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JEL Classification: D63, G28, C72

Keywords: UEFA Financial Fair Play, financial regulation, competitive balance, investment tournaments.

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Financial Restrictions and Competitive Balance in Sports Leagues*

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Abstract

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JEL Classification: C72, D63, G28, Z23

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1 Introduction

Sports are an important part of global culture, being a favorite pastime or even an occupation for many. Advances in technology and international outreach, increased coverage through multi-year broadcasts and sponsorship deals, as well as fan loyalty contributing to resilience, have led to the tremendous and consistent growth of major leagues, turning them into multi-billion-dollar enterprises. For example, the annual revenues of the National Football League (NFL) surpassed $13 billion in 2016, with an eye on reaching the ambitious goal of $25 billion by the year 2027, a goal set by NFL Commissioner Roger Goodell. While the American major leagues dominate the list of professional sports by revenue, 14 out of the top 20 are association football (soccer) groups—most of which are, unsurprisingly, located in Europe—with Major League Soccer taking only a modest 18th place. According to the Club Licensing Benchmarking Report for the 2016 financial year of the Union of European Football Associations (UEFA), the administrative body for association football in Europe, club revenues have tripled this century, reaching a mark of €18.5 billion in 2016.

Naturally, the main objective of a sports club is to win any tournament that it participates in, and gifted players are the main instrument by which this goal is typically achieved (Smith and Szymanski, 1997). Club managers make heavy use of leverage to magnify their purchasing power so that they can attract the best and brightest talent. Their competitors are affected indirectly by this strategy as their likelihood of winning falls, forcing them into a financial "arms race." Clearly, clubs operate in a "winner takes all" world; it is therefore not surprising that many of them have reported repeated losses in recent years. Novel measures, such as financial regulations, were recently introduced by UEFA to improve the "financial health" of European football clubs, and their effects are still unclear (the basic economics of UEFA's Financial Fair Play (FFP) Regulations were covered in Franck (2014)). In our model, we have found how these regulations shift the alignment of forces in European football; moreover, we analyze their impact on clubs' credit risk and the entry of new investors into the football market.

That UEFA imposes financial restrictions on clubs participating in all-European tournaments is by no means unique. All major American leagues have financial restrictions such as salary caps on participating clubs, first introduced in the National Basketball Association (NBA) in the mid-80s. They could either be hard (as in the case of the National Hockey League, where a salary cap is introduced annually for each team with no exemptions) or soft (as in the case of Major League Baseball, where teams whose total payroll exceeds a certain figure (determined annually) have to pay "luxury tax"—also known as a competitive balance tax—on the excess amount). Fort and Quirk (1995) predicted that the NBA's "sharing cap" was going to improve competitive balance. Even though their own data did not indicate a significant change in the standard deviation of win percentages, they attributed this contradiction to various exemptions from the soft cap, such as the Larry Bird exception.[1]

A professional sports team has specific features: while the objective of any team is to win tournaments and they therefore can be considered as win-maximizers (in contrast with "ordinary" firms that are usually studied in academic literature as profit-maximizers), they also need opponents (preferably stronger ones) in order to improve their economic performance. This observation comes into conflict with standard models of economic competition, where any firm will benefit from driving competitors out of business and staying alone in the market. While the idea that financial restrictions level the playing field and improve competitive balance might seem intuitive, our paper is, to the best of our knowledge, the first to demonstrate this in a general equilibrium model with strategic actors.

As we have noted before, every team prefers to play with stronger opponents. In other words, a weak team imposes a negative externality on its stronger competitors, and the commercial appeal of a sports league increases with its competitive balance. In a famous paper, Rottenberg (1956) studied US baseball leagues and came up with the similar observation that "in baseball no team can be successful unless its competitors also survive and prosper sufficiently so that the differences in the quality of play among teams are not too great." Likewise, Neale (1964) noted that "the closer the standings, and within any range of standings the more frequently the standings change, the larger will be the gate receipts." He also presented the "Louis-Schmeling Paradox": the heavyweight champion of the world needs a strong contender to earn more money, although the uncertainty of the outcome will rise and the chances of defending a title will fall. Many methods were employed to measure the competitive balance of sports leagues, including the dispersion of winning probabilities within a league (as was used in Scully (1989)), the Herfindahl-Hirschman Index (HHI) of the concentration of championship titles, and the Gini coefficient (in Fort and Quirk (1995)). We will use the Gini coefficient in this paper.

Prior to the introduction of FFP, there has been evidence of significant decrease in the competitive balance of major European leagues. Szymanski (2001) conducted a natural experiment on the effect of growing inequality within English clubs by comparing same-division fixtures in both The Football Association Challenge Cup and the Premier League from 1982 to 1998 and concluded that the "relative increase in inequality has led to a relative decrease in attendance." Using a variety of metrics, Pawlowski et al. (2010) documented a persistent decline in competitive balance in the five leading European leagues and attributed it to the redistribution scheme of the UEFA Champions League's TV revenues, but without establishing a causal link. Surprisingly, there are very few

[1] Restrictions imposed in major US leagues are conceptually simpler than FFP. This is arguably because there is a variety of organizational forms for football clubs in Europe, which operate in different legal environments. In some countries, they are corporations that have all the reporting duties and are subject to all the procedures applicable to private firms: e.g., in a bankruptcy; in others, they are public associations, for which bankruptcy procedures are different and sometimes non-existent.

papers that address the relationship between financial rules and competitive balance. Based on a hand-collected dataset of match results, player market values, and "investor payments," Birkhauser et al. (2016) pointed at the post-FFP increasing imbalance of major leagues, accrediting it to higher barriers to entry for new investors. At the same time, Caglio et al. (2016) explored changes in leverage before and after the introduction of FFP and looked at the connection between leverage and sporting success. Their results are mixed and dependent on the league, and the main conclusion is that the "[FFP Regulations] have been associated with consistent but still weak effects on the financial sustainability of the European football clubs."

Peeters and Szymanski (2014) modeled FFP by hardening budget constraints and simulated its impact on four of the five largest European leagues under the assumption that the regulations were active in the 2010 and 2011 seasons. They compared the break-even rule with the uniform salary cap and inferred that "conventional salary caps are a superior device to improve competitive balance in national leagues." Madden (2015) used a framework of a large league with two types of clubs differing in terms of their fan base and "generosity" (or pure consumption benefits from ownership) of investors, and the main implication of his model was the "Pareto dis-improvement for all fans of the league as well as a fall in owner utilities and player wages given relatively elastic talent supply." Franck and Lang (2014) showed that money injected by owners induced clubs to pursue riskier strategies; consequently, the FFP would limit such opportunities. Under certain model parameters, they demonstrated that clubs would implement optimal strategies from the social welfare perspective. They also noticed that FFP regulations might decrease the bankruptcy risk of football clubs and improve competitive balance. However, neither empirical evidence nor theoretical proof of these claims was presented. We provide a strict framework to show that FFP will have a positive impact on competitive balance, and, from a social welfare perspective, the distribution of winning probabilities under FFP is preferable to the distribution without FFP.

In some respects, FFP is similar to the regulations in the banking sector. Banks similarly do not conform to standard models of economic competition, because the failure of one bank will have disastrous consequences for the others: i.e., it may ignite a banking panic. For instance, the collapse of Lehman Brothers in September 2008 escalated the financial crisis and significantly worsened credit conditions all over the world. As a result, its competitors, including Goldman Sachs and Morgan Stanley, were put on the verge of bankruptcy and were saved only after various government actions.

Freixas and Rochet (2008) noted that "a bank failure may spread to other banks (interbank loans account for a significant proportion of banks' balance sheets) and similarly endanger the solvency of non financial firms". They also noted that "... when depositors are insured ..., moral hazard appears. Bankers have incentives to take too much risk and to keep operating (at the expense of the deposit insurance fund) in situations in which liquidation would be efficient."

Similarly, football club managers are incentivized to gamble on success: borrow money, buy stars, and in the event of loss and bankruptcy, rely on the owner's financial aid.

A new set of banking regulations, Basel III, was introduced in response to the financial crisis in the late 2000s. These measures (including capital requirements and leverage constraints) aim to "improve the banking sector's ability to absorb shocks arising from financial and economic stress, whatever the source," and "improve risk management and governance."[2] Admati et al. (2013) examined "claims that high capital requirements are costly and would affect credit markets adversely," and found that "better capitalized banks suffer fewer distortions in lending decisions and would perform better." Similarly, FFP imposes a maximum loss requirement, and club owners may cover these losses only by equity injections. Critics argue that it would harm the football industry by forgoing benefits from "external" investments. In contrast, we show that new regulations would increase the investment attractiveness of "poor" teams and further level the playing field in European football. Consequently, as the outcomes of football matches become more uncertain, the revenues of clubs will grow.

