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Salomon Faure and Hans Gersbach

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Centre for Economic Policy Research
33 Great Sutton Street, London EC1V 0DX, UK
Tel: +44 (0)20 7183 8801
www.cepr.org

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Abstract

We examine monetary architectures in which money is solely created by the public and lent by the central bank to the private sector. We compare them to today's fractional-reserve system in which money is created mainly by commercial banks. We use a simple general equilibrium setting and determine under which conditions these architectures yield the same welfare and stability outcomes and under which conditions they do not. We show, in particular, that the decentralized sovereign money system yields the same level of money creation and allocation of commodities as the fractional-reserve monetary system if the central bank solely pursues interest-rate policy.

JEL Classification: D50, E4, E5, G21

Keywords: money creation, Chicago Plan, full-reserve banking, 100% reserve banking, monetary architecture, monetary system, Capital regulation, reserve requirement, monetary policy, price rigidities

Salomon Faure - sfaure@ethz.ch
ETH Zurich

Hans Gersbach - hgersbach@ethz.ch
ETH Zurich and CEPR

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Money Creation in Different Architectures*

Salomon Faure
CER-ETH – Center of Economic Research at ETH Zurich
Zürichbergstrasse 18
8032 Zurich, Switzerland
salomon.faure@gmail.com

Hans Gersbach
CER-ETH – Center of Economic Research at ETH Zurich and CEPR
Zürichbergstrasse 18
8032 Zurich, Switzerland
hgersbach@ethz.ch

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Abstract

We examine monetary architectures in which money is solely created by the central bank and issued or lent to the private sector. We compare them to today's fractional-reserve system in which money is mainly created by commercial banks. We use a simple general equilibrium setting and determine under which conditions these architectures yield the same welfare and stability outcomes and under which conditions they do not. Moreover, we show that the decentralized sovereign money system yields the same level of money creation and same allocation of commodities as the fractional-reserve system, when the central bank solely pursues an interest-rate policy.

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1 Introduction

Motivation and approach

Today, in the vast majority of countries, most money is created by commercial banks in the form of bank deposits, when commercial banks grant loans to firms or households, for instance. These deposits are only partially covered by central bank reserves and the system is thus often called "fractional-reserve banking". This type of modern banking roots in the behavior of goldsmiths in the seventeenth century. As goldsmiths noticed that deposit holders would very rarely withdraw all their deposits—which were claims on gold at the time—they started to lend a portion of the deposits to new customers, thereby increasing their profits. However, these goldsmiths also became prone to bank runs, as they were not able to hand out the gold if a large share of deposit holders wanted to withdraw their deposits at the same time[1]. Thus, this strategy was subject to a trade-off between liquidity and profitability.

Concerns about the performance and stability of the current monetary system triggered proposals that forbid commercial banks to create money, which we will discuss below. Most recently, in Switzerland, the monetary system was subject to a popular initiative, called the "Vollgeldinitiative". The idea was to replace the monetary system currently at work in Switzerland by a new monetary system, in which money can only be created by the central bank and banks are required to hold deposits off-balance sheet as well as to maintain at all times an amount of central bank reserves sufficient to cover all outstanding deposits.

To examine monetary architecture, we will use a simple general equilibrium setting in which banks and bond financing coexist in the presence of macroeconomic risk. Money can only be created by the central bank, when it grants loans to commercial banks. In one monetary system, deposits are held directly at the central bank. We call this system a "centralized deposit system". In the other alternative monetary system, deposits are held at commercial banks. We call this system a "decentralized deposit system". In both models, banks make profits by granting loans to firms and, thus, demand some amount of money from the central bank. The central bank sets the interest rate charged for borrowing central bank money. Central bank lending is either unconstrained, or commercial banks are subject to capital requirements, or the central bank imposes reserve requirements combined with haircuts

[1] See e.g. Ferguson (2008).

on borrowing. The two monetary architectures we examine are illustrated in Figure 1. We compare these monetary architectures among themselves and to the fractional-reserve

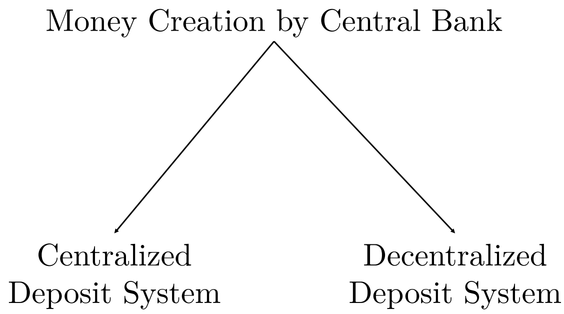

Figure 1: The two monetary architectures examined in our paper.

system. On purpose, we focus on simple benchmarks to decide when these architectures yield the same or different outcomes and when they replicate first-best allocations or yield inefficient outcomes. For this exercise, we consider both cases, when prices are flexible and when they are rigid.

Main insights

We first look at situations when prices are flexible. Then, the decentralized deposit system replicates the allocations of the fractional-reserve monetary system. There is a clear intuition for this result. If the central bank only uses the interest rate as a policy instrument, it does not matter whether a bank creates loans and deposits and later refinances its interbank liabilities – created in the payment process – at the central bank, or directly borrows central bank money in a sovereign money architecture and lends this money to the private sector. The default risks are also the same under both systems.

Except when capital requirements are set at low levels, the decentralized deposit system achieves the first-best allocation. Since banks can scale up their balance sheets easily by money creation, we obtain too large and too risky banking systems, comprising, however, the possibility of a banking crisis, when capital requirements are too low. In contrast, when deposits are centralized, only equilibria that implement the first-best allocation emerge, since there is no default risk of banks.

We next look at situations when prices are rigid. Due to the knife-edge property of our model, we obtain situations in which equilibria are either first-best, represent a too large and too risky banking system or lead to unlimited or vanishing money creation and banking activity. Again, the equilibria under the current architecture and for the decentralized deposit scheme with central bank money creation are identical. With rigid prices, however, both monetary systems are much more fragile. The banking system may contract excessively when central bank lending rates are too large, or it may grow excessively in the opposite case. Excessive growth of banks' balance sheets can always be avoided by imposing sufficiently high capital requirements.

The centralized deposit system does not involve equilibria with large and overly risky bank balance sheets. However, there are circumstances when a decentralized deposit system can still support an active banking system, while the centralized system may not be able to do the same. The reason is that the profitability of banks from the perspective of shareholders is higher in the decentralized system, since banks can leverage themselves.

While we are focusing on the role of capital requirements on constraining attempts of banks to expand their balance sheets excessively, we show that reserve requirements combined with haircuts on borrowing from the central bank will achieve the same effects in all situations. Moreover, we focus on money that is lent to commercial banks by the central bank. We obtain equivalent results if the central bank issues money directly to the private sector against bonds or investment goods, and the gains from money creation are distributed as lump-sum transfers to households.

Relation to the literature

At the beginning of the twentieth century, a new monetary architecture was described by six notable economists—Paul H. Douglas, Irving Fisher, Frank D. Graham, Earl J. Hamilton, Willford I. King, and Charles R. Whittlesey—in an article entitled "A Program for Monetary Reform" (Douglas et al. (1939)). In this monetary system described, money is only created by the central bank and deposits are fully covered by central bank reserves. As explained by Allen (1993), Irving Fisher was particularly involved in the ensuing debate (Fisher (1936)) and submitted the proposal, called the "Chicago Plan", to President Roosevelt in 1941. But the suggested reform was not implemented, although it received the overwhelming support of about four hundred economists. In the Chicago Plan and the monetary architectures we describe in our paper, money is created by a central authority and deposits are entirely covered by central bank reserves.[2]

The interest in alternative monetary architectures was rcently revived by the runs on financial institutions that took place during the recent financial crisis. Kotlikoff (2010) advocates a switch to a monetary architecture similar to the one described in the Chicago Plan. He calls it "Limited Purpose Banking". Benes and Kumhof (2012) compared two DSGE models of the US economy—one under the current monetary architecture and one under the Chicago Plan. They suggest that under the alternative monetary system, credit fluctuations can be better controlled, bank runs are eliminated, and public and private debt is reduced.

In this paper, we use a simple model to compare the current monetary architecture to the two presented monetary systems in which money is only created by the central bank. We choose the most simple set-up, in the hope that the outcome difference will be minimal and that the results can establish some simple benchmarks. This should help to understand considerable differences in more sophisticated models. The characteristics of our model yield results with knife-edge properties. For example, either money collapses to zero, is unlimited, or is at the optimal level. This allows a simple description of the mechanisms at work, and of the appropriate capital regulation and monetary policy responses. It will be the starting point for smoother variations of the model and extensions and generalizations along the line of Magill and Quinzii (1992) for instance.[3]

Our paper also draws on the literature why fiat money can have positive value in a finite-horizon model when, first, there are sufficiently large penalties when debts to governments—such as tax liabilities—are not paid and, second, there are sufficiently large gains from using and trading money, see for instance Shubik and Wilson (1977), Dubey and Geanakoplos (2003a,b), Shapley and Shubik (1977), Kiyotaki and Moore (2003), and Shubik and Tsomocos (1992). Moreover, we use a model in which financial markets and banks can coexist and we follow Bolton and Freixas (2000) in this regard. They show that safe firms borrow from the bond market, whereas riskier firms are financed by banks. Based on these insights we construct our model on the assumption that there are two different types of firms. The first type encompasses small and opaque firms, which are risky and need to be monitored

[2]Differences can occur in the way the central bank injects money into the economy. In the Chicago Plan, money is created when the central bank buys government bonds, whereas in the model we describe in our paper, the central bank grants loans to banks, which then lend the money to firms.

[3]The introduction of risk-averse households, transaction costs, costs of monitoring and deposit creation may entail smoother versions in which the amount of money created may respond more smoothly to changes in interest rates, for instance.

by banks to get financing. The second type assembles large firms, which are safe and can obtain financing directly from households through bond issues.

Structure of the paper

Section 2 describes different monetary architectures corresponding to general equilibrium models with money creation by the central bank only. Section 3 characterizes the first-best allocation and describes the potential sources of inefficiencies. Section 4 examines, under flexible and rigid prices, the impact of policies, such as the central bank policy rate and a minimum-equity-ratio requirement, for each monetary architecture on the existence of equilibria with banks and on their welfare. Section 5 concludes.

2 The Model

Our approach is based on a simple general equilibrium setting. Our model has two periods, one investment good, two production sectors and one consumption good. Households are initially endowed with the investment good. We use $t = 0$ and $t = 1$ to denote the first and the second period, respectively. There are firms in both sectors, commercial banks, the central bank and the government. Households own firms and commercial banks.

We next describe the details of the model. We use bold characters for real variables to distinguish them from nominal variables. Furthermore, we differentiate individual quantities from aggregate quantities by using lower case letters for the former and capitals for the latter.

2.1 Agents

2.1.1 Entrepreneurs

In each production sector, firms use a specific technology to transform the investment good obtained in period $t = 0$ into the consumption good in period $t = 1$. These firms are owned by households and run by entrepreneurs, who only play a passive role and simply maximize shareholder value.

One technology is called the “moral hazard technology”. We refer to this technology hereafter as “sector MT” or simply “MT”. The entrepreneurs using this technology are subject to moral hazard, and repayment, and hence financing, is only possible if these firms are monitored by a banker.[4] The aggregate amount of the investment good invested in MT is denoted by $\mathbf{K_M} \in [\mathbf{0}, \mathbf{W}]$, where we use $\mathbf{W} > 0$ to denote the total amount of the investment good available in the first period. An investment of $\mathbf{K_M}$ units produces $\mathbf{K_M R_M}$ units of the consumption good, where $\mathbf{R_M} > 0$ denotes the real gross rate of return.[5]

The other technology is called the "frictionless technology". We refer to this technology hereafter as "sector FT" or simply "FT". The entrepreneurs using this technology are not subject to any moral hazard. The aggregate amount of investment good invested in FT is denoted by $\mathbf{K_F} \in [\mathbf{0}, \mathbf{W}]$, and we use $\mathbf{f}(\mathbf{K_F})$ to denote the output of sector FT in terms of the consumption good.[6] We assume $\mathbf{f}(\mathbf{0}) = \mathbf{0}$, $\mathbf{f}' > 0$ and $\mathbf{f}'' < 0$.

Positive profits from both production sectors are distributed to shareholders, i.e. to households. We use Π_F and Π_M to denote the profits of firms in FT and MT, respectively, and we use $\max(\Pi_F, 0)$ and $\max(\Pi_M, 0)$ to denote their shareholders' value.

2.1.2 Bankers

We use $b \in [0, 1]$ to label banks. Bankers do not play any active role, but simply aim at maximizing their shareholders' value. At the very beginning of the first period, banks are founded by households' investment in bank equity and, thus, households become the owners of banks. We assume that each bank obtains athe same amount of equity financing. Households invest in bank equity by paying with bank deposits they receive from selling their investment good.

We use e_B and E_B to denote the amount of equity investment in an individual bank and in aggregate, respectively. As banks constitute a set of measure equal to 1, $E_B = e_B$. We will first focus on cases with $E_B > 0$ and, thus, on situations in which banks are founded[7]

[4]Typically, in practice, sector MT consists of small or opaque firms that cannot obtain financing directly from households.

[5]We refer to a real gross rate of return—which we also call "real gross rate" or simply "gross rate"—as being the number of units of output (in terms of the consumption good) that is produced by using one unit of the investment good in the first period. Similarly, we refer to a nominal gross rate of return—which we also call "nominal gross rate" or simply "gross rate"—as being the amount of deposits that has to be reimbursed per unit of nominal investment in the first period.

[6]To obtain such a production function, one can assume a continuum of firms, each having one project, and projects have heterogeneous productivities.

[7]Typically, some minimal amount of equity investment is required to apply for a banking license.

and can engage in lending activities.[8] For the sake of simplicity, we assume that the moral hazard problem can be perfectly alleviated by banks when they grant loans to firms in MT and that there is no monitoring cost.[9] Banks grant loans to firms in MT, and charge them a lending gross rate, which we denote by R_L. The individual and the aggregate amount of loans is denoted by l^b_M and L_M, respectively. We use $\alpha^b_M := l^b_M/L_M$ to denote the quotient of lending by an individual bank b and the amount of average lending by banks.[10]

In this paragraph as well, we assume that the monetary architecture is a decentralized deposit system. When bank b lends l^b_M to firms in MT, it provides $d^b_M = l^b_M$ deposits to them. We denote aggregate private deposits by $D_M = L_M$. The deposits of firms in MT are distributed across banks according to the distribution d^b_M or, equivalently, according to α^b_M. Firms will then use these deposits to buy the investment good from households and in this process, households will obtain the deposits, which they will use to buy the consumption good in the second period.[11] We assume that households spread their accounts, and therefore their deposit holdings, evenly across banks. We use d_H to denote the amount of deposits held by households at an individual bank and R_D to denote the gross rate of return on deposits.

Limited liability protects bank owners from losses that are larger than their equity investment. Positive profits from banks are distributed to their shareholders. We use Π^b_B to denote the profits of Bank b, $\max(\Pi^b_B, 0)$ to denote its shareholders' value, and $\max(\Pi^b_B, 0)/E_B$ to denote its gross rate of return on equity.

2.1.3 Households

There is a continuum of identical and risk-neutral[12] households represented by $[0, 1]$. The consumption good is consumed solely by households in the second period. We can simplify our analysis by considering a representative household, which owns $\mathbf{W}$ units of the investment

[8] The case $E_B = 0$ will be discussed in Subsection A.1.2.

[9] Typically, banks monitor borrowers and enforce contractual obligations. In the language of typical moral hazard settings a la Holmström and Tirole (1997), our assumption means that, through their monitoring, banks can pledge the entire output from MT firms to depositors.

[10] Banks constitute a set of measure 1, and thus aggregate lending L_M is equal to average lending per bank.

[11] In practice, private deposits and central bank deposits are claims on banknotes. However, we do not explicitly introduce banknotes and coins into our model, as we assume that all payments are solely settled with deposits.

[12] A stronger degree of household risk-aversion would entail more complicated portfolio decisions. This issue is left to future research.

good at the beginning of the first period, as well as all firms in both sectors of production. The representative household exchanges a share of its endowment of the investment good against deposits from firms in MT. Then, it makes a portfolio choice of deposits, bank equity E_B, and bonds, which it buys from firms in FT against its remaining endowment of the investment good.[13] The representative household obtains dividends from its ownership of firms and of banks, together with the repayments from the reimbursement of bonds. It will use the deposits it receives in this process to buy the consumption good.

2.2 Macroeconomic Shock, Contracts, Prices and Assumptions

At the beginning of period $t = 1$ and thus after the allocation of the investment good to the production sectors in period $t = 0$, the output of the two technologies is affected by a macroeconomic shock $s \in \{l, h\}$. More specifically, we assume that this macroeconomic shock affects the real gross rate of return in sector MT only.[14] In particular, if $\mathbf{K_M}$ units of the investment good are invested in MT, sector MT produces $\mathbf{K_M R_M^s}$ with probability σ^s in state s ($0 < \sigma^s < 1$ for both states $s \in \{l, h\}$), where $\mathbf{R_M^s}$ represents the real gross rate of return in the state of the economy s. We assume that $0 < \mathbf{R_M^l} < \mathbf{R_M^h}$.

We assume that all contracts and thus all nominal gross rates that are to be repaid during period $t = 1$ can be made contingent on the realization of the macroeconomic shock. This reflects our assumption of complete markets.[15] In particular, banks offer state-contingent loans with nominal lending gross rates $(R_L^s)_{s\in\{l,h\}}$ to firms in MT and use their monitoring technology to ensure repayment from those entrepreneurs who are plagued by moral hazard (see Subsection 2.1.1). The lending interest rates are given by $(R_L^s - 1)_{s\in\{l,h\}}$.

