DISCUSSION PAPER SERIES

DP13285
(v. 2)

THE NETWORK ORIGINS OF THE GAINS FROM TRADE

Maarten Bosker and Bastian Westbrock

INTERNATIONAL TRADE AND REGIONAL ECONOMICS

ISSN 0265-8003

THE NETWORK ORIGINS OF THE GAINS FROM TRADE

Maarten Bosker and Bastian Westbrock

Discussion Paper DP13285
First Published 31 October 2018
This Revision 07 March 2019

Centre for Economic Policy Research
33 Great Sutton Street, London EC1V 0DX, UK
Tel: +44 (0)20 7183 8801
www.cepr.org

THE NETWORK ORIGINS OF THE GAINS FROM TRADE

Abstract

In this paper, we develop a network perspective on the welfare gains from trade in today's internationally fragmented supply chains. Towards this end, we study a Ricardian trade model featuring trade in final and intermediate products, and introduce a novel comparative statics approach to decompose the total welfare effects of an arbitrary trade cost shock into several meaningful, easily quantifiable, components. This approach uncovers a unique feature of supply chain trade: the gains from trade are not so much determined by a country's own access to the technologies and markets of its direct trading partners, but rather by its supply chain exposure to countries further up- and downstream in the global supply chain. We develop a set of simple statistics to measure each country's supply chain exposure, show how it predicts the gains from trade, and identify each country's key trade intermediaries, i.e., other nations that primarily leverage its supply chain exposure.

JEL Classification: F10, F11

Keywords: global production network, Supply Chains, Gains from trade, network diffusion, network exposure, trade intermediation

Maarten Bosker - bosker@ese.eur.nl
Erasmus University Rotterdam and CEPR

Bastian Westbrock - b.westbrock@uu.nl
Utrecht University

The network origins of the gains from trade

Maarten Bosker* and Bastian Westbrock‡

March 2019

Abstract

In this paper, we develop a network perspective on the welfare gains from trade in today's internationally fragmented supply chains. Towards this end, we study a Ricardian trade model featuring trade in final and intermediate products, and introduce a novel comparative statics approach to decompose the total welfare effects of an arbitrary trade cost shock into several meaningful, easily quantifiable, components. This approach uncovers a unique feature of supply chain trade: the gains from trade are not so much determined by a country's own access to the technologies and markets of its direct trading partners, but rather by its *supply chain exposure* to countries further up- and downstream in the global supply chain. We develop a set of simple statistics to measure each country's supply chain exposure, show how it predicts the gains from trade, and identify each country's key *trade intermediaries*, i.e., other nations that primarily leverage its supply chain exposure.

Keywords: global supply chains, gains from trade, network diffusion, network exposure, trade intermediation (JEL codes: F10, F11)

1 Introduction

Global supply chains are a defining feature of the modern world economy. This paper argues that their emergence has had important implications for our understanding of the

*Erasmus University Rotterdam, The Netherlands, Tinbergen Institute and CEPR. bosker@ese.eur.nl

†Erasmus University Rotterdam, The Netherlands. bastian.westbrock@gmail.com.

‡This paper is a much extended version of an earlier working paper circulated under the title *'A theory of trade in a global production network.'* We thank a number of people whose inputs have significantly improved the quality of our paper: Costas Arkolakis, Daron Acemoglu, Joppe de Ree, Julian Emami Namini, Pol Antràs, Treb Allen, Vincent Rebeyrol, Yuan Zi, and Yves Zenou. We are also grateful to numerous comments and suggestions received by seminar participants in Bielefeld, Brussels, Groningen, Leuven, Munich, Oxford, Paris, Rotterdam, Stockholm, and Utrecht.

origins of the welfare gains from trade. In particular, we show that these gains are no longer primarily determined by a country's own access to the technologies and markets of its direct trade partners (its own geography and technology). Instead, they crucially depend on a country's precise position in the global production network, that can be captured by a set of simple statistics closely linked to concepts from network theory.

We develop this network perspective on the gains from trade within the confines of a standard Ricardian trade model à la Armington (1969) and Eaton and Kortum (2002) that features trade in final and intermediate products, and that embeds a rich set of potential production sequences between countries, ranging from a simple linear supply chain to an arbitrary complex network with loops. We first characterize how an arbitrary trade cost shock along any number of trade routes affects real per capita income in each and every country. This reveals that the comparative statics predictions of the model can, in fact, be expressed in terms of a *network diffusion model* that describes how the *local effects* of the shock, i.e. the well-known goods supply, factor demand, and import competition effects in the countries directly involved in the affected trade routes, diffuse to all remaining countries. This diffusion happens through two, very different, channels:

(i) the *general equilibrium multipliers* that determine how the goods and factor price changes emerging from the local effects reverberate between the goods and factor markets in each and every country,

(ii) *supply chain diffusion* that defines how the local effects of a shock propagate to countries that are only indirectly exposed to this shock through a chain of two or more links in the global supply chain.

While both channels are an integral part of any modern general-equilibrium theory of input-output trade, our ability to set the supply chain diffusion channels apart from the general equilibrium multipliers is new. We show how to derive two simple statistics that measure supply chain diffusion in a theory-consistent manner: one capturing each country's *supply chain exposure* to shocks occurring in the more upstream stages of production, and another measuring each country's *supply chain exposure* to downstream shocks.[1] The remainder of the paper then highlights the role of a country's up- and downstream exposure in determining the welfare gains from trade. We do this by means of three specific counterfactual exercises.

[1]These supply chain diffusion channels also play an important role in a macroeconomic literature on the role of national input-output chains (Acemoglu et al., 2015; Baqaee, 2016). Note, however, that our diffusion model is richer than its closed-economy precursors in that it contains the general equilibrium multipliers as an additional set of diffusion channels.

First, a unilateral export cost reduction along a single trade route. In a world without traded intermediate inputs, this cost reduction would simply reduce labor demand in all nations but the exporter, which is nothing but the second Hicksian law of comparative statics. In a global production network, in contrast, parts of the exporter's gains spill over to its supply chain partners. These spillovers can in fact be very sizable, and we derive a set of conditions on the local network structure around a trade link for which the Hicksian law is even overturned.

Second, a uniform trade cost reduction along all trade routes worldwide. One may expect this 'equal opportunity' cost reduction to lead to equally sized welfare gains in all nations, as all countries improve their access to other markets alike. Yet, we show that the logic only goes through in the absence of international production linkages. In their presence, each country's welfare gains are proportional to its *supply chain exposure*, with countries operating in the downstream stages of production gaining most.

Finally, we perform what could be regarded as the flipside of the classic gains from trade analysis. We isolate one nation after the other from the global production network and ask by how much, and through which channels, the remaining nations are affected. Our findings show that, next to the nations that are important because of their own value added to the network or the size of their own markets, there is a group of countries that primarily intermediates other nations' demand or valued added. It is these trade partners, the important *trade intermediaries*, that explain most of the cross-country variation in network exposure.

In sum, our paper sheds new light on one of the classic questions in trade theory: where do the gains from trade come from? We show that, in today's global supply chains, the established determinants of these gains, notably a country's own 'geography and technology', are superseded by concepts that are central to the theory of social and economic networks: positive externalities, network centrality, and intermediation. It is, thus, not surprising that our measures for supply chain exposure and trade intermediation are closely related to measures of diffusion centrality (Bonacich, 1987; Banerjee et al., 2013), respectively bridging capital (e.g., Ballester et al., 2006), from that literature.[2]

Of course, our paper is not the first to study the economics of global supply chains. Already the early theories of Ethier (1979) and Dixit and Grossman (1982) have made clear that the associated gains from specialization have important implications for the location of production, and for the sensitivity of incomes to changes in trade barriers or

[2]See Jackson (2017) for a comprehensive survey of the social capital and network centrality measures.

factor costs. Our paper is less ambitious than recent extensions to this work, notably Yi (2003), Baldwin and Venables (2013), Costinot et al. (2013), Fally and Hillberry (2015) or Antràs and de Gortari (2017), in that we do not study the building up of a supply chain or the endogenous sorting of countries into the different production steps. What sets our paper apart from these studies is, however, our use of a comparative statics approach that shows exactly how the network structure of production matters for the welfare gains from small-scale trade cost shocks, enabling us to develop some simple statistics to measure the relevant dimensions of the network in this regard.

This aspect of our paper also makes it closely related to a recent group of network studies in macroeconomics. Responding to the foundational works of Hulten (1978) and Lucas (1977) who argued that, under the assumptions of an efficient and frictionless market, the exact micro structure of production does not matter for aggregate economic outcomes, these studies are occupied with the question of how relaxing these assumptions changes the picture (e.g., Acemoglu et al., 2012, 2015; Baqaee, 2016; Grassi, 2017; Baqaee and Farhi, 2017).[3] It should be clear that the Ricardian trade model studied in this paper, featuring imperfect mobility of goods and factors across space, is a case in point. Our findings in fact suggest that, in the presence of such frictions, the micro structure of international production determines how the aggregate economic gains from a trade cost reduction are distributed over the different geographical units that are part of this network.

On the methodological side, our classic comparative statics approach complements two alternative approaches to quantify the gains from trade in general equilibrium: the 'sufficient statistics approach' of Arkolakis et al. (2012) and the 'exact hat algebra' of Dekle et al. (2008) and Costinot and Rodríguez-Clare (2014). Also our approach requires no more than readily observable macroeconomic variables, such as gross trade flows and total production values, as well as estimates of the model's main elasticity parameters, to put numbers on our counterfactual predictions. A disadvantage of our first-order effect approximations might be that they are not suited for analyzing the consequences of quantitatively large shocks, as the omitted higher-order terms can be very sizable in the presence of production linkages.[4] Against this, however, also stand some clear advantages. In contrast to the 'sufficient statistics approach', our counterfactual predictions solely rely on observable macroeconomic variables in the initial equilibrium, for any type of trade cost shock,

[3] See Arkolakis and Ramanarayanan (2009) for one of few applications of Hulten's (1978) foundational theorem in an international economics context.

[4] A point that is clear at least since Yi (2003) and which has been raised more recently in Baqaee and Farhi (2017) for a closed-economy context.

and not just for the special case of a country's entire isolation. And, in contrast to the 'exact hat algebra', it goes without the need to numerically solve a system of non-linear equations. The closed form expressions underlying our approach in fact allow us to cast several of our findings into general propositions.[5] Even more important, they allow us to decompose the total effect of a trade cost shock into several meaningful components, revealing in particular the importance of the supply chain diffusion channels.[6]

Finally, our paper speaks to a group of studies developing measures that describe the exact position that different countries and sectors occupy in global supply chains. In particular, it provides a sound general equilibrium foundation for some of the measures developed there. Notably, the downstreamness measure of Antràs and Chor (2013), proves a valid sufficient statistic for predicting the welfare effects of a global trade cost reduction in our model. And, Hummels et al. (2001)'s measure of vertical specialization trade can be interpreted as a meaningful statistic for the importance of a country as a trade intermediary. As such, our findings support the usefulness of these measures as inputs in policy evaluations or welfare analyses.

We develop our arguments as follows: Section 2 presents our basic framework where countries trade varieties of one final and one intermediate product only. This stylized model provides the simplest possible way to bring our network perspective on the gains from trade across. Section 3 sets out our comparative statics approach and introduces the concepts of supply chain diffusion and exposure. Their importance for understanding the welfare gains from trade is then highlighted in Sections 4-6. The formal proofs of all our statements are delegated to Appendices A.1-A.7. Finally, Appendix A.8 confirms the generality of our findings in a model featuring multiple sectors of production and a general input-output structure, similar to e.g., di Giovanni et al. (2014), Caliendo and Parro (2015), Ossa (2015) or Blaum et al. (2018).

[5] It also makes our approach computationally much less intensive.

[6] Based on the sufficient statistics approach or the equilibrium in changes, one can distinguish between the different adjustment margins at the ports of call. This includes, for example, the distinction between the extensive and the intensive expansion of the importer's basket of goods, the role of firm selection or, what is most relevant for our paper, the role of imported intermediate inputs. Yet, even the intermediate inputs margin is too coarse for our purposes, as it lumps up the benefits that a country derives from the improved access to its direct suppliers, which we subsume under the local effects, with the access to the value added of producers further upstream in the global supply chains.

2 A simple model of the global production network

Consider a world economy consisting of n countries, indexed by $i \in \mathcal{N} = \{1, 2, ..., n\}$, that produce and trade two types of products: the varieties of a final product, $\eta^f \in \mathcal{H}^f$, and the varieties of an intermediate input, $\eta^i \in \mathcal{H}^i$.

Preferences. Consumers have CES preferences over the different varieties of the final product. In particular, a consumer in country i chooses to consume an amount $q_i(\eta^f) \geq 0$ of every $\eta^f \in \mathcal{H}^f$ so as to

$$\begin{aligned} \max u_i &= \left(\int_{\eta^f \in \mathcal{H}^f} q_i(\eta^f)^{\frac{\alpha-1}{\alpha}} \right)^{\frac{\alpha}{\alpha-1}} \\ \text{subject to} \quad & \int_{\eta^f \in \mathcal{H}^f} q_i(\eta^f) p_i(\eta^f) \leq w_i \end{aligned} \tag{1}$$

where $p_i(\eta^f)$ denotes the price of a η^f-variety, w_i the common wage rate in country i, and $\alpha > 1$ the elasticity with which consumers substitute between final goods varieties.

Technologies. Producers employ labor and the different varieties of the intermediate product, and use a two-tier CES production technology. They substitute between labor and intermediate inputs at an elasticity $\beta \geq 0$, $\beta \neq 1$, and between the different intermediate input varieties at elasticity α. Thus, in order to ship $q_{ij}^t \geq 0$ units to a buyer in country j, a producer of a final (intermediate) goods variety η^t, $t \in \{f, i\}$, employs $l_i^t \geq 0$ units of labor and $q_i^t(\eta^i) \geq 0$ inputs of variety $\eta^i \in \mathcal{H}^i$ so as to

$$\begin{aligned} \min \ c_i^t &= \left(l_i^t w_i + \int_{\eta^i \in \mathcal{H}^i} q_i^t(\eta^i) p_i(\eta^i) \right) \\ \text{subject to} \quad q_{ij}^t &\leq \frac{\mu_{ij}(\eta^t)}{\tau_{ij}} \left(\kappa_i^l \left(l_i^t\right)^{\frac{\beta-1}{\beta}} + \kappa_i^i \left(\int_{\eta^i \in \mathcal{H}^i} q_i^t(\eta^i)^{\frac{\alpha-1}{\alpha}} \right)^{\frac{\alpha}{\alpha-1}\frac{\beta-1}{\beta}} \right)^{\frac{\beta}{\beta-1}} \end{aligned} \tag{2}$$

Here, $\mu_{ij}(\eta^t) > 0$ denotes the seller-buyer specific total productivity of labor and intermediate inputs, $\kappa_i^l > 0$ the relative productivity of labor, and $\kappa_i^i \geq 0$ the relative productivity of intermediate inputs, which we also interpret as an inverse 'coordination cost' in the use of (foreign) intermediate inputs. If $\kappa_i^i = 0$ then no intermediates are used in country i. Finally, $\tau_{ij} \in [1, \infty]$ measures a origin-destination-specific 'iceberg' trade cost parameter, which may be prohibitively high ($\tau_{ij} = \infty$) and which satisfies the triangular inequality: $\tau_{ij} \leq \tau_{ik}\tau_{kj}$ for all $i, j, k \in \mathcal{N}$.

Market structure. We assume that competition is perfect in each market and that all markets clear. For the product markets, this means that a buyer in country i pays

$$p_i(\eta^t) \;=\; \min\left\{c^t_{ji} \equiv p_j\tau_{ji}/\mu_{ji}(\eta^t) \mid j \in \mathcal{N}\right\} \tag{3}$$

for a variety η^t, whereby we follow Eaton and Kortum (2002) and let the identity of the lowest-cost supplier be determined by an i.d. draw for $\mu_{ji}(\eta^t)$ from a Fréchet distribution featuring an exporter-country j specific scale parameter and a shape parameter γ.[7]

For the labor markets, this means that total fixed labor supply, $l_i w_i$, equals total labor demand, l^d_i, in each country:

$$l_i w_i \;=\; l^d_i(\mathbf{p}, \mathbf{w}, \mathbf{q}) \;\;\equiv\;\; \lambda_i \sum_{j\in\mathcal{N}} \pi_{ij}(e^f_j + e^i_j) \tag{4}$$

A specific feature of our model is that it unites the two key forces of the Ricardian trade literature. On the one hand, the endogenously determined 'length' of the sequence of production steps performed in a country, which is key to the vertical-specialization branch of this literature (e.g. Dixit and Grossman, 1982; Costinot et al., 2013). In our model, this is captured by the endogenous labor cost share λ_i, or 'value added' share, of each country's typical producer. Along with its complement, the value added share of the producer's intermediate input suppliers, $1-\lambda_i$, the labor cost share is in fact given by

$$\lambda_i \;=\; \frac{(\kappa^l_i)^\beta w_i^{1-\beta}}{p_i^{1-\beta}} \qquad \text{and} \quad 1-\lambda_i \;=\; \frac{(\kappa^i_i)^\beta (p^i_i)^{1-\beta}}{p_i^{1-\beta}} \tag{5}$$

where p_i denotes the producer price for an optimal input bundle and p^i_i the price for a composite intermediate input. Labor and intermediate inputs might, thus, be complementary

[7] In particular, $\mu_{ji}(\eta^t)$ is drawn from the the Fréchet distribution

$$\Pr\left(\mu_{ji}(\eta^t) < x\right) = \exp\left[-\left(\frac{x}{\nu_j}\right)^{1-\gamma}\right]$$

with as parameters, on the one hand, the shape parameter γ, which supersedes the elasticity of substitution α from now on and which we assume to satisfy $\gamma > \max\{\alpha, 2\}$ and $\gamma \geq \beta$, and on the other hand the country j-specific scale parameter $\nu_j > 0$. Based on this assumption, the 'mean productivity' of country j's producers, μ_j, follows as

$$\mu_j \equiv \left(\frac{\gamma-\alpha}{\gamma-1}\,\Gamma\right)^{\frac{\gamma-1}{\alpha-1}} \nu_j$$

where Γ is the Gamma function.

($\beta < 1$) or substitutable ($\beta > 1$) in our model, with perfect substitutability ($\beta \to \infty$), perfect complementarity ($\beta = 0$), and the Cobb-Douglas case with fixed input shares ($\beta \to 1$) as the limit cases.

On the other hand, our model features the horizontal specialization of countries into the production of different varieties on the same step of the sequence. This central feature of the analysis in, for example, Eaton and Kortum (2002) is captured by the endogenously determined trade share of each country j's products in country i's total expenditure on final as well as on intermediate products. Since trade costs are identical for both product types, these shares are both given by

$$\pi_{ji} \equiv \frac{x_{ji}^t}{e_i^t} = \frac{\mu_j \, p_j^{1-\gamma} \tau_{ji}^{1-\gamma}}{(p_i^t)^{1-\gamma}} \tag{6}$$

where $e_i^t \equiv \sum_{i \in \mathcal{N}} x_{ji}^t$ denotes i's total expenditure on final (intermediate) products, x_{ji}^t the value of the final (intermediate) products bought from j, μ_j the 'mean productivity' of country j's producers as defined in footnote 7, and $1 - \gamma$ the 'trade elasticity'.

Equilibrium. Appendix A.1 proves that an equilibrium exists for our model, and it is locally unique when the following two additional regularity conditions are met: first, all countries should add value to the global production network: $\lambda_i \in (0\,,\,1]$ for all $i \in \mathcal{N}$. Second, the system of labor demand functions, $l_i^d(\mathbf{p}, \mathbf{w}, \mathbf{q})$ $i \in \mathcal{N}$, should be locally invertible around an equilibrium point.

Substantively, the first condition requires us to assume that the bilateral trade costs, τ_{ij}, and the country-specific 'coordination costs', $1/\kappa_i^i$, are sufficiently high, when labor and intermediate inputs are substitutes ($\beta > 1$), and sufficiently low in case they are complements ($\beta < 1$). Loosely speaking, the global production network should be sufficiently 'sparse' ('dense') to ensure that all producers employ at least some local labor. To satisfy the second condition, we impose that the cross-price elasticities of the labor demand system lie in the unit interval: $\partial\, l_i^d / \partial(l_j w_j) \in (0, 1)$ for all $j \neq i$. In other words, we assume that the interaction between different nations' labor demands is dominated by the classic foreign income multiplier (Samuelson, 1943) and less so by, on the one hand, the labor demand complementarities that naturally arise in a production network and, on the other hand, the competition for market shares in each country's product markets.[8]

[8] The intuition behind the labor demand complementarities should be clear: a higher wage in country j puts negative pressure on the demand for products of any country i that sources intermediate inputs from it. Yet, as known at least since Rader (1968), any form of complementarity between the equations of a

Given a parameter constellation that meets these two conditions, the following holds:

Proposition 1. *Consider our regular Ricardian economy with traded intermediate products and flexible input cost shares. There is at least one strictly positive equilibrium* $(\mathbf{p}, \mathbf{p^f}, \mathbf{p^i}, \mathbf{w}, \mathbf{l^d}, \mathbf{q})$, *which is locally unique (up to normalization) and which admits for local comparative statics analysis.*

3 Network diffusion and the gains from trade

So far, we have described a fairly standard Ricardian trade model that has been used in many papers before ours to look at the welfare gains of various types of (trade cost) shocks. In this section, we introduce our novel network perspective on these welfare gains. We show that the model's comparative statics predictions can, in fact, be expressed as a *network diffusion model* that describes exactly how the *local effects* of a trade cost shock, i.e. the well-known goods supply, factor demand, and import competition effects in the countries directly involved in the affected trade routes, diffuse to all remaining countries.

In particular, consider an arbitrary, but small, proportional shock to any number of elements of the trade cost matrix, $\mathbf{T} = (\tau_{ij}) \in \mathbb{R}^{n \times n}_{++}$. In a first-order approximation, each country's real per capita income effects can be decomposed into a wage effect and a price effect,

$$d\ln(u_i) \;\; = \;\; d\ln(w_i) \, - \, d\ln(p_i^f)$$

which are themselves defined by the total derivative of labor market equation (4) and the total derivative of the implicitly defined price indices $\tilde{p}_i, \tilde{p}_i^f$, and $\tilde{p}_i^i$. Solving this system of equations results in the following matrix expressions:

Definition 1 (diffusion model). *The welfare effects of an arbitrary, but small, trade cost shock* $\mathbf{d}\ln(\mathbf{T}) = (d\ln(\tau_{ij})) \in \mathbb{R}^{n \times n}$ *are given by the following linear mapping of the shock's local effects* $\boldsymbol{\delta}^{\mathbf{l^d}}$ *and* $\boldsymbol{\delta}^{\mathbf{p}}$, *as defined by the column vectors in (8), into the vectors of wages*

demand system jeopardizes the very idea of analyzing it, because the conditions for uniqueness and stability of a fixed point are violated (see also Adao et al., 2017). We additionally require that $\partial l_i^d / \partial(l_j w_j) < 1$ for all $j \neq i$. The reason is that the labor demand system must also satisfy $\sum_{k \in \mathcal{N}} \partial l_k^d / \partial(l_j w_j) = 1$. Thus, in order for a wage increase to indeed raise domestic demand, such as expected from an income multiplier, the cross-price elasticities of the labor demand system should not be too large.

and prices:

$$\begin{aligned}\mathbf{d}\ln(\mathbf{w}) &= \boldsymbol{\Phi}^{\mathbf{mult}}_{\mathbf{i}^*}\,\boldsymbol{\Phi}^{\mathbf{de}}\,\boldsymbol{\delta}^{\mathbf{ld}} \\ \mathbf{d}\ln(\tilde{\mathbf{p}}^{\mathbf{f}}) &= \boldsymbol{\Phi}^{\mathbf{se}}\,\boldsymbol{\delta}^{\mathbf{p}} + \boldsymbol{\Phi}^{\mathbf{tot}}\,\mathbf{d}\ln(\mathbf{w})\end{aligned} \tag{7}$$

with the general equilibrium multipliers $\boldsymbol{\Phi}^{\mathbf{mult}}_{\mathbf{i}^*}$ *and* $\boldsymbol{\Phi}^{\mathbf{tot}}$*, as defined in (9), and the supply chain exposure matrices* $\boldsymbol{\Phi}^{\mathbf{de}}$ *and* $\boldsymbol{\Phi}^{\mathbf{se}}$*, as defined in (14), as coefficients.*

A trade cost shock has, in a first instance, a direct impact on the exporters and importers involved in the affected trade links. We call these the local effects of the shock and formally define them as follows:

Definition 1 (local effects). *The local price and demand effects of a trade cost shock are given by:*

$$\begin{aligned}\boldsymbol{\delta}^{\mathbf{p}} &\equiv \underbrace{\left[\boldsymbol{\Pi}^{\mathbf{T}} \circ (\mathbf{d}\ln(\mathbf{T}))^{\mathbf{T}}\right]\mathbf{1}}_{\text{Supplier access}} \\ \boldsymbol{\delta}^{\mathbf{ld}} &\equiv (1-\gamma)\Bigg(\underbrace{\left[\boldsymbol{\Pi} \circ \mathbf{d}\ln(\mathbf{T})\right](\mathbf{e}^{\mathbf{f}}+\mathbf{e}^{\mathbf{i}})}_{\text{Market access}} - \underbrace{\boldsymbol{\Pi}(\mathbf{E}^{\mathbf{f}}+\mathbf{E}^{\mathbf{i}})\left[\boldsymbol{\Pi}^{\mathbf{T}} \circ (\mathbf{d}\ln(\mathbf{T}))^{\mathbf{T}}\right]\mathbf{1}}_{\text{Import competition}} \\ &\qquad + \underbrace{(\mathbf{X}^{\mathbf{f}}+\mathbf{X}^{\mathbf{i}})\,\mathbf{d}\ln(\tilde{\mathbf{p}})}_{\text{Exporter's productivity}} - \underbrace{\boldsymbol{\Pi}(\mathbf{E}^{\mathbf{f}}+\mathbf{E}^{\mathbf{i}})\,\boldsymbol{\Pi}^{\mathbf{T}}\,\mathbf{d}\ln(\tilde{\mathbf{p}})}_{\text{Competitors' productivity}}\Bigg) \\ &\quad + \underbrace{(\mathbf{X}^{\mathbf{f}}+\mathbf{X}^{\mathbf{i}})\,\mathbf{d}\ln(\boldsymbol{\lambda})}_{\text{Exporter's offshoring}} - \underbrace{\boldsymbol{\Pi}\,(\mathbf{E}^{\mathbf{f}}+\mathbf{E}^{\mathbf{i}})\,\mathbf{d}\ln(\boldsymbol{\lambda})}_{\text{Importer's offshoring}}\end{aligned} \tag{8}$$

where $\circ$ *denotes the Hadamard (pointwise) product, which we give priority in the order of operations,* $\mathbf{1}$ *denotes a column vector of ones,* $\boldsymbol{\Pi}$ *the full matrix of expenditure shares,* $\mathbf{E}^{\mathbf{t}}$*,* $\mathbf{X}^{\mathbf{t}}$*, and* $\boldsymbol{\Lambda}$*,* $\mathbf{t} \in \{\mathbf{f}, \mathbf{i}\}$*, the diagonal matrices corresponding to the column vectors of every country's final (intermediate) goods expenditures,* $\mathbf{e}^{\mathbf{t}}$*, outputs,* $\mathbf{x}^{\mathbf{t}}$*, and labor cost shares,* $\boldsymbol{\lambda}$*, respectively, and* $\mathbf{d}\ln(\tilde{\mathbf{p}})$ *denotes the shock's impact on producer prices, a detailed expression for which can be found in Appendix A.1 equation (34). Finally,* $\mathbf{Z}^{\mathbf{T}}$ *is our notation for the transpose of a matrix* $\mathbf{Z}$*.*

These local effects are in fact all well-known. Any neoclassic trade theory considers the direct impact of a trade cost shock on the exporter's *market access* into the importer country, the importer's *supplier access* to the former's products, and the intensity of *competition* in the importer market. We additionally subsume two further channels under the local demand effects, which are only active in the presence of international production linkages. These are the *productivity channel* (cf. Grossman and Rossi-Hansberg, 2008; Rodríguez-Clare, 2010), which captures the fact that a trade cost shock alters the exporter's production costs, relative to that of its competitors, through its impact on every country's supplier access to intermediate products. Finally, the *offshoring* or *production fragmentation* channel (cf. Dixit and Grossman, 1982; Costinot et al., 2013),

$$\mathbf{d}\ln(\boldsymbol{\lambda}) \;=\; (\beta - 1)\,\mathbf{d}\ln(\tilde{\mathbf{p}})$$

which captures the labor demand consequences in the exporter and importer countries that are due to the fact that a trade cost shock changes the relative costs of labor vis-à-vis intermediate inputs.[9]

Diffusion channels. The matrices in equation system (7), in turn, define how a trade cost shock affects every country indirectly, through a variety of diffusion channels.[10] In fact, the sole function of the matrices is to determine how the total effect of a shock is distributed among all countries. No more and no less. This is what the following result states:

Lemma 1. *The worldwide total welfare effect of an arbitrary, but small, trade cost shock* $\mathbf{d}\ln(\mathbf{T})$ *is given by*

$$\sum_{i\in\mathcal{N}} l_i w_i d\ln(u_i) \;\;=\;\; \sum_{i\in\mathcal{N}} (e_i^f + e_i^i)\,\delta_i^p$$

The Lemma, which is proven in Appendix A.4, is essentially an application of Hulten (1978)'s Theorem. It states that all we need to know from a total world-welfare perspective is by how much a trade cost shock in- or decreases the value of all imports on the immediately affected trade routes. In other words, only the magnitude of the local supplier access effects, $\boldsymbol{\delta}^{\mathbf{p}}$, matters from a world-welfare perspective. The matrices in (7) are of no further relevance.[11]

---

[9] In fact, a look at expression (34) tells us that we might subsume these two channels under the group of supply chain diffusion channels. We abstain from doing so to simplify the exposition.

[10] In fact, the equations in (7) essentially define a diffusion model that generalizes precursors in the macroeconomic literature on production networks (Acemoglu et al., 2015; Baqaee, 2016) and the social network literature (Bonacich, 1987; Banerjee et al., 2013).

[11] The intuition is the following: the effect of a trade cost shock on wages does not matter from a world-welfare perspective, as wages merely redistribute incomes in a Walrasian economy. Also, it follows from

They are, however, crucial for understanding how the total gains or losses are distributed across countries through two very different types of diffusion channels: first, we have the *general equilibrium multipliers* that capture the interdependencies between the goods and factor markets in our model. As such, they are active in any general equilibrium framework of the world economy, regardless of whether countries share production linkages or not. In particular, the *terms of trade multiplier*, $\mathbf{\Phi^{tot}}$, captures the elasticity with which the final goods prices in each country respond to factor price changes in all other nations or, in other words, the spillovers from the labor to the goods markets. The *foreign trade multiplier*, $\mathbf{\Phi^{mult}_{i^*}}$, on the other hand, measures the elasticity with which one country's wage rate responds to a labor demand shock in any other country, hereby capturing all the international labor market spillovers in our model. Formally,

Definition 1 (general equilibrium multipliers). *The terms of trade multiplier and the foreign trade multiplier are given by*

$$\begin{aligned} \mathbf{\Phi^{tot}} &\equiv \frac{\partial \ln(\tilde{\mathbf{p}}^{\mathbf{f}})}{\partial \ln(\mathbf{w})} \in \mathbb{R}^{n\times n}_{++} \\ \mathbf{\Phi^{mult}_{i^*}} &\equiv \frac{\partial \ln(\tilde{\mathbf{w}})}{\partial \ln(\mathbf{l^d})} \in \mathbb{R}^{n\times n}_{++} \end{aligned} \tag{9}$$

with detailed expressions provided in Appendix A.1 (33) and (39).

The second class of channels only emerges in a global supply chain. It is based on one of the fundamental consequences of production fragmentation, namely that trade in value added is 'decoupled' from trade in products. While a trade cost shock alters the flow of goods between countries, the value added content embodied in these goods neither fully belongs to the exporters nor is it fully absorbed by the importers. The consequences of the shock are, instead, shared by all countries that contributed to the goods' production and by all countries that ultimately consume them.

Naturally, the importance of these *supply chain diffusion channels* depends on the precise structure of the global production network, before the shock. This structure is fully captured by two simple statistics in our model: first, the Leontief inverse matrix relating labor demand in each nation to the final goods expenditures in every other location. This

the Envelope Theorem that the implications of the shock for the allocation of inputs is of no more than second-order importance. What remains is the first-order effect on the costs of importing or, differently put, δ_i^p.

inverse can be expressed in terms of the following infinite matrix series[12]

$$\mathbf{l^d} \;=\; \mathbf{\Lambda} \sum_{\mathbf{h=0}}^{\infty} \left[\mathbf{\Pi}(\mathbf{I}-\mathbf{\Lambda})\right]^{\mathbf{h}} \mathbf{\Pi}\, \mathbf{e^f} \tag{10}$$

Second, the Ghosh (1958) inverse matrix relating every country's final goods expenditures to the value added shares, λ_j, of every other nation,[13]

$$\mathbf{e^f} \;=\; \mathbf{LW}\, \mathbf{\Pi^T} \sum_{\mathbf{h=0}}^{\infty} \left[(\mathbf{I}-\mathbf{\Lambda})\mathbf{\Pi^T}\right]^{\mathbf{h}} \boldsymbol{\lambda} \tag{12}$$

whereby we use **LW** to denote the diagonal matrix $(l_i w_i) \in \mathbb{R}^{n\times n}_{++}$.

