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DP13277

**VALUATION IN THE PUBLIC AND  
PRIVATE SECTORS: TAX, RISK, DEBT  
CAPACITY, AND THE COST OF CAPITAL**

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Habib

**FINANCIAL ECONOMICS**



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Discussion Paper DP13277  
Published 23 October 2018  
Submitted 22 October 2018

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Tel: +44 (0)20 7183 8801  
[www.cepr.org](http://www.cepr.org)

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## Abstract

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JEL Classification: G18

Keywords: Public sector, private sector, cost of capital, Valuation, tax, risk, debt capacity

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## Acknowledgements

We thank Charles Cuny, Paul Groot, Jean-Charles Rochet, Kristian Rydqvist, and seminar participants at the Université Paris-Dauphine and the European Financial Management Association meetings for helpful comments.

# **Valuation in the Public and Private Sectors: Tax, Risk, Debt Capacity, and the Cost of Capital**

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This version: October 2018

## **Abstract**

The public and private sector costs of capital differ in the presence of taxes, because taxes are a cost to the private but not the public sector. We use a quasi-arbitrage approach to show how to include taxes in a comparison of capital costs. We find that taxes induce distortions that generate a systematic private sector preference for assets with rapid tax depreciation, high debt capacity, and low risk. We examine the implications of that preference for privatization, government outsourcing, and regulation. Our approach facilitates the analysis of transactions such as pure risk transfers, otherwise difficult using standard discounting methods.

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## 1. Introduction

The important economic role of the government and the large fraction of investment that it accounts for have given rise to the question of what the respective roles of the public and private sectors should be. The privatization of public assets, the private financing of public infrastructure, the contracting out of government services, the granting of government guarantees, and the forward purchase of goods and services by the government all involve comparisons of the value of assets or cash flows under public and private ownership. They therefore involve comparisons between the public and the private sector costs of capital.

These two costs of capital differ in the presence of taxes, even for the same asset or cash flow: taxes are a cost to the private sector but not to the public sector. We use a novel approach based on quasi-arbitrage to derive the discount rates and establish the valuation procedures that should be used by the public and private sectors in the presence of corporate and personal taxes. To concentrate on the effect of tax, we abstract from other important considerations. Specifically, we abstract from differences in risk-bearing ability by assuming that both sectors have access to the same perfectly elastic capital market. We also abstract from differences in agency by assuming that both sectors generate the same cash flow.

The public sector should discount pre-tax cash flows at pre-tax discount rates, whereas the private sector should discount after-tax cash flows at after-tax discount rates. The reason is that taxes are a cost to private sector investors, who individually receive, in the form of government benefits, only a very small fraction of the incremental tax payments that they make following a particular investment decision. The private sector therefore adjusts for tax both in the cash flows that it receives from a given project and in the discount rate that reflects the opportunity cost of investing in that project. In contrast, taxes incurred by public investment are but a transfer within the public sector, for example from the public sector corporation that has undertaken a given project to the general government budget.

This implies that the same project will appear to have different values depending on whether it is undertaken by the public or the private sector. This difference is illusory, as a difference in valuation that is due entirely to tax considerations amounts only to a wealth transfer between the public and the private sector; the total wealth of the country remains unchanged when only domestic investors and corporations are involved. The apparent difference in valuation has possibly misleading implications for whether a project should be undertaken by the

public or the private sector. We analyse these implications for the choice between public and private ownership, transactions between the public and the private sector, and regulation.

To determine the public and private costs of capital, we adopt a quasi-arbitrage argument similar to that used by Modigliani and Miller (1958, 1963). The assumption of a perfectly elastic capital market means that risk-sharing considerations of the type examined by Stapleton and Subrahmanyam (1978) do not arise. To compare the two sectors' costs of capital, we transform the private sector's after-tax discount rate into its pre-tax equivalent, which the private sector would use if it were to discount pre-tax rather than after-tax cash flows. We show how the private sector's pre-tax cost of capital differs from that of the public sector when debt and equity or real and financial investments are taxed differently. We further show that the difference in discount rates corresponds to the change in tax payments that follows the transfer of the asset from public to private ownership.

Using assumptions that correspond to standard features of tax systems, we find that taxes induce a systematic preference by the private sector for assets with fast tax depreciation, high debt capacity, and low systematic risk. We argue, however, that tax-induced differences in valuation between the public and the private sector should not constitute a motive for transferring an asset between the two sectors without a difference in efficiency. We show how to evaluate such decisions correctly and briefly extend our discussion to include the difference between domestic tax, which accrues to the government, and foreign tax, which does not. We also analyse transactions such as pure risk transfers, which can be accommodated in the quasi-arbitrage approach but are difficult to deal with in more standard approaches based on discounting.

The paper proceeds as follows. Section 2 reviews the literature. Section 3 presents the basic assumptions. Section 4 analyses the benchmark case with no taxes. Section 5 introduces corporate and personal taxes and derives the costs of capital with taxes. Section 6 examines the distortions introduced by tax and discusses the adjustments to valuation procedures that these distortions make necessary. Section 7 illustrates the effects of the distortions through a number of examples. Section 8 concludes.

## **2. Literature review**

The central role of taxes in our analysis naturally relates our work to the very extensive body of work, surveyed by Auerbach (2002) and Graham (2013), on the effect of taxes on

considerations as varied as capital structure, organizational form, payout and compensation policies, risk and earnings management, and financial innovation. Benninga and Sarig (2003) and Cooper and Nyborg (2008) examine the effect of taxes on the private sector costs of debt and equity. Our concern is with the effect of taxes in introducing a wedge between public and private sector costs of capital and valuations.

Our work is closest to that of Huizinga and Nielsen (2001), who compare under certainty the importance of tax and efficiency considerations to privatization decisions. They conclude that tax effects can be of the same order of magnitude as efficiency effects; indeed, ‘taxation of capital income forms an intervention which, other things equal, renders public production more attractive’ (Huizinga and Nielsen, 2001, p. 411). Our work differs from Huizinga and Nielsen’s in considering risk, in introducing a capital market in which risk is priced, in distinguishing between a risky claim, equity, and a risk-free claim, debt, and in abstracting from efficiency considerations. Whereas Huizinga and Nielsen seek to identify the tax system that achieves the best trade-off between tax and efficiency considerations, we aim to analyse the distortions induced by taxes even if there should be no difference in efficiency whatsoever.

We do not deny the importance of efficiency considerations but wish to limit the difference between the public and the private sector to a single consideration, the tax system; we therefore assume equality of private and public sector pre-tax cash flows. As an example of taxes’ importance, consider Baumol’s (1968, p. 790) observation that, in the presence of a 50% corporate tax rate, a required rate of return of  $r$  for the private sector implies a corresponding rate of  $2r$  for the public sector. We formalize and considerably extend Baumol’s (1968) observation.

The importance of efficiency, information, and contractibility in determining the choice between public or private ownership, financing, or service provision is reflected in the extensive literature that analyses such issues. To take but very few examples, Schmidt (1996) and Hart, Shleifer, and Vishny (1997) examine the desirability of privatization and Bentz, Grout, and Halonen (2005), Engel, Fischer, and Galetovic (2013), and Iossa and Martimort (2015) that of public–private partnerships used to finance public infrastructure privately and to contract out government services.<sup>1</sup> As just noted, we abstract from efficiency and similar considerations to focus on taxes.

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<sup>1</sup> Lucas and McDonald (2010) discuss the pricing of government guarantees, a topic that, perhaps surprisingly, appears to receive relatively little attention.

It is for this reason that we assume that the investment to be made by either the public or the private sector is not so large as to affect interest rates and risk premia. This distinguishes our work from that which determines the discount rate applicable to government investment in the presence of taxes when the size of government investment alters the interest rates. The studies by Harberger (1968), Sandmo and Drèze (1971), and Marchand and Pestiau (1984) are examples of such work in the absence of risk and Bailey and Jensen (1972) in its presence.

Another consideration that we abstract from is the public sector's greater or lesser ability to diversify risk and, consequently, its lower or higher risk premium, respectively. This is an important issue, which receives much attention but which, like investment size, does not arise in the infinitely elastic global capital market setting in which we conduct our analysis. Arrow (1965), Hirshleifer (1966), Diamond (1967), Arrow and Lind (1970), Bailey and Jensen (1972), and Stapleton and Subrahmanyam (1978) lay the basis for the formal analysis of the government's capacity for risk bearing; their results are used by Bazelon and Smetters (1999), Spackman (2004), and Lucas and Phaup (2010), among others, to compare the public and private sector costs of capital in the presence of risk.

The last consideration that we abstract from is that stemming from the presence of externalities, such as those imposed by the present generation on future generations with whom no trade is possible. Marglin (1963), Baumol (1968), Dasgupta, Mäler, and Barrett (1999), Caplin and Leahy (2004), and Dasgupta (2008) are among those who derive the social discount rate to be used in such circumstances. In short, in our setting, there should be no difference whatsoever between public and private sector discount rates and valuations in the absence of taxes.

### **3. Assumptions**

We consider an economy with two dates, 0 and 1. We focus on a single country. The country has a citizenry, a government (the public sector), and private firms (the private sector). Both government and firms can invest in real projects, but only firms can issue both debt and equity to fund these projects; the government can issue only debt.<sup>2</sup> We make the following assumptions:

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<sup>2</sup> We do not allow the government to issue direct equity claims on the project as this would amount to privatization.

Assumption 1. No difference in efficiency between the public and the private sectors;

Assumption 2. No externalities;

Assumption 3. Full information;

Assumption 4. Free access to the global capital market, which is perfect, complete, and infinitely elastic; projects are therefore ‘small’ in that they affect neither the equilibrium interest rate nor the price of risk; citizens, the government, and firms can all borrow and lend at the risk-free rate;

Assumption 5. The capital market is in the equilibrium described by the global capital asset pricing model (CAPM); there is neither inflation uncertainty nor real exchange rate uncertainty.<sup>3</sup>

Assumptions 1 and 2 together imply the equality of public and private sector pre-tax cash flows. Assumption 3 provides citizens and shareholders with the information necessary for portfolio rebalancing to hedge investment risk.<sup>4</sup> Assumptions 2 and 4 ensure that citizens and shareholders have access to identical hedging opportunities on terms left unchanged by investment.<sup>5</sup> Assumption 5 has these terms set by the global CAPM.

Assumptions 1 to 5 will be shown in Section 4 to imply the equality of public and private sector discount rates and valuations in the absence of taxes. We note that, even if the pre-tax cash flows were in fact to differ because Assumption 1 were not to hold, the valuation *procedures* that we derive would remain valid under Assumptions 2 to 5, which define the environment in which investment risk is hedged.

#### **4. The benchmark case: no taxes**

To highlight better the central role of taxes in our analysis, we initially consider a benchmark case with no taxes. We show that the public and private sector discount rates and valuations are identical under Assumptions 1 to 5.

