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JEL Classification: N/A

Keywords: Financial conditions index; Coincident economic index; Dynamic factor model; Markov switching; Business cycle; Bayesian inference

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Modeling of Economic and Financial Conditions for Nowcasting and Forecasting Recessions: A Unified Approach

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This paper puts forward a unified framework for the joint estimation of the indexes that can broadly capture economic and financial conditions together with their cyclical regimes of recession and expansion. We do this by utilizing a dynamic factor model together with Markov regime switching dynamics of model parameters that specifically exploit the temporal link between the cyclical behavior of economic and financial factors. This is achieved by constructing the cycle in the financial factor using the cycle in the economic factor together with phase shifts. The resulting framework allows the financial cycle to potentially lead/lag the business cycle in a systematic manner and exploits the information in economic and financial variables for estimation of both economic and financial conditions as well as their cyclical behavior in an efficient way. We examine the potential of the model using a mixed frequency and mixed time span ragged-edge dataset for Turkey. Comparison of our framework with more conventional polar cases imposing a single common cyclical dynamics as well as independent cyclical dynamics for economic and financial conditions reveal that the proposed specification provides precise estimates of economic and financial conditions and it delivers quite accurate probabilities of recessions that match with stylized facts. We further conduct a recursive real-time exercise of nowcasting and forecasting business cycle turning points. The results show convincing evidence of superior predictive power of our specification by signaling oncoming recessions (expansions) as early as 3.5 (3.4) months ahead of the actual realization.

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1 Introduction

Monitoring economic and financial activity and anticipating economic downturns in a timely manner is of key importance for economic agents. Accordingly, several econometric models have been put forward to construct accurate measures of economic and financial conditions that can capture and predict economic downturns. Such methodologies involve modeling the comovement in the behavior of a large number of economic and financial variables following the notion of indicators of economic and financial conditions involving business and financial cycles.

Perhaps surprisingly, economic and financial indicators are often measured in isolation of each other. Typically, the link between the two measures are established by pre-selecting financial variables according to their predictive capability of macroeconomic aggregates, i.e. GDP or industrial production. Such a sequential procedure using individual variables often obscures the link between economic and financial cycles by not necessarily fully exploiting the common information in economic and financial variables on signaling future recessions. The reason is that the individual predictive capability of key aggregates does not necessarily translate into the predictive capability of economic conditions and, even more crucially, recessions. However, the recent global recession together with its underlying financial roots has made understanding the impact of financial conditions on (the turning points of) real activity a key requirement for predicting the evolution of both developed and emerging economies.

In this paper, we propose a methodology where we jointly estimate economic and financial conditions by exploiting the intertemporal link between their cyclical behavior explicitly. Specifically, our methodology combines a dynamic factor model for the joint modeling of economic and financial variables with mixed frequencies together with Markov regime switching model parameters for modeling the cyclical behavior embedded in economic and financial conditions. Crucially, our specification for regime dynamics allows for imperfect synchronization between the cycles embodied in economic and financial conditions/factors by explicitly estimating the phase shifts between the cyclical regimes. This enables us to fully exploit a rich dataset of economic and financial variables for estima-

tion of economic and financial conditions and, even more importantly, for nowcasting and forecasting economic downturns accurately in real time.

We employ our methodology using a dataset of emerging markets, most notably, of Turkey. In this sense, another contribution of our paper is to extend current methodologies that are mostly suitable for datasets of developed countries, especially for the US, to the case of emerging markets with distinct complications that plague econometric inference. While in most of the cases there is lack of data, existing datasets are typically subject to structural breaks as these countries have undergone massive structural transformations and have more noisy structures due to relatively more shallow financial markets. We circumvent these obstacles by integrating flexible structures into the modeling framework that can take potential structural breaks and extreme observations into account.

Our results using a mixed frequency dataset (with different time spans for the earliest case) starting from January 1999 until November 2017 indicate that the model can nicely capture the recessions at the onset of 2000's and 2008 together with a single structural break in September 2001, which is in line with stylized facts. As for the phase shifts, the estimation results reveal that financial indicators enter recessions (expansions), on average, 3.5 (3.4) months earlier than the recession (expansion) of economic indicator. Finally, we conduct a recursive nowcasting/forecasting exercise in real time and document that the recent recession can be predicted much more timely using our methodology compared to a model with independent cycles and to a model with single common cycle.

The comovement in the behavior of a large number of economic series was noted by Burns and Mitchell (1946) in their quest to define a 'business cycle'. Following the seminal papers of Sargent and Sims (1977) and Stock and Watson (1989) who construct a monthly coincident indicator of (US) real activity summarizing the behavior of a large number of macroeconomic series representing broadly the demand and supply sides and labor market conditions, dynamic factor models have been the major workhorse of the empirical research on business cycles. Diebold and Rudebusch (1996), Chauvet (1998), Kim and Nelson (1998), among others, integrate a Markov mixture structure of Hamilton (1989) into the dynamic factor structure to capture distinct dynamics of the different phases of the business cycle, i.e. expansions and recessions, endogenously. A recent generation of the factor

models include a large number of variables with mixed frequencies and potentially mixed time span thanks to the unobserved components modeling framework that can handle missing values in a statistically optimal way, see for example, Mariano and Murasawa (2003) and Aruoba *et al.* (2009) who develop monthly and weekly coincident indicators using such datasets for US. A similar approach is followed by Banbura *et al.* (2013), though not for the extraction of the business cycle but for 'nowcasting' and forecasting key macroeconomic aggregates using multiple factors, see Bok *et al.* (2018) for a recent review on this vein of research.

While papers focusing on modeling and nowcasting/forecasting economic conditions are abundant, the research on financial conditions remain relatively limited prior to great recession of 2008, see Kaminsky and Reinhart (1999) for example. However, the worldwide financial crisis of 2008 has demonstrated that developments in financial markets may have unforeseen effects on the overall functioning of the economic system by deepening the link between financial and economic conditions; see Borio (2012); Gourinchas and Obstfeld (2012), among others. The rapidly growing complexity of financial systems and the evolving nature of their interaction with real markets pose many new challenges for policymakers and private agents alike. Financial prices are important not only because they reflect wealth, which enters into key consumption and investment relationships, but also because they incorporate market expectations of future price and output development; see, for example, Mayes and Virén (2001). Consequently, understanding the behavior of key financial variables such as credits, asset prices, their volatilities, interest rate spreads, and risk indicators of various sorts and establishing their link with economic conditions has gained importance; see for example Claessens *et al.* (2012) for an extensive analysis on cyclical behavior of those variables. Therefore, several financial conditions indexes (FCI) has been developed using such key financial variables to examine the role of financial factors in determining future real activity. The literature on financial condition indexes is summarized by Hatzius *et al.* (2010), who provide an extensive review and a comparison of alternative indexes, typically published by financial industry and central banks, that are available for the US and the EU. In their study, they construct an FCI for US that includes a large array of risk measures and conventional financial variables such as interest rates

and asset prices, and show that their measure of financial conditions is tightly related to future economic conditions. Other examples include Wacker *et al.* (2012) who construct several indexes of financial conditions for major non-Euro Area economies.

A number of papers consider estimating the predictive capability of the leading indicators involving many financial variables in predicting the recessions using US GDP and/or coincident and leading indicators provided by the Conference Board. Using quarterly GDP and the US leading indicator, Hamilton and Perez-Quiros (1996) provide a framework in Markov regime Switching Vector AutoRegression (MS-VAR) context to model the lead/lag relation between the regimes of corresponding variables where they allow for the turning points of (the growth rate of) the leading indicator to lead that of the US GDP. Their findings suggest that the leading indicator can predict the business cycle peaks and troughs one quarter ahead. Paap *et al.* (2009) extend this model by allowing for distinct lead times for peaks and troughs using the coincident indicator of the Conference Board in place of the GDP and conclude that this lead time escalates to almost 12 months for business cycle peaks. Finally, Çakmaklı *et al.* (2011) and Çakmaklı *et al.* (2013) provide a framework allowing for distinct phase shifts in the timing of multiple regimes and together with regime dependent correlations and volatilities. They conclude that the 12 months lead time shrinks considerably to 6 months in predicting more severe recessions such as the recent global crisis of 2008-9 compared to previous mild recessions.

Our methodology combines several modeling approaches in a unified framework. First, following Aruoba *et al.* (2009) and Banbura *et al.* (2013) we employ a dynamic factor model framework using a real time ragged-edge Turkish dataset comprised by the variables with mixed frequencies and with varying time spans to circumvent the problems related to lack of data often confronted for emerging markets. Second, departing from these studies, we incorporate Markov regime dynamics into the dynamic factor model structure. Third, we allow for potential phase shifts of the business cycle for modeling the financial cycle by explicitly estimating the temporal link between the cyclical dynamics of the coincident and financial indicators/factors. We use simulation based Bayesian inference for joint estimation of all of these features in a unified framework. Bayesian inference provides an appealing and tractable way for econometric inference as it takes the uncertainty related

to unobserved features including factors, regimes and phase shifts in addition to missing values and model parameters into account. This might be even more crucial for emerging markets, where data are scarce and potentially more noisy.

Our model differs from competing approaches in various ways. An alternative specification could be to proceed with the unobserved components modeling framework by specifying common or idiosyncratic cycle components of the series that are modeled using trigonometric specifications, as in the similar cycle model of Koopman and Harvey (1997). Koopman *et al.* (2016) provide such a decomposition of business and financial cycle using a panel of economic and financial time series of developed countries. In this case, one still needs to identify the regimes for the estimated cycles for capturing the phase shifts for specific phases of recessions and expansions. Our motivation for adopting discrete (Markov) regime switching is that the conventional discrete turning point approach is intrinsically embedded in these models, which also enables us to estimate distinct phase shifts varying according to the type of regimes.

Despite the large number of applications of construction of indexes of economic and/or financial conditions in the context of the US or the countries of the euro area[1], there are relatively few applications for emerging market economies. While Atabek *et al.* (2005) construct a composite leading indicator for the Turkish economy using seven demand, supply and policy indicators over the period 1987-2005, Aruoba and Sarikaya (2013) develops a monthly indicator of real economic activity using multiple indicators at mixed frequencies by employing the dynamic factor model proposed in Aruoba *et al.* (2009). Our model clearly differs from those as we provide a unified framework for joint estimation of coincident and financial indicators together with their potentially imperfectly synchronized cyclical regimes due to phase shifts of the business cycle.

The remainder of this paper is as follows. Section 2 presents the model and data. Section 3 describes the estimation approach. Section 4 presents the empirical results and discusses real-time estimation and forecasting. Finally, Section 5 concludes.

[1]See Banbura *et al.* (2013) for a discussion of the different applications.

2 The Model

In this section we present the dynamic factor model for extraction of the coincident economic index (CEI) and financial conditions index (FCI) from a broad set of variables with mixed frequency and mixed time span. The cyclical phases of the indicators, i.e. recessions and expansions, are captured endogenously by a single Markov process. The key feature of the models is that we estimate the intertemporal links between the cyclical regimes of the CEI and FCI by means of the phase shifts of the single common cycle, i.e. the business cycle.

Let $X_{i,t}$ denote the observation of the ith variable in period t for $i = 1, \ldots, N$. We assume that the variable potentially has a deterministic or stochastic trend together with higher frequency components, factors, denoted as F_t, that are common across all variables. Without loss of generality, assuming a deterministic trend, we can write

$$X_{i,t} = \gamma_{i,0} + \gamma_{i,1} t + \gamma_{i,2} F_t + e_{i,t}. \tag{1}$$

We transform variables with required transformations to remove the trend and to achieve stationarity. The resulting specification is as follows

$$y_{i,t} = \gamma_{i,1} + \lambda_i f_t + \varepsilon_{i,t}, \tag{2}$$

where λ_i is the loading of the ith (transformed) variable, $y_{i,t}$, on the common factor(s) f_t. We allow the idiosyncratic component to follow an autoregressive dynamics as

$$\psi_i(L)\varepsilon_{it} = \epsilon_{i,t}. \tag{3}$$

The fact that we use a broad dataset involving stock and flow variables with missing observations implies some care in the handling of the data. Here we follow the practice in Banbura *et al.* (2013), which we discuss briefly only for case of quarterly data being used for the estimation of monthly factors and refer Banbura *et al.* (2013) for further details. Consider the transformation of variables measured at the quarterly frequency, denoted by $X^Q_{i,t}$ and by $y^Q_{i,t}$ as its first difference, to the monthly frequency. For stock variables, this would imply missing observations for all periods excluding the corresponding period of the observation. For flow variables, however, temporal aggregation should be taken

into account. Specifically, for the differenced variables, the transformation to the higher frequency is implemented as follows

$$\begin{aligned} y_{i,t}^{Q} &= X_{i,t}^{Q} - X_{i,t-3}^{Q} = \textstyle\sum_{k=0}^{2} X_{i,t-k} - \sum_{k=0}^{2} X_{i,t-k-3} \\ &= y_t + 2y_{t-1} + 3y_{t-2} + 2y_{t-3} + y_{t-4}. \end{aligned} \tag{4}$$

For the log-differenced variables, we use the approximation in (Mariano and Murasawa, 2003) in line with Banbura *et al.* (2013), which allows us to use (4) also for those variables.

So far, the model is similar to the methodologies employed for the developed countries as in Aruoba *et al.* (2009) and Banbura *et al.* (2013). Departing from these studies we add two modifications to the general framework to capture the characteristics that are specific to emerging markets. First, as is the case for many of the emerging markets, Turkey has undergone a massive structural change and moderation at the onset of the 2000's, similar to the Great Moderation[2]. To reflect such a moderation we allow for a single structural break in the variances of the variables as

$$\sigma_{i,t}^2 = \sigma_{i,1}^2 \, \mathbb{I}\left[t < \tau\right] + \sigma_{i,2}^2 \, \mathbb{I}\left[t > \tau\right], \tag{5}$$

where τ is the period of the structural break to be estimated and $\mathbb{I}[.]$ denotes the indicator function, which takes the value 1 if the condition in brackets is true and 0 otherwise.

Second, data for emerging market economies often embrace more aberrant observations compared to those for developed economies with deeper financial markets. Considering this, we model the distribution of the variables, $\epsilon_{i,t}$ with a $t-$distribution with variance $\sigma_{i,t}^2$ and ν degrees freedom. We note that the $t-$distribution with ν degrees of freedom is essentially a scale mixture of normal distribution that leads to the following specification

$$\epsilon_{i,t} = \xi_t^{-1/2} \sigma_{i,t} \zeta_t, \tag{6}$$

where ζ_t follows a standard normal distribution. When ξ_t follows a Gamma distribution with $\Gamma(\frac{\nu}{2}, \frac{\nu}{2})$, then $\epsilon_{i,t}$ follows a Student's $t-$distribution with ν degrees of freedom and accordingly $\epsilon_{i,t}|\xi_t \sim N(0, \sigma_{i,t}^2/\xi_t)$, see Geweke (1993), Geweke (2005) for textbook exposi-

[2]The Great Moderation refers to the reduction in the volatility of many key macroeconomic aggregates experienced in US and in many developed countries during the mid-1980's.

tions and Curdia *et al.* (2014) for an application in the context of structural macroeconomic models.

We conclude the specification of the factor structure in the data by describing the assumptions required for the identification of the factors, since both factors and the loadings are unobserved as in (2). First, we set the number of factors as two and the loadings of the financial (coincident) variables that loads on the first (second) factor as zero to identify the first factor as CEI and the second as FCI. Second, we standardize the dataset and we restrict the unconditional variance of the factors to be one for identification of the scale and location of the factors following Sargent and Sims (1977), Stock and Watson (1989) and Stock and Watson (1993), for example[3].

