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## DYNAMIC LEGISLATIVE BARGAINING WITH VETO POWER: THEORY AND EXPERIMENTS

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## Abstract

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JEL Classification: C72, C73, C78, D71, D72, D78

Keywords: Dynamic Legislative Bargaining, Endogenous Status Quo, Veto Power, Markov perfect equilibrium, Laboratory experiments



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# Dynamic Legislative Bargaining with Veto Power: Theory and Experiments*

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June 13, 2020

**Abstract**

In many domains, committees bargain over a sequence of policies and a policy remains in effect until a new agreement is reached. In this paper, I argue that, in order to assess the consequences of veto power, it is important to take into account this *dynamic* aspect. I analyze an infinitely repeated divide-the-dollar game with an endogenous status quo policy. I show that, irrespective of legislators' patience and the initial division of resources, policy eventually gets arbitrarily close to full appropriation by the veto player; that increasing legislators' patience or decreasing the veto player's ability to set the agenda makes convergence to this outcome slower; and that the veto player supports reforms that decrease his allocation. These results stand in sharp contrast to the properties of models where committees bargain over a single policy. The main predictions of the theory find support in controlled laboratory experiments.

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**Keywords:** Dynamic Legislative Bargaining; Distributive Politics; Standing Committees; Endogenous Status Quo; Veto Power; Markov Perfect Equilibrium; Laboratory Experiments

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# 1 Introduction

A large number of important voting bodies grant one or several of their members the right to block decisions even when a proposal has secured the necessary majority—a *veto right*. One prominent example is the United Nations Security Council (UNSC), where any of the five permanent members can prevent the adoption of a proposal with a negative vote. Another example is the U.S. President's ability to veto congressional decisions or the European Parliament's power to block legislation proposed by the European Commission. Additionally, in assemblies with asymmetric voting weights and complex voting procedures, veto power may arise implicitly: this is the case of the U.S. in the International Monetary Fund and the World Bank governance bodies (Leech and Leech 2004).

The existence of veto power raises a frequent concern among practitioners and in the public opinion: although the formal veto right only grants the power to block undesirable decisions, it could *de facto* allow veto members to impose their ideal decision on the rest of the committee. This concern was expressed by the delegates of the smaller countries when the founders of the UN met in San Francisco in June 1945 (Russell 1958), and has been a crucial point of contention in the ongoing discussion over UNSC reform (Blum 2005). A similar debate has arisen regarding the IMF and WB voting weights (Woods 2000).

This paper investigates, theoretically and experimentally, the consequences of veto power in a dynamic bargaining setting where the location of the current status quo policy is determined by the policy implemented in the previous period. In each of an infinite number of periods, one of three legislators—one of whom is a veto player—is recognized to propose the allocation of a fixed endowment. The proposal is implemented if it receives at least two affirmative votes, including the vote of the veto player. Otherwise, the status quo policy prevails and the endowment is allocated as it was in the previous period. In this sense, the status quo policy evolves endogenously. In this setting, I answer three basic questions: To what extent is the veto player able to leverage his veto power into favorable outcomes, both in the short and in the long run? How does this depend on the legislators' patience and the initial agreement? What are the effects of institutional measures meant to reduce the power of the veto player?

Formal models have mostly investigated veto power from the *static* perspective of an *ad hoc* committee bargaining over a single policy: committee members come to the table with an *exogenous* status quo policy and negotiations end when they reach an agreement for a reform. This is not a realistic description of many bargaining environments. Committees are often *dynamic*: their members bargain over a sequence of policies—that is, the committee is *standing*—and a policy remains in effect until a new agreement is reached—that is, the status quo is *endogenous*. These are key features of European legislation on environmental standards or competition policy. Similarly, Supreme Court opinions remain in force unless revisited and some UNSC resolutions—for example, on disputed borders or economic sanctions—and IMF decisions—for example, on its basket of global reserve currencies—hold until explicitly addressed by a new vote. In this dynamic setting, the status quo policy—which determines the bargaining advantage of veto players—is the product of past decisions rather than being exogenous. This suggests that, in order to assess veto members' incentive to uphold the status quo and the balance of power between veto and non-veto members in a committee, it is important to take into account the inherently dynamic process by which the status quo policy is generated. This is exactly what I do in this paper.

In particular, I fully characterize a Markov Perfect Equilibrium (MPE) and prove it exists for any discount factor and any initial divisions of the resources.[1] In this MPE, the veto player is able to gradually converge to his ideal policy, irrespective of the legislators' patience and the initial division of resources. At the same time, it takes an infinite number of periods to converge to this long run outcome. This happens because the bargaining

[1] The only general existence results for dynamic bargaining games apply to settings with stochastic shocks to preferences and the status quo (Duggan and Kalandrakis 2012) or a non-collegial voting rule, i.e., no veto power (Anesi and Duggan 2018). As these features are not present in my model, proving existence is a necessary step of the analysis. Moreover, if I were to consider a model with stochastic shocks to preferences and the status quo, the results in Duggan and Kalandrakis (2012) would guarantee existence of an equilibrium but would not provide a characterization of its outcomes and dynamics, or comparative statics with respect to patience, the initial division of the dollar or recognition probabilities.

power of a patient non-veto player decreases with the share held by the veto player in the status quo. For this reason, the veto player's coalition partner demands a *premium* to vote in favor of an allocation that increases the veto player's share: the veto player has to compensate his coalition partner with a short term gain in stage utility for the long term loss in future bargaining power. This premium is always positive and, thus, some benefits accrue to non-veto players in all periods of the game. The speed of convergence to the veto player's ideal outcome is decreasing in the discount factor of the committee, as the premium demanded by non-veto legislators increases in their patience. Interestingly, when the status quo policy prescribes dispersed benefits, the veto player supports reforms that decrease his allocation, moving the status quo policy further away from his ideal policy. In particular, he is willing to move to an allocation where both he and one non-veto player have a smaller share. This occurs because the future status quo policy affects the future leverage the veto player has when he is the proposer: in this event, he needs to secure the vote of just one non-veto player, and he will, thus, build a coalition with the non-veto player who demands the least. These results are not a feature of a particular equilibrium. In fact, I show that complete appropriation by the veto player is the only absorbing outcome in any *continuous* and *consistent* MPE — that is, in any MPE whose associated strategies are continuous and do not lead to choice behavior inconsistent with standard criteria in decision theory (e,g., the Weak Axiom of Revealed Preferences) — and that this outcome is only reached asymptotically in any *symmetric* MPE.

This dynamic model suggests that giving a committee member the power to oppose does not deprives completely other members of their bargaining power in the short run but it guarantees a strong leverage on long run outcomes. Therefore, I analyze an institutional mechanism to weaken veto power and promote more equitable outcomes for longer: reducing the agenda setting power of the veto player. I show that, as long as the veto player maintains some proposal power, this measure does not prevent complete expropriation of non-veto players in the long run. At the same time, assigning monopolistic agenda setting powers to non-veto players is effective in preventing their complete expropriation, as the veto player cannot improve on his initial allocation. I characterize an MPE to ensure existence and then show that these outcomes are shared by any continuous and consistent MPE of the game.

These results stand in sharp contrast to the properties of models with ad hoc committees. To highlight the effect of dynamic considerations, I analyze a benchmark model, where a committee makes a single decision which determines allocations in the current and all future periods, without the possibility of further reforms. In this setting, outcomes are strongly affected by legislators' patience and the initial division of resources. Moreover, the veto player never supports a proposal decreasing his allocation.

It would be challenging to evaluate the ability of this complex theoretical model to predict empirical behavior using observational data. Instead, I test the predictions from the theoretical analysis with laboratory experiments, which allow a tight control of the decision environment and the evolution of the status quo policy. I consider an experimental design that varies legislators' long run incentives, comparing legislatures with different degrees of patience. The theory is consistent with many features of the data: the vast majority of policies give a positive amount only to a minimal winning coalition (i.e., the veto player and one non-veto player); the allocation to the veto player gradually increases over time, more rapidly in less patient committees; allocations which give most resources to the veto player are the only stable policies; both veto and non-veto proposers expropriate resources from one non-veto player and share the spoils with a coalition partner; veto proposers share resources with the coalition partner more evenly in more patient committees; voting behavior is selfish, with both veto and non-veto legislators more likely to support proposals which are more generous to themselves and less likely to support proposals which are more generous to the other non-proposer.

## 2 Related Literature

This paper contributes primarily to the theoretical literature on the consequences of veto power.[2] In particular, Winter (1996) shows that the share of resources to veto players is decreasing in the cost of delaying an agreement, so that the share of resources to non-veto players declines to zero as the cost of delay becomes negligible, that is, as legislators become infinitely patient. A common limitation of this literature, and the main point of departure with my paper, is the focus on static settings: the legislative interaction ceases once the legislature has reached a decision, and policy cannot be modified after its initial introduction. In this paper, the legislature makes multiple decisions and the status quo policy is not exogenously specified but is rather the product of policy makers' past decisions.

In this sense, this study belongs to a recent literature on legislative policy making with an endogenous status quo and farsighted legislators (Baron 1996, Kalandrakis 2004, 2010, Penn 2009, Diermeier and Fong 2011, Bowen and Zahran 2012, Richter 2014, Dziuda and Loeper 2016). Four papers in this literature explore the consequences of veto power: Duggan et al. (2008), Anesi and Seidmann (2015), Anesi and Duggan (2017) and Diermeier et al. (2017). I discuss each of them in detail below.

Duggan et al. (2008) model the specific institutional details of the American presidential veto and limit their analysis to numerical computations. Anesi and Seidmann (2015) consider unanimous voting, that is, committees where all legislators have the power to oppose. They show that the unique stationary MPE payoffs coincide with the unique stationary SPE payoffs in the equivalent model with ad hoc committees (i.e., à la Baron and Ferejohn 1989). Anesi and Duggan (2017) consider the *finite framework* introduced by Anesi (2010), where the set of feasible policies is finite, legislators have strict preferences and are sufficiently

[2] A large number of studies build on models of legislative bargaining à la Baron and Ferejohn (1989) to examine the role of veto power in specific environments, e.g., the case of the U.S. Presidential veto. See, among, others, Romer and Rosenthal 1978, Matthews 1989, Diermeier and Myerson 1999, McCarty 2000, Groseclose and McCarty 2001, Callander and Krehbiel 2014, Dragu et al. 2014). Less related to the non-cooperative approach of this paper, Lucas (1992) and Ray and Vohra (2015) discuss cooperative solutions for bargaining games with veto players.

patient. They show that, if there is a veto player with positive recognition probability, then starting from any given alternative, there is a unique absorbing point which the equilibrium process transitions to. While they do not characterize this point and their framework is not nested with mine, the equilibrium outcomes I characterize are consistent with their result: there is a unique absorbing outcome, the veto player's ideal policy, which is independent of the initial alternative.[3]

Diermeier et al. (2017, DES) consider a model where legislators allocate a set of indivisible, identical objects among themselves and are sufficiently patient.[4] A key element of their analysis is the notion of a *protocol*, which might be any finite sequence of players (possibly with repetition) ending with a veto player. The protocol to be used is realized at the beginning of each period and prescribes the sequence of proposers within that period. A bargaining period ends as soon as a proposal is accepted or after the last player in the protocol had his proposal rejected. DES focus on *protocol-free MPEs*, that is, MPEs where the function which maps the status quo allocation and the realization of the bargaining protocol into the current period's allocation does not depend on the bargaining protocols. They show that, in any protocol-free MPE, the set of stable allocations coincides with the unique von Neumann-Morgenstern-stable set.[5] In committees with three players, one veto player and simple majority (as in this paper), this set is composed of all allocations which give the same amount to the two non-veto players. I show that, when the objects legislators bargain over are infinitely divisible, mutual protection by non-veto players is not assured and the veto

[3]Diermeier and Fong (2011) study the finite framework with a persistent agenda setter. This is related to a special case of the model with heterogeneous proposal power I investigate in Section 5. They find that legislators without agenda setting power mutually protect each other and the persistent agenda setter is unable to fully expropriate them. See the Appendix for an MPE where full expropriation by a persistent agenda setter is the only stable allocation even when, as in Diermeier and Fong (2011), the set of feasible policies is finite and the legislators' discount factor approaches 1.

[4]Specifically, they assume $\delta > 1 - \frac{1}{b+2}$, where $b$ is the number of available indivisible objects.

[5]In the Appendix, I show that, dropping the requirement of protocol independence and using the protocols commonly assumed in the legislative bargaining literature (and in this paper), can lead to MPEs where full expropriation by the veto player is the only stable allocation even when, as in DES, the set of feasible policies is finite and the legislators' discount factor approaches 1.

player might be able to fully expropriate non-veto players regardless of their patience and the initial division of the dollar. Moreover, in contrast with DES, I characterize both stable outcomes and the transition to these policies for any level of patience, and I offer results on the effect of legislators' patience and recognition probabilities on bargaining outcomes.

Finally, this paper contributes to the literature on laboratory experiments testing models of legislative bargaining (McKelvey 1991, Frechette et al. 2003, Frechette et al. 2005a,b,c, Diermeier and Morton 2005, Diermeier and Gailmard 2006, Frechette 2009, Drouvelis et al. 2010, Miller and Vanberg 2013, 2015, Agranov and Tergiman 2014, Baranski and Kagel 2013, Tergiman 2015, Nunnari and Zapal 2016, Cook and Woon Forthcoming, Fréchette and Vespa 2017). In particular, Wilson and Herzberg (1987), Haney et al. (1992), Kagel et al. (2010), and Agranov and Tergiman (2019) provide experimental evidence on the consequences of veto power in *ad hoc committees*. All this work focuses on static environments where resources are allocated only once.[6] More closely related to this paper, the experiments presented in Battaglini and Palfrey (2012), Battaglini et al. (2012), Agranov et al. (2016), Baron et al. (2017), Agranov et al. (2020), and Battaglini et al. (Forthcoming) investigate models of legislative bargaining with *standing committees*, where resources are allocated repeatedly. As in the current paper, in Battaglini and Palfrey (2012) and Baron et al. (2017), the status quo policy evolves endogenously: if an agreement is not reached, resources are allocated as in the previous period.[7] None of these papers considers the effect of veto power.[8]

[6]Less related to the private value environment of this paper, Guarnaschelli et al. (2000), Goeree and Yariv (2011), Bouton et al. (2017), and Elbittar et al. (Forthcoming) study the consequences of veto power for the aggregation of information in common value environments.

[7]In Battaglini et al. (2012), Agranov et al. (2016), Agranov et al. (2020), and Battaglini et al. (Forthcoming) the status quo policy is exogenous and time-invariant. The linkage between periods is represented by the stock of a durable public good the committee can invest in (Battaglini et al. 2012, Agranov et al. 2016), the allocation of proposal power (Agranov et al. 2020), or the available budget (Battaglini et al. Forthcoming).

[8]The sole exception is Battaglini et al. (2012) who consider a treatment with unanimous voting. Contrary to the divide-the-dollar game with endogenous status quo studied in this paper, in Battaglini et al. (2012), resources can be allocated both to private transfers and to investment in a durable public good; and the status quo policy does not depend on past decisions but is always zero investment in the public good and an even share of the budget to each committee member's private consumption.

# 3 Model and Equilibrium Notion

Three agents repeatedly bargain over a legislative outcome $\mathbf{x}^t$ for each period of an infinite horizon, $t = 1, 2, \ldots$. One of the three agents is endowed with the power to veto any proposed outcome in every period. I denote the veto player with the subscript $v$ and the two non-veto players with the subscript $j = \{1, 2\}$. The possible outcomes in each period are all possible divisions of a fixed resource among the three players.

**The Bargaining Protocol.** At the beginning of each period, one agent is randomly selected to propose a new policy, $\mathbf{z} \in \Delta$. Each agent has the same probability of being recognized as policy proposer, that is $\frac{1}{3}$. This proposal is voted up or down by the committee. A proposal passes if it gets the support of the veto player and at least one other committee member. If a proposal passes, $\mathbf{x}^t = \mathbf{z}$ is the implemented policy at $t$. If a proposal is rejected, the policy implemented is the same as it was in the previous period, $\mathbf{x}^t = \mathbf{x}^{t-1}$. Thus, the previous period's decision, $\mathbf{x}^{t-1}$, serves as the *status quo policy* in period $t$. The initial status quo $\mathbf{x}^0$ is exogenously specified.

**Stage Utilities.** Agent $i$ derives stage utility $u_i = x_i$ from the implemented policy $\mathbf{x}^t$. Players discount the future with a common factor $\delta \in [0, 1)$, and their payoff in the game is given by the discounted sum of stage payoffs.

**Strategies and Equilibrium Notion.** In what follows, I look for a *stationary Markov perfect equilibrium* (MPE). In this type of equilibrium, strategies depend only on payoff-relevant effects of past behavior (Maskin and Tirole 2001). I define the state in period $t$ as the status quo policy, or the previous period's decision, $\mathbf{s}^t = \mathbf{x}^{t-1}$. In an MPE, agents behave identically in different periods with the same state $\mathbf{s}$, even if that state arises from different histories. In this dynamic game, the *expected utility* of agent $i$ from the allocation implemented in period $t$ does not only depend on his stage utility, but also on the discounted expected flow of future stage utilities, given a set of strategies. The *continuation value*, $v_i(\mathbf{s})$, is the expected payoff of legislator $i$ when the state is $\mathbf{s}$ before the proposer is selected. We can write the expected utility of legislator $i$ from the allocation implemented in period $t$, $\mathbf{x}^t$, as:

$$U_i(\mathbf{x}^t) = x_i^t + \delta v_i(\mathbf{x}^t)$$

As is standard in models of bargaining, I require that agents use stage-undominated voting strategies—that is, they vote yes if and only if their expected utility from the status quo is not greater than their expected utility from the proposal.[9]

# 4 Equilibrium Analysis

In this Section, I propose natural conditions on strategies, and show that these conditions define an equilibrium. First, equilibrium proposals involve *minimal winning coalitions*, such that at most one non-veto player receives a positive amount in each period. Second, the proposer proposes the *acceptable* allocation—that is, an allocation that defeats the status quo—that maximizes his current share of the dollar. The set of allocations each agent prefers to the status quo policy changes with the discount factor, as legislators take more or less into account the impact of the current allocation on future periods. Not surprisingly, this has important consequences for the dynamics of the game.

I first discuss the case when the proposer is a non-veto player, and then the case when the proposer is a veto player. To help with the exposition, I partition the space of possible divisions of the dollar into two subsets, $\overline{\Delta}$ and $\Delta \backslash \overline{\Delta}$. Define $\overline{\Delta} \subset \Delta$ as the set of states $\mathbf{x} \in \Delta$ in which at least one non-veto legislator gets zero. Define the *demand* of legislator $i$, $d_i$, as the minimum amount he requires to accept a proposal $\mathbf{x} \in \overline{\Delta}$.

## 4.1 Non-Veto Proposer

When a non-veto player is proposing, he needs to secure the vote of the veto player in order to change the current status quo. If the non-veto proposer wants to maximize his current share of the dollar, he will propose the veto player's demand to the veto player,

[9]This restriction rules out uninteresting equilibria where voting decisions constitute best responses solely due to the fact that a single vote cannot change the outcome.

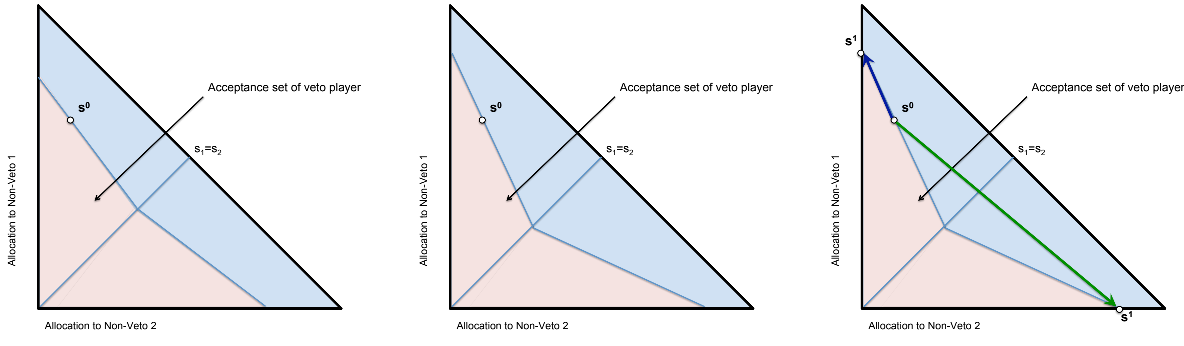
and assign the remainder to himself. Therefore, to characterize the equilibrium proposal strategies of a non-veto player, we need to identify the acceptance set of the veto player.

A perfectly impatient veto player values only his current allocation and, thus, only supports proposals that give him as much as the status quo or more. On the other hand, a patient veto player is not indifferent between all states in which he receives the same allocation, and might be better off with allocations that reduce his current share when these decrease his future coalition building costs. In particular, he is willing to move from an interior allocation where he gets a higher share, to an allocation where both he and one non-veto player have a smaller share. This occurs because the future status quo policy affects the future leverage the veto player has when he is the proposer. In this event, he needs to secure the vote of just one non-veto player, and he will, thus, build a coalition with the non-veto player who demands the least. As shown below, the demand of each non-veto player is an increasing function of what he gets in the status quo and, therefore, a veto player's coalition building costs with status quo $\mathbf{s}$ are a positive function of $\min\{s_1, s_2\}$. Thus, a veto player prefers an allocation $\mathbf{s}'$ where he gets $s'_v$ and $\min\{s'_1, s'_2\} = s'_{nv}$ to an alternative allocation $\mathbf{s}''$ with $s''_v = s'_v$ but $\min\{s''_1, s''_2\} = s''_{nv} > s'_{nv}$.

Figure 2 depicts the acceptance set of a patient veto player for two different values of $\delta > 0$. The vertical dimension represents the share to (non-veto) player 1, while the horizontal dimension represents the share to (non-veto) player 2. The remainder is the share that goes to the veto player. In the Appendix, I characterize the amount the veto player demands to accept a proposal that brings the status quo into $\overline{\Delta}$—where one non-veto player gets nothing—as:

$$d_v = \max\{s_v - \tfrac{\delta}{3-2\delta}\underline{s_{nv}}, 0\} \tag{1}$$

where $\underline{s_{nv}}$ is the allocation of the poorer non-veto player in the status quo. The reduction accepted by the veto player increases with his discount factor $\delta$ and the share to the poorer




(a) (b) (c)

Figure 1: Veto's acceptance set and non-veto's proposal strategies for state $\mathbf{s}^0$: (a) $A_v(\mathbf{s}^0)$ when $\delta = \delta_1 > 0$; (b) $A_v(\mathbf{s}^0)$ $\delta = \delta_2 > \delta_1$, (c) equilibrium proposal of non-veto 1 (blue arrow) and non-veto 2 (green arrow).

non-veto player $\underline{s_{nv}}$. A veto player does not accept any division of the dollar that gives him less than the status quo when $\mathbf{s} \in \overline{\Delta}$. Note also that the reduction a veto player is willing to accept could be more than what he has in the status quo, in which case his demand is bounded below by 0.

The non-veto proposer proposes the acceptable policy that maximizes his current allocation. These are depicted in the right-most panel of Figure 2. A non-veto proposer completely expropriates the other non-veto player, gives the veto player his demand, and allocates the remainder to himself. When the state is in $\overline{\Delta}$, the non-veto proposer can only get $1 - s_v$, but when the state is in $\Delta \backslash \overline{\Delta}$ he can extract an higher amount, namely $1 - d_v$.

## 4.2 Veto Proposer

When the veto player desires to pass a proposal with a minimal winning coalition, he is not bound to include any specific legislator. Thus, he selects the legislator who accepts the highest increase to the veto player's share—that is, the legislator with the lowest demand—as his coalition partner. When legislators are perfectly impatient, the veto player builds a coalition with the *poorer* non-veto player—the non-veto player who receives the least in the status quo—giving him as much as he is granted by the status quo. A perfectly impatient non-veto player accepts this proposal. A patient non-veto player does not.

In fact, the bargaining power of a patient non-veto player decreases with the share held

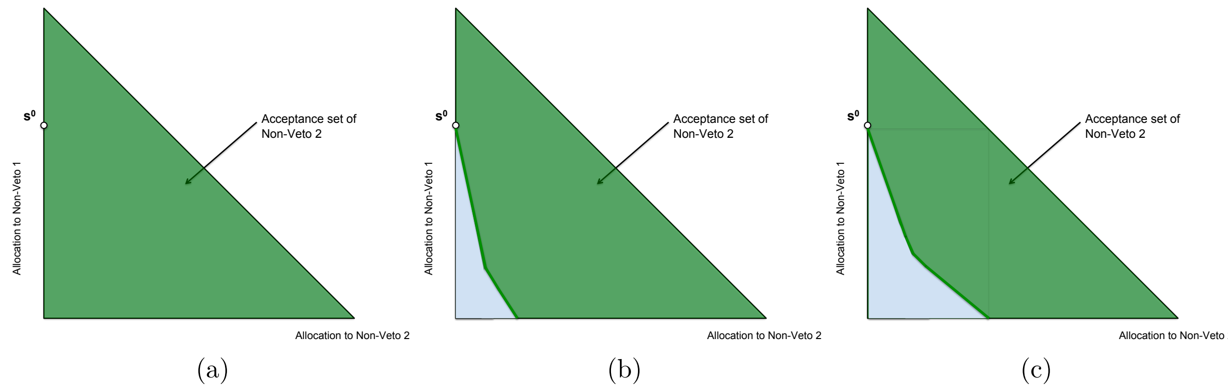



Figure 2: Non-veto 2's acceptance sets for state $\mathbf{s}^0$ where $s_1 > s_2$: (a) $A_2(\mathbf{s}^0)$ when $\delta = 0$, (b) $A_2(\mathbf{s}^0)$ when $\delta = \delta_1 > 0$; (c) $A_2(\mathbf{s}^0)$ $\delta = \delta_2 > \delta_1$.

by the veto player in the status quo. For this reason, a patient non-veto player prefers an allocation $\mathbf{s}' \in \overline{\Delta}$ where he gets $s'_j = 0$ and the veto player gets $s'_v$ to an alternative allocation $\mathbf{s}'' \in \overline{\Delta}$ with $s''_j = s'_j$ but $s''_v > s'_v$. The difference between these allocations arises when he is recognized in $t+1$, as he will gain the support of the veto player only for proposals that give him no more than $1 - s_v$. Figure 3 depicts the acceptance set of the poorer non-veto player for a state $\mathbf{s}^0 \in \overline{\Delta}$ and three increasing values of the discount factor.

With $\delta > 0$, the veto player's coalition partner demands a *premium* to vote in favor of an allocation that increases the veto player's share: the veto player has to compensate his coalition partner with a short term gain in stage utility for the long term loss in future bargaining power. The Appendix shows that the demand of the poorer non-veto player for states $\mathbf{s} \in \overline{\Delta}$ is:

$$d_{nv} \quad = \tfrac{\delta}{3-2\delta}\overline{s_{nv}} \tag{2}$$

where $\overline{s_{nv}}$ is the allocation to the richer non-veto player in the status quo.

Some properties of $d_{nv}$ are worth noting. First, $d_{nv}$ is smaller than $\overline{s_{nv}}$ for any $\delta \in [0, 1)$. This means that, as long as $\delta < 1$, the veto proposer can increase his share, as he can assign himself $1 - d_{nv} > s_v = 1 - \overline{s_{nv}}$. Since the veto player does not accept any reduction to his allocation once $\mathbf{s} \in \overline{\Delta}$, the allocation to the veto player displays a *ratchet effect*: it can only

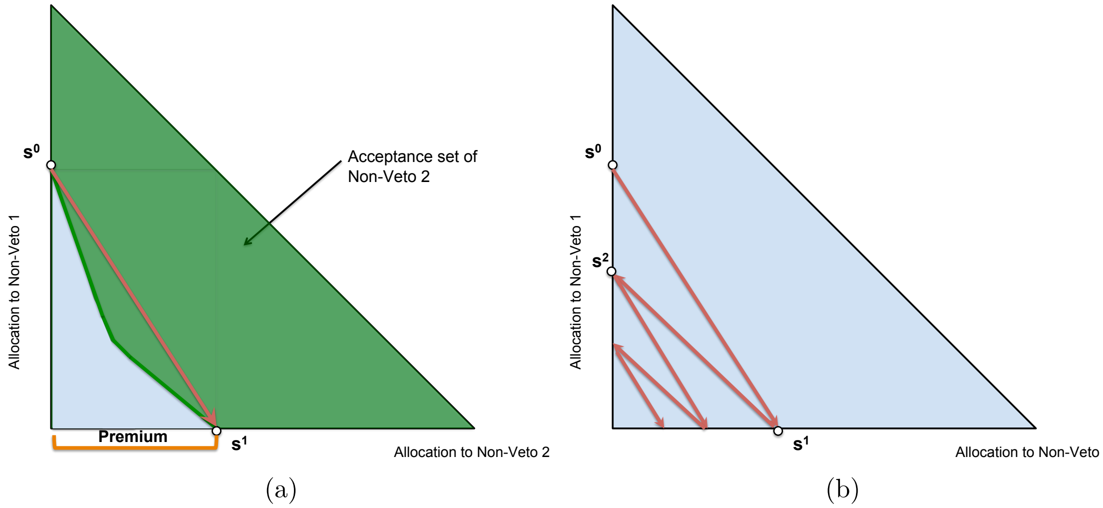



Figure 3: Veto's equilibrium proposal strategy for state $\mathbf{s}^0$ and $\delta > 0$.

stay constant or increase.

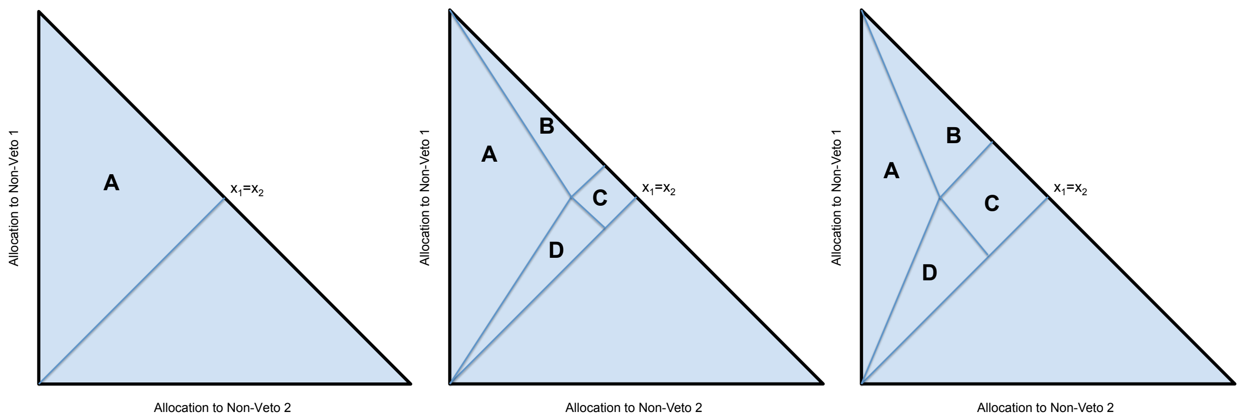
Second, the premium paid by the veto player to his coalition partner is monotonically increasing in $\delta$ and linearly increasing in $\overline{s_{nv}}$: $d_{nv}$ converges to $\overline{s_{nv}}$ as $\delta$ converges to 1, and to 0 as $\delta$ converges to 0. This implies that the ratchet effect is slower with more patient legislators. With $\delta = 0$, the premium is 0 and the veto player is able to steer the status quo policy to his ideal point in at most two proposals, as he can pass any $\mathbf{x} \in \Delta$ when the poorer non-veto player has zero. With $\delta \in (0, 1)$, the premium is always positive and convergence to the veto player's ideal point happens only asymptotically. Figure 4(b) shows how the state would evolve when the veto player always proposes.

When the allocations to the two non-veto players are close, the veto player mixes between coalition partners. This is necessary to guarantee that the proposer's choice of a partner is a best response to what they demand: if the veto player always picked the poorer non-veto player as coalition partner, this player would become the most expensive coalition partner.

## 4.3 Theoretical Results

Proposition 1 provides a summary of the discussion above:

**Proposition 1** *For any $\delta \in [0, 1)$ and any $\mathbf{s}^0 \in \Delta$ there exists an MPE such that:*




(a) (b) (c)

Figure 4: Partition of $\Delta$ into regions with different equilibrium strategies for allocations where $s_1 \geq s_2$: (a) $\delta$=0; (b) $\delta = \delta_1 > 0$; (c) $\delta = \delta_2 > \delta_1$. In A and B, veto proposer builds a coalition with poorer non-veto player; in C and D, veto proposer mixes between coalition partners; in B, and C veto player is willing to accept nothing when he is not proposing.