Consider a project that can be undertaken either by the government or by a private firm. The project requires an investment of 1 at date 0 and has a random payoff of  $C$  at date 1. The

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<sup>3</sup> See Grauer, Litzenberger, and Stehle (1976) for an analysis of the global CAPM. The assumption that there is neither inflation uncertainty nor real exchange rate uncertainty is made for simplicity only: the presence of either type of uncertainty would leave our results unchanged under the assumption of infinitely elastic capital markets.

<sup>4</sup> See the formal analysis in Sections 4 and 5.

<sup>5</sup> Assumption 2 is necessary because externalities may introduce a difference between citizens’ social and shareholders’ market discount rates; recall the discussion in the literature review.

cash flows from the project are identical for both the government and the firm because of Assumptions 1 and 2. The specific division of project cash flows among the citizens of the country, or between the citizens of the country and foreign citizens, may nonetheless differ in the two cases in which the government or the firm undertakes the project: in the absence of portfolio rebalancing, the project's costs and benefits would be limited to the firm's shareholders in the latter case, and they would extend to all the country's citizens in the former case.

We define  $R_F$  as the global risk-free rate,  $R_M$  as the return on the global market portfolio,  $\beta$  as the beta of the project's cash flow  $C$  with respect to the global market portfolio, and  $P$  as the global market risk premium. Letting  $E[.]$  denote expectation, we have:

$$\beta \equiv \frac{\text{cov}[C, R_M]}{\text{var}[R_M]} \quad (1)$$

and:

$$P \equiv E[R_M] - R_F. \quad (2)$$

Our first result considers valuation by the private firm.

**Proposition 1:** If the project is undertaken by the private sector, it has required return  $R_F + \beta P$  and net present value (NPV):

$$NPV(\text{private}) = -1 + \frac{E[C]}{1 + R_F + \beta P}. \quad (3)$$

Proof: Hamada (1969).

We now turn to the valuation procedure to be used by the government. We start with an equilibrium situation in which the citizens of the country hold their optimal portfolio allocations. The government must decide whether to undertake the project. This and subsequent decisions are evaluated incrementally to the initial equilibrium.

Should the government decide to undertake the project, it would fund it entirely with debt borrowed at the rate  $R_F$ .<sup>6</sup> The government can borrow at the risk-free rate to fund even a risky project, because it is assumed to have an independent source of revenue that would cover any shortfall in the cash flows from the project; state-owned corporations are an example.<sup>7</sup> The government's decision to undertake the project implies that citizens will owe the risk-free cash flow  $1 + R_F$  at date 1 and receive the risky cash flow  $C$  at that date. We show in Appendix 1 that:

**Proposition 2:** If the project is undertaken by the public sector, it has required return  $R_F + \beta P$  and net present value:

$$NPV(\text{public}) = -1 + \frac{E[C]}{1 + R_F + \beta P}. \quad (4)$$

Propositions 1 and 2 establish the result that, in the absence of taxes and under Assumptions 1 to 5, the required return and NPV of the project are the same for the public and private sectors. This enables us to focus our analysis below solely on the effect of taxes.

The proof of Proposition 2 uses a quasi-arbitrage argument.<sup>8</sup> A consensus citizen hedges the market risk of the risky cash flow to re-establish his initial, optimal portfolio allocation. He does so by decreasing his holding of the global market portfolio and increasing his holding of the risk-free asset. Assumption 3 ensures that he has the cash flow and rate of return information necessary for rebalancing. Assumptions 4 and 5 set the terms on which rebalancing occurs; these are determined by the global CAPM. They are identical to those available to a shareholder of the private firm, who is confronted with the same hedging problem as the citizen: the shareholder will also receive  $C$  and owe  $1 + R_F$ . The citizen and the shareholder therefore use the same discount rate. As the cash flows are equal in the two sectors, following Assumptions 1 and 2, the NPVs are equal too. This establishes the result that the public and private sectors should use the

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<sup>6</sup> Recall from Footnote 2 that the government does not issue equity in the project.

<sup>7</sup> This assumption is made necessary in this part of the analysis by the assumed absence of taxes, which precludes the taxation of the citizens to honour the government's obligations. An alternative assumption would be that of lump sum taxes. These have no effect at the margin, in contrast to the proportional taxes considered later.

<sup>8</sup> See Modigliani and Miller (1958, 1963).

same discount rate to obtain the same NPV when evaluating the same project. This result bears some similarity to Modigliani and Miller's (1958) capital structure irrelevance theorem: debt or equity financing in their case and public or private sector ownership in our case have no effect on firm or project discount rates and valuations in the absence of taxes.

The project should be undertaken if it has positive NPV. While the total wealth increases as a result, it is not necessarily the case that all the citizens of the country benefit directly from the project. The benefits of the project may accrue primarily to some citizens, and the costs may be borne primarily by other citizens. Nonetheless, the value created by the project should make possible the compensation of the citizens who are adversely affected by the project. Thus, if the allocation of wealth was deemed to be socially optimal prior to the undertaking of the project, the government could redistribute wealth to re-establish the desired allocation (Kaldor, 1939; Hicks, 1940). There will be no risk-sharing consequences of the way in which the project is financed, as citizens can trade market risk back to their optimal allocations and other risks are diversified. This justifies the undertaking of the project as well as the reliance on a consensus citizen to determine the public sector cost of capital.

## 5. Public and private discount rates with taxes

We now introduce taxes, which are the main focus of our analysis. We define  $T_{PD}$  and  $T_{PE}$  as the investor tax rates on debt and equity, respectively.<sup>9</sup> We denote as  $T_C$  the statutory corporate tax rate and as  $T_C^R$  the effective corporate tax rate: the tax rules are such that  $T_C$  and  $T_C^R$  will generally differ. We focus on the role of tax depreciation in lowering the effective tax rate on real investment below the rate on financial investment; we therefore also refer to  $T_C$  as the tax rate on financial investment and to  $T_C^R$  as the tax rate on real investment.<sup>10</sup> In

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<sup>9</sup> Note that in general not all investors – whether shareholders or citizens – will have tax rates  $T_{PD}$  and  $T_{PE}$ . The conditions for a well-defined equilibrium with multiple tax brackets and debt and equity finance are given by Miller (1977) and Schaefer (1982). An important condition is that investors are subject to short-selling constraints, because tax arbitrage among investors in different tax brackets would otherwise be possible (Schaefer, 1982).

<sup>10</sup> To motivate this, contrast the manner in which corporate investments in financial assets such as bonds and real assets such as plant and equipment are depreciated. A firm that buys a bond and holds it to maturity is liable to pay corporate tax on every cash flow that it receives save the repayment of the principal at maturity. Thus, a firm depreciates an investment in a bond only in its final year. By contrast, a firm depreciates an investment in plant and equipment over the life of the equipment. This discrepancy in depreciation schedules reduces the effective tax rate on real investment below that on financial investment.

accordance with conventional notation,  $R_M$  is now the post-corporate-tax return on the global market portfolio and  $P \equiv E[R_M] - R_F$  is now the post-corporate-tax global market risk premium. We denote:

$$\beta' \equiv \frac{\text{cov}[(1-T_C^R)C, R_M]}{\text{var}[R_M]} = (1-T_C^R)\beta \quad (5)$$

as the post-corporate-tax beta corresponding to the pre-corporate-tax beta  $\beta$  in (1). We derive the discount rate for the public sector followed by that for the private sector.

### 5.1 The public sector discount rate

The derivation of the public sector discount rate with taxes is similar to that in Section 4 above without taxes. As in Section 4, the consensus citizen, now taxpayer, will receive the net cash flow  $C - (1 + R_F)$  from the government at date 1. This cash flow has a beta equal to  $\beta$ . The taxpayer will hedge the market risk of that cash flow, because his prior allocation of risk was optimal and he wishes to re-establish it. He will do so by reducing his holding of the market portfolio and lending the amount thus obtained. Although the former transaction entails foregoing after-tax return  $(1 - T_{PE})R_M$  and the latter has after-tax return  $(1 - T_{PD})R_F$  to the taxpayer, the government, in making its decision regarding whether to undertake the project, considers the pre-tax returns, as it recognizes that the ensuing change in government tax receipts will, in the final analysis, accrue to the taxpayers themselves.

A direct analogy to Proposition 2 implies that the public sector discount rate and valuation are the same as in the no-tax case:

**Proposition 3:** The public sector has required return on the project  $R_{PU} \equiv R_F + \beta P$ . It is used to discount the pre-tax expected cash flow from the project  $E[C]$ .

Note that both the taxpayer and the government use the post- rather than the pre-corporate-tax global market risk premium, computed using the post-corporate-tax expected return  $E[R_M]$  on the global market portfolio as the relevant benchmark. That is the pre-investor-tax expected

return on risky assets that is available in the capital market. Further note that the government pays no corporate tax: the cash inflow from the project is  $C$  not  $(1-T_C^R)C$ .

The required return given by Proposition 3 differs from the results of Harberger (1968), Sandmo and Drèze (1971), Bailey and Jensen (1972), and Marchand and Pestieau (1984) in that these authors' assumption of inelastic capital markets makes the public sector discount rate an average of households' and firms' discount rates. These are not the same, because a wedge is introduced between households' marginal rate of time preference and firms' marginal productivity of capital by the presence of taxes. Households' and firms' discount rates are weighted by the derivatives of consumption and investment with respect to public sector borrowing to account for the displacement of consumption and investment by public sector projects. No such displacement occurs under Assumption 4's infinitely elastic capital market.

## 5.2 The private sector discount rate

We now derive the private sector discount rate. It differs from the public sector discount rate because, unlike the government acting on behalf of taxpayers in Section 5.1, shareholders do not account for the benefits derived from any change in government tax receipts that may result from the undertaking of the project. Only a very small fraction of any such change would accrue to or be borne by the shareholders of the firm undertaking the investment. Shareholders therefore consider after- rather than before-tax returns on the transactions that they too will enter into to hedge the market risk of the project. These returns are  $(1-T_{PD})R_F$  on the risk-free asset and  $(1-T_{PE})R_M$  on the market portfolio. Note that, like the taxpayers in Section 5.1, shareholders use the post- rather than the pre-corporate-tax global market risk premium, computed using the post-corporate-tax return  $R_M$  on the global market portfolio.

The shareholders of a private firm need not all reside in the same country, which need not be the country in which the firm undertakes the project. The investor tax rates  $T_{PD}$  and  $T_{PE}$  are therefore those of the consensus shareholder and the corporate tax rates  $T_C$  and  $T_C^R$  those of the country in which the investment is made. In Section 6 below, we briefly discuss how the incidence of taxes by country should affect the government's analysis of private sector valuations. We assume that a fraction  $1-L$  of the project is financed by equity and the

remaining fraction  $L$  is financed by risk-free debt. The project has an after-corporate-tax beta equal to  $\beta'$  in (5). We show in Appendix 2 that the private sector evaluates the project as follows:

**Proposition 4:** The after-tax required return on equity is:

$$(1-T_{PD})R_F + \frac{\beta'}{1-L} \left[ (1-T_{PE})E[R_M] - (1-T_{PD})R_F \right]; \quad (6)$$

it is used to discount the expected after-tax cash flow to equity:

$$(1-T_{PE}) \left( (1-T_C^R)E[C-1] - (1-T_C)LR_F \right) + (1-L). \quad (7)$$

The after-tax required return on the project is:

$$(1-L) \left( (1-T_{PD})R_F + \frac{\beta'}{1-L} \left[ (1-T_{PE})E[R_M] - (1-T_{PD})R_F \right] \right) + L(1-T_{PE})(1-T_C)R_F; \quad (8)$$

it is used to discount the expected after-tax cash flow from the project:

$$(1-T_{PE})(1-T_C^R)E[C-1] + 1. \quad (9)$$

The after-tax required return on the project has the pre-tax equivalent:

$$\begin{aligned} R_{PR} \equiv R_F + \beta P + L \left[ \frac{T_C^R - T_C}{1-T_C^R} \right] R_F + \beta \left[ \frac{T_{PD} - T_{PE}}{1-T_{PE}} \right] R_F \\ + (1-L) \left[ \frac{(1-T_{PD}) - (1-T_{PE})(1-T_C^R)}{(1-T_{PE})(1-T_C^R)} \right] R_F; \end{aligned} \quad (10)$$

it is used to discount the expected pre-tax cash flow from the project  $E[C]$ .

The first two parts of Proposition 4 establish the result that the private sector will discount after-tax cash flows at after-tax discount rates. This is true for the cash flow to equity, (7), discounted at the cost of equity, (6), as well as for the cash flow from the project, (9), discounted at the cost of capital, (8). It is in contrast to the public sector, which was shown in Proposition 3 to discount pre-tax cash flows at the pre-tax cost of capital. Such a contrast is not surprising, for the public sector does not pay taxes whereas the private sector does. The last part of Proposition 4 expresses the private sector after-tax cost of capital on a pre-tax basis; it makes the comparison of the private and public sector costs of capital possible.

### 5.3 The relation to the existing results: CAPM, WACC, and Miller

A first insight into the formulas derived in Proposition 4 can be gained by considering the special case of a tax-exempt investor,  $T_{PD} = T_{PE} = 0$ . The after-tax required return on equity (6) becomes:

$$R_F + \frac{\beta'}{1-L} [E[R_M] - R_F]. \quad (11)$$

This is the standard CAPM formula for the cost of equity, which uses the leveraged after-tax equity beta:  $\beta'/(1-L)$  equals  $(1-T_C^R)\beta/(1-L)$ . The required return on equity is used to discount the after-tax cash flow to equity (7), which becomes:

$$(1-T_C^R)E[C] + T_C^R + T_C LR_F - (1+R_F)L. \quad (12)$$

The term  $T_C^R$  represents the depreciation tax shield and the term  $T_C LR_F$  the interest tax shield;  $(1+R_F)L$  is the payment to debtholders.

The after-tax required return on the project (8) becomes:

$$(1-L) \left( R_F + \frac{\beta'}{1-L} [E[R_M] - R_F] \right) + L(1-T_C)R_F. \quad (13)$$

This is the standard weighted average cost of capital, WACC, an average of the cost of equity and the after-tax cost of debt  $(1-T_C)R_F$ . The WACC is used to discount the after-tax cash flow from the project (9), which becomes:

$$(1-T_C^R)E[C] + T_C^R. \quad (14)$$

The term  $T_C LR_F$ , representing the interest tax shield, no longer appears, as its effect is now incorporated into the WACC through the after-tax cost of debt.

We now return to the case of investor tax,  $T_{PE} > 0$  and  $T_{PD} > 0$ . We can restate the after-corporate- and investor-tax cost of equity (6) on an after-corporate- but before-investor-tax basis by dividing (6) by  $1-T_{PE}$ . This is the level of taxation at which the CAPM is generally expressed, adapted to the case in which the consensus shareholder is not a tax-exempt institution. The after-corporate-tax cost of equity is then:

$$\frac{1-T_{PD}}{1-T_{PE}}R_F + \frac{\beta'}{1-L} \left[ E(R_M) - \frac{1-T_{PD}}{1-T_{PE}}R_F \right]. \quad (15)$$

The adjustment to the risk-free rate is that of the Miller (1977) equilibrium. Recall that the Miller equilibrium is characterized by  $(1-T_{PD}) = (1-T_C)(1-T_{PE})$ : there is a tax advantage neither to equity nor to debt financing. This implies that the after-corporate-tax required return on a risk-free project,  $(1-T_C)R_F$ , equals  $\left[ (1-T_{PD}) / (1-T_{PE}) \right] R_F$ , exactly as in equation (15) above. Note that (15) further makes explicit the way in which the Miller adjustment affects the equity market risk premium, thereby extending the adjustment to the case of a risky project.

#### 5.4 The difference between the public and the private sector discount rates: a first look

We examine the difference between the public and the private sector pre-tax discount rates. We subtract the public sector discount rate,  $R_{PU}$ , defined in Proposition 3, from the private sector discount rate,  $R_{PR}$ , defined in Proposition 4. We call the resulting difference, either as such or divided by the risk-free rate, the ‘tax wedge’. We denote the tax wedge’s second variant  $\tau$ ; it consists of three components,  $\tau = \tau_1 + \tau_2 + \tau_3$ :

$$\tau_1 \equiv L \left[ \frac{T_C^R - T_C}{1 - T_C^R} \right], \quad (16)$$

$$\tau_2 \equiv \beta \left[ \frac{T_{PD} - T_{PE}}{1 - T_{PE}} \right], \quad (17)$$

and

$$\tau_3 \equiv (1-L) \left[ \frac{(1-T_{PD}) - (1-T_{PE})(1-T_C^R)}{(1-T_{PE})(1-T_C^R)} \right]. \quad (18)$$

Observe that (1) the sign of each component depends only on the tax rates, (2) the magnitudes of  $\tau_1$  and  $\tau_3$  depend on the debt capacity, measured by leverage  $L$ , and (3) the magnitude of  $\tau_2$  depends on the project pre-corporate-tax beta  $\beta$ .

We now consider the three components in turn. The first component,  $\tau_1$ , in (16) represents the combined effect of the interest and depreciation tax shields. The tax rate on financial investment  $T_C$  is the rate at which the interest tax shield is granted; the tax rate on real

investment  $T_C^R$  reflects the benefits of the depreciation tax shield. Where depreciation lowers the tax rate on real investment below that on financial investment,  $T_C^R < T_C$ , depreciation causes profit to be taxed at a lower rate ( $T_C^R$ ) than is shielded by leverage ( $T_C$ ). This is of benefit to the private sector, the pre-tax discount rate of which is lowered below its public sector counterpart,  $\tau_1 < 0$ . It is the more so the higher is the debt capacity of the project  $L$ . The private sector therefore favours assets with high debt capacity and rapid tax depreciation; all else being equal, the private sector does so the more the larger is the tax rate on financial investment  $T_C$  at which the interest tax shield is granted.

The preceding conclusion about  $\tau_1$  assumes that the tax system allows a tax loss generated by the combined effect of the depreciation and interest tax shields to be used to protect other taxable income. For that to be the case, the high debt capacity and low tax assets that give rise to the tax loss have to be pooled for tax purposes with other assets that generate taxable profit.<sup>11</sup> We return to this issue in Section 5.8 below.

The second component,  $\tau_2$ , represents the effect of the difference in investor tax on the risk premium received by investors. To take on the incremental risk of a project, investors rebalance the remainder of their portfolios away from equity and towards debt. If the investor tax rate on debt is higher than that on equity,  $T_{PD} > T_{PE}$ , as is the case in most tax systems, this rebalancing is costly to investors. The private sector pre-tax discount rate is therefore raised above its private sector counterpart,  $\tau_2 > 0$ , because the public sector does not suffer this tax penalty when it rebalances risk. The larger the beta of the project and the more extensive the rebalancing required, the larger this effect. Thus, if tax is the only factor, high beta assets will be valued less by the private sector than by the public sector. The private sector will therefore be reluctant to take risky assets away from the public sector.

The third component,  $\tau_3$ , recalls Miller's (1977) net tax discrimination term between debt and equity,  $(1 - T_{PD}) - (1 - T_{PE})(1 - T_C)$ , from which  $\tau_3$ 's numerator differs in including the tax rate on real ( $T_C^R$ ) rather than financial ( $T_C$ ) investment. It represents the possible tax benefit (if

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<sup>11</sup> An example is the UK privatized water companies, which are funded with large amounts of debt and are generally held in combination with other assets rather than as stand-alone firms (see *Financial Times*, 23 January 2018, page 9).

negative) or cost (if positive) to choosing equity over debt financing. While it is zero in a Miller (1977) equilibrium, the net tax discrimination term is positive in most tax systems, implying that equity financing is costlier than debt financing.<sup>12</sup> This raises the private sector cost of capital above its public sector counterpart,  $\tau_3 > 0$ . The greater the reliance on equity financing  $1 - L$ , the larger is the effect. Assets that need to be financed heavily with equity will therefore be valued less by the private sector than by the public sector. This reinforces the preference of the private sector for assets that can be funded heavily with debt.<sup>13</sup>

Our analysis can be viewed as complementing Miller's (1977) in that it accounts for both the combined effect of the interest and depreciation tax shields ( $\tau_1$ ) and the effect of the difference in investor tax on the risk premium ( $\tau_2$ ) in addition to the differential tax effect of equity and debt financing ( $\tau_3$ ) analysed by Miller (1977). That taxes affect the private sector pre-tax cost of capital  $R_{PR}$  but not the public sector cost of capital  $R_{PU}$  is, as previously noted, a consequence of the fact that taxes are a transfer to taxpayers but a loss to shareholders.<sup>14</sup>

### 5.5 Public and private sector discount rates and asset preferences

We now examine the combined effect of all three components of the tax wedge,  $\tau = \tau_1 + \tau_2 + \tau_3$ . As indicated by (16)–(18), the wedge depends on the tax rates, debt capacity  $L$ , and asset beta  $\beta$ . In line with our discussion of tax rates in Section 5.4, we make the following assumption, which holds for most standard tax systems:

$$\text{Assumption 6. } T_C^R < T_C, T_{PD} > T_{PE} \text{ and } (1 - T_{PD}) - (1 - T_{PE})(1 - T_C^R) > 0.$$

The first part of the assumption states that depreciation lowers the corporate tax rate on real investment below that on financial investment. The second part states that the investor tax rate on debt is greater than that on equity. The third part asserts that the combination of the effective corporate tax rate and the investor tax rate on equity exceeds the investor tax rate on debt.