Next, we proceed with the specification of the evolution of factors, f_t, which are comprised by (the growth rate of) the coincident economic and financial conditions indexes. We specify an autoregressive process for the factors with intercept parameter depending on the cyclical regime of the corresponding factor. Specifically, in case of first-order autoregressive dynamics for the factors, our assumptions imply the model specification

$$f_t = \alpha_{\mathbb{S}_t} + \Phi f_{t-1} + \eta_t \quad \eta_t \sim N(0, \Sigma), \tag{7}$$

where

$$f_t = \begin{pmatrix} f_{1,t} \\ f_{2,t} \end{pmatrix}, \mathbb{S}_t = \begin{pmatrix} S_{1,t} \\ S_{2,t} \end{pmatrix}, \alpha_{\mathbb{S}_t} = \begin{pmatrix} \alpha_{1,S_{1,t}} \\ \alpha_{2,S_{2,t}} \end{pmatrix}, \eta_t = \begin{pmatrix} \eta_{1,t} \\ \eta_{2,t} \end{pmatrix}, \Phi = \begin{pmatrix} \phi_{1,1} & \phi_{1,2} \\ \phi_{2,1} & \phi_{2,2} \end{pmatrix}, \Sigma = \begin{pmatrix} \sigma^2_{f_1} & \sigma_{1,2} \\ \sigma_{2,1} & \sigma^2_{f_2} \end{pmatrix}.$$

Here $S_{l,t}$, $l = 1, 2$ are latent binomial variables taking the value 0 (1), if $f_{l,t}$ is in expansion (recession) at time t representing the cyclical regimes embedded in economic and financial factors. We assume that $S_{1,t}$ and $S_{2,t}$ are governed by the first-order Markov processes with transition probabilities as

$$\begin{aligned} Pr(S_{l,t} = 0 \mid S_{l,t-1} = 0) &= q_l \\ Pr(S_{l,t} = 1 \mid S_{l,t-1} = 1) &= p_l \quad \text{for } l = 1, 2. \end{aligned} \tag{8}$$

[3]Alternatively, Bernanke *et al.* (2005) and Bańbura and Modugno (2014), among others, set the upper $N \times k$ part of the matrix of factor loadings to identity, where N (k) is the number of variables (factors), to set the factor orientation according to the order of the variables. Such a strategy is prone to the ordering of variables which might even be more sensitive in our application for emerging markets. See also Del Negro and Otrok (2008) and Bai and Wang (2015) for alternative identification schemes.

In line with the regime specifications as expansion and recession, we restrict $\alpha_0 > \alpha_1$, an assumption which we discuss further when we specify the prior distributions. As our methodology involves the joint estimation of the factors, we need to specify the intertemporal links between the cyclical dynamics of the CEI and FCI. Note that (7) already indicates a linear association between the factors by means of cross-autoressive coefficients. Essentially, we are after the nonlinear associations between these factors by specifying the link between the cyclical regimes.

Without loss of generality, we assume that $f_{1,t}$, i.e. the CEI, is the 'reference series' and we define the properties of $S_{2,t}$, the regime indicator of $f_{2,t}$, i.e. the FCI, relative to $S_{1,t}$. Different specifications of the relation between the two Markov processes $S_{1,t}$ and $S_{2,t}$ imply different types of relations between the cycles of the two indicators. We start the analysis with two extreme cases. First, we can assume that the cycles embedded in economic and financial conditions are independent. Note that this specification still does not rule out the synchronization of the cycles completely as the two cycles are in the same regime with probability

$$\Pr(S_{2,t} = S_{1,t}) = p_1 p_2 + q_1 q_2 > 0. \tag{9}$$

Second, we can assume that the cycle in both indicators are identical, that is,

$$S_{2,t} = S_{1,t}, \tag{10}$$

or, put differently, there is a single cycle governing both indexes. Following Harding and Pagan (2006), we refer this case as 'perfect synchronization' (PS).

In practice, the relation between the two cycles of economic and financial conditions may not be perfect but still higher than the expected level under independence as shown in (9). In fact, as stated in Hatzius *et al.* (2010), financial conditions often lead business cycle, thus, neither perfect synchronization nor independence may be adequate representations of the relation between these two cycles. Following Paap *et al.* (2009) and Çakmaklı *et al.* (2011), we model the intermediary cases to allow for the cycle in the FCI to lead/lag the cycle in CEI by $\kappa_{S_{1,t}}$ periods, i.e.

$$S_{2,t-\kappa_{S_{1,t}}} = S_{1,t}. \tag{11}$$

Specifically, to specify the cycle in the FCI, we assume that the regime indicator $S_{1,t}$ itself is shifted but allow the amount of phase shift to be different across regimes. The subscript $S_{1,t}$ to κ indicates that the regime indicator is shifted by a possibly different number of time periods for each regime. Hence, this specification involves a separate regime shift parameter κ_j for expansions and recessions. To put things differently, we assume that the lead/lag time is different per regime, such that each regime in the other series starts later or earlier by κ_j periods. This specification is denoted as imperfect synchronization of the cycles with 'asymmetric' phase shifts (APS). When $\kappa_{S_{1,t}} \equiv \kappa$, that is, when phase shifts are restricted to be identical across regimes, the model boils down to the case of imperfect synchronization with 'symmetric' phase shifts (SPS) of Hamilton and Perez-Quiros (1996).

The specification in (11) is not sufficient though, as it may lead to situations where for some time periods $S_{2,t}$ is assigned multiple values or is not defined at all. In these cases, the regime with the larger amount of phase shift is assigned to such conflicting periods ensuring that $S_{2,t}$ is assigned only a single regime and each regime starts with a phase shift of κ_j periods relative to $S_{1,t}$. This implies that in case of $\kappa_0 \leq \kappa_1$ when a transition from recession to expansion takes place, then recessions in FCI start κ_0 periods before recessions in CEI and end κ_1 periods earlier. Consequently, recessions in FCI are $\kappa_1 - \kappa_0$ periods shorter than recessions in CEI.

Combining (2),(3) and (7) together with (5)-(11) we can specify the final model as

$$\begin{aligned}
y_{i,t} &= \lambda_i f_t + \varepsilon_{i,t} \\
\psi(L)\varepsilon_{i,t} &= \epsilon_{i,t} \quad \epsilon_{i,t} \sim t(0, \nu, \sigma^2_{i,t}) \\
\sigma^2_{i,t} &= \sigma^2_{i,1}\, \mathbb{I}\,[t < \tau] + \sigma^2_{i,2}\, \mathbb{I}\,[t > \tau] \;\text{ for } i = 1, \ldots, N \\
f_t &= \alpha_{\mathbb{S}_t} + \Phi f_{t-1} + \eta_t \quad \eta_t \sim N(0, \Sigma) \\
S_{2,t-\kappa_{S_{1,t}}} &= S_{1,t}.
\end{aligned} \tag{12}$$

2.1 Data

We use a comprehensive set of variables for the estimation of the CEI and FCI using our model framework described in (12). However, many of the variables suffer from missing observations. The presence of missing observations stems from the facts that we use data

with mixed frequency leading to periodically missing observations, we use data with mixed time span leading to successive periods with missing observations, and finally, we use data exhibiting lags in their releases leading to missing observations at the end of the dataset called as ragged-edge. Following our focus on the extraction of measures of coincident and financial conditions together with their cyclical regimes of recessions and expansions and given the information content of the dataset that mostly involves monthly and quarterly variables, we design the model to estimate 'monthly' indicators.

For construction of the CEI, we follow the common practice of choosing variables that broadly represent different aspects of the economy, see for example Stock and Watson (1989) or Kim and Nelson (1998). These include variables such as industrial production, employment, trade and sales, measures of income, etc.. In addition, we make use of variables related to the trade balance considering the small open economy characteristics of Turkey. An important remark is on the use of GDP, which is the prominent measure of economic conditions in conventional applications. We note that the national accounts in Turkey have undergone a substantial revision in 2016 and the discussion of the accuracy of this revision has not reached a consensus. This is due the fact that not only the levels but also the growth rates of old and new series substantially diverge, see the discussion in Yilmaz *et al.* (2017). Therefore, we exclude this series in our analysis to preclude any potential bias in our analysis[4]. In the final set of coincident variables, we include the industrial production index (*ip*) and the purchasing manager index (*pmi*) representing the production side of the economy, total non-agricultural employment (*empna*) representing labor markets, the trade and services turnover index (*traserv*) and the retail sales volume index (*retails*) representing trade and sales and finally, the total export and import quantity indexes (*export* and *import*) which are less prone to the nominal fluctuations.

While the dataset used for construction of economic indicators is settled, this is not the case for construction of financial indicators. Typically, common practice on selecting variables for construction of such indexes involves choosing those series that represent financial side of the economy together with the ability to predict future real activity, see

[4]Still, our robustness checks suggest that the model estimates with old/new GDP series are very similar. The results are available upon request.

for example, Hatzius *et al.* (2010). The predictive ability is often measured in terms of success of predictive regressions with a quadratic loss function, i.e. the mean squared forecast error criterion. Still, a few studies using individual series examine the predictive ability to predict US recessions using econometric methods suited for binary variables of NBER recession dates, see Estrella and Mishkin (1997) and Kauppi and Saikkonen (2008) on predictive ability of interest rate spreads for predicting recessions and Estrella and Mishkin (1998) for a similar analysis using various financial variables. In our analysis, first, we construct a large dataset comprised of the variables representing stock markets, bond markets, interest rates and related spreads, variables related to banking system including debt stock and credits, foreign exchange rates, measures related to the monetary base, various financial risk measures and economic confidence indicators, and rates related to major international markets. A brief description of these variables together with the set of economic variables is provided in Appendix A. We, then, take the advantage of our unified modeling approach of constructing both indexes jointly and we conduct an analysis using several combinations of variables from each group. We evaluate the variables based on the conformity of the findings with the stylized facts and based on ability to predict recessions. The final set of variables includes the stock market index (BIST100) in real terms (*rbist*), price-earnings ratio of the portfolio (*P-E*) used for computing the BIST100, the MSCI emerging market index (*MSCIem*)[5], realized volatility of BIST100 representing stock markets (*VOL*), the treasury auction rate (*TAuc*) representing (sovereign) bond markets, various spreads including the term spread (*TermS*) computed as the spread between the interest rate on deposits - up to 1 year and more and the interest rate on deposits up to 1 month[6], the TET spread (*TETS*) computed as the difference between the 3-month interest rate on deposits and 3-month LIBOR capturing the credit risk in financial markets, the spread between the JP Morgan Emerging Markets Bond Index[7] and the 1-month interest rate on deposits (*EMBI-Tr*) representing various sources of risk, the Central Bank of the

[5]MSCI emerging market index is a broad stock market index encompassing all emerging markets serving as a measure of the risk appetite to emerging economies.

[6]We use the interest rates on deposits rather than the sovereign bond (zero-coupon) yields for computing the term spread. This is mainly due to the fact that short-term sovereign bonds possess limited liquidity.

[7]JP Morgan Emerging Markets Bond Index is a broad bond market index encompassing all emerging markets serving as a measure of the cost of funding for emerging markets.

Republic of Turkey (CBRT)'s gross foreign exchange reserves in real terms (*FXRes*), the confidence index of CBRT (*Conf*) and banking sector credit loans (*Cred*), which have been a central focus of the discussions surrounding the predictability of recessions given the failure to predict the recent global crisis, see Gadea and Perez-Quiros (2015), for example.

3 Estimation

The model specified in (12) is a special case of the unobserved components model together with (Markov) regime dependent parameters, as neither the factors, i.e. economic and financial indicators nor regimes and the phase shifts are observed. Since we conduct a joint estimation strategy of all these unobserved components taking the uncertainty related to these components fully into account, classical inference is not feasible due to discrete nature of the phase shift. Therefore, we adopt a Bayesian approach for estimation and inference and we make use of Markov Chain Monte Carlo (MCMC) techniques. Specifically, we use Metropolis within Gibbs sampling together with data augmentation (see Metropolis *et al.*, 1953; Geman and Geman, 1984; Tanner and Wong, 1987) for posterior inference. In Section 3.1 we derive the likelihood function of the model, while we discuss the specifications of the prior distributions in Section 3.2. In Section 3.3 we outline the resulting algorithm for simulating from the posterior distribution. Full details on the model specification and conditional posterior distributions given in Sections B and C of the supplementary material for the sake of brevity.

3.1 Likelihood Function

The fact that the dynamic factor model involves regime dependent parameters governed by a Markov process, we need to derive the complete data likelihood function. To do this, first, we cast the model in (12) into state-space form as

$$\begin{aligned} \mathbf{y}_t &= \mathbf{H}\boldsymbol{\beta}_t + \boldsymbol{\varepsilon}_t & \boldsymbol{\varepsilon}_t|\xi_t \sim N(\mathbf{0}, \mathbf{R}_t) \\ \boldsymbol{\beta}_t &= \boldsymbol{\alpha}_{\mathbb{S}_t} + \mathbf{F}\boldsymbol{\beta}_{t-1} + \boldsymbol{\eta}_t & \boldsymbol{\eta}_t|\xi_t \sim N(\mathbf{0}, \boldsymbol{\Omega}_t), \end{aligned} \tag{13}$$

where $\mathbf{y}_t = (y_{1,t}, \ldots, y_{i,t}, \ldots, y_{N,t})'$, $\mathbf{H}$ is comprised by the factor loadings with the specific location and form depending on the frequency and on the type as flow and stock

of the corresponding variable. $\mathbf{R}_t$ is the diagonal matrix with conditional variances of the variables on the diagonal. The state vector $\boldsymbol{\beta}_t$ includes $f_t = (f_{1,t}, f_{2,t})'$, i.e. factors representing the coincident and financial indicators, as well as error components $\varepsilon_{i,t}$ as idiosyncratic factors and their lags. $\mathbf{F}$ is comprised by the autoregressive coefficients of the coincident and financial factors as well as idiosyncratic factors and accordingly $\boldsymbol{\Omega}_t$ includes the variances (and covariances) of these factors. The time variation in $\mathbf{R}_t$ as well as $\boldsymbol{\Omega}_t$ stems from the fact that we allow for a single structural change for the variances of the variables. Notice that these variances are scaled by the Gamma distributed elements of $\xi_t = (\xi_{1,t}, \ldots, \xi_{i,t}, \ldots, \xi_{N,t})'$ leading to a $t-$distribution as discussed earlier. Finally, regime dependent parameters, $\boldsymbol{\alpha}_{\mathbb{S}_t}$ includes $\alpha_{1,S_{1,t}}$ and $\alpha_{2,S_{2,t}}$. Conditional on the model parameters and regimes we can proceed with standard inference of the linear Gaussian state-space models by running the Kalman filter. Still, before running the Kalman filter a slight modification to the system is required for handling missing observations. This is simply achieved by creating a selection matrix, $\mathbf{W}_t$, that is a diagonal matrix with the ith diagonal element taking the value 1 if $y_{i,t}$ is observed and 0 otherwise. Kalman filter, then, can be run by replacing $\mathbf{y}_t$, $\mathbf{H}$ and $\mathbf{R}$ with $\mathbf{y}_t^* = \mathbf{W}_t\mathbf{y}_t$, $\mathbf{H}^* = \mathbf{W}_t\mathbf{H}$ and $\mathbf{R}_t^* = \mathbf{W}_t\mathbf{R}_t\mathbf{W}_t^{'}$, respectively as

$$\begin{aligned}
\boldsymbol{\beta}_{t|t-1} &= \boldsymbol{\alpha}_{\mathbb{S}_t} + \mathbf{F}\boldsymbol{\beta}_{t-1|t-1} \\
\mathbf{P}_{t|t-1} &= \mathbf{F}\mathbf{P}_{t-1|t-1}\mathbf{F}^{'} + \boldsymbol{\Sigma} \\
\mathbf{v}_{t|t-1} &= \mathbf{y}_t - \mathbf{H}^*\boldsymbol{\beta}_{t|t-1} \\
\mathbf{V}_{t|t-1} &= \mathbf{H}^*\mathbf{P}_{t|t-1}\mathbf{H}^{*'},
\end{aligned} \tag{14}$$

to compute the prediction error, $\mathbf{v}_{t|t-1}$, and its variance, $\mathbf{V}_{t|t-1}$. Let $\mathbf{y}^T = \{\mathbf{y}_1, \ldots, \mathbf{y}_i, \ldots, \mathbf{y}_T\}$ and $\mathbb{S}^T = \{\mathbb{S}_1, \ldots, \mathbb{S}_i, \ldots, \mathbb{S}_T\}$, then, the complete data likelihood can be written as

$$f(\mathbf{y}^T, \mathbb{S}^T|\boldsymbol{\theta}) = \left(\prod_{i=1}^{2}\prod_{j=1}^{2} p_{ij}^{T_{ij}}\right)\prod_{t=1}^{T}\left(\frac{1}{\sqrt{2\pi}}\right) \mid \mathbf{V}_{t|t-1} \mid^{-\frac{1}{2}} \exp\left(-\frac{1}{2}\sum_{t=1}^{T}\mathbf{v}_{t|t-1}{}'\mathbf{V}_{t|t-1}{}^{-1}\mathbf{v}_{t|t-1}\right), \tag{15}$$

where T_{ij} is the number of transitions from regime i to regime j and $P = \{p_{ij}\}_{i,j=1}^2$ is the matrix with transition probabilities. $\theta = (\text{vec}(\Phi)', \alpha', \lambda', \sigma^{2\prime}, \psi', \text{vec}(P)', \kappa, \text{vec}(\Sigma)')'$ represent all model parameters with $\alpha = (\alpha_{1,0}, \alpha_{1,1}, \alpha_{2,0}, \alpha_{2,1})'$, $\lambda = (\lambda_1', \ldots, \lambda_i', \ldots, \lambda_N')'$ where $\lambda_i = (\lambda_{i,1}, \lambda_{i,2})'$, $\sigma^2 = (\sigma_1^{2\prime}, \ldots, \sigma_i^{2\prime}, \ldots, \sigma_N^{2\prime})'$ where $\sigma_i^2 = (\sigma_{i,1}^2, \sigma_{i,2}^2)'$ and $\psi = (\psi_1', \ldots, \psi_i', \ldots, \psi_N')'$ where $\psi_i = (\psi_{i,1}, \ldots, \psi_{i,p})'$ where p is the lag order of the autore-

gressive process for the idiosyncratic factors and finally $\kappa = (\kappa_1, \kappa_2)'$. The likelihood function conditional only on the model parameters can be obtained by summing (15) over all the possible states

$$f(\mathbf{y}^T|\boldsymbol{\theta}) = \sum_{S_{1,1}=1}^{2} \sum_{S_{2,1}=1}^{2} \cdots \sum_{S_{T,1}=1}^{2} f(\mathbf{y}^T, \mathbb{S}^T|\boldsymbol{\theta}). \tag{16}$$

3.2 Prior Distributions

We use diffuse prior for the abundance of the parameters in order to let the data be decisive for estimation results. For the discrete parameters this can be achieved using proper priors but for the continuous parameters this strategy leads to use of improper priors.