- *All proposals give a positive allocation at most to a minimal winning coalition.*
- *For some* $\mathbf{s} \in \Delta \backslash \overline{\Delta}$, *the veto proposer mixes between possible coalition partners. For the remaining* $\mathbf{s} \in \Delta$, *the veto proposer proposes* $d_{nv}$ *to the poorer non-veto player.*
- *For all* $\mathbf{s} \in \Delta$, *the non-veto proposers propose* $d_v$ *to the veto player.*
- *For all* $\mathbf{s} \in \overline{\Delta}$, $d_v = s_v$ *and* $d_{nv} \geq s_{nv}$, *that is, the veto player demands his status quo allocation, non-veto players demand weakly more.*

In the Appendix, I give the exact statement of the equilibrium proposal and voting strategies for each region of $\Delta$, and show that these strategies and the associated value functions constitute part of an MPE. Moreover, I show that the MPE from Proposition 1 is continuous in $\delta$ and $\mathbf{s}$, meaning that a small change in the discount factor or a small change in the status quo imply a small change in proposal strategies and, by extension, to the equilibrium transition probabilities. An immediate implication of the continuity of transition probabilities is the fact that continuation values and expected utilities are continuous.

**Proposition 2** *The continuation value functions,* $V_i$, *and the expected utility functions,* $U_i$, *induced by the equilibrium in Proposition 1 are continuous.*

Proposition 3 discusses the long run implications of the equilibrium from Proposition 1:

**Proposition 3** *For any* $\delta \in [0, 1)$ *and any* $\mathbf{s}^0 \in \Delta$*, there exists an MPE such that the status quo policy eventually gets arbitrarily close to the veto player's ideal policy, that is,* $\forall\ \varepsilon > 0$ $\lim_{t\to\infty} Pr[x_v^t > 1 - \epsilon] = 1$.

Proposition 4 addresses the speed of convergence to this long run outcome:

**Proposition 4** *In the MPE characterized in the proof of Proposition 1, if legislators are impatient,* $\delta = 0$*, it takes at most two rounds of proposals by the veto player to converge to his ideal policy. If legislators are patient,* $\delta \in (0, 1)$*, convergence to this state does not happen in a finite number of periods, and the higher the discount factor the slower the convergence.*

# 5 Heterogeneous Recognition Probabilities

In some settings, the veto player is an outsider with reduced ability to set the agenda—for example, the U.S. President—while in others the veto player has a privileged position to set the agenda—for example, committee chairs in the U.S. Congress. In this Section, I relax the assumption of symmetric recognition probabilities and explore an institutional measure that could, in principle, reduce the leverage of the veto player and promote more equitable outcomes: manipulating the recognition probability of the veto player. In particular, I characterize sufficient conditions on the discount factor and the recognition probabilities under which the veto player is able to eventually appropriate all resources for any initial status quo. In these cases, the speed of convergence to this outcome is increasing in the probability the veto player sets the agenda and decreasing in the legislators' patience.

Denote by $p_v$ the probability the veto player is recognized as the proposer in each period, with $p_{nv} = \frac{1-p_v}{2}$ being the probability a non-veto player is recognized. Proposition 5 shows that, when $p_v \in (0, 1/2]$ or $p_v = 1$, there exists an MPE of this dynamic game that has the same features as the one characterized in Proposition 1: all proposals entail positive distribution to, at most, a minimal winning coalition and the status quo allocation converges to the ideal point of the veto player irrespective of the discount factor and the initial allocation of resources. When $p_v \in (0.5, 1)$, an MPE with these features exists as long as the discount factor is below a threshold, $\overline{\delta}(p_v)$.

**Proposition 5** *Consider the game with heterogenous recognition probabilities. If* $p_v \in \left(0, \frac{1}{2}\right] \cup \{1\}$*, then for any* $\delta \in [0, 1)$ *and any* $\mathbf{s}^0 \in \Delta$*, there exists an MPE such that the status quo policy eventually gets arbitrarily close to the veto player's ideal point. If* $p_v \in \left(\frac{1}{2}, 1\right)$*, then for any* $\delta \leq \overline{\delta}(p_v)$ *and any* $\mathbf{s}^0 \in \Delta$*, there exists an MPE such that the status quo policy eventually gets arbitrarily close to the veto player's ideal point.*[10]

As in the case with even recognition probabilities, this result hinges on the fact that the veto player is able to move the status quo to $\overline{\Delta}$—the set of allocations where at least one non-veto player gets zero—as soon as he proposes and that, once an allocation is in this absorbing set, the veto player is able to increase his share whenever he proposes. When the sufficient conditions in Proposition 5 are met, the veto player's proposal power influences the speed of convergence to this policy both directly and indirectly. The direct effect is given by the change in the frequency at which the veto player can increase his allocation—which happens only when he proposes. The indirect effect is given by the change in the amount the veto player can extract from the non-veto players when he proposes. The probability of recognition of the veto player affects the continuation value of the status quo policy for all legislators, and, thus, it affects how much they demand to support a policy change.

Consider status quo allocations where one non-veto player has nothing. As $p_v$ increases, the poorer non-veto player is less likely to be recognized and, thus, he is less concerned about the endowment of the richer non-veto player, which represents the resources he can appropriate when he has the power to set the agenda. This reduces the premium he demands to support an allocation that increases the share to the veto player. In the proof of Proposition

[10]The threshold $\overline{\delta}(p_v)$ is characterized in the proof of Proposition 5. This MPE does not exist only for a small fraction of parameters: the lowest value of $\overline{\delta}$ is 0.875, reached when $p_v \approx 0.857$. Thus, for $\delta \in [0, 0.875)$, the MPE exists for any $p_v \in (0, 1]$ and any $\mathbf{s}^0 \in \Delta$. For a discussion of the intuition behind the irregular shape of the existence set, see the proof of Proposition 5.

5, I show that this premium is monotonically decreasing in $p_v$. When $p_v = 1$, the poorer non-veto player does not demand a premium and supports any allocation. Thus, with a higher $p_v$, the veto player is more likely to increase his share in each period, and he can also extract more from the non-veto players when he is the proposer.

The limit case where the veto player does not have any chance to set the agenda is effective in avoiding full expropriation of non-veto players and, thus, merits discussion. Consider the dynamic bargaining game with $p_v = 0$. In the MPE I fully characterize in the proof of Proposition 5, each non-veto proposer offers to the veto player his status quo allocation and takes the remainder for himself. As a consequence, the veto player receives the amount prescribed by the initial agreement in every period of the game and he is never able to increase his allocation. Proposition 6 summarizes this discussion:

**Proposition 6** *In the MPE from Proposition 5, (a) if $\delta = 0$ or $p_v = 1$, it takes at most two rounds of proposals by the veto player to converge to his ideal policy; (b) if $\delta \in (0,1)$ and $p_v \in (0,1)$, convergence to this state does not happen in a finite number of periods and convergence is slower the higher is $\delta$ and the lower is $p_v$; (c) if $p_v = 0$, the veto player receivers the amount prescribed in the initial status quo $s_v^0$, in all periods.*

In spite of this discontinuities long run outcomes and the speed of convergence, the MPE from Proposition 5 is continuous in $\delta$, $\mathbf{s}$ and $p_v$, meaning that, in the set of parameters for which the equilibrium exists, a small change in the discount factor, the status quo or recognition probabilities imply a small change in legislators' value functions and expected utilities.

**Proposition 7** *The continuation value functions, $V_i$, and the expected utility functions, $U_i$, induced by the equilibrium in Proposition 5 are continuous.*

# 6 Uniqueness of Equilibrium Outcomes

The legislative game studied in this paper is an infinite horizon dynamic game with a plethora of subgame perfect Nash equilibria and, thus, an equilibrium-selection issue. As standard in the literature on dynamic legislative bargaining, I do not consider equilibria involving stage-dominated or non-stationary strategies.[11] Even so, it is still possible that other MPEs of this game exist.[12] In this Section, I show that the results presented in Sections 4 and 5 are not a feature of a particular equilibrium but hold in larger classes of equilibria. Before I state the formal results, I need to introduce some additional definitions:

**Definition 1 (Symmetric MPE)** *For any* $\mathbf{s} = \{s_v, s_1, s_2\} \in \Delta$*, let* $\tilde{\mathbf{s}}(\mathbf{s}) = \{s_v, s_2, s_1\}$ *be the policy permuting the allocations to non-veto players and keeping the same allocation to the veto player. We say that an MPE is symmetric if* $\mu_i[\mathbf{x}|\mathbf{s}] = \mu_{-i}[\tilde{\mathbf{x}}(\mathbf{x})|\tilde{\mathbf{s}}(\mathbf{s})]$*,* $\mu_v[\mathbf{x}|\mathbf{s}] = \mu_v[\tilde{\mathbf{x}}(\mathbf{x})|\tilde{\mathbf{s}}(\mathbf{s})]$*,* $A_i(\mathbf{s}) = A_{-i}(\tilde{\mathbf{s}}(\mathbf{s}))$*, and* $A_v(\mathbf{s}) = A_v(\tilde{\mathbf{s}}(\mathbf{s}))$*, where* $i = \{1,2\}$*,* $-i = \{1,2\} \setminus \{i\}$*,* $\mu_i[\mathbf{x}|\mathbf{s}]$ *is the probability player* $i$ *proposes* $\mathbf{x}$ *when the status quo is* $\mathbf{s}$ *and* $A_i(\mathbf{s})$ *is the set of policies that player* $i$ *weakly prefers to status quo* $\mathbf{s}$*.*

**Definition 2 (Continuous MPE)** *We say that an MPE is continuous if the continuation value functions induced by equilibrium strategies,* $V_i(\mathbf{s})$*, are continuous at any policy* $\mathbf{s} \in \Delta$ *for any* $i = \{v, 1, 2\}$*.*

**Definition 3 (Consistent MPE)** *Fix an MPE. Let* $A(\mathbf{s})$ *be the set of proposals which beat status quo* $\mathbf{s}$ *and let* $\mathbf{x}^a(\mathbf{s})$ *be the equilibrium proposal of* $a \in \{1,2\}$ *given status quo* $\mathbf{s}$*. We say that a proposal strategy is consistent if, for any pair of status quo policies* $\mathbf{s}$ *and* $\mathbf{s}'$ *and any proposer* $a \in \{1,2\}$*,* $\mathbf{x}^a(\mathbf{s}) \in A(\mathbf{s}')$ *and* $\mathbf{x}^a(\mathbf{s}) \neq \mathbf{x}^a(\mathbf{s}')$ *implies* $\mathbf{x}^a(\mathbf{s}') \notin A(\mathbf{s})$*. A consistent MPE is an MPE in consistent proposal strategies.*

[11] In large legislatures, non-stationary strategies that depend on the history are implausible because of legislators' turnout, extraordinary commitment, coordination, and/or communications requirements.

[12] For example, Anesi and Duggan (2018) show that, in a large class of models of dynamic bargaining with an endogenous quo, MPEs are indeterminate. While their result does not apply to the framework of this paper (because the voting rule is collegial), it highlights the fact that constructive techniques may fail to identify other plausible outcomes. In the Appendix, I consider the game with homogeneous recognition probabilities and characterize a class of MPEs where an allocation (or a pair of allocations) giving a positive amount to all players is stable. The set of such allocations is non-empty for any $\delta > 0.68$ and grows with $\delta$. As $\delta$ goes to 1, there exists an MPE where $Y = \{(x_v, x_1, x_2), (x_v, x_2, x_1\}$ is an irreducible absorbing set for any $\mathbf{x} \in \Delta$ such that $\min\{x_v, x_1, x_2\} > 0$ and $x_1 + x_2 > 0.25$. Moreover, in subsequent work, Sethi and Verriest (2019) show that, when agents are sufficiently patient, the veto player holds sufficient proposal power, and the initial allocations to non-veto players are sufficiently similar, there exists an MPE where the veto player is only able to partially expropriate non-veto players.

**Definition 4 (Irreducible Absorbing Set)** *Fix an MPE. The set $Y \subseteq \Delta$ is an absorbing set if once the committee implements policy $\mathbf{y} \in Y$, it never transitions to policy $\mathbf{x} \notin Y$ with positive probability. The set $Y$ is an irreducible absorbing set if $Y$ is an absorbing set and there does not exist a proper subset $Y' \subset Y$ such that $Y'$ is an absorbing set.*

Continuity requires that a small change in the status quo implies a small change in proposal strategies, value functions and expected utilities. Symmetry requires that non-veto player's (the veto player's) proposal and voting strategies are symmetric (identical) for each pair of states permuting the allocations to non-veto players. Consistency is adapted from Forand (2014) and requires that proposal strategies not lead to choice behavior that is inconsistent with standard criteria in decision theory (e.g., the Weak Axiom of Revealed Preferences): consider the policy chosen under status quo $\mathbf{s}$, $\mathbf{x}^a(\mathbf{s})$; if the same policy is still available under a new status quo $\mathbf{s}'$ but it is not chosen, that is, $\mathbf{x}^a(\mathbf{s}) \in A(\mathbf{s}')$ and $\mathbf{x}^a(\mathbf{x}) \neq \mathbf{x}^a(\mathbf{x}')$, then, it must be the case that the policy chosen under this new status quo was not available under the old status quo, that is, $\mathbf{x}^a(\mathbf{s}') \notin A(\mathbf{s})$.[13]

Proposition 8 considers the broader class of subgame perfect Nash equilibria (a superset of MPEs) and shows that no allocation other than full appropriation by the veto player can be stable when the discount factor is sufficiently low.

**Proposition 8** *In any subgame perfect Nash equilibrium of the dynamic legislative bargaining game with heterogeneous recognition probabilities, (a) full appropriation by the veto player is absorbing for any $\delta \in [0,1)$; (b) $V_v(\mathbf{x}) \geq \frac{x_v}{1-\delta}$; (c) $V_i(\mathbf{x}) \geq \frac{p_{nv}x_i}{1-\delta p_{nv}}$ for $i = \{1,2\}$; and (d) no allocation other than full appropriation by the veto player is absorbing for any $\delta \in \left[0, \frac{1}{2-p_{nv}}\right)$.*

**Corollary 1** *Consider the dynamic legislative bargaining game with equal recognition probabilities. If $\delta < 3/5$, there is no subgame perfect Nash equilibrium where an allocation other than full appropriation by the veto player is absorbing.*

[13] These conditions have already been used to refine equilibria of dynamic games of elections and bargaining (see, e.g., Battaglini and Coate 2007, 2008, Diermeier and Fong 2011, Battaglini et al. 2012, Forand 2014).

Proposition 9 considers the class of stationary Markov perfect equilibria. It shows that, in any irreducible absorbing set which includes more than one policy, the veto player must receive the same allocation in all these policies.[14]

**Proposition 9** *Consider any MPE of the dynamic legislative bargaining game with* $p_v \in [0,1]$*. For any set* $Y \subseteq \Delta$*,* $Y$ *is an irreducible absorbing set with respect to this MPE only if all elements of* $Y$ *give the same allocation to the veto player.*

Proposition 10 imposes symmetry and shows that one of the key features of the MPEs from Proposition 1 and 5 extends to any MPE in this broader class: as long as non-veto players have some proposal power and care, even minimally, about future policies, convergence to full appropriation by the veto player is only asymptotic in any symmetric MPE.

**Proposition 10** *In any symmetric MPE of the dynamic legislative bargaining game, (a)* $V_i(\mathbf{x}) \geq \frac{p_i \max\{x_1,x_2\}}{1-\delta p_i}$ *for* $i = \{1,2\}$*; and (b) for any* $\delta \in (0,1)$*,* $\mathbf{s}^0 \neq \{1,0,0\}$*, and* $p_v \in (0,1)$*, the committee does not converge in finite time to full appropriation of the dollar by the veto player.*

This is the intuition behind the proof, which is presented in the Appendix. From Proposition 8(a), we know that $\mathbf{z} = (1,0,0)$ is an absorbing outcome in any subgame perfect Nash equilibrium (and, thus, in any MPE). Therefore, proposal $\mathbf{z}$ gives an expected utility of 0 to either veto player. This means that such a proposal defeats a status quo $\mathbf{s} \neq \mathbf{z}$ only if $s_j + \delta V_j(\mathbf{s}) \leq 0$ for at least one non-veto player $j = \{1,2\}$. Since allocations are non-negative, a non-veto player whose status quo allocation is positive will never support a proposal which leads to $\mathbf{z}$. Thus, $\mathbf{z}$ defeats the status quo only if there is at least one non-veto players for which $s_j = 0$ and $V_j(\mathbf{s}) = 0$. Proposition 10(a) shows that, as long as non-veto players are not perfectly impatient and have some proposal power, $V_j(\mathbf{s})$ is strictly positive for any $\mathbf{s} \neq \mathbf{z}$. The intuition is that, when he proposes, a non-veto player can swap his allocation

[14] Note that this is the continuous policies analogous of what Anesi and Duggan (2017) showed for a dynamic legislative bargaining game with a finite set of feasible policies and strict utilities.

with the other non-veto player's allocation and give the same as in the status quo to the veto player. In a symmetric MPE, this proposal will pass with the support of the veto player. As a consequence, no proposer will be able to reform a status quo policy which allocates a positive amount to some non-veto player with a policy which allocates zero resources to non-veto players.

Proposition 11 shows that, in continuous and consistent MPEs, long run equilibrium outcomes are insensitive to the distribution of agenda setting power, the discount factor and the initial division of the dollar, as long as the veto player has some ability to set the agenda.

**Proposition 11** *In any consistent and continuous MPE of the game with $\delta \in [0, 1)$, $\mathbf{s}^0 \in \Delta$, and $p_v \in (0, 1]$, the unique irreducible absorbing set is a singleton and its only element is full appropriation of the dollar by the veto player.*

The proof is presented in the Appendix but I sketch here the argument. Above I established that any irreducible absorbing set of an MPE must be composed by policies giving the same amount, $k \in [0, 1]$, to the veto player (Proposition 9) and that $Y = \{(1, 0, 0)\}$ is an irreducible absorbing set (Proposition 8(a)) in any MPE. Thus, to prove the statement from Proposition 11, it is sufficient to show that any set of policies giving the same amount $k < 1$ to the veto player cannot be an irreducible absorbing set. I then show that for any status quo policy in the irreducible absorbing set of a consistent MPE, each player proposes the same policy (Lemma 6) and the equilibrium proposals of non-veto players are minimal winning coalitions (Lemma 7). This allows me to characterize the continuation value each non-veto player derives from any policy in the irreducible absorbing set as a function of $\delta$, $p_v$ and two parameters. At this point, I can derive a contradiction. Let $Y$ be an irreducible absorbing set and assume $k < 1$. Since $U_v(\mathbf{y}) = \frac{k}{1-\delta}$ for any $\mathbf{y} \in Y$ and, in any MPE, $U_v(\mathbf{s}) \geq \frac{s_v}{1-\delta}$, the veto player would be strictly better off moving to a policy outside of $Y$ where he receives a higher allocation. Using the continuation values of non-veto players and continuity, I show that when the veto player proposes he can always find a proposal which allocates to himself strictly more than $k$ and that is weakly preferred to the status quo by at least one non-veto player.

Finally, Proposition 12 tackles the limit, yet interesting, case where the veto player has no power to propose and shows that this institutional measure is effective in preventing the expropriation of non-veto players in the class of continuous and consistent MPEs.

**Proposition 12** *In any consistent and continuous MPE of the game with $p_v = 0$, the allocation to the veto player in any period is the amount in the initial status quo, $s_v^0$.*

In the proof, which is presented in the Appendix, I first show that, in this case, the veto player's continuation value from any policy is strictly increasing in his allocation. Thus, the veto player never accepts a reduction to his allocation. Since the veto player never proposes, this means that, to prove Proposition 11, it is sufficient to rule out that equilibrium proposals by either non-veto player increase the allocation to the veto player. Proceeding towards a contradiction, I show that, if this were the case, consistency would imply that one non-veto player proposes the same increased allocation to the veto player for an interval of status quo policies; and that, for the same interval of policies, the other non-veto player is better off offering the status quo allocation to the veto player. In turn, this would lead to discontinuous continuation values.

# 7 The Effect of Multiple Decisions

To highlight the effect of repeated interaction, I analyze a benchmark model where an ad hoc committee composed of three players—one veto player and two non-veto players—bargains over a single decision. At the beginning of each period of an infinite horizon, one committee member is randomly selected to propose an agreement, $\mathbf{x} \in \Delta$. If the proposal is supported by the veto player and at least one non-veto player, then negotiations end with outcome $\mathbf{x}$ in the current and every subsequent period. Otherwise, a status quo policy, $\mathbf{s} \in \Delta$, is implemented in the current period and there is a call for new proposals. Legislators discount the future with a common factor, $\delta \in [0,1]$.[15]

**Proposition 13** *Consider bargaining in ad hoc committees with status quo* $\mathbf{s} \in \Delta$. *Assume, without loss of generality,* $s_1 \geq s_2$. *In the unique stationary equilibrium, there is an immediate agreement and proposals are such that:*

$$x^\star_{nv}(\mathbf{s}) = \begin{cases} \frac{(3-3\delta)(3-\delta)}{9(1-\delta)+\delta^2} s_2 + \frac{(3-3\delta)\delta}{9(1-\delta)+\delta^2} s_1 & \text{if } s_1 \geq \frac{3-2\delta}{3-3\delta} s_2 \\ \frac{3-3\delta}{6-5\delta}(s_1+s_2) & \text{if } s_1 < \frac{3-2\delta}{3-3\delta} s_2 \end{cases}$$

$$x^\star_{v}(\mathbf{s}) = \begin{cases} s_v + \frac{\delta^2}{9(1-\delta)+\delta^2} s_2 + \frac{\delta(3-2\delta)}{9(1-\delta)+\delta^2} s_1 & \text{if } s_1 \geq \frac{3-2\delta}{3-3\delta} s_2 \\ s_v + \frac{\delta}{6-5\delta}(s_1+s_2) & \text{if } s_1 < \frac{3-2\delta}{3-3\delta} s_2 \end{cases}$$

$$\mu^\star_1(\mathbf{s}) = \begin{cases} 0 & \text{if } s_1 \geq \frac{3-2\delta}{3-3\delta} s_2 \\ \frac{\delta s_1 - (3-2\delta)(s_1-s_2)}{\delta(s_1+s_2)} & \text{if } s_1 < \frac{3-2\delta}{3-3\delta} s_2 \end{cases}$$

*where* $x^\star_{nv}$ *is the amount proposed by the veto player to one non-veto player,* $x^\star_v$ *is the amount proposed by each non-veto player to the veto player, and* $\mu^\star_1(\mathbf{s}) \in [0, 1/2]$ *is the probability that the veto player proposes* $x^\star_{nv}$ *to non-veto player 1. Note that* $x^\star_{nv}$ *is strictly decreasing in* $\delta$, *with* $\lim_{\delta\to 1} x^\star_{nv} = 0$*; and that* $x^\star_v$ *is weakly larger than* $s_v$ *and strictly increasing in* $\delta$, *with* $\lim_{\delta \to 1} x^\star_v = 1$.

Proposition 13 shows that bargaining in ad hoc committees displays these properties:

- There exists $\mathbf{s} \in \Delta$ and $\delta \in [0,1]$ such that any $\mathbf{x} \in \Delta$ is an equilibrium outcome, i.e., outcomes crucially depends on patience and the initial division of the dollar.
- Full appropriation by the veto player is an equilibrium outcome only if $\delta = 1$ or $s_v = 1$.

[15] Winter (1996) studies veto power in ad hoc committees but a direct comparison is hindered by his assumption that failure to reach an agreement is worse for all legislators than every possible bargaining outcome (i.e., $\mathbf{s} = \{0,0,0\} \notin \Delta$). The conclusions on the role of patience do not depend on this assumption (see Propositions 2 and A.1 in Winter 1996). Banks and Duggan (2006) show that, in this setting, a stationary equilibrium exists and that there are no stationary equilibria without immediate agreement but do not offer comparative static results on legislators' patience and the status quo policy.

- The value of the game for the veto player is strictly increasing in legislators' patience.
- The veto player never supports a reform which goes against his immediate preferences. More generally, any committee member's preferences over policies only depend on the allocation to oneself, not on the entire distribution of resources.

These results illuminate the role of multiple decisions as they stand in sharp contrast to the properties of bargaining in standing committees I presented above.

# 8 Experimental Design

The theory provides sharp empirical implications, in particular on the shadow of the future in standing committees, that is, on how different degrees of patience affects legislators' bargaining behavior and the allocation of resources. In the remainder of the paper, I assess the empirical validity of these theoretical predictions with the use of controlled laboratory experiments, which have some important advantages over field data when studying a highly structured dynamic environment such as the one in this paper (Falk and Heckman 2009).

The experiments were conducted at the Rady Behavioral Laboratory between November 2012 and February 2013. Subjects were undergraduate students from the University of California San Diego and were recruited from a database of volunteer subjects. Eight sessions were run, using a total of 96 subjects. No subject participated in more than one session.

The experimental treatment is the discount factor, that is, the degree of patience of the committee. I conduct four sessions with *low patience* committees ($\delta = 0.50$), and four sessions with *high patience* committees ($\delta = 0.75$). Discount factors were induced by a random termination rule: after each round of the same game, a fair die was rolled by the experimenter at the front of the room, with the outcome determining whether the game continued to another round (with probability $\delta$). This is a standard technique used in the experimental literature to preserve the incentives of infinite horizon games in the laboratory (Roth and Murnighan 1978, Fréchette and Yuksel 2017).[16]

[16]The length of a game ranged from 1 to 13 rounds. To ensure the same number of expected rounds (40),

| Treatment | $\delta$ | Sessions | Committees | Subjects |
|---|---|---|---|---|
| High Patience | 3/4 | 4 | 160 | 48 |
| Low Patience | 1/2 | 4 | 320 | 48 |

Table 1: Experimental design.

All sessions were conducted with 12 subjects, divided into 4 committees of 3 members each—one veto player and two non-veto players. Veto players were selected randomly at the beginning of the session, with their role as veto players remaining fixed throughout the session. Committees stayed the same throughout the rounds of a given game, and subjects were randomly rematched into committees between games. The exogenous amount of resources in each round was 60 experimental units (corresponding to $2). At the beginning of each game, an initial status quo was randomly chosen by the computer among all vectors of three non-negative integers which sum to 60. After being informed of the initial status quo, each committee member was prompted to enter a *provisional proposal*. After all members had entered a provisional proposal, one was selected at random to become the *proposed budget*. This proposal was then voted on against the status quo, which was referred to as the *standing budget*. The proposed budget defeated the standing budget with the approval of the veto player and at least one non-veto player. Whichever budget passed the voting stage was the policy that was implemented in that round, each member received earnings accordingly, and the budget that just passed became the new status quo. Instructions were read aloud and subjects were required to correctly answer all questions on a short comprehension quiz before the experiment was conducted. The experiments were conducted via computers.[17] Table 1 summarizes the experimental design.

each of the high patience sessions lasted for 10 games and each of the low patience sessions for 20 games.

[17]Sample instructions are available in the Appendix. The computer program used in the experiment was an extension to the open source software Multistage.

# 9 Experimental Results

Unless otherwise noted, in this Section, I use random effects panel regressions with standard errors clustered at the session level to compare policy outcomes and bargaining behavior between different treatments Clustering at the session level accounts for potential interdependencies between observations that come from random re-matching of subjects between games in a session.[18]

## 9.1 Policy Outcomes and Dynamics

The evolution of policies over time provides a clear picture of outcome dynamics, since it provides a synthetic description of aggregate behavioral data on both proposal making and voting. One way to represent the data compactly is to cluster policies in seven regions. The D regions correspond to dictatorial allocations where one committee member receives the lion's share of the budget: D1, D2 and DV are the regions where, respectively, committee member 1, committee member 2 or the veto player receives at least 2/3 of the budget, that is, 40 out of 60 tokens. The U region consists of universal allocations, where all committee members receive at least 1/4 of the budget (15 tokens out of 60) and, thus, the budget is equally, or nearly equally, shared. Finally, the C regions correspond to the remaining allocations, where only two committee members receive a substantial share of the budget, while the third committee member is assigned a negligible share: C12 is the coalition composed of committee member 1 and committee member 2; C1V is the coalition composed of committee member 1 and the veto player; and C2V is the coalition composed of committee member 2 and the veto player.

Before discussing the results, it is useful to recall the theoretical predictions. If the status quo is in region U or C12, the MPE from Proposition 1 predicts that policy moves immediately to a region where the budget is shared by the members of a minimal winning coalition (that is, to region C1V or C2V), regardless of the identity of the proposer. If

[18]See Fréchette (2012) for a discussion.

the status quo is in region C1V or C2V, the the MPE from Proposition 1 predicts that the status quo is maintained or that policy moves to the opposite side of $\Delta$. In this latter case, if the veto player is proposing and the initial status quo gives him enough, policy transitions to region DV. Finally, if the status quo lies in region DV, policy almost always stays there. The predicted evolution of policies between these regions is very similar between the two treatments. The theory does predict sharp differences between high and low patience committees for finer details of behavior and I investigate them below.

The overall frequency of each region and the transition probabilities between each pair of regions for the two treatments is summarized in Table 2. For each panel, the last row gives the overall outcome frequencies, excluding the initial status quo policies, which were decided randomly by the computer to start each game. Each cell in the other seven rows gives the probability of moving to a policy in the column region when starting from a policy in the row region. I highlight four results from this table.[19]

**Finding 1:** *Consistent with the theory, most policies give a positive amount of resources to the veto player and to, at most, one non-veto player.* In both high and low patience committees, around 88% of all policies give a substantial share to the veto player. Moreover, only around 22% of all policies give a substantial share to both non-veto players (28% in high patience committees and 17% in low patience committees).

**Finding 2:** *Consistent with the theory, allocations which give most resources to the veto player are an absorbing state.* The chance of leaving region DV is around 2% in high patience committees and around 3% in low patience committees. This is the only absorbing state: the second most resilient region is U, which survives 80% of the time in high patience committees and 55% of the time in low patience committees (meaning that the status quo policy transitions to another region, respectively, 20% and 45% of the time).

[19] Table 6 in the Appendix shows that these results are robust to a different classification of outcomes, which adopts a stricter definition of dictatorial and universal allocations. In Table 6, I define as dictatorial an allocation which gives at least 3/4 of the budget (45 out of 60 tokens) to a single committee member; and I define as universal an allocation which gives at least 3/10 of the budget (18 tokens out of 60) to each committee member.

Panel A: High Patience

| Status Quo (t) | Status Quo (t+1) D1 | D2 | DV | C12 | C1V | C2V | U |
|---|---|---|---|---|---|---|---|
| Dictator 1 | 0.45 | 0.03 | 0.03 | 0.04 | 0.05 | 0.22 | 0.18 |
| Dictator 2 | 0.12 | 0.40 | 0.07 | 0.07 | 0.14 | 0.05 | 0.14 |
| Dictator V | 0.00 | 0.00 | 0.98 | 0.00 | 0.01 | 0.01 | 0.01 |
| Coalition 1 + 2 | 0.02 | 0.02 | 0.00 | 0.67 | 0.05 | 0.07 | 0.16 |
| Coalition 1 + V | 0.00 | 0.00 | 0.16 | 0.00 | 0.51 | 0.28 | 0.05 |
| Coalition 2 + V | 0.01 | 0.00 | 0.13 | 0.00 | 0.26 | 0.56 | 0.04 |
| Universal | 0.01 | 0.01 | 0.03 | 0.00 | 0.08 | 0.09 | 0.80 |
| **Frequency** | 0.05 | 0.02 | 0.36 | 0.04 | 0.13 | 0.16 | 0.23 |

Panel B: Low Patience

| Status Quo (t) | Status Quo (t+1) D1 | D2 | DV | C12 | C1V | C2V | U |
|---|---|---|---|---|---|---|---|
| Dictator 1 | 0.22* | 0.10** | 0.11 | 0.00** | 0.11 | 0.30 | 0.17 |
| Dictator 2 | 0.14 | 0.32 | 0.16* | 0.00** | 0.16 | 0.06 | 0.16 |
| Dictator V | 0.00 | 0.00 | 0.97 | 0.01 | 0.00 | 0.00 | 0.03 |
| Coalition 1 + 2 | 0.07 | 0.15 | 0.02 | 0.29* | 0.15** | 0.24** | 0.07 |
| Coalition 1 + V | 0.01 | 0.03 | 0.20 | 0.00 | 0.43 | 0.22* | 0.10 |
| Coalition 2 + V | 0.02 | 0.02 | 0.28** | 0.00 | 0.20 | 0.42 | 0.06 |
| Universal | 0.05 | 0.01 | 0.04 | 0.00 | 0.23 | 0.13 | 0.55** |
| **Frequency** | 0.05 | 0.05* | 0.43 | 0.02** | 0.15 | 0.15 | 0.15 |

Table 2: Policy frequencies and transition probabilities. Notes: ** and * indicate difference with *High Patience* is significant, respectively, at 1% and at 5% level (see p-values in Table 5).

**Finding 3:** *Consistent with the theory, when the status quo lies in region C1V or C2V (giving a negligible amount of resources to one non-veto player), resources continue to be shared by a minimal winning coalition or policy transitions to region DV.* When the status quo shares resources between the members of a minimal winning coalition, the policy implemented in that round lies in region C1V, C2V or DV 95% of the time in high patience committees and 86% of the time in low patience committees.