A trade cost shock changes the value added flows through the supply chain. It thus directly affects the coefficient matrices, $\mathbf{\Pi}(\mathbf{I}-\mathbf{\Lambda})$ and $(\mathbf{I}-\mathbf{\Lambda})\mathbf{\Pi^T}$, in (10) and (12). The full impact of the shock, however, depends on how these direct effects scale up in a non-trivial way, as a product might traverse the same trade route multiple times while being processed. To keep track of these effect magnifications, we extend on (and add to) a collection of results from the network literature (Ballester et al., 2006; Temurshoev, 2010), that allows us to express —in the spirit of comparative statics analysis— the shock's full effect on the Leontief and Ghosh inverse matrices in terms of the initial value of these matrices, and the direct impact on their coefficients. Our most valuable extension tackles the additional complication that these coefficients are endogenously dependent on the ex-ante prices of all goods and production factors so that a shock to any cell of these matrixes potentially triggers an adjustment in all others.[14]

[12]To arrive at the identity, make repeated use of the expression for labor demand in (4).

[13]The identity follows from the elementary relationship

$$[\mathbf{I}-(\mathbf{I}-\mathbf{\Lambda})\mathbf{\Pi^T}]^{-1}\boldsymbol{\lambda} = \mathbf{1} \tag{11}$$

combined with the market clearing conditions $\mathbf{LW} = \mathbf{E^f}$ and $\mathbf{\Pi^T 1} = \mathbf{1}$. Identity (11), in turn, stems from the series of simple identities

$$\boldsymbol{\lambda} = \mathbf{1} - \mathbf{1} + \boldsymbol{\lambda} = \mathbf{1} - \mathbf{\Pi^T 1} + \mathbf{\Lambda \Pi^T 1} = \left[\mathbf{I}-(\mathbf{I}-\mathbf{\Lambda})\mathbf{\Pi^T}\right]\mathbf{1}$$

Assuming invertibility, which is needed anyway for Proposition 1, we immediately arrive at (11).

[14]In order to circumvent this problem, which is nothing but a violation of the requirements for the Nonsubstitution Theorem, prior research has worked with either Leontief's original formulation of an input-output model ($\beta = \gamma = 0$) or with Long and Plosser (1983)'s Cobb-Douglas version ($\beta = \gamma = 1$) both of which result in a fixed coefficient matrix. It should be clear, however, that both are ill-suited in our international economics context, since they are at odds with any of the reported estimates for the trade elasticity.

For a small shock to any number of elements of an (endogenous) coefficient matrix $\mathbf{Z}$, we can verify that[15]

$$\mathbf{d}\sum_{\mathbf{h=0}}^{\infty}\mathbf{Z^h} \;=\; \sum_{\mathbf{h=0}}^{\infty}\mathbf{Z^h}\,\mathbf{dZ}\sum_{\mathbf{h=0}}^{\infty}\mathbf{Z^h} \tag{13}$$

A very useful implication of identity (13) is that it allows us to separate the local effects of a trade cost shock from its network effects. In particular, we interpret the first summand of the matrix series on the left-hand side of the local effects matrix $\mathbf{dZ}$ (defined in (8)), the identity matrix $\mathbf{I}$, as the coefficient on this local effect. The second and higher-order summands, $\mathbf{Z}, \mathbf{Z^2}, ...$, then define what we call a country's *supply chain exposure* to this shock. Countries are in fact exposed in two ways:

Definition 1 (supply chain exposure). *Country i's supply chain exposure to a trade cost shock is given by the elements in row i of the $n \times n$-matrices:*

$$\text{Upstream exposure:} \quad \mathbf{\Phi^{se}} - \mathbf{I} \equiv \sum_{\mathbf{h=1}}^{\infty}\left[\mathbf{\Pi^T}(\mathbf{I}-\mathbf{\Lambda})\right]^{\mathbf{h}} \tag{14}$$

$$\text{Downstream exposure:} \quad \mathbf{\Phi^{de}} - \mathbf{\Lambda} \equiv \mathbf{\Lambda}\sum_{\mathbf{h=1}}^{\infty}\left[\mathbf{\Pi}(\mathbf{I}-\mathbf{\Lambda})\right]^{\mathbf{h}}$$

whereby in the absence of production linkages: $\mathbf{\Phi^{se}} = \mathbf{\Phi^{de}} = \mathbf{I}$.

An element ij of matrix $(\mathbf{\Phi^{de}} - \mathbf{\Lambda})$ measures the extent to which country i is exposed to a local demand shock in country j 'further down' in the global supply chain. Likewise, $(\mathbf{\Phi^{se}} - \mathbf{I})$ measures how country i's prices respond to a local supply shock in country j

[15] Identity (13) is an immediate consequence of the derivative rule for inverse matrices. Even though (13) is valid for any endogenous coefficient matrix, it is worth noting that the analysis becomes significantly simpler for a uniform elasticity specification of our model ($\beta = \gamma$), as the coefficient matrices in (10) and (12) only feature exogenous trade cost and technology parameters in this case. In a linear supply chain, for example, with country 1 at the top and the final goods producing nation 3 at the bottom, the labor demand function of country 1 can be written as

$$\begin{aligned} l_1^d &= \lambda_1 * \pi_{12}(1-\lambda_2) * \pi_{23}(1-\lambda_3) * \sum_j \pi_{3j} e_j^f \\ &= (\kappa_1^l)^\gamma w_1^{1-\gamma} * (\mu_1 \tau_{12}^{1-\gamma}(\kappa_2^i)^\gamma) * (\mu_2 \tau_{23}^{1-\gamma}(\kappa_3^i)^\gamma) * \sum_j (\mu_3 \tau_{3j}^{1-\gamma}) l_j\, w_j (p_j^f)^{\gamma-1} \end{aligned}$$

We will thus return to this specification at several points in the paper, since it allows us to also quantify the effects of certain inframarginal trade cost variations.

'further up' in the chain.

In sum, the presence of international production linkages adds two new diffusion channels determining how a (trade cost) shock affects every country's welfare regardless of whether its own trade routes are directly impacted by the shock or not. In the remainder of the paper, we show how the presence of these two supply chain diffusion channels changes our understanding of the origins of the gains from trade, and study three particular counterfactual trade cost scenarios for this purpose.[16] Besides deriving several general propositions on the role of supply chain exposure, we also take our diffusion model to the data using the following empirical approach.

3.1 Empirical implementation

Empirically quantifying the counterfactual welfare effects of any, exogenously given, trade cost variation, $\mathbf{d}\ln(\mathbf{T})$, is easily done using (7). As becomes clear from the expressions for the local effects and the different diffusion channels (see above and Appendix A.1), all we need is data on bilateral import shares, $\mathbf{\Pi}$, total outputs, $\mathbf{x^f} + \mathbf{x^i}$, national incomes, $\mathbf{Lw}$, and estimates for the model's elasticity parameters β and γ. The only remaining missing pieces of information can be inferred from the equilibrium identities: $x_i^f + x_i^i = e_i^f + e_i^i$, $l_i w_i = e_i^f$, and $\lambda_i = l_i w_i / (x_i^f + x_i^i)$.

For our own illustrations, we use data from the following sources. First, the CEPII Trade and Production Database that provides all the information we need to explore the evolving importance of the supply chain diffusion channels over the period 1980-2006. Second, for the more recent period 2000-2011, we combine data on bilateral trade flows

[16] Some additional general properties of diffusion model (7) are worth mentioning: first, while Lemma 1 has made clear that the worldwide sum of local price effects is positive for an arbitrary trade cost reduction, the local demand effects neutralize each other. That is, it holds $\mathbf{1^T}\boldsymbol{\delta}^{\mathbf{l^d}} = 0$. See Appendix A.4 for proof.

Second, regarding the diffusion channels, the supply chain channels do —in the spirit of production sharing— no more than spreading the local effects of a shock across the different countries, while keeping the sum of effects constant. The foreign trade multiplier, on the other hand, amplifies any initial effect differences. This can be seen from the fact that the supply chain exposure matrices (and the terms of trade matrix) are mean preserving transformations, i.e., for

$$\mathbf{Z} \in \left\{ \mathbf{\Phi^{de}} \; ; \; \mathbf{LW}\big(\mathbf{\Phi^{se}} - \mathbf{I}\big)[\mathbf{E^i}]^{-1} \; ; \; \mathbf{LW}\mathbf{\Phi^{tot}}\,[\mathbf{LW}]^{-1} \right\} \text{ it holds } \quad \mathbf{1^T Z} = \mathbf{1^T}$$

The foreign trade multiplier, on the other hand, is mean amplifying, i.e.,

$$\mathbf{1^T}\,\mathbf{LW}\,\mathbf{\Phi^{mult}_{i^*}}[\mathbf{E^f}]^{-1} > \mathbf{1^T_{i^*}}$$

where $\mathbf{1^T_{i^*}}$ is a row vector of ones with a zero in element i^*.

from UN Comtrade, total manufacturing output from the UN Industrial Statistics, and total manufacturing GDP from the World Development Indicators. In both data sets, we solely focus on the manufacturing sectors defined by ISIC rev.3 categories 15-37. Data availability leads to yearly variation in the countries included in the sample. To be included in a particular year, a country must report its total value added, the value of its total production, and at least one import or export flow from or to another nation.[17]

As for the model's elasticity parameters, we kept things simple and fixed $\gamma = 5$ for all our illustrations, which lies in the middle of the range of the available trade elasticity estimates by Eaton and Kortum (2002), Romalis (2007), and Caliendo and Parro (2015). Since no reliable point estimate is available for β, we treat it as a floating parameter and report results using different values from the set $\{.001, .5, 1.001, 1.5, ..., 4.5, 5\}$.

4 The externalities of local trade cost shocks

Our first counterfactual exercise stresses the potential magnitude of the supply chain diffusion channel. In particular, we study the simplest possible shock: a one-sided export cost reduction.[18] Naturally, this improves the exporter's access to the importer's market and the importer's access to the products of the former. Our ambition is to go beyond these direct effects and to compare the exporter's and importer's gains with the size of the *network externalities* on third countries.

In a world without production linkages: For comparison, consider an export cost reduction in a world without production linkages. This is our result:

Proposition 2. *Suppose only final goods are traded ($\kappa_i^i = 0$). An export cost reduction between exporter i and importer j (a) lowers the wage in all nations relative to the exporter, i.e., $d\ln(w_k) < d\ln(w_i)$ for $k \neq i$, and (b) lowers the average welfare of all countries but the exporter, i.e., $d\ln(u_i) > 0$ and $\frac{1}{n-1}\sum_{k\neq i} l_k w_k \, d\ln(u_k) < 0$, if γ is sufficiently large.*

The result, which is proven in Appendix A.5, is essentially an application of the second Hicksian law of comparative statics. Because the export cost reduction shifts country j's

[17] The minimum, median and maximum number of countries included in a particular year is 64, 87, and 93 for the CEPII data, and 85, 89, and 96 for our own collected data set. Missing countries are predominantly small developing economies. Our sample period ends in 2011, because this is the last year in which the UN Industrial Statistics report China's total manufacturing production. Results for the years 2012-2015 without China are available upon request.

[18] One might think of the easing of a country's export regulations or an import tariff reduction by its trade partner.

demand away from the products of all other nations and towards country i, it puts the labor demand in all nations but the exporter's under pressure. Thus, wages, and typically also real incomes, in all nations but i decline.[19]

In a world with production linkages: Things are different in a global production network. The reason is essentially that the exporter's own gains partially spill over to other nations, so that the tight connection between the exporter's product and labor markets is broken. In fact, nothing in our model speaks against a scenario where workers in third countries benefit more than the exporter's own workers. The following Proposition (proven in Appendix A.5) sketches two such scenarios, establishing a set of sufficient conditions on the ex-ante observable trade and input cost shares of the exporter, the importer, and the countries in the 'local neighborhood' around them, so that most of the demand gains spill over to either an intermediate goods supplier of the exporter, or to the importer, or one of its customers.

Proposition 3. *Consider an export cost reduction between exporter i and importer j in a global production network:*

(i) Suppose that exporter i is an 'intermediary' and a major supplier of j (λ_i is sufficiently small and π_{ij} is sufficiently large), country k is an upstream supplier of i ($\pi_{ki} > 0$), and importer j is a 'conventional' open economy (λ_j is sufficiently large). Then, the export cost reduction increases the wage of supplier k by more than of exporter i, $d\ln(w_k) > d\ln(w_i)$.

(ii) Suppose that exporter i is a major supplier to the world (π_{il} is sufficiently large for all $l \in \mathcal{N}$) and importer j is an 'intermediary' to downstream country k (λ_j and λ_k are sufficiently small, and for all $l \in \mathcal{N}_j \cup N_k \backslash \{j,k\} \equiv \{\mathcal{N}\backslash\{j,k\} \,|\, \pi_{jl} > 0 \text{ or } \pi_{kl} > 0\}$ it is λ_l sufficiently large). Then, the export cost reduction increases the wage of either country j or k by more than of exporter i, $d\ln(w_l) > d\ln(w_i)$ for at least one $l \in \{j,k\}$.

Moreover, in either case (i) or (ii), the export cost reduction increases the average welfare of all nations but the exporter, $\frac{1}{n-1}\sum_{l\neq i} l_l w_l \, d\ln(u_l) > 0$ and $d\ln(u_i) < 0$.

The first scenario involves an exporter that —due to its small value added share in

[19] That real incomes decline on average in countries $k \neq i$, despite the improved supplier access of country j's consumers, is due to the magnifying effect of the foreign trade multiplier: the export cost reduction lowers the demand for products from any $k \neq i$ by at least the import competition effect $\delta_k^{l^d} = -(\gamma-1)\pi_{kj}\pi_{ij}e_j^f$. The sum of the labor income losses in $k \in \mathcal{N}\backslash\{i\}$, thus, unambiguously overshoots the direct benefits to consumers in j, $\delta_j^p e_j^f = \pi_{ij}e_j^f$, if the trade diverting effect is sufficiently strong, i.e., if γ is sufficiently large, in particular $\gamma > (2-\pi_{ij})/(1-\pi_{ij})$.

production— primarily passes the additional demand created by the export cost reduction on to one of its upstream suppliers. The second scenario instead, involves an importer, or one of its downstream customers, that takes advantage of the additional (indirect) inflow of intermediate products from the exporter to improve its productivity in all its sales markets. And, as these productivity gains scale up in the country's domestic supply chain, it can even capture market shares of the exporter. In either scenario, the exporter should ideally be a major supplier of the importer, so that no single other country is severely hurt by the intensified competition in the importer market. Thus, downstream exposure is the important diffusion channel generating the positive externality in the first scenario, whereas it is upstream exposure in the latter.

The precise conditions on the observable trade shares and input cost shares required for Proposition 3 can be found in Appendix A.5. Underneath them, we essentially impose some deeper restrictions on the trade cost and factor productivity parameters of our model. This is particularly clear in a Cobb-Douglas specification for our model ($\beta = 1$), where λ_i, λ_j, and λ_k become exogenous parameters. Another tight characterization for the conditions leading to sizable supply chain spillovers is provided in the following example. The example, moreover, shows that the positive labor market spillovers might even reach countries more than two steps up- or down the supply chain from the directly affected nations:

Example 1 (linear supply chain). Consider an archetypical linear supply chain: a product is assembled in n sequential production steps. Country 1 is at the top of the chain and country $n > 1$ at the bottom, so that each $1 \leq i \leq n$ adds $w_i l_i$ to the value of the product before country n sells the final output to all $j \in \mathcal{N}$.

The welfare effects of an export cost reduction between countries i and $i+1$, $1 \leq i < n$, in the middle of the chain are give by

$$d\ln(w_j) = \begin{cases} 0 & \text{for } j \leq i \\ (1-\beta)/\beta & \text{otherwise} \end{cases}$$

$$d\ln(u_j) = \begin{cases} \frac{\beta-1}{\beta} + \frac{e^i_{i+1}}{\beta\, x_n} & \text{for } j \leq i \\ \frac{e^i_{i+1}}{\beta\, x_n} & \text{otherwise} \end{cases}$$

where $e^i_{i+1} = \sum_{j=1}^{i} w_j l_j$ and $x_n = \sum_{j=1}^{n} w_j l_j$ (see Appendix A.5 for the derivation). The cost reduction, thus, affects all countries upstream to the exporter and all countries downstream to the importer alike (if $\beta \neq 1$). In particular, when $\beta > 1$, the cost reduction shifts the demand for value added from the downstream to the upstream stages of production, such that all countries upstream to the exporter gain as much as the exporter itself (which is treated as the reference country here, i.e., $d\ln(w_i) = 0$). When $\beta < 1$, on the other hand, nominal wages and real incomes increase relatively more in the downstream countries $j > i$.[20]

Quantitative predictions: To verify that such '*positive spillover links*' —trade routes where an export cost reduction results in at least one positive labor demand or welfare externality that even exceeds the exporter's own gains— are not just a mere theoretical possibility, we explored their presence in the data. Towards this end, we imposed a 1% unilateral export cost reduction on each active trade route ($\pi_{ij} > 0$) in our two datasets.

We find a surprisingly large number of such links: across all years and different β-specifications used, roughly a half (a quarter) of all active trade routes qualify as a positive welfare (wage) spillover link.[21] Even when leaving any positive welfare (wage) externality on the directly affected importer aside, the share of active trade routes associated with a positive *third-country externality* on a supplier located further upstream from the exporter or a buyer further downstream from the importer still amounts to 11% (5%).

Table 1 focuses on these third-country externalities. It reports, for both the first (1980-1995) and second (1996-2011) half of our sample period, the seven countries that are most often found on the exporter side of these positive spillover links, together with the share of their active trade links that qualify as a positive welfare (wage) spillover link (in columns 1 and 3), the average percentage of third countries experiencing a positive welfare (wage) externality due to a trade cost reduction on those links (in columns 2 and 4).

Not surprisingly, the typical exporter involved in a positive spillover link is —just as required for Proposition 3— a large open economy. In the earlier 1980-1995 period, these are primarily the large industrialized economies: Japan, the US, the larger European economies, and Canada. In the later 1996-2011 period, other countries start to appear, whereby China and several other of the larger emerging economies stand out (South Korea, Brazil, Indonesia, Russia).[22] On the importer's side, instead, we typically find countries

[20] Costinot et al. (2013) analyze a linear supply chain where producers employ a Leontief technology ($\beta = 0$) at each step. Our export cost reduction is closest to what the authors call the effects of 'routinization'. Similar to their main finding, an export cost reduction affects countries very differently at the bottom and the top of the chain. What our analysis adds to their findings is that the sign of these externalities also crucially depends on the elasticity of input substitution, β.

[21] The share of trade routes with a sizable real income externality exceeds that with a sizable labor demand externality by a factor two, simply because the former contains the obvious cases where only the importer benefits from the better access to the exporter's products.

[22] The prime reason is that China, and other emerging economies, have turned into the preferred trading

Table 1: Positive spillover links: third-country externalities

	WELFARE		WAGES		
Countries	% positive spillover links (1)	% 3rd countries affected (2)	% positive spillover links (3)	% 3rd countries affected (4)	% upstream spillovers (5)
			1980 - 2011		
All	10.7	13.4	4.7	15.8	50.1
			1980 - 1995		
All	9.6	16.2	2.9	26.7	54.4
Top 7 - exporters (total # positive wage spillover links)					
Japan	99.9	70.3	97.4	51.3	68.6
US	97.7	24.8	53.0	7.8	40.9
Germany	93.2	13.9	32.7	6.7	31.0
France	59.1	3.3	8.1	1.9	47.0
UK	54.4	2.7	6.7	1.6	54.0
Canada	47.1	2.1	4.4	1.3	66.8
Italy	39.5	2.3	4.1	1.7	44.7
			1996 - 2011		
All	11.5	11.6	6.0	11.8	49.2
Top 7 - exporters (total # positive wage spillover links)					
China	95.3	29.7	75.4	25.3	46.3
Japan	98.9	40.4	78.8	23.1	46.1
US	93.2	11.3	41.2	6.0	39.8
Germany	66.9	6.7	29.1	6.7	31.6
France	45.5	3.9	16.1	4.4	49.1
South Korea	47.8	5.4	12.4	7.1	44.0
Italy	40.6	4.9	11.7	5.9	36.7

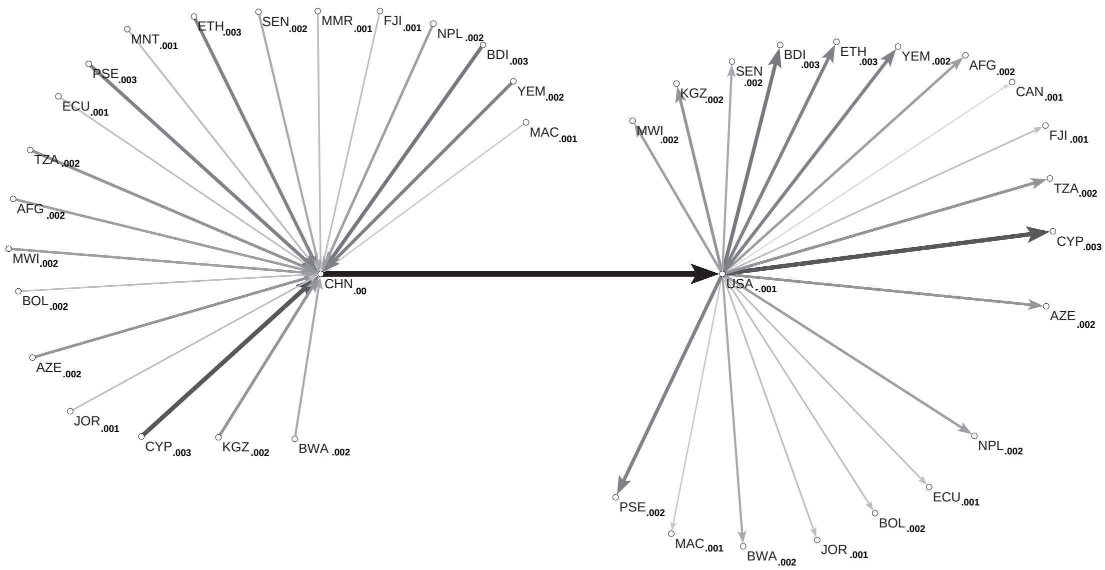
NOTES: The total number of trade links ($\pi_{ij} \geq 0$) in the years covered by our two data sets is 283,780 of which 235,655 are active ($\pi_{ij} > 0$). The numbers reported in columns 1 and 3 are averages across all active links, and across all 11 different values used for $\beta \in \{.001, .5, 1.001, 1.5, ..., 4.5, 5\}$. The numbers reported in columns 2, 4 and 5 are averages across all active links generating a positive welfare (column 2) or wage externality (columns 4 and 5) in at least one third country, and across all 11 different values used for β. In column 5, we classify a wage externality as primarily driven by the upstream (downstream) diffusion channel when the combined market access and competition effect on the third country's labor demand, relative to that in the exporter, is larger (smaller) than the combined productivity and offshoring effect, relative to that in the exporter — see (8).

that are either themselves relatively unimportant as a trading partner for other nations or, less often, trade primarily with other, relatively unimportant, nations.

Figure 1 provides a more detailed picture of the externalities generated by an export cost reduction on one such link. It shows the 20 largest positive up- and downstream wage externalities resulting from a counterfactual 1% export cost reduction on the China-US link in 2011. Notably, the predicted wage gains for China (normalized to zero) as well as the predicted losses to the US (due to the more intense competition with Chinese imports)

partners for many smaller developing countries that often trade little with other nations.

Figure 1: The wage externalities of a China-US trade cost reduction



NOTES: The figure illustrates the importance of the up- and downstream channel in determining the wage externalities of a 1% trade cost reduction on all Chinese exports into the US, when using the latest year in our data (2011) and setting $\beta = 5$. All predictions are relative to China's predicted wage effect, which is normalized to zero. In total, wages in 65 countries go up more than China's, with half of these countries primarily benefiting as intermediate input suppliers to China. The other half primarily gains as buyers of US intermediate products, benefiting from the US' improved access to Chinese supplies. Arrows pointing towards China depict the 20 largest 'upstream wage externalities', and arrows pointing away from the US depict the 20 largest 'downstream wage externalities' instead. See the notes to Table 1 for how we define a wage externality as being primarily driven by a country's up- or downstream exposure.

are both smaller than the wage gains in no less than 65 other countries, whereby —as is the case more generally (see column 5 of Table 1)— the up- and downstream diffusion channels (depicted by the arrows pointing towards China and away from the US respectively) tend to be equally important. Also, many of the third countries experiencing the largest wage gains are typically —as already suggested by Proposition 3— smaller countries that are not significantly hurt by the intensified import competition in the US.

4.1 The externalities of multiple trade cost shocks

The previous arguments can be easily extended to quantify the externalities of a bilateral, or even a multilateral, trade cost reduction. The reason is that, in a first-order approximation, the total effect of a shock to multiple cells of the trade cost matrix is simply the sum of effects of each constituent cell-specific shock.

Even more can be said, however, in the uniform elasticity specification of our model ($\beta =$

γ), where we can also look at an *inframarginal* trade cost shock and the interaction between the constituent shocks on different trade routes.[23] A particularly interesting question here is whether one trade cost reduction stirs or rather stifles the incremental gains from another cost reduction (see, e.g., Aghion et al., 2007). Our results are the following: as long as no intermediate products are traded along the directly affected trade routes, the predictions of Proposition 2 are a viable approximation. In the presence of traded intermediate products, things are again very different and the following applies:

Proposition 4. *Consider trade in a global production network, and suppose a uniform-elasticity specification for our model ($\beta = \gamma$). The gains from an inframarginal export cost reduction between exporter k and importer j are increasing in the presence of another 'positive spillover' cost reduction between another country i and j. That is,*

$$d_{kj+ij}\ln(w_k) \quad > \quad d_{kj}\ln(w_k) \,+\, d_{ij}\ln(w_k)$$

if λ_k is sufficiently large and the cost reduction along ij satisfies the requirements of Proposition 3 Part (i).

The result (proven in Appendix A.5) provides another theoretical foundation for the so called 'building bloc' hypothesis (e.g., Bhagwati, 1993; Baldwin, 1995): in the presence of production linkages, the incremental gains from two export cost reductions (to the same importer) are greater than the sum of the gains of each cost reduction individually. The logic behind this lies in the *supermodularity* of the Leontief and Ghosh inverses in (10) and (12) with regard to multiple sizable shocks to their coefficient matrices. Intuitively, as any single trade cost reduction facilitates the flow of intermediate products through the supply chain, it leverages every country's supply chain exposure and, thus, its incremental gains from a cost reduction of its own. And, although the conditions in Proposition 4 are rather restrictive, the logic can be easily extended to a group of two or more arbitrary country pairs. As long as an export cost reduction between any one of the countries involved in the group yields a positive first-order externality (on another country k), every actual trade cost reduction between them becomes a building bloc.[24]

[23]When $\beta = \gamma$, all elements of the inverse matrices in (10) and (12) only feature exogenous trade cost and technology parameters (see also footnote 14), allowing us to also consider inframarginal shocks to one, or more, of these elements. Note, however, that we still need to make the implicit assumption that the shock is small enough for our first-order approximations of the resulting wage and price effects to be viable.

[24]Against the backdrop of Ornelas (2005) or Aghion et al. (2007), this is rather surprising. In their

5 The gains from a global trade cost shock

In the previous section, we saw that a local trade cost shock can, in the presence of cross-border production linkages, trigger a sizable welfare externality in a country that is only indirectly exposed to this shock. Here, we show that these network externalities are even all that matters for understanding the welfare effects of a global trade cost shock.

In particular, we look at a proportional trade cost decline along all trade routes, such as triggered for example by a global innovation in transportation or communication technologies. Intuitively, one might expect this cost reduction to improve the economic prospects of all countries alike. As each country scales up on its initial access to (foreign) suppliers and markets, it is tempting to conclude that also the welfare gains are proportional to the initial level of income in each nation. Yet, it turns out that this logic only holds true in the absence of production linkages.

In a world without production linkages: The following result, which is proven in Appendix A.6, extends on the logic of Lemma 1 and establishes the irrelevance of all the diffusion channels in (7) in determining each and every country's welfare response to such a global trade cost reduction.

Proposition 5. *In the absence of production linkages ($\kappa_i^i = 0$ for all $i \in \mathcal{N}$), the per capita income gains from a global trade cost reduction $\mathbf{dT} = -x\mathbf{T}$ are proportional to the initial level of welfare in each nation, i.e., $\mathbf{d}\ln(\mathbf{u}) = x\,\boldsymbol{\delta}^{\mathbf{P}} = x\,\mathbf{1}$.*

Since all countries improve their market access alike, the cost reduction leaves wages in all countries unaffected (i.e., $\mathbf{d}\ln(\mathbf{w}) = \mathbf{0}$). What remains is the effect on each country's supplier access, which is the result of the shipping cost decline between consumers and their final goods suppliers, and which gives rise to a proportional welfare increase.[25]

In a world with production linkages: The above logic of 'demand neutrality' no longer applies in a global production network. Even though all countries still improve their market access alike, the labor demand effects of a global trade cost reduction, $\mathbf{dT} =$

models (without international supply chain linkages), the presence of a positive first-order externality is precisely the circumstance under which the second-order externality of a trade agreement becomes negative.

[25] The result can be generalized in several ways: first, Proposition 5 carries fully over to a global trade cost reduction in a world with production linkages, as long as only the shipping costs on final goods are affected. Second, the result also holds in approximation for a truly international trade cost reduction, where only the costs of importing and exporting of final products are affected. In both cases, the trade cost reduction is still 'demand neutral' leading to a proportional welfare increase in all nations.

$-x\mathbf{T}$, can be very different depending on a country's exposure to the increased amount of intermediates flowing through the global value chain. In particular, they depend on a country's upstream exposure to every other nation, as clearly illustrated by the *productivity effects*:[26]

$$\mathbf{d}\ln(\tilde{\mathbf{p}}) \;=\; -x\,(\mathbf{I}-\mathbf{\Lambda})\,\mathbf{\Phi}^{\mathbf{se}}\,\mathbf{1} \tag{15}$$

What is still not clear however is whether supply chain exposure is a blessing or a curse. All workers benefit, to a lesser or greater extent, from the productivity gains of their domestic producers. But, as a global trade cost reduction also improves the productivity of their foreign competitors and, at the same, triggers input substitution at home, it also puts labor demand in each nation under pressure. These two sides to supply chain exposure are summarized in the following expression, showing the net welfare effect of a global trade cost reduction in each nation:[27]

$$\begin{aligned}\mathbf{d}\ln(\mathbf{u}) \;=\;& x\,\mathbf{\Phi}^{\mathbf{se}}\,\mathbf{1} \;+\; x\left(\mathbf{I}-\mathbf{\Phi}^{\mathbf{tot}}\right)\mathbf{\Phi}^{\mathbf{mult}}_{\mathbf{i^*}}\Bigg[\underbrace{(1-\beta)\left(\mathbf{E}^{\mathbf{f}} \;-\; \mathbf{\Phi}^{\mathbf{de}}\,\mathbf{\Pi}\,\mathbf{E}^{\mathbf{f}}\right)}_{\text{Own and customers' offshoring}} \\ &+\; \underbrace{(\gamma-1)\left(\mathbf{E}^{\mathbf{f}}+\mathbf{\Phi}^{\mathbf{de}}\mathbf{\Pi}\,\mathbf{E}^{\mathbf{i}}-\mathbf{\Phi}^{\mathbf{de}}\mathbf{\Pi}\,(\mathbf{E}^{\mathbf{f}}+\mathbf{E}^{\mathbf{i}})\mathbf{\Pi}^{\mathbf{T}}\right)}_{\text{Own, customers, and competitors' productivity}}\Bigg]\,(\mathbf{I}-\mathbf{\Lambda})\,\mathbf{\Phi}^{\mathbf{se}}\,\mathbf{1}\end{aligned} \tag{16}$$

The first summand in (16) shows the unambiguously positive effect on consumer prices, that solely depends a country's upstream exposure to every other country. Not suprisingly, thus, this effect can be alternatively be written as $x\mathbf{\Pi}^{\mathbf{T}}\mathbf{d}$, where $\mathbf{d}$ denotes the vector of every country's 'downstreamness' as defined in Antràs and Chor (2013) or Antràs and Chor (2017),[28] or as $x\mathbf{b}$, where $\mathbf{b}$ is the vector of Bonacich (1987) centralities corresponding to

[26] The intuition behind the expression in (15) is the following: the *local price effect* of a global trade cost reduction on producer prices is proportional to their initial intermediate input shares, $d\ln(p_i) = -x(1-\lambda_i)$. Yet, producers are also indirectly exposed to the local price effects of their direct and indirect upstream suppliers, as summarized in the row entries of matrix $(\mathbf{I}-\mathbf{\Lambda})\mathbf{\Phi}^{\mathbf{se}}[\mathbf{I}-\mathbf{\Lambda}]^{-1}$.

[27] Expression (16) follows immediately from the application of the elementary relationships $(i)\,\mathbf{X}^{\mathbf{f}}+\mathbf{X}^{\mathbf{i}} = \mathbf{E}^{\mathbf{f}}+\mathbf{E}^{\mathbf{i}}$ and $(ii)\,\mathbf{I} = \mathbf{\Phi}^{\mathbf{de}}\left(\mathbf{I}-\mathbf{\Pi}(\mathbf{I}-\mathbf{\Lambda})\right)[\mathbf{\Lambda}]^{-1}$ to reorganize the wage effects in diffusion model (7).

[28] Simply note that since $\mathbf{\Pi}^{\mathbf{T}}\mathbf{1} = \mathbf{1}$, it also is

$$\mathbf{\Phi}^{\mathbf{se}}\,\mathbf{1} \;=\; \mathbf{\Pi}^{\mathbf{T}}\sum_{\mathbf{h=0}}^{\infty}\left[(\mathbf{I}-\mathbf{\Lambda})\mathbf{\Pi}^{\mathbf{T}}\right]^{\mathbf{h}}\mathbf{1} \;=\; \mathbf{\Pi}^{\mathbf{T}}\mathbf{d}$$

the network represented by matrix $\mathbf{\Pi}^{\mathbf{T}}(\mathbf{I}-\mathbf{\Lambda})$.