We use the notation $\mathbb{E}[X]$ to denote the expected value of some random variable X. We make the following two assumptions throughout our paper:

[13]As the representative household is the only source of financing for firms in FT, the household has no preference for a particular capital structure, and we could also assume that firms in FT would be solely financed by equity, without affecting our results.

[14]This assumption is not crucial for our results in the sense that they would not change qualitatively if the macroeconomic shock affected both sectors of production, but the analysis would just be more complicated in that case.

[15]However, the market is incomplete in the following two respects: Payments must be made with deposits, and households cannot invest directly in all firms, as firms in one sector have to be financed by financial intermediaries.

Assumption 1

$$\mathbf{f'(W)} < \mathbb{E}[\mathbf{R^s_M}] < \mathbf{f'(0)}.$$

where $\mathbf{f'(0)}$ denotes $\lim_{\mathbf{K_M}\to\mathbf{0}} \mathbf{f'(K_M)}$. Assumption 1 implies that the total output is not maximized by investing the entire amount of the investment good in only one of the sectors.

Assumption 2

$$R^s_D = R^s_{CB} \qquad \textit{for all states } s \in \{l, h\}.$$

Assumption 2 essentially means that there is a perfect pass-through between the deposit gross rate and the central bank policy gross rate.[16]

The prices of the consumption good and the investment good in terms of monetary units are denoted by $(p^s_C)_{s\in\{l,h\}}$ and p_I, respectively. As the production, in terms of the consumption good of firms in FT, does not depend on the state of the world, the real gross rate of return on bonds, which we denote by $\mathbf{R_F}$, is risk-free.

2.3 The Monetary Architectures, Public Authorities and the Origin of Money

There are two different monetary architectures regarding the form of deposit holdings:

(1) Centralized deposit system

In this monetary architecture only the central bank can create money and all agents have an account at the central bank. Banks cannot default against households, as deposits are not held at commercial banks. Depositing and lending are thus completely decoupled. As an alternative and equivalent formulation, we could assume that deposits and the corresponding reserves are held off-balance sheet by banks. In a centralized deposit system, banks are thus solely financed by equity and by a loan from the central bank.

[16] This assumption can be rationalized by arbitrage opportunities for banks which would exist in case $R^s_D < R^s_{CB}$ or $R^s_D > R^s_{CB}$ for some state of the world, either by borrowing central bank money from other banks or by borrowing central bank money from the central bank and depositing it at other banks.

(2) Decentralized deposit system

In this monetary architecture only the central bank can create money and all deposits are fully covered by central bank reserves, but only commercial banks have an account at the central bank. Banks can default against households, and depositing and lending are not entirely decoupled. Banks are thus financed by equity, by a loan from the central bank, and by households' deposits.

Banks demand money from the central bank in order to lend to entrepreneurs plagued by moral hazard. Borrowing from the central bank is either unconstrained, or commercial banks are subject to capital requirements, or the central bank imposes reserve requirements combined with haircuts on borrowing.

In the two monetary architectures that we will describe in more detail in Subsections A.1 and A.2, we will assume that money is created at the beginning of period $t = 0$ by the central bank, when it grants loans to banks. Banks can also borrow from the central bank in order to make dividend payments to households, to settle households' payments of the lump-sum taxes used to bail-out defaulting banks, or in order to comply with a reserve requirement.

Money has value in our (finite-horizon) model since there are sufficiently large penalties for defaulting against the issuer of money and it is interest bearing. Moreover, we assume that money is essential to buy investment and consumption goods. Hence, equilibria in which both sectors receive investment goods necessarily involve money creation. The functioning of the monetary architecture is ensured by two governmental authorities—a government and a central bank. They play three roles. First, banks can obtain central bank deposits at the policy gross rates $(R^s_{CB})_{s\in\{l,h\}}$[17] at any time. Second, if a bank defaults on a liability to a governmental authority, the government levies sufficiently high penalties on the responsible bankers to deter bankers from defaulting. As a result, banks will avoid any strategy that entails a default on obligations to the central bank. However, banks may choose to default on households' deposits in some state of the economy. In such situations, the government's third role consists in ensuring the safety of households' deposits by bailing out banks that default on households' deposits. The bail-out is financed by lump-sum taxes on all households. In practice, the use of deposits as money requires these deposits to be safe. At a later stage, a third public authority, which we call "bank regulators", will impose capital requirements

[17] $(R^s_{CB} - 1)_{s\in\{l,h\}}$ are the policy interest rates.

and reserve requirements combined with haircuts.

We now examine the equilibria implemented for different regulations and policies—the reserve requirement together with the haircut, the capital requirement, and the central bank policy gross rates—and for each combination of these regulations and policies, we determine the associated level of welfare expressed in terms of household consumption. We assume that the central bank and the bank regulators aim at maximizing the households' welfare.

2.4 Sequence of Events

In Appendix A we describe every detail of these events as this ensures the consistency of the evolution of stocks and flows across three stages. In the main text here we focus on two ingredients of the model that are central for the definition and examination of equilibria.

First, an equilibrium with banks (and thus positive lending to Sector MT) requires $\mathbf{R}_{\mathbf{M}}^{\mathbf{s}} p_C^s = R_L^s p_I$ and thus entrepreneurs in Sector MT make zero profit. This a direct consequence of the linear MT technology. Second, the expression of Bank b's profits as follows:

$$\begin{aligned}\Pi_B^{b,s} &= (1-\alpha_M^b)L_M R_{CB}^s + \alpha_M^b L_M R_L^s - d_H R_D^s \\ &= (1-\alpha_M^b)L_M R_{CB}^s + \alpha_M^b L_M R_L^s - (L_M - E_B)R_D^s \\ &= \alpha_M^b L_M (R_L^s - R_{CB}^s) + L_M(R_{CB}^s - R_D^s) + E_B R_D^s. \end{aligned} \tag{1}$$

This is the central element for our analysis and has three terms. The first term is the profit from money creation and loan activities of bank b. As all deposits created will moove in the payment process to other banks, the bank has to settle the liabilities by central bank money. The intermediation margin $R_L^s - R_{CB}^s$ applies to these actvities The second term are the consequences from deposits of other banks moving to bank b. The intermediation margin $R_{CB}^s - R_D^s$ applies to these actvities. The third term stems from the reduction of bank debt since some of the deposits are transformed into bank equity. The gross rate of return on equity is equal to shareholders' value per unit of equity, and it is denoted by $R_E^{b,s} = \frac{\max(\Pi_B^{b,s},0)}{e_B}$.

Finally, it may be useful to display all the interactions in figures: Figure 2 summarizes the agents' interactions during period $t = 0$. Figure 3 summarizes the agents' interactions during period $t = 1$.

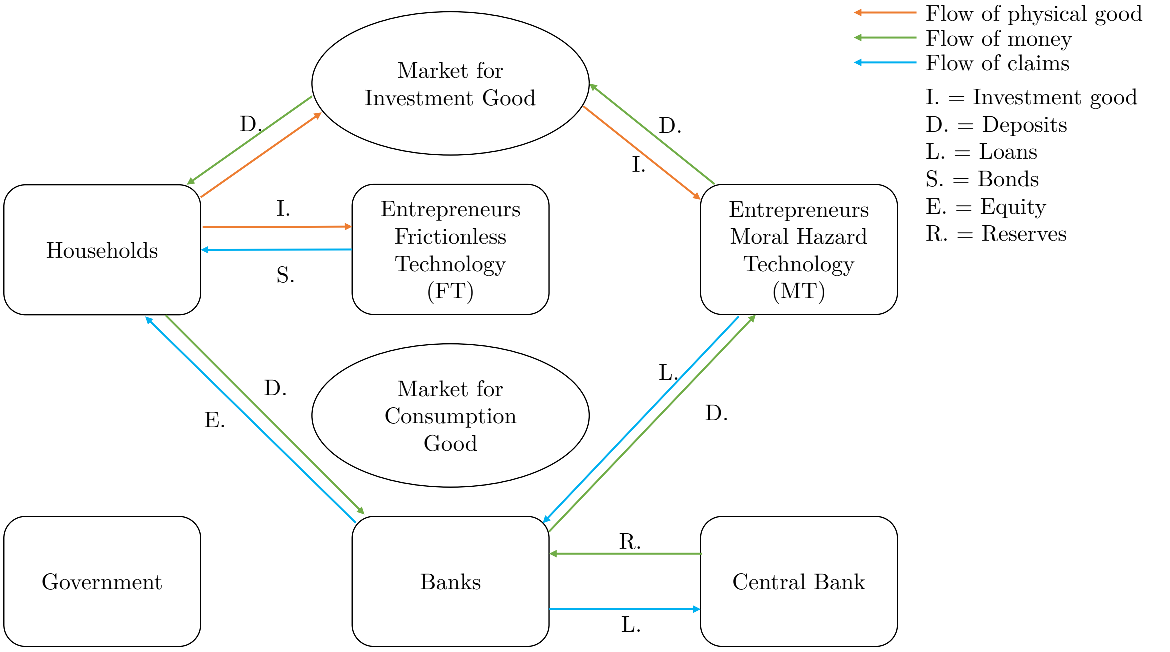

Figure 2: Interactions between agents during period $t = 0$.

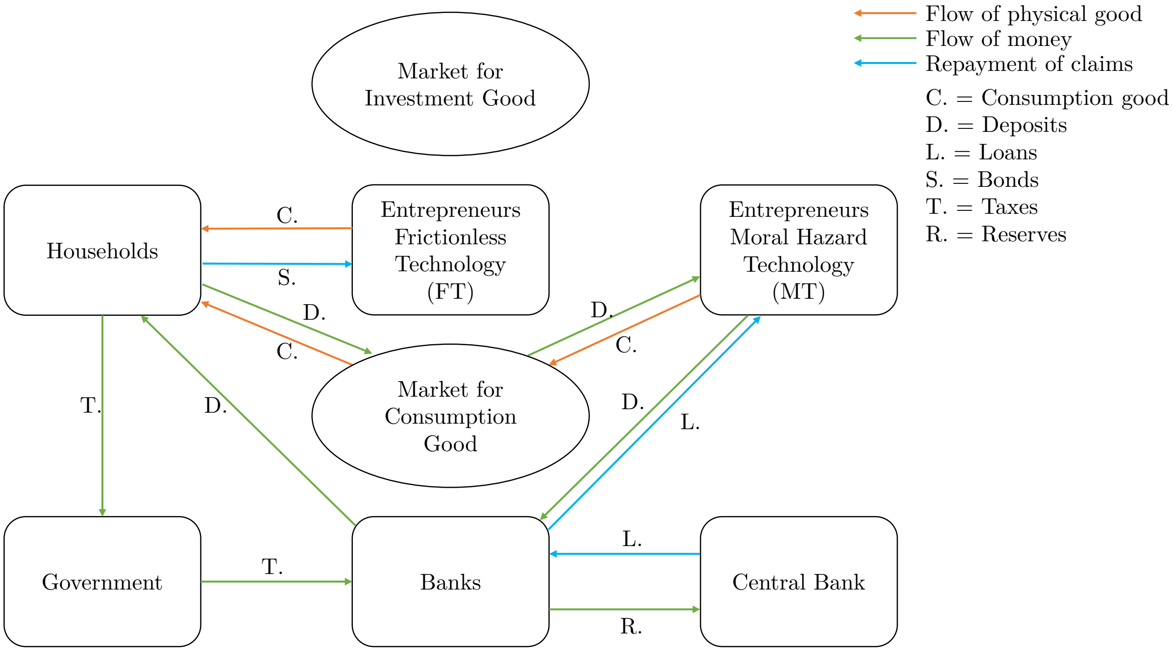

Figure 3: Interactions between agents during period $t = 1$.

2.5 Definition of an Equilibrium with Banks

In the end, the only difference between a decentralized and a centralized deposit system is that in a centralized deposit system, banks never fail, while in a decentralized deposit system, banks may fail. In a symmetric equilibrium with banks, all banks take the same decision regarding money demand and, thus, have identical balance sheets in equilibrium. Moreover, the policy gross rates $(R^s_{CB})_{s\in\{l,h\}}$ are set by the central bank, so equilibria with banks are dependent on the following choice.

Definition 1

Given the central bank policy gross rates $(R^s_{CB})_s$, a symmetric equilibrium with banks in the decentralized (resp. centralized) deposit system as described in Subsection A.1 (resp. A.2) is defined as a tuple

$$\begin{aligned}\mathcal{E} := \Big(&(R^s_E)_s, (R^s_D)_s, (R^s_L)_s, \mathbf{R_F},\\ &p_I, (p^s_C)_s, E_B, D_H, (\tilde{D}^s_H)_s, L_M, S_F,\\ &\mathbf{K_M}, \mathbf{K_F}\Big)\end{aligned}$$

consisting of positive and finite gross rates of return, prices, savings, bank deposits D_H in period $t = 0$, bank deposits $(\widetilde{D}^s_H)_s$ in period $t = 1$, and the corresponding physical investment allocation, such that

- *households hold some private deposits $D_H > 0$,*[18]
- *households maximize their expected utility*

$$\begin{aligned}&\max_{\{D_H,E_B,S_F\}} \left\{ E_B\mathbb{E}\left[\frac{R^s_E}{p^s_C}\right] + D_H\mathbb{E}\left[\frac{R^s_D}{p^s_C}\right] + \mathbf{f}(S_F)\right\}\\ &s.t. \quad E_B + D_H + p_I S_F = p_I\mathbf{W},\end{aligned}$$

  taking gross rates of return $(R^s_E)_s$, $(R^s_D)_s$ and prices p_I, $(p^s_C)_s$ as given,
- *firms in MT and FT, as well as each bank $b \in [0, 1]$, maximize their expected share-*

[18] As deposits are the only means of payment, we rule out knife-edge equilibria with banks in which, at the end of period $t = 0$, money demand is zero.

holders' value,[19] *given respectively by*

$$
\begin{aligned}
&\max_{\mathbf{K_M}\in[\mathbf{0},\mathbf{W}]}\left\{\mathbb{E}\left[\max\left(\mathbf{K_M}\left(\mathbf{R^s_M}-\frac{R^s_L}{p^s_C}p_I\right),0\right)\right]\right\}\\
&s.t.\ \mathbf{R^s_M}p^s_C = R^s_L p_I \text{ for } s\in\{l,h\},\\
&\max_{\mathbf{K_F}\in[\mathbf{0},\mathbf{W}]}\{\mathbb{E}[\max(\mathbf{f}(\mathbf{K_F})-\mathbf{K_F}\mathbf{R_F},0)]\},\\
\text{and}\quad &\max_{\alpha^b_M\geq 0}\left\{\mathbb{E}\left[\max\left(\alpha^b_M L_M\frac{R^s_L-R^s_{CB}}{p^s_C}+L_M\frac{R^s_{CB}-R^s_D}{p^s_C}+E_B\frac{R^s_D}{p^s_C},0\right)\right]\right\},
\end{aligned}
$$

taking gross rates of return $(R^s_D)_s$, $(R^s_L)_s$ *and* $\mathbf{R_F}$, *as well as prices* p_I *and* $(p^s_C)_s$ *as given,*

- *all banks choose the same level of money demand,*
- *banks may default or may not default (resp. never default), and*
- *markets for investment and consumption goods clear in each state.*

Henceforth, for ease of presentation, an equilibrium with banks given $(R^s_{CB})_s$ is a symmetric equilibrium with banks given $(R^s_{CB})_s$ in the sense of Definition 1.

In Appendix B, we show that a reserve requirement combined with a haircut has the same effect on money demand as a capital requirement. Therefore, without loss of generality, we consider neither reserve requirements nor haircuts, and we only use the minimum-equity-ratio requirement $\varphi^{reg}\in[0,1)$ which banks have to comply with, i.e. it holds that $\varphi^{reg}\leq\varphi$. We note that, by allowing φ^{reg} to take the value 0, we also consider situations where banks do not have to comply with any capital requirement.

Finally, we often will use the following fact:

Fact 1

If individual banks demand more money than the average bank and the central bank fulfills all demands of money, no finite money creation can exist.

[19] In our setting, the maximization of profits by firms and by banks in nominal terms is qualitatively equivalent to the maximization of profits in real terms. Details are available on request.

3 Welfare

The social planner's problem is given by

$$\max_{(\mathbf{K_M},\mathbf{K_F})} \mathbb{E}[\mathbf{K_M R_M^s} + \mathbf{f}(\mathbf{K_F})]$$
$$\text{s.t.} \quad \mathbf{K_M} + \mathbf{K_F} = \mathbf{W}.$$

Households' utility is clearly maximized at $\mathbf{K_F^{FB}} := \mathbf{f}'^{-1}(\mathbb{E}[\mathbf{R_M^s}])$.

Throughout the paper, we examine the equilibria arising from different policies. For this purpose, it is useful to introduce three types of situations which can give rise to inefficiencies:

(*i*) Money demand is positive and limited, but aggregate investment is distorted between sectors,

(*ii*) money demand is zero, and physical investment occurs only in sector FT, and

(*iii*) money demand unlimited and the central bank cannot restrain banks; the monetary system collapses, and physical investment remains viable in sector FT only.[20]

We now analyze the role of different policies on equilibria with banks, the allocations of investment good across sectors, and finally welfare in different monetary architectures.