**Finding 4:** *Contrary to the theory, the survival rate of allocations giving a substantial amount to both non-veto players is positive and greater in more patient committees.* The diagonal of the transition matrices suggests that there is status quo inertia (at least within the boundaries of these regions) and this is true also for status quo policies which do not assign resources primarily to a minimal winning coalition — that is, policies where resources are mostly shared between the two non-veto players or policies where every committee members receive a non-negligible share. This inertia is statistically stronger in more patient committees: the chance a status quo in region C12 survives is 29% in low patience committees and 67% in high patience committees; the chance a status quo in region U survives is 55% in low patience committees and 80% in high patience committees. Moreover, while they represent only a small fraction of policies in both treatments, allocations in region C12 are more frequent in more patient committees (4% versus 2%) and this difference is statistically significant.

## 9.2 Veto Player's Allocation

From the transition probabilities in Table 2, we can see that the policies slowly transition to the DV region, regardless of the initial status quo and degree of patience and that, once there, they do not leave this region. The transition to DV can happen directly: with the exception of region C12 in committees with high patience, there is a positive probability of moving to region DV starting from any region. More frequently, the transition to DV happens indirectly: there is a substantial probability of moving to region DV when the status quo lies in a region where exactly one non-veto player has a negligible share of the budget—that

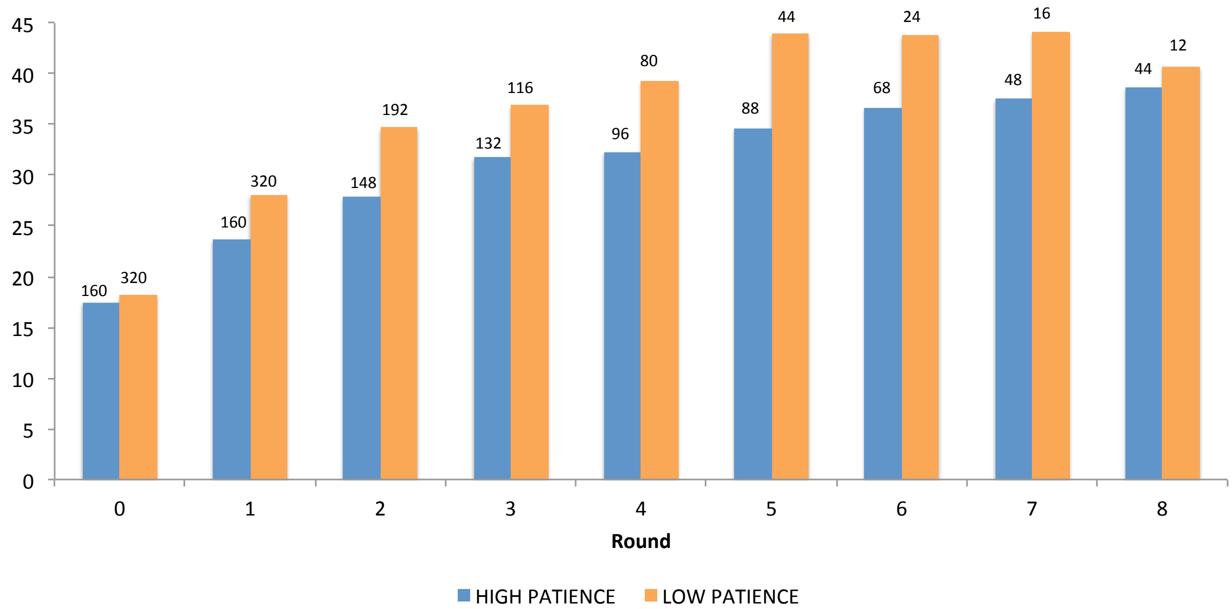



Figure 5: Average allocation to veto player. Numbers on bars are observations (committees).

is, regions C1V and C2V—and policies move to these regions at substantial rates starting from any other region (the only exception being region DV, which is an absorbing state).[20] Since this is one of the main empirical implications of the theory, it is interesting to give a closer look to the evolution of the allocation to the veto player.

Figure 5 shows the evolution of the average allocation to the veto player as the number of rounds played in the same game grows, separately for the two treatments. The first data point on the left is the average allocation to the veto player in the initial status quo policy randomly drawn by the computer in all games of the same treatment.[21] The duration of each game is stochastic: the number of observations available for each round is different and higher rounds have fewer observations.[22]

[20]Starting from regions C1V and C2V, policies move to region DV 14% of the time with high patience and 24% of the time with low patience. The probability of moving to region C1V or C2V starting from any region other than DV is at least 12% (in high patience committees, starting from region C12) and as large as 41% (in low patience committees, starting from region D1).

[21]The initial allocation to the veto player is not statistically different between the two treatments (p-value: 0.593).

[22]Figure 5 shows only rounds for which we have at least 12 committees for each treatment. This covers 93% of all observations for high patience committees and 96% of all observations for low patience committees.

| A: VETO PROPOSER | HIGH PATIENCE | | LOW PATIENCE | |
|---|---|---|---|---|
| | ALL | ACC | ALL | ACC |
| Mean Premium to Proposer | 8.34 | 7.71 | 12.65* | 12.77** |
| Mean Premium to Rich Non-Veto | -15.09 | -16.99 | -19.15** | -20.74** |
| Mean Premium to Poor Non-Veto | 6.75 | 9.28 | 6.51 | 7.97 |
| Mean Premium to Coalition Partner | 7.11 | 8.97 | 5.00 | 5.79* |
| Observations | 844 | 178 | 824 | 215 |

| B: NON-VETO PROPOSER | HIGH PATIENCE | | LOW PATIENCE | |
|---|---|---|---|---|
| | ALL | ACC | ALL | ACC |
| Mean Premium to Proposer | 8.12 | 9.12 | 10.09* | 11.04 |
| Mean Premium to Veto | -0.83 | 3.86 | -0.35 | 6.81** |
| Mean Premium to Other Non-Veto | -7.29 | -12.98 | -9.74* | -17.85** |
| Mean Premium to Coalition Partner | 0.38 | 3.46 | 2.04 | 6.74** |
| Observations | 1688 | 291 | 1648 | 316 |

Table 3: Proposing behavior. Notes: For each treatment, the first column is for all observed proposals, the second column for proposals that are voted on and accepted; Rich Non-Veto (Poor Non-Veto) is the non-veto player who receives the most (least) in the status quo; Coalition Partner is the non-proposing player who receives the most in the proposal; ** and * indicate difference with *High Patience* is significant, respectively, at 1% and at 5% level.

**Finding 5:** *Consistent with the theory, there is a ratchet effect in the allocation to the veto player, slower in more patient committees.* The allocation to the veto player gradually increases over time in both treatments with a veto player. The allocation to the veto player is larger in lower patience committees in all rounds.[23]

## 9.3 Proposal Making

The experimental data is very rich and allow us to test the finer predictions of the model. To investigate the origin of the dynamic patterns described above, I decompose

[23]The difference between the High Patience and the Low Patience series is positive for all rounds and significant at the 5% level for round 1 (p-value: 0.041), round 2 (p-value: 0.010), round 3 (p-value: 0.032), round 4 (p-value: 0.037), round 5 (p-value: 0.010), round 6 (p-value: 0.011) and at the 10% level for round 7 (p-value: 0.051). The lack of significance for round 8 (p-value: 0.650) can be due to the random termination rule, which means the number of observations for high rounds is small in both treatments. The existence of a ratchet effect is confirmed by the Tobit regressions presented in Table 8 in the Appendix.

the determinants of the transition probabilities and analyze in detail proposal and voting behavior. Regarding proposing behavior, the model predicts that both veto and non-veto proposers completely expropriate one non-veto player; that veto proposers are forced to share resources more evenly with the other non-veto player in more patient committees; and that non-veto proposers give the veto player no more than what is granted by the current status quo.

Table 3 shows how proposers allocate resources among committee members. To compare proposals made at different status quo policies, I look at the *premium* proposed to each committee member, rather than at the absolute amount. The premium to a member is the difference between the amount proposed to that member by the agenda setter and the amount granted to that same member by the status quo policy. If the premium to a member is positive, this means the proposer is suggesting an increase to that member's allocation.

**Finding 6:** *As predicted by the theory, both veto and non-veto proposers expropriate resources from one non-veto player and share the spoils with a coalition partner.* Regardless of their degree of patience and their role, proposers expropriate resources from a non-veto player and redistribute the spoils towards themselves and a coalition partner. In particular, veto proposers expropriate resources from the non-veto player who is allocated the largest amount in the status quo and give a significant premium to themselves and to the other non-veto player; and non-veto player expropriates the other non-veto player and give a significant premium to themselves and to the veto player.[24]

**Finding 7:** *As predicted by the theory, veto proposers share resources more evenly with coalition partners in more patience committees. In general, both veto and non-veto proposers are less greedy and more generous with other committee members in more patient committees.* The premium to a veto proposer and the premium to the non-veto coalition partner are, respectively, smaller and larger in high patience than in low patience committees. Moreover,

[24]As shown in Table 9 in the Appendix, this is true also when I restrict the analysis to status quo polices which gives a non-negligible amount to both non-veto players, that is, policies in regions U and C12.

| | HIGH PATIENCE | | LOW PATIENCE | |
|---|---|---|---|---|
| | (1) | (2) | (3) | (4) |
| Premium Me | 0.016** | 0.019** | 0.013** | 0.016** |
| | (0.002) | (0.000) | (0.001) | (0.001) |
| Premium Other Non-Proposer | -0.007** | 0.002 | -0.007** | -0.005** |
| | (0.001) | (0.003) | (0.001) | (0.001) |
| Constant | 0.535** | 0.493** | 0.535** | 0.545** |
| | (0.021) | (0.019) | (0.039) | (0.015) |
| Voter Type | Veto | Non-Veto | Veto | Non-Veto |
| Proposer Type | Non-Veto | Veto | Non-Veto | Veto |
| Pseudo-R2 | 0.2488 | 0.4136 | 0.3413 | 0.4798 |
| Observations | 566 | 556 | 560 | 528 |

Table 4: Probability of supporting a proposal: panel random fixed effects estimates with standard errors clustered at the session level. Notes: ** and * indicate, respectively, significant at 1% and at 5% level.

veto (non-veto) proposers expropriate a lower amount from the rich non-veto player (the other non-veto player) when they are more patient; and non-veto proposers offer a larger amount to their coalition partner (the veto player) when they are more patient. As detailed in Table 3, these differences are statistically significant.

## 9.4 Voting Decisions

To investigate the determinants of voting behavior, I run regressions for the likelihood of voting in favor of a proposal using premium to oneself and premium to the other non-proposer as the explanatory variables. I do this separately for different roles of proposer and voter—whether they are a veto or non-veto player—and for different treatments—columns 1 and 2 analyze behavior in high patience committees; columns 3 and 4 analyze behavior in low patience committees. Table 4 shows the results.[25]

**Finding 8:** *Subjects vote selfishly and are less likely to support proposals which are more generous to the other non-proposer.* Regardless of patience and role, committee members are

[25] I exclude proposers from the analysis. Excluding votes between identical allocations, subjects vote in favor of their own proposal 91% in High Patience and 94% in Low Patience.

more likely to support a proposal when it offers a larger premium to themselves. Interestingly, the probability a non-veto player supports a proposal by the veto player does not depend on (high patience committees) or decreases in (low patience committees) the premium offered to the other non-veto player. The probability the veto player supports a proposal by a non-veto player decreases in the premium offered to the other non-veto player in both treatments.

# 10 Conclusions

This paper studies the consequences of veto power in a bargaining game with an evolving status quo policy. As the importance of the right to block a decision crucially depends on the status quo, a static analysis cannot draw general conclusions about the effect of veto power on policy capture by the veto player. Instead of making ad hoc assumptions on the status quo policy, I study veto power by exploring the inherently dynamic process via which the location of the current status quo is determined. I prove that there exists an equilibrium of this dynamic game such that the veto player is eventually able to extract all resources, irrespective of the discount factor and the initial agreement, and that this is the unique stable outcome in a class of MPEs. This result shows that, in the long run, the right to veto is extremely powerful, especially if coupled with proposal power. This is true even when non-veto legislators are patient, and take into account the loss in future bargaining power implied by making concessions to veto players in the current period. At the same time, institutional measures can be effective in promoting more equitable outcomes, at least in the short run: reducing the veto player's ability to set the agenda decreases the speed of convergence to the veto player's ideal policy, and assigning monopolistic agenda setting power to non-veto players prevents the veto player from expropriating other legislators. The main predictions of the theory find support in the behavior of committees bargaining in controlled laboratory experiments: outcomes evolve according to the predicted transition probabilities, albeit with stronger persistence; the allocation to the veto player gradually increases over time; and patient committees exhibit significantly different proposal and voting behavior than impatient committees.

While the results in this paper certainly add to our understanding of the incentives present in real world legislatures, the setup is intentionally very simple and uses a number of specific assumptions. There are many possible directions for next steps in this research. First, while I have limited the analysis to committees with three legislators and one veto player, it would certainly be interesting to extend the asymptotic result of full appropriation by the veto player(s) to legislatures with an arbitrary number of veto and non-veto legislators. The existence proofs for the equilibria proposed in this paper rely on constructing the equilibrium strategies, and the associated continuation values, for any allocation of the dollar, $\mathbf{s} \in \Delta$. It is a challenging task to extend this existence result and to characterize an MPE with a higher number of legislators, as the dimensionality of the state space increases and tractability is quickly lost. In the Appendix, I introduce two assumptions to preserve the analytical tractability of the model: I assume that only veto players are able to make proposals and I restrict the set of feasible allocations to those with, at most, two types of non-veto players, a subset who receives zero and a subset who receives the same, non-negative amount. This allows me to study the effect of competing veto powers, committee size and majority requirements on veto players' ability to appropriate resources in the short and in the long run. I show that these institutional measures do not prevent complete expropriation of non-veto players in the long run but can affect short run outcomes. Future research could explore the dynamics of a larger legislature using numerical methods, a solution often adopted in the literature on dynamic models with endogenous status quo (Baron and Herron 2003, Penn 2009, Battaglini and Palfrey 2012, Duggan et al. 2008). Second, this study analyzes a divide-the-dollar game where legislators' preferences are purely conflicting. This is a natural starting point to analyze the consequences of veto power in a dynamic setting as it lays bare the incentives at work. However, legislative committees make decisions on many policy domains where agents' preferences are partially aligned. Extending the policy space beyond the pure distributive setting—either considering a unidimensional policy space or allowing resources to be allocated to a pubic good—is an important direction for future work. Finally, on the experimental side, one interesting possibility is to allow for unrestricted communication among committee members. Recent experimental studies on dynamic bargaining show that communication affects the prevailing norm of fairness (Baron et al. 2017) and makes it easier to sustain non-stationary, history-dependent strategies (Agranov et al. 2020).

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# Appendices


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# A Proofs of Propositions from Section 4

## Proof of Proposition 1

Before proceeding to the proof, I introduce two formal definitions:

**Definition 5 (Markov Strategy)** *A Markov strategy is a pair of functions, $\sigma_i(\mathbf{s}) = (\mu_i[\cdot|\mathbf{s}], A_i(\mathbf{s}))$, where $\mu_i[\mathbf{z}|\mathbf{s}]$ represents the probability that legislator $i$ makes the proposal $\mathbf{z}$ when recognized, conditional on the state being $\mathbf{s}$; and $A_i(\mathbf{s})$ represents the allocations for which $i$ votes yes when the state is $\mathbf{s}$.*

**Definition 6 (Legislators' Demands)** *For an MPE, non-veto legislator $j$'s demand when the state is $\mathbf{s}$ is the minimum amount $d_j(\mathbf{s}) \in [0,1]$ such that for a proposal $\mathbf{x} \in \overline{\Delta}$ with $x_j = d_j(\mathbf{s})$, $x_v = 1 - d_j(\mathbf{s})$, we have $U_j(\mathbf{x}) \geq U_j(\mathbf{s})$. Similarly, veto legislator $v$'s demand when the state is $\mathbf{s}$ is the minimum amount $d_v(\mathbf{s}) \in [0,1]$ such that for a proposal $\mathbf{x} \in \overline{\Delta}$ with $x_v = d_v(\mathbf{s})$, $x_j = 1 - d_v(\mathbf{s})$, for $j = 1, 2$, we have $U_v(\mathbf{x}) \geq U_v(\mathbf{s})$.*

The results of Proposition 1 follow from the existence of an MPE with the following minimal winning coalition proposal strategies for all $\mathbf{s} \in \Delta$, where $s_1 \geq s_2$:

- Case A $\left(s_1 \leq 1 - \frac{3-\delta}{3-2\delta}s_2,\ s_1 \geq \frac{3-\delta}{3-2\delta}s_2\right)$:

$$\begin{aligned}
\mathbf{x}^v &= [1-d_2, 0, d_2], \mathbf{x}^1 = [d_v, 1-d_v, 0], \mathbf{x}^2 = [d_v, 0, 1-d_v] \\
d_v &= s_v - \frac{\delta s_2}{3-2\delta} \\
d_2 &= \frac{\delta}{3-2\delta}s_1 + \frac{(3-\delta)}{(3-2\delta)}s_2
\end{aligned}$$

- Case B $\left( s_1 > 1 - \frac{3-\delta}{3-2\delta} s_2,\ s_1 \geq \frac{27-27\delta+3\delta^2+\delta^3}{(3-2\delta)(3-\delta)^2} s_2 + \frac{\delta^2}{(3-\delta)^2} \right)$ :

$$\begin{aligned} \mathbf{x}^v &= [1-d_2, 0, d_2], \mathbf{x}^1 = [d_v, 1-d_v, 0], \mathbf{x}^2 = [d_v, 0, 1-d_v] \\ d_v &= 0 \\ d_2 &= \frac{9-12\delta+3\delta^2}{(3-2\delta)^2} s_2 + \frac{\delta}{(3-2\delta)} \end{aligned}$$

- Case C $\left( s_1 > \frac{6-3\delta}{2(3-\delta)} - s_2,\ s_1 < \frac{27-27\delta+3\delta^2+\delta^3}{(3-2\delta)(3-\delta)^2} s_2 + \frac{\delta^2}{(3-\delta)^2} \right)$ :

$$\begin{aligned} \mathbf{x}^v &= \begin{cases} [1-d_2, d_2, 0] & \text{w/ } \Pr = 1-\mu_v^C \\ [1-d_2, 0, d_2] & \text{w/ } \Pr = \mu_v^C \end{cases}, \mathbf{x}^1 = [d_v, 1-d_v, 0], \mathbf{x}^2 = [d_v, 0, 1-d_v] \\ d_v &= 0 \\ d_2 &= \frac{(-3+\delta)(3\delta^2-12\delta+9)}{(-3+2\delta)(\delta^2-15\delta+18)}(s_1+s_2) + \frac{(-3+\delta)(6\delta-4\delta^2)}{(-3+2\delta)(\delta^2-15\delta+18)} \\ \mu_v^C &= \frac{(\delta^3+3\delta^2-27\delta+27)s_1 + (2\delta^3-15\delta^2+36\delta-27)s_2 - 2\delta^3 + 3\delta^2}{\delta\left[(3\delta^2-12\delta+9)(s_1+s_2) + 6\delta - 4\delta^2\right]} \end{aligned}$$

- Case D $\left( s_1 \leq \frac{6-3\delta}{2(3-\delta)} - s_2,\ s_1 < \frac{3-\delta}{3-2\delta} s_2 \right)$ :

$$\begin{aligned} \mathbf{x}^v &= \begin{cases} [1-d_2, d_2, 0] & \text{w/ } \Pr = 1-\mu_v^D \\ [1-d_2, 0, d_2] & \text{w/ } \Pr = \mu_v^D \end{cases}, \mathbf{x}^1 = [d_v, 1-d_v, 0], \mathbf{x}^2 = [d_v, 0, 1-d_v] \\ d_v &= s_v - \frac{\delta s_2}{3-2\delta} \\ d_2 &= \frac{(3-\delta)(9-6\delta-\delta^2)}{(3-2\delta)(\delta^2-15\delta+18)} s_1 + \frac{(3-\delta)(9-6\delta+\delta^2)}{(3-2\delta)(\delta^2-15\delta+18)} s_2 \\ \mu_v^D &= \frac{(\delta^3-6\delta^2+27\delta-27)s_1 + (-\delta^3+12\delta^2-36\delta+27)s_2}{\delta\left[(\delta^2+6\delta-9)s_1 + (-\delta^2+6\delta-9)s_2\right]} \end{aligned}$$

It is tedious but straightforward to check that, if players play the proposal strategies in cases A-D and these proposals pass, their continuation values are as follows:

- Case A

$$
\begin{aligned}
v_v(\mathbf{s}) &= \frac{1}{1-\delta} - \frac{2-\delta}{(3-\delta)(1-\delta)}s_1 - \frac{1}{(1-\delta)}s_2 && (3)\\
v_1(\mathbf{s}) &= \frac{(3-3\delta+\delta^2)}{(3-\delta)^2(1-\delta)}s_1 + \frac{(3-\delta)}{(3-\delta)^2(1-\delta)}s_2 && (4)\\
v_2(\mathbf{s}) &= \frac{(3-2\delta)}{(3-\delta)^2(1-\delta)}s_1 + \frac{(6-5\delta+\delta^2)}{(3-\delta)^2(1-\delta)}s_2 && (5)
\end{aligned}
$$

- Case B

$$
\begin{aligned}
v_v(\mathbf{s}) &= \frac{1}{(1-\delta)(3-\delta)} - \frac{(3-4\delta+\delta^2)}{(3-2\delta)(1-\delta)(3-\delta)}s_2\\
v_1(\mathbf{s}) &= \frac{(3\delta-4\delta^2+\delta^3)}{(3-\delta)^2(1-\delta)(3-2\delta)}s_2 + \frac{(9-15\delta+9\delta^2-2\delta^3)}{(3-\delta)^2(1-\delta)(3-2\delta)}\\
v_2(\mathbf{s}) &= \frac{(3-2\delta)}{(3-\delta)^2(1-\delta)} + \frac{(3-4\delta+\delta^2)}{(3-\delta)^2(1-\delta)}s_2
\end{aligned}
$$

- Case C

$$
\begin{aligned}
v_v(\mathbf{s}) &= \frac{-9-7\delta^2+15\delta+\delta^3}{(-1+\delta)(-3+\delta)(\delta^2-15\delta+18)}(s_1+s_2) + \frac{2\delta^2+18-\delta^3-15\delta}{(-1+\delta)(-3+\delta)(\delta^2-15\delta+18)}\\
v_1(\mathbf{s}) &= -\frac{\delta^4-13\delta^3+48\delta^2-63\delta+27}{(-1+\delta)(-3+\delta)(\delta^2-15\delta+18)\delta}s_1 - \frac{6\delta^3-33\delta^2+54\delta-27}{(-1+\delta)(-3+\delta)(\delta^2-15\delta+18)\delta}s_2 +\\
&\quad + \frac{-8\delta^3+24\delta^2-18\delta}{(-1+\delta)(-3+\delta)(\delta^2-15\delta+18)\delta}\\
v_2(\mathbf{s}) &= -\frac{\delta^4-13\delta^3+48\delta^2-63\delta+27}{(-1+\delta)(-3+\delta)(\delta^2-15\delta+18)\delta}s_2 - \frac{6\delta^3-33\delta^2+54\delta-27}{(-1+\delta)(-3+\delta)(\delta^2-15\delta+18)\delta}s_1 +\\
&\quad + \frac{-8\delta^3+24\delta^2-18\delta}{(-1+\delta)(-3+\delta)(\delta^2-15\delta+18)\delta}
\end{aligned}
$$

- Case D

$$\begin{aligned}
v_v(\mathbf{s}) &= \frac{\delta^3 - 23\delta^2 + 63\delta - 45}{(-1+\delta)(-3+\delta)(\delta^2 - 15\delta + 18)} s_1 + \frac{\delta^3 - 15\delta^2 + 51\delta - 45}{(-1+\delta)(-3+\delta)(\delta^2 - 15\delta + 18)} s_2 + \\
&\quad + \frac{18\delta^2 + 54 - \delta^3 - 63\delta}{(-1+\delta)(-3+\delta)(\delta^2 - 15\delta + 18)} \\
v_1(\mathbf{s}) &= -\frac{\delta^4 - 21\delta^3 + 72\delta^2 - 81\delta + 27}{(-1+\delta)(-3+\delta)(\delta^2 - 15\delta + 18)\delta} s_1 - \frac{2\delta^3 - 15\delta^2 + 36\delta - 27}{(-1+\delta)(-3+\delta)(\delta^2 - 15\delta + 18)\delta} s_2 \\
v_2(\mathbf{s}) &= -\frac{-2\delta^3 - 9\delta^2 + 36\delta - 27}{(-1+\delta)(-3+\delta)(\delta^2 - 15\delta + 18)\delta} s_1 - \frac{\delta^4 - 17\delta^3 + 66\delta^2 - 81\delta + 27}{(-1+\delta)(-3+\delta)(\delta^2 - 15\delta + 18)\delta} s_2
\end{aligned}$$

On the basis of these continuation values, we obtain players' expected utility functions, $U_i(\mathbf{x}) = x_i + \delta V_i(\mathbf{x})$. The reported demands are in accordance with Definition 2. In particular, $d_i$, $i = 1, 2$ and $d_v$ can be easily derived from the following equations:

$$\begin{aligned}
s_i + \delta V_i(\mathbf{s}) &= d_i + \delta V_i([1 - d_i, d_i, 0]) \\
s_v + \delta V_v(\mathbf{s}) &= d_v + \delta V_v([d_v, 1 - d_v, 0])
\end{aligned}$$

The demands for non-veto player 1 are never part of a proposed allocation and have therefore been omitted in the statement of the equilibrium proposal strategies above but we will use them in the remainder of the proof. In cases C and D, the mixing of the veto player is such that $d_1 = d_2$. In the other two cases, $d_1$ is as follows:

- Case A $\left(s_1 \leq 1 - \frac{3-\delta}{3-2\delta} s_2,\ s_1 \geq \frac{3-\delta}{3-2\delta} s_2\right)$ :

$$d_1 = \frac{(4\delta^2 - 12\delta + 9)}{(3 - 2\delta)^2} s_1 + \frac{(3\delta - \delta^2)}{(3 - 2\delta)^2} s_2$$

- Case B $\left(\ s_1 > 1 - \frac{3-\delta}{3-2\delta} s_2,\ s_1 \geq \frac{27 - 27\delta + 3\delta^2 + \delta^3}{(3-2\delta)(3-\delta)^2} s_2 + \frac{\delta^2}{(3-\delta)^2}\right)$ :

$$d_1 = \frac{(27 - 63\delta + 51\delta^2 - 17\delta^3 + 2\delta^4)}{(3 - 2\delta)^3} s_1 + \frac{(3\delta^2 - 4\delta^3 + \delta^4)}{(3 - 2\delta)^3} s_2 + \frac{9\delta - 15\delta^2 + 9\delta^3 - 2\delta^4}{(3 - 2\delta)^3}$$

Furthermore, all reported non-degenerate mixing probabilities are well defined. On the basis of the expected utility functions, $U_i$, we can then construct equilibrium voting strategies, $A_i^*(\mathbf{s}) = \{\mathbf{x}|U_i(\mathbf{x}) \geq U_i(\mathbf{s})\}$, $i = \{v, 1, 2\}$, for all $\mathbf{s} \in \Delta$. These voting strategies are obviously stage-undominated. Then, to prove Proposition 1 it suffices to verify the optimality of proposal strategies. To do so, we make use of five lemmas. We seek to establish an equilibrium with proposals that allocate a positive amount to at most one non-veto player. Lemma 1 shows that the expected utility function for these proposals satisfies some continuity and monotonicity properties. Lemma 2 proves that minimal winning coalition proposals are optimal among the set of feasible proposals in $\overline{\Delta}$. Lemma 3 establishes that the equilibrium demands of the veto player and one non-veto player sum to less than unity and that the demands of the two non-veto players are (weakly) ordered in accordance to the ordering of allocations under the state $\mathbf{s}$. Lemma 4 then establishes that the proposal strategies for legislators $i = \{v, 1, 2\}$ in Proposition 1 maximize $U_i(\mathbf{x})$ over all $\mathbf{x} \in W(\mathbf{s}) \cap \overline{\Delta}$, where $W(\mathbf{s})$ is the set of all proposals that beat $\mathbf{s}$ in the voting stage. These proposals would then maximize $U_i(\mathbf{x})$ over all $\mathbf{x} \in W(\mathbf{s})$ if there is no $\mathbf{x} \in W(\mathbf{s}) \cap \Delta/\overline{\Delta}$ that accrues $i$ higher utility. We establish that this is indeed the case in Lemma 5.

**Lemma 1** *Consider a Markov Perfect strategy profile with expected utility $U_i(\mathbf{s})$, $\mathbf{s} \in \overline{\Delta}$, determined by the continuation values in equations (3)-(5). Then, for all $\mathbf{x} = (x, 1-x, 0) \in \Delta$ (a) $U_i(\mathbf{x})$, $i = \{v, 1, 2\}$ is continuous and differentiable with respect to $x$, (b) $U_v(\mathbf{x})$ is strictly increasing in $x$, while $U_1(\mathbf{s})$ and $U_2(\mathbf{s})$ is strictly decreasing in $x$.*

**Proof.** An allocation $\mathbf{x} = (x, 1-x, 0) \in \Delta$ belongs to case A in Proposition 2. Therefore we can write $U_i(\mathbf{x}) = x_i + \delta V_i(\mathbf{x})$ as follows:

$$U_v(\mathbf{x}) = x + \frac{\delta}{1-\delta} - \frac{\delta(2-\delta)}{(3-\delta)(1-\delta)}(1-x) \tag{6}$$

$$U_1(\mathbf{x}) = 1 - x + \delta\frac{(3-3d+\delta^2)}{(3-\delta)^2(1-\delta)}(1-x) \tag{7}$$

$$U_2(\mathbf{x}) = \delta\frac{(3-2\delta)}{(3-\delta)^2(1-\delta)}(1-x) \tag{8}$$

$U_i(\mathbf{x})$ is linear and continuous in $x$ for $i = \{v, 1\}$, establishing part (a) of the Lemma. Regarding part (b):

$$
\begin{aligned}
\frac{\partial U_v(\mathbf{x})}{\partial x} &= 1 + \frac{\delta(2-\delta)}{(3-\delta)(1-\delta)} > 0 \\
\frac{\partial U_1(\mathbf{x})}{\partial x} &= -\left(1 + \delta\frac{(3-3d+\delta^2)}{(3-\delta)^2(1-\delta)}\right) < 0 \\
\frac{\partial U_2(\mathbf{x})}{\partial x} &= -\delta\frac{(3-2\delta)}{(3-\delta)^2(1-\delta)} < 0
\end{aligned}
$$

$\frac{\partial U_v(\mathbf{x})}{\partial x} > 0$ for any $\delta \in [0, 1)$, since both the numerator and the denominator of $\frac{\delta(2-\delta)}{(3-\delta)(1-\delta)}$ are positive for any $\delta \in [0, 1)$; $\frac{\partial U_1(\mathbf{x})}{\partial x} < 0$ for any $\delta \in [0, 1)$, since both the numerator and the denominator of $\frac{(3-3d+\delta^2)}{(3-\delta)^2(1-\delta)}$ are positive for any $\delta \in [0, 1)$; and $\frac{\partial U_2(\mathbf{x})}{\partial x} < 0$ for any $\delta \in [0, 1)$, since both the numerator and the denominator of $\frac{(3-2\delta)}{(3-\delta)^2(1-\delta)}$ are positive for any $\delta \in [0, 1)$.

■

By the definition of demands and the monotonicity established in part (b) of Lemma 1 we immediately deduce:

**Lemma 2** *Consider a Markov Perfect strategy profile with expected utility, $U_i(\mathbf{x})$, for $\mathbf{x} \in \overline{\Delta}$, $i = \{v, 1, 2\}$, given by (6)-(8). Every minimal winning coalition proposal of the veto player $x(v, i, d_i(\mathbf{s}))$, $i = \{1, 2\}$ is such that $x(v, i, d_i(\mathbf{s})) \in \arg\max\{U_v(\mathbf{x})|\mathbf{x} \in \overline{\Delta}, U_i(\mathbf{x}) \geq U_i(\mathbf{s})\}$; similarly, every minimal winning coalition proposal of a non-veto player $x(i, v, d_v(\mathbf{s}))$, $i = \{1, 2\}$ is such that $x(i, v, d_i(\mathbf{s})) \in \arg\max\{U_i(\mathbf{x})|\mathbf{x} \in \overline{\Delta}, U_v(\mathbf{x}) \geq U_v(\mathbf{s})\}$.*

**Lemma 3** *For all $\mathbf{s} \in \Delta$, the demands reported in Proposition 1 are such that (a) $s_i \geq s_j \Rightarrow d_i \geq d_j$, $i, j = \{1, 2\}$, and (b) $d_i + d_v \leq 1$, $i = \{1, 2\}$.*

**Proof.** *Part (a).* Since we focus on the half of $\Delta$ in which $s_1 \geq s_2$, we want to prove that $d_1 \geq d_2$. In cases C and D the mixed strategy of the veto player is such that $d_1 = d_2$, so we focus on cases A and B.