The remaining summands describe the conflicting wage effects. When $\beta > 1$ ($\beta < 1$) workers of all nations suffer (benefit) from the offshoring of their local value added, but benefit (suffer) from the offshoring in other countries that buy their intermediate products. In other words, the offshoring channel favors countries in the upstream stages of the global supply chain, if and only if $\beta > 1$, as they are less affected by this channel at home, while they sell to countries that are more severely affected. On the other hand, since $\gamma > 1$, workers benefit from the productivity increases of their domestic employers, while they are hurt by the productivity increases of their competitors abroad. Thus, it is a country's downstreamness in the global supply chain that tends to be in its advantage, because downstream countries 'fetch' more of the productivity gains in the upstream stages of production while they suffer less from the productivity gains in other countries.

These possibly conflicting effects of a country's up- and downstream exposure, make it impossible to unambiguously sign the welfare effects in (16). Nevertheless, the following examples, and our empirical implementation of (16), show that the productivity channels tend to prevail, making a global trade cost reduction typically in the advantage of downstream countries.

Example 1 (linear supply chain). Suppose countries are lined up in a linear chain with country 1 at the top and country n at the bottom. Expanding on Example 1 in Section 4, the welfare effects of a global trade cost reduction are given by

$$\begin{aligned} d\ln(w_j) &= \frac{\beta-1}{\beta}(1-j) \\ d\ln(u_j) &= \frac{\beta-1}{\beta}(n+1-j) + \frac{\sum_{k=2}^{n} e_k^i}{\beta\, x_n^f} \end{aligned} \tag{17}$$

for $1 \leq j \leq n$. Welfare is, thus, unambiguously increasing in a country's upstream exposure, or downstreamness, if and only if $\beta < 1$. This can be viewed as a special case of the general formula (16), acknowledging that in a linear supply chain each country's share in its sales markets is fixed at one, which is equivalent to assuming $\gamma = 1$ in (16).

Example 2 (Eaton & Kortum). Consider the Eaton and Kortum (2002) specification of our model, with $\beta = 1$ and identical labor cost shares in each country ($\lambda_i = \lambda$ for all

$i \in \mathcal{N}$). In this case, the general formula (16) simplifies to[29]

$$\mathbf{d}\ln(\mathbf{u}) \;=\; x\,\mathbf{\Phi^{se}}\,\mathbf{1} \tag{18}$$

Hence, what remains is the benefit to consumers, which depends on each country's upstream exposure to every other nation. Given identical labor cost shares, this benefit can be further simplified to: $\mathbf{d}\ln(\mathbf{u}) = (x/\lambda)\mathbf{1}$.

Example 3 (Long & Plosser). The same welfare effect, $\mathbf{d}\ln(\mathbf{u}) = x\mathbf{\Phi^{se}}\mathbf{1}$, as in Example 2 emerges —by immediate inspection of formula (16)— from the canonical input-output model in macroeconomics: the Long and Plosser (1983) model with $\beta = \gamma = 1$.

Example 4 (Leontief). Finally, for perfectly complementary inputs ($\beta = 0$) we obtain, as shown in Appendix A.6, the following welfare effect:

$$\mathbf{d}\ln(\mathbf{u}) \;=\; x\,[\mathbf{\Lambda}]^{-\mathbf{1}}\mathbf{1}$$

Again, welfare improves most in countries enjoying better upstream exposure, i.e., those with a higher intermediate input cost share.

Quantitative predictions: As each of the examples makes at least one rather restrictive assumption, either concerning the structure of the production network or the model's elasticity parameters, we also verified the importance of upstream exposure in determining the predicted welfare effects of a global trade cost reduction, empirically. To do so, we imposed a 1% trade cost reduction on all active trade routes in each of the 39 years covered

[29]The intuition is the following: when all producers use the same share of labor in production, the trade cost reduction has the same *local price effect* on each country's producers, $d\ln(p_i) = -x(1-\lambda)$. As a result, no country obtains a competitive edge from its relative up- or downstreamness in the production network. More concretely, when $\lambda_i = \lambda$, it holds

$$\mathbf{d} \;=\; [\mathbf{LW}]^{-\mathbf{1}}\mathbf{\Phi^{de}}\mathbf{\Pi}\,(\mathbf{e^f}+\mathbf{e^i})$$

or, differently put, 'downstreamness' is equal to 'upstreamness' in the terminology of Antràs et al. (2012). This ensures that the conflicting labor demand effects in (16) cancel each other out. To verify the identity, simply note that

$$\mathbf{d} \;=\; \frac{1}{\lambda}\mathbf{1} \;=\; [\mathbf{LW}]^{-1}\mathbf{\Lambda}\sum_{\mathbf{h=0}}^{\infty}\left[\mathbf{\Pi}(\mathbf{I}-\mathbf{\Lambda})\right]^{\mathbf{h}}\mathbf{\Pi}\,\mathbf{e^f}\frac{1}{\lambda} \;=\; [\mathbf{LW}]^{-\mathbf{1}}\mathbf{\Phi^{de}}\,\mathbf{\Pi}(\mathbf{e^f}+\mathbf{e^i})$$

The final expression is upstreamness.

by our data, and calculated the resulting welfare effects based on (16). Our findings can be summarized as follows:

First, in line with Examples 2-4, the international fragmentation of production works in the advantage of each and every nation. Across all years and β-specifications used, the average predicted per capita income gain is 3.2%, with a minimum gain that is always strictly larger than the 1%-effect we would expect in the absence of production linkages between countries.[30] Moreover, the effect magnification through supply chain linkages has increased over time, as illustrated in Figure 2 by the persistently increasing average welfare gain over the years in our sample.[31]

Second, the predicted welfare gains —although positive in every country— differ significantly by country, suggesting substantial differences in supply chain exposure. The interquartile range in each year's welfare predictions in Figure 2 is typically around 0.6 percentage points, with a difference of about 3ppt between the smallest and the largest welfare effect. Figure 3 explores this variation in more detail. It 'zooms in' onto the most recent year in our data, 2011, and plots each country's predicted welfare effect against its entry in vector $x\,\mathbf{\Phi}^{\mathbf{se}}\,\mathbf{1}$, which fully captures the consumer price effects of a global trade cost reduction and which is even the sole determinant of these welfare effects in Examples 2 and 3 above. The countries with the largest predicted welfare effects are typically found in South-East Asia, followed by several European countries. Their gains are roughly 1.5-2ppt larger than those of the least-benefiting countries in Central Asia, Africa, or South America.

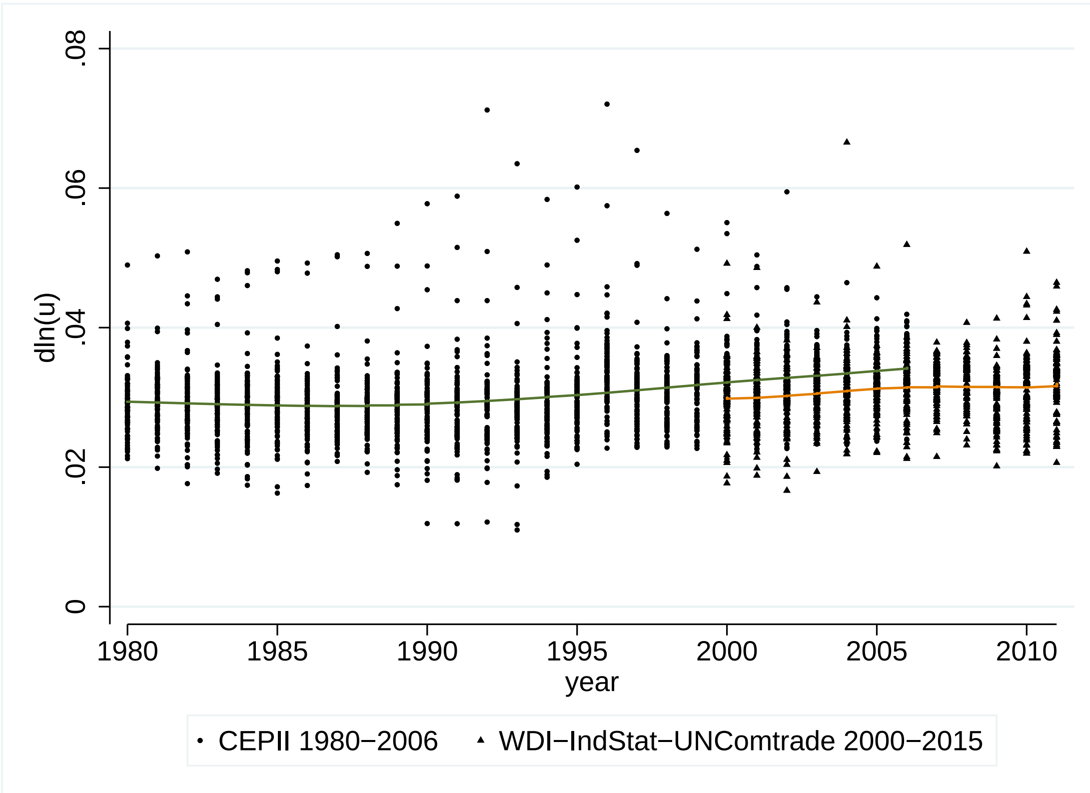
Finally, in line with Examples 2-4, it is a country's *upstream exposure* that seems to be

[30] At first sight, an average welfare effect of 3.2% seems at odds with Lemma 1. There, we concluded that the global supply chain does no more than distributing the total gains from trade. The fact is, however, that the average effect of a 1% global trade cost reduction is, according to this Lemma, larger than one percent, because it is

$$\frac{\sum_{i\in\mathcal{N}} l_i w_i d\ln(u_i)}{\sum_{i\in\mathcal{N}} l_i w_i} \;=\; 0.01\frac{\sum_{i\in\mathcal{N}}(e_i^f + e_i^i)}{\sum_{i\in\mathcal{N}} e_i^f} \;>\; 0.01$$

[31] Figure 2 uses a model specification with $\beta = \gamma = 5$. The average income gains are larger for all our other β-specifications. For example, the average welfare gain is 3.25% across all years when using $\beta = 1$, compared to an average 3.1% when $\beta = 5$. In the years covered by both our data sets, we always find larger welfare gains in the CEPII data. This can be partly explained by it covering about 10-20 fewer countries. In addition, the UN Comtrade data is based on import flows that, unlike the CEPII data, have not been cross-checked with export flows. As a result some import shares, typically of developing countries, are (much) lower than in the CEPII data, making these countries look less exposed to the supply chain and, thus, also less benefiting from a global trade cost reduction. However, the trend in the predicted welfare effects over the years covered in both datasets (2000-2006) is virtually identical.

Figure 2: The gains from a global trade cost reduction, 1980-2011



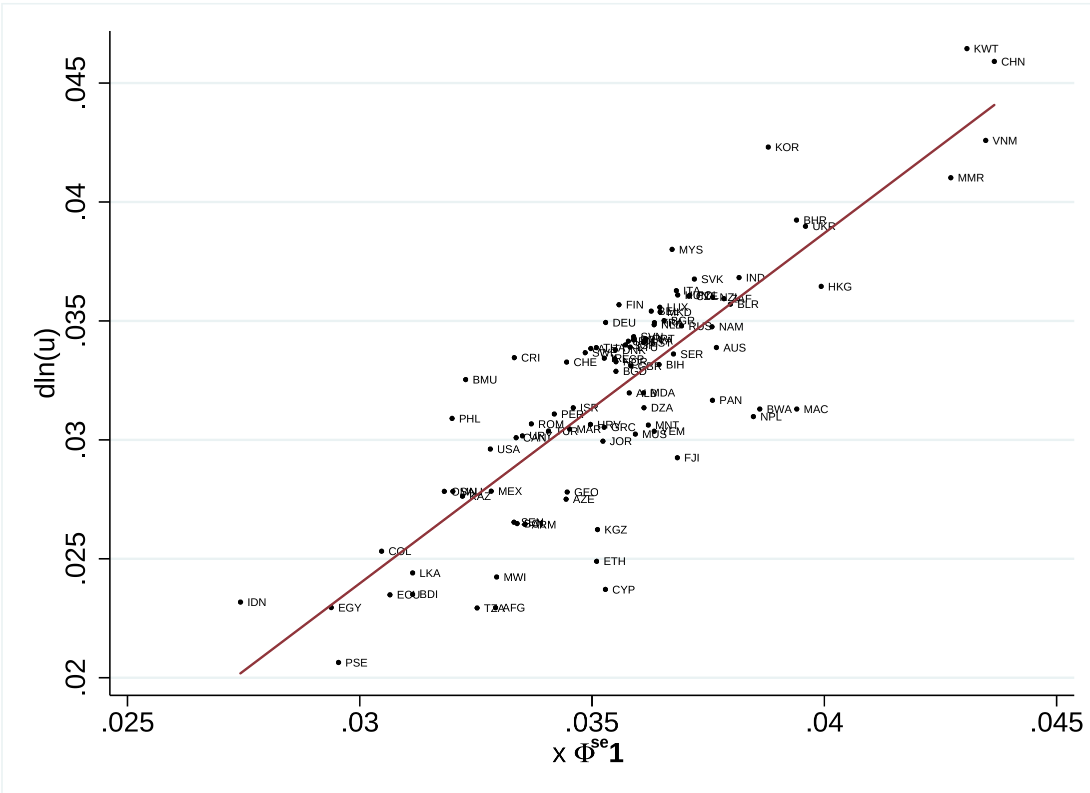
NOTES: The figure shows the real per capita income effects (in%) of a 1% global trade cost reduction in a model specification with $\beta = \gamma = 5$. In the CEPII data, the average welfare effects increase from 2.9% in 1980 to 3.5% in 2006; in our hand-collected data, from 3.0% in 2001 to 3.2% in 2011.

the predominant driver of the predicted welfare effects. In fact, the regression line fitted through the points of Figure 3 has a slope of 1.47 (SE 0.10), suggesting that a country's upstream exposure has an even stronger effect on these welfare effects than the one-to-one relationship implied for by the pure consumer price effect. Put differently, upstream exposure also appears to be the key determinant of the labor market responses to a global trade cost shock in our data.

6 Key trading partners

The previous section showed that a country's exact position in the global production network is a source of absolute advantage: even when trade costs decline at the same rate everywhere, countries enjoying better (upstream) supply chain exposure experience larger welfare gains. Here, we unravel the origins of this exposure. We look at a country's trading

Figure 3: The gains from a global trade cost reduction: cross-country variation in 2011



NOTES: The figure plots each country's predicted real per capita income gains following a 1% global trade cost reduction against a country's entry in the upstream exposure vector $x\,\mathbf{\Phi}^{se}\,\mathbf{1}$ (using 2011 data and assuming $\beta = \gamma = 5$). The regression line through the points has a slope of 1.47 (SE 0.10).

partners for this purpose, because they are the ones that link a country to suppliers further upstream and to buyers further downstream.

In particular, building on an extant network literature, we measure the importance of a country as a trading partner in terms of the welfare losses inflicted on other nations, when the country hypothetically weakens all its in- and outgoing trade relationships.[32] If one is

[32]Other studies measuring the importance of 'key players' to a network can be found in such diverse fields as the literature on network robustness (Foti et al., 2013; Goyal and Vigier, 2014), shock diffusion in domestic production networks (Acemoglu et al., 2012), information diffusion (Ballester et al., 2006; Banerjee et al., 2013), disaster impact analysis (Hertel et al., 2014), R&D policy (König et al., 2014), or conflict theory (König et al., 2017). The concept can even be traced back to an early regional economics literature, that defined the value of a sector for a national supply chain by the forward and backward linkages that are severed when the sector is disconnected from the rest of the economy (Rasmussen, 1956).

Also, note the close relationship between our analysis here and classic gains from trade analyses that look at the 'flipside' of what we are interested in here, namely at the loss in the isolated nation itself. To determine this loss, the linear approximations in Formulas (19) or (20) are of little use, as this loss is certainly more than marginal. Yet, we can calculate it based on the approach in Arkolakis et al. (2012), merely interested in the importance of a country for the world as a whole, our first result shows that all we need to know is its total value of production, $x^i_f + x^i_i$. This follows as an immediate corollary of Lemma 1:

Corollary 1. *The worldwide total welfare effect of country i's partial isolation,* $\mathbf{dT} = x(\mathbf{I_i T} + \mathbf{T\,I_i})$*, where* $0 < x \leq 1$ *and* $\mathbf{I_i}$ *is a matrix of zeros with a single one in the diagonal element ii, is given by*

$$\sum_{j \in \mathcal{N}} l_j w_j d\ln(u_j) = -x\left(x^f_i + x^i_i \;+\; e^f_i + e^i_i\right) = -2x\left(x^f_i + x^i_i\right) \tag{19}$$

For our purposes of unraveling the origins of countries' supply chain exposure, this measure is too coarse however. For one thing, countries are differently dependent on the isolated nation, leading to potentially (very) different welfare effects across countries. For another, a country's total output does not reveal anything about the important channels underlying the welfare losses in other nations. These two missing pieces are delivered by the following 'key trading partner' formula.

Definition 2 (key trade partners). *Suppose that* $\beta = \gamma$.[33] *The effect of country i's entire isolation from the global supply chain on the per capita incomes in any country* $j \neq i$

which yields, in the case of $\beta = \gamma$, a predicted loss of:

$$d_{-i}\ln(u_i) = \frac{1}{\gamma - 1}\ln\left(\frac{x^f_{ii}}{x^f_i + x^i_i - x^i_{ii}}\right)$$

[33] As noted earlier already, the uniform elasticity specification allows us to invoke an infra-marginal trade cost shock on our model, such as the entire isolation of a country. The precise effect on the Leontief and Ghosh inverse matrices in (10) and (12) is derived in Lemma 6 of Appendix A.3 (property 3.). Note also that we again make the implicit assumption that the impact of a country's isolation is at the same time small enough so that our first-order approximation of the resulting welfare effects in other nations is viable.

is defined by

$$\mathbf{d}\ln(\tilde{\mathbf{p}}^{\mathbf{f}}) = \boldsymbol{\Phi}^{\mathbf{se}}\,\boldsymbol{\Pi}^{\mathbf{T}}\Bigg(\underbrace{\frac{1}{y_i}\boldsymbol{\lambda}_{\mathbf{i}}}_{\substack{\textit{(i) ctr. i's local}\\ \textit{value added}}} + \underbrace{\frac{1}{y_i}(\mathbf{1}-\boldsymbol{\lambda})_{\mathbf{i}}}_{\substack{\textit{(ii) intermediated}\\ \textit{value added}}} \Bigg) + \boldsymbol{\Phi}^{\mathbf{tot}}\,\mathbf{d}\ln(\mathbf{w})$$

$$\mathbf{d}\ln(\mathbf{w}) = (1-\gamma)\,\boldsymbol{\Phi}^{\mathbf{mult}}_{\mathbf{i^*}}\boldsymbol{\Phi}^{\mathbf{de}}\Bigg[\underbrace{\mathbf{e}^{\mathbf{f}}_{\mathbf{i}}}_{\substack{\textit{(iii) ctr. i's}\\ \textit{local demand}}} + \underbrace{\frac{1}{y_i}\mathbf{e}^{\mathbf{f}}_{-\mathbf{i}}}_{\substack{\textit{(iv) intermediated}\\ \textit{demand}}} - \underbrace{\boldsymbol{\Pi}\,\mathbf{LW}\,\mathbf{I}_{-\mathbf{i}}\Big(\mathbf{d}\ln(\tilde{\mathbf{p}}^{\mathbf{f}}) - \boldsymbol{\Phi}^{\mathbf{tot}}\mathbf{d}\ln(\mathbf{w})\Big)}_{\substack{\textit{softer import competition due to foregone}\\ \textit{(v) local and (vi) intermediated value added}}} \Bigg] \tag{20}$$

where $\mathbf{I}_{-\mathbf{i}} = \mathbf{I} - \mathbf{I}_{\mathbf{i}}$, $\mathbf{e}^{\mathbf{f}}_{\mathbf{i}} \equiv (x^f_{1i}, x^f_{2i}, ..., x^f_{ni})^T$, *and* $\mathbf{e}^{\mathbf{f}}_{-\mathbf{i}} \equiv \mathbf{I}_{\mathbf{i}}\sum_{\mathbf{h=0}}^{\infty}[\boldsymbol{\Pi}(\mathbf{I}-\boldsymbol{\Lambda})]^{\mathbf{h}}\mathbf{x}^{\mathbf{f}}_{-\mathbf{i}}$. *Furthermore,* $\boldsymbol{\lambda}_{\mathbf{i}} \equiv (0, 0, ..., \lambda_i, ..., 0)^T$, $(\mathbf{1}-\boldsymbol{\lambda})_{\mathbf{i}} \equiv (0, 0, ..., 1-\lambda_i, ..., 0)^T$, *and* $y_i > 0$ *is a scale factor defined in Appendix A.7.*

The formula, which is developed in Appendix A.7, distinguishes a total of six different channels —next to the general equilibrium multipliers— through which welfare in other nations is affected. Three channels are active regardless of whether supply chain linkages are present or not: first, workers from all nations lose access to the isolated country's final demand, putting their wages under pressure (effect iii). Second, consumers forego access to the isolated country's local value added, implying a higher price for their consumption bundle (effect i). And third, all countries lose a competitor in their sales markets, which in contrast to the previous two channels, is associated with a positive impact on their welfare (effect v). In other words, the isolated country is to some extent dispensable, because others can fill the gap it leaves on the world markets.

In a global supply chain, there are three additional effects at work: first, workers from all nations need to accept additional wage cuts, because they now also lose access to the final demands that the isolated country intermediated from elsewhere (effect iv). The ideal measure for this 'intermediated demand' is

$$\mathbf{e^f_{-i}} \equiv \mathbf{I_i}\sum_{\mathbf{h=0}}^{\infty}[\mathbf{\Pi}(\mathbf{I}-\mathbf{\Lambda})]^{\mathbf{h}}\mathbf{x^f_{-i}}$$
$$\text{with} \qquad \mathbf{x^f_{-i}} \equiv (\sum_{k\neq i} x^f_{1k}, \sum_{k\neq i} x^f_{2k}, ..., \sum_{k\neq i} x^f_{nk})^T$$

which, pre-multiplied by the full matrix $\mathbf{\Phi^{de}}$ —as in (20)—, immediately gives us a measure of 'bridging capital' from the social network literature (Ballester et al., 2006; Jackson, 2017). Moreover, it is also very closely related to the *vertical specialization trade* measure of Hummels et al. (2001). As such, formula (20) suggests that a country's degree of vertical specialization can be interpreted as meaningful statistic for its importance as a trade intermediary.[34]

Second, consumers have to endure further rounds of price increases, because of the foregone access to the foreign value added incorporated in the isolated country's products (effect ii). This foregone value added can be easily measured by the country's intermediate goods share in production: $(\mathbf{1}-\boldsymbol{\lambda})_{\mathbf{i}}$. Third, and finally, all remaining countries benefit, because also their competitors lose access to that intermediated value added and, thus, competition in world markets is further relaxed (effect vi).

Linking trade intermediation and supply chain exposure: Above, we established that the value of a country as a trading partner is determined by the size of its domestic final goods markets and its local value added, on the one hand, and its 'capacity' to provide indirect access to the demand and value added of other nations, on the other hand. The following result shows that only the latter contributes to other nations' supply chain exposure and, thus, following Section 5, to their predicted welfare gains from a global

[34] To make the link with Hummels et al. (2001) clear, note that their measure can be written as (see above for the definition of $\mathbf{I_{-i}}$ and $\mathbf{x^f_{-i}}$):

$$v_i = \mathbf{1_i^T}(\mathbf{I}-\mathbf{\Lambda})\,\mathbf{x^f_{-i}} + \mathbf{1_i^T}(\mathbf{I}-\mathbf{\Lambda})\mathbf{\Pi}(\mathbf{I}-\mathbf{\Lambda})\,\mathbf{I_{-i}}\sum_{\mathbf{h=0}}^{\infty}[\mathbf{\Pi}(\mathbf{I}-\mathbf{\Lambda})]^{\mathbf{h}}\mathbf{x^f}$$

For comparison, pre-multiplying $\mathbf{e^f_{-i}}$ by the direct exposure of every country to nation i, i.e., the second summand in matrix $\mathbf{\Phi^{de}}$, we get

$$\mathbf{1^T\Pi}\,(\mathbf{I}-\mathbf{\Lambda})\,\mathbf{e^f_{-i}} = \mathbf{1_i^T}(\mathbf{I}-\mathbf{\Lambda})\mathbf{x^f_{-i}} + \mathbf{1_i^T}(\mathbf{I}-\mathbf{\Lambda})\textstyle\sum_{\mathbf{h=1}}^{\infty}[\mathbf{\Pi}(\mathbf{I}-\mathbf{\Lambda})]^{\mathbf{h}}\mathbf{x^f_{-i}}$$

Thus, the major difference between the two measures is that ours also includes domestically consumed intermediate products, as long as they are used in the production of final goods consumed elsewhere.

trade cost reduction:

Proposition 6. *Suppose $\beta = \gamma$. The relative welfare gains from a global trade cost reduction* $\mathbf{dT}$ *are solely determined by the intermediation capacities of a country's trading partners. That is, let* $\mathbf{dT^{int}_{-i}}$ *denote the (partial) shutdown of the intermediation channels ii, iv, and vi in Formula (20). Then, for any two nations $j, k \in \mathcal{N}$, it holds:*

$$d_{\mathbf{T}} \ln(u_j) \; - \; d_{\mathbf{T}} \ln(u_k) \quad = \quad -\,\frac{1}{2} \sum_{i \in \mathcal{N}} \left(d_{\mathbf{T^{int}_{-i}}} \ln(u_j) \; - \; d_{\mathbf{T^{int}_{-i}}} \ln(u_k) \right)$$

For the proof (presented in Appendix A.7), we simply take advantage of the additive separability of the total effect of a small shock to every element of the trade cost matrix.

Quantitative predictions: To illustrate the usefulness of our key trading partner formula, we empirically identified each country's most important trade intermediaries in our dataset. Figure 4 positions all countries present in the most recent year in our data, 2011, in a network graph. The size of a node indicates the overall importance of a country as a 'trade intermediary'; an arrow ij the specific importance of country i as an intermediary for country j.

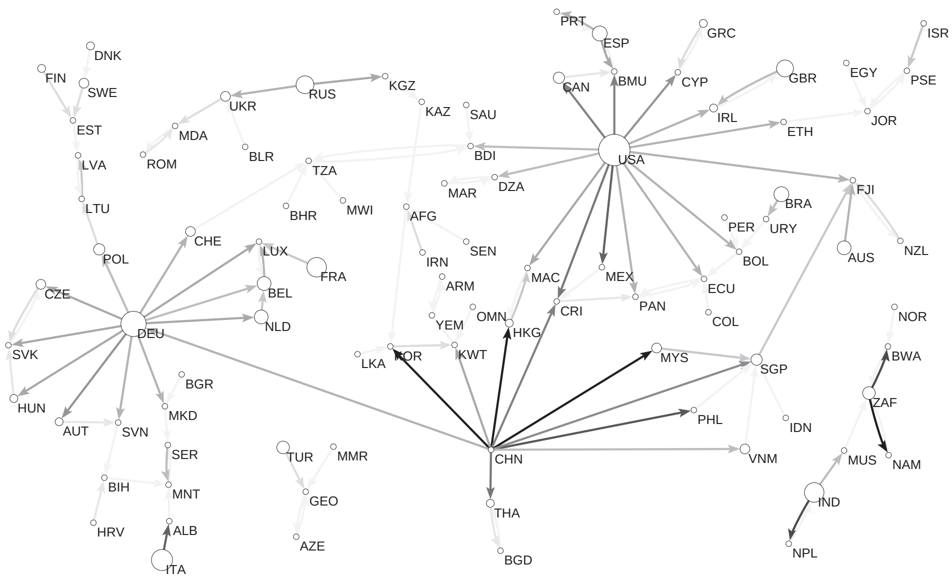
There are two striking observations: first, quite different from the network of overall key trading partners shown in Figure 5 of Appendix A.7, trade intermediation is a geographically confined phenomenon.[35] Second, it is typically the larger economy that intermediates for its smaller neighbors.

Not surprisingly then, each country is its own most important trade intermediary.[36] Furthermore, most of the important international 'intermediation ties' root from the same few countries and end in their immediate neighborhood. The key intermediaries in Europe and Asia stand out in this regard. China, as an extreme example, holds the strongest intermediation ties of all nations with its neighbors in South East Asia. Yet, China is, as illustrated by its small node size, only of minor importance for the world as a whole,

[35] The difference between the figures becomes particularly clear in the bottom panel of Figure 5, where the top two trading partners, the U.S. and Germany, sell their own value added and source their own consumption goods from a number of locations significantly further away than the ones shown in Figure 4. Nevertheless, as the welfare losses of the average country's isolation can for 67% be attributed to their foregone intermediation capacities, the network of overall key trade partners in the top panel of Figure 5 looks quite similar to the key intermediary network shown in Figure 4.

[36] This rather obvious pattern is omitted from Figure 4, where we only show the *international* 'intermediation ties'. Nevertheless, these international ties are still responsible for 52% of the typical country's supply chain exposure.

Figure 4: Key Intermediaries by Nation



NOTES: The figure highlights the intermediation capacities of 96 in the year 2011. The network is based on the quantified welfare effects of a 1% trade cost increase on all a country's in- and outgoing trade links, $\mathbf{dT^{int}_{-i}} = .01(\mathbf{I_i T} + \mathbf{T\,I_i})$, whereby we focus on the intermediation channels (ii), (iv), and (vi) of Formula (20). Node sizes indicate country i's importance as an intermediary for all $j \in \mathcal{N}\backslash\{i\}$. Arrows indicate that either (a) i contributes most to the welfare in j, among all $l \neq i$, or that (b) country pair ij belongs to the top 50 of all 'intermediation ties' in terms of effect size.

because of the many other countries that are severely hurt by the intense competition with the foreign value added embodied in Chinese exports (channel (vi) in Formula (20)). Only the U.S. is an example of a key intermediary that is not just important for countries in the Americas, but also for several countries beyond its immediate geographic neighborhood.

These empirical findings are in line with the empirical pattern found elsewhere (e.g. Daudin et al., 2011; Johnson and Noguera, 2012), leading Baldwin and Lopez-Gonzalez (2014) to divide the world into Factories Asia, America, and Europe. Our analysis adds some important meaning to these stylized facts, because it shows how the geographical concentration of intermediation ties matters for where the gains from trade materialize. In fact, in light of Proposition 6, it is not surprising that many of the largest beneficiaries of a global trade cost reduction are located in Europe and East Asia (see Figure 3), where countries benefit from their proximity to, sometimes multiple, key intermediaries. At the same time, the low density of intermediation ties in the Americas explains why the top gainer on that continent, Brazil, ranks only 31st in the world.

7 Conclusion

In this paper, we develop a novel network perspective on the origins of the gains from trade. Most importantly, we show that in today's integrated global value chains, a country's access to the technologies and markets of its direct trading partners is no longer key to understanding the welfare effects of trade policies, technological progress or other types of natural or man-made trade cost shocks. What matters, instead, is a country's exposure to such shocks through the entire network structure of production linkages.

More specifically, we find that the up- or downstreamness of a country in the global supply chain, relative to the location of the shock, is key in this respect. The relative importance of the two depends, however, on the specific type of shock: under a global shock, such as for example a global innovation in information or transportation technology, countries operating in the downstream stages of the supply chain tend to experience the largest welfare gains. In contrast, under a local shock, such as for example a bilateral trade agreement, both the upstream suppliers as well as the downstream customers of the directly affected nations might experience the largest gains, whereby the relative magnitude of these up- and downstream externalities crucially depends on whether it is the exporter or the importer involved in the agreement that is the more important intermediary of other nations' demand and supply.

Beyond the immediate importance of these insights, we believe that our paper opens up several interesting avenues for future research. First, our comparative statics approach to disentangle the different welfare channels of a (trade cost) shock might also prove useful in other models of input-output trade or economic geography. The more so, as each of these channels can be easily quantified using no more than readily available macroeconomic data and estimates of the model's elasticity parameters. In this regard, our approach might even inspire new empirical strategies. The system of total derivatives that lies at its heart establishes a linear relationship between observable 'sufficient statistics', with the elasticity parameters as coefficients. Using a large scale shock as a quasi-experimental setting, one could possibly estimate these coefficients based on fairly straightforward methods or, alternatively, use the system to create theoretically founded exposure measures that 'connect' the shock to the data. In light of our focus on network effects, it seems particularly worthwhile to complement existing studies on the direct impact in countries experiencing the shock (e.g., Autor et al., 2013; Caliendo and Parro, 2015) by estimating the size of the network externality in countries that are only indirectly affected.

Second, our findings add to a number of recent policy questions regarding global supply chains. For one, it is only one step from the system of total derivatives underlying our counterfactual approach to the first-order conditions for an optimal tariff regime. Even though we view our current findings on this topic as no more than stepping stones, they already add some insights to earlier papers stressing the need for new supply chain trade policies (e.g., Antràs and Staiger, 2012; Ornelas and Turner, 2012). In particular, we identified a surprisingly large number of trade routes, where most of the welfare gains from a trade cost reduction do not materialize in the directly involved countries, but rather in their up- or downstream trade partners. Moreover, our analysis puts a number of trade partners into the spotlight, not so much because of the size of their own markets, but because of their importance as trade intermediaries determining their trade partners' indirect exposure to shocks in the global production network.

