DISCUSSION PAPER SERIES

DP12971

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FINANCIAL ECONOMICS

ISSN 0265-8003

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Discussion Paper DP12971
Published 03 June 2018
Submitted 03 June 2018

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JEL Classification: F31, G15

Keywords: Stochastic discount factor, Exchange Rates, Market Segmentation, market incompleteness, financial intermediaries

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Model-Free International Stochastic Discount Factors*

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April 2018

Abstract

We provide a theoretical characterization of international stochastic discount factors (SDFs) in incomplete markets under different degrees of market segmentation. Using 40 years of data on a cross-section of countries, we estimate model-free SDFs and factorize them into permanent and transitory components. We find that large permanent SDF components help to reconcile the low exchange rate volatility, the exchange rate cyclicality, and the forward premium anomaly. However, integrated markets entail highly volatile and almost perfectly comoving international SDFs. In contrast, segmented markets can generate less volatile and more dissimilar SDFs. In quest of relating the SDFs to economic fundamentals, we document strong links between proxies of financial intermediaries' risk-bearing capacity and model-free international SDFs. We interpret this evidence through the lens of an economy with two building blocks: limited participation by households and financiers who face an intermediation friction.

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*We thank Federico Gavazzoni, Tarek Hassan, Matteo Maggiori, Thomas Maurer, Anna Pavlova, Carolin Pflueger, Alireza Tahbaz-Salehi, Ngoc-Khanh Tran, Raman Uppal and seminar and conference participants at the International Conference of the French Finance Association (Valence), SoFiE Annual Meeting at NYU, the International Finance Conference at Cass Business School, the SITE workshop "New Models of Financial Markets", the Chicago Booth International Macro Finance Conference 2017, the ECB/Banca d'Italia/Norges Bank "Financial Determinants of Foreign Exchange Rates" conference, the 2018 Econometric Society Meetings in Philadelphia, Boston University, University of Tilburg, VU Amsterdam/Tinbergen Institute, Université Catholique de Louvain, and McGill University for helpful comments. All errors are ours.

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The standard models in international finance are at odds with some of the most salient features of exchange rates and international asset prices. These inconsistencies are usually cast as three asset pricing puzzles: (i) the low exchange rate volatility puzzle documented by Obstfeld and Rogoff (2001) and Brandt, Cochrane, and Santa-Clara (2006), (ii) the counter-cyclicality puzzle of Kollmann (1991) and Backus and Smith (1993), and (iii) the forward premium anomaly of Hansen and Hodrick (1980) and Fama (1984).

One way to address these challenges is to depart from the assumption of separable utility while maintaining market completeness. For instance, Colacito and Croce (2011, 2013) rely on recursive preferences with highly correlated long-term consumption components, in order to explain the three puzzles mentioned above.[1] One pervasive feature of models that impose complete and integrated markets is that international stochastic discount factors (SDFs) are almost perfectly correlated. Indeed, it is well known that under such assumptions, the rate of appreciation of the real exchange rate (X) has to be equal to the ratio of foreign (M_f) and domestic (M_d) SDFs: $X = M_f/M_d$, an identity we refer to as the asset market view of exchange rates. Consequently, to match the low exchange rate volatility in the data, international SDF comovement needs to be high.

The common understanding in international financial economics is that a departure from market completeness relaxes the straightjacket of the asset market view. In incomplete markets, the exchange rate return is in general different from the ratio of international SDFs, a feature that can be captured by a stochastic exchange rate wedge as in Backus, Foresi, and Telmer (2001). Recent work by Lustig and Verdelhan (2016) shows, however, that the three puzzles cannot be jointly explained in an international consumption CAPM, even in presence of a non zero exchange rate wedge.

In this paper, we address these challenges by taking a different approach: rather than imposing distributional assumptions on the underlying economic fundamentals or agents' utilities, we develop a parsimonious model-free framework to uncover the relationship between stochastic wedges, international SDFs, and international market structures. More specifically, we characterize SDFs in incomplete markets by allowing for varying degrees of asset market segmentation. This approach enables us to not only study in a systematic way how the extent and nature of market segmentation shapes stochastic wedges that measure deviations from the asset market view, but also to reveal fundamental forces that are central to addressing the three exchange rate puzzles and may be obscured by the various assumptions built into economic models commonly used in the literature.

As our first theoretical result, we show that the asset market view is always satisfied, even in integrated but incomplete international financial markets, with respect to a particular pair of international SDFs. We then prove that there exists an even larger family of international SDF pairs satisfying the asset market view if and only if the risks spanned by portfolios of domestic and foreign returns are identical.

Given the multitude of SDFs pricing returns in incomplete markets, we explore minimum

[1]Other examples include the habit model of Heyerdahl-Larsen (2014) and Stathopoulos (2017) to generate sizable currency risk premia. Farhi and Gabaix (2016) rely on a complete market economy with time-additive preferences and a time-varying probability of rare consumption disasters. Gabaix and Maggiori (2015) provide a theory of exchange rate determination based on capital flows in segmented financial markets with heterogenous trading technologies. Hassan (2013), Hassan and Mano (2017) and Colacito, Croce, Gavazzoni, and Ready (2018) emphasize the importance of heterogeneous exposure to global shocks to match currency risk premia.

dispersion SDFs, which minimize different notions of variability, e.g., the Hansen and Jagannathan (1991) SDF when we minimize the SDF variance, or minimum entropy SDFs. We prove that minimum entropy SDFs always imply the validity of the asset market view in integrated international markets. This finding is key, because it implies that a Backus, Foresi, and Telmer (2001)–type stochastic wedge can only arise with respect to any other pair of minimum dispersion SDFs, including the tradable minimum variance SDFs. Importantly, we also show that the stochastic wedge with respect to the minimum variance SDF pair is always interpretable as a relative measure of the amount of unspanned exchange rate risks in international financial markets. However, we also find that the wedges resulting from these unspanned exchange rate risks are quantitatively small in the data under integrated markets. Therefore, an economically intuitive and empirically relevant way to break the strong link between exchange rates and international SDF ratios is to introduce market segmentation. Our results document the tight connection between deviations from the asset market view and market segmentation.

Our empirical analysis identifies the conditions under which international SDFs successfully address the exchange rate puzzles and it highlights the importance of financial market segmentation as a key ingredient to quantitatively match salient features of international asset returns. More specifically, we examine the implications of different international market structures, from segmented domestic and foreign markets to highly integrated markets, in which (risk-free) short- and long-term bonds and stocks are traded internationally. This allows us to quantify the trade-offs between a larger domestic and foreign SDF dispersion necessary to price a wider set of returns, the three exchange rate puzzles, and a deviation from the asset market view. We further adopt the insights of Bansal and Lehmann (1997), Alvarez and Jermann (2005) and Hansen and Scheinkman (2009), in order to factorize international SDFs into a permanent and a transient component using long-term bonds.

We study eight benchmark currencies, namely the US dollar, the British pound, the Swiss franc, the Japanese yen, the euro, the Australian dollar, the Canadian dollar, and the New Zealand dollar over the sample period spanning January 1975 to December 2015. We document several stylized facts about model-free SDFs, which are independent of the dispersion measure used. First, permanent (martingale) components of domestic and foreign SDFs across markets are highly volatile, irrespective of the degree of market segmentation, to the point that they actually dominate the overall SDF variability. The co-movement of permanent SDF components and long-term bond returns is positive, in order to match the typically negative local risk premia of long-term bonds. These features are consistent with previous evidence for the US market in, e.g., Alvarez and Jermann (2005). Second, we find that international SDFs are almost perfectly correlated in integrated markets, which is mainly due to the nearly perfect correlation of permanent components. In segmented markets, however, this correlation is greatly reduced. For example, while the average correlation among international SDFs with the US SDFs is almost 1 in integrated market, the correlation is, on average, only 0.4 in segmented markets. In a next step, we then ask whether these SDFs are successful at explaining the exchange rate behavior.

In a setting where equity and long-term bond markets are segmented and investors are allowed to trade internationally only the riskless short-term bonds, we find the ensuing minimum dispersion SDFs to jointly address the three exchange rate puzzles. The low exchange rate volatility is explained by means of a volatile wedge between exchange rates and the ratio of foreign and domestic SDFs. The cyclicality puzzle is addressed because cross-country differences in transient SDF components are only weakly related to exchange rate returns. Carry trade premia are also in line with those observed in the data, because the international pricing constraints on risk-free bonds effectively force domestic and foreign SDFs to correctly reproduce the cross-section of currency risk premia. Moreover, the SDFs in this setting exhibit on average volatilities only slightly higher than the local Sharpe ratios, whereas the correlation between domestic and foreign SDFs is far from perfect, ranging from 12% to 65%, giving rise to large deviations from the asset market view.

As most of the international finance literature assumes that markets are integrated, we compare the segmented market case to an integrated market economy. Under integrated international bond and stock markets, we show that the corresponding SDFs again address the three exchange rate puzzles. However, this comes at the cost of significantly larger dispersions than under integrated short-term bond markets alone. Moreover, the correlations between international minimum dispersion SDFs are nearly perfect and deviations from the asset market view uniformly small. These SDF properties are necessary in order to match the cross-sections of international equity premia and long-term bond risk premia, together with the low exchange rate volatility. Compared to the integrated short-term bond markets case, the increase in minimum variance SDFs dispersion is on average 30%, where the highest variances are observed for the funding currencies. These large SDF dispersions suggest that it may be difficult to explain exchange rate puzzles in a structural model with integrated international bond and equity markets, while satisfying standard SDF bounds. We hence conclude that market segmentation is essential to produce SDFs that can quantitatively match these features. Our results also show that market segmentation is central for producing a substantially lower degree of co-movement or similarity between international minimum dispersion SDFs. To measure SDF similarity in our incomplete markets setting, we propose a novel SDF similarity index which complements existing measures of SDF co-dependence, such as Brandt, Cochrane, and Santa-Clara (2006) and Chabi-Yo and Colacito (2017). Consistent with our earlier results, we find that SDF similarity in the data is maximal for integrated markets. In contrast, when markets are segmented, SDF similarity is significantly lower.

In quest of relating our model-free SDFs to economic fundamentals, we follow the motivation in Gabaix and Maggiori (2015), who study an economy with segmented short-term bond markets, in which households' asset demand is met by financial intermediaries. Specifically, we first empirically examine the relationship between model-free SDFs and different proxies of financial intermediaries' risk-bearing capacity, such as the VIX, a model-free implied volatility index extracted from S&P500 options, and a composite measure of the capital ratio of He, Kelly, and Manela (2017) and the broker-dealer leverage of Adrian, Etula, and Muir (2014). We find that proxies of financial constraints and intermediaries' wealth alone can explain up to 50% of the variation of our model-free SDFs. This model-free evidence supports the notion of the important role played by intermediaries in international financial markets.

Finally, we interpret our empirical results through the lens of a parsimonious theoretical framework of financial intermediation in international markets, which features two key building blocks: Limited participation by households and an intermediation friction on financiers. We assume that households can trade linear portfolios of basic assets internationally, but they cannot trade more complex assets with non-linear payoffs. In contrast, financial intermediaries have the knowledge to invest in risky complex assets and supply these assets to households. More specifically, we assume that financial intermediaries are able to synthetically replicate the entropy SDF, which is a nonlinear transformation of returns, but they face a Value-at-Risk constraint. Since minimum entropy SDFs load on higher-order moments of international returns, they are an ideal hedging vehicle for households to insure against unspanned exchange rates risks in incomplete financial markets. Borrowing from the good-deal bounds approach in Cochrane and Saá-Requejo (2000), we determine the equilibrium quantity and price for intermediated entropy payoffs. We then estimate intermediaries' supply and households' demand for the entropy SDFs and we illustrate the effects of Value-at-Risk constraints and wealth shocks in the data. Finally, we document that the resulting good-deal bounds SDF, which prices both the original returns in segmented markets and the intermediated entropy payoff, displays a strong link to empirical proxies of risk-bearing capacity of financiers.

Literature Review: Our paper contributes to the literature that studies the ability of market incompleteness to address various puzzles in international finance. Bakshi, Cerrato, and Crosby (2018) and Lustig and Verdelhan (2016) study SDFs in incomplete markets to accommodate the weak link between exchange rates and macroeconomic fundamentals. The former impose "good-deal" bounds on the international SDFs of a hypothetical integrated market economy, in order to generate international SDFs with a low degree of co-movement. The latter introduce a stochastic wedge between foreign and domestic SDFs and conclude that incomplete markets cannot jointly address the three exchange rate puzzles under a consumption CAPM framework. Distinct from these papers, we provide a theoretical characterization of minimum dispersion SDFs under different assumptions about the underlying market structure and while allowing SDFs to be factorized into transient and permanent components. We then study empirically the implications of various market structures for the properties of international SDFs and the exchange rate puzzles. In this context, we show that long-run SDF components are instrumental to reconcile stylized exchange rate facts in incomplete markets, especially the Backus and Smith (1993) puzzle. Further, we document that a low co-movement of international minimum dispersion SDFs only arises when markets feature some degree of segmentation.

Another strand of the literature studies structural models of exchange rate determination in segmented markets. Chien, Lustig, and Naknoi (2015) show that while limited stock market participation can reconcile highly correlated international SDFs with a low correlation in consumption growth, it is less successful in addressing the Backus and Smith (1993) puzzle. Alvarez, Atkeson, and Kehoe (2009) explain the Backus and Smith (1993) puzzle in a general equilibrium model with financial frictions and endogenous market participation. Gabaix and Maggiori (2015) study the disconnect puzzle and violations of the uncovered interest rate parity condition (UIP), in a setting where specialized financiers intermediate households' asset demands in segmented markets. In the data, Maggiori, Neiman, and Schreger (2017) document extreme segmentation in international lending markets at the currency level. We contribute to this literature by jointly addressing the three exchange rate puzzles under a model-free approach and by quantifying empirically the trade-off between large SDF dispersions and international co-movement in integrated versus segmented international financial markets. We further document strong links between model-free SDFs and proxies of financial intermediaries' constraints. Lastly, we provide a theoretical model-free framework to study the impact of intermediation frictions and wealth shocks on intermediated assets and to quantify their effects in the data.

Maurer and Tran (2016) construct minimum variance SDFs in incomplete continuous-time models. They show that minimum variance SDFs satisfy the asset market view in integrated markets if and only if there is no jump risk. Our approach is different and model-free, as we explicitly consider various minimum dispersion SDFs in discrete time, without imposing distributional assumptions on returns. Notably, we theoretically show that the asset market view always holds for minimum entropy SDFs in integrated markets, irrespective of the degree of incompleteness. We also prove that the asset market view holds with respect to any other minimum dispersion SDF pair, including minimum variance SDFs, if and only if the spaces of returns traded by domestic and foreign investors coincide.[2] Finally, we show that the stochastic exchange rate wedges associated with international minimum variance SDFs can be interpreted as a measure of the relative amount of unspanned exchange rate risks in domestic and foreign markets.

The SDF factorization in permanent and transient components has been empirically employed in various studies of international asset pricing. Chabi-Yo and Colacito (2017) make use of co-entropies to characterize the horizon properties of SDF co-movement. Lustig, Stathopoulos, and Verdelhan (2016) explore international long-term bond premia and conclude that the bond return parity condition holds when nominal exchange rates are stationary. The large co-movement of permanent components in these studies is a direct consequence of the assumed validity of the asset market view. Indeed, we demonstrate that regardless of the underlying degree of market incompleteness, an almost perfect co-movement of permanent SDF components emerges in all our integrated international economies. While we document that the underlying SDF dispersion in integrated markets may be difficult to explain using structural models with integrated (complete or incomplete) markets, we also quantify the SDF dispersion and similarity trade-offs in segmented market settings.

Finally, our paper also contributes to the recent literature on financial intermediaries and their effect on international asset prices. For example, Haddad and Muir (2018) show that financial intermediaries' health matters more for assets that households are less willing to hold directly, such as CDS, sovereign bonds, and FX. He and Krishnamurthy (2018) provide anecdotal evidence of the importance of intermediaries in FX markets. Malamud and Schrimpf (2017) posit a theoretical model to explain several no arbitrage violations where intermediaries trade with customers in over-the-counter FX markets. Hébert (2017) documents the impact of financial intermediaries for international money markets in the presence of externalities. Maggiori (2017) documents that

[2] In a continuous-time setting with non-degenerate multivariate Brownian shocks, absence of jump risk is equivalent to the assumption of domestic and foreign asset return spaces being the same.

heterogeneity in financial development can rationalize the special role of the United States as the main risk taker in the global financial architecture. Different from these papers, we do not make any assumptions about investors' utility functions, nor the economic fundamental processes. We estimate the supply and demand of intermediated assets in a model-free way and illustrate how wealth shocks and a tightening of Value-at-Risk constraints affect the demand and supply of these assets.

Outline of the paper: The rest of the paper is organized as follows. Section 1 provides the theoretical framework for studying model-free SDFs in international financial markets. Section 2 describes our data and our main empirical findings. Section 3 explores the empirical relation between international model-free SDFs and proxies of financial intermediaries' wealth and proposes a theoretical framework to rationalize our findings. Section 4 contains robustness checks regarding the SDF factorization into permanent and transitory components. Section 5 concludes. An online appendix provides additional extensions and results omitted in the paper.

1 Preference-Free SDFs in International Markets

Consider an economy consisting of two countries, one domestic (d) and one foreign (f), each with its own currency. Investors in each country can trade in a set of assets denominated in their respective currencies, with $\mathbf{R}_d = (R_{d0}, \ldots, R_{dK_d})'$ and $\mathbf{R}_f = (R_{f0}, \ldots, R_{fK_f})'$ denoting the returns in the domestic and foreign countries, respectively. We use R_{d0} and R_{f0} to indicate the corresponding risk-free returns in the two markets.

Let M_d and M_f denote generic domestic and foreign SDFs that price the vectors of returns $\mathbf{R}_d$ and $\mathbf{R}_f$ from the perspective of domestic and foreign investors (i.e., in the domestic and foreign currencies), respectively. This means that the corresponding Euler equations are given by

$$\begin{aligned} \mathbb{E}(M_d \mathbf{R}_d) &= \mathbf{1} \\ \mathbb{E}(M_f \mathbf{R}_f) &= \mathbf{1}, \end{aligned} \tag{1}$$

where $\mathbf{1}$ denotes the vector of ones of appropriate size. Finally, let X denote the gross exchange rate return, with the exchange rate defined as the domestic currency price of one unit of the foreign currency.[3]

Definition 1. International financial markets are *integrated* if

$$\mathrm{span}(\mathbf{R}_d) = \mathrm{span}(\mathbf{R}_f X),$$

where $\mathrm{span}(\mathbf{R}_d)$ and $\mathrm{span}(\mathbf{R}_f X)$ denote the linear spans of portfolio returns generated by domestic returns and foreign returns converted in domestic currency, respectively.

According to the above definition, when international financial markets are integrated, each foreign return is tradable by domestic investors through exchange rate markets and vice versa. In

[3]Throughout the paper, unless noted explicitly, we work with real SDFs, returns, and exchange rates.

contrast, when $\mathrm{span}(\mathbf{R}_d) \neq \mathrm{span}(\mathbf{R}_f X)$, markets are (fully or partially) segmented, in the sense that certain returns are accessible to investors in one country, but not the other. Note that the notion of market integration in Definition 1 is distinct from market completeness: international markets can be integrated even if the collection of assets available in each country does not span all the uncertainty in that country.

Definition 2. The *asset market view of exchange rates* holds with respect to a pair of SDFs (M_d, M_f) if

$$X = \frac{M_f}{M_d}. \tag{2}$$

It is well-known that whenever domestic and foreign markets are complete and integrated, the asset market view of exchange rates holds with respect to the unique pair of SDFs that satisfy the pricing restrictions (1). This means that one can unambiguously back out the implied changes in exchange rates from the two countries' stochastic discount factors. More generally, under incomplete markets, the multiplicity of international SDFs satisfying (1) may lead to violations of (2), even in integrated markets. In this case, deviations from the asset market view can be parameterized by a stochastic wedge η between exchange rate returns and the ratio of foreign and domestic SDFs (see, e.g., Backus, Foresi, and Telmer (2001)):

$$X = \frac{M_f}{M_d} \exp(\eta). \tag{3}$$

Crucially, since M_f and M_d are not uniquely determined, each choice of SDF pairs may lead to a potentially different wedge.

In the remainder of this section, we study how the international market structure and the choice of SDFs jointly determine the extent and nature of deviations from the asset market view, as measured by the stochastic wedge η.

1.1 Minimum-Dispersion SDFs

Since market incompleteness implies the pair of SDFs M_d and M_f satisfying (1) is not unique, we start our analysis by constructing a family of SDFs for each country. For country $i \in \{d, f\}$, we define the *minimum-dispersion SDF* with parameter $\alpha \neq 1$ as:[4]

$$\begin{aligned} M_i^*(\alpha) = \arg\min_{M_i} \quad & \frac{1}{\alpha(\alpha-1)} \log \mathbb{E}[M_i^\alpha] \\ \text{s.t.} \quad & \mathbb{E}[M_i \mathbf{R}_i] = \mathbf{1} \\ & M_i > 0. \end{aligned} \tag{4}$$

The pricing restriction $\mathbb{E}[M_i \mathbf{R}_i] = \mathbf{1}$ in (4) ensures that M_i prices all assets available to investors in country i, while the positivity constraint $M_i > 0$ ensures that it is indeed an admissible SDF.

As a first observation, note that the family of SDFs defined in equation (4) subsumes various SDFs commonly used in the literature. For instance, setting $\alpha = 2$ leads to the minimum variance SDF underlying the well-known Hansen and Jagannathan (1991) bounds, whereas $\alpha = 0$ and $\alpha = 1/2$ yield

[4]The case $\alpha = 0$ is defined by continuity: $\lim_{\alpha\to 0} \frac{1}{\alpha(\alpha-1)} \log \mathbb{E}[M_i^\alpha] = \mathbb{E}[-\log M_i]$.

the minimum entropy and minimum Hellinger divergence SDFs, respectively.[5] Clearly, all SDFs in this family coincide with one another when markets are complete.

While SDFs are not directly observable from the data, by construction, different values of α assign different weights to the various moments of the asset return distribution.[6] Our first result leverages this property to provide a formal link between the family of minimum dispersion SDFs and the distribution of asset returns. To this end, we consider the following optimal portfolio problem (Orłowski, Sali, and Trojani, 2016):

$$\begin{aligned} R_i^*(\alpha) = \arg\max_{\lambda_i} \quad & -\frac{1}{\alpha}\log\mathbb{E}\left[R_{\lambda_i}^{\alpha/(\alpha-1)}\right] \\ \text{s.t.} \quad & R_{\lambda_i} = (1-\sum_{k=1}^{K_i}\lambda_{ik})R_{i0} + \sum_{k=1}^{K_i}\lambda_{ik}R_{ik} \\ & R_{\lambda_i} > 0, \end{aligned} \tag{5}$$

where λ_{ik} denotes the portfolio weight of asset k. We have the following result:

Proposition 1. *The minimum dispersion SDF of country i with parameter $\alpha \neq 1$ satisfies*

$$M_i^*(\alpha) = \frac{R_i^*(\alpha)^{1/(\alpha-1)}}{\mathbb{E}\left[R_i^*(\alpha)^{\alpha/(\alpha-1)}\right]}, \tag{6}$$

where $R_i^*(\alpha)$ is the optimal portfolio return which solves the optimization problem (5) with corresponding parameter α. Furthermore,

$$\frac{1}{\alpha(\alpha-1)}\log\mathbb{E}[M_i^\alpha] \geq -\frac{1}{\alpha}\log\mathbb{E}\left[R_{\lambda_i}^{\alpha/(\alpha-1)}\right] \tag{7}$$

for any stochastic discount factor M_i and any portfolio return R_{λ_i}.

Equation (6) in Proposition 1 provides a characterization of the entire family of minimum dispersion SDFs in terms of the returns of a corresponding family of optimal portfolios. For instance, it implies that the minimum entropy and minimum variance SDFs are equal to $M_i^*(0) = 1/R_i^*(0)$ and $M_i^*(2) = R_i^*(2)/\mathbb{E}[R_i^*(2)^2]$, respectively. More importantly for our purposes, equation (6) provides us with a closed-form expression for model-free SDFs that can be estimated directly from the data. This relationship will serve as the basis of our empirical analysis in Section 2.

In addition to equation (6), Proposition 1 establishes a family of lower bounds (7) for all admissible SDFs M_i in terms of the returns of any portfolio in country i, formalizing the idea that each value of α imposes a restriction on a specific moment of M_i. Note that this family contains some of the well-known bounds in the literature as special cases. For instance, when $\alpha = 2$, inequality (7) reduces to the well-known variance bound of Hansen and Jagannathan (1991), whereas setting $\alpha = 0$ leads to the entropy bounds (see e.g., Stutzer (1995), Bansal and Lehmann (1997), and Alvarez and Jermann (2005)).

[5]In the following, we focus on minimum variance and minimum entropy SDFs. We discuss minimum Hellinger divergence SDFs in the online appendix.

[6]For instance, as argued by Almeida and Garcia (2012), among others, whereas the minimum-entropy SDF delivers a tight upper bound on the maximal expected log return with respect to the available traded assets, the minimum-variance SDF delivers a tight upper bound on the maximal Sharpe ratio.

1.2 The Asset Market View

With the above result in hand, we now turn to establishing the link between international market structure and the choice of SDFs on the one hand and the extent and nature of deviations from the asset market view on another.

Proposition 2. *Suppose international financial markets are integrated. Then,*

(a) The asset market view of exchange rates holds with respect to minimum entropy SDFs, i.e., $X = M_f^*(0)/M_d^*(0)$.

(b) The asset market view of exchange rates holds with respect to any minimum dispersion SDF with dispersion parameter $\alpha \notin \{0, 1\}$ *if and only if span*$(\mathbf{R}_d) =$ *span*$(\mathbf{R}_f)$.

Statement (a) of the above result establishes that, as long as financial markets are integrated, minimum entropy SDFs are always consistent with the asset market view of exchange rates. Notice that this relationship holds without imposing any other restrictions on the distribution of asset returns or the extent of market incompleteness. Intuitively, it follows from the fact that domestic and foreign minimum entropy SDFs are *numéraire invariant*, in the sense that they can always be expressed in the other currency units by a simple change of numéraire via the exchange rate return. This finding has important implications for our understanding of Backus, Foresi, and Telmer (2001)-type wedges in equation (3), as it implies that, as long as international markets are integrated, the stochastic wedge η corresponding to minimum entropy SDFs is equal to zero, even if the domestic and foreign markets are incomplete. A fundamental consequence of this result is that the only way to break the strong link between exchange rates and international minimum entropy SDFs is to introduce some form of market segmentation.

The tight link between exchange rates and minimum entropy SDFs in integrated markets is further clarified in Proposition 2(b), which establishes that $X = M_f^*(\alpha)/M_d^*(\alpha)$ for any $\alpha \neq 0$ if and only if the spaces of traded risks in domestic and foreign countries are identical. Together with statement (a), we thus obtain that the minimum entropy SDF pair is the *single* numéraire invariant pair of SDFs in all economies where the spaces of traded domestic and foreign risks are different. Conversely, the existence or absence of a stochastic wedge (3) in integrated markets is unrelated to the extent of domestic or foreign market incompleteness: whereas the asset market view always holds with respect to the minimum entropy SDF pair, wedges arise for any other minimum dispersion SDF pair, unless the risks spanned in one country are identical to the risks spanned in another country.[7]

We conclude this discussion by noting that Proposition 2(a) also implies that the minimum entropy SDFs in the two countries are given by the same transformation of a common linear combination of returns in the domestic and foreign currencies. More specifically, the juxtaposition of Propositions 1 and 2(a) implies that $M_d^*(0) = 1/R_d^*(0)$ and $M_f^*(0) = X/R_d^*(0)$. This means that, under integrated international markets, $M_d^*(0)$ and $M_f^*(0)$ are always consistent with the idea of perfect sharing of the financial risks reflected in portfolios of returns in domestic and foreign currencies.

[7] Indeed, in the special case where markets are integrated and complete, the asset market view holds precisely because the returns in the two countries exhibit identical spans.

However, note that, since minimum entropy SDFs are nonlinear transformations of asset returns, this risk sharing is in general not attainable by portfolios of traded returns.

1.3 Unspanned Exchange Rate Risks

Our results in the previous subsection emphasize that minimum variance SDFs are generally inconsistent with the asset market view of exchange rates, even when international markets are fully integrated. Our next result characterizes the extent of this inconsistency in terms of the amount of unspanned exchange rate risk in international financial markets.

Corollary 1. *Suppose international financial markets are integrated but incomplete. Then,*

$$X = \frac{M_f^*(2)}{M_d^*(2)} \exp(\eta(2)),$$

where the stochastic wedge $\eta(2)$ is given by

$$\eta(2) = \log\left(\frac{1 + [M_f^*(0) - M_f^*(2)]/M_f^*(2)}{1 + [M_d^*(0) - M_d^*(2)]/M_d^*(2)}\right). \tag{8}$$

Corollary 1 illustrates that the deviation from the asset market view with respect to minimum variance SDFs is determined by the ratio of the relative projection errors of foreign and domestic minimum entropy SDFs on the space of foreign and domestic returns. Note that the difference between minimum entropy and minimum variance SDFs, $M_i^*(0) - M_i^*(2)$, can be interpreted as the amount of *unspanned exchange rate risk* that cannot be replicated using basic asset returns.[8,9] Therefore, Corollary 1 suggests that stochastic wedges in the Backus, Foresi, and Telmer (2001) decomposition can be regarded as a measure of the relative amount of unspanned domestic and foreign exchange rate risks.

Viewed through the lens of Proposition 1, departures from the asset market view with respect to minimum variance SDFs can also be expressed in terms of asset returns. More specifically, equation (6) implies that exchange rate returns can be decomposed as

$$X = \frac{R_d^*(2)}{R_f^*(2)} \cdot \frac{1 + [R_d^*(0) - R_d^*(2)]/R_d^*(2)}{1 + [R_f^*(0) - R_f^*(2)]/R_f^*(2)}, \tag{9}$$

where $R_i^*(\alpha)$ denotes the optimal portfolio return under dispersion parameter α in optimization problem (5). Recalling that $R_i^*(2)$ and $R_i^*(0)$ are the returns of maximum Sharpe ratio and maximum growth portfolios in country i, equation (9) provides a direct characterization of exchange rates in terms of the tradable risk-return trade-offs in international financial markets. The exchange rate return is larger when the domestic maximum Sharpe ratio return is higher than the foreign maximum Sharpe ratio return. This effect is captured by the ratio $R_d^*(2)/R_f^*(2)$ on the right-hand side of equation (9) and can be interpreted as a tradable exchange rate effect due to the mean-variance trade-off between domestic and foreign markets. The exchange rate return is also higher when the excess

[8] This follows from the numéraire invariance property of minimum entropy SDFs and the fact that $M_i^*(0) - M_i^*(2)$ is orthogonal to the space of traded returns in country i.