- Case A:

$$\begin{aligned}
\frac{(4\delta^2-12\delta+9)}{(3-2\delta)^2}s_1+\frac{(3\delta-\delta^2)}{(3-2\delta)^2}s_2 &\geq \frac{\delta}{3-2\delta}s_1+\frac{(3-\delta)}{(3-2\delta)}s_2\\
s_1 &\geq \frac{3-\delta}{3-2\delta}s_2
\end{aligned}$$

- Case B:

$$\begin{aligned}
&\frac{(27-63\delta+51\delta^2-17\delta^3+2\delta^4)}{(3-2\delta)^3}s_1+\frac{(3\delta^2-4\delta^3+\delta^4)}{(3-2\delta)^3}s_2+\frac{9\delta-15\delta^2+9\delta^3-2\delta^4}{(3-2\delta)^3}\\
\geq\ & \frac{9-12\delta+3\delta^2}{(3-2\delta)^2}s_2+\frac{\delta}{(3-2\delta)}\\
s_1 \geq\ & \frac{27-27\delta+3\delta^2+\delta^3}{(3-2\delta)(3-\delta)^2}s_2+\frac{\delta^2}{(3-\delta)^2}
\end{aligned}$$

*Part (b).* Since we focus on the half of the $\Delta$ in which $s_1 \geq s_2$, by part (a) of the same Lemma, it is enough to prove that $d_1 + d_v \leq 1$.

- Case A:

$$\begin{aligned}
s_v-\frac{\delta s_2}{(3-2\delta)}+\frac{(4\delta^2-12\delta+9)}{(3-2\delta)^2}s_1+\frac{(3\delta-\delta^2)}{(3-2\delta)^2}s_2 &\leq 1\\
s_v+s_1+\frac{\delta^2}{(3-2\delta)^2}s_2 &\leq 1
\end{aligned}$$

which holds for any $\delta \in [0,1)$, because $s_v + s_1 + s_2 = 1$ and $\frac{\delta^2}{(3-2\delta)^2} \in [0,1)$. To see this notice that $\frac{\delta^2}{(3-2\delta)^2}$ is monotonically increasing in $\delta$ and is equal to 1 when $\delta = 1$.

- Case B:

$$\frac{(27-63\delta+51\delta^2-17\delta^3+2\delta^4)s_1}{(3-2\delta)^3}+\frac{(3\delta^2-4\delta^3+\delta^4)s_2}{(3-2\delta)^3}+\frac{9\delta-15\delta^2+9\delta^3-2\delta^4}{(3-2\delta)^3}\leq 1$$

Notice that $\frac{(27-63\delta+51\delta^2-17\delta^3+2\delta^4)}{(3-2\delta)^3} \geq \frac{(3\delta^2-4\delta^3+\delta^4)}{(3-2\delta)^3}$ for any $\delta \in [0,1)$, so the LHS has an upper bound when $s_1 = 1$ and $s_2 = 0$. Therefore, we can prove the following inequality:

$$
\begin{aligned}
\frac{(27-63\delta+51\delta^2-17\delta^3+2\delta^4)}{(3-2\delta)^3}+\frac{9\delta-15\delta^2+9\delta^3-2\delta^4}{(3-2\delta)^3} &\leq 1\\
\frac{(3-2\delta)^3}{(3-2\delta)^3} &\leq 1
\end{aligned}
$$

- Case C:

$$
\begin{aligned}
\frac{(-3+\delta)(3\delta^2-12\delta+9)}{(-3+2\delta)(\delta^2-15\delta+18)}(s_1+s_2)+\frac{(-3+\delta)(6\delta-4\delta^2)}{(-3+2\delta)(\delta^2-15\delta+18)} &\leq 1\\
(s_1+s_2) &\leq \frac{(\delta-6)(-3+2\delta)}{(-3+\delta)^2}
\end{aligned}
$$

which holds for any $\delta \in [0,1)$, since $s_v+s_1+s_2 = 1$ and $\frac{(\delta-6)(-3+2\delta)}{(-3+\delta)^2} \geq 1$ for any $\delta \in [0,1)$. To see this notice that $\frac{(\delta-6)(-3+2\delta)}{(-3+\delta)^2}$ is monotonically decreasing in $\delta$ and it is equal to $5/4$ when $\delta = 1$.

- Case D:

$$
\begin{aligned}
(1-s_1-s_2)-\frac{\delta s_2}{3-2\delta}-\frac{(3-\delta)(9-6\delta-\delta^2)}{(3-2\delta)(\delta^2-15\delta+18)}s_1-\frac{(3-\delta)(9-6\delta+\delta^2)}{(3-2\delta)(\delta^2-15\delta+18)}s_2 &\leq 1\\
-\frac{-30\delta^2+54\delta-27+3\delta^3}{(-3+2\delta)(\delta^2-15\delta+18)}s_1-\frac{-9\delta^2+36\delta-27}{(-3+2\delta)(\delta^2-15\delta+18)}s_2 &\leq 0
\end{aligned}
$$

which holds for any $\delta \in [0,1]$ because the coefficients of $s_1$ and $s_2$ in the right hand side are always non-positive: they are strictly increasing in $\delta$ and are equal to 0 for $\delta = 1$. ■

We now show that equilibrium proposals are optimal over feasible alternatives in $\overline{\Delta}$.

**Lemma 4** $\mu_i[\mathbf{z}|\mathbf{s}] > 0 \Rightarrow \mathbf{z} \in \arg\max\{U_i(\mathbf{x})|\mathbf{x} \in W(s) \cap \overline{\Delta}\}$, *for all* $\mathbf{z}, \mathbf{s} \in \Delta$.

**Proof.** All equilibrium proposals take the form of minimal winning coalition proposals: $\mathbf{x}(v, j, d_j(\mathbf{x}))$ when the veto player is proposing and $\mathbf{x}(j, v, d_v(\mathbf{x}))$ when a non-veto player is proposing. Also, whenever $\mu_v[\mathbf{x}(v,1,d_1)|\mathbf{s}] > 0$ and $\mu_v[\mathbf{x}(v,2,d_2)|\mathbf{s}] > 0$, we have $d_1 = d_2$

so that $U_v(\mathbf{x}(v,1,d_1)) = U_v(\mathbf{x}(v,2,d_2))$. Thus, in view of Lemma 2 it suffices to show that if $\mu_i[\mathbf{x}(i,j,dj)|\mathbf{s}] = 1$, then $U_i(\mathbf{x}(i,j,d_j)) = U_i(\mathbf{x}(i,h,d_h))$, $h \neq i,j$, i.e. proposer $i$ has no incentive to coalesce with player $h$ instead of $j$. This is immediate for a non-veto player, since only coalescing with the veto player guarantees the possibility to change the state. To show that - for the veto player - if $\mu_v[\mathbf{x}(v,j,dj)|\mathbf{s}] = 1$, then $U_v(\mathbf{x}(v,j,d_j)) = U_v(\mathbf{x}(v,h,d_h))$, $j \neq h$, it suffices to show $d_h \geq d_j$ by part (b) of Lemma 1. In Proposition 1 we have $s_1 \geq s_2$, (by part (a) of Lemma 3) $d_1 \geq d_2$, and when $d_1 \neq d_2$, we have $\mu_v[\mathbf{x}(v,1,d_1)|\mathbf{s}] = 0$ which gives the desired result. ■

We conclude the proof by showing that optimum proposal strategies cannot belong in $\Delta/\overline{\Delta}$. In particular, we show that if an alternative in $\Delta/\overline{\Delta}$ beats the status quo by majority rule, then for any player $i$ we can find another alternative in $\overline{\Delta}$ that is also majority preferred to the status quo and improves $i$'s utility.

**Lemma 5** *Assume* $\mathbf{x} \in W(\mathbf{s}) \cap \Delta/\overline{\Delta}$*; then for any* $i = v,1,2$ *there exists* $\mathbf{y} \in W(\mathbf{s}) \cap \overline{\Delta}$ *such that* $U_i(\mathbf{y}) \geq U_i(\mathbf{s})$.

**Proof.** Consider first the veto player, $i = v$. Let $\mathbf{x} \in W(\mathbf{s}) \cap \Delta/\overline{\Delta}$. Consider first the case $\mathbf{x} \in A_v^*(\mathbf{s})$. Then, $\mathbf{x}$ is weakly preferred to $s$ by $v$ and at least one $i$, $i = 1,2$. Now set $\mathbf{y} = \mathbf{x}(v,j,d_j(\mathbf{x}))$, where $d_j(\mathbf{x})$ is the applicable demand from Proposition 1. We have $U_j(\mathbf{x}(v,j,d_j(\mathbf{x}))) \geq U_j(\mathbf{x})$, by the definition of demand. From part (b) of Lemma 3 have $d_v(\mathbf{x}) + d_j(\mathbf{x}) \leq 1$ and as a result $x_v(v,j,d_j(\mathbf{x})) = 1 - d_j(\mathbf{x}) \geq d_v(\mathbf{x})$; hence, $U_v(\mathbf{x}(v,j,d_j(\mathbf{x}))) \geq U_v(\mathbf{x})$, which follows from the weak monotonicity in part (b) of Lemma 1. Thus, $\mathbf{y} = \mathbf{x}(v,j,d_j(\mathbf{x})) \in W(\mathbf{s})$ (because is supported by $v$ and $j$), and we have completed the proof for this case. Now consider the case $\mathbf{x} \notin A_v^*(\mathbf{s})$, i.e. $U_v(\mathbf{s}) > U_v(\mathbf{x})$. Part (a) of Lemma 3 ensures that $d_v(\mathbf{s}) + d_j(\mathbf{s}) \leq 1$, hence proposal $\mathbf{y} = \mathbf{x}(v,j,d_j(\mathbf{s}))$ has $x_v(v,j,d_j(\mathbf{s})) = 1 - d_j(\mathbf{s}) \geq d_v(\mathbf{s})$. Then $U_v(\mathbf{y}) \geq U_v(\mathbf{s}) > U_v(\mathbf{x})$, and $\mathbf{y} \in W(\mathbf{s}) \cap \overline{\Delta}$.

Now consider a non veto player, $i = 1,2$. Let $\mathbf{x} \in W(\mathbf{s}) \cap \Delta/\overline{\Delta}$. Consider first the case $\mathbf{x} \in A_i^*(\mathbf{s})$. Then, $\mathbf{x}$ is weakly preferred to $s$ by $v$ and (at least) $i$. Now set $\mathbf{y} = \mathbf{x}(i,v,d_v(\mathbf{x}))$, where $d_v(\mathbf{x})$ is the applicable demand from Proposition 1. We have $U_v(\mathbf{x}(i,v,d_v(\mathbf{x}))) \geq U_v(\mathbf{x})$, by the definition of demand. From part (b) of Lemma 3 have $d_v(\mathbf{x}) + d_i(\mathbf{x}) \leq 1$ and as a result $x_i(i, v, d_v(\mathbf{x})) = 1 - d_v(\mathbf{x}) \geq d_i(\mathbf{x})$; hence, $U_i(\mathbf{x}(i, v, d_v(\mathbf{x}))) \geq U_i(\mathbf{x})$, which follows from the weak monotonicity in part (b) of Lemma 1. Thus, $\mathbf{y} = \mathbf{x}(i, v, d_v(\mathbf{x})) \in W(\mathbf{s}) \cap \overline{\Delta}$ (because is supported by $v$ and $i$), and we have completed the proof for this case. Finally, consider the case $\mathbf{x} \notin A_i^*(\mathbf{s})$, i.e. $U_i(\mathbf{s}) > U_i(\mathbf{x})$. Part (a) of Lemma 3 ensures that $d_v(\mathbf{s}) + d_i(\mathbf{s}) \leq 1$, hence proposal $\mathbf{y} = \mathbf{x}(i, v, d_v(\mathbf{s}))$ has $x_i(i, v, d_v(\mathbf{s})) = 1 - d_v(\mathbf{s}) \geq d_i(\mathbf{s})$. Then $U_i(\mathbf{y}) \geq U_i(\mathbf{s}) > U_i(\mathbf{x})$, and $\mathbf{y} \in W(\mathbf{s}) \cap \overline{\Delta}$, which completes the proof. ■

As a result of Lemmas 4 and 5, equilibrium proposals are optimal over the entire range of feasible alternatives which completes the proof.

## Proof of Proposition 2

The result of Proposition 2 follows once we establish that the proposal strategies in the equilibrium from Proposition 1 are weakly continuous in the status quo $\mathbf{s}$, i.e., that in equilibrium a small change in the status quo implies a small change in proposal strategies and, by extension, to the equilibrium transition probabilities. Formally, we want to show that the equilibrium proposal strategies $\mu_i^*$ in the proof of Proposition 1 are such that for every $\mathbf{s} \in \Delta$ and every sequence $\mathbf{s}_n \in \Delta$ with $\mathbf{s}_n \to \mathbf{s}$, $\mu_i^*[\cdot|\mathbf{s}_n]$ converges weakly to $\mu_i^*[\cdot|\mathbf{s}]$.

The equilibrium is such that $\mu_i^*[\cdot|\mathbf{s}]$ $i = \{1, 2\}$ has mass on only one point $\mathbf{x}(i, v, d_v(\mathbf{s}))$ and that $\mu_v^*[\cdot|\mathbf{s}]$ has mass on at most two points $\mathbf{x}(v, 1, d_1(\mathbf{s}))$, and $\mathbf{x}(v, 2, d_2(\mathbf{s}))$. It suffices to show that these proposals (when played with positive probability) and associated mixing probabilities are continuous in $\mathbf{s}$. Continuity holds in the interior of Cases A-D in Proposition 1, so it remains to check the boundaries of these cases. In order to distinguish the various applicable functional forms we shall write $d_i^w$ and $\mu_v^w[\cdot|\mathbf{s}]$ where $w = \{A, B, C, D\}$ identifies the case for which the respective functional form applies.

- Boundary of Cases A and B: at the boundary (as in the interior of the two cases) we have $\mu_v^A[\mathbf{x}(v, 1, d_2)|\mathbf{s}] = \mu_v^B[\mathbf{x}(v, 1, d_2)|\mathbf{s}] = 0$; at the boundary we have $s_1 = 1 - \frac{3-\delta}{3-2\delta} s_2$, then:

$$
\begin{aligned}
d_v^A &= d_v^B = 0 \\
d_1^A &= d_1^B = 1 - \frac{9 - 12\delta - 3\delta^2}{(3-2\delta)^2} s_2 \\
d_2^A &= d_2^B = \frac{9 - 12\delta - 3\delta^2}{(3-2\delta)^2} s_2 + \frac{\delta}{(3-2\delta)}
\end{aligned}
$$

- Boundary of Cases B and C: at the boundary we have $s_1 = \frac{27-27\delta+3\delta^2+\delta^3}{(3-2\delta)(3-\delta)^2} s_2 + \frac{\delta^2}{(3-\delta)^2}$; then:

$$
\begin{aligned}
\mu_v^B[\mathbf{x}(v,1,d_2)|\mathbf{s}] &= \mu_v^C[\mathbf{x}(v,1,d_2)|\mathbf{s}] = 1 \\
d_v^B &= d_v^C = 0 \\
d_1^B &= d_1^C = \frac{9 - 12\delta + 3\delta^2}{(3-2\delta)^2} s_2 + \frac{\delta}{(3-2\delta)} \\
d_2^B &= d_2^C = \frac{9 - 12\delta - 3\delta^2}{(3-2\delta)^2} s_2 + \frac{\delta}{(3-2\delta)}
\end{aligned}
$$

- Boundary of Cases C and D: at the boundary we have $s_1 = 1 - \frac{3-\delta}{3-2\delta} s_2$; then:

$$
\begin{aligned}
\mu_v^C[\mathbf{x}(v,1,d_2)|\mathbf{s}] &= \mu_v^D[\mathbf{x}(v,1,d_2)|\mathbf{s}] = \\
&= \frac{(-36\delta^3 + 3\delta^4 + 153\delta^2 - 270\delta + 162)s_2 + 15\delta^3 - 2\delta^4 - 72\delta^2 + 135\delta - 81}{\left[(-12\delta^2 + 3\delta^3 + 9\delta)s_2 - 9\delta^2 - 2\delta^3 + 36\delta - 27\right]\delta} \\
d_v^C &= d_v^D = 0 \\
d_1^C &= d_1^D = d_2^C = d_2^D = \\
&= \frac{(-3+\delta)(-12\delta^2 + 3\delta^3 + 9\delta)}{(2\delta-3)^2(\delta^2 - 15\delta + 18)} s_2 + \frac{(-3+\delta)(-2\delta^3 - 9\delta^2 + 36\delta - 27)}{(2\delta-3)^2(\delta^2 - 15\delta + 18)}
\end{aligned}
$$

- Boundary of Cases D and A: at the boundary we have $s_1 = \frac{3-\delta}{3-2\delta}s_2$; then:

$$\begin{aligned}
\mu_v^D[\mathbf{x}(v,1,d_2)|\mathbf{s}] &= \mu_v^A[\mathbf{x}(v,1,d_2)|\mathbf{s}] = 1 \\
d_v^D &= d_v^A = s_v - \frac{\delta s_2}{3-2\delta} \\
d_1^D &= d_1^A = d_2^D = d_2^A = \frac{(3-\delta)^2}{(3-2\delta)^2}s_2
\end{aligned}$$

## Proof of Proposition 3

The result derives from the features of the MPE characterized in the proof of Proposition 1. In this MPE, once we reach allocations in the absorbing set $\overline{\Delta}$, which happens after at most one period, the veto player is able to increase his share whenever he has the power to propose, and keeps a constant share when not proposing. For any $\varepsilon$ and any starting allocation $\mathbf{s}^0$, there exists a number of proposals by the veto player—which depends on $\delta$—such that the veto player's allocation in the status quo will be at least $1-\varepsilon$ for all subsequent periods. Let this number of proposals be $n^*(\varepsilon, \delta, \mathbf{s}^0)$. Since each player has a positive probability of proposing in each period, the probability that in infinitely many periods the veto player proposes less than $n^*(\varepsilon, \delta, \mathbf{s}^0)$ is zero.

# B Proofs of Propositions from Section 5

## Proof of Proposition 4

This result follows directly from the equilibrium demand of the poorer non-veto player in the absorbing set $\overline{\Delta}$, $d_{nv}(\mathbf{s},\delta) = \frac{\delta}{3-2\delta}\overline{s_{nv}}$. When $\delta = 0$, this demand is zero. This means that, when the status quo is in $\overline{\Delta}$—a set that is reached in at most one period—the poorer non-veto supports any proposal by the veto player. The veto player can thus pass his ideal outcome as soon $\mathbf{s} \in \overline{\Delta}$ and he proposes. On the other hand, when $\delta \in (0,1)$, this is not possible, and the poorer non-veto player always demands a positive share of the dollar to support any allocation that makes the veto player richer. The convergence in this case is only asymptotic as the non-veto player's demand is always positive as long as the allocation to the richer non-veto is positive, that is as long as the poorer veto player does not have the whole dollar in the status quo.[26]

## Proof of Proposition 5

As for Proposition 1, we focus on the allocations in which $s_1 \geq s_2$. The other cases are symmetric. Consider the following equilibrium proposal strategies (all supported by a minimal winning coalition) and demands (as defined in the proof of Proposition 1):

- CASE A: $s_1 \leq 1 - \frac{2-\delta(1-p_v)}{2-\delta(1+p_v)}s_2$; $s_1 \geq \frac{2-\delta(1-p_v)}{2-\delta(1+p_v)}s_2$

$$
\begin{aligned}
\mathbf{x}^v &= [1-d_2^A, 0, d_2^A], \mathbf{x}^1 = [d_v^A, 1-d_v^A, 0], \mathbf{x}^2 = [d_v^A, 0, 1-d_v^A] \\
d_v^A &= s_v - \frac{2p_v\delta}{2-(1+p_v)\delta}s_2 \\
d_2^A &= \frac{\delta(1-p_v)}{2-\delta(1+p_v)}s_1 + \frac{2-\delta(1-p_v)}{2-\delta(1+p_v)}s_2 \\
d_1^A &= \frac{-4p_v\delta + 4 + 2p_v\delta^2 - 4\delta + p_v^2\delta^2 + \delta^2}{(2-\delta(1+p_v))^2}s_1 + \frac{-p_v^2\delta^2 - \delta^2 - 2p_v\delta + 2\delta + 2p_v\delta^2}{(2-\delta(1+p_v))^2}s_2
\end{aligned}
$$

[26] Notice that when the initial division of the dollar—which is assumed to be exogenous—assigns the whole dollar to the veto player, then the status quo will never be changed and the veto player gets the whole dollar in every period.

- CASE B: $s_1 > 1 - \frac{2-\delta(1-p_v)}{2-\delta(1+p_v)} s_2; s_1 \geq \frac{p_v^3\delta^3 - 2p_v^2\delta^3 + p_v\delta^3 + p_v^2\delta^2 - 2p_v\delta^2 + \delta^2 - 4\delta + 4}{(2-(1+p_v)\delta)(2-(1-p_v)\delta)(1-p_v\delta)} s_2 + \frac{p_v^3\delta^3 - p_v\delta^3 - 2p_v^2\delta^2 + 2p_v\delta^2}{(2-(1+p_v)\delta)(2-(1-p_v)\delta)(1-p_v\delta)}$

$$\begin{aligned}
\mathbf{x}^v &= [1-d_2^B, 0, d_2^B], \mathbf{x}^1 = [d_v^B, 1-d_v^B, 0], \mathbf{x}^2 = [d_v^B, 0, 1-d_v^B] \\
d_v^B &= 0 \\
d_2^B &= \frac{-2p_v\delta^2 + 2\delta^2 + 2p_v\delta - 6\delta + 4}{(2-\delta(1+p_v))^2} s_2 + \frac{p_v^2\delta^2 - \delta^2 - 2p_v\delta + 2\delta}{(2-\delta(1+p_v))^2} \\
d_1^B &= \frac{16\delta - 8 + 2p_v^2\delta^2 + 8p_v\delta - 16p_v\delta^2 - 10\delta^2 - 2p_v^3\delta^3 + 2\delta^3 - 2p_v^2\delta^3 + 10p_v\delta^3 - 2p_v\delta^4 + 2p_v^3\delta^4}{(-2+p_v\delta+\delta)^3} s_1 + ... \\
&+ \frac{6p_v\delta^3 + 2p_v^3\delta^3 - 8p_v^2\delta^3 + 4p_v^2\delta^2 - 4p_v\delta^2 + 4p_v^2\delta^4 - 2p_v\delta^4 - 2p_v^3\delta^4}{(-2+p\delta+\delta)^3} s_2 + ... \\
&+ \frac{-\delta^3 - 7p_v\delta^3 + 5p_v^2\delta^3 + 4p_v\delta^2 + 4p_v\delta - 8p_v^2\delta^2 + 4\delta^2 + 2p_v\delta^4 - 4\delta + 3p_v^3\delta^3 - 2p_v^3\delta^4}{(-2+p\delta+\delta)^3}
\end{aligned}$$

- CASE C: $s_1 > \frac{2-\delta}{2-\delta(1-p_v)} - s_2; s_1 < \frac{p_v^3\delta^3 - 2p_v^2\delta^3 + p_v\delta^3 + p_v^2\delta^2 - 2p_v\delta^2 + \delta^2 - 4\delta + 4}{(2-(1+p_v)\delta)(2-(1-p_v)\delta)(1-p_v\delta)} s_2 + \frac{p_v^3\delta^3 - p_v\delta^3 - 2p_v^2\delta^2 + 2p_v\delta^2}{(2-(1+p_v)\delta)(2-(1-p_v)\delta)(1-p_v\delta)}$

$$\begin{aligned}
\mathbf{x}^v &= \begin{cases} [1-d_2^C, d_2^C, 0] & \text{w/ } \Pr = 1-\mu_v^C \\ [1-d_2^C, 0, d_2^C] & \text{w/ } \Pr = \mu_v^C \end{cases}, \mathbf{x}^1 = [d_v^C, 1-d_v^C, 0], \mathbf{x}^2 = [d_v^C, 0, 1-d_v^C] \\
d_v^C &= 0 \\
d_1^C &= d_2^C = \frac{(p_v\delta-1)(-p_v\delta^2+\delta^2+p_v\delta-3\delta+2)}{(p_v\delta+\delta-2)(p_v^2\delta^2-2p_v\delta-\delta+2)}(s_1+s_2) + \frac{(p_v\delta-1)(p_v^2\delta^2+2\delta-\delta^2-2p_v\delta)}{(p_v\delta+\delta-2)(p_v^2\delta^2-2p_v\delta-\delta+2)} \\
\mu_v^C &= \frac{(p_v^3\delta^3 - 2p_v^2\delta^3 + p_v\delta^3 + p_v^2\delta^2 - 2p_v\delta^2 + \delta^2 - 4\delta + 4)s_1}{2p_v\delta((-p_v\delta^2+\delta^2+p_v\delta-3\delta+2)s_1 + (-p_v\delta^2+\delta^2+p_v\delta-3\delta+2)s_2 + p_v^2\delta^2 + 2\delta - \delta^2 - 2p_v\delta)} + \\
&+ \frac{(-p_v^3\delta^3 + p_v\delta^3 + p_v^2\delta^2 - 4p_v\delta^2 - \delta^2 + 4p_v\delta + 4\delta - 4)s_2}{2p_v\delta((-p_v\delta^2+\delta^2+p_v\delta-3\delta+2)s_1 + (-p_v\delta^2+\delta^2+p_v\delta-3\delta+2)s_2 + p_v^2\delta^2 + 2\delta - \delta^2 - 2p_v\delta)} + \\
&+ \frac{p_v^3\delta^3 - p_v\delta^3 - 2p_v^2\delta^2 + 2p_v\delta^2}{2p_v\delta((-p_v\delta^2+\delta^2+p_v\delta-3\delta+2)s_1 + (-p_v\delta^2+\delta^2+p_v\delta-3\delta+2)s_2 + p_v^2\delta^2 + 2\delta - \delta^2 - 2p_v\delta)}
\end{aligned}$$

- CASE D: $s_1 \leq \frac{2-\delta}{2-\delta(1-p_v)} - s_2; s_1 < \frac{2-\delta(1-p_v)}{2-\delta(1+p_v)} s_2$

$$\begin{aligned}
\mathbf{x}^v &= \begin{cases} [1-d_2^D, d_2^D, 0] & \text{w/ } \Pr = 1-\mu_v^D \\ [1-d_2^D, 0, d_2^D] & \text{w/ } \Pr = \mu_v^D \end{cases}, \mathbf{x}^1 = [d_v^D, 1-d_v^D, 0], \mathbf{x}^2 = [d_v^D, 0, 1-d_v^D] \\
d_v^D &= s_v - \frac{2p_v\delta}{2-(1+p_v)\delta} s_2 \\
d_1^D &= d_2^D = \frac{(p_v\delta-1)(2p_v^2\delta^3 - 2p_v\delta^3 - 3p_v^2\delta^2 + p_v\delta^2 - 2\delta^2 + 3p_v\delta + 7\delta - 6)}{(-3+2\delta)(p_v\delta+\delta-2)(p_v^2\delta^2 - 2p_v\delta - \delta + 2)} s_1 + \\
&+ \frac{(p_v\delta-1)(p_v^2\delta^3 - 2p_v\delta^3 + \delta^3 - 3p_v^2\delta^2 + 3p_v\delta^2 - 4\delta^2 + 3p_v\delta + 7\delta - 6)}{(-3+2\delta)(p_v\delta+\delta-2)(p_v^2\delta^2 - 2p_v\delta - \delta + 2)} s_2 \\
\mu_v^D &= \frac{(4p_v^3\delta^4 - 4p_v^2\delta^4 - 6p_v^3\delta^3 + 4p_v^2\delta^3 + 2\delta^3 + 3p_v^2\delta^2 - 11\delta^2 + 20\delta - 12)s_1}{T} + \\
&+ \frac{(-p_v^3\delta^4 + p_v\delta^4 - 6p_v\delta^3 - 2\delta^3 + 3p_v^2\delta^2 + 14p_v\delta^2 + 11\delta^2 - 12p_v\delta - 20\delta + 12)s_2}{T}
\end{aligned}$$

where $T = 2p_v\delta[(2p_v^2\delta^3 - 2p_v\delta^3 - 3p_v^2\delta^2 + p_v\delta^2 - 2\delta^2 + 3p_v\delta + 7\delta - 6)s_1 +$ $+ (p_v^2\delta^3 - 2p_v\delta^3 + \delta^3 - 3p_v^2\delta^2 + 3p_v\delta^2 - 4\delta^2 + 3p_v\delta + 7\delta - 6)s_2]$, and $\mu_v^C$, $\mu_v^D$ are the probabilities that the veto player coalesces with non-veto player 2 in cases C, and D respectively. These are well defined probability in [0,1] such that $d_1^C = d_2^C$ and $d_1^D = d_2^D$, or such that $s_1 + \delta v_1(\mathbf{s}, \mu_v, d_2) = s_2 + \delta v_2(\mathbf{s}, \mu_v, d_2)$.