For the phase shifts parameters, $\boldsymbol{\kappa} = (\kappa_1, \kappa_2)$, we use a uniform prior assigning equal probability to each value of $\boldsymbol{\kappa}$ in a predefined set

$$f(\boldsymbol{\kappa}) \propto \begin{cases} 1 & \text{for all } (\kappa_1, \kappa_2) \in \mathcal{C}, \\ 0 & \text{otherwise.} \end{cases} \tag{17}$$

The set $\mathcal{C} = \{(\kappa_1, \kappa_2) \in \mathbb{Z}^2 \mid -c \leq \kappa_j \leq c \text{ for } j = 1, 2, \ |\kappa_1 - \kappa_2| \leq d\}$ specifies the restrictions imposed on κ_1 and κ_2. Specifically, we set $c = 8$ and $d = 6$ implying that κ_1 and κ_2 are restricted to lie in the interval $[-8, 8]$ and their difference is restricted not to exceed 6.[8] Note that setting $d = 0$ implies identical κ parameters which reduces the model in (12) to the model of Hamilton and Perez-Quiros (1996) and setting additionally $c = 0$ leads to the model with single common cycle. See Çakmaklı *et al.* (2011) for more details.

For the transition probabilities, we use an informative Beta prior such that 95% highest posterior density interval covers the domain of 0.9 to 1 to match the duration of the recession and expansions with stylized facts.

The prior for the regime dependent intercept parameters α is specified using improper distributions with sign restrictions as

$$f(\alpha_l) = \begin{cases} 1 & \text{if } \alpha_l \in \{\alpha_l \in \mathbb{R}^2 \mid \alpha_{l,0} > \alpha_{l,1}\} \\ 0 & \text{elsewhere.} \end{cases} \tag{18}$$

for $l = 1, 2$ to identify expansions and recessions as discussed in Section 2. For the matrix

[8] We experiment with various setups. Results are quite similar and available upon request. Setting these values to sensibly small values that are not affecting the results facilitates the computation substantially.

of autoregressive coefficients of common factors, Φ, and for the vector of autoregressive coefficients of idiosyncratic factors, ψ, we use flat priors

$$f(\Phi) \propto 1 \quad \text{and} \quad f(\psi_i) \propto 1 \quad \text{for } i = 1, \ldots, N \tag{19}$$

if the condition that characteristic roots of Φ and ψ lie outside the unit circle holds and 0 otherwise.

For the factor loading parameters we also use flat priors

$$f(\lambda_i) \propto 1 \quad \text{for } i = 1, \ldots, N \tag{20}$$

For the variance parameters of the variables as well as factors we use noninformative Jeffrey's priors of the form

$$\begin{aligned} f(\sigma^2_{k,i}) &\propto \sigma^{-2}_{k,i} \quad \text{for } k = 1, 2 \text{ and } i = 1, \ldots, N \\ f(\Sigma) &\propto |\Sigma|^{-1} \end{aligned} \tag{21}$$

see Geisser (1965). For the distribution of the structural break parameter, τ, we use a discrete uniform distribution assigning equal probability for all time periods but the first and the last 12 observations, that is, we trim the first and last year of the sample period.

3.3 Posterior simulation scheme

The posterior distribution is proportional to the product of the likelihood in (16) together with the prior specifications described in (17)-(21). For inference of the posterior distribution we use Metropolis within Gibbs algorithm that leads to the following sampling scheme. Starting with initializing the parameters, at step (m) of the iteration

1. Sample f^T from $p(f^T|y^T, \Phi^{(m-1)}, \Sigma^{(m-1)}, \mathbb{S}^{T(m-1)})$
2. Sample $\mathbb{S}^T$ from $p(\mathbb{S}^T|f^{T(m)}, \Phi^{(m-1)}, \Sigma^{(m-1)})$
3. Sample α from $f(\alpha|y^T, \mathbb{S}^{T(m)}, \Phi^{(m-1)}, \sigma^{2(m-1)}, \lambda^{(m-1)}, \psi^{(m-1)})$
4. Sample Φ from $f(\Phi|y^T, \mathbb{S}^{T(m)}, \alpha^{(m)}, \sigma^{2(m-1)}, \lambda^{(m-1)}, \psi^{(m-1)})$
5. Sample Σ from $f(\Sigma|y^T, \mathbb{S}^{T(m)}, \alpha^{(m)}, \Phi^{(m)}, \sigma^{2(m-1)}, \lambda^{(m-1)}, \psi^{(m-1)})$
6. Sample κ from $f(\kappa|y^T, \mathbb{S}^{(m)}_1, \alpha^{(m)}, \Phi^{(m)}, \sigma^{2(m-1)}, \lambda^{(m-1)}, \psi^{(m-1)})$
7. Sample λ from $f(\lambda|y^T, f^{T(m)}, \sigma^{2(m-1)}, \psi^{(m-1)}, \tau^{(m-1)})$
8. Sample σ^2 from $f(\sigma^2|y^T, f^{T(m)}, \lambda^{(m)}, \psi^{(m-1)}, \tau^{(m-1)})$
9. Sample ψ from $f(\psi|y^T, f^{T(m)}, \lambda^{(m)}, \sigma^{2(m)}, \tau^{(m-1)})$

10. Sample τ from $f(\tau|y^T, f^{T(m)}, \lambda^{(m)}, \sigma^{2(m)}, \psi^{(m)})$
11. Sample P from $f(P|S_1^{(m)})$
12. Repeat (1)-(11) M times.

Our model specification implies that the unobserved regimes are linked to the variables through the common factors of economic and financial indicators. Therefore, direct sampling of $\mathbb{S}^T$ conditional on observed data requires the factor to be integrated out which is not feasible in our case. The fact that our model specification involves potential phase shifts precludes efficient simulation techniques such as Gerlach *et al.* (2000). Accordingly, we sample the regimes conditional on factors in step (2). However, in steps (3)-(6) any factor related parameters are sampled conditional on data rather than factors using Metropolis steps to alleviate potential autocorrelation in the draws that could slow down the convergence of the algorithm.

4 Empirical Findings

In this section, we report our empirical findings for competing models that vary according to the relation between the cyclical components of the coincident economic index (CEI) and the financial conditions index (FCI) estimated in a unified framework specified in (12). We first conduct an analysis on the cross-autoregressive parameters of the CEI and FCI. Posterior odds ratios using mildly informative priors indicate that zero is inside Highest Posterior Density Interval (HPDI) and therefore we exclude these parameters.

We, first, display findings of the full-sample estimation. In the next section, we provide a detailed analysis on the performance of the competing models in real time forecasting of business cycle turning points. The competing models involve (i) the model with independent cycles for the CEI and FCI, (ii) the model with Perfectly Synchronized cycles for the CEI and FCI (PS), (iii) the model with Imperfectly Synchronized cycles due to Symmetric Phase Shifts (IS-SPS) between the cyclical components of the CEI and the FCI, (iv) the model with Imperfectly Synchronized phase synchronized due to Asymmetric Phase Shifts (IS-APS) between the cyclical components of the CEI and the FCI.

Tables 1-3 display the parameter estimates related to the measurement equation in

(12). In Table 1, we present the factor loadings on the economic and financial variables used to construct the CEI and the FCI.

[Insert Table 1 about here]

We observe that all of the eight variables used to construct the CEI load positively on the common factor due to the procyclicality of selected variables. We note that the magnitude of the factor loadings tend to be similar across different models for the bulk of the variables such as industrial production (*ip*), imports, retail sales volume index (*retails*) or total employment less agricultural employment (*empna*). Still, loading of the trade and services turnover index (*traserv*) shows some variation when cycles are completely independent, reflecting the effects of joint estimation of the cycles rather than estimating two independent cyclical processes.

Turning to the factor loadings for the FCI, there is much less variation in the magnitudes of the factor loadings across different models. Variables that are related to various risk sources such as the volatility of the return on the markets index BIST100 (*VOL*), the Treasury auction rate (*TAuc*) and the spread between the 3-month rate the 3-month LIBOR Rate (*TETS*), have sizable negative loadings on the common factor. Moreover, the distributions related to these loadings have relatively small standard deviations leading to quite precise estimates. These results seem intuitive in that greater volatility on local stock markets or an increase in *TETS* is likely to signal adverse developments in financial markets and the related distress in the real economy. An important finding is on the loading of the credit-related variable that has been subject to intense discussions after the recent global crisis of 2008-9. Similar to the risk-related variables, we find that banking sector credit loans (*Cred*) have a negative loading. By contrast, an increase in the real value of stock market index (*rbist*) or the MSCI Emerging Markets Index (*MSCIem*) tends to signal favorable developments and hence, lead to an increase in the FCI.

Table 2 provides estimates of the conditional variances of economic and financial variables together with the timing of the structural break in these variances.

[Insert Table 2 and Figure 1 about here]

The first panel of Table 2 shows that the posterior mode for the breakpoint parameter τ is estimated as September 2001 regardless of the model specification. Figure 1 shows

the posterior density of the break parameter for the model IS-APS. We observe that almost all of the posterior mass is located around the years 2001 and 2002 reflecting the precision in the break parameter estimate. For the variance parameters, we can track a general reduction in the shock variances of almost all variables following the break date of September 2001. This date corresponds to the ending of the financial crisis of 2000-2 that erupted in the midst of an IMF-sponsored stabilization program and that led to fundamental reforms in Turkey's banking and financial sectors. For economic variables, the largest declines in the shock variances occur for the variable trade and financial services index (*traserv*). This occurs because the 2000-2 banking and financial crisis was associated with large capital flow reversals and the consequent changes in Turkey's trade and current account balance. Financial variables exhibit even larger declines in the estimated shock variances. Especially, variables related to interest rates are subject to sizable reductions in variances. This finding reflects the financial turbulence and the large increases in the sovereign risk that Turkey endured during the banking and financial crisis of 2000-2.

Finally, we display the estimates of the autoregressive coefficients related to idiosyncratic factors in Table 3.

[Insert Table 3 about here]

Our experience with the Turkish dataset reveals the importance of modeling the dynamics of the idiosyncratic factors for identification of the common factors. This is mostly related to sticky variables such as employment (*empna*) where the dynamic process in idiosyncratic factors is nearly nonstationary, suggesting the importance of robust inference of this type of models for handling such processes as well.

4.1 Coincident economic index and financial conditions index

In this section, we introduce the coincident economic index and financial conditions indexes that we estimate using our model summarized in (12), that is, the general specification allowing for asymmetric phase shifts between the cyclical components of the economic and financial factors. We estimate the model using growth rates (of nonstationary variables), which facilitates estimation. We then retrieve the indexes in their levels using the idea proposed in Stock and Watson (1989) by reverse engineering the level using the growth rates,

which facilitates interpretation. In Figure 3 we display these indexes together with the dates of recessions indicated by the grey shaded area computed using the BBQ algorithm[9].

[Insert Figure 3 about here]

It is seen that the CEI nicely tracks the business cycle and it accurately predicts the economic downturns that occurred during both the 2000-1 and 2008-9 recessions. Moreover, we can observe the accelerated expansion of the Turkish economy between 2002 and 2008 and right after the 2008-9 crisis, which is replaced by a more mediocre growth after 2012. The FCI displays similar behavior but with a clear lead of cyclical regimes of recessions and expansions by several months. While both the CEI and the FCI show downturns during these recessions, these are amplified further for the FCI, reflecting the relatively volatile nature of the financial variables that are used to constitute it.

Table 4 reports estimates of the parameters related to the evolution of (the growth rates of) the CEI and FCI estimated using the four competing models differing according to the type of synchronization between the cyclical components of the CEI and FCI.

[Insert Table 4 about here]

We observe that the estimates of intercepts and therefore, the mean of the monthly growth rates in recessions differ substantially from the mean of the monthly growth rates in expansions indicating that the regimes are identified quite precisely. For the models with imperfect and perfect synchronization of the evolution of CEI (FCI), the estimates of the intercepts in recessions vary between -0.57% (-0.79%) and -0.61% (-0.81%) and those in expansions vary between 0.062% (0.087%) and 0.069% (0.097%). However, for the model with independent dynamics for the cyclical components, estimates of the intercepts during recessions and expansions is somewhat different with values of -1.09% and -0.92% during recessions, and of 0.04% and 0.15 during expansions for the CEI and the FCI, respectively.

[9] As Turkey lacks a business cycle dating committee as opposed to US (NBER dating committee) we use the dates estimated by the BBQ algorithm as reference recession dates. The BBQ algorithm which is a nonparamatric procedure used for dating business cycle turning points based on the definition of a recession as two consecutive quarters decline in economic activity. The algorithm is proposed by Bry and Boschan (1971) and simplified by Harding and Pagan (2002a) and Harding and Pagan (2002b). This approach uses the aggregate real GDP series or the monthly industrial production growth rate according to the choice of frequency. The resulting recession dates are identified as the period between October 2000 until June 2001 and between April 2008 until March 2009 in our sample.

This implies that estimates of intercepts of the model with independent cycles capture typically more severe aspects of the recessions than those for the other three models that impose some sort of synchronization between cycles.

The second panel of Table 4 displays the estimates of transition probabilities. We observe that the probability of remaining in recessions is less than the probability of remaining in expansions for all four models, reflecting the fact that expansions last longer than recessions. Based on the posterior estimates of the transition probabilities, the duration of expansions is predicted to be 35 months while the duration of recessions is given by 15 months for the models with perfect or imperfect synchronization. By contrast, the model with independent cycles yields a slightly lower probability of remaining in recessions, with an implied duration of recessions being equal to 14 months. This indicates the concordance of cyclical behavior of financial and economic conditions in the sense that while the amplitudes seem to differ between the cyclical components of the two indexes, the length of the cycles are quite similar for the specifications with independent cycles and imperfectly synchronized cycles.

