)\tilde{\mu}\Theta}\mathbb{E}_t\{q_{t+1} - \sigma[y_{t+1} + (1-\xi)\tilde{c}_{t+1}]\}. \end{aligned} \tag{50}$$

The last constraint that we need for the optimal policy problem is the Euler equation of borrowers to keep track of the multiplier on the collateral constraint. Using the resource constraint and the expression for the borrower rate, we can write

$$y_t+(1-\xi)\tilde{c}_t = -\sigma^{-1}\left[\frac{1-(1-\beta_b\gamma_d)\tilde{\mu}}{1-\tilde{\mu}}\left(i_t+\frac{1-\xi}{\xi}\psi\kappa_t\right)-\mathbb{E}_t\pi_{t+1}+\frac{\tilde{\mu}}{1-\tilde{\mu}}(\mu_t-\beta_b\gamma_d\mathbb{E}_t\mu_{t+1})\right] + \mathbb{E}_t[y_{t+1}+(1-\xi)\tilde{c}_{t+1}]. \quad (51)$$

Given a specification of monetary policy (in terms of the interest rate i_t) and macroprudential policy (in terms of the LTV ratio θ_t), and exogenous processes for the housing demand shock u_t^h and the capital requirement κ_t, equations (45)-(51), constitute a system of seven equations in seven unknowns (y_t, π_t, $\tilde{c}$, $\tilde{h}_t$, q_t, d_t^b, and μ_t) that characterises the equilibrium.

D.7 Calibration of ψ

To calibrate ψ, we use the linearized Euler equation for borrowers (38), which we can write in terms of 'gaps' as:

$$y_t+(1-\xi)\tilde{c}_t = -\sigma^{-1}\left[\frac{1-(1-\beta_b\gamma_d)\tilde{\mu}}{1-\tilde{\mu}}\left(i_t+\frac{1-\xi}{\xi}\psi\kappa_t\right)-\mathbb{E}_t\pi_{t+1}+\frac{\tilde{\mu}}{1-\tilde{\mu}}(\mu_t-\beta_b\gamma_d\mathbb{E}_t\mu_{t+1})\right] + \mathbb{E}_t[y_{t+1}+(1-\xi)\tilde{c}_{t+1}].$$

The evidence from the Macroeconomic Assessment Group cited in the paper gives an estimated range for $\partial y_t/\partial\kappa_t$ in partial equilibrium. From the expression above, we obtain

$$\frac{\partial y_t}{\partial \kappa_t} = -\sigma^{-1}\frac{1-(1-\beta_b\gamma_d)\tilde{\mu}}{1-\tilde{\mu}}\frac{1-\xi}{\xi}\psi$$

The steady state Lagrange multiplier on the borrowing constraint is

$$\tilde{\mu} = \frac{1-\beta_b/\beta_s}{1-\beta_b\gamma_d}$$

Given the values of the other parameters, we can use this expression to, infer the value for $\psi = \Psi\tilde{\kappa}$ that matches the empirical estimate. In particular, we have

$$\tilde{\mu} = \frac{1-\beta_b/\beta_s}{1-\beta_b\gamma_d} = \frac{1-0.985/0.995}{1-0.985\times 0.7} \approx 0.032.$$

Plugging into the partial derivative above together with the other parameter values, we get

$$\frac{\partial y_t}{\partial \kappa_t} = -1\times\frac{1-(1-0.985\times 0.7)\times 0.032}{1-0.032}\times\frac{1-0.57}{0.57}\times\Psi\times 0.1 = -0.077\times\Psi$$

The estimated range of the effects of a change in capital requirements on annual GDP in the MAG report is $[-0.05\%, -0.35\%]$. Converting this result to the effects on quarterly GDP gives a range of $[-0.0125\%, -0.0875\%]$. Setting $\Psi = 0.5$ in the formula above implies an effect of -0.0385, roughly in the middle of the estimated interval. Since we assume $\tilde{\kappa} = 0.1$, this implies $\psi = 0.05$ as reported in the text.

E Computational Details

In this section, we describe how we implement the optimal time-consistent monetary and macro-prudential policy plans in the linear-quadratic approximation that we have derived so far, taking into account occasionally binding constraints.

E.1 Timing Protocol and Equilibrium Properties

The definition of 'optimal discretion' that we use requires the following protocol:

- At the start of each period nature reveals the values of all shocks.
- Today's policymaker acts as a Stackelberg leader with respect to (a) private agents today; (b) future policymakers and private agents. The policymaker acts after shocks are revealed, so takes them as given.
- Today's policymaker is constrained by the behavior of private agents today: outcomes today must be compatible with the decisions of private agents. The policymaker is not constrained by the decisions of private agents in the future (that is a problem for future policymakers).
- Today's policymaker recognises that future policymakers will act according to a feedback rule that determines outcomes for the endogenous variables. That feedback rule is Markovian in the sense that it determines outcomes as a function of the minimum number of variables that can determine equilibrium in period t: the state vector plus the value of today's shocks. Today's policymaker takes the feedback rule as given.
- We use the first order conditions of the policymaker's problem to solve for a fixed point: the future policymaker's feedback rule and today's decisions have the same form.