Given Assumption 6, we show that taxes induce a private sector preference for projects with a low asset beta and high debt capacity. Formally, the tax wedge  $\tau$  has partial derivatives:

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<sup>12</sup> That the net tax discrimination term is zero in a Miller equilibrium (1977) will be recalled from the discussion at the end of the previous section.

<sup>13</sup> Recall the discussion of  $\tau_1$  above.

<sup>14</sup> Compare the discussion at the beginning of Sections 5.1 (public sector) and 5.2 (private sector).

$$\frac{\partial \tau}{\partial \beta} = \frac{T_{PD} - T_{PE}}{1 - T_{PE}} > 0$$

and

$$\frac{\partial \tau}{\partial L} = \frac{(1 - T_{PE})(1 - T_C) - (1 - T_{PD})}{(1 - T_{PE})(1 - T_C^R)} < 0.$$

Thus, based on taxes alone and compared with the public sector, the private sector shows a greater preference for projects or cash flows that have low risk and a high debt capacity.

In some cases, this preference will result in an advantage for private sector ownership, as Table 1 illustrates. Panel A considers high debt capacity projects,  $L = 0.8$ . For projects that combine a high debt capacity with a low asset beta, the tax wedge  $\tau$  is negative, indicating that the private sector has a tax-induced advantage over the public sector in undertaking these projects. Panel B shows that projects with low debt capacity always have a tax disadvantage for the private sector, regardless of their asset beta. This is because the tax wedge  $\tau$  is always positive, increasing the private sector's discount rate above that of the public sector. The table also illustrates that the tax advantage of the private sector stems only from  $\tau_1$ , the component of the tax wedge due to the difference between the effective and the statutory corporate tax rates.

The private sector is therefore in a position to offer the public sector high prices for assets with rapid tax depreciation schedules and high debt capacity. Since only low beta assets have high debt capacity and given that high leverage naturally implies low equity, the disadvantage from the positive  $\tau_2$  and  $\tau_3$  does not eliminate the advantage from the negative  $\tau_1$ . We discuss in Section 6 whether such tax-induced high prices constitute a valid reason for the public sector to transfer assets to the private sector.

**Table 1: The relative tax advantage of the public sector in terms of the pre-tax discount rate**

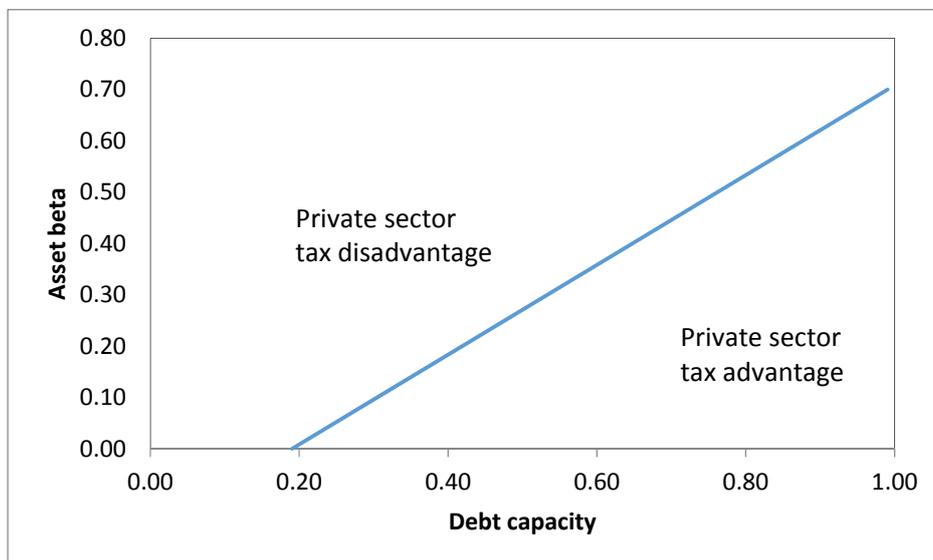
The table shows the pre-tax discount rate advantage of the public sector,  $\tau$ , and its three components.  $\beta$  is the asset beta. The tax rates are  $T_C = 30\%$ ,  $T_C^R = 20\%$ ,  $T_{PD} = 30\%$ , and  $T_{PE} = 15\%$ . The debt capacity,  $L$ , is 80% in Panel A and 20% in Panel B.

<b>Panel A:</b>				
$L = 0.8$				
$\beta$	0.80	0.60	0.40	0.20
$\tau_1$	-0.1000	-0.1000	-0.1000	-0.1000
$\tau_2$	0.1412	0.1059	0.0706	0.0353
$\tau_3$	0.0059	0.0059	0.0059	0.0059
$\tau$	0.0471	0.0118	-0.0235	-0.0588
<b>Panel B:</b>				
$L = 0.2$				
$\beta$	0.80	0.60	0.40	0.20
$\tau_1$	-0.0500	-0.0500	-0.0500	-0.0500
$\tau_2$	0.1412	0.1059	0.0706	0.0353
$\tau_3$	0.0176	0.0176	0.0176	0.0176
$\tau$	0.1088	0.0735	0.0382	0.0029

Figure 1 generalizes Table 1 in that it shows, for a range of values for debt capacity  $L$  and asset beta  $\beta$ , the areas in which there is a tax advantage ( $\tau < 0$ ) or disadvantage ( $\tau > 0$ ) to the private sector. The blue line represents the combinations  $(L, \beta)$  for which the tax wedge is zero,  $\tau(L, \beta) = 0$ : the private and public sector discount rates are equal on that line.

**Figure 1: Combinations of asset beta and debt capacity and the tax advantage/disadvantage of the private sector**

The figure shows the areas of tax advantage/disadvantage for the private sector, as measured by the tax wedge  $\tau$ . The tax rates are  $T_C = 30\%$ ,  $T_C^R = 20\%$ ,  $T_{PD} = 30\%$ , and  $T_{PE} = 15\%$ .



The area of tax advantage for the private sector is at the bottom right of the figure; it corresponds to assets with a high debt capacity and a low beta.<sup>15</sup> Conversely, the area of tax disadvantage for the private sector is at the top left of the figure; it corresponds to assets with a low debt capacity and a high beta. The boundary between the two areas is a straight line that cuts the x-axis at a strictly positive value;<sup>16</sup> assets with a debt capacity below that value always have a tax disadvantage for the private sector, regardless of how low risk they may be.

<sup>15</sup> Assets with a high debt capacity and a high beta are unlikely to exist in practice.

<sup>16</sup> These two properties are easily derived by solving  $\tau(L, \beta) = 0$  for  $\beta(L)$ .

Besides  $\tau$ , the tax wedge  $\tau R_F$  depends on the risk-free rate  $R_F$ . A larger risk-free rate increases the tax wedge, both in absolute value,  $|\tau R_F|$ , and relative to the public sector discount rate,  $|\tau R_F|/(R_F + \beta P)$ . A larger risk-free rate therefore strengthens the private sector's preference for assets with negative tax wedge,  $\tau < 0$ , that is, assets with rapid tax depreciation schedule, low beta, and high debt capacity. This observation may provide a possible explanation for the lesser willingness recently of the private sector to invest in public infrastructure (Gomes, 2018): one feature of the post-credit-crisis environment has been the low level of interest rates; as the risk-free rate has fallen, so has the tax wedge  $|\tau R_F|$ , thereby reducing the private sector's willingness to invest in assets with negative tax wedge, such as some types of infrastructure.

## 5.6 The relation to Baumol's result

Our analysis can be related to Baumol's (1968) observation that a 50% corporate tax rate doubles the government's 'hurdle rate' compared with that of the private sector.<sup>17</sup> Specifically, Baumol argues that, if a private sector firm obtains an (after-tax) return of  $r$  by using some given resources, the government should displace the firm as a user of these resources only if it can obtain a (pre-tax) return of at least  $2r$ .

We express Baumol's analysis in the notation of our model. Baumol considers a risk-free ( $\beta = \beta' = 0$ ), all-equity ( $L = 0$ ) project, in an environment with no investor tax ( $T_{PD} = T_{PE} = 0$ ) and equal corporate tax on real and financial investment ( $T_C^R = T_C$ ). His result may be interpreted as stating that, when the firm discounts the after-tax cash flow  $0.5 \times E[C] + 0.5$  from (7) (recall  $T_C^R = 0.5$ ) at the discount rate  $R_F$  from (6) (recall  $\beta' = L = 0$ ), the government, in seeking to compare its NPV with that of the firm, must discount the before-tax cash flow  $E[C]$  at the rate from (10):<sup>18</sup>

$$R_F + \left[ \frac{1 - (1 - 0.5)}{1 - 0.5} \right] R_F = 2R_F.$$

Hence, our results do indeed reproduce Baumol's for the special case that he considers.

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<sup>17</sup> Recall the discussion of Baumol (1968) in the literature review.

<sup>18</sup> Recall from Section 3 that Assumptions 1 and 2 together imply the equality of public and private sector pre-tax cash flows.

We stress that the use of the discount rate  $2R_F$  by the government is for comparison purposes only; it is not for valuation purposes: recall from Proposition 3 that the value of the project to the government is obtained by discounting the pre-tax project cash flow  $E[C]$  at the pre-tax discount rate  $R_F$ , resulting in project NPV  $-1 + (E[C]/(1 + R_F))$ .<sup>19</sup> We discuss why the comparison of public and private sector projects requires a change in the discount rate in Section 6.

### 5.7 The relation between the private sector discount rate and the public sector cash flow

We now turn our attention from the wedge introduced by taxes to the private sector discount rate,  $R_{PR} = R_{PU} + \tau R_F$ , to the corresponding change in public sector tax revenues: taxes paid by a private sector firm are taxes received by the government, and taxes saved by the firm are foregone by the government. We do so because, as we shall see in Section 6, such information is relevant to the decision regarding whether an asset should be owned or a service provided by the public or the private sector. We show in Appendix 3:

**Proposition 5:** The value to the government of the change in tax revenues at date 1 resulting from a private sector project with zero NPV is given by  $\Delta TAX = \tau R_F$ .

Comparing Propositions 4 and 5, it is clear that, for initial investment normalized to 1, the tax wedge and the change in tax revenues are identical.<sup>20</sup> Formally, we can write:

**Corollary 1:** The wedge between the private and the public sector costs of capital equals the value of the change in tax revenues:  $R_{PR} = R_{PU} + \Delta TAX$ .

To understand the intuition for this result, consider a zero-NPV project.<sup>21</sup> Using (8) and (9), we have:

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<sup>19</sup> Recall that the project that Baumol considers has  $\beta = \beta' = 0$ .