Also, past studies have argued, and shown, that the emergence of global supply chains has opened up new transmission channels for foreign shocks, with important implications for the international synchronization of business cycles (e.g., Arkolakis and Ramanarayanan, 2009; Caselli et al., 2017). As our approach helps to single out, and easily quantify, the different channels by which a shock diffuses across nations —in particular, the general equilibrium effects vs. supply chain diffusion—, it has the potential to shed further light on their relative importance in dampening or exacerbating international business cycles.

Finally, recent evidence points to falling labor cost shares or, more generally, to falling domestic value added shares in many countries and sectors in recent decades (e.g, Karabarbounis and Neiman, 2013; Timmer et al., 2014). At least part of this decline appears to be related to the parallel drop in trade and communication costs, leading to increased production fragmentation (e.g., Hasan et al., 2007; Fort, 2017). The basic model adopted in this paper, as well as the extension presented in Appendix A.8, allow for such flexible labor cost shares that respond to changes in the cost of sourcing intermediate inputs. Hence, these models have the potential to add valuable qeneral-equilibrium insights regarding the welfare consequences of these declines. Crucial in this respect would be, however, to first have a well-identified estimate of the elasticity of substitution between labor and intermediates in production.

A Appendix

A.1 Equilibrium and comparative statics

Here, we verify Proposition 1 stating that the model of Section 2 has a locally unique equilibrium with well-behaved comparative statics properties. Towards this end, we follow the standard solution approach for production economies and reduce the economy, in a first instance, into a pure 'labor exchange economy' where the prices of all products, outputs, and trade flows are expressed in terms of their labor content (see Taylor (1938) and Wilson (1980) and more recently also Alvarez and Lucas (2007) or Adao et al. (2017)). The equilibrium of this reduced economy is, subsequently, solved for by a wage vector $\mathbf{w}$ and the corresponding price, quantity, and labor demand vectors that clear all labor markets.

Equilibrium definition: Let $\boldsymbol{\Omega} \in \mathcal{U}$ denote the parameters of the original economy and let $(\mathbf{p}, \mathbf{p^f}, \mathbf{p^i}, \mathbf{w})$ denote its variables. The producer price index of each country can be represented by a vector function $\mathbf{p} = \mathbf{f}(\mathbf{p}, \mathbf{w}, \boldsymbol{\Omega})$, with row entries

$$f_i(\mathbf{p}, w_i, \boldsymbol{\Omega}) \equiv \left((\kappa_i^l)^\beta w_i^{1-\beta} + (\kappa_i^i)^\beta \Big(\sum_{j \in \mathcal{N}} p_j^{1-\gamma} \mu_j \tau_{ji}^{1-\gamma} \Big)^{\frac{1-\beta}{1-\gamma}} \right)^{\frac{1}{1-\beta}} \tag{21}$$

As will be shown in Lemma 2, this function fixes an implicitly defined price vector, $\tilde{\mathbf{p}}(\mathbf{w}, \boldsymbol{\Omega})$, and accordingly the vectors of composite goods prices, $\tilde{\mathbf{p}}^{\mathbf{f}}(\mathbf{w}, \boldsymbol{\Omega})$ and $\tilde{\mathbf{p}}^{\mathbf{i}}(\mathbf{w}, \boldsymbol{\Omega})$. This, however, requires us to make the following additional parameter restriction ensuring that all nations add value to the global supply chain, i.e., $\lambda_i(\mathbf{w}) \in (0, 1]$,

Assumption 1. *Let $\bar{\omega} \equiv \max\{\omega_{ij} \,|\, ij \in \mathcal{N} \times \mathcal{N}\}$ and $\underline{\omega} \equiv \min\{\omega_{ij} \,|\, ij \in \mathcal{N} \times \mathcal{N}\}$ for any parameter ω_{ij}. It either holds*[37]

$$\text{(a)} \quad 1 < \beta < \gamma \qquad \text{or} \quad \text{(b)} \qquad (\bar{\kappa}^i)^\beta \big(n \bar{\mu} \underline{\tau}^{1-\gamma} \big)^{\frac{\beta-1}{\gamma-1}} < 1 \tag{22}$$

Subsequently, we make use of the labor market-clearing condition

$$\mathbf{Lw} \;\; = \;\; \mathbf{l^d}(\mathbf{w}) \equiv \boldsymbol{\Lambda} \sum_{\mathbf{h}=\mathbf{0}}^{\infty} \big[\boldsymbol{\Pi}(\mathbf{I} - \boldsymbol{\Lambda}) \big]^{\mathbf{h}} \boldsymbol{\Pi} \, \mathbf{Lw} \tag{23}$$

to fix $\mathbf{w}$. Lemmas 3 and 4 show that this condition defines a *locally* unique $\mathbf{w}$. This, however, requires us to assume that $\mathbf{l^d}(\mathbf{w})$ is at least locally invertible, which is ensured by the following:

[37] Part (b) mirrors the familiar constraint on κ_i^i from models with a Cobb-Douglas production technology ($\beta = 1$), which in this case simplifies to $\kappa_i^i < 1$. In the more general case of a CES technology, the constraint is combined with a condition that is common to the social network literature (e.g., Ballester et al., 2006) and that puts a limit on the number of actors and the strength of links in a network, the latter of which is captured by $\mu_i \tau_{ij}^{1-\gamma}$ in our case. Loosely speaking, Part (b) demands that the global network structure of production is not 'too dense' when $\beta > 1$, and not 'too sparse' when $\beta < 1$. Alternatively, as stated in Part (a), all nations add value to the production network regardless of its density, when intermediate inputs primarily substitute away other intermediate inputs and less so local labor ($1 < \beta < \gamma$).

Assumption 2. *In some open neighborhood around an equilibrium vector* $\mathbf{w}$, *let* $\partial\, l_i^d/\partial(l_j w_j) \in$ (09._) *for all* $j \neq i$.[38]

Jacobian matrices: For our arguments, it will also prove useful to determine the partial derivatives of (21) and (23). Concerning the former, note that $\partial \ln(f_i)/\partial \ln(p_j) = \pi_{ji}(1-\lambda_i)$. The Jacobian matrix of (21) is, thus, given by

$$\frac{\partial \ln(\mathbf{f})}{\partial \ln(\mathbf{p})} \quad = \quad (\mathbf{I}-\mathbf{\Lambda})\mathbf{\Pi}^{\mathbf{T}} \tag{24}$$

Concerning the Jacobian matrix of $\ln \mathbf{l^d}(\mathbf{w})$, Appendix A.2 derives the following expression

$$\begin{aligned}\frac{\partial \ln(\mathbf{l^d})}{\partial \ln(\mathbf{w})} \quad = \quad & (1-\beta)\,\mathbf{I} \;+\; \left[\mathbf{LW}\right]^{-\mathbf{1}}\mathbf{\Phi^{de}}\Bigg((\beta-\gamma)\,(\mathbf{X^f}+\mathbf{X^i})\frac{\partial \ln(\tilde{\mathbf{p}})}{\partial \ln(\mathbf{w})} \\ & + (\gamma-\beta)\mathbf{\Pi}\,\mathbf{E^i}\frac{\partial \ln(\tilde{\mathbf{p}}^{\mathbf{i}})}{\partial \ln(\mathbf{w})} \;+\; (\gamma-1)\mathbf{\Pi}\,\mathbf{E^f}\,\frac{\partial \ln(\tilde{\mathbf{p}}^{\mathbf{f}})}{\partial \ln(\mathbf{w})} \;+\; \mathbf{\Pi}\,\mathbf{E^f}\Bigg)\end{aligned} \tag{25}$$

where $\partial \ln(\tilde{\mathbf{p}})/\partial \ln(\mathbf{w})$ denotes the Jacobian matrix of the implicit function $\tilde{\mathbf{p}}(\mathbf{w}, \mathbf{\Omega})$, presented in (26), and

$$\frac{\partial \ln(\tilde{\mathbf{p}}^{\mathbf{f}})}{\partial \ln(\mathbf{w})} \quad = \quad \frac{\partial \ln(\tilde{\mathbf{p}}^{\mathbf{i}})}{\partial \ln(\mathbf{w})} \quad = \quad \mathbf{\Pi}^{\mathbf{T}}\frac{\partial \ln(\tilde{\mathbf{p}})}{\partial \ln(\mathbf{w})}$$

Proof of Proposition 1: We are now in the position to prove the claim. Extending on Theorem 1 of Alvarez and Lucas (2007), who prove existence of a unique positive equilibrium price vector under the Cobb-Douglas assumption ($\beta = 1$), we first show that equilibrium prices are positive as well, when technologies are given by a more general CES function.

Lemma 2. *Suppose that producer prices satisfy (21),* **A1** *is satisfied,* $\mathbf{p} \in \mathbb{R}^n_+$, *and* $\mathbf{w} \in \mathbb{R}^n_{++}$. *There exists an implicit function* $\tilde{\mathbf{p}}(\mathbf{w}, \mathbf{\Omega}) : \mathbb{R}^n_{++} \times \mathcal{U} \rightarrow \mathbb{R}^n_{++}$ *that satisfies (i)* $\tilde{\mathbf{p}} = \mathbf{f}(\tilde{\mathbf{p}}, \mathbf{w}, \mathbf{\Omega})$ *and that has (ii) partial derivatives given by*

$$\begin{aligned}\frac{\partial \ln(\tilde{\mathbf{p}})}{\partial \ln(\mathbf{w})} \quad &= \quad \sum_{\mathbf{h}=\mathbf{0}}^{\infty}\left[(\mathbf{I}-\mathbf{\Lambda})\mathbf{\Pi}^{\mathbf{T}}\right]^{\mathbf{h}}\mathbf{\Lambda} \\ \frac{\partial \ln(\tilde{\mathbf{p}})}{\partial\,\mathbf{\Omega}} \quad &= \quad \sum_{h=0}^{\infty}\left[(\mathbf{I}-\mathbf{\Lambda})\mathbf{\Pi}^{\mathbf{T}}\right]^{\mathbf{h}}\frac{\partial \ln(\mathbf{f})}{\partial \mathbf{\Omega}}\end{aligned} \tag{26}$$

Proof : We verify that $\mathbf{f}(\mathbf{p})$, as defined in (21):

[38] Note that **A2** does not require all nations to trade with each other. All we need is that there is between any country i and every other $j \in \mathcal{N}$ either (i) a chain of outgoing 'value added flows',

$$\lambda_i \pi_{il_1}(1-\lambda_{l_1}) \times \pi_{l_1 l_2}(1-\lambda_{l_2}) \times \ldots \times \pi_{l_k j}$$

or (ii) a third country k such that both i and j have a chain of outgoing value added flows to that country.

1. is an *endomorphic* function on the compact and complete space $\mathcal{P} \subset \mathbb{R}^n_{++}$ (i.e., $\mathbf{f} : \mathcal{P} \to \mathcal{P}$),
2. is a *contraction* mapping, and
3. the Jacobian matrix of $\ln(\mathbf{p}) - \ln \mathbf{f}\big(\mathbf{ln}(\mathbf{p})\big)$ is invertible.

Existence of a unique $\mathbf{p} \in \mathcal{P}$ such that $\mathbf{p} = \mathbf{f}(\mathbf{p}, \mathbf{w}, \mathbf{\Omega})$ —and thus an implicit function $\tilde{\mathbf{p}}(\mathbf{w}, \mathbf{\Omega})$— then follows from the Contraction Mapping Theorem.

1.) To confirm the *endomorphism*, note first that (21) is monotonically increasing in τ_{ji}. Thus, a conservative upper bound for $f_i(\mathbf{p}, w_i)$ is given by

$$f_i(\mathbf{p}) \;\; \leq \;\; \bar{p}_i \equiv (\kappa_i^l)^{\frac{\beta}{1-\beta}} w_i$$

A lower bound for $f_i(\mathbf{p})$ is, on the other hand, given by

$$f_i(\mathbf{p}) \; \geq \; \underline{p}_i \;\; \equiv \;\; \Big((\kappa_i^l)^\beta w_i^{1-\beta} + (\kappa_i^i)^\beta \big(\bar{\mu} \underline{\tau}^{1-\gamma} \sum_{j \in \mathcal{N}} \underline{p}_j^{1-\gamma}\big)^{\frac{1-\beta}{1-\gamma}} \Big)^{\frac{1}{1-\beta}} \tag{27}$$

in combination with

$$\sum_{i \in \mathcal{N}} \underline{p}_i^{1-\gamma} \;\; = \;\; g(\sum_{i \in \mathcal{N}} \underline{p}_i^{1-\gamma}) \; \equiv \; \sum_{i \in \mathcal{N}} \Big((\kappa_i^l)^\beta w_i^{1-\beta} \; + \; (\kappa_i^i)^\beta \big(\bar{\mu} \underline{\tau}^{1-\gamma} \sum_{i \in \mathcal{N}} \underline{p}_i^{1-\gamma}\big)^{\frac{1-\beta}{1-\gamma}} \Big)^{\frac{1-\gamma}{1-\beta}} \tag{28}$$

Here, $\underline{\tau}$ denotes the lowest trade cost and $\bar{\mu}$ the highest factor productivity among all $i, j \in \mathcal{N}$.

Thus, the remaining question is whether $\underline{p}_i > 0$ for all countries? As is easily verified, under the conditions of **A1** Part (a), i.e., $1 < \beta < \gamma$, function $g(\cdot)$ satisfies:

$$\begin{aligned} g \;\; &> \;\; 0 \\ g' \;\; &= \;\; \sum_{i \in \mathcal{N}} \underline{p}_i^{\beta-\gamma} \big(\sum_{i \in \mathcal{N}} \underline{p}_i^{1-\gamma}\big)^{\frac{\gamma-\beta}{1-\gamma}} (\kappa_i^i)^\beta \big(\bar{\mu} \underline{\tau}^{1-\gamma}\big)^{\frac{1-\beta}{1-\gamma}} > 0 \\ g'' \;\; &= \;\; \frac{\beta - \gamma}{\gamma} \sum_{i \in \mathcal{N}} \underline{p}_i^{2\beta-\gamma-1} \big(\sum_{i \in \mathcal{N}} \underline{p}_i^{1-\gamma}\big)^{\frac{2\gamma-\beta-1}{1-\gamma}} \; (\kappa_i^l)^\beta w_i^{1-\beta} \, (\kappa_i^i)^\beta \big(\bar{\mu} \underline{\tau}^{1-\gamma}\big)^{\frac{1-\beta}{1-\gamma}} < 0 \end{aligned}$$

on the domain $\sum_{i \in \mathcal{N}} \underline{p}_i^{1-\gamma} \in (0\,,\,\infty)$. Hence, g has a unique fixed point $\underline{x} > 0$. By (27), we then find a $\underline{p}_i(\underline{x})$, which by (28) satisfies $\sum_{i \in \mathcal{N}} \underline{p}_i(\underline{x})^{1-\gamma} = \underline{x}$ as well as $\bar{p}_i > \underline{p}_i(\underline{x}) > 0$.

When $\beta \geq \gamma$ or $\beta < 1$, in contrast, $\underline{p}_i > 0$ can be established as follows. Start from the inequality

$$f_i(\mathbf{p}) \; \geq \; \underline{p} \;\; = \;\; \Big((\bar{\kappa}^l)^\beta \underline{w}^{1-\beta} + (\bar{\kappa}^i)^\beta \big(\bar{\mu} \underline{\tau}^{1-\gamma} \sum_{j \in \mathcal{N}} \underline{p}^{1-\gamma}\big)^{\frac{1-\beta}{1-\gamma}} \Big)^{\frac{1}{1-\beta}} \tag{29}$$

where $\underline{w} > 0$ denotes the lowest wage rate among all i. The corresponding function $h(\cdot) \equiv \sum_{j \in \mathcal{N}} \underline{p}^{1-\gamma}$ then satisfies $h(\cdot) > 0$ as well as

$$h' \;\; = \;\; (\bar{\kappa}^i)^\beta \big(n \bar{\mu} \underline{\tau}^{1-\gamma}\big)^{\frac{1-\beta}{1-\gamma}}$$

Obviously, $0 < h' < 1$ under the conditions of **A1** Part (b). Hence, h has a unique fixed point given by

$$\underline{y} \;\equiv\; n\underline{p}^{1-\gamma} = \left(\underline{w}^{1-\beta} \frac{n^{\frac{1-\beta}{1-\gamma}}(\bar{\kappa}^l)^\beta}{1-(\bar{\kappa}^i)^\beta(n\bar{\mu}\underline{\tau}^{1-\gamma})^{\frac{1-\beta}{1-\gamma}}}\right)^{\frac{1-\gamma}{1-\beta}} \tag{30}$$

From this, a lower bound for $f_i(\mathbf{p})$ is given by $\underline{p}(\underline{y})$, which satisfies $\bar{p} = \max_{i\in\mathcal{N}}\{\bar{p}_i\} > \underline{p}_i(\underline{x}) \geq \underline{p}(\underline{y}) > 0$. For the *endomorphism*, it remains to be seen that if $\mathcal{P}$ is defined as the compact and complete space $\mathcal{P} \equiv [\underline{p}(\underline{y})\,,\,\bar{p}]^n$ then, because $f_i(\mathbf{p})$ is monotonically increasing in p_j for all $j \in \mathcal{N}$, $\mathbf{f}$ maps $\mathcal{P}$ onto itself.

2.) To establish the *contraction* property of $\mathbf{f}(\mathbf{p})$, note that for any two log-linearized price vectors $\ln(\mathbf{p}), \ln(\mathbf{p}') \in \ln\mathcal{P}$ it holds

$$\ln(\mathbf{p}) + \big(\ln(\bar{p}) - \ln(\underline{p})\big)\mathbf{1} \geq \ln(\mathbf{p}') \tag{31}$$

Thus, let $s \equiv (\ln(\bar{p}) - \ln(\underline{p}))$ denote the sup norm of $\ln(\mathbf{f}) : \ln\mathcal{P} \to \ln\mathcal{P}$. We get

$$\begin{aligned}
\ln\mathbf{f}\big(\ln(\mathbf{p}')\big) - \ln\mathbf{f}\big(\ln(\mathbf{p})\big) \;&\leq\; \ln\mathbf{f}\big(\ln(\mathbf{p}) + s\mathbf{1}\big) - \ln\mathbf{f}\big(\ln(\mathbf{p})\big)\\
&= \frac{\partial\ln\mathbf{f}\big(\ln(\mathbf{p}) + s\mathbf{z}\big)}{\partial\ln(\mathbf{p})}\, s\mathbf{1}\\
&= (\mathbf{I} - \mathbf{\Lambda})\mathbf{\Pi}^{\mathbf{T}} s\mathbf{1}\\
&= s(\mathbf{I} - \mathbf{\Lambda})\mathbf{1}
\end{aligned}$$

The inequality in the first line follows from (31) and the identity in line two from the Mean Value Theorem applied to an interior point $s\mathbf{z} = (sz_1, sz_2, ..., sz_n)$, $z_i \in (0,1)$, between $\ln(\mathbf{p})$ and $\ln(\mathbf{p}) + s\mathbf{1}$. To continue to the third line, note that $\partial\ln(\mathbf{f})/\partial\ln(\mathbf{p})$ is nothing but the Jacobian matrix of producer prices evaluated at $\ln(\mathbf{p}) + s\mathbf{z}$. The expression is given in (24). Finally, in line four, we use that the sum of import shares in each country satisfies, by definition, $\mathbf{\Pi}^{\mathbf{T}}\mathbf{1} = \mathbf{1}$. The contraction property follows, now, from the fact that $\mathbf{I} - \mathbf{\Lambda} \leq (1 - \underline{\lambda})\mathbf{I} < \mathbf{I}$ so that

$$\ln\mathbf{f}\big(\ln(\mathbf{p}')\big) - \ln\mathbf{f}\big(\ln(\mathbf{p})\big) \;\leq\; s(1 - \underline{\lambda})\mathbf{1} < s\mathbf{1}$$

where $1 - \underline{\lambda}$ denotes the *modulus* of $\ln(\mathbf{f})$ given by

$$\underline{\lambda} \;=\; \frac{(\bar{w})^{1-\beta}(\underline{\kappa}^l)^\beta}{\underline{p}^{1-\beta}} = \frac{\bar{w}^{1-\beta}(\underline{\kappa}^l)^\beta}{\underline{w}^{1-\beta}(\bar{\kappa}^l)^\beta}\big(1 - (\bar{\kappa}^i)^\beta(n\bar{\mu}\underline{\tau}^{1-\gamma})^{\frac{1-\beta}{1-\gamma}}\big) \tag{32}$$

3.) Existence of an implicit, continuously differentiable function $\tilde{\mathbf{p}}(\mathbf{w}, \mathbf{\Omega})$ follows finally from the fact that the Jacobian matrix of $\ln(\mathbf{p}) - \ln\mathbf{f}\big(\mathbf{ln}(\mathbf{p})\big)$,

$$\mathbf{I} - \frac{\partial\ln(\mathbf{f})}{\partial\ln(\mathbf{p})} \;=\; \mathbf{I} - (\mathbf{I} - \mathbf{\Lambda})\mathbf{\Pi}^{\mathbf{T}}$$

is invertible, since the row sum norm of $\partial\ln(\mathbf{f})/\partial\ln(\mathbf{p})$ is clearly smaller than one. For the same reason can the matrix inverse $[\mathbf{I} - (\mathbf{I} - \mathbf{\Lambda})\mathbf{\Pi}^{\mathbf{T}}]^{-1}$ also be expressed in terms of the converging Neumann series shown in (26). ■

Several things follow from Lemma 2. First, based on the partial derivatives in (26), we define the *terms of trade multiplier* as

$$\mathbf{\Phi^{tot}} \;\equiv\; \frac{\partial \ln(\tilde{\mathbf{p}}^{\mathbf{f}})}{\partial \ln(\mathbf{w})} \;=\; \mathbf{\Phi^{se}}\,\mathbf{\Pi^T}\,\mathbf{\Lambda} \tag{33}$$

Moreover, the *direct effect* of a trade cost shock on producer prices is given by

$$\mathbf{d}\ln(\tilde{\mathbf{p}}) \;\equiv\; \frac{\partial \ln(\tilde{\mathbf{p}})}{\partial \ln(\mathbf{T})} \circ (\mathbf{d}\ln(\mathbf{T}))^{\mathbf{T}}\,\mathbf{1} \;=\; (\mathbf{I}-\mathbf{\Lambda})\,\mathbf{\Phi^{se}}\left[\mathbf{\Pi^T} \circ (\mathbf{d}\ln(\mathbf{T}))^{\mathbf{T}}\right]\mathbf{1} \tag{34}$$

such that in the absence of production linkages: $\mathbf{\Phi^{tot}} = \mathbf{\Pi^T}\,\mathbf{\Lambda}$ and $\mathbf{d}\ln(\tilde{\mathbf{p}}) = \mathbf{0}$. Second, Lemma 2 implies that the proof of equilibrium existence reduces to finding a vector $\mathbf{w}$ that satisfies (23). This is established in the following lemma:

Lemma 3. *Suppose that* **A1** *is satisfied. There exists at least one* $\mathbf{w} \in \mathbb{R}^n_{++}$ *that satisfies* $\mathbf{Lw} = \mathbf{l^d}\big(\mathbf{w}, \tilde{\mathbf{p}}(\mathbf{w}), \tilde{\mathbf{p}}^{\mathbf{f}}(\mathbf{w}), \tilde{\mathbf{p}}^{\mathbf{i}}(\mathbf{w})\big)$.

Proof: We verify that the system of excess demand functions:

$$\mathbf{Wz}(\mathbf{w}) = \mathbf{\Lambda}\sum_{\mathbf{h=0}}^{\infty}\big[\mathbf{\Pi}(\mathbf{I}-\mathbf{\Lambda})\big]^{\mathbf{h}}\mathbf{\Pi}\,\mathbf{Lw} \;-\; \mathbf{Lw} \tag{35}$$

satisfies the following properties: for all rows i of $\mathbf{z}(\mathbf{w})$

1. $z_i(\mathbf{w})$ is continuous on the domain $\mathbf{w} \in \mathbb{R}^n_{++}$,
2. $z_i(\mathbf{w})$ is homothetic,
3. $\sum_{i\in\mathcal{N}} w_i z_i(\mathbf{w}) = 0$ (Walras' Law),
4. for all $\mathbf{w} \in \mathbb{R}^n_{++}$, there is a $k \in \mathbb{R}_{++}$ such that $z_i(\mathbf{w}) > -k$,
5. if $\mathbf{w} \to \mathbf{w}^0$, where $w^0_{-i} \neq 0$ and $w^0_i = 0$ for some i, then $z_i(\mathbf{w}) \to \infty$.

Existence of a 'wage equilibrium' then follows from Proposition 17.C.1 of Mas-Collel et al. (1995, p.585).

1.) As becomes clear from the expressions in (5) and (6), all entries in $\mathbf{\Pi}$ and $\mathbf{\Lambda}$ are the products of augmented wage rates, $w_i^{1-\beta}$, and the implicitly defined functions $\tilde{p}_i(\mathbf{w})$, both of which are continuously differentiable on $\mathbf{w} \in \mathbb{R}^n_{++}$. The continuity of $z_i(\mathbf{w})$, thus, hinges on the continuity of the Neumann series in (35). Note, however, that for any $\mathbf{w} \in \mathbb{R}^n_{++}$ the column norm of matrix $\mathbf{\Pi}(\mathbf{I}-\mathbf{\Lambda})$ satisfies the inequality $|\mathbf{\Pi}(\mathbf{I}-\mathbf{\Lambda})| < 1-\underline{\lambda} < 1$, where $1-\underline{\lambda}$ is defined in (32). The Neumann series, thus, converges uniformly as $h \to \infty$ and, based on the Uniform Limit Theorem, $z_i(\mathbf{w})$ is continuous.

2.) Note, first, that $\tilde{\mathbf{p}}(\mathbf{w})$ is homothetic. To see this, start from the partial derivatives of this function given in (26) and make use of the elementary identity in (11) to find for a proportional wage change, $\mathbf{d}\ln(\mathbf{w}) = \mathbf{1}$, that $(\partial\ln(\tilde{\mathbf{p}})/\partial\ln(\mathbf{w}))\,\mathbf{d}\ln(\mathbf{w}) = \mathbf{1}$. Hence, by Euler's Theorem, $\tilde{\mathbf{p}}(\mathbf{w})$

is homothetic. It remains to be seen that, as a consequence of this, $\mathbf{\Pi}$ and $\mathbf{\Lambda}$ are both homogeneous of degree 0 and, in turn, the same holds for $\mathbf{z}(\mathbf{w})$.

3.) To verify Walras' Law, note that the condition $\mathbf{Wz} = \mathbf{0}$ is nothing but labor market clearing (4) in each country.

4.) Consider a $\mathbf{w} \in \mathbb{R}^n_{++}$. Because the total output in each i is greater than zero, it must hold $z_i(\mathbf{w}) > -l_i$. Thus, a lower bound k for $z_i(\mathbf{w})$ is given by $k = \max_{k\in\mathcal{N}}\{l_k\}$.

5.) Suppose that $\mathbf{w} \to \mathbf{w}^0$, where $w^0_{-i} > 0$, and $w^0_i = 0$, and denote by $\underline{y}$ and $\bar{y}$ the smallest, respectively highest, value of a variable y_i among all $i \in \mathcal{N}$. It holds for any i

$$
\begin{aligned}
z_i(\mathbf{w}) &\geq \frac{1}{w_i}\lambda_i \min_{j\in\mathcal{N}}\{\pi^f_{ij}\, l_j w_j\} \ - \max_{k\in\mathcal{N}}\{l_k\} \\
&\geq \frac{1}{w_i}\frac{(\kappa^l_i)^\beta w_i^{1-\beta}}{\underline{p}^{1-\beta}} \frac{((\kappa^l_i)^\beta w_i^{1-\beta})^{\frac{1-\gamma}{1-\beta}}\underline{\mu}\bar{\tau}^{1-\gamma}}{\underline{p}^{1-\gamma} n\bar{\mu}\underline{\tau}^{1-\gamma}} \min_{j\in\mathcal{N}}\{w_j l_j\} - \max_{k\in\mathcal{N}}\{l_k\} \qquad (36)\\
&\geq \frac{1}{w_i}\frac{\big((\kappa^l_i)^\beta w_i^{1-\beta}\big)^{(1+\frac{1-\gamma}{1-\beta})}\underline{\mu}\bar{\tau}^{1-\gamma}}{\big((\bar{\kappa}^l)^\beta \underline{w}^{1-\beta}\big)^{(1+\frac{1-\gamma}{1-\beta})} n\bar{\mu}\underline{\tau}^{1-\gamma}} \left(1 - (\bar{\kappa}^i)^\beta (n\bar{\mu}\underline{\tau}^{1-\gamma})^{\frac{1-\beta}{1-\gamma}}\right)^{(1+\frac{1-\gamma}{1-\beta})} \min_{j\in\mathcal{N}}\{w_j l_j\} - \max_{k\in\mathcal{N}}\{l_k\}
\end{aligned}
$$

The first inequality follows the identity $x^f_{ij} = \pi_{ij} l_j w_j$, and the fact that the total output of i is larger than $\min\{x^f_{ij}\}$ (if $x^f_{ij} = 0$ for all $j \in \mathcal{N}$ we can make a similar argument for an $x^i_{ij} > 0$). The second inequality follows, in turn, from the definition of $\underline{y}$ and $\bar{y}$ in combination with fact that

$$
\pi_{ij} \ \geq \ \frac{p_i^{1-\gamma}\underline{\mu}\bar{\tau}^{1-\gamma}}{\underline{p}^{1-\gamma} n\bar{\mu}\underline{\tau}^{1-\gamma}} \qquad \text{and} \quad \frac{p_i^{1-\gamma}}{\underline{p}^{1-\gamma}} \ \geq \ \Big(\frac{(\kappa^l_i)^\beta w_i^{1-\beta}}{\underline{p}^{1-\beta}}\Big)^{\frac{1-\gamma}{1-\beta}}
$$

Finally, inequality three follows from the definition of $\underline{p}$ in (29) and (30).

As w_i converges to $w^0_i = 0$ and w_{-i} to $w^0_{-i} > 0$, we obviously get the identity $\lim(\underline{w}) = \lim(w_i)$ in the final line of (36). Thus, $z_i(\mathbf{w})$ grows unboundedly and, by Proposition 17.C.1 of Mas-Collel et al. (1995, p.585), we have thus established existence of an equilibrium $\mathbf{w}$. ■

So far, we have seen that an interior equilibrium exists under any parameter constellation $\mathbf{\Omega} \in \mathcal{U}$ that satisfies **A1**. The final remaining problem is that multiple $\mathbf{w}$ might be consistent with $\mathbf{Lw} = \mathbf{l^d}(\mathbf{w})$. Even worse, it is not clear whether we can perform comparative statics for any of our equilibrium points.

The feasibility of the latter is verified in the following lemma:

Lemma 4. *Suppose that* **A2** *is satisfied in an equilibrium point* $\mathbf{w}$. *Define* $\bar{\mathbf{w}} \equiv \frac{1}{|\mathbf{w}|}\mathbf{w}$, *where* $|\mathbf{w}| \equiv \sum_{i\in\mathcal{N}} w_i$. *Also, for an arbitrary matrix* $\mathbf{Z}$, *define* $\mathbf{Z^{-i^*}}$ *to be the matrix that follows after removing row* i^* *and column* i^* *from* $\mathbf{Z}$, *while* $\mathbf{Z^{+i^*}}$ *is the matrix that results from the insertion of vectors of zeros before row* i^* *and column* i^*. *Then:*

1. *There is a* locally *unique* $\bar{\mathbf{w}}$ *that satisfies* $\bar{\mathbf{W}}\mathbf{z}(\bar{\mathbf{w}}) = \mathbf{0}$.

2. *For any $\mathbf{\Omega}' \in \mathcal{U}'$, where $\mathcal{U}' \subset \mathcal{U}$ is an open set containing $\mathbf{\Omega}$, there exists an implicit function $\mathbf{g} : \mathcal{U}' \to \mathbb{R}^n_{++}$ that satisfies (i) $\mathbf{G}(\mathbf{\Omega})\mathbf{z}(\mathbf{g}(\mathbf{\Omega})) = \mathbf{0}$, where $\mathbf{G}$ is the diagonal matrix corresponding to $\mathbf{g}$, and (ii) that has partial derivatives given by*

$$\frac{\partial \ln(\mathbf{g})}{\partial \mathbf{\Omega}} = \frac{1}{\zeta}\left\{ \sum_{\mathbf{h=0}}^{\infty} \left[\frac{\mathbf{1}}{\zeta} \left\{ \frac{\partial \ln(\mathbf{l^d})}{\partial \ln(\bar{\mathbf{w}})} + (\zeta - \mathbf{1})\mathbf{I} \right\}^{-\mathbf{i}^*} \right]^{\mathbf{h}} \right\}^{+\mathbf{i}^*} \frac{\partial \ln(\mathbf{l^d})}{\partial \mathbf{\Omega}} \tag{37}$$

where $\zeta \geq \max\{\beta, \gamma\}$.[39]

Proof: Following up on Lemma 3, we know that $\bar{\mathbf{W}}\mathbf{z}(\bar{\mathbf{w}})$ is homogeneous of degree 0 and satisfies Walras' Law. Hence, we are free to fix $\bar{w}_{i^*} = 1$ and to remove row i^* and column i^* from $\bar{\mathbf{W}}\mathbf{z}(\bar{\mathbf{w}})$, which by Walras' Law are redundant. We show here that the reduced system $\{ -\bar{\mathbf{W}}\mathbf{z}(\bar{\mathbf{w}})\}^{-\mathbf{i}^*} = \mathbf{0}^{-\mathbf{i}^*}$ satisfies the conditions of the Implicit Function Theorem, i.e.,

1. $\mathbf{z}^{-i^*} : \mathbb{R}^{n-1}_{++} \times \mathcal{U} \to \mathbb{R}^{n-1}_{++}$ is continuously differentiable in $\mathbf{w}^{-i^*}$, and

2. in an equilibrium point $\bar{\mathbf{w}}$, the (log-linearized and transformed) Jacobian matrix of the reduced system

$$\frac{\partial \ln\{-\bar{\mathbf{W}}\mathbf{z}\}^{-\mathbf{i}^*}}{\partial \ln\{\bar{\mathbf{w}}\}^{-\mathbf{i}^*}} = \left\{ \mathbf{I} - \frac{\partial \ln(\mathbf{l^d})}{\partial \ln(\bar{\mathbf{w}})} \right\}^{-\mathbf{i}^*} \tag{38}$$

   is invertible, where $\partial \ln(\mathbf{l^d})/\partial \ln(\bar{\mathbf{w}})$ is defined in (25).