4 Comparison Between Different Monetary Architectures

In Appendix D, we derive the optimal choices of the agents in the monetary architectures described in Subsections A.1 and A.2. We then evaluate the impact of the interest-rate policy pursued by the central bank and capital requirements on these optimal choices such that we can derive necessary and sufficient conditions for equilibria with banks. A general description of equilibria with banks is given in Proposition 2 in Subsection 4.1, for which we use a characterization of equilibria in reduced form that is also introduced in Subsection 4.1.

[20]Essentially, no equilibrium with banks and finite money creation exists. However, there exists an equilibrium in which no household offers equity to banks, all investment goods are channeled to sector FT, and no lending to sector MT occurs.

In Subsection and we finally provide a description of equilibria with banks under flexible and rirgid prices, respectively. Moreover, we will use the results of Faure and Gersbach (2016) to compare these monetary architectures to a monetary architecture with inside money creation, i.e. a system where banks create money by issuing deposits while granting loans.

4.1 Equilibria with Banks

From households' and firms' privately-optimal choices that we found in Lemma 2 and 3, respectively, and from the conditions for an equilibrium with banks, we can immediately derive a reduced form of an equilibrium with banks.

Proposition 1
Given the central bank policy gross rates $(R^s_{CB})_s$, an equilibrium with banks can be written as a function of $\mathbf{W}$, $\mathbf{R_F}$, $(\mathbf{R^s_M})_s$, $(R^s_L)_s$, φ, and p_I as follows:

$$\begin{aligned}
&R^s_E = \max\left(\mathbf{R^s_M}\left(\frac{R^s_L - R^s_{CB}}{\varphi p_I R^s_L} + \frac{R^s_{CB}}{p_I R^s_L}\right), 0\right), \quad R^s_D = R^s_{CB}, \quad p^s_C = \frac{R^s_L p_I}{\mathbf{R^s_M}},\\
&\hat{E}_B = \varphi p_I\big(\mathbf{W} - \mathbf{f}'^{-1}(\mathbf{R_F})\big), \quad \hat{D}_H = (1-\varphi)p_I\big(\mathbf{W} - \mathbf{f}'^{-1}(\mathbf{R_F})\big),\\
&\hat{D}^s_H = p_I\big(\mathbf{W} - \mathbf{f}'^{-1}(\mathbf{R_F})\big)R^s_{CB}, \quad L_M = p_I\big(\mathbf{W} - \mathbf{f}'^{-1}(\mathbf{R_F})\big), \quad \hat{S}_F = \mathbf{f}'^{-1}(\mathbf{R_F}),\\
&\mathbf{K_M} = (\mathbf{W} - \mathbf{f}'^{-1}(\mathbf{R_F})), \quad \mathbf{K_F} = \mathbf{f}'^{-1}(\mathbf{R_F}).
\end{aligned}$$

We use $\mathcal{R}\Big(\mathbf{W}, \mathbf{R_F}, (\mathbf{R^s_M})_s, (R^s_L)_s, \varphi, p_I\Big)$ to denote an equilibrium with banks given as above.

We note that banks' money demand will further constraint $\mathbf{R_F}$, $(\mathbf{R^s_M})_s$, (R^s_L), φ, and p_I to form an equilibrium with banks. In other words, any equilibrium with banks can be written in reduced form $\mathcal{R}\big(\mathbf{W}, \mathbf{R_F}, (\mathbf{R^s_M})_s, (R^s_L)_s, \varphi, p_I\big)$. However, for some arbitrary constellation $(\mathbf{W}, \mathbf{R_F}, (\mathbf{R^s_M})_s, (R^s_L)_s, \varphi, p_I)$ leading to a reduced form $\mathcal{R}\big(\mathbf{W}, \mathbf{R_F}, (\mathbf{R^s_M})_s, (R^s_L)_s, \varphi, p_I\big)$, the corresponding $\mathcal{E}$ may not be an equilibrium with banks.

The results in Appendix D allow us to characterize all equilibria with banks in the different architectures, described in Subsections A.1 and A.2, for all possible policy rates and capital requirements. We obtain

Proposition 2

Suppose that prices are flexible and that the central bank sets policy gross rates $(R^s_{CB})_s$ *and potentially a capital requirement* $\varphi^{reg} \in [0,1)$.

(1) In a decentralized deposit system, equilibria with banks can take either of the following two forms:

(1.a) $\mathcal{R}\Big(\mathbf{W}, \mathbf{R_F}, (\mathbf{R^s_M})_s, (R^s_L)_s, \varphi, p_I\Big)$ *for arbitrary values* $\mathbf{W} > 0$, $\mathbf{R^s_M} > 0$, $p_I > 0$, $\varphi \in (0,1)$, *and* $(R^s_L)_s$ *such that*

$$\mathbf{R_F} = \mathbb{E}[\mathbf{R^s_M}], \quad \mathbb{E}\left[\mathbf{R^s_M}\frac{R^s_{CB}}{R^s_L}\right] = \mathbb{E}[\mathbf{R^s_M}] \quad \textit{and} \quad \varphi^{reg} \geq \max_{s\in\{l,h\}}\left(1 - \frac{R^s_L}{R^s_{CB}}\right).$$

In these equilibria, no bank defaults.

(1.b) $\mathcal{R}\left(\mathbf{W}, \mathbf{R_F}, (\mathbf{R^s_M})_s, (R^s_L)_s, \varphi, p_I\right)$ *for arbitrary values* $\mathbf{W} > 0$, $\mathbf{R^s_M} > 0$, $p_I > 0$, *and* $(R^s_L)_s$ *such that*

$$\mathbf{R_F} = \mathbf{f}'(\mathbf{0}) - \max\left(0, \mathbf{f}'(\mathbf{0}) - p_I\mathbb{E}\left[\mathbf{R^s_M}\frac{R^s_{CB}}{R^s_L}\right]\right)$$

and

$$\mathbb{E}[\mathbf{R^s_M}] < \mathbb{E}\left[\mathbf{R^s_M}\frac{R^s_{CB}}{R^s_L}\right] \quad \textit{and} \quad \varphi^{reg} = \varphi = \max_{s\neq s'\in\{l,h\}}\left\{\frac{\sigma^s}{\sigma^{s'}}\frac{R^{s'}_L}{R^s_L}\frac{\mathbf{R^s_M}}{\mathbf{R^{s'}_M}}\frac{R^s_L - R^s_{CB}}{R^{s'}_{CB}}\right\}.$$

In these equilibria, banks default in some state $s \in \{l,h\}$.

(2) In a centralized deposit system, equilibria with banks take the following form:

$\mathcal{R}\Big(\mathbf{W}, \mathbf{R_F}, (\mathbf{R^s_M})_s, (R^s_L)_s, \varphi, p_I\Big)$ *for arbitrary values* $\mathbf{W} > 0$, $\mathbf{R^s_M} > 0$, $p_I > 0$, $\varphi \in (0,1)$, *and* $(R^s_L)_s$ *such that*

$$\mathbf{R_F} = \mathbb{E}[\mathbf{R^s_M}], \quad \mathbb{E}[\mathbf{R^s_M}] = \mathbb{E}\left[\mathbf{R^s_M}\frac{R^s_{CB}}{R^s_L}\right] \quad \textit{and} \quad \varphi \geq \max\left(\max_{s\in\{l,h\}}\left(1 - \frac{R^s_L}{R^s_{CB}}\right), \varphi^{reg}\right).$$

In these equilibria, no bank defaults.

The proof of Proposition 2 is given in Appendix E.1.

4.2 Flexible Prices

We start with the following proposition:

Proposition 3
Suppose that the monetary system is a decentralized deposit system and prices are flexible. Suppose that the central bank policy gross rates are given by $(R^s_{CB})_{s\in\{l,h\}}$. *Depending on the level of the capital requirement, we obtain different possible equilibria:*

(1) When no capital requirement is imposed, all equilibria with banks are efficient.

(2) When the capital requirement is positive but sufficiently low, i.e.

$$0 < \varphi^{reg} < \frac{\max_{s\in\{l,h\}}\Big(\sigma^s \mathbf{R}^{\mathbf{s}}_{\mathbf{M}}\Big)}{\mathbb{E}[\mathbf{R}^{\mathbf{s}}_{\mathbf{M}}]},$$

there are efficient and inefficient equilibria with banks.

(3) A sufficiently high capital requirement

$$\varphi^{reg} \geq \frac{\max_{s\in\{l,h\}}\Big(\sigma^s \mathbf{R}^{\mathbf{s}}_{\mathbf{M}}\Big)}{\mathbb{E}[\mathbf{R}^{\mathbf{s}}_{\mathbf{M}}]}$$

removes the inefficient equilibria with banks and only efficient equilibria with banks remains.

The proof of Proposition 3 is given in Appendix E.1. We deduce the following proposition directly from Proposition 2.

Proposition 4
Suppose that the monetary system is a centralized deposit system with flexible prices. Then, there are only efficient equilibria with banks, independently of the central bank policy gross rates $(R^s_{CB})_s$ *and the capital requirement* $\varphi^{reg} \in [0, 1)$.

We summarize the comparison between the different monetary architectures with flexible prices in Table 1. We include the results from Faure and Gersbach (2016) who characterize the equilibria for the same model for today's fractional reserve system in which deposits are created by commercial banks via deposit/loan creation to firms and in which commercial banks have to settle their interbank liabilities through electronic central bank money. This system is called inside money creation[21]

Policy Rates	$R^s_{CB} > 0$		
Capital Requirements	No	Low	High
Inside Money Creation	F	F, D	F
Decentralized	F	F, D	F
Centralized	F	F	F

Table 1: Comparison between the monetary architectures with flexible prices; F: first-best allocation, D: distorted investment allocation.

Several remarks are in order. First, we note that the equilibrium outcomes with inside money creation are identical to the ones in a decentralized deposit system.[22]

There is a clear intuition for this result. If the central bank only uses the interest rate as a policy instrument, it does not matter whether a bank creates loans and deposits and later refinances its interbank liabilities – created in the payment process – at the central bank, or directly borrows central bank money in a sovereign money architecture and lends this money to the private sector. The default risks are also the same under both systems.

Second, we note that in the monetary architecture with inside money creation as well as in the decentralized deposit system, equilibria with a distorted investment allocation appear when low capital requirements are implemented. These equilibria with banks are equilibria in which banks default in one of the states. The bail-out of households' deposits by the government authorities creates a distortion that makes investment in banking—equity or deposit investment—more attractive than it ultimately is.

The reason why these distorted equilibria appear under low capital requirements is that capital requirements limit money demand in the decentralized deposit system and money

[21] The corresponding paper is available at `cepr.org/active/publications/discussion_papers/dp.php?dpno=11368`.

[22] We note that all results heavily depend on our assumption that banks' demand for money is not rationed. Further results with possible rationing schemes are available on request.

creation in the monetary architecture analyzed by Faure and Gersbach (2016). In these equilibria, banks hold the minimum-required equity, which constrains their ability of money creation (resp. money demand) to the average level in the monetary architecture with inside money creation (resp. in the decentralized deposit system). Then, equilibria emerge in which banks default in one state and make enough profits in the other state, so that the expected real return on equity equals the expected real return on deposits.

Third, these distorted equilibria can be removed in the decentralized deposit system as well as in the monetary architecture with inside money creation by the implementation of capital requirements that are sufficiently high, as shown in Proposition 3. The reason is that the equity ratio has to be sufficiently low for commercial banks to default, which is necessary for these equilibria to emerge.

Fourth, there is no distorted equilibrium with banks in the centralized deposit system. The reason is that banks cannot fail in any state of the world, because of heavy penalties for defaulting against the central bank and because households do not hold deposits at commercial banks. Therefore, in all equilibria with banks that exist in a centralized deposit system, the equity ratio is high enough to prevent any default and there is no inefficient equilibrium with banks.

Fifth, we focused on money that is lent to commercial banks by central banks. We obtain the equivalent results if the central banks issues money directly to the private sector against bonds or investment goods, and the gains from money creation are distributed as lump-sum transfers to households.

4.3 Rigid Prices

We consider the most simple case of rigid prices and set $p_I = p_c^s = 1$ for all states which implies $R_L^s = \mathbf{R_M^s}$ for all states. Using this property in the formulas and proof for Proposition 2, we find the conditions under rigid prices. This yields the following two propositions:

Proposition 5

Suppose that the monetary system is a decentralized deposit system, and prices are rigid. Suppose that the central bank policy gross rates are given by $(R^s_{CB})_s$.

(1) *There exist efficient equilibria with banks if and only if*

$$\mathbb{E}[R^s_{CB}] = \mathbb{E}[\mathbf{R^s_M}]$$

and the potential capital requirement $\varphi^{reg} \in [0, 1)$ *is sufficiently high, i.e.*

$$\varphi^{reg} \geq \max_{s \in \{l,h\}} \left(1 - \frac{\mathbf{R^s_M}}{R^s_{CB}}\right).$$

(2) *There are inefficient equilibria with banks if and only if*

$$\mathbb{E}[\mathbf{R^s_M}] < \mathbb{E}[R^s_{CB}]$$

and the capital requirement $\varphi^{reg} \in [0, 1)$ *is equal to*

$$\varphi^{reg} = \frac{\sigma^s}{\sigma^{s'}} \frac{\mathbf{R^s_M} - R^s_{CB}}{R^{s'}_{CB}}.$$

for some state $s \neq s'$.

In all other cases, there is no equilibrium with banks and only the inefficient equilibria without banks remain.

Proposition 6

Suppose that the monetary system is a centralized deposit system, and prices are rigid. Suppose that the central bank policy gross rates are given by $(R^s_{CB})_s$. *Then, there are efficient equilibria with banks if and only if*

$$\mathbb{E}[R^s_{CB}] = \mathbb{E}[\mathbf{R^s_M}],$$

independently of the potential capital requirement $\varphi^{reg} \in [0, 1)$. *Independently of the central bank policy gross rates* $(R^s_{CB})_s$ *and of a potential capital requirement* $\varphi^{reg} \in [0, 1)$, *there never is any inefficient equilibrium with banks.*

When prices are rigid, we can summarize the comparison between different monetary architectures in Table 1 for the cases where $R^s_{CB} = \mathbf{R^s_M}$ in both states $s \in \{l, h\}$ and where $R^s_{CB} \geq \mathbf{R^s_M}$ and $R^{s'}_{CB} > \mathbf{R^{s'}_M}$ for some states $s \neq s'$, as well as in Table 2 for the case where $\mathbf{R^s_M} > R^s_{CB}$ for some state s. We again include the results from Faure and Gersbach (2016) for the inside money creation architecture in the same model.

Policy Rates	$R^s_{CB} = \mathbf{R^s_M}$	$R^s_{CB} \geq \mathbf{R^s_M}$ and $R^{s'}_{CB} > \mathbf{R^{s'}_M}$
Capital Requirements	$\varphi^{reg} \in [0, 1)$	$\varphi^{reg} \in [0, 1)$
Inside Money Creation	F	B
Decentralized	F	B
Centralized	F	B

Table 2: Comparison between the monetary architectures with rigid prices when either $R^s_{CB} = \mathbf{R^s_M}$ in all states $s \in \{l, h\}$ or $R^s_{CB} \geq \mathbf{R^s_M}$ and $R^{s'}_{CB} > \mathbf{R^{s'}_M}$ in some states $s \neq s'$. We use F to denote equilibria with banks implementing the first-best allocation and B to denote a breakdown of equilibria with banks.

Policy Rates	$\mathbb{E}[R^s_{CB}] < \mathbb{E}[\mathbf{R^s_M}]$	$\mathbb{E}[R^s_{CB}] = \mathbb{E}[\mathbf{R^s_M}]$		$\mathbb{E}[\mathbf{R^s_M}] < \mathbb{E}[R^s_{CB}]$		
Capital Requirements	$\varphi^{reg} \in [0, 1)$	No / Low	High	No / Low	Precise Value	High
Inside Money Creation	B	B	F	B	D	B
Decentralized	B	B	F	B	D	B
Centralized	B	F	F	B	B	B

Table 3: Comparison between the monetary architectures with rigid prices when $R^s_{CB} < \mathbf{R^s_M}$ for some state s. We use F to denote equilibria with banks implementing the first-best allocation, D to denote equilibria with banks implementing a distorted investment allocation, and B to denote a breakdown of equilibria with banks.

Several remarks are in order. First, we note that the equilibrium outcomes with inside money creation are identical to the ones in a decentralized deposit system for the same reasons as explained in Subsection 4.2.

Second, there cannot exist inefficient equilibria with banks in the centralized deposit system for the same reasons as explained in Subsection 4.2. There are equilibria with banks if and only if $\mathbb{E}[R^s_{CB}] = \mathbb{E}[\mathbf{R^s_M}]$. The latter equality is required to make investment in deposits as profitable as investing in bank equity. In these equilibria with banks, the equity ratio has to be high enough to prevent any default against the central bank.

Third, when $R^s_{CB} = \mathbf{R^s_M}$ in all states $s \in \{l, h\}$, equity is automatically as profitable as deposits and banks do not default. Thus, households basically have the choice between two investment technologies, which implements the first-best allocation. However, when $R^s_{CB} \geq \mathbf{R^s_M}$ for all states $s \in \{l, h\}$ with at least one strict inequality, banking is not profitable and banks neither demand nor create any amount of money. Hence, in these constellations, there cannot exist any equilibrium with banks.

Fourth, when $\mathbb{E}[R^s_{CB}] < \mathbb{E}[\mathbf{R^s_M}]$, the return on bank equity is higher than the return on deposits, which cannot be an equilibrium since money creation would be infinite. Similarly, $\mathbb{E}[\mathbf{R^s_M}] < \mathbb{E}[R^s_{CB}]$ can only be an equilibrium with banks if banks default, as otherwise, the return on equity would be lower than the return on deposits, which cannot hold in equilibrium.

Fifth, in the decentralized deposit system or in the architecture with inside money creation, if there exists a state s for which banking is profitable, which means that $\mathbf{R^s_M} > R^s_{CB}$ for some state s, all banks would like to create more money than the average bank, as potential losses in the other state $s' \neq s$ would be limited because of shareholder liability. In such situations, equilibria with banks cannot exist without a capital requirement. The minimum-equity-ratio requirement puts a constraint on money demand or money creation. Only capital requirements that are high enough can implement efficient equilibria with banks. Moreover, for efficient equilibria with banks to exist, the relation $\mathbb{E}[R^s_{CB}] = \mathbb{E}[\mathbf{R^s_M}]$ has to hold. Otherwise, households cannot be indifferent between equity and deposits which, in turn, is an equilibrium condition.

Sixth, in the decentralized deposit system or in the architecture with inside money creation, the equity ratio has to be equal to the capital requirement to keep money demand or money creation at the average level, when $\mathbb{E}[R^s_{CB}] > \mathbb{E}[\mathbf{R^s_M}]$. Then, only a unique value of the capital requirement can make bank equity as profitable as deposits. Thus, for any other capital requirement, there is no equilibrium with banks.

Finally, capital requirements that are high enough to avoid bank defaults for any money demand or any money created eliminate all possible equilibria with banks when $\mathbb{E}[R^s_{CB}] >$

$\mathbb{E}[\mathbf{R}^{\mathbf{s}}_{\mathbf{M}}]$. Without banks' default, $\mathbb{E}[R^s_{CB}] > \mathbb{E}[\mathbf{R}^{\mathbf{s}}_{\mathbf{M}}]$ would imply that the return on equity would be lower than the return on deposits, which we excluded in an equilibrium with banks.

We can summarize the role of capital requirements in the case where $R^s_{CB} < \mathbf{R}^{\mathbf{s}}_{\mathbf{M}}$ for some s—i.e. the case where banking is profitable in some state s—as follows: In the centralized deposit system, default against households is not possible, investment allocation is always first-best, and capital requirements have no impact on investment allocation. In the decentralized deposit system (resp. the architecture with inside money creation), capital requirements limit banks' demand for money (resp. banks' money creation), an adequately-chosen capital requirement can improve welfare, and capital requirement that are too high are counter-productive, as they are not compatible with a viable banking system.

5 Outlook

We have limited ourselves to a simple structure that allows to compare today's monetary architecture with alternatives. Hereby, our goal was establishing some simple benchmarks. Building on our analysis, numerous extensions are possible. Apart from smoother versions of our model with two concave production functions, introducing more fiscal considerations—government expenditures and government bonds—can help answer the question whether alternative monetary architectures could generate more seignorage for the public sector with the same type of monetary policy. Moreover, introducing maturity transformations and more sophisticated financial markets will shed more light on similarities and potential differences between today's monetary architecture and alternatives.

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A Sequence of Events (solely for the Referee)

A.1 A Decentralized Deposit System

We first describe in detail the timeline of events for a monetary architecture with a decentralized deposit system.

A.1.1 Period $\mathbf{t = 0}$

It is convenient to describe the sequence of economic activities via the balance sheets of households and banks. The economy starts with the balance sheets given in Table 4.

Households		Bank b	
$p_I \mathbf{W}$	E_H	0	0

Table 4: Decentralized deposit system: The agents' balance sheets at the beginning of period $t = 0$.

E_H denotes the households' equity in nominal terms, which represents the ownership of the investment good at the beginning of period $t = 0$.[23]

Either banks are not founded because no household invests in bank equity and the only possible allocation is given in Subsection A.1.2, or households found banks by pledging to convert a predefined share $\varphi \in (0, 1]$ of their initial deposits D_M into an amount $E_B = \varphi D_M$ of bank equity before production. When banks are founded, the gross rate of return on equity is equal to the shareholders' value per unit of equity, and it is denoted by $R_E^{b,s} = \max(\Pi_B^{b,s}, 0)/e_B$. In this section, we focus on the case where banks are founded, unless specified otherwise.

Each bank b has some demand for money. The amount of loans that the central bank supplies is denoted by l_{CB}^b. The lending gross rate charged for money borrowed from the central bank is denoted by $(R_{CB}^s)_{s \in \{l,h\}}$. Banks lend the amount $l_M^b = l_{CB}^b = \alpha_M^b L_M$ to firms in MT, which use the deposits to buy an amount of $\mathbf{K_M} = L_M/p_I$ of investment goods from households. Households invest in FT by buying $\mathbf{S_F}$ bonds denominated in real terms at the gross rate of return $p_C^s \mathbf{R_F}$, meaning that such a bond costs one unit of investment good and promises the delivery of $\mathbf{R_F}$ units of the consumption good once production has taken place.[24] Finally,

[23]Note that households also own firms in sectors MT and FT and may receive dividends from these firms' profits.

[24]In practice, such bonds are called "inflation-indexed bonds". Using bonds denominated in nominal terms does not change the results qualitatively but complicates the analysis, as one has to verify that firms do not default.

at the end of period $t = 0$, households pay for the equity E_B pledged at the beginning of period $t = 0$ with deposits, which destroys money by reducing the amount of deposits in the economy. The resulting amount of deposits is denoted by d_H for an individual bank and $D_H = L_M - E_B$ for the aggregate banking system. The total amount of reserves held by any bank is denoted by d_{CB} and fulfills $d_{CB} = d_H + e_B$. At all times, banks hold reserves that are at least equal to their deposit liabilities. The balance sheets showing positions before the macroeconomic shock takes place are given in Table 5.

Households		Firms in MT		Bank b		Central Bank	
$p_I \mathbf{S_F}$	E_H	$p_I \mathbf{K_M}$	L_M	l^b_M	l^b_{CB}	L_{CB}	D_{CB}
D_H				d_{CB}	d_H		
E_B					e_B		

Table 5: Decentralized deposit system: The agents' balance sheets at the end of period $t = 0$.

A.1.2 Period $\mathbf{t = 1}$

In period $t = 1$ we distinguish two cases: either no bank is founded by households, or banks are founded by households. The latter case can again be divided into two subcases: either no bank defaults, or some banks default.

A.1.2.1 Case I: No Bank is founded.

If no bank is founded, we have $E_B = 0$. This could constitute an equilibrium, as no household can found a bank individually. We would call this an *equilibrium without banks*. In such circumstances, no money is created, the central bank is inactive, no investment in MT is possible, and the investment good is entirely allocated to sector FT, i.e.[25]

$$\mathbf{K}^*_\mathbf{M} = \mathbf{0} \qquad \text{and} \qquad \mathbf{K}^*_\mathbf{F} = \mathbf{W},$$

where * denotes the equilibrium value of the respective variable. This is an inefficient allocation, as households are risk-neutral and Assumption 1 stipulates that $\mathbf{f}'(\mathbf{W}) < \mathbb{E}[\mathbf{R}^\mathbf{s}_\mathbf{M}]$.

[25]Note that no bank deposits are needed to buy the output from sector FT, as bonds are in real terms and are repaid in terms of the output.

A.1.2.2 Case II: Banks are founded.

When banks are founded, they can borrow some amount of money from the central bank and then grant loans to firms in MT. We can considerably simplify the description of period $t = 1$ with the observation given by the following Lemma[26]

Lemma 1
An equilibrium with banks, and hence with positive lending to Sector MT, requires

$$\mathbf{R^s_M} p^s_C = R^s_L p_I$$

and implies $\Pi^s_M = 0$ *for* $s \in \{l, h\}$.

Lemma 1 is a direct consequence of the MT technology. If, for some state s, $\mathbf{R^s_M} p^s_C > R^s_L p_I$, firms in MT would demand an infinite amount of loans, as their shareholders' value per loan unit would be positive in one state, be at least zero in the other state,[27] and scale with the level of borrowing. If $\mathbf{R^s_M} p^s_C < R^s_L p_I$ for both states of the world, firms would forgo borrowing from banks.[28]

A.1.2.2.1 Subcase II.a: No Bank defaults.

Suppose next that no bank defaults. The macroeconomic state s is realized. Firms produce and repayments, which are contingent on the state s, fall due. Using bank balance sheets in Table 5, the equations $l^b_M = \alpha^b_M L_M$ and $D_H = D_{CB} - E_B$ as well as Assumption 2, we derive the expression of bank b's profits as follows:[29]

$$\begin{aligned}\Pi^{b,s}_B &= l^b_M R^s_L + d_{CB} R^s_{CB} - l^b_{CB} R^s_{CB} - d_H R^s_D \\ &= \alpha^b_M L_M (R^s_L - R^s_{CB}) + E_B R^s_{CB} \qquad (2)\end{aligned}$$

and the real gross rate of return on equity is given by

$$R^{b,s}_E = \max\Big(\alpha^b_M \frac{R^s_L - R^s_{CB}}{\varphi} + R^s_{CB}, 0\Big).$$

[26] This observation allows us to rule out considerations in which firms in MT would make positive profits or go bankrupt.

[27] Since entrepreneurs running firms in sector MT do not have any wealth, they have zero profit if they cannot repay and thus default against banks.

[28] Other arguments could be used to derive the zero profit condition in sector MT. As banks monitor entrepreneurs running firms in sector MT, they can offer them state-contingent repayment gross rates of return, and are thus able to extract the entrepreneurs' entire surplus.

[29] Note that profits are non-negative here, as we have assumed that banks do not default. In the case of a default by bank b, $\Pi^{b,s}_B$ will be negative, but the shareholders' value will be equal to zero, and bank shareholders will not be affected by the magnitude of $\Pi^{b,s}_B$, as they are protected by limited liability.

Profits from firms in the real (good) sector are given by

$$\Pi_M^s = \mathbf{K_M}(\mathbf{R_M^s} p_C^s - R_L^s p_I),$$
$$\Pi_F^s = (\mathbf{f}(\mathbf{K_F}) - \mathbf{K_F}\mathbf{R_F}) p_C^s.$$

The balance sheets are given in Table 6, where R_H^s denotes the resulting gross rate of return on household ownership of the investment good and of both production technologies.

Households	
$p_C^s \mathbf{R_F}\mathbf{S_F}$	$E_H R_H^s$
$D_H R_D^s$	
$E_B R_E^s$	
Π_F^s	

Bank b	
$d_{CB} R_{CB}^s$	$l_{CB}^b R_{CB}^s$
$l_M^b R_L^s$	$d_H R_D^s$
	$e_B R_E^{b,s}$

Table 6: Decentralized deposit system: The agents' balance sheets at the beginning of period $t = 1$ in the case where no bank defaults.

The dividend payment and repayment processes are described in detail in Appendix C. The resulting balance sheets at the end of period $t = 1$ are given in Table 7.

Households	
$p_C^s \mathbf{K_M}\mathbf{R_M^s}$	$E_H R_H^s$
$p_C^s \mathbf{f}(\mathbf{K_F})$	

Bank b	
0	0

Table 7: Decentralized deposit system: The agents' balance sheets at the end of period $t = 1$ in the case where no bank defaults.

A.1.2.2.2 Subcase II.b: Some Banks default.

Finally, we consider the scenario where some banks default. In this case, the description of period $t = 1$ has to be changed as follows:

The macroeconomic state s is realized. Firms produce, and repayments fall due. Two cases can occur. First, if $-d_H R_D^s \leq \Pi_B^{b,s} < 0$, bank b defaults on households but not on the central bank. Second, if $\Pi_B^{b,s} \geq 0$, bank b does not default. We note that the case $\Pi_B^{b,s} < -d_H R_D^s < 0$

cannot occur, as banks would default on households and the central bank. Due to the heavy penalties incurred for defaulting against governmental authorities, banks will avoid the latter case under all circumstances. Consider now a non-defaulting bank b. If $R^s_{CB} > R^s_L$ for some state s, there exists then an upper bound on α^b_M given by

$$\alpha^b_M \leq \alpha^s_{DH} := \frac{R^s_{CB} - (1-\varphi)R^s_D}{R^s_{CB} - R^s_L},$$

such that this bank does not default on households in state s. α^s_{DH} is the critical amount of loans at which a bank is just able to pay back depositors in state s. α^s_{DH} is obtained from Equation (2) by setting $\Pi^{b,s}_B = 0$ and using $\varphi = E_B/L_M$.

From now on, consider a defaulting bank b. If $R^s_{CB} > R^s_L$ for some state s, there exists a lower bound α^s_{DH} and an upper bound α^s_{DCB} for α^b_M given by

$$\alpha^s_{DH} < \alpha^b_M \leq \alpha^s_{DCB} := \frac{R^s_{CB}}{R^s_{CB} - R^s_L},$$

which mark the two default points. For any $\alpha^b_M \in (\alpha^s_{DH}, \alpha^s_{DCB}]$, bank b defaults against households, but not against the central bank in state s. For $\alpha^b_M > \alpha^s_{DCB}$, the bank would default against households *and* the central bank in state s. In state s, the critical amount of loans at which a bank is just able to pay back the central bank is represented by α^s_{DCB}. We obtain α^s_{DCB} from Equation (2) by setting $\Pi^{b,s}_B = -D_H R^s_D$. The lump-sum tax levied to bail out bank b in state s is denoted by $t^{b,s}$. If bank b defaults against households in state s, $t^{b,s} = -\Pi^{b,s}_B$ and if bank b does not default against households in state s, $t^{b,s} = 0$. Aggregate tax payments by households in state s are then given by

$$T^s = \int_{b\in[0,1]} t^{b,s} \mathrm{d}b.$$

Furthermore, we use $\Pi^{+,s}_B$ to denote the aggregate profits of non-defaulting banks in state s. The possible balance sheets are given in Table 8.

In Table 8, the labels b_d and b_n denote banks defaulting and not defaulting against households, respectively. Banks may have to borrow reserves from the central bank in order to be able to settle households' payment of the lump-sum taxes used to bail out defaulting banks. As to the balance sheets in Table 8, two remarks are in order. First, lump-sum taxation of households to bail out defaulting banks is directly equivalent to a bail-out of defaulting banks by the central bank. Second, a bail-out of defaulting banks destroys an amount of money equal to the size of the bail-out.

Banks that do not default against households potentially need more money to be able to pay dividends to the households and, thus, borrow an amount $\max(e_B R^{b,s}_E - d_{CB} R^s_{CB}, 0)$ from the

Households	
$p_C^s \mathbf{R_F S_F}$	$E_H R_H^s$
$D_H R_D^s - T^s$	
$\Pi_B^{+,s}$	
Π_F^s	

Bank b_n	
$d_{CB} R_{CB}^s$	$l_{CB}^{b_n} R_{CB}^s + T^s$
$l_M^{b_n} R_L^s$	$d_H R_D^s - T^s$
	$\Pi_B^{b_n,s}$

Bank b_d	
$d_{CB} R_{CB}^s + t^{b_d,s}$	$l_{CB}^{b_d} R_{CB}^s + T^s$
$l_M^{b_d} R_L^s$	$d_H R_D^s - T^s$

Table 8: Decentralized deposit system: The agents' balance sheets at the end of period $t = 1$ if some banks default.

central bank. Banks, then, pay dividends to households,[30] which buy the consumption good produced by firms in MT. These firms receive money from households and repay their loans to banks. Firms in FT repay their bonds directly in the form of the consumption goods they have produced. Finally, at the end of period $t = 1$, banks repay their loans to the central bank and money is thereby destroyed. The resulting balance sheets are given in Table 7.

A.2 A Centralized Deposit System

We now describe the timeline of events, if the monetary architecture is a centralized deposit system. As the description of the timeline of events is similar to the one for a decentralized deposit system in Subsection A.1, we refer to Subsection A.1 for details and we solely focus on the differences.[31]

A.2.1 Period $\mathbf{t = 0}$

The central bank grants loans to banks and banks grant loans to firms in MT, as in the process described in Subsection A.1. However, firms in MT now hold deposits at the central bank and use them to buy some amount of investment good from households. Households use some of the acquired deposits d_B to invest into bank equity: $e_B = d_B = \varphi L_M$. They also invest in firms in FT, as in the investment process described in Subsection A.1. The balance sheets at the end of period $t = 0$ are given in Table 9.

[30] Banks pay dividends in anticipation of the repayment of loans by firms in Sector MT.

[31] Throughout the paper, we use the same notations as in Subsection A.1 for variables that are identical to the ones in Subsection A.1, and do not redefine them.

Households	
$p_I\mathbf{S_F}$	E_H
D_H	
E_B	

Firms in MT	
$p_I\mathbf{K_M}$	L_M

Bank b	
l^b_M	l^b_{CB}
d_B	e_B

Central Bank	
L_{CB}	D_H
	D_B

Table 9: Centralized deposit system: The agents' balance sheets at the end of period $t = 0$.

A.2.2 Period $\mathbf{t = 1}$

As banks do not default against households in a centralized deposit system, the analysis of period $t = 1$ is shorter than the one given in Subsection A.1. After production has taken place and the macroeconomic shock has realized, banks ask for an amount of loan $\max(e_B R^{b,s}_E - d_B R^s_{CB}, 0)$ from the central bank to be able to pay dividends to shareholders. Then, banks pay dividends to households and households buy the consumption good. Firms repay their loans and bonds, and banks repay their loans to the central bank.