Remember that the veto player proposes with probability $p_v$ and each non-veto player with probability $(1-p_v)/2$. If proposers use the proposal strategies above and these proposals pass, players' continuation values in cases A-D are as follows:

- Case A

$$\begin{aligned}
v_v(\mathbf{s}) &= \frac{1}{(1-\delta)} - \frac{(1-p_v)(2-\delta)}{(1-\delta)[2-\delta(1-p_v)]} s_1 - \frac{1}{(1-\delta)} s_2 \\
v_1(\mathbf{s}) &= \frac{(1-p_v)[2-\delta-\delta p_v(3-2\delta)]}{2(1-\delta)(1-p_v\delta)[2-(1-p_v)\delta]} s_1 + \frac{(1-p_v)}{2(1-\delta)(1-p_v\delta)} s_2 \\
v_2(\mathbf{s}) &= \frac{p_v^2\delta - \delta - 2p_v + 2}{2(1-\delta)(1-p_v\delta)[2-(1-p_v)\delta]} s_1 + \frac{-2p_v^2\delta^2 + 2p_v\delta^2 + p_v^2\delta - 4p_v\delta - \delta + 2p_v + 2}{2(1-\delta)(1-p_v\delta)[2-(1-p_v)\delta]} s_2
\end{aligned}$$

- Case B

$$
\begin{aligned}
v_v(\mathbf{s}) &= \frac{2p_v(p_v\delta^2+3\delta-\delta^2-p_v\delta-2)}{(p_v\delta+2-\delta)(-1+\delta)(-2+p_v\delta+\delta)}s_2+\frac{2p_v(-p_v\delta+2-\delta)}{(p_v\delta+2-\delta)(-1+\delta)(-2+p_v\delta+\delta)} \\
v_1(\mathbf{s}) &= -\frac{(-2p_v\delta^3+2p_v^2\delta^3-2p_v^2\delta^2+6p_v\delta^2-4p_v\delta)(-1+p_v)}{2(-2+p_v\delta+\delta)(-1+\delta)(p_v\delta+2-\delta)(p_v\delta-1)}s_2+ \\
&\quad -\frac{(-4+2p_v\delta^3+2p_v^2\delta^3-8p_v\delta^2+8p_v\delta-3p_v^2\delta^2-\delta^2+4\delta)(-1+p_v)}{2(-2+p_v\delta+\delta)(-1+\delta)(p_v\delta+2-\delta)(p_v\delta-1)} \\
v_2(\mathbf{s}) &= -\frac{2p_v^2\delta^2-2p_v\delta^2-2p_v^2\delta+6p_v\delta-4p_v}{2(-1+\delta)(p_v\delta+2-\delta)(p_v\delta-1)}s_2-\frac{-p_v^2\delta+\delta+2p_v-2}{2(p_v\delta+2-\delta)(-1+\delta)(p_v\delta-1)}
\end{aligned}
$$

- Case C

$$
\begin{aligned}
v_v(\mathbf{s}) &= \frac{(p_v^2\delta^3+2p_v\delta^2-p_v\delta^3-p_v^2\delta^2-p_v\delta-3\delta+\delta^2+2)p_v}{(-1+\delta)(p_v\delta+2-\delta)(p_v^2\delta^2-2p_v\delta-\delta+2)}(s_1+s_2)+ \\
&\quad \frac{(2\delta-p_v^2\delta^3+p_v\delta^3-p_v^2\delta^2-p_v\delta^2+4p_v\delta-4)p_v}{(-1+\delta)(p_v\delta+2-\delta)(p_v^2\delta^2-2p_v\delta-\delta+2)} \\
v_1(\mathbf{s}) &= -\frac{-4+\delta^3-5\delta^2+8\delta-2p_v^2\delta^4+2p_v^3\delta^4-2p_v^3\delta^3+3p_v^2\delta^3+2p_v\delta^3-p_v^2\delta^2-6p_v\delta^2+4p_v\delta}{2\delta(p_v^2\delta^2-2p_v\delta-\delta+2)(p_v\delta+2-\delta)(-1+\delta)}s_1+ \\
&\quad -\frac{4-\delta^3+5\delta^2-8\delta-p_v^2\delta^2+p_v^2\delta^3}{2\delta(p_v^2\delta^2-2p_v\delta-\delta+2)(p_v\delta+2-\delta)(-1+\delta)}s_2+ \\
&\quad -\frac{-4p_v\delta+4\delta+4p_v^2\delta^2-4\delta^2+\delta^3+p_v\delta^3-p_v^2\delta^3-p_v^3\delta^3}{2\delta(p_v^2\delta^2-2p_v\delta-\delta+2)(p_v\delta+2-\delta)(-1+\delta)} \\
v_2(\mathbf{s}) &= -\frac{4-\delta^3+5\delta^2-8\delta-p_v^2\delta^2+p_v^2\delta^3}{2\delta(p_v^2\delta^2-2p_v\delta-\delta+2)(p_v\delta+2-\delta)(-1+\delta)}s_1 \\
&\quad -\frac{-4+\delta^3-5\delta^2+8\delta-2p_v^2\delta^4+2p_v^3\delta^4-2p_v^3\delta^3+3p_v^2\delta^3+2p_v\delta^3-p_v^2\delta^2-6p_v\delta^2+4p_v\delta}{2\delta(p_v^2\delta^2-2p_v\delta-\delta+2)(p_v\delta+2-\delta)(-1+\delta)}s_2 \\
&\quad -\frac{-4p_v\delta+4\delta+4p_v^2\delta^2-4\delta^2+\delta^3+p_v\delta^3-p_v^2\delta^3-p_v^3\delta^3}{2\delta(p_v^2\delta^2-2p_v\delta-\delta+2)(p_v\delta+2-\delta)(-1+\delta)}
\end{aligned}
$$

- Case D

$$\begin{aligned}
v_v(\mathbf{s}) &= \frac{H}{(-1+\delta)(p_v\delta+2-\delta)(p_v^2\delta^2-2p_v\delta-\delta+2)(-3+2\delta)}s_1+\\
&+\frac{I}{(-1+\delta)(p_v\delta+2-\delta)(p_v^2\delta^2-2p_v\delta-\delta+2)(-3+2\delta)}s_2+\\
&+\frac{12-6p_v\delta-20\delta+11\delta^2+7p_v\delta^2+3p_v^3\delta^3-2\delta^3-2p_v\delta^3-3p_v^2\delta^3+2p_v^2\delta^4-2p_v^3\delta^4}{(-1+\delta)(p_v\delta+2-\delta)(p_v^2\delta^2-2p_v\delta-\delta+2)(-3+2\delta)}\\
v_1(\mathbf{s}) &= -\frac{J}{2\delta(p_v^2\delta^2-2p_v\delta-\delta+2)(-3+2\delta)(p_v\delta+2-\delta)(-1+\delta)}s_1\\
&-\frac{K}{2\delta(p_v^2\delta^2-2p_v\delta-\delta+2)(-3+2\delta)(p_v\delta+2-\delta)(-1+\delta)}s_2\\
v_2(\mathbf{s}) &= -\frac{L}{2\delta(p_v^2\delta^2-2p_v\delta-\delta+2)(-3+2\delta)(p_v\delta+2-\delta)(-1+\delta)}s_1\\
&-\frac{M}{2\delta(p_v^2\delta^2-2p_v\delta-\delta+2)(-3+2\delta)(p_v\delta+2-\delta)(-1+\delta)}s_2
\end{aligned}$$

where $H = -11\delta^2+20\delta+2\delta^3+6p_v-12+6p_v^3\delta^2-9p_v^2\delta+5p_v^2\delta^3+4p_v\delta^3+3p_v^2\delta^2-12p_v\delta^2+5p_v\delta+2p_v^3\delta^4-2p_v^2\delta^4-7p_v^3\delta^3$, $I = 6p_v+16\delta+6p_v^3\delta^2-9p_v^2\delta+6p_v^2\delta^3+3p_v\delta^3-p_v^2\delta^2-12p_v\delta^2+9p_v\delta+2p_v^3\delta^4-2p_v^2\delta^4-6p_v^3\delta^3-12+\delta^3-7\delta^2$, $J = -12p_v^3\delta^4+10p_v^2\delta^4+9p_v^3\delta^3-9p_v^2\delta^2-21p_v\delta^3+18p_v\delta^2+12+6\delta^4p_v+4p_v^3\delta^5-4p_v^2\delta^5-24\delta^3+51\delta^2-44\delta+4\delta^4$, $K = -p_v^3\delta^4+\delta^4p_v-3p_v\delta^3-9p_v^2\delta^2-4p_v\delta^2+12p_v\delta+2p_v^2\delta^3+p_v^2\delta^4+3p_v^3\delta^3+6\delta^3-15\delta^2-\delta^4-12+20\delta$, $L = -2p_v^3\delta^4+3p_v^3\delta^3+6p_v^2\delta^3-9p_v^2\delta^2-3p_v\delta^3-8p_v\delta^2+12p_v\delta-12+2\delta^4p_v+2\delta^3-11\delta^2+20\delta$, and $M = -11p_v^3\delta^4+5\delta^4p_v-21p_v\delta^3-9p_v^2\delta^2+22p_v\delta^2-4p_v^2\delta^3+11p_v^2\delta^4+9p_v^3\delta^3+4p_v^3\delta^5-4p_v^2\delta^5-20\delta^3+47\delta^2+3\delta^4+12-44\delta$.

One can show that these equilibrium strategies and the associated value functions are part of an MPE, using the same strategy employed in the proof of Proposition 1. The only difference with the proof of Proposition 1 is in the proof of Lemma 3 (b). With heterogenous recognition probabilities, $d_v+d_2$ is not always less than or equal to 1 when $p_v \in (0.5,1)$. This condition is what determines the bound on $\delta$ in the statement of Proposition 5. In particular, the binding case is the allocation where $s_1 = s_2 = 0.5$. This is the case in which non-veto players are most demanding, as it can be proven by inspection of $d_2$ in the four cases above. Setting $s_1 = s_2 = 0.5$ and solving for $d_v^C + d_2^C \leq 1$, we obtain the bound $\delta < \overline{\delta} = \frac{1+3p_v-\sqrt{1+6p_v-7p_v^2}}{4p_v^2}$. Figure 6 shows the space of $(p_v,\delta)$ for which the MPE in Proposition 5 exists.

The irregular shape of the set of parameters for which the MPE from Proposition 5 exists merits discussion. The condition for existence of the MPE from Proposition 5 is determined by the ability of the veto player to convince a non-veto player to completely expropriate the other non-veto player. The allocation demanded by a non-veto player to accept such a proposal changes with $\delta$, $p_v$ and $\mathbf{s}$ and the available resources are not always sufficient to meet this demand. To understand the intuition behind the irregular shape of the existence set, consider the status quo allocation where the dollar is split evenly between the two non-veto players, $\mathbf{s} = (0, 1/2, 1/2)$. This is the status quo allocation where the veto player's bargaining power is the lowest and it is toughest to trigger convergence to full appropriation by the veto player. In the MPE from Proposition 4, a non-veto player proposes to appropriate the whole dollar; and the veto player offers a randomly chosen non-veto player the minimum amount he is willing to accept to completely expropriate the other non-veto player. The most tempting offer of this kind is the whole dollar. Non-veto player 1 prefers $\mathbf{x} = (0, 1, 0)$ to $\mathbf{s} = (0, 1/2, 1/2)$ if and only if

$$\begin{aligned}
1 + \delta V_1((0,1,0)) &\geq \frac{1}{2} + \delta V_1((0,1/2,1/2)) \\
1 + \delta V_1((0,1,0)) &\geq \frac{1}{2} + \delta \left[\frac{1 + \delta V_1((0,1,0))}{2} + \frac{\delta V_1((0,0,1))}{2}\right] \\
(1-\delta)\left(1 + \delta V_1((0,1,0))\right) &\geq \delta^2 V_1((0,0,1))
\end{aligned}$$

where $V_1((0,1,0))$ is the continuation value of non-veto player 1 from the allocation which gives him the whole dollar; and $V_1((0,0,1))$ is the continuation value of non-veto player 1 from the allocation which gives the whole dollar to the other non-veto player. The temptation to accept is increasing in $V_1((0,1,0))$ and decreasing in $V_1((0,0,1))$. First, note that $V_1((0,0,1)) > V_1((0,1,0))$, that is, the continuation value of being the poorer non-veto player is larger than the continuation value of being the richer non-veto player. This is because, when the status quo lies in $\overline{\Delta}$, the poor non-veto player can pass a proposal swapping the non-veto players' allocations and the veto player proposes a positive amount only to the poorer non-veto player. Second, both $V_1((0,0,1))$ and $V_1((0,1,0))$ are strictly increasing in $\delta$. This is because, as $\delta$ grows, the amount the veto player offers to his coalition partner increases. Moreover, $V_1((0,0,1))$ grows faster in $\delta$ than $V_1((0,1,0))$, because the poorer non-veto player benefits sooner of the veto player's increased generosity. This means that the condition in equation (9) becomes more difficult to satisfy — that is, the best feasible offer the veto player can make to a non-veto player in state (0,1/2,1/2) becomes less tempting — as $\delta$ grows. Third, both $V_1((0,0,1))$ and $V_1((0,1,0))$ are strictly decreasing in $p_v$. This is because, as $p_v$ grows, both the probability that a non-veto player proposes and the amount the veto player offers to his coalition partner decrease. Moreover, the speed at which $V_1((0,0,1))$ and $V_1((0,1,0))$ decrease in $p_v$ is different and it changes with $p_v$. This is because the probability that the poorer non-veto player receives a positive amount at the end of the period while the richer non-veto player gets 0 is increasing in $p_v$ (as this happens exactly when the veto player proposes). At the same time, what the poorer non-veto player gains in this case, that is, the amount the veto player offers to the coalition partner, decreases with $p_v$. As a consequence, $V_1((0,0,1)) - V_1((0,1,0))$ is non-monotonic in $p_v$. For low values of $p_v$, the first effect dominates and $V_1((0,0,1)) - V_1((0,1,0))$ grows in $p_v$, reducing the ability of the veto player to convince a non-veto player to expropriate the other. For large values of $p_v$, the second effect dominates and $V_1((0,0,1)) - V_1((0,1,0))$ decreases in $p_v$, making it easier for the veto player to bribe a non-veto player. This effect of $p_v$ on the veto player's ability to convince a non-veto player to completely expropriate the other complements the effect of $\delta$ for low values of $p_v$ but it counteracts it (and eventually dominates it) for high values of $p_v$. Finally, as $p_v$ goes to 1, both $V_1((0,0,1))$ and $V_1((0,1,0))$ go to 0. This is because, when non-veto players have no chance to set the agenda, non-veto players cannot improve on their current allocation and, thus, the non-veto player who is completely expropriated is willing to accept any allocation proposed by the veto player. This means that, in the limit, the veto player is able to convince a non-veto player to completely expropriate the other regardless of $\delta$ and $\mathbf{s}$.

## Proof of Proposition 6

The results in part (a) and (b) follow directly from the equilibrium demand of the poorer non-veto player in the absorbing set $\overline{\Delta}$, that is, $d_{nv}(\mathbf{s}, \delta) = \frac{\delta(1-p_v)}{2-\delta(1+p_v)} s_{\overline{nv}}$. When $\delta = 0$, this demand is zero. This means that, when the status quo is in $\overline{\Delta}$—a set that is reached in at most one period—the poorer non-veto player supports any proposal by the veto player. The veto player can thus pass his ideal outcome as soon $\mathbf{s} \in \overline{\Delta}$ and he proposes. On the other hand, when $\delta \in (0,1)$, this is not possible, and the poorer non-veto player always demands a positive share of the dollar to support any allocation that makes the veto player richer. The convergence in this case is only asymptotic and the speed of the convergence is inversely related to $d_{nv}(\mathbf{s}, \delta)$, which is strictly increasing in $\delta$ for

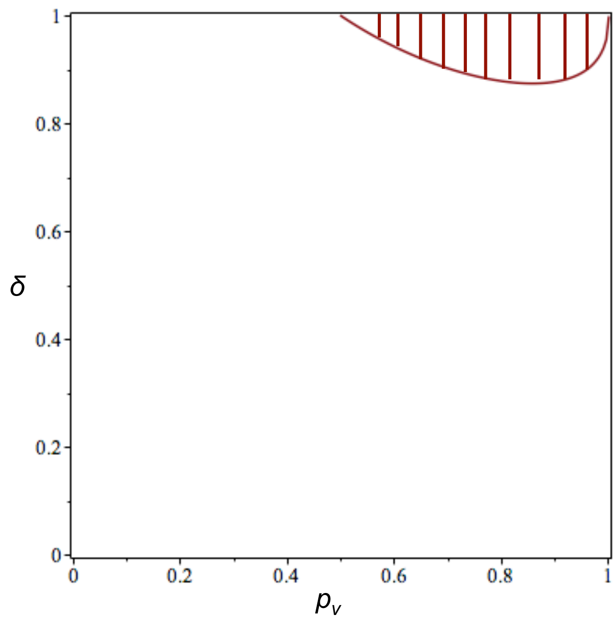


Figure 6: Existence of MPE from Proposition 5. The shaded area represents the pairs of $\delta$ and $p_v$ for which the MPE does not exist.

any $p_v \in (0,1)$, strictly increasing in $s_{\overline{nv}}$ for any $p_v \in (0,1)$ and any $\delta \in (0,1]$, strictly decreasing in $p_v$ for any $\delta \in (0,1)$. The result in part (c) follows directly from the equilibrium demand of the veto player. When $p_v = 0$, all $\mathbf{s} \in \Delta$ belong to Case A. In this case, we have, $d_v^A = s_v - \frac{2p_v\delta}{2-(1+p_v)\delta}s_2$ which equals $d_v^A = s_v$ if $p_v = 0$. This means that, for any $\mathbf{s} \in \Delta$, either non-veto proposer offers $s_v$ to the veto player. Since the veto player has never a chance to propose he gets $s_v^0$ in all periods.

## Proof of Proposition 7

Continuity of the expected utility functions hold once we establish continuity of the continuation value functions. Continuity of the continuation value functions holds in the interior of Cases A–D in the proof of Proposition 5, so it remains to check the boundaries of these cases. In order to distinguish the various applicable functional forms we shall write $V_i^w$ where $w = \{A, B, C, D\}$ identifies the case for which the respective functional form applies.

- Boundary of Cases A and B: at the boundary we have $s_1 = 1 - \frac{2-\delta(1-p_v)}{2-\delta(1+p_v)} s_2$, then:

$$\begin{aligned}
V_v^A &= V_v^B = \frac{2p_v}{(1-\delta)(2-\delta(1-p_v))} - \frac{2p_v s_2}{2-\delta(1+p_v)} \\
V_1^A &= V_1^B = \frac{1}{1-\delta} - V_v^A - V_2^A \\
V_2^A &= V_2^B = \frac{p_v s_2}{1-\delta p_v} + \frac{(1-p_v)(2-\delta(1+p_v))}{2(1-\delta)(2-\delta(1-p_v))(1-\delta p_v)}
\end{aligned}$$

- Boundary of Cases B and C: at the boundary we have
$s_1 = \frac{p_v^3\delta^3 - 2p_v^2\delta^3 + p_v\delta^3 + p_v^2\delta^2 - 2p_v\delta^2 + \delta^2 - 4\delta + 4}{(2-(1+p_v)\delta)(2-(1-p_v)\delta)(1-p_v\delta)} s_2 + \frac{p_v^3\delta^3 - p_v\delta^3 - 2p_v^2\delta^2 + 2p_v\delta^2}{(2-(1+p_v)\delta)(2-(1-p_v)\delta)(1-p_v\delta)}$; then:

$$\begin{aligned}
V_v^B &= V_v^C = \frac{2p_v}{(1-\delta)(2-(1-p_v)\delta)} - \frac{2p_v s_2}{2-\delta(1+p_v)} \\
V_1^B &= V_1^C = \frac{p_v\delta(1-p_v)s_2}{(1-\delta p_v)(2-\delta(1+p_v))} - \frac{1}{2}\frac{(-1+p_v)(2\delta^2 p_v - 3\delta p_v - \delta + 2)}{(1-\delta)(2-\delta(1-p_v))(1-\delta p_v)} \\
V_2^B &= V_2^C = \frac{1}{1-\delta} - V_v^B - V_1^B
\end{aligned}$$

- Boundary of Cases C and D: at the boundary we have $s_1 = \frac{2-\delta}{2-\delta(1-p_v)} - s_2$; then:

$$\begin{aligned}
V_v^C &= V_v^D = \frac{p_v(1+\delta)}{(1-\delta)(2-(1-p_v)\delta)} \\
V_1^C &= V_1^D = \frac{s_2}{\delta} + \frac{2\delta^2 + (p-5)\delta + 2}{2(\delta-1)(2+(-1+p_v)\delta)\delta} \\
V_2^C &= V_2^D = \frac{1}{1-\delta} - V_v^C - V_1^C
\end{aligned}$$

- Boundary of Cases D and A: at the boundary we have $s_1 = \frac{2-\delta(1-p_v)}{2-\delta(1+p_v)} s_2$; then:

$$\begin{aligned}
V_v^D &= V_v^A = \frac{1}{1-\delta} - \frac{2(2-\delta-p_v)}{(1-\delta)(2-\delta(1-p_v))} s_2 \\
V_1^D &= V_1^A = \frac{1}{1-\delta} - V_v^D - V_2^D \\
V_2^D &= V_2^A = \frac{s_2}{1-\delta}
\end{aligned}$$

# C Proofs of Propositions from Section 6

## Proof of Proposition 8

**Part (a)** If $Y = \{(1,0,0)\}$ is an irreducible absorbing set, then $V_v((1,0,0)) = \frac{1}{1-\delta}$ and $U_v((1,0,0)) = 1 + \delta V_v((1,0,0)) = \frac{1}{1-\delta}$. Since the amount of resources available in any period is 1, $V_v(\mathbf{x}) \leq \frac{1}{1-\delta}$ for any $\mathbf{x} \in \Delta$ in any subgame perfect Nash equilibrium of the game. Thus, if $\mathbf{x}$ is such that $x_v < 1$, we have $U_v(\mathbf{x}) = x_v + \delta V_v(\mathbf{x}) < \frac{1}{1-\delta}$. This means that the veto player is strictly worse off moving to a policy outside of $Y$ and, thus, a) he will never propose a policy outside of $Y$ and b) he will veto any policy outside of $Y$ proposed by a non-veto player.

**Part (b)** Consider any allocation $\mathbf{s} \in \Delta$. In any SPE, the veto player can unilaterally implement the status quo in the current and all following periods, regardless of the identity of the proposer. The payoff from this strategy is $s_v$ and this establishes that $V_v(\mathbf{s}) \geq \frac{s_v}{1-\delta}$ in any SPE.

**Part (c)** Consider any allocation $\mathbf{s} \in \Delta$. In any SPE, a non-veto player can unilaterally implement the status quo whenever he proposes. The payoff from this strategy is $s_i$ in the history in which $i$ proposes in this and all following periods. Since we are looking for a lower bound, suppose that at any other history, player $i$ gets zero. We have established that, in any SPE,

$$V_i(\mathbf{s}) \quad \geq \quad p_{nv}s_i + p_{nv}^2\delta s_i + p_{nv}^3\delta^2 s_i + \ldots + p_{nv}^t\delta^{t-1}s_i + \ldots = \frac{p_{nv}s_i}{1-\delta p_{nv}}$$

where $p_{nv}$ is the probability either non-veto player proposes and $s_i$ is the allocation to $i$ in $\mathbf{s}$.

**Part (d)** Assume $\mathbf{s} \in \Delta$ is an absorbing allocation. Then, $U_i(\mathbf{s}) = \frac{s_i}{1-\delta}$ for $i = \{v, 1, 2\}$. Since, as we established above, $U_v(\mathbf{x}) \geq \frac{x_v}{1-\delta}$ for any $\mathbf{x} \in \Delta$, the veto player supports any reform of $\mathbf{s}$ as long as he is offered at least $s_v$. We want to show that, when $\delta < \frac{1}{2-p_{nv}}$, either non-veto player prefers a feasible allocation other than $\mathbf{s}$ where the veto player receives at least $s_v$ to $\mathbf{s}$. This means that this non-veto player will find it optimal to reform the status quo, with the support of the veto player, as soon as he has a chance to propose and, thus, that $\mathbf{s}$ cannot be absorbing. The non-veto player who has the least incentive to maintain the policy in $\mathbf{s}$ forever is the one with the lowest amount. Without loss of generality, assume $s_1 \geq s_2$ and consider the incentive of non-veto player 2 to reform $\mathbf{s}$. Non-veto player 2 can move to allocation $(s_v, 0, s_1 + s_2)$ with the support of the veto player. Non-veto player 2 strictly prefers allocation $\mathbf{x} = (s_v, 0, s_1 + s_2)$ to $(s_v, s_1, s_2)$ if:

$$\frac{s_2}{1-\delta} < s_1 + s_2 + \delta\left(\frac{p_{nv}(s_1+s_2)}{1-\delta p_{nv}}\right) \leq s_1 + s_2 + \delta V_2(\mathbf{x}) \tag{9}$$

Note that the lower bound of the"temptation" to reform the status quo is increasing in $s_1$. This means that the inequality will be hardest to satisfy when $s_1 = s_2$. In this case, the inequality becomes:

$$\begin{aligned} \frac{s_2}{1-\delta} &< 2s_2 + \delta\left(\frac{p_{nv}(2s_2)}{1-\delta p_{nv}}\right) \\ \frac{s_2}{1-\delta} &< \frac{2s_2}{1-\delta p_{nv}} \\ \frac{1}{2} &< \frac{1-\delta}{1-\delta p_{nv}} \\ \delta &< \frac{1}{2-p_{nv}} \end{aligned} \tag{10}$$

This establishes that when $\delta < \frac{1}{2-p_{nv}}$ there is no $\mathbf{s} \in \Delta$ other than (1,0,0) which is absorbing.

## Proof of Proposition 9

Fix an MPE of the dynamic legislative bargaining game. Consider an irreducible absorbing set with respect to this MPE, $Y$ with $|Y| = k \geq 2$, and enumerate the elements of $Y$ as $\mathbf{y}_1, \ldots, \mathbf{y}_k$. For each $h = 1, \ldots, k$, let $Y_h \subseteq Y \backslash \{\mathbf{y}_h\}$ denote the policies that occur with positive probability given status quo $\mathbf{y}_h$ and enumerate the elements of $Y_h$ as $\mathbf{z}_{h,1}, \mathbf{z}_{h,2}, \ldots, \mathbf{z}_{h,n_h}$. For each $h = 1, \ldots, k$ and each $l = 1, \ldots, n_h$, the veto player must get a weakly higher dynamic payoff from $\mathbf{z}_{h,l}$ than from $\mathbf{y}_h$. If this was not true, the veto player would block transition to policy $\mathbf{z}_{h,l}$ and, thus, $\mathbf{z}_{h,l}$ would not occur with positive probability given status quo $\mathbf{y}_h$. We have:

$$U_v(\mathbf{z}_{h,l}) \geq U_v(\mathbf{y}_h)$$

Let $\mathbf{m}$ be a solution to $\max_{h\in\{1,\ldots,k\}} U_v(\mathbf{y}_h)$, so that $\mathbf{y}_m$ maximizes the veto player's dynamic payoff over $Y$, i.e., $U_v(\mathbf{y}_m) = \max_{h=1,\ldots,k} V_v(\mathbf{y}_h)$. Then for all $l = 1, \ldots, n_m$, we must have:

$$U_v(\mathbf{z}_{m,l}) = U_v(\mathbf{y}_m)$$

Thus, all the policies that occur with positive probability following $\mathbf{y}_m$ also maximize the veto player's dynamic payoff. Since $Y$ is an irreducible absorbing set, this argument in fact implies that for all $h = 1, \ldots, k$, we have $U_v(\mathbf{y}_h) = U_v(\mathbf{y}_m)$, so that the dynamic payoff of the veto player is constant on $Y$, and we can denote this by $\overline{U}$. Denote with $y_{i,v}$ the amount allocated to the veto player in policy $\mathbf{y}_i$. Choosing any $\mathbf{y}_i, \mathbf{y}_j \in Y$, we then have:

$$\begin{aligned} y_{i,v} + \delta\overline{U} &= U_v(\mathbf{y}_i) \\ &= U_v(\mathbf{y}_j) \\ &= y_{j,v} + \delta\overline{U} \end{aligned}$$

which implies $y_{i,v} = y_{j,v}$.

## Proof of Proposition 10

**Part (a)** We want derive a lower bound on $V_i(\mathbf{s})$, $i = \{1, 2\}$, that is, on the continuation value of the game starting at policy $\mathbf{s} \in \Delta$ for non-veto player $i$. Non-veto player $i$ can adopt the following proposal strategy: propose an allocation $\mathbf{y}$ such that $y_i = \max\{s_1, s_2\}$, $y_{-i} = \min\{s_1, s_2\}$, and $y_v = s_v$, where $-i$ refers to the other non-veto player. In other words, non-veto player $i$ can propose the status quo policy (when this policy gives $i$ the largest amount to a non-veto player) or propose to swap his allocation with the allocation of the other non-veto player and keep the allocation to the veto player unchanged (when the status quo policy gives $i$ the lowest amount to a non-veto player). In a symmetric MPE, both proposals will pass with the support of the veto player (who receives the same allocation and the same continuation value from the status quo and from either proposal above). After the current period, as long as non-veto player $i$ is the proposer (that is, in the history where non-veto player $i$ is the proposer in every following period), he can propose and implement the status quo allocation $\mathbf{y}$. The payoff from this proposal strategy is $\max\{s_1, s_2\}$

in the history in which $i$ proposes in this and all following periods. Since we are looking for a lower bound, suppose that at any other history, player $i$ gets zero (the lowest possible amount). We have established that:

$$V_i(\mathbf{s}) \quad \geq \quad p_i \max\{s_1, s_2\} + p_i^2 \delta \max\{s_1, s_2\} + p_i^3 \delta^2 \max\{s_1, s_2\} + \ldots + p_i^t \delta^{t-1} \max\{s_1, s_2\} + \ldots = \frac{p_i \max\{s_1, s_2\}}{1 - \delta p_i}$$

where $p_i$ is the probability non-veto player $i$ proposes and $\max\{s_1, s_2\}$ is the largest allocation to a non-veto player in allocation $\mathbf{s}$.

**Part (b)** From Proposition 8(a), we know that $\mathbf{z} = (1, 0, 0)$ is an absorbing outcome. Therefore, the continuation value of $\mathbf{z}$ for non-veto player $j$, $j = \{1, 2\}$, is $V_j(\mathbf{z}) = 0$. In any symmetric MPE in stage-undominated voting strategies, non-veto player $i$ votes in favor of proposal $\mathbf{x}$ against status quo $\mathbf{s}$ if and only if:

$$x_j + \delta V_j(\mathbf{x}) \geq s_j + \delta V_j(\mathbf{s})$$
$$x_j \geq s_j + \delta \left(V_j(\mathbf{s}) - V_j(\mathbf{x})\right)$$

The status quo policy can be changed only with the approval of 1 non-veto player. Proposal $\mathbf{z}$ gives $z_j + \delta V_j(\mathbf{z}) = 0$ to any non-veto player $j = \{1, 2\}$. This means that such a proposal defeats a status quo $\mathbf{s} \neq \mathbf{z}$ only if the following condition holds for at least 1 non-veto player:

$$s_j + \delta V_j(\mathbf{s}) \leq 0 \tag{11}$$

Since allocations are constrained to be non-negative, we have $s_j \geq 0$ and $V_j(\mathbf{s}) \geq 0$. This means that (11) will never be satisfied for a non-veto player whose status quo allocation is positive: this legislator will never support a proposal which leads to absorbing outcome $\mathbf{z}$. Thus, the absorbing outcome $\mathbf{z}$ defeats the status quo only if there is at least 1 non-veto players for which $s_j = 0$ and $V_j(\mathbf{s}) = 0$.

Consider a status quo allocation $\mathbf{s} \neq (1, 0, 0)$ which gives 0 to one non-veto player. Without loss of generality, let $s_1 > s_2 = 0$. Given the discussion above, these are the only status quo policies we have not ruled out as conducive to the absorbing state $\mathbf{z}$.

The lower bound on $V_2(\mathbf{s})$ derived in part (a) is strictly positive for any legislator who is not perfectly impatient, $\delta > 0$, and has some positive probability to propose, $p_2 > 0$. This means that, as long as legislators are not perfectly impatient and everybody has some agenda setting power, there is no $\mathbf{s} \neq \mathbf{z}$ where (11) is satisfied for one non-veto player, or, in other words, no proposer will be able to bring the status quo policy from an outcome which allocates a positive amount to some non-veto player to an outcome which allocates zero resources to non-veto players.

## Proof of Proposition 11

Proposition 9 shows that any irreducible absorbing set must be composed by policies giving the same amount to the veto player. Proposition 8(a) shows that $Y = \{(1, 0, 0)\}$ is an irreducible absorbing set. Thus, to prove the statement from Proposition 11 it is sufficient to show that any set of policies giving the same amount $k < 1$ to the veto player cannot be an absorbing set.

The proof uses two lemmas.

**Lemma 6** *Consider any consistent MPE of the dynamic legislative bargaining game with $p_v \in (0, 1]$ and let $Y \subseteq \Delta$ be an irreducible absorbing set with respect to this MPE. For any status quo $\mathbf{y} \in Y$, player $i = \{v, 1, 2\}$ proposes the same policy.*

**Proof.** From Proposition 9, we know that any $\mathbf{y} \in Y$ gives the same allocation to the veto player. This means that, for any status quo policy $\mathbf{y} \in Y$, we have $y_v = k$, $y_1 = \rho_y(1 - k)$, and $y_2 = (1 - \rho_y)(1 - k)$, where $k \in [0, 1]$ is the same for all policies in the set and $\rho_y \in [0, 1]$ is specific to policy $\mathbf{y}$. Thus, for any $\mathbf{y} \in Y$, we have $U_v(\mathbf{y}) = \frac{k}{1-\delta}$ and $A_v(\mathbf{y}) = \{\mathbf{x} \in \Delta | x_v \geq k\}$. This means that a proposing non-veto player faces the same acceptance set for any status quo policy in $Y$. Therefore, by consistency, each non-veto player proposes the same policy for any status quo policy in $Y$. Denote this policy with $\mathbf{x}^i$, $i = \{1, 2\}$.

We now argue that also the veto player faces the same acceptance set for any policy in $\mathbf{y}$ and, in particular, that for any status quo policy $\mathbf{y} \in Y$, the set of proposals that are supported by at least one non-veto player, that is, $A_1(\mathbf{y}) \cup A_2(\mathbf{y})$, encompasses all policies where the veto player has $y_v = k$.