<sup>20</sup> If the initial investment were  $K$ , then the change in tax revenues would be  $\tau R_F K$ ; the wedge between the discount rates would, however, remain  $\tau R_F$ .

$$-1 + \frac{(1-T_{PE})(1-T_C^R)E[C-1]+1}{\left\{ 1+(1-L)\left( (1-T_{PD})R_F + \frac{\beta'}{1-L}\left[ (1-T_{PE})E[R_M] - (1-T_{PD})R_F \right] \right) \right\} + L(1-T_{PE})(1-T_C)R_F} = 0 \quad (19)$$

$$\Leftrightarrow E[C] = 1 + (1-L) \left[ \frac{1-T_{PD}}{(1-T_{PE})(1-T_C^R)} R_F + \beta \left[ E[R_M] - \left[ \frac{1-T_{PD}}{1-T_{PE}} \right] R_F \right] + L \left[ \frac{1-T_C}{1-T_C^R} \right] \right] \quad (20)$$

$$\Leftrightarrow E[C] = 1 + R_F + \beta P + \tau R_F, \quad (21)$$

where we have used (16)–(18). The effect of taxes is to change the cash flow required for the project to have zero NPV from  $E[C] = 1 + R_F + \beta P$  in the absence of taxes to (21) in their presence. The difference between these two values, the tax wedge  $\tau R_F$ , is the tax paid by the firm and received by the government if positive or the tax saved by the firm and foregone by the government if negative. In the present case of unit initial investment, the cash flow required to have zero NPV equals the cost of capital.

The result in Proposition 5 is obtained by having the government hedge the incremental cash flows associated with the changes in tax payments resulting from the project: recall that the tax system makes the government a ‘partner’ of the firm in sharing both the rewards and the risks of the project. Thus, like shareholders who hedge the after-tax cash flows from the project (recall Proposition 4), the government acting on behalf of taxpayers hedges the tax cash flows from the project. Unlike shareholders who hedge on an after-tax basis, the government hedges on a pre-tax basis.<sup>22</sup> This difference makes the equality between the tax wedge and the change in tax revenues non-trivial, albeit intuitive.

Proposition 5 and Corollary 1 imply that, in any transaction between the public and the private sector that is materially affected by tax considerations, the impact of tax on the private sector’s discount rate is, in the present case of unit initial investment, identical to its impact on

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<sup>21</sup> We consider a zero-NPV project because we wish to keep the beta constant: when a private sector project has non-zero NPV, the undertaking of the project changes the beta because it changes the value of the tax shield relative to the operating cash flow. Although it is possible to derive a formula for the NPV that accounts for the change in the beta, this complicates the analysis without adding much insight.

<sup>22</sup> Compare the proofs of propositions 4 and 5.

the public sector's tax revenue.<sup>23</sup> We discuss the implications of this result for the analysis of transactions between the public and the private sector in Section 6.

### 5.8 Details of the tax system and multiple periods

Under Assumption 6, and as illustrated in Table 1 for example, it is clear that any tax advantage that the private sector may have over the public sector,  $\tau < 0$ , derives not from investor but from corporate tax:  $\tau_1 < 0$ , whereas  $\tau_2 > 0$  and  $\tau_3 > 0$ . The private sector's tax advantage is attributable to the combination of depreciation and leverage: depreciation causes profit to be taxed at a lower rate ( $T_C^R$ ) than it is shielded by leverage ( $T_C$ ).

For a firm to be able to profit from that difference, however, it must be able to use the tax loss generated by depreciation and leverage to offset the tax payments elsewhere in the corporation. This will be immediate when the asset being depreciated is a relatively small part of a larger profitable firm. In contrast, when the tax loss is generated by a large collection of assets, for example by a regulated utility, the assets and losses must be consolidated into another profitable firm. Tax systems differ regarding the conditions under which such consolidation is allowed. Details of the relevant tax system therefore affect the extent to which this primary source of private sector tax advantage operates.

Details of the tax system also affect the tax efficiency of the hedging transactions used to neutralize the risk of a project. Recall that investors hedge the risk of a project by selling equities, taxed at  $T_{PE}$ , and buying debt, taxed at  $T_{PD}$ . If hedging were instead achieved by shorting stock index futures contracts, a transaction that may attract a different tax rate, then the private sector tax disadvantage will vary, possibly decreasing if the investor tax rate on stock index futures is lower than that on equities.<sup>24</sup>

The derivation in Section 5 assumes a single period, yet the life of most assets spans multiple periods. The tax rate on real investment  $T_C^R$  therefore should be viewed as an average of the rates paid over the life of an asset. These rates generally vary over time. For example, it may be possible to take large tax benefits early in the life of an asset, using a high tax rate corporation for

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<sup>23</sup> If the initial investment were  $K$ , then the impact on tax revenues would be  $K$  times that on the discount rate (see Footnote 20).

<sup>24</sup> Both  $\partial \tau_2 / \partial T_{PE} > 0$  and  $\partial \tau_3 / \partial T_{PE} > 0$  from (17) and (18).

that purpose, and then to transfer the asset to a low tax rate corporation once the tax benefits have been exhausted. Whether such a transfer is possible, and how much it lowers the effective tax rate on real investment, depends on the details of the tax rules and on the extent to which tax transfer markets are allowed to operate (Cooper and Franks, 1983).

## **6. Implications for transactions between the public and the private sector**

The preceding analysis has shown that there is a tax-induced wedge between public and private discount rates for the same asset with the same pre-tax expected future cash flows and that this wedge equals the value of the incremental tax revenues received or foregone by the public sector in the case in which the asset is owned by the private sector. This naturally implies that the resulting discount rate-induced difference in valuations between the public and the private sector does not constitute a legitimate motive for transactions between the public and the private sector: the difference in valuation amounts only to redistribution via the tax system; a higher private sector valuation means foregone public sector tax revenues, and a lower valuation means additional tax revenues. Thus, Proposition 5 and Corollary 1 imply that an asset has the same value to both public and private sectors when (i) both sectors generate the same cash flow from the asset and (ii) the public sector fully internalizes all incremental tax effects: no third party is affected by the changes in tax revenues that result from a change in ownership of the asset.<sup>25</sup>

The difference in valuation nonetheless introduces a preference in the private sector for certain types of asset, specifically those with rapid tax depreciation, a high debt capacity, and low risk. Unless adjusted for, this preference may distort public sector decisions regarding matters as varied as the privatization of government assets, the regulation of private utilities, and transactions that commit the government to making fixed payments to the private sector. In this section, we discuss such distortions and show how the government should analyse the aforementioned transactions to avoid disadvantaging its citizens. We further distinguish between the two cases in which all incremental taxes affect only the domestic government and in which they affect other governments as well.

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<sup>25</sup> An example of such a party might be a foreign government. We return to that issue in Section 6.4.

To illustrate the type of error that may be made, consider a government contemplating the sale of an asset to the private sector. The apparently intuitive procedure that would have the government discount the cash flows associated with the asset at the government's cost of capital  $R_{pU}$  and compare the value thus obtained with the price offered by the private sector would be incorrect. Although that value represents the true value of the asset to the government, as shown in Proposition 3, it is not the value to be compared with the price offered by the private sector. This is because such a comparison ignores the change in tax revenues that the sale of the asset necessarily produces. An asset generally provides both a depreciation tax shield and an interest tax shield; *ceteris paribus*, these can be expected to decrease the government tax revenues. Failure on the part of the government to account for the tax implication of the sale of the asset would systematically distort the government decisions regarding the sale of tangible assets with a high debt capacity and a low beta, regardless of whether the private sector is actually more efficient than the public sector in the sense of generating a higher pre-tax cash flow from the asset.

### 6.1 Privatization with full internalization of all incremental taxes

When the public sector fully internalizes all the incremental tax effects, Proposition 5 holds. There are then two alternative procedures for making privatization decisions. Both involve the government evaluating the privatization decision as if it – the government – were subject to taxation in exactly the same manner as private firms. The first procedure reflects taxation in the discount rate and the second taxation in the cash flow. They are:

1. Discount the public sector cash flow  $E[C]$  at the private sector discount rate  $R_{pR}$  in (10) and compare it with the price offered by the private sector.
2. Subtract the value of the change in tax revenue that would result from privatization  $\Delta TAX = \tau R_F$  from the public sector cash flow  $E[C]$ , discount it at the public sector discount rate  $R_{pU}$ , and compare it with the price offered by the private sector.

Both procedures entail the government incurring the same cost ( $\tau > 0$ ) or enjoying the same benefit ( $\tau < 0$ ) as the private firm, which pays tax but also enjoys depreciation and interest tax shields.

The two procedures are equivalent from Corollary 1: the tax wedge in the discount rate equals the value of the change in the government tax revenues, that is, minus the change in the private sector cash flow.<sup>26</sup> It may nonetheless be useful to use both approaches, as these will complement each other, when the errors that necessarily will be made in estimating the discount rates and cash flows are not the same. By way of example, contrast the sale and leaseback of a real estate asset with an investment in a nuclear power plant. The greater uncertainty about the cost of capital of the latter suggests a greater need to rely on the cash flow in that case.

To illustrate the need for the change in discount rate mandated by the first procedure, consider a government's decision regarding whether to privatize an asset that, in accordance with Assumptions 1 and 2, has identical pre-tax cash flows  $E[C]$  for both the public and the private sector. Normalize the present value (PV) to the private sector to 1 and follow the same steps as in Section 5.7 to obtain (21),  $E[C] = 1 + R_F + \beta P + \tau R_F$ . Further assume that the asset markets are competitive so that 1 would be the price paid by an acquiring firm to the government.

Now compute the value of the asset to the government. From Proposition 3, that value is:

$$\frac{E[C]}{1 + R_F + \beta P} = \frac{1 + R_F + \beta P + \tau R_F}{1 + R_F + \beta P} = 1 + \frac{\tau R_F}{1 + R_F + \beta P}, \quad (22)$$

where we have used (21). This is the true value of the project to the government, which, depending on the sign of  $\tau$ , may be higher or lower than 1, the receipt from privatization. However, a positive  $\tau$  is no more a reason for the asset to remain in government hands than a negative  $\tau$  is a reason for the asset to be privatized: both the firm and the government generate the same pre-tax cash flow  $E[C]$ , and both can source financing in the same infinitely elastic capital market, which both taxpayers and shareholders can use to hedge the risk of the asset. The reason for the firm's PV being lower or higher than the government's is because the firm, unlike the government, pays tax; this alters both the cash flow and the discount rate. The purpose of using the adjustment in (10) is to account for this tax when comparing the two sectors' valuations. Indeed, having the government use  $1 + R_F + \beta P + \tau R_F$  from (10) as a discount rate, the government computes the PV:

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<sup>26</sup> We further show this by way of an example in the following.

$$\frac{E[C]}{1 + R_F + \beta P + \tau R_F} = \frac{1 + R_F + \beta P + \tau R_F}{1 + R_F + \beta P + \tau R_F} = 1. \quad (23)$$

The comparison is no longer distorted by the failure to account for the firm's tax payments. The general formula in (10) generalizes Baumol's analysis discussed in Section 5.6 in that (10) incorporates risk, leverage, investor tax, and the differential taxation of debt and equity and of real and financial investment.