Indeed, we observe the presence of imperfect synchronization when we focus on the estimates of the lead/lag parameters. The bottom panel of Table 4 shows that the posterior mean of the phase shift parameter for the model with imperfect synchronization with symmetric phase shifts between the cyclical components of CEI and FCI is around 3.2 months indicating the leading capacity of the FCI for the business cycle embedded in the CEI. When we further relax the assumption of symmetric phase shifts to allow for distinct phase shifts for recession and expansion regimes, the parameters, κ_1 and κ_2, are estimated as 3.48 and 3.44 months, respectively. This suggests that asymmetries in terms of lead periods of the FCI over the business cycle are less pronounced for an emerging economy like Turkey relative to those for a developed economy like the US. Indeed, (Paap *et al.*, 2009) and Çakmaklı *et al.* (2013) find that the lead time for recessions is 12 months while the lead time for expansions is 4 months based on the estimation of an asymmetric phase shift model using the Conference Board's monthly Composite Coincident Index (CCI) and the Leading Indicators (CLI) in the US. These findings suggest that financial conditions serve as a leading indicator not only for the developed economies as in US but also for

emerging markets such as Turkey. However, for emerging markets the lead time of financial conditions in leading the business cycle is shorter than that of the US. This is due to the relatively more volatile markets in an emerging economy such as Turkey, which limit the foresight and accuracy of expectations of economic agents. Figure 2 displays the posterior distribution for the phase shift parameters κ_1 and κ_2.

[Insert Figure 2 about here]

From Figure 2, we observe that the mean of the posterior joint distribution of κ_1 and κ_2 are approximately between 3 and 4 for both parameters. By contrast, for the symmetric phase shift model displayed in this figure, the mean of the posterior distribution is slightly greater than 3, suggesting that the differences between the APS and SPS specifications are minor. An important finding regarding the precision of the lead/lag parameter estimates is that the restriction arising from the symmetric phase shifts which impose identical lead/lag parameters pays-off in the case of Turkey, which has experienced a limited number of recessions in the sample period. While the joint distribution of these parameters in the case of the APS model is quite wide assigning non-zero probabilities to quite distinct values, the distribution of the same parameters from the SPS model is concentrated around the posterior mean, due to the symmetry restriction penalizing less relevant values.

Finally, we observe from the marginal likelihoods reported at the bottom of Table 4 that both the model with independent cycles and with perfect synchronization perform worse than the models with imperfect synchronization, confirming our findings on the conformity of the cyclical behavior of economic and financial conditions. While the model with independent cycles has the lowest marginal likelihood value performing even worse than the model with perfect synchronization of the cycles, the models with imperfect synchronization dominate the model with perfect synchronization. In line with the earlier findings on precision of estimates the phase shift parameters, the model with imperfect synchronization together with symmetric phase shifts attains the highest value of marginal likelihood. This is due to the fact that the estimates of the distinct phase shift parameters in the case of asymmetric phase shifts are quite similar, essentially implying the specification with symmetric phase shifts. However, additional uncertainty brought by the flexible specification of asymmetric phase shifts is penalized by the marginal likelihood

metric leading this specification to have a lower value of marginal likelihood compared to the specification where the phase shift parameters are restricted to be identical.

We now examine the behavior of the different model specifications based on their ability to determine turning points and to identify recessionary episodes. Figure 4 shows recessionary episodes for the Turkish economy based on the BBQ algorithm together with the recession probabilities implied by the models.

[Insert Figure 4 about here]

First, we observe that the specifications of imperfect synchronization yield quite similar results in line with earlier findings. Consistent with Figure 3 and nonzero estimates of phase shifts between the cyclical components of CEI and FCI, the smoothed probabilities of being in recession for FCI precede the smoothed probabilities of being in recession for the CEI in both cases of 2000-1 and 2008-9 recessions at the onset when entering recessions as well as when leaving recessions. Moreover, the timing of the recessions for CEI in the sense that the periods where smoothed probabilities of being in recession for CEI exceed 0.5 perfectly match with the periods of recessions computed by the BBQ algorithm. This implies that the FCI together with its cyclical components estimated using our model specification not only measures the current financial conditions but also perfectly serves as an early warning indicator for the oncoming downturns of economic activity.

When we consider the model with perfect synchronization we observe that it has some success in capturing the cyclical turning points specifically at the onset of the 2008-9 recessions. However, it can be clearly seen that due to the leading capability of the financial variables, it produces false signals of recessions at the onset of the 2000-1 recession as the periods when smoothed probabilities exceed 0.5 precede the periods of actual recessions. Even more pronounced, the model produces false signals of expansions towards the end of both recessions during the transition periods from recession to expansion in the sense that model implied probabilities decline to levels below 0.5 much earlier than the actual periods of expansionary periods following recessions. This indicates that blending economic variables together with financial variables for estimation of indicators of economic activity and its cyclical turning points often yields false signals of this cyclical behavior. This is due to the fact that economic and financial variables have distinct characteristics in

terms of the cyclicality with respect to business cycle. Indeed, this is the focal point of our model for construction of the indicators of economic and financial conditions. Finally, considering the model with independent cycles for CEI and FCI, we observe the poor performance of CEI in capturing the cyclical behavior of economic activity. First, it misses the 2000-1 recession completely producing smoother probabilities below 0.5 over the course of these periods and, second, it enters the 2008-9 recession with a substantial lag and similarly, it leaves the recession quite earlier than the actual period of trough. On the other hand, it seems that the financial cycle can still be captured by the FCI produced by this specification as smoothed probabilities in this case are very similar to the smoothed probabilities computed using the model with imperfect synchronization.

Comparing these two graphs of smoothed probabilities implied by the models of perfect synchronization of the cycles and independent cycles together with that of the models with imperfect synchronization reveal the efficiency of our modeling framework. In the first case with perfect synchronization with both financial and economic variables, the model produces signals of recessions but the timing of such recessions are erroneous, while in the second case of independent cycles when only economic variables are used, it produces no signals at all. On the other hand, using our framework which takes the intermediary case of imperfect synchronization into account, we make an efficient use of financial variables in dating business cycle turning points most accurately.

4.2 Predicting business cycle turning points in real-time

In previous sections, we show that the model specification allowing for imperfect synchronization between financial and business cycles is able to capture, first, economic and financial conditions summarized by the two corresponding indexes/factors, and second, business cycle turning points quite timely and accurately using the full sample of variables *ex post*. However, economic agents are often interested in predicting economic downturns before they are actually realized. Therefore, we also assess the efficacy of the model in signaling business cycle turning points *ex ante*. To do this, we conduct a recursive forecasting exercise for examining the predictive ability of the models in predicting business cycle turning points over the evaluation period starting from December 2006 until Novem-

ber 2017. To obtain the predictions in real-time we first restructure the dataset leading to a ragged-edge in each period of recursive estimation as many of the variables are not available timely due to the delays in releases. Specifically, while many of macroeconomic variables including *ip*, *import*, *export* and *retails* are released with a lag of 2-months, other variables including *empna* and *traserv* are released with lags of 3 and 4 months, respectively. *pmi* is the most timely variable with a lag of release date as only 1 month. On the other hand, financial variables are released quite timely with just a few exceptions including *FXRes*, *P-E*, *MSCIem* and *EMBI-Tr* that are released with a lag of 2 months.

For comparison of the real-time predictive ability of the models in predicting business cycle turning points we make use of the metric of turning point forecast errors (TPFE) using predictive probabilities of being in recession. To obtain these probabilities we first compute the predictive distribution of the regime indicator of being in a recession, $f(S_{1,t_0+h} = 1|\theta, Y^{t_0})p(\theta|Y^{t_0})$, where $p(\theta|Y^{t_0})$ is the posterior distribution of model parameters given the observations until t_0. To do this, we use the posterior simulator for obtaining a sample from the distribution of the model parameters $\{\theta^{(m)}\}_{m=1}^{M}$ and to obtain a sample of predictive distribution of regime indicators $\{S_{1,t_0+h}^{(m)}\}_{m=1}^{M}$ using this sample afterwards, where M is a large number of draws from the posterior distribution. Finally, predictive recession probabilities for period $t_0 + h$ are computed using the sample average as $\bar{S}_{1,t_0+h} = M^{-1} \sum_{m=1}^{M} S_{1,t_0+h}^{(m)}$. The TPFE, is, then, given by

$$\text{TPFE}(h) = \frac{1}{T_2 - h - T_1 + 2} \sum_{t=T_1}^{T_2+1-h} (BC_{t+h} - \bar{S}_{1,t+h})^2, \tag{22}$$

where BC_{t+h} is the indicator function that equals to 1 if the economy is in recession at time $t + h$ and 0 otherwise, according to the BBQ algorithm. T_1 and T_2 correspond to the first and terminal dates of the evaluation period, respectively. In our forecasting exercise, we compute the TPFE for forecast horizons of $h = 1, 2, \ldots, 9$ to evaluate the predictive ability of competing models for various horizons.

Table 5 displays the TPFE differences of competing models from the model specification that allows for imperfect synchronization between the cyclical regimes of coincident economic indicator and financial conditions index with asymmetric phase shifts.

[Insert Table 5 about here]

Two clear-cut conclusions can be drawn from Table 5. First, the model specifications of popular polar cases with independent cycles for CEI and FCI and with perfect synchronization of the cycles of CEI and FCI perform much worse than our general model specification as can be seen in the third and fourth columns of Table 5. Essentially, the specification with independent cycles performs worst with sizable differences in TPFEs compared to TPFEs produced by our specification. The differences are as high as 4 in short horizons due to false signals produced by the model specification with independent cycles. These differences gradually decline to values around 1 as the forecast horizon increases due to the deteriorating predictions across all models. Equal predictive accuracy tests of Clark and McCracken (2005) indicate that these sizable differences are significant at least at 5% significance level up to 5 months of forecast horizon and significant at 10% significance level for the forecast horizon of 6 month. The specification with perfect synchronization of the cycles also perform worse than the counterpart with imperfect synchronization of the cycles with asymmetric phase shifts at all horizons. Consistent with the estimate of the phase shift parameter indicating the lead time of financial cycle as around 3 months, the large differences between TPFE reduces to more mediocre values for the forecast horizons of 4 months and longer. Equal predictive accuracy tests in this case point to the significance of these difference for the forecast horizons up to 2 months. Note that due to the data availability we keep the evaluation sample short. This, in turn, leads to limited degrees of freedom deteriorating the power of this type of test. Second, in line with the full-sample results model specifications with imperfect synchronization of cycles yield similar results regardless whether the phase shifts are restricted to be identical or not. Still, we can observe some pattern in the sense that the specification with symmetric phase shifts perform better at short horizons up to 4 months but this picture reverses at long horizons with an increasing discrepancy. However, statistical tests of equal predictive accuracy uniformly concludes the insignificance of these differences. This finding reflects the fact that the estimated phase shift parameters in the specification with asymmetric phase shifts are not different from each other, and close to the estimates of the phase shift parameter for the specification with symmetric phase shift.

Figure 5 displays the performance of the models in predicting the economic downturns with a focus on the 2008-9 recession.

[Insert Figure 5 about here]

Specifically, we display the posterior probabilities of being in recession for a given vintage T, before the terminal date, i.e. in-sample estimates of the probabilities, and after the terminal date of the vintage, i.e. predictive probabilities of being in recession up to nine months ahead. These probabilities are computed for data vintages between the period December 2006, T_1, until January 2011 comprising the periods just before, during and after the 2008-9 recession. The vertical axis shows the specific vintage, T, used to compute the posterior probabilities while the horizontal axis shows time, t, starting from January 2007 to February 2011. Each row of the graphs represents the values of the posterior probabilities of a recession over time, $Pr(S_{1,t} = 1|\mathbf{y}^T)$ for $t = T_1, T_1 + 1, \ldots, T, T + 1, \ldots, T + 9$, based on the vintage as indicated on the vertical axis. Values of the recession probabilities greater than 0.5 are represented by the shades of red color getting darker as the probabilities are getting closer to 1. Probabilities smaller than 0.5 are represented by the shades of the blue color getting darker as the probabilities are getting closer to 0. As it is shown in the bars next to the graphs providing the link between the values of probabilities and the shades of the colors values around 0.5 are represented with light colors of green and yellow. If for a particular vintage the color changes from blue to red in a certain month and remain red thereafter, then this month is considered as a business cycle peak, i.e as the onset of the recession. A change from red to blue similarly represents a business cycle trough, the onset of the expansion. We indicate the periods of the 2008-9 recession identified according to the BBQ algorithm on horizontal axis with the red marker as the peak and the blue marker as the trough of the cycle. Looking across the columns of these graphs shows how the assessment of the probability of a recession changes across the different data releases. We also add the red and blue markers on the vertical axis, however, these are representing the release date of the GDP or industrial production series, when, in real-time, the BBQ algorithm computed using these vintages starts to indicate the recession date. We include these markers on the vertical axis to compare our methodology with more conventional methods in terms of generating recession signals in

a timely manner.

Figure 5 provides insights on the dynamics of the competing models through the lens of 2008-9 recession. First, we consider the onset of the recession, i.e. the business cycle peak, which is (ex post) dated as April 2008 by the BBQ algorithm. When we focus on the January 2008 vintage, we observe that the model specification with imperfect synchronization of cycles with asymmetric phase shifts starts to deliver signals with predictive probabilities approaching to 0.4 for around April 2008. Focusing on the model specification with imperfect synchronization of cycles but with symmetric phase shifts we observe similar signals but for the February 2008 vintage rather than the January 2008 vintage. A striking finding is on the signals delivered by the specification with perfect synchronization of the cycles. Essentially, for the vintage of January 2008, this model produces signals of oncoming recession already starting almost from December 2007 with recessions probabilities wandering around 0.4-0.5. However, in line with our in sample findings, these 'false' early signals are due to the fact that this model captures the financial cycle rather than the business cycle. This once more reflect the overwhelming impact of financial variables suppressing the impact of economic variables in estimation of the cyclical phases of 'economic' conditions. By contrast, by modeling the imperfect synchronization between these two cyclical components, we are able to capture the turning points of both the financial and the business cycle in a timely and accurate manner by efficiently linking financial variables together with the economic variables. This is seen from the evidence surrounding the business cycle peak of April 2008. Finally, the model with independent cycles display the poorest performance of signaling recessions. The first signals using this specification emerge as late as in April 2008 and are interrupted later on until August 2008. Note that, in many cases the model with independent cycles constitutes the conventional methodology of measuring business cycle as discussed in the Introduction.

Next, we consider the performance of the models in predicting the oncoming expansion, i.e. business cycle trough, which is (ex post) dated as March 2009 by the BBQ algorithm. Focusing on the January 2009 vintage, it is seen that the model specification with imperfect synchronization of cycles with asymmetric phase shifts and with symmetric phase shifts deliver similar signals in the sense that the prediction of recession probabilities for March

and April 2009 gradually declines to values around 0.6-0.7 which reduces well below 0.5 with the release of the March 2009 vintage. For the model specification with perfect synchronization of the cycles, signals of oncoming expansion are delivered for January 2009 much earlier than the actual date of the trough. Even for the vintages released later than March 2009 in-sample estimates of recession probabilities indicates the end of recession as early as January 2009 confirming the finding that this model essentially conveys information about the financial cycle rather than the business cycle. Finally, the model with independent cycles perform worst as in the previous case providing false signals of the trough much earlier than the actual realization.

On top of the evaluation of the predictive performance of the parametric methods, we further conduct a real-time dating exercise of recessions using the nonparametric BBQ algorithm on GDP and IP series. Notice that the timing of the BBQ algorithm relies on the release dates of these variables. Indeed, the algorithm sets the starting (final) date of the recession as April 2008 (March 2009) only in September 2008 (September 2009) considerably 'lagging' the business cycle. On the contrary, our model specification with imperfect synchronization of cycles with asymmetric phase shifts is able to signal the oncoming recession (expansion) as early as January 2008 (January 2009) essentially 'leading' the business cycle thanks to modeling financial and economic conditions jointly in a unified framework together with the link between the cyclical components of these indicators.

5 Conclusion

Tracking economic and financial conditions in a timely and systematic manner is central for accurate predictions of economic downturns and for resolving economic and financial uncertainty. Not surprisingly, many central banks and policy makers construct such indexes of economic and financial conditions for anticipating economic downturns in a timely manner. Interestingly, economic and financial conditions are often constructed independently of each other, thereby missing the important link between the cyclical components of these measures. This is a key deficiency of this approach, as it is widely accepted that

many financial variables serve as important leading indicators of business cycle phases, i.e. of recessions and expansions.