1.) Simply note that the excess demand function (35) is continuously differentiable in $\mathbf{w}$. The same thus holds for the reduced system.

2.) The Jacobian matrix (38) is invertible, if its row norm is unequal zero, i.e., if

$$\frac{\partial \ln\{-\bar{\mathbf{W}}\mathbf{z}\}^{-\mathbf{i}^*}}{\partial \ln\{\bar{\mathbf{w}}\}^{-\mathbf{i}^*}} \mathbf{1}^{-\mathbf{i}^*} = \left\{ \mathbf{I} - \frac{\partial \ln(\mathbf{l^d})}{\partial \ln(\bar{\mathbf{w}})} \right\}^{-\mathbf{i}^*} \mathbf{1}^{-\mathbf{i}^*} \neq \mathbf{0}^{-\mathbf{i}^*}$$

Moreover, under the same condition, we can also express the matrix inverse of $\{\mathbf{I} - \partial \ln(\mathbf{l^d})/\partial \ln(\bar{\mathbf{w}})\}^{-\mathbf{i}^*}$ by the Neumann series shown in (37), where we additionally adopted the following affine transformation of the Jacobian matrix

$$\frac{\partial \ln\{-\bar{\mathbf{W}}\mathbf{z}\}^{-\mathbf{i}^*}}{\partial \ln\{\bar{\mathbf{w}}\}^{-\mathbf{i}^*}} = \left\{ \zeta\mathbf{I} - \left(\frac{\partial \ln(\mathbf{l^d})}{\partial \ln(\bar{\mathbf{w}})} + (\zeta - 1)\mathbf{I} \right) \right\}^{-\mathbf{i}^*}$$

which we adjusted by a scalar $\zeta \geq \max\{\beta, \gamma\}$ that is sufficiently large, so that $\{\partial \ln(\mathbf{l^d})/\partial \ln(\bar{\mathbf{w}}) + (\zeta - 1)\mathbf{I}\}^{-i^*}$ has *all* its entries strictly positive. The norm condition follows immediately from the homogeneity of $\bar{\mathbf{W}}\mathbf{z}(\bar{\mathbf{w}})$ in combination with the gross substitutes property ($\partial \ln(l_i^d)/\partial \ln(w_j) > 0$ for all i and $j \neq i$) implied by **A2**. ■

[39] The assumption $\zeta \geq \max\{\beta, \gamma\}$ ensures that the diagonal elements of $\partial \ln(\mathbf{l^d})/\partial \ln(\bar{\mathbf{w}}) + (\zeta - 1)\,\mathbf{I}$ are strictly positive. The off-diagonal elements are positive by **A2**.

Based on Lemma 4, we define the *foreign trade multiplier* as

$$\mathbf{\Phi^{mult}_{i^*}} \quad \equiv \quad \frac{1}{\zeta}\left\{\sum_{h=0}^{\infty}\left[\frac{1}{\zeta}\Big\{\frac{\partial \ln(\mathbf{l^d})}{\partial \ln(\mathbf{w})} + (\zeta-1)\mathbf{I}\Big\}^{-\mathbf{i^*}}\right]^{\mathbf{h}}\right\}^{+\mathbf{i^*}} [\mathbf{LW}]^{-\mathbf{1}} \in \mathbb{R}^{n\times n}_{++} \tag{39}$$

where $\partial \ln(\mathbf{l^d})/\partial \ln(\mathbf{w})$ is given in (25).

A.2 The Jacobian matrix of labor demand

Here, we derive the Jacobian matrix (25) of the log-linearized labor demand system

$$\ln(\mathbf{l^d}) = \ln(\boldsymbol{\lambda}) + \ln\left(\sum_{\mathbf{h=0}}^{\infty}\left[\mathbf{\Pi}(\mathbf{I}-\mathbf{\Lambda})\right]^{\mathbf{h}}\mathbf{\Pi}\,\mathbf{Lw}\right) \tag{40}$$

The Jacobian of the (log) labor cost share is given by

$$\frac{\partial \ln(\lambda_i)}{\partial \ln(w_j)} = (\beta-1)\frac{\partial \ln(\tilde{p}_i)}{\partial \ln(w_j)}$$

for any cell $j \neq i$. Including the diagonal elements,

$$\frac{\partial \ln(\boldsymbol{\lambda})}{\partial \ln(\mathbf{w})} \quad = \quad (1-\beta)\left(\mathbf{I} - \frac{\partial \ln(\tilde{\mathbf{p}})}{\partial \ln(\mathbf{w})}\right) \tag{41}$$

where $\partial \ln(\tilde{\mathbf{p}})/\partial \ln(\mathbf{w})$ is defined in (26).

Turning to the Jacobian of the output vector, the total derivative of the Neumann series in (40) can be determined with the help of Lemma 6 Property (i) (Appendix A.3). Based on this, entry ij of the Jacobian is given by

$$\frac{\partial \ln\left(x_i^f + x_i^i\right)}{\partial \ln(w_j)} \quad = \quad \frac{1}{x_i^f + x_i^i}\left(\sum_{k\in\mathcal{N}} z_{ik}\,\frac{\partial \sum_{l\in\mathcal{N}} \pi_{kl}(1-\lambda_l)}{\partial \ln(w_j)} \sum_{m\in\mathcal{N}} z_{lm}x_m^f + \sum_{k\in\mathcal{N}} z_{ik}\,\frac{\partial\, x_k^f}{\partial \ln(w_j)}\right)$$

where z_{ij} denotes cell ij of matrix $\sum_{\mathbf{h=0}}^{\infty}\left[\mathbf{\Pi}(\mathbf{I}-\mathbf{\Lambda})\right]^{\mathbf{h}}$ and where

$$\sum_{m\in\mathcal{N}} z_{lm}x_m^f \quad = \quad x_l^f + x_l^i$$

Concerning the different summands, it holds

$$\frac{\partial\, x_k^f}{\partial \ln(w_j)} \quad = \quad (1-\gamma)\frac{\partial \ln(\tilde{p}_k)}{\partial \ln(w_j)}x_k^f \; - \; (1-\gamma)\sum_{k\in\mathcal{N}} x_{kl}^f\frac{\partial \ln(\tilde{p}_l^f)}{\partial \ln(w_j)} \; + \; \pi_{kj}l_jw_j$$

or, in vector notation,

$$\frac{\partial\, \mathbf{x^f}}{\partial \ln(\mathbf{w})} \quad = \quad (1-\gamma)\mathbf{X^f}\frac{\partial \ln(\tilde{\mathbf{p}})}{\partial \ln(\mathbf{w})} \; - \; (1-\gamma)\mathbf{\Pi}\,\mathbf{LW}\frac{\partial \ln(\tilde{\mathbf{p}}^{\mathbf{f}})}{\partial \ln(\mathbf{w})} \; + \; \mathbf{\Pi}\,\mathbf{LW}$$

Furhermore, making use of the identity $\sum_{k\in\mathcal{N}} \pi^i_{ik}(1-\lambda_k)(x^f_k + x^i_k) = x^i_i$, it holds

$$\begin{aligned}
\frac{\partial \sum_{k\in\mathcal{N}} \pi_{ik}(1-\lambda_k)}{\partial \ln(w_j)}(x^f_k + x^i_k) &= (1-\gamma)\frac{\partial \ln(\tilde{p}_i)}{\partial \ln(w_j)} x^i_i \\
&- (1-\gamma)\sum_{k\in\mathcal{N}} \pi_{ik}\frac{\partial \ln(\tilde{p}^i_k)}{\partial \ln(w_j)}(1-\lambda_k)(x^f_k + x^i_k) \\
&+ (1-\beta)\sum_{k\in\mathcal{N}} \pi_{ik}(1-\lambda_k)\left(\frac{\partial \ln(\tilde{p}^i_k)}{\partial \ln(w_j)} - \frac{\partial \ln(\tilde{p}_k)}{\partial \ln(w_j)}\right)(x^f_k + x^i_k)
\end{aligned}$$

Thus, in sum, the Jacobian matrix of the output vector is given by

$$\begin{aligned}
\frac{\partial \ln\left(\mathbf{x^f} + \mathbf{x^i}\right)}{\partial \ln(\mathbf{w})} &= [\mathbf{X^f} + \mathbf{X^i}]^{-\mathbf{1}} \sum_{\mathbf{h=0}}^{\infty} \left[\mathbf{\Pi}(\mathbf{I}-\mathbf{\Lambda})\right]^{\mathbf{h}} \\
&\Bigg((1-\gamma)\mathbf{X^i}\frac{\partial \ln(\tilde{\mathbf{p}})}{\partial \ln(\mathbf{w})} - (1-\gamma)\mathbf{\Pi}(\mathbf{I}-\mathbf{\Lambda})(\mathbf{X^f}+\mathbf{X^i})\frac{\partial \ln(\tilde{\mathbf{p}}^{\mathbf{i}})}{\partial \ln(\mathbf{w})} \\
&+ (1-\beta)\mathbf{\Pi}(\mathbf{I}-\mathbf{\Lambda})(\mathbf{X^f}+\mathbf{X^i})\left(\frac{\partial \ln(\tilde{\mathbf{p}}^{\mathbf{i}})}{\partial \ln(\mathbf{w})} - \frac{\partial \ln(\tilde{\mathbf{p}})}{\partial \ln(\mathbf{w})}\right) \\
&+ (1-\gamma)\mathbf{X^f}\frac{\partial \ln(\tilde{\mathbf{p}})}{\partial \ln(\mathbf{w})} - (1-\gamma)\mathbf{\Pi}\,\mathbf{LW}\frac{\partial \ln(\tilde{\mathbf{p}}^{\mathbf{f}})}{\partial \ln(\mathbf{w})} + \mathbf{\Pi}\,\mathbf{LW}\Bigg)
\end{aligned} \tag{42}$$

Combining (41) and (42) and making use of the following expansion for (41):

$$\begin{aligned}
(\beta-1)\frac{\partial \ln(\tilde{\mathbf{p}})}{\partial \ln(\mathbf{w})} &= (\beta-1)\left[\mathbf{X^f}+\mathbf{X^i}\right]^{-\mathbf{1}} \sum_{\mathbf{h=0}}^{\infty} \left[\mathbf{\Pi}(\mathbf{I}-\mathbf{\Lambda})\right]^{\mathbf{h}} \\
&\quad * \Big(\mathbf{I} - \mathbf{\Pi}(\mathbf{I}-\mathbf{\Lambda})\Big)\left(\mathbf{X^f}+\mathbf{X^i}\right)\frac{\partial \ln(\tilde{\mathbf{p}})}{\partial \ln(\mathbf{w})}
\end{aligned}$$

we arrive at the simpler expression in (25).

Lemma 5. *The Jacobian matrix of the labor demand system (25) satisfies:*

1. $\left(\partial \ln(\mathbf{l^d})/\partial \ln(\mathbf{w})\right)\mathbf{1} = \mathbf{1}$, *and*
2. $\mathbf{1^T}\mathbf{LW}\left(\partial \ln(\mathbf{l^d})/\partial \ln(\mathbf{w})\right)[\mathbf{LW}]^{-\mathbf{1}} = \mathbf{1^T}$

Proof: Part (1.) follows immediately from the homotheticity of the labor demand function in combination with Euler's Theorem.

Concerning Part (2.), note that

$$
\begin{aligned}
\mathbf{1^T LW}\,\frac{\partial \ln(\mathbf{l^d})}{\partial \ln(\mathbf{w})}\,[\mathbf{LW}]^{-\mathbf{1}} \;&=\; (1-\beta)\mathbf{1^T} \;+\; (\beta-\gamma)\mathbf{1^T}(\mathbf{X^f}+\mathbf{X^i})\frac{\partial \ln(\tilde{\mathbf{p}})}{\partial \ln(\mathbf{w})}\,[\mathbf{LW}]^{-\mathbf{1}} \\
&+\; (\gamma-\beta)\mathbf{1^T X^i}\frac{\partial \ln(\tilde{\mathbf{p}})}{\partial \ln(\mathbf{w})}\,[\mathbf{LW}]^{-\mathbf{1}} \;+\; (\gamma-1)\mathbf{1^T X^f}\frac{\partial \ln(\tilde{\mathbf{p}})}{\partial \ln(\mathbf{w})}\,[\mathbf{LW}]^{-\mathbf{1}} \;+\; \mathbf{1^T} \\
&=\; (1-\beta)\mathbf{1^T} \;+\; (\beta-1)(\mathbf{x^f})^{\mathbf{T}}\frac{\partial \ln(\tilde{\mathbf{p}})}{\partial \ln(\mathbf{w})}\,[\mathbf{LW}]^{-\mathbf{1}} \;+\; \mathbf{1^T} \\
&=\; (1-\beta)\mathbf{1^T} \;+\; (\beta-1)\mathbf{1^T}\,\mathbf{LW}\,[\mathbf{LW}]^{-\mathbf{1}} \;+\; \mathbf{1^T}
\end{aligned}
$$

where in lines one and two, we make use of the properties of $\mathbf{\Phi^{de}}$ summarized in Lemma 8 (Appendix A.4) combined with the identities $\mathbf{LW}[\mathbf{X^f}+\mathbf{X^i}]^{-\mathbf{1}} = \mathbf{\Lambda}$, $\mathbf{1^T\Pi} = \mathbf{1^T}$, and $\mathbf{1^T E^t \Pi^T} = \mathbf{1^T X^t}$. Line three simplifies, and line four follows from the properties for $\mathbf{\Phi^{se}}$ in Lemma 8. The chain of identities, thus, leads to $\mathbf{1^T}$. ■

A.3 Exact comparative statics for an inverse matrix

In this appendix, we expand on a collection of results from the regional science and social network literature with the ambition to establish an exact functional relationship between a *new* inverse matrix, $[\mathbf{I}-\mathbf{Z'}]^{-\mathbf{1}}$, the *initial* matrix, $[\mathbf{I}-\mathbf{Z}]^{-\mathbf{1}}$, and the imposed change $\mathbf{dZ} = \mathbf{Z'}-\mathbf{Z}$.

Part (1.) of the following result is altogether and new and provides the foundation for our analysis of a *small* shock to any number of elements of an exogenous or endogenous matrix $\mathbf{Z}$. Part (2.) concerns a *large* shock to *two* elements of an exogenous $\mathbf{Z}$-matrix. Part (3.), finally, extends on the 'Key Player' analysis of Ballester et al. (2006) and Temurshoev (2010) and looks at arbitrary large shocks to row i and column i of an exogenous $\mathbf{Z}$-matrix.

Lemma 6. *Consider square matrices $\mathbf{Z}$ and $\mathbf{Z'}$ and a scalar $x \in \mathbb{R}$, such that $[\mathbf{I}-\mathbf{Z}]^{-\mathbf{1}}$ and $[\mathbf{I}-\mathbf{Z'}]^{-\mathbf{1}}$ exist:*

1. *For $\mathbf{Z'} = \mathbf{Z}+x\mathbf{dZ}$ with $x \to 0$, it holds*

$$x \lim_{x\to 0} \frac{1}{x}\big[\mathbf{I}-\mathbf{Z}-x\mathbf{dZ}\big]^{-\mathbf{1}} \;=\; \big[\mathbf{I}-\mathbf{Z}\big]^{-\mathbf{1}} + x\,\big[\mathbf{I}-\mathbf{Z}\big]^{-\mathbf{1}}\mathbf{dZ}\,\big[\mathbf{I}-\mathbf{Z}\big]^{-\mathbf{1}} \tag{43}$$

2. *For $\mathbf{Z'} = \mathbf{Z}+xz_{ij}\mathbf{I_{ij}}+yz_{kl}\mathbf{I_{kl}}$, where $\mathbf{I_{ij}}$ is a square matrix with a one in element ij and zero everywhere else and $x, y \in \mathbb{R}$, it holds*

$$
\begin{aligned}
\big[\mathbf{I}-\mathbf{Z}-xz_{ij}\mathbf{I_{ij}}+yz_{kl}\mathbf{I_{kl}}\big]^{-\mathbf{1}} \;&=\; \big[\mathbf{I}-\mathbf{Z}\big]^{-\mathbf{1}} + \frac{1}{\psi}\,\big[\mathbf{I}-\mathbf{Z}\big]^{-\mathbf{1}}\,\mathbf{dZ}\,\big[\mathbf{I}-\mathbf{Z}\big]^{-\mathbf{1}} \\
&+ \frac{xz_{ij}yz_{kl}}{\psi}\,\big[\mathbf{I}-\mathbf{Z}\big]^{-\mathbf{1}}\left(\sum_{h=0}^{\infty} z_{jk}^{[h]}\mathbf{I_{il}} + \sum_{h=0}^{\infty} z_{li}^{[h]}\mathbf{I_{kj}} - \sum_{h=0}^{\infty} z_{lk}^{[h]}\mathbf{I_{ij}} - \sum_{h=0}^{\infty} z_{ji}^{[h]}\mathbf{I_{kl}}\right)\big[\mathbf{I}-\mathbf{Z}\big]^{-\mathbf{1}}
\end{aligned}
\tag{44}
$$

where $\psi \in \mathbb{R}_{++}$ is defined in (50) and $\sum_{h=0}^{\infty} z_{ij}^{[h]}$ denotes entry ij of matrix $[\mathbf{I}-\mathbf{Z}]^{-\mathbf{1}}$.

3. *For $\mathbf{Z'} = \mathbf{I_{xi}ZI_{yi}}$, where $\mathbf{I_{xi}} \equiv (\mathbf{I}+x\mathbf{I_i})$ and $\mathbf{I_{yi}} \equiv (\mathbf{I}+y\mathbf{I_i})$, $x, y \in \mathbb{R}$, $\mathbf{I_i}$ denotes a square matrix with a one in element ii and zero everywhere else, and where $\sum_{h=0}^{\infty} z_{ii}^{[h]}$ denotes*

element ii of matrix $[\mathbf{I}-\mathbf{Z}]^{-1}$, *it holds*

$$\left[\mathbf{I}-\mathbf{I_{xi}}\mathbf{Z}\mathbf{I_{yi}}\right]^{-\mathbf{1}} \;=\; \mathbf{I} \;+\; \mathbf{I_{xi}} \quad \Bigg(\quad \left[\mathbf{I}-\mathbf{Z}\right]^{-\mathbf{1}} \;+\; \frac{x+y+xy}{1-\sum_{h=1}^{\infty} z_{ii}^{[h]}(x+y+xy)} \\ * \quad \left[\mathbf{I}-\mathbf{Z}\right]^{-\mathbf{1}}\mathbf{Z}\mathbf{I_i}\mathbf{Z}\left[\mathbf{I}-\mathbf{Z}\right]^{-\mathbf{1}}\Bigg)\mathbf{I_{yi}} \tag{45}$$

Before we proceed to the proof, let us first review a number of results on the exact solution for the inverse of a sum of matrices:

Henderson and Searle (1981). Let $\mathbf{X}$ be a nonsingular square matrix, and $\mathbf{U}, \mathbf{Y}$ and $\mathbf{V}$ be (possibly rectangular) matrices such that $\mathbf{UYV}$ is a square matrix. It holds

$$\left[\mathbf{X}+\mathbf{UYV}\right]^{-\mathbf{1}} \;=\; \mathbf{X}^{-\mathbf{1}} \;-\; \mathbf{X}^{-\mathbf{1}}\mathbf{U}\left[\mathbf{I}+\mathbf{YVX}^{-\mathbf{1}}\mathbf{U}\right]^{-\mathbf{1}}\mathbf{YVX}^{-\mathbf{1}} \tag{46}$$

The following identities are useful special cases:

Minabe (1966, p.58). By successive application of (46) for a nonsingular square matrix $\mathbf{X}$ and a square matrix $\mathbf{Y}$, such that all characteristic roots μ of $\mathbf{X}^{-\mathbf{1}}\mathbf{Y}$ satisfy $|\mu| < 1$, we get

$$\left[\mathbf{X}-\mathbf{Y}\right]^{-\mathbf{1}} \;=\; \mathbf{X}^{-\mathbf{1}} \;+\; \sum_{\mathbf{h}=\mathbf{1}}^{\infty}\left(\mathbf{X}^{-\mathbf{1}}\mathbf{Y}\right)^{\mathbf{h}}\mathbf{X}^{-\mathbf{1}} \tag{47}$$

Neumann's series expansion. Expanding on Minabe (1966), we get for $\mathbf{X}=\mathbf{I}$

$$[\mathbf{I}-\mathbf{Y}]^{-\mathbf{1}} \;=\; \mathbf{I}+\sum_{\mathbf{h}=\mathbf{1}}^{\infty}(\mathbf{Y})^{\mathbf{h}} \;=\; \sum_{\mathbf{h}=\mathbf{0}}^{\infty}\mathbf{Y}^{\mathbf{h}} \tag{48}$$

Sherman and Morrison (1950). For $s \in \mathbb{R}$, a column vector $\mathbf{u}$, and a row vector $\mathbf{v}^T$ of identical length

$$\left[\mathbf{X}+y\mathbf{u}\mathbf{v}^{\mathbf{T}}\right]^{-\mathbf{1}} \;=\; \mathbf{X}^{-\mathbf{1}} - \frac{y}{1+y\mathbf{v}^{\mathbf{T}}\mathbf{X}^{-\mathbf{1}}\mathbf{u}}\mathbf{X}^{-\mathbf{1}}\mathbf{u}\mathbf{v}^{\mathbf{T}}\mathbf{X}^{-\mathbf{1}} \tag{49}$$

Equipped with these results, we are ready to the prove Lemma 6:

Proof of part 1. Applying (47) for $\mathbf{X}=\mathbf{I}-\mathbf{Z}$ and $\mathbf{Y}=x\mathbf{dZ}$, we get

$$\left[\mathbf{I}-\mathbf{Z}-x\mathbf{dZ}\right]^{-\mathbf{1}} \;=\; \left[\mathbf{I}-\mathbf{Z}\right]^{-\mathbf{1}} \;+\; \sum_{\mathbf{h}=\mathbf{1}}^{\infty}\left(\left[\mathbf{I}-\mathbf{Z}\right]^{-\mathbf{1}}x\mathbf{dZ}\right)^{\mathbf{h}}\left[\mathbf{I}-\mathbf{Z}\right]^{-\mathbf{1}}$$

Suppose, now, that $x \to 0$. Then, in the limit, all characteristic roots of $\left[\mathbf{I}-\mathbf{Z}\right]^{-\mathbf{1}} x\mathbf{dZ}$ clearly satisfy $\lim_{x\to 0^+} |\mu| < 1$. We moreover obtain

$$\lim_{x\to 0}\frac{1}{x}\sum_{\mathbf{h=1}}^{\infty}\left(\left[\mathbf{I}-\mathbf{Z}\right]^{-\mathbf{1}} x\mathbf{dZ}\right)^{\mathbf{h}}\left[\mathbf{I}-\mathbf{Z}\right]^{-\mathbf{1}} \;=\; \left[\mathbf{I}-\mathbf{Z}\right]^{-\mathbf{1}}\mathbf{dZ}\left[\mathbf{I}-\mathbf{Z}\right]^{-\mathbf{1}}$$

Thus, expression (43) is nothing but the partial derivative rule for an inverse matrix $[\mathbf{I}-\mathbf{Z}]^{-\mathbf{1}}$.

Proof of part 2. Applying Sherman and Morrison (1950) twice, first for $\mathbf{X} = \mathbf{I} - \mathbf{Z}'$ with $\mathbf{Z}' = \mathbf{Z} + xz_{ij}\mathbf{I_{ij}}$, $y = -yz_{kl}$, $\mathbf{u} = (u_1 = 0, u_2 = 0, ..., u_k = 1, u_{k+1} = 0, u_n = 0)$, and $\mathbf{v} = (v_1 = 0, v_2 = 0, ..., v_l = 1, v_{l+1} = 0, v_n = 0)$, we get

$$\left[\mathbf{I}-\mathbf{Z}'-yz_{kl}\mathbf{I_{kl}}\right]^{-\mathbf{1}} \;=\; \left[\mathbf{I}-\mathbf{Z}'\right]^{-\mathbf{1}} + \left[\mathbf{I}-\mathbf{Z}'\right]^{-\mathbf{1}}\frac{yz_{kl}}{1-yz_{kl}\mathbf{v^T}\left[\mathbf{I}-\mathbf{Z}'\right]^{-\mathbf{1}}\mathbf{u}}\mathbf{I_{kl}}\left[\mathbf{I}-\mathbf{Z}'\right]^{-\mathbf{1}}$$

Next, for $\mathbf{X} = \mathbf{I} - \mathbf{Z}$, $s = -xz_{ij}$, $\mathbf{u} = (u_1 = 0, u_2 = 0, ..., u_i = 1, u_{i+1} = 0, u_n = 0)$, and $\mathbf{v} = (v_1 = 0, v_2 = 0, ..., v_j = 1, v_{j+1} = 0, v_n = 0)$, to get

$$\begin{aligned}
= \;& \left[\left[\mathbf{I}-\mathbf{Z}\right]^{-\mathbf{1}} + \frac{xz_{ij}}{1-xz_{ij}\sum_{h=0}^{\infty} z_{ji}^{[h]}}\left[\mathbf{I}-\mathbf{Z}\right]^{-\mathbf{1}}\mathbf{I_{ij}}\left[\mathbf{I}-\mathbf{Z}\right]^{-\mathbf{1}}\right] \\
& * \left(\mathbf{I} + \frac{yz_{kl}}{1-yz_{kl}\sum_{h=0}^{\infty} z_{lk}^{[h]} - \frac{xz_{ij}yz_{kl}}{1-xz_{ij}\sum_{h=0}^{\infty} z_{ji}^{[h]}}\sum_{h=0}^{\infty} z_{li}^{[h]}\sum_{h=0}^{\infty} z_{jk}^{[h]}}\right. \\
& \left. * \,\mathbf{I_{kl}}\left[\left[\mathbf{I}-\mathbf{Z}\right]^{-\mathbf{1}} + \frac{xz_{ij}}{1-xz_{ij}\sum_{h=0}^{\infty} z_{ji}^{[h]}}\left[\mathbf{I}-\mathbf{Z}\right]^{-\mathbf{1}}\mathbf{I_{ij}}\left[\mathbf{I}-\mathbf{Z}\right]^{-\mathbf{1}}\right]\right)
\end{aligned}$$

Reordering and simplifying, we get

$$\begin{aligned}
= \;& \left[\mathbf{I}-\mathbf{Z}\right]^{-\mathbf{1}} + \frac{xz_{ij}(1-yz_{kl}\sum_{h=0}^{\infty} z_{lk}^{[h]})}{\psi}\left[\mathbf{I}-\mathbf{Z}\right]^{-\mathbf{1}}\mathbf{I_{ij}}\left[\mathbf{I}-\mathbf{Z}\right]^{-\mathbf{1}} \\
& + \frac{yz_{kl}(1-xz_{ij}\sum_{h=0}^{\infty} z_{ji}^{[h]})}{\psi}\left[\mathbf{I}-\mathbf{Z}\right]^{-\mathbf{1}}\mathbf{I_{kl}}\left[\mathbf{I}-\mathbf{Z}\right]^{-\mathbf{1}} \\
& + \frac{xz_{ij}yz_{kl}}{\psi}\left(\sum_{h=0}^{\infty} z_{jk}^{[h]}\left[\mathbf{I}-\mathbf{Z}\right]^{-\mathbf{1}}\mathbf{I_{il}}\left[\mathbf{I}-\mathbf{Z}\right]^{-\mathbf{1}} + \sum_{h=0}^{\infty} z_{li}^{[h]}\left[\mathbf{I}-\mathbf{Z}\right]^{-\mathbf{1}}\mathbf{I_{kj}}\left[\mathbf{I}-\mathbf{Z}\right]^{-\mathbf{1}}\right)
\end{aligned}$$

where

$$\psi = (1-xz_{ij}\sum_{h=0}^{\infty} z_{ji}^{[h]})(1-yz_{kl}\sum_{h=0}^{\infty} z_{lk}^{[h]}) - xz_{ij}yz_{kl}\sum_{h=0}^{\infty} z_{li}^{[h]}\sum_{h=0}^{\infty} z_{jk}^{[h]} \tag{50}$$

Expression (44) follows immediately after a final sorting step.

Proof of part 3. Applying (46) for $\mathbf{X} = \mathbf{I}$, $\mathbf{Y} = -\mathbf{Z}$, $\mathbf{U} = \mathbf{I_{xi}}$, and $\mathbf{V} = \mathbf{I_{yi}}$, we get

$$\begin{aligned} \left[\mathbf{I} - \mathbf{I_{xi}} \mathbf{Z} \mathbf{I_{yi}}\right]^{-1} &= \mathbf{I} + \mathbf{I_{xi}} \left[\mathbf{I} - \mathbf{Z} \mathbf{I_{xi}} \mathbf{I_{yi}}\right]^{-1} \mathbf{Z} \mathbf{I_{yi}} \\ &= \mathbf{I} + \mathbf{I_{xi}} \left[\mathbf{I} - \mathbf{Z} - \mathbf{Z}(x + y + xy)\mathbf{I_i}\right]^{-1} \mathbf{Z} \mathbf{I_{yi}} \end{aligned} \tag{51}$$

Applying (46) again, this time for $\mathbf{X} = \mathbf{I} - \mathbf{Z}$, $\mathbf{Y} = \mathbf{I_i}$, $\mathbf{U} = -\mathbf{Z}(x + y + xy)\mathbf{I_i}$, and $\mathbf{V} = \mathbf{I}$, we get

$$\begin{aligned} \left[\mathbf{I} - \mathbf{Z} - \mathbf{Z}(x + y + xy)\mathbf{I_i}\right]^{-1} &= \left[\mathbf{I} - \mathbf{Z}\right]^{-1} + \left[\mathbf{I} - \mathbf{Z}\right]^{-1} \mathbf{Z}(x + y + xy) \\ &* \quad \mathbf{I_i} \left[\mathbf{I} - \mathbf{I_i} \left[\mathbf{I} - \mathbf{Z}\right]^{-1} \mathbf{Z}(x + y + xy)\mathbf{I_i}\right]^{-1} \mathbf{I_i} \left[\mathbf{I} - \mathbf{Z}\right]^{-1} \end{aligned} \tag{52}$$

where we have made use of the fact that $\mathbf{I_i}$ is idempotent, i.e., $\mathbf{I_i} = \mathbf{I_i}\mathbf{I_i}$. Finally, we can write

$$\mathbf{I_i} \left[\mathbf{I} - \mathbf{I_i} \left[\mathbf{I} - \mathbf{Z}\right]^{-1} \mathbf{Z}(x + y + xy)\mathbf{I_i}\right]^{-1} \mathbf{I_i} = \frac{1}{1 - \sum_{h=1}^{\infty} z_{ii}^{[h]} (x + y + xy)} \mathbf{I_i} \tag{53}$$

where $\sum_{h=1}^{\infty} z_{ii}^{[h]}$ denotes element ii of matrix $\mathbf{Z}[\mathbf{I} - \mathbf{Z}]^{-1}$. Hence, combining (51)-(53), we obtain the desired expression (45).