Consider status quo $\mathbf{s} \in Y$ such that $s_v = 1 - k$, $s_1 = \rho_s(1 - k)$, $s_2 = (1 - \rho_s)(1 - k)$ and a proposal $\mathbf{x} \in Y$ such that $x_v = k$, $x_1 = \rho_x(1-k)$, $x_2 = (1-\rho_x)(1-k)$. We want to show that , $\forall \rho_s, \rho_x \in [0,1]$, $\mathbf{x} \in A_1(\mathbf{s}) \cup A_2(\mathbf{s})$. Let $z = 1-k$ denote the sum of allocations to non-veto players in any policy in the irreducible absorbing set. Non-veto player 1 accepts $\mathbf{x}$ if and only if:

$$\begin{aligned} \rho_x z + \delta V_1(\mathbf{x}) &\geq \rho_s z + \delta V_1(\mathbf{s}) \\ (\rho_x - \rho_s) z &\geq \delta \left[ V_1(\mathbf{s}) - V_1(\mathbf{x}) \right] \end{aligned} \tag{12}$$

Non-veto player 2 accepts $\mathbf{x}$ if and only if:

$$\begin{aligned} (1-\rho_x) z + \delta V_2(\mathbf{x}) &\geq (1-\rho_s) z + \delta V_2(\mathbf{s}) \\ -(\rho_x - \rho_s) z &\geq \delta \left[ V_2(\mathbf{s}) - V_2(\mathbf{x}) \right] = -\delta \left[ V_1(\mathbf{s}) - V_1(\mathbf{x}) \right] \\ (\rho_x - \rho_s) z &\leq \delta \left[ V_1(\mathbf{s}) - V_1(\mathbf{x}) \right] \end{aligned} \tag{13}$$

The equality in the second line follows from the fact that $V_v(\mathbf{s}) = V_v(\mathbf{x})$ and $\sum_{i=\{v,1,2\}} V_i(\mathbf{s}) = \sum_{i=\{v,1,2\}} V_i(\mathbf{x})$. It is easy to see that, for any $\rho_s, \rho_x \in [0,1]$, at least one condition is always satisfied. This means that the proposing veto player faces the same acceptance set for any status quo policy in $Y$. Therefore, by consistency, the veto player proposes the same policy for any status quo policy in $Y$. Denote this policy with $\mathbf{x}^v$. ■

**Lemma 7** *Consider any consistent MPE of the dynamic legislative bargaining game with $p_v \in (0,1]$ and let $Y \subseteq \Delta$ be an irreducible absorbing set with respect to this MPE: (a) $V_i(\mathbf{y})$, the continuation value function of player $i = \{v,1,2\}$ from policy $\mathbf{y}$, is constant over set $Y$; (b) the equilibrium proposals of non-veto players when the status quo is $\mathbf{y} \in Y$ are MWC, that is, the proposer offers exactly 0 to the other non-veto player.*

**Proof.** From Lemma 6, we know that $\forall \mathbf{y} \in Y$, each player makes the same proposal, $\mathbf{x}^i, i = \{v,1,2\}$. As a consequence, $V_i(\mathbf{y}), i = \{v,1,2\}$ is constant in set $Y$. Since $U_1(\mathbf{x^1}) = x_1^1 + \delta V_1(\mathbf{x^1})$ does not depend on the amount allocated to non-veto player 2 in $\mathbf{x^1}$, non-veto player 1 finds most advantageous to propose a policy which allocates the whole amount not allocated to the veto player to himself (as before, call this amount, which is identical for all policies in set $Y$, $z$). Similarly, since $U_2(\mathbf{x^2}) = x_2^2 + \delta V_2(\mathbf{x^2})$ does not depend on the amount allocated to non-veto player 1 in $\mathbf{x^2}$, non-veto player 2 finds most advantageous to propose a policy which allocates $z$ to himself. ■

We are now ready to prove the statement from Proposition 11. Fix a consistent MPE. Consider the irreducible absorbing set with respect to this MPE, $Y \subseteq \Delta$. The Lemmas above show that $|Y| \leq 3$. The three policies which can belong to $Y$ are the unique equilibrium proposals made by each player for any status quo in $Y$. Denote with $\mathbf{x}^i, i = \{1, 2\}$, each non-veto player's equilibrium proposal for a status quo policy in the irreducible absorbing set, $Y$. In Lemma 7, we showed that $\mathbf{x}^1 = \{1 - z, z, 0\}$, $\mathbf{x}^2 = \{1 - z, 0, z\}$, where $z \in [0, 1]$ is the allocation to the veto player in any policy in $Y$. Denote the veto player's equilibrium proposal with $\mathbf{x}^v = \{1 - z, \rho z, (1 - \rho)z\}$, where $\rho \in [0, 1]$. We can characterize the continuation value each non-veto player derives from any policy in the irreducible absorbing set as a function of $z$, $\rho$, $\delta$ and $p_v$:

$$\begin{aligned}
V_1 &= \frac{1-p_v}{2}\left[z + \delta V_1\right] + \frac{1-p_v}{2}\left[0 + \delta V_1\right] + p_v\left[\rho z + \delta V_1\right] = \frac{1 + (2\rho - 1)p_v}{2(1-\delta)} z \\
V_2 &= \frac{1-p_v}{2}\left[z + \delta V_2\right] + \frac{1-p_v}{2}\left[0 + \delta V_2\right] + p_v\left[(1-\rho) z + \delta V_2\right] = \frac{1 - (2\rho - 1)p_v}{2(1-\delta)} z
\end{aligned}$$

We, thus, have:

$$\begin{aligned}
U_1(\mathbf{x^1}) &= z + \delta \frac{1 + (2\rho - 1)p_v}{2(1-\delta)} z \\
U_1(\mathbf{x^2}) &= 0 + \delta \frac{1 + (2\rho - 1)p_v}{2(1-\delta)} z \\
U_2(\mathbf{x^1}) &= 0 + \frac{1 - (2\rho - 1)p_v}{2(1-\delta)} z \\
U_2(\mathbf{x^2}) &= z + \frac{1 - (2\rho - 1)p_v}{2(1-\delta)} z
\end{aligned}$$

Above we showed that all policies in an irreducible absorbing set give the veto player the same allocation, $k \in [0, 1]$. Assume, towards a contradiction, $k < 1$. First, note that, since $U_v(\mathbf{y}) = \frac{k}{1-\delta}$ for any $\mathbf{y} \in Y$ and, in any MPE, $U_v(\mathbf{s}) \geq \frac{s_v}{1-\delta}$, the veto player would be strictly better off moving to a policy outside of the irreducible absorbing set where he receives a higher allocation. We want to show that, when the veto player proposes, he can implement such policies with the support of a non-veto player and, thus, that policies such that $k < 1$ cannot belong to an irreducible absorbing set.

Consider the status quo policy $\mathbf{x^1} \in Y$. Since the veto player does not receive the whole pie (that is, $k < 1$) and, thus, $z > 0$, we have $U_1(\mathbf{x^1}) > U_1(\mathbf{x^2})$ and $U_2(\mathbf{x^2}) > U_2(\mathbf{x^1})$. This means that, by continuity, when the veto player proposes and the status quo is either $\mathbf{x^1}$ or $\mathbf{x^2}$ (an event which happens with probability 1), he can always find a proposal which allocates to himself strictly more than $k$ and that is weakly preferred to the status quo by at least one veto player. Without loss of generality, consider status quo $\mathbf{x^1} \in Y$. If the veto player proposes $\mathbf{x^2} = \{1 - z, 0, z\}$, the dynamic payoff non-veto player 2 receives from the proposal is strictly larger than the dynamic payoff he receives from the status quo, $U_2(\mathbf{x^2}) > U_2(\mathbf{x^1})$. Consider proposal $\mathbf{w} = \{1-z+\epsilon, 0, z-\epsilon\}$, which shocks $\mathbf{x^2}$ by redistributing $\epsilon$ from non-veto player 2 to the veto player. By continuity of the continuation values, there is $\epsilon > 0$ such that $U_2(\mathbf{w}) \geq U_2(\mathbf{x}^1)$ and, thus, $\mathbf{w}$ can be approved with the support of non-veto player 2.

## Proof of Proposition 12

The proof uses a series of lemmas.

**Lemma 8** *In any continuous and consistent MPE of the dynamic bargaining game where* $p_v = 0$ *and* $\min\{p_1, p_2\} > 0$*: (a)* $U_v(\mathbf{s})$ *depends only on* $s_v$*; (b)* $U_v(\mathbf{s})$ *is strictly monotone in* $s_v$*; (c)* $U_v(\mathbf{s})$ *is strictly increasing in* $s_v$*.*

**Proof.** Remember that $U_v(\mathbf{x}) = x_v + \delta V_v(\mathbf{x})$, the dynamic payoff of the veto player from policy $\mathbf{x}$.

*Part (a).* We want to show that $\forall \mathbf{x}, \mathbf{x}' \in \Delta U_v(\mathbf{x}) = U_v(\mathbf{x}') \Leftrightarrow x_v = x'_v$.

First, we prove that $U_v(\mathbf{x}) = U_v(\mathbf{x}') \Rightarrow x_v = x'_v$.

Fix an equilibrium and let $\mathbf{x}^a(\mathbf{s})$ be the equilibrium proposal of agenda setter $a \in \{1, 2\}$ given status quo $\mathbf{s}$. Then, for all $i \in \{1, 2, v\}$,

$$V_i(\mathbf{s}) = \sum_{a\in\{1,2\}} p_a \left[x_i^a(\mathbf{s}) + \delta V_i\left(\mathbf{x}^a(\mathbf{s})\right)\right] = \sum_{a\in\{1,2\}} p_a U_i\left(\mathbf{x}^a(\mathbf{s})\right)$$

so that $U_i(\mathbf{x}) = x_i + \delta \sum_{a\in\{1,2\}} p_a U_i\left(\mathbf{x}^a(\mathbf{x})\right)$.

Now fix $\mathbf{x}$ and $\mathbf{x}'$ such that $U_v(\mathbf{x}) = U_v(\mathbf{x}')$. Then, $A(\mathbf{x}) = A(\mathbf{x}')$ and, in any consistent MPE, $\mathbf{x}^a(\mathbf{x}) = \mathbf{x}^a(\mathbf{x}')$ for all $j \in \{1, 2\}$. Hence:

$$U_v(\mathbf{x}) - U_v(\mathbf{x}') = x_v + \delta \sum_{a \in \{1,2\}} p_a U_v\left(\mathbf{x}^a(\mathbf{x})\right) - \left[x'_v + \delta \sum_{a \in \{1,2\}} p_a U_v\left(\mathbf{x}^a(\mathbf{x}')\right)\right] = x_v - x'_v$$

Since $U_v(\mathbf{x}) - U_v(\mathbf{x}') = 0$, we have $x_v = x'_v$.

Second, we prove that $U_v(\mathbf{x}) = U_v(\mathbf{x}') \Leftarrow x_v = x'_v$.

Fix an equilibrium, a pair of policies $\mathbf{x}$ and $\mathbf{x}'$ such that $x_v = x'_v$, and assume, towards a contradiction, that $U_v(\mathbf{x}) \neq U_v(\mathbf{x}')$. In any stationary equilibrium, $U_v(\mathbf{x}) \neq U_v(\mathbf{x}')$ and $x_v = x'_v$ imply $\mathbf{x} \neq \mathbf{x}'$ (because, otherwise, the continuation values from the two policies and, thus, the dynamic payoffs would be the same). This means that $x_v = x'_v < 1$, since there are no two distinct policies which give 1 to the veto player. Without loss of generality, assume $U_v(\mathbf{x}) < U_v(\mathbf{x}')$. Because in any MPE, $V_v(\mathbf{s}) \geq \frac{s_v}{1-\delta}$ and $V_v(\mathbf{s}) \leq \frac{1}{1-\delta}$, we have $V_v((1,0,0)) = \frac{1}{1-\delta}$. Since $U_v(\mathbf{x}') = x'_v + \delta V_v(\mathbf{x}')$ and $V_v(\mathbf{x}') \leq \frac{1}{1-\delta}$, $x'_v < 1$ implies $U_v(\mathbf{x}') < \frac{1}{1-\delta}$. To see this, note that, even when $V_v(\mathbf{x}')$ equals its upper bound, $U_v(\mathbf{x}')$ can equal $\frac{1}{1-\delta}$ only if $x'_v = 1$. We have $U_v((1,0,0)) = \frac{1}{1-\delta} > U_v(\mathbf{x}') > U_v(\mathbf{x})$. By continuity of $V_v$ and hence of $U_v$, there exists $\alpha \in (0,1)$ such that $U_v(\alpha(1,0,0) + (1-\alpha)\mathbf{x}) = U_v(\mathbf{x}')$. But as we proved above, $U_v(\mathbf{x}) = U_v(\mathbf{x}') \Rightarrow x_v = x'_v$. This implies that, if $U_v(\alpha(1,0,0) + (1-\alpha)\mathbf{x}) = U_v(\mathbf{x}')$, then $(\alpha(1,0,0) + (1-\alpha)\mathbf{x})_v = x'_v$, which is a contradiction since it rewrites as $\alpha + (1-\alpha)x_v = x'_v$, or, equivalently, $\alpha = \alpha x_v$ (since $x_v = x'_v$), or, equivalently, $x_v = 1$ (since $\alpha \in (0,1)$).

*Part (b).* To see that $U_v(\mathbf{s})$ is strictly monotone in $x_v$, take $\mathbf{x}, \mathbf{x}' \in X$ with $x_v < x'_v$ and assume, towards a contradiction, that $U_v(\mathbf{x}) = U_v(\mathbf{x}')$. Then, by Lemma 1, we have $x_v = x'_v$, a contradiction.

*Part (c).* To prove that $U_v(\mathbf{s})$ is strictly increasing in $x_v$, it suffices to show that it is not strictly decreasing. Assume, towards a contradiction, that $U_v(\mathbf{x})$ is strictly decreasing in $x_v$. Since $U_v((1,0,0)) = \frac{1}{1-\delta}$, this implies that $U_v(\mathbf{x}) > \frac{1}{1-\delta}$ $\forall \mathbf{x} \in \Delta \setminus \{(1,0,0)\}$. This is not feasible since

$U_v(\mathbf{x}) \leq \frac{1}{1-\delta}$ $\forall \mathbf{x} \in \Delta$ (since the largest allocation the veto can receive in any period is 1). ■

**Lemma 9** *In any continuous and consistent MPE of the dynamic bargaining game where* $p_v = 0$ *and* $\min\{p_1, p_2\} > 0$*,* $V_i(\mathbf{s})$ *depends only on* $s_v$ *for any* $i \in \{v, 1, 2\}$ *and any policy* $\mathbf{s} \in \Delta$*.*

**Proof.** Denoting the equilibrium proposal by agenda setter $a \in \{1, 2\}$ for status quo $\mathbf{s} \in \Delta$ with $\mathbf{x}^a(\mathbf{s})$, the value function of player $i \in \{v, 1, 2\}$ from policy $\mathbf{s} \in \Delta$ can be written as:

$$V_i(\mathbf{s}) = \sum_{a \in \{1,2\}} p_a \left[x_i^a(\mathbf{s}) + \delta V_i(\mathbf{x}^a(\mathbf{s}))\right] \tag{14}$$

Since, by Lemma 8, $U_v(\mathbf{s}) = U_v(\mathbf{s}') \Leftrightarrow s_v = s'_v$, the set of policies that beat the status quo, $A(\mathbf{s})$, only depends on $s_v$. By consistency of proposal strategies, this implies that $\mathbf{x}^a(\mathbf{s})$ depends only on $s_v$ for any status quo $\mathbf{s} \in \Delta$ and any agenda setter $a \in \{1, 2\}$. In turn, this implies that also $V_i(\mathbf{s})$ depends only on $s_v$ for any $i \in \{v, 1, 2\}$ for any status quo $\mathbf{s} \in \Delta$. ■

**Lemma 10** *In any continuous and consistent MPE of the dynamic bargaining game where* $p_v = 0$ *and* $\min\{p_1, p_2\} > 0$*, equilibrium proposals are MWC, that is, the proposer offers exactly 0 to the other non-veto player.*

**Proof.** For any status quo $\mathbf{s} \in \Delta$, the equilibrium proposal of proposer $a \in \{1, 2\}$ solves $\max_{\mathbf{z} \in A(\mathbf{s})} \mathbf{z}_a + \delta V_a(\mathbf{z})$. Denote the proposing non-veto player with $a \in \{1, 2\}$ and the non-proposing veto player with $-a = \{1, 2\} \setminus \{a\}$. Assume, towards a contradiction, that $\mathbf{x} = \mathbf{x}^a(\mathbf{s})$ is such that $x_{-a} > 0$ and consider an alternative proposal $\mathbf{y}$ such that $y_v = x_v$, $y_a = x_a + x_{-a} > x_a$ and $y_{-a} = 0$. If $\mathbf{x} \in A(\mathbf{s})$, then also $\mathbf{y} \in A(\mathbf{s})$. This is because the set of policies that beat the status quo, $A(\mathbf{s}) = \{\mathbf{z} \in \Delta | U_v(\mathbf{z}) \geq U_v(\mathbf{s})\}$ depend only on $z_v$ and $s_v$. Moreover, since $V_a(\mathbf{z})$ only depend on $z_v$, we have $V_a(\mathbf{x}) = V_a(\mathbf{y})$. This means that the dynamic payoff the proposer derives from $\mathbf{y}$ is strictly larger than the dynamic payoff the proposer derives from $\mathbf{x}$. Since they both belong to the set of acceptable policies, it cannot be the case that $\mathbf{x}$ solves $\max_{\mathbf{z} \in A(\mathbf{s})} \mathbf{z}_a + \delta V_a(\mathbf{z})$. ■

**Lemma 11** *Consider a continuous and consistent MPE of the dynamic bargaining game where* $p_v = 0$ *and* $\min\{p_1, p_2\} > 0$*. Denote with* $\mathbf{x}^a(\mathbf{s})$ *the equilibrium proposal of* $a = \{1, 2\}$ *under status quo* $\mathbf{s} \in \Delta$*. If* $\mathbf{x} = \mathbf{x}^a(\mathbf{s})$ *is such that* $x_v > s_v$ *for some* $a \in \{1, 2\}$ *and some* $\mathbf{s} \in \Delta$*, then,* $\forall \mathbf{s}' \in \Delta$ *such that* $s'_v \in [s_v, x_v]$*,* $\mathbf{x}^a(\mathbf{s}') = \mathbf{x}^a(\mathbf{s})$*.*

**Proof.** Assume that $\mathbf{x} = \mathbf{x}^a(\mathbf{s})$ is such that $x_v > s_v$ for some $a \in \{1, 2\}$ and $\mathbf{s} \in \Delta$ and fix $\mathbf{s}' \in \Delta$ such that $s'_v \in [s_v, x_v]$. Since $x_v \geq s'_v$ and $U_v(\mathbf{s})$ is strictly increasing in $s_v$, we have $\mathbf{x}^a(\mathbf{s}) \in A(\mathbf{s}')$. Moreover, $s'_v \geq s_v$ implies $A(\mathbf{s}') \subseteq A(\mathbf{s})$ and hence $\mathbf{x}^a(\mathbf{s}') \in A(\mathbf{s})$. Thus, by consistency, $\mathbf{x}^a(\mathbf{s}') = \mathbf{x}^a(\mathbf{s})$. ■

**Lemma 12** *Consider a continuous and consistent MPE of the dynamic bargaining game where $p_v = 0$ and $\min\{p_1, p_2\} > 0$. Denote with $\mathbf{x}^a(\mathbf{s})$ the equilibrium proposal of non-veto player $a = \{1, 2\}$ under status quo $\mathbf{s} \in \Delta$, and with $\mathbf{x}^{-a}(\mathbf{s})$ the equilibrium proposal of the other non-veto player under the same status quo. If $\mathbf{y} = \mathbf{x}^a(\mathbf{s})$ is such that $y_v > s_v$ for some $a \in \{1, 2\}$ and some $\mathbf{s} \in \Delta$, then, $\mathbf{x} = \mathbf{x}^{-a}(\mathbf{s})$ is such that $x_v = s_v$.*

**Proof.** Assume, towards a contradiction, that, for some $a \in \{1, 2\}$ and $\mathbf{s} \in \Delta$, $\mathbf{y} = \mathbf{x}^a(\mathbf{s})$ is such that $y_v > s_v$ and $\mathbf{x} = \mathbf{x}^{-a}(\mathbf{s})$ is such that $x_v > s_v$ (Note that, because non-veto proposers cannot pass a proposal decreasing the allocation to the veto player for any status quo, this suffices.) Without loss of generality, assume that $x_v < y_v$. By Lemma 11, $\forall \mathbf{s}' \in \Delta$ such that $s'_v \in [s_v, x_v]$, $\mathbf{x}^a(\mathbf{s}') = \mathbf{x}^a(\mathbf{s})$ and $\mathbf{x}^{-a}(\mathbf{s}') = \mathbf{x}^{-a}(\mathbf{s})$. This means that $\forall \mathbf{s}' \in \Delta$ such that $s'_v \in [s_v, x_v]$, $V_{-a}(\mathbf{s}')$ is constant. Consider $\mathbf{x}' \in \Delta$ such that $x'_v \in [s_v, x_v)$ and $x'_{-a} = x_{-a} + (x_v - x'_v)$. Since $x_v - x'_v > 0$ and $V_{-a}(\mathbf{x}) = V_{-a}(\mathbf{x}')$, we have: $x'_{-a} + \delta V_{-a}(\mathbf{x}') = x_{-a} + (x_v - x'_v) + \delta V_{-a}(\mathbf{x}') > x_{-a} + \delta V_{-a}(\mathbf{x})$. However, this is a contradiction since $x_{-a} + \delta V_{-a}(\mathbf{x}) = \max_{\mathbf{z} \in A(\mathbf{s})} z_{-a} + \delta V_{-a}(\mathbf{z})$. ■

Finally, we are in the position to argue that, $\forall \mathbf{s} \in \Delta$ and $\forall a \in \{1, 2\}$, $\mathbf{x} = \mathbf{x}^a(\mathbf{s})$ is such that $x_v = s_v$. Since the veto player cannot propose, this shows that, in any period of a consistent and continuous MPE, the veto player gets $s_v^0$. Notice that because $U_v(\mathbf{s})$ is strictly increasing in $s_v$, to prove Proposition 12 it is sufficient to rule out $x_v > s_v$ (as the veto player never accepts a reduction to this allocation). Suppose, towards a contradiction, that for some $a \in \{1, 2\}$ and some $\mathbf{s} \in \Delta$, we have $\mathbf{x} = \mathbf{x}^a(\mathbf{s})$ such that $x_v > s_v$.

We can assume, without loss of generality, that $s_v = 1 - s_a$. (To see why this is without loss of generality, note that, if there is $\mathbf{s} \in \Delta$ such that $x_v > s_v$, then, since by consistency the equilibrium proposal depends on only on $s_v$, there has to be $\mathbf{s}' \in \Delta$ with $s'_v = s_v$ and $s'_a = 1 - s'_v$ such that $\mathbf{x}^a(\mathbf{s}') = \mathbf{x}^a(\mathbf{s})$.) By Lemma 10, we have $x_{-a} = 0$ and, thus, $x_v = 1 - x_a$. Hence $x_v > s_v$ implies $x_a < s_a$. Because $a$ proposes $\mathbf{x} = \mathbf{x}^a(\mathbf{s})$ when the status quo is $\mathbf{s}$, we have $x_a + \delta V_a(\mathbf{x}) \geq s_a + \delta V_a(\mathbf{s})$

and because $x_a < s_a$, we must have $V_a(\mathbf{x}) > V_a(\mathbf{s})$.

We now argue that, $\forall \mathbf{s}' \in \Delta$ such that $s'_v \in [s_v, x_v)$, $V_a(\mathbf{s}')$ is constant in $\mathbf{s}'$. Remember that we denote $\mathbf{x} = \mathbf{x}^a(\mathbf{s})$. Let $\mathbf{x}' = \mathbf{x}^a(\mathbf{s}')$ and $\mathbf{y}' = \mathbf{x}^{-a}(\mathbf{s}')$. Since $x_v > s_v$, by Lemma 11 we have that $\forall \mathbf{s}' \in \Delta$ such that $s'_v \in [s_v, x_v)$, $\mathbf{x}' = \mathbf{x}$. Thus, $\forall \mathbf{s}' \in \Delta$ such that $s'_v \in [s_v, x_v)$, $x'_v = x_v > s'_v$ and, by Lemma 12, $y'_v = s'_v$. Because, by Lemma 10 $y'_a = 0$ $\forall \mathbf{s}' \in \Delta$ and because, by Lemma 9 $V_a(\mathbf{s}')$ depends only on $s'_v$, we can express as follows $V_a(\mathbf{s}')$ $\forall \mathbf{s}' \in \Delta$ such that $s'_v \in [s_v, x_v)$:

$$\begin{aligned} V_a(\mathbf{s}') &= p_a \left[x'_a + \delta V_a(\mathbf{x}')\right] + p_{-a} \left[y'_a + \delta V_a(\mathbf{y}')\right] \\ &= p_a \left[x_a + \delta V_a(\mathbf{x})\right] + p_{-a} \left[0 + \delta V_a(\mathbf{s}')\right] \\ &= \tfrac{p_a}{1-\delta p_{-a}} \left[x_a + \delta V_a(\mathbf{x})\right]. \end{aligned} \tag{15}$$

Now consider a sequence $(\mathbf{s}^n)_{n=1}^{\infty}$ defined by $s^n_v = \frac{1}{n} s_v + \frac{n-1}{n} x_v$ and $s^n_a = 1 - s^n_v$. Because, $\forall \mathbf{s}' \in \Delta$ such that $s'_v \in [s_v, x_v)$, $V_a(\mathbf{s}')$ is constant in $s'_v$, $V_a(\mathbf{s}^n) = V_a(\mathbf{s})$ $\forall n \in \mathbb{N}$. Hence $\lim_{n\to\infty} V_a(\mathbf{s}^n) = V_a(\mathbf{s}) < V_a(\mathbf{x})$. Thus $V_a$ is not continuous at $x_v$, a contradiction.

# D Proofs of Propositions from Section 7

## Proof of Proposition 13

Banks and Duggan (2006) show that there are no stationary equilibria of this game with delay. This means that, in any stationary equilibrium, each legislator proposes only policies that are accepted. Let $V_i$ denote the continuation value for player $i = \{v, 1, 2\}$.

Negotiations end following an agreement. This means that, if accepted, proposal $\mathbf{x} \in \Delta$ gives to player $i = \{v, 1, 2\}$ a dynamic payoff equal to $\frac{x_i}{1-\delta}$. Thus, player $i$ supports any proposal offering him an amount weakly greater than $(1-\delta)s_i + \frac{\delta}{1-\delta}V_i$. It is straightforward to show that in any stationary equilibrium, each legislators offers to exactly one other committee member (which, together with the proposer, forms a minimal winning coalition) the amount which makes this member indifferent between accepting and rejecting the proposal and keeps the remainder for himself.

Let the proposal made by player $i = \{v, 1, 2\}$ when recognized be denoted by $\mathbf{x}^i = \{x_v^i, x_1^i, x_2^i\}$. If non-veto player $i = \{1, 2\}$ is recognized, he offers $x_v^i$ such that $\frac{x_v^i}{1-\delta} = s_v + \delta V_v$ to the veto player. If the veto player is recognized, he offers $x_j^v$ such that $\frac{x_j^v}{1-\delta} = s_j + \delta V_j$ to one non-veto player $j = \{1, 2\}$. Denoting with $\mu_1(\mathbf{s}) \in [0, 1]$ the probability that the veto player chooses non-veto player 1 as coalition partner given status quo $(\mathbf{s})$ and with $x_{nv}$ the amount the veto player offers to this coalition partner, we can express the continuation values as follows:

$$
\begin{aligned}
V_v &= \frac{1}{3}\left(\frac{1 - x_{nv}}{1-\delta}\right) + \frac{2}{3}\left(\frac{x_v}{1-\delta}\right) \\
V_1 &= \frac{1}{3}\left(\frac{1 - x_v}{1-\delta}\right) + \frac{1}{3}\mu_1(\mathbf{s})\left(\frac{x_{nv}}{1-\delta}\right) \\
V_2 &= \frac{1}{3}\left(\frac{1 - x_v}{1-\delta}\right) + \frac{1}{3}(1 - \mu_1(\mathbf{s}))\left(\frac{x_{nv}}{1-\delta}\right)
\end{aligned}
$$

In equilibrium, (a) either the veto player chooses one non-veto player with certainty and, even if he is always chosen as coalition partner, this non-veto player is the cheapest coalition partner; or (b) the veto player chooses either non-veto player with positive probability and his proposal strategy is such that the veto player is indifferent between choosing either non-veto player as coalition partner. Since $s_1 \geq s_2$, non-veto player 2 is potentially a cheaper coalition partner. However, if non-veto player 2 is always chosen as coalition partner and $s_1$ is sufficiently close to $s_2$, non-veto player 2 requires a higher amount than non-veto player 1 to support a proposal. It is easy to verify that, if $\mu_1^\star(\mathbf{s})$ is as characterized in the statement of Proposition 13, then the amount that makes non-veto player 2 indifferent between accepting and rejecting is strictly smaller than the amount that makes non-veto player 1 indifferent if $s_1 \geq \frac{3-2\delta}{3-3\delta}s_2$; and that instead these two amounts are the same if $s1 < \frac{3-2\delta}{3-3\delta}s_2$. Using this probability distribution over coalition partners in the continuation values above and solving the system of two equations and two unknowns formed by $\frac{x_v}{1-\delta} = s_v + \delta V_v$ and $\frac{x_2}{1-\delta} = s_2 + \delta V_2$ , we get the equilibrium proposals in the statement of the proposition.

# E Committee Size and Majority Requirement

In this Appendix, I study committees with $n$ legislators, $k \leq n$ veto players and $m = n - k$ non-veto players (with $m$ even). A proposal defeats the status quo if it receives the concurring support of the $k$ veto players and $q \in \left[0, \frac{2}{3}m\right]$ non-veto players. This includes a wide array of voting rules, from oligarchies where the coalition of all veto players can change the status quo without the approval of any non-veto player ($q = 0$), to qualified majorities where the status quo is defeated only with the approval of more than 50% of legislators ($k + q > n/2$). This more general setup allows me to investigate whether expanding the committee or changing the majority requirement can reduce the leverage of the veto player(s) and promote more equitable outcomes.

In order to preserve the analytical tractability of the model, I introduce two assumptions. First, having explored the impact of recognition probabilities in smaller committees, I assume that only veto players are able to make proposals and that each veto player proposes with the same probability. Second, I restrict the set of feasible allocations to those with, at most, two types of non-veto players: a subset who receives zero and a subset who receives the same, non-negative amount. In particular, a feasible allocation is summarized by $\mathbf{s} = \{s_{v1}, s_{v2}, \ldots, s_{vk}, \underline{m}, s_{\overline{m}}\}$, where $s_{vi}$, $i = \{1, 2, \ldots, k\}$, denotes the share to veto player $i$, $\underline{m}$ denotes the number of *poor* non-veto players (whose share is $s_{\underline{m}} = 0$), and $s_{\overline{m}} \geq 0$ denotes the share to each of the $(m - \underline{m})$ *rich* non-veto players.[27] I denote with $s_{nv} = (m - \underline{m})s_{\overline{m}}$ the total share of resources allocated to non-veto players in allocation $\mathbf{s}$.

The presence of multiple veto players or qualified majorities do not prevent the complete expropriation of the resources initially allocated to non-veto players. Proposition 14 shows that this dynamic game has an MPE in which the $k$ veto players extract all the surplus in finite time.

**Proposition 14** *Consider the game with $n$ legislators, $k$ veto players, and $q$ non-veto players needed for passage. For any $n \geq 3$, any $k \leq n$, any $q \in \left[0, \frac{2(n-k)}{3}\right]$, any $\delta \in [0, 1)$, and any $\mathbf{s}^0 \in \Delta$, there exists an MPE such that the committee transitions to an absorbing state where the $k$ veto*

[27] This assumption does not restrict the number of legislators who receive a positive allocation and it allows proposers to give different amounts to different legislators. The role of this assumption is to simplify the identification of the cheapest coalition: the proposer randomizes among coalition partners with the same status quo allocation but does not need to employ a different mixing probability for each feasible allocation.

*players get the whole pie in at most three periods.*[28]

Complete appropriation by veto players is possible because a veto proposer can always pass an allocation that increases his allocation. The committee converges to this outcome in finite time because poor non-veto players do not demand a premium and support any allocation of resources. This means that the veto player can expropriate non-veto players completely as soon as the status quo gives zero to at least $q$ non-veto legislators.

At the same time, non-veto players might enjoy positive resources in the initial periods and larger majority requirements reduce the speed of convergence to the absorbing state where non-veto players hold nothing. This is because rich non-veto players do demand a premium to support an allocation which increases the allocation to veto players, and this premium is increasing in the discount factor and in the fraction of the resources to non-veto players. The cumulative demand of the $(q - \underline{m})$ rich non-veto players needed for a minimal winning coalition is less than the cumulative amount to non-veto players in the status quo. Thus, the proposing veto player can increase his allocation and increase the number of poor non-veto players in the future status quo.[29] However, with a larger $q$ or a lower $m$, it takes more periods to have at least $q$ poor non-veto players and for veto players to appropriate all resources.

Moreover, the presence of other legislators with veto power reduces the amount a single legislator can extract. A non-proposing veto player asks more than what he receives in the status quo to support the expropriation of non-veto players. He demands this premium because a higher current allocation to another veto player decreases the amount he is able to extract when he proposes in the future. In particular, when the non-veto players are completely expropriated, the policy moves to a *gridlock region* where no future proposer will be able to pass a policy he prefers to the status quo. In order to offset this loss and gain their vote, the proposing veto player has to share part of the amount he expropriates from the non-veto players with the other $k - 1$ veto players. Nonetheless, the proposing veto player gets a greater share of the resources expropriated from the non-veto players for any $\delta \in [0, 1)$.

[28]For the case where veto players are decisive ($q = 0$), Proposition 16 in the proof of Proposition 14 below proves an analogous result for the more general setup where $p_v \in (0, 1]$ and all vectors of non-negative transfers which sum to 1 are feasible agreements.

[29]This is because $\underline{m}^{t+1} = m - (q - \underline{m}^t) = \underline{m}^t + (m - q) > \underline{m}^t$.

Proposition 15 summarizes this discussion.

**Proposition 15** *In the MPE from Proposition 14: (a) the number of periods with positive resources to non-veto players is weakly increasing in $q$; (b) the cumulative value of the game for non-veto players is weakly increasing in $\delta$, $q$ and $s^0_{nv}$, and weakly decreasing in $m$; (c) the share to each veto player in the absorbing state is strictly larger than his starting share, unless $\mathbf{s}^0$ is an absorbing state (that is, $s^0_{nv} = 0$) or $\delta = 0$.*

## Proof of Proposition 14

**Veto Players are Decisive:** $q = 0$

When the coalition of veto players is decisive (that is, $q = 0$) we can consider a more general setup where we relax the two assumptions employed in this Section: each veto player proposes with chance $\frac{p_v}{k}$, where $p_v \in (0, 1]$; each non-veto player proposes with chance $\frac{1-p_v}{n-k}$; and the space of possible agreements is composed of all vectors of non-negative transfers to the $n$ legislators which sum to 1. We can prove the following result:

**Proposition 16** *Consider the game with $n$ legislators and $k$ decisive veto players. For any $n \geq 3$, any $k \leq n$, any $p_v \in (0, 1]$, any $\delta \in [0, 1)$, and any $\mathbf{s}^0 \in \Delta$, there exists an MPE such that the committee transitions to an absorbing state where the $k$ veto players get the whole pie as soon as one veto player proposes. In the absorbing state, the share to each veto player is strictly larger than his starting share, unless $\mathbf{s}^0$ is an absorbing state or $\delta = 0$.*

The result of Proposition 14 for $q = 0$ is a special case of the more general result in Proposition 16 above, for the case where $p_v = 1$. Below, we prove Proposition 16.

In this game an allocation is $\mathbf{s} = [s_{v1}, s_{v2}, \ldots, s_{vk}, s_1, s_2, \ldots, s_{n-k}]$, where $s_{vi}$, $i = 1, 2, \ldots, k$, denotes the share to a veto player and $s_i$, $i = 1, 2, \ldots, (n-k)$ denotes the share to a non-veto player. The unique minimal winning coalition is composed of all veto players. The result of Proposition 16 follows from the existence of a symmetric MPE where veto players propose a positive allocation to the members of the minimal winning coalition and non-veto players propose a positive allocation to these members and to themselves. In particular, consider the following proposal strategies for all status quo policies $\mathbf{s} \in \mathbf{\Delta}$:

- When the proposer is veto player $vi$, she offers $d_{vj}^{v}(\mathbf{s}) = s_{vj} + \frac{\delta p_v}{k(1-\delta+p_v\delta)}\sum_{i=1}^{n-k} s_i$ to each of the other $(k-1)$ veto players, 0 to all non-veto players, and $1-\sum_{j\neq i} d_{vj}^{v}(\mathbf{s}) = s_{vi} + \left(1-\frac{(k-1)\delta p_v}{k(1-\delta+p_v\delta)}\right)\sum_{i=1}^{n-k} s_i$ to herself.

- When the proposer is non-veto player $i$, he offers $d_{vi}^{nv} = s_{vi}$, $i = 1, 2, \ldots, k$, to the $k$ veto players, 0 to all other non-veto players, and $1-\sum_i s_{vi} = \sum_{i=1}^{n-k} s_i$ to herself.

If these proposals pass, allocations which do not give anything to any non-veto players are absorbing states and one of these allocations is reached as soon as one veto player proposes. Remember that a veto player is selected to propose with probability $p_v/k$ and a non-veto player with probability $(1-p_v)/(n-k)$. Therefore, if legislators play the proposal strategies above and these proposals pass, legislators' continuation values for allocation $\mathbf{s} \in \Delta$ are as follows:

$$\begin{aligned}
v_{vi}(\mathbf{s}) &= (1-p_v)\left[s_{v1} + \delta v_{v1}(\mathbf{s})\right] + \frac{p_v}{k}\left[\frac{1-\sum_{j\neq i} d_{vj}^{v}(\mathbf{s})}{1-\delta}\right] + \frac{p_v(k-1)}{k}\left[\frac{d_{v1}^{v}}{1-\delta}\right] \\
&= \frac{s_{vi}}{(1-\delta)} + \frac{p_v\sum_{i=1}^{n-k} s_i}{(1-\delta)k(1-\delta+p_v\delta)} \\
v_i(\mathbf{s}) &= \frac{1-p_v}{n-k}\left[\sum_{i=1}^{n-k} s_i + \delta v_i(\mathbf{s})\right] = \frac{(1-p_v)\left(\sum_{i=1}^{n-k} s_i\right)}{(n-k)-\delta(1-p_v)}
\end{aligned}$$

On the basis of these continuation values, we obtain veto players' expected utility functions, $U_{vi}(\mathbf{x}) = x_{vi} + \delta v_{vi}(\mathbf{x})$. The reported demands for veto players as a function of the proposer's type are in accordance with Definition 2. In particular, $d_{vi}^{v}$and $d_{vi}^{nv}$, $i = 1, 2, \ldots, k$, can be easily derived from the following equations:

$$\begin{aligned}
s_{vi} + \delta v_{vi}\left(s_{vi}, \sum_{i=1}^{n-k} s_i\right) &= \frac{d_{vi}^{v}}{1-\delta} \\
s_{vi} + \delta v_{vi}\left(s_{vi}, \sum_{i=1}^{n-k} s_i\right) &= d_{vi}^{nv} + \delta v_{vi}\left(d_{vi}^{nv}, \sum_{i=1}^{n-k} s_i\right)
\end{aligned}$$

By the definition of demands, supporting the proposals outlined above is an equilibrium voting strategy. Finally, we need to prove that those proposal strategies are optimal, given the continuation values. First, note that the expected utility a veto player derives from a policy $\mathbf{x}$ is an increasing and linear function of what $\mathbf{x}$ assigns to her and of what $\mathbf{x}$ assigns to all non-veto players. Second, since $\frac{p}{k(1-\delta+p_v\delta)} \in [p_v/k, 1/k]$ is always smaller than 1, the optimal proposal for player $vi$ is the one that maximizes $x_{vi}$, subject to being approved, that is, subject to giving the other veto players at least $d^v_{vj}$. The unique proposal that maximizes this objective function subject to this constraint is the one that gives exactly $d^v_{vj}$ to the other veto player, 0 to the non-veto players, and the remainder to the proposer.

**Veto Players are not Decisive:** $q \in \left(0, \frac{2}{3}m\right]$

In this case, a proposal passes if it receives the support of the $k$ veto players, plus at least $q \in \left[1, \frac{2}{3}m\right]$ non-veto legislators. Remember that we denote with $s_{nv} = (m - \underline{m})s_{\overline{m}}$ the total share of resources allocated to non-veto players in allocation $\mathbf{s}$. The results of Proposition 14 follow from the existence of a symmetric MPE with the following minimal winning coalition proposal strategies for all feasible allocations:

- Case A $\underline{m} \geq q$:
  - The proposing veto player offers $d^A_{vi} = s_{vi} + \frac{\delta}{k} s_{nv}$ to the other $(k-1)$ veto players, the remainder to himself, and 0 to everybody else. The proposal passes with the support of the veto players and the poor non-veto players, (who are indifferent between the status quo and the proposal because $d^A_{\underline{m}} = 0$). At the beginning of the following period, the status quo allocation is such that $\underline{m} = m > q$. This means that the new status quo belongs to case A.

- Case B $q > \underline{m} \geq 2q - m$:
  - The proposing veto player offers $d^B_v = s_{vi} + \frac{\delta}{k} \frac{(m-\underline{m})-2(q-\underline{m})}{(m-\underline{m})-\delta(q-\underline{m})} s_{nv}$ to the other $(k-1)$ veto players, $d^B_{\overline{m}} = \frac{1}{(m-\underline{m})-\delta(q-\underline{m})} s_{nv}$ to $(q - \underline{m})$ randomly chosen rich non-veto players, the remainder to himself, and 0 to everybody else. The proposal passes with the support of the veto players, the $(q - \underline{m})$ rich veto players who are offered a positive amount and the poor non-veto players (who are indifferent between the status quo and the proposal because $d^B_{\underline{m}} = 0$). At the beginning of the following period, the status quo allocation

$\mathbf{s}^{t+1}$ is such that $\underline{m}^{t+1} = \underline{m}^t + (m-q)$. Since $\underline{m}^t \geq 2q - m$, this means that $\underline{m}^{t+1} \geq q$, or that the new status quo belongs to case A.

- Case C $\underline{m} < 2q - m$:
  - The proposing veto player offers $d_v^C = s_{vi} + \frac{\delta}{k}\frac{(m-q)^2(1-\delta)^2}{[m-\underline{m}-\delta(q-\underline{m})]^2}s_{nv}$ to the other $(k-1)$ veto players, $d_{\overline{m}}^C = \frac{(q-\underline{m})-\delta(2q-\underline{m}-m)}{(q-\underline{m})[m-\underline{m}-\delta(q-\underline{m})]}s_{nv}$ to $(q-\underline{m})$ randomly chosen rich non-veto players, the reminder to himself, and 0 to everybody else. The proposal passes with the support of the veto players, the $(q-\underline{m})$ rich veto players who are offered a positive amount and the poor non-veto players (who are indifferent between the status quo and the proposal because $d_{\underline{m}}^C = 0$). At the beginning of the following period, the status quo allocation $\mathbf{s}^{t+1}$ is such that $\underline{m}^{t+1} = \underline{m}^t + (m-q)$. Since $\underline{m}^t < 2q - m$, we have $\underline{m}^{t+1} < q$. Moreover, since $q < \frac{2}{3}m$, we have $\underline{m}^{t+1} \geq 2q - m$. This means that the new status quo belongs to case B.

If players play the proposal strategies in cases A-C and these proposals pass, their continuation values are as follows:

- Case A

$$\begin{aligned}
v_{vi}(\mathbf{s}) &= \frac{s_{vi}}{1-\delta} + \frac{s_{nv}}{(1-\delta)k} \\
v_{\underline{m}}(\mathbf{s}) &= 0 \\
v_{\overline{m}}(\mathbf{s}) &= 0
\end{aligned}$$

- Case B

$$\begin{aligned}
v_{vi}(\mathbf{s}) &= \frac{s_{vi}}{1-\delta} + \frac{(m-q)}{(1-\delta)k[m-\underline{m}-\delta(q-\underline{m})]}s_{nv} \\
v_{\underline{m}}(\mathbf{s}) &= 0 \\
v_{\overline{m}}(\mathbf{s}) &= \frac{(q-\underline{m})}{(m-\underline{m})}\frac{s_{nv}}{(m-\underline{m})-\delta(q-\underline{m})}
\end{aligned}$$

- Case C

$$
\begin{aligned}
v_{vi}(\mathbf{s}) &= \frac{s_{vi}}{1-\delta} + \frac{(m-q)[(1-\delta)(m-\underline{m}) + \delta^2(m-q)]}{(1-\delta)k[m-\underline{m}-\delta(q-\underline{m})]^2} s_{nv} \\
v_{\underline{m}}(\mathbf{s}) &= 0 \\
v_{\overline{m}}(\mathbf{s}) &= \frac{(q-\underline{m})}{(m-\underline{m})} \frac{s_{nv}}{(m-\underline{m}) - \delta(q-\underline{m})}
\end{aligned}
$$

On the basis of these continuation values, we obtain players' expected utility functions, $U_{vi}(\mathbf{x}) = x_{vi} + \delta v_{vi}(\mathbf{x})$, $U_{\underline{m}}(\mathbf{x}) = 0 + \delta v_{\underline{m}}(\mathbf{x})$, and $U_{\overline{m}}(\mathbf{x}) = x_{\overline{m}} + \delta v_{\overline{m}}(\mathbf{x})$. The reported demands are in accordance with Definition 2. In particular, $d^A_{vi}$, $d^B_{vi}$, $d^C_{vi}$, $i = 1, 2, \ldots, k$, $d^A_{\underline{m}}$, $d^B_{\underline{m}}$, $d^C_{\underline{m}}$, $d^B_{\overline{m}}$, and $d^C_{\overline{m}}$ can be derived from the following indifference conditions:

$$
\begin{aligned}
s_{vi} + \delta v^A_{vi}(s_{vi}, s_{nv}) &= \frac{d^A_{vi}}{1-\delta} \\
s_{vi} + \delta v^B_{vi}(s_{vi}, s_{nv}) &= d^B_{vi} + \delta v^A_{vi}\left(d^B_{vi}, (q-\underline{m})d^B_{\overline{m}}\right) \\
s_{vi} + \delta v^C_{vi}(s_{vi}, s_{nv}) &= d^C_{vi} + \delta v^B_{vi}\left(d^C_{vi}, (q-\underline{m})d^C_{\overline{m}}\right) \\
0 + \delta v^A_{\underline{m}} &= d^A_{\underline{m}} + \delta v^A_{\underline{m}} \\
0 + \delta v^B_{\underline{m}} &= d^B_{\underline{m}} + \delta v^A_{\underline{m}} \\
0 + \delta v^C_{\underline{m}} &= d^C_{\underline{m}} + \delta v^B_{\underline{m}} \\
s_{\overline{m}} + \delta v^B_{\overline{m}}(s_{nv}, \underline{m}) &= d^B_{\overline{m}} + \delta v^A_{\overline{m}} \\
s_{\overline{m}} + \delta v^C_{\overline{m}}(s_{nv}, \underline{m}) &= d^C_{\overline{m}} + \delta v^B_{\overline{m}}\left((q-\underline{m})d^C_{\overline{m}}, \underline{m} + (m-q)\right)
\end{aligned}
$$

By the definition of demands, supporting the proposals outlined above is an equilibrium voting strategy. Finally, we need to prove that those proposal strategies are optimal, given the continuation values. In case A, $U_{vi}(\mathbf{x})$ is a linear and positive function of both $x_{vi}$ and $x_{nv}$. Since $\frac{1}{1-\delta} \geq \frac{\delta}{k(1-\delta)}$ for any $k \geq 1$, and $\delta \in [0, 1]$, $U_{vi}(\mathbf{x})$ is maximized when $x_{vi}$ is as large as possible. Similarly, in cases B and C, $U_{vi}(\mathbf{x})$ is a linear and positive function of both $x_{vi}$ and $x_{nv}$. Since the coefficient of $x_{vi}$, $\frac{1}{1-\delta}$, is greater than or equal to the coefficient of $x_{nv}$ for any $k \geq 1$, $\delta \in [0, 1]$, $m \geq 1$, $\underline{m} \leq m$, and $q < m$, $U_{vi}(\mathbf{x})$ is maximized when $x_{vi}$ is as large as possible. This means that the expected utility of the proposing veto player is maximized when $x_{vi}$ is as large as possible. The proposal strategies above are the acceptable proposals which give the largest possible share to the proposer (because they make the agents of a minimal winning coalition barely indifferent between accepting and rejecting).

## Proof of Proposition 15

This result in part (a) follows from the equilibrium demand of poor non-veto players which demand zero to support any allocation. This means that, when the status quo is such that $\underline{m} > q$, the proposing veto player can expropriate the non-veto players completely with the support of the poor non-veto players and the other veto players. When in the initial status quo $\underline{m}^0 < q$, the proposer cannot extract completely the rich non-veto players because he needs the support of $(q - \underline{m}^0)$ rich non-veto players who demand a positive allocation. However, the number of cheap coalition partners at the beginning of the second period will be larger: $\underline{m}^1 = m - (q - \underline{m}^0) = \underline{m}^0 + (m - q) > \underline{m}^0$, which holds $\forall q < m$. When $q > \underline{m}^0 \geq 2q - m$, we have $\underline{m}^1 > q$. In this case, in the second period the proposer does not need the support of any rich non-veto player and we converge to the absorbing state where non-veto players have zero resources. When $\underline{m}^0 < 2q - m$, we have $\underline{m}^1 < q$ so, in the second period, the proposer needs to muster the costly support of $(q - \underline{m}^1)$ rich non-veto players. However, since $q < \frac{2}{3}m$, we have $\underline{m}^2 \geq q$: in the third period the proposer does not need the support of any rich non-veto player and we converge to the absorbing state where non-veto players have zero resources.

The result in part (b) follows from investigating the sum of all non-veto players' value functions evaluated at the initial status quo, $\mathbf{s}^0$. Since the value of any initial allocation to poor non-veto players is 0 and the value to each non-veto player is the same, this sum is given by $(m - \underline{m})v_{\overline{m}}(\mathbf{s}^0)$ or by:

$$\sum_{i \in nv} v_i(\mathbf{s}^0) = (m - \underline{m})v_{\overline{m}}(\mathbf{s}^0) = \begin{cases} 0, & \text{if } q < \underline{m}^0 \\ \frac{q - \underline{m}^0}{m - \underline{m}^0 - \delta(q - \underline{m}^0)} s^0_{nv}, & \text{if } q > \underline{m}^0 \end{cases}$$

When $q < \underline{m}^0$, this value is constant. When $q > \underline{m}^0$, this value is increasing in $q$, $\delta$ and $s^0_{nv}$ and decreasing in $m$.

The result in part (c) follows from the equilibrium demand of veto players: $\frac{\delta}{k}$ and $\frac{\delta}{k} \frac{(m-q)^2(1-\delta)^2}{[m - \underline{m} - \delta(q - \underline{m})]^2}$ are strictly positive for any $k \geq 1$, $\delta \in (0, 1)$, and $q < m$; $\frac{\delta}{k} \frac{(m - \underline{m}) - 2(q - \underline{m})}{(m - \underline{m}) - \delta(q - \underline{m})}$ (the demand in case B) is strictly positive for any $\delta \in (0,1)$, $q < m$, and $\underline{m} < q$ (which is true in case B). This means that each veto player gets a positive fraction of the resources expropriated from non-veto players regardless of the identity of the proposer.

# F MPEs Where Positive Allocations to Non-Veto Players Are Stable

**Proposition 17** *Consider any* $\mathbf{x} \in \Delta$ *such that (a) min*$\{x_v, x_1, x_2\} > 0$*, (b)* $x_2 + \frac{\delta(x_1 - x_2)}{2} > \overline{x}(\delta) = \frac{(3-2\delta)^2}{(3-\delta)^2}$ *and (c)* $x_1 + x_2 > \overline{\overline{x}}(\delta) = \frac{54-72\delta+18\delta^2+4\delta^3}{(3-\delta)(18-15\delta+\delta^2)}$*. There exists an MPE of the legislative bargaining game where* $Y = \{(x_v, x_1, x_2), (x_v, x_2, x_1)\}$ *is an irreducible absorbing set.*

In the proof of Proposition 17, I construct a class of MPEs where an allocation (or a pair of allocations) giving a positive amount to all players is stable. In particular, I show that, for each set $Y$ satisfying the sufficient conditions in the statement of Proposition 17, there exists an MPE of the legislative bargaining game such that (a) if $\mathbf{s}^0 \in Y$, the policy is never changed (if $x_1 = x_2$ and, thus, $|Y| = 1$) or alternates forever between allocations in $Y$ (if $x_1 \neq x_2$ and, thus $|Y| = 2$); and (b) if $\mathbf{s}^0 \notin Y$, the policy converges asymptotically to full extraction by the veto player as in the MPE from Proposition 1. Notice that the set of allocations satisfying the sufficient conditions in the statement of Proposition 17 is non-empty for any $\delta > 0.68$ and grows with $\delta$.

**Corollary 2** *Consider any* $\mathbf{x} \in \Delta$ *such that (a) min*$\{x_v, x_1, x_2\} > 0$*, (b)* $x_1 = x_2 = x_{nv}$*, (c)* $x_{nv} > \frac{(3-2\delta)^2}{(3-\delta)^2}$ *and (d)* $x_{nv} > \frac{27-36\delta+9\delta^2+2\delta^3}{(3-\delta)(18-15\delta+\delta^2)}$*. There exists an MPE of the legislative bargaining game where* $\mathbf{x}$ *is absorbing. In particular, as* $\delta$ *goes to 1, for any* $\mathbf{x} \in \Delta$ *such that* $x_{nv} > 1/4$*, there exists an MPE of the legislative bargaining game where* $\mathbf{x}$ *is absorbing.*

**Example 1** *Assume* $\delta = 0.75$*. In this case,* $\overline{x}(\delta) = 0.44$ *and* $\overline{\overline{x}}(\delta) = 0.72$*. Note that* $x_2 + \frac{3(x_1-x_2)}{8} > 0.44$ *implies* $x_1 + x_2 > 0.72$ *as long as* $x_1 \geq 1/25$*. Since neither inequality is satisfied when* $x_1 < 1/25$*, there exists an MPE where* $Y = \{(x_v, x_1, x_2), (x_v, x_2, x_1\}$ *is an irreducible absorbing set for any* $\mathbf{x} \in \Delta$ *such that min*$\{x_v, x_1, x_2\} > 0$ *and* $\frac{5}{8}x_2 + \frac{3}{8}x_1 > 0.44$*.*

**Example 2** *Assume* $\delta = 0.95$*. In this case,* $\overline{x}(\delta) = 0.29$*, and* $\overline{\overline{x}}(\delta) = 0.55$*. Note that* $x_2 + \frac{95(x_1-x_2)}{200} > 0.29$ *implies* $x_1 + x_2 > 0.55$ *as long as* $x_1 \geq 1/8$*. Since neither inequality is satisfied when* $x_1 < 1/8$*, there exists an MPE where* $Y = \{(x_v, x_1, x_2), (x_v, x_2, x_1\}$ *is an irreducible absorbing set for any* $\mathbf{x} \in \Delta$ *such that min*$\{x_v, x_1, x_2\} > 0$ *and* $\frac{21}{40}x_2 + \frac{19}{40}x_1 > 0.29$*.*

**Example 3** *Assume* $\delta = 0.99$. *In this case,* $\overline{x}(\delta) = 0.26$, *and* $\overline{\overline{x}}(\delta) = 0.51$. *Note that* $x_2 + \frac{99(x_1-x_2)}{200} > 0.26$ *implies* $x_1+x_2 > 0.51$ *Then,there exists an MPE where* $Y = \{(x_v, x_1, x_2), (x_v, x_2, x_1\}$ *is an irreducible absorbing set for any* $\mathbf{x} \in \Delta$ *such that min*$\{x_v, x_1, x_2\} > 0$ *and* $\frac{101}{200}x_2+\frac{99}{200}x_1 > 0.26$.

**Proof.** Assume $s_1 \geq s_2$ (without loss of generality) and consider the following strategies: (a) if $\mathbf{s} \in Y = \{(s_v, s_1, s_2), (s_v, s_2, s_1)\}$, then a veto proposer offers either allocation in $Y$ with the same chance; non-veto proposer 1 offers $(s_v, s_1, s_2)$; non-veto proposer 2 offers $(s_v, s_2, s_1)$; (b) if $\mathbf{x} \notin Y$, players follow the proposing and voting strategies from the MPE characterized in the proof of Proposition 1.

This is the strategy of the proof. First we want to show that when the policy is in $Y$, it does not move out of this set. Second, we want to show that when the policy is not in $Y$, policy does not move in this set. To prove the first statement, I show that, given the strategies described above and the associated value functions, there is no allocation outside $Y$ which the veto player and at least one non-veto player prefer to a status quo is in $Y$. In particular, I derive the minimum amount a non-veto player needs to be offered in order to support a proposal outside of $Y$ (and label it $d_2^\star$); then, I derive the minimum amount a veto player needs to be offered in order to support a proposal outside of $Y$ (and label it $d_v^\star$); finally, I show that, when the assumptions in the statement of Proposition 17 are satisfied, there is no feasible allocation outside $Y$ which both the veto player and one non-veto player are willing to support (that is, $d_2^\star+d_v^\star > 1$). To prove the second statement, I show that, when the policy is not in $Y$ the veto player is better off vetoing any attempt to bring the policy in $Y$.

Consider an allocation $\mathbf{s} = (s_v, s_1, s_2) \notin Y$. Given the conjectured equilibrium strategies, the evolution of policies follows the MPE from Proposition 1 (with all future policies lying in $\overline{\Delta}$ and asymptotic convergence to full extraction by the veto player). Thus, the continuation values and expected utilities are those from the proof of Proposition 1.

Consider an allocation $\mathbf{s} = (s_v, s_1, s_2) \in Y$.

Given the conjectured equilibrium strategies, expected utilities from $\mathbf{s}$ are:

$$
\begin{aligned}
U_v(\mathbf{s}) &= \frac{s_v}{1-\delta} \\
U_1(\mathbf{s}) &= s_1 + \delta\left[\frac{s_1+s_2}{2(1-\delta)}\right] = \frac{2s_1 - \delta(s_1-s_2)}{2(1-\delta)} \\
U_2(\mathbf{s}) &= s_2 + \delta\left[\frac{s_1+s_2}{2(1-\delta)}\right] = \frac{2s_2 + \delta(s_1-s_2)}{2(1-\delta)}
\end{aligned}
$$

**Part 1: if $\mathbf{s}^t \in Y$, then $\mathbf{s}^{t+1} \in Y$.**

Consider status quo $\mathbf{s} \in Y$. Since $U_1(\mathbf{s}) \geq U_2(\mathbf{s})$, we can focus on the minimum amount required by non-veto player 2 to support a policy outside $Y$. Note that, given the conjectured equilibrium strategies, the continuation value of any allocation outside $Y$ is the continuation value from the MPE in Proposition 1. These continuation values are such that expected utilities are strictly increasing in one's own allocation for any $\delta \in [0, 1)$ so we can focus on allocations in $\overline{\Delta}$.

The amount that makes player 2 indifferent between $\mathbf{s} \in Y$ and an allocation $x \in \overline{\Delta}$ (or player 2's *demand*, using the definition introduced in the proof of Proposition 1) is the amount $d_2$ such that:

$$
\begin{aligned}
\frac{2s_2 + \delta(s_1-s_2)}{2(1-\delta)} &= d_2 + \delta v_2^A(\mathbf{d}) \\
\frac{2s_2 + \delta(s_1-s_2)}{2(1-\delta)} &= d_2 + \delta\left[\frac{3-3\delta+\delta^2}{(3-\delta)^2(1-\delta)}\right] d_2 \\
d_2^\star &= (2s_2 + \delta(s_1-s_2))\frac{(3-\delta)^2}{2(3-2\delta)^2}
\end{aligned}
$$

where $v_2^A(\mathbf{d})$ is the continuation value of allocation $\mathbf{d} = (1-d_2, 0, d_2)$ from the MPE in Proposition 1. Note that $d_2^\star(s_1, s_2, \delta)$ is strictly increasing in $s_1, s_2, \delta$.

The amount that makes the veto player indifferent between $\mathbf{s} \in Y$ and an allocation $x \in \overline{\Delta}$ (or the veto player's *demand*) is the amount $d_v$ such that:

$$
\begin{aligned}
\frac{1-s_1-s_2}{1-\delta} &= d_v + \delta v_v^A(\mathbf{d}) \\
\frac{1-s_1-s_2}{1-\delta} &= \frac{1}{1-\delta} - \frac{2-\delta}{(3-\delta)(1-\delta)}(1-d_v)
\end{aligned}
$$

where $v_v^A(\mathbf{d})$ is the continuation value of allocation $\mathbf{d} = (d_v, 0, 1-d_v)$ from the MPE in Proposition 1. Since negative allocations are not feasible, we have:

$$
d_v^\star = \begin{cases} 0 & \text{if } s_1 + s_2 > \frac{2-\delta}{3-\delta} \\ \frac{(1-s_1-s_2)(3-\delta)-1}{2-\delta} & \text{if } s_1 + s_2 \leq \frac{2-\delta}{3-\delta} \end{cases}
$$

which is weakly decreasing in $s_1, s_2, \delta$.

If $d_2^\star + d_v^\star > 1$, then there is no feasible allocation outside $Y$ which can count on the support of the veto player and at least one non-veto player. We have two cases to consider.

**Case A:** $s_1 + s_2 > \frac{2-\delta}{3-\delta} \in \left[\frac{1}{2}, \frac{2}{3}\right]$

$$
\begin{aligned}
d_2^\star + d_v^\star &> 1 \\
d_2^\star + 0 &> 1 \\
(2s_2 + \delta(s_1 - s_2))\frac{(3-\delta)^2}{2(3-2\delta)^2} &> 1 \\
s_2 + \frac{\delta(s_1 - s_2)}{2} &> \frac{(3-2\delta)^2}{(3-\delta)^2} = \overline{x}(\delta)
\end{aligned}
\tag{16}
$$

For any $\delta \in [0, 1]$, the LHS is largest when $s_1 = s_2 = 1/2$. The RHS is continuous and strictly decreasing in $\delta$; it goes to 1 as $\delta$ goes to 0 and it goes to 1/4 as $\delta$ goes to 1. Thus, the inequality in equation (16) cannot be satisfied for any $\mathbf{s} \in \Delta$ when $\delta < \frac{9}{7} - \frac{3\sqrt{2}}{7} \approx 0.68$.

**Case B:** $s_1 + s_2 \leq \frac{2-\delta}{3-\delta} \in \left[\frac{1}{2}, \frac{2}{3}\right]$

$$\begin{aligned}
d_2^\star + d_v^\star &> 1 \\
(2s_2 + \delta(s_1 - s_2))\frac{(3-\delta)^2}{2(3-2\delta)^2} + \frac{(1-s_1-s_2)(3-\delta)-1}{2-\delta} &> 1 \\
\left(\frac{\delta(3-\delta)^2}{2(3-2\delta)^2} - \frac{3-\delta}{2-\delta}\right)s_1 + \left(\frac{(2-\delta)(3-\delta)^2}{2(3-2\delta)^2} - \frac{3-\delta}{2-\delta}\right)s_2 &> 0 \qquad (17)
\end{aligned}$$

The coefficients of $s_1$ and $s_2$ in the LHS are non-positive for any $\delta \in [0,1]$. Thus, the inequality can never be satisfied. This means that no MPE in the class constructed in this proof exists when $s_1 + s_2 \leq \frac{2-\delta}{3-\delta}$ or (regardless of $\delta$) when $s_1 + s_2 \leq 1/2$.

**Part 2: if $\mathbf{s}^t \notin Y$, then $\mathbf{s}^{t+1} \notin Y$.**

Consider a status quo allocation $\mathbf{s} \notin Y$. A sufficient condition for policy to evolve according to the MPE in Proposition 1 and never moving in set $Y$ is that the veto player blocks any attempt to move policy inside $Y$. From the continuation values in the proof of Proposition 1, we know that the $\mathbf{s} \notin Y$ which gives the lowest expected utility to the veto player is $(0, 1/2, 1/2)$. Thus, if the veto player prefers $(0, 1/2, 1/2)$ to allocations in $Y$, then he prefers any allocation outside $Y$ to allocations in $Y$. The veto player prefers $(0, 1/2, 1/2)$ to either policy in $Y$ when the following inequality is satisfied:

$$\begin{aligned}
0 + \delta v_v^C(0, 1/2, 1/2) &> \frac{1-s_1-s_2}{1-\delta} \\
\frac{9\delta - 5\delta^3}{(3-\delta)(18-15\delta+\delta^2)} &> 1 - s_1 - s_2 \\
s_1 + s_2 &> 1 - \frac{9\delta - 5\delta^3}{(3-\delta)(18-15\delta+\delta^2)} = \frac{54 - 72\delta + 18\delta^2 + 4\delta^3}{(3-\delta)(18-15\delta+\delta^2)} = \overline{\overline{x}}(\delta) \qquad (18)
\end{aligned}$$

where $v_v^C(0, 1/2, 1/2)$ is the continuation value of allocation $(0, 1/2, 1/2)$ from the MPE in Proposition 1. Notice that the RHS is strictly decreasing in $\delta$ and converges to $1/2$ as $\delta$ goes to 1.

For any $\delta \in [0,1]$, if equation (18) is satisfied, then $s_1 + s_2 > \frac{2-\delta}{3-\delta}$ so equation (17) is irrelevant. The sufficient conditions in the statement of Proposition 17 are exactly equation (16) and equation (18).

For Corollary 2, note that when $s_1 = s_2 = s_{nv}$, equation (16) becomes:

$$s_{nv} > \frac{(3-2\delta)^2}{(3-\delta)^2} \tag{19}$$

Similarly, when when $x_1 = x_2 = x_{nv}$, equation (18) becomes

$$s_{nv} > \frac{27 - 36\delta + 9\delta^2 + 2\delta^3}{(3-\delta)(18 - 15\delta + \delta^2)} \tag{20}$$

The RHS of equation (19) is strictly decreasing in $\delta$ and goes to 1/4 as $\delta$ goes to 1. The RHS of equation (20) is strictly decreasing in $\delta$ and goes to 1/4 as $\delta$ goes to 1. Thus, as $\delta$ goes to 1, any $\mathbf{s} \in \Delta$ such that $s_{nv} \in (1/4, 1/2)$ satisfies the sufficient conditions for MPE existence. ■

# G MPEs Where Only Full Appropriation by Veto Player Is Stable in DES

## Protocol with Veto Player as Persistent Proposer ($p_v = 1$)

Consider the bargaining protocol where the veto player has monopolistic agenda setting power ($p_v = 1$). Contrary to the setup in this paper and, in line with the assumptions of Diermeier, Egorov and Sonin (2017), here I assume that in each period, players bargain over the allocation of $b$ indivisible objects, where $b = 2$. I show that, in spite of this difference, there is an MPE of the legislative bargaining game where full extraction by the veto player is the only stable allocation for any $\delta \in [0, 1]$. In the analysis by Diermeier, Egorov and Sonin (2017), this equilibrium is refined away by the assumption that the bargaining protocol is randomly selected from a set of potential protocols at the beginning of each round and by the focus on equilibria which do not depend on the protocol selected. Using the language from Diermeier, Egorov and Sonin (2017), here I assume that there is only one feasible bargaining protocol, where the only proposer in each period is the veto player.