Finally, Allen and Gale (2000) studied the interconnectedness of the banking system and the possibility of contagion. They suggested that regulation can play an essential role: "by intervening appropriately, the central bank can ensure that the inefficient allocation associated with contagion can be avoided." Morris and Shin (2008) considered a maximum leverage constraint that "has the potential to prevent the buildup in leverage that leaves the system vulnerable to a sudden reversal." In our paper, we explore the effect of an alternative restriction—a cap on borrowing—on the behavior of football clubs and the overall level of risk in European football.

The rest of the paper is organized as follows. Section 2 provides a very brief overview of FFP. Section 3 introduces our formal model, Section 4 contains the analysis, while Section 5 investigates the impact of financial restrictions on entry incentives. Section 6 concludes.

2 UEFA Financial Fair Play

The club tournaments with the largest TV audience, the UEFA Champions League and the UEFA Europa League, are held in Europe and administered by the UEFA. Approximately 350 million spectators watched the 2016–2017 UEFA Champions League final between Real Madrid and Juventus FC, compared with a Super Bowl audience of less than half that number. It comes as no surprise that these competitions attract multinational corporations as sponsors, such as Heineken (paying €70 million annually), UniCredit, Ford, MasterCard, Sony Computer Entertainment Europe, and Gazprom. UEFA also receives hefty fees from broadcasters—the latest deal brings in €2.5 billion per year, while the new one for 2018–2021 is expected to contribute a record €3.2

[2] http://www.bis.org/bcbs/basel3.htm

billion per year, consistent with UEFA's decision to guarantee half the slots (and the revenue that goes with them) to the four highest-ranked leagues. UEFA later shares these revenues with participating clubs; according to the official website, "a total of €2.04 billion will be distributed to clubs competing in the 2018/19 UEFA Champions League and the 2018 UEFA Super Cup." Moreover, "each of the 32 clubs that qualify for the group stage can expect to receive a group stage allocation of €15.25 million."[3] Additionally, performance bonuses are paid in the group stage: teams will receive €2.7 million for every win and €0.9 million for every draw, and clubs get additional bonuses for advancing to the next stage of the tournament. To sum up, clubs' payoffs from participating in such competitions are estimated to be in tens of millions of euros, which may constitute a decent share of their revenues (especially for teams with relatively small budgets).

Generous payoffs from participation and progression in UEFA competitions prompted club managers to take on huge loans to buy new players with the goal of placing in a national championship (in order to qualify for European tournaments) and successfully perform at those competitions later. Unsurprisingly, this high-risk strategy resulted in a number of failures, with externalities affecting other clubs, sponsors, and fans. We are familiar with a number of cases where things went awry after implementing such a strategy. A prime example is the Rangers, a football club from Glasgow, Scotland, with a glorious history. At the end of the 80s, the new owners of the Rangers adopted the novel strategy of borrowing heavily to buy international stars such as Gascoigne, Amoruso, and Laudrup. These and many other purchases were intended to help with conquering the European cups, but the Rangers never succeeded. In 2011, they were knocked out in the early qualifying stages, and the Rangers was placed into administration in 2012 because of increasing financial problems. Later that year, the club failed to reach an agreement with its creditors and entered the process of liquidation. The Scottish Premier League (SPL) clubs voted to relegate the Rangers—who were previously Scottish champions a record 54 times—to the fourth tier, the country's lowest professional league. The club was also banned from European competitions for three years and lost its strongest players. Without the Rangers, SPL lost its most famous rivalry—the Glasgow derby between the Celtic and the Rangers—and with it, a significant part of its commercial appeal to broadcasters and sponsors. Other remarkable examples include Parma FC from Parma, Italy and Leeds United from Leeds, England, which experienced similar problems a decade ago. Overall, UEFA reported in 2010 that half of Europe's top-division clubs were losing money.

To fight the adverse effects of excessive risk-taking in European football, the UEFA's Executive Committee approved the concept of FFP in September 2009, "with its principal objectives being:

- to introduce more discipline and rationality in club football finances;
- to decrease pressure on salaries and transfer fees and limit inflationary effect;

[3]https://www.uefa.com/uefachampionsleague/news/newsid=2562033.html

- to encourage clubs to compete with(in) their revenues;
- to encourage long-term investments in the youth sector and infrastructure;
- to protect the long-term viability of European club football; and
- to ensure clubs settle their liabilities on a timely basis."[4]

The FFP regulations require clubs entering UEFA competitions to pay off their obligations in a timely manner (the "no overdue payable rule") and live within their revenues (the "break-even requirement"). The second rule established a maximum deficit amount for relevant expenses over relevant income for a monitoring period, currently €5 million for the three previous consecutive seasons (or €30 million where losses are covered by contributions from equity participants); relevant expenses do not include some costs such as investments in infrastructure or youth academies. Assuming that the deficit of a football club increases its debt level, the "break-even requirement" can be modeled as a cap on borrowing. Sanctions that can be taken against a club include "a warning, a reprimand, a fine, a deduction of points, the withholding of revenues from a UEFA competition, the prohibition on registering new players in UEFA competitions, the restriction on the number of players that a club may register for participation in UEFA competitions, the disqualification from competitions in progress and/or the exclusion from future competitions and the withdrawal of a title or award."

However, these regulations have already encountered significant criticism, the main points of which were that, first, the effect of FFP on competitive balance is uncertain and FFP will freeze the current hierarchy (i.e., relatively small clubs will not be able to compete with bigger clubs, if clubs can only spend within their own means); second, clubs will sign questionable sponsorship contracts to bypass FFP regulations; and third, different tax rates across Europe create unequal conditions for clubs from various leagues. While addressing the second and third major concerns are beyond the scope of this paper, we will show that, at least theoretically, financial restrictions do lead to an increase in competitive balance.

The FFP regulations have been in place for several years, and UEFA claims that they "have transformed football finances, creating a more stable and sustainable financial position for European top-division clubs. The highlights of the latest report (the Club Licensing Benchmarking Report for the 2016 financial year) state that European clubs generated more than €2.3 billion in operating profits over the last three years, compared with the €0.8 billion in combined losses over the last three unregulated years (2010–2012). Net bottom line losses (equal to operating losses/profits, or "the break-even result," without adjustments for relevant expenses and income) have been cut by 84% since the introduction of the FFP in 2011. Foreign investors, the majority of whom came

[4] `http://www.uefa.org/protecting-the-game/financial-fair-play/`

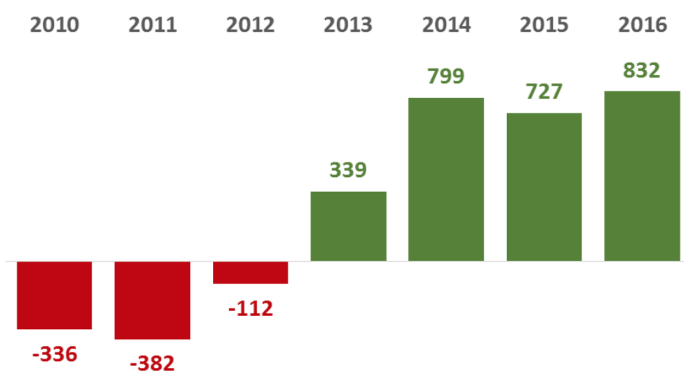

Figure 1: Change in operating profits

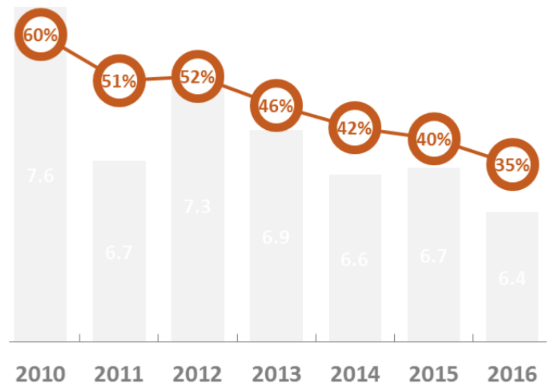

Figure 2: Change in debt/revenue ratio

Source: UEFA, Club Licensing Benchmarking Report, Financial Year 2016

from China, set a record in 2016 by taking over 9 clubs. In line with our theoretical results, the combined net debt has fallen by almost €1.2 billion in the last seven years and its ratio to revenues has decreased from 65% in 2010 to 35% to 2016 (see Figure 2).

3 Setup

There are n clubs competing in a tournament. Each club $i \in 1, \ldots, n$ is given a budget W_i and chooses how much to spend on the squad so that total spending does not exceed the budget plus the amount the club borrows at a risk-adjusted interest rate, $D_i \geq 0$. Clubs are heterogeneous: $W_1 < W_2 < \ldots < W_{n-1} < W_n$.

As is standard in the sports competition literature, we assume that each club maximizes its probability of winning. Given club i's spending m_i, the probability of club i winning the tournament is given by the following formula (from Tullock (1980)):

$$P_i(m_1, \ldots, m_n) = \frac{m_i}{\sum_{j=1}^{n} m_j}.$$

The Tullock rent-seeking function encompasses two main stylized facts that connect clubs' spending and performance: more spending by a club increases the probability of winning, while spending by competitors decreases this probability (as was shown in Smith and Szymanski (1997) for the case of the English Football League).

We assume that m_i, club i's spending, is the main variable of choice, which determines both the revenue and cost components of the club's budget. The revenue comes from three sources. The first source is matchday revenues $u \cdot m_i \sum_{j=1}^{n} m_j$, where u is a positive constant that parametrizes the customer's willingness to pay, which in turn depends on, e.g., the country's GDP per capita. For simplicity, we assume this particular functional form; nevertheless, the qualitative results would remain true for any functional form that encompasses assumptions that games are more interesting when clubs spend more on players and matchday revenues increase more as a result of the club's own investments than the investments of competitors. The second source, broadcasting revenues $B_C \cdot \mathbb{I}\,[\text{team } i \text{ is a winner}] + B_L \cdot m_i \sum_{j=1}^{n} m_j$, typically consists of "merit money" distributed according to the team's final league position and the league's overall attractiveness. The third source is prize money $S \cdot \mathbb{I}\,[\text{team i is a winner}]$ that only accrues to the winner of the tournament.[5]

Part of the expenses are wage costs ωm_i, which are proportional to the club's spending. Spending then determines the amount that club i has to borrow to balance the budget, $D_i = m_i - W_i$.

A higher amount of indebtedness increases the probability of a club's bankruptcy. Specifically, we assume that given debt D_i, club i faces an (exogenous) probability of bankruptcy given by the following formula:

$$P(\text{bankruptcy}|D_i) = 1 - \exp\left(-\lambda \frac{D_i}{W_i}\right),\ \lambda = \text{const} > 0$$

Bankruptcies are independent across clubs, and the recovery rate is zero.

The debt produces two types of costs for club i. First, the debt has to be repaid with risk-adjusted interest. Clubs have access to a competitive debt market of risk-averse lenders with squared root utility:

$$0 \cdot \left[1 - \exp\left(-\lambda \frac{D_i}{W_i}\right)\right] + \sqrt{r(D_i)D_i} \cdot \left[\exp\left(-\lambda \frac{D_i}{W_i}\right)\right] = \sqrt{rD_i}$$

We receive

$$r(D_i) = r \exp\left(2\lambda \frac{D_i}{W_i}\right)$$

where $r > 0$ is the risk-free rate or opportunity cost of a lender. Second, the cost of bankruptcy for club i is proportional to the level of debt D_i:

$$c_i(D_1, \ldots, D_n) = \sum_{j=1}^{n} cD_i \cdot \mathbb{I}\,[\text{team } j \text{ is bankrupt}]$$

Note that this formula assumes an external effect: a bankruptcy by club j is a cost for all clubs.

Summing up, each club maximizes its probability of success, while satisfying the constraint that

[5] Our assumptions imply that the matchday revenues and broadcasting revenues are described by the same function of clubs' spending. Similarly, the "merit money" in the broadcasting pool and the prize money are conditional on the same event. In an abstract mathematical model, the second source of revenues is redundant. However, we retain it so as to align with standard accounting practices in sports.

expected revenues must be greater or equal than expected costs

$$\begin{aligned}
P_i(m_1,\ldots,m_n) &\;\rightarrow\; \max_{m_i} \\
\textbf{s.t.}\,(B_C+S)\cdot\mathbb{I}\left[\text{team i is a winner}\right]+(B_L+u)m_i\sum_{j=1}^{n}m_j &\;\geq\; \sum_{j=1}^{n}cD_i\cdot\mathbb{P}\left[\text{team } j \text{ is bankrupt}\right]+ \\
&\quad\;\; \omega m_i + r(D_i)D_i\cdot\mathbb{P}\left[\text{team } i \text{ is not bankrupt}\right] \\
m_i &\;=\; W_i+D_i
\end{aligned}$$

We will consider two different cases: under the first scenario, clubs will be allowed to borrow without any limits. Under the second, there will be an exogenous constraint on debt.

4 Analysis

Setting $\Pi = B_C + S, t = B_L + u$, the problem transforms into:

$$\begin{aligned}
\frac{m_i}{\sum_{j=1}^{n}m_j} &\;\longrightarrow\; \max_{m_i} \\
\textbf{s.t.}\,\Pi\cdot\frac{m_i}{\sum_{j=1}^{n}m_j}+tm_i\sum_{j=1}^{n}m_j &\;\geq\; cD_i\sum_{j=1}^{n}\left[1-\exp\left(-\lambda\frac{D_j}{W_j}\right)\right]+r\exp\left(\lambda\frac{D_i}{W_i}\right)D_i+\omega m_i \\
m_i &\;=\; W_i+D_i
\end{aligned}$$

The constraint must be binding in equilibrium; otherwise, a team can deviate, increase its investment slightly (as long as the budget constraint permits), and therefore increase its own probability of winning. With n clubs, the problem transforms into the non-linear system of n equations:

$$\Pi\cdot\frac{W_i+D_i}{\sum_{j=1}^{n}(W_j+D_j)}+t(W_i+D_i)\sum_{j=1}^{n}(W_j+D_j) = cD_i\sum_{j=1}^{n}\left[1-\exp\left(-\lambda\frac{D_j}{W_j}\right)\right]+r\exp\left(\lambda\frac{D_i}{W_i}\right)D_i+$$
$$\omega(W_i+D_i)\ \text{ for each } i\in 1,\ldots,n$$

This system has solution $D_i = k\cdot W_i$; i.e., each team borrows a fixed proportion of its budget, or, in other words, leverage is the same across all teams. Therefore, equilibrium probabilities of winning in unconstrained model $P_i(W_1,\ldots,W_n) = \dfrac{W_i}{\sum_{j=1}^{n}W_j}$ increase in clubs' budget.

In line with FFP regulations, we now impose a constraint on the amount of debt a club may have: $D_i < D_{FFP}$. Let us denote equilibrium debt levels of unconstrained teams as $(D_i^*)_{i=1}^n$. There are three possible cases:

1. The constraint is not binding: $D_i^* < D_{FFP}$; for each club, this case is trivial, as nothing changes.

2. In the new equilibrium, the constraint is binding for all clubs: $D_i^* > D_{FFP}$; each team will borrow exactly D_{FFP} and the new probability of winning is

$$\frac{W_i + D_{FFP}}{\sum_{j=1}^n W_j + nD_{FFP}},$$

and the change in probabilities is given by

$$\frac{W_i + D_{FFP}}{\sum_{j=1}^n W_j + nD_{FFP}} - \frac{W_i}{\sum_{j=1}^n W_j} = \frac{nD_{FFP}(\overline{W} - W_i)}{(\sum_{j=1}^n W_j + nD_{FFP})\sum_{j=1}^n W_j},$$

where $\overline{W}$ is the average budget. Thus, teams with budgets lower than the average will benefit from such a constraint. Obviously, the overall debt load decreases, as $nD_{FFP} < \sum_{i=1}^n D_i^*$.

3. The first $n_0 < n$ teams are unconstrained; the other $n - n_0$ teams are not, and they borrow D_{FFP}. Note that the first teams to become constrained are the "rich" ones as they borrow the most in the unconstrained model. However, nothing fundamentally changes, as the first n_0 teams still borrow the fixed proportion of their budget: $D_i = \widetilde{k}W_i$, $i \in 1, \ldots, n_0$, but the constant is different from the original k. The equation to determine the threshold $\widetilde{k}$ is

$$\Pi \cdot \frac{\widetilde{k} + 1}{\sum_{j=1}^n W_j + \widetilde{k}\sum_{j=1}^{n_0} W_j + (n - n_0)D_{FFP}} + t(\widetilde{k} + 1)(\sum_{j=1}^n W_j + \widetilde{k}\sum_{j=1}^{n_0} W_j +$$
$$(n - n_0)D_{FFP}) = c\widetilde{k} \cdot \left[n - n_0 \exp(-\lambda\widetilde{k}) - \sum_{j=n_0+1}^{n} \exp\left(-\lambda\frac{D_{FFP}}{W_j}\right)\right] + r\exp(\lambda\widetilde{k})\widetilde{k} + \omega(\widetilde{k} + 1) \tag{1}$$

For constrained teams $i \in n_0 + 1, \ldots, n : D_{FFP} > \widetilde{k}W_i$ or $D_{FFP}/W_i > \widetilde{k}$, it then follows automatically from equation 1 that their revenues exceed costs:

$$\Pi \cdot \frac{D_{FFP}/W_i + 1}{\sum_{j=1}^n W_j + \widetilde{k}\sum_{j=1}^{n_0} W_j + (n - n_0)D_{FFP}} + t(\frac{D_{FFP}}{W_i} + 1)(\sum_{j=1}^n W_j + \widetilde{k}\sum_{j=1}^{n_0} W_j +$$
$$(n - n_0)D_{FFP}) > c\frac{D_{FFP}}{W_i} \cdot \left[n - n_0 \exp(-\lambda\widetilde{k}) - \sum_{j=n_0+1}^{n} \exp\left(-\lambda\frac{D_{FFP}}{W_j}\right)\right] +$$
$$r\frac{D_{FFP}}{W_i}\exp(\lambda\frac{D_{FFP}}{W_i}) + \omega(\frac{D_{FFP}}{W_i} + 1)$$

Lemma 1 *Equation 1 has a unique solution* $\widetilde{k} > 0$.

The proof of Lemma 1 is in the appendix. We have not ruled out various solutions for specific levels of D_{FFP}; although from Lemma 1 we know that $\widetilde{k}$ is unique for a fixed number n_0 of unconstrained teams, this number might itself be non-unique. To deal with this ambiguity, we first provide the following auxiliary result, where D^z is the debt level for the solution with z unconstrained teams.

Lemma 2 *Suppose that for one level of D_{FFP} there are two solutions $\widetilde{k}, \widehat{k}$ with $n_{01} < n_{02}$ number of unconstrained teams. It must then be true that $\widetilde{k} > \widehat{k}$ or, in other words, leverage decreases with the number of unconstrained teams. Additionally, $D^{n_{01}} > D^{n_{02}}$.*

Using the above, we prove the monotonicity of budget constraints in the number of teams bound by financial restrictions and consequently the uniqueness of an equilibrium.

Lemma 3 *If $S + B_C > (u + B_L)\left(\sum_{j=1}^{n}(W_j + D_j)\right)^2$, the number of unconstrained teams n_0 is unique for a given D_{FFP}.*

We will discuss the implications of these conditions further. With the results of Lemma 3, we describe how to search for a (constrained) equilibrium in our model:

1. Set $n_0 := n$.
2. Solve the system with n_0 unconstrained teams: $D_i = \widetilde{k}W_i$, $i \in 1, \ldots, n_0; \text{if}\, n_0 < n : D_i = D_{FFP}$, $i \in n_0 + 1, \ldots, n$.
3. If $D_i \leq D_{FFP}$, $i \in 1, \ldots, n_0$, we have found the solution $n_0, \widetilde{k}$.
4. If for some $i \in 1, \ldots, n_0 : D_i > D_{FFP}$, we set $n_0 := n_0 - 1$ and go back to the second step of the algorithm (if $n_0 = 0$, we stop, and the solution for each club is $D_i = D_{FFP}$).

We now want to examine changes in debt levels and winning probabilities for this case. Let us denote the debt level of the initial (pre-regulation) equilibrium as D^n and the debt level of the equilibrium with n_0 constrained teams as D^{n_0}. The proof of the next result is provided in the appendix.

Lemma 4 *Compared to the case where there are no restrictions, for $r/n > c$ and $S + B_C > (u + B_L)\left(\sum_{j=1}^{n}(W_j + D_j)\right)^2$, the overall debt level falls: $D^{n_0} < D^n$, where $n_0 < n$.*

The above-mentioned constraint implies that the share of sponsorship/commercial and "merit" TV revenues is greater than gate receipts and "egalitarian" TV revenue, which is true for most of the top European leagues (usually, no more than 40–50% of TV rights sales is distributed equally, with the German Bundesliga having the highest share at 65%):

	ENG	GER	ESP	ITA	FRA
Broadcasting	46%	28%	37%	51%	34%
Sponsorship/Commercial	30%	41%	25%	21%	35%
Gate Receipts	16%	18%	18%	10%	15%

Table 1: Revenue mix of European leagues. *Source:* UEFA, Club Licensing Benchmarking Report, Financial Year 2016

Moreover, we show in the next lemma that the direction of change in the *individual* level of debt is clear: for constrained teams, it obviously falls, and for unconstrained teams, it rises. With the help of this result, we can improve the search for the constrained equilibrium of the model; if $j : D_{FFP} < kW_j$, then team j would be constrained, and we can start searching for an equilibrium not from n unconstrained teams but from $j - 1$.

Lemma 5 *The change in individual leverage is positive for unconstrained clubs: $\widetilde{k} > k$.*

Changes in individual leverage across the league can be summarized in the following picture. It can also be shown with our model (see accompanying graph) that the ratio of debt to revenue $D/\left(\Pi + t\,(W+D)^2\right)$ will also decrease after total debt falls past some level D^*_d.

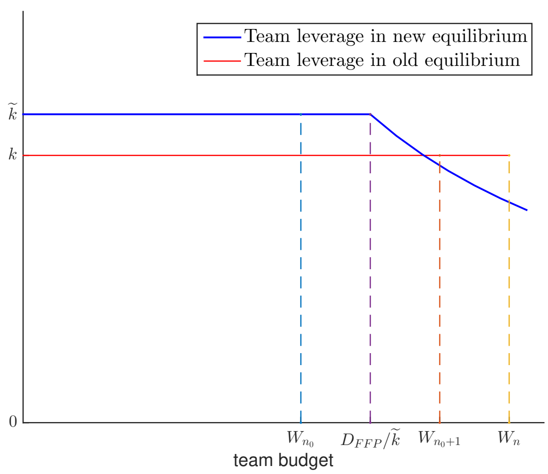

Figure 3: Change in individual leverage across the league

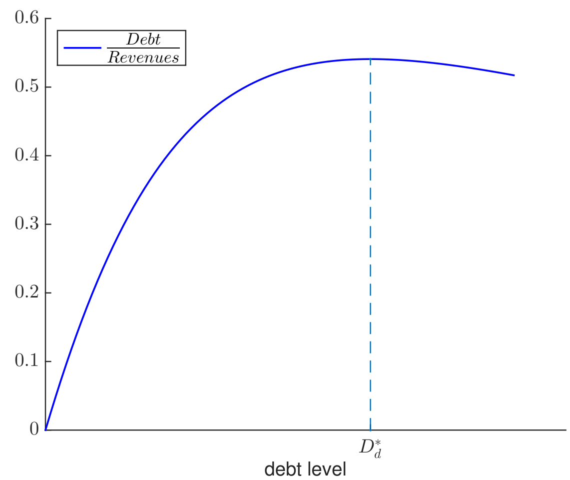

Figure 4: Ratio of debt/revenue to combined debt

Additionally, we introduce two risk characteristics of the equilibrium with n_0 unconstrained teams: the expected number of teams in default

$$E^{n_0} = \sum_{j=1}^{n} \mathbb{E}\mathbb{I}\left[\text{team } j \text{ is bankrupt}\right] = \sum_{j=1}^{n}\left[1 - \exp\left(-\lambda \frac{D_j}{W_j}\right)\right] =$$

$$= n - \sum_{j=1}^{n} \exp\left(-\lambda \frac{D_j}{W_j}\right) = n - n_0 \exp(-\lambda \widetilde{k}) - \sum_{j=n_0+1}^{n} \exp\left(-\lambda \frac{D_{FFP}}{W_j}\right),$$

and the solvent state (or "best-case scenario," where every team repays its debt) with corresponding probability

$$P^{n_0} = \prod_{j=1}^{n} \exp\left(-\lambda \frac{D_j}{W_j}\right) = \exp\left(-\lambda \sum_{j=1}^{n} \frac{D_j}{W_j}\right) = \exp\left(-\lambda n_0 \widetilde{k} - \lambda \sum_{j=n_0+1}^{n} \frac{D_{FFP}}{W_j}\right).$$

We could use the foregoing to describe "systemic risk," as the number of bankrupt teams is a random variable in our setting. These two figures clearly depend on the leverage across the clubs—the second is a monotonically decreasing function of its average. If average individual leverage falls, the probability of the solvent state increases, and from a straightforward application of Jensen's lemma it follows that the expected number of teams in default decreases as well. However, with the introduction of financial restrictions, the average individual leverage could rise in our model. In example 1 below, we show that for the same parameters, the impact on average leverage might depend on the distribution of club budgets.

Example 1 Fix parameters for model with two teams:

$\lambda = 0.1$, $\Pi = 100$, t $= 0.001$, r $= 1$, c $= 0.2$, and $\omega = 0.5$. The equations determining the debt levels before and after financial restrictions are imposed are as follows:

$$\frac{\Pi}{W_1 + W_2} + t(k+1)^2(W_1 + W_2) = ck(2 - 2e^{-\lambda k}) + rke^{\lambda k} + \omega(k+1)$$

$$\frac{\Pi(\widetilde{k}+1)}{W_1 + W_2 + \widetilde{k}W_1 + D_{FFP}} + t(\widetilde{k} + 1)(W_1 + W_2 + \widetilde{k}W_1 + D_{FFP}) =$$
$$c\widetilde{k}(2 - e^{-\lambda\widetilde{k}} - e^{-\lambda D_{FFP}/W_2}) + r\widetilde{k}e^{\lambda\widetilde{k}} + \omega(\widetilde{k}+1)$$

Fixing D_{FFP}, the average leverage could move in opposite directions for various distributions of team budgets.

	$\widetilde{k}$	k	Average leverage with restrictions	Difference
$W_1 = 20, W_2 = 50, D_{FFP} = 30$	0.7711	0.7076	0.6855	< 0
$W_1 = 5, W_2 = 50, D_{FFP} = 30$	1.3586	0.9328	0.9793	> 0

Table 2: Change in average leverage

Next, we will examine the impact of FFP on individual winning probabilities: the probability of winning was $\frac{W_i}{\sum_{j=1}^{n} W_j}$ for all teams; now, it is $\frac{W_i + D_{FFP}}{\sum_{j=1}^{n} W_j + \widetilde{k}\sum_{j=1}^{n_0} W_j + (n - n_0)D_{FFP}}$ for constrained teams and $\frac{W_i + \widetilde{k}W_i}{\sum_{j=1}^{n} W_j + \widetilde{k}\sum_{j=1}^{n_0} W_j + (n - n_0)D_{FFP}}$ for unconstrained teams. The following lemma states the results of changes in individual probabilities, and its proof is provided in the appendix.

Lemma 6 *The change in winning probabilities is strictly positive for all teams for which the constraint was not binding, and negative for relatively rich teams (the ones that had budgets higher than the average of constrained teams' budgets).*

We can combine the results of the three previous lemmas in the following proposition:

Proposition 1 *(i) Financial restrictions decrease overall debt load. The average indebtedness of clubs might increase or decrease as a result of financial restrictions.*

(ii) Financial restrictions increase the winning probabilities of poor (below-average) clubs and decrease the winning probabilities of relatively rich clubs.

Although "poor" clubs will win more often than before, it is not clear how the new regulations will affect "middle" teams and, consequently, the overall distribution of winning probabilities. We proceed to observe how this distribution will be affected by FFP. We can strengthen the second result using a Lorenz curve and the concept of Lorenz dominance.

Definition 1 *A Lorenz curve is a graphical representation of the distribution of winning probabilities, where for any proportion p between zero and one, the ordinate of the corresponding point on the Lorenz curve for a given distribution is the proportion of teams that accrues to the first $100p$ percent of clubs, which are sorted in increasing order of winning probability. If for any point p, the Lorenz curve of distribution 1 lies higher than the Lorenz curve of distribution 2, then distribution 1 is said to Lorenz-dominate (be more equitable than) distribution 2.*

Our next result, Proposition 2. From this result we get that teams become more equal in their probabilities of winning. The quantitative measure of inequality is the Gini coefficient, where 0 means perfect equality and 1 implies complete inequality. A straight line would represent the Lorenz curve for a distribution of teams which are equally likely to win the tournament—in other words, it is the line of "perfect equality." The Gini coefficient is the ratio of the area between the Lorenz curve and the line of perfect equality over the area under the line of perfect equality. It is easy to see that the Lorenz dominance of distribution 1 over distribution 2 implies $Gini_1 < Gini_2$. In their paper, Fort and Quirk (1995) measure the competitive balance of various sports leagues by comparing the Gini coefficients of corresponding distributions of winning probabilities, where a lower Gini coefficient corresponds to an increased competitive balance.

Proposition 2 *The distribution of winning probabilities with financial restrictions Lorenz-dominates the distribution of winning probabilities without financial restrictions. In other words, financial restrictions increase the competitive balance in a sports league.*

Thus, with regard to competitive balance, FFP restrictions work in the same way as a salary cap (a limit on each team's total salary expenditure as a percentage of the average team's revenue). In the case of two win-maximizing teams (where team 1 is "rich" and team 2 is "poor"), Dobson and Goddard (2011) show that "if team 1's salary expenditure is constrained while team 2 is unconstrained, competitive inequality is reduced in comparison with the case of no payroll cap. If both teams' salary expenditures are constrained, competitive inequality is eliminated altogether.

Suppose we have two distributions F, G with finite support $[a; b]$. Second-order stochastic dominance is equivalent to generalized Lorenz curve dominance (as shown in Thistle (1989)), where a generalized Lorenz curve is the usual Lorenz curve scaled by the mean of distribution, and its dominance can be defined similarly to Lorenz dominance. From the social welfare perspective, any planner with increasing and concave utility would prefer distribution F over distribution G if F second-order stochastically dominates distribution G. Noting that the mean for all distributions of winning probabilities equal to $\frac{1}{n}$ and using Proposition 2, we have proved the next result.

Proposition 3 *The distribution of winning probabilities with financial restrictions second-order stochastically dominates the distribution of winning probabilities without restrictions. From the social welfare perspective, any planner with increasing and concave utility would prefer having financial restrictions to not having financial restrictions.*

5 Market Entry

In this section, we will study the effects of the new regulations on individuals or organizations considering the purchase of a football club. We decompose the investor's problem into a two-stage game: at the first stage, he chooses which club he will acquire a stake in; at the second stage, he will make a decision about the level of borrowing (the behavior of the investor at this stage is completely described in the previous section). At the first stage, we have only one player, the investor, whose set of actions consists of choosing a team $i \in 1, \ldots, n$. As clubs in our model have zero expected profits, it is natural to assume that the investor wants to buy a club with the largest winning probability P_i, which is going to be the investor's "value of project;" we assume that his payoff $\mu(P_i)$ is weakly convex in P_i : $\mu(\cdot)' > 0$, $\mu(\cdot)'' > 0$. We can model this game similarly to the "investment tournaments" of Schwarz and Severinov (2010), who showed that when a decision-maker's payoff is weakly convex in the value of project, the optimal strategy is "to invest all resources in each time period into one favorite alternative." In our game, the investor is constrained (he has funds I) and wants to acquire a stake in the club of at least $\alpha \geq \alpha_0$. We assume that clubs have valuations $f(W_i)$, where f is an increasing function; in other words, clubs with higher budgets are more valuable as they are expected to win more frequently. We can write down the investor's problem as follows:

$$\begin{aligned} P_i(m_1, \ldots, m_n) &\rightarrow \max_{i \in 1, \ldots, n} \\ \mathbf{s.t.}\, \alpha f(W_i) &\leq I \\ \alpha &\geq \alpha_0 \end{aligned}$$

In our model, $W_1 < \ldots < W_n$, and corresponding winning probabilities $P_1 < \ldots < P_n$, before and after FFP (as we have found in Theorem 1). FFP would also not affect the investor's budget constraints, so the investor would still choose the same team $i = \max_{j \in 1,\ldots,n} \left[j : W_j \le f^{-1}\left(\frac{I}{\alpha_0}\right)\right]$. Although FFP does not impact the ranking of teams in terms of winning probabilities, their value is influenced by the new regulations. According to Lemma 5, as a result of FFP, the winning probabilities of unconstrained teams have increased, while the effect on constrained and relatively richer teams is the opposite. This lemma can be summarized in the following figure (where j is such that $W_j < \overline{W(n_0+1,n)}$, but $W_{j+1} > \overline{W(n_0+1,n)}$).

Figure 5: Impact of FFP on winning probability of team i

Though the effect on "middle" teams is uncertain, it is clear that the investor's utility from investing in teams $\in 1,\ldots,n_0 \cup M$ increased while utility from investing in teams $\in n_0+1,\ldots,n \setminus M$ decreased, where $M = \{l \in n_0+1,\ldots,j : P_l$ increased after FFP$\}$. All in all, as our teams sorted in budgets, we can state the following result:

Proposition 4 *Financial restrictions increase incentives for investment into "poorer" teams.*

This result reflects the fact that the "attractiveness" of rich teams will fall and the "attractiveness" of poor teams will rise. Additionally, if we assume that investors have their own reservation utilities (as they usually do), then the change in winning probabilities due to FFP might throw some investors in "rich" clubs out of the game, while attracting new investors to "poor" teams, which will further level the playing field in European football.

6 Conclusion

We construct a game-theoretic model to illustrate how the capital structure of a football club is affected by its own and its peers' investment decisions. On the one hand, the increase by one team of its debt implies an increase in the winning probability for that team and an increase in ticket sales for all teams (i.e., more investments into the line-up leads to a stronger team, and fans prefer to watch matches with such teams). On the other hand, debt growth leads to higher interest payments (as the interest rate increases in the amount of debt) and, more importantly, to the increased probability of bankruptcy, which in turn magnifies the expected bankruptcy penalties for all teams. We have shown that in the equilibrium, each team equalizes its expected revenues and costs and borrows a fixed proportion of its budget; leverage is the same for all clubs.

Next, we introduced FFP regulations by imposing a constraint on the amount of debt. In the resulting equilibrium, constrained teams borrowed their maximum allowed amount, while unconstrained teams continued to borrow a fixed ratio of their budget (though the leverage is different from the leverage in the equilibrium without FFP). We showed that for unconstrained teams, the winning probability is greater than it was before. Under reasonable assumption on a revenue structure of European football league overall debt load falls, but the effect on "systemic risk" in the league is mixed. We show that, at least theoretically, the competitive balance in sports competitions improves after introducing financial restrictions. Regulations such as salary caps or FFP improve investors' incentives to bring money to clubs other than those in the top financial tier, thus further leveling the playing field.

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Appendix

Proof of Lemma 1

Proof. Derivative of left-hand side with respect to $\widetilde{k}$ is positive for $\widetilde{k} \geq 0$:

$$\Pi \cdot \frac{\sum_{j=n_0+1}^{n} W_j + (n-n_0)D_{FFP}}{(\sum_{j=1}^{n} W_j + \widetilde{k}\sum_{j=1}^{n_0} W_j + (n-n_0)D_{FFP})^2} + t(\widetilde{k}+1)\sum_{j=1}^{n_0} W_j +$$
$$+t(\sum_{j=1}^{n} W_j + \widetilde{k}\sum_{j=1}^{n_0} W_j + (n-n_0)D_{FFP}) > 0$$

Derivative of right-hand side with respect to $\widetilde{k}$ is also positive for $\widetilde{k} \geq 0$:

$$c \cdot \left[n - n_0 \exp(-\lambda\widetilde{k}) - \sum_{j=n_0+1}^{n} \exp\left(-\lambda \frac{D_{FFP}}{W_j}\right)\right] + cn_0\lambda\widetilde{k} \cdot \exp(-\lambda\widetilde{k})$$
$$+1 + r\exp(\lambda\widetilde{k}) + \lambda r \exp(\lambda\widetilde{k})\widetilde{k} + \omega > 0$$

Right-hand side equals ω for $\widetilde{k} = 0$, while left-hand side:

$$\frac{\Pi}{\sum_{j=1}^{n} W_j + (n-n_0)D_{FFP}} + t(\sum_{j=1}^{n} W_j + (n-n_0)D_{FFP}) > \omega,$$

if we assume that the unconstrained team makes enough to cover its wage costs in a zero-leverage environment. Left- and right-hand sides both increase in $\widetilde{k} > 0$, but the rate of growth of the right-hand side is higher $\Rightarrow$ two curves must intersect once, and a unique solution $\widetilde{k}^* > 0$ for the equation exists. ■

Proof of Lemma 2

Proof. Teams $i \in n_{01+1}, \ldots, n_{02}$ stay unconstrained in the second case, but become constrained in the first case. Consequently, the following inequalities must hold for each $i \in n_{01+1}, \ldots, n_{02}$:

$$\begin{aligned} \widetilde{k}W_i &> D_{FFP} \\ \widehat{k}W_i &< D_{FFP} \end{aligned}$$

It immediately follows that $\widetilde{k} > \widehat{k}$. Turning to debt levels:

$$\begin{aligned} D^{n_{01}} &= \widetilde{k}\sum_{i=1}^{n_{01}} W_i + (n-n_{01})D_{FFP} \\ D^{n_{02}} &= \widehat{k}\sum_{i=1}^{n_{02}} W_i + (n-n_{02})D_{FFP} \end{aligned} \tag{2}$$

Thus, the difference is $D^{n_{01}} - D^{n_{02}} = (\widetilde{k} - \widehat{k}) \sum_{i=1}^{n_{01}} W_i + (n_{02} - n_{01}) D_{FFP} - \widehat{k} \sum_{i=n_{01}+1}^{n_{02}} W_i > 0$, as $\widetilde{k} > \widehat{k}$ and $\widehat{k} W_i < D_{FFP},\ i \in n_{01+1}, \dots, n_{02}$ (these teams are unconstrained). ■

Proof of Lemma 3

Proof. Suppose that some number of unconstrained teams n_{01} with corresponding leverage $\widetilde{k}$ and debt level $D^{n_{01}}$ clear the budget constraint $BC(n_{01}, \widetilde{k}, D^{n_{01}})$ of teams $i \in 1, \dots, n_0$. Scaled by $1/W_i$, we get

$$\begin{aligned} BC(n_{01}, \widetilde{k}, D^{n_{01}}) \quad = \quad & \left(1 + \frac{1}{\widetilde{k}}\right) \cdot \left[\frac{\Pi}{\sum_{j=1}^{n} W_j + D^{n_{01}}} + t \left(\sum_{j=1}^{n} W_j + D^{n_{01}} \right) - \omega \right] - \\ & c \cdot \left[n - n_{01} \exp(-\lambda \widetilde{k}) - \sum_{j=n_{01}+1}^{n} \exp\left(-\lambda \frac{D_{FFP}}{W_j} \right) \right] - r \exp[\lambda \widetilde{k}] = 0 \end{aligned}$$

Next, if other figures $n_{02}, \widehat{k}, D^{n_{02}}$ are solutions to our problem, then it must be that $BC(n_{02}, \widehat{k}, D^{n_{02}}) = 0$. Without loss of generality, $n_{02} > n_{01}$. From previous lemma, we know that $\widetilde{k} > \widehat{k}, D^{n_{01}} > D^{n_{02}}$. As $1 + \frac{1}{\widetilde{k}} < 1 + \frac{1}{\widehat{k}}$, and if $S + B_C > (u + B_L) \left(\sum_{j=1}^{n} (W_j + D_j) \right)^2$ or $t < \frac{\Pi}{(\sum_{j=1}^{n} W_j + D)^2}$, then function $\frac{\Pi}{\sum_{j=1}^{n} W_j + D} + t(\sum_{j=1}^{n} W_j + D)$ is decreasing in D, and we get

$$\begin{aligned} \left(1 + \frac{1}{\widetilde{k}}\right) \cdot \left[\frac{\Pi}{\sum_{j=1}^{n} W_j + D^{n_{01}}} + t \left(\sum_{j=1}^{n} W_j + D^{n_{01}} \right) - \omega \right] \quad < \\ \left(1 + \frac{1}{\widehat{k}}\right) \quad \cdot \quad \left[\frac{\Pi}{\sum_{j=1}^{n} W_j + D^{n_{02}}} + t \left(\sum_{j=1}^{n} W_j + D^{n_{02}} \right) - \omega \right] \end{aligned}$$

Also, it simple to see that $-r \exp[\lambda \widetilde{k}] < -r \exp[\lambda \widehat{k}], c n_{01} \exp[-\lambda \widetilde{k}] < c n_{01} \exp[-\lambda \widehat{k}]$ and $(n_{02} - n_{01}) \exp(-\lambda \widehat{k}) - \sum_{j=n_{01}+1}^{n_{02}} \exp\left(-\lambda \frac{D_{FFP}}{W_j} \right) > 0$, as $\widehat{k} W_j < D_{FFP}$ for $j \in 1, \dots, n_{02}$. We thus get $BC(n_{02}, \widehat{k}, D^{n_{02}}) > BC(n_{01}, \widetilde{k}, D^{n_{01}}) = 0$, or that the number of unconstrained teams n_0 is unique in equilibrium given the constraint D_{FFP}. ■

Proof of Lemma 4

Proof. We will examine how the overall debt level changes compared to the case with n teams. Debt levels are as follows:

$$D^{n_0} = \widetilde{k} \sum_{j=1}^{n_0} W_j + (n - n_0) D_{FFP}$$

$$D^n = k\sum_{j=1}^{n} W_j$$

Change in debt is as follows:

$$D^{n_0} - D^n = \left((n-n_0)D_{FFP} - k\sum_{j=n_0+1}^{n} W_j\right) + (\widetilde{k}-k)\sum_{j=1}^{n_0} W_j$$

If $\widetilde{k} < k$, then the first term is negative, as if for some team $i \in n_0+1,\dots,n : \widetilde{k}W_i < kW_i < D_{FFP}$, then we would have more than n_0 teams unconstrained because $W_1 < \ldots < W_n \Rightarrow$ for $\widetilde{k} < k$, it follows that $D^{n_0} - D^n < 0$.