This protocol is consistent with the implementation of the numerical simulations.

In the baseline policy configuration there is a single (monetary) policymaker, setting the short-term policy rate (i) to minimize the 'flexible inflation targeting' component of the welfare-based loss function. Though the model contains endogenous state variables, none of these state variables constrain the ability of future policymakers to deliver the optimal output-inflation allocation by an appropriate setting of the nominal interest rate. As a result, time-consistent monetary policy satisfies the static trade-off condition (11).

When the LTV tool is active, we assume that the timing protocol above is modified so that, within each period t, the macro-prudential policymaker sets θ_t before the monetary policymaker sets i_t.[42] Once again, the endogenous state variables do not constrain monetary policymakers from delivering the optimal output-inflation allocation by an appropriate setting of the nominal interest rate. To solve for optimal time-consistent policy with active LTV, we therefore add the static trade-off condition (11) to the set of linearized constraints derived in Appendix D. The macro-prudential policymaker then sets θ_t to minimize $\mathcal{L}_t^{MaP}$ taking the behavior of the monetary policymaker as a constraint.

E.2 Dealing with Occasionally Binding Constraints

We follow the approach in Holden and Paetz (2012), which is very similar to 'OccBin' (Guerrieri and Iacoviello, 2015) and particularly convenient in our case. Let x_t denote the vector of endogenous variables. We can rewrite the equations that characterize the solution of an optimal policy problem (equilibrium conditions describing the private sector behavior plus first order conditions of the optimal policy problem) as

$$H_F \mathbb{E}_t x_{t+1} + H_C x_t + H_B x_{t-1} = \Psi_\epsilon \epsilon_t + \Psi_\delta \delta_t. \tag{52}$$

The matrices H_F, H_C, and H_B collect the coefficients on the endogenous variables. The matrix Ψ_ϵ collects the coefficient on the exogenous shocks, which are in the vector ϵ_t. The vector δ_t contains the shocks that we introduce to impose the occasionally binding constraints, weighted by the matrix Ψ_δ. We add these shocks to the model equations with occasionally binding constraints.

Consider for example the borrowing constraint. We include to its expression the 'shock' δ_t^d. When the borrowing constraint binds, $\delta_t^d = 0$, so that $d_t^b = (1-\gamma_d)\left[\theta_t + q_t + (1-\xi)\tilde{h}_t\right] + \gamma_d\left(d_{t-1}^b - \pi_t\right)$, and the borrowing constraint determines the level of debt. When the borrowing constraint is slack, we choose $\delta_t^d > 0$ so that the multiplier on the constraint is exactly zero ($\mu_t = 0$ or $\tilde{\mu}_t = -\tilde{\mu}^{ss}$). In that case, $d_t^b < (1-\gamma_d)\left[\theta_t + q_t + (1-\xi)\tilde{h}_t\right] + \gamma_d\left(d_{t-1}^b - \pi_t\right)$, and the level of debt is less than the borrowing constraint.

E.2.1 Comparison with OccBin

As mentioned, our approach shares several similarities with 'OccBin' (Guerrieri and Iacoviello, 2015). For example, when the collateral constraint does not bind, the shock δ_t^d ensures that the Lagrange multiplier μ_t is exactly equal to zero. Adding this shock is effectively analogous to the OccBin approach of defining different sets of model equations that apply whether the constraints are binding or not. One advantage of our approach is that it scales up easily as the number of occasionally binding constraints grows.[43] In addition, our approach also allows us to check for the uniqueness of the solution.

[42] Both policymakers make their decisions after the shocks have been revealed, but before private agents make their decisions.

[43] Incorporating N occasionally binding constraints using OccBin requires specifying 2^N alternative sets of model equations, whereas in our approach we need to add N 'shocks' (and possibly up to N auxiliary equations/variables such as d^{gap}).