It is easy to see that the second procedure achieves an identical result. Indeed, using Proposition 5 and (21), we have

$$\frac{E[C] - \Delta TAX}{1 + R_F + \beta P} = \frac{E[C] - \tau R_F}{1 + R_F + \beta P} = \frac{1 + R_F + \beta P + \tau R_F - \tau R_F}{1 + R_F + \beta P} = 1. \quad (24)$$

## 6.2 Regulation

Suppose that the purpose of regulation is to limit a private sector utility's return to its cost of capital and that such a limitation is achieved by having the government set regulated prices in such a way that the utility's pre-tax cash flow equals the utility's capital charge. If the utility's capital is normalized to 1, then the government would set the price such that the utility's pre-tax cash flow equals, by the same derivation as for (21),

$$E[C_{PR}] = 1 + R_F + \beta P + \tau R_F. \quad (25)$$

If the utility were owned by the government instead, and again assuming that the price is set to cover the utility's capital charge, then the price would be set such that the cash flow equals

$$E[C_{PU}] = 1 + R_F + \beta P. \quad (26)$$

The government will be deemed to be too stringent in its regulatory policy,  $E[C_{PR}] < E[C_{PU}]$ , if  $\tau < 0$ ; it will be deemed to be too lax,  $E[C_{PR}] > E[C_{PU}]$ , if  $\tau > 0$ . However, neither assessment would be correct. Instead, the government simply reflects in its regulatory pricing policy the tax benefits ( $\tau < 0$ ) or costs ( $\tau > 0$ ) that ownership of the regulated utility confers, or imposes, on the private sector. Thus, tangible-asset-rich utilities with rapid tax depreciation, low risk, and high debt capacity, such as water companies, will appear to be

penalized, whereas intangible-asset-rich utilities with little depreciation, high risk, and low debt capacity, such as telecoms, will appear to be privileged. Both impressions would be mistaken.

### 6.3 Risk transfers and fixed payments

Many transactions between the public and the private sector involve either explicit or implicit risk transfers. These often take the form of fixed payments. In public–private partnerships for example, a private firm will assume the risks of financing, building, and maintaining an asset such as a school or a hospital in return for a series of fixed payments from the government agency that will use the asset. An oft-stated motivation for such a transaction is the private sector’s often greater efficiency in controlling costs, a consequence of the private sector’s generally higher-powered incentives: the fixed payments received by the private sector make it a residual claimant to any cost saving, thereby incentivizing it to engage in vigorous cost cutting.

Transactions such as these have the same safe cash flow  $C$  paid and received by the government and the firm, respectively. Essentially the same issues as in our analysis of privatization in Section 6.1 arise: the discount rate  $R_{PU} = R_F$  ( $\beta = 0$  as  $C$  is safe) that the government should use to determine the cost of the guarantee differs from the rate  $R_{PR} = R_F + \tau R_F$  that it should use if it should wish to compare the cost of private sector provision with that of public sector provision.

### 6.4 Privatization with partial internalization of incremental taxes

Our analysis thus far has assumed that the taxes the effect of which the wedge  $\tau$  is intended to capture are paid and received domestically: any tax benefit that investment in an asset provides is received by domestic firms and investors, whose gain is the domestic government’s loss; the same is true of any tax cost.<sup>27</sup>

When foreign investors or tax authorities are involved, however, either as recipients or as payers of tax, then the use of the tax wedge  $\tau$  no longer accounts for the entirety of the tax effects: a positive  $\tau$  would be too high if part of the tax were paid to foreign tax authorities, and a negative  $\tau$  would be (algebraically) too low if part of the tax saved were at the expense

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<sup>27</sup> The single exception is corporate taxes paid on the global market portfolio; see the discussion after Proposition 3.

of foreign tax authorities. For example, if a privatized asset acquired by a foreign firm were to decrease the foreign firm's tax payments in its home country, then even a difference in valuation due entirely to tax redistribution might justify the decision to privatize. This is because the privatizing government would enjoy the benefit (higher privatization proceeds) but not bear the cost (lower tax receipts) of the tax shield that ownership of the privatized asset provides. Such considerations can be accounted for by adjusting the tax wedge  $\tau$  to distinguish between foreign and domestic taxes, but the formal analysis of such adjustments is beyond the scope of the present paper.

## 7. Examples

In this section, we illustrate the distortions discussed in Section 6 with a number of examples. We emphasize that tax considerations constitute only one aspect of the analysis of transactions between the public and the private sectors. The purpose of the two procedures introduced in Section 6 is to remove the distortions introduced by tax, thereby permitting the necessary focus on those motives for private–public transactions that are legitimate, efficiency gains in particular.

We consider in some detail two types of assets for which financing may be viewed as illustrating the most extreme forms of tax distortion, the first to the benefit and the second to the detriment of the private sector. These are (i) independent power plants (IPPs) and (ii) nuclear power plants. The private financing of IPPs through project finance has proven to be pervasive; we show that it falls into the area of the highest tax advantage for the private sector. In contrast, the privatization of nuclear power generation has proven to be extremely difficult; we show that it falls into the area of the highest tax penalty for the private sector. We then briefly discuss other types of assets and services.

For all these, we examine the variation of the tax wedge  $\tau$  in the asset beta and debt capacity, the former depending on the operating leverage and systematic cash flow variability and the latter on the total cash flow variability and asset tangibility.<sup>28</sup> We continue to assume that Assumption 6 holds but now distinguish between those assets for which the difference between the statutory and the effective tax rates is small and those for which it is large.<sup>29</sup> We

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<sup>28</sup> See Brealey, Myers, and Allen (2017, Chapter 9).

<sup>29</sup> The tax rates on real and financial investment generally vary with the type of investment, because depreciation

relate the resulting tax wedge to the observed tendency to have an asset owned or a service provided by the public or the private sector.

### 7.1 Independent power plants

IPPs have a low asset beta and high debt capacity, a consequence of the long-term ‘take-or-pay’ contracts that IPPs enter into with electricity distributors (Brealey, Cooper, and Habib, 2000). Such contracts effectively guarantee an IPP’s revenues or its margin over fuel costs, thereby decreasing the IPP’s cash flow volatility and in turn decreasing its asset beta and increasing its debt capacity. IPPs are therefore located at the bottom-right corner in Figure 1 above, indicating a tax-induced funding cost advantage for the private sector:  $\tau_1 < 0$  in (16) is large in magnitude because the debt financing  $L$  is large, and  $\tau_2 > 0$  in (17) and  $\tau_3 > 0$  in (18) are small because the asset beta  $\beta$  and equity financing  $1 - L$  are small, respectively. The magnitude of  $\tau_1$  is further increased by the tax advantage to real investment enjoyed by high-capital-cost coal- or gas-fired power plants:  $T_C^R$  is markedly below  $T_C$ .

That IPPs are the archetypical asset funded by project finance is consistent with the private sector seeking to exploit IPPs’ high debt capacity: a near defining characteristic of project finance is the high leverage that it entails. Nevertheless, as argued in Section 6, the private sector’s tax gains are the public sector’s losses; a government therefore would be mistaken in concluding from the private sector’s lower cost of capital,  $\tau < 0$ , the desirability of encouraging IPPs for electricity generation: doing so neglects the consequent decrease in tax revenues. This is not to say that resorting to IPPs must always be eschewed but simply that the government must not partake in a transaction that may be designed solely to arbitrage the tax system. When agency considerations confer an efficiency advantage on the private sector, then the gains from project-financed IPPs can be large (Brealey, Cooper, and Habib, 1996, 2000).

### 7.2 Nuclear power plants

Nuclear power plants have very high fixed and low variable costs; they therefore have a very high operating leverage, implying a high asset beta and low debt capacity (Taylor, 2007).

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schedules, investment tax credits, and other provisions of the tax code vary across assets.

Nuclear power plants' debt capacity is further depressed by the uncertainty represented by future decommissioning costs, which can be extremely high (Ansolabehere et al., 2003). Nuclear power plants are therefore located in the top-left corner of Figure 1, indicating a tax-induced funding cost penalty to the private sector:  $\tau_2 > 0$  in (17) and  $\tau_3 > 0$  in (18) are large because the asset beta  $\beta$  and equity financing  $1 - L$  are large, respectively. Although the extremely high capital costs of nuclear power plants decrease the tax rate on real investment  $T_C^R$  considerably below tax on financial investment  $T_C$ , the magnitude of  $\tau_1 < 0$  in (16) is small because debt financing  $L$  is small.

That the privatization of the nuclear electricity generator of England and Wales occurred well after that of the coal generators, that it necessitated a very high level of debt forgiveness on the part of the UK Government as well as other forms of government support, and that it ultimately resulted in what at least one observer described as 'the financial collapse of the nuclear industry' (Taylor, 2007) are consistent with our conclusion of a tax-induced penalty for the private sector,  $\tau > 0$ ; this is despite the generous depreciation allowances that nuclear power plants enjoy. That the tax penalty is in fact illusory in that it corresponds to increased government tax revenues does not particularly lighten the tax burden borne by the privatized nuclear power industry. Although there may well be valid reasons for the government assisting the industry in lowering its cost of capital, for example by guaranteeing the take-off price of electricity or by insuring decommissioning costs, it should be cognizant of the fact that the resulting increase in debt capacity will decrease the government tax revenues; the government would, in fact, be engaging in tax arbitrage against itself and its taxpayers. Furthermore, guarantees and insurance might defeat the very purpose of privatization, the provision of high-powered incentives. Again, efficiency considerations should be paramount.

### 7.3 Other types of assets and services

Table 2 extends the analysis to other types of assets and services, specifically a toll road, commercial/industrial and residential real estate, and routine services such as garbage collection; it also reproduces the information pertaining to IPPs and nuclear power plants. For all such assets and services, the table shows stylized measures of (i) risk as measured by the asset beta, (ii) depreciation-induced tax benefits as measured by the difference between the statutory and the

effective tax rate, and (iii) debt capacity. The choice of assets and services considered is motivated by (a) the growing recourse to private finance to fund infrastructure projects such as toll roads (as well as the previously discussed IPPs) throughout the world (Gatti, 2008), (b) the sale of government real estate assets,<sup>30</sup> and (c) the contracting out of government services.

**Table 2: Characteristics of different assets and services**

Type of Asset	Asset Beta	$T_C - T_C^R$	Debt Capacity
Independent Power Plant	Low	Medium	High
Nuclear Power Plant	High	High	Low
Toll Road	Medium	Medium	Medium
Comm./Ind. Real Estate	Low/Medium	Medium	High/Medium
Residential Real Estate	Low/Medium	Low	High/Medium
Routine Services	Low	Low	High

Consider toll roads first. Although some such roads appear to benefit from government guarantees similar to those granted IPPs through ‘take-or-pay’ contracts, most toll roads do not. Toll roads therefore have a higher asset beta and lower debt capacity than IPPs but a smaller asset beta and higher debt capacity than nuclear power plants with very high operating leverage. Toll roads also enjoy a tax advantage to real investment, albeit one lower than that enjoyed by very-high-cost nuclear power plants. In terms of the overall tax-induced funding advantage or disadvantage, toll roads therefore fall somewhere between tax-advantaged IPPs and tax-disadvantaged nuclear power plants:  $\tau_{IPP} < \tau_{toll} < \tau_{nuclear}$  with  $\tau_{IPP} < 0$  and  $\tau_{nuclear} > 0$ . This may account for the lack of an apparent pattern in toll road financing, whether by the public or by the private sector.

Now compare commercial/industrial and residential real estate. They are similar in having a low to medium asset beta, a characteristic that favours reliance on debt finance. They differ in that investment in commercial and industrial real estate receives generous depreciation allowances, in contrast to investment in residential real estate, which receives only very limited allowances in most countries. This favours investment in commercial/industrial real estate:  $\tau_1$  in

<sup>30</sup> An example is the sale of military housing in the UK.

(16) is strictly negative for commercial/industrial real estate but zero for residential real estate, and  $\tau_2$  in (17) and  $\tau_3$  in (18) are more or less identical. That the privatization of government real estate in the UK concerned mainly commercial real estate, such as office buildings, is consistent with a tax-induced funding cost advantage for commercial/industrial real estate.<sup>31</sup>

The payments received and costs incurred in the provision of routine services generally share the low asset beta and high debt capacity of the IPP. As such costs are for the most part expensed in the year in which they are incurred, they do not give rise to any depreciation allowance:  $\tau_1 = 0$  in (16) because  $T_C = T_C^R$ . As it is further the case that  $\tau_2 \approx 0$  in (17) and  $\tau_3 \approx 0$  in (18) because asset beta  $\beta$  and equity financing  $1 - L$  are small, respectively, taxes bring neither an advantage nor a disadvantage to the provision of routine services,  $\tau \approx 0$ . This implies that the growing incidence of contracting out, whereby the public sector contracts out the provision of government services, such as garbage collection, to the private sector, is likely to be motivated not by tax distortions but by efficiency considerations.

While there undoubtedly are multiple reasons for the transactions that we have discussed, the overall pattern that they display is to a large extent consistent with the tax-induced advantages and disadvantages that our analysis has identified. This suggests that governments by and large do not adjust for tax distortions in their transactions with the private sector. One reason is that the cost of tax distortions, decreased tax revenue, is borne by foreign tax authorities (see Section 6.4); we doubt that this is the case for more than a relatively small fraction of transactions. Another, more likely, reason is that the adjustment necessary for accounting for the distortions, the computation of  $\tau$  in Section 5.4, is not easy to perform. At least some government transactions, then, must in fact be unfavourable to the government in that they decrease the tax revenue more than the benefits that they may provide, through gains in efficiency for example. Perhaps this is true of some public–private partnerships (see Section 6.3). The fixed payments received by the private partners endow the assets thus financed with a very high debt capacity. If these assets further enjoy very high depreciation tax shields, then the partnerships provide the private partners with very high tax savings. As these are at the expense of the government, however, government agencies should consider eschewing private–public partnerships that do not bring demonstrable efficiency gains.

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<sup>31</sup> We exclude the privatization of public sector housing, sold to tenants at subsidized prices.

## **8. Conclusion**

Using a new approach based on a quasi-arbitrage argument similar to that of Modigliani and Miller, we have established the existence of a tax-induced wedge between the public and the private sector discount rates and examined its variation with asset risk and debt capacity. Even if the public and private sectors should be equally efficient, the private sector might be willing to pay higher prices than the public sector for some assets solely because of their embedded tax advantages; conversely, the private sector might require higher returns than the public sector for holding other, tax-disadvantaged assets. In the case in which the government captures all the tax effects, the differences in valuations derived solely from taxation should not affect the government's choice between public and private sector ownership or provision: any apparent gain will be entirely illusory and represent not value creation but value transfer through the tax system. If the government should fail to adjust for such a transfer, then it might enter into transactions that concurrently decrease government tax revenue and distort private sector incentives.

We have shown how the government should include the preceding considerations in its analysis of privatization (or nationalization), regulation, and transactions, such as risk transfers and fixed payments. Interestingly, the pattern of ownership and transactions that would result from the failure to account for tax-induced distortions bears some resemblance to the prevailing patterns. In some cases at least, the government appears not to adjust for tax distortions, thereby unwittingly engaging in tax arbitrage against itself. This may be particularly true of transactions with which the government is unfamiliar or those that involve extensive financial engineering.

## References

- Ansola-behere, S., J. Deutch, M. Driscoll, P.E. Gray, J.P. Holdren, P.L. Joskow, R.K. Lester, E.J. Moniz, and N.E. Todreas, 2003, *The Future of Nuclear Power*, MIT Press, Cambridge, MA.
- Arrow, K., 1965, Criteria for Social Investment, *Water Resources Research* 1, 1–8.
- Arrow, K., and R.C. Lind, 1970, Uncertainty and the Evaluation of Public Sector Investment Decisions, *American Economic Review* 60, 364–378.
- Auerbach, A.J., 2002, Taxation and Corporate Financial Policy, in A.J. Auerbach and M. Feldstein (eds.), *Handbook of Public Economics*, Elsevier, Amsterdam.
- Bailey, M.J., and M.C. Jensen, 1972, Risk and the Discount Rate for Public Investment, in M.C. Jensen (ed.), *Studies in the Theory of Capital Markets*, Frederick A. Praeger, New York, NY.
- Baumol, W.J., 1968, On the Social Rate of Discount, *American Economic Review* 58, 788–802.
- Bazon, C., and K. Smetters, 1999, Discounting Inside the Washington D.C. Beltway, *Journal of Economic Perspectives* 13, 213–228.
- Benninga, S., and O. Sarig, 2003, Risk, Return, and Values in the Presence of Differential Taxation, *Journal of Banking and Finance* 27, 1123–1138.
- Bentz, A., P.A. Grout, and M.A. Halonen, 2005, What Should Governments Buy from the Private Sector—Assets or Services? Mimeo, University of Bristol.
- Brealey, R.A., I.A. Cooper, and M. Habib, 1996, Using Project Finance to Fund Infrastructure Investments, *Journal of Applied Corporate Finance* 9, 25–38.
- Brealey, R.A., I.A. Cooper, and M. Habib, 1997, Investment Appraisal in the Public Sector, *Oxford Review of Economic Policy* 13, 12–28.
- Brealey, R.A., I.A. Cooper, and M. Habib, 2000, The Financing of Large Engineering Projects, in R. Miller and D.R. Lessard (eds.), *The Strategic Management of Large Engineering Projects*, MIT Press, Cambridge, MA.
- Brealey, R.A., S.C. Myers, and F. Allen, 2017, *Principles of Corporate Finance*, 12<sup>th</sup> Edition, McGraw-Hill, New York, NY.
- Caplin, A., and J. Leahy, 2004, The Social Discount Rate, *Journal of Political Economy* 112, 1257–1268.
- Cooper, I.A., and J.R. Franks, 1983, The Interaction of Financing and Investment Decisions When the Firm has Unused Tax Credits, *Journal of Finance* 38, 571–583.

- Cooper, I.A., and K.G. Nyborg, 2008, Tax-adjusted discount rates with investor taxes and risky debt, *Financial Management* 37, 365–379.
- Dasgupta, P., 2008, Discounting Climate Change, *Journal of Risk and Uncertainty* 37, 141–169.
- Dasgupta, P., K.-G. Mäler, and S. Barrett, 1999, Intergenerational Equity, Social Discount Rates and Global Warming, in P. Portney and J. Weyant (eds), *Discounting and Intergenerational Equity*, Resources for the Future, Washington, DC.
- Diamond, P.A., 1967, The Role of a Stock Market in a General Equilibrium Model with Technological Uncertainty, *American Economic Review* 57, 759–776.
- Engel, E., R. Fischer, and A. Galetovic, 2013, The Basic Public Finance of Public–Private Partnerships, *Journal of the European Economic Association* 11, 83–111.
- Gatti, S., 2008, *Project Finance in Theory and Practice*, Academic Press, Cambridge, MA.
- Gomes, J., 2018, Why Private Investment in Public Infrastructure is Declining, <http://knowledge.wharton.upenn.edu/article/private-investment-in-public-infrastructure/>
- Graham, J.R., 2013, Do Taxes Affect Corporate Decisions? A Review, in G.M. Constantinides, M. Harris, and R.M. Stulz (eds), *Handbook of the Economics of Finance*, Elsevier, Amsterdam.
- Grauer, F.L.A., R.H. Litzenberger, and R.E. Stehle, 1976, Sharing Rules and Equilibrium in an International Capital Market under Uncertainty, *Journal of Financial Economics* 3, 233–256.
- Hamada, R.S., 1969, Portfolio Analysis, Corporation Finance, and Market Equilibrium, *Journal of Finance* 24, 13–31.
- Harberger, A.C., 1968, On Measuring the Social Opportunity Cost of Public Funds, in *Proceedings of the Committee on Water Resources and Economic Development of the West; The Discount Rate in Public Investment Evaluation*, Western Agricultural Research Council, Denver, CO.
- Hart, O.D., A. Shleifer, and R.W. Vishny, 1997, The Proper Scope of Government: Theory and an Application to Prisons, *Quarterly Journal of Economics* 112, 1127–1161.
- Hicks, J.R., 1940, The Valuation of Social Income, *Economica* 7, 105–124.
- Hirshleifer, J., 1966, Investment Decision under Uncertainty: Applications of the State-Preference Approach, *Quarterly Journal of Economics* 80, 252–277.
- Huizinga, H., and S.B. Nielsen, 2001, Privatization, Public Investment, and Capital Income Taxation, *Journal of Public Economics* 82, 399–414.

- Iossa, E., and D. Martimort, 2015, The Simple Microeconomics of Public–Private Partnerships, *Journal of Public Economic Theory* 17, 4–48.
- Kaldor, N., 1939, Welfare Propositions of Economists and Interpersonal Comparisons of Utility, *Economic Journal* 49, 549–552.
- Lucas, D., and R. McDonald, 2010, Valuing Government Guarantees: Fannie and Freddie Revisited, in D. Lucas (ed.), *Measuring and Managing Federal Financial Risk*, U. of Chicago Press, Chicago, IL.
- Lucas, D., and M. Phaup, 2010, The Cost of Risk to the Government and Its Implications for Federal Budgeting, in D. Lucas (ed.), *Measuring and Managing Federal Financial Risk*, U. of Chicago Press, Chicago, IL.
- Marchand, M., and P. Pestieau, 1984, Discount Rates and Shadow Prices for Public Investment, *Journal of Public Economics* 24, 153–169.
- Marglin, S.A., 1963, The Social Rate of Discount and the Optimal Rate of Investment, *Quarterly Journal of Economics* 77, 95–111.
- Miller, M.H., 1977, Debt and Taxes, *Journal of Finance* 32, 261–276.
- Modigliani, F., and M.H. Miller, 1958, The Cost of Capital, Corporate Finance, and the Theory of Investment, *American Economic Review* 48, 261–297.
- Modigliani, F., and M.H. Miller, 1963, Corporate Income Taxes and the Cost of Capital: A Correction, *American Economic Review* 53, 433–443.
- Sandmo, A. and J.H. Drèze, 1971, Discount Rates for Public Investment in Closed and Open Economies, *Economica* 38, 395–412.
- Schaefer, S.M., 1982, Taxes and Security Market Equilibrium, in W.F. Sharpe and C.M. Cootner (eds), *Financial Economics: Essays in Honor of Paul Cootner*, Prentice-Hall, Englewood Cliffs, NJ.
- Schmidt, K.M., 1996, The Costs and Benefits of Privatization: An Incomplete Contracts Approach, *Journal of Law, Economics, and Organization* 12, 1–24.
- Spackman, M., 2004, Time Discounting and the Cost of Capital in Government, *Fiscal Studies* 25, 467–518.
- Stapleton, R.C., and M.G. Subrahmanyam, 1978, Capital Market Equilibrium in a Mixed Economy, Optimal Public Sector Investment Decision Rules, and the Social Rate of Discount, *Quarterly Journal of Economics* 92, 399–411.