[9] In our empirical specification in Section 2, we consider bonds and stocks to be basic assets that investors can trade.

return of the domestic maximal growth return in integrated markets, relative to the maximum Sharpe ratio tradable return, is larger than the corresponding foreign excess return. This effect is summarized by the second ratio on the right-hand side of (9) and directly reflects the risk-return trade-offs between domestic and foreign markets due to the higher moments of returns.

The above interpretation of unspanned exchange rate risks can be extended to minimum variance SDFs obtained in segmented markets, because we can always embed a segmented international market into a nesting integrated market.[10] Indeed, consider two subvectors of domestic and foreign returns from a given integrated market, which give rise to corresponding minimum variance SDFs $M^*_{i,S}(2)$ $(i = d, f)$. In such settings, $M^*_d(0) - M^*_{d,S}(2)$ is orthogonal to the space of traded domestic returns in segmented markets, because the latter space is contained in the space of domestic returns in the embedding integrated market. Therefore, $M^*_d(0) - M^*_{d,S}(2)$ is interpretable as an unspanned domestic exchange rate risk in segmented markets. Similarly, $M^*_f(0) - M^*_{f,S}(2)$ is interpretable as an unspanned foreign exchange rate risk in segmented markets.

1.4 SDF Similarity and the Asset Market View

An important literature in international finance quantifies the amount of SDF co-dependence needed to explain the low volatility puzzle when the asset market view holds. For example, Brandt, Cochrane, and Santa-Clara (2006) introduce a SDF correlation index that is decreasing in the exchange rate variance.[11] In a related vein, Chabi-Yo and Colacito (2017) propose an index based on co-entropies, which incorporates higher-moment dependence and is decreasing in the exchange rate entropy. In the spirit of these papers, we propose a novel class of SDF similarity indices, which can be calculated in real time.[12]

Definition 3. International SDF similarity is defined as follows:

$$S(M_d, M_f) := \frac{\mathbb{E}[\min(M_d, M_f)]}{\min(\mathbb{E}[M_d], \mathbb{E}[M_f])} \,. \tag{10}$$

The similarity index (10) has some appealing properties. First, by construction, $S(M_d, M_f) = S(M_f, M_d)$, i.e., (10) is a symmetric measure. Moreover, the index is bounded between 0 and 1, i.e. $0 \leq S(M_d, M_f) \leq 1$ and $S(M_d, M_f) = 1$ if and only if $M_d = M_f$ with probability one, i.e., international SDFs are identical.[13]

Hence, the two SDFs are more dissimilar when the expected minimum SDF is lower. This can be interpreted as follows. First, as each expected SDF is a bond price, in order to obtain a minimum SDF near zero in some states, SDFs need to be sufficiently volatile. Second, in order to obtain a

[10] If a domestic (foreign) return R_d (R_f) traded in segmented markets is such that R_d/X ($R_f X$) is not traded in the foreign (domestic) market, we can always add the latter return to the foreign (domestic) return space to obtain the smallest integrated international market containing the initial segmented market.

[11] Formally, this property holds for SDF correlations smaller than the minimum between the ratios of the two SDF volatilities.

[12] Our SDF similarity index is related to Chernoff (1952) divergence, which measures the distance between two probability distributions.

[13] In the empirically highly unlikely case where $M_d > M_f$ or $M_f > M_d$ almost surely, the similarity index is also equal to one. In these cases, one can always transform the SDFs to have a normalized expectation. We discuss these cases in the online appendix.

minimum SDF near to zero with a sufficiently large probability, SDFs need to be negatively associated. Therefore, settings of small SDF similarities tend to arise in economies with volatile and negatively associated SDFs. Obviously, this is in sharp contrast to the implications of settings of volatile SDFs that satisfy the asset market view, as in this case the only way to obtain a low exchange rate volatility is by means of (strongly) positively associated domestic and foreign SDFs.

While the definition of similarity index (10) is completely independent of the validity of the asset market view, a quite useful property arises in settings where the asset market view holds. In this case, equation (10) is directly computable from at-the-money currency option and bond prices alone.

Proposition 3. *If the asset market view of exchange rates holds, then*

$$\frac{\mathbb{E}[M_d] - \mathbb{E}[M_d \max(0, 1 - X)]}{\min(\mathbb{E}[M_d], \mathbb{E}[M_f])} = S(M_d, M_f) = \frac{\mathbb{E}[M_f] - \mathbb{E}[M_f \max(0, 1 - (1/X))]}{\min(\mathbb{E}[M_d], \mathbb{E}[M_f])} , \tag{11}$$

where $\max(0, 1 - X)$ $(\max(0, 1 - (1/X)))$ *is the payoff of an at-the-money put option on the spot exchange rate change* X $(1/X)$*.*

Under the conditions of Proposition 3, index (10) can be directly estimated from the prices of at-the-money put options on spot exchange rates and short-term bond prices, without relying on time series information about the remaining set of traded asset returns. This feature offers a simple way of measuring empirically the *conditional* similarity dynamics and the cross-sectional similarity patterns of international SDFs.[14]

Whenever there is a deviation from the asset market view, an asymmetry in the pricing properties of domestic and foreign exchange rate options emerges. In such settings, we define the following SDF similarity index:

$$\overline{S}(M_d, M_f) := \min(S(M_d, \tilde{M}_f), S(M_f, \tilde{M}_d)) , \tag{12}$$

where $\tilde{M}_f := M_d X$ and $\tilde{M}_d := M_f/X$. Thus, $\overline{S}(M_d, M_f)$ is the *minimal* similarity of domestic and foreign SDFs M_d and M_f with respect to their numéraire invariant foreign and domestic SDF counterparts $\tilde{M}_f$ and $\tilde{M}_d$.

By construction, $\overline{S}(M_d, M_f)$ is a symmetric similarity index, satisfying $0 \leq \overline{S}(M_d, M_f) \leq 1$. Moreover, whenever the market view holds, $\overline{S}(M_d, M_f) = S(M_d, M_f)$, i.e., index (12) still measures the similarity of M_d and M_f. Under a deviation from the asset market view, the index equals one if and only if $M_d = M_d X$ and $M_f = M_f/X$ with probability one, i.e., the exchange rate return is constant and equal to one.

The explicit link between index (12) and the prices of a domestic and a foreign at-the-money put option on the spot exchange rate return is provided next.

Proposition 4. *Whenever the risk-free domestic and foreign returns* $\tilde{R}_{f0} = R_{f0} X$ *and* $\tilde{R}_{d0} = R_{d0}/X$ *are traded, it follows:*

$$\overline{S}(M_d, M_f) = \frac{\min\left(\mathbb{E}[M_d] - \mathbb{E}[M_d \max(0, 1 - X)], \mathbb{E}[M_f] - \mathbb{E}[M_f \max(0, 1 - (1/X))]\right)}{\min(\mathbb{E}[M_d], \mathbb{E}[M_f])} , \tag{13}$$

[14] In the online appendix, we compare our similarity index to other similarity indices used in the literature.

where $\max(0, 1 - X)$ $(\max(0, 1 - (1/X)))$ *is the payoff of an at-the-money put option on the spot exchange rate change* X $(1/X)$*.*

The significance of the above result is twofold. First, whenever the prices of domestic and foreign exchange rate at-the-money options are both observable, a comparison of indices (10) and (12) provides direct evidence on the existence of asset market view deviations, in terms of, e.g., their dynamic properties or their cross-sectional patterns across currencies. Second, comparing these deviations to those implied by theoretical models which imply a violation of the asset market view, such as models featuring segmented markets, provides a practical tool to empirically assess some of the implications of these models.[15]

1.5 SDF Decomposition

Before we turn to the empirical analysis, it is useful to decompose the SDFs into transitory and permanent components, as this will further shed light on the SDF properties needed to address exchange rate behavior. To this end, we follow Alvarez and Jermann (2005), Hansen and Scheinkman (2009), and Hansen (2012), who show that SDF processes can be factorized into permanent and transitory components. The permanent component is a martingale which is used to characterize pricing over long investment horizons. The transitory component is related to the return on a discount bond of (asymptotically) long maturity. Therefore, for $i = d, f$, we decompose international SDFs into martingale (permanent) and transient components:[16]

$$M_i = M_i^P M_i^T. \tag{14}$$

Theoretically, M_i^P satisfies the martingale normalization $\mathbb{E}[M_i^P] = 1$, while the transient component can be written as the inverse of the return of the infinite maturity bond ($R_{i\infty}$), i.e., $M_i^T := 1/R_{i\infty}$. In this setting, the normalization of the permanent component is ensured by requiring the return on the infinite maturity bond to be priced by SDF $M_i = M_i^P / R_{i\infty}$. Equivalently, $R_{i\infty}$ is defined as one of the components of return vector $\mathbf{R}_i$ in problem (4). Tradability of $R_{i\infty}$ obviously impacts the form of minimum dispersion SDFs and increases the SDF variability.

Moreover, from expression (14), exchange rate wedges and persistent international SDF components are related via the following factorization:

$$\frac{M_f^P}{M_d^P} \frac{R_{d\infty}}{R_{f\infty}} \exp(\eta) = X. \tag{15}$$

The goal of our next section is to estimate each component of the LHS of equation (15) in both integrated and segmented markets and to study their properties.

[15] In cases where option quotes are available only in the domestic or the foreign currency, Propositions 3 and 4 show that in general, i.e., in presence of deviations from the asset market view, the index computed from one of the equalities in (11) is always an upper bound for the similarity index $\overline{S}(M_d, M_f)$.

[16] There are two important issues. First, in general, the decomposition in Alvarez and Jermann (2005) is not unique. Nevertheless, it can be unique under additional stochastic stability assumptions. The complete theoretical framework characterizing existence and uniqueness of the factorization is treated in Hansen and Scheinkman (2009) and Qin and Linetsky (2017), among others. Second, in the data, we do not observe an infinite maturity bond return. While in the main text we approximate this return using a long-term bond of finite maturity, in Section 4 we compute non-parametric estimates of permanent components using the sieve approach in Christensen (2017), which is based on the Hansen and Scheinkman (2009) eigenvalue decomposition. We find that both approaches lead to virtually identical results.

2 Empirical Analysis

Using our model-free minimum-dispersion SDF approach, we can now characterize and quantify properties of international SDFs under different assumptions about the degree of segmentation between domestic and foreign arbitrage-free financial markets. Full market integration arises when investors have access to all assets, both in domestic and foreign markets: the risk-free return, the aggregate equity return, and the long-term bond return.[17] Market segmentation, on the other hand, arises when international investors do not have access to all the assets. This framework allows us to empirically measure the impact of different market structures on stochastic wedges and to quantify their importance in addressing the three exchange rate puzzles. We are also able to dissect international SDFs into permanent and transient components and single out the ingredients necessary to match international asset returns.

We seek to answer a number of questions. First, how much dispersion is necessary for SDFs to be able to match internationally unconditional risk premia in integrated versus segmented markets? Second, in order to successfully address all three exchange rate puzzles, how are transient and permanent SDF components connected to exchange rate wedges? Third, how much international SDF co-movement does a given market structure imply? Studying these questions helps us to identify market structures that can be consistent not only with the well-known exchange rate puzzles, but also with a plausible degree of SDF comovement.

2.1 Data

In our empirical analysis, we take the United States (US) as the domestic market and the United Kingdom (UK), Switzerland (CH), Japan (JP), the European Union (EU), Australia (AU), Canada (CA) or New Zealand (NZ) as the foreign markets.[18] We use monthly data between January 1975 and December 2015 from Datastream.[19] The resulting seven exchange rates are expressed with respect to the USD as the domestic currency. We compute equity returns from the corresponding MSCI country indices' prices and risk-free rates from one-month LIBOR rates. We follow Alvarez and Jermann (2005) and proxy transient SDF components by the inverse of the bond return with the longest maturity available, i.e., the ten-year (government) bond in our case.[20]

Finally, over-the-counter currency options data is obtained from J.P. Morgan. Due to data limitations, our options sample starts in April 1993 and ends in April 2013. For every currency pair, we study one-month maturity at-the-money plain-vanilla European put options, quoted versus the US dollar. We deflate all domestic returns and exchange rates by the corresponding domestic Consumer Price Index in order to obtain real quantities. We report summary statistics of all variables in the online appendix.

[17] We consider stocks and bonds to be the basic assets that international investors can trade.

[18] We use Germany as a proxy for EU, especially prior to the introduction of the Euro.

[19] Throughout the paper, the sample period ranges from January 1988 to December 2015 for New Zealand, due to data availability on the long-term bonds.

[20] In order to study whether the ten-year bond return is a valid proxy for the (unobservable) infinite maturity bond return, a model-based approach may be also applied, e.g., based on a family of affine term structure models on countries' yields. Lustig, Stathopoulos, and Verdelhan (2016) do not obtain significant differences between the yields of a hypothetical infinite maturity bond and a ten-year bond in such a setting.

2.2 Integrated Markets

We start our analysis with integrated markets, in which investors can trade without restrictions domestic and foreign short- and long-term bonds ($R_{i0}, R_{i\infty}$), and equity indices (R_{i1}). The tradable vector of returns in market $i = d, f$ reads $\mathbf{R}_i = (R_{i0}, R_{i1}, R_{i\infty}, \tilde{R}_{i0}, \tilde{R}_{i\infty}, \tilde{R}_{i1})'$, where $\tilde{R}_{d0,t+1} := R_{f0,t+1}X_{t+1}$ ($\tilde{R}_{f0,t+1} := R_{d0,t+1}/X_{t+1}$) is the domestic (foreign) currency return of the foreign (domestic) risk-free asset, $\tilde{R}_{d\infty,t+1} = R_{f\infty,t+1}X_{t+1}$ ($\tilde{R}_{f\infty,t+1} = R_{d\infty,t+1}/X_{t+1}$) is the domestic (foreign) currency return of the foreign (domestic) long-term bond, and $\tilde{R}_{d1,t+1} := R_{f1,t+1}X_{t+1}$ ($\tilde{R}_{f1,t+1} := R_{d1,t+1}/X_{t+1}$) is the domestic (foreign) currency return of the foreign (domestic) aggregate equity return. The estimated optimal portfolio return in market $i = d, f$ is given by:

$$\begin{aligned} R_{\hat{\lambda}_i^*,t+1} \quad = \quad & R_{i0,t+1} + \hat{\lambda}_{i1}^*(R_{i1,t+1} - R_{i0,t+1}) + \hat{\lambda}_{i2}^*(R_{i\infty,t+1} - R_{i0,t+1}) \\ & + \hat{\lambda}_{i3}^*(\tilde{R}_{i0,t+1} - R_{i0,t+1}) + \hat{\lambda}_{i4}^*(\tilde{R}_{i\infty,t+1} - R_{i0,t+1}) + \hat{\lambda}_{i5}^*(\tilde{R}_{i1,t+1} - R_{i0,t+1}) \,. \end{aligned} \tag{16}$$

Based on these returns, the time series of estimated minimum dispersion SDFs is obtained in closed-form from equation (6):

$$\hat{M}_{i,t+1}^* = \frac{R_{\hat{\lambda}_i^*,t+1}^{-1/(1-\alpha)}}{\hat{\mathbb{E}}_i\left[R_{\hat{\lambda}_i^*,t+1}^{-\alpha/(1-\alpha)}\right]} \,, \tag{17}$$

where $\hat{\mathbb{E}}_i[\cdot]$ denotes expectations under the empirical return distributions and estimated portfolio weights in equation (16) are the unique solution of the exactly identified set of empirical moment conditions:

$$\hat{\mathbb{E}}_i\left[R_{\hat{\lambda}_i^*}^{-1/(1-\alpha)}(R_{ik} - R_{i0})\right] = 0 \,, \tag{18}$$

with $k = 1, ..., K_i$ and $K_d = K_f = 5$.[21] We estimate parameter vector $\hat{\lambda}_i^*$ in (18) using the exactly identified (generalized) method of moments.

2.2.1 Properties of Minimum Dispersion SDFs

Since the domestic and foreign risk-free rate, bond return, and equity return are all priced by domestic and foreign minimum dispersion SDFs, the risk premia of these returns are all matched by construction. In particular, the currency risk premia are also exactly matched and the forward premium anomaly is implicitly incorporated by the pricing properties of minimum dispersion SDFs. We report the summary statistics of minimum dispersion SDFs in Table 1 for each bilateral pair vis-à-vis the US.

As expected, since the risk-free return R_{i0} is priced by the minimum dispersion SDF M_i^*, average minimum dispersion SDFs are virtually the same across different dispersion measures. The sample volatilities, on the other hand, display more dispersion across different values of α and are the lowest for $\alpha = 2$, by construction. The US NZ pair features the lowest volatilities (0.6 and 0.53, respectively), while the US CH pair has the highest volatility (0.87 and 0.83). One important first finding is that

[21] Recall that equation (18) represents the first-order conditions (FOCs) associated with optimization problem (5).

Table 1. Properties of SDFs (Integrated Markets)

The table reports joint sample moments of the SDF and its components. Panel A reports statistics with respect to the minimum-entropy SDFs ($\alpha = 0$) and Panel B for minimum variance SDFs ($\alpha = 2$), $i = d, f$, $j = d, f$, $i \neq j$. The SDFs are derived when international trading is unrestricted, i.e. the financial markets are integrated. There is a US domestic SDF for each bilateral trade. We use monthly data from January 1975 to December 2015.

	US	**UK**	**US**	**CH**	**US**	**JP**	**US**	**EU**	**US**	**AU**	**US**	**CA**	**US**	**NZ**
	Panel A: $\alpha = 0$ (minimum entropy)													
$\mathbb{E}[M_i]$	0.982	0.973	0.982	0.990	0.982	0.991	0.982	0.980	0.982	0.966	0.982	0.973	0.982	0.956
Std(M_i)	0.841	0.872	0.979	0.926	0.740	0.694	0.690	0.681	0.919	0.951	0.726	0.720	0.639	0.557
Std(M_i^T)	0.120	0.122	0.120	0.061	0.120	0.091	0.120	0.068	0.120	0.107	0.120	0.111	0.120	0.091
Std(M_i^P)	0.917	0.948	1.048	0.951	0.814	0.707	0.774	0.725	1.029	1.065	0.823	0.827	0.681	0.625
$\sqrt{\text{Entropy}(M_i)}$	0.684	0.703	0.795	0.753	0.687	0.636	0.604	0.585	0.732	0.702	0.618	0.616	0.581	0.519
corr(M_i^T,M_i^P)	-0.454	-0.498	-0.407	-0.233	-0.519	-0.155	-0.549	-0.502	-0.411	-0.636	-0.506	-0.607	-0.317	-0.634
corr(M_i,M_j)		0.992		0.989		0.989		0.985		0.992		0.994		0.981
	Panel B: $\alpha = 2$ (minimum variance)													
$\mathbb{E}[M_i]$	0.982	0.973	0.982	0.990	0.982	0.991	0.982	0.980	0.982	0.966	0.982	0.973	0.982	0.956
Std(M_i)	0.739	0.754	0.873	0.834	0.699	0.658	0.639	0.622	0.776	0.791	0.659	0.655	0.600	0.535
Std(M_i^T)	0.120	0.122	0.120	0.061	0.120	0.091	0.120	0.068	0.120	0.107	0.120	0.111	0.120	0.091
Std(M_i^P)	0.803	0.824	0.930	0.853	0.763	0.670	0.711	0.659	0.839	0.874	0.733	0.735	0.632	0.595
corr(M_i^T,M_i^P)	-0.517	-0.587	-0.455	-0.268	-0.552	-0.169	-0.597	-0.564	-0.500	-0.775	-0.566	-0.683	-0.340	-0.665
corr(M_i,M_j)		0.989		0.988		0.989		0.984		0.988		0.993		0.979

each of these volatilities exceeds by a large amount the equity Sharpe ratios in each country, which indicates a clearly tightened Hansen and Jagannathan (1991) bound in integrated markets.[22]

To understand in more detail the properties driving the SDF dispersion, we decompose the SDFs into their transitory and permanent components. There are two interesting observations. First, the largest part of the SDF dispersion is generated by the permanent component, regardless of the country or dispersion measure considered. This is in line with the US evidence in Alvarez and Jermann (2005). Second, the correlation between domestic and foreign SDFs are virtually perfect, which is a first broad indication of a very high SDF similarity under market integration.[23] Indeed, while this high co-movement is fully expected for the minimum entropy SDFs, due to the low exchange rate volatility and their consistency with the asset market view in this setting, our findings show that it is a general feature of market integration, as it emerges also for the minimum variance SDFs. To illustrate, the lowest correlation between minimum dispersion SDFs, which is as large as 97.9%, is obtained for the US and NZ pair using the minimum variance SDFs.

Remark: Notice that it is in general always possible to construct international SDFs which feature low correlations, even in integrated markets, by adding to a given pair of domestic and foreign SDFs two almost perfectly negatively correlated components that are orthogonal to the spaces of domestic and foreign returns. In this way, one obtains two new SDFs, with lower correlations, at the cost of higher SDF dispersions; see also Bakshi et al. (2018). However, by construction, these new SDFs have to imply strongly negatively correlated unspanned exchange rate risks. This feature may be hard to reconcile with economic intuition suggesting that unspanned exchange rate risks jointly increase in

[22] The annualized Sharpe ratios are 48, 34, 37, 19, 45, 32, 31, and 5 for Switzerland, Eurozone, United Kingdom, Japan, US, Australia, Canada and New Zealand, respectively.

[23] Lustig, Stathopoulos, and Verdelhan (2016) also document high correlations of permanent components internationally.

periods of global market distress or economic recessions.[24] We discuss these issues in greater detail in the online appendix.

2.2.2 Exchange Rate Volatilities and Wedges

The high SDF co-movement under integrated markets is related to the volatility puzzle in Brandt, Cochrane, and Santa-Clara (2006), who show that when the asset market view holds, international SDFs need to be almost perfectly correlated to match the low exchange rate volatility. Our results indicate that such high correlations arise more broadly also under a violation of the market view.

In our incomplete market setting, the high SDF correlation stems from the properties of the permanent SDF components. This seemingly perfect co-movement can be understood by rearranging terms in equation (15), to get the identity:

$$X_{t+1}\frac{R_{f\infty,t+1}}{R_{d\infty,t+1}} = \frac{M^P_{f,t+1}}{M^P_{d,t+1}}e^{\eta_{t+1}} \,. \tag{19}$$

As the variability of the LHS of equation (19) in the data is rather low, identity (19) can hold either under a low variability of both the ratio of permanent SDF components and the wedge, under a strong negative co-movement between the ratio of permanent SDF components and the wedge, or under a combination of these effects. A direct implication of these properties is that under the market view (i.e., $\eta_{t+1} = 0$), permanent components need to be almost perfectly positively related. In contrast, in a setting where the market view is not satisfied, a trade-off between the co-movement of the permanent SDF components and the long-run cyclicality of exchange rate wedges can emerge.

It follows that the high SDF dispersions in Table 1 can be empirically consistent with identity (19) only in presence of a sufficiently large wedge dispersion or when permanent SDF components are strongly positively correlated. To study this trade-off in more detail, we estimate the wedge using the closed-form expressions for minimum dispersion SDFs:

$$X_{t+1}\exp(-\eta_{t+1}) = \frac{R^{1/(1-\alpha)}_{\hat{\lambda}^*_d,t+1}R^{-1/(1-\alpha)}_{\hat{\lambda}^*_f,t+1}}{\hat{\mathbb{E}}_d\left[R^{-\alpha/(1-\alpha)}_{\hat{\lambda}^*_d,t+1}\right]^{-1}\hat{\mathbb{E}}_f\left[R^{-\alpha/(1-\alpha)}_{\hat{\lambda}^*_f,t+1}\right]} \,, \tag{20}$$

where the optimal returns are given in equation (16). Table 2 reports the wedge properties for minimum variance SDFs only, because the wedge resulting from minimum entropy SDFs vanishes by construction (see Proposition 2). Consistent with the above intuition, we obtain a small wedge dispersion in all cases, which is an order of magnitude smaller than the domestic and foreign SDF dispersions. For instance, while the wedge dispersions implied by minimum variance SDFs are larger and in a few cases comparable to the volatility of exchange rates, they still are an order of magnitude smaller than the dispersion of minimum variance SDFs themselves.[25]

[24] For example, Farhi et al. (2015) document that out-of-the-money put options of high interest rate currencies jointly become more expensive after 2008. Similarly, Mueller et al. (2017) document high option-implied correlations during recession periods.

[25] Lustig and Verdelhan (2016) show that in order to match the low exchange rate volatility puzzle using equation (3), the wedge needs to co-vary positively (negatively) with the domestic (foreign) SDF, i.e., it needs to be pro-cyclical. Given the very low wedge dispersion, the cyclicality properties are not particularly insightful in the integrated market setting and we do not report them here to save space.

Table 2. Wedge Summary Statistics (Integrated Markets)

This table reports the annualized mean, standard deviation, skewness and kurtosis of the wedge, defined as $\eta_{t+1} = \log\left(\frac{X_{t+1}M^*_{d,t+1}(2)}{M^*_{f,t+1}(2)}\right)$, for $\alpha = 2$, i.e. for the minimum variance SDFs. The domestic currency is the US dollar. The SDFs account for the fact that domestic investors can trade any foreign asset. We use monthly data from January 1975 to December 2015, except for New Zealand, for which the data starts from January 1988.

	$\mathbb{E}[\eta]$	Std(η)	Sk(η)	K(η)
UK	-0.007	0.059	-0.259	11.62
CH	-0.019	0.120	-6.146	68.40
JP	-0.009	0.083	-4.483	36.18
EU	0.000	0.064	-1.130	11.47
AU	0.005	0.075	2.110	24.82
CA	-0.001	0.034	-0.538	5.772
NZ	-0.031	0.216	-15.61	268.3

Given the low wedge dispersion and exchange rate variability, we expect a large co-movement between martingale SDF components. Table 3 reports the average correlations across different exchange rate pairs, showing that there is almost perfect co-movement across all currency pairs. Therefore, we conclude that in integrated markets minimum dispersion SDFs are very similar and highly disperse, mainly due to their very similar and highly volatile permanent components.

Table 3. Correlation of Permanent SDF Components Across Exchange Rate Parities

This table reports the correlation between permanent components of domestic and foreign SDFs for $\alpha = 0$ (minimum entropy) and $\alpha = 2$ (minimum variance). The domestic SDF is the US one, whereas the foreign SDFs are those for the UK, CH, JP, EU, AU, CA and NZ. We use monthly data from January 1975 to December 2015, except for New Zealand, for which the data starts from January 1988. Standard errors are computed using a circular block bootstrap of size 10 with $10,000$ simulations and reported in square brackets. *** denotes significance at the 1% level.

	UK	**CH**	**JP**	**EU**	**AU**	**CA**	**NZ**
$\alpha = 0$	0.972^{***}	0.984^{***}	0.981^{***}	0.978^{***}	0.987^{***}	0.976^{***}	0.968^{***}
	[0.019]	[0.007]	[0.004]	[0.007]	[0.006]	[0.005]	[0.006]
$\alpha = 2$	0.968^{***}	0.982^{***}	0.977^{***}	0.974^{***}	0.980^{***}	0.973^{***}	0.965^{***}
	[0.009]	[0.003]	[0.004]	[0.006]	[0.005]	[0.004]	[0.005]

2.2.3 Backus and Smith (1993) Puzzle

We can now study exchange rate cyclicality by means of the following Backus and Smith (1993)-type regressions:

$$
\begin{aligned}
m_{f,t+1} - m_{d,t+1} &= \delta + \beta x_{t+1} + u_{t+1}, \\
m^U_{f,t+1} - m^U_{d,t+1} &= \delta^U + \beta^U x_{t+1} + u^U_{t+1},
\end{aligned}
$$

for $U = T, P$, where we regress both the log difference between foreign and domestic SDFs and their transient and permanent components on the log exchange rate return x_{t+1}.

When the asset market view holds, the population point estimate from these regressions based on the overall SDFs is exactly one. This case emerges for the minimum entropy SDFs under integrated markets. More generally, when risk-free returns are traded internationally, Lustig and Verdelhan (2016) show that the same finding holds also under a deviation from the market view. Therefore, we expect similar results also for minimum variance SDFs. Finally, since the SDF variability in Table 1 is dominated by the permanent component, we anticipate analogous implications for regressions using the persistent SDF components.

Table 4 reports Backus-Smith (1993) estimated coefficients for the various country pairs. For all dispersion measures, the regressions with $m_{f,t+1} - m_{d,t+1}$ and $m^P_{f,t+1} - m^P_{d,t+1}$ produce estimated coefficients that are positive, highly significant, and close to one, with estimates based on the permanent component that are almost indistinguishable from the total SDF estimates. Turning to the regressions with transitory SDF components, we obtain estimated coefficients that are statistically not different from zero. Hence, we conclude that also the cyclicality puzzle can be explained in a setting of integrated markets, by a transitory component that is largely unrelated to exchange rate changes.

In summary, the integrated, but incomplete, market setting is consistent with the three exchange

Table 4. Backus and Smith (1993)-Type Regressions (Integrated Markets)

This table reports the point estimates of a linear regression of the log difference between foreign and domestic SDFs on the log real exchange rate return: $m_{f,t+1} - m_{d,t+1} = \delta + \beta x_{t+1} + u_{t+1}$, where small-cap letters denote quantities in logs. We additionally report point estimates of a linear regression of the log difference of each component of the SDF on the log change in the real exchange rate: $m^U_{f,t+1} - m^U_{d,t+1} = \delta^U + \beta^U x_{t+1} + u^U_{t+1}$, where $U = P, T$ for permanent and transitory components, respectively. We use monthly data from January 1975 to December 2015, except for New Zealand, for which the data starts from January 1988. Standard errors are reported in square brackets. *** highlights significance at the 1% level.