B Appendix – Reserve Requirements, Haircuts, and Capital Requirements

A bank b grants loans l^b_M taking into account that it has to comply with three regulations at the end of period $t = 0$: A minimum-equity-ratio requirement $\varphi^{reg} \leq \varphi^b$ ($\varphi^{reg} \in (0, 1)$), where we define $\varphi^b := e_B/l^b_M$, requires each bank to hold more equity at the end of period $t = 0$ than the fraction φ^{reg} of its total assets. A minimum-reserve requirement $r^{reg} \leq r^b$ ($r^{reg} \in (0, 1)$), where we define $r^b := d^b_{CB}/d_H$, requires each bank at the end of period $t = 0$ to hold more central bank reserves than the fraction r^{reg} of its deposits. In order to comply with the reserve requirement, some banks may have to borrow from the central bank just before the end of period $t = 0$. Finally, a haircut regulation $h \in (0, 1)$ requires each bank at the end of period $t = 0$ to hold more loans to sector MT than a multiple $\frac{1}{1-h}$ of its central bank liabilities.

We now investigate the impact of a minimum-reserve requirement r^{reg} coupled with a haircut regulation h on money demand α^b_M by a bank b.

Proposition 7
A combination of a minimum-reserve requirement r^{reg} and a haircut regulation h imposes

the following constraint on money demand by bank b:

$$\alpha_M^b \leq \frac{1 - r^{reg}(1-\varphi)}{h}.$$

Equilibria with banks exist if and only if the equity ratio φ fulfills

$$1 - \frac{1-h}{r} \leq \varphi.$$

The proof of Proposition 7 is given in Appendix E.1. We also examine the impact of a minimum-equity-ratio requirement φ^{reg} on money demand by bank b:

Proposition 8
Suppose the average capital structure in the economy is φ and $\varphi^{reg} \leq \varphi$. Then, the capital requirement φ^{reg} imposes an upper bound on money demand of a bank b:

$$\alpha_M^b \leq \frac{\varphi}{\varphi^{reg}} \quad \textit{for all banks } b \in [0,1].$$

The proof of Proposition 8 is given in Appendix E.1. Propositions 7 and 8 directly show the impact of the minimum-reserve requirement coupled with the haircut regulation on money demand by commercial banks can be identical with the one of the minimum-equity-ratio requirement:

Corollary 1
A combination of a reserve requirement r^{reg} and a haircut regulation h imposes exactly the same constraint on banks' behavior as a minimum-equity-ratio φ^{reg} if and only if

$$\varphi^{reg} = \frac{\varphi h}{1 - r^{reg}(1-\varphi)}.$$

In this case, the condition on the bank capital structure for which an equilibrium with banks exists then writes

$$\varphi^{reg} \leq \varphi,$$

or alternatively,

$$1 - \frac{1-h}{r^{reg}} \leq \varphi.$$

C Appendix – Dividend Payments and Repayments in the Decentralized Deposit System

Banks potentially need an additional amount of money for paying dividends to households and, thus, borrow an amount

$$\max(e_B R_E^{b,s} + d_H R_D^s - d_{CB} R_{CB}^s, 0) = \max\left(e_B \alpha_M^b \frac{R_L^s - R_{CB}^s}{\varphi}, 0\right)$$

from the central bank. Then, banks pay the dividends to households,[32] which buy the consumption goods produced by the firms in MT. These firms are paid with money from households. At this stage, the balance sheets of banks are given in Table 10, where

$$\begin{aligned} l_{CB_2}^{b,s} &= l_{CB}^b R_{CB}^s + \max\left(e_B \alpha_M^b \frac{R_L^s - R_{CB}^s}{\varphi}, 0\right), \\ d_{CB_2}^{b,s} &= \alpha_M^b \left(d_{CB} R_{CB}^s + \max\left(e_B \frac{R_L^s - R_{CB}^s}{\varphi}, 0\right)\right), \quad \text{and} \\ d_M^{b,s} &= \alpha_M^b \big(\Pi_B^s + d_H R_{CB}^s\big). \end{aligned}$$

Households	
$p_C^s \mathbf{R_F S_F}$	$E_H R_H^s$
$p_C^s \mathbf{R_M^s K_M}$	
Π_F^s	

Bank b	
$l_M^b R_L^s$	$d_M^{b,s}$
$d_{CB_2}^{b,s}$	$l_{CB_2}^{b,s}$

Table 10: Decentralized deposit system: The balance sheets of agents after firms in MT have bought some amount of investment good from households, if no bank defaults.

Firms in MT then repay their loans to banks. They are able to repay their debt as $d_M^{b,s} = \alpha_M^b(\Pi_B^s + D_H R_{CB}^s) = \alpha_M^b L_M R_L^s = l_M^b R_L^s$. Firms in FT repay their bonds directly with the consumption goods they have produced. Finally, at the end of period $t = 1$, banks repay their loans to the central bank and money is thereby destroyed. They are able to repay their debt as $d_{CB_2}^{b,s} = l_{CB_2}^{b,s}$.

[32]Banks pay the dividends in anticipation of the repayment of the loans by the firms in sector MT.

D Appendix – Individual Optimal Choices

D.1 Choices Independent of the Monetary System

We first deal with the households' investment behavior. The correspondences representing households' optimal choices for different constellations of prices and interest rates are as follows:

Lemma 2

The representative household's optimal portfolio choices are represented by three correspondences[33] *denoted by*

$$\begin{aligned}
\hat{\mathbf{S}}_{\mathbf{F}} &: \mathbb{R}^8_{++} \to \mathcal{P}(\mathbb{R}_+ \cup \{+\infty\}),\\
\hat{E}_B &: \mathbb{R}^8_{++} \times [0, \mathbf{W}] \to \mathcal{P}(\mathbb{R}_+ \cup \{+\infty\}),\\
\hat{D}_H &: \mathbb{R}^8_{++} \times \mathbb{R}_+ \times [0, \mathbf{W}] \to \mathcal{P}(\mathbb{R}_+ \cup \{+\infty\}),
\end{aligned}$$

[33] For a set X, we use $\mathcal{P}(X)$ to denote the power set of X.

and given by

$$\begin{aligned}&\Big(\hat{\mathbf{S}}_{\mathbf{F}}\big(\mathbf{W},(R_E^s)_s,(R_D^s)_s,p_I,(p_C^s)_s\big)\\&\hat{E}_B\big(\mathbf{W},(R_E^s)_s,(R_D^s)_s,p_I,(p_C^s)_s,\hat{\mathbf{S}}_{\mathbf{F}}\big),\\&\hat{D}_H\big(\mathbf{W},(R_E^s)_s,(R_D^s)_s,p_I,(p_C^s)_s,\hat{\mathbf{S}}_{\mathbf{F}},\hat{E}_B\big)\Big)=\end{aligned}\tag{3}$$

$$\begin{cases}\big(\{\mathbf{W}\},\{0\},\{0\}\big)\\ \qquad\qquad \textit{if } \max\left(\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right],\mathbb{E}\left[\frac{R_E^s}{p_C^s}\right]\right)\le\frac{\mathbf{f}'(\mathbf{W})}{p_I},\\ \big(\{0\},\{0\},\{p_I\mathbf{W}\}\big)\\ \qquad\qquad \textit{if } \max\left(\frac{\mathbf{f}'(\mathbf{0})}{p_I},\mathbb{E}\left[\frac{R_E^s}{p_C^s}\right]\right)<\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right],\\ \big(\{0\},\{p_I\mathbf{W}\},\{0\}\big)\\ \qquad\qquad \textit{if } \max\left(\frac{\mathbf{f}'(\mathbf{0})}{p_I},\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]\right)<\mathbb{E}\left[\frac{R_E^s}{p_C^s}\right],\\ \big(\{0\},[0,p_I\mathbf{W}],\{p_I\mathbf{W}-E_B\}\big)\\ \qquad\qquad \textit{if } \frac{\mathbf{f}'(\mathbf{0})}{p_I}<\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]=\mathbb{E}\left[\frac{R_E^s}{p_C^s}\right],\\ \left(\left\{\mathbf{f}'^{-1}\left(p_I\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]\right)\right\},\{0\},\{p_I\left(\mathbf{W}-S_F\right)\}\right)\\ \qquad\qquad \textit{if } \max\left(\frac{\mathbf{f}'(\mathbf{W})}{p_I},\mathbb{E}\left[\frac{R_E^s}{p_C^s}\right]\right)<\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]\le\frac{\mathbf{f}'(\mathbf{0})}{p_I},\\ \left(\left\{\mathbf{f}'^{-1}\left(p_I\mathbb{E}\left[\frac{R_E^s}{p_C^s}\right]\right)\right\},\{p_I\left(\mathbf{W}-S_F\right)\},\{0\}\right)\\ \qquad\qquad \textit{if } \max\left(\frac{\mathbf{f}'(\mathbf{W})}{p_I},\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]\right)<\mathbb{E}\left[\frac{R_E^s}{p_C^s}\right]\le\frac{\mathbf{f}'(\mathbf{0})}{p_I},\\ \left(\left\{\mathbf{f}'^{-1}\left(p_I\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]\right)\right\},[0,p_I\left(\mathbf{W}-S_F\right)],\{p_I\left(\mathbf{W}-S_F\right)-E_B\}\right)\\ \qquad\qquad \textit{if } \frac{\mathbf{f}'(\mathbf{W})}{p_I}<\mathbb{E}\left[\frac{R_E^s}{p_C^s}\right]=\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]\le\frac{\mathbf{f}'(\mathbf{0})}{p_I}.\end{cases}\tag{4}$$

The proof of Lemma 2 is given in Appendix E.2. We next turn to the firms' behavior.

Lemma 3
Demand for the investment good by firms in MT and FT are represented by two correspondences denoted by $\hat{\mathbf{K}}_{\mathbf{M}}:\mathbb{R}_{++}\to\mathcal{P}([0,W])$ *and* $\hat{\mathbf{K}}_{\mathbf{F}}:\mathbb{R}^2_{++}\to\mathcal{P}([\mathbf{0},\mathbf{W}])$ *respectively, and*

given by

$$\hat{\mathbf{K}}_{\mathbf{M}}(\mathbf{W}) = [\mathbf{0}, \mathbf{W}]$$

$$\textit{and} \quad \hat{\mathbf{K}}_{\mathbf{F}}(\mathbf{W}, \mathbf{R}_{\mathbf{F}}) = \begin{cases} \{\mathbf{0}\} & \textit{if } \mathbf{f}'(\mathbf{0}) \leq \mathbf{R}_{\mathbf{F}}, \\ \{\mathbf{W}\} & \textit{if } \mathbf{R}_{\mathbf{F}} \leq \mathbf{f}'(\mathbf{W}), \\ \{\mathbf{f}'^{-1}(\mathbf{R}_{\mathbf{F}})\} & \textit{otherwise.} \end{cases}$$

The proof of Lemma 3 is given in Appendix E.2. We note that in sector MT firms are indifferent between any investment level $\mathbf{K}_{\mathbf{M}}$, as the condition $\mathbf{R}^{\mathbf{s}}_{\mathbf{M}} p^s_C = R^s_L p_I$ for $s \in \{l, h\}$ in Lemma 1 implies that these firms make zero profit at any level of $\mathbf{K}_{\mathbf{M}}$.

D.2 Decentralized Deposit System

D.2.1 Without Capital Requirements

We now determine banks' demand for money in a decentralized deposit system. In circumstances where the optimal demand for money is unlimited, we denote the amount of issued loans (relative to the average) by "∞". We then obtain the following proposition:[34]

Lemma 4
Suppose that the monetary system is a decentralized deposit system and no capital requirement is imposed. Then, the individually optimal amounts of loans by a bank b are represented by

[34]Throughout the paper, we use the notations s_d and s_n, where $s_d, s_n \in \{l, h\}$ and $s_d \neq s_n$, to denote the two different states. If in some state banks default, this state will be denoted by s_d. The state free of defaults is denoted by s_n.

the correspondence $\hat{\alpha}_M : \mathbb{R}^8_{++} \times (0,1) \to \mathcal{P}(\mathbb{R} \cup \{+\infty\})$ *given by*

$$\hat{\alpha}_M\big((\mathbf{R^s_M})_s, (R^s_L)_s, (R^s_{CB})_s, (p^s_C)_s, \varphi\big) =$$
$$\begin{cases} \{+\infty\} & \textit{if } R^s_L \geq R^s_{CB} \textit{ for all states } s \in \{l,h\} \\ & \textit{with at least one strict inequality,} \\ \{\alpha^{s_d}_{DCB}\} & \textit{if } (\mathbb{E}\left[\frac{R^s_L}{p^s_C}\right] \geq \mathbb{E}\left[\frac{R^s_{CB}}{p^s_C}\right] \textit{ and } R^{s_d}_L < R^{s_d}_{CB}) \textit{ or} \\ & \textit{if } (\mathbb{E}\left[\frac{R^s_L}{p^s_C}\right] < \mathbb{E}\left[\frac{R^s_{CB}}{p^s_C}\right],\ R^{s_n}_{CB} < R^{s_n}_L, \textit{ and } \varphi < \frac{\sigma^{s_n}}{\sigma^{s_d}} \frac{\mathbf{R^{s_d}_M}}{\mathbf{R^{s_n}_M}} \frac{R^{s_d}_L}{R^{s_n}_L} \frac{R^{s_n}_L - R^{s_n}_{CB}}{R^{s_d}_{CB} - R^{s_d}_L}), \\ [0,+\infty) & \textit{if } R^s_L = R^s_{CB} \textit{ for all states } s \in \{l,h\}, \\ \{0, \alpha^{s_d}_{DCB}\} & \textit{if } \mathbb{E}\left[\frac{R^s_L}{p^s_C}\right] < \mathbb{E}\left[\frac{R^s_{CB}}{p^s_C}\right],\ R^{s_n}_{CB} < R^{s_n}_L, \textit{ and } \varphi = \frac{\sigma^{s_n}}{\sigma^{s_d}} \frac{\mathbf{R^{s_n}_M}}{\mathbf{R^{s_d}_M}} \frac{R^{s_d}_L}{R^{s_n}_L} \frac{R^{s_n}_L - R^{s_n}_{CB}}{R^{s_d}_{CB} - R^{s_d}_L}, \\ \{0\} & \textit{if } (R^s_L \leq R^s_{CB} \textit{ for all states } s \in \{l,h\} \\ & \textit{with at least one strict inequality) or} \\ & \textit{if } (\mathbb{E}\left[\frac{R^s_L}{p^s_C}\right] < \mathbb{E}\left[\frac{R^s_{CB}}{p^s_C}\right],\ R^{s_n}_{CB} < R^{s_n}_L, \textit{ and } \frac{\sigma^{s_n}}{\sigma^{s_d}} \frac{\mathbf{R^{s_n}_M}}{\mathbf{R^{s_d}_M}} \frac{R^{s_d}_L}{R^{s_n}_L} \frac{R^{s_n}_L - R^{s_n}_{CB}}{R^{s_d}_{CB} - R^{s_d}_L} < \varphi). \end{cases}$$

The proof of Lemma 4 is given in Appendix E.2.

D.2.2 With Capital Requirements

When banks have to comply with a capital requirement in a decentralized deposit system, banks' behavior is given by the following lemma.