For the special case of $x = y = -1$, i.e., the total isolation of a country, the combination of (51)-(53) yields the simpler expression

$$\left[\mathbf{I} - \mathbf{I_{-i}} \mathbf{Z} \mathbf{I_{-i}}\right]^{-1} = \mathbf{I_i} + \left[\mathbf{I} - \mathbf{Z}\right]^{-1} - \frac{1}{\sum_{h=0}^{\infty} z_{ii}^{[h]}} \left[\mathbf{I} - \mathbf{Z}\right]^{-1} \mathbf{I_i} \left[\mathbf{I} - \mathbf{Z}\right]^{-1}$$

since

$$\begin{aligned} \left[\mathbf{I} - \mathbf{I_{-i}} \mathbf{Z} \mathbf{I_{-i}}\right]^{-1} &= \mathbf{I} + \mathbf{I_{-i}} \left(\left[\mathbf{I} - \mathbf{Z}\right]^{-1} - \left[\mathbf{I} - \mathbf{Z}\right]^{-1} \mathbf{Z} \frac{1}{\sum_{h=0}^{\infty} z_{ii}^{[h]}} \mathbf{I_i} \left[\mathbf{I} - \mathbf{Z}\right]^{-1} \right) \mathbf{Z} \mathbf{I_{-i}} \\ &= \mathbf{I_i} + \mathbf{I_{-i}} \left[\mathbf{I} - \mathbf{Z}\right]^{-1} \mathbf{I_{-i}} - \left[\mathbf{I} - \mathbf{Z}\right]^{-1} \frac{1}{\sum_{h=0}^{\infty} z_{ii}^{[h]}} \mathbf{I_i} \left[\mathbf{I} - \mathbf{Z}\right]^{-1} \\ &+ \mathbf{I_{-i}} \left[\mathbf{I} - \mathbf{Z}\right]^{-1} \mathbf{I_i} + \mathbf{I_i} \left[\mathbf{I} - \mathbf{Z}\right]^{-1} \mathbf{I_{-i}} \\ &= \mathbf{I_i} + \left[\mathbf{I} - \mathbf{Z}\right]^{-1} - \frac{1}{\sum_{h=0}^{\infty} z_{ii}^{[h]}} \left[\mathbf{I} - \mathbf{Z}\right]^{-1} \mathbf{I_i} \left[\mathbf{I} - \mathbf{Z}\right]^{-1} \end{aligned}$$

where, for the second line, we used $\mathbf{I_{-i}}[\mathbf{I} - \mathbf{Z}]^{-1}\mathbf{Z}\mathbf{I_i} = \mathbf{I_{-i}}[\mathbf{I} - \mathbf{Z}]^{-1}\mathbf{I_i} = [\mathbf{I} - \mathbf{Z}]^{-1}\mathbf{I_i} - \mathbf{I_i} \sum_{h=0}^{\infty} z_{ii}^{[h]}$ and similar $\mathbf{I_i}[\mathbf{I} - \mathbf{Z}]^{-1}\mathbf{Z}\mathbf{I_{-i}} = \mathbf{I_i}[\mathbf{I} - \mathbf{Z}]^{-1}\mathbf{I_{-i}} = \mathbf{I_i}[\mathbf{I} - \mathbf{Z}]^{-1} - \mathbf{I_i} \sum_{h=0}^{\infty} z_{ii}^{[h]}$. ■

Expanding on the identity of Sherman and Morrison (1950), one can furthermore trace back an arbitrary *large* shock to any number of elements of an inverse matrix in a sequence of $k \geq 1$ functional mappings:

Let $\mathbf{X}$ be an invertible square matrix. Moreover, let $\mathbf{I_{i_s j_s}}$ be a square matrix of the same dimension with a one in entry $i_s j_s$ and zero everywhere else, and let $x_s \in \mathbb{R}$. Define the endomorphic function

$$\mathbf{f_{i_s j_s}}\left(\mathbf{X^{-1}}\right) \equiv \mathbf{X^{-1}} + y_{i_s j_s}\mathbf{X^{-1}}\mathbf{I_{i_s j_s}}\mathbf{X^{-1}}$$

with a scalar $y_{i_s j_s}$ which is the output of the function $y_{i_s j_s} \equiv g(x_s, \mathbf{X^{-1}}, \mathbf{I_{i_s j_s}})$. We can then write

$$\left[\mathbf{I} - \mathbf{Z} - \sum_{\mathbf{s=1}}^{\mathbf{k}} x_s \mathbf{I_{i_s j_s}}\right]^{-1} = \mathbf{f_{i_k j_k}}\Bigg(\mathbf{f_{i_{k-1} j_{k-1}}}\left(\mathbf{f_{i_{k-2} j_{k-2}}}\left(\ldots\mathbf{f_{i_1 j_1}}([\mathbf{I} - \mathbf{Z}]^{-1})\right)\right)\Bigg)$$

whereby, at any step $s \geq 1$, $\mathbf{f_{i_s j_s}}$ can be written in the form

$$\mathbf{f_{i_s j_s}}\left([\mathbf{I} - \mathbf{Z}]^{-1}\right) = [\mathbf{I} - \mathbf{Z}]^{-1} + [\mathbf{I} - \mathbf{Z}]^{-1}\mathbf{Y_s}[\mathbf{I} - \mathbf{Z}]^{-1}$$

with the matrix $\mathbf{Y_s}$ given by $\mathbf{Y_s} \equiv z_{i_s j_s} \sum_{\mathbf{t_e} \in \mathbf{C_e(s)}} \sum_{\mathbf{t_i} \in \mathbf{C_i(s)}} \mathbf{I_{i_{t_e} j_{t_i}}}$, where $C_e(s)$ $(C_i(s))$ denotes the set of exporter (importer) countries involved in any link up until step s and the scalar $z_{i_s j_s}$ is the output of the function $z_{i_s j_s} \equiv h\big((x_t)_{t=1,\ldots,s}, [\mathbf{I} - \mathbf{Z}]^{-1}, (\mathbf{I_{i_t j_t}})_{\mathbf{t=1,\ldots,s}}\big)$.

A.4 Properties of the diffusion model

We first describe some basic properties of diffusion model (7) and, subsequently, turn to the proof of Lemma 1. The vector of local demand effects, $\boldsymbol{\delta}^{\mathbf{ld}}$, satisfies:

Lemma 7. *It is* $\mathbf{1^T}\,\boldsymbol{\delta}^{\mathbf{ld}} = 0$ *for any* $\mathbf{dT}$.

Proof. Note first that the sum of the market access effects cancels against the sum of the import competition effects, since

$$\mathbf{1^T}\left[\boldsymbol{\Pi} \circ \mathbf{dT}\right](\mathbf{e^f} + \mathbf{e^i}) \quad = \quad (\mathbf{e^f} + \mathbf{e^i})^{\mathbf{T}}\left[\boldsymbol{\Pi}^{\mathbf{T}} \circ (\mathbf{dT})^{\mathbf{T}}\right]\mathbf{1}$$

Concerning the productivity effects, we get

$$\begin{aligned}
& (1-\gamma)\,\mathbf{1^T}\Big((\mathbf{X^f} + \mathbf{X^i})\,\mathbf{d}\ln(\tilde{\mathbf{p}}) - \boldsymbol{\Pi}(\mathbf{E^f} + \mathbf{E^i})\,\boldsymbol{\Pi}^{\mathbf{T}}\,\mathbf{d}\ln(\tilde{\mathbf{p}})\Big) \\
= \; & (1-\gamma)\,\mathbf{1^T}\Big((\mathbf{X^f} + \mathbf{X^i}) \; - \; (\mathbf{E^f} + \mathbf{E^i})\,\boldsymbol{\Pi}^{\mathbf{T}}\Big)\mathbf{d}\ln(\tilde{\mathbf{p}}) \\
= \; & (1-\gamma)\,\mathbf{1^T}\Big((\mathbf{X^f} + \mathbf{X^i}) \; - \; (\mathbf{X^f} + \mathbf{X^i})\Big)\mathbf{d}\ln(\tilde{\mathbf{p}})
\end{aligned}$$

because in equilibrium it holds $(\mathbf{X^f} + \mathbf{X^i})\,\mathbf{1} = \boldsymbol{\Pi}(\mathbf{E^f} + \mathbf{E^i})\,\mathbf{1}$. For the same reason, the offshoring effects cancel against each other as well. □

The diffusion channels of model (7) satisfy the following properties:

Lemma 8. *The supply chain exposure matrices and the terms of trade matrix are mean preserving transformations. That is, for*

$$\mathbf{Z} \in \left\{ \mathbf{\Phi^{de}} \; ; \; \mathbf{LW}\big(\mathbf{\Phi^{se}} - \mathbf{I}\big)[\mathbf{E^i}]^{-1} \; ; \; \mathbf{LW\Phi^{tot}}\,[\mathbf{LW}]^{-1} \right\} \text{ it holds } \quad \mathbf{1^T Z} = \mathbf{1^T}.$$

The foreign trade multiplier is mean amplifying. That is

$$\mathbf{1^T\,LW\,\Phi^{mult}_{i^*}}[\mathbf{E^f}]^{-1} \quad > \quad \mathbf{1^T_{i^*}}$$

where $\mathbf{1^T_{i^*}}$ is a row vector of ones with a zero in element i^*.

Proof. Concerning $\mathbf{\Phi^{de}}$, the identity follows immediately from the elementary identity in (11). Concerning $\mathbf{\Phi^{se}}$ and $\mathbf{\Phi^{tot}}$, all we need is to note that $\mathbf{1^T LW}\big(\mathbf{\Phi^{se}} - \mathbf{I}\big)[\mathbf{E^i}]^{-1}$ and $\mathbf{1^T LW\Phi^{tot}}\,[\mathbf{LW}]^{-1}$ are the transposes of $[\mathbf{LW}]^{-1}\mathbf{l^d} = \mathbf{1}$.

Concerning $\mathbf{\Phi^{mult}_{i^*}}$, note that by Lemma 5 it is $\mathbf{1^T}\,(\partial \mathbf{l^d}/\partial(\mathbf{Lw})) = \mathbf{1^T}$. Hence, set $\zeta = 1$, consider an arbitrary reference country i^*, and define $y \equiv \max\{\partial l^d_{i^*}/\partial(l_j w_j) \mid j \in \mathcal{N}\backslash\{i^*\}\}$, which by **A2** satisfies $0 < y < 1$. Then

$$\mathbf{1^T\,\Phi^{mult}_{i^*}} \quad \geq \quad \mathbf{1^T_{i^*}} + (1-y)\mathbf{1^T_{i^*}} + (1-y)^2\mathbf{1^T_{i^*}} - \dots = \frac{1}{y}\mathbf{1^T_{i^*}}$$

The claim immediately follows from $0 < y < 1$. □

Proof of Lemma 1: The lemma is an application of Hulten (1978)'s theorem. In the terminology of our model (7), just note that the total income effect can be written as

$$\begin{aligned} \mathbf{1^T\,LW\,d}\ln(\mathbf{u}) &= \mathbf{1^T\,LW\,d}\ln(\mathbf{w}) - \mathbf{1^T}\Big(\mathbf{LW\Phi^{se}}\,\boldsymbol{\delta}^{\mathbf{p}} + \mathbf{LW\Phi^{tot}\,d}\ln(\mathbf{w})\Big) \\ &= -\,\mathbf{1^T}\big(\mathbf{E^f} + \mathbf{E^i}\big)\,\boldsymbol{\delta}^{\mathbf{p}} \end{aligned}$$

whereby the second line follows from Lemma 8 in combination with the equilibrium relationship $\mathbf{E^f} = \mathbf{LW}$. ■

A.5 Local shocks

Proof of Proposition 2 Part 1. Suppose that $\mathbf{\Lambda} = \mathbf{I}$ in diffusion model (7). The wage effect of a unilateral export cost reduction is then given by

$$\mathbf{d}\ln(\mathbf{w}) \quad = \quad (\gamma - 1)\,\mathbf{\Phi^{mult}_{i^*}}[\mathbf{LW}]^{-1}\big(\mathbf{x^f_{ij}} - \pi_{ij}\mathbf{x^f_j}\big)$$

where $\mathbf{x^f_{ij}} = (0, 0, ..., x^f_{ij}, 0, ..., 0)^T$ and $\mathbf{x^f_j} = (x^f_{1j}, x^f_{2j}, ..., x^f_{ij}, x^f_{i+1j}, ..., x^f_{nj})^T$.

Taking i as the reference country, i.e., $i^* = i$, and noting that $\mathbf{\Phi^{mult}_i}$ has all its entries positive (except for the zeros in row i and column i), we immediately arrive at $d\ln(w_k) < 0$ for all $k \neq i$.

Part 2. Keeping i as the reference country, it immediately follows that $d\ln(u_i) > 0$, because the wages and, thus, the prices in any $k \neq i$ decline. Moreover, the average effect on the wages in $k \neq i$ can be written as

$$\sum_{k \neq i} l_k w_k \, d\ln(w_k) \;=\; \sum_{j \neq i} \sum_{k \neq i} \frac{\partial(l_k^d)}{\partial(l_j w_j)} \, d(l_j w_j) \;+\; \sum_{k \neq i} \delta_k^{l^d}$$

Making use of Lemma 5 Part (2.), this line can be written as

$$\sum_{k \neq i} l_k w_k \, d\ln(w_k) \;=\; \sum_{j \neq i} \big(1 - \frac{\partial(l_i^d)}{\partial(l_j w_j)}\big) \, d(l_j w_j) \;+\; \sum_{k \neq i} \delta_k^{l^d}$$

Hence,

$$\sum_{k \neq i} l_k w_k \, d\ln(w_k) \;\leq\; \frac{\sum_{k \neq i} \delta_k^{l^d}}{\max_{j \neq i} \big\{ \frac{\partial(l_i^d)}{\partial(l_j w_j)} \big\}} \;<\; \sum_{k \neq i} \delta_k^{l^d} \;=\; -\sum_{k \neq i} (\gamma - 1) \pi_{kj} \pi_{ij} e_j^f$$

where the second inequality follows from Assumption **A2**. Thus,

$$\sum_{k \neq i} l_k w_k \, d\ln(u_k) \;<\; -\sum_{k \neq i} (\gamma - 1) \pi_{kj} \pi_{ij} e_j^f \;+\; \pi_{ij} e_j^f$$

whereby the right-hand side is smaller zero, if $\gamma > (2 - \pi_{ij})/(1 - \pi_{ij})$. ■

Proof of Proposition 3. Suppose an arbitrary $\mathbf{\Lambda}$ in diffusion model (7). The claim holds, when for at least one country k the wage effect of a unilateral export cost reduction $d\tau_{ij} = -x\tau_{ij}$ is given by

$$d\ln(w_k) - d\ln(w_i) \;=\; \sum_{l \in \mathcal{N}} \phi_{kl}^{mult} \, d_{ij} l_l^d \;>\; 0$$

whereby we treat exporter i as the reference country, i.e., $d\ln(w_i) = 0$, and where

$$d_{ij} l_l^d \;\equiv\; \sum_{m \in \mathcal{N}} \phi_{lm}^{de} \, \delta_m^{l^d}$$

denotes the *direct labor demand effect* of the export cost reduction.

In Part (i), we verify in three steps that the inequality holds for Part (i) of the proposition, when λ_i, λ_j, and π_{ij} satisfy the conditions in (54), (55), (61), and (62).

In Part (ii), we verify in three steps that the inequality holds for Part (ii) of the proposition, when λ_j, π_{il} for all $l \in \mathcal{N}$, and $\lambda_{l'}$ for all $l' \in \mathcal{N}_j \cup N_k \backslash \{j,k\} \equiv \{\mathcal{N} \backslash \{j,k\} \,|\, \pi_{jl'} > 0 \text{ or } \pi_{k'l} > 0\}$ satisfy the conditions in (58), (59), (60), (61), and (62).

Finally, in Part (iii), we show that in either one of the above cases it also holds $\frac{1}{n-1} \sum_{l \neq i} l_l w_l \, d\ln(u_l) > 0$ and $d\ln(u_i) < 0$.

(Part ia) The direct demand effect is positive in k. By Lemmas 7 and 8 (Appendix A.4), there must be at least one l with $d_{ij} l_l^d > 0$ (except for in the knife edge case, where all $d_{ij} l_l^d = 0$ and where the claim, thus, holds with weak equality, i.e., $dw_k - dw_i \geq 0$).

The question is which this country is? Suppose that λ_j is sufficiently large (i.e., country j is a conventional open economy). The *local* demand effects (8) are, then, approximately given by

$$\begin{aligned} \delta_i^{l^d} - \epsilon_i^d &= (\gamma-1)(1-\pi_{ij})\pi_{ij}\big(e_j^f + e_j^i\big) \\ \delta_l^{l^d} - \epsilon_l^d &= -(\gamma-1)\pi_{ij}\pi_{lj}\big(e_j^f + e_j^i\big) \end{aligned}$$

for $l \in \mathcal{N}\backslash\{i\}$, where ϵ_i^d and ϵ_l^d are all arbitrarily small, because $\delta_i^{l^d}$ and $\delta_l^{l^d}$ monotonically converge to the right-hand side expressions as $\lambda_j \to 1$ (there are no *productivity effects*).

Since $\gamma > 1$, $0 < \pi_{ij} < 1$, and $0 \leq \pi_{lj} < 1$, it immediately follows that

$$\delta_i^{l^d} > 0 \qquad \text{and} \quad \delta_l^{l^d} \leq 0 \tag{54}$$

Suppose, now, that λ_i is sufficiently small in addition. Then, ϕ_{il}^{de} becomes arbitrarily small for all $l \in \mathcal{N}$, and so does the *direct labor demand effect* in country i. Hence, by Lemmas 7 and 8, there must be at least one $k \neq i$, with $\pi_{ki} > 0$, such that

$$d_{ij} l_k^d > 0 \tag{55}$$

(ib) The direct demand effects are arbitrarily small for $l \notin \{i, k\}$. Denote by $d_{ij}\ln(\underline{l}^d)$ the smallest direct labor demand effect (in logarithms), i.e.,

$$d_{ij}\ln(\underline{l}^d) \quad \equiv \quad \min\left\{ d_{ij}\ln(l_l^d) \,|\, l \notin \{i,k\} \right\}$$

which by Lemmas 7 and 8 satisfies $d_{ij}\ln(\underline{l}^d) < 0$. The claim follows immediately from the fact that, when $\pi_{lj} \to 0$ for $l \notin \{i,k\}$ and $\pi_{ij} \to \pi_{ij}^0 > 0$ (and $\pi_{kj} \to \pi_{kj}^0 > 0$), then it holds $\lim d_{ij}\ln(\underline{l}^d) = 0$.[40]

(ic) Country k's trade multiplier is bounded from below. Denote by

$$l_{kk}^d \quad \equiv \quad \frac{\partial \ln(l_k^d)}{\partial \ln(w_k)}$$

the own price elasticity of k's labor demand, which by Lemma 5 Part (1.) (Appendix A.2) satisfies $l_{kk}^d < 1$. Moreover, denote by $\underline{l}_{li}^d$ the smallest cross price elasticity of country l's labor demand with respect to the wage in country i, i.e,

$$\underline{l}_{li}^d \equiv \min\left\{ \frac{\partial \ln(l_l^d)}{\partial \ln(w_i)} \mid l \in \mathcal{N}\backslash\{i,k\} \right\}$$

[40] A positive π_{ij}^0 poses no problem, because of the positive local effect in country i ($d_{ij}\delta_i^{l^d} > 0$). A positive π_{kj}^0 poses no problem, because the *direct demand effect* is positive in k ($d_{ij} l_k^d > 0$), and thus if any $l \in \mathcal{N}\backslash\{i,k\}$ sells to k then this sale implies a positive demand effect for l.

which by Assumption **A2** satisfies $\underline{l}^d_{li} > 0$. Applying Lemma 5 Part (1.), again, the wage effect in country k is bounded from below by

$$
\begin{aligned}
d\ln(w_k) \;\; > \;\; & \frac{1}{\zeta} d_{ij}\ln(l^d_k) \; + \; \frac{1}{\zeta^2}\Big(\big((\zeta-1)+l^d_{kk}\big)\, d_{ij}\ln(l^d_k) \; + \; \big(1-l^d_{kk}\big)\, d_{ij}\ln(\underline{l}^d) \Big) \\
+ \;\; & \frac{1}{\zeta^3}\big((\zeta-1)+l^d_{kk}\big)\Big(\big((\zeta-1)+l^d_{kk}\big)\, d_{ij}\ln(l^d_k) \; + \; \big(1-l^d_{kk}\big)\, d_{ij}\ln(\underline{l}^d) \Big) \\
+ \;\; & \frac{1}{\zeta^3}\big(1-l^d_{kk}\big)\big((\zeta-1)+1-\underline{l}^d_{li}\big)\, d_{ij}\ln(\underline{l}^d) \\
+ \;\; & \frac{1}{\zeta^4}\big((\zeta-1)+l^d_{kk}\big)\Big(\big((\zeta-1)+l^d_{kk}\big)\Big[\big((\zeta-1)+l^d_{kk}\big)\, d_{ij}\ln(l^d_k) \\
& \quad + \big(1-l^d_{kk}\big)\, d_{ij}\ln(\underline{l}^d) \Big) \; + \; \big(1-l^d_{kk}\big)\big((\zeta-1)+1-\underline{l}^d_{li}\big)\, d_{ij}\ln(\underline{l}^d) \Big) \\
+ \;\; & \frac{1}{\zeta^4}\big(1-l^d_{kk}\big)\big((\zeta-1)+1-\underline{l}^d_{li}\big)^2\, d_{ij}\ln(\underline{l}^d) \; + \; \ldots
\end{aligned}
$$

since the right-hand side is the wage effect in country k, given that the direct effects in all other countries $l \in \mathcal{N}\backslash\{i,k\}$ are at their minimum: $d_{ij}\ln(l^d_l) = d_{ij}\ln(\underline{l}^d)$.

The claim follows immediately from the fact that for any $\zeta \geq \beta$, the inequality simplifies to

$$
d\ln(w_k) \;\; > \;\; \frac{1}{1-l^d_{kk}} d_{ij}\ln(l^d_k) \; + \; \frac{1}{\underline{l}^d_{li}}\, d_{ij}\ln(\underline{l}^d) \; > \; 0 \tag{56}
$$

where the final inequality follows from steps (i) and (ii) of the proof.

(Part iia) The direct demand effect is negative for exporter i. Suppose that for all $l \in \mathcal{N}_j\backslash\{k\} \equiv \{\mathcal{N}\backslash\{j,k\}\,|\,\pi_{jl} > 0\}$ and all $l \in \mathcal{N}_k \equiv \{\mathcal{N}\backslash\{k\}\,|\,\pi_{kl} > 0\}$ it holds that λ_l is sufficiently large. The *local* demand effects (8) are, then, approximately given by

$$
\begin{aligned}
\delta^{l^d}_i - \epsilon^d_i \;\; = \;\; & \Big((\gamma-1)(1-\pi_{ij}) \; + \; (\gamma-1)\big(d\ln(p_j)\pi_{jj} + d\ln(p_k)\pi_{kj}\big) \; - \; (\beta-1)d\ln(p_j) \Big)\pi_{ij}\big(e^f_j + e^i_j\big) \\
+ \;\; & \Big((\gamma-1)\big(d\ln(p_j)\pi_{jk} + d\ln(p_k)\pi_{kk}\big) \; - \; (\beta-1)d\ln(p_k) \Big)\pi_{ik}\big(e^f_k + e^i_k\big) \\
+ \;\; & (\gamma-1)\sum_{l\notin\{j,k\}} \Big(d\ln(p_j)\pi_{jl} + d\ln(p_k)\pi_{kl} \Big)\pi_{il}\big(e^f_l + e^i_l\big)
\end{aligned} \tag{57}
$$

$$
\begin{aligned}
\delta^{l^d}_j - \epsilon^d_j \;\; = \;\; & \Big(-\,(\gamma-1)\pi_{ij} \; + \; (\gamma-1)\big(d\ln(p_j)\pi_{jj} + d\ln(p_k)\pi_{kj}\big) \; - \; (\beta-1)d\ln(p_j) \Big)\pi_{jj}\big(e^f_j + e^i_j\big) \\
+ \;\; & \Big((\gamma-1)\big(d\ln(p_j)\pi_{jk} + d\ln(p_k)\pi_{kk}\big) \; - \; (\beta-1)d\ln(p_k) \Big)\pi_{jk}\big(e^f_k + e^i_k\big) \\
+ \;\; & (\beta-\gamma)\big(e^f_j + e^i_j\big)d\ln(p_j) \; + \; (\gamma-1)\sum_{l\notin\{j,k\}} \Big(d\ln(p_j)\pi_{jl} + d\ln(p_k)\pi_{kl} \Big)\pi_{jl}\big(e^f_l + e^i_l\big)
\end{aligned}
$$

$$\begin{aligned}
\delta_k^{l^d} - \epsilon_k^d &= \Big((\gamma-1)\big(d\ln(p_j)\pi_{jj} + d\ln(p_k)\pi_{kj}\big) \;-\; (\beta-1)d\ln(p_j)\Big)\pi_{kj}\big(e_j^f + e_j^i\big) \\
&+ \Big((\gamma-1)\big(d\ln(p_j)\pi_{jk} + d\ln(p_k)\pi_{kk}\big) \;-\; (\beta-1)d\ln(p_k)\Big)\pi_{kk}\big(e_k^f + e_k^i\big) \\
&+ (\beta-\gamma)\big(e_k^f + e_k^i\big)d\ln(p_k) \;+\; (\gamma-1)\sum_{l\notin\{j,k\}}\Big(d\ln(p_j)\pi_{jl} + d\ln(p_k)\pi_{kl}\Big)\pi_{kl}\big(e_l^f + e_l^i\big) \\
\delta_l^{l^d} - \epsilon_l^d &= \Big(-\;(\gamma-1)\pi_{ij} \;+\; (\gamma-1)d\ln(p_j)\pi_{jj} \;-\; (\beta-1)d\ln(p_j)\Big)\pi_{lj}\big(e_j^f + e_j^i\big) \\
&+ \Big((\gamma-1)\big(d\ln(p_j)\pi_{jk} + d\ln(p_k)\pi_{kk}\big) \;-\; (\beta-1)d\ln(p_k)\Big)\pi_{lk}\big(e_k^f + e_k^i\big) \\
&+ (\gamma-1)\sum_{m\notin\{j,k\}}\Big(d\ln(p_j)\pi_{jm} + d\ln(p_k)\pi_{km}\Big)\pi_{lm}\big(e_m^f + e_m^i\big))
\end{aligned}$$

for $l \in \mathcal{N}\backslash\{i,j,k\}$, where ϵ_i^d, ϵ_j^d, ϵ_k^d, and ϵ_l^d are all arbitrarily small and where

$$\begin{aligned}
d\ln(p_j) + \epsilon_j^p &= -\frac{\pi_{ij}(1-\lambda_j)}{1-(1-\lambda_j)\pi_{jj}} \\
d\ln(p_k) + \epsilon_k^p &= \big(d\ln(p_j) + \epsilon_j^p\big)\frac{\pi_{jk}(1-\lambda_k)}{1-(1-\lambda_k)\pi_{kk}}
\end{aligned}$$

Suppose, in addition, that π_{ij} is sufficiently large and λ_j sufficiently small. In particular, suppose that

$$\pi_{ij} \;>\; \frac{(\gamma-1)(1-(1-\lambda_j)\pi_{jj})}{(\gamma-\beta)+(\beta-1)\lambda_j+(\gamma-1)\frac{(1-\lambda_j)\pi_{jk}(1-\lambda_k)\pi_{kj}}{(1-(1-\lambda_k)\pi_{kk})}} \tag{58}$$

Then, $\delta_i^{l^d} < 0$ and $\delta_l^{l^d} \leq 0$. Moreover, when $\pi_{li} \to 0$, $\pi_{l'j} \to 0$, and $\pi_{l''k} \to 0$ for all $l \neq i$, $l' \notin \{i,j\}$, and $l'' \notin \{i,j,k\}$, then

$$\begin{aligned}
\phi_{ii}^{de} - \epsilon_i^\phi &= \frac{\lambda_i}{1-\pi_{ii}(1-\lambda_i)} \\
\phi_{ij}^{de} - \epsilon_j^\phi &= \frac{\pi_{ij}(1-\lambda_j)}{1-\pi_{jj}(1-\lambda_j)}\phi_{ii}^{de} \\
\phi_{ik}^{de} - \epsilon_k^\phi &= \frac{\pi_{jk}(1-\lambda_k)}{1-\pi_{kk}(1-\lambda_k)}\phi_{ij}^{de}
\end{aligned}$$

and country k's and l's access in country j become negligible in addition ($\pi_{kj} \to 0$ and $\pi_{lj} \to 0$ in (57)).

Thus, an upper bound for the direct labor demand effect of exporter i is given by

$$\begin{aligned}
d_{ij} l_i^d &= \sum_{l \in \mathcal{N}} \phi_{il}^{de}\, \delta_l^{l^d} \\
&\le \frac{\phi_{ij}^{de}\big(e_j^f + e_j^i\big)}{(1-\lambda_j)\big(1-\pi_{jj}(1-\lambda_j)\big)} \Big[(\gamma-1)\big(1-(1-\lambda_j)\pi_{jj}\big)^2 \;-\; (\gamma-1)\pi_{ij}\lambda_j \\
&\qquad\qquad - (\gamma-\beta)\lambda_j(1-\lambda_j)\pi_{ij}\Big] \\
&+ \phi_{ij}^{de}\pi_{jk}\big(e_k^f + e_k^i\big)\Big[(\gamma-1)\frac{d\ln(p_k)}{1-\pi_{kk}(1-\lambda_k)} \;-\; (\gamma-1)\big(\frac{\pi_{ik}}{1-\pi_{kk}(1-\lambda_k)} + d\ln(p_j)\pi_{jk}\big) \\
&\qquad - (\beta-1)\frac{\pi_{ik}(1-\lambda_k)}{1-\pi_{kk}(1-\lambda_k)} \;-\; (\gamma-\beta)\frac{\lambda_k\, d\ln(p_k)}{1-\pi_{kk}(1-\lambda_k)}\Big]
\end{aligned} \tag{59}$$

which is *smaller zero*, when π_{ij} and π_{ik} satisfy the additional requirements

$$\begin{aligned}
\pi_{ij} &> \frac{(\gamma-1)(1-(1-\lambda_j)\pi_{jj})^2}{(\gamma-1)\lambda_j + (\gamma-\beta)\lambda_j(1-\lambda_j)} \\
\pi_{ik} &> d\ln(p_k)\frac{(\gamma-1)(1-(1-\lambda_k)\pi_{kk}) \;-\; (\gamma-1)(1-\lambda_k)^2 \;-\; (\beta-1)\lambda_k(1-\lambda_k)}{(1-\lambda_k)\big((\gamma-\beta)+(\beta-1)\lambda_k\big)}
\end{aligned} \tag{60}$$

(iib) The direct demand effects are positive in either j or k and arbitrarily small for all $l \notin \{i,j,k\}$. Note that, since $\delta_i^{l^d} < 0$ and $\delta_l^{l^d} \le 0$, it must either be $\delta_j^{l^d} > 0$ or $\delta_k^{l^d} > 0$. This, however, also implies that, when $\pi_{l'j} \to 0$ and $\pi_{l''k} \to 0$ for all $l' \notin \{i,j\}$ and $l'' \notin \{i,j,k\}$ then it must either be $d_{ij} l_j^d > 0$ or $d_{ij} l_k^d > 0$. Moreover, when $\pi_{ll'''} \to 0$ for all $l''' \in \mathcal{N}_k \cup \{i\}$ then it also holds $\lim d_{ij} \ln(\underline{l}^d) = 0$.

(iic) Importer j's trade multiplier is bounded from below. The proof is analogous to Part (ic).

Part (iii) Keeping i as the reference country, the real income effect in i can be written as

$$d\ln(u_i) \;-\; \epsilon_i^u \;=\; -\sum_{l \ne i} \phi_{il}^{tot}\, d\ln(w_l)$$

regardless of the scenario (i) or (ii), where ϵ_i^u is arbitrarily small, because $\lim \phi_{ij}^{se} = 0$, when either $\lambda_j \to 1$ or $\lambda_i \to 1$ or $\lambda_k \to 1$ for $k \in \mathcal{N}_j$.

By Part (ic) of the proof, a negative $d\ln(w_l)$ is bounded from below by $d_{ij}\ln(\underline{l})^d/\underline{l}_{li}^d$, and a positive $d\ln(w_l)$ by the right-hand side of inequality (56). Hence,

$$\begin{aligned}
d\ln(u_i) &< -\phi_{ik}^{tot}\left(\frac{1}{1-l_{kk}^d} d_{ij}\ln(l_k^d) \;+\; \frac{1}{\underline{l}_{li}^d}\, d_{ij}\ln(\underline{l})^d\right) \;-\; \sum_{l \notin \{i,k\}} \phi_{il}^{tot}\frac{1}{\underline{l}_{li}^d}\, d_{ij}\ln(\underline{l}^d) \qquad (61) \\
&< -\frac{\phi_{ik}^{tot}}{1-l_{kk}^d} d_{ij}\ln(l_k^d) \;-\; \frac{1}{\underline{l}_{li}^d}\, d_{ij}\ln(\underline{l}^d) \\
&< 0 \qquad (62)
\end{aligned}$$

where the second inequality follows from the elementary identity in (11), implying that the *terms of trade matrix* satisfies $\mathbf{\Phi^{tot}1} = \mathbf{1}$. Thus, $d\ln(u_i) < 0$ by steps (i) and (ii) in Part 1 of the proof. Finally, because the average welfare effect must be positive (Lemma 1 Appendix A.4), we also get $\sum_{k \neq i} d\ln(u_k) > 0$. ■

Example 1. A linear supply chain. A unilateral export cost reduction between countries i and $i+1$, $1 \leq i < n$, has the following *direct demand effect*:

$$\mathbf{\Phi^{de}}\,\boldsymbol{\delta}^{\mathbf{ld}} = \begin{pmatrix} 1 & 1-\lambda_2 & \prod_{s=2}^{3}(1-\lambda_s) & \dots & \dots & \prod_{s=2}^{n}(1-\lambda_s) \\ 0 & \lambda_2 & \lambda_2(1-\lambda_3) & \dots & \dots & \lambda_2\prod_{s=3}^{n}(1-\lambda_s) \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & \lambda_{i+1} & \lambda_{i+1}(1-\lambda_{i+2}) & \dots & \lambda_{i+1}\prod_{s=i+2}^{n}(1-\lambda_s) \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & \dots & \dots & \lambda_n \end{pmatrix} * (1-\beta) \begin{pmatrix} 0 \\ \dots \\ -\pi_{i,i+1}(e^f_{i+1} + e^i_{i+1})d\ln(\tilde{p}_{i+1}) \\ x_{i+1}d\ln(\tilde{p}_{i+1}) - \pi_{i+1,i+2}(e^f_{i+2} + e^i_{i+2})d\ln(\tilde{p}_{i+2}) \\ \dots \\ x_n d\ln(\tilde{p}_n) \end{pmatrix}$$

where

$$\begin{aligned} &\pi_{j,j+1} = 1 \quad \text{for } 1 \leq j \leq n\ , \\ &x_j = e^i_{j+1} \quad \text{for } 1 \leq j < n\,, \quad x_n = \sum_{j\in\mathcal{N}} e^f_j\ , \\ &e^i_1 = 0\ , \quad e^i_j = \sum_{k<j} e^f_k \quad \text{for } 1 < j \leq n\ , \\ &e^f_j = \lambda_j x_j \quad \text{for } 1 < j \leq n\,, \quad e^f_1 = x_1 \quad \text{, and} \\ &d\ln(\tilde{p}_j) = \begin{cases} 0 & \text{for } j \leq i \\ -\ \prod_{s=i+1}^{j}(1-\lambda_s) & \text{otherwise} \end{cases} \end{aligned}$$

Simplifying, we get

$$\mathbf{\Phi^{de}}\,\boldsymbol{\delta}^{\mathbf{ld}} = (\beta-1)\begin{pmatrix} e^f_1(1+d\ln(\tilde{p}_n)) \\ e^f_2(1+d\ln(\tilde{p}_n)) \\ \dots \\ e^f_i(1+d\ln(\tilde{p}_n)) \\ e^f_{i+1}\,d\ln(\tilde{p}_n) \\ \dots \\ e^f_n\,d\ln(\tilde{p}_n) \end{pmatrix}$$

where we have made use of the fact that

$$x_j d\ln(\tilde{p}_j) - \pi_{j,j+2}(e^f_{j+1} + e^i_{j+1}) d\ln(\tilde{p}_{j+1}) \quad = \quad 0$$

for $i < j < n$.