Practically, to solve for the values of δ_t (with $t = 1, \dots$) that impose the occasionally binding constraints, we use the rational expectations solution of the model (52)[44]

$$x_t = Bx_{t-1} + \Phi_\epsilon \epsilon_t + \sum_{i=0}^{\infty} F^i \Phi_\delta \mathbb{E}_t \delta_{t+i} \tag{53}$$

where B, F, Φ_ϵ and Φ_δ are functions of the coefficient matrices H_F, H_C, H_B, Ψ_ϵ and Ψ_δ in (52). The solution in (53) is valid for any expected shock sequence $\{\delta_{t+i}\}_{i=0}^{\infty}$, as long the shocks do not increase at a rate faster than (the inverse of) the maximum eigenvalue of F. This point is important because our solution must cope with the fact that the equilibrium in period t may be affected by the expectation that the constraints are binding in period(s) $s > t$.

E.2.2 Incorporating Bounds on Policy Instruments

In the same vein as for the collateral constraint, we can append a 'shock' δ_t^i to the flexible inflation targeting criterion (11) to impose the zero bound on the nominal interest rate for our baseline ('flexible inflation targeting') policy assumption. This approach works well when the behavior of the model with a binding constraint is equivalent to its behavior with the targeting rule featuring the time-varying shock.[45]

More generally, however, discretionary solutions with instrument bounds give rise to first order conditions with time-varying coefficients. In particular, the coefficients that capture the marginal effects of the current state variables on expected future control variables will not be constant during a period in which the policy instruments are constrained. The reason is that the instrument constraints alter the ability of the policymaker to affect the current state of the economy.[46] In the context of the simulations we consider in the text, this consideration applies to the assumption that there is an upper bound on θ_t: we assume that the LTV limit cannot exceed 100%.

To deal with this issue, Brendon et al. (2011) develop an algorithm to solve for the equilibrium allocations of a linear model subject to instrument constraints under perfect foresight. The algorithm casts the problem into a discrete time dynamic programming problem, creating a set of first order conditions that account for the number of periods that the instrument bound(s) are expected to bind. This approach therefore generates coefficients (the matrix of derivatives of expected future controls with respect to current states) that vary during the period over which

[44] We obtain the rational expectation solution of the model with the Anderson and Moore (1985) algorithm.

[45] If we assumed that the bounds on policy instruments never bind, the marginal effects of allocations at date t on expected outcomes at date $t + 1$ would incorporate (in equilibrium) policy responses that are a linear function of the state vector.

[46] For example, consider the marginal effect of debt on expected inflation. When unconstrained by the zero bound, changes in the short-term nominal interest rate affect debt via the borrowers' budget constraints, and hence future inflation (and other endogenous variables). However, when constrained by the zero bound, the marginal effect of changes in current debt on expected inflation do not include any response by the policymaker.

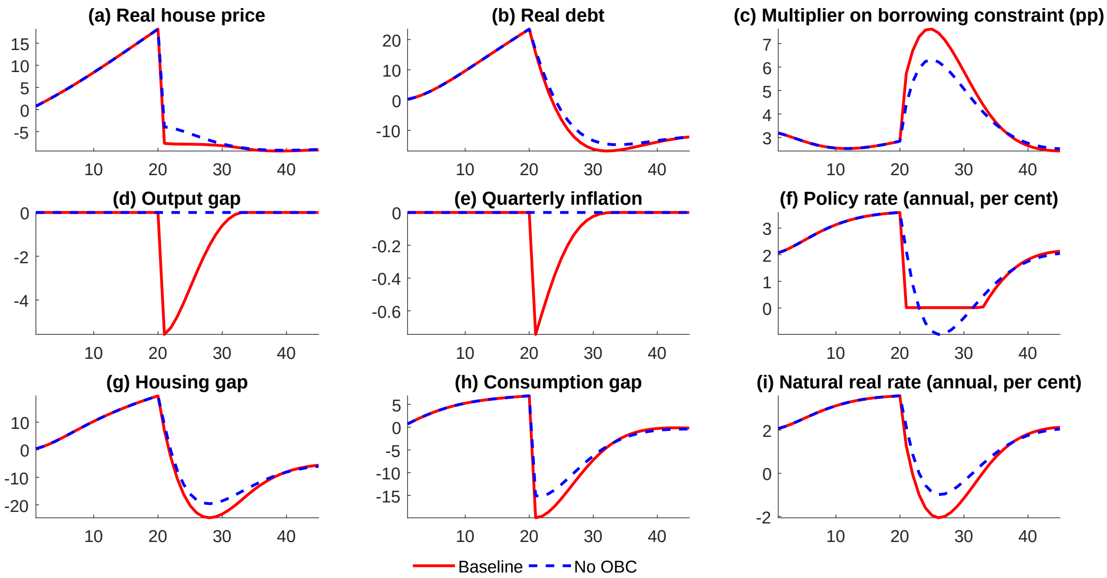

Figure 8: Credit supply scenario under flexible inflation targeting with (solid red line) and without (dashed blue line) occasionally binding constraints.

the instruments are constrained. We use this algorithm to compute the equilibrium in our model in the relevant cases.[47]

E.2.3 The Baseline Simulation and OBCs

To simulate the model, we first assume that none of the occasionally binding constraints binds. We produce this first simulation by setting $\delta_t = 0\ \forall t$. Then, from a given initial condition x_0 and a realization of the shocks ϵ_1, we compute $x_t = Bx_{t-1} + \Phi\epsilon_t$ for $t = 1, \ldots, H$, for some simulation horizon H.