Lemma 5
Suppose that banks have to comply with a minimum-equity-ratio $\varphi^{reg} \in [0,1)$*. If* $R^s_D = R^s_{CB}$ *in all states* $s \in \{l,h\}$*, the privately optimal amounts of loans by an individual bank are represented by the correspondence* $\hat{\alpha}^{reg}_M : \mathbb{R}^8_{++} \times [\varphi^{reg}, 1) \to \mathcal{P}(\mathbb{R}_+ \cup \{+\infty\})$ *given by*

$$
\hat{\alpha}_M^{reg}\big((\mathbf{R^s_M})_s, (R_L^s)_s, (R_{CB}^s)_s, (p_C^s)_s, \varphi\big) =
$$

$$
\begin{cases}
\{\frac{\varphi}{\varphi^{reg}}\} & \textit{if } (R_L^s \geq R_{CB}^s \textit{ for all states } s = l, h \\
& \textit{with at least one strict inequality}), \\
& \textit{if } (\mathbb{E}\left[\frac{R_L^s}{p_C^s}\right] > \mathbb{E}\left[\frac{R_{CB}^s}{p_C^s}\right],\ R_L^{s_d} < R_{CB}^{s_d},\ R_{CB}^{s_n} < R_L^{s_n},\ \textit{and } \frac{\varphi}{\varphi^{reg}} \leq \alpha_{DCB}^{s_d}), \\
& \textit{if } (\mathbb{E}\left[\frac{R_L^s}{p_C^s}\right] = \mathbb{E}\left[\frac{R_{CB}^s}{p_C^s}\right],\ R_L^{s_d} < R_{CB}^{s_d},\ R_{CB}^{s_n} < R_L^{s_n}, \\
& \textit{and } \alpha_{DH}^{s_d} < \frac{\varphi}{\varphi^{reg}} \leq \alpha_{DCB}^{s_d}),\ \textit{or} \\
& \textit{if } (\mathbb{E}\left[\frac{R_L^s}{p_C^s}\right] < \mathbb{E}\left[\frac{R_{CB}^s}{p_C^s}\right],\ R_L^{s_d} < R_{CB}^{s_d},\ R_{CB}^{s_n} < R_L^{s_n},\ \alpha_{DH}^{s_d} < \frac{\varphi}{\varphi^{reg}} < \alpha_{DCB}^{s_d}, \\
& \textit{and } \varphi^{reg} < \frac{\sigma^{s_n}}{\sigma^{s_d}} \frac{\mathbf{R^{s_n}_M}}{\mathbf{R^{s_d}_M}} \frac{R_L^{s_d}}{R_L^{s_n}} \frac{R_L^{s_n} - R_{CB}^{s_n}}{R_{CB}^{s_d}}) \\
\{\alpha_{DCB}^{s_d}\} & \textit{if } (\mathbb{E}\left[\frac{R_L^s}{p_C^s}\right] < \mathbb{E}\left[\frac{R_{CB}^s}{p_C^s}\right],\ R_L^{s_d} < R_{CB}^{s_d},\ R_{CB}^{s_n} < R_L^{s_n},\ \alpha_{DCB}^{s_d} \leq \frac{\varphi}{\varphi^{reg}}, \\
& \textit{and } \varphi < \frac{\sigma^{s_n}}{\sigma^{s_d}} \frac{\mathbf{R^{s_n}_M}}{\mathbf{R^{s_d}_M}} \frac{R_L^{s_d}}{R_L^{s_n}} \frac{R_L^{s_n} - R_{CB}^{s_n}}{R_{CB}^{s_d} - R_L^{s_d}}),\ \textit{or} \\
& \textit{if } (\mathbb{E}\left[\frac{R_L^s}{p_C^s}\right] \geq \mathbb{E}\left[\frac{R_{CB}^s}{p_C^s}\right],\ R_L^{s_d} < R_{CB}^{s_d},\ R_{CB}^{s_n} < R_L^{s_n},\ \textit{and } \alpha_{DCB}^{s_d} \leq \frac{\varphi}{\varphi^{reg}}) \\
\left[0, \frac{\varphi}{\varphi^{reg}}\right] & \textit{if } (R_L^s = R_{CB}^s \textit{ for all states } s = l, h),\ \textit{or} \\
& \textit{if } (\mathbb{E}\left[\frac{R_L^s}{p_C^s}\right] = \mathbb{E}\left[\frac{R_{CB}^s}{p_C^s}\right],\ R_L^{s_d} < R_{CB}^{s_d},\ R_{CB}^{s_n} < R_L^{s_n},\ \textit{and } \frac{\varphi}{\varphi^{reg}} \leq \alpha_{DH}^{s_d}) \\
\{0, \frac{\varphi}{\varphi^{reg}}\} & \textit{if } (\mathbb{E}\left[\frac{R_L^s}{p_C^s}\right] < \mathbb{E}\left[\frac{R_{CB}^s}{p_C^s}\right],\ R_L^{s_d} < R_{CB}^{s_d},\ R_{CB}^{s_n} < R_L^{s_n},\ \alpha_{DH}^{s_d} < \frac{\varphi}{\varphi^{reg}} < \alpha_{DCB}^{s_d}, \\
& \textit{and } \varphi^{reg} = \frac{\sigma^{s_n}}{\sigma^{s_d}} \frac{\mathbf{R^{s_n}_M}}{\mathbf{R^{s_d}_M}} \frac{R_L^{s_d}}{R_L^{s_n}} \frac{R_L^{s_n} - R_{CB}^{s_n}}{R_{CB}^{s_d}}) \\
\{0, \alpha_{DCB}^{s_d}\} & \textit{if } (\mathbb{E}\left[\frac{R_L^s}{p_C^s}\right] < \mathbb{E}\left[\frac{R_{CB}^s}{p_C^s}\right],\ R_L^{s_d} < R_{CB}^{s_d},\ R_{CB}^{s_n} < R_L^{s_n},\ \alpha_{DCB}^{s_d} \leq \frac{\varphi}{\varphi^{reg}}, \\
& \textit{and } \varphi = \frac{\sigma^{s_n}}{\sigma^{s_d}} \frac{\mathbf{R^{s_n}_M}}{\mathbf{R^{s_d}_M}} \frac{R_L^{s_d}}{R_L^{s_n}} \frac{R_L^{s_n} - R_{CB}^{s_n}}{R_{CB}^{s_d} - R_L^{s_d}}), \\
\{0\} & \textit{if } (R_L^s \leq R_{CB}^s \textit{ for all states } s = l, h \\
& \textit{with at least one strict inequality})\ \textit{or} \\
& \textit{if } (\mathbb{E}\left[\frac{R_L^s}{p_C^s}\right] < \mathbb{E}\left[\frac{R_{CB}^s}{p_C^s}\right],\ R_L^{s_d} < R_{CB}^{s_d},\ R_{CB}^{s_n} < R_L^{s_n},\ \alpha_{DCB}^{s_d} \leq \frac{\varphi}{\varphi^{reg}}, \\
& \textit{and } \frac{\sigma^{s_n}}{\sigma^{s_d}} \frac{\mathbf{R^{s_n}_M}}{\mathbf{R^{s_d}_M}} \frac{R_L^{s_d}}{R_L^{s_n}} \frac{R_L^{s_n} - R_{CB}^{s_n}}{R_{CB}^{s_d} - R_L^{s_d}} < \varphi), \\
& \textit{if } (\mathbb{E}\left[\frac{R_L^s}{p_C^s}\right] < \mathbb{E}\left[\frac{R_{CB}^s}{p_C^s}\right],\ R_L^{s_d} < R_{CB}^{s_d},\ R_{CB}^{s_n} < R_L^{s_n},\ \alpha_{DH}^{s_d} < \frac{\varphi}{\varphi^{reg}} < \alpha_{DCB}^{s_d}, \\
& \textit{and } \frac{\sigma^{s_n}}{\sigma^{s_d}} \frac{\mathbf{R^{s_n}_M}}{\mathbf{R^{s_d}_M}} \frac{R_L^{s_d}}{R_L^{s_n}} \frac{R_L^{s_n} - R_{CB}^{s_n}}{R_{CB}^{s_d}} < \varphi^{reg}),\ \textit{or} \\
& \textit{if } (\mathbb{E}\left[\frac{R_L^s}{p_C^s}\right] < \mathbb{E}\left[\frac{R_{CB}^s}{p_C^s}\right],\ R_L^{s_d} < R_{CB}^{s_d},\ R_{CB}^{s_n} < R_L^{s_n},\ \textit{and } \frac{\varphi}{\varphi^{reg}} \leq \alpha_{DH}^{s_d}).
\end{cases}
$$

The proof of Lemma 5 is given in Appendix E.2.

D.3 Centralized Deposit System

Only the objective function of banks changes compared to the decentralized deposit system. As heavy penalties are imposed on bankers whose bank defaults against the central bank, and as households don't deposit at individual banks anymore, banks cannot default anymore. Thus, their profits are equal to their shareholders' value. Expected shareholders' value of any bank b is given by

$$l_M^b \mathbb{E}\left[\frac{R_L^s}{p_C^s}\right] + (d_B - l_{CB}^b)\mathbb{E}\left[\frac{R_{CB}^s}{p_C^s}\right] = l_M^b \left(\mathbb{E}\left[\frac{R_L^s}{p_C^s}\right] - \mathbb{E}\left[\frac{R_{CB}^s}{p_C^s}\right]\right) + e_B \mathbb{E}\left[\frac{R_{CB}^s}{p_C^s}\right].$$

We thus directly obtain

Lemma 6
Suppose that the monetary system is a centralized deposit system and no capital requirement is imposed. Then, the individually optimal amounts of loans by a bank b are represented by the correspondence $\hat{l}_M : \mathbb{R}^6_{++} \to \mathcal{P}(\mathbb{R} \cup \{+\infty\})$ given by

$$\hat{l}_M\big((R_L^s)_s, (R_{CB}^s)_s, (p_C^s)_s\big) = \begin{cases} +\infty & \text{if } \mathbb{E}\left[\frac{R_L^s}{p_C^s}\right] > \mathbb{E}\left[\frac{R_{CB}^s}{p_C^s}\right], \\ [0, +\infty) & \text{if } \mathbb{E}\left[\frac{R_L^s}{p_C^s}\right] = \mathbb{E}\left[\frac{R_{CB}^s}{p_C^s}\right], \quad \text{and} \\ 0 & \text{if } \mathbb{E}\left[\frac{R_L^s}{p_C^s}\right] < \mathbb{E}\left[\frac{R_{CB}^s}{p_C^s}\right]. \end{cases}$$

The proof of Lemma 6 is given in Appendix E.2.

E Appendix – Proofs

E.1 Proof of Propositions

Proof of Proposition 2
The procedure to find all equilibria with banks is the same for any monetary architecture. We will therefore refer to this procedure in the various cases of Proposition 2. The procedure is as follows:

We first suppose that $\mathcal{E}^*$ is an equilibrium with banks.

Then, we note that a direct consequence of a minimum-equity ratio requirement $\varphi^{reg} \in [0,1)$ is that $\varphi^* \in [\varphi^{reg}, 1)$. In addition, in any symmetric equilibrium with banks, the amount of money borrowed from the central bank has to be equal to $\alpha_M^* = 1$, because of the symmetry. Depending on the monetary architecture and whether a capital requirement is imposed or not, we use Lemma 4, 5, or 6 and we obtain that given gross rates of return $(R_L^{s*})_s$, policy choices $(R_{CB}^s)_s$, prices $(p_C^{s*})_s$, and the equity ratio φ^*, all banks $b \in [0,1]$ choose a lending level $\alpha_M^b \in \hat{\alpha}_M\big((\mathbf{R_M^s})_s, (R_L^{s*})_s, (R_{CB}^s)_s, (p_C^{s*})_s, \varphi^*\big)$ as given by the Lemmas mentioned previously. The only gross rates of return in these Lemmas realizing $\alpha_M^* = 1$ are given by relations which we will call the *banks' incentive compatibility conditions.*

Moreover, $\Pi_M^{s*} = 0$ for all states $s = l, h$ in any monetary architecture, which translates into

$$\mathbf{R_M^s} p_C^{s*} = R_L^{s*} p_I^*$$

for all states $s = l, h$.

Given the gross rates of return $(R_E^{s*})_s$ and $(R_D^{s*})_s$ as well as prices p_I^* and $(p_C^{s*})_s$, households choose bond investments $\mathbf{S_F^*} \in \hat{\mathbf{S}}_\mathbf{F}\big(\mathbf{W}, (R_E^{s*})_s, (R_D^{s*})_s, p_I^*, (p_C^{s*})_s\big)$, equity provisions to banks $E_B^* \in \hat{E}_B\big((\mathbf{W}, R_E^{s*})_s, (R_D^{s*})_s, p_I^*, (p_C^{s*})_s, \mathbf{S_F^*}\big)$, and deposit holdings $D_H^* \in \hat{D}_H\big(\mathbf{W}, (R_E^{s*})_s, (R_D^{s*})_s, p_I^*, (p_C^{s*})_s, \mathbf{S}$
These correspondences are given in Lemma 2 in Appendix D. Only the fourth and the seventh case of the definition of the correspondences $\hat{\mathbf{S}}_\mathbf{F}$, $\hat{E}_B$ and $\hat{D}_H$ correspond to positive levels of equity and deposits, which are required in any equilibrium with banks. We call the conditions given by the fourth and the seventh case the *households' incentive compatibility conditions.*

Finally, $\mathbf{R_F^*}$ can be determined using Lemma 3 and equating the demand for the investment good $\mathbf{K_F^*}$ to its supply $\mathbf{S_F^*}$. We call the relation determining $\mathbf{R_F^*}$ the *firm's production plan.* With the help of the equity ratio φ^*, we can then rewrite all equilibrium variables.

It is straightforward to verify that for any monetary architecture and equilibrium with banks that the tuples found in each case constitute equilibria with banks as defined in Subsection 2.5. Suppose first that the monetary system is a decentralized deposit system and that $\varphi^{reg} = 0$.

Using Lemma 4, the banks' incentive compatibility constraint is given by

$$R_L^{s*} = R_{CB}^s$$

for all states $s \in \{l, h\}$. A direct consequence of this relation, Assumption 2, and the expression of profits directly below Equation (2) is that

$$R_E^{s*} = R_D^{s*} = R_L^{s*} = R_{CB}^s \tag{5}$$

for all states $s \in \{l, h\}$. The assumption $\mathbf{f'(W)} < \mathbb{E}[\mathbf{R_M^s}] < \mathbf{f'(0)}$ together with $\mathbf{R_M^s} p_C^{s*} = R_L^{s*} p_I^*$ rule out the fourth case of the households' incentive compatibility conditions. From the firms' production plan and again since $\mathbf{f'(W)} < \mathbb{E}[\mathbf{R_M^s}] < \mathbf{f'(0)}$, we obtain $\mathbf{R_F^*} = \mathbb{E}[\mathbf{R_M^s}]$. Suppose now that the monetary system is a decentralized deposit system and that $\varphi^{reg} > 0$.

Using Lemma 5, the banks' incentive compatibility condition is given by

either Case a) $(R_L^{s*} = R_{CB}^s$ for all states $s \in \{l, h\})$,

or Case b) $(\mathbb{E}\left[\frac{R_L^{s*}}{p_C^{s*}}\right] = \mathbb{E}\left[\frac{R_{CB}^s}{p_C^{s*}}\right]$, $R_L^{l*} < R_{CB}^l$, $R_{CB}^h < R_L^{h*}$, and $\alpha_{DH}^l \geq \frac{\varphi^*}{\varphi^{reg}})$,

or Case c) $(\mathbb{E}\left[\frac{R_L^{s*}}{p_C^{s*}}\right] = \mathbb{E}\left[\frac{R_{CB}^s}{p_C^{s*}}\right]$, $R_L^{h*} < R_{CB}^h$, $R_{CB}^l < R_L^{l*}$, and $\alpha_{DH}^h \geq \frac{\varphi^*}{\varphi^{reg}})$,

or Case d) $(\mathbb{E}\left[\frac{R_L^{s*}}{p_C^{s*}}\right] < \mathbb{E}\left[\frac{R_{CB}^s}{p_C^{s*}}\right]$, $R_L^{l*} < R_{CB}^l$, $R_{CB}^h < R_L^{h*}$, $\alpha_{DH}^l < 1$,

and $\varphi^* = \varphi^{reg} = \frac{\sigma^h}{\sigma^l} \frac{\mathbf{R_M^h}}{\mathbf{R_M^l}} \frac{R_L^{l*}}{R_L^{h*}} \frac{R_L^{h*} - R_{CB}^h}{R_{CB}^l})$,

or Case e) $(\mathbb{E}\left[\frac{R_L^{s*}}{p_C^{s*}}\right] < \mathbb{E}\left[\frac{R_{CB}^s}{p_C^{s*}}\right]$, $R_L^{h*} < R_{CB}^h$, $R_{CB}^l < R_L^{l*}$, $\alpha_{DH}^h < 1$,

and $\varphi^* = \varphi^{reg} = \frac{\sigma^l}{\sigma^h} \frac{\mathbf{R_M^l}}{\mathbf{R_M^h}} \frac{R_L^{h*}}{R_L^{l*}} \frac{R_L^{l*} - R_{CB}^l}{R_{CB}^h})$,

or Case f) $(R_L^{s*} \geq R_{CB}^s$ for all states $s \in \{l, h\}$ with at least one strict

inequality, and $\varphi^* = \varphi^{reg})$,

or Case g) $(\mathbb{E}\left[\frac{R_L^{s*}}{p_C^{s*}}\right] = \mathbb{E}\left[\frac{R_{CB}^s}{p_C^{s*}}\right]$, $R_L^{l*} < R_{CB}^l$, $R_{CB}^h < R_L^{h*}$, $\alpha_{DH}^l < 1$,

and $\varphi^* = \varphi^{reg})$,

or Case h) $(\mathbb{E}\left[\frac{R_L^{s*}}{p_C^{s*}}\right] = \mathbb{E}\left[\frac{R_{CB}^s}{p_C^{s*}}\right]$, $R_L^{h*} < R_{CB}^h$, $R_{CB}^l < R_L^{l*}$, $\alpha_{DH}^h < 1$,

and $\varphi^* = \varphi^{reg})$,

or Case i) $(\mathbb{E}\left[\frac{R_L^{s*}}{p_C^{s*}}\right] < \mathbb{E}\left[\frac{R_{CB}^s}{p_C^{s*}}\right]$, $R_L^{l*} < R_{CB}^l$, $R_{CB}^h < R_L^{h*}$, $\alpha_{DH}^l < 1$,

and $\varphi^* = \varphi^{reg} < \frac{\sigma^h}{\sigma^l} \frac{\mathbf{R_M^h}}{\mathbf{R_M^l}} \frac{R_L^{l*}}{R_L^{h*}} \frac{R_L^{h*} - R_{CB}^h}{R_{CB}^l})$,

or Case j) $(\mathbb{E}\left[\frac{R_L^{s*}}{p_C^{s*}}\right] < \mathbb{E}\left[\frac{R_{CB}^s}{p_C^{s*}}\right]$, $R_L^{h*} < R_{CB}^h$, $R_{CB}^l < R_L^{l*}$, $\alpha_{DH}^h < 1$,

and $\varphi^* = \varphi^{reg} < \frac{\sigma^l}{\sigma^h} \frac{\mathbf{R_M^l}}{\mathbf{R_M^h}} \frac{R_L^{h*}}{R_L^{l*}} \frac{R_L^{l*} - R_{CB}^l}{R_{CB}^h})$,

or Case k) $(\mathbb{E}\left[\frac{R_L^{s*}}{p_C^{s*}}\right] > \mathbb{E}\left[\frac{R_{CB}^s}{p_C^{s*}}\right]$, $R_L^{l*} < R_{CB}^l$, $R_{CB}^h < R_L^{h*}$, and $\varphi^* = \varphi^{reg})$,

or Case l) $(\mathbb{E}\left[\frac{R_L^{s*}}{p_C^{s*}}\right] > \mathbb{E}\left[\frac{R_{CB}^s}{p_C^{s*}}\right]$, $R_L^{h*} < R_{CB}^h$, $R_{CB}^l < R_L^{l*}$, and $\varphi^* = \varphi^{reg})$.