To arrive at the net wage effect, multiply the direct effects with the following trade multiplier

$$\mathbf{\Phi^{mult}_{i^*}} \quad \equiv \quad \frac{1}{\beta}\left\{\sum_{\mathbf{h=0}}^{\infty}\left[\{\mathbf{Z}\}^{-\mathbf{i}^*}\right]^{\mathbf{h}}\right\}^{+\mathbf{i}^*}[\mathbf{E^f}]^{-\mathbf{1}}$$

where

$$\mathbf{Z} \quad \equiv \quad \frac{1}{x_n}\begin{pmatrix} e^f_1 & e^f_2 & ... & e^f_n \\ e^f_1 & e^f_2 & ... & e^f_n \\ ... & ... & ... & ... \\ e^f_1 & e^f_2 & ... & e^f_n \end{pmatrix}$$

and where $\mathbf{Z}\,\mathbf{1} = \mathbf{1}$.[41] Hence, we get

$$\mathbf{\Phi^{mult}_{i^*}} \quad \equiv \quad \frac{1}{\beta}\left\{\mathbf{I}^{-\mathbf{i}^*} \;+\; \frac{1}{e^f_{i^*}/x_n}\mathbf{Z}^{-\mathbf{i}^*}\right\}^{+\mathbf{i}^*}[\mathbf{E^f}]^{-\mathbf{1}}$$

[41]The trade multiplier follows from the general formula (39), setting $\zeta = \beta$ and considering that in a linear supply chain it holds

$$\begin{aligned}
\frac{\partial \ln(\mathbf{l^d})}{\partial \ln(\mathbf{w})} \quad &= \quad (1-\beta)\,\mathbf{I} \;+\; \left[\mathbf{LW}\right]^{-\mathbf{1}}\mathbf{\Phi^{de}}\Big((\beta-1)\,(\mathbf{X^f}+\mathbf{X^i})\frac{\partial \ln(\tilde{\mathbf{p}})}{\partial \ln(\mathbf{w})} \\
&\qquad + (1-\beta)\mathbf{\Pi}\,\mathbf{E^i}\,\mathbf{\Pi^T}\frac{\partial \ln(\tilde{\mathbf{p}})}{\partial \ln(\mathbf{w})} \;+\; \mathbf{\Pi}\,\mathbf{E^f}\Big) \\
&= \quad (1-\beta)\,\mathbf{I} \;+\; \left[\mathbf{LW}\right]^{-\mathbf{1}}\mathbf{\Phi^{de}}\Big((\beta-1)\,\mathbf{X^f}\frac{\partial \ln(\tilde{\mathbf{p}})}{\partial \ln(\mathbf{w})} \;+\; \mathbf{\Pi}\,\mathbf{E^f}\Big) \\
&= \quad (1-\beta)\,\mathbf{I} \;+\; \beta\,\mathbf{Z}
\end{aligned}$$

where, in line one, we make use of the fact that in a linear supply chain the sales shares are fixed at $\pi_{j,j+1} = 1$ for $1 \leq j \leq n$ (i.e., it is as if $\gamma = 1$). For line two, we use the identity

$$(\beta-1)\,\mathbf{X^i}\frac{\partial \ln(\tilde{\mathbf{p}})}{\partial \ln(\mathbf{w})} \quad = \quad (1-\beta)\mathbf{\Pi}\,\mathbf{E^i}\,\mathbf{\Pi^T}\frac{\partial \ln(\tilde{\mathbf{p}})}{\partial \ln(\mathbf{w})}$$

and, for line three,

$$\mathbf{X^f}\frac{\partial \ln(\tilde{\mathbf{p}})}{\partial \ln(\mathbf{w})} \quad = \quad \mathbf{\Pi E^f}$$

which is, again, (only) valid for the linear supply chain looked at here.

Taking exporter i as the reference country, the net wage effect in j is thus given by

$$\begin{aligned} d\ln(w_j) &= \begin{cases} \frac{\beta-1}{\beta\, e_i^f}\big(e_{i+1}^i \;+\; x_n\, d\ln(\tilde{p}_n)\big) & \text{for } j \leq i \\ \frac{\beta-1}{\beta\, e_i^f}\big(e_i^i \;+\; x_n\, d\ln(\tilde{p}_n)\big) & \text{otherwise} \end{cases} \\ &= \begin{cases} 0 & \text{for } j \leq i \\ (1-\beta)/\beta & \text{otherwise} \end{cases} \end{aligned}$$

where line two follows from the identities $x_n\, d\ln(\tilde{p}_n) = -e_{i+1}^i$ and $e_{i+1}^i = e_i^i + e_i^f$.

To arrive at the real income effect, make repeated use of the alternative expressions for e_j^f and e_j^i and the identities

$$\phi_{ij}^{tot} \;=\; \lambda_j \prod_{s=j+1}^{n} (1-\lambda_s) \;=\; \frac{e_j^f}{x_n} \qquad \text{and} \qquad \phi_{ij}^{se} \;=\; \prod_{s=j}^{n}(1-\lambda_s) \;=\; \frac{e_j^i}{x_n}$$

to get

$$d\ln(u_j) \;=\; \begin{cases} \frac{\beta-1}{\beta} + \frac{e_{i+1}^i}{\beta\, x_n} & \text{for } j \leq i \\ \frac{e_{i+1}^i}{\beta\, x_n} & \text{otherwise} \end{cases}$$

Proof of Proposition 4. Under a uniform-elasticity specification ($\beta = \gamma$), the Leontief inverse in (10) can be written in terms of exogenous parameters only. Specifically, we can write

$$\begin{aligned} \mathbf{P}^{\gamma-\mathbf{1}}\,[\mathbf{I}-\mathbf{\Pi}(\mathbf{I}-\mathbf{\Lambda})]^{-\mathbf{1}}\,(\mathbf{P^i})^{\mathbf{1}-\gamma} &= [\mathbf{I}-\mathbf{M}\mathbf{T}^{\mathbf{1}-\gamma}(\mathbf{K^i})^{\gamma}]^{-\mathbf{1}} \\ &\equiv [\mathbf{I}-\mathbf{Z}]^{-\mathbf{1}} \end{aligned}$$

where $\mathbf{M}$ denotes the diagonal matrix of total factor productivities, $\mathbf{M} = (\mu_i)_{i\in\mathcal{N}}$, $\mathbf{T}$ the trade cost matrix, and $\mathbf{K^i}$ the diagonal matrix of intermediate input productivities, $\mathbf{K^i} = (\kappa_i^i)_{i\in\mathcal{N}}$. Similar, the Ghosh inverse in (12) can be written as

$$\begin{aligned} (\mathbf{P^i})^{\mathbf{1}-\gamma}[\mathbf{I}-(\mathbf{I}-\mathbf{\Lambda})\mathbf{\Pi^T}]^{-\mathbf{1}}\mathbf{P}^{\gamma-\mathbf{1}} &= [\mathbf{I}-(\mathbf{K^i})^{\gamma}(\mathbf{T}^{\mathbf{1}-\gamma})^{\mathbf{T}}\mathbf{M^T}]^{-\mathbf{1}} \\ &\equiv [\mathbf{I}-\mathbf{Z^T}]^{-\mathbf{1}} \end{aligned}$$

By Lemma 6 Part (2.), the effect of an inframarginal increase of the matrix coefficients z_{ij} and z_{kj}, at rates $x > 0$ and $y > 0$ respectively, is given by

$$\begin{aligned} & \mathbf{d_{ij+kj}}\big([\mathbf{I}-\mathbf{Z}]^{-\mathbf{1}}\mathbf{Z}\big) \;-\; \mathbf{d_{ij}}\big([\mathbf{I}-\mathbf{Z}]^{-\mathbf{1}}\mathbf{Z}\big) \;-\; \mathbf{d_{kj}}\big([\mathbf{I}-\mathbf{Z}]^{-\mathbf{1}}\mathbf{Z}\big) \\ =\;& \mathbf{d_{ij+kj}}[\mathbf{I}-\mathbf{Z}]^{-\mathbf{1}} \;-\; \mathbf{d_{ij}}[\mathbf{I}-\mathbf{Z}]^{-\mathbf{1}} \;-\; \mathbf{d_{kj}}[\mathbf{I}-\mathbf{Z}]^{-\mathbf{1}} \\ =\;& \big[\mathbf{I}-\mathbf{Z}\big]^{-\mathbf{1}}\bigg(\frac{xz_{ij}}{\psi_{ij+kj}}\mathbf{I_{ij}} \;+\; \frac{yz_{kj}}{\psi_{ij+kj}}\mathbf{I_{kj}} \;-\; \frac{xz_{ij}}{\psi_{ij}}\mathbf{I_{ij}} \;-\; \frac{yz_{kj}}{\psi_{kj}}\mathbf{I_{kj}}\bigg)\big[\mathbf{I}-\mathbf{Z}\big]^{-\mathbf{1}} \qquad (63) \\ =\;& \frac{\psi_{ij}-\psi_{ij+kj}}{\psi_{ij}\psi_{ij+kj}}\mathbf{d_{ij}}\big([\mathbf{I}-\mathbf{Z}]^{-\mathbf{1}}\mathbf{Z}\big) \;+\; \frac{\psi_{kj}-\psi_{ij+kj}}{\psi_{kj}\psi_{ij+kj}}\mathbf{d_{kj}}\big([\mathbf{I}-\mathbf{Z}]^{-\mathbf{1}}\mathbf{Z}\big) \end{aligned}$$

Line two follows from the fact that a shock to the coefficient matrix does not affect the first summand in the Neumann series corresponding to the matrix inverses. Line two applies Lemma 6 Part (2.) for the special case of $l = j$. Line three is nothing but a simplification step, whereby Assumption **A2** implies that the scale factors satisfy $\psi_{ij} - \psi_{ij+kj} > 0$ and $\psi_{kj} - \psi_{ij+kj} > 0$. Thus, we arrive at the conclusion that the Leontief and Ghosh inverses are *supermodular*.

It remains to be seen that, for a 'positive spillover' link ij as defined in Proposition 3 Part (i), the wage externality of $\mathbf{d_{ij}}([\mathbf{I} - \mathbf{Z}]^{-1}\mathbf{Z})$ is positive, i.e., $d_{ij}\ln(w_k) > 0$. Furthermore, when λ_k is sufficiently large it follows from Proposition 2 that $\partial \ln(l_k^d)/\partial \ln(\tau_{kj}) > 0$. We can, thus, apply the arguments of Proposition 3 Part (a) to arrive at $d_{kj}\ln(w_k) > 0$. Combined with (63), we finally arrive at the conclusion that also the labor demand functions are *supermodular*, i.e., it holds $d_{ij+kj} ln(w_k) > d_{ij}\ln(w_k) + d_{kj}\ln(w_k)$. ■

A.6 A global shock

Proof of Proposition 5: Suppose that $\mathbf{\Lambda} = \mathbf{I}$ in diffusion model (7). A global trade cost reduction $\mathbf{dT} = -x\mathbf{T}$ materializes in the following vectors of local effects

$$\begin{aligned} \boldsymbol{\delta}^{\mathbf{p}} &= \left[\mathbf{\Pi}^{\mathbf{T}} \circ (\mathbf{d}\ln(\mathbf{T}))^{\mathbf{T}}\right]\mathbf{1} \; = \; -x\,\mathbf{1} \\ \boldsymbol{\delta}^{\mathbf{l^d}} &= (\gamma - 1)\Big(\left[\mathbf{\Pi} \circ \mathbf{d}\ln(\mathbf{T})\right]\mathbf{e^f} \; - \; \mathbf{\Pi}\,\mathbf{e^f}\left[\mathbf{\Pi}^{\mathbf{T}} \circ (\mathbf{d}\ln(\mathbf{T}))^{\mathbf{T}}\right]\mathbf{1}\Big) \; = \; \mathbf{0} \end{aligned} \tag{64}$$

Thus, $\mathbf{d}\ln(\mathbf{w}) = \mathbf{0}$ and $\mathbf{d}\ln(\mathbf{u}) = x\,\mathbf{1}$. ■

Example 4. When $\beta = 0$ the wage effects of a global trade cost reduction, $\mathbf{dT} = -x\mathbf{T}$, are given by

$$\mathbf{d}\ln(\mathbf{w}) \;\; = \;\; [\mathbf{LW}]^{-1}\mathbf{\Phi}^{\mathbf{de}}\,\mathbf{Z}\,\mathbf{d}\ln(\tilde{\mathbf{p}}) \; + \; \frac{\partial\ln(\mathbf{l^d})}{\partial\ln(\mathbf{w})}\,\mathbf{d}\ln(\mathbf{w})$$

where $\mathbf{Z}$ is defined as

$$\mathbf{Z} \;\; \equiv \;\; \gamma\Big(\mathbf{\Pi}(\mathbf{E^f} + \mathbf{E^i})\mathbf{\Pi}^{\mathbf{T}} \; - \; \mathbf{X^f} + \mathbf{X^i}\Big) \; + \; \Big(\mathbf{\Pi}\,(\mathbf{E^f} + \mathbf{E^i}) \; - \; \mathbf{\Pi}(\mathbf{E^f} + \mathbf{E^i})\mathbf{\Pi}^{\mathbf{T}}\Big)$$

By inspection of the Jacobian matrix in (25), combined with the identity

$$\begin{aligned} & \frac{\partial\ln(\tilde{\mathbf{p}})}{\partial\ln(\mathbf{w})} \; = \; \mathbf{\Lambda} \; + \; (\mathbf{I} - \mathbf{\Lambda})\mathbf{\Pi}^{\mathbf{T}}\frac{\partial\ln(\tilde{\mathbf{p}})}{\partial\ln(\mathbf{w})} \\ \Leftrightarrow \;\; & \mathbf{E^f} \; - \; \mathbf{E^f}\mathbf{\Pi}^{\mathbf{T}}\frac{\partial\ln(\tilde{\mathbf{p}})}{\partial\ln(\mathbf{w})} \; = \; (\mathbf{E^f} + \mathbf{E^i})\Big(\frac{\partial\ln(\tilde{\mathbf{p}})}{\partial\ln(\mathbf{w})} \; - \; \mathbf{\Pi}^{\mathbf{T}}\frac{\partial\ln(\tilde{\mathbf{p}})}{\partial\ln(\mathbf{w})}\Big) \end{aligned}$$

it also is

$$\frac{\partial\ln(\mathbf{l^d})}{\partial\ln(\mathbf{w})} \;\; = \;\; \mathbf{I} \; + \; [\mathbf{LW}]^{-1}\mathbf{\Phi}^{\mathbf{de}}\,\mathbf{Z}\,\frac{\partial\ln(\tilde{\mathbf{p}})}{\partial\ln(\mathbf{w})}$$

where moreover

$$\mathbf{d}\ln(\tilde{\mathbf{p}}) \;=\; -x\,\frac{\partial \ln(\tilde{\mathbf{p}})}{\partial \ln(\mathbf{w})}(\mathbf{I}-\boldsymbol{\Lambda})[\boldsymbol{\Lambda}]^{-\mathbf{1}}\mathbf{1}$$

Thus, we get

$$[\mathbf{LW}]^{-\mathbf{1}}\boldsymbol{\Phi}^{\mathbf{de}}\,\mathbf{Z}\,\frac{\partial \ln(\tilde{\mathbf{p}})}{\partial \ln(\mathbf{w})}\,\mathbf{d}\ln(\mathbf{w}) \;=\; x\,[\mathbf{LW}]^{-\mathbf{1}}\boldsymbol{\Phi}^{\mathbf{de}}\,\mathbf{Z}\,\frac{\partial \ln(\tilde{\mathbf{p}})}{\partial \ln(\mathbf{w})}(\mathbf{I}-\boldsymbol{\Lambda})[\boldsymbol{\Lambda}]^{-\mathbf{1}}\mathbf{1}$$

Suppose, now, that $\mathbf{Z}$ is invertible. Then, because $[\mathbf{LW}]^{-\mathbf{1}}$, $\boldsymbol{\Phi}^{\mathbf{de}}$, and $\partial \ln(\tilde{\mathbf{p}})/\partial \ln(\mathbf{w})$ are all invertible as well (see their definitions), we get

$$\mathbf{d}\ln(\mathbf{w}) \;=\; x\,(\mathbf{I}-\boldsymbol{\Lambda})[\boldsymbol{\Lambda}]^{-\mathbf{1}}\mathbf{1}$$

or, after making use of the property $\boldsymbol{\Phi}^{\mathbf{tot}}(\mathbf{I}-\boldsymbol{\Lambda})[\boldsymbol{\Lambda}]^{-\mathbf{1}} = \boldsymbol{\Phi}^{\mathbf{se}} - \mathbf{I}$, in real income terms,

$$\begin{aligned}
\mathbf{d}\ln(\mathbf{u}) &= x\,\boldsymbol{\Phi}^{\mathbf{se}}\mathbf{1} \;+\; x\,(\mathbf{I}-\boldsymbol{\Lambda})[\boldsymbol{\Lambda}]^{-\mathbf{1}}\mathbf{1} \;-\; x\,\boldsymbol{\Phi}^{\mathbf{tot}}(\mathbf{I}-\boldsymbol{\Lambda})[\boldsymbol{\Lambda}]^{-\mathbf{1}}\mathbf{1}\\
&= x\,\boldsymbol{\Phi}^{\mathbf{se}}\mathbf{1} \;+\; x\,(\mathbf{I}-\boldsymbol{\Lambda})[\boldsymbol{\Lambda}]^{-\mathbf{1}}\mathbf{1} \;-\; x\,(\boldsymbol{\Phi}^{\mathbf{se}}-\mathbf{I})\mathbf{1}\\
&= x\,[\boldsymbol{\Lambda}]^{-\mathbf{1}}\mathbf{1}
\end{aligned}$$

A.7 Key trading partners

To derive Formula (20), let us first spell out the impact of isolating country i on the Leontief inverse (10), which for the special case of $\beta = \gamma$ is given by

$$[\mathbf{I} - \mathbf{M}\,\mathbf{T}^{\mathbf{1}-\gamma}(\mathbf{K}^{\mathbf{i}})^{\gamma}]^{-\mathbf{1}}\mathbf{M}\,\mathbf{T}^{\mathbf{1}-\gamma} \equiv [\mathbf{I}-\mathbf{Y}]^{-\mathbf{1}}\mathbf{Z} \tag{65}$$

where $\mathbf{M}$ denotes the diagonal matrix of total factor productivities, $(\mu_i) \in \mathbb{R}^{n\times n}$, $\mathbf{T}$ the trade cost matrix, and $\mathbf{K}^{\mathbf{i}}$ the diagonal matrix of intermediate goods productivities. Making use of Lemma 6 Part (3.) (in particular, the expression given in the proof), the fact that $\mathbf{I_i}$ is an idempotent matrix, i.e., $\mathbf{I_i} = \mathbf{I_i}\mathbf{I_i}$, and the identity $\mathbf{I_i}[\mathbf{I}-\mathbf{Y}]^{-\mathbf{1}}\mathbf{I_i} = \sum_{h=0}^{\infty} y_{ii}^{[h]}\,\mathbf{I_i}$, where $\sum_{h=0}^{\infty} y_{ii}^{[h]}$ denotes entry ii of matrix $[\mathbf{I}-\mathbf{Y}]^{-\mathbf{1}}$, the impact is given by

$$\begin{aligned}
\mathbf{d_{-i}}[\mathbf{I}-\mathbf{Y}]^{-\mathbf{1}}\mathbf{Z} &= -\,(1-\gamma)\Big([\mathbf{I}-\mathbf{I_{-i}}\mathbf{Y}\mathbf{I_{-i}}]^{-\mathbf{1}}\mathbf{I_{-i}}\mathbf{Z}\mathbf{I_{-i}} - [\mathbf{I}-\mathbf{Y}]^{-\mathbf{1}}\mathbf{Z}\Big) \qquad (66)\\
&= (1-\gamma)[\mathbf{I}-\mathbf{Y}]^{-\mathbf{1}}\frac{1}{\sum_{h=0}^{\infty} y_{ii}^{[h]}}\mathbf{I_i}[\mathbf{I}-\mathbf{Y}]^{-\mathbf{1}}\mathbf{Z}\mathbf{I_{-i}} \;+\; (1-\gamma)[\mathbf{I}-\mathbf{Y}]^{-\mathbf{1}}\mathbf{Z}\mathbf{I_i}
\end{aligned}$$

Going from here to the price and labor demand effects, note that when $\beta = \gamma$ the column vector of producer prices can be explicitly solved for by

$$\begin{aligned}
\mathbf{p}^{\mathbf{1}-\gamma} &= (\mathbf{K}^{\mathbf{l}})^{\gamma}\mathbf{w}^{\mathbf{1}-\gamma} \;+\; \mathbf{Y}^{\mathbf{T}}\,\mathbf{p}^{\mathbf{1}-\gamma}\\
\Leftrightarrow\quad \mathbf{p}^{\mathbf{1}-\gamma} &= [\mathbf{I}-\mathbf{Y}^{\mathbf{T}}]^{-\mathbf{1}}(\mathbf{K}^{\mathbf{l}})^{\gamma}\mathbf{w}^{\mathbf{1}-\gamma} \qquad (67)
\end{aligned}$$

Hence, applying (66) and (65) onto the vector of consumer prices, $(\mathbf{p}^{\mathbf{f}})^{\mathbf{1}-\gamma} = \mathbf{Z}^{\mathbf{T}}\,\mathbf{p}^{\mathbf{1}-\gamma}$, and making use of the fact that

$$\mathbf{d_{-i}}\big(\mathbf{Z^T}[\mathbf{I}-\mathbf{Y^T}]^{-\mathbf{1}}\mathbf{K}^{\gamma}\mathbf{w}^{\mathbf{1}-\gamma}\big) \;=\; \big(\mathbf{d_{-i}}[\mathbf{I}-\mathbf{Y}]^{-\mathbf{1}}\mathbf{Z}\big)^{\mathbf{T}}\,\mathbf{K}^{\gamma}\mathbf{w}^{\mathbf{1}-\gamma} \;+\; \mathbf{\Phi^{tot}}\mathbf{d}\ln(\mathbf{w})$$

we arrive at the following price effect

$$\begin{aligned}
\mathbf{d}\ln(\mathbf{p^f}) \;&=\; \Bigg(\mathbf{I_{-i}\Pi^T}\big[\mathbf{I}-(\mathbf{I}-\mathbf{\Lambda})\mathbf{\Pi^T}\big]^{-\mathbf{1}}\frac{1}{\sum_{h=0}^{\infty}y_{ii}^{[h]}}\mathbf{I_i}\big[\mathbf{I}-(\mathbf{I}-\mathbf{\Lambda})\mathbf{\Pi^T}\big]^{-\mathbf{1}} \\
&\quad + \mathbf{I_i\Pi^T}\big[\mathbf{I}-(\mathbf{I}-\mathbf{\Lambda})\mathbf{\Pi^T}\big]^{-\mathbf{1}}\Bigg)\boldsymbol{\lambda} \;+\; \mathbf{\Phi^{tot}}\mathbf{d}\ln(\mathbf{w}) \\
&=\; \mathbf{I_{-i}\Pi^T}\sum_{\mathbf{h=0}}^{\infty}\big[(\mathbf{I}-\mathbf{\Lambda})\mathbf{\Pi^T}\big]^{\mathbf{h}}\frac{1}{\sum_{h=0}^{\infty}y_{ii}^{[h]}}\Bigg(\underbrace{\mathbf{I_i}\sum_{\mathbf{h=1}}^{\infty}[(\mathbf{I}-\mathbf{\Lambda})\mathbf{\Pi^T}]^{\mathbf{h}}}_{\text{intermediated value added}} \;+\; \underbrace{\mathbf{I_i}}_{\substack{\text{ctr. } i\text{'s} \\ \text{value added}}}\Bigg)\boldsymbol{\lambda} \\
&\quad + \mathbf{1_i} \;+\; \mathbf{\Phi^{tot}}\mathbf{d}\ln(\mathbf{w})
\end{aligned}$$

where, in line three, we decompose the term in line one into the channels (i) and (ii) of Formula (20) and, in line four, we make use of the homotheticity of consumer prices, implying that $\mathbf{\Pi^T}\big[\mathbf{I}-(\mathbf{I}-\mathbf{\Lambda})\mathbf{\Pi^T}\big]^{-\mathbf{1}}\boldsymbol{\lambda}=\mathbf{1}$. The resulting expression $\mathbf{I_i1}\equiv\mathbf{1_i}$ is eventually omitted from Formula (20), because (a) we ignore the welfare effects in the isolated country i itself and (b) we ignore the relaxed import competition in country i, since no other country $j\neq i$ is going to sell in i anyhow.

Finally, applying (65) and (66) onto the labor demand equation (10), we find a wage effect of

$$\begin{aligned}
\mathbf{d}\ln(\mathbf{w}) \;&=\; (1-\gamma)\;\mathbf{\Phi^{mult}_{i^*}}\sum_{\mathbf{h=0}}^{\infty}\big[\mathbf{\Pi}(\mathbf{I}-\mathbf{\Lambda})\big]^{\mathbf{h}} \\
&\quad * \Bigg[\underbrace{\frac{1}{\sum_{h=0}^{\infty}y_{ii}^{[h]}}\mathbf{I_i}\sum_{\mathbf{h=0}}^{\infty}\big[\mathbf{\Pi}(\mathbf{I}-\mathbf{\Lambda})\big]^{\mathbf{h}}\mathbf{\Pi\,I_{-i}Lw}}_{\text{intermediated demand}} \;+\; \underbrace{\mathbf{\Pi\,I_iLw}}_{\text{ctr } i\text{'s demand}} \\
&\quad - \underbrace{\mathbf{\Pi\,LW\,I_{-i}}\Big(\mathbf{d}\ln(\mathbf{p^f}) \;-\; \mathbf{\Phi^{tot}}\mathbf{d}\ln(\mathbf{w})\Big)}_{\substack{\text{productivity losses and} \\ \text{softer import competition}}}\Bigg]
\end{aligned}$$

whereby the different summands are nothing but channels (iii)-(vi) of Formula (20).

Proof of Proposition 6 Starting from diffusion model (7), the wage and price effects of a (partial) isolation of a country, i.e., $\mathbf{d_{-i}T}=x(\mathbf{I_iT}\;+\;\mathbf{T\,I_i})$, $0<x\leq 1$, can for $\beta=\gamma$ be written

as:

$$\begin{aligned}
\mathbf{d_{-i}}\ln(\mathbf{w}) &= (1-\gamma)\,\boldsymbol{\Phi}^{\mathbf{mult}}_{\mathbf{i^*}}\,\boldsymbol{\Phi}^{\mathbf{de}}\Bigg[\underbrace{\big[\boldsymbol{\Pi}\circ\mathbf{d_{-i}}\ln(\mathbf{T})\big]\,(\mathbf{e^f}+\mathbf{e^i})}_{\substack{\text{Foregone local (iii) and}\\ \text{intermediated demand (iv)}}} \;-\; \underbrace{\boldsymbol{\Pi}(\mathbf{E^f}+\mathbf{E^i})\,\boldsymbol{\delta^p}}_{\substack{\text{Softer import}\\ \text{competition (v+vi)}}} \\
&\quad + \Bigg(\underbrace{\boldsymbol{\Pi}\,(\mathbf{E^f}+\mathbf{E^i})}_{\text{Productivity losses (i+ii)}} \;-\; \underbrace{\boldsymbol{\Pi}(\mathbf{E^f}+\mathbf{E^i})\,\boldsymbol{\Pi}^{\mathbf{T}}}_{\text{Rivals' productivity losses (v+vi)}}\Bigg)\,(\mathbf{I}-\boldsymbol{\Lambda})\,\boldsymbol{\Phi}^{\mathbf{se}}\,\boldsymbol{\delta^p}\Bigg] \\
\mathbf{d_{-i}}\ln(\tilde{\mathbf{p}}^{\mathbf{f}}) &= \underbrace{\boldsymbol{\Phi}^{\mathbf{se}}\boldsymbol{\delta^p}}_{\substack{\text{Foregone value}\\ \text{added (i+ii)}}} \;+\; \boldsymbol{\Phi}^{\mathbf{tot}}\,\mathbf{d}\ln(\mathbf{w})
\end{aligned}$$

where

$$\boldsymbol{\delta^p} \;=\; \underbrace{\big[\boldsymbol{\Pi}^{\mathbf{T}}\circ(\mathbf{d_{-i}}\ln(\mathbf{T}))^{\mathbf{T}}\big]\,\boldsymbol{\lambda}}_{\text{local value added (i)}} \;+\; \underbrace{\big[\boldsymbol{\Pi}^{\mathbf{T}}\circ(\mathbf{d_{-i}}\ln(\mathbf{T}))^{\mathbf{T}}\big]\,(\mathbf{1}-\boldsymbol{\lambda})}_{\text{intermediated value added (ii)}}$$

and where the different channels are enumerated according to the distinction in Formula (20).