	US/UK		**US/CH**		**US/JP**		**US/EU**	
	$\alpha = 0$	$\alpha = 2$	$\alpha = 0$	$\alpha = 2$	$\alpha = 0$	$\alpha = 2$	$\alpha = 0$	$\alpha = 2$
β	1.000^{***}	1.022^{***}	1.000^{***}	1.233^{***}	1.000^{***}	1.107^{***}	1.000^{***}	1.055^{***}
	[0.000]	[0.0261]	[0.000]	[0.043]	[0.000]	[0.033]	[0.000]	[0.027]
β^P	1.085^{***}	1.065^{***}	0.951^{***}	1.183^{***}	1.083^{***}	1.189^{***}	0.956^{***}	1.011^{***}
	[0.068]	[0.0742]	[0.044]	[0.064]	[0.053]	[0.065]	[0.046]	[0.056]
β^T	-0.084	-0.084	0.049	0.049	-0.083	-0.083	0.044	0.044
	[0.068]	[0.068]	[0.044]	[0.044]	[0.053]	[0.053]	[0.046]	[0.046]

	US/AU		**US/CA**		**US/NZ**	
	$\alpha = 0$	$\alpha = 2$	$\alpha = 0$	$\alpha = 2$	$\alpha = 0$	$\alpha = 2$
β	1.000^{***}	1.117^{***}	1.000^{***}	1.031^{***}	1.000^{***}	1.409^{***}
	[0.000]	[0.029]	[0.000]	[0.022]	[0.000]	[0.095]
β^P	1.005^{***}	1.122^{***}	1.027^{***}	1.059^{***}	1.006^{***}	1.415^{***}
	[0.049]	[0.059]	[0.089]	[0.093]	[0.038]	[0.098]
β^T	-0.005	-0.005	-0.028	-0.028	-0.006	-0.006
	[0.049]	[0.049]	[0.089]	[0.089]	[0.037]	[0.037]

rate puzzles and with small or inexistent deviations from the asset market view. In this framework, martingale SDF components across countries are highly volatile and almost perfectly correlated, which is an indication of a large SDF similarity, while differences in transient SDF components are disconnected from exchange rate variations. The large SDF dispersion of integrated market settings is potentially a challenge for existing asset pricing models that assume the validity of the asset market view. One natural way to lower the dispersion and high correlation of international SDFs is to introduce some form of market segmentation.

2.3 Segmented Markets

To lower SDF dispersion and co-movement, we now allow investors to trade internationally only the risk-free bonds. Hence, the vector of tradable real gross returns in the domestic (US) market reads $\mathbf{R}_d = (R_{d0}, R_{d1}, R_{d\infty}, \tilde{R}_{d0})'$, where $\tilde{R}_{d0,t+1} := R_{f0,t+1}X_{t+1}$ is the domestic currency return of the foreign risk-free asset. Similarly, the vector of tradable real gross returns in the foreign market reads $\mathbf{R}_f = (R_{f0}, R_{f1}, R_{f\infty}, \tilde{R}_{f0})'$, where $\tilde{R}_{f0,t+1} := R_{d0,t+1}/X_{t+1}$ is the foreign currency return of the domestic risk-free asset. Besides matching the risk premia on the domestic returns, minimum dispersion SDFs are still forced to match the risk premia on returns $\tilde{R}_{d0}$ and $\tilde{R}_{f0}$. Therefore, they exactly match the exchange rate risk premium in the data, implicitly incorporating the forward premium anomaly.

Compared to the integrated market case, estimation of minimum dispersion SDFs in this economy is based on a reduced number of moment conditions, as the equity and long-term bonds are not traded internationally anymore, i.e., $K_d = K_f = 3$ in the set of moment conditions (18). Hence, the estimated optimal portfolio return in market $i = d, f$ reads:

$$R_{\hat{\lambda}_i^*,t+1} = R_{i0,t+1} + \hat{\lambda}_{i1}^*(R_{i1,t+1} - R_{i0,t+1}) + \hat{\lambda}_{i2}^*(R_{i\infty,t+1} - R_{i0,t+1}) + \hat{\lambda}_{i3}^*(\tilde{R}_{i0,t+1} - R_{i0,t+1})\,.$$

The closed-form expression for the estimated minimum dispersion SDF follows by plugging this optimal return into equation (17).

2.3.1 Properties of Minimum Dispersion SDFs

Table 5 documents how market segmentation affects the properties of minimum dispersion SDFs. Due to the reduced set of pricing restriction on returns, the variability of minimum dispersion SDF decreases considerably, relative to the full integration case, across all currency pairs. For instance, the SDF variability in Switzerland drops by 40%, in Japan by 50%, and similarly for New Zealand.

As in the integrated market case, permanent SDF components are in all cases very volatile and positively correlated with the long-term bond return. Because of the lower dispersion of the permanent SDF components, these correlations are larger in absolute value, in order to match the negative long-term bond risk premia in local currencies.[26] More importantly, the correlation between

[26] The correlation between the transient and permanent SDF components for an SDF M can be expressed as:

$$\text{Corr}(M^T, M^P) = \frac{\mathbb{E}[M] - \mathbb{E}[M^T]}{\sqrt{\text{Var}(M^T)}\sqrt{\text{Var}(M^P)}}\,. \tag{21}$$

When the distribution of the transient SDF component is fixed by an observable proxy, the numerator is also fixed and the correlation increases in absolute value whenever the volatility of the permanent SDF component decreases.

Table 5. Properties of SDFs (Segmented Markets)

The table reports joint sample moments of the SDF and its components. Panel A reports statistics with respect to the minimum-entropy SDFs ($\alpha = 0$) and Panel B for minimum variance SDFs ($\alpha = 2$), $i = d, f$, $j = d, f$, $i \neq j$. There is a US domestic SDF for each bilateral trade. We use monthly data from January 1975 to December 2015.

	US	**UK**	**US**	**CH**	**US**	**JP**	**US**	**EU**	**US**	**AU**	**US**	**CA**	**US**	**NZ**
	Panel A: $\alpha = 0$ (minimum entropy)													
$\mathbb{E}[M_i]$	0.982	0.973	0.982	0.990	0.982	0.991	0.982	0.979	0.982	0.966	0.983	0.973	0.983	0.956
Std(M_i)	0.611	0.723	0.769	0.674	0.722	0.364	0.645	0.487	0.603	0.821	0.634	0.514	0.539	0.380
Std(M_i^T)	0.120	0.122	0.120	0.061	0.120	0.091	0.120	0.068	0.120	0.107	0.120	0.111	0.120	0.091
Std(M_i^P)	0.707	0.824	0.843	0.698	0.800	0.379	0.732	0.535	0.702	0.931	0.729	0.641	0.585	0.466
$\sqrt{\text{Entropy}(M_i)}$	0.520	0.550	0.659	0.598	0.666	0.362	0.575	0.461	0.515	0.702	0.533	0.457	0.486	0.359
corr(M_i^T,M_i^P)	-0.586	-0.567	-0.503	-0.310	-0.527	-0.280	-0.578	-0.680	-0.593	-0.726	-0.566	-0.781	-0.372	-0.846
corr(M_i,M_j)		0.122		0.374		0.460		0.646		0.585		0.396		0.536
	Panel B: $\alpha = 2$ (minimum variance)													
$\mathbb{E}[M_i]$	0.983	0.973	0.982	0.990	0.982	0.991	0.982	0.980	0.982	0.966	0.982	0.973	0.982	0.956
Std(M_i)	0.555	0.581	0.699	0.630	0.681	0.359	0.608	0.471	0.551	0.728	0.568	0.474	0.512	0.366
Std(M_i^T)	0.120	0.122	0.120	0.061	0.120	0.091	0.120	0.068	0.120	0.107	0.120	0.111	0.120	0.091
Std(M_i^P)	0.637	0.666	0.766	0.651	0.747	0.376	0.684	0.518	0.634	0.816	0.648	0.579	0.549	0.446
corr(M_i^T,M_i^P)	-0.652	-0.719	-0.554	-0.340	-0.564	-0.301	-0.617	-0.716	-0.656	-0.829	-0.638	-0.866	-0.393	-0.886
corr(M_i,M_j)		0.166		0.276		0.492		0.586		0.480		0.451		0.503

US and foreign SDFs is on average much lower than in the integrated market setting and much more volatile across currency pairs: for $\alpha = 0$, the largest correlation is only 65% for the USD/EUR pair, while the smallest one is as low as 12% for the USD/GBP pair. This evidence reflects more dissimilar domestic and foreign SDFs linked to economically relevant asset market view deviations across currencies.

2.3.2 Exchange Rate Volatility and Wedges

Intuitively, the presence of more pronounced asset market view deviations in segmented markets suggests a larger exchange rate wedge variability. Table 6 reports summary wedge statistics that support this intuition. Overall, we obtain wedges that are similarly volatile as minimum dispersion SDFs, with non-trivial higher moments that reflect the non-normality of exchange rate returns in a way that depends on the choice of minimum dispersion SDFs in incomplete markets.[27]

Because of these deviations from the asset market view, the wedge now displays a non-trivial cyclicality with respect to the minimum dispersion SDFs. This is depicted in Table 7, where we observe a pro-cyclicality that is explained by a pronounced positive (negative) wedge correlation with the permanent components of domestic (foreign) SDFs. The co-movement with the transient component is instead typically weaker and of opposite sign.

[27] Especially for UK, CH and AU markets, we obtain wedges with fatter tails and opposite signs for skewness under parameter choices $\alpha = 2$ and $\alpha = 0$, respectively. This last feature is a consequence of the fact that the minimum variance SDF does not capture the higher moment features linked to extreme exchange rate movements as strongly as minimum entropy SDFs.

Table 6. Wedge Summary Statistics (Segmented Markets)

The table reports the annualized sample mean, standard deviation, skewness and kurtosis of the wedge in equation (3), i.e. $\eta_{t+1} = \log\left(\frac{M_{d,t+1}(\alpha)X_{t+1}}{M_{f,t+1}(\alpha)}\right)$, for dispersion measures $\alpha = 0, 2$. The SDFs account for the fact that domestic investors can trade internationally only the short-term risk-free bond. We use monthly data from January 1975 to December 2015, except for New Zealand, for which the data starts from January 1988.

	$\alpha = 0$ **(minimum entropy)**				$\alpha = 2$ **(minimum variance)**			
	$\mathbb{E}[\eta]$	Std(η)	Sk(η)	K(η)	$\mathbb{E}[\eta]$	Std(η)	Sk(η)	K(η)
UK	0.003	0.636	-0.646	13.55	0.042	0.814	1.074	9.239
CH	-0.006	0.682	-0.367	6.270	-0.021	0.826	-0.019	3.724
JP	-0.123	0.545	1.446	8.938	-0.149	0.612	-0.259	5.417
EU	-0.048	0.439	0.265	4.026	-0.059	0.517	-0.554	5.011
AU	0.104	0.581	-0.181	5.573	0.129	0.716	1.051	6.714
CA	-0.036	0.490	0.148	9.963	-0.040	0.561	0.305	5.082
NZ	-0.020	0.413	0.362	4.556	-0.029	0.442	0.178	3.834

Table 7. Correlation Between Wedge and SDFs (Segmented Markets)

This table reports the correlation between the wedge η, the (log) domestic and foreign minimum entropy SDFs ($\alpha = 0$), as well as the log permanent and transient components of minimum entropy SDFs. Log SDFs are denoted by $m_i := \log M_i$ and log SDF components by $m_i^U := \log M_i^U$ ($i = d, f$ and $U = T, P$). We use monthly data from January 1975 to December 2015, except for New Zealand, for which the data starts from January 1988. Standard errors (SE) are computed using a circular block bootstrap of size 10 with $10,000$ simulations and reported in square brackets. *** denotes significance at the 1% level.

	corr(η, m_i)	SE	corr(η, m_i^P)	SE	corr(η, m_i^T)	SE
US	0.658^{***}	[0.039]	0.651^{***}	[0.039]	-0.356^{***}	[0.049]
UK	-0.617^{***}	[0.052]	-0.602^{***}	[0.057]	0.338^{***}	[0.053]
US	0.541^{***}	[0.026]	0.569^{***}	[0.029]	-0.431^{***}	[0.038]
CH	-0.594^{***}	[0.051]	-0.585^{***}	[0.053]	0.077	[0.054]
US	0.728^{***}	[0.039]	0.759^{***}	[0.042]	-0.532^{***}	[0.058]
JP	-0.201^{***}	[0.054]	-0.200^{***}	[0.056]	-0.029	[0.061]
US	0.552^{***}	[0.047]	0.546^{***}	[0.050]	-0.273^{***}	[0.058]
EU	-0.296^{***}	[0.084]	-0.324^{***}	[0.909]	0.402^{***}	[0.052]
US	0.257^{***}	[0.062]	0.204^{***}	[0.065]	0.087	[0.057]
AU	-0.685^{***}	[0.049]	-0.714^{***}	[0.044]	0.756^{***}	[0.030]
US	0.606^{***}	[0.077]	0.605^{***}	[0.076]	-0.342^{***}	[0.055]
CA	-0.426^{***}	[0.120]	-0.470^{***}	[0.111]	0.565^{***}	[0.069]
US	0.523^{***}	[0.031]	0.441^{***}	[0.040]	0.278^{***}	[0.039]
NZ	-0.465^{***}	[0.075]	-0.508^{***}	[0.071]	0.606^{***}	[0.057]

In summary, the economy with internationally traded risk-free bonds alone can incorporate both the forward premium anomaly and the low exchange rate volatility, by means of a volatile exchange rate wedge and low correlations between domestic and foreign SDFs.

2.3.3 Backus and Smith (1993) Puzzle

In line with the evidence in the last section, SDFs incorporating martingale components can support the low co-movement of cross-sectional differences in consumption growth and exchange rate returns, i.e., the exchange rate cyclicality properties. Table 8 quantifies these relations by reporting the point estimates of Backus-Smith (1993)-type regressions of log differences in minimum dispersion SDFs and martingale SDF components on real log exchange rate returns.[28]

Consistent with the findings in Lustig and Verdelhan (2016), the population point estimate from these regressions for the overall SDFs is one also under a deviation from the market view, whenever risk-free returns are traded internationally. Given the large fraction of permanent SDF variability in Table 5, similar implications have to hold also for regressions using the persistent SDF components in a setting with internationally traded risk-free bonds alone.

Table 8 shows that indeed all point estimates in the Backus-Smith (1993)-type regressions are significantly different from zero and never significantly different from the target value of one.

Table 8. Backus and Smith (1993)-Type Regressions (Segmented Markets)

This table reports the point estimates of a linear regression of the log difference between foreign and domestic SDFs on the log real exchange rate return: $m_{f,t+1} - m_{d,t+1} = \delta + \beta\, x_{t+1} + u_{t+1}$, where small-cap letters denote quantities in logs. We additionally report point estimates of a linear regression of the log difference of the permanent component of the SDF on the log real exchange rate return: $m^P_{f,t+1} - m^P_{d,t+1} = \delta^P + \beta^P x_{t+1} + u^P_{t+1}$. We use monthly data from January 1975 to December 2015, except for New Zealand, for which the data starts from January 1988. Standard errors are reported in square brackets. ** and *** highlight significance at the 5% and 1% level, respectively.

	US/UK		**US/CH**		**US/JP**		**US/EU**	
	$\alpha = 0$	$\alpha = 2$	$\alpha = 0$	$\alpha = 2$	$\alpha = 0$	$\alpha = 2$	$\alpha = 0$	$\alpha = 2$
β	0.801^{***}	1.0683^{***}	0.709^{***}	1.059^{***}	0.937^{***}	1.114^{***}	0.935^{***}	0.994^{***}
	[0.274]	[0.352]	[0.253]	[0.307]	[0.229]	[0.245]	[0.184]	[0.217]
β^P	0.886^{***}	1.153^{***}	0.660^{**}	1.009^{***}	1.019^{***}	1.197^{***}	0.892^{***}	0.949^{***}
	[0.316]	[0.391]	[0.276]	[0.329]	[0.249]	[0.276]	[0.213]	[0.245]

	US/AU		**US/CA**		**US/NZ**	
	$\alpha = 0$	$\alpha = 2$	$\alpha = 0$	$\alpha = 2$	$\alpha = 0$	$\alpha = 2$
β	0.981^{***}	1.087^{***}	0.974^{***}	1.062^{***}	0.875^{***}	1.045^{***}
	[0.231]	[0.284]	[0.312]	[0.357]	[0.186]	[0.199]
β^P	0.986^{***}	1.093^{***}	1.002^{***}	1.089^{**}	0.881^{***}	1.051^{***}
	[0.262]	[0.315]	[0.383]	[0.429]	[0.202]	[0.215]

[28] We do not report the regression of log differences in transitory SDF components since estimates remain the same, regardless of the degree of market segmentation and the dispersion measure used.

In summary, the market setting with internationally traded risk-free bonds alone is consistent with the three exchange rate puzzles and with large deviations from the asset market view. In this setup, martingale SDF components across countries are volatile and only weakly correlated, while differences in transient SDF components are disconnected from exchange rate variations.

2.4 Unspanned Exchange Rate Risk Component

Recall from Corollary 1 that the stochastic wedge in incomplete markets measures the amount of unspanned exchange rate risks. While the asset market view holds in integrated markets for minimum entropy SDFs, the latter cannot be traded using only basic assets. The minimum variance SDF, however, can be traded. In the following, we want to quantify the amount of unspanned exchange rate risk both in integrated and in segmented long-term bond and equity markets.

Figure 1 depicts a scatter plot of the relative unspanned exchange rate risk and the level of minimum entropy SDFs in integrated (Panel A) and segmented (Panel B) markets. Interestingly, we notice a strong U-shaped relation in integrated markets, implying that the unspanned exchange rate risk is larger both when SDFs are high and low, i.e. in bad and in good times. Intuitively, high SDF values can be linked to specific market events, such as the October 1987 crash. However, in segmented markets, a less pronounced U-shaped relationship between unspanned risks and the level of SDFs emerges: Here, unspanned risks are particularly large during bad times, i.e., when the marginal utility of wealth is high.

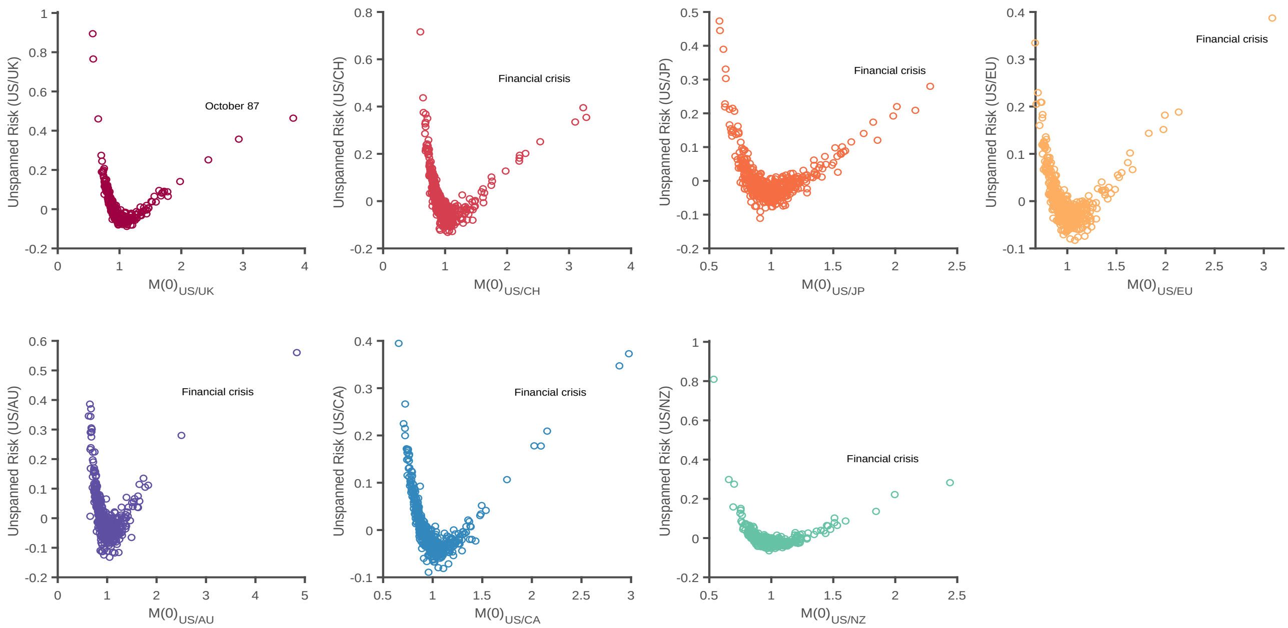
To uncover the time-series behavior of unspanned risks, we plot in Figure 2 the average unspanned exchange rate risk across the different currency pairs vis-à-vis the USD, both in integrated and segmented markets. There are two noteworthy observations. First, substantial spikes occur during major financial market events. Second, while in integrated markets the largest spikes are positive, in segmented markets they tend to be both negative and positive. This evidence is confirmed in Figure 3, where we plot the empirical distribution of the average US unspanned risks in integrated (left panel) and segmented (right panel) markets. Indeed, the distribution of unspanned risks in integrated markets is clearly more positively skewed. This positive skewness reflects the occurrences in which the minimum entropy SDFs substantially exceed the minimum variance SDFs, indicating tail risk in international asset returns.

2.5 International Long-Term Bond and Equity Risk Premia

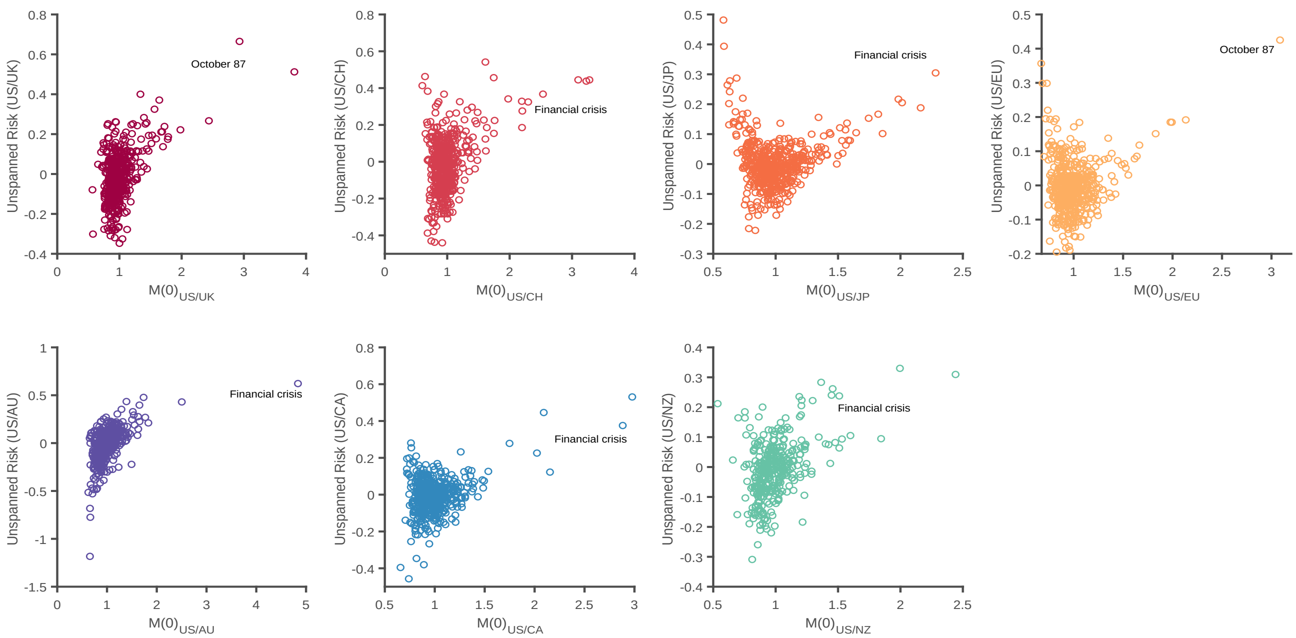
The fact that SDFs are systematically different in integrated versus segmented markets can be intuitively understood by analyzing the distinct set of assets assumed to be traded internationally in each setting. While both frameworks lead to potential "resolutions" of the three exchange rate puzzles, their different degree of segmentation implies variations in international asset prices.

Figure 4 reports the cross-sections of (real) international long-term bond, equity, and currency risk premia in the data, together with the risk premia implied by minimum entropy SDFs for the economy with internationally traded risk-free bonds alone. The latter setting is not constrained to price the cross-sections of international long-term bond and equity returns.

Figure 1. Unspanned exchange rate risk



(a) Panel A: Integrated markets



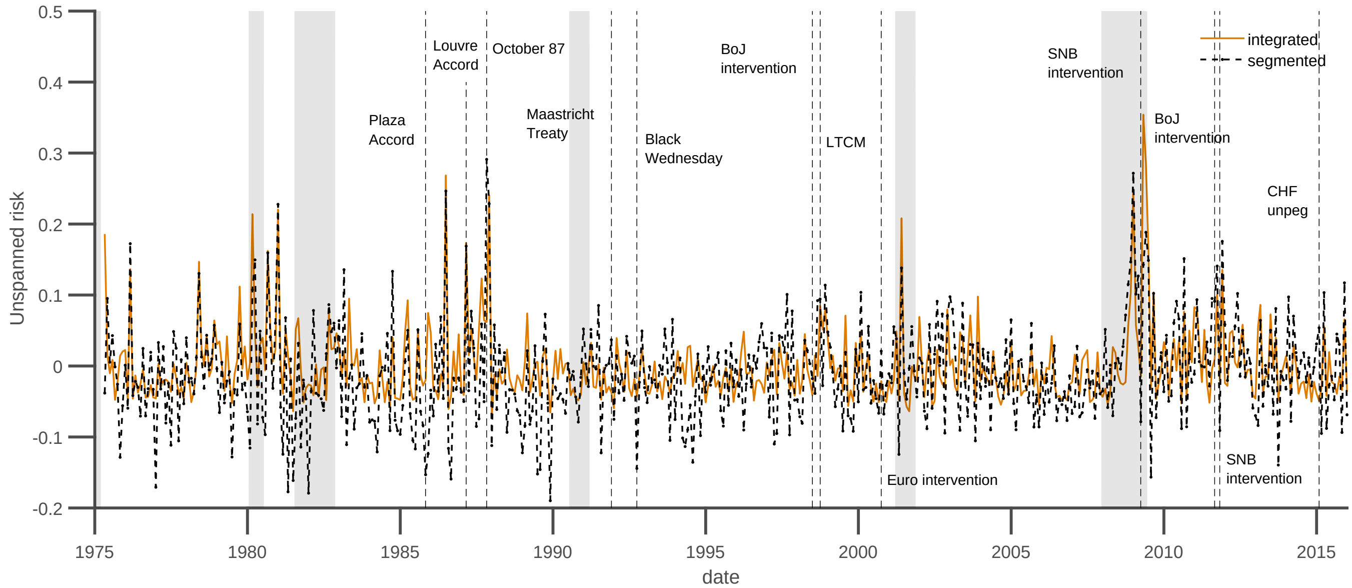
(b) Panel B: Segmented markets

The figure plots the unspanned exchange rate risk component, computed as $(M_d^*(0) - M_d^*(2))/M_d^*(0)$ versus the corresponding minimum entropy US SDFs, both in integrated markets (upper panel) and segmented markets (lower panel). Data is monthly and runs from January 1975 to December 2015.

In our sample, international bond risk premia in USD are monotonically decreasing in the average interest rate differential.[29] While the bond risk premia implied by minimum entropy SDFs are similarly monotonic, they also systematically overstate the actual risk premia, especially

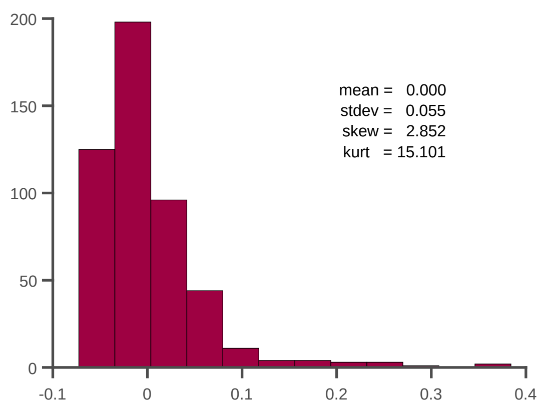
[29] With the exception of the New Zealand dollar, however this arises as the sample period is shorter in this case.

Figure 2. Time-series of unspanned exchange rate risk

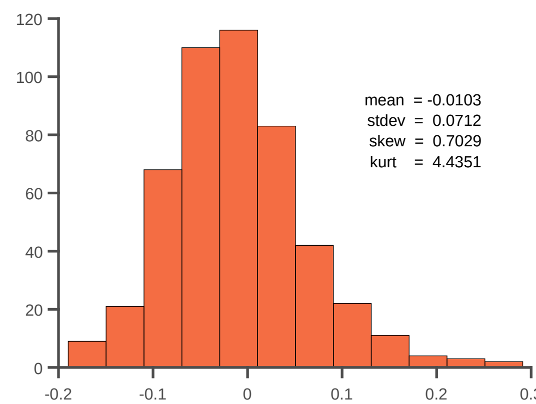


The figure plots the average unspanned component of all currency pairs vis-à-vis the USD, both in segmented and integrated markets. Data is monthly and runs from January 1975 to December 2015. Gray shaded areas highlight recessions as defined by the NBER.

Figure 3. Distribution of unspanned exchange rate risk



(a) Panel A: Integrated markets

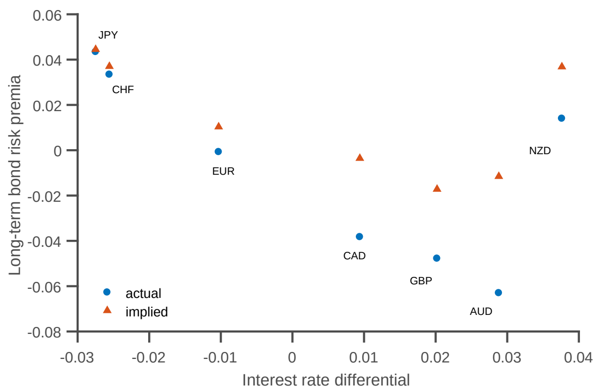


(b) Panel B: Segmented markets

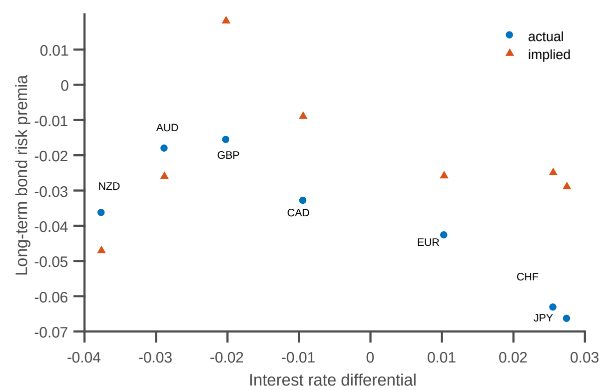
Panel A plots the distribution of the average unspanned US FX risk in integrated markets. Panel B plots the distribution of the average unspanned US FX risk in segmented markets. Data is monthly and runs from January 1975 to December 2015.

for investment currencies. Analogously, the foreign risk premia of US bonds are systematically overstating those in the data, particularly for funding currencies. Interestingly, the minimum entropy bond risk premia in Figure 4 follow to a good extent the pattern of minimum entropy currency risk premia reported in the bottom panels of Figure 4, which by construction exactly match the one observed in the data. Nevertheless, the actual long-term bond risk premia in Panel A of Figure 4

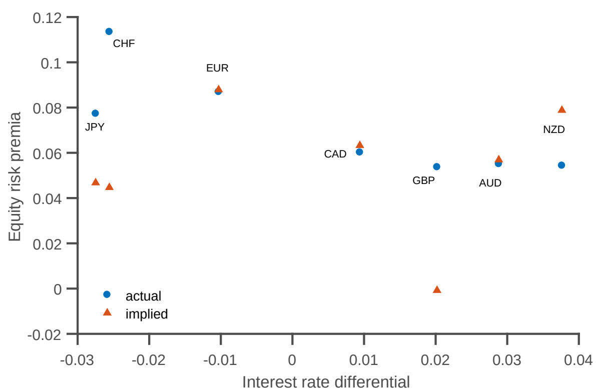
Figure 4. International long-term bond, equity risk and currency premia ($\alpha = 0$)



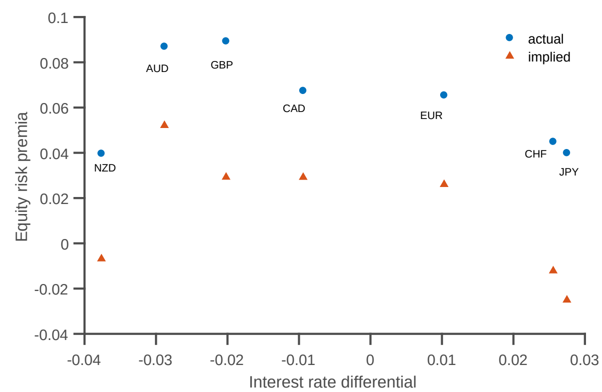
(a) Panel A: Domestic investor



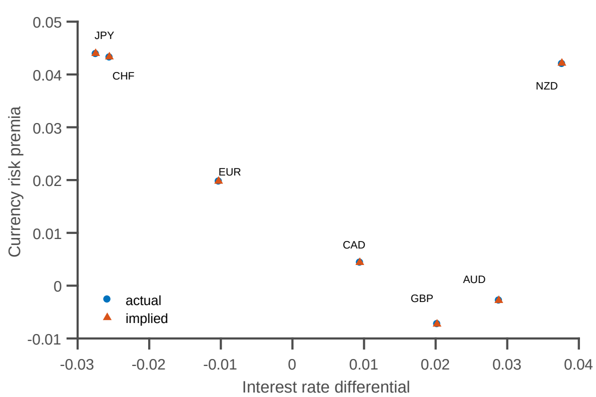
(b) Panel B: Foreign investor



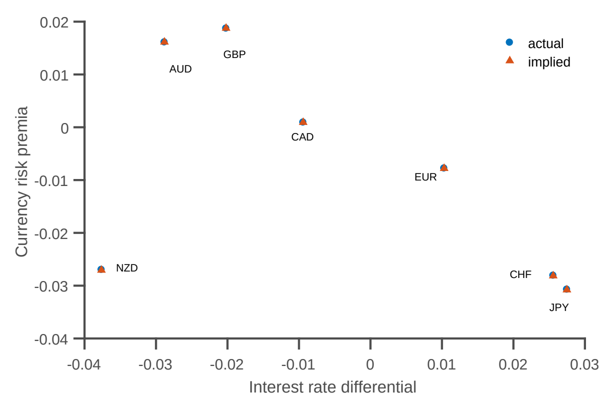
(c) Panel C: Domestic investor



(d) Panel D: Foreign investor



(e) Panel E: Domestic investor



(f) Panel F: Foreign investor

The figure plots for $i = d, f$ international risk premia against the average interest rate differential, computed as the difference between foreign and domestic nominal one-month LIBOR rates. The top panels plot the long-term bond risk premia $E[\tilde{R}_{i\infty,t+1}] - E[R_{i0,t+1}]$ and the risk premia $-\text{cov}(M_{i,t+1}/E[M_{i,t+1}], \tilde{R}_{i\infty,t+1} - R_{i0,t+1})$. The middle panels plot the observed international equity premium $E[\tilde{R}_{i1,t+1}] - E[R_{i0,t+1}]$ and the equity risk premia $-\text{cov}(M_{i,t+1}/E[M_{i,t+1}], \tilde{R}_{i1,t+1} - R_{i0,t+1})$. The bottom panels plot the observed exchange rate risk premium $\mathbb{E}[\tilde{R}_{i0,t+1}] - \mathbb{E}[R_{i0,t+1}]$ and the risk premium $-\text{cov}(M_{i,t+1}/\mathbb{E}[M_{i,t+1}], \tilde{R}_{i0,t+1} - R_{i0,t+1})$. Panels A, C, and E report risk premia for domestic investors; Panel B, D, and F for foreign investors. Data is monthly and runs from January 1975 to December 2015, except for New Zealand, for which the sample starts in January 1988.

decrease more as the interest rate differential increases. Finally, the economy with internationally traded risk-free bonds alone implies international average equity premia that are underestimated by minimum entropy risk premia, with large biases arising especially for CH, JP, UK and US (Figure 4, middle panel).

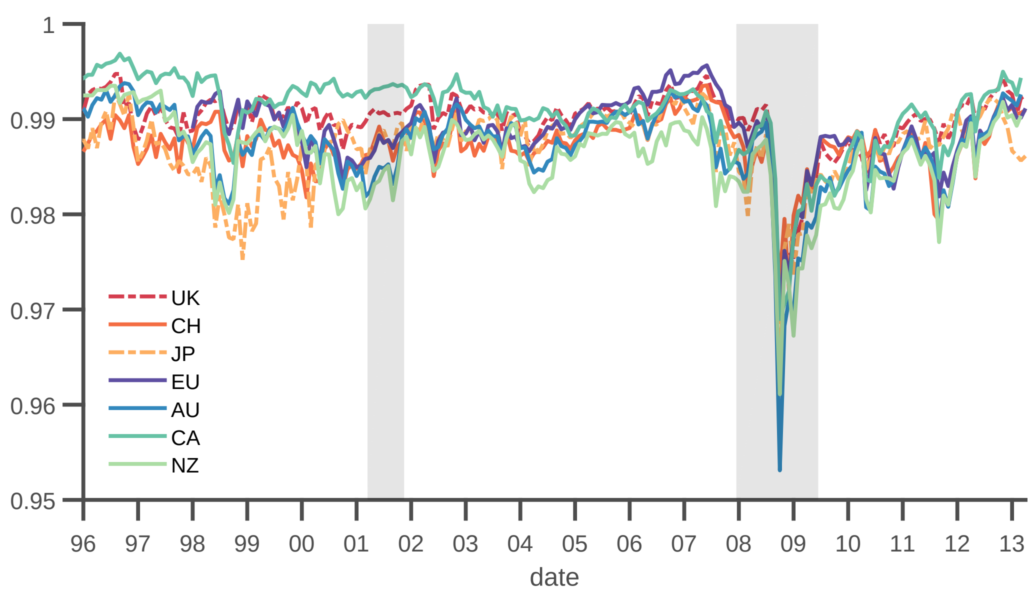
In summary, these differences in international asset risk premia in the two market settings are related to the significantly distinct dispersion and co-movement properties of international SDFs arising in the presence of deviations from the asset market view.

2.6 SDF Similarity

The significantly lower correlations between international SDFs in segmented, compared to integrated, markets will also impact SDF similarity. To gauge SDF similarity in the two market settings, we implement equation (13) using currency options.

Figure 5 plots the time-series of option-implied SDF similarity indices $\overline{S}(M_d, M_f)$, using in all cases the USD as the domestic currency, while Table 9 reports similarity summary statistics.[30]

Figure 5. Option-implied SDF similarity



The figure plots option-implied SDF similarity measures from equation (11). Data is monthly and starts in January 1996 and ends in December 2013. AU = Australia, NZ = New Zealand, JP = Japan, CH = Switzerland, UK = United Kingdom, CA = Canada, EU = Eurozone. The US is always the domestic country. Gray shaded areas highlight recessions as defined by the NBER.

We find that option-implied similarity indices across currencies feature a large degree of co-movement and are on average near to their upper bound of one. Moreover, they exhibit interesting time-varying patterns, with a tendency to simultaneously decrease in periods of global financial market turmoil. The decrease in similarity during periods of crisis is a natural consequence of the progressively more expensive at-the-money put exchange rate options; see again Proposition 3. These

[30] To take unconditional expectations as in equation (13), we can either average the values of $\overline{S}(M_d, M_f)$ directly or we can take averages for each component in the expression. Both yield very similar numbers.

Table 9. SDF Similarity Summary Statistics

Panel A provides descriptive statistics for SDF similarity measures computed from equation (10). Data starts in April 1993 and ends in April 2013. Panel B reports the average for the SDF similarity in integrated (I) and segmented (II) markets. Minimum entropy and minimum variance are obtained for $\alpha = 0$ and $\alpha = 2$, respectively. AU = Australia, NZ = New Zealand, JP = Japan, CH = Switzerland, UK = United Kingdom, CA = Canada, EU = Eurozone. The US is always the domestic country.

		UK	**CH**	**JP**	**EU**	**AU**	**CA**	**NZ**
	Panel A : Option-Implied SDF Similarity Index							
	Mean	0.990	0.987	0.987	0.989	0.987	0.991	0.987
	Std	0.003	0.003	0.004	0.004	0.005	0.004	0.005
	Skewness	-2.624	-1.412	-1.753	-1.501	-2.810	-2.198	-1.388
	Kurtosis	13.85	7.449	8.982	8.143	17.39	10.21	7.321
	Panel B : Nominal Minimum Dispersion SDF Similarity Index							
	I : Integrated Markets							
$\alpha = 0$	$S(M_d, M_f)$	0.991	0.987	0.989	0.988	0.987	0.991	0.987
	$\bar{S}(M_d, M_f)$	0.991	0.987	0.989	0.988	0.987	0.991	0.987
$\alpha = 2$	$S(M_d, M_f)$	0.990	0.986	0.989	0.988	0.986	0.990	0.987
	$\bar{S}(M_d, M_f)$	0.991	0.987	0.988	0.987	0.986	0.991	0.986
	II : Segmented Markets							
$\alpha = 0$	$S(M_d, M_f)$	0.952	0.928	0.945	0.971	0.961	0.973	0.949
	$\bar{S}(M_d, M_f)$	0.991	0.987	0.988	0.988	0.986	0.991	0.987
$\alpha = 2$	$S(M_d, M_f)$	0.950	0.921	0.944	0.970	0.959	0.971	0.947
	$\bar{S}(M_d, M_f)$	0.991	0.987	0.988	0.988	0.986	0.991	0.986

options offer protection against a depreciation of the foreign currency relative to the USD. Consistent with the evidence in Figure 5, the average option-implied SDF similarity across countries in Table 9 (Panel A) is about 0.988 and the cross-sectional standard deviation is very low.

We can now compare the degree of option-implied SDF similarity to the similarity induced by model-free minimum entropy SDFs. To this end, recall that option-implied similarity index $\overline{S}(M_d, M_f)$ is identical to SDF-based similarity index $S(M_d, M_f)$ only when the asset market view holds. In order to obtain SDF-based similarity indices corresponding to the nominal option-implied indices, we first convert real minimum dispersion SDFs to nominal SDFs, by deflating real SDFs by the corresponding inflation rate.[31] We report the resulting SDF-based indices $\overline{S}(M_d, M_f)$ and $S(M_d, M_f)$ in Table 9 (Panel B), both for the integrated and the segmented market settings of the previous sections.

We find that across all currencies, SDF-based indices $\overline{S}(M_d, M_f)$ match almost perfectly the corresponding option-implied similarities in the data, independent of the assumed market segmentation settings. This suggests that all minimum dispersion SDFs fit unconditionally quite accurately the put option prices underlying the expression for index $\overline{S}(M_d, M_f)$ in Proposition 4.

Whenever international trading is unrestricted, the SDF similarity index $S(M_d, M_f)$ closely follows

[31] In the data, we find that SDF similarities computed from real and nominal SDFs are virtually identical.

index $\overline{S}(M_d, M_f)$, because of the documented small deviations from the asset market view under integrated markets. In contrast, the SDF similarity index $S(M_d, M_f)$ under internationally traded risk-free bonds alone is lower, about 0.941 on average across countries and dispersion measures. In summary, the unconditional option-implied similarity is reproduced quite well both by the integrated and the segmented market setting, which however imply very different SDF-similarities.

Our minimum dispersion SDFs also allow us to compute indices of international SDF co-dependence, without imposing the validity of the asset market view. We focus for brevity on the co-entropy-based index of Chabi-Yo and Colacito (2017):

$$\rho_{M_f,M_d} = 1 - \frac{L[M_f/M_d]}{L[M_f] + L[M_d]}, \tag{22}$$

where $L[x] \equiv \log(\mathbb{E}[x]) - \mathbb{E}[\log(x)]$ defines the entropy of a positive random variable x. Summary statistics for various minimum dispersion SDFs are reported in Table 10.

Consistent with the previous results, SDF co-entropies are close to being perfect whenever international financial markets are integrated, i.e., when the asset market view holds with respect to minimum entropy SDFs and when deviations from it are small under the remaining minimum dispersion SDFs. In these settings, co-entropies across countries are never less than 95%. A different picture emerges in the segmented markets case. Co-entropies are particularly low for the funding currencies, with values below 33% and 20% for Switzerland and Japan, respectively.

Table 10. International Co-Entropies

The table reports the values of co-entropy as in Chabi-Yo and Colacito (2017), defined as $\rho_{M_f,M_d} = 1 - \frac{L[M_f/M_d]}{L[M_f]+L[M_d]}$, with $L[x] \equiv \log(\mathbb{E}[x]) - \mathbb{E}[\log(x)]$ being the entropy of the positive random variable x. We provide estimates using the minimum dispersion SDFs derived in integrated and segmented markets. Data starts in April 1993 and ends in April 2013. Minimum entropy and variance are obtained for $\alpha = 0$ and $\alpha = 2$, respectively. AU = Australia, NZ = New Zealand, JP = Japan, CH = Switzerland, UK = United Kingdom, CA = Canada, EU = Eurozone. The US is always the domestic country.

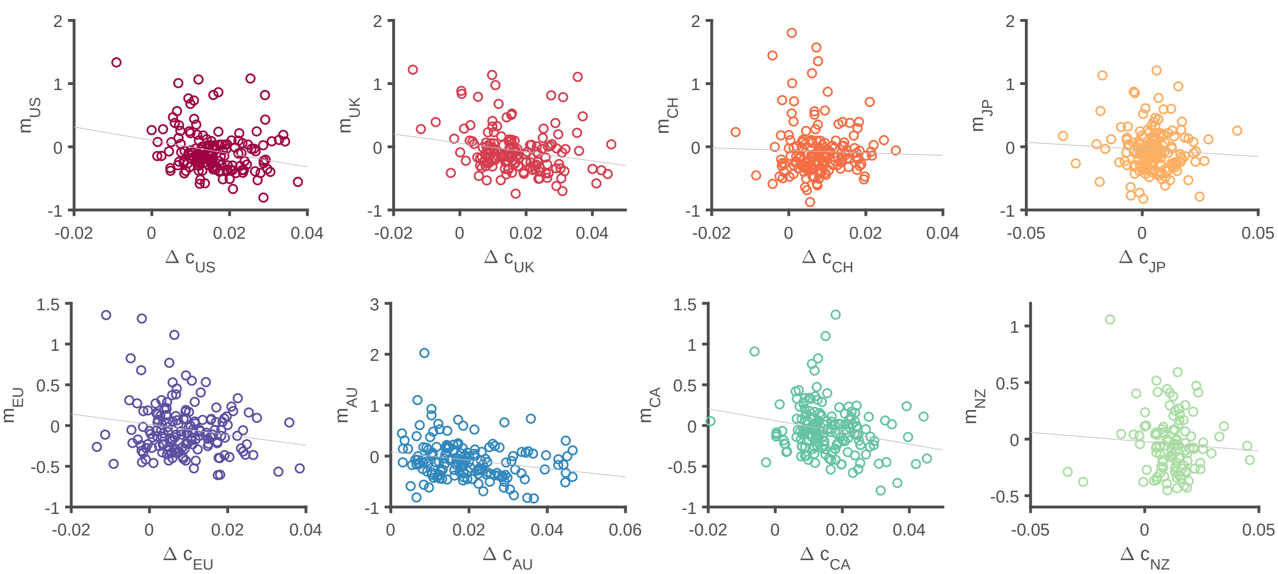
		UK	**CH**	**JP**	**EU**	**AU**	**CA**	**NZ**
$\alpha = 0$	Segmented	0.351	0.415	0.239	0.740	0.748	0.816	0.493
	Integrated	0.983	0.988	0.980	0.962	0.979	0.982	0.982
$\alpha = 2$	Segmented	0.295	0.327	0.197	0.710	0.717	0.794	0.471
	Integrated	0.977	0.982	0.952	0.955	0.964	0.975	0.817

In summary, while minimum dispersion SDFs both in integrated and segmented markets are able to replicate the three exchange rate puzzles, the properties of these SDFs are very different. The large risk compensations which are implied by minimum dispersion SDFs under integrated markets are difficult to square with the optimal risk-return tradeoff attainable by households in international financial markets. In the next section, we rationalize these findings in a model-free framework where sophisticated investors intermediate complex assets in possibly segmented markets.

3 Intermediaries in International Financial Markets

With our model-free SDFs at hand, the next natural step is to relate them to economic fundamentals. As a first illustration, we depict in Figure 6 scatter plots between our minimum entropy SDFs (in logs) and log consumption growth for each of the eight countries in our sample. Not surprisingly, we find the correlation between the two series to be close to zero or mildly negative.

Figure 6. International SDFs and consumption growth



The figure plots (log) minimum entropy SDFs for the eight countries (y-axis) together with (log) consumption growth (x-axis) and the fitted least-square line. Data is quarterly and runs from January 1975 to December 2015, except for New Zealand, for which the sample starts in January 1988.

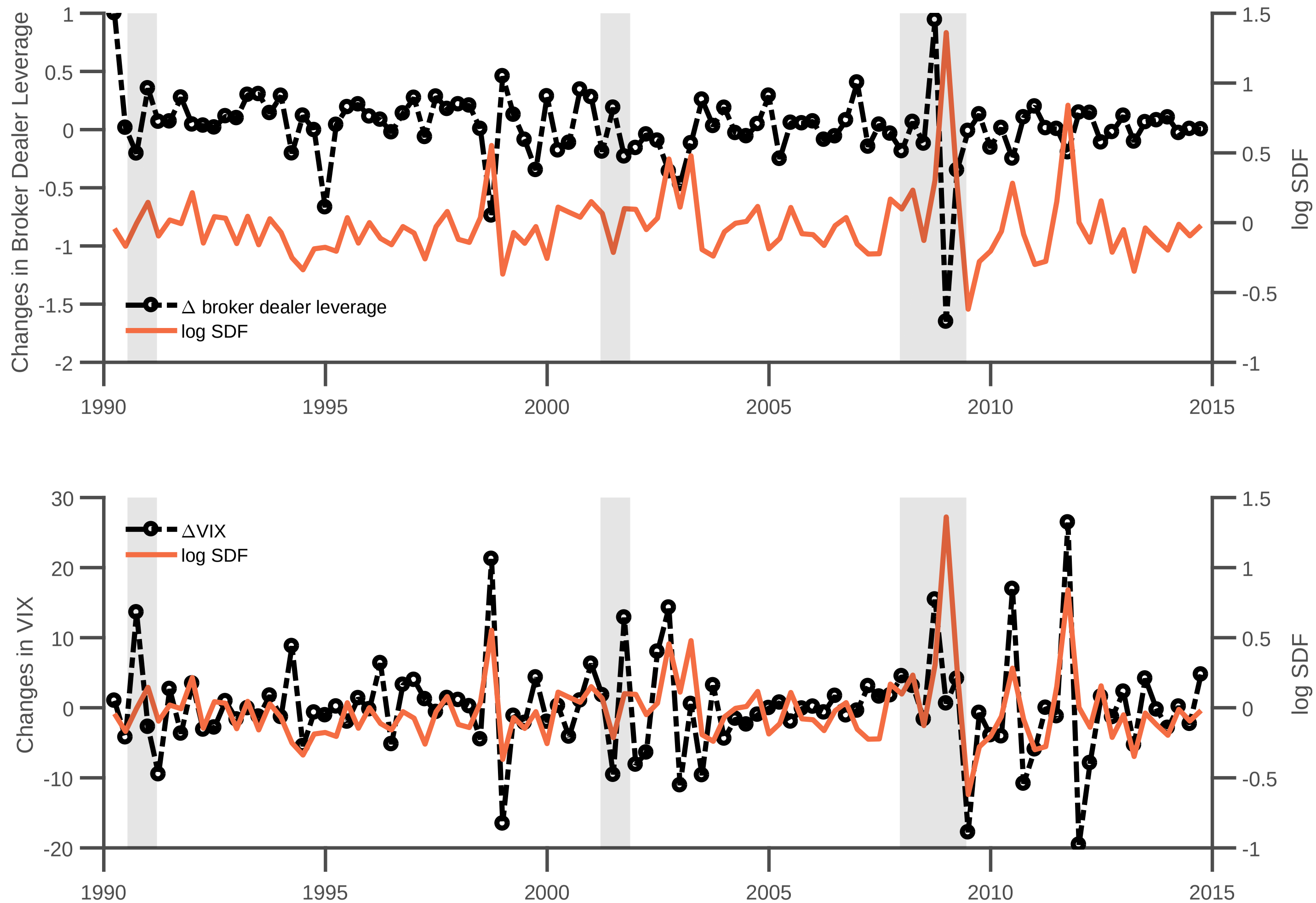
A growing empirical research documents the importance of financial intermediaries for asset prices and the role of these intermediaries seems particularly important in markets that feature complex financial assets, such as credit default swaps, sovereign bonds, and FX (see, e.g., Haddad and Muir (2018)).[32]

In the following, we link model-free SDFs to various measures of financial intermediaries' constraints. To motivate this analysis, we plot in Figure 7 the average US SDF alongside a proxy of intermediary constraints. To this end, we rely on two measures: the broker-dealer leverage of Adrian, Etula, and Muir (2014) and the intermediary equity measure of He, Kelly, and Manela (2017).[33] We find the two series to feature a clear negative co-movement. As a second empirical proxy of intermediaries' risk-bearing capacity, we use the VIX, an implied volatility index from S&P500 options. The lower

[32] He and Krishnamurthy (2018) review several examples of how financial intermediaries matter for the FX market.

[33] The former measure is defined as the ratio between broker dealers' total financial assets and the difference between total financial assets and liabilities available from the Federal Reserve Flow of Funds tables. The latter is defined as the aggregate value of market equity divided by the sum of the aggregate market equity and the aggregate book debt of primary dealers who serve as counterparties of the Federal Reserve Bank of New York. Following Haddad and Muir (2018), we standardize both measures and take the average, to reflect the mean of the risk bearing capacity measures used in the literature.

Figure 7. US Stochastic Discount Factors, broker-dealer leverage and VIX



The figure plots the average log US SDF together with changes in broker dealer leverage (upper panel) and VIX (lower panel). Data is quarterly and runs from January 1990 to December 2015. Gray shaded areas highlight recessions as defined by the NBER.

panel of Figure 7 depicts the average SDF together with changes in VIX.[34] Again, we notice a strong correlation between the two series: They both tend to increase in bad times and are low in good times. We now analyze the relationship between minimum dispersion SDFs and intermediaries' constraints more formally via linear regressions.

Table 11 reports point estimates and $t-$statistics of linear regressions of our model-free SDFs on changes in broker-dealer leverage, VIX and TIV (Treasury yield implied volatility).[35] To save space, we only report regression results for US SDFs and we defer results for foreign SDFs to the online appendix. The upper panel depicts estimated coefficients for broker-dealer leverage. The results strengthen our findings from Figure 7: Estimated coefficients are statistically significant for all currency pairs (except the USDAUD) and negative. This implies that a drop in the risk-bearing capacity of financial intermediaries is associated with high values of SDFs. Similarly, we find that tighter constraints (as measured by VIX and TIV) also imply higher values of SDFs. Interestingly, the constraint proxies are able to explain a significant amount of the variation in the model-free SDFs, with R^2 of almost 50% for some currency pairs.

The multivariate regressions validate our earlier findings: The SDFs which we estimated without imposing any modeling assumptions, other than no-arbitrage, significantly load on measures of intermediaries' constraints. Overall, this evidence is consistent with the intuition that the large dispersions of some model-free SDFs in integrated bond and stock markets may be interpreted as a higher risk compensation available only to a subset of specialized intermediaries, which are subject to various forms of limits-to-arbitrage, captured, for example, by Value-at-Risk constraints.

3.1 A Framework of Financial Intermediation in International Financial Markets

Motivated by our empirical evidence, we now study a simple model-free framework of financial intermediation in international financial markets. We are inspired by the work of Gabaix and Maggiori (2015), where domestic and foreign households can only trade their own local short-term bond and do not have direct access to sovereign bonds. In their model, the role of financiers is to intermediate the market for sovereign bonds and thereby provide access to the FX carry trade.

Similar to these authors, we assume a heterogeneous agents economy with limited participation. Domestic and foreign households can trade linear portfolios of the underlying asset returns $\mathbf{R}_d$ and $\mathbf{R}_f$ in domestic and foreign currency. In addition, financial intermediaries are able to synthetically replicate non-linear payoffs of the underlying returns in each market (such as option payoffs) and sell these assets to households. In doing so, they face an intermediation friction in form of a Value-at-Risk constraint.

3.1.1 Pricing of Unspanned Risks

Consistent with Corollary 1, in integrated markets the minimum entropy SDF $M_i^*(0)$ is a natural payoff for hedging exchange rate risks that are unspanned by linear portfolio returns. Intuitively,

[34] The VIX is used as a proxy of global intermediaries' leverage constraints in the works of Adrian and Shin (2014), Miranda-Agrippino and Rey (2015) or Bruno and Shin (2015), among many others.

[35] These results are based on SDFs from Table 1. The results for SDFs from segmented markets are quantitatively very similar. We report these results in the online appendix.

Table 11. Financial Intermediaries' Constraints and SDFs in Integrated Markets

The table reports estimated coefficients from regressing minimum entropy US SDFs derived in Section 2.2 on changes in broker-dealer leverage and VIX or TIV: $M_{t+1} = \alpha + \beta_k \; \Delta \text{bd leverage}_{t+1} + \beta_v \; \Delta\text{VIX}_{t+1}$ or $\Delta\text{TIV}_{t+1} + \epsilon_{t+1}$, where $\Delta \text{bd leverage}_{t+1}$ are changes in broker-dealer leverage and ΔVIX_{t+1} are changes in the VIX. t-statistics are calculated according to Newey and West (1987) and reported in parenthesis. Data runs from January 1990 to December 2015.

	US/UK	**US/CH**	**US/JP**	**US/EU**	**US/AU**	**US/CA**	**US/NZ**
constant	-0.004	-0.041	-0.009	-0.012	0.002	0.015	-0.029
t-stat	(-0.18)	(-1.66)	(-0.45)	(-0.68)	(0.06)	(0.62)	(-1.26)
bdlev	-0.467	-0.542	-0.322	-0.391	-0.362	-0.407	-0.277
t-stat	(-4.15)	(-4.50)	(-2.91)	(-3.19)	(-1.59)	(-3.92)	(-1.83)
Adj. R^2	20.89%	20.92%	12.13%	19.96%	10.85%	24.11%	10.84%
constant	-0.015	-0.053	-0.016	-0.021	-0.006	0.006	-0.035
t-stat	(-0.51)	(-1.68)	(-0.74)	(-0.97)	(-0.22)	(0.20)	(-1.51)
ΔVIX	0.026	0.029	0.016	0.016	0.013	0.014	0.012
t-stat	(8.72)	(7.21)	(4.11)	(3.90)	(3.50)	(6.93)	(3.11)
Adj. R^2	35.13%	33.31%	17.36%	17.05%	7.21%	14.19%	11.27%
constant	-0.014	-0.052	-0.016	-0.021	-0.006	0.006	-0.035
t-stat	(-0.48)	(-1.67)	(-0.70)	(-0.94)	(-0.20)	(0.21)	(-1.56)
ΔTIV	0.055	0.054	0.024	0.023	0.034	0.019	0.002
t-stat	(3.11)	(2.50)	(1.41)	(1.92)	(2.82)	(1.75)	(0.17)
Adj. R^2	6.00%	3.99%	0.66%	0.57%	1.42%	0.28%	-1.02%
constant	-0.006	-0.043	-0.010	-0.013	0.001	0.014	-0.030
t-stat	(-0.25)	(-1.67)	(-0.50)	(-0.71)	(0.03)	(0.56)	(-1.27)
bdlev	-0.366	-0.428	-0.259	-0.334	-0.316	-0.360	-0.232
t-stat	(-3.90)	(-4.80)	(-2.18)	(-2.38)	(-1.30)	(-2.97)	(-1.42)
ΔVIX	0.023	0.026	0.014	0.013	0.010	0.011	0.010
t-stat	(7.37)	(6.50)	(4.00)	(3.28)	(2.89)	(5.97)	(2.80)
Adj. R^2	47.59%	45.99%	24.79%	31.12%	15.06%	32.43%	18.44%
constant	-0.002	-0.038	-0.008	-0.011	0.003	0.016	-0.029
t-stat	(-0.09)	(-1.58)	(-0.40)	(-0.62)	(0.10)	(0.65)	(-1.26)
bdlev	-0.478	-0.553	-0.327	-0.396	-0.369	-0.411	-0.278
t-stat	(-4.76)	(-5.27)	(-2.97)	(-3.06)	(-1.55)	(-3.80)	(-1.83)
ΔTIV	0.059	0.058	0.027	0.026	0.037	0.023	0.004
t-stat	(3.19)	(2.44)	(1.72)	(1.80)	(1.95)	(1.68)	(0.42)
Adj. R^2	28.19%	26.04%	13.31%	21.24%	12.85%	25.14%	9.97%

this payoff directly loads on the higher moments of international portfolio returns.[36] Therefore, the non-linear transformation of portfolio returns embedded in minimum entropy SDFs can serve as insurance-like assets against adverse shocks in international financial markets.

In order to span the set of prices that households are willing to pay to get access to non-linear entropy payoffs, we follow Cochrane and Saá-Requejo (2000). To this end, we parameterize the

[36]Confirming this intuition, recall from Figure 2 that the amount of unspanned exchange rate risk reflected in minimum entropy SDFs is particularly elevated during bad times.

admissible pricing rules for unspanned risks, set by the financier in domestic and foreign markets, using the following family of SDFs:[37]

$$M_i(\nu) := M_i^*(2) + \nu(M_i^*(0) - M_i^*(2)) \,, \tag{23}$$

where $\nu \in [-1, 1]$ and $M_i^*(0) - M_i^*(2)$, $i = d, f$, is the projection error of the minimum entropy SDF on the space of traded (domestic or foreign) returns. The parameter choice for ν ensures that the SDF in equation (23) has a volatility which is bounded from below by the volatility of the minimum variance SDF and from above by the volatility of the minimum entropy SDF.[38] In particular, setting $\nu = 0$ yields the minimum variance SDF, whereas $\nu = 1$ produces the minimum entropy SDF. Therefore, the maximal Sharpe ratio under SDFs (23) is constrained in a way that it reflects the maximum reward-for-risk attainable by a log utility investor in integrated markets.[39]

Under pricing rule (23), the $\nu-$dependent price of the entropy payoff is:

$$P_i(\nu) := \mathbb{E}[M_i(\nu)M_i^*(0)] = \mathbb{E}\left[(M_i^*(2))^2\right] + \nu\mathbb{E}\left[(M_i^*(0) - M_i^*(2))^2\right] \,, \tag{24}$$

since we can always rewrite the minimum entropy SDF as $M_i^*(0) = M_i^*(2) + (M_i^*(0) - M_i^*(2))$.