Note first that in Cases f) to l), the expected real gross rate of return on equity achieved by any bank b when choosing $\alpha_M^b = 1$ is higher than the expected real gross rate of return on equity when choosing $\alpha_M^b = 0$. Since the latter is equal to the expected real deposit gross rate, we can conclude that in all cases f) to l) the expected real gross rate of return on equity is larger than the expected real deposit gross rate. Moreover, for Cases a) to e), the expected real gross rate of return on equity is equal to the expected real deposit gross rate. Cases f) to l) are not compatible with the households' incentive compatibility conditions. Therefore, Cases f) to l) do not correspond to possible equilibria with banks.

In Cases a) to c), the assumption $\mathbf{f}'(\mathbf{W}) < \mathbb{E}[\mathbf{R}^{\mathbf{s}}_{\mathbf{M}}] < \mathbf{f}'(\mathbf{0})$ together with $\mathbb{E}[\mathbf{R}^{\mathbf{s}}_{\mathbf{M}}] = \mathbb{E}\left[\frac{R^{s*}_E}{p^{s*}_C}\right] = \mathbb{E}\left[\frac{R^{s*}_D}{p^{s*}_C}\right]$ rule out the fourth case of households' incentive compatibility conditions. From the firms' production plan, we obtain $\mathbf{R}^*_{\mathbf{F}} = \mathbb{E}[\mathbf{R}^{\mathbf{s}}_{\mathbf{M}}]$.

Cases d) and e) are both compatible with the households' incentive compatibility conditions. From the firms' production plan, we obtain

$$\mathbf{R}^*_{\mathbf{F}} = \mathbf{f}'(\mathbf{0}) - \max\left(0, \mathbf{f}'(\mathbf{0}) - p^*_I \mathbb{E}\left[\mathbf{R}^{\mathbf{s}}_{\mathbf{M}} \frac{R^s_{CB}}{R^{s*}_L}\right]\right).$$

Suppose now that the monetary system is a centralized deposit system and that $\varphi^{reg} = 0$. Using Lemma 6, the banks' incentive compatibility condition is given by

$$\mathbb{E}\left[\frac{R^{s*}_L}{p^{s*}_C}\right] = \mathbb{E}\left[\frac{R^s_{CB}}{p^{s*}_C}\right].$$

The real gross rate of return on equity is given by

$$\frac{R^{s*}_E}{p^{s*}_C} = \frac{R^{s*}_L - R^s_{CB}}{\varphi^* p^{s*}_C} + \frac{R^s_{CB}}{p^{s*}_C}. \tag{6}$$

The households' incentive compatibility conditions imply

$$\mathbb{E}\left[\frac{R^s_{CB}}{p^{s*}_C}\right] = \mathbb{E}\left[\frac{R^{s*}_E}{p^{s*}_C}\right],$$

which translates together with (6) into

$$\mathbb{E}[\mathbf{R}^{\mathbf{s}}_{\mathbf{M}}] = p^*_I \mathbb{E}\left[\frac{R^s_{CB}}{p^{s*}_C}\right].$$

Moreover, banks cannot default against the central bank, which can be written as $R^{s*}_E \geq 0$ for all states $s \in \{l, h\}$. Therefore, this inequality, together with Equality (6) gives

$$\varphi^* \geq \max_{s \in \{l,h\}} \left(1 - \frac{R^{s*}_L}{R^s_{CB}}\right).$$

The assumption $\mathbf{f}'(\mathbf{W}) < \mathbb{E}[\mathbf{R}^{\mathbf{s}}_{\mathbf{M}}] < \mathbf{f}'(\mathbf{0})$ together with $\mathbf{R}^{\mathbf{s}}_{\mathbf{M}} p^{s*}_C = R^{s*}_L p^*_I$ rule out the fourth case in households' incentive compatibility conditions. From the firms' production plan and again since $\mathbf{f'(W)} < \mathbb{E}[\mathbf{R^s_M}] < \mathbf{f'(0)}$, we obtain $\mathbf{R^*_F} = \mathbb{E}[\mathbf{R^s_M}]$. Suppose finally that the monetary system is a centralized deposit system and that $\varphi^{reg} > 0$. As capital requirements have no impact on banks' incentive compatibility conditions, the results found in the previous case continue to apply. □

Proof of Proposition 3

Suppose that the monetary system is a decentralized deposit system. From Proposition 2, we deduce that when no capital requirement is imposed, there are only efficient equilibria with banks. We still have to prove that there are inefficient equilibria with banks in this case. We derive a necessary and sufficient condition for the existence of inefficient equilibria.

Let $\varphi^{reg} \in (0,1)$ be a minimum equity ratio requirement and $(R^s_{CB})_{s\in\{l,h\}}$ the central bank policy gross rates. In the following, we define an equilibrium with banks and default in state s'. We define

$$R^s_L := R^s_{CB}(1+\epsilon), \tag{7}$$

where $\epsilon > 0$ is sufficiently large. We also define

$$R^{s'}_L := \varphi^{reg}\frac{\sigma^{s'}}{\sigma^s}\frac{R^s_L}{\epsilon}\frac{\mathbf{R^{s'}_M}}{\mathbf{R^s_M}}\frac{R^{s'}_{CB}}{R^s_{CB}}. \tag{8}$$

Equation (8) implies that

$$\varphi^{reg} = \frac{\sigma^s}{\sigma^{s'}}\frac{R^{s'}_L}{R^s_L}\frac{\mathbf{R^s_M}}{\mathbf{R^{s'}_M}}\frac{R^s_L - R^s_{CB}}{R^{s'}_{CB}}.$$

When ϵ is sufficiently large, Equation (8) implies that $R^{s'}_L < R^{s'}_{CB}$. In order to fulfill Definition 1 the following inequality has to be shown. This inequality therefore is a sufficient and necessary condition for the existence of an inefficient equilibrium with banks given central bank policy rates $(R^s_{CB})_{s\in\{l,h\}}$ and the minimum-equity-ratio requirement φ^{reg}.

$$\frac{\mathbb{E}[\mathbf{R^s_M}]}{p_I} = \mathbb{E}\left[\frac{R^s_L}{p^s_C}\right] < \mathbb{E}\left[\frac{R^s_{CB}}{p^s_C}\right] = \frac{1}{p_I}\mathbb{E}\left[\mathbf{R^s_M}\frac{R^s_{CB}}{R^s_L}\right],$$

which is equivalent to

$$\begin{aligned}\sigma^s\mathbf{R^s_M}\frac{R^s_{CB}}{R^s_L} + \sigma^{s'}\mathbf{R^{s'}_M}\frac{R^{s'}_{CB}}{R^{s'}_L} &= \sigma^s\frac{\mathbf{R^s_M}}{1+\epsilon} + \sigma^s\mathbf{R^s_M}\frac{\epsilon}{\varphi^{reg}}\frac{R^s_{CB}}{R^s_L}\\ &= \sigma^s\mathbf{R^s_M}\left(\frac{1+\frac{\epsilon}{\varphi^{reg}}}{1+\epsilon}\right) > \sigma^s\mathbf{R^s_M} + \sigma^{s'}\mathbf{R^{s'}_M}.\end{aligned}$$

By rearranging terms, the previous inequality rewrites

$$\epsilon \left(\frac{1}{\varphi^{reg}} - 1 - \frac{\sigma^{s'} \mathbf{R}_{\mathbf{M}}^{\mathbf{s}'}}{\sigma^{s} \mathbf{R}_{\mathbf{M}}^{\mathbf{s}}} \right) > \frac{\sigma^{s'} \mathbf{R}_{\mathbf{M}}^{\mathbf{s}'}}{\sigma^{s} \mathbf{R}_{\mathbf{M}}^{\mathbf{s}}}.$$

As ϵ is sufficiently large, this inequality holds if and only if

$$\varphi^{reg} < \frac{\sigma^{s} \mathbf{R}_{\mathbf{M}}^{\mathbf{s}}}{\mathbb{E}[\mathbf{R}_{\mathbf{M}}^{\mathbf{s}}]},$$

which concludes the proof. □

Proof of Proposition 7

Suppose that a minimum-reserve requirement $r^{reg} \in (0, 1)$ and a haircut regulation $h \in (0, 1)$ are imposed on each bank b at the end of period $t = 0$.

Then, a bank b_i has to borrow the amount $\max(0, r^{reg} d_H - d_{CB}^{b_i})$ of central bank money at the end of period $t = 0$ in order to fulfill the reserve requirement r^{reg}. The maximum amount of reserves which bank b_i can borrow from the central bank is given by $(1 - h) l_M^{b_i}$.[35] Therefore, the following constraint should hold in equilibrium at the end of period $t = 0$:

$$\max(0, r^{reg} d_H - d_{CB}^{b_i}) \leq (1 - h) l_M^{b_i},$$

which is equivalent to

$$\alpha_M^{b_i} \leq \frac{1 - r^{reg}(1 - \varphi)}{h},$$

where $\alpha_M^{b_i} \leq 1$.

Similarly, a bank b_j has to borrow the amount $r^{reg} d_H$ of central bank money at the end of period $t = 0$ to fulfill the reserve requirement r^{reg}. The maximum amount of reserves which bank b_j can borrow from the central bank is given by $(1 - h) l_M^{b_j}$. Therefore, the following constraint should hold in equilibrium at the end of Period $t = 0$:

$$r^{reg} d_H + l_{CB}^{b_j} \leq (1 - h) l_M^{b_j},$$

which is equivalent to

$$\alpha_M^{b_j} \leq \frac{1 - r^{reg}(1 - \varphi)}{h},$$

[35] Note that banks are indifferent between borrowing any lower reserve level as long as it fulfills the reserve requirement, as the gross rate of return charged for central bank liabilities is equal to the gross rate of return for holding central bank reserves.

where $\alpha_M^{b_j} \geq 1$. We note that the constraint for each bank b is given by

$$\alpha_M^b \leq \frac{1 - r^{reg}(1-\varphi)}{h}.$$

$\square$

Proof of Proposition 8

Let $\mathcal{E}^*$ be an equilibrium with banks for which a minimum-equity ratio φ^{reg} is imposed on banks at the end of Period $t = 0$. If $\alpha_M^b \geq 1$ for some bank $b \in [0, 1]$, the minimum-equity ratio imposes the following constraint on lending α_M^b:

$$\frac{E_B^*}{\alpha_M^b L_M^*} \geq \varphi^{reg}, \qquad \text{or equivalently} \qquad \alpha_M^b \leq \frac{\varphi^*}{\varphi^{reg}}.$$

If $\alpha_M^b \leq 1$, the previous constraint becomes

$$\frac{E_B^*}{L_M^*} \geq \varphi^{reg}, \qquad \text{or equivalently} \qquad \varphi^* \geq \varphi^{reg}.$$

$\square$

E.2 Proof of Lemmas

Proof of Lemma 2

Suppose first that $\max\left(\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right], \mathbb{E}\left[\frac{R_E^s}{p_C^s}\right]\right) \leq \frac{\mathbf{f}'(\mathbf{W})}{p_I}$. Define the auxiliary function

$$g_1(\mathbf{S_F}) := \mathbf{f}(\mathbf{W}) - \left(\mathbf{f}(\mathbf{S_F}) + p_I(\mathbf{W} - \mathbf{S_F}) \max\left(\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right], \mathbb{E}\left[\frac{R_E^s}{p_C^s}\right]\right)\right).$$

It is easy to verify that, for all $\mathbf{S_F} \in [0, \mathbf{W})$, $g_1'(\mathbf{S_F}) < 0$. Moreover, $g_1(\mathbf{W}) = 0$. Therefore, $g_1(\mathbf{S_F}) > 0$ for all $\mathbf{S_F} \in [0, \mathbf{W})$, which establishes the first case in Equation (4).

Suppose now that $\max\left(\frac{\mathbf{f}'(\mathbf{0})}{p_I}, \mathbb{E}\left[\frac{R_E^s}{p_C^s}\right]\right) < \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]$. Next we consider the function

$$g_2(\mathbf{S_F}) := p_I \mathbf{W} \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right] - \left(\mathbf{f}(\mathbf{S_F}) + p_I(\mathbf{W} - \mathbf{S_F})\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]\right),$$

which shares similar properties to g_1: for all $\mathbf{S_F} \in [0, \mathbf{W}]$, $g_2'(\mathbf{S_F}) > 0$, $g_2(0) = 0$, and thus $g_2(\mathbf{S_F}) > 0$ for all $\mathbf{S_F} \in (0, \mathbf{W}]$. Accordingly, we can apply an analog argument to g_2 as previously for g_1 and obtain the second case of Equation (4). With similar arguments we also obtain the third and fourth cases.

Suppose finally that $\max\left(\frac{\mathbf{f}'(\mathbf{W})}{p_I}, \mathbb{E}\left[\frac{R_E^s}{p_C^s}\right]\right) < \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right] \leq \frac{\mathbf{f}'(\mathbf{0})}{p_I}$. Now we consider

$$\begin{aligned} g_3(\mathbf{S_F}) := \mathbf{f}\left(\mathbf{f}'^{-1}\left(p_I \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]\right)\right) + p_I\left(\mathbf{W} - \mathbf{f}'^{-1}\left(p_I \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]\right)\right)\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right] \\ - \left(\mathbf{f}(\mathbf{S_F}) + p_I(\mathbf{W} - \mathbf{S_F})\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]\right). \end{aligned}$$

We observe that g_3 is strictly convex (in $\mathbf{S_F}$), $g_3'(0) = -\mathbf{f}'(\mathbf{0}) + p_I \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right] \leq 0$, and $g_3'(\mathbf{W}) = -\mathbf{f}'(\mathbf{W}) + p_I \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right] > 0$. Hence, on $[\mathbf{0}, \mathbf{W}]$, g_3 takes the minimum at $\mathbf{S_F} = \mathbf{f}'^{-1}\left(p_I \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]\right)$, and it holds that $g_3\left(\mathbf{f}'^{-1}\left(p_I \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]\right)\right) = 0$. Therefore, $g_3(\mathbf{S_F}) > 0$ for all $\mathbf{S_F} \neq \mathbf{f}'^{-1}\left(p_I \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]\right)$, which proves the fifth case in Equation (4). With similar arguments we also obtain the last two cases. □

Proof of Lemma 3

Demand for the investment good by firms in MT and FT are directly derived from the following shareholders' value-maximization problems:

$$\begin{aligned} &\max_{\mathbf{K_M} \in [\mathbf{0}, \mathbf{W}]} \left\{\mathbb{E}\left[\max\left(\mathbf{K_M}\left(\mathbf{R_M^s} - \frac{R_L^s}{p_C^s} p_I\right), 0\right)\right]\right\} \\ &\text{s.t. } \mathbf{R_M^s} p_C^s = R_L^s p_I \text{ for all states } s \in \{l, h\} \\ \text{and} \quad &\max_{\mathbf{K_F} \in [\mathbf{0}, \mathbf{W}]} \{\mathbb{E}[\max(\mathbf{f}(\mathbf{K_F}) - \mathbf{K_F}\mathbf{R_F}, 0)]\}. \end{aligned}$$

□

Proof of Lemma 4

Let $b \in [0,1]$ denote a bank. As $R_D^s = R_{CB}^s$ in all states $s \in \{l,h\}$ (by Assumption 2), the expected shareholders' value of Bank b is given by

$$\mathbb{E}\left[\max\left(\alpha_M^b L_M \frac{R_L^s - R_{CB}^s}{p_C^s} + E_B \frac{R_{CB}^s}{p_C^s}, 0\right)\right].$$

Suppose that $\mathbb{E}\left[\frac{R_L^s}{p_C^s}\right] < \mathbb{E}\left[\frac{R_{CB}^s}{p_C^s}\right]$.

- Suppose first that $R_L^s \leq R_{CB}^s$ for all states $s \in \{l,h\}$ with at least one strict inequality. In this case, bank b's expected shareholders' value is decreasing in the volume of loans. Therefore, $\alpha_M^b = 0$.
- Suppose now that $R_L^l < R_{CB}^l$ and $R_{CB}^h < R_L^h$. For these constellations, Figure 4 depicts three typical cases representing the expected shareholders' value per unit of equity as a function of α_M^b. The three different cases are given by the comparison between the capital ratio φ and $\frac{\sigma^h}{\sigma^l} \frac{\mathbf{R_M^h}}{\mathbf{R_M^l}} \frac{R_L^l}{R_L^h} \frac{R_L^h - R_{CB}^h}{R_{CB}^l - R_L^l}$.

  For $\alpha_M^b \leq \alpha_{DH}^l$, bank b does not default against it depositors, and its expected shareholders' value is decreasing with α_M^b, as illustrated in Figure 4. However, for $\alpha_{DH}^l < \alpha_M^b$, bank b defaults against it depositors in the bad state. Then, bank b can further increase expected shareholders' value by granting more loans, as illustrated in Figure 4. The reason is that shareholders are protected by limited liability and due to depositors' bail-out by the government, the deposit gross rate of return of bank b received by households in the bad state is R_L^l.