Taylor, S., 2007, *Privatisation and Financial Collapse in the Nuclear Industry: The Origins and Causes of the British Energy Crisis of 2002*, Routledge, New York, NY.

## Appendix 1: Proof of Proposition 2

We use a quasi-arbitrage argument to prove Proposition 2. Consider the consensus citizen who will receive the cash flow  $C - (1 + R_F)$  from the project. The citizen will hedge the systematic risk of that cash flow, as his prior allocation of risk was optimal and he wishes to conserve it. He achieves this by selling an amount  $\alpha$  of the market portfolio and investing the proceeds in the risk-free asset.<sup>32</sup> We assume that foreign investors are the net counterparties to the consensus citizen's trades.

We decompose the market return into its expectation and a random part with expectation zero:  $R_M = E[R_M] + \varepsilon$ . The condition that the citizen's two transactions, which have zero net cost, have zero wealth impact in all states at date 1 defines the government's required return.

The cash flow from the real investment with beta  $\beta$  can be written as:  $C = E[C] + \beta\varepsilon$ . The combination of this cash flow with the sale of an amount  $\alpha$  of the market portfolio and the offsetting purchase of  $\alpha$  of the risk-free asset results in the cash flow at date 1:

$$E[C] + \beta\varepsilon - (1 + R_F) - \alpha(1 + E[R_M]) - \alpha\varepsilon + \alpha(1 + R_F). \quad (\text{A1.1})$$

Setting  $\alpha = \beta$  eliminates the random terms and hedges the investor's payoff. The transaction is self-financing and results in a deterministic cash flow at date 1:

$$E[C] - (1 + R_F) - \beta(E[R_M] - R_F). \quad (\text{A1.2})$$

This expression must equal zero for the project to have zero NPV. This establishes the required return part of Proposition 2. A simple extension gives the NPV in the proposition.

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<sup>32</sup> The consensus citizen need not short sell equity but need only sell part of his holdings of the market portfolio.

## Appendix 2: Proof of Proposition 4

The proof is similar to that of Proposition 2, with the difference that after-tax returns and cash flows are used in place of their pre-tax equivalents. The consensus shareholder hedges the after-tax cash flow:

$$(1-T_{PE})\left[(1-T_C^R)E[C-1]-(1-T_C)LR_F\right]+(1-L)+\beta'(1-T_{PE})(R_M-E[R_M]). \quad (\text{A2.1})$$

The first term is the after-tax profit. The term  $1-L$  is the initial investment in equity, recovered at date 1. The last term is the after-tax, zero-mean, random component of the cash flow.

The consensus shareholder hedges this cash flow by selling an amount  $\alpha$  of the market portfolio and buying the same amount of the risk-free asset. His after-tax payoff is:

$$(1-T_{PE})\left[(1-T_C^R)E[C-1]-(1-T_C)LR_F\right]+(1-L)+\beta'(1-T_{PE})(R_M-E[R_M]) \\ -\alpha(1-T_{PE})(R_M-E[R_M])-\alpha\left[1+(1-T_{PE})E[R_M]\right]+\alpha\left[1+(1-T_{PD})R_F\right]. \quad (\text{A2.2})$$

Setting  $\alpha = \beta'$  eliminates the random terms. The expected cash flow corresponding to a zero-NPV project is  $E[C]$  such that the resulting payoff equals the payoff that the shareholder would have obtained had he invested the amount  $1-L$  in the risk-free asset rather than in the project:

$$(1-T_{PE})\left[(1-T_C^R)E[C-1]-(1-T_C)LR_F\right]-\beta'\left[(1-T_{PE})E[R_M]-(1-T_{PD})R_F\right]= \\ (1-T_{PD})(1-L)R_F. \quad (\text{A2.3})$$

The first term on the LHS is the after-tax profit from the real project. The second term is the after-tax expected return to the hedge position of size  $\beta'$ . The RHS is the after-tax opportunity cost of the investment.

The preceding equation can be rearranged in three different ways to obtain:

1. *The after-tax required rate of return on equity:*

$$\frac{(1-T_{PE})\left[(1-T_C^R)E[C-1]-(1-T_C)LR_F\right]}{1-L} = \\ (1-T_{PD})R_F + \frac{\beta'}{1-L}\left[(1-T_{PE})E[R_M]-(1-T_{PD})R_F\right], \quad (\text{A2.4})$$

where the RHS is (6) in Proposition 4; the sum of the numerator and the denominator on the LHS is the after-tax cash flow to equity (7) in the proposition: this is because it is the total cash flow

received by the shareholder that must be discounted, the sum of the net profit made (numerator) and the initial investment recovered (denominator).

2. *The after-tax required rate of return on the project:*

$$\frac{(1-T_{PE})(1-T_C^R)E[C-1]}{1} = (1-T_{PD})(1-L)R_F + \beta'[(1-T_{PE})E[R_M] - (1-T_{PD})R_F] + (1-T_{PE})(1-T_C)LR_F, \quad (\text{A2.5})$$

where the RHS is (8) in Proposition 4; the sum of the numerator and the denominator on the LHS is the after-tax cash flow from the project (9) in the proposition.

3. *The pre-tax required return on the project:*

$$\frac{E[C-1]}{1} = \left[ \frac{1-T_{PD}}{(1-T_{PE})(1-T_C^R)} \right] (1-L)R_F + \frac{\beta'}{1-T_C^R} \left[ E[R_M] - \left( \frac{1-T_{PD}}{1-T_{PE}} \right) R_F \right] + \left[ \frac{1-T_C}{1-T_C^R} \right] LR_F, \quad (\text{A2.6})$$

where the RHS is (10) in Proposition 4; the sum of the numerator and the denominator on the LHS is the pre-tax cash flow from the project  $E[C]$ .

### Appendix 3: Proof of Proposition 5

We define the auxiliary variables  $\theta_1 \equiv (1-T_{PE})(1-T_C^R)$ ,  $\theta_2 \equiv (1-T_{PE})(1-T_C)$ ,  $\theta_3 \equiv (1-T_{PD})$ , and  $\theta_4 \equiv (1-T_{PE})$ . We note that:

$$\tau_1 = L \left[ \frac{\theta_2}{\theta_1} - 1 \right], \tau_2 = \beta \left[ 1 - \frac{\theta_3}{\theta_4} \right], \tau_3 = (1-L) \left[ \frac{\theta_3}{\theta_1} - 1 \right], \quad (\text{A3.1})$$

from (16)–(18). The consensus shareholder has after-tax cash flow, hedged and net of investment cost:<sup>33</sup>

$$\begin{aligned} CF(\text{aftertax}) = & \theta_1 E[C-1] - \beta \theta_1 E[R_M] + \beta(1-T_{PD})(1-T_C^R)R_F \\ & - \theta_2 LR_F - \theta_3(1-L)R_F. \end{aligned} \quad (\text{A3.2})$$

The preceding after-tax cash flow can be decomposed into the sum of the corresponding pre-tax cash flow and the incremental tax paid (or saved) by the shareholder as a consequence of having undertaken the project and entered into its associated hedging transaction:

$$CF(\text{aftertax}) = CF(\text{pretax}) + \Delta TAX, \quad (\text{A3.3})$$

where:

$$CF(\text{pretax}) = E[C-1] + (\beta - \beta')Z - \beta'(E[R_M] - R_F) - LR_F - (1-L)R_F, \quad (\text{A3.4})$$

$$\begin{aligned} \Delta TAX = & -(1-\theta_1)E[C-1] - \beta(1-\theta_1)Z + \beta'T_{PE}Z \\ & - \beta'(T_{PD}R_F - T_{PE}E[R_M]) + (1-\theta_2)LR_F + T_{PD}(1-L)R_F, \end{aligned} \quad (\text{A3.5})$$

with:

$$Z \equiv R_M - E[R_M]. \quad (\text{A3.6})$$

Note that both the pre-tax cash flow and the incremental tax are stochastic: both depend on  $Z$ , the difference between the realized return on the market portfolio and its expectation. The consensus shareholder's hedging of the post-tax cash flow does not hedge the entirety of the pre-tax cash flow, for these two quantities differ by the amount of tax. The pre-tax cash flow therefore retains partial dependence on  $Z$ , as does tax, its own dependence on  $Z$  reflecting the role of the tax system as a risk-sharing mechanism between the public and the private sector.

The tax received by the public sector is the negative of that paid by the private sector, (A3.5). Similar to the case of a public sector project, the government hedges the stochastic part

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<sup>33</sup> The RHS of (A3.2) is the LHS–RHS difference of (A2.3).

of the tax on behalf of taxpayers. It does so with a synthetic forward contract that sells one dollar of the market portfolio and buys one dollar of bonds. This has zero net cost and a payoff at date 1 equal to  $R_F - E[R_M] - Z$ . Thus, the public sector exchanges at date 1 the stochastic cash flow  $Z$  for the deterministic cash flow  $R_F - E[R_M]$ .

Again considering a project that has zero NPV to the private sector, that is, a project for which the after-tax, hedged, net cash flow (A3.2) equals zero, the expected cash flow  $E[C-1]$  is such that:

$$E[C-1] = L \begin{bmatrix} \theta_2 \\ \theta_1 \end{bmatrix} R_F + \beta E[R_M] - \beta \begin{bmatrix} \theta_3 \\ \theta_4 \end{bmatrix} R_F + (1-L) \begin{bmatrix} \theta_3 \\ \theta_1 \end{bmatrix} R_F. \quad (\text{A3.7})$$

Substituting  $Z = (R_F - E[R_M])$  and (A3.7) into the negative of (A3.5) yields:

$$\Delta TAX = (\tau_1 + \tau_2 + \tau_3) R_F. \quad (\text{A3.8})$$