3.1.2 Households

Note that since $\mathbb{E}\left[(M_i^*(0))^2\right] = P_i(1) > P_i(\nu)$ for any $\nu \in [-1, 1)$, unsophisticated log utility households have a positive, but downward sloping demand for the entropy payoff. Their demand for the entropy payoff is determined by the following first-order condition:

$$\mathbb{E}[R^{-1}_{\lambda_i^*(\nu)}(R_i(\nu) - R_{i0})] = 0 \,, \tag{25}$$

where the portfolio return including the intermediated entropy return is given by

$$R_{\lambda_i^*(\nu)} := R_{\lambda_i^*} + \mu_i(\nu)(R_i(\nu) - R_{i0}) > 0 \,, \tag{26}$$

with $R_i(\nu) := M_i^*(0)/P_i(\nu)$ and $R_{\lambda_i^*}$ the maximum growth return in the market consisting of return vector $\mathbf{R}_i$. For simplicity, we hold the demand for tradable basic assets constant. In this way, we can isolate the marginal impact of adding the entropy payoff to the set of traded returns, without incorporating additional substitution effects induced by re-balancing the weights for the other assets.[40]

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[37] Cochrane and Saá-Requejo (2000) set up orthogonal decompositions of the unspanned asset payoff and the stochastic discount factor. Their solution for the optimal SDF subject to good deal bounds generates lower and upper bounds for the price of the unspanned payoff; see Proposition 1 and Lemma 1 in their paper.

[38] This is a direct consequence of the fact that the minimum variance SDF is orthogonal to any projection error on the space of traded returns.

[39] In this sense, we follow the intuition set forth in Cochrane and Saá-Requejo (2000) to preclude "too good to be true" deals through financial intermediation.

[40] However, the general case where households can re-balance all portfolio weights is straightforward and produces virtually unchanged results.

3.1.3 Intermediaries

For simplicity, we assume that financiers trade exclusively in the market for intermediated entropy payoffs and in the short-term bond market. The financial intermediary enters the market with no capital on her own and she takes positions of $q_0(\nu)/\mathbb{E}[M_i^*(0)]$ bonds and $-q_0(\nu)/P_i(v)$ units of the entropy payoff. Moreover, she also faces an intermediation friction in form of a Value-at-Risk constraint. In each period, after taking positions, but before shocks are realized, the financier can divert a portion $\Gamma(\nu)q_0(\nu)$ of the funds she intermediates, where $q_0(\nu) > 0$ and

$$\Gamma(\nu) := \gamma \text{VaR}(R_{i0} - R_i(\nu)), \tag{27}$$

with $\gamma \geq 0$ and $\text{VaR}(R_{i0} - R_i(\nu))$ being the Value-at-Risk of excess return $R_{i0} - R_i(\nu)$. The constrained optimization problem of the financier reads as follows:

$$\max_{q_0} \mathbb{E}[R_{i0} - R_i(\nu)]q_0(\nu) \quad \text{s.t.} \quad \mathbb{E}[R_{i0} - R_i(\nu)]q_0(\nu) \geq \Gamma(\nu)q_0^2(\nu). \tag{28}$$

Since in this case the constraint always binds, we immediately obtain the intermediaries' optimal policy:

$$q_0(\nu) = \frac{\mathbb{E}[R_{i0} - R_i(\nu)]}{\Gamma(\nu)} = \frac{\mathbb{E}[R_{i0} - R_i(\nu)]}{\gamma \left(R_{i0} + \frac{\text{VaR}(-M_i^*(0))}{P_i(\nu)} \right)} . \tag{29}$$

Given that the financier can only sell the entropy payoff to households, the optimal policy $q_0(\nu)$ needs to be non-negative, so that $\mathbb{E}[R_{i0} - R_i(\nu)] \geq 0$, i.e., shorting the unspanned exchange rate risk needs to be profitable on average. Moreover, it follows that the financiers' optimal supply $q_0(\nu)$ is decreasing in the Value-at-Risk of the entropy payoff and increasing in the price $P_i(v)$ of this payoff, giving rise to an upward sloping supply for intermediated entropy payoffs.

3.2 Equilibrium supply and demand for entropy payoffs

By matching the households' demand implied by their optimal return (26) with the financier's supply schedule (29), we can now clear the market for intermediated entropy payoffs. For simplicity, we focus on the demand for entropy payoffs of log-utility households, so that for any fixed level of wealth W_i invested by households in the optimal return $R_{\lambda_i^*}$, the optimal price parameter ν^* is such that:

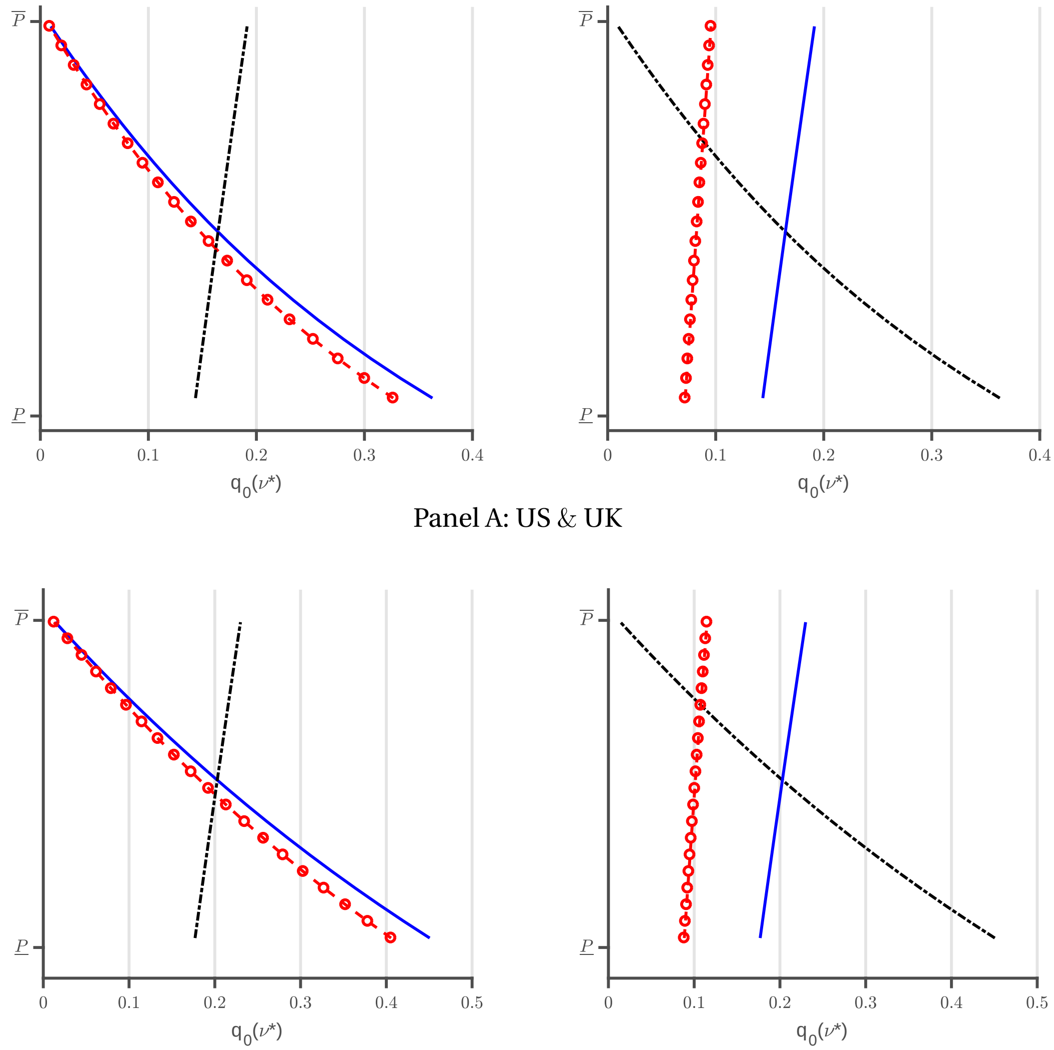
$$W_i \mu_i(\nu^*) = q_0(\nu^*) . \tag{30}$$

Market clearing determines the SDF which prices all assets, including the intermediated entropy payoff:

$$M_i(\nu^*) = M_i^*(2) + \nu^*(M_i^*(0) - M_i^*(2)) . \tag{31}$$

This SDF depends more strongly on the unspanned component $M_i^*(0) - M_i^*(2)$ when, ceteris paribus, the Value-at-Risk of the entropy payoff is higher. Intuitively, when financiers are constrained, their optimal supply schedule shrinks, pushing the equilibrium price (and hence the parameter ν^*) up. On the other hand, a drop in households' wealth W_i entails a lower demand for insurance against adverse shocks in international financial markets, which reduces the unspanned component of SDF $M_i(\nu^*)$.[41]

Figure 8. Matching entropy supply and demand in integrated markets



Panel A: US & UK

Panel B: US & CH

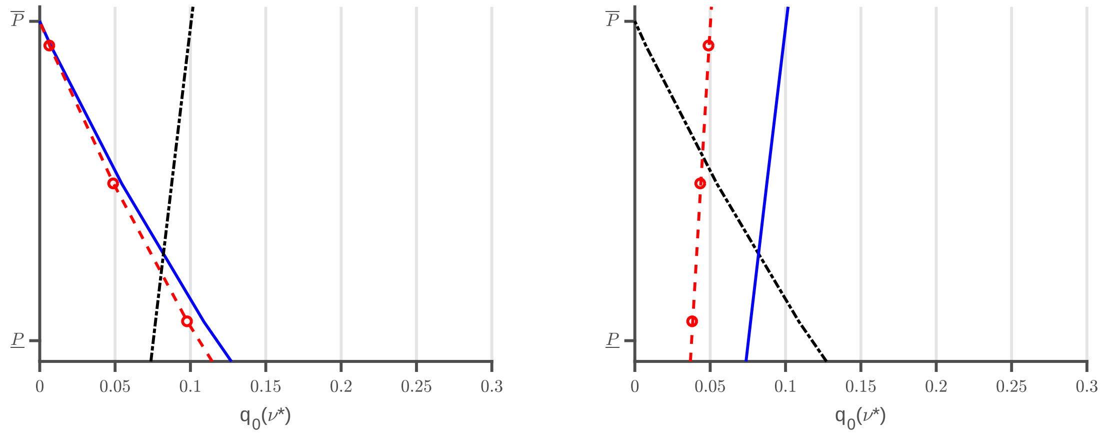
The figure plots the supply and demand curves for the entropy payoff derived in equation (30) for a fixed level of wealth, normalized to 1, under market integration. The left panel additionally plots the shift in demand when households' wealth suffers a 10% decrease (dashed line with markers), whereas the right panel shows the shift in the supply curve when intermediaries' VaR increases by 50% (dashed line with markers). The upper panel illustrates the US and UK pair, whereas the lower panel the US and CH pair.

We can illustrate the effects of trading frictions and wealth shocks on the demand and supply for entropy payoffs in the data. To this end, we estimate the optimal demand and supply curves using the model-free SDFs for different currency pairs. We report the results under integrated markets in Figure 8 for the US and UK pair (upper panel) and for the US and CH pair (lower panel).[42] The left panels show the effect of a 10% drop in households' wealth whereas the right panels plot the effect of

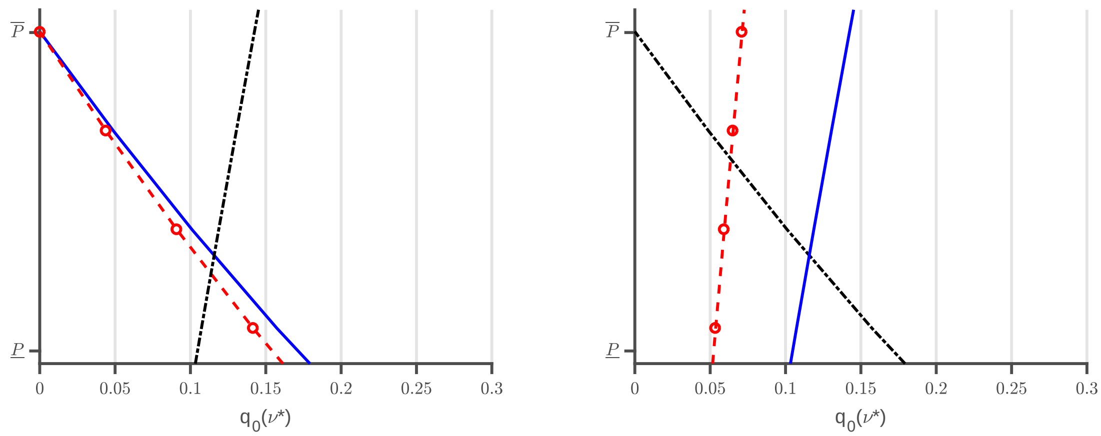
[41]More generally, one could also consider situations in which financiers' wealth depends on the valuation of financial assets other than just the intermediated payoff. In such settings, a drop in the valuation of these assets naturally decreases intermediaries' wealth, which effectively leads to a tighter Value-at-Risk constraint and thus reduces the supply for the intermediated payoff.

[42]Other currency pairs look very similar.

Figure 9. Matching entropy supply and demand in segmented markets



Panel A: US & UK



Panel B: US & CH

The figure plots the supply and demand curves for the entropy payoff derived in equation (30) for a fixed level of wealth, normalized to 1, under market segmentation. The left panel additionally plots the shift in demand when households' wealth suffers a 10% decrease (dashed line with markers), whereas the right panel shows the shift in the supply curve when intermediaries' VaR increases by 50% (dashed line with markers). The upper panel illustrates the US and UK pair, whereas the lower panel the US and CH pair.

an increase in the Value-at-Risk of 50%.[43] Quite intuitively, we find that the demand curve shifts to the left as the level of wealth decreases (left panel), whereas the inelastic supply drops as the Value-at-Risk becomes larger (right panel). Note that the shift in the demand curve is not parallel, because households' demand is positive only for prices of the entropy payoff that are below their valuation, denoted by $\overline{P}$, in the market with no financial intermediaries.[44] In other words, households will price the new entropy payoff using their optimal minimum entropy SDF, $M_i^*(0)$. Analogously, we denote by $\underline{P}$ the valuation of the entropy payoff by a Sharpe ratio maximizer, who uses the minimum variance SDF, $M_i^*(2)$, to price this additional payoff.

[43]Between 2008 and 2010, the median Value-at-Risk has more than doubled for financial intermediaries; see, e.g., Adrian, Stackman, and Vogt (2017). The population-weighted average drop in household wealth was around 9.5% during the Great Recession; see, e.g., Mian and Sufi (2014).

[44]For price levels above $\overline{P}$, households would theoretically like to short the entropy payoff. However, sophisticated intermediaries are the only market participants able to attain this payoff.

Figure 9 depicts the demand and supply curves in segmented markets. A few observations are worth mentioning. First, notice that the supply is as inelastic as the supply in integrated markets. Second, households' valuation $\overline{P}$ of the minimum entropy payoff is different when markets are segmented. Therefore, the demand for the entropy payoff is now zero at a price level different from the corresponding price level under market integration.

In summary, different degrees of market segmentation have distinct implications for the equilibrium quantity of intermediated entropy payoffs: the demand curve are more affected by international market segmentation, while the supply curve remains largely inelastic. This has direct implications on the dependence of SDF (31) on spanned and unspanned risks in incomplete international financial markets.

It is interesting to examine the links between the SDFs in (31) and proxies of financial intermediaries' constraints in the data. Table 12 reports the estimated coefficients from regressing these SDFs on changes in broker-dealer leverage and VIX. We find that the estimates are highly statistically significant and consistent with our previous findings. Indeed, SDFs are higher both when the risk-bearing capacity of intermediaries declines or when they become more financially constrained. For instance, under integrated markets (Panel A), these two variables can explain 40% of the variation in SDFs. We find the results to be similar under segmented markets (Panel B).

Table 12. Financial Intermediaries' Constraints and SDFs

The table reports estimated coefficients from regressing the SDFs derived in equation (31) on changes in broker-dealer leverage and changes in the VIX: $M_{t+1} = \alpha + \beta_k\ \Delta\text{bd leverage}_{t+1} + \beta_v\ \Delta\text{VIX}_{t+1} + \epsilon_{t+1}$. t-statistics are calculated according to Newey and West (1987) and reported in parenthesis. Data runs from January 1990 to December 2015.

Panel A: Integrated markets				
	domestic		foreign	
	US/UK	US/CH	UK	CH
constant	-0.002	-0.041	0.000	-0.036
t-stat	(-0.07)	(-1.60)	(0.01)	(-1.43)
bdlev	-0.364	-0.441	-0.344	-0.466
t-stat	(-4.43)	(-5.53)	(-4.17)	(-5.40)
ΔVIX	0.551	0.604	0.560	0.623
t-stat	(4.40)	(4.29)	(4.73)	(5.05)
Adj. R^2	39.67%	39.66%	39.25%	43.18%
Panel B: Segmented markets				
	domestic		foreign	
	US/UK	US/CH	UK	CH
constant	-0.003	-0.020	0.009	-0.038
t-stat	(-0.13)	(-0.87)	(0.76)	(-2.36)
bdlev	-0.275	-0.248	-0.123	-0.236
t-stat	(-3.88)	(-3.22)	(-1.91)	(-2.96)
ΔVIX	0.345	0.337	0.397	0.439
t-stat	(4.36)	(2.32)	(4.57)	(5.76)
Adj. R^2	33.64%	22.73%	31.24%	31.17%

Table 13. Financial Intermediaries' Constraints and Unspanned Components

The table reports estimated coefficients from regressing the unspanned component in Equation (31), $M^*_d(0) - M^*_d(2)$, for domestic markets, on changes in broker-dealer leverage and changes in the VIX: $(M^*_d(0) - M^*_d(2)) = \alpha + \beta_k\ \Delta\text{bd leverage}_{t+1} + \beta_v\ \Delta\text{VIX}_{t+1} + \epsilon_{t+1}$. t-statistics are calculated according to Newey and West (1987) and reported in parenthesis. Data runs from January 1990 to December 2015.

	US/UK	US/CH	US/JP	US/EU	US/AU	US/CA	US/NZ
Panel A: Integrated markets							
constant	-0.016	-0.007	-0.008	-0.003	0.011	-0.008	0.002
t-stat	(-1.01)	(-0.29)	(-0.78)	(-0.21)	(0.37)	(-0.63)	(0.12)
bdlev	-0.166	-0.166	-0.148	-0.203	-0.640	-0.270	-0.192
t-stat	(-2.46)	(-2.68)	(-4.38)	(-2.34)	(-2.09)	(-3.13)	(-2.13)
ΔVIX	-0.056	0.093	-0.053	-0.038	-0.232	-0.040	-0.127
t-stat	(-1.40)	(0.73)	(-2.08)	(-1.01)	(-3.57)	(-1.38)	(-3.16)
Adj. R^2	16.89%	7.35%	21.83%	26.56%	35.06%	38.20%	23.95%
Panel B: Segmented markets							
	US/UK	US/CH	US/JP	US/EU	US/AU	US/CA	US/NZ
constant	0.004	-0.007	-0.001	0.000	0.041	0.014	-0.002
t-stat	(0.22)	(-0.26)	(-0.05)	(0.01)	(1.26)	(0.80)	(-0.14)
bdlev	-0.256	-0.402	-0.196	-0.179	-0.562	-0.224	-0.148
t-stat	(-5.25)	(-5.37)	(-5.54)	(-1.75)	(-1.38)	(-2.08)	(-1.01)
ΔVIX	0.258	0.469	0.023	0.002	-0.172	-0.065	-0.067
t-stat	(3.21)	(4.50)	(0.62)	(0.04)	(-1.59)	(-1.66)	(-1.06)
Adj. R^2	29.02%	33.82%	20.30%	13.00%	16.20%	19.34%	5.44%

Next, we study the dependence of each component of SDFs (31) on proxies of financial intermediary constraints. Since our earlier results show that spanned components are higher both when the risk-bearing capacity of the intermediaries declines or when they become more financially constrained, we focus on the unspanned component instead. Panel A of Tables 13 and 14 reports the results under integrated markets. We find that the domestic and foreign unspanned components are negatively related to broker-dealer leverage, in a statistically significant way. In contrast, the estimated coefficients for the VIX are in most cases insignificant, or they imply unspanned risks that decrease with the tightness of the Value-at-Risk constraint. Panel B of Tables 13 and 14 shows that, under segmented markets, domestic and foreign unspanned components are again decreasing in the broker dealer leverage. However, the loadings on the VIX are consistent with our earlier findings, as they are either insignificant or significantly positive. Overall, the explanatory power of proxies of financial intermediaries' constraints for unspanned risks under segmented markets is on average quite large, with adjusted R^2s up to 42% for foreign markets.

In summary, both the spanned and unpanned components of good-deal bounds SDFs (31) under segmented markets display a strong link to empirical proxies of risk-bearing capacity of financiers.

We conclude this discussion by computing the correlations of the unspanned components in the two market settings. The scatter plots in Figure 10 (left panel) highlight a strong positive relationship between domestic and foreign unspanned components under market integration. Some of the implications of segmentation become apparent in the right panels, where we notice that the

Table 14. Financial Intermediaries' Constraints and Unspanned Components

The table reports estimated coefficients from regressing the unspanned component in Equation (31), $M_f^*(0) - M_f^*(2)$, for foreign markets, on changes in broker-dealer leverage and changes in the VIX: $(M_f^*(0) - M_f^*(2)) = \alpha + \beta_k\ \Delta\text{bd leverage}_{t+1} + \beta_v\ \Delta\text{VIX}_{t+1}\ + \epsilon_{t+1}$. t-statistics are calculated according to Newey and West (1987) and reported in parenthesis. Data runs from January 1990 to December 2015.

Panel A: Integrated markets							
	UK	CH	JP	EU	AU	CA	NZ
constant	-0.022	-0.004	-0.004	-0.002	0.008	-0.010	0.002
t-stat	(-1.42)	(-0.19)	(-0.41)	(-0.15)	(0.29)	(-0.91)	(0.17)
bdlev	-0.118	-0.186	-0.181	-0.212	-0.664	-0.244	-0.137
t-stat	(-2.46)	(-2.68)	(-3.69)	(-2.43)	(-2.09)	(-3.23)	(-2.14)
ΔVIX	-0.051	0.043	-0.054	-0.063	-0.264	-0.035	-0.082
t-stat	(-1.37)	(0.43)	(-2.19)	(-2.09)	(-3.97)	(-1.24)	(-3.29)
Adj. R^2	9.30%	8.95%	27.90%	28.67%	35.34%	36.95%	24.65%
Panel B: Segmented markets							
	UK	CH	JP	EU	AU	CA	NZ
constant	-0.002	0.021	0.016	0.008	0.012	0.016	0.017
t-stat	(-0.10)	(0.93)	(0.69)	(0.44)	(0.41)	(0.64)	(1.08)
bdlev	-0.427	-0.488	-0.486	-0.398	-0.841	-0.423	-0.328
t-stat	(-3.52)	(-3.35)	(-3.71)	(-2.82)	(-2.56)	(-3.63)	(-4.76)
ΔVIX	0.180	0.295	0.215	0.119	-0.108	0.134	0.070
t-stat	(2.98)	(2.12)	(3.47)	(2.89)	(-1.73)	(2.11)	(1.87)
Adj. R^2	41.11%	34.49%	39.59%	33.57%	40.50%	33.14%	31.60%

unspanned components exhibit, with the exception of some outliers, no or even negative correlation.

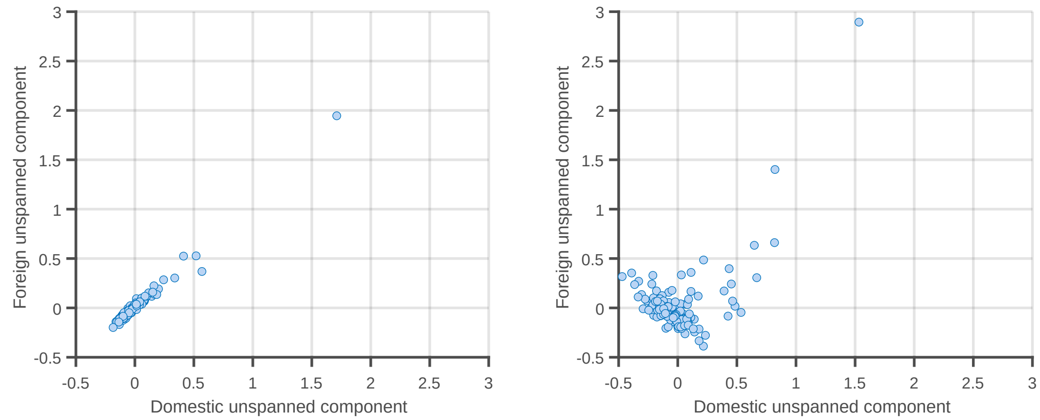
4 Robustness of Long-Run SDF Factorizations

In our empirical analysis, we factorize minimum dispersion SDFs into transitory and permanent components using as a proxy for the unobservable return of infinite maturity bonds the long, but finite-horizon bond return. While this is of course an approximation, note that this SDF decomposition can impact our main analysis only in terms of the exchange rate cyclicality patterns and the corresponding Backus and Smith (1993) regression results. Intuitively, given the dominating role of martingale components in minimum dispersion SDFs, it is unlikely that this approximation error can substantially change our findings. Indeed, model-based evidence from estimated affine term structure models tends to support this intuition.[45] However, to address these concerns in more detail, we follow Christensen (2017) and estimate non-parametrically the solution of the Perron-Frobenius eigenfunction problem that uniquely characterizes the long-run SDF factorization in Hansen and Scheinkman (2009). We defer exhaustive estimation details to the online appendix.

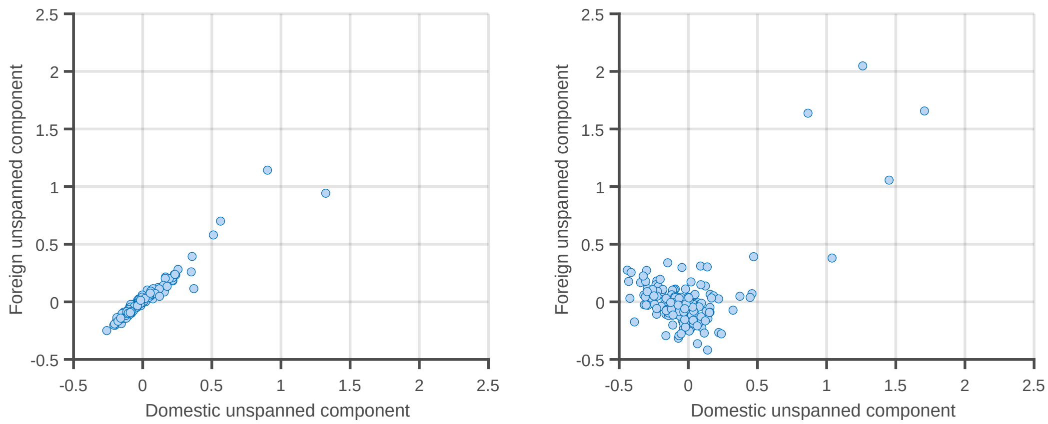
We report in Figure 11 the time-series of the estimated permanent SDF components identified

[45] See again Lustig, Stathopoulos, and Verdelhan (2016), who do not obtain large differences between the yields of a hypothetical infinite maturity bond and a ten-year bond in such settings.

Figure 10. Correlations of international unspanned components



Panel A: US & UK



Panel B: US & CH

The figure plots the correlation between domestic and foreign unspanned components, computed as $M_i^*(0) - M_i^*(2)$, under market integration (left panels) and under market segmentation (right panels). The upper panel illustrates the US and UK pair, whereas the lower panel the US and CH pair. Data runs from January 1990 to December 2015.

with the approach in Section 2.2, together with those estimated with the non-parametric methodology.[46]

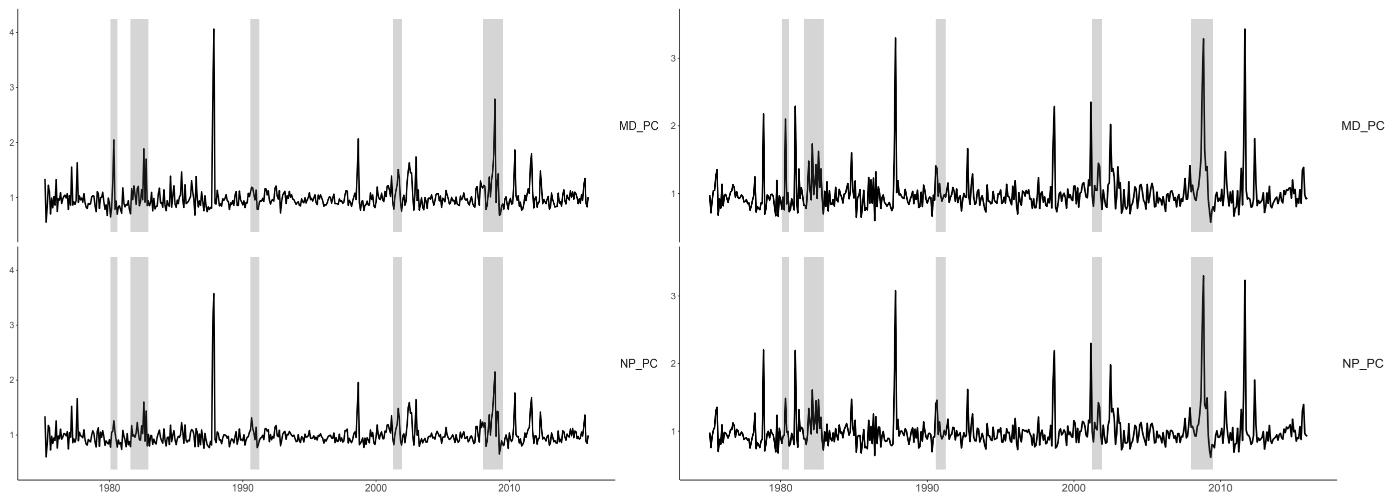
The two series exhibit similar time series properties, with large common spikes during crises and recession periods, and are almost perfectly correlated.[47] While the unconditional dispersion of the two permanent components is similar and generates in both cases the largest fraction of the overall SDF volatility, the non-parametric component has a consistently slightly lower volatility than the overall SDF volatility. This implies a positive risk premium for the infinite maturity bond return resulting from the non-parametric SDF factorization.[48] Given the dominating role of permanent

[46]For the sake of brevity, we only report results for two different currency pairs. The results look virtually the same for all other currency pairs.