With the initial simulation in hand, we then check whether the relevant variables violate the assumption that the constraints never bind. For example, we check whether the implied trajectory of the multiplier on the borrowing constraint is always positive ($\tilde{\mu}_t > -\tilde{\mu}^{ss}\ \forall t$) and whether the path of the policy rate is always positive ($i_t > i^{ZLB}\ \forall t$). If the initial simulation violates any of these assumptions, we invoke the quadratic programming procedure discussed in the previous section to enforce the occasionally binding constraints.

To illustrate the effects of the occasionally binding constraints, Figure 8 shows the outcome of our baseline credit supply scenario when the policymaker pursues a flexible inflation targeting monetary policy rule. The figure shows two variants of the simulation. The dashed blue lines ignore the occasionally binding constraints. In this case, the nominal interest rate and the multiplier on the borrowing constraint can in principle take negative values (the former does,

[47] We are grateful to Matt Waldron for helpful discussions on these issues and for sharing his code to implement the algorithm in Brendon et al. (2011).

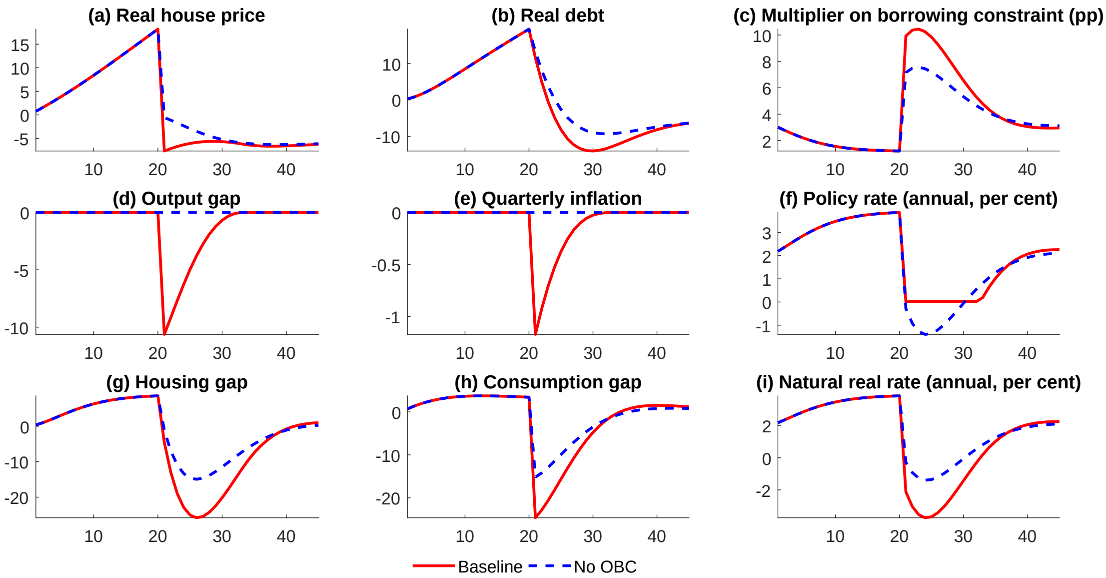

Figure 9: Housing demand scenario under flexible inflation targeting with (solid red line) and without (dashed blue line) occasionally binding constraints.

the latter does not). The solid red lines replicates the simulation that respects the occasionally binding constraints reported in the text.

The results demonstrate the importance of applying the occasionally binding constraints. When disregarded, the policymaker is able to fully stabilize the output gap and inflation. However, achieving that stabilization requires quite large fluctuations in the nominal interest rate. Indeed, the collapse in housing demand generates a long period in which the nominal interest rate is negative. The distributional consequences, however, remain severe, with the consumption and housing gaps both falling by about 15%, although less than when the ZLB binds.

When we impose the constraints, the monetary policymaker cannot stabilize the output gap and inflation following the deleveraging shock. The economy enters a recession and inflation falls sharply while the nominal interest rate remains constrained by the ZLB.

Figure 9 repeats this analysis for the housing demand shock variant of the experiment. The same pattern emerges. The reversal of the housing boom causes the ZLB to bind so that a recession occurs (solid red lines). Allowing the policy rate to become negative—and track the natural rate r_t^n—avoids the recession (dashed blue lines).