This paper fills this gap by proposing a unified framework for joint estimation of the Coincident Economic Index (CEI) and Financial Conditions Index (FCI) by modeling the cyclical behavior of these indexes allowing for imperfect synchronization together with asymmetric phase shifts between the cyclical regimes. We estimate our model using a dataset with mixed frequencies and mixed time span to construct the CEI and FCI for Turkey and for dating cyclical regimes of the Turkish business cycle over the period starting from January 1999 until November 2017. The results from the full-sample estimation show that these indexes as well as the model-implied recession probabilities are able to capture stylized facts of the Turkish economy quite precisely and match perfectly with the dates of recessions computed using the nonparametric BBQ algorithm. Strikingly, we document the leading capacity of the FCI in leading the business cycle phases by showing that the financial cycle enters recessions on average 3.5 months earlier than that of the business cycle, while this lead time becomes on average 3 months for entering expansions. We further conduct a real-time recursive forecasting exercise for predicting the recessions over the periods starting from January 2006 until the end of the sample and provide convincing evidence on the superior nowcasting and forecasting ability of our specification that clearly outperforms competing parametric models with perfect synchronization of cycles as well as independent cycles and nonparametric BBQ algorithm.

Our model provides a prototype for joint estimation of the CEI and the FCI together with their cyclical components in a data-rich environment of variables with mixed frequencies and mixed time-span. Crucially, it serves as an effective early warning indicator of oncoming recessions exploiting joint behavior of the forward-looking financial variables efficiently. Therefore, the framework would also be useful for other emerging markets with similar characteristics as well as for advanced economies such as US for the joint construction of the CEI and the FCI in high frequencies such as at weekly or even at daily frequency.

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Table 1: Estimates of factor loadings

		Imperfect synchronization with APS	Imperfect synchronization with SPS	Perfect synchronization of cycles	Independent cycle
Economic variables					
ip	$\lambda_{1,1}$	0.392 (0.096)	0.394 (0.096)	0.391 (0.094)	0.392 (0.098)
import	$\lambda_{2,1}$	0.288 (0.074)	0.288 (0.075)	0.285 (0.072)	0.285 (0.079)
export	$\lambda_{3,1}$	0.115 (0.054)	0.113 (0.054)	0.109 (0.052)	0.109 (0.053)
retails	$\lambda_{4,1}$	0.257 (0.154)	0.263 (0.155)	0.265 (0.151)	0.265 (0.148)
pmi	$\lambda_{5,1}$	0.159 (0.199)	0.171 (0.200)	0.175 (0.194)	0.197 (0.195)
empna	$\lambda_{6,1}$	0.205 (0.135)	0.212 (0.135)	0.223 (0.134)	0.245 (0.138)
traserv	$\lambda_{7,1}$	0.111 (0.501)	0.069 (0.500)	0.041 (0.511)	0.333 (0.195)
Financial variables					
rbist	$\lambda_{8,2}$	0.654 (0.064)	0.654 (0.064)	0.652 (0.063)	0.661 (0.063)
FXRes	$\lambda_{9,2}$	0.178 (0.065)	0.178 (0.065)	0.179 (0.065)	0.179 (0.066)
Conf	$\lambda_{10,2}$	0.561 (0.075)	0.565 (0.076)	0.566 (0.076)	0.567 (0.076)
TermS	$\lambda_{11,2}$	0.013 (0.020)	0.013 (0.021)	0.013 (0.020)	0.012 (0.021)
VOL	λ_{12}	−0.202 (0.082)	−0.202 (0.083)	−0.201 (0.082)	−0.203 (0.082)
P-E	$\lambda_{13,2}$	0.276 (0.115)	0.275 (0.114)	0.274 (0.114)	0.278 (0.114)
TAuc	$\lambda_{14,2}$	−0.274 (0.079)	−0.275 (0.079)	−0.276 (0.079)	−0.278 (0.080)
TETS	$\lambda_{15,2}$	−0.064 (0.036)	−0.065 (0.038)	−0.064 (0.037)	−0.065 (0.039)
Cred	$\lambda_{16,2}$	−0.240 (0.095)	−0.239 (0.095)	−0.235 (0.096)	−0.242 (0.096)
MSCIem	$\lambda_{17,2}$	0.763 (0.087)	0.764 (0.088)	0.762 (0.087)	0.771 (0.088)
EMBI-Tr	$\lambda_{18,2}$	0.064 (0.026)	0.064 (0.027)	0.064 (0.026)	0.064 (0.027)

Note: The table shows posterior means and standard deviations (in parentheses) of the factor loading parameters in the measurement equations in (12) for competing models estimated using the data for the periods starting from January 1999 until November 2017. The competing models are constituted by the model with Imperfectly Synchronized phase synchronized due to Asymmetric Phase Shifts (IS-APS) between the cyclical components of the CEI and the FCI, the model with Imperfectly Synchronized cycles due to Symmetric Phase Shifts (IS-SPS) between the cyclical components of the CEI and the FCI, the model with Perfectly Synchronized cycles (PS) for the CEI and FCI and the model with independent cycles for the CEI and FCI. Posterior results are based on 110 000 draws from the posterior distribution where the first 10 000 draws are discarded as burn-in sample.

Table 2: Estimates of conditional variances of the variables

		Imperfect synchronization with APS	Imperfect synchronization with SPS	Perfect synchronization of cycles	Independent cycle
Most likely break date	τ	2001 : 09	2001 : 09	2001 : 09	2001 : 09
Economic variables					
ip	$\sigma^2_{1,1}$	1.032 (0.313)	1.027 (0.313)	1.027 (0.311)	1.024 (0.315)
	$\sigma^2_{1,2}$	0.755 (0.111)	0.753 (0.111)	0.752 (0.111)	0.750 (0.110)
import	$\sigma^2_{2,1}$	1.957 (0.613)	1.951 (0.617)	1.954 (0.616)	1.949 (0.611)
	$\sigma^2_{2,2}$	0.570 (0.082)	0.569 (0.083)	0.569 (0.083)	0.567 (0.083)
export	$\sigma^2_{3,1}$	1.277 (0.388)	1.275 (0.389)	1.278 (0.391)	1.285 (0.393)
	$\sigma^2_{3,2}$	0.589 (0.073)	0.590 (0.074)	0.591 (0.075)	0.589 (0.073)
retails	$\sigma^2_{4,1}$	1.654 (2.185)	1.655 (2.483)	1.650 (2.388)	1.653 (2.528)
	$\sigma^2_{4,2}$	0.887 (0.154)	0.884 (0.154)	0.884 (0.154)	0.881 (0.153)
pmi	$\sigma^2_{5,1}$	1.640 (1.944)	1.658 (2.073)	1.645 (1.939)	1.649 (1.961)
	$\sigma^2_{5,2}$	0.885 (0.156)	0.882 (0.156)	0.882 (0.155)	0.874 (0.155)
empna	$\sigma^2_{6,1}$	1.624 (2.016)	1.614 (1.898)	1.635 (2.567)	1.623 (1.987)
	$\sigma^2_{6,2}$	0.874 (0.114)	0.873 (0.114)	0.870 (0.114)	0.860 (0.113)
traserv	$\sigma^2_{7,1}$	1.604 (2.219)	1.583 (2.291)	1.593 (2.232)	1.630 (2.212)
	$\sigma^2_{7,2}$	0.080 (0.031)	0.082 (0.073)	0.081 (0.032)	0.846 (0.209)
Financial Variables					
rbist	$\sigma^2_{8,1}$	1.564 (0.530)	1.563 (0.536)	1.576 (0.538)	1.546 (0.526)
	$\sigma^2_{8,2}$	0.281 (0.057)	0.282 (0.058)	0.283 (0.057)	0.278 (0.056)
FXRes	$\sigma^2_{9,1}$	3.551 (1.137)	3.542 (1.145)	3.541 (1.137)	3.545 (1.134)
	$\sigma^2_{9,2}$	0.490 (0.063)	0.489 (0.063)	0.489 (0.063)	0.490 (0.063)
Conf	$\sigma^2_{10,1}$	0.933 (0.316)	0.917 (0.312)	0.915 (0.316)	0.902 (0.308)
	$\sigma^2_{10,2}$	0.586 (0.081)	0.584 (0.082)	0.583 (0.082)	0.585 (0.082)
TermS	$\sigma^2_{11,1}$	11.990 (6.602)	11.963 (6.641)	11.950 (6.593)	11.983 (6.547)
	$\sigma^2_{11,2}$	0.033 (0.013)	0.033 (0.014)	0.033 (0.014)	0.033 (0.014)
VOL	$\sigma^2_{12,1}$	1.179 (0.313)	1.177 (0.310)	1.181 (0.311)	1.179 (0.314)
	$\sigma^2_{12,2}$	0.937 (0.104)	0.938 (0.104)	0.938 (0.104)	0.937 (0.104)
P-E	$\sigma^2_{13,1}$	2.023 (1.141)	2.020 (1.140)	2.024 (1.147)	2.017 (1.139)
	$\sigma^2_{13,2}$	0.674 (0.298)	0.673 (0.300)	0.674 (0.299)	0.675 (0.297)
TAuc	$\sigma^2_{14,1}$	1.638 (0.470)	1.631 (0.472)	1.630 (0.471)	1.625 (0.468)
	$\sigma^2_{14,2}$	0.800 (0.111)	0.799 (0.112)	0.799 (0.111)	0.800 (0.110)
TETS	$\sigma^2_{15,1}$	10.589 (5.701)	10.584 (5.742)	10.605 (5.759)	10.555 (5.628)
	$\sigma^2_{15,2}$	0.077 (0.016)	0.077 (0.017)	0.077 (0.017)	0.077 (0.017)
Cred	$\sigma^2_{16,1}$	1.629 (2.037)	1.622 (2.324)	1.637 (2.272)	1.634 (2.028)
	$\sigma^2_{16,2}$	0.770 (0.099)	0.769 (0.099)	0.771 (0.099)	0.769 (0.099)
MSCIem	$\sigma^2_{17,1}$	1.596 (2.078)	1.593 (2.274)	1.593 (1.995)	1.598 (2.109)
	$\sigma^2_{17,2}$	0.536 (0.088)	0.536 (0.088)	0.536 (0.088)	0.532 (0.088)
EMBI-Tr	$\sigma^2_{18,1}$	6.624 (3.606)	6.591 (3.589)	6.624 (3.631)	6.630 (3.587)
	$\sigma^2_{18,2}$	0.052 (0.013)	0.052 (0.013)	0.052 (0.013)	0.052 (0.014)

Note: The table shows posterior means and standard deviations (in parentheses) of the variances of the idiosyncratic components in the measurement equations in (12) for competing models estimated using the data for the periods starting from January 1999 until November 2017. Posterior results are based on 110 000 draws from the posterior distribution where the first 10 000 draws are discarded as burn-in sample. See Table 1 for further details.

Table 3: Autoregressive coefficients of the idiosyncratic factors of economic variables

		Imperfect synchronization with APS	Imperfect synchronization with SPS	Perfect synchronization of cycles	Independent cycle
ip	$\psi_{1,1}$	−0.306 (0.101)	−0.312 (0.100)	−0.319 (0.099)	−0.314 (0.090)
	$\psi_{1,2}$	−0.059 (0.091)	−0.063 (0.090)	−0.069 (0.090)	−0.058 (0.087)
	$\psi_{1,3}$	−0.023 (0.079)	−0.025 (0.079)	−0.029 (0.078)	−0.020 (0.079)
import	$\psi_{2,1}$	−0.462 (0.090)	−0.465 (0.090)	−0.470 (0.089)	−0.465 (0.085)
	$\psi_{2,2}$	−0.106 (0.097)	−0.110 (0.097)	−0.115 (0.096)	−0.105 (0.093)
	$\psi_{2,3}$	0.013 (0.083)	0.010 (0.083)	0.008 (0.082)	0.015 (0.079)
export	$\psi_{3,1}$	−0.595 (0.070)	−0.595 (0.070)	−0.595 (0.070)	−0.594 (0.070)
	$\psi_{3,2}$	−0.327 (0.078)	−0.327 (0.078)	−0.328 (0.077)	−0.327 (0.078)
	$\psi_{3,3}$	−0.074 (0.068)	−0.075 (0.068)	−0.076 (0.068)	−0.075 (0.068)
retails	$\psi_{4,1}$	−0.337 (0.122)	−0.340 (0.122)	−0.342 (0.123)	−0.343 (0.122)
	$\psi_{4,2}$	−0.191 (0.124)	−0.191 (0.124)	−0.191 (0.124)	−0.191 (0.124)
	$\psi_{4,3}$	−0.076 (0.117)	−0.076 (0.117)	−0.076 (0.117)	−0.070 (0.117)
pmi	$\psi_{5,1}$	−0.143 (0.124)	−0.143 (0.124)	−0.142 (0.124)	−0.147 (0.124)
	$\psi_{5,2}$	−0.122 (0.119)	−0.122 (0.119)	−0.122 (0.120)	−0.124 (0.120)
	$\psi_{5,3}$	0.095 (0.117)	0.096 (0.117)	0.096 (0.117)	0.099 (0.118)
empna	$\psi_{6,1}$	0.126 (0.087)	0.125 (0.087)	0.121 (0.087)	0.118 (0.088)
	$\psi_{6,2}$	0.254 (0.085)	0.252 (0.085)	0.248 (0.085)	0.250 (0.084)
	$\psi_{6,3}$	−0.188 (0.084)	−0.187 (0.085)	−0.188 (0.085)	−0.188 (0.084)
traserv	$\psi_{7,1}$	1.867 (0.179)	1.864 (0.197)	1.856 (0.179)	−0.016 (0.176)
	$\psi_{7,2}$	−1.395 (0.275)	−1.395 (0.281)	−1.382 (0.274)	0.143 (0.165)
	$\psi_{7,3}$	0.437 (0.142)	0.437 (0.146)	0.430 (0.142)	0.111 (0.169)

Note: The table shows posterior means and standard deviations (in parentheses) of the autoregressive coefficients of the idiosyncratic factors of economic variables in the measurement equations in (12) for competing models estimated using the data for the periods starting from January 1999 until November 2017. Posterior results are based on 110 000 draws from the posterior distribution where the first 10 000 draws are discarded as burn-in sample. See Table 1 for further details.

Table 4: Posterior means and standard deviations (in parentheses) of parameters in the transition equations of CEI and FCI for competing models

		Imperfect synchronization with APS	Imperfect synchronization with SPS	Perfect synchronization of cycles	Independent cycle
Intercepts	$\alpha_{1,0}$	0.069 (0.029)	0.066 (0.027)	0.062 (0.027)	0.037 (0.069)
	$\alpha_{1,1}$	−0.571 (0.265)	−0.610 (0.270)	−0.588 (0.258)	−1.086 (0.110)
	$\alpha_{2,0}$	0.097 (0.040)	0.090 (0.032)	0.087 (0.037)	0.153 (0.080)
	$\alpha_{2,1}$	−0.787 (0.187)	−0.813 (0.195)	−0.790 (0.194)	−0.918 (0.107)
Transition	p_1	0.972 (0.011)	0.973 (0.011)	0.972 (0.011)	0.973 (0.010)
probabilities	q_1	0.934 (0.024)	0.933 (0.024)	0.932 (0.024)	0.929 (0.026)
	p_2				0.971 (0.011)
	q_2				0.932 (0.024)
Autoregressive	$\phi_{1,1}$	0.344 (0.193)	0.356 (0.195)	0.397 (0.198)	0.406 (0.198)
coefficients	$\phi_{2,2}$	0.320 (0.188)	0.322 (0.191)	0.326 (0.184)	0.309 (0.180)
Variances	$\sigma^2_{f_1}$	0.845 (0.143)	0.835 (0.148)	0.804 (0.159)	0.796 (0.162)
	$\sigma^2_{f_2}$	0.862 (0.126)	0.860 (0.128)	0.860 (0.123)	0.872 (0.117)
Phase shifts	κ_1	3.483 (2.026)	3.198 (1.878)		
	κ_2	3.444 (2.312)			
Log-marginal likelihood		−837.39	−814.98	−858.57	−895.89

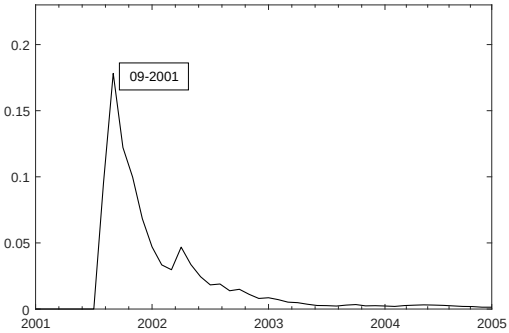
Note: The table shows posterior means and standard deviations (in parentheses) of the parameters in the transition equation definining the autoregressive process for CEI and FCI in (7) for competing models estimated using the data for the periods starting from January 1999 until November 2017. Log-marginal likelihood values are computed for the full models together with measurement equations. Posterior results are based on 110 000 draws from the posterior distribution where the first 10 000 draws are discarded as burn-in sample. See Table 1 for further details.

Table 5: Turning point forecast error differences to imperfect synchronization of cycles with APS-model

Horizon h	Imperfect synchronization with SPS	Perfect synchronization of cycles	Independent cycles
1	−0.178	0.756**	3.915**
	(0.483)	(0.387)	(2.168)
2	−0.100	0.681**	3.844**
	(0.483)	(0.374)	(2.013)
3	0.004	0.486	3.595**
	(0.561)	(0.603)	(1.737)
4	−0.078	0.077	2.794***
	(0.480)	(0.675)	(1.111)
5	0.037	0.117	2.188***
	(0.482)	(0.687)	(0.726)
6	0.141	0.228	1.528*
	(0.427)	(0.555)	(0.743)
7	0.241	0.365	1.231
	(0.434)	(0.325)	(0.828)
8	0.191	0.345	1.014
	(0.421)	(0.418)	(0.962)
9	0.320	0.224	0.654
	(0.423)	(0.503)	(1.053)

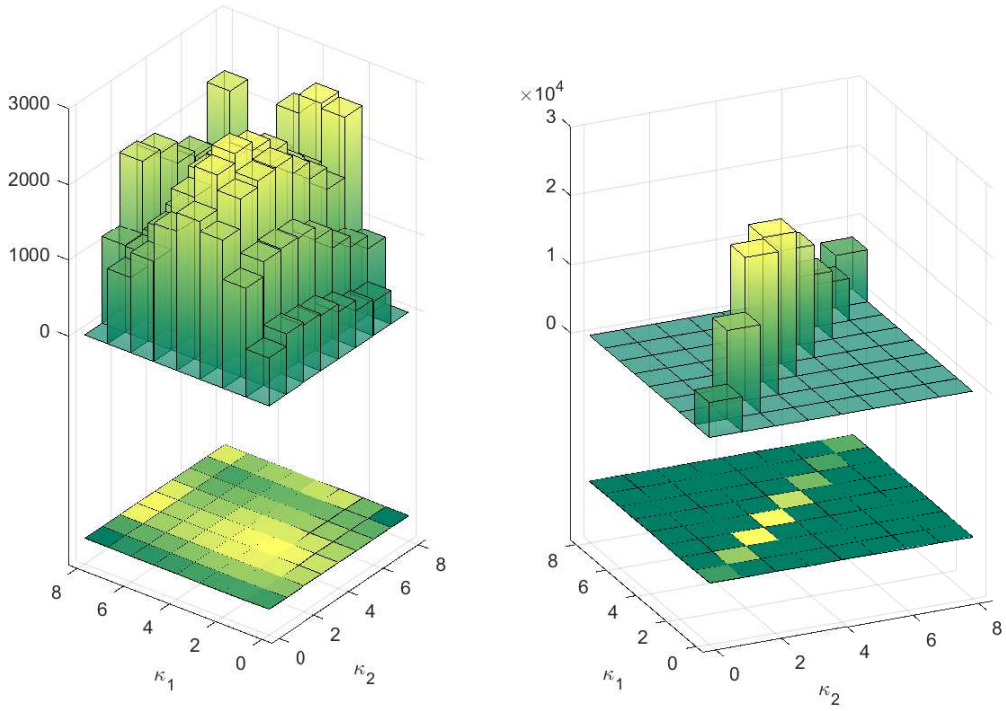
Note: The table shows prediction error differences of the models with (i) imperfect synchronization of cycles with symmetric phase shifts, (ii) perfect synchronization of cycles and (iii) independent cycles from the model with imperfect synchronization of cycles with asymmetric phase shifts. Pairwise comparisons are carried out using OOS-t test, see Clark and McCracken (2005); McCracken (2007) for details. The comparisons involve the competing models against the baseline model. '***' indicates significance at 1%, '**' indicates significance at 5%, '*' indicates significance at 10% with the critical values provided in McCracken (2007) for the recursive scheme. A positive (negative) value with asterisk indicates statistical significance for superior (inferior) performance of the the model with imperfect synchronization of cycles with asymmetric phase shifts.

Figure 1: Posterior density of the break point parameter, τ, for the structural break in conditional variances of variables



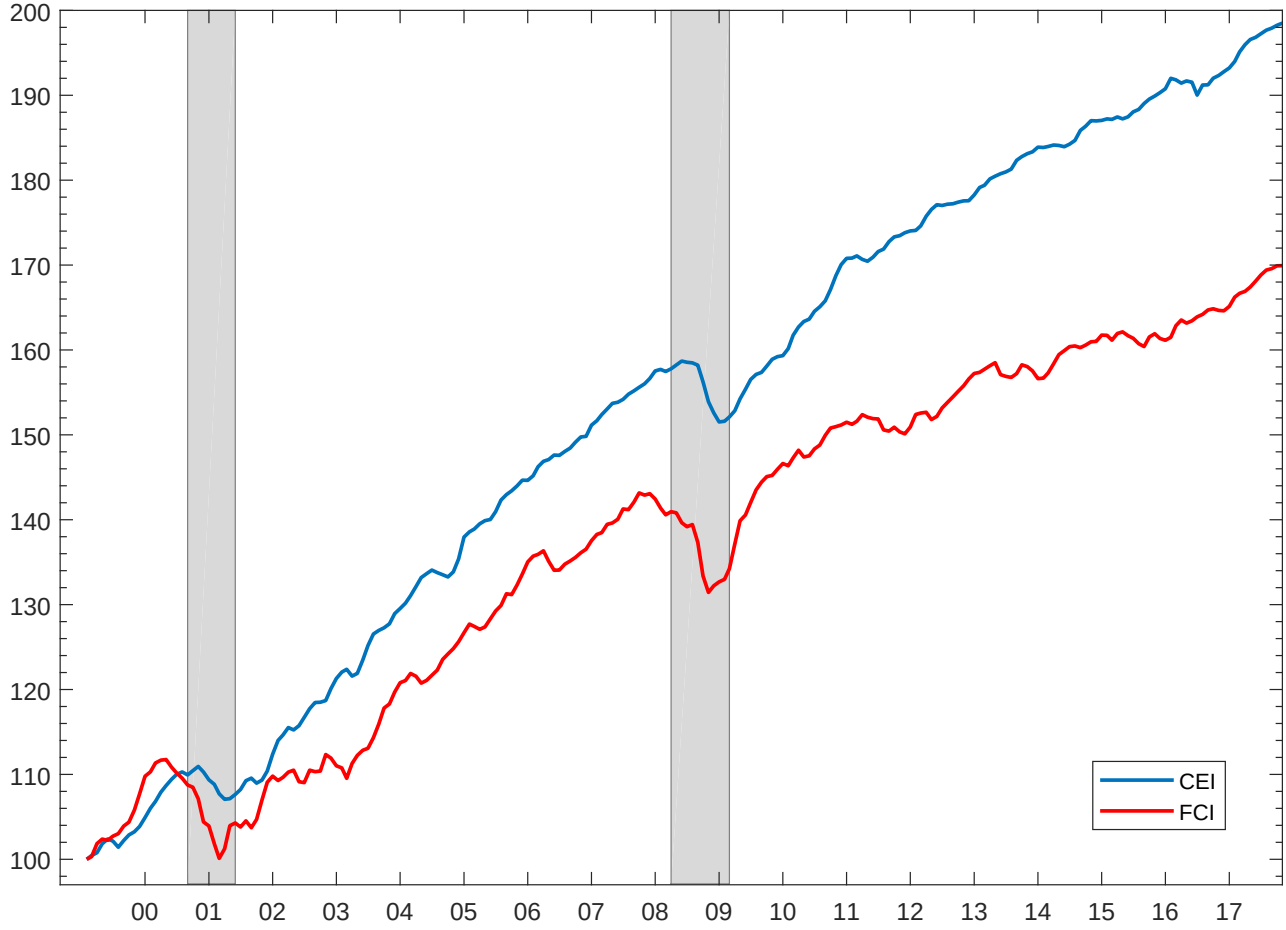
Note: The graph displays the posterior distribution of the break date, τ, in conditional variances of variables estimated for the model of imperfect synchronization of cycles with asymmetric phase shifts using the data for the periods starting from January 1999 until November 2017. Posterior results are based on 110 000 draws from the posterior distribution where the first 10 000 draws are discarded as burn-in sample.

Figure 2: Histograms of the phase shift parameters κ_1 and κ_2 for models with imperfect synchronization with asymmetric and symmetric phase shifts



Note: The graph displays the posterior distribution of the phase shift parameters, κ_1 and κ_2, between the cyclical components of Coincident Economic Index (CEI) and Financial Conditions Index (FCI) estimated for the model with imperfect synchronization of cyclical components of CEI and FCI with asymmetric phase shifts in the left panel and with symmetric phase shifts on the right panel using the data for the periods starting from January 1999 until November 2017. Posterior results are based on 110 000 draws from the posterior distribution where the first 10 000 draws are discarded as burn-in sample.

Figure 3: Estimate of Coincident Economic Index and Financial Conditions Index



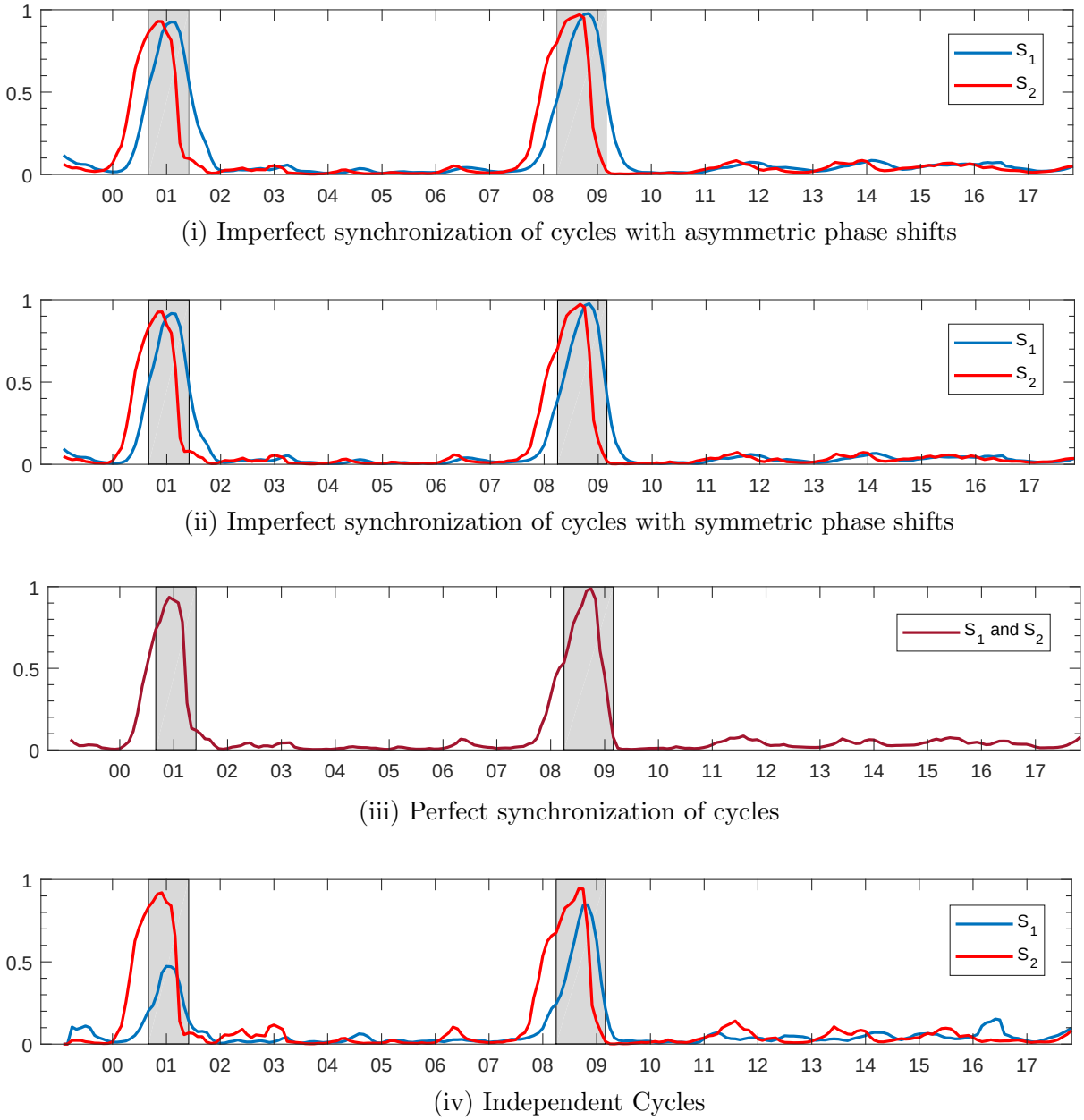
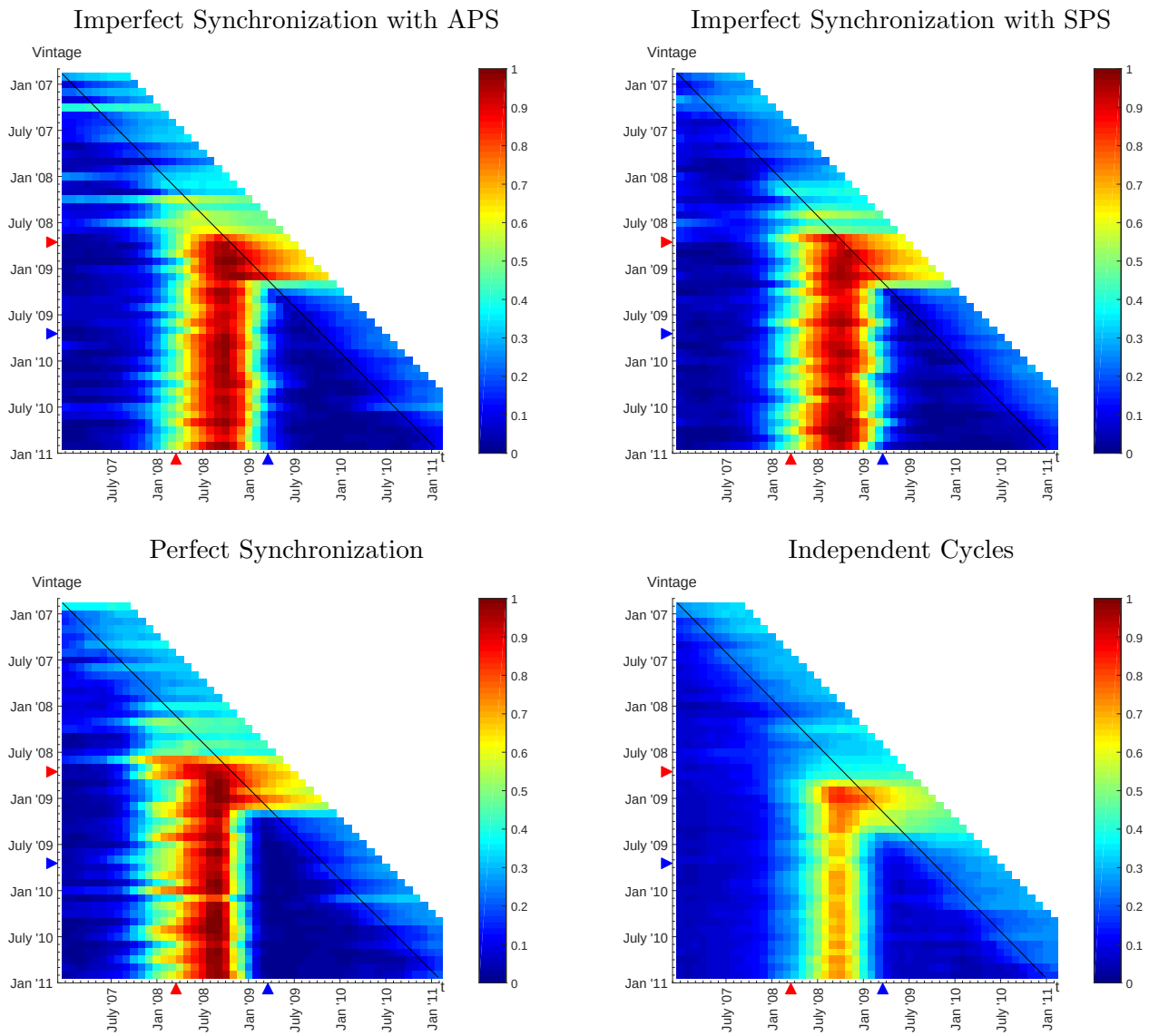


Figure 4: In sample - posterior recession probabilities in four separate models with volatility break.
Note: The graphs represent the posterior recession probabilities in four separate models with a single structural break in the variances of the measurement equations in (2), estimated for monthly growth rates of CEI and FCI for period January 1999 - November 2017, with alternative types of relationships between the real economy and financial conditions cycles. S_1 and S_2 represent recessions(=1) and expansions(=0) for CEI and FCI, respectively. The shaded areas shows recessionary episodes for Turkish economy based on the non-parametric business cycle dating algorithm proposed by Harding and Pagan (2002a,b). Posterior results are based on 110 000 draws from the posterior distribution where the first 10 000 draws are discarded as burn-in sample.

Figure 5: Real time nowcasting/forecasting exercise: In sample estimates and out-of-sample predictions of recession probabilities for the 2008-9 recession



Note: The graphs display the recession probabilities in an expanding horizon, where at every point on the vertical axes, the latest data vintage (each starts at January 1999 and ends at the indicated date) is used to compute in-sample estimates and out-of-sample predictions for 9 months ahead. Values of the recession probabilities which are bigger than 0.5 are represented by the shades of red color getting darker as the probabilities are getting closer to 1 and values less than 0.5 are represented by the shades of the blue color getting darker as the probabilities are getting closer to 0 as shown in the bars next to the graphs. On the vertical axes, the announcement date of GDP, II. quarter-2008 (September 10,2008) and II. quarter-2009 (September 10,2009), which give clues about the peak and through for 2008-9 recession, are marked by red and blue pointers, respectively. On the horizontal axes, the pointers mark the dates of start and end of the recession itself.

Supplementary Material for 'Modeling of Economic and Financial Conditions for Nowcasting and Forecasting Recessions: A Unified Approach'

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Appendix A The data set

Table 1: Set of Economic Variables: Series labels and their descriptions

Series Label	Description
ip	Industrial production index
import	Import quantity index
export	Export quantity index
empna	Total employment less agricultural employment
traserv	Trade and services turnover index
retails	Retail sales volume index
pmi	Purchasing manager index

Table 2: Set of Economic Variables: The transformations, adjustments, periods, frequencies and sources of coincident series

Series Label	T	Start	End	SA&NSA	Freqency	Source
ip	3	1986:7	2017:12	SA	M	TURKSTAT[1]
import	3	1997:1	2017:12	SA	M	TURKSTAT
export	3	1997:1	2017:12	SA	M	TURKSTAT
empna	3	2005:1	2017:11	SA	M	TURKSTAT
traserv	3	2005:3	2017:10	SA	Q	TURKSTAT
retails	3	2010:1	2017:12	SA	M	TURKSTAT
pmi	3	2011:1	2018:1	SA	M	ICI[2]

Note: T indicates the transformation of variable to ensure stationarity (1=level, 2=first difference, 3=first difference of logarithm). SA and NSA denote the adjustment to remove potential seasonality from series, where SA stands for Seasonally Adjusted or NSA for Not Seasonally Adjusted. M and Q denote frequency of the series, where M stands for Monthly and Q for Quarterly.

[1] TURKSTAT : Turkish Statistical Institute

[2] ICI : Istanbul Chamber of Industry

Table 3: Set of financial variables: Series labels and their descriptions

Series Label	**Description**
FXRes	Real Central Bank's Gross Foreign Exchange Reserves
goldres	Central Bank's Gross Gold Reserves
m1	Money Stock : M1
m2	Money Stock : M2
m3	Money Stock : M3
rm1	Real Money Stock : M1
rm2	Real Money Stock : M2
rm3	Real Money Stock : M3
bist100tra	Stock Exchange Trading Volume on the Istanbul Stock Exchange
rbist	Real Stock Price Index on the Istanbul Stock Exchange
VOL	Volatility on the Istanbul Stock Exchange 100
P-E	Price-Earning Ratio on the Istanbul Stock Exchange 100
liv	Cost of Living Index for Wage Earners
ppi	Producer Price Index
Conf	Real Confidence Index
embi	JP Morgan Emerging Markets Bond Index-Turkey
EMBI-Tr	Spread between JP Morgan Emerging Markets Bond Index-Turkey and 1-month Interest Rate on deposits
MSCIem	MSCI-Emerging Market Index
TETS	Spread between the 3-month Interest Rate on deposits and 3-month London Interbank Offered Rate
TermS	Spread between the 1-year and 1-month Interest Rate on Deposits
intbnk	Interbank Overnight Interest Rate
int1m	Interest Rate on Deposits - up to 1 month
int3m	Interest Rate on Deposits - up to 3 months
int6m	Interest Rate on Deposits - up to 6 months
int1y	Interest Rate on Deposits - up to 1 year
int1y_m	Interest Rate on Deposits - up to 1 year and more
discount	Discount Rate
TAuc	Treasury Auction Rate
cds	Credit Default Swap for Turkey 5-year Bond
dbeta	Downside Beta-Bist100 and MSCI World Index
exrate	Average USD-TRY Nominal Exchange Rate
exratecpi	CPI-based Effective Real Exchange Rate (base year=2003)
curac	Current Account Balance/ Nominal GDP (in $)
finac	Balance Of Payments-Financial Account/Nominal GDP (in $)
intdebt	Real Internal Debt Stock
Cred	Banking Sector Credit Loans
bnksec	Banking Sector-Securities at fair value through profit/loss, Securities available for sale, and securities to be held till maturity-real value
elpro	Gross Electricity Production
bullp	Gold Price Growth Rate (in $)
euribor3m	Euro Interbank Offered Rate-3 month
libor3m	London Interbank Offered Rate-3 month
efunr	Effective Federal Funds Rate
tedsprd	TED Spread: Spread between 3-month US Treasury bill and 3-month LIBOR
vix	CBOE Volatility Index: VIX growth rate

Table 4: Set of financial variables: The transformations, adjustments, periods, frequencies and sources of coincident series

Series Label	T	Start	End	SA&NSA	Source
FXRes	3	1990:2	2017:12	NSA	CBRT[3]
goldres	3	1990:2	2018:1	NSA	CBRT
m1	3	1990:1	2018:1	SA	CBRT
m2	3	1986:2	2018:1	SA	CBRT
m3	3	1986:2	2018:1	SA	CBRT
rm1	3	1990:1	2018:1	SA	CBRT
rm2	3	1986:2	2018:1	SA	CBRT
rm3	3	1986:2	2018:1	SA	CBRT
bist100tra	3	1998:2	2018:1	NSA	Bloomberg
rbist	3	1986:3	2018:2	NSA	Bloomberg
VOL	3	1988:2	2018:1	NSA	ISE[4]
P-E	2	1988:2	2017:12	NSA	ISE
liv	3	1996:2	2018:1	SA	CBRT
ppi	3	1994:2	2017:12	SA	CBRT
Conf	3	1988:1	2018:1	NSA	CBRT
embi	3	1999:8	2018:1	NSA	World Bank
EMBI-Tr	2	1996:6	2017:12	NSA	World Bank
MSCIem	2	1996:6	2017:12	NSA	World Bank
TETS	2	1996:6	2017:12	NSA	World Bank
TermS	2	1996:6	2017:12	NSA	World Bank
intbnk	2	1990:1	2018:2	NSA	OECD Statistics
int1m	2	2002:8	2017:12	NSA	TDM[5]
int3m	2	2002:8	2017:12	NSA	TDM
int6m	2	2002:8	2017:12	NSA	TDM
int1y	2	2002:8	2017:12	NSA	TDM
int1y_m	2	2002:8	2017:12	NSA	TDM
discount	2	1964:1	2017:12	NSA	IFS[6]
TAuc	3	1994:6	2018:1	NSA	TREASURY
cds	3	2000:11	2018:2	NSA	Bloomberg
dbeta	2	1987:1	2018:2	NSA	Thomson One
exrate	3	1990:1	2018:2	NSA	CBRT
exratecpi	3	1994:1	2017:12	NSA	BIS[7]
curac	1	1992:1	2017:9	SA	CBRT
finac	1	1992:1	2017:9	SA	TREASURY
intdebt	3	1998:1	2017:12	NSA	TREASURY
Cred	3	1998:1	2018:1	NSA	CBRT
bnksec	3	1986:1	2017:12	NSA	CBRT
elpro	3	1999:1	2018:1	SA	TETC[8]
bullp	3	1998:1	2018:1	NSA	CBRT
euribor3m	2	1999:1	2018:2	NSA	FRED[9]
libor3m	2	1986:2	2018:2	NSA	FRED
efunr	2	1954:8	2018:2	NSA	FRED
tedsprd	3	1986:1	2018:2	NSA	FRED
vix	3	2004:2	2018:2	NSA	FRED

Note: T indicates the transformation of variable to ensure stationarity (1=level, 2=first difference, 3=first difference of logarithm). SA and NSA denote the adjustment to remove potential seasonality from series, where SA stands for Seasonally Adjusted or NSA for Not Seasonally Adjusted. All series are at monthly frequency. Higher frequency series are converted to monthly frequency by averaging the observations. The volatility of Istanbul Stock Exchange, *bistvol*, is computed as the realized volatility of the Index on a daily basis for 21 trading days. The downside beta for Turkey, *dbeta*, is computed using Istanbul Stock Exchange Index-100 and MSCI World Index. For further details, see Bawa and Lindenberg (1977).

[3] Central Bank of Republic of Turkey
[4] Istanbul Stock Exchange
[5] Turkey Data Monitor
[6] International Financial Statistics
[7] Bank for International Settlements
[8] Turkish Electricity Transmission Company
[9] Federal Reserve Bank of St. Louis Economic Database

Appendix B Econometric Model

In this section we provide details about econometric model. In the next section we discuss Bayesian inference of model parameters in detail. The econometric models is as follows

$$\begin{aligned} y_{i,t} &= \lambda_i f_t + \varepsilon_{i,t} \\ \psi(L)\varepsilon_{i,t} &= \epsilon_{i,t} \quad \epsilon_{it} \sim t(0, \nu, \sigma^2_{i,t}) \\ \sigma^2_{i,t} &= \sigma^2_{i,1}\, \mathbb{I}\,[t < \tau] + \sigma^2_{i,2}\, \mathbb{I}\,[t > \tau] \;\text{ for } i = 1, \ldots, N \\ f_t &= \alpha_{\mathbb{S}_t} + \Phi f_{t-1} + \eta_t \quad \eta_t \sim N(0, \Sigma) \\ S_{2,t-\kappa_{S_{1,t}}} &= S_{1,t}. \end{aligned} \tag{B.1}$$

For the autoregressive dynamics of the idiosyncratic factors we use an AR(3) specificication for those of the coincident variables. For the financial variables we assume that the idiosyncratic factors are temporally independent. The resulting model can be cast into a state-space form as

$$\begin{aligned} \mathbf{y}_t &= \mathbf{H}\boldsymbol{\beta}_t + \boldsymbol{\varepsilon}_t & \boldsymbol{\varepsilon}_t|\xi_t \sim N(\mathbf{0}, \mathbf{R}_t) \\ \boldsymbol{\beta}_t &= \boldsymbol{\alpha}_{\mathbb{S}_t} + \mathbf{F}\boldsymbol{\beta}_{t-1} + \boldsymbol{\eta}_t & \boldsymbol{\eta}_t|\xi_t \sim N(\mathbf{0}, \boldsymbol{\Omega}_\mathbf{t}), \end{aligned} \tag{B.2}$$

where

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_2 \end{bmatrix}, \boldsymbol{\beta}_t = \begin{bmatrix} \boldsymbol{\beta}_{1,t} \\ f_{2,t} \end{bmatrix}, \mathbf{R}_t = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_{2,t} \end{bmatrix}, \boldsymbol{\alpha}_{\mathbb{S}_t} = \begin{bmatrix} \boldsymbol{\alpha}_{1,S_{1,t}} \\ \alpha_{2,S_{2,t}} \end{bmatrix}, \mathbf{F} = \begin{bmatrix} \mathbf{F}_1 & \mathbf{F}_{1,2} \\ \mathbf{F}_{2,1} & \phi_{2,2} \end{bmatrix} \boldsymbol{\Omega}_\mathbf{t} = \begin{bmatrix} \boldsymbol{\Omega}_{1,t} & \boldsymbol{\Omega}_{1,2} \\ \boldsymbol{\Omega}_{2,1} & \sigma^2_{f_2} \end{bmatrix}.$$

More specifically,

$$\mathbf{H}_1 = \begin{bmatrix} \lambda_{1,1} & 1 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\ \lambda_{2,1} & 0 & 0 & 0 & 1 & \ldots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & & \ddots & \vdots & \vdots & \vdots \\ \lambda_{7,1} & 0 & 0 & 0 & 0 & \ldots & 1 & 0 & 0 \end{bmatrix} \mathbf{H}_2 = \begin{bmatrix} \lambda_{8,2} \\ \lambda_{9,2} \\ \vdots \\ \lambda_{18,2} \end{bmatrix} \boldsymbol{\beta}_{1,t} = \begin{bmatrix} f_{1,t} \\ \varepsilon_{1,t} \\ \varepsilon_{1,t-1} \\ \varepsilon_{1,t-2} \\ \varepsilon_{2,t} \\ \varepsilon_{2,t-1} \\ \varepsilon_{2,t-2} \\ \vdots \\ \varepsilon_{7,t} \\ \varepsilon_{7,t-1} \\ \varepsilon_{7,t-2} \end{bmatrix} \mathbf{R}_{2,t} = \begin{bmatrix} \sigma^2_{8,t}/\xi_t & 0 & \ldots & 0 \\ 0 & \sigma^2_{9,t}/\xi_t & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \sigma^2_{18,t}/\xi_t \end{bmatrix}$$

$$\boldsymbol{\alpha}_{1,S_{1,t}} = \begin{bmatrix} \alpha_{1,S_{1,t}} \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \mathbf{F}_1 = \begin{bmatrix} \phi_{1,1} & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & \psi_{1,1} & \psi_{1,2} & \psi_{1,3} & \dots & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & \psi_{7,1} & \psi_{7,2} & \psi_{7,3} \\ 0 & 0 & 0 & 0 & \dots & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 1 & 0 \end{bmatrix}$$

$$\mathbf{Q}_{1,t} = \begin{bmatrix} \sigma^2_{f_1} & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & \sigma^2_{1,t}/\xi_t & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 0 & \sigma^2_{7,t}/\xi_t & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \end{bmatrix}$$

The contemporaneous and temporal link between CEI and FCI in **linear** form is through the specifications of the $\boldsymbol{\Omega}_{1,2}$ and $\mathbf{F}_{1,2}, \mathbf{F}_{2,1}$ respectively. As we model the **nonlinear** link between CEI and FCI through their relation between the cyclical components, we set these matrices to zero to improve identification. Bayes factors computed using mildly informative prior favors these restrictions as well.