[47]Correlations between permanent components under the two approaches are 0.96, 0.98, 0.98, 0.98, 0.98, 0.95 and 0.95 for the USDGBP, USDCHF, USDJPY, USDEUR, USDAUD, USDCAD and USDNZD pairs, respectively.

[48]This finding for the sign of the risk premium of infinite maturity bond returns is consistent with the empirical evidence in Bakshi and Chabi-Yo (2012). The volatilities of the non-parametric permanent SDF components are reported in the online appendix.

Figure 11. Minimum entropy and non-parametric permanent components



(a) USDGBP (b) USDCHF

The figure plots the time-series of the minimum entropy permanent component (upper panel) and the non-parametric estimates of the permanent component (lower panel). The left panel depicts the domestic permanent components for the USDGBP currency pair, whereas the right panel illustrates the domestic permanent components of the USDCHF pair. Data is monthly and runs from January 1975 to December 2015. Gray shaded areas highlight recessions as defined by the NBER.

SDF components under both approaches, these findings imply overall unchanged exchange rate cyclicality patterns and Backus and Smith (1993) regression results, using either of the two proxies for the permanent SDF components.[49]

5 Conclusion

In this paper, we study the asset pricing implications of different degrees of market segmentation in incomplete international financial markets. Using a parsimonious model-free framework, our main contributions can be summarized as follows.

First, we theoretically show that the minimum entropy SDF pair always implies the market view of exchange rates in integrated markets. In contrast, a stochastic wedge arises for any other pair of minimum dispersion SDFs when the spaces of traded domestic and foreign returns are different. Additionally, we infer that the stochastic wedge with respect to the minimum variance SDF pair is always interpretable as a relative measure of the amount of unspanned exchange rate risks in international financial markets.

Second, we estimate minimum dispersion SDFs by positing two necessary ingredients to explain international macro finance puzzles: large permanent SDF components that are correlated across countries and internationally traded risk-free bonds. Under integrated markets, we obtain negligible stochastic wedges and highly volatile SDFs exhibiting almost perfect international correlation. Introducing market segmentation, however, is of crucial importance to lower both international SDF dispersion and their co-movement. Given the strong support for market segmentation in the data

[49]Detailed results are available from the authors upon request.

(see, e.g., Maggiori et al. (2017)), this last evidence offers a promising characterization of the empirical properties of international SDFs.

Third, we document a strong empirical link between model-free SDFs and proxies of financial intermediaries' constraints. This evidence is in line with the intuition that the large reward-for-risk ratios implied by model-free SDFs under integrated markets may reflect a higher risk compensation accessible only by a subset of sophisticated financial intermediaries. We rationalize this evidence with a simple model-free framework of financial intermediation, in which households are limited to trade only linear portfolios of domestic or foreign assets, whereas financially constrained intermediaries are able to synthetically replicate non-linear payoffs of international asset returns. The model-free SDFs derived in this way jointly price intermediated unspanned exchange rate risks and the original returns. We illustrate quantitatively the impact of Value-at-Risk constraints and wealth shocks on the supply and demand curves for intermediated exchange rate risks and on the structure of the corresponding model-free SDFs. We also document a strong link between intermediated exchange rate risks in segmented markets and empirical proxies of financial intermediaries' constraints.

Our findings speak to a growing intermediary asset pricing literature, that emphasizes the important role played by financiers in international financial markets. Limited participation by households and intermediation frictions generate a wedge between households' and intermediaries' valuations. We show that such wedges can be coherently incorporated by model-free international SDFs. In a nutshell, we document that these SDFs: (i) price international asset returns accurately, (ii) are largely unrelated to macroeconomic fundamentals such as consumption growth, and (iii) are explained to a significant degree by proxies of financial intermediaries constraints. Modeling and analyzing in greater detail the differences between intermediaries' and households' consumption-based SDFs using a structural approach is an important and challenging topic for future research.

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Appendix A Proofs and Derivations

Proof of Proposition 1. For $\alpha \neq 1$, consider the following minimum divergence problem:[50]

$$\min_{M_i} \frac{1}{\alpha(\alpha-1)} \log \mathbb{E}[M_i^\alpha],$$

subject to $\mathbb{E}(M_i \mathbf{R}_i) = \mathbf{1}$ and $M_i > 0$. Therefore, in order to determine the minimum dispersion SDF, we can focus on the following Lagrange function:

$$\mathfrak{L}(M_i, \mu_i) = \frac{\mathbb{E}[M_i^\alpha]}{\alpha(\alpha-1)} - \mu_{i0}\mathbb{E}[M_i R_{i0} - 1] - \sum_{k=1}^{K_i} \mu_{ik}\mathbb{E}[M_i (R_{ik} - R_{i0})],$$

with $\mu_i \in \mathbb{R}^{K_i+1}$ the vector of multipliers for the pricing constraints. The first order conditions read:

$$\frac{M_i^\alpha}{\alpha-1} = M_i \left(\mu_{i0} R_{i0} + \sum_{k=1}^{K_i} \mu_{ik} (R_{ik} - R_{i0}) \right) =: M_i \mu_{i0} R_{\lambda_i},$$

where we applied the renormalization $\lambda_i := \mu_{ik}/\mu_{i0}$, $k = 0, \ldots, K_i$. Taking expectations yields

$$\mathbb{E}[M_i^\alpha] = (\alpha - 1)\,\mu_{i0}. \tag{A-1}$$

Using (A-1) and plugging into the first order conditions, it follows

$$M_i = ((\alpha-1)\,\mu_{i0} R_{\lambda_i})^{1/(\alpha-1)}.$$

Therefore $M_i > 0$ if and only if $R_{\lambda_i} > 0$. Hence, the optimal SDF M_i^* and optimal return $R_{\lambda_i^*}$ are such that

$$\log \mathbb{E}[(M_i^*)^\alpha] = \log \mathbb{E}[R_{\lambda_i^*}^{\alpha/(\alpha-1)}]^{1-\alpha}.$$

In summary, we get the following inequality:

$$\frac{\log \mathbb{E}[M_i^\alpha]}{\alpha(\alpha-1)} \geq \frac{\log \mathbb{E}[M_i^{*\alpha}]}{\alpha(\alpha-1)} = -\frac{1}{\alpha} \log \mathbb{E}[R_{\lambda_i^*}^{\alpha/(\alpha-1)}]\,. \tag{A-2}$$

The optimal SDF is given by

$$M_i^* = R_{\lambda_i^*}^{1/(\alpha-1)} / \mathbb{E}[R_{\lambda_i^*}^{\alpha/(\alpha-1)}],$$

with

$$R_{\lambda_i^*} = R_{i0} + \sum_{k=1}^{K_i} \lambda_{ik}^* (R_{ik} - R_{i0}) > 0\,,$$

such that for any $k = 1, \ldots, K_i$

$$\mathbb{E}[R_{\lambda_i^*}^{1/(\alpha-1)} (R_{ik} - R_{i0})] = 0.$$

Equivalently, the bound of the RHS of inequality (A-2) is obtained by solving the following maximization problem:

$$\max_{\lambda_i} -\frac{1}{\alpha} \log \mathbb{E}[R_{\lambda_i}^{\alpha/(\alpha-1)}]$$

[50] The case $\alpha = 1$ can be treated separately in a similar way, but we do not consider it here as it is not necessary for the dispersion measures used in the main text.

such that

$$R_{\lambda_i} = R_{i0} + \sum_{k=1}^{K_i} \lambda_{ik} (R_{ik} - R_{i0}) > 0.$$

Indeed, this maximization problem has for $k = 1, \dots, K_i$ the first order conditions:

$$0 = \frac{\partial \log \mathbb{E}[R_{\lambda_i}^{\alpha/(\alpha-1)}]}{\partial \lambda_i} |_{\lambda_i = \lambda_i^*} = \mathbb{E}\left[R_{\lambda_i^*}^{1/(\alpha-1)} (R_{ik} - R_{i0})\right], \tag{A-3}$$

which are identical to the pricing constraints for λ_i in the solution of the primal SDF optimization problem. This concludes the proof. □

Proof of Proposition 2. Proof of part (a). From equation (6), for $i = d, f$ and $\alpha = 0$, we get the minimum entropy SDF: $M_i^* = R_{\lambda_i^*}^{-1}$, with optimal weights, λ_i^*, that uniquely solve for $k = 1, \dots, K_i$, the first-order conditions of the optimization problem given in (5):

$$\mathbb{E}\left[R_{\lambda_i^*}^{-1} (R_{ik} - R_{i0})\right] = 0. \tag{A-4}$$

Therefore, $\tilde{M}_f^* := R_{\lambda_d^*}^{-1} X = \tilde{R}_{\lambda_f^*}^{-1}$ where $\tilde{R}_{\lambda_f^*} = R_{\lambda_d^*}/X$ is a foreign return, which solves for $k = 1, \dots, K_i$, the following first-order conditions:

$$\mathbb{E}\left[\tilde{R}_{\lambda_f^*}^{-1} \left(\tilde{R}_{fk} - \tilde{R}_{f0}\right)\right] = 0. \tag{A-5}$$

Similarly, $\tilde{M}_d^* := R_{\lambda_f^*}^{-1}/X = \tilde{R}_{\lambda_d^*}^{-1}$ where $\tilde{R}_{\lambda_d^*} = R_{\lambda_f^*} X$ is a foreign return, which solves for $k = 1, \dots, K$, the following first-order conditions:

$$\mathbb{E}\left[\tilde{R}_{\lambda_d^*}^{-1} \left(\tilde{R}_{dk} - \tilde{R}_{d0}\right)\right] = 0. \tag{A-6}$$

As $\tilde{R}_{\lambda_f^*}^{-1}$ solves the moment conditions (A-5), it is the minimum entropy SDF for return vector $\tilde{\mathbf{R}}_f$. Hence, if $R_{\lambda_d^*}^{-1}$ is the minimum entropy SDF for return vector $\mathbf{R}_d$, i.e., it solves the moment condition (A-4), then $\tilde{R}_{\lambda_f^*}^{-1}$ is the minimum entropy SDF for return vector $\tilde{\mathbf{R}}_f$. Symmetric arguments show that $\tilde{M}_d^* = \tilde{R}_{\lambda_d^*}^{-1}$ is the minimum entropy SDF for return vector $\tilde{\mathbf{R}}_d$ whenever $M_f^* = R_{\lambda_f^*}^{-1}$ is the minimum entropy SDF for return vector $\mathbf{R}_f$. This concludes the proof for part (a).

Proof of part (b). Note first that M_f is a foreign SDF for returns $\mathbf{R}_f$ if and only if M_f/X is a domestic SDF for return $\mathbf{R}_f X$. Second, M_f is a foreign minimum dispersion SDF for parameter $\alpha \in \mathbb{R}\backslash\{1\}$ if and only if $M_f^{\alpha-1} \in span(\mathbf{R}_f)$. Third, under the same parameter α, M_f/X is a domestic minimum dispersion SDF for return $\mathbf{R}_f X$ if and only if:

$$(M_f/X)^{\alpha-1} \in span(\mathbf{R}_f X) \left[\iff M_f^{\alpha-1} \in span(\mathbf{R}_f X^{\alpha})\right]. \tag{A-7}$$

As a consequence, generic foreign and domestic SDFs M_f and M_f/X are both minimum dispersion SDFs if and only if the following spanning constraint holds:

$$span(\mathbf{R}_f) \subset span(\mathbf{R}_f X^{\alpha}) \left[\iff span(\mathbf{R}_f X^{-\alpha/2}) \subset span(\mathbf{R}_f X^{\alpha/2})\right]. \tag{A-8}$$

By applying identical arguments as above for domestic and foreign minimum dispersion SDFs M_d and M_dX, it follows:

$$span(\mathbf{R}_d) \subset span(\mathbf{R}_d X^{-\alpha}) \ \left[\iff span(\mathbf{R}_d X^{\alpha/2}) \subset span(\mathbf{R}_d X^{-\alpha/2})\right] . \tag{A-9}$$

Note that these conditions always hold for minimum entropy SDFs ($\alpha = 0$), which always satisfy the market view in integrated markets because of their numéraire invariance shown in part (a). Therefore, we let $\alpha \neq 0$ and consider the following identity, which always holds by definition under integrated markets: $span(\mathbf{R}_d X^{-\alpha/2}) = span(\mathbf{R}_f X^{\alpha/2})$. Hence, under integrated markets, conditions (A-8)–(A-9) are equivalent to following constraints on the spaces of domestic and foreign tradable returns: (i) $span(\mathbf{R}_f X^{-\alpha/2}) \subset span(\mathbf{R}_d X^{-\alpha/2})$ and (ii) $span(\mathbf{R}_d X^{\alpha/2}) \subset span(\mathbf{R}_f X^{\alpha/2})$. However, joint conditions (i) and (ii) are equivalent to the single spanning condition $span(\mathbf{R}_f) = span(\mathbf{R}_d)$, i.e., the requirement of identical domestic and foreign spaces of tradable returns in integrated markets. This concludes the proof. □

Proof of Proposition 3. Let the asset market view hold, i.e., $X = M_f/M_d$. It then follows:

$$\mathbb{E}[\min(M_d, M_f)] = \mathbb{E}[M_d \min(1, X)] = \mathbb{E}[M_d] - \mathbb{E}[M_d(1 - \min(1, X))] .$$

Moreover,

$$\mathbb{E}[M_d(1 - \min(1, X))] = \mathbb{E}[M_d(1 + \max(-1, -X))] = \mathbb{E}[M_d \max(0, 1 - X)] .$$

The second equality in the statement of the proposition follows by symmetry. This concludes the proof. □

Proof of Proposition 4. By definition, we have:

$$S(M_d, M_dX) = \frac{\mathbb{E}[M_d \min(1, X)]}{\min(\mathbb{E}[M_d], \mathbb{E}[M_dX])} . \tag{A-10}$$

Whenever the domestic return $\tilde{R}_{d0} := R_{f0}X$ is priced, it follows:

$$1 = \mathbb{E}_t[M_{d,t+1}\tilde{R}_{d0,t+1}] \iff \frac{1}{R_{f0,t+1}} = \mathbb{E}_t[M_{d,t+1}X_{t+1}] , \tag{A-11}$$

as $R_{f0,t+1}$ is the foreign risk-free rate. Taking unconditional expectations, we obtain $\mathbb{E}[M_dX] = \mathbb{E}[1/R_{f0}] = \mathbb{E}[M_f]$. Therefore,

$$S(M_d, M_dX) = \frac{\mathbb{E}[M_d \min(1, X)]}{\min(\mathbb{E}[M_d], \mathbb{E}[M_dX])} = \frac{\mathbb{E}[M_d] - \mathbb{E}[M_d \max(0, 1 - X)]}{\min(\mathbb{E}[M_d], \mathbb{E}[M_f])} ,$$

with the same arguments as in the proof of Proposition 3. By applying an identical approach with the foreign return $\tilde{R}_{f0} := R_{d0}/X$, we obtain:

$$S(M_f, M_f/X) = \frac{\mathbb{E}[M_f] - \mathbb{E}[M_f \max(0, 1 - (1/X))]}{\min(\mathbb{E}[M_d], \mathbb{E}[M_f])} .$$

This concludes the proof. □

Model-Free International Stochastic Discount Factors – Not for Publication –

This online appendix consists of six sections: Section OA-1 presents summary statistics of our data used. Section OA-2 discusses the properties of model-free SDFs when the Hellinger divergence is used, i.e. $\alpha = 1/2$. In Section OA-3, we discuss SDFs factorizations into transitory and permanent components when using non-parametric estimates. Section OA-4 compares our similarity measure to other measures of similarity in the literature and derives SDF similarity in two commonly used asset pricing models. Section OA-5 discusses the trade-off between SDF correlation and volatility. And finally Section OA-6 presents tables omitted in the main paper.

OA-1 Summary Statistics Data

We provide in Table OA-1 summary statistics for the different time-series. Panel A reports bond market summary statistics. We find that the CHF and the JPY feature low interest rates, in line with the intuition that they act as funding currencies in the carry trade, whereas the remaining currencies can be regarded as investment ones. Cross-sectional differences across countries arise with respect to unconditional long-term bond risk premia. To illustrate, (nominal) long-term risk premia in all countries are negative, but while in Japan and Switzerland they are -0.3% and -1.02%, respectively, in the remaining countries they range between -2.07% (EU) and -6.06% (Australia). The fact that nominal returns on long-term bonds in local currencies are negative has been documented also in Lustig, Stathopoulos, and Verdelhan (2016). There are cross-sectional differences also with respect to unconditional equity premia, especially in the case of Japan relative to all other countries, which exhibits a substantially lower unconditional equity premium of 3.49% per year. New Zealand features the lowest cross-sectional average equity premia, but this is also a consequence of the restricted sample period. Switzerland displays the lowest market volatility with 15.42%, while the Euro-zone has the largest one (20.08%). These numbers imply a Sharpe ratio of 48% for Switzerland, which is close to the one in the US, and a much lower Sharpe ratio for Japan, 19%.[1] The unconditional average returns on exchange rates against the US dollar also display cross-sectional variation. The highest (positive) average return is obtained for the Swiss Franc ($+2.96\%$), while the lowest (negative) average return follows for the Australian dollar (-0.86%). The cross-section of unconditional exchange rate volatilities does not exhibit significant variation, even though funding currencies, i.e., the Swiss franc and the Japanese yen, feature a higher volatility (12.12% and 11.32%, respectively), whereas the lowest is encountered for the Canadian dollar (6.78%). The last Panel reports inflation statistics for the countries in our sample. The highest average inflation rates are observed in New Zealand (5.57%) and in the UK (4.74%), while the lowest ones are those for Japan (1.57%) and Switzerland (1.76%).[2]

[1] Using the whole sample period in the case of New Zealand would yield a Sharpe ratio close to the Japanese one.

[2] Note that the sample includes observations associated with The Great Inflation of the 1970s and early 1980s, also known as stagflation, when markets in general exhibited high inflation rates.

Table OA-1. Data Summary Statistics

The table provides descriptive statistics for nominal domestic returns, exchange rates and CPI inflation for Switzerland, the Euro-zone (Germany before the introduction of the euro), the United Kingdom, Japan, the US, Australia, Canada and New Zealand. The sample period spans January 1975 to December 2015 (January 1988 to December 2015 for New Zealand) and the sampling frequency is monthly. Returns and inflation rates are annualized and displayed in percentages. In Panel A we report the annualized average returns for one-month risk-free bonds and ten-year government bonds. Panel B reports mean excess returns on equity, their volatility and the corresponding Sharpe ratios, computed as the ratio between the excess return and the return standard deviation. Panel C reports the annualized mean and standard deviation of exchange rates returns with respect to the US dollar. Panel D reports the average CPI inflation and its standard deviation.

	CHF	**EUR**	**GBP**	**JPY**	**USD**	**AUD**	**CAD**	**NZD**
				Panel A: Bonds				
1M	2.81	4.33	7.39	2.61	5.36	8.25	6.31	6.68
10Y	1.79	2.26	3.23	2.31	1.91	2.19	2.04	3.94
				Panel B: Excess stock returns				
Mean	7.39	6.89	6.17	3.49	7.08	5.71	5.15	0.78
Std	15.42	20.08	16.88	18.28	15.71	17.76	16.77	18.23
SR	48	34	37	19	45	32	31	5
				Panel C: Exchange rates				
Mean	2.96	0.03	-0.65	2.85	–	-0.86	-0.48	0.76
Std	12.12	10.56	10.20	11.32	–	10.93	6.78	11.92
				Panel D: Inflation				
Mean	1.76	2.25	4.80	1.60	3.71	4.86	3.73	5.63
Std	1.24	1.60	2.12	1.78	1.28	1.22	1.45	1.71

OA-2 Minimum Hellinger Divergence SDFs

In this section, we illustrate the properties of minimum Hellinger stochastic discount factors. These SDFs are obtained for $\alpha = 1/2$ in the minimization problem in equation (4) in the main paper and are given by:

$$M_i^*(1/2) = R_{\lambda_i^*}^{-2} / \mathbb{E}(R_{\lambda_i^*}^{-1}). \tag{1}$$

More generally, we obtain the following Hellinger bound:[3]

$$\log \mathbb{E}\left[M_i^{1/2}\right] \leq \frac{\log \mathbb{E}[R_{\lambda_i}^{-1}]}{2}. \tag{2}$$

There is a natural link between Hellinger bounds and the transitory component of SDFs. Namely, Hellinger bounds induce tight constraints on the first moment of transitory SDF components. To see this, notice that inequality (2) holds for any traded return, and we hence obtain the following constraint:

$$\log \mathbb{E}\left[(M_i^*(1/2))^{1/2}\right] \leq \frac{\log \mathbb{E}[R_{i\infty}^{-1}]}{2}. \tag{3}$$

Therefore, the Hellinger minimum dispersion SDF directly reveals information about the expected size of transient SDF components, and vice-versa.

Using expression (1), we can calculate minimum Hellinger SDFs in both fully integrated markets and a market where long-term bond and equity markets are segmented. Summary statistics are reported in Table OA-2.

Table OA-2. Properties of Minimum Hellinger SDFs

The table reports joint sample moments of the minimum SDFs and its components. Panel A reports statistics with respect in integrated whereas Panel B reports statistics from segmented markets. There is a US domestic SDF for each bilateral trade. We use monthly data from January 1975 to December 2015.

	US	UK	US	CH	US	JP	US	EU	US	AU	US	CA	US	NZ
							Panel A: Integrated							
$\mathbb{E}[M_i]$	0.982	0.973	0.982	0.990	0.982	0.991	0.982	0.980	0.982	0.966	0.982	0.973	0.982	0.956
Std(M_i)	0.784	0.805	0.925	0.880	0.720	0.677	0.661	0.647	0.823	0.843	0.688	0.682	0.620	0.547
Std(M_i^T)	0.120	0.122	0.120	0.061	0.120	0.091	0.120	0.068	0.120	0.107	0.120	0.111	0.120	0.091
Std(M_i^P)	0.856	0.882	0.991	0.903	0.791	0.688	0.741	0.687	0.908	0.943	0.775	0.779	0.657	0.612
$\sqrt{\text{Hellinger}(M_i)}$	0.706	0.725	0.823	0.780	0.695	0.646	0.599	0.599	0.752	0.768	0.634	0.631	0.590	0.526
corr(M_i^T,M_i^P)	-0.483	-0.534	-0.428	-0.243	-0.531	-0.152	-0.570	-0.524	-0.461	-0.716	-0.533	-0.641	-0.324	-0.646
corr(M_i,M_j)		0.990		0.989		0.989		0.984		0.990		0.994		0.980
							Panel B: Segmented							
$\mathbb{E}[M_i]$	0.983	0.973	0.983	0.990	0.982	0.991	0.982	0.980	0.982	0.966	0.982	0.973	0.982	0.956
Std(M_i)	0.577	0.631	0.730	0.650	0.702	0.363	0.625	0.479	0.572	0.762	0.592	0.494	0.525	0.372
Std(M_i^T)	0.120	0.122	0.120	0.061	0.120	0.091	0.120	0.068	0.120	0.107	0.120	0.111	0.120	0.091
Std(M_i^P)	0.668	0.728	0.803	0.673	0.776	0.377	0.709	0.527	0.664	0.864	0.682	0.612	0.567	0.457
$\sqrt{\text{Hellinger}(M_i)}$	0.533	0.568	0.676	0.610	0.674	0.363	0.587	0.466	0.528	0.717	0.547	0.466	0.496	0.363
corr(M_i^T,M_i^P)	-0.617	-0.640	-0.526	-0.318	-0.540	-0.272	-0.593	-0.683	-0.623	-0.780	-0.602	-0.814	-0.377	-0.862
corr(M_i,M_j)		0.154		0.351		0.469		0.627		0.538		0.430		0.523

Compared to the results for the minimum variance and minimum entropy SDFs reported in Table 2 of the main paper, we notice that minimum Hellinger SDFs provide a conservative alternative between the minimum variance and minimum entropy SDFs. This is intuitive since minimum Hellinger SDFs account for higher-order moments, but they do not exhibit dispersions as large as the minimum entropy ones.

[3] Kitamura, Otsu, and Evdokimov (2013) emphasize the optimal robustness features of Hellinger-type dispersion measures.

Table OA-3. Wedge Summary Statistics (Minimum Hellinger)

This table reports the annualized mean, standard deviation, skewness and kurtosis of the wedge η, for $\alpha = 0.5$, as implied by integrated and segmented markets. The domestic currency is the US dollar. The wedge is $\eta_{t+1} = \log\left(\frac{X_{t+1}M_{d,t+1}}{M_{f,t+1}}\right)$. We use monthly data from January 1975 to December 2015.

	Integrated				**Segmented**			
	$\mathbb{E}[\eta]$	Std(η)	Sk(η)	K(η)	$\mathbb{E}[\eta]$	Std(η)	Sk(η)	K(η)
UK	0.000	0.022	-0.395	6.336	0.005	0.665	-0.207	6.847
CH	-0.001	0.026	-1.277	8.426	-0.007	0.713	-0.174	4.647
JP	0.000	0.023	-1.286	7.608	-0.124	0.554	1.053	6.713
EU	0.000	0.021	-0.065	5.814	-0.048	0.455	0.058	3.629
AU	0.000	0.019	0.424	8.003	0.106	0.614	0.050	4.898
CA	0.000	0.013	-0.488	6.080	-0.036	0.507	0.093	5.655
NZ	-0.001	0.023	-2.695	17.23	-0.021	0.419	0.265	4.045

From identity (19) in the main paper, the wedge dispersions implied by minimum Hellinger divergence SDFs in integrated markets are very small, which is a consequence of the fact that these SDFs are those most related to the minimum entropy SDFs. When only short-term bonds can be traded internationally (segmented markets), the wedge plays a bigger role, especially since its volatility is comparable to that of the SDFs themselves.

Table OA-4. Backus and Smith (1993)-Type Regressions

This table reports the point estimates of a linear regression of the log difference between foreign and domestic minimum Hellinger SDFs on the log real exchange rate return: $m_{f,t+1} - m_{d,t+1} = \delta + \beta x_{t+1} + u_{t+1}$, where small-cap letters denote quantities in logs. We additionally report point estimates of a linear regression of the log difference of each component of the SDF on the log change in the real exchange rate: $m^U_{f,t+1} - m^U_{d,t+1} = \delta^U + \beta^U x_{t+1} + u^U_{t+1}$, where $U = P, T$ for permanent and transitory components, respectively. Standard errors are reported in square brackets. *** highlights significance at the 1% level.

	US/UK	**US/CH**	**US/JP**	**US/EU**	**US/AU**	**US/CA**	**US/NZ**
	`Panel A: Integrated Markets`						
$\boldsymbol{\beta}$	1.060^{***}	1.079^{***}	1.056^{***}	1.050^{***}	1.042^{***}	1.044^{***}	1.055^{***}
	[0.009]	[0.009]	[0.009]	[0.008]	[0.007]	[0.008]	[0.009]
$\boldsymbol{\beta^P}$	1.145^{***}	1.030^{***}	1.139^{***}	1.006^{***}	1.047^{***}	1.072^{***}	1.061^{***}
	[0.067]	[0.045]	[0.053]	[0.046]	[0.049]	[0.090]	[0.038]
$\boldsymbol{\beta^T}$	-0.084	0.049	-0.083	0.044	-0.005	-0.028	-0.006
	[0.068]	[0.044]	[0.053]	[0.046]	[0.049]	[0.089]	[0.037]
	`Panel B: Segmented Markets`						
$\boldsymbol{\beta}$	0.918^{***}	0.849^{***}	1.003^{***}	0.975^{***}	1.031^{***}	1.039^{***}	0.933^{***}
	[0.287]	[0.265]	[0.222]	[0.191]	[0.244]	[0.323]	[0.189]
$\boldsymbol{\beta^P}$	1.003^{***}	0.799^{***}	1.086^{***}	0.931^{***}	1.037^{***}	1.067^{***}	0.939^{***}
	[0.329]	[0.288]	[0.253]	[0.219]	[0.275]	[0.395]	[0.205]

From Table OA-4, we find that when using the minimum Hellinger SDFs, the Backus-Smith regressions yield qualitatively the same results as for the other minimum dispersion SDFs: the slope coefficient is equal to one when regressing the overall and the permanent components, whereas transient components are disconnected from the exchange rates.

Table OA-5. Minimum Hellinger SDFs Similarity Summary Statistics

The table reports in Panel A the average for the SDF similarity as implied by (I) integrated and (II) segmented markets. Minimum Hellinger SDFs are obtained for $\alpha = 0.5$ Panel B reports the values of co-entropy as in Chabi-Yo and Colacito (2017), defined as $\rho_{M_f,M_d} = 1 - \frac{L[M_f/M_d]}{L[M_f]+L[M_d]}$, with $L[x] \equiv \log(\mathbb{E}[x]) - \mathbb{E}[\log(x)]$ being the entropy of the positive random variable x. Data starts in April 1993 and ends in April 2013. AU = Australia, NZ = New Zealand, JP = Japan, CH = Switzerland, UK = United Kingdom, CA = Canada, EU = Eurozone. The US is always the domestic country.

	UK	**CH**	**JP**	**EU**	**AU**	**CA**	**NZ**
Panel A: Nominal Minimum Dispersion SDF Similarity Index							
I : Integrated Markets							
$S(M_d, M_f)$	0.990	0.987	0.988	0.988	0.986	0.985	0.987
$\bar{S}(M_d, M_f)$	0.991	0.987	0.989	0.988	0.986	0.992	0.987
II : Segmented Markets							
$S(M_d, M_f)$	0.951	0.926	0.945	0.970	0.960	0.972	0.947
$\bar{S}(M_d, M_f)$	0.991	0.987	0.988	0.988	0.986	0.993	0.985
Panel B: International Co-Entropies							
Segmented	0.328	0.390	0.228	0.727	0.735	0.806	0.479
Integrated	0.980	0.986	0.978	0.958	0.976	0.977	0.979

OA-3 Non-parametric Estimates of SDF Components

In this section, we discuss the SDF factorization into transitory and permanent components. The seminal work of Hansen and Scheinkman (2009) characterizes SDFs decompositions in Markovian environments with a Perron-Frobenius eigenfunction problem. The permanent and transitory components are constructed from a given SDF process, the Perron-Frobenius eigenfunction and its eigenvalue. The eigenvalue determines the average yield on long-horizon payoffs and the eigenfunction characterizes the dependence of the prices on the Markov state. Sufficient conditions for existence of such an eigenfunction are provided in Hansen and Scheinkman (2009). Moreover, under an additional ergodicity assumption, the SDF factorization is unique and boils down to the factorization in Alvarez and Jermann (2005), which decomposes the SDF into discounting at the rate of return on the zero-coupon bond of asymptotically long maturity (the long bond) and a further risk adjustment via the martingale component.