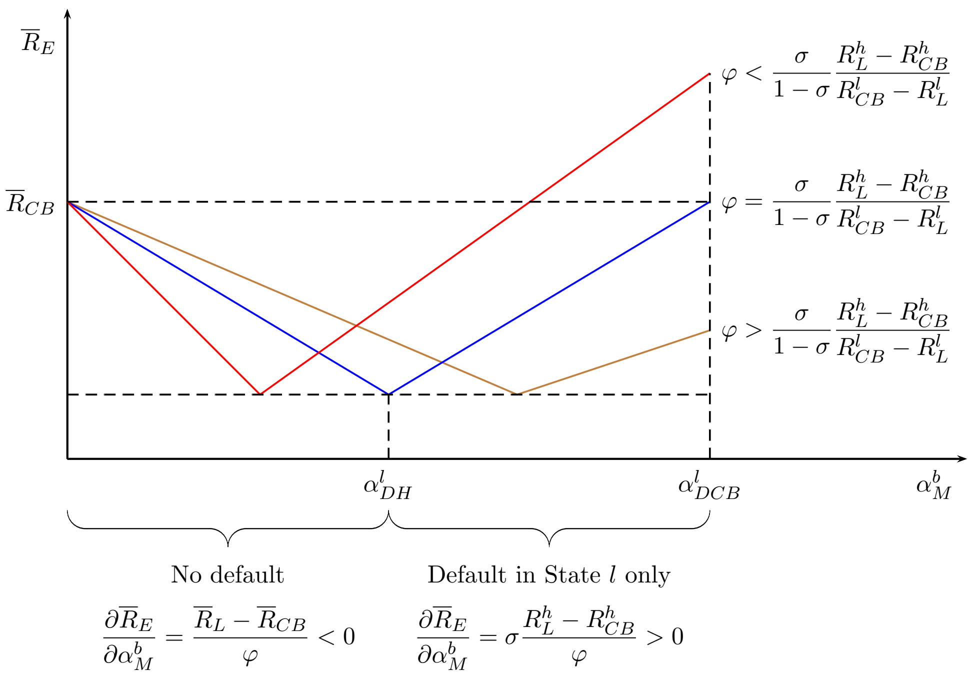

Figure 4: Expected gross rate of return on equity of a bank b as a function of α_M^b when $\mathbb{E}\left[\frac{R_L^s}{p_C^s}\right] < \mathbb{E}\left[\frac{R_{CB}^s}{p_C^s}\right]$ and $R_{CB}^h < R_L^h$ for three typical cases given by the comparison between the capital ratio φ and $\frac{\sigma^h}{\sigma^l}\frac{\mathbf{R_M^h}}{\mathbf{R_M^l}}\frac{R_L^l}{R_L^h}\frac{R_L^h - R_{CB}^h}{R_{CB}^l - R_L^l}$. α_{DH}^l and the corresponding areas of default and no default are depicted for $\varphi = \frac{\sigma^h}{\sigma^l}\frac{\mathbf{R_M^h}}{\mathbf{R_M^l}}\frac{R_L^l}{R_L^h}\frac{R_L^h - R_{CB}^h}{R_{CB}^l - R_L^l}$.

However, money demand levels $\alpha_M^b > \alpha_{DCB}^l$ cannot be optimal for bank b, as it would default against the central bank and would be subject to heavy penalties. Therefore, bank b compares expected shareholders' value with $\alpha_M^b = 0$ given by

$$E_B \mathbb{E}\left[\frac{R_{CB}^s}{p_C^s}\right]$$

and expected shareholders' value with $\alpha_M^b = \alpha_{DCB}^l$ given by

$$\sigma^h \left(\alpha_{DCB}^l L_M \frac{R_L^h - R_{CB}^h}{p_C^h} + E_B \frac{R_{CB}^h}{p_C^h}\right).$$

This comparison leads to the threshold of the equity ratio φ

$$\frac{\sigma^h}{\sigma^l}\frac{\mathbf{R_M^h}}{\mathbf{R_M^l}}\frac{R_L^l}{R_L^h}\frac{R_L^h - R_{CB}^h}{R_{CB}^l - R_L^l},$$

below which bank b chooses $\alpha_M^b = \alpha_{DCB}^l$ and above which it chooses $\alpha_M^b = 0$.

– Suppose now that $R^h_L < R^h_{CB}$ and $R^l_{CB} < R^l_L$. Analogously to the previous case,

$$\frac{\sigma^l}{\sigma^h}\frac{\mathbf{R^l_M}}{\mathbf{R^h_M}}\frac{R^h_L}{R^l_L}\frac{R^l_L - R^l_{CB}}{R^h_{CB} - R^h_L}$$

is the equity ratio below which bank b chooses $\alpha^b_M = \alpha^h_{DCB}$ and above which it chooses $\alpha^b_M = 0$.

Suppose now that $\mathbb{E}\left[\frac{R^s_L}{p^s_C}\right] = \mathbb{E}\left[\frac{R^s_{CB}}{p^s_C}\right]$.

– Suppose first that $R^s_L = R^s_{CB}$ for all states $s \in \{l, h\}$. In this case, bank b cannot influence its expected shareholders' value by varying its amount of loans. Therefore, $[0, +\infty)$ constitutes the set of bank b's optimal choices.

– Suppose now that $R^l_L < R^l_{CB}$ and $R^h_{CB} < R^h_L$. In this case, for $\alpha^b_M \leq \alpha^l_{DH}$, bank b does not default against its depositors, and its expected shareholders' value is constant and equal to $E_B\mathbb{E}\left[\frac{R^s_{CB}}{p^s_C}\right]$. However, for $\alpha^l_{DH} < \alpha^b_M$, Bank b defaults on depositors in the bad state. Then, bank b can further increase expected shareholders' value by granting more loans. The reason is that shareholders are protected by limited liability and due to depositors' bail-out by the government, the gross rate of return of the deposits of bank b received by households in the bad state is still R^l_D. However, levels of money demand $\alpha^b_M > \alpha^l_{DCB}$ cannot be optimal for bank b, as it would default against the central bank and would be subject to heavy penalties. Therefore, bank b chooses the highest level of lending for which it does not default against the central bank. This means that bank b chooses $\alpha^b_M = \alpha^l_{DCB}$.

– Suppose now that $R^h_L < R^h_{CB}$ and $R^l_{CB} < R^l_L$. Analogously to the previous case, bank b chooses $\alpha^b_M = \alpha^h_{DCB}$.

Suppose finally that $\mathbb{E}\left[\frac{R^s_L}{p^s_C}\right] > \mathbb{E}\left[\frac{R^s_{CB}}{p^s_C}\right]$.

– Suppose first that $R^s_{CB} \leq R^s_L$ for all states $s \in \{l, h\}$ with at least one strict inequality. In this case, bank b can increase expected shareholders' value by granting more loans. Accordingly, its choice is $\alpha^b_M = +\infty$.

– Suppose now that $R^l_L < R^l_{CB}$ and $R^h_{CB} < R^h_L$. In this case, for $\alpha^b_M \leq \alpha^l_{DH}$, bank b does not default against its depositors, and it can increase expected shareholders' value by increasing its lending level. However, for $\alpha^l_{DH} < \alpha^b_M$, Bank b defaults on depositors in the bad state. Then, bank b can further increase expected shareholders' value by granting more loans. The reason is that shareholders are protected by limited liability and due to depositors' bail-out by the government, the gross rate of return of deposits of bank b received by households in the bad state is still R^l_L. However, levels of money demand $\alpha^b_M > \alpha^l_{DCB}$ cannot be optimal for bank b, as it would default against the central bank and would be subject to heavy penalties. Therefore, bank b chooses the highest level of lending for which it does not default against the central bank. This means that bank b chooses $\alpha^b_M = \alpha^l_{DCB}$.

– Suppose finally that $R_L^h < R_{CB}^h$ and $R_{CB}^l < R_L^l$. Analogously to the previous case, bank b chooses $\alpha_M^b = \alpha_{DCB}^h$.

We can summarize the choices of lending levels by banks, given productivities of firms in MT $(\mathbf{R}_\mathbf{M}^\mathbf{s})_s$, gross rates $(R_L^s)_s$, policy choices $(R_{CB}^s)_s$, prices $(p_C^s)_s$, and their equity ratio φ, with the correspondence $\hat{\alpha}_M\big((\mathbf{R}_\mathbf{M}^\mathbf{s})_s, (R_L^s)_s, (R_{CB}^s)_s, (p_C^s)_s, \varphi\big)$ given in Lemma 4. $\square$

Proof of Lemma 5

Let $b \in [0,1]$ denote a bank and assume that a minimum equity ratio $\varphi^{reg} \leq \varphi$ is imposed on banks at the end of period $t = 0$. Using Proposition 8 as well as the property $R_D^s = R_{CB}^s$ for all states $s \in \{l,h\}$, bank b's maximization problem simplifies to

$$\max_{\alpha_M^b \in \left[0, \frac{\varphi}{\varphi^{reg}}\right]} \left\{ \mathbb{E}\left[\max\left(\alpha_M^b L_M \frac{R_L^s - R_{CB}^s}{p_C^s} + E_B \frac{R_{CB}^s}{p_C^s}, 0 \right) \right] \right\}.$$

As the arguments used in this proof to investigate the impact of lending on shareholders' value are similar to the ones given in the proof of Lemma 4, we refer the reader to the proof of Lemma 4 for further details.

Suppose that $\mathbb{E}\left[\frac{R_L^s}{p_C^s}\right] < \mathbb{E}\left[\frac{R_{CB}^s}{p_C^s}\right]$.

– Suppose first that $R_L^s \leq R_{CB}^s$ for all states $s \in \{l,h\}$ with at least one strict inequality. In this case, expected shareholders' value of bank b is decreasing in the volume of loans. Therefore, its choice is $\alpha_M^b = 0$.

– Suppose now that $R_L^l < R_{CB}^l$ and $R_{CB}^h < R_L^h$.

  – Suppose first that $\alpha_{DCB}^l \leq \frac{\varphi}{\varphi^{reg}}$. Then, the equity ratio requirement does not impose an additional constraint on bank b, and its optimal choice of money demand is

$$\begin{aligned}
&\alpha_M^b = 0 && \text{if } \frac{\sigma^h}{\sigma^l}\frac{\mathbf{R}_\mathbf{M}^\mathbf{h}}{\mathbf{R}_\mathbf{M}^\mathbf{l}}\frac{R_L^l}{R_L^h}\frac{R_L^h - R_{CB}^h}{R_{CB}^l - R_L^l} < \varphi, \\
&\alpha_M^b \in \{0, \alpha_{DCB}^l\} && \text{if } \varphi = \frac{\sigma^h}{\sigma^l}\frac{\mathbf{R}_\mathbf{M}^\mathbf{h}}{\mathbf{R}_\mathbf{M}^\mathbf{l}}\frac{R_L^l}{R_L^h}\frac{R_L^h - R_{CB}^h}{R_{CB}^l - R_L^l}, \\
\text{and}\quad &\alpha_M^b = \alpha_{DCB}^l && \text{if } \varphi < \frac{\sigma^h}{\sigma^l}\frac{\mathbf{R}_\mathbf{M}^\mathbf{h}}{\mathbf{R}_\mathbf{M}^\mathbf{l}}\frac{R_L^l}{R_L^h}\frac{R_L^h - R_{CB}^h}{R_{CB}^l - R_L^l}.
\end{aligned}$$

  – Suppose now that $\alpha_{DCB}^l > \frac{\varphi}{\varphi^{reg}}$. Then either $\alpha_{DH}^l < \frac{\varphi}{\varphi^{reg}}$ and expected shareholders' value of bank b is decreasing for $\alpha_M^b \in [0, \alpha_{DH}^l]$ and increasing for $\alpha_M^b \in [\alpha_{DH}^l, \frac{\varphi}{\varphi^{reg}}]$, or $\alpha_{DH}^l \geq \frac{\varphi}{\varphi^{reg}}$ and expected shareholders' value is decreasing for $\alpha_M^b \in [0, \frac{\varphi}{\varphi^{reg}}]$. Therefore, if $\alpha_{DH}^l \geq \frac{\varphi}{\varphi^{reg}}$, the choice of bank b is $\alpha_M^b = 0$. Suppose that $\alpha_{DH}^l < \frac{\varphi}{\varphi^{reg}}$. Then the choice of bank b can be derived by comparison between expected shareholders' value for $\alpha_M^b = 0$ and for $\alpha_M^b = \frac{\varphi}{\varphi^{reg}}$. Using the expression of profits directly below Equation (2) and rearranging terms establishes that the choice for bank b is

$$\begin{aligned} \alpha_M^b &= 0 && \text{if } \frac{\sigma^h}{\sigma^l}\frac{\mathbf{R_M^h}}{\mathbf{R_M^l}}\frac{R_L^l}{R_L^h}\frac{R_L^h - R_{CB}^h}{R_{CB}^l} < \varphi^{reg}, \\ \alpha_M^b &\in \{0, \tfrac{\varphi}{\varphi^{reg}}\} && \text{if } \varphi^{reg} = \frac{\sigma^h}{\sigma^l}\frac{\mathbf{R_M^h}}{\mathbf{R_M^l}}\frac{R_L^l}{R_L^h}\frac{R_L^h - R_{CB}^h}{R_{CB}^l}, \\ \text{and} \quad \alpha_M^b &= \tfrac{\varphi}{\varphi^{reg}} && \text{if } \varphi^{reg} < \frac{\sigma^h}{\sigma^l}\frac{\mathbf{R_M^h}}{\mathbf{R_M^l}}\frac{R_L^l}{R_L^h}\frac{R_L^h - R_{CB}^h}{R_{CB}^l}. \end{aligned}$$

– The analysis for $R_L^h < R_{CB}^h$ and $R_{CB}^l < R_L^l$ is similar to the previous one.

Suppose now that $\mathbb{E}\left[\frac{R_L^s}{p_C^s}\right] = \mathbb{E}\left[\frac{R_{CB}^s}{p_C^s}\right]$.

– Suppose first that $R_L^s = R_{CB}^s$ for all states $s \in \{l, h\}$. Then, any choice of $[0, \frac{\varphi}{\varphi^{reg}}]$ constitutes the set of bank b's optimal choices.
– Suppose now that $R_L^l < R_{CB}^l$ and $R_{CB}^h < R_L^h$.
  – Suppose now that $\alpha_{DH}^l < \frac{\varphi}{\varphi^{reg}}$. Then, the expected shareholders' value of bank b is constant for all $\alpha_M^b \in [0, \alpha_{DH}^l]$ and increasing in α_M^b in the interval $[\alpha_{DH}^l, \frac{\varphi}{\varphi^{reg}}]$. Therefore, bank b chooses $\alpha_M^b = \min(\alpha_{DCB}^l, \frac{\varphi}{\varphi^{reg}})$.
  – Suppose now that $\alpha_{DH}^l \geq \frac{\varphi}{\varphi^{reg}}$. Then, bank b's expected shareholders' value is constant for all $\alpha_M^b \in [0, \frac{\varphi}{\varphi^{reg}}]$. Therefore, any choice of $[0, \frac{\varphi}{\varphi^{reg}}]$ constitutes the set of bank b's optimal choice.
– The analysis for $R_L^h < R_{CB}^h$ and $R_{CB}^l < R_L^l$ is similar to the previous case.

Suppose finally that $\mathbb{E}\left[\frac{R_L^s}{p_C^s}\right] > \mathbb{E}\left[\frac{R_{CB}^s}{p_C^s}\right]$.

– Suppose first that $R_L^s \geq R_{CB}^s$ for all states $s \in \{l, h\}$ with at least one strict inequality. In this case, bank b can increase expected shareholders' value by granting more loans. Therefore, its choice is $\alpha_M^b = \frac{\varphi}{\varphi^{reg}}$.
– Suppose now that $R_L^l < R_{CB}^l$ and $R_{CB}^h < R_L^h$. In this case, bank b can increase expected shareholders' value by granting more loans. Therefore, its choice is $\alpha_M^b = \min(\alpha_{DCB}^l, \frac{\varphi}{\varphi^{reg}})$.
– The analysis for $R_L^h < R_{CB}^h$ and $R_{CB}^l < R_L^l$ is analog to the previous one.

We can summarize our findings with the correspondence $\hat{\alpha}_M^{reg}$ given in Lemma 5. □

Proof of Lemma 6

Let $b \in [0, 1]$ denote a bank. As $R_D^s = R_{CB}^s$ for all states $s \in \{l, h\}$, the expected shareholders' value of bank b is given by

$$\mathbb{E}\left[\max\left(l_M^b \frac{R_L^s - R_{CB}^s}{p_C^s} + E_B \frac{R_{CB}^s}{p_C^s}, 0\right)\right].$$

We first observe that the bank's money demand does not depend on the capital structure of bank b. We then distinguish the three cases:

- Suppose first that $\mathbb{E}\left[\frac{R_L^s}{p_C^s}\right] < \mathbb{E}\left[\frac{R_{CB}^s}{p_C^s}\right]$. The expected shareholders' value of bank b decreases with l_M^b. Therefore, bank b's optimal choice is $l_M^b = 0$.
- Suppose now that $\mathbb{E}\left[\frac{R_L^s}{p_C^s}\right] = \mathbb{E}\left[\frac{R_{CB}^s}{p_C^s}\right]$. The expected shareholders' value of bank b does not vary with l_M^b. Therefore, any $l_M^b \in [0, +\infty)$ is optimal.
- Suppose now that $\mathbb{E}\left[\frac{R_L^s}{p_C^s}\right] > \mathbb{E}\left[\frac{R_{CB}^s}{p_C^s}\right]$. The expected shareholders' value of bank b increases with l_M^b. Therefore, bank b's optimal choice is $+\infty$.

We can summarize the choices of lending levels by banks given the gross rates $(R_L^s)_s$, the policy choices $(R_{CB}^s)_s$, and the prices $(p_C^s)_s$ with the correspondence $\hat{l}_M\big((R_L^s)_s, (R_{CB}^s)_s, (p_C^s)_s\big)$ given in Lemma 6. $\square$