Appendix C Conditional Posterior Distributions

In this appendix, we derive the posterior distributions used in the sampling scheme described in Section 3.3 which is the following

1. Sample f^T from $p(f^T|y^T, \Phi^{(m-1)}, \Sigma^{(m-1)}, \mathbb{S}^{T(m-1)})$
2. Sample $\mathbb{S}^T$ from $p(\mathbb{S}^T|f^{T(m)}, \Phi^{(m-1)}, \Sigma^{(m-1)})$
3. Sample α from $f(\alpha|y^T, \mathbb{S}^{T(m)}, \Phi^{(m-1)}, \sigma^{2(m-1)}, \lambda^{(m-1)}, \psi^{(m-1)})$
4. Sample Φ from $f(\Phi|y^T, \mathbb{S}^{T(m)}, \alpha^{(m)}, \sigma^{2(m-1)}, \lambda^{(m-1)}, \psi^{(m-1)})$
5. Sample Σ from $f(\Sigma|y^T, \mathbb{S}^{T(m)}, \alpha^{(m)}, \Phi^{(m)}, \sigma^{2(m-1)}, \lambda^{(m-1)}, \psi^{(m-1)})$
6. Sample κ from $f(\kappa|y^T, \mathbb{S}_1^{(m)}, \alpha^{(m)}, \Phi^{(m)}, \sigma^{2(m-1)}, \lambda^{(m-1)}, \psi^{(m-1)})$
7. Sample λ from $f(\lambda|y^T, f^{T(m)}, \sigma^{2(m-1)}, \psi^{(m-1)}, \tau^{(m-1)})$
8. Sample σ^2 from $f(\sigma^2|y^T, f^{T(m)}, \lambda^{(m)}, \psi^{(m-1)}, \tau^{(m-1)})$
9. Sample ψ from $f(\psi|y^T, f^{T(m)}, \lambda^{(m)}, \sigma^{2(m)}, \tau^{(m-1)})$
10. Sample τ from $f(\tau|y^T, f^{T(m)}, \lambda^{(m)}, \sigma^{2(m)}, \psi^{(m)})$
11. Sample P from $f(P|S_1^{(m)})$
12. Repeat (1)-(11) M times.

C.1 Sampling of f_t Conditional on the discrete regimes and model parameters the system (B.2) is linear Gaussian state-space model and therefore standard inference of the model can be carried out. This involves first running the Kalman filter forwards and running the simulation smoother backwards. The Kalman filter prediction steps are given in (14) in the main text. The remaining part of the Kalman filter is the updating steps, given as:

$$\begin{aligned} \boldsymbol{\beta}_{t|t} &= \boldsymbol{\beta}_{t|t-1} + \mathbf{K}_t \mathbf{v}_{t|t-1} \\ \mathbf{P}_{t|t} &= \mathbf{P}_{t|t-1} - \mathbf{K}_t \mathbf{H}^* \mathbf{P}_{t|t-1} \end{aligned} \tag{C.1}$$

where $\mathbf{K}_t = \mathbf{P}_{t|t-1}\mathbf{H}^{*'}\mathbf{V}_{t|t-1}^{-1}$ is the Kalman Gain. Once Kalman filter is ran forward and we can run a simulation smoother using the filtered values for drawing smoothed states as in Carter *et al.* (1994) and Frühwirth-Schnatter (1994). As this has become a standard practice in many applications, here we do not provide a detailed analysis but refer to standard textbooks such as Durbin and Koopman (2012).

C.2 Sampling of S_1^T To sample the discrete regime we employ a single-move sampler using the posterior density of $S_{1,t}$ as

$$f(S_{1,t}|S_1^{-t}, f^T, \theta) \propto f(S_{1,t}|S_{1,t-1}, \theta) f(S_{1,t+1}|S_{1,t}, \theta) \prod_{s=t-\kappa_{\min}}^{t+1+\kappa_{\max}} f(f_s|f^{s-1}, \mathbb{S}^s, \theta) \tag{C.2}$$

due to the Markov structure where $\kappa_{\max} = \max\{\kappa_1, \kappa_2\}$, $\kappa_{\min} = \min\{\kappa_1, \kappa_2\}$ and $X^t = \{X^1, \ldots, X^t\}$, $X^{-t} = \{X^1, \ldots, X^{t-1}, X^{t+1}, , \ldots, X^T\}$.
Conditional on the factors, $f(f_s|f^{s-1}, \mathbb{S}^s, \theta)$ follows a Gaussian distribution derived from the standard regression framework with Gaussian error terms in (C.4). The term $f(S_{1,t+1}|S_{1,t}, \boldsymbol{\theta})$ drops out at $t = T$. For $t = 1$, the term $S_{1,1}$ can be sampled from

$$f(S_{1,1}|S_1^{-1}, f^T, \theta) \propto f(S_{1,1}|\theta) f(S_{1,2}|S_{1,1}, \theta) \prod_{s=\max(0,1-\kappa_{\min})}^{2+\kappa_{\max}} f(f_s|f^{s-1}, \mathbb{S}^s, \theta) \tag{C.3}$$

where the unconditional density $f(S_{1,1}|\boldsymbol{\theta})$ follows a binomial density with probability $(1-p_1)/(2-p_1-q_1)$ derived from the ergodic probabilities of the Markov chain. Sampling of the state variables can be implemented by starting from the most recent value of S_1^T and sampling the states backward in time, one after another. After each step, the tth element of S_1^T is replaced by its most recent draw.

We proceed with parameters that are related to the evolution of the common factors. For these parameters, we setup Metropolis Hastings samplers with candidates derived using the transition equations. The autoregressive process for the factors can be written as

$$f_{l,t} = (1 - S_{l,t})\alpha_{l,0} + S_{l,t}\alpha_{l,1} + \phi_{l,l} f_{l,t-1} + \eta_{l,t} \quad \eta_{l,t} \sim N(0, \sigma_{f_l}^2) \text{ for } l = 1, 2 \tag{C.4}$$

C.3 Sampling of $\boldsymbol{\alpha}_l$ for $l = 1, 2$ We use a Metropolis Hastings (MH) step to sample $\boldsymbol{\alpha}_l = (\alpha_{l,0}, \alpha_{l,1})'$. For obtaining an efficient candidate density, we first restructure (C.4) as

$$\sigma_{f_l}^{-1}(f_{l,t} - \phi_{l,l} f_{l,t-1}) = \sigma_{f_l}^{-1}\left((1 - S_{l,t})\alpha_{l,0} + S_{l,t}\alpha_{l,1}\right) + \sigma_{f_l}^{-1}\eta_{l,t} \text{ for } l = 1, 2 \tag{C.5}$$

to form a regression as

$$Y_t = X_t \boldsymbol{\alpha}_l + v_{l,t} \qquad v_{l,t} \sim N(0,1)$$

To sample $\boldsymbol{\alpha}_i = (\alpha_{i,0}, \alpha_{i,1})'$ from the candidate density, we use an multivariate normal distribution with mean $(X'X)^{-1}X'Y$ and variance $(X'X)^{-1}$, where $Y = (Y_2, \ldots, Y_T)'$ and $X = (X_2', \ldots, X_T')'$. As discussed in Section 3.2. in the main text, we impose restrictions on the elements on α_1 by sampling the parameters from the corresponding truncated distribution for identification of regimes. Once a draw is obtained from the candidate density, we setup a Metropolis Hastings step that evaluates the acceptance probability based on the distribution conditional on data rather than the factors by running the Kalman filter and computing the likelihood.

C.4 Sampling of $\phi_{l,l}$ and $\sigma^2_{f_l}$ for $l = 1, 2$ In order to impose unit unconditional variance for the identification of the factors we sample $\phi_{l,l}$ and σ^2_l jointly using the fact that $\sigma^2_{f_l} = (1 - \phi^2_{l,l})$ in case of unit unconditional variance. We use a MH step to sample $\phi_{l,l}$ and σ^2_l jointly. As in the previous case, for obtaining an efficient candidate density, we first restructure (C.4) as

$$\sigma^{-1}_{f_l}(f_{l,t} - \alpha_{l,S_{l,t}}) = \sigma^{-1}_{f_l} f_{l,t-1}\phi_{l,l} + \sigma^{-1}_{f_l}\eta_{l,t} \tag{C.6}$$

to form a regression as

$$Y_t = X_t \phi_{l,l} + v_{l,t} \qquad v_{l,t} \sim N(0,1)$$

To sample $\phi_{l,l}$ and σ^2_l from the candidate density, we use an multivariate normal distribution with mean $(X'X)^{-1}X'Y$ and variance $(X'X)^{-1}$, where $Y = (Y_2, \ldots, Y_T)'$ and $X = (X_2', \ldots, X_T')'$. Stationary is imposed by sampling the $\phi_{l,l}$ from the truncated distribution ensuring that $\phi_{l,l} < 1$. Using these variables we optimize the density using the restriction that $\sigma^2_{f_l} = (1 - \phi^2_{l,l})$ conditional on the factors to obtain a candidate draw for $\phi_{l,l}$ and $\sigma^2_{f_l}$. Once a draw is obtained we setup a Metropolis Hastings step that evaluates the acceptance probability based on the distribution conditional on data rather than the factors by running the Kalman filter and computing the likelihood.

C.5 Sampling of lead parameters $\boldsymbol{\kappa}$ As κ_1 and κ_2 parameters can only take discrete values we can compute the posterior probabilities for all $\boldsymbol{\kappa} \in \mathcal{C}$, where $\mathcal{C}$ defines restrictions and synchronization types. We sample $\boldsymbol{\kappa}$ from multinomial distribution, both (κ_1, κ_2) parameters at once conditional on data rather than factors using a MH step. We can minimize the computational cost by using only the part that is related to financial cycle S_2 as the shifts in S_1 and thus distinct values of $\boldsymbol{\kappa}$ is reflected distinct values of S_2. Therefore, we decompose the Kalman filter recursion and the simulation smoother into parts for obtaining the kernel distribution $\boldsymbol{\kappa}$ which improves the computational efficiency substantially.

Next, we proceed with parameters that are related to the measurement equation. The autoregressive process for the factors can be written as

$$\begin{aligned} y_{i,t} &= \lambda_i f_t + \varepsilon_{i,t} \\ \psi(L)\varepsilon_{i,t} &= \epsilon_{i,t} \quad \epsilon_{it}|\xi_{i,t} \sim N(0, \sigma^2_{i,t}/\xi_{i,t}) \quad \xi_{i,t} \sim \Gamma(\tfrac{\nu}{2}, \tfrac{\nu}{2}) \\ \sigma^2_{i,t} &= \sigma^2_{i,1}\, \mathbb{I}\,[t < \tau] + \sigma^2_{i,2}\, \mathbb{I}\,[t > \tau] \;\text{ for } i = 1, \ldots, N \end{aligned} \tag{C.7}$$

We first sample ξ_t using Gamma distribution update as

$$f(\xi_{i,t}|y_{i,t}, f_t, \sigma^2_{i,1}, \sigma^2_{i,2}, \psi_i(L), \lambda_i) \sim \begin{cases} \Gamma(\frac{v+1}{2}, \frac{v+\sigma^{-2}_{i,1}\psi_i(L)(y_{i,t}-\lambda_i f_t)^2}{2}) & \text{for} \quad t < \tau \\ \Gamma(\frac{v+1}{2}, \frac{v+\sigma^{-2}_{i,2}\psi_i(L)(y_{i,t}-\lambda_i f_t)^2}{2}) & \text{for} \quad t \geq \tau \end{cases} \tag{C.8}$$

, see for example Albert and Chib (1993), to transform the system to follow a Gaussian distribution. Let $a_{i,t} \equiv \xi^{1/2}_{i,t}\epsilon_{i,t}$ and $e_{i,t} \equiv \xi^{1/2}_{i,t}\varepsilon_{i,t}$ denote the scaled error terms that follow Gaussian distributions.

C.6 Sampling of λ_i To sample λ_i we first transform the measurement equation by pre-multiplying with $\psi_i(L)$, $\xi_{i,t}$ and $\sigma^{-1}_{i,t}$ as

$$\sigma^{-1}_{i,t}\xi^{1/2}_{i,t}\Big(\psi_i(L)y_{i,t}\Big) = \sigma^{-1}_{i,t}\xi^{1/2}_{i,t}\Big(\psi_i(L)f_t\Big)\lambda_i + \sigma^{-1}_{i,t}\Big(\psi_i(L)e_{i,t}\Big) \tag{C.9}$$

to form a regression as

$$Y_t = X_t\lambda_i + v_{i,t} \qquad v_{i,t} \sim N(0,1)$$

To sample λ_i, we use a normal distribution with mean $(X'X)^{-1}X'Y$ and variance $(X'X)^{-1}$, where $Y = (Y_{k_i+1}, \ldots, Y_T)'$ and $X = (X'_{k_i+1}, \ldots, X'_T)'$. The lag structure of $\psi(L)$, k_i, is 3 for the economic variables whereas it is set to zero for the financial variables where $\psi(L)$ becomes an identity matrix.

C.7 Sampling of $\sigma^2_{i,1}$ and $\sigma^2_{i,2}$ Following the transformation in the previous step we can sample $\sigma^2_{i,1}$ and $\sigma^2_{i,2}$ from an inverse-Gamma distributions with scale parameters $\left(\sum_{t=4}^{\tau-1} a^2_{i,t}\right)$ and $\left(\sum_{t=\tau}^{T} a^2_{i,t}\right)$ and degrees of freedom $(\tau - (k_i + 1))$ and $(T - \tau + 1)$, respectively.

C.8 Sampling of $\psi_i(L)$ To sample $\psi_i(L)$ we first transform the measurement equations by pre-multiplying it with $\sigma^{-1}_{i,t}$. For the regression equations regarding to economic variables with 3 lags of idiosyncratic factors, we can write

$$\sigma^{-1}_{i,t} e_{i,t} = \sigma^{-1}_{i,t} e_{i,t-1}\psi_{i,1} + \sigma^{-1}_{i,t} e_{i,t-2}\psi_{i,2} + e_{i,t-3}\psi_{i,3} + \sigma^{-1}_{i,t} a_{i,t} \tag{C.10}$$

to form a regression as

$$Y_t = X_t \Psi_i + v_{i,t} \qquad v_{i,t} \sim N(0,1)$$

where $\Psi_i = (\psi_{i,1}, \psi_{i,2}, \psi_{i,3})'$. To sample Ψ_i, we use a normal distribution with mean $(X'X)^{-1}X'Y$ and variance $(X'X)^{-1}$, where $Y = (Y_{k_i+1}, \ldots, Y_T)'$ and $X = (X'_{k_i+1}, \ldots, X'_T)'$.

C.9 Sampling of τ The conditional posterior density of τ is as follows:

$$\begin{aligned} f(\tau|y^T, f^T, \theta) \propto \mathbb{I}[b+4 \leq \tau \leq T-b] \times \prod_{i=1}^{N} \Big((\sigma^{-1}_{i,1})^{(\tau-3)} (\sigma^{-1}_{i,2})^{(T-\tau+2)} \Big) \times \\ \exp\Big(-\frac{1}{2} \sum_{i=1}^{\hat{n}_1} \Big(\sigma^{-2}_{i,1} \sum_{t=4}^{\tau-1} a^2_{i,t} + \sigma^{-2}_{i,2} \sum_{t=\tau}^{T} a^2_{i,t} \Big) \Big) \end{aligned} \tag{C.11}$$

where N is the number of variables. We can sample τ as discrete values from the range $[b+4 \leq \tau \leq T-b]$ where $b = 12$ denoting the first and last 12 observations.

C.10 Sampling of p_i and q_i The conditional posterior densities of the transition parameters are given by

$$\begin{aligned} f(p_i \mid S_i) &\propto p_i^{T_{00}+N_{00}-1}(1-p_i)^{T_{01}+N_{01}-1} \\ f(q_i \mid S_i) &\propto q_i^{T_{10}+N_{10}-1}(1-q_i)^{T_{11}+N_{11}-1} \end{aligned} \tag{C.12}$$

where T_{ij} denotes the number of transitions from state i to state j and N_{ij} denotes the corresponding parameters regarding to prior distribution. This corresponds to the kernel of a Beta distribution. Therefore, the transition probabilities can be sampled from a Beta distribution with parameters $T_{ij} + N_{ij}$.

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