We follow Christensen (2017) and estimate the solution to the Perron-Frobenius eigenfunction problem in Hansen and Scheinkman (2009) from time-series data on state variables and a given SDF process. For simplicity, we consider as a state variable the return on the long maturity bond and as a benchmark SDF the minimum entropy SDF estimated in Section 2.2 in the main paper. By deriving the eigenvalue and eigenfunction, we can reconstruct the time-series of the estimated non-parametric permanent and transitory components and compare them to our estimates based on the return of long, but finite horizon bond returns.

The estimation procedure builds upon a sieve approach, in which the infinite-dimensional eigenfunction problem is approximated by a low-dimensional matrix eigenvector problem. We use Hermite polynomials as basis functions and choose the smoothing parameter, i.e., the degree of the polynomial, k, equal to five. Under some regularity conditions, the solution to the eigenfunction and eigenvalue problem is unique and pinned down by the basis functions. We refer the interested reader to Christensen (2017) for the complete set of derivations and proofs. Here, $b^k \in L^2$ are the basis functions for the state variable X, $m(\cdot)$ denotes the SDF, ρ is the largest real eigenvalue and c_k is a vector in $\mathbb{R}^k$ such that the eigenfunction ϕ satisfies $\phi_k(x) = b^k(x)'c_k$. The sample counterparts of the matrices $\mathbf{G}_k$ and $\mathbf{M}_k$ are obtained by substituting the expected values with the sample averages, for a given time-series of the state variable $\{X_0, X_1, \ldots, X_n\}$ and a given SDF process. In our case, the state variable is the return on the ten-year maturity bond and the SDF is given by the minimum entropy projection derived in Section 2.2 in the main paper.

Finally, given the eigenvector ρ and eigenfunction ϕ that solve the Perron-Frobenius problem, the permanent and transient components from time t to $t+\tau$ are given by:

$$\frac{M^P_{t+\tau}}{M^P_t} = \rho^{-\tau}\frac{M_{t+\tau}}{M_t}\frac{\phi(X_{t+\tau})}{\phi(X_t)}, \qquad \frac{M^T_{t+\tau}}{M^T_t} = \rho^{\tau}\frac{\phi(X_t)}{\phi(X_{t+\tau})}. \tag{4}$$

We summarize in Table OA-6 the properties of the non-parametric estimates of the permanent and transient components for the domestic minimum entropy SDFs. Consistent with our earlier results in the main paper, we find that the permanent component accounts for most of the variability of the overall SDF, whereas the transient component accounts for a much smaller fraction, which on average is close to the annualized volatility of our proxy for the long-term bond. These findings are supplemented by the almost perfect correlation between our permanent components and the corresponding non-parametric estimates. This is further illustrated by the scatter plots in Figure OA-1.

Table OA-6. Properties of SDFs components (Nonparametric estimates)

The table reports the annualized volatility of the non-parametric estimates of the domestic minimum-entropy SDF components ($\alpha_d = 0$). We use monthly data from January 1975 to December 2015.

	USDGBP	**USDCHF**	**USDJPY**	**USDEUR**	**USDAUD**	**USDCAD**	**USDNZD**
Std(M_d^P)	0.786	0.970	0.725	0.683	0.907	0.660	0.601
Std(M_d^T)	0.126	0.065	0.057	0.039	0.143	0.140	0.112

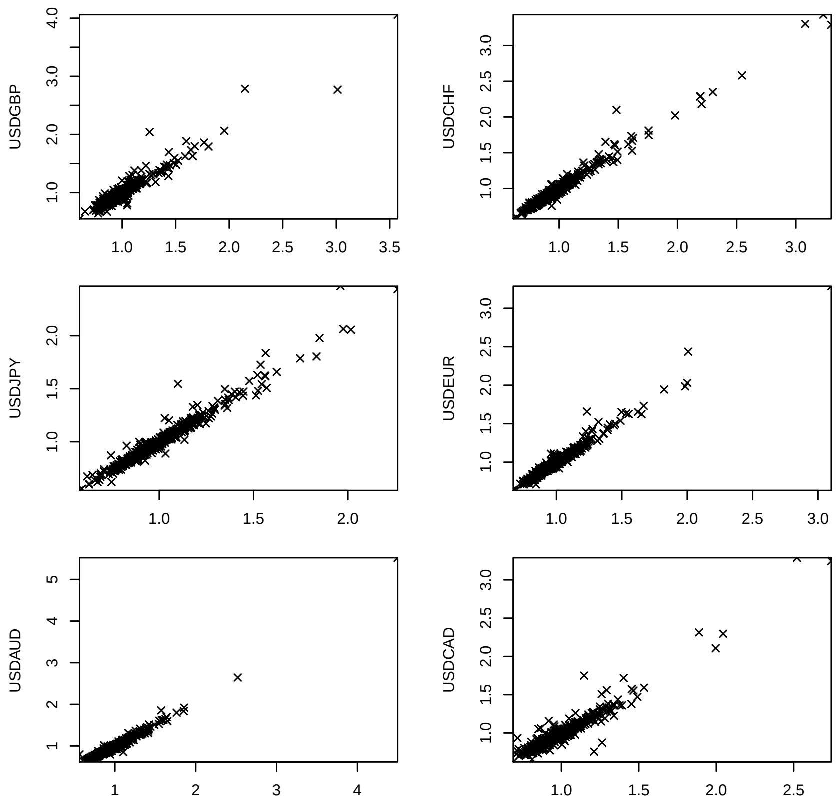
OA-4 Relation to Other SDF Similarity Indices

In the following, we compare our SDF similarity index to other indices used in the literature. The normalizing constant $\min(\mathbb{E}[M_d], \mathbb{E}[M_f])$ in the denominator of index (10) in the main paper ensures both an upper bound of one and the model-free index representation in Proposition 3, which is based on the price of an at-the-money put on the *spot* exchange rate. Another approach is to directly normalize the SDFs in definition (10) in the main paper, which implies the following similarity index:

$$S(M_d/\mathbb{E}[M_d], M_f/\mathbb{E}[M_f]) := \mathbb{E}[\min(M_d/\mathbb{E}[M_d], M_f/\mathbb{E}[M_f])]\,. \tag{5}$$

This index preserves all key properties of index (10) in the main paper, but it is not computable from the prices of options on spot exchange rates. It can be computed from the price of a single at-the-money put on the *forward* exchange rate, whenever option quotes on forward exchange rates are

Figure OA-1. Minimum dispersion vs. non-parametric permanent SDF components



The figure plots the correlation between the non-parametric estimate of the permanent component (x-axis) and the minimum entropy permanent component (y-axis). Data is monthly and runs from January 1975 to December 2015.

available.[4]

A relevant computation issue arises for the power similarity indices introduced in Orłowski, Sali, and Trojani (2016):

$$S_\alpha(M_d, M_f) := \frac{\mathbb{E}[M_d^\alpha M_f^{1-\alpha}]}{\mathbb{E}[M_d]^\alpha \mathbb{E}[M_f]^{1-\alpha}}\,, \tag{6}$$

where $\alpha \in (0,1)$ parameterizes the index family. This family includes for $\alpha = 1/2$ the Hellinger similarity index proposed in Bakshi, Gao, and Panayotov (2017).[5] A key property of these indices is that under the asset market view, they can be computed from option information alone, whenever a continuum of out-of-the money exchange rate options with arbitrary strike price $K > 0$ is traded,

[4]The following relations between the two indices hold: $0 \le S(M_d/\mathbb{E}[M_d], M_f/\mathbb{E}[M_d]) \le S(M_d, M_f) \le 1$. Moreover, under validity of the asset market view, the same arguments as in the proof of Proposition 3 in the main paper yield:

$$\mathbb{E}[\min(M_d/\mathbb{E}[M_d], M_f/\mathbb{E}[M_f])] = 1 - \frac{1}{\mathbb{E}[M_d]}\mathbb{E}[M_d \max(0, 1 - X\mathbb{E}[M_d]/\mathbb{E}[M_f])]\,.$$

[5]Note that:

$$S_{1/2}(M_d, M_f) = \mathbb{E}[(M_d/\mathbb{E}[M_d])^{1/2}(M_f/\mathbb{E}[M_f])^{1/2}] = 1 - \frac{\mathbb{E}\left[((M_d/\mathbb{E}[M_d])^{1/2} - (M_f/\mathbb{E}[M_f])^{1/2})^2\right]}{2}\,.$$

The expectation on the RHS corresponds to Bakshi, Gao, and Panayotov (2017)'s Hellinger distance.

i.e., under a complete exchange rate option market.[6] Unfortunately, the intrinsic incompleteness of exchange rate option markets makes such a computation of similarity index (6) very challenging. However, due to the following inequality:

$$S(M_d, M_f)\frac{\min(\mathbb{E}[M_d], \mathbb{E}[M_f])}{\mathbb{E}[M_d]^{\alpha}\mathbb{E}[M_f]^{1-\alpha}} \leq S_{\alpha}(M_d, M_f)\,, \tag{7}$$

we can always compute with Proposition 3 in the main paper a lower bound for similarity index (6), using the price of a single at-the-money put on the spot exchange rate.[7]

OA-4.1 Similarity in Benchmark Models

The index $\overline{S}(M_d, M_f)$ measures similarity, rather than SDF co-movement or risk sharing. In the following, we illustrate this using some well-known benchmark models.

Lognormal SDFs and Exchange Rates. Whenever (M_d, X) and $(M_f, 1/X)$ are jointly lognormal, similarity can be directly computed from the Black and Scholes (1973)–Garman and Kohlhagen (1983) model-implied prices of the corresponding at-the-money put options:

$$\mathbb{E}[M_d \max(0, 1 - X)] = \mathbb{E}[M_d]\mathcal{N}(-d_2) - \mathbb{E}[M_f]\mathcal{N}(-d_1) =: BS_d(\sigma_x)\,, \tag{8}$$

$$\mathbb{E}[M_f \max(0, 1 - (1/X))] = \mathbb{E}[M_f]\mathcal{N}(d_1) - \mathbb{E}[M_d]\mathcal{N}(d_2) =: BS_f(\sigma_x)\,, \tag{9}$$

where

$$d_1 := \frac{-\log(\mathbb{E}[M_d]/\mathbb{E}[M_f]) + \frac{\sigma_x^2}{2}}{\sigma_x}\ ;\ d_2 := \frac{-\log(\mathbb{E}[M_d]/\mathbb{E}[M_f]) - \frac{\sigma_x^2}{2}}{\sigma_x}\,, \tag{10}$$

with σ_x the volatility of log exchange rate returns. Hence, $\overline{S}(M_d, M_f)$ is maximal (minimal) for $\sigma_x \downarrow 0$ ($\sigma_x \uparrow \infty$). Given the identity of physical and implied volatilities under joint log-normality, these arbitrage-free settings are unlikely to generate simultaneously a low exchange rate volatility and a low option-implied SDF similarity. For example, such settings include jointly lognormal specifications of domestic and foreign SDFs under the asset market view.

Rare Disasters. One way to break the link between the realized and implied volatilities is to introduce time-varying rare disasters. We borrow from Farhi and Gabaix (2016) who model an economy with traded and nontraded goods in complete international financial markets. In this model, a disaster may happen in the world consumption of the tradable good with probability p_t. The exogenous SDF in units of the world numéraire, is given as follows: if no disaster happens in $t+1$, $M_{t+1}^* = \exp(-R)$, for a parameter $R > 0$ which is related to the subjective discount rate and the expected growth rate of the tradable good. If a disaster happens, then $M_{t+1}^* = \exp(-R)B_{t+1}^{-\gamma}$, where $B_{t+1} > 0$ models the size of world disasters and γ is the coefficient of relative risk aversion. A key parameter in Farhi and Gabaix (2016) is a country's resilience to disasters, defined by:

$$H_{it} := H_{i*} + \hat{H}_{it} := p_t \mathbb{E}_t^D[B_{t+1}^{-\gamma} F_{it+1} - 1], \tag{11}$$

where F_{it+1} is the future country's recovery rate and $\mathbb{E}_t^D[\cdot]$ denotes the expectation conditional on a disaster state.[8] Thus, a relatively safe country has a high resilience H_{it}, while a relatively risky country

[6]Using standard replication formulas for nonlinear payoffs, this result follows from the identity:

$$S_{\alpha}(M_d, M_f) = \frac{1}{\mathbb{E}[M_d]^{\alpha}\mathbb{E}[M_f]^{1-\alpha}}\mathbb{E}[M_d X^{1-\alpha}]\,.$$

[7]The bound follows from the inequality $x^{\alpha}y^{1-\alpha} \geq \min(x, y)$ for $\alpha \in (0, 1)$.

[8]By definition, H_{i*} and $\hat{H}_{it}$ are the constant and the time-varying components of a country's i resilience.

has a low resilience. The stochastic discount factor for the nontraded good in country i depends on the time-varying resilience component $\hat{H}_{it}$. It is given by:

$$M_{it+1} = M^*_{t+1} \cdot \frac{\omega_{it+1}}{\omega_{it}} \cdot \frac{r_{ei} + \phi_{H_i} + \hat{H}_{it+1}}{r_{ei} + \phi_{H_i} + \hat{H}_{it}} , \tag{12}$$

where r_{ei} is the sum of the country's "steady state" interest rate for $\hat{H}_{it} = 0$ and a constant investment depreciation rate λ, ϕ_{H_i} is the resilience speed of mean reversion and ω_{it} the country's i export productivity. The price of a put option on the exchange rate follows from the asset market view and the joint lognormality of $r_{ed} + \phi_{H_d} + \hat{H}_{dt+1}$, $r_{ef} + \phi_{H_f} + \hat{H}_{ft+1}$ conditional on a no disaster state.[9] Therefore, the similarity index is available in closed-form.

Proposition OA-1. *In the Farhi and Gabaix (2016) model, the similarity index (10) in the main paper is given by:*

$$\overline{S}_t(M_d, M_f) = S(M_d, M_f) = \frac{\mathbb{E}_t[M_{d,t+1}] - \mathbb{E}_t[M_{d,t+1} \max(0, 1 - X_{t+1})]}{\min(\mathbb{E}_t[M_{d,t+1}], \mathbb{E}_t[M_{f,t+1}])} , \tag{13}$$

where the price of the domestic at-the-money put on the exchange rate is given by:

$$\begin{aligned} \mathbb{E}_t[M_{d,t+1} \max(0, 1 - X_{t+1})] &= (1 - p_t) BS_d(\sigma_x) \\ &\quad + p_t \mathbb{E}^D_t \left[G_{d,t} F_{d,t+1} \max\left(0, 1 - \frac{F_{f,t+1} G_{f,t}}{F_{d,t+1} G_{d,t}}\right)\right] , \end{aligned} \tag{14}$$

with

$$G_{i,t} = \exp(g_{\omega_i}) \frac{r_{ei} + \phi_{H_i} + \frac{1+H_{i*}}{1+H_{it}} \exp(-\phi_{H_i}) \hat{H}_{it}}{r_{ei} + \phi_{H_i} + \hat{H}_{it}} , \tag{15}$$

and g_{ω_i} the constant growth rate of country i's productivity in no-disaster states.

Proof: See Appendix.

The first term on the RHS of equation (14) is the Black and Scholes (1973)–Garman and Kohlhagen (1983) price of a put option, weighted by the probability of no disaster. The second term is the put price conditional on a common disaster in domestic and foreign productivity, weighted by the probability of a world disaster. This term reflects the price for ensuring a large depreciation of foreign versus domestic non-traded goods, due to a larger decrease in foreign productivity when disasters occur. This component can create a disconnect between the minimal similarity index in Proposition 4 in the main paper and the exchange rate volatility σ_x, which may be useful to induce low option-implied SDF similarities together with low exchange rate volatilities encountered in the data.

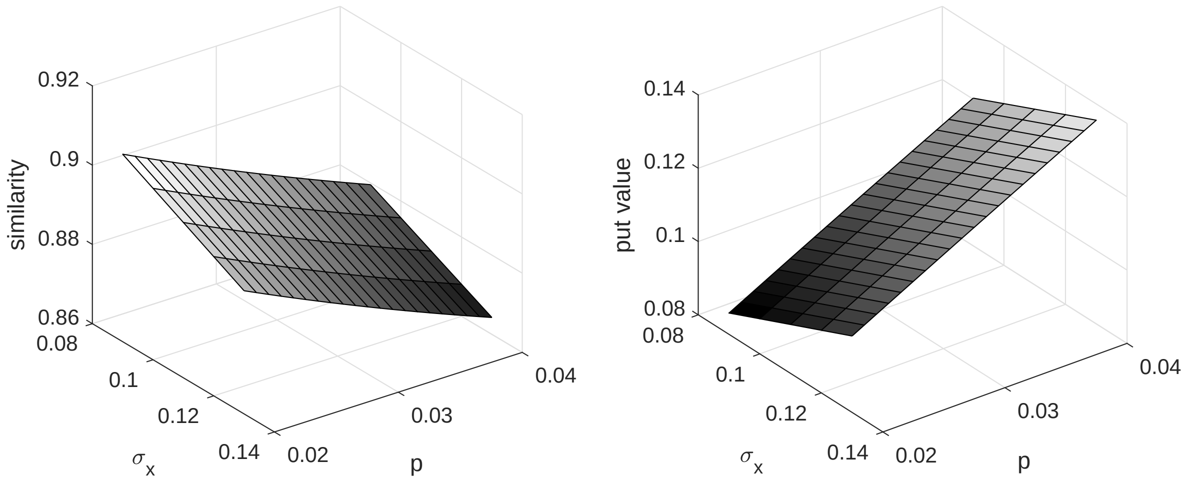
We can use the calibration parameters in Farhi and Gabaix (2016) to illustrate the range of similarity indices attainable in this model. We set $g_{\omega_d} = g_{\omega_f} = 0$, $\sigma_x \in [0.09, 0.13]$, $p_t \in [0.02, 0.04]$ and consider for simplicity the case where resilience in both countries is at its time-invariant level, i.e., $H_{it} = H_{i*}$ $(i = d, f)$, so that $G_{d,t} = G_{f,t} = 1$. Conditional on a disaster event, log recovery rates across countries are IID normally distributed, $\ln F_{it+1} \sim II\mathcal{N}(-\frac{\sigma^2_{F_i}}{2}, \sigma^2_{F_i})$, with calibrated standard deviation $\sigma_{F_i} = 0.16$. Therefore,

$$\mathbb{E}_t[F_{d,t+1} \max(0, 1 - F_{f,t+1}/F_{d,t+1})] = \mathcal{N}(\sigma_{F_i}/\sqrt{2}) - \mathcal{N}(-\sigma_{F_i}/\sqrt{2}) , \tag{16}$$

and the similarity index in Proposition OA-1 follows in closed-form. To gauge the effect of world disasters on the similarity index, we plot in Figure OA-2 (left panel) the model-implied similarity index

[9] See also Proposition 6 in Farhi and Gabaix (2016).

Figure OA-2. SDF Similarity in the Farhi and Gabaix (2016) model



The left panel plots the similarity index given in Proposition OA-1 as a function of exchange rate volatility (σ_x) and the disaster probability (p). The right panel plots the price of the at-the-money put option also as a function of the exchange rate volatility and the disaster probability. Calibrated values are as in Farhi and Gabaix (2016).

in Proposition OA-1 as a function of the disaster probability (p) and the volatility of exchange rates (σ_x). We notice that, on average, the similarity is around 0.88. As expected, when we increase the volatility of exchange rates (σ_x) or increase the probability of rare disasters (p), the similarity falls. This is explained by the fact that the price of the put option in the right panel of Figure OA-2 concomitantly increases when the probability of world disasters (the volatility of exchange rates) increases.

OA-5 Unspanned Exchange Rate Risks and Minimum Dispersion SDFs

As discussed in Section 1.3 of the main paper, given a set of traded domestic (foreign) returns $\mathbf{R}_d$ ($\mathbf{R}_f$), unspanned exchange rate risks in domestic (foreign) currency arise when there are exchange rate-dependent payoffs that are not replicable by a linear portfolio return. In the following, we discuss the conditions under which this unspanned risk can be used to construct domestic and foreign SDFs that feature low correlation in integrated but incomplete markets.

OA-5.1 A Family of SDFs with Unspanned Risks and Low International Correlations

Nonlinear transformations of a portfolio return R_{λ_d} (R_{λ_f}) in integrated but incomplete markets typically generate unspanned exchange rate risks. In particular, the difference $M_i^*(\alpha) - M_i^*(2)$ ($i = d, f$) between any minimum dispersion SDF for $\alpha \neq 2$ and the minimum variance SDF generates an unspanned payoff in local currency, which is orthogonal to the space of traded returns. As a consequence, for any $\nu \in \mathbb{R}$ a strictly positive random variable of the form

$$M_i^*(\nu, \alpha) := M_i^*(2) + \nu u_i^*(\alpha) , \tag{17}$$

is a stochastic discount factor pricing returns $\mathbf{R}_i$, where $u_i^*(\alpha) := M_i^*(\alpha) - M_i^*(2)$ is an unspanned risk component. Given the almost perfect co-movement of domestic and foreign unspanned SDF components in integrated markets, documented previously in the main body of the paper, a simple model-free approach to construct international SDFs with lower correlations is to add to domestic and foreign minimum variance SDFs unspanned components with opposing signs. In other words, for any $\nu \geq 0$ we can define the following pair of domestic and foreign SDFs:

$$M_d^*(\nu,\alpha) := M_d^*(2) + \nu u_d^*(\alpha)\ ,\ M_f^*(-\nu,\alpha) := M_f^*(2) - \nu u_f^*(\alpha)\ . \tag{18}$$

The correlation between these SDFs is:

$$\begin{aligned}\rho(\nu,\alpha) &= \frac{Cov(M_d^*(2),M_f^*(2)) + \nu Cov(M_f^*(2),u_d^*(\alpha)) - \nu Cov(M_d^*(2),u_f^*(\alpha)) - \nu^2 Cov(u_d^*(\alpha),u_f^*(\alpha))}{\sqrt{(Var(M_d^*(2)) + \nu^2 Var(u_d^*(\alpha)))(Var(M_f^*(2)) + \nu^2 Var(u_f^*(\alpha)))}} \\ &\approx \frac{Cov(M_d^*(2),M_f^*(2)) - \nu^2 Cov(u_d^*(\alpha),u_f^*(\alpha))}{\sqrt{(Var(M_d^*(2)) + \nu^2 Var(u_d^*(\alpha)))(Var(M_f^*(2)) + \nu^2 Var(u_f^*(\alpha)))}}\ ,\end{aligned}$$

since $u_i^* \perp M_i^*$ and because empirically the spans of domestic and foreign returns in integrated markets are not too different, due to the low exchange rate volatility. Recalling the almost perfect co-movement of unspanned components implied by minimum dispersion SDFs in integrated international financial markets, it further follows:

$$\rho(\nu,\alpha) \approx \frac{Cov(M_d^*(2),M_f^*(2)) - \nu^2\sqrt{Var(M_d^*(\alpha) - M_d^*(2))Var(M_f^*(\alpha) - M_f^*(2))}}{\sqrt{(Var(M_d^*(2)) + \nu^2 Var(M_d^*(\alpha) - M_d^*(2)))(Var(M_f^*(2)) + \nu^2 Var(M_f^*(\alpha) - M_f^*(2)))}}\ .$$

As the minimum dispersion international SDF correlations are typically positive, the right hand side of the last equality is decreasing in ν. In parallel, the volatility of SDF $M_i^*(\nu,\alpha)$ is increasing in ν:

$$Var(M_i^*(\nu,\alpha)) = Var(M_i^*(-\nu,\alpha)) = Var(M_i^*(2)) + \nu^2 Var(M_i^*(\alpha) - M_i^*(2))\ . \tag{19}$$

Therefore, it is always possible to decrease the correlation $\rho(\nu,\alpha)$, at the cost of highering the SDF variances $Var(M_d^*(\nu,\alpha))$ and $Var(M_f^*(-\nu,\alpha))$. This trade-off is depicted in Figure OA-3 where we plot SDF correlation (left panels) and the corresponding SDF variances (right panels). When we set ν equal to zero, SDF correlation is almost perfect and variance is minimized. Increasing the ν parameter leads to a decrease in the correlation between domestic and foreign SDFs in integrated markets, while simultaneously increasing the underlying SDF dispersion. Specifically, international correlations of 0.5 entail an increase between 0.15 and 0.2 in the SDF volatility.

In particular, for $\alpha = 0$ (the minimum entropy SDF) and $\nu = 1$, it follows:

$$M_d^*(1,0) = M_d^*(0)\ ,\ M_f^*(-1,0) = 2M_f^*(2) - M_f^*(0)\ . \tag{20}$$

By construction, both these SDFs have a volatility identical to the volatility of the minimum entropy SDF in each market, while the international correlation is decreased to 0.5.

OA-5.2 Relation to Good-Deal-Bounds SDFs

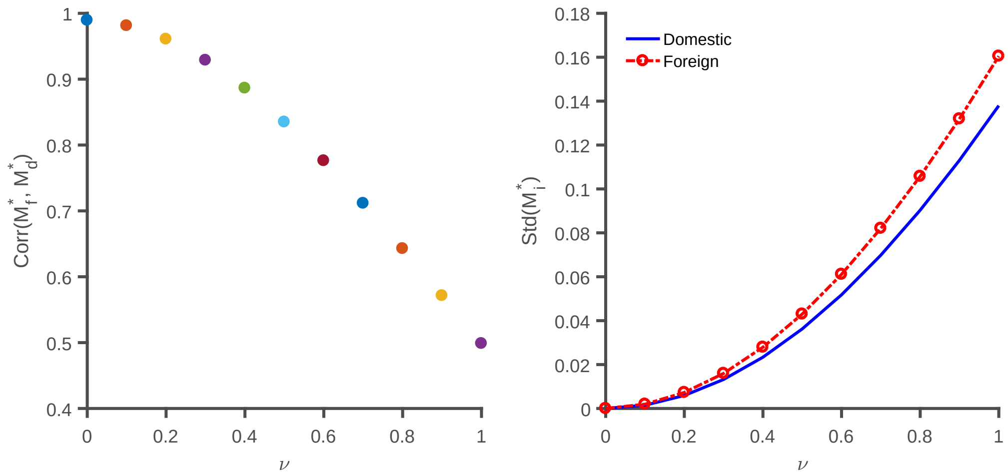
In the following, we assume a setting where households cannot trade the unspanned component because they only have access to basic assets. Financial intermediaries, on the other hand, can sell/buy the unspanned component to households in international financial markets.

Given a generic unspanned risk u_i orthogonal to the set of returns $\mathbf{R}_i$, i.e., such that $\mathbb{E}[u_i\mathbf{R}_i] = 0$, stochastic discount factors of the form:[10]

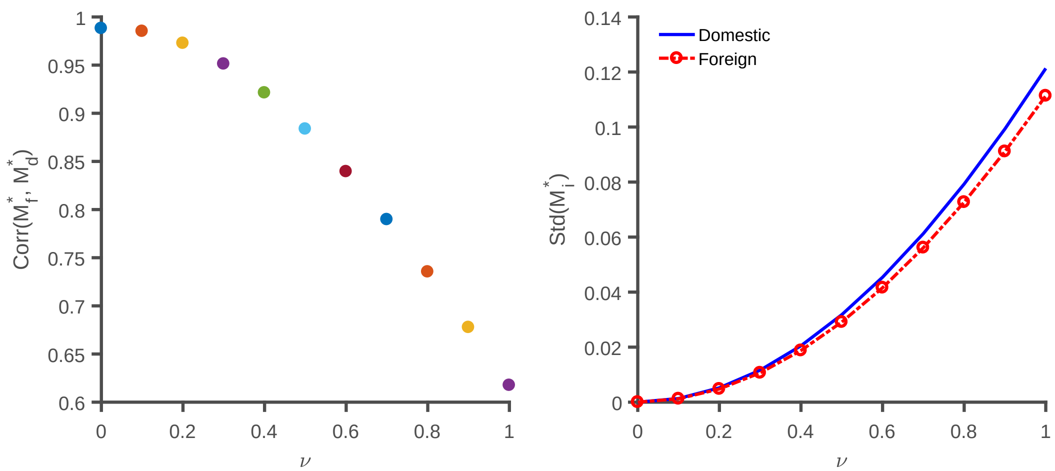
$$M_i^+(\nu) = M_i^*(2) + \nu u_i\ ;\ M_i^-(\nu) = M_i^*(2) - \nu u_i\ , \tag{21}$$

[10]Note that in this case, the unspanned risk u_i is not restricted to any specification, as opposed to the previous u_i^*. The only condition that has to be satisfied is the orthogonality one, with respect to the set of tradable returns.

Figure OA-3. International SDF correlation and volatility trade-off



Panel A: US & UK



Panel B: US & CH

The figure plots the correlation between the domestic and foreign SDFs derived as in equation (18) (left panel) and the corresponding increase in volatility (right panel), for different values of parameter ν. The upper panel illustrates the US and UK pair, whereas the lower panel the US and CH pair.

where $\nu \geq 0$, are in the family of good-deal-bound SDFs introduced in Cochrane and Saá-Requejo (2000). More specifically, $M_i^+(\nu)$ ($M_i^-(\nu)$) is the unique stochastic discount factor that prices the original returns $\mathbf{R}_i$ and such that the price $\mathbb{E}[M_i^+(\nu)u_i] = \nu\mathbb{E}[u_i^2]$ ($\mathbb{E}[M_i^-(\nu)u_i] = -\nu\mathbb{E}[u_i^2]$) of the unspanned risk is maximal (minimal) under a binding upper bound on the SDF volatility:

$$SR = \sqrt{Var(M_i^+(\nu))} = \sqrt{Var(M_i^-(\nu))}\,. \tag{22}$$

A key feature of these SDFs is that they provide the largest ask price that a financial intermediary could require in order to sell the unspanned risk to an investor in the original market, and vice versa the lowest bid price for buying. Moreover, this new market setting, in which the intermediary enables the trading of unspanned risk, is also robust to implausible good-deals.

From the intermediary's perspective, whether she will be willing to sell or buy payoff u_i at a given price, depends on her attitudes to risk and possible financing constraints. In most cases, the sign of the implied risk premium on an intermediary's short or long position needs to be positive in order to ensure a positive gain from intermediation. This already gives a necessary condition for the emergence of an intermediary's short or long position. For instance, in order to sell the unspanned risk at the highest price under the corresponding good deal bound, a necessary condition for a short intermediary position is[11]

$$\nu \frac{\mathbb{E}[u_i^2]}{\mathbb{E}[M_i^*(2)]} > 0. \tag{23}$$

Hence, using the ask price $\nu\mathbb{E}[u_i^2]$, the excess return is always positive since $\nu > 0$. Equivalently, the condition for a long intermediary position is $-(-\nu\mathbb{E}[u_i^2])$, so we obtain again the relationship in (23). Therefore, these quantities represent the buying and selling prices for which is optimal for the financier to trade.

