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JEL Classification: D42, D82, L42

Keywords: Loyalty discounts; As-efficient competitor; Price-cost tests, Sacrifice of profