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THE HALF LIFE OF ECONOMIC INJUSTICE

David K Miles

FINANCIAL ECONOMICS AND MACROECONOMICS AND GROWTH

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Elizabeth by the grace of God Queene of England, do give and graunt to our trustie and welbeloved servant Sir Humfrey Gilbert of Compton...and to his heires and assignes for ever, free libertie and licence from time to time and at all times for ever hereafter, to discover, finde, search out, and view such remote, heathen and barbarous lands, countreys and territories not actually possessed of any Christian prince or people...and the same to have, hold, occuppie and enjoy to him, his heires and assignes for ever

Queen Elizabeth I charter of 1578 to Sir Humfrey Gilbert granting him permission to own all lands he found in North America.

This land is your land. This land is my land

From California to the New York island;

From the red wood forest to the Gulf Stream waters

This land was made for you and me.

Woody Guthrie

1   The half life of economic injustice

There seems little doubt that in the past of nearly all countries a great many durable assets were acquired by means that we would judge unfair. The history of ownership of much of the land of many countries is littered with examples of one group forcibly taking control of areas. Few people would accept that Queen Elizabeth I of England had justice on her side in granting to Sir Humfrey Gilbert (and his half brother Sir Walter Raleigh) the right to ownership in perpetuity of any land he found in America not occupied by Europeans. Backing the winning side in a battle for control of the state - picking the person who would become king - has been a common route to amassing capital. But for many countries these things happened a long time in the past - many decades, often several centuries, ago. Is it plausible that such unfairness in acquisition of assets from the past could still have a material impact on income and wealth today? This paper addresses that question. It presents a framework for assessing the extent to which unfairness in ownership of assets in the past affects unfairness in incomes and wealth today. It sets out to explore conditions under which unjust acquisition of assets in the relatively distant past can have a lasting, significant effect upon incomes and wealth within countries where unjustly acquired assets were used. It shows that in some cases unfairness dwindles over time so that even if the pattern of ownership of assets 100 years ago was dramatically unjust it has a small impact upon ownership today. With illustrative calibrations I show that the half life of the effects of many types of unjust acquisition of assets is relatively low. But things can be very different when we consider certain assets - notably human capital - where some parts of the stock could also be considered as having been acquired unjustly. Things can also be very different if savings rates of those with little wealth are consistently very low.

The question I address could be put simply: how much of today's income – GDP – is a result of economic transactions which are a direct legacy of past unjust acquisition of wealth (or capital)? This question is central to the judgements people make now on the legitimacy of market outcomes that reflect the current pattern of ownership of assets. There is a long tradition of questioning the

justice of outcomes in market economies based on the argument that assets were unjustly acquired in the relatively distant past and that this undermines the legitimacy of the distribution of much of wealth and incomes today. This view is exemplified in the work of Marx but finds expression in many (non-Marxist) thinkers; it is one of the justifications for highly re-distributive taxation.

To answer the question requires two things: first, a principle to determine what is, and what is not, just acquisition of wealth or a just source of income; second, a means of using that principle to estimate what fraction of wealth and income today is unjust because of past unfairness. I use a principle put forward by Robert Nozick to provide the first of these things and then use calculations based on standard neoclassical models of economic growth to illustrate its implications for the scale of unfairness today. The issue I address is fundamental to the legitimacy of the pattern of income and wealth today. It has received most attention from philosophers and historians - see, for example, (Waldron 1992) and (Linklater 2013) - but rather less by economists. There is a substantial literature on how imposition of institutions by foreign countries can have long run - and damaging - effects on former colonies (see for example (Acemoglu et al. 2001) and (Acemoglu et al. 2002)). But the question of whether assets appropriated unjustly have a lasting impact on the income and wealth within the country where those assets are used is a different one. That question of how past unjust acquisition within an economy affects the pattern of income and wealth today is an economic question with significant political and moral implications.

A simple example: Suppose at some point in the past within a country some real assets – physical productive assets like cattle, machines, land or buildings – were unjustly acquired. Assume the capital was looted from another country. That may have happened several decades, maybe centuries, ago. Suppose also that we agree that those assets can be described as unjust wealth and income derived from those assets as unjust income. How much of today's assets and income is unjust as a result of that past unjust acquisition is less clear. One way towards an answer is to assume a process for growth and production and see what impact past acquisition of assets has on wealth and income today.

Suppose total output is produced by an aggregate Cobb Douglas production function combining capital (which depreciates) and labor (which grows with population and also labor enhancing technical progress). Assume also a fixed saving rate out of all income. This is a Solow growth model. With constant rates of depreciation (at δ), saving (s), labor force growth (n) and labor enhancing productivity (g) the economy will converge to a steady state capital income ratio (K/Y) of $s/(\delta + n + g)$ and both income and capital will then grow at rate $(n + g)$. Either with or without those stolen assets then the economy will eventually converge on the same capital stock and income level. In that sense in the limit the impact of the past acquisition of the unjust assets on capital and income becomes zero. The speed of convergence to that steady state affects how much of the past unjust acquisition of assets affects incomes and wealth (capital) at a given point later. That speed of convergence depends on s, n, g and δ. It also depends on the share of capital income in GDP (π).

Suppose that the level of capital without the theft of assets was only 30% of steady state K (given L at the time) and that the amount of capital stolen was large and enough to get to steady state K/L. Thus immediately after the unjust acquisition, just capital was only 30% of total capital

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and unjust capital was 70% of capital. We can readily calculate the level of income and capital T years later both including and excluding the unjust acquisition of assets[1].

As an illustration suppose we take s=0.18; n=0.005; g=0.015; δ =0.03 and $\pi = 0.3$. This set of parameters is not unreasonable for developed economies (see below). Then 50 years after the theft of assets, income and capital had there been no past injustice are approximately 95% and 86% respectively of what they are with it. After 100 years the ratios are approximately 99% and 97%.

This simple calculation might lead one to conclude that gross injustices in the way that assets were acquired a few generations ago has ceased to be of much significance. They will have generated real benefits to past generations who enjoyed the fruits of expropriated assets but have largely ceased to affect living standards today. Of course it is clear that in assuming there is a unique steady state to which the economy converges then we know without any calculations that in this example in the long run injustice shrinks to zero. The calibration serves to show that with this model convergence seems likely to be rapid enough to mean historic injustice has shrunk to small amounts within a lifetime. But how are things different if assets are unjustly acquired from the domestic population rather than stolen from foreigners, as in this simple example? How much difference does it make if income earned - and then saved - from unjust assets increases both physical *and* human capital? How significant are different assumptions about the substitutability between factors? How significant are assumptions such as a common saving rate out of capital and labor income? More fundamentally, what philosophical principles would allow us to identify these ratios as revealing the scale of just and unjust income today?

This paper explores all these issues.

I continue to use neoclassical growth models where there is a unique steady state to which the economy converges. But it turns out that this does *not* mean that half lives of injustice need be low, or even finite. I show that there are two limiting cases under which injustice never fades at all - unlike in the example above where it falls quite fast. I show that the half life of injustice tends to the infinite when assets are unfairly expropriated by one group within an economy from another in the same economy and when savings from those only with labor income are very low. That also happens when the remuneration of labor largely represents a return to human capital and when some part of human capital is accumulated from unjust incomes. But I also find that if injustice in the distribution of wealth and income only reflects ownership of physical assets, then unless saving from those who have little capital is very low the injustice generally fades at a pace which makes it relatively insignificant within a few generations. Things can be significantly different - and injustice last very much longer - if there can be injustice in the ownership of human capital as well as in physical capital, which is partly a philosophical question.

Since saving out of labor income and the significance of human capital have likely changed a lot over time it is possible that the rate at which injustice fades may be very different now from what it was in the past. This is an issue I explore.

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[1] The ratio of just capital to total capital at time T is the level of capital which would have been reached without the initial stolen capital relative to the (steady state) level that was acheived with the stolen assets. It is given by: $[(\lambda_0{}^{1-\pi} - 1)e^{-(1-\pi)(n+g+\delta)T} + 1]^{1/(1-\pi)}$

where λ_0 is the inital ratio of unjust (i.e. stolen) capital to total capital. If at time t this ratio is λ_t then with Cobb Douglas production the ratio of just income to total income is λ_t^{π}

I begin with the philosophical issue of what just and unjust ownership of assets might mean. I use principles that draw on Nozick's ideas to build a framework for modelling how one form of economic injustice evolves. I then implement the ideas using first an overlapping generations growth model where output is produced with physical capital and labor. I consider different assumptions about how (and crucially from whom) assets may have been unjustly acquired in the past and also consider the effects of different savings rates out of different sources of income. In Section 4 I introduce human capital before drawing conclusions.

2   The notion of just income and wealth

In *Anarchy State and Utopia* (Nozick 1974) Robert Nozick puts forward 3 rules for a just distribution of resources:

1.   Justice of acquisition: If you acquired something justly, then it is just to own it

2.   Justice of transfer: If someone who justly owns something freely transfers that property to another, then it is just for that other person to own it

3.   Rectification of injustices: If someone unjustly "owns" something (by unjust acquisition or transfer), then the situation ought to be rectified (e.g., by restoring the property to its rightful owner).

The principles are controversial; they locate justice, or fairness, in the process which generated today's pattern of income and wealth - not by reference to the distribution of resources itself. The Nozick principles are not consistent with other notions of distributive justice – most obviously those put forward by John Rawls (Rawls 1999), which focus on the distribution of resources and specifically on whether inequality is in the interests of the least well-off group. Amartya Sen calls Nozick's principles (and those of Rawls) an example of a transcendental theory of justice (Sen 2009) - that is one that puts forth a single notion of a completely just outcome and which does not allow competing and partially conflicting notions of what is fair. He argues that such transcendental theories do not help much with real world problems of what is better or worse (more or less just). The Nozick principles are also clearly not consistent with the idea that unequal outcomes for the distribution of economic resources that reflect factors beyond the influence of individuals are not justifiable (see (Dworkin 2002)) - the guiding idea behind so called *luck egalitarianism*.

It is, however, not necessary for our purposes to believe that the Nozick principles are an exhaustive (necessary and sufficient) set of conditions to generate just outcomes. You could take a much more pluralistic view of justice - as does Sen - and still accept the central idea used in this paper, namely that if we can agree that some assets have been acquired unjustly the income derived from them is unjust, as is the ownership of assets acquired out of saving from that income[2]. This principle does not say that this is the only source of unfairness in the distribution of resources. But it does rest very much on the Nozickian idea that how the distribution of income and wealth came about is central to the notion of justice, or fairness. Merely noting that income and wealth is unequal - or for that matter equal - is insufficient information on which to base judgements on its justice or

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[2]This is recognised in the laws of many countries, for example by confiscating assets acquired out of the proceeds of criminal activity.

fairness. (Nozick 1981) brings out the essential idea when he describes a historical theory of justice - such as he advocates in *Anarchy State and Utopia* - as one that:

"..holds that a distribution is just, whatever its pattern, if it grows out an earlier just distribution in specified ways. One closest relative version of this theory adds that the current distribution also must not be as closely related ...to previously unjust distributions"

In this paper I rely on the notion that if you acquired something unjustly the income that stems from that asset is unjust, as is the income subsequently earned from assets acquired from saving out of that unjust income. To be clear, this does not imply that there are no other causes of injustice in distribution besides the lingering effects of past unjustly acquired assets. It does mean that one component of unjust outcomes in the ownership of wealth and incomes reflects past unjust acquisition of assets. Knowing how big that component is of what we might call unjust economic outcomes is significant given the attention it has been paid in discussions of unfairness. Much of the radical critique of the differences in incomes between developed and less developed countries rests on the belief that past imperial appropriation of resources set back development of poor nations and kept them poor - the belief that we are rich because they are poor.

I am asking how great is the total stock of unjust assets and how much of aggregate income is derived from that stock. Focusing on the aggregate amount of unjust wealth and income, and its evolution, is directly relevant to the question of how much of today's total wealth and income is due to ill-gotten gains from the past. I do not focus on how the stock of unjust capital is distributed - it could be that just one tyrant holds it all or that a large proportion of the population used their weight of numbers to dispossess a minority of their assets. It is not clear what one should make of the way in which unjust assets are distributed. Should one count as less unjust a situation where expropriated assets are more, rather than less, equally distributed amongst the expropriators? In focusing on aggregate injustice in overall wealth and incomes this question does not arise. In focusing on aggregates I also do not attempt to model the buying and selling of assets that have in the past been unjustly acquired. It is in the spirit of Nozick to consider such transactions as besides the point if they do not rectify past injustice. The ownership of an asset (and the income from it) that has been unjustly acquired by agent X is not transformed into just ownership by its being bought by agent Y out of income fairly earned by Y. The point is it was not X's to sell. Creation of *new* assets from saving out of Y's legitimate income that might be otherwise indistinguishable from existing assets unjustly acquired in the past is, in contrast, just. The way in which I estimate the evolution of just and unjust income and wealth reflects this principle.

The Nozickian principles do not imply that there should be a minimal state which does little re-distribution. That is because the third principle (of rectification of injustices) may well require a state to intervene to shift the distribution of income and wealth and that might mean significant tax levied on some elements of income. If a significant proportion of today's stock of capital assets reflects unjust acquisition then, based on Nozick's principle of the rectification of injustices, a significant tax on the income derived from it could be warranted. Knowing how much of today's wealth and incomes comes from past unjust acquisition of assets (not subsequently rectified) is clearly relevant here.

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Using the Nozick principles, and in the absence of rectification required by principle 3, we can ask how much injustice might there be in today's income and wealth. Clearly that depends on whether wealth (or capital) used today was justly acquired in the past. There is no doubt that in the past much wealth was indeed unjustly acquired – it is hard, for example, to view the ownership of most land in Tudor England as just when much of it reflected whether powerful families backed one or other challenger to the throne in past armed struggles and of how well the church's interests matched those of the king[3]. Ownership of land across significant parts of the USA or Australia in the mid nineteenth century is equally hard to see as being the result of just acquisition – much of it was certainly not acquired in conformity with Nozick's principles 1 and 2.

Given that capital (including land) was not very justly distributed a century or more ago in many developed economies it is of significance to assess the implications that has for the proportion of income and wealth that is today unjust. In the next several sections I explore that question using a standard model of economic growth in which output is produced with capital and labor which are paid their marginal product. Initially I assume that labor is owned by individuals who always have rights to all the wages and salaries they earn from providing it, unless some part of capital that boosts wages is stolen from outside the economy. When injustice involves capital being expropriated from one group within an economy from another within the same economy then there is no unjust labor income [4]. Although this appears unexceptional it is not entirely self-evident – labor income obviously depends on human capital and its acquisition can depend upon income from physical assets that have been unjustly acquired. I return to this issue below but initially assume that all labor income is just in the Nozickian sense – that is that the labor power of anyone is theirs to use as they see fit and that wages earned from hiring it out are rightfully their own.

Monopoly power plays no role in this paper - I assume factors get paid their marginal product; it is the ownership of productive factors that is unfair not the way they are compensated. So if oligarchs steal factories and land and oil fields, the returns to those assets is assumed to be the same as if they had been allocated widely across the population. This ignores oligarchs' influence over the state which might allow monopoly power on pricing. There are also many other forms of injustice that affect economic outcomes and about which the models used here are silent; discrimination on grounds of race, gender and religion have had a significant impact on the distribution of income and wealth - I focus only on injustice from the past expropriation of assets and not from ongoing discrimination on such grounds.

There is one sense in which the models used here are likely to overstate the half life of injustice. This is because I do not allow for the tax and welfare system to redistribute resources in ways that partially rectify past economic injustices. The question I address is what might be the lasting impact of injustices on the assumption that in competitive markets factors get paid their marginal products without redistributive fiscal policies.

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[3] (Linklater 2013) says that around 60% of the twelve million acres of farmland in England on the eve of the Tudor era in the mid fifteenth century was owned by the crown, by the church and some 30 dukes, earls and barons. After Henry Tudor won the battle of Bosworth, where King Richard III was killed, and established the Tudor dynasty, there was a major shift in ownership of assets in England. There was a further massive transfer of assets from church to king under his son Henry VIII some forty years later.

[4] I do not explicitly model the impact of slavery - self evidently a gross injustice. But the lingering effects of past slavery can create ongoing injustice in part because some part of the capital that existed in the past was created from the fruits of slave labor. The analysis here is relevant to that.

The models I use do not have multiple equilibria; nor is there a role for luck. Both of those things may generate long lasting unfairness and it might be assumed that by ignoring them (using a model with a unique and stable steady state) one precludes very long-lived effects from historic injustice. This turns out not to be the case. One of the aims of the paper is to explore the conditions under which initial injustice never fades even though an economy might have a unique steady state. I do, however, have little to say on how institutional structure may embed unfairness; in what follows I assume that institutions are such as to preserve current ownership rights on property (while not rectifying past injustice) and are consistent with factors being paid their marginal products.

3   The Model

A natural way to model an economy where assets vary across people and over time is in a setting with overlapping generations who have accumulated stocks of assets that vary with their age. I use a version of the Blanchard OLG model (Blanchard 1985) where there is a probability of death in any period that is the same for all agents, denoted d. The population is large enough so that its aggregate structure is predictable. The length of life of each agent is uncertain and varies greatly; its expected value is $1/d$. I assume that agents maximize a lifetime utility function where utility in each period is a function of consumption. An agent born at time s maximizes

$$\sum_{t=s}^{\infty}[(1-d)/(1+\rho)]^{t-s}[C_{st}^{1-\phi}/1-\phi] \tag{1}$$

ρ is the discount rate (rate of time preference); $1/\phi$ is the intertemporal elasticity of substitution. I denote the population at time t by P_t. Births at time t are assumed to be $(n+d)P_{t-1}$ so that the population (and labor force) grow at constant rate n. If we normalize the number of births at time t=1 to be 1 then P_1 is given by $\sum_{j=0}^{\infty}[(1-d)/(1+n)]^j = (1+n)/(n+d)$. Births and population grow at rate n; if $n=0$ then births are constant at 1 and population is constant at $1/d$. P_t is equal to $(1+n)^t/(n+d)$.

There is time-related labor-enhancing productivity growth at rate g. There is age decay in labor supplied at rate v - we can think of this as either a decline in hours worked with age or in the effectiveness with which people work. The labor supply of each agent measured in effective units grows at rate $(1+g)/(1+v)$. With steady population growth the age structure is unchanging so that the age-related decline in individual labor supply is offset by the birth of new agents. In aggregate the effective labor force grows at rate $n+g$.

I assume insurance-saving contracts are offered at actuarially fair rates so that a unit of saving at time t pays out an amount conditional on an agent surviving into the next period of $(1+r_t)/(1-d)$ where the return on assets during time t is denoted r_t . If an agent does not survive the financial intermediary keeps the wealth of $1+r_t$; this means the payouts exactly match its available resources. The period to period budget constraint for an agent born at time s, whose units of labor supplied at time t is $1/(1+v)^{t-s}$, is:

$$A_{st+1} = (A_{st} + w_t/(1+v)^{t-s} - C_{st})(1+r_t)/(1-d) \qquad (2)$$

A_{st} is the stock of (non-human) assets at period t of the agent born in period s ; w_t is the wage per unit labor supplied at time t. On a balanced growth path the interest rate is constant (at r) and the wage per unit supplied of labor grows at rate g. Imposing a transversality condition (that wealth does not explode) gives the lifetime budget constraint at time s, when effective labor supply of a member of the new cohort is 1, of

$$\sum_{t=s}^{\infty} C_{st}[(1-d)/(1+r)]^{t-s} = A_{st} + w_s \sum_{t=s}^{\infty}[(1+g)(1-d)/((1+v)(1+r))]^{t-s} \qquad (3)$$

The first order condition from this optimization problem implies:

$$C_{st}/C_{st-1} = [(1+r)/(1+\rho)]^{1/\phi} \qquad (4)$$

combining these two equations implies

$$C_{st} = [H_{st} + A_{st}][1 - (1-d)(1+r)^{1/\phi-1}(1+\rho)^{-1/\phi}] \qquad (5)$$

Where H_{st} is human capital at time t for the person born at time s. This is the value of future wages discounted at the rate $(1-d)/(1+r)$ and is given by

$$H_{st} = (w_t/(1+v)^{t-s})/[1 - (1+g)(1-d)/((1+v)(1+r))] \qquad (6)$$

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A key feature of this model is that there is a common propensity to consume out of comprehensive wealth (the sum of human capital and non-human assets) for all agents. This will generate a common saving rate out of labor income across agents that is equal for people of different ages and also a common saving rate out of income from assets - though the two savings rates are not generally equal to each other. Because consumption is linear in human and other capital it aggregates easily so that the way the aggregate economy evolves over time is tractable. Births at date 1 are normalized to 1 so births at date t are $(1+n)^{t-1}$. For any aggregate variable X_t we define it as

$(1+n)^{t-1} \sum_{s=t}^{-\infty} X_{st}[(1-d)/(1+n)]^{t-s}$

Aggregate labor income at time t is given by:

$(1+n)^{t-1} \sum_{s=t}^{-\infty} w_t[(1-d)/((1+n)(1+v))]^{t-s} = w_t(1+n)^{t-1}[1/\{1 - (1-d)/((1+n)(1+v))\}]$

The aggregate labor supply at time t, denoted L_t, is $(1+n)^{t-1}/\{1 - (1-d)/((1+n)(1+v))\}$. If $v = 0$ this is equal to population; if $v > 0$ then L is proportional to P but smaller than it.

I assume economies are closed so that aggregate assets held by individuals (A_t) equal the aggregate capital stock (K_t). Capital depreciates at rate δ and the gross rate of return on capital is $\delta + r$.

Aggregate income - GDP - is given by $L_t w_t + K_t(r + \delta)$ where $K_t = (1+n)^{t-1} \sum\limits_{s=t}^{-\infty} A_{st}[(1-d)/(1+n)]^{t-s}$

Pulling together these results we can now write aggregate savings, the difference between GDP and consumption, as:

$S_t = K_t(r + \delta) + L_t w_t - C_t = K_t(r+\delta)[1 - \{1 - (1-d)(1+r)^{1/\phi-1}(1+\rho)^{-1/\phi}\}/(r+\delta)]$
$+ L_t w_t[1 - \{1 - (1-d)(1+r)^{1/\phi-1}(1+\rho)^{-1/\phi}\}/(1-(1+g)(1-d)/((1+v)(1+r)))]$

This simplifies greatly if we have log preferences $(\phi = 1)$ and if g and v are approximately equal. In this case :

$S_t = K_t(r+\delta)[1 - [(\rho+d)/((1+\rho)((r+\delta))] + L_t w_t[1 - (1+r)(\rho+d)/((1+\rho)(r+d))]$

In this case the balanced growth path saving rate out of gross capital income, which I will denote s_π, is $1 - (\rho+d)/((1+\rho)/(r+\delta))$ and the saving rate out of labor income, denoted s_L, is $1 - (\rho+d)(1+r)/((1+\rho)(r+d)) = (r-\rho)(1-d)/((1+\rho)(r+d))$. Blanchard shows that $r > \rho$ if $d > 0$ which means that the propensity to save out of labor income is positive. Since it is plausible that $\delta > d$ (i.e. on average people live longer than machines) the propensity to save out of capital income would be higher than the propensity to save out of labor income. So savings rates out of labor and capital income are positive but not necessarily equal.

In the more general case (where $\phi \neq 1; v \neq g$) the OLG model with a constant probability of death implies that the relation between consumption, the present value of future labor incomes and non-human wealth (capital) remains linear. This allows straightforward aggregation and generates aggregate saving which is an additive function of aggregate capital income and of aggregate labor income, which is of the form assumed in the models used in this paper.

I assume output is produced with a Constant Elasticity of Substitution production function which combines labor and capital. Markets for capital and labor are competitive and factors paid their marginal product. Aggregation from production at the level of individual enterprises is possible and I focus on total output produced from aggregate capital and labor; the fraction of the aggregate stock of factor inputs that has been unjustly acquired is central to the analysis, though such factors are indistinguishable from justly acquired factors in terms of their productivity. Capital is accumulated when gross investment (which equals saving in a closed economy) exceeds depreciation. Initially I assume all capital is physical capital. To keep the capital to effective labor ratio constant net investment must also match the growth in the labor force plus productivity growth. On a balanced growth path the saving rates out of capital income (s_π) and out of labor income (s_L) will be constant, though as noted above not necesasrily equal. Since the results on unfainress turn out to be not very

sensitive to whether we have reached a balanced growth path, I will assume these two saving rates are constant from now on. Output (GDP) at time t is denoted Y_t and is given by:

$$Y_t = \left[\alpha K_t^\beta + (1-\alpha)L_t^\beta\right]^{1/\beta} \tag{7}$$

where $1/(1-\beta)$ is the elasticity of substitution between capital (K) and effective units of labor (L); α is a share parameter.

The aggregate effective labor force evolves according to:

$$L_t = L_{t-1}(1+n+g) \tag{8}$$

In a later section I will introduce human capital that adds to the stock of effective labor at a rate that depends upon investment.

The share of returns to capital in total output (π) is:

$$\pi_t = \alpha(K_t/Y_t)^\beta \tag{9}$$

The share of labor income is $(1-\pi)$.

A special case of the CES function is when the elasticity of substitution is unity, $\beta = 0$, in which case α is the share of capital income and output is produced with the Cobb Douglas function $Y_t = K_t^\alpha L_t^{1-\alpha}$

Capital evolves according to:

$$K_t = K_{t-1}(1-\delta) + s_\pi \pi_t Y_{t-1} + s_L(1-\pi_t)Y_{t-1} \tag{10}$$

The interest rate in the economy (r_t) is the return to capital net of depreciation, which is:

$$r_t = \alpha(Y_t/K_t)^{1-\beta} - \delta \tag{11}$$

In a steady state capital, income and the effective stock of labor all grow at rate $n+g$ and the capital stock satisfies

$$K_t(n+g+\delta) = s_\pi \pi_t Y_t + s_L(1-\pi_t)Y_t \tag{12}$$

The steady state capital to income ratio, $(K/Y)^{ss}$, is:

$$(K/Y)^{ss} = s/(n+g+\delta) \tag{13}$$

the steady state level of the capital to labor ratio is:

$$(K/L)^{ss} = (s/(n + g + \delta))^{1/(1-\alpha)} \qquad (14)$$

where s is the weighted average of s_π and s_L with weights $\pi, 1 - \pi$. In steady state π is constant and so is s. (If $s_\pi = s_L = s$ or if $\beta = 0$ then the aggregate saving rate is constant even out of steady state). There is a direct relation between the saving rate and the steady state net rate of return. With an aggregate saving rate s then in steady state $r_t = r = \alpha(s/(n + g + \delta))^{\beta-1} - \delta$.

When we calibrate the model we chose a value for s which gives a steady state net of depreciation return that is in line with the empirical evidence and which is consistent with the first order conditions from the OLG model outlined above and plausible values for ρ and d.

I assume that at some initial time t=0 the capital stock is made up of a stock of capital which has been unjustly acquired, K_0^U, and a stock of capital which has been justly acquired, K_0^J

$$K_0 = K_0^U + K_0^J \qquad (15)$$

Let the proportion of capital which has been unjustly acquired be λ_0 so that:

$$K_0^U = \lambda_0 K_0 \qquad (16)$$

The ratio of unjust capital to total capital at time t is λ_t.

The evolution of the stock of just and unjust capital, and the fraction of income that is unjust, depends on how we view income and saving generated as a result of the original stock K_0^U. In the following sections we consider several cases, starting with the one illustrated in the introduction to the paper where unjust capital at time 0 has been unfairly acquired from foreigners and brought to the expropriators's economy to be used. This is a case of expropriation of assets from abroad and the Nozickian principle suggests that the capital *and* labor income derived from the assets is unjust - as is income from future assets accumulated out of savings from unjust income.

3.1 Assets expropriated from foreigners

If assets are expropriated from foreigners then all the income derived from those assets - the direct return on that capital and the extra labor income generated by that capital - is unjust. In this case it is straightforward to compare how capital and income evolves with and without the unjust capital. Initial unjust capital is $\lambda_0 K_0$. Initial just income is:

$$Y_0^J = \left[\alpha((1 - \lambda_0)K_0)^\beta + (1 - \alpha)L_0^\beta\right]^{1/\beta} \qquad (17)$$

Just capital evolves according to:

$$K_t^J = K_{t-1}^J(1 - \delta) + s_\pi \pi_t Y_{t-1}^J + s_L(1 - \pi_t)Y_{t-1}^J \qquad (18)$$

unjust income is defined as:

$$Y_t^U = Y_t - Y_t^J \tag{19}$$

Unjust capital is simply

$$K_t^U = K_t - K_t^J \tag{20}$$

If we start in a steady state then K_t and Y_t grow at rate $(n + g)$

In calculating fair income in period t we are finding the level of output if the productive capital stock was just the fair capital of $(1 - \lambda_t)K_t$; in the Cobb Douglas case, fair labor income is $(1-\pi)(1-\lambda_t)^\alpha Y_t$ and fair capital income is $\pi(1-\lambda_t)^\alpha Y_t$. The expressions are similar for the CES case though capital and labor shares will vary over time until we reach a steady state.

The evolution of just output and just capital is exactly as it would be if the unjust capital was all repatriated at time 0. We can compare income and capital with and without repatriation and those ratios tell us how much of today's capital and income is just - given no historic repatriation.

In this model just income and capital evolve exactly as if all unjust capital was removed at the initial time. (Of course if repatriation of capital had occurred in the past consumption of subsequent generations would have been lower than if there was no repatriation.) Because capital and income converge on the steady state values from any initial level of capital we know that just income and capital in the limit converge on the steady state paths so the impact of initial unjust capital on wealth and incomes will ultimately always shrink to insignificance. The speed with which that happens depends on $n, g, \delta, \beta, s_\pi, s_L$.

For the Cobb Douglas case ($\beta = 0$) there is a closed form expression for the ratio of just to unjust capital at time t. If the initial ratio of total capital at time 0 to steady state capital at that time is denoted χ then the ratio of unjust capital to total steady state capital at time 0 is $\lambda_0\chi$. For the Cobb Douglas case this means that at time t the ratio of just to total capital (in a continuous time version of the model) is then given by:

$$K_t^J/K_t = \left\{ [(((1-\lambda_0)\chi)^{1-\alpha} - 1)e^{-(1-\alpha)(n+g+\delta)t} + 1]/[(\chi^{1-\alpha} - 1)e^{-(1-\alpha)(n+g+\delta)t} + 1] \right\}^{1/(1-\alpha)} \tag{21}$$

and unjust income to total income at time t is

$$Y_t^J/Y_t = (K_t^J/K_t)^\alpha \tag{22}$$

In the case of assets expropriated from foreigners and then used in the expropriator's economy (but not in general) the ratio of unjust income to total income depends only on the overall saving

rate and not on the relative magnitude of saving out of capital and income[5]. This is no longer true if we treat labor income that depends on past acquisition of unjust capital as itself fair - a case we turn to in the next section. In that case the relative size of s_π and s_L turns out to matter a great deal for the rate at which historic unfairness fades.

If initial capital (just and unjust aggregated) is close to the steady state level then the ratio K_t^J/K_t is very close to when χ is set to 1 and for the Cobb-Douglas this means that as the time periods become small:

$$K_t^J/K_t = [((1 - \lambda_0)^{1-\alpha} - 1)e^{-(1-\alpha)(n+g+\delta)t} + 1]^{1/(1-\alpha)} \tag{23}$$

In the tables below I will show how just and unjust capital shares and income shares evolve given some specific values of key parameters. Those parameters are chosen to match broad features of developed economies. Before presenting the results I describe how the parameters are set.

3.1.1 Calibration of parameters

n : US population rose at an average annual rate of around 1.2% in the 100 years from 1911 to 2011. It has grown slightly more slowly recently. For other developed economies (large European economies and Japan) population growth over the past hundred years has been slower than in the US and averages near 0.5% a year. I set population growth at 0.5% a year in the base case and also show the impact of faster growth.

g : Annual growth in labor productivity in many developed economies over the past century has averaged around 2%. But it has slowed recently. In the base case I take labor productivity growth of 1.5% but show the impact of higher and lower growth.

δ_K : On a steady state growth path $\delta_K = (I^k/K) - (n + g)$. Using US data on the average ratio of non residential capital investment to the non residential capital stock since 1929, and using the values of $n + g$ as above, implies a depreciation rate of just above 6%. For the US Davis & Heathcote (2005) use a quarterly value for depreciation of business capital of 0.0136 (annual of around 5.4%). Deprecation on non residential capital of around 6% seems plausible. Residential property depreciates at lower rates. As with non-residential physical capital, on a steady state growth path the depreciation of residential capital is given by the investment to capital stock ratio minus $(n+g)$. Using US data on the average ratio of residential capital investment to the residential capital stock since 1929, and using our values of $n + g$ as above, implies a depreciation rate of only around 1.25% a year. This seems slightly lower than estimates based on the difference between gross and net US residential investment which give a figure near 2%. Fraumeni (1997) reports that in the US structures depreciate at a rate between 1.5% and 3% a year. Van Nieuwerburgh & Weill (2010) use 1.6% in their simulations. Davis & Heathcote (2005) use a quarterly value for depreciation of the housing stock of 0.0035 (annual of around 0.014). Hornstein (2009) suggests a figure of 1.5%.

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[5] When assets are expropriated from foreigners and used in the domestic economy this is exactly true when factor shares are constant (as with Cobb Douhglas). When $\beta \neq 0$ there is a second order effect of the relative size of s_π and s_L on the rate at which injustice fades.

Depreciation on residential structures at 2% a year looks plausible. Land used for residential and other commercial purposes has a lower depreciation rate, but it is probably not zero. Agricultural land will become much less productive if not mainlined so its depreciation rate is not zero but likely lower than for residential property.

Overall a weighted average of depreciation on all physical capital (including land) might plausibly be between 2% and 4%. I take 3% as a central value and I also consider higher and lower values.

α : I set α (the share parameter in the production function) to the typical share of capital in private domestic value added in developed economies in recent years. This figure is around 0.3, (Rognlie 2016).

Using these figures for α, n, g, δ we can assess what rate of of saving s is needed to generate a

rate of return, r, which is plausible given the historical evidence. In our model all assets (land, non-residential capital, structures, housing) generate the same return. Jordà et al. (2017) provide data on the real returns on a range of assets, including claims on corporate (equities and bonds), over the period 1870-2015 for 16 advanced economies. Returns on equities and housing look similar and average about 7% a year - though they are a little lower pre-1950. Bonds generate a lower real return which averages about 2.5% over the whole sample. The equally weighted average of the real return of these three asset classes is close to 6%. Piazzesi & Schneider (2016) show data for the ratio of US house prices to rents that averages about 13 over 1960-2015. The implied yield of about 8% exceeds the real return by depreciation of structures and other property expenses, but real capital gains need to be added; the net figure is probably somewhat under 8%. Returns on land used for non-residential purposes are likely to be lower. A figure of 5-6% seems reasonable for the past weighted average of returns on all real assets. I use a figure of 5%. If the net return is 0.05 and $\delta = 0.03$, we require a saving rate to satisfy: $0.05 + 0.03 = \alpha(K/Y)^{\beta-1} = \alpha(s/(n+g+\delta))^{\beta-1}$

For the Cobb Douglas case this implies $s = 0.1875$ and steady state K/Y is 3.75. The saving rate is a little below the OECD countries average gross capital formation rate relative to GDP since 1960 and also a little below that ratio for the US (both average close to 22% over the period 1960-2017). The implied ratio K/Y is a little low for a comprehensive measure of physical assets. But when we vary β , and adjust s to hit the same target for the rate of return, we can easily generate much higher values for steady state K/Y.

A saving rate of 0.1875 is plausible given the OLG-perpetual youth model. With log utility, and when the effects of declining labor with age is offset by time-related productivity growth $(v = g)$, that OLG model generates an optimal steady state saving rate out of labor income of $1 - (d+\rho)(1+r)/((r+d)(1+\rho))$. Assuming an average adult life of 50 years means $d = 0.02$. To generate a saving rate out of labor income of 0.1875 with an r of 0.05 would then imply a plausible value for the annual discount rate (ρ) of 0.036. In that OLG model the saving rate out of capital income is $1 - (d+\rho)/((1+\rho)(r+\delta))$ With a depreciation rate (δ) of 0.03 the saving rate out of gross capital income would then be 0.32. If the depreciation rate were to be 0.02 $(\delta = d)$ the saving rate out capital income would be 0.23. If the depreciation rate were slightly lower than the mortality rate $(\delta = 0.017; d = 0.02)$ the saving rates out of all income would be equal at 0.1875.

	$\beta = 0$		$\beta = 0.2$		$\beta = -0.2$	
	K_t^U/K_t	Y_t^U/Y_t	K_t^U/K_t	Y_t^U/Y_t	K_t^U/K_t	Y_t^U/Y_t
Time						
t=0	0.70	0.30	0.7	0.37	0.7	0.27
t=20	0.39	0.14	0.44	0.21	0.37	0.11
t=50	0.14	0.05	0.19	0.08	0.13	0.03
t=75	0.06	0.02	0.10	0.04	0.05	0.01
t=125	0.01	0.003	0.02	0.01	0.01	0.002
half life	24	18	29	24	22	16

Table 1: Assumes a common saving rate out of capital and labor income to deliver r=5% at depreciation rate of 0.03; n=0.005; g=0.015; half life is the number of years to reduce injustice to half its initial value

3.1.2 Results - assets expropriated from abroad

Table 1 shows the evolution of unjust capital and incomes using the calibration strategy described above and assuming an initial ($t = 0$) high level of unjust capital relative to total capital. We set that at 0.7, but this is illustrative in the sense that it is the rate of decay of injustice that matters which is almost independent[6] of the initial value of λ. In the Cobb Douglas case unfairness declines fairly quickly. After 50 years the proportion of capital that is unjust falls to 20% of its initial level (from 70% of total capital to 14%). The half life of capital injustice - the time taken for the share of capital that is unjust to fall to half its initial value - is 24 years. The half life of economic injustice - that is the time taken for the proportion of GDP that is unjust to fall to half its initial value - is 18 years.

The lower is the elasticity of substitution (more negative is β) the smaller is the steady state capital ratio, saving rate and share of capital income in GDP. Not surprisingly the impact of unjust capital ownership on income is correspondingly lower. The opposite is true if the elasticity is higher (the more positive is β). What accounts for the impact of varying β is that because we vary the saving rate to generate the same rate of return (aligned to historic data at 5%) we get a shift in the shares of capital and labor. At $\beta = 0.2$ (elasticity of substitution of 1.25) the profit share is around 42% of GDP and the capital to income ratio is 5.2; at $\beta = -0.2$ (elasticity of substitution of 0.83) the profit share is 24% and the capital output ratio is 3. Table 1 shows that the effects of varying β are

not trivial. At an elasticity of substitution of 0.83 ($\beta = -0.2$) the share of unjust income at 50 years on from a point where 70% of capital was unjust is around 3%; at an elasticity of 1.25 ($\beta = 0.2$) it is nearly three times as great at just over 8%. But elasticities of substation a long way from 1 generate implausible levels of the capital to income ratio and of required savings rates for a steady growth path with a rate of return of around 5%. Evidence on the elasticity of substitution is mixed on whether it is above or below 1 (see (Blume & Durlauf 2015)). (Karabarbounis & Neiman 2013) estimate it as 1.25. But that estimate is high relative to (Chirinko 2008), or (León-Ledesma et al. 2010). Unless the elasticity is a long way above 1 it does not have a big impact and after 80 years unjust income always shrinks to very low levels.

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[6] When $\beta \neq 0$ and $s_\pi \neq s_L$ there is a second order effect of the initial extent of unfair ownership of capital and income on half lives.

	$\chi = 0.75$		$\chi = 0.6$		$\chi = 0.5$	
	K_t^U/K_t	Y_t^U/Y_t	K_t^U/K_t	Y_t^U/Y_t	K_t^U/K_t	Y_t^U/Y_t
Time						
t=0	0.70	0.30	0.70	0.30	0.70	0.30
t=20	0.36	0.12	0.33	0.11	0.31	0.10
t=50	0.12	0.04	0.11	0.03	0.10	0.03
t=75	0.05	0.02	0.04	0.01	0.04	0.01
t=125	0.01	0.003	0.01	0.002	0.01	0.002
half life	21	16	19	14	17	13

Table 2: Assumes a common saving rate to deliver r=5% at depreciation rate of 0.03; n=0.005; g=0.015. Cobb Douglas case.

	$n+g = 0.03$		$n+g = 0.01$		$\delta = 0.05$		$\delta = 0.02$	
	K_t^U/K_t	Y_t^U/Y_t	K_t^U/K_t	Y_t^U/Y_t	K_t^U/K_t	Y_t^U/Y_t	K_t^J/K_t	Y_t^J/Y_t
Time								
t=0	0.70	0.30	0.70	0.30	0.70	0.30	0.70	0.30
t=20	0.35	0.12	0.44	0.16	0.31	0.10	0.44	0.16
t=50	0.10	0.03	0.20	0.06	0.07	0.02	0.20	0.07
t=75	0.04	0.01	0.10	0.03	0.02	0.01	0.10	0.03
t=125	0.005	0.001	0.02	0.008	0.002	0.001	0.03	0.008
half life	20	16	29	23	17	14	30	23

Table 3: Assume common saving rate to deliver r=5%. For the cases n+g=0.03 and n+g=0.01 I set the depreciation rate at 0.03. When depreciation is at either 0.05 or 0.02 I set n+g=0.02. Cobb Douglas production.

Table 2 shows the impact of starting from a position with overall capital well below the steady state level relative to income. This has a negligible impact on the rate of decay of injustice.

Table 3 varies the value of $n + g$ and, separately, varies the depreciation rate δ. These have more of an impact on the decay of injustice than whether we start from a steady state capital to income ratio. Faster (slower) growth of the effective labor force and higher (lower) depreciation both accelerate (slow down) the decay in injustice. But in all cases injustice remains small after 70 years.

The conclusion to be drawn from Tables 1-3 is that when capital is expropriated from foreigners and taken back to the expropriator's country, the half life of the resulting injustice (where injustice is the extra income and wealth enjoyed by the expropriator) may be relatively short - a few decades rather than a few centuries. Things are different when assets are expropriated from one group by another within the same country - the case we consider in the next section.

3.2   Expropriation by one group from another within the same country

Unjustly acquiring assets from foreigners and taking them home is historically of much less significance than getting them from people who continue to live and work in the same place as the

assets remain - in most cases neither the people from whom assets are unjustly acquired nor the assets themselves may move far, though the income from them may do so. In the case of imperial acquisitions the capital - often largely in the form of land, buildings and infrastructure - was rarely sent back to the imperial power though the income derived from using it, usually with labor from the colony, often was. In that case to count as unjust part of labor income of those whose wages are higher because they combine their labor with unjust capital (relative to what it would be if that capital was removed) is not reasonable. Indeed there is something particularly unfair about labelling the labor income of someone who works with (but does not own) some expropriated capital as "unjust" when they might have a legitimate right to part of that capital. In this case instead of counting as unjust all income that depends on such capital (as is done in the calculations above and as may be appropriate when that capital was legitimately the property of people outside the economy who are deprived of its use) it is more appropriate to count as unjust only the income derived directly from ownership of that capital – including income earned on any additions to capital from saving the capital income on unjust assets.

In this case the evolution of unjust capital depends positively on the saving rate out of capital income but not on the saving rate out of labor income.

In allowing for this I will now assume that initial total capital (the sum of just and unjust capital) is at the steady state level since the results above show that the evolution of unfairness is insensitive to whether total capital is initially below the steady state level. So now initial unjust capital is a fraction λ_0 of initial steady state capital (K_0^{ss}): $K_0^U = \lambda_0 K_0^{ss}$ Just capital at time t = 0 is $(1 - \lambda_0)K_0^{ss}$. The evolution of unjust capital is given by:

$$K_t^U = K_{t-1}^U(1 - \delta) + s_\pi \lambda_{t-1}(\pi_t Y_{t-1}) \tag{24}$$

The share of profits in GDP is $\pi_t = \alpha(K_t/Y_t)^\beta$. Since we start in a steady state the overall capital-income ratio (K_t/Y_t) is constant at $s/(n + g + \delta)$. s is a weighted average of s_π and s_L; the share of capital income, π, is constant at $\alpha(s/(n + g + \delta))^\beta$. Substituting this into the previous equation, noting that $K_t/K_{t-1} = n + g$ and simplifying allows us to write the evolution of the ratio of unjust to total capital stock as:

$$\lambda_t = \lambda_{t-1} \left[(1 - \delta) + s_\pi \alpha(s/(n + g + \delta))^{\beta-1}\right] /(1 + n + g) \tag{25}$$

If $s_L = 0$ then $s = \pi s_\pi$. In steady state $\pi = \alpha(s/(n + g + \delta))^\beta$ and using this in the previous equation implies that when $s_L = 0$ $\lambda_t = \lambda_{t-1}$. So in the extreme case of zero saving out of labor income there is never any reduction in injustice. This makes intuitive sense: when some part of capital is unfairly appropriated by one group within the economy from another within the same economy it doe not affect wages and does not make some part of labor income unjust. It affects the fairness with which total capital income is allocated. If there is no saving out of labor income, capital is only accumulated out of capital income and since initially a fraction of that λ_0 is unjust

	$\beta = 0$		$\beta = 0.2$		$\beta = -0.2$	
	K_t^U/K_t	Y_t^U/Y_t	K_t^U/K_t	Y_t^U/Y_t	K_t^U/K_t	Y_t^U/Y_t
Time						
t=0	0.70	0.21	0.7	0.29	0.7	0.17
t=20	0.36	0.11	0.40	0.17	0.34	0.08
t=50	0.13	0.04	0.17	0.07	0.11	0.03
t=75	0.05	0.02	0.10	0.03	0.04	0.01
t=125	0.01	0.003	0.02	0.01	0.01	0.002
half life - years	21	21	25	26	20	19

Table 4: Asssumes a common saving rate out of capital and labor income to deliver r=5% at depreciation rate of 0.03; n=0.005; g=0.015.

(and so replaces a fraction λ_0 of worn out capital in steady state) the share λ never falls. But so long as $s_L > 0$ then λ will fall over time.

Just income is now defined as total labor income plus that part of total capital income that is the share going to owners of just capital $((1 - \lambda_t)\pi_t)$. The evolution of the share of just income is governed by:

$$Y_t^J/Y_t = (1 - \lambda_t)\pi_t + (1 - \pi_t) = (1 - \lambda_t)\alpha(s/(n + g + \delta))^\beta + (1 - \pi_t) \qquad (26)$$

For the special case of Cobb Douglas production and a common saving rate $(s = s_\pi = s_L, \beta = 0)$ the process for λ simplifies to $\lambda_t = \lambda_{t-1}\left[(1 - \delta) + \alpha(n + g + \delta)\right]/(1 + n + g)$

Table 4 shows the evolution of unjust capital and income for three values of β. As before we assume that the overall saving rate is sufficient to generate a net of depreciation return on capital of 5%. Table 5 shows the impact of varying the value of $n + g$ and of δ. In Tables 4 and 5 I set $s_\pi = s_L$

Comparing tables 4 and 5 with tables 1 and 3 shows that if we assume $s_\pi = s_L$ there is a somewhat faster decline in unfairness in the ownership of capital when the labor income from working with unjust assets is itself considered just compared to a situation where that part of labor income reflecting the use of unjust capital is itself unjust.

But things look different when $s_\pi > s_L$. Varying the ratio of saving out of capital income to saving out of labor income matters since all saving out of labor income generates fair capital and future income and that is not true of all saving out of capital income. In the OLG-perpetual youth model, assuming log utility, the saving rate out of capital income could be substantially greater than saving out of labor income if the decline in labor hours with age was significantly below the growth in labor productivity $(g > v)$ so that labor income increased with age. A higher depreciation rate on capital would also raise s_π relative to s_L.

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	$n+g=0.03$		$n+g=0.01$		$\delta=0.05$		$\delta=0.02$	
	K_t^U/K_t	Y_t^U/Y_t	K_t^U/K_t	Y_t^U/Y_t	K_t^U/K_t	Y_t^U/Y_t	K_t^J/K_t	Y_t^J/Y_t
Time								
t=0	0.70	0.21	0.70	0.21	0.70	0.21	0.70	0.30
t=20	0.32	0.10	0.41	0.12	0.27	0.08	0.41	0.12
t=50	0.09	0.03	0.18	0.05	0.06	0.02	0.18	0.05
t=75	0.03	0.01	0.09	0.03	0.02	0.01	0.09	0.03
t=125	0.004	0.001	0.02	0.006	0.002	0.001	0.02	0.007
half life	18	18	26	26	16	16	26	26

Table 5: Assume common saving rate to deliver r=5% Cobb Douglas production. For the cases n+g=0.03 and n+g=0.01 I set the depreciation rate at 0.03. When depreciation is at either 0.05 or 0.02 we set n+g=0.02

	$s_\pi=0.29$		$s_\pi=0.43$		$s_\pi=0.50$	
	$s_L=0.14$		$s_L=0.08$		$s_L=0.05$	
	K_t^U/K_t	Y_t^U/Y_t	K_t^U/K_t	Y_t^U/Y_t	K_t^U/K_t	Y_t^U/Y_t
Time						
t=0	0.70	0.21	0.7	0.21	0.7	0.21
t=20	0.42	0.13	0.52	0.16	0.58	0.17
t=50	0.19	0.06	0.33	0.10	0.43	0.13
t=75	0.10	0.03	0.23	0.07	0.34	0.10
t=125	0.03	0.01	0.10	0.03	0.21	0.06
half life	27	27	46	46	76	76

Table 6: The weighted average saving rate is so as to deliver r=5% Cobb Douglas production.

Table 6 shows the evolution of unfairness when the saving rates on capital and labor income differ significantly. I set the weighted average saving rates so that in steady state they deliver an amount of capital to generate a net return on capital of 5%. For the Cobb Douglas case this requires an overall gross saving rate of 18.75%. Table 6 shows the Cobb-Douglas case with a constant capital share of $\alpha = 0.3$ and so we require that $0.1875 = s = \alpha s_\pi + (1-\alpha)s_L$ Results are similar when β is somewhat above or below zero and within ranges suggested by the evidence.

Table 6 shows that we can get much more long lasting impacts of initial unjust assets if saving out of capital is much higher than saving out of labor income. When $s_\pi = 0.43; s_L = 0.08$ the half life of injustice is 46 years, more than twice as high as when $s_\pi = s_L = 0.1875$. If the saving rate out of capital income is ten times that out of labor income the half life rises to 76 years. Even then after 100 years unjust shares - even if they start at 70% of total capital - have fallen a great deal so that unjust capital is down to around 25% of total capital and unjust income is around 8% of GDP. Provided there is a non trivial amount of saving out of labor income, unjust capital will dwindle a great deal in a few generations. It is plausible that today such saving is indeed non trivial. Pension arrangements mean that many workers in recent decades automatically make contributions out of labor income; one could also interpret some part of taxes out of labor income as helping accumulate (or preserve) public sector assets such as roads, parks, schools and hospitals. Although we do not model taxes and public spending explicitly one can see some part of overall national saving and

asset accumulation as reflecting tax financed government saving. Since labor income taxes raise a substantial part of total tax revenue this means that saving out of labor income is of significance even if direct saving by workers were to be small. In fact such direct saving - even putting to one side pension contributions - is clearly not small since many households build up ownership of housing out of paying off mortgage debt largely from labor income; the housing stock in developed economies is substantially financed out of saving from owner occupiers - a group making up 60% to 70% of households in most economies in recent decades. But in the more distant past average saving out of

labor income may have been very low.

Only if saving out of labor income is very small is the half life more than 75 years. 75 years ago is near the end of the second world war. If as much as 70% of capital was then unjust, *and* if saving out of labor income was also non-trivial, the impact on incomes and wealth today is unlikely to be more than a few percent. Only in extreme cases - a saving rate out of capital income ten times that of saving out of labor income - do we get a fraction of GDP that is unjust as high as 10%.

It is of course rather arbitrary to take 1945 as the year in which the stock of unjust capital is inherited and it is extreme to assume that thereafter unjust capital and income are simply a result of its use and not of new injustices. But it is not entirely arbitrary since the second world war marked the beginning of the end of the era of empire for many rich countries and was also followed by more left leaning governments that saw the expansion of the welfare state and the rise of distributive taxation. France and the UK are cases in point. But of course there is nothing in the calculations that hinges on the choice of a particular start point. It is however likely that the half life of injustice over the past 75 is different - and probably much lower - than in the more distant past. In the post second world war period average saving rates out of labor income have been non-negligible in rich (and some middle income) countries. In the more distant past many more people who relied only on their wages struggled to make ends meet and their saving rate was likely lower; because of that the half life of injustice was greater. But whether the half life of injustice is really much lower in the modern, developed world depends also on the significance of human capital, an issue I turn to in the next section.

4 Human capital

Much of human capital is the result of people using their own time to study. There is no question that this element of human capital is justifiably their own – according both to common sense and Nozick's principles; there is as much a claim to the fruits of that human capital as there is to that of innate ability. But some significant part of human capital formation is due to other resources used - computers, books, school buildings, university laboratories, and the labor of teachers. How much of human capital in modern developed economies could have been accumulated through the

use of income from assets that have been unjustly acquired? That may be hard to figure out – but in principle, and to maintain consistency with the treatment of physical capital, that part of human capital formation that reflects saving from income is unjust to the extent that such income is unjust.

I now explore the implications of that idea. I assume that output (income) is a result of combining effective units of aggregate labor input (E) and capital. Effective labor input is a function of raw (innate) labor power (L) and human capital (H). Here I follow (Mankiw et al. 1992) in using a model for the evolution of human capital that has the same structure as that for physical capital.

$$Y_t = \left[\alpha K_t^\beta + (1-\alpha)E_t^\beta\right]^{1/\beta} \tag{27}$$

$$E_t = [\eta(H_t^\varphi) + (1-\eta)(L_t^\varphi)]^{1/\varphi} \tag{28}$$

η is a share parameter between human capital and innate ability in generating effective labor E; φ is a substitutability parameter. As with physical capital, the stock of human capital has just and unjust parts:

$$H_t = H_t^U + H_t^J \tag{29}$$

I assume the evolution of H_t^U and H_t^J follows:

$$H_t^U = (1-\delta_H)H_{t-1}^U + s_H Y_{t-1}^U \tag{30}$$

$$H_t^J = (1-\delta_H)H_{t-1}^J + s_H Y_{t-1}^J \tag{31}$$

δ_H is the depreciation rate for human capital; s_H is the saving (or investment) rate out of income for the accumulation of human capital. I assume the saving-investment rate into human capital is the same out of labor income and out of capital income.

Denote the share of human capital that is unjust as $\lambda_t^{UH} = H_t^U/H_t$. The share of physical capital that is unjust is denoted by $\lambda_t^{UK} = K_t^U/K_t$. I focus on steady states where the growth of Y, H and K are all at rate $n + g$. I shall assume for the moment that saving to accumulate physical capital is at the same rate out of all income at rate s (so that $s_\pi = s_L$); the depreciation rate of physical capital is denoted δ_K (until now this has been denoted by δ but we now want to distinguish it from the depreciation rate of human capital).

In steady state $H/Y = s_H/(\delta_H + n + g); K/Y = s/(\delta_K + n + g)$; shares of capital and of labor (i.e. the income share of the effective labor force E) are then constant at $\pi, 1 - \pi$ where $\pi = \alpha(K/Y)^\beta = \alpha(s/(\delta_K + n + g))^\beta$

Unjust and just income are given by:

$$Y_t^U = \left[\theta\lambda_t^{UH}(1-\pi) + \pi\lambda_t^{UK}\right] Y_t \tag{32}$$

$$Y_t^J = \left[(1-\theta\lambda_t^{UH})(1-\pi) + \pi(1-\lambda_t^{UK})\right] Y_t \tag{33}$$

where θ is the share of returns to human capital in the overall remuneration of the effective stock of labor and is constant in steady state. (In the Cobb Douglas case for the stock of effective labor $\varphi = 0$; $E_t = (H_t^\eta L_t^{1-\eta})$ and $\theta = \eta$). I assume here that the part of labor income that is a return to innate ability $((1-\theta)(1-\pi)Y)$ plus that part due to justly acquired human capital $(\theta(1-\lambda_t^{UH})(1-\pi)Y)$ are just; the remainder of labor remuneration $(\theta\lambda_t^{UH}(1-\pi)Y)$ is unjust.

It is straightforward to show that in this economy the evolution of $\lambda_t^{UH}; \lambda_t^{UK}$ is given by:

$$\lambda_t^{UK} = \left[\lambda_{t-1}^{UK}(1-\delta_K + \pi(\delta_K + n + g)) + \theta\lambda_{t-1}^{UH}(1-\pi)(\delta_K + n + g)\right]/(1+n+g) \tag{34}$$

$$\lambda_t^{UH} = \left[\lambda_{t-1}^{UH}(1-\delta_H + \theta(1-\pi)(\delta_H + n + g)) + \lambda_{t-1}^{UK}(\pi)(\delta_H + n + g)\right]/(1+n+g) \tag{35}$$

This is a homogenous system of first order linear difference equations. Both of the eigenvalues of this simultaneous difference equation system are less than one provided that $\theta < 1$, in which case from any initial state of injustice - assuming no new exogenous shocks to injustice - unfairness eventually declines to zero. But the rate of decline of injustice is sensitive to the proportion of the effective stock of labor that is accounted for by human capital.

As was the case without human capital, the half life of injustice depends on shares of capital and labor in income; it is also a decreasing function of depreciation rates and the growth of the labor force (n) and of exogenous (non human capital related) labor productivity growth (g). In addition the split between returns to human capital (H) and innate ability (L) now matters.

The properties of the decay rate of injustice are illustrated in the tables below. To construct these we use the same parameter estimates for the common variables we introduced above (δ, n, g) and assume Cobb Douglas for the outer production function combining capital and labor ($\beta = 0$).

The new parameters once we introduce human capital are: $\delta_H, \eta, \varphi\ s_H$. I now briefly describe how they are set.

δ_H — a lower limit might be (1/ life expectation) i.e. about 0.0125; but much human capital is acquired later in life and some things you learn are quickly forgotten (though unlike physical depreciation δ_H is plausibly lower the more you use H). I set δ_H equal to either .02 or .04, either side of the assumed rate of deprecation of physical assets (which I now keep constant at 0.03 for all simulations). $\delta_H = 0.02$ would mean that some productivity enhancing skill learned at age 20 would

dwindle to half its effectiveness by age 54, unless topped up by re-training; if $\delta_H = 0.04$ it would fall to half its effectiveness by age 37 .

s_H- Based on OECD measures of ever widening definitions of spending on education and training, plausible limits for s_H might be 0.05 and 0.15. The choice of s_H should be consistent with a plausible ratio of H/K. (Mankiw et al. 1992) quote an estimate that around one half of the US capital stock was human capital. They also argue that between 50% and 70% of total labor income represents a return to human capital. With a labor share of total income of around 70% that would suggest returns to human and physical capital might be the same order of magnitude. For given s_H, δ_H we have the steady state $H/Y = s_H/(\delta_H + n + g)$. If K/Y is around 4 and H and K are comparable in size, this means that with $\delta_H = .02$ we would need s_H to be near 16%; if $\delta_H = .04$. the saving rate for human capital out of total income would need to be 24%. These saving rates into human capital formation seem high - OECD data on total education spending in schools, colleges and universities plus estimated spending on training at between 5% and 10% of GDP for most developed countries - so I also explore implications of much lower values for s_H.

I initially set parameter η so as to generate a steady state net of depreciation return to human capital at the same rate as the return on physical capital - a level we have set to 0.05. This implies:

$$\eta = (0.05 + \delta_H)(H/E)^{1-\varphi} / \left[(1-\alpha)Y/E \right]$$

The share of human capital in total labor remuneration is:

$$\theta = \eta(H/E)^\varphi = (0.05 + \delta_H)(H/Y)/(1-\alpha) = s_H(0.05 + \delta_H)/((\delta_H + n + g)(1-\alpha))$$

Note in this case where $\beta = 0$, α is the share of profits in income. Thus a key parameter for the evolution of unfairness, namely θ the share of human capital returns in total labor remuneration, depends only upon the share of returns to capital in income, saving for human capital and its rate of depreciation, and $n + g$.

Tables 7-9 show results for the rate of decay of unfairness when some part of the stocks of both physical and human capital might have been unjustly acquired. Initially I assume that there is a common return on all capital – human and physical - at a net of depreciation rate of 5%.

Table 7 uses a high value for s_h (0.2) and a relatively low value for δ_H (0.02). This generates a share of returns to human capital in total labor remuneration of 0.5. Table 8 uses a higher depreciation rate (0.04) and a lower saving rate (0.15) and gives a share of labor remuneration that goes to human capital of 0.32.

The main message from Tables 7 and 8 is that the rate of decay in unfairness is materially less rapid when there can be unfairness in some part of the stock of human capital relative to a world where only physical capital can be unjustly acquired. That decay rate is faster the higher is the depreciation rate of human capital and the lower is saving into human capital. If 70% of both physical and human capital is initially unjust (and assuming $\delta_H = 0.02, \delta_K = 0.03, s_H = 0.2$) then

	K_t^U/K_t	H_t^U/H_t	Y_t^U/Y_t		K_t^U/K_t	H_t^U/H_t	Y_t^U/Y_t
Time							
t=0	0.70	0.70	0.46		0.7	0.00	0.21
t=20	0.51	0.54	0.34		0.37	0.09	0.14
t=50	0.31	0.34	0.22		0.17	0.10	0.09
t=75	0.21	0.24	0.15		0.10	0.08	0.06
t=125	0.10	0.11	0.07		0.04	0.04	0.03
half life- years	44	49	47		22	-	38

Table 7: Saving rates deliver r=5% on both physical and human capital; depreciation rate is set at 0.03 for physical capital and at 0.02 for human capital. The saving rate for human capital is set at 0.2 and for physical capital is 0.1875. n=0.005; g=0.015. The implied share of human capital remuneration in overall remuneration of labor is 0.5

	K_t^U/K_t	H_t^U/H_t	Y_t^U/Y_t		K_t^U/K_t	H_t^U/H_t	Y_t^U/Y_t
Time							
t=0	0.70	0.70	0.37		0.7	0.00	0.21
t=20	0.44	0.41	0.23		0.37	0.11	0.14
t=50	0.21	0.19	0.11		0.15	0.09	0.07
t=75	0.11	0.10	0.06		0.08	0.06	0.04
t=125	0.03	0.03	0.02		0.02	0.02	0.01
half life- years	29	27	28		22	-	32

Table 8: Saving rates deliver r=5% on both physical and human capital; depreciation rate is set at 0.03 for physical capital and at 0.04 for human capital. The saving rate for human capital is set at 0.15 and for physical capital is 0.1875. n=0.005; g=0.015. The implied share of human capital remuneration in overall remuneration of labor is 0.32

	K_t^U/K_t	H_t^U/H_t	Y_t^U/Y_t		K_t^U/K_t	H_t^U/H_t	Y_t^U/Y_t
Time							
t=0	0.70	0.70	0.63		0.7	0.00	0.21
t=20	0.64	0.65	0.58		0.39	0.11	0.17
t=50	0.56	0.57	0.51		0.20	0.14	0.15
t=75	0.46	0.50	0.52		0.16	0.14	0.13
t=125	0.41	0.42	0.37		0.12	0.12	0.11
t=200	0.3	0.31	0.27		0.09	0.09	0.08
half life- years	161	167	161		24	-	126

Table 9: Saving rates deliver r=5% on physical capital. The net rate of return on human capital is assumed to be 10% The depreciation rate is set at 0.03 for physical capital and at 0.02 for human capital; The saving rate for human capital is set at 0.20 and for physical capital is 0.1875. n=0.005; g=0.015. The implied share of human capital remuneration in overall remuneration of labor is 0.85

around 46% of income is initially unjust. After 75 years the amount of income that is unfair is still a substantial 15%. The decay rates for unfairness with only unjust physical capital are much faster.

If human capital is more important the decay of unfairness can be significantly lower. In table 9 I show results where we drop the assumption of common returns on human and physical capital. I now assume human capital is more productive and has a net rate of return of 10% - twice that on physical capital. One possible reason is that credit restrictions have a bigger impact on individual's ability to accumulate human capital than they do on companies acquisition of physical capital; another factor might be companies inability to appropriate much of the returns to their investing in the human capital of their work force. For either reason returns on human capital may be high. Now injustice lasts a lot longer - and can build up. The second block of results in Table 9 shows that if initially there is no injustice in human capital it can become significant in a few decades.

Table 9 shows that the half life of injustice is over 160 years if we start with injustice in both physical and human capital. In this calibration human capital accounts for 85% of the returns to labor - a figure that may be implausibly high for the economy of 100 or more years ago, but does not seem so implausible today. Even if there is no initial injustice in human capital, the overall half life of income injustice is 126 years.

In the next section I consider what happens to the decay rate of unfairness in a limiting case where human capital is the sole source of returns to labor.

4.1 Share of human capital in total labor remuneration approaches 1:

The share of returns to human capital in labor remuneration can be made very large if we are prepared to assume that the return to human capital, and the saving rate out of income to accumulation of human capital, are high. With a common (net of depreciation) rate of return on capital of 5%, a

depreciation rate for human capital of 0.02 and a saving rate of 0.2 the share of human capital in total labor income is 50%. If we assume instead a 10% net of depreciation return to investment in human capital, a depreciation rate of only 0.015 and the same (high) saving rate of 0.2 of income devoted to human capital investment, the share of human capital (θ) rises to 94%. In this case when φ is zero (elasticity of substitutability between H and $L = 1$) the share parameter η from the production function for effective labor is also 0.94. (If we set $\varphi = 0.2$. so elasticity of substitution is 1.25, the share parameter η is only 0.59 even though the share of human capital remuneration in total labor income is still 0.94). In a world in which physical labor power is in many sectors becoming less relevant it is plausible that the returns to human capital take a rising share of total labor remuneration. One could imagine a supremely well trained person with an exceptionally high stock of human capital who only had to work a few minutes a week to earn a large salary. So very high values of θ are not wildly implausible in many sectors and its whole-economy average value may be rising over time.

In the limiting case where human capital accounts for all returns to labor one eigenvalue of the dynamic system for the evolution of unjust physical and human capital becomes 1; the other is given by

$$[1 - (\pi\delta_H + (1 - \pi)\delta_K)] / (1 + n + g) < 1$$

The unit eigenvalue means that the impact of initial unjust shares of either capital or of human capital never dies away. There is a convergence of unjust shares on steady state values. The process for the evolution of the unjust shares of capital and human capital is given by:

$$
\begin{bmatrix} \lambda_t^{UK} \\ \lambda_t^{UH} \end{bmatrix} = \begin{bmatrix} 1 & -[(1-\pi)(\delta_K + n + g)]/[\pi(\delta_H + n + g)] \\ 1 & 1 \end{bmatrix} \begin{bmatrix} c1 \\ c2 * [[1 - (\pi\delta_H + (1-\pi)\delta_K)]/(1+n+g)]^t \end{bmatrix}
$$

where

$c1 = [\lambda_0^{UK}\pi(\delta_H + n + g) + \lambda_0^{UH}(1 - \pi)(\delta_K + n + g))]/[(1-\pi)(\delta_K + n + g) + \pi(\delta_H + n + g)]$

$c2 = (\lambda_0^{UH} - \lambda_0^{UK})/[1 + (1 - \pi)(\delta_K + n + g)/(\pi(\delta_H + n + g))]$

There are several properties of this model, some of which are surprising:

1. $\lambda_t^{UH}, \lambda_t^{UK}$ both converge on a common value of $c1$ - which is also the level to which overall income unfairness, Y_t^U/Y_t converges. This level is an increasing function of $\lambda_0^{UH}, \lambda_0^{UK}$. The steady state value $c1$ is a weighted average of initial physical capital unfairness λ_0^{UK} and of initial human capital unfairness λ_0^{UH}; the weights depend on labor and capital income shares and on the relative sizes of human capital and physical capital depreciation rates (δ_H, δ_K).

2. No matter how small is λ_0^{UH} we get infinitely long lasting unfairness if $\lambda_0^{UK} > 0$. If $\lambda_0^{UH} = 0$ then $\lambda_t^{UH}, \lambda_t^{UK}$ and Y_t^U/Y_t converge to $\lambda_0^{UK}/[1 + (1 - \pi)(\delta_K + n + g)/(\pi(\delta_H + n + g))]$

3. No matter how small is λ_0^{UK} we get infinitely long lasting unfairness if $\lambda_0^{UH} > 0$. If $\lambda_0^{UK} = 0$ then $\lambda_t^{UH}, \lambda_t^{UK}$ and Y_t^U/Y_t converge to
$\lambda_0^{UH}/[1 + \pi(\delta_H + n + g)/((1 - \pi)(\delta_K + n + g))]$

4. With zero initial human capital unfairness we get long run income unfairness in excess of initial income unfairness if $\delta_H > \delta_K$; unchanging unfairness if $\delta_H = \delta_K$ and declining unfairness that converges on a level below $\lambda_0^{UK}(\pi)$ if $\delta_H < \delta_K$.

5. If $\delta_H = \delta_K$, Y_t^U/Y_t is constant at $\lambda_0^{UH}(1 - \pi) + \lambda_0^{UK}(\pi)$ and $\lambda_t^{UH}, \lambda_t^{UK}$ both converge on that level. So $\lambda_t^{UH}, \lambda_t^{UK}$ converge on the weighted average initial levels of inequality in human and physical capital, while the overall level of income unfairness never changes.

6. If $\lambda_0^{UH} = \lambda_0^{UK}$ unfairness in human capital, physical capital and in overall income never change.

4.2   Is the decay rate of injustice falling?

Even if injustice does eventually die away there obviously arise new sources of injustice (in the Nozickian sense). Are there reasons to believe that the rate at which injustice dies away is itself falling? If that were true then with an unchanged arrival rate of new injustices the average level of injustice would still be falling as the seriousness of new injustices declined as their half life fell. I showed above that either of two conditions is sufficient for initial injustice *never* to go away: saving out of labor income is zero or human capital accounts for all of labor remuneration. In the distant past, when many people lived on subsistence wages, saving out of labor income may have been very small and injustice very slow to decline. One reason saving was low was that for the great majority of the population there was no period of retirement - people worked as long as they were able. In terms of the OLG model of perpetual youth this corresponds to a near zero value v that generates near zero value of s_L.[7] The spectacular rise in labor incomes over the past 200 years in developed economies, and the rise in institutions that generate savings out of labor incomes (company pensions, amortizing mortgages) would suggest that this sort of long lived injustice would have declined - half lives should be lower now that in 1800. But it is also likely that the significance of human capital in overall labor remuneration is higher now than in the past, and may be rising. So with these countervailing effects it is far from clear that the half life on injustice is declining.

Table 10 illustrates the offsetting forces at work by considering two scenarios. In the first human capital is not a major part of overall labor remuneration - the aggregate saving rate out of income into human capital formation is 0.05 and the depreciation rate is 0.05 (so that something learned at age 20 has dwindled to half in terms of its value by age 33). Perhaps this reflected life expectancy and education levels a hundred and more years ago. I set the return on human capital a bit above that on physical capital - at 7.5%, relative to 5% on physical capital. But I suppose also that in this world most workers were hardly able to save - and the saving rate out of capital income was 10 times that out of labor income. This combination of forces would generate a proportion of labor income due to human capital of only 13%; a saving rate out of capital income of nearly 50% and

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[7] With log utility the steady state saving rate out of labor income is given by
$1 - [(\rho + d)(1 + v)(1 + r)/(1 + \rho)]/[(1 + v)(1 + r) - (1 + g)(1 - d)]$

With $d = 0.02; r = 0.05; g = 0.015; \rho = 0.036$
this saving rate is zero when $v = 0.0015$.
That value of v implies virtually no change in labor supply with age.

	K_t^U/K_t	H_t^U/H_t	Y_t^U/Y_t		K_t^U/K_t	H_t^U/H_t	Y_t^U/Y_t
	$\delta_H = 0.05, s_H = 0.05, s_L = 0.05, s_\pi = 0.50$				$\delta_H = 0.02, s_H = 0.15, s_L = s_\pi = 0.1875$		
Time							
t=0	0.70	0.70	0.27		0.7	0.70	0.53
t=20	0.60	0.35	0.21		0.56	0.58	0.43
t=50	0.46	0.19	0.16		0.40	0.42	0.31
t=75	0.37	0.14	0.12		0.30	0.32	0.24
t=125	0.24	0.09	0.08		0.18	0.19	0.14
t=200	0.12	0.05	0.04		0.08	0.09	0.06
half life- years	81	21	65		62	68	65

Table 10: Saving rates deliver r=5% on physical capital; depreciation rate is set at 0.03 for physical capital. n=0.005; g=0.015. The implied share of human capital remuneration in overall remuneration of labor is 0.13 for the first block of results and 0.64 for the second

out of labor income of only 5%. The first block of columns in Table 10 shows how injustice evolves in this economy.

Now imagine an economy more like that of advanced countries today. Human capital is much more important: there is a common saving rate out of all income of 15% for human capital formation, a rate of return higher than on physical capital at 10% as opposed to 5% and a lower depreciation of that human capital of 0.02 a year. But there is also much more significant savings out of labor income - at a rate equal to that out of capital income. This is the second block of results in Table 10, where human capital accounts for 64% of labor remuneration and where saving out of both labor and capital incomes for physical investment is at the same rate of 0.1875 (generating a 5% return on physical capital).

The half lives of overall economic injustice in both cases is 65 years. In the more modern economy the half life of injustice in ownership of physical capital is lower; but that is offset by a much higher half life of injustice in human capital. There is no change in the overall rate of decay of injustice as one moves from the distant past to the present.

5   Conclusions

I have drawn on an idea developed at length in the work of Nozick - that ownership of resources now that depends on past unjust ownership of resources is itself unjust - and used it to explore the impact today of an unjust pattern of the ownership of assets in the past.

I suppose that at some point in the past the total capital stock (or wealth) was composed of two parts – one part that was justly acquired and one part that was unjustly acquired. I present a framework to assess how much of wealth and income today is due to the existence of unjustly acquired capital. In doing this I am assuming that all future income derived from any wealth that has been unjustly acquired is itself unjust; any capital accumulated from income that derives from unjust capital is also unjust. This means that a large stock of unjust capital and income has potentially long lasting effects. But I also assume that all economic activity connected with just capital – the

generation of wages and of capital income and the saving from such incomes to create new capital – is always just.

Four factors emerge as significant determinants of the extent to which past unjust acquisition of capital (land, buildings, machines and human capital) creates continuing injustice in incomes and assets: saving rates out of capital and labor income; depreciation of capital; the elasticity of substitution in production between capital and labor; the growth of the effective labor force (the sum of population growth and labor enhancing productivity). A fifth factor - the extent to which human capital can be seen as unjustly acquired - is particularly significant.

Using plausible assumptions on productivity, saving, depreciation and population growth I find that if we ignore human capital (or assume that it is all justly accumulated) the lingering impact of an assumed very large initial unjust ownership of wealth is very likely to be fairly small after several decades, *unless* saving by those with no capital income is very low.

In the more distant past, when subsistence wages were paid to most workers, savings of those with no capital income may have been very low and in that world the longevity of injustice would have been great. But if we continue to ignore human capital, the half life of injustice in a world where people do save out of labor income - as is the case in most developed economies today - is not likely to be much above 50 years. Based on that, the notion that the wealth and income of the rich countries today significantly depends upon unjust acquisition of wealth from before several decades ago does not look very plausible[8].

Things look different when we take account of human capital and the possibility that a significant part of the stock of human capital may have been built up from savings out of unjust income. This is not implausible - the children of the aristocracy and of the robber barons of the past (and of today's oligarchs) typically have had an unusually good education. If human capital were to be the dominant factor in remuneration of labor the half life of injustice can become long - and stretch well beyond 100 years. In the limit, if human capital becomes the overwhelming source of the productive capacity of labor, initial injustice may never go away.

Human capital has become more important in modern economies and saving out of labor income is probably much higher than in the distant past. These two factors pull in different directions meaning it is not obvious whether the half life of injustice is higher or lower than in the past. But it is likely that for much of the period up to the early twentieth century saving out of labor income for the great majority of the population was very low - probably negligible. If that saving rate out of labor income was negligible the half life of injustice would have been very long. Such saving is now not negligible. The fraction of labor remuneration due to human capital would have to rise to nearly 1 to offset the effect of that so as to keep the half life of injustice from falling. While human capital now very likely does account for a much higher share of labor income than a hundred and

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[8] The converse may not be true - the wealth of rich countries today might not be because of unjustly acquiring assets from poor ones in the distant past, but those poor countries may be poor today in part because of that. There is evidence that this may be true if we include political institutions in a wide definition of assets. To put the point simply - Belgium today may not be much richer because of assets looted from Congo in the nineteenth century, but the dire poverty of the Democratic Republic of Congo today may be a lasting effect of that.

more years ago, its share has not approached 100%, so on balance it seems likely that the half life of injustice is now lower in developed economies than in the world of 100 or more years ago.

The notion of a just distribution of resources is highly contentious. In many ways this paper uses a narrow conception of just and unjust outcomes. It sets out a framework for thinking through the evolution of one type of injustice and presents illustrative results based on a calibrated version of a standard growth model. The value of such an exercise does not require that we believe that the only source of injustice is from the past distribution of assets. One would want to know how past injustice of this sort affects today's outcomes even if one took the view of Sen that "we can have a strong sense of injustice on may different grounds, and yet not agree on one particular ground as being the dominant reason for the diagnosis of injustice" ((Sen 2009)). That there may be many reasons for unjust economic outcomes beyond the lingering effects of past injustices is likely - see, for example, (Zingales 2012). But knowing how significant is the impact of past unjust outcomes is still fundamental to judging the legitimacy of today's outcomes.

Yet the estimates of the half life of injustice presented here would be underestimates if those who acquire great material wealth can shape political and legal institutions in their favour *and* if those institutions (including laws) show much greater durability than physical or human capital. Angus Deaton has pointed out that hereditary landowners in Britain 200 years ago were not only rich, but they also controlled parliament through a severely limited franchise. After 1815 the Corn Laws kept wheat prices high - to the benefit of land owners and much to the cost of the majority of the population. Those laws lasted 31 years until repealed in 1846 - a rate of depreciation somewhat higher than the rates assumed in this paper.

The models of growth and distribution that I have used to illustrate the mechanisms whereby injustice in acquisition lingers are simple - I assume competitive markets, that factors are paid marginal products and that the conditions for aggregation of capital and labor into an all economy production function exist. These are strong assumptions. How the persistence of injustice in economic outcomes is affected by relaxation of these assumptions seems a fruitful subject for further work.

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