OA-5.3 Relation to Bakshi, Cerrato, and Crosby (2018)

In a recent paper, Bakshi, Cerrato, and Crosby (2018) show how to construct international SDFs in incomplete integrated markets that feature a low degree of correlation. Their approach is akin to the good deal bound SDFs in Cochrane and Saá-Requejo (2000) and aligned with our discussion about low SDF correlation in Section OA-5.1 using unspanned entropy payoffs in domestic and foreign currency.

Bakshi, Cerrato, and Crosby (2018) construct their domestic and foreign SDFs as an exchange-rate dependent transformation of the minimum covariance SDFs arising in a new fictitious economy with bounded good deals and a common numéraire for domestic and foreign investors. More specifically, let

$$M_d := \frac{M_z^*}{\sqrt{X}} + \frac{U_z^d}{\sqrt{X}} \text{ ; and } M_f := M_z^*\sqrt{X} + U_z^f\sqrt{X}\,, \tag{24}$$

with $M_z^* \geq 0$ the minimum variance SDF pricing the common set of fictitious returns $\mathbf{Z} := \mathbf{R}_d/\sqrt{X} = \mathbf{R}_f\sqrt{X}$ and with the two unpanned risks U_z^d and U_z^f that are orthogonal to these returns. By construction, domestic and foreign SDFs M_d and M_f derived in this way price returns $\mathbf{R}_d$ and $\mathbf{R}_f$, respectively.[12]

It follows that in order to minimize the covariance between M_d and M_f in equation (24), one can equivalently minimize the second cross-moment of unspanned risks U_z^d and U_z^f. However, second cross-moment minimization without imposing a bound on the individual second moments of unspanned risks can imply SDFs with implausibly large Sharpe ratios. Therefore, unpanned risks are determined as the solution of the following constrained optimization problem with an additional good-deal bound Θ^2:

$$\begin{aligned}
(U_z^{d*}, U_z^{f*}) := \arg \min_{(U_z^d, U_z^f)} \quad & \mathbb{E}[U_z^d U_z^f] \\
\text{s.t.} \quad & \mathbb{E}[(U_z^d - U_z^f)^2] \leq \Theta^2, \\
& \mathbb{E}[U_z^d \mathbf{Z}] = \mathbb{E}[U_z^f \mathbf{Z}] = \mathbf{0}, \\
& \min(U_z^d, U_z^f) \geq -M_z^*\,.
\end{aligned} \tag{25}$$

In this problem, the first inequality constraint is the bound on good deals, while the last inequality constraint ensures nonnegativity of SDFs M_d and M_f. Since, by the Cauchy-Schwartz inequality,

[11] The expected excess return from the short position actually reads $\nu\frac{\mathbb{E}[u_i^2]}{\mathbb{E}[M_i^*(2)]} - \mathbb{E}[u_i]$, but since the risk-free bond is within the initial set of traded returns $\mathbf{R}_i$, then $\mathbb{E}[u_i] = 0$. Analogously, the expected excess return from the long position is $\mathbb{E}[u_i] + \nu\frac{\mathbb{E}[u_i^2]}{\mathbb{E}[M_i^*(2)]}$, however as the unspanned risk is zero in expectation, we retrieve equation (23).

[12] By definition, the identity $\mathbf{R}_d/\sqrt{X} = \mathbf{R}_f\sqrt{X}$ always holds under market integration. However, note that $\frac{M_z^*}{\sqrt{X}}$ and $M_z^*\sqrt{X}$ are not tradable SDFs in domestic and foreign currency, respectively.

$-\sqrt{\mathbb{E}[(U_z^d)^2]\mathbb{E}[(U_z^f)^2]}$ is a lower bound for the objective function in optimization problem (25), we can focus on an optimal solution of the form $U_z^{d*} = \frac{\Theta}{2} \cdot U_z^*$ and $U_z^{f*} = -\frac{\Theta}{2} \cdot U_z^*$, where U_z^* is a random variable such that

$$\mathbb{E}[(U_z^*)^2] \leq 1 \, , \tag{26}$$

$$\mathbb{E}[U_z^* \mathbf{Z}] = \mathbf{0} \, , \tag{27}$$

$$\min\left(\frac{\Theta}{2}U_z^*, -\frac{\Theta}{2}U_z^*\right) \geq -M_z^* \, . \tag{28}$$

Without loss of generality, we can also assume that $U_z^* = d \cdot \delta_z^*$, where $\mathbb{E}[(\delta_z^*)^2] = 1$ and $d \in [0,1]$. In this case, one can then determine the optimal d^* as the largest $d \in [0,1]$ such that inequality (28) holds pointwise across all random states. In summary, the minimal covariance optimal SDFs from equation (24) then read as follows:

$$M_d^* = \left(M_z^* + \frac{\Theta d^*}{2} \cdot \delta_z^*\right)\frac{1}{\sqrt{X}} \; ; \; M_f = \left(M_z^* - \frac{\Theta d^*}{2} \cdot \delta_z^*\right)\sqrt{X} \, . \tag{29}$$

These SDFs arise from an exchange rate-dependent transformation of the following SDFs for the fictitious economy with return vector $\mathbf{Z}$:

$$M_z^* + \frac{\Theta d^*}{2} \cdot \delta_z^* \; ; \; M_z^* - \frac{\Theta d^*}{2} \cdot \delta_z^* \, . \tag{30}$$

Note that these last SDFs are the Cochrane and Saá-Requejo (2000) good-deal bound SDFs that imply the largest and lowest price for an unspanned payoff δ_z^* in the fictitious economy with return vector $\mathbf{Z}$, subject to a maximum Sharpe ratio bound equal to $\sqrt{Var(M_z^*) + \frac{(\Theta d^*)^2}{4} Var(\delta_z^*)}$.

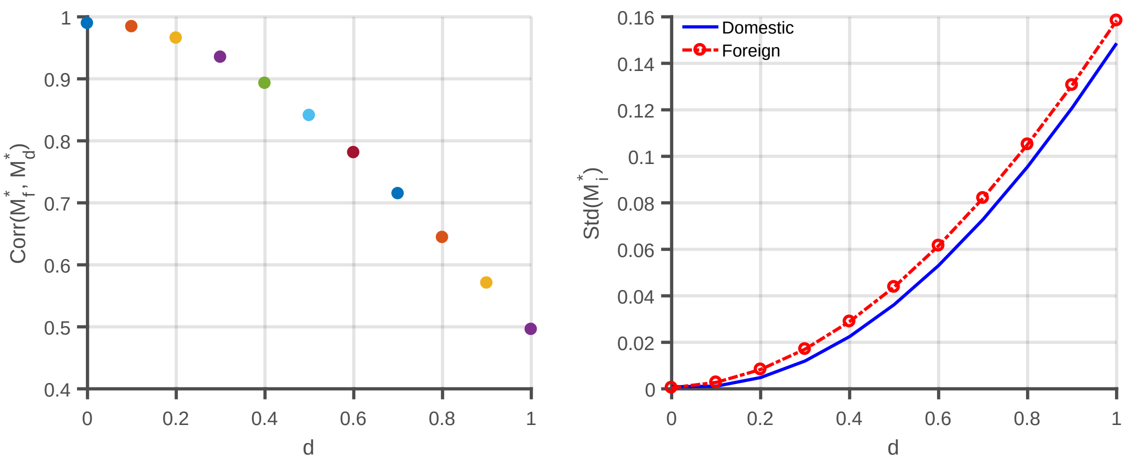
In their approach, Bakshi, Cerrato, and Crosby (2018) use as unpanned payoff the normalized residual from the projection of one on the space of returns $\mathbf{Z}$.[13] Using their approach, we can illustrate the resulting tradeoff between SDF volatility and correlation in Figure OA-4, where we plot the SDF correlation (left panels) and the corresponding SDF variances (right panels) for various choices of parameter d^* and a fixed Θ. Notice that the conclusions are comparable with our earlier results presented in Figure OA-3, where we directly applied the Cochrane and Saá-Requejo (2000) good-deal bounds pricing approach with unspanned entropy payoffs in the original domestic and foreign numéraires.

What is the relation between Bakshi, Cerrato, and Crosby (2018) minimum covariance SDFs and our minimum dispersion SDFs in the main text? We find that the minimum covariance SDFs in local currency are very highly correlated with our minimum variance SDF projections for $\Theta = 0$. This feature holds despite the fact that they are not formally tradable in domestic and foreign markets, because they are not a linear combination of returns in local currencies. Note that, in contrast to domestic and foreign minimum variance SDFs, Bakshi, Cerrato, and Crosby (2018) SDFs for $\Theta = 0$ satisfy by construction the asset market view.

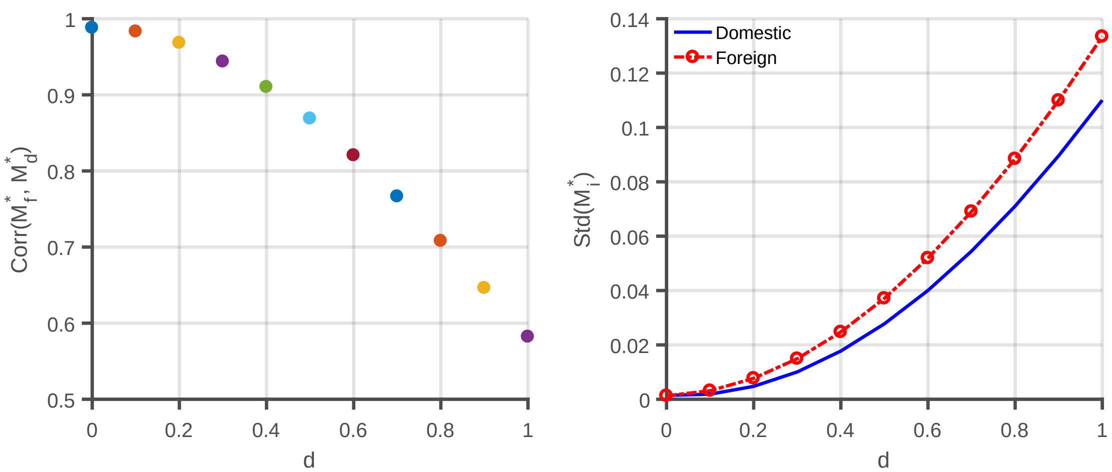
Very different economic features arise between the unspanned SDF components arising under our approach – which are based on unspanned minimum entropy SDF components – and the unspanned SDF components under Bakshi, Cerrato, and Crosby (2018) approach for $\Theta > 0$. We illustrate this in Figure OA-5, where we show that the unconditional correlation between unspanned components under the two approaches is virtually zero. Importantly, while unspanned minimum entropy SDF components typically spike both in domestic and foreign markets during periods of large economic distress or market uncertainty (as depicted in Figure 1 in the main paper), Bakshi, Cerrato, and Crosby (2018) unspanned components are muted on these dates. They also do not seem to price in any apparent way the macro-economic risk related to, e.g., US recessions.

[13]This choice of the unspanned payoff also implies $\mathbb{E}[\delta_z^*] = 0$ and $\mathbb{E}[(\delta_z^*)^2] = Var(\delta_z^*)$.

Figure OA-4. International SDF correlation and volatility trade-off



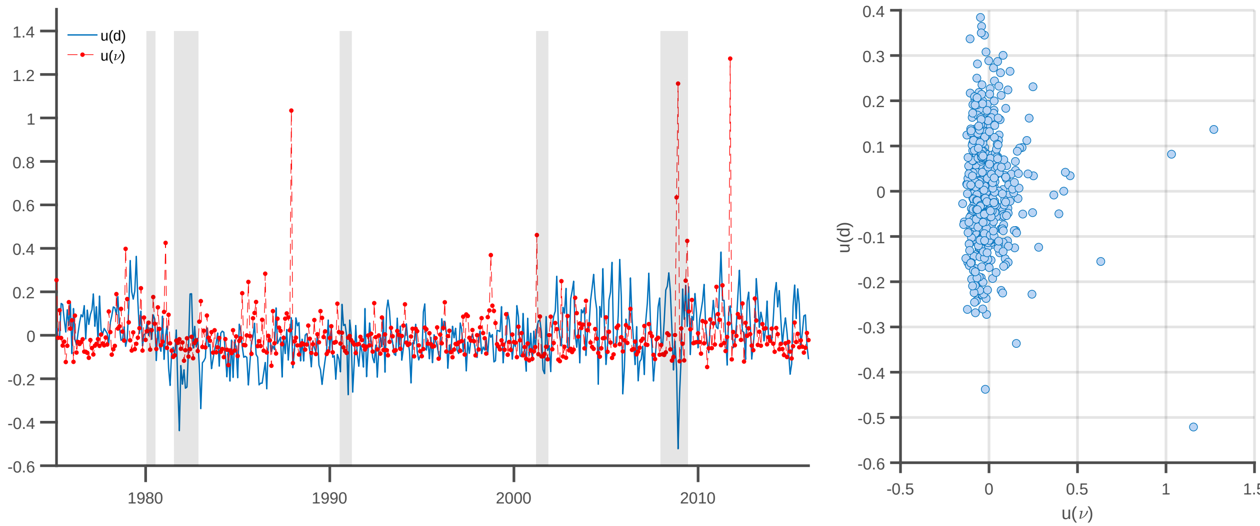
Panel A: US & UK



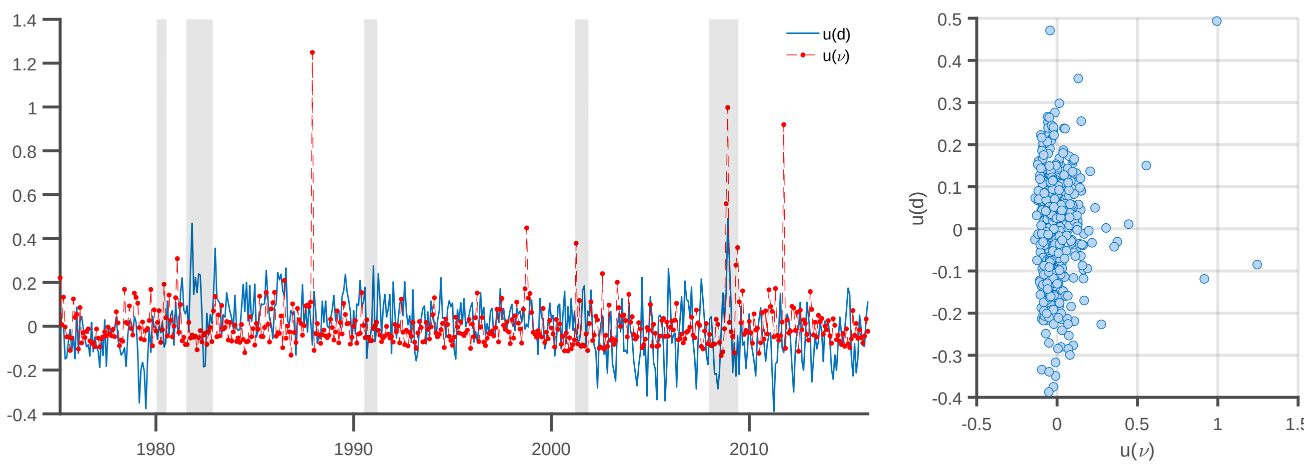
Panel B: US & CH

The figure plots the correlation between the domestic and foreign SDFs derived as in equation (23) in Bakshi, Cerrato, and Crosby (2018) (left panel) and the corresponding increase in volatility with respect to the minimum variance SDF (right panel) for different values of parameter d. The upper panel illustrates the US and UK pair, whereas the lower panel the US and CH pair.

Figure OA-5. International SDF correlation and volatility trade-off



Panel A: US unspanned components



Panel B: CH unspanned components

The figure plots in the left panel the time series of the minimum entropy unspanned component (dashed line with marker) and the unspanned component derived as in Bakshi, Cerrato, and Crosby (2018). The right panel plots a scatter of the two unspanned components. We assume that $d = \nu = 1$. The upper panel depicts domestic US components, whereas the lower panel Swiss counterparts. Data is monthly and runs from January 1975 to December 2015. Gray shaded areas highlight recessions as defined by the NBER.

OA-6 Omitted Tables

The main text presents results where we regress model-free SDFs on proxies of intermediary wealth when SDFs are estimated in integrated markets. In this section, we present regression results when SDFs are estimated in segmented markets (see Section 2.2 in the main paper) in Table 11. Comparing the results, we find them to be very similar across different market structures.

Table OA-7. Financial Intermediaries' Constraints and SDFs in Integrated Markets

The table reports estimated coefficients from regressing foreign minimum entropy SDFs derived in Section 2.3 in the main paper on changes in broker-dealer leverage and VIX or TIV: $M_{t+1} = \alpha + \beta_k\ \Delta\text{bd leverage}_{t+1} + \beta_v\ \Delta\text{VIX}_{t+1}$ or $\Delta\text{TIV}_{t+1} + \epsilon_{t+1}$. t-statistics are calculated according to Newey and West (1987) and reported in parenthesis.

	UK/US	**CH/US**	**JP/US**	**EU/US**	**AU/US**	**CA/US**	**NZ/US**
constant	-0.005	-0.032	-0.003	-0.008	0.000	0.019	-0.023
t-stat	(-0.20)	(-1.34)	(-0.15)	(-0.47)	(0.01)	(0.71)	(-1.12)
bdlev	-0.432	-0.569	-0.355	-0.400	-0.351	-0.403	-0.250
t-stat	(-3.70)	(-4.48)	(-3.18)	(-3.37)	(-1.66)	(-3.99)	(-1.81)
Adj. R^2	18.24%	23.50%	16.03%	22.10%	11.60%	23.61%	11.25%
constant	-0.014	-0.045	-0.011	-0.017	-0.008	0.009	-0.029
t-stat	(-0.52)	(-1.41)	(-0.50)	(-0.81)	(-0.25)	(0.30)	(-1.33)
ΔVIX	0.026	0.030	0.017	0.016	0.011	0.013	0.011
t-stat	(9.10)	(8.53)	(4.45)	(4.67)	(3.36)	(6.83)	(4.18)
Adj. R^2	36.05%	34.88%	20.14%	18.87%	5.47%	12.80%	12.20%
constant	-0.013	-0.044	-0.010	-0.017	-0.007	0.010	-0.029
t-stat	(-0.49)	(-1.41)	(-0.47)	(-0.79)	(-0.24)	(0.31)	(-1.39)
ΔTIV	0.055	0.061	0.027	0.024	0.027	0.020	0.003
t-stat	(3.14)	(2.91)	(1.73)	(2.00)	(2.51)	(1.90)	(0.34)
Adj. R^2	6.08%	5.41%	1.32%	0.87%	0.64%	0.42%	-0.98%
constant	-0.007	-0.034	-0.004	-0.009	0.000	0.018	-0.024
t-stat	(-0.27)	(-1.34)	(-0.21)	(-0.51)	(-0.01)	(0.65)	(-1.12)
bdlev	-0.330	-0.455	-0.290	-0.342	-0.314	-0.359	-0.208
t-stat	(-3.74)	(-4.60)	(-2.42)	(-2.49)	(-1.41)	(-2.98)	(-1.36)
ΔVIX	0.023	0.026	0.015	0.013	0.008	0.010	0.010
t-stat	(8.17)	(8.22)	(4.30)	(4.01)	(2.56)	(5.78)	(3.76)
Adj. R^2	46.33%	49.44%	30.42%	34.48%	14.37%	30.89%	19.58%
constant	-0.003	-0.030	-0.002	-0.007	0.001	0.019	-0.023
t-stat	(-0.12)	(-1.25)	(-0.09)	(-0.41)	(0.05)	(0.74)	(-1.12)
bdlev	-0.442	-0.581	-0.360	-0.405	-0.356	-0.408	-0.251
t-stat	(-4.38)	(-5.31)	(-3.25)	(-3.26)	(-1.63)	(-3.82)	(-1.81)
ΔTIV	0.058	0.065	0.030	0.027	0.029	0.024	0.005
t-stat	(3.34)	(2.93)	(2.09)	(2.18)	(1.78)	(1.73)	(0.62)
Adj. R^2	25.54%	30.24%	18.05%	23.77%	12.75%	24.79%	10.46%

Table OA-8. Financial Intermediaries' Constraints and SDFs in Segmented Markets

The table reports estimated coefficients from regressing US minimum entropy SDFs derived in Section 2.3 in the main paper on changes in broker-dealer leverage and VIX or TIV: $M_{t+1} = \alpha + \beta_k\ \Delta\text{bd leverage}_{t+1} + \beta_v\ \Delta\text{VIX}_{t+1}$ or $\Delta\text{TIV}_{t+1} + \epsilon_{t+1}$. t-statistics are calculated according to Newey and West (1987) and reported in parenthesis.

	US/UK	US/CH	US/JP	US/EU	US/AU	US/CA	US/NZ
constant	0.002	-0.024	-0.014	-0.011	0.001	0.009	-0.015
t-stat	(0.10)	(-1.20)	(-0.73)	(-0.56)	(0.04)	(0.36)	(-0.79)
bdlev	-0.348	-0.313	-0.271	-0.398	-0.383	-0.401	-0.315
t-stat	(-3.47)	(-3.38)	(-2.61)	(-3.05)	(-3.43)	(-3.72)	(-3.24)
Adj. R^2	24.49%	13.33%	10.55%	23.74%	27.54%	27.88%	22.20%
constant	-0.006	-0.031	-0.021	-0.020	-0.008	-0.001	-0.022
t-stat	(-0.25)	(-1.32)	(-0.93)	(-0.81)	(-0.32)	(-0.02)	(-1.05)
ΔVIX	0.013	0.015	0.012	0.015	0.013	0.014	0.011
t-stat	(5.41)	(2.46)	(3.53)	(3.71)	(5.26)	(7.21)	(3.20)
Adj. R^2	19.21%	16.69%	10.98%	16.55%	17.67%	18.24%	14.62%
constant	-0.006	-0.032	-0.021	-0.020	-0.008	0.000	-0.022
t-stat	(-0.24)	(-1.35)	(-0.93)	(-0.81)	(-0.32)	(0.00)	(-1.04)
ΔTIV	0.023	0.004	0.013	0.024	0.023	0.029	0.011
t-stat	(1.91)	(0.40)	(0.77)	(2.42)	(1.99)	(2.43)	(1.02)
Adj. R^2	1.58%	-0.97%	-0.42%	1.10%	1.42%	2.36%	-0.39%
constant	0.001	-0.025	-0.015	-0.012	0.000	0.008	-0.015
t-stat	(0.05)	(-1.20)	(-0.75)	(-0.58)	(0.00)	(0.31)	(-0.83)
bdlev	-0.300	-0.256	-0.226	-0.347	-0.336	-0.351	-0.276
t-stat	(-2.75)	(-2.56)	(-2.00)	(-2.41)	(-2.74)	(-2.85)	(-2.75)
ΔVIX	0.011	0.013	0.010	0.012	0.011	0.011	0.009
t-stat	(4.87)	(2.11)	(3.41)	(2.98)	(4.53)	(6.40)	(2.58)
Adj. R^2	36.83%	25.19%	17.95%	34.00%	38.26%	39.02%	31.08%
constant	0.003	-0.024	-0.014	-0.010	0.002	0.010	-0.014
t-stat	(0.14)	(-1.19)	(-0.69)	(-0.50)	(0.08)	(0.41)	(-0.77)
bdlev	-0.353	-0.314	-0.273	-0.403	-0.388	-0.407	-0.318
t-stat	(-3.53)	(-3.34)	(-2.62)	(-2.94)	(-3.42)	(-3.68)	(-3.26)
ΔTIV	0.026	0.007	0.015	0.028	0.026	0.032	0.013
t-stat	(2.46)	(0.49)	(1.02)	(1.81)	(2.40)	(2.76)	(1.42)
Adj. R^2	27.02%	12.59%	10.46%	25.71%	29.96%	31.38%	22.37%

Table OA-9. Financial Intermediaries' Constraints and SDFs in Segmented Markets

The table reports estimated coefficients from regressing foreign minimum entropy SDFs derived in Section 2.3 in the main paper on changes in broker-dealer leverage and VIX or TIV: $M_{t+1} = \alpha + \beta_k \; \Delta\text{bd leverage}_{t+1} + \beta_v \; \Delta\text{VIX}_{t+1}$ or $\Delta\text{TIV}_{t+1} + \epsilon_{t+1}$. t-statistics are calculated according to Newey and West (1987) and reported in parenthesis.

	UK/US	**CH/US**	**JP/US**	**EU/US**	**AU/US**	**CA/US**	**NZ/US**
constant	-0.009	-0.037	0.003	-0.003	0.003	0.007	-0.025
t-stat	(-0.83)	(-2.06)	(0.19)	(-0.21)	(0.09)	(0.59)	(-1.62)
bdlev	-0.127	-0.305	-0.020	-0.234	-0.206	-0.174	-0.110
t-stat	(-1.23)	(-2.38)	(-0.46)	(-4.54)	(-1.16)	(-3.08)	(-1.08)
Adj. R^2	3.94%	13.10%	-0.86%	9.41%	4.19%	12.47%	3.19%
constant	-0.012	-0.044	0.003	-0.009	-0.002	0.003	-0.027
t-stat	(-1.10)	(-2.09)	(0.17)	(-0.51)	(-0.08)	(0.26)	(-1.89)
ΔVIX	0.014	0.019	0.005	0.009	0.006	0.006	0.005
t-stat	(7.15)	(8.48)	(1.96)	(2.28)	(1.93)	(2.45)	(1.78)
Adj. R^2	31.41%	29.61%	4.00%	7.59%	1.17%	7.83%	3.08%
constant	-0.011	-0.043	0.003	-0.009	-0.002	0.003	-0.027
t-stat	(-1.02)	(-2.01)	(0.17)	(-0.52)	(-0.05)	(0.27)	(-1.93)
ΔTIV	0.032	0.040	0.008	0.006	0.027	0.010	0.007
t-stat	(2.38)	(1.87)	(0.83)	(0.54)	(2.33)	(0.87)	(1.08)
Adj. R^2	6.09%	4.67%	-0.41%	-0.86%	1.10%	-0.06%	-0.59%
constant	-0.010	-0.038	0.003	-0.004	0.002	0.007	-0.025
t-stat	(-1.00)	(-2.15)	(0.17)	(-0.24)	(0.07)	(0.55)	(-1.63)
bdlev	-0.068	-0.228	0.001	-0.201	-0.187	-0.152	-0.093
t-stat	(-1.10)	(-3.05)	(0.02)	(-3.44)	(-1.03)	(-2.50)	(-0.88)
ΔVIX	0.014	0.018	0.005	0.008	0.004	0.005	0.004
t-stat	(6.51)	(8.35)	(1.92)	(1.80)	(1.53)	(2.52)	(1.70)
Adj. R^2	32.07%	36.56%	3.00%	14.12%	4.35%	17.00%	4.99%
constant	-0.008	-0.035	0.003	-0.003	0.004	0.007	-0.024
t-stat	(-0.79)	(-2.00)	(0.21)	(-0.18)	(0.12)	(0.60)	(-1.59)
bdlev	-0.133	-0.313	-0.022	-0.236	-0.211	-0.176	-0.112
t-stat	(-1.48)	(-2.85)	(-0.51)	(-4.52)	(-1.14)	(-3.07)	(-1.06)
ΔTIV	0.033	0.043	0.008	0.008	0.029	0.011	0.008
t-stat	(2.41)	(1.90)	(0.83)	(0.75)	(1.87)	(1.21)	(1.13)
Adj. R^2	10.62%	18.69%	-1.26%	8.76%	5.62%	12.85%	2.74%

Proofs Online Appendix

Proof of Proposition OA-1. Given the underlying market completeness, the asset market view holds and the exchange rate return is given by:

$$X_{t+1} = \frac{M_{f,t+1}}{M_{d,t+1}} = \frac{\omega_{ft+1}/\omega_{ft}}{\omega_{dt+1}/\omega_{dt}} \cdot \frac{\frac{r_{ef}+\phi_{H_f}+\hat{H}_{ft+1}}{r_{ef}+\phi_{H_f}+\hat{H}_{ft}}}{\frac{r_{ed}+\phi_{H_d}+\hat{H}_{dt+1}}{r_{ed}+\phi_{H_d}+\hat{H}_{dt}}} . \tag{31}$$

If follows that conditional on a disaster the exchange rate is:

$$X_{t+1} = \frac{M_{f,t+1}}{M_{d,t+1}} = \exp(g_{\omega_f} - g_{\omega_d})\frac{F_{ft+1}}{F_{dt+1}} \cdot \frac{\frac{r_{ef}+\phi_{H_f}+\frac{1+H_{f*}}{1+H_{ft}}\exp(-\phi_{H_f})\hat{H}_{ft}}{r_{ef}+\phi_{H_f}+\hat{H}_{ft}}}{\frac{r_{ed}+\phi_{H_d}+\frac{1+H_{d*}}{1+H_{dt}}\exp(-\phi_{H_d})\hat{H}_{dt}}{r_{ed}+\phi_{H_d}+\hat{H}_{dt}}} , \tag{32}$$

where g_{ω_i} is a constant productivity growth of country i in no-disaster times. Similarly, conditional on no disasters, the exchange rate is:

$$X_{t+1} = \exp(g_{\omega_f} - g_{\omega_d})\frac{\frac{r_{ef}+\phi_{H_f}+\hat{H}_{ft+1}}{r_{ef}+\phi_{H_f}+\hat{H}_{ft}}}{\frac{r_{ed}+\phi_{H_d}+\hat{H}_{dt+1}}{r_{ed}+\phi_{H_d}+\hat{H}_{dt}}} , \tag{33}$$

which is lognormally distributed, as it is the ratio of two jointly lognormal variables. If follows that conditional on a no disaster event M_d and X are jointly lognormal and we can apply the Black and Scholes (1973)–Garman and Kohlhagen (1983) formula. In summary, we obtain

$$\mathbb{E}[M_{d,t+1}\max(0, 1 - X_{t+1})] = (1 - p_t)BS_d(\sigma_x) + p_t\mathbb{E}_t^D[M_{d,t+1}\max(0, 1 - X_{t+1})] ,$$

where $\mathbb{E}_t^D[\cdot]$ denotes expectations conditional on a disaster event. The explicit computation of the expectation on the RHS yields

$$\mathbb{E}_t^D[M_{d,t+1}\max(0, 1 - X_{t+1})] = \mathbb{E}_t\left[G_{d,t}F_{d,t+1}\max\left(0, 1 - \frac{G_{f,d}F_{f,t+1}}{G_{d,t}F_{d,t+1}}\right)\right] ,$$

where

$$G_{i,t} = \exp(g_{\omega_i}) \cdot \frac{r_{ei} + \phi_{H_i} + \frac{1+H_{i*}}{1+H_{it}}\exp(-\phi_{H_i})\hat{H}_{it}}{r_{ei} + \phi_{H_i} + \hat{H}_{it}} . \tag{34}$$

This concludes the proof. □

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