Hence, isolating one country after the other from the rest and summing up the effects gives

$$\sum_{i\in\mathcal{N}}\big[\boldsymbol{\Pi}\circ\mathbf{d_{-i}}\ln(\mathbf{T})\big] \;=\; 2x\,\boldsymbol{\Pi}$$

that is, we emulate the welfare effects of a global trade cost increase. In contrast, if we only sum up the *local* channels, we get $\mathbf{d^{loc}}\ln(\mathbf{u})$ with

$$\begin{aligned}
\mathbf{d^{loc}}\ln(\mathbf{w}) &= (1-\gamma)\,\boldsymbol{\Phi}^{\mathbf{mult}}_{\mathbf{i^*}}\,\boldsymbol{\Phi}^{\mathbf{de}}\Bigg[\big[\boldsymbol{\Pi}\circ\mathbf{d}\ln(\mathbf{T})\big]\,\mathbf{e^f} \;+\; \boldsymbol{\Pi}(\mathbf{E^f}+\mathbf{E^i})(\mathbf{I}-\boldsymbol{\Lambda})\,\boldsymbol{\Phi}^{\mathbf{se}}\,\boldsymbol{\delta^{loc}} \\
&\qquad -\boldsymbol{\Pi}\,(\mathbf{E^f}+\mathbf{E^i})\Big(\boldsymbol{\delta^{loc}} \;+\; \boldsymbol{\Pi}^{\mathbf{T}}(\mathbf{I}-\boldsymbol{\Lambda})\,\boldsymbol{\Phi}^{\mathbf{se}}\,\boldsymbol{\delta^{loc}}\Big)\Bigg] \\
\mathbf{d^{loc}}\ln(\tilde{\mathbf{p}}^{\mathbf{f}}) &= \boldsymbol{\Phi}^{\mathbf{se}}\boldsymbol{\delta^{loc}} \;+\; \boldsymbol{\Phi}^{\mathbf{tot}}\,\mathbf{d^{loc}}\ln(\mathbf{w})
\end{aligned}$$

and where $\boldsymbol{\Pi}\circ\mathbf{d}\ln(\mathbf{T}) = 2x\,\boldsymbol{\Pi}$ and $\boldsymbol{\delta^{loc}} = 2x\,\boldsymbol{\Pi}^{\mathbf{T}}\boldsymbol{\lambda}$. Hence, to round up the proof, it remains to be seen that

$$\boldsymbol{\Phi}^{\mathbf{se}}\,\boldsymbol{\delta^{loc}} \;=\; \boldsymbol{\delta^{loc}} \;+\; \boldsymbol{\Pi}^{\mathbf{T}}(\mathbf{I}-\boldsymbol{\Lambda})\,\boldsymbol{\Phi}^{\mathbf{se}}\,\boldsymbol{\delta^{loc}} \;=\; 2x\,\mathbf{1}$$

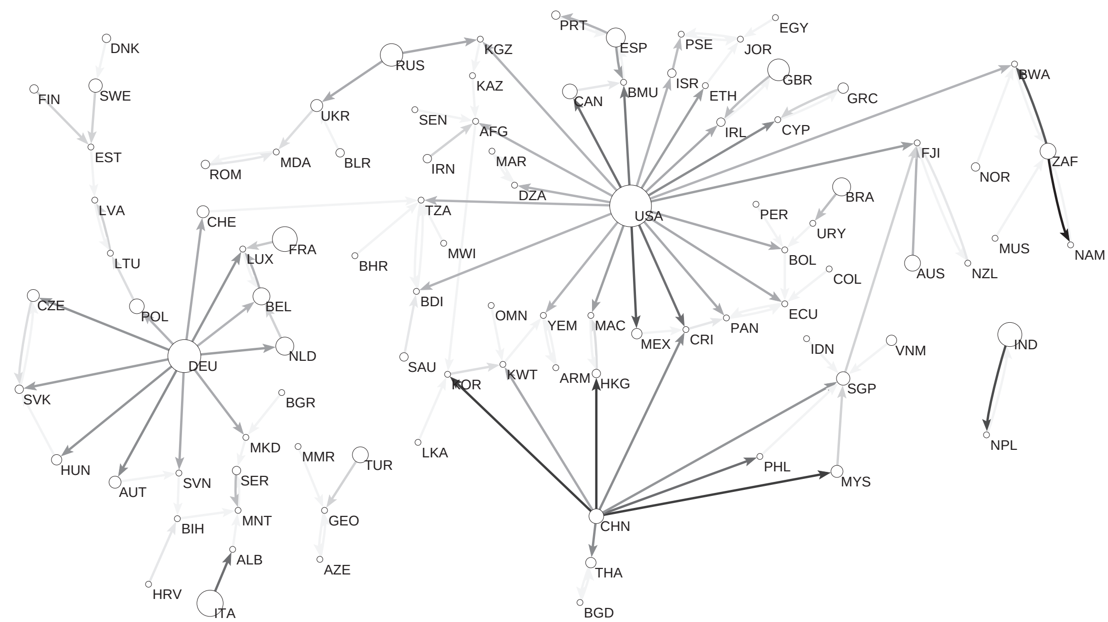
which is an immediate consequence of the elementary identity (11). We get

$$\begin{aligned}
\mathbf{d^{loc}}\ln(\mathbf{w}) &= 2x\,(1-\gamma)\,\boldsymbol{\Phi}^{\mathbf{mult}}_{\mathbf{i^*}}\,\boldsymbol{\Phi}^{\mathbf{de}}\Big[\boldsymbol{\Pi}(\mathbf{e^f}+\mathbf{e^i}) \;-\; \boldsymbol{\Pi}(\mathbf{e^f}+\mathbf{e^i})\Big] \;=\; \mathbf{0} \\
\mathbf{d^{loc}}\ln(\tilde{\mathbf{p}}^{\mathbf{f}}) &= 2x\,\mathbf{1}
\end{aligned}$$

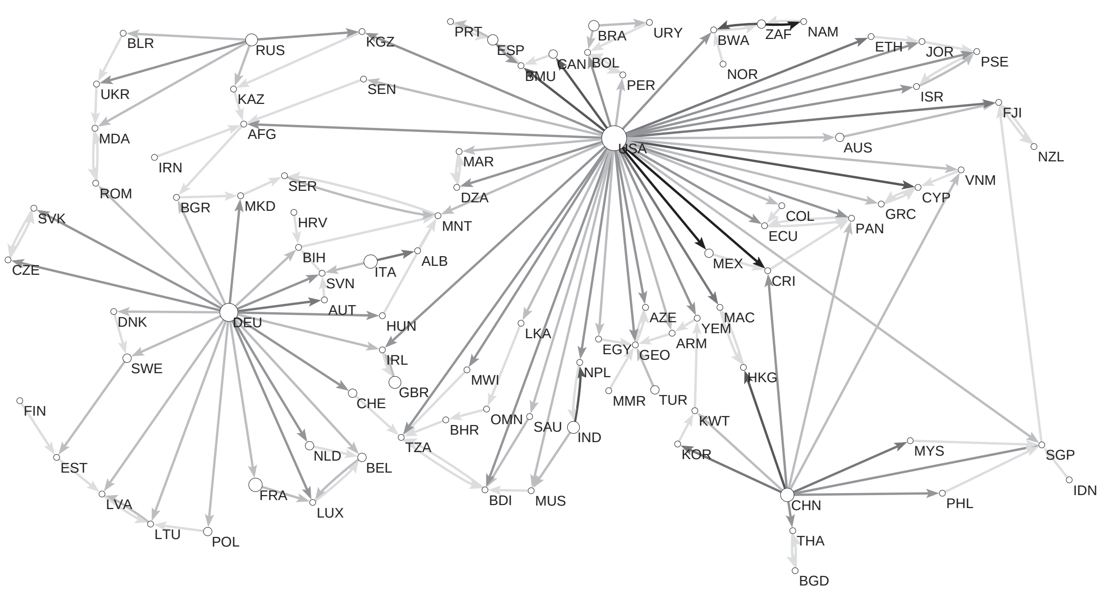
which was to be shown. ■

Figure 5: Key Players by Nation

(a) Key Players



(b) Key Players in terms of local value added and final demand



NOTES: The upper (lower) panel shows, for each country in our 2011 dataset, its most important trade partner (contributor of local value added and local market access). Underlying the figure is a 1% trade cost increase on all of a country's in- and outgoing trade links, $\mathbf{dT^{int}_{-i}} = .01(\mathbf{I_i T} + \mathbf{T I_i})$, and, in case of the lower panel, an isolation of the *local welfare channels* highlighted in Formula (20).

A.8 Generalization

Here, we extend on the simple 'diffusion model' (7) from the main text by deriving a more general version for a world economy with an arbitrary number of products. So, suppose there are s different sectors, indexed by $s \in \mathcal{S} = \{1, ..., s\}$, that each produce one product either used for consumption or production.

Preferences: As in the basic model, consumers have CES preferences and they maximize the following nested function

$$\begin{aligned} \max u_i &= \left(\sum_{s \in \mathcal{S}} \kappa_i^{sf} (q_i^{sf})^{\frac{\beta^f - 1}{\beta^f}} \right)^{\frac{\beta^f}{\beta^f - 1}} \\ \text{where for } t = f \qquad q_i^{st} &= \left(\sum_{j \in \mathcal{N}} (q_{ji}^{st})^{\frac{\gamma^s - 1}{\gamma^s}} \right)^{\frac{\gamma^s}{\gamma^s - 1}} \\ \text{subject to} \qquad & \sum_{s \in \mathcal{S}} \sum_{j \in \mathcal{N}} q_{ji}^{sf} p_{ji}^{sf} \leq w_i \end{aligned}$$

where q_i^{sf} denotes the CES aggregator for sector-s varieties from different nations, β^f the consumer elasticity of product substitution, $\beta^f \geq 0$, $\beta^f \neq 1$, and γ^s measures the elasticity of variety substitution (in an Armington interpretation of the model), respectively the shape of the productivity distribution (in an Eaton & Kortum interpretation), whereby in either case $\gamma^s > 1$.

Utility maximization leads to the matrix of consumer expenditures $\mathbf{E^f} \in \mathbb{R}^{ns \times n}$, with elements given by

$$e_j^{sf} = \lambda_j^{sf} w_j = \frac{(\kappa_j^{sf})^{\beta^f} (p_j^{sf})^{1-\beta^f}}{(p_j^f)^{1-\beta^f}}$$

for all rows $i, i' \in \mathcal{N}$.

Technologies: In order to ship q_{ij}^{st} units to a sector-u buyer in country k, $u \in \mathcal{S} \cup \{f\}$, $k \in \mathcal{N}$, the sector-s producers in country i purchase inputs so that

$$\begin{aligned} \min\ c_i^s &= l_i^s w_i + \sum_{t \in \mathcal{S}} \sum_{j \in \mathcal{N}} q_{ji}^{ts} p_{ji}^{ts} \\ \text{subject to} \qquad q_{ik}^{su} \leq f_{ik}^{su} &\equiv \frac{\mu_i^s}{\tau_{ik}^{su}} \left(\kappa_i^{ls} (l_i^s)^{\frac{\beta^s - 1}{\beta^s}} + \sum_{t \in \mathcal{S}} \kappa_i^{ts} (q_i^{ts})^{\frac{\beta^s - 1}{\beta^s}} \right)^{\frac{\beta^s}{\beta^s - 1}} \end{aligned}$$

where q_i^{ts} denotes the CES aggregator for sector-t varieties (presented above), β^s the sector-s elasticity of factor substitution, $\beta^s \geq 0$, $\beta^s \neq 1$, and $\tau_{ik}^{su} \in [1, \infty]$ a country- and sector-pair specific trade cost.

Cost minimization leads to the matrix of intermediate input expenditures $\mathbf{E} \in \mathbb{R}^{ns \times ns}$ and the column vector of labor costs $\mathbf{e^l} \in \mathbb{R}^{ns\times 1}$, with elements

$$e_j^{st} \;=\; \lambda_j^{st} x_j^t \;=\; \frac{(\kappa_j^{st})^{\beta^t}(p_j^{st})^{1-\beta^t}}{(p_j^t)^{1-\beta^t}} x_j^t\,, \quad e_j^{lt} \;=\; \lambda_j^{lt} x_j^t \;=\; \frac{(\kappa_j^{lt})^{\beta^t}(w_j)^{1-\beta^t}}{(p_j^t)^{1-\beta^t}} x_j^t$$

where x_j^t measures the total output of all sector-t firms in country j (in values).

Market structure: Just as in the simpler model, competition is perfect in each market and all markets clear. For the product markets, this means that a sector-t buyer in country j pays $p_{ij}^{st} = p_i^s \tau_{ij}^{st}$ for a sector-s output from country i. Moreover, a typical producer sells q_{ij}^{st} units so that $\sum_{t\in\mathcal{S}\cup\{f\}}\sum_{j\in\mathcal{N}} q_{ij}^{st} = f_i^s$. The matrix of expenditure shares on product varieties is, thus, given by $\mathbf{\Pi} \in \mathbb{R}^{ns\times ns}$ for producers and $\mathbf{\Pi^f} \in \mathbb{R}^{ns\times n}$ for consumers, with elements given by

$$\pi_{ij}^{st} \;=\; \frac{\mu_i^s (p_i^s)^{1-\gamma^s}(\tau_{ij}^{st})^{1-\gamma^s}}{(p_j^{st})^{1-\gamma^s}}$$

for $t \in \mathcal{S}\cup\{f\}$. In the labor markets, perfect competition and market clearing implies

$$l_i w_i \;=\; \sum_{s\in\mathcal{S},t\in\mathcal{S}\cup\{f\}}\sum_{j\in\mathcal{N}} \lambda_i^{ls}\, p_{ij}^{st} q_{ij}^{st} \equiv \sum_{s\in\mathcal{S}} \lambda_i^{ls}\, x_i^s$$

which closes the model. Totally differentiating the above identities leads to the following system of equations describing how an arbitrary trade cost shock diffuses to all nations:

Definition 3. *The welfare effects of an arbitrary, but small, trade cost shock* $\mathbf{d}\ln(\mathbf{T}) = (d\ln(\tau_{ij}^{st})) \in \mathbb{R}^{ns\times ns}$ *in any country* i *are given by*

$$\begin{aligned}\mathbf{d}\ln(\mathbf{w}) \;&=\; \mathbf{\Phi^{mult}_{i^*}} \sum_{\mathbf{s\in S}} \big(\mathbf{\Lambda^{ls}} + \sum_{\mathbf{t\in S}} (\mathbf{\Phi^{ts}})^{\mathbf{de}}\big)\boldsymbol{\delta}^{\mathbf{ls}} \\ \mathbf{d}\ln(\mathbf{\tilde{p}^f}) \;&=\; \mathbf{\Phi^{se}} \circ \boldsymbol{\delta}^{\mathbf{p}} \;+\; \sum_{\mathbf{t\in S}} (\mathbf{\Phi^{t}})^{\mathbf{tot}}\,\mathbf{d}\ln(\mathbf{w})\end{aligned}$$

where the local price and demand effects are given by[42]

$$\boldsymbol{\delta}^{\mathbf{p}} \equiv \underbrace{(\mathbf{d}\ln(\mathbf{T}))^{\mathbf{T}}\,\mathbf{1}}_{\textit{Supplier access}}$$

$$\begin{aligned}
\boldsymbol{\delta}^{\mathbf{ls}} \equiv\; & \underbrace{\sum_{t\in S\cup\{f\}} (1-\gamma^s)\left[\boldsymbol{\Pi}^{\mathbf{st}} \circ \mathbf{d}\ln(\mathbf{T})^{\mathbf{st}}\right]\mathbf{e}^{\mathbf{st}}}_{\textit{Market access}} - \underbrace{\sum_{t\in S\cup\{f\}} (1-\gamma^s)\boldsymbol{\Pi}^{\mathbf{st}}\mathbf{E}^{\mathbf{st}}\left[(\boldsymbol{\Pi}^{\mathbf{st}})^{\mathbf{T}} \circ (\mathbf{d}\ln(\mathbf{T})^{\mathbf{st}})^{\mathbf{T}}\right]\mathbf{1}}_{\textit{Import competition}} \\
+\; & \underbrace{\sum_{t\in S\cup\{f\}} (1-\gamma^s)\mathbf{X}^{\mathbf{st}}\,\mathbf{d}\ln(\tilde{\mathbf{p}}^{\mathbf{s}})}_{\textit{Exporter's productivity}} - \underbrace{\sum_{t\in S\cup\{f\}} (1-\gamma^s)\boldsymbol{\Pi}^{\mathbf{st}}\mathbf{E}^{\mathbf{st}}\,(\boldsymbol{\Pi}^{\mathbf{st}})^{\mathbf{T}}\,\mathbf{d}\ln(\tilde{\mathbf{p}}^{\mathbf{s}})}_{\textit{Competitors' productivity}} \\
-\; & \underbrace{\left(\sum_{t\in S\cup\{f\}} (1-\beta^s)\mathbf{X}^{\mathbf{st}}\,\mathbf{d}\ln(\tilde{\mathbf{p}}^{\mathbf{s}}) + (1-\beta^f)\boldsymbol{\Pi}^{\mathbf{sf}}\mathbf{E}^{\mathbf{sf}}\,\mathbf{d}\ln(\tilde{\mathbf{p}}^{\mathbf{f}})\right)}_{\textit{Exporter's offshoring}} + \underbrace{\sum_{t\in S\cup\{f\}} (1-\beta^t)\boldsymbol{\Pi}^{\mathbf{st}}\mathbf{E}^{\mathbf{st}}\,\mathbf{d}\ln(\tilde{\mathbf{p}}^{\mathbf{st}})}_{\textit{Importer's offshoring}}
\end{aligned}$$

with $\sum_{s\in S}\boldsymbol{\delta}^{\mathbf{ls}} = 0$ *and where* $\mathbf{Z}^{\mathbf{st}}$ *denotes the submatrix containing only the s-rows and t- columns of an arbitrary matrix* $\mathbf{Z}$*,* $\boldsymbol{\Lambda}^{\mathbf{ls}}$ *the diagonal matrix corresponding to vector* $\boldsymbol{\lambda}^{\boldsymbol{ls}}$*, and where the diffusion of the local effects is determined by the matrices*

$$\begin{aligned}
\textit{Upstream exposure:} \quad & \boldsymbol{\Phi}^{\mathbf{se}} \equiv (\boldsymbol{\Lambda}^{\mathbf{f}})^{\mathbf{T}}(\boldsymbol{\Pi}^{\mathbf{f}})^{\mathbf{T}} \sum_{\mathbf{h}=\mathbf{0}}^{\infty}\left[\boldsymbol{\Lambda}^{\mathbf{T}} \circ \boldsymbol{\Pi}^{\mathbf{T}}\right]^{\mathbf{h}} \\
\textit{Donwstream exposure:} \quad & (\boldsymbol{\Phi}^{\mathbf{st}})^{\mathbf{de}} \equiv \boldsymbol{\Lambda}^{\mathbf{ls}}\,\boldsymbol{\Pi}^{\mathbf{st}}\,\boldsymbol{\Lambda}^{\mathbf{st}} + \boldsymbol{\Lambda}^{\mathbf{ls}}\left[\boldsymbol{\Pi}^{\mathbf{s}} \circ \boldsymbol{\Lambda}^{\mathbf{s}}\right]\sum_{\mathbf{h}=\mathbf{0}}^{\infty}\left[\boldsymbol{\Pi} \circ \boldsymbol{\Lambda}\right]^{\mathbf{h}}\left[\boldsymbol{\Pi}^{\mathbf{t}} \circ \boldsymbol{\Lambda}^{\mathbf{t}}\right] \\
\textit{Terms of trade multiplier:} \quad & (\boldsymbol{\Phi}^{\mathbf{t}})^{\mathbf{tot}} = \boldsymbol{\Phi}^{\mathbf{se}}\,\boldsymbol{\Lambda}^{\mathbf{lt}}
\end{aligned}$$

and where the trade multiplier $\boldsymbol{\Phi}_{\mathbf{i}^*}^{\mathbf{mult}}$ *is the same as in (39), but with*

$$\begin{aligned}
\frac{\partial\ln(\mathbf{l}^{\mathbf{d}})}{\partial\ln(\mathbf{w})} =\; & \sum_{s\in S}(1-\beta^s)\,[\mathbf{LW}]^{-\mathbf{1}}\boldsymbol{\Lambda}^{\mathbf{ls}} \sum_{t\in S\cup\{f\}} \mathbf{X}^{\mathbf{st}} + \left[\mathbf{LW}\right]^{-\mathbf{1}}\sum_{\mathbf{s}\in\mathbf{S}}\left(\boldsymbol{\Lambda}^{\mathbf{ls}} + \sum_{\mathbf{t}\in\mathbf{S}}(\boldsymbol{\Phi}^{\mathbf{ts}})^{\mathbf{de}}\right) \\
+\; & \left(\sum_{t\in S\cup\{f\}}(\beta^s-\gamma^s)\mathbf{X}^{\mathbf{st}}\frac{\partial\ln(\tilde{\mathbf{p}}^{\mathbf{s}})}{\partial\ln(\mathbf{w})} - \sum_{t\in S\cup\{f\}}(\beta^t-\gamma^s)\boldsymbol{\Pi}^{\mathbf{st}}\mathbf{E}^{\mathbf{st}}\frac{\partial\ln(\tilde{\mathbf{p}}^{\mathbf{st}})}{\partial\ln(\mathbf{w})}\right. \\
+\; & \left.(\beta^f-1)\,\boldsymbol{\Pi}^{\mathbf{sf}}\,\mathbf{E}^{\mathbf{sf}}\frac{\partial\ln(\tilde{\mathbf{p}}^{\mathbf{f}})}{\partial\ln(\mathbf{w})} + \boldsymbol{\Pi}^{\mathbf{sf}}\,\mathbf{E}^{\mathbf{sf}}\right)
\end{aligned}$$

[42]The expressions for the *importer's offshoring* and the *exporter's offshoring* follow from the partial derivatives of the cost shares,

$$\begin{aligned}
\mathbf{d}\boldsymbol{\lambda}^{\boldsymbol{st}} &= (1-\beta^t)\boldsymbol{\Lambda}^{\mathbf{st}}\mathbf{d}\ln(\tilde{\mathbf{p}}^{\mathbf{st}}) - (1-\beta^t)\boldsymbol{\Lambda}^{\mathbf{st}}\,\mathbf{d}\ln(\tilde{\mathbf{p}}^{\mathbf{t}}) \\
\mathbf{d}\boldsymbol{\lambda}^{\boldsymbol{sf}} &= (1-\beta^f)\,\boldsymbol{\Lambda}^{\mathbf{sf}}\mathbf{d}\ln(\tilde{\mathbf{p}}^{\mathbf{sf}}) - (1-\beta^f)\boldsymbol{\Lambda}^{\mathbf{sf}}\,\mathbf{d}\ln(\tilde{\mathbf{p}}^{\mathbf{f}}) \\
\mathbf{d}\boldsymbol{\lambda}^{\boldsymbol{lt}} &= -(1-\beta^t)\boldsymbol{\Lambda}^{\mathbf{lt}}\,\mathbf{d}\ln(\tilde{\mathbf{p}}^{\mathbf{t}})
\end{aligned}$$

in combination with the expansion trick developed in Appendix A.2.

and the price effects given by

$$\begin{aligned}
\mathbf{d}\ln(\tilde{\mathbf{p}}^{\mathbf{s}}) &= \left[(\mathbf{\Lambda}^{\mathbf{s}})^{\mathbf{T}} \circ (\mathbf{\Pi}^{\mathbf{s}})^{\mathbf{T}}\right] \sum_{\mathbf{h}=\mathbf{0}}^{\infty} \left[\mathbf{\Lambda}^{\mathbf{T}} \circ \mathbf{\Pi}^{\mathbf{T}}\right]^{\mathbf{h}} \circ (\mathbf{d}\ln(\mathbf{T}))^{\mathbf{T}}\,\mathbf{1} \\
\mathbf{d}\ln(\tilde{\mathbf{p}}^{\mathbf{st}}) &= \left[(\mathbf{\Pi}^{\mathbf{st}})^{\mathbf{T}} \circ (\mathbf{d}\ln(\mathbf{T})^{\mathbf{st}})^{\mathbf{T}}\right]\mathbf{1} + (\mathbf{\Pi}^{\mathbf{st}})^{\mathbf{T}}\,\mathbf{d}\ln(\tilde{\mathbf{p}}^{\mathbf{s}}) \\
\frac{\partial \ln(\tilde{\mathbf{p}}^{\mathbf{s}})}{\partial \ln(\mathbf{w})} &= \mathbf{\Lambda}^{\mathbf{ls}} + \left[(\mathbf{\Lambda}^{\mathbf{s}})^{\mathbf{T}} \circ (\mathbf{\Pi}^{\mathbf{s}})^{\mathbf{T}}\right] \sum_{\mathbf{h}=\mathbf{0}}^{\infty} \left[\mathbf{\Lambda}^{\mathbf{T}} \circ \mathbf{\Pi}^{\mathbf{T}}\right]^{\mathbf{h}} \mathbf{\Lambda}^{\mathbf{l}} \\
\frac{\partial \ln(\tilde{\mathbf{p}}^{\mathbf{st}})}{\partial \ln(\mathbf{w})} &= (\mathbf{\Pi}^{\mathbf{st}})^{\mathbf{T}}\, \frac{\partial \ln(\tilde{\mathbf{p}}^{\mathbf{s}})}{\partial \ln(\mathbf{w})}
\end{aligned}$$

with $\sum_{\mathbf{s}\in\mathbf{S}} \mathbf{\Lambda}^{\mathbf{st}}\mathbf{d}\ln(\tilde{\mathbf{p}}^{\mathbf{st}}) = \mathbf{d}\ln(\tilde{\mathbf{p}}^{\mathbf{t}})$.

Hence, the general model inherits all the network properties of the simpler setting. First, a local trade cost shock $d\ln\tau_{ij}^{st} > 0$ will diffuse up- and downstream to the buyers and suppliers of the immediately affected sector-countries, with a labor demand externality that might even exceed the exporter's and importer's own demand effects when λ_i^{ls} and/or λ_i^{lt} is sufficiently small (Proposition 3). Second, the welfare effects of a global trade cost shock, $\mathbf{dT} = x\mathbf{T}$, are crucially dependent on a country's *upstream* exposure, which is typically in a country's advantage. This is unambiguously the case under, for example, the Eaton & Kortum specification with $\beta^t = 1$, $\lambda_j^{st} = \lambda_{j'}^{s't'}$, and $\lambda_j^{lt} = \lambda_{j'}^{lt'}$ for all $j, j' \in \mathcal{N}$ and $s, t, s', t' \in \mathcal{S} \cup \{f\}$, in which case $\mathbf{d}\ln(\mathbf{u}) = x\mathbf{\Phi}^{\mathbf{se}}\mathbf{1}$. Third, the intermediation capacities of a country's trading partners are a crucial determinant of its supply chain exposure, and even its sole determinant when $\gamma^s = \beta^t$ for all $s, t \in \mathcal{S} \cup \{f\}$ (Proposition 6).

References

Acemoglu, D., Akcigit, U., and Kerr, W. (2015). Networks and the macroeconomy: An empirical exploration. NBER working paper.

Acemoglu, D., Carvalho, V. M., Ozdaglar, A., and Tahbaz-Salehi, A. (2012). The Network Origins of Aggregate Fluctuations. *Econometrica*, 80(5):1977–2016.

Adao, R., Costinot, A., and Donaldson, D. (2017). Nonparametric counterfactual predictions in neoclassical models of international trade. *The American Economic Review*, 107(3):633–689.

Aghion, P., Antràs, P., and Helpman, E. (2007). Negotiating free trade. *Journal of International Economics*, 73:1–30.

Alvarez, F. and Lucas, R. E. (2007). General equilibrium analysis of the Eaton-Kortum model of international trade. *Journal of Monetary Economics*, 54:1726–1768.

Antràs, P. and Chor, D. (2013). Organizing the global value chain. *Econometrica*, 81(6):2127–2204.

Antràs, P. and Chor, D. (2017). On the measurement of upstreamness and downstreamness in global value chains. NBER Working Paper No. 24185.

Antràs, P., Chor, D., Fally, T., and Hillberry, R. (2012). Measuring the Upstreamness of Production and Trade Flows. *The American Economic Review*, 102(3):412–416.

Antràs, P. and de Gortari, A. (2017). On the Geography of Global Value Chains. NBER working paper 23456.

Antràs, P. and Staiger, R. W. (2012). Offshoring and the role of trade agreements. *American Economic Review*, 102(7):3140–83.

Arkolakis, C., Costinot, A., and Rodríguez-Clare, A. (2012). New Trade Models, Same Old Gains? *The American Economic Review*, 102(1):94–130.

Arkolakis, C. and Ramanarayanan, A. (2009). Vertical specialization and international business cycle synchronization. *Scandinavian Journal of Economics*, 111(4):655–680.

Armington, P. S. (1969). A theory of demand for products distinguished by place of production. *International Monetary Fund Staff Papers*, 16:159–178.

Autor, D. H., Dorn, D., and Hanson, G. H. (2013). The China Syndrome: Local Labor Market Effects of Import Competition in the United States. *The American Economic Review*, 103(6):2121–68.

Baldwin, R. (1995). A domino theory of regionalism. In Baldwin, R., Haapararanta, P., and Kiander, J., editors, *Expanding membership of the European Union*. Cambridge University Press.

Baldwin, R. and Lopez-Gonzalez, J. (2014). Supply-chain Trade: A Portrait of Global Patterns and Several Testable Hypotheses. *The World Economy*, 38:1682–1721.

Baldwin, R. and Venables, A. J. (2013). Spiders and Snakes: Off-shoring and Agglomeration in the Global Economy. *Journal of International Economics*, 90:245–254.

Ballester, C., Calvó-Armengol, A., and Zenou, Y. (2006). Who's Who in Networks. Wanted: The Key Player. *Econometrica*, 74(5):1403–1417.

Banerjee, A., Chandrasekhar, A. G., Duflo, E., and Jackson, M. O. (2013). The diffusion of microfinance. *Science*, 341(6144):1236498.

Baqaee, D. R. (2016). Cascading failures in production networks. unpublished manuscript.

Baqaee, D. R. and Farhi, E. (2017). The macroeconomic impact of microeconomic shocks: Beyond hulten's theorem. Technical report, National Bureau of Economic Research.

Bhagwati, J. (1993). Regionalism and multilateralism: an overview. In de Melo, J. and Panagariya, A., editors, *New Dimensions in Regional Integration*. Cambridge University Press, Cambridge, UK.

Blaum, J., Lelarge, C., and Peters, M. (2018). The gains from input trade with heterogeneous importers. *American Economic Journal: Macroeconomics*, 10(4):77–127.

Bonacich, P. (1987). Power and centrality: A family of measures. *American Journal of Sociology*, 92:1170–1182.

Caliendo, L. and Parro, F. (2015). Estimates of the Trade and Welfare Effects of NAFTA. *The Review of Economic Studies*, 82(1):1–44.

Caselli, F., Koren, M., Lisicky, M., and Tenreyro, S. (2017). Diversification through trade. Technical report, working paper.

Costinot, A. and Rodríguez-Clare, A. (2014). Trade theory with numbers: Quantifying the consequences of globalization. In *Handbook of international economics*, volume 4, pages 197–261. Elsevier.

Costinot, A., Vogel, J., and Wang, S. (2013). An Elementary Theory of Global Supply Chains. *The Review of Economic Studies*, 80:109–144.

Daudin, G., Rifflart, C., and Schweisguth, D. (2011). Who produces for whom in the world economy? *Canadian Journal of Economics*, 44(4):1403–1437.

Dekle, R., Eaton, J., and Kortum, S. (2008). Global Rebalancing with Gravity: Measuring the Burden of Adjustment. *IMF Staff Papers*, 55(3):511–540.

di Giovanni, J., Levchenko, A. A., and Zhang, J. (2014). The Global Welfare Impact of China: Trade Integration and Technological Change. *American Economic Journal: Macroeconomics*, 6(3):153–183.

Dixit, A. K. and Grossman, G. M. (1982). Trade and protection with multistage production. *The Review of Economic Studies*, 49(4):583–594.

Eaton, J. and Kortum, S. (2002). Technology, Geography, and Trade. *Econometrica*, 70(5):1741–1779.

Ethier, W. J. (1979). Internationally decreasing costs and world trade. *Journal of International Economics*, 9:1–24.

Fally, T. and Hillberry, R. (2015). A coasian model of international production chains.

Fort, T. C. (2017). Technology and production fragmentation: Domestic versus foreign sourcing. *The Review of Economic Studies*, 84(2):650–687.

Foti, N. J., Pauls, S., and Rockmore, D. N. (2013). Stability of the world trade web over time–an extinction analysis. *Journal of Economic Dynamics and Control*, 37(9):1889–1910.

Ghosh, A. (1958). Input-output approach in an allocation system. *Economica*, 25(97):58–64.

Goyal, S. and Vigier, A. (2014). Attack, defence, and contagion in networks. *The Review of Economic Studies*, 81(4):1518–1542.

Grassi, B. (2017). Io in io: Competition and volatility in input-output networks. Technical report.

Grossman, G. M. and Rossi-Hansberg, E. (2008). Trading Tasks: A Simple Theory of Offshoring. *The American Economic Review*, 98(5):1978–1997.

Hasan, R., Mitra, D., and Ramaswamy, K. V. (2007). Trade reforms, labor regulations, and labor-demand elasticities: Empirical evidence from india. *The Review of Economics and Statistics*, 89(3):466–481.

Henderson, H. V. and Searle, S. R. (1981). On deriving the inverse of a sum of matrices. *SIAM Review*, 23(1):53–60.

Hertel, T., Hummels, D., and Walmsley, T. (2014). The vulnerability of the asian supply chain to localized disasters. *Asia and Global Production Networks. Implications for Trade, Incomes and Economic Vulnerability, Cheltenham, UK: Edward Elgar and Asian Development Bank*, pages 81–111.

Hulten, C. R. (1978). Growth accounting with intermediate inputs. *The Review of Economic Studies*, 45(3):511–518.

Hummels, D., Ishii, J., and Yi, K.-M. (2001). The nature and growth of vertical specialization in world trade. *Journal of International Economics*, 54:75–96.

Jackson, M. O. (2017). A typology of social capital and associated network measures. mimeo.

Johnson, R. C. and Noguera, G. (2012). Proximity and production fragmentation. *The American Economic Review*, 102(3):407–411.

Karabarbounis, L. and Neiman, B. (2013). The global decline of the labor share. *The Quarterly Journal of Economics*, 129(1):61–103.

König, M., Liu, X., and Zenou, Y. (2014). R&d networks: Theory, empirics and policy implications.

König, M. D., Rohner, D., Thoenig, M., and Zilibotti, F. (2017). Networks in conflict: Theory and evidence from the great war of africa. *Econometrica*, 85(4):1093–1132.

Long, J. B. and Plosser, C. I. (1983). Real business cycles. *Journal of political Economy*, 91(1):39–69.

Lucas, R. E. (1977). Understanding business cycles. In *Carnegie-Rochester conference series on public policy*, volume 5, pages 7–29. Elsevier.

Mas-Collel, A., Whinston, M., and Green, J. (1995). *Microeconomic Theory.* Oxford University Press, New York.

Minabe, N. (1966). Growth and International Trade in a Simple Dynamic Leontieff Model. *The Canadian Journal of Economics and Political Science*, 32(1):55–65.

Ornelas, E. (2005). Trade creating free trade areas and the undermining of multilateralism. *European Economic Review*, 49(7):1717–1735.

Ornelas, E. and Turner, J. L. (2012). Protection and international sourcing. *The Economic Journal*, 122(559):26–63.

Ossa, R. (2015). Why Trade Matters After All. *Journal of International Economics*, 97(2):266–277.

Rader, T. (1968). Normally, factor inputs are never gross substitutes. *Journal of Political Economy*, 76(1):38–43.

Rasmussen, P. N. (1956). *Studies in Inter-Sectoral Relations*. Amsterdam: North-Holland.

Rodríguez-Clare, A. (2010). Offshoring in a ricardian world. *American Economic Journal: Macroeconomics*, 2(2):227–58.

Romalis, J. (2007). NAFTA's and CUSFTA's Impact on International Trade. *The Review of Economics and Statistics*, 89(3):416–435.

Samuelson, P. A. (1943). A fundamental multiplier identity. *Econometrica, Journal of the Econometric Society*, pages 221–226.

Sherman, J. and Morrison, W. (1950). Adjustment of an inverse matrix corresponding to a change in one element of a given matrix. *Annals of Mathematical Statistics*, 21:124–127.

Taylor, F. (1938). The guidance of production in a socialist state. In Lange, O., Taylor, F., and Lippincott, B., editors, *On the economic theory of socialism*, pages 39—-54. University of Minnesota Press.

Temurshoev, U. (2010). Identifying optimal sector groupings with the hypothetical extraction method. *Journal of Regional Science*, 50(4):872–890.

Timmer, M. P., Erumban, A. A., Los, B., Stehrer, R., and de Vries, G. J. (2014). Slicing up global value chains. *The Journal of Economic Perspectives*, 28(2):99–118.

Wilson, C. A. (1980). On the general structure of ricardian models with a continuum of goods: applications to growth, tariff theory, and technical change. *Econometrica: Journal of the Econometric Society*, pages 1675–1702.

Yi, K.-M. (2003). Can Vertical Specialization Explain the Growth of World Trade? *Journal of Political Economy*, 111(1):52–102.