Consider the following strategies: when $\mathbf{s} = (0, 1, 1)$, the veto player proposes (0,0,2) or (0,2,0) with equal chance and this proposal is supported by the non-veto player who is offered a positive amount; when $\mathbf{s} \neq (0, 1, 1)$, the veto player proposes (2,0,0) and the proposal is supported by the non-veto player who has 0 in the status quo. In this MPE, the only irreducible absorbing set is $\{(2, 0, 0)\}$, that is, full expropriation by the veto player, which is reached in, at most, two periods.

The continuation values are as follows:

$$V_i(2, 0, 0) = V_i(1, 0, 1) = V_i(1, 1, 0) = V_i(0, 2, 0) = V_i(0, 0, 2) = 0$$

for $i = \{1, 2\}$.

$$V_1(0, 1, 1) = \frac{1}{2} [2 + \delta V_1(0, 2, 0)] + \frac{1}{2} [0 + \delta V_1(0, 0, 2)] = 1$$
$$V_2(0, 1, 1) = \frac{1}{2} [2 + \delta V_1(0, 0, 2)] + \frac{1}{2} [0 + \delta V_1(0, 2, 0)] = 1$$

Non-veto player 1 accepts proposal (0,2,0) when the status quo is (0,1,1) if and only if:

$$\begin{aligned} 2 + \delta V_i(0,2,0) &\geq 1 + \delta V_1(0,1,1) \\ 1 &\geq \delta \end{aligned}$$

Thus, this MPE exists for any $\delta \in [0,1]$.

## Protocol with Random Selection of Proposer ($p_v = \frac{1}{3}$)

Consider the bargaining protocol where the proposer is randomly selected in each period ($p_v = 1/3$). Contrary to the setup in this paper and, in line with the assumptions of Diermeier, Egorov and Sonin (2017), here I assume that 1) in each period, players bargain over the allocation of $b$ indivisible objects, where $b = 3$; and 2) a reform which simply shuffles the allocations to non-veto players is costly and always rejected by the veto player, that is, it is impossible to move from allocation $(s_v, s_1, s_2)$ to allocation $(s_v, s_2, s_1)$. I show that, in spite of these differences, there is an MPE of the legislative bargaining game where full extraction by the veto player is the only stable allocation. In the analysis by Diermeier, Egorov and Sonin (2017), this equilibrium is refined away by the assumption that the bargaining protocol is randomly selected from a set of potential protocols at the beginning of each round and by the focus on equilibria which do not depend on the protocol selected. Using the language from Diermeier, Egorov and Sonin (2017), here I assume that there are three feasible bargaining protocols — one where the only proposer is the veto player, one where the only proposer is non-veto player 1, and one were the only proposer is non-veto player 2 — and that each protocol is equally likely to be selected at the beginning of each round.

Below, I describe proposal and voting strategies for each feasible allocation and show that, together with the continuation values, these strategies constitute a symmetric MPE of the legislative bargaining game where (3,0,0) is the only absorbing outcome. Without loss of generality, I focus on the portion of the simplex where $s_1 \geq s_2$ and consider the following proposal and voting strategies for status quo $\mathbf{s} = (s_v, s_1, s_2)$:

- when $\mathbf{s} = (3,0,0)$: everybody proposes $\mathbf{s}$; the veto player blocks any reform; non-veto players support any reform;

- when $\mathbf{s} = (2, 1, 0)$: the veto player and non-veto player 2 propose (3,0,0); non-veto player 1 proposes $\mathbf{s}$; the veto player and non-veto player 1 support only reforms which increase one's own allocations; non-veto player 2 supports any reform;

- when $\mathbf{s} = (1, 2, 0)$: the veto player and non-veto player 2 propose (2,0,1); non-veto player 1 proposes $\mathbf{s}$; everybody supports only reforms which increase one's own allocation;

- when $\mathbf{s} = (1, 1, 1)$ and $\delta < 0.908$: the veto player proposes (1,2,0) and (1,0,2) with equal chance; non-veto player 1 proposes (1,2,0); non-veto player 2 proposes (1,0,2); the veto player supports any reform which gives him at least 1; non-veto players support only reforms which increase one's own allocation;

- when $\mathbf{s} = (1, 1, 1)$ and $\delta \geq 0.908$: the veto player proposes (0,3,0) and (0,0,3) with equal chance; non-veto player 1 proposes (0,3,0); non-veto player 2 proposes (0,0,3); the veto player supports any reform; non-veto players support only reforms which give them everything;

- when $\mathbf{s} = (0, 3, 0)$: the veto player and non-veto player 2 propose (1,0,2); non-veto player 1 proposes $\mathbf{s}$; everybody supports only reforms which increase one's own allocation;

- when $\mathbf{s} = (0, 2, 1)$: the veto player proposes (0,3,0) with probability $(1 - \mu)$ and (0,0,3) with probability $\mu$; non-veto player 1 proposes (0,3,0); non-veto player 2 proposes (0,0,3); the veto player supports any reform; non-veto players only support reforms which increase one's own allocation; $\mu = 1$ if $\delta \leq 0.5787$ and $\mu = \frac{27-21\delta^2+10\delta^3}{2\delta(7\delta^2-18\delta+27)} \in [1/2, 1]$ if $\delta > 0.5787$.

Given these strategies, the continuation values of the non-veto players are:

$$
\begin{aligned}
V_1(3,0,0) &= V_2(3,0,0) = 0 \\
V_1(1,2,0) &= V_2(1,0,2) = \frac{1}{3}\left[2+\delta V_1(1,2,0)\right] + \frac{2}{3}\left[0+\delta V_1(2,0,1)\right] = \frac{2}{3-\delta} \\
V_1(1,0,2) &= V_2(1,2,0) = \frac{1}{3}\left[0+\delta V_1(1,0,2)\right] + \frac{2}{3}\left[1+\delta V_1(2,1,0)\right] = \frac{6}{(3-\delta)^2} \\
V_1(2,1,0) &= V_2(2,0,1) = \frac{1}{3}\left[1+\delta V_1(2,1,0)\right] + \frac{2}{3}[0] = \frac{1}{3-\delta} \\
V_1(2,0,1) &= V_2(2,1,0) = 0 \\
V_1(0,3,0) &= V_2(0,0,3) = \frac{1}{3}\left[3+\delta V_1(0,3,0)\right] + \frac{2}{3}\left[0+\delta V_1(1,0,2)\right] = \frac{27-6\delta+3\delta^2}{(3-\delta)^3} \\
V_1(0,0,3) &= V_2(0,3,0) = \frac{1}{3}\left[0+\delta V_1(0,0,3)\right] + \frac{2}{3}\left[2+\delta V_1(1,2,0)\right] = \frac{12}{(3-\delta)^2} \\
V_1(0,2,1) &= V_2(0,1,2) = \frac{2-\mu}{3}\left(3+\delta V_1(0,3,0)\right) + \frac{1+\mu}{3}\left(0+\delta V_1(0,0,3)\right) \\
V_1(0,1,2) &= V_2(0,2,1) = \frac{1+\mu}{3}\left(3+\delta V_1(0,3,0)\right) + \frac{2-\mu}{3}\left(0+\delta V_1(0,0,3)\right) \\
V_1(1,1,1) &= V_2(1,1,1) = \begin{cases} \frac{1}{2}\left[2+\delta V_1(1,2,0)\right] + \frac{1}{2}\left[0+\delta V_1(1,0,2)\right] = \frac{9}{(3-\delta)^2} & \text{if } \delta < 0.908 \\[2ex] \frac{1}{2}\left[3+\delta V_1(0,3,0)\right] + \frac{1}{2}\left[0+\delta V_1(0,0,3)\right] = \frac{9(9-2\delta+\delta^2)}{2(3-\delta)^3} & \text{if } \delta \geq 0.908 \end{cases}
\end{aligned}
$$

The continuation values of the veto player are:

$$
\begin{aligned}
V_v(3,0,0) &= \frac{3}{1-\delta} \\
V_v(1,2,0) &= V_v(1,0,2) = \frac{1}{3}\left[1+\delta V_v(1,2,0)\right] + \frac{2}{3}\left[2+\delta V_v(2,1,0)\right] = \frac{15-4\delta+\delta^2}{(1-\delta)(3-\delta)^2} \\
V_v(2,1,0) &= V_v(2,0,1) = \frac{1}{3}\left[2+\delta V_v(2,1,0)\right] + \frac{2}{3}\left[3+\delta V_v(3,0,0)\right] = \frac{8-2\delta}{(1-\delta)(3-\delta)} \\
V_v(0,3,0) &= V_v(0,0,3) = \frac{1}{3}\left[0+\delta V_v(0,3,0)\right] + \frac{2}{3}\left[1+\delta V_v(1,2,0)\right] = \frac{18+6\delta}{(1-\delta)(3-\delta)^2} \\
V_v(1,1,1) &= \begin{cases} 1+\delta V_v(1,2,0) = \frac{3(3+\delta^2)}{(1-\delta)(3-\delta)^2} & \text{if } \delta < 0.908 \\[2ex] 0+\delta V_v(0,3,0) = \frac{18\delta+6\delta^2}{(1-\delta)(3-\delta)^2} & \text{if } \delta \geq 0.908 \end{cases}
\end{aligned}
$$

It is straightforward to show that, given these continuation values, the proposing and voting strategies above are optimal for status quo policies where exactly one non-veto players receives 0. The crucial steps are: A) proving that the veto player prefers state (1,2,0) to state (0,3,0) when both are in the acceptance set of non-veto player 1 (or, equivalently, that the veto player prefers state (1,0,2) to state (0,0,3) when both are in the acceptance set of non-veto player 2); B) proving that non-veto player 1 supports proposal (1,2,0) when the status quo is (1,1,1) if and only if $\delta < 0.908$ (or, equivalently, that non-veto player 2 supports proposal (1,0,2) when the status quo is (1,1,1) if and only if $\delta < 0.908$); C) proving that both non-veto player 1 and the veto player support proposal (1,2,0) when the status quo is (1,1,1) and $\delta < 0.908$ (or equivalently that both non-veto player 2 and the veto player support proposal (1,0,2) when the status quo is (1,1,1) and $\delta < 0.908$); D) proving that both non-veto player 1 and the veto player support proposal (0,3,0) when the status quo is (1,1,1) and $\delta \geq 0.908$ (or, equivalently, that both non-veto player 2 and the veto player support proposal (0,0,3) when the status quo is (1,1,1) and $\delta \geq 0.908$); E) proving that non-veto player 1 supports proposal (0,3,0) when the status quo is (0,2,1) (or, equivalently, that non-veto player 2 supports proposal (0,0,3) when the status quo is (0,1,2)); F) proving that non-veto player 1 supports proposal (0,3,0) when the status quo is (0,1,2) (or, equivalently, that non-veto player 2 supports proposal (0,0,3) when the status quo is (0,2,1)).

### Step A

The veto player prefers state (0,3,0) to state (1,2,0) if:

$$\begin{aligned}
\delta V_v(0,3,0) &> 1+\delta V_v(1,2,0) \\
\delta\left(\frac{18+6\delta}{(1-\delta)(3-\delta)^2}\right) &> 1+\delta\left(\frac{15-4\delta+\delta^2}{(1-\delta)(3-\delta)^2}\right) \\
\frac{3\delta(6+\delta)-9}{(1-\delta)(3-\delta)^2} &> 0
\end{aligned}$$

which holds for any $\delta > 0.464$. In other words, when the status quo is (1,1,1) and $\delta \geq 0.908$ so non-veto player 1 is willing to support both (0,3,0) and (1,2,0), the veto player prefers to bring the status quo to (0,3,0).

**Step B**

Player 1 supports (1,2,0) when the status quo is (1,1,1) if and only if

$$\begin{aligned} 1+\delta V_1(1,1,1) &\leq 2+\delta V_1(1,2,0) \\ 1+\frac{9\delta}{(3-\delta)^2} &\leq 2+\frac{2\delta}{3-\delta} \\ \delta &\leq \frac{9}{9+\delta} \end{aligned}$$

which is satisfied for any $\delta < 0.908$.

**Step C**

Step A shows that player 1 supports (1,2,0) when the status quo is (1,1,1) and $\delta < 0.908$. When the status quo is (1,1,1) and $\delta < 0.908$, the veto player supports (1,2,0) if and only if:

$$\begin{aligned} 1+\delta V_v(1,2,0) &\geq 1+\delta V_1(1,1,1) \\ V_v(1,2,0) &\geq V_1(1,1,1) \\ \frac{15-4\delta+\delta^2}{(1-\delta)(3-\delta)^2} &\geq \frac{3(3+\delta^2)}{(1-\delta)(3-\delta)^2} \\ \frac{15-4\delta+\delta^2}{3(3+\delta^2)} &\geq 1 \end{aligned}$$

which is satisfied for any $\delta < 0.908$.

**Step D**

Non-veto player 1 supports proposal (0,3,0) when the status quo is (1,1,1) if and only if:

$$\begin{aligned} 3+\delta V_1(0,3,0) &\geq 1+\delta V_1(1,1,1) \\ 3+\delta\left(\frac{27-6\delta+3\delta^2}{(3-\delta)^3}\right) &\geq 1+\delta\left(\frac{9(9-2\delta+\delta^2)}{2(3-\delta)^3}\right) \end{aligned}$$

which is true for any $\delta \in [0,1]$.

The veto player supports proposal (0,3,0) over the status quo (1,1,1) if and only if:

$$\begin{aligned} 0 + \delta V_v(0,3,0) &\geq 1 + \delta V_v(1,1,1) \\ 0 + \delta V_v(0,3,0) &\geq 1 + \delta[0 + \delta V_v(0,3,0)] \\ V_v(0,3,0) &\geq \frac{1}{\delta(1-\delta)} \\ \frac{18+6\delta}{(1-\delta)(3-\delta)^2} &\geq \frac{1}{\delta(1-\delta)} \end{aligned}$$

which is true for any $\delta \in (0.642, 1]$.

## Step E

When $\delta \leq 0.587$, $\mu = 1$ and we have:

$$V_1(0,2,1) = \frac{1}{3}\left(3 + \delta V_1(0,3,0)\right) + \frac{2}{3}\left(0 + \delta V_1(0,0,3)\right)$$

Player 1 supports proposal (0,3,0) when the state is (0,2,1) if and only if:

$$\begin{aligned} 3 + \delta V_1(0,3,0) &\geq 2 + \delta V_1(0,2,1) \\ 3 + \delta V_1(0,3,0) &\geq 2 + \delta\left[\frac{1}{3}\left(3 + \delta V_1(0,3,0)\right) + \frac{2}{3}\left(0 + \delta V_1(0,0,3)\right)\right] \\ 1 + \delta V_1(0,3,0) &\geq \delta + \frac{\delta^2}{3} V_1(0,3,0) + \frac{2\delta^2}{3} V_1(0,0,3) \\ 1 - \delta + \left(\frac{3\delta - \delta^2}{3}\right) V_1(0,3,0) &\geq \frac{2\delta^2}{3} V_1(0,0,3) \\ 1 - \delta + \left(\frac{3\delta - \delta^2}{3}\right)\left(\frac{27 - 6\delta + 3\delta^2}{(3-\delta)^3}\right) &\geq \frac{2\delta^2}{3}\left(\frac{12}{(3-\delta)^2}\right) \\ 1 - \delta + \left(\frac{3\delta - \delta^2}{3}\right)\left(\frac{27 - 6\delta + 3\delta^2}{(3-\delta)^3}\right) - \frac{2\delta^2}{3}\left(\frac{12}{(3-\delta)^2}\right) &\geq 0 \end{aligned}$$

which is satisfied for any $\delta \in [0, 1]$.

When $\delta > 0.587$, $\mu = \frac{27-21\delta^2+10\delta^3}{2\delta(7\delta^2-18\delta+27)}$ and we have:

$$\begin{aligned}
V_1(0,2,1) &= \frac{1+\left(\frac{27-21\delta^2+10\delta^3}{2\delta(7\delta^2-18\delta+27)}\right)}{3}\left(3+\delta V_1(0,3,0)\right) + \frac{2-\left(\frac{27-21\delta^2+10\delta^3}{2\delta(7\delta^2-18\delta+27)}\right)}{3}\left(0+\delta V_1(0,0,3)\right) \\
&= \frac{3}{2}\frac{(8\delta^6-27\delta^5+85\delta^4-62\delta^3+18\delta^2+153\delta+81)}{\delta(7\delta^2-18\delta+27)(3-\delta)^2}
\end{aligned}$$

Player 1 supports proposal (0,3,0) when the state is (0,2,1) if and only if:

$$\begin{aligned}
3+\delta V_1(0,3,0) &\geq 2+\delta V_1(0,2,1) \\
3+\delta\left(\frac{27-6\delta+3\delta^2}{(3-\delta)^3}\right) &\geq 2+\delta\left(\frac{3}{2}\frac{(8\delta^6-27\delta^5+85\delta^4-62\delta^3+18\delta^2+153\delta+81)}{\delta(7\delta^2-18\delta+27)(3-\delta)^2}\right) \\
1+\delta\left(\frac{27-6\delta+3\delta^2}{(3-\delta)^3}\right) - \delta\left(\frac{3}{2}\frac{(8\delta^6-27\delta^5+85\delta^4-62\delta^3+18\delta^2+153\delta+81)}{\delta(7\delta^2-18\delta+27)(3-\delta)^2}\right) &\geq 0
\end{aligned}$$

which is satisfied for any $\delta \in [0,1]$.

## Step F

When $\delta \leq 0.587$, $\mu = 1$ and we have:

$$V_1(0,1,2) = \frac{2}{3}\left(3+\delta V_1(0,3,0)\right) + \frac{1}{3}\left(0+\delta V_1(0,0,3)\right)$$

Player 1 supports proposal (0,3,0) when the state is (0,1,2) if and only if:

$$\begin{aligned}
3+\delta V_1(0,3,0) &\geq 1+\delta V_1(0,1,2) \\
3+\delta V_1(0,3,0) &\geq 1+\delta\left[\frac{2}{3}\left(3+\delta V_1(0,3,0)\right)+\frac{1}{3}\left(0+\delta V_1(0,0,3)\right)\right] \\
2+\delta V_1(0,3,0) &\geq 2\delta+\frac{2\delta^2}{3}V_1(0,3,0)+\frac{\delta^2}{3}V_1(0,0,3) \\
2-2\delta+\left(\frac{3\delta-2\delta^2}{3}\right)V_1(0,3,0) &\geq \frac{\delta^2}{3}V_1(0,0,3) \\
2-2\delta+\left(\frac{3\delta-2\delta^2}{3}\right)\left(\frac{27-6\delta+3\delta^2}{(3-\delta)^3}\right) &\geq \frac{\delta^2}{3}\left(\frac{12}{(3-\delta)^2}\right) \\
2-2\delta+\left(\frac{3\delta-2\delta^2}{3}\right)\left(\frac{27-6\delta+3\delta^2}{(3-\delta)^3}\right) - \frac{\delta^2}{3}\left(\frac{12}{(3-\delta)^2}\right) &\geq 0
\end{aligned}$$

which is satisfied for any $\delta \in [0,1]$.

When $\delta > 0.587$, $\mu = \frac{27-21\delta^2+10\delta^3}{2\delta(7\delta^2-18\delta+27)}$ and we have:

$$
\begin{aligned}
V_1(0,1,2) &= \frac{2-\left(\frac{27-21\delta^2+10\delta^3}{2\delta(7\delta^2-18\delta+27)}\right)}{3}(3+\delta V_1(0,3,0)) + \frac{1+\left(\frac{27-21\delta^2+10\delta^3}{2\delta(7\delta^2-18\delta+27)}\right)}{3}(0+\delta V_1(0,0,3)) \\
&= \frac{3}{2}\frac{(6\delta^6-23\delta^5+103\delta^4-118\delta^3+36\delta^2+333\delta-81)}{(\delta(7\delta^2-18\delta+27)(3-\delta)^2}
\end{aligned}
$$

Player 1 supports proposal (0,3,0) when the state is (0,1,2) if and only if:

$$
\begin{aligned}
3+\delta V_1(0,3,0) &\geq 1+\delta V_1(0,1,2) \\
3+\delta\left(\frac{27-6\delta+3\delta^2}{(3-\delta)^3}\right) &\geq 1+\delta\left(\frac{3}{2}\frac{(6\delta^6-23\delta^5+103\delta^4-118\delta^3+36\delta^2+333\delta-81)}{(\delta(7\delta^2-18\delta+27)(3-\delta)^2}\right) \\
2+\delta\left(\frac{27-6\delta+3\delta^2}{(3-\delta)^3}\right)-\delta\left(\frac{3}{2}\frac{(6\delta^6-23\delta^5+103\delta^4-118\delta^3+36\delta^2+333\delta-81)}{(\delta(7\delta^2-18\delta+27)(3-\delta)^2}\right) &\geq 0
\end{aligned}
$$

which is satisfied for any $\delta \in [0,1]$.

# H Additional Experimental Results

| | **Status Quo (t+1)** | | | | | | |
|---|---|---|---|---|---|---|---|
| **Status Quo (t)** | D1 | D2 | DV | C12 | C1V | C2V | U |
| Dictator 1 | 0.018 | 0.000 | 0.096 | 0.003 | – | 0.468 | 0.513 |
| Dictator 2 | 0.979 | 0.425 | 0.042 | 0.000 | 0.844 | 0.658 | 0.698 |
| Dictator V | – | – | 0.725 | 0.282 | 0.265 | 0.265 | 0.643 |
| Coalition 1 + 2 | 0.153 | 0.051 | – | 0.022 | 0.004 | 0.008 | 0.234 |
| Coalition 1 + V | – | 0.089 | 0.524 | – | 0.298 | 0.014 | 0.079 |
| Coalition 2 + V | 0.605 | 0.389 | 0.004 | – | 0.292 | 0.087 | 0.420 |
| Universal | 0.240 | – | 0.353 | – | 0.073 | 0.286 | 0.002 |
| **Frequency** | 0.676 | 0.027 | 0.430 | 0.008 | 0.521 | 0.554 | 0.261 |

Table 5: Policy frequencies and transition probabilities, p-values for treatment effects.

Panel A: High Patience

| | **Status Quo (t+1)** | | | | | | |
|---|---|---|---|---|---|---|---|
| **Status Quo (t)** | D1 | D2 | DV | C12 | C1V | C2V | U |
| Dictator 1 | 0.29 | 0.10 | 0.00 | 0.04 | 0.10 | 0.33 | 0.14 |
| Dictator 2 | 0.16 | 0.37 | 0.00 | 0.16 | 0.21 | 0.00 | 0.11 |
| Dictator V | 0.00 | 0.00 | 0.99 | 0.00 | 0.01 | 0.00 | 0.00 |
| Coalition 1 + 2 | 0.02 | 0.00 | 0.00 | 0.61 | 0.10 | 0.12 | 0.15 |
| Coalition 1 + V | 0.00 | 0.00 | 0.12 | 0.00 | 0.57 | 0.26 | 0.06 |
| Coalition 2 + V | 0.00 | 0.00 | 0.11 | 0.00 | 0.29 | 0.56 | 0.04 |
| Universal | 0.00 | 0.00 | 0.00 | 0.00 | 0.08 | 0.10 | 0.82 |
| **Frequency** | 0.01 | 0.01 | 0.24 | 0.07 | 0.26 | 0.24 | 0.16 |

Panel B: Low Patience

| | **Status Quo (t+1)** | | | | | | |
|---|---|---|---|---|---|---|---|
| **Status Quo (t)** | D1 | D2 | DV | C12 | C1V | C2V | U |
| Dictator 1 | 0.09 | 0.03 | 0.13* | 0.03 | 0.22 | 0.25* | 0.25 |
| Dictator 2 | 0.09 | 0.29 | 0.09* | 0.03** | 0.23 | 0.09 | 0.20 |
| Dictator V | 0.00 | 0.00 | 0.98 | 0.01 | 0.01 | 0.00 | 0.01 |
| Coalition 1 + 2 | 0.03 | 0.09 | 0.01 | 0.24** | 0.21 | 0.29* | 0.13 |
| Coalition 1 + V | 0.00 | 0.00 | 0.13 | 0.01 | 0.54 | 0.27 | 0.05 |
| Coalition 2 + V | 0.00 | 0.00 | 0.17 | 0.00 | 0.31 | 0.46 | 0.06 |
| Universal | 0.00 | 0.00 | 0.02 | 0.00 | 0.21 | 0.06 | 0.71 |
| **Frequency** | 0.01 | 0.02** | 0.24 | 0.03** | 0.31 | 0.26 | 0.12 |

Table 6: Policy frequencies and transition probabilities, stricter definitions of D and U regions. Notes: ** and * indicate difference with *High Patience* is significant, respectively, at 1% and at 5% level (see p-values in Table 7).

| | Status Quo (t+1) | | | | | | |
|---|---|---|---|---|---|---|---|
| **Status Quo (t)** | D1 | D2 | DV | C12 | C1V | C2V | U |
| Dictator 1 | 0.201 | 0.245 | 0.013 | 0.671 | – | 0.034 | – |
| Dictator 2 | 0.358 | 0.223 | 0.038 | 0.003 | 0.753 | 0.176 | 0.764 |
| Dictator V | – | – 0.488 | 0.206 | 0.737 | – | 0.206 | |
| Coalition 1 + 2 | 0.641 | – | – | 0.000 | 0.173 | 0.044 | 0.418 |
| Coalition 1 + V | – | – | 0.933 | 0.858 | 0.709 | 0.635 | 0.699 |
| Coalition 2 + V | – | – | 0.130 | – | 0.867 | 0.205 | 0.293 |
| Universal | – | – | – | – | 0.305 | 0.495 | 0.261 |
| **Frequency** | 0.810 | 0.008 | 0.733 | 0.001 | 0.272 | 0.217 | 0.662 |

Table 7: Policy frequencies and transition probabilities, stricter definitions of D and U regions, p-values for treatment effects.

| | (1) | (2) | (3) |
|---|---|---|---|
| Round | 2.545** | 3.185** | 2.531** |
| | (0.000) | (0.001) | (0.000) |
| Constant | 22.707** | 23.360** | 20.647** |
| | (1.022) | (1.301) | (2.326) |
| Sample | All | $\delta = 0.50$ | $\delta = 0.75$ |
| Pseudo-R2 | 0.0210 | 0.0223 | 0.0254 |
| Observations | 2148 | 1144 | 1004 |

Table 8: Tobit estimates of allocation to the veto player (out of 60 units). The unit of analysis is a committee in a round and include the initial status quo exogenously assigned by the computer (coded as policy outcome in round 0). Standard errors clustered by sessions in parentheses. ** significant at the 1% level.

| **A: VETO PROPOSER** | HIGH PATIENCE | | LOW PATIENCE | |
|---|---|---|---|---|
| | ALL | ACC | ALL | ACC |
| Mean Premium to Proposer | 10.63 | 9.13 | 13.25 | 14.58** |
| Mean Premium to Rich Non-Veto | -8.59 | -7.91 | -13.17 | -14.76** |
| Mean Premium to Poor Non-Veto | -2.04 | -1.22 | -0.09 | 0.18 |
| Mean Premium to Coalition Partner | 3.95 | 4.19 | 1.89 | 3.00* |
| Observations | 239 | 32 | 169 | 33 |

| **B: NON-VETO PROPOSER** | HIGH PATIENCE | | LOW PATIENCE | |
|---|---|---|---|---|
| | ALL | ACC | ALL | ACC |
| Mean Premium to Proposer | 4.03 | 2.74 | 6.97** | 8.16** |
| Mean Premium to Veto | 1.19 | 3.93 | 5.43** | 8.16* |
| Mean Premium to Other Non-Veto | -5.22 | -6.68 | -12.40** | -16.32** |
| Mean Premium to Coalition Partner | 1.87 | 4.22 | 5.72** | 8.11* |
| Observations | 478 | 74 | 338 | 75 |

Table 9: Proposing behavior in regions U and C12. Notes: For each treatment, the first column is for all observed proposals, the second column for proposals that are voted on and accepted; Rich Non-Veto (Poor Non-Veto) is the non-veto player who receives the most (least) in the status quo; Coalition Partner is the non-proposing player who receives the most in the proposal; ** and * indicate difference with *High Patience* is significant, respectively, at 1% and at 5% level.

# I Experimental Instructions (High Patience Treatment)

Thank you for agreeing to participate in this experiment. During the experiment we require your complete, undistracted attention and ask that you follow instructions carefully. Please turn off your cell phones. Do not open other applications on your computer, chat with other students, or engage in other distracting activities, such as reading books, doing homework, etc. You will be paid for your participation in cash, at the end of the experiment. Different participants may earn different amounts. What you earn depends partly on your decisions, partly on the decisions of others, and partly on chance. It is important that you do not talk or in any way try to communicate with other participants during the experiments.

Following the instructions, there will be a practice session and a short comprehension quiz. All questions on the quiz must be answered correctly before continuing to the paid session. At the end you will be paid in private and you are under no obligation to tell others how much you earned. Your earnings are denominated in FRANCS which will be converted to dollars at the rate of 60 FRANCS to 1 DOLLAR.

This an experiment in committee decision making. The experiment will take place over a sequence of 10 matches. We begin the match by dividing you into 4 committees of 3 members each. Each of you is assigned to exactly one of these committees. You will be given a temporary Committee Member Number (either 1, 2 or 3) and you are not told the identity of the other members of your committee. One of the members of your committee is selected at random by the computer to be the Veto Player for this committee. The Committee Member Number of the Veto Player will be displayed on your computer. For example, if you are Committee Member Number 1 and the Veto Player for this committee in this match is Committee Member Number 1, then you are the Veto Player in your committee in this match. In each match, your committee will make budget decisions over a sequence of several rounds.

In each round, your committee has a budget of 60 francs. Your committee must decide how to divide this budget into private allocations A1, A2, and A3. These private allocations A1, A2, and A3 have all to be greater than or equal to 0 and must add up to exactly 60. If your committee budget decision is (A1, A2, A3), then A1 francs go directly to member 1's earnings, A2 francs go to member 2's earnings, and A3 francs go to member 3's earnings.

Here is the procedure for how your committee makes budget decisions. At the beginning of the first round, the computer randomly selects an initial budget decision (A1, A2, A3) and displays it on your computer as what we call the Standing Budget. Next, each of you makes a provisional proposal for an alternative budget decision you would like your committee to consider. (You may propose the Standing Budget itself if you wish.) Your proposal can be any budget decision—that is, any three non-negative numbers (including 0s) that add up to exactly 60. After all three members of your committee have chosen provisional proposals, one of these provisional proposals is selected at random by the computer to be the Proposed Budget. The Proposed Budget will be displayed on your computer, along with the number of the Committee Member who proposed it. The committee then conducts a vote between the Standing Budget and the Proposed Budget. The Proposed Budget passes only if the Veto Player and at least one other committee member vote in its favor. If the Veto Player votes against the Proposed Budget, the Standing Budget wins. If the Veto Player votes in favor of the Proposed Budget but the two other committee members vote against it, the Standing Budget wins. Your earnings in this round are determined by your private allocation in whichever budget decision wins in the voting stage.

One important aspect of your committee's budget decision is that it is inertial. That is, the budget decision that prevails in round 1 becomes the Standing Budget in round 2 and will thus determine the private allocations in round 2 if your committee does not agree on a different budget decision. Every round, the budget decision of your committee determines both your earnings in this round and the Standing Budget for the following round.

The total number of rounds in a match will depend on the rolling of a fair 8-sided die. When the first round ends, we roll it to decide whether to move on to the second round. If the die comes up a 1 or a 2 we do not go on to round 2, and the match is over. We will describe in a moment what happens after a match is over. If the die comes up a 3, 4, 5, 6, 7, or 8, we continue to the next round. In round 2, your Committee Member Number, the members of your committee and the identity of the Veto Player all stay the same. Round 2 proceeds just as round 1, with the exception that the Standing Budget in round 2 is whatever the committee decision was in round 1. Therefore, if the original Standing Budget won the voting stage in round 1, this continues as the Standing Budget in round 2. But if the Proposed Budget in round 1 won the voting stage, then it replaces the original Standing Budget and becomes the new Standing Budget for round 2. The proposal and voting process then follows the same rules as round 1. Once again, each member types in a proposal, the computer then randomly selects one of them to be the Proposed Budget and a vote is taken between the round 2 Standing Budget and the Proposed Budget. After round 2 is over, we roll the 8-sided die again to determine whether to move on to a third round. We continue to more rounds, until a 1 or a 2 is rolled at the end of a round and the match ends. It is important to remember that your Committee Member Number, the members of your committee, and the identity of the Veto Player all stay the same in all rounds of the match. In round T, the Standing Budget is always whatever the committee decision was in round T-1.

After the first match ends, we move to match 2. In this new match, you are reshuffled randomly into 4 new committees of 3 members each. Your assigned a new Committee Member Number (1, 2, or 3). The computer randomly selects a Standing Budget for each committee for round 1, and randomly selects a Veto Player for each committee. The match then proceeds the same way as match 1. This continues for 10 matches. After match 10, the experiment is over. Your total earnings for the experiment are the sum of your earnings over all rounds and all matches.