If $\widetilde{k} > k$, then we will prove by contradiction. Let us assume that $D^{n_0} > D^n$. Denote $\left[n - n_0\exp(-\lambda\widetilde{k}) - \sum_{j=n_0+1}^{n}\exp\left(-\lambda\frac{D_{FFP}}{W_j}\right)\right]$ as E^{n_0}. The equations to find $\widetilde{k}$ and k are:

$$\begin{aligned}
\Pi\cdot\frac{\widetilde{k}+1}{\sum_{j=1}^{n}W_j + D^{n_0}} + t(\widetilde{k}+1)\left(\sum_{j=1}^{n}W_j + D^{n_0}\right) &= c\widetilde{k}\cdot E^{n_0} + r\widetilde{k}\exp[\lambda\widetilde{k}] + \omega(\widetilde{k}+1)\\
\Pi\cdot\frac{k+1}{\sum_{j=1}^{n}W_j + D^{n}} + t(k+1)\left(\sum_{j=1}^{n}W_j + D^{n}\right) &= ck\left[n - n\exp(-\lambda k)\right] + rk\exp[\lambda k] + \omega(k+1),
\end{aligned}$$

which we can rewrite as

$$\begin{aligned}
1+\frac{1}{\widetilde{k}} &= \frac{c\cdot E^{n_0}. + r\exp[\lambda\widetilde{k}]}{\frac{\Pi}{\sum_{j=1}^{n}W_j + D^{n_0}} + t(\sum_{j=1}^{n}W_j + D^{n_0}) - \omega}\\
1+\frac{1}{k} &= \frac{c\left[n - n\exp(-\lambda k)\right] + r\exp[\lambda k]}{\frac{\Pi}{\sum_{j=1}^{n}W_j + D^{n}} + t(\sum_{j=1}^{n}W_j + D^{n}) - \omega}
\end{aligned}$$

If $\widetilde{k} > k > 0 \Rightarrow 1+\frac{1}{\widetilde{k}} < 1+\frac{1}{k} \Rightarrow$

$$\frac{c\cdot E^{n_0} + r\exp[\lambda\widetilde{k}]}{\frac{\Pi}{\sum_{j=1}^{n}W_j + D^{n_0}} + t(\sum_{j=1}^{n}W_j + D^{n_0}) - \omega} < \frac{c\left[n - n\exp(-\lambda k)\right] + r\exp[\lambda k]}{\frac{\Pi}{\sum_{j=1}^{n}W_j + D^{n}} + t(\sum_{j=1}^{n}W_j + D^{n}) - \omega}$$

We will show that $c\cdot E^{n_0}. + r\exp[\lambda\widetilde{k}] > c\left[n - n\exp(-\lambda k)\right] + r\exp[\lambda k] > 0$, which is equivalent to

$$r\exp[\lambda\widetilde{k}] + c\left[n - n_0\exp(-\lambda\widetilde{k}) - \sum_{j=n_0+1}^{n}\exp\left(-\lambda\frac{D_{FFP}}{W_j}\right)\right] > r\exp[\lambda k] + c\left[n - n\exp(-\lambda k)\right] \Leftrightarrow$$

$$\frac{r}{cn}\left(\exp[\lambda\widetilde{k}] - \exp[\lambda k]\right) + \frac{n_0}{n}\left(\exp[-\lambda k] - \exp[-\lambda\widetilde{k}]\right) > \frac{1}{n}\left(\sum_{j=n_0+1}^{n}\exp\left(-\lambda\frac{D_{FFP}}{W_j}\right) - (n-n_0)\exp[-\lambda k]\right)$$

As $\dfrac{r}{cn} > 1$ and $k < \widetilde{k}$, then the right-hand side is greater than $\exp[\lambda\widetilde{k}] - \exp[\lambda k] > \lambda\exp[\lambda k]\left(\widetilde{k} - k\right)$. The left-hand side is smaller than

$$\frac{1}{n}\sum_{j=n_0+1}^{n}\lambda\exp\left[-\lambda\frac{D_{FFP}}{W_j}\right]\left(k - \frac{D_{FFP}}{W_j}\right) < \frac{\lambda\exp\left[-\lambda\dfrac{D_{FFP}}{W_n}\right]}{n}\left((n-n_0)k - \sum_{j=n_0+1}^{n}\frac{D_{FFP}}{W_j}\right)$$

All that is left to prove is that $\widetilde{k} - k > \dfrac{n-n_0}{n}\left(k - \dfrac{1}{n-n_0}\sum_{j=n_0+1}^{n}\dfrac{D_{FFP}}{W_j}\right)$. Let us denote $\overline{W(a,b)} = \dfrac{1}{b-a+1}\sum_{j=a}^{b}W_j$ and return to the debt assumption $D^{n_0} > D^n$, which can be rewritten as $kn\overline{W(1,n)} < \widetilde{k}(n\overline{W(1,n)} - (n-n_0)\overline{W(n_0+1,n)}) + (n-n_0)D_{FFP}$ or $\widetilde{k} - k > \dfrac{n-n_0}{n}\left(\widetilde{k}\dfrac{\overline{W(n_0+1,n)}}{\overline{W(1,n)}} - \dfrac{D_{FFP}}{\overline{W(1,n)}}\right) > \dfrac{n-n_0}{n}\left(k - \dfrac{1}{n-n_0}\sum_{j=n_0+1}^{n}\dfrac{D_{FFP}}{W_j}\right)$, as

$$\widetilde{k} - \frac{D_{FFP}}{\overline{W(n_0+1,n)}} > k - \frac{1}{n-n_0}\sum_{j=n_0+1}^{n}\frac{D_{FFP}}{W_j} > \frac{\overline{W(1,n)}}{\overline{W(n_0+1,n)}}\left(k - \frac{1}{n-n_0}\sum_{j=n_0+1}^{n}\frac{D_{FFP}}{W_j}\right)$$

because $\overline{W(1,n)} \leq \overline{W(n_0+1,n)}$ and harmonic mean $\dfrac{n-n_0}{\sum_{j=n_0+1}^{n}\dfrac{D_{FFP}}{W_j}}$ is less than arithmetic mean $\overline{W(n_0+1,n)}$.

Finally, returning to the original inequality, it must be that

$$\frac{\Pi}{\sum_{j=1}^{n}W_j + D^{n_0}} + t(\sum_{j=1}^{n}W_j + D^{n_0}) > \frac{\Pi}{\sum_{j=1}^{n}W_j + D^{n}} + t(\sum_{j=1}^{n}W_j + D^{n}) > \omega$$

As $S + B_C > (u + B_L)\left(\sum_{j=1}^{n}(W_j + D_j)\right)^2$ or $t < \dfrac{\Pi}{(\sum_{j=1}^{n}W_j + D)^2}$, then function $\dfrac{\Pi}{\sum_{j=1}^{n}W_j + D} + t(\sum_{j=1}^{n}W_j + D)$ is decreasing in D $\Rightarrow$

the above inequality is wrong. We obtained the contradiction and therefore $D^{n_0} < D^n$. The proof is completed. ■

Proof of Lemma 5

Proof.

Suppose that $k > \widetilde{k} \Rightarrow D^n = k\sum_{j=1}^{n}W_j > D^{n_0} = \widetilde{k}\sum_{j=1}^{n_0}W_j + (n-n_0)D_{FFP} \Rightarrow$

$$\left(1 + \frac{1}{\widetilde{k}}\right)\left[\frac{\Pi}{\sum_{j=1}^{n}W_j + D^{n_0}} + t(\sum_{j=1}^{n}W_j + D^{n_0}) - \omega\right] > \left(1 + \frac{1}{k}\right)\left[\frac{\Pi}{\sum_{j=1}^{n}W_j + D^{n}} + t(\sum_{j=1}^{n}W_j + D^{n}) - \omega\right]$$

and

$$c\left(n - n_0 \exp[-\lambda\widetilde{k}] - \sum_{j=n_0+1}^{n} \exp[-\lambda \frac{D_{FFP}}{W_j}]\right) + r\exp[\lambda\widetilde{k}] < c\left(n - n\exp[-\lambda k]\right) + r\exp[\lambda k]$$

In other words, budget constraint $BC(n_{01}, \widetilde{k}, D^{n_{01}}) > BC(n, k, D^n)$, so k and $\widetilde{k}$ cannot be solutions at the same time. As we obtained a contradiction, our premise is wrong $\Rightarrow k < \widetilde{k}$.

■

Proof of Lemma 6

Proof.

1. The difference in probabilities for an unconstrained team is

$$\frac{\widetilde{k}\sum_{j=n_0+1}^{n} W_j - (n-n_0)D_{FFP}}{\sum_{j=1}^{n} W_j(\sum_{j=1}^{n} W_j + \widetilde{k}\sum_{j=1}^{n_0} W_j + (n-n_0)D_{FFP})}$$

   As $D_{FFP} - \widetilde{k}W_j < 0$, $j = n_0+1, \ldots, n$ (they are all constrained). This difference is positive or, in other words, unconstrained (relatively "poor") teams benefit from the introduction of such a constraint.

2. The difference in probabilities for a constrained team is

$$\frac{(D_{FFP} - \widetilde{k}W_i)\sum_{j=1}^{n_0} W_j + D_{FFP}(\sum_{j=n_0+1}^{n} W_j - (n-n_0)W_i)}{\sum_{j=1}^{n} W_j(\sum_{j=1}^{n} W_j + \widetilde{k}\sum_{j=1}^{n_0} W_j + (n-n_0)D_{FFP})}$$

   As $D_{FFP} - \widetilde{k}W_i < 0$ (ith team is constrained), we see that if $\sum_{j=n_0+1}^{n} W_j - (n-n_0)W_i < 0$ or $W_i > \overline{W(n_0+1, n)}$ - the average budget among constrained teams, then the probability of winning decreases. In other words, it decreases for relatively "rich" teams, but for other teams among the constrained teams it is unclear how the probability of winning will change.

■

Proof of Proposition 2

Proof. Total investments before FFP: $\sum_{i=1}^{n}(W_i + D_i^*) = (k+1)\sum_{i=1}^{n} W_i$
Total investments after FFP (assuming the first n_0 clubs are unconstrained): $\sum_{i=1}^{n} W_i + \widetilde{k}\sum_{i=1}^{n_0} W_i + (n-n_0)D_{FFP}$
Winning probabilities before FFP increase in budget, and therefore the teams can be sorted in this order. For the same probabilities after FFP, we can sort the teams in the same order as

the probability of winning of the unconstrained team ($\frac{\widetilde{k}W_i}{\sum_{i=1}^n m_i}$) trivially increases in budget (we use m_i to denote an investment by the ith team under FFP regulations); the same is true for a constrained team as the probability equates to $\frac{W_i + D_{FFP}}{\sum_{i=1}^n m_i}$. We need to show that $\frac{(\widetilde{k}+1)W_{n_0}}{\sum_{i=1}^n m_i} < \frac{W_{n_0+1} + D_{FFP}}{\sum_{i=1}^n m_i}$ to justify our ordering for all teams; this inequality holds as $\frac{W_{n_0+1} - W_{n_0} + D_{FFP} - \widetilde{k}W_{n_0}}{\sum_{i=1}^n m_i} > 0$ because $W_{n_0+1} > W_{n_0}$ and $D_{FFP} - \widetilde{k}W_{n_0} > 0$ (team n_0 was unconstrained).

Before FFP, the proportion G_k of winning teams (as we study the discrete distribution) was

$$\frac{W_1}{\sum_{i=1}^n W_i}, \frac{W_1 + W_2}{\sum_{i=1}^n W_i}, \dots, \frac{\sum_{i=1}^{n-1} W_i}{\sum_{i=1}^n W_i}, 1$$

Let us denote the corresponding ratios after FFP as

$$F_i = \frac{\sum_{j=1}^i m_j}{\sum_{j=1}^n m_j}, \forall i = 1, \dots, n$$

1. For $i \in 1, \dots, n_0$, obviously $F_i > \frac{\sum_{j=1}^i W_j}{\sum_{j=1}^n W_j} = \frac{W_1}{\sum_{j=1}^n W_j} + \dots + \frac{W_i}{\sum_{j=1}^n W_j}$, as we have proved that the difference in probabilities of winning for an unconstrained team is positive or that $\frac{m_k}{\sum_{j=1}^n m_j} > \frac{W_k}{\sum_{i=1}^n W_i}$, $k = 1, \dots, n_0$.

2. For $m \in n_0 + 1, \dots, n$, let us compute the difference of F_m and G_m:

$$F_m - G_m = \frac{\sum_{i=1}^m W_i + \widetilde{k}\sum_{i=1}^{n_0} W_i + (m-n_0)D_{FFP}}{\sum_{i=1}^n W_i + \widetilde{k}\sum_{i=1}^{n_0} W_i + (n-n_0)D_{FFP}} - \frac{\sum_{i=1}^m W_i}{\sum_{i=1}^n W_i} =$$

$$= \frac{\widetilde{k}\sum_{i=1}^{n_0} W_i \sum_{i=m+1}^n W_i + D_{FFP}((m-n_0)\sum_{i=1}^n W_i - (n-n_0)\sum_{i=1}^m W_i)}{(\sum_{i=1}^n W_i + \widetilde{k}\sum_{i=1}^{n_0} W_i + (n-n_0)D_{FFP})\sum_{i=1}^n W_i}$$

Considering the numerator:

$$\widetilde{k}\sum_{i=1}^{n_0} W_i \sum_{i=m+1}^{n} W_i + D_{FFP}\left((m-n_0)\sum_{i=1}^{n} W_i - (n-n_0)\sum_{i=1}^{m} W_i\right) =$$

$$= \sum_{i=1}^{n_0} W_i \left(\widetilde{k}\sum_{i=m+1}^{n} W_i - (n-m)D_{FFP}\right) + (n-m)D_{FFP}\sum_{i=1}^{n_0} W_i +$$

$$+ D_{FFP}\left((m-n_0)\sum_{i=1}^{n} W_i - (n-n_0)\sum_{i=1}^{m} W_i\right) = \sum_{i=1}^{n_0} W_i \left(\widetilde{k}\sum_{i=m+1}^{n} W_i - (n-m)D_{FFP}\right) +$$

$$+D_{FFP}\left((n-m)\sum_{i=1}^{n_0}W_i+(m-n_0)\sum_{i=1}^{n}W_i-(n-n_0)\sum_{i=1}^{m}W_i\right)=$$

$$=|\text{sums up to } n_0 \text{ cancel out}|=\sum_{i=1}^{n_0}W_i\left(\widetilde{k}\sum_{i=m+1}^{n}W_i-(n-m)D_{FFP}\right)+$$

$$+D_{FFP}\left((m-n_0)\sum_{i=n_0+1}^{n}W_i-(n-n_0)\sum_{i=n_0+1}^{m}W_i\right)$$

The first term is positive as teams $i \in m+1,\ldots,n$ are constrained, i.e., $\widetilde{k}W_i > D_{FFP}$.
The second term is positive only if $\frac{\sum_{i=n_0+1}^{m}W_i}{m-n_0} < \frac{\sum_{i=n_0+1}^{n}W_i}{n-n_0}$ or if $\overline{W(n_0+1,m)} < \overline{W(n_0+1,n)}$. However, the budget set is ordered: $W_1 < W_2 < \ldots < W_{n-1} < W_n \Rightarrow$ this inequality for average budgets holds as long as $m < n$. Finally, we obtain $F_m > G_m$, $m \in n_0+1,\ldots n-1$. For $m=n:\ F_n = G_n = 1$.
Therefore, we obtain the result that "new" distribution F *Lorenz-dominates* "old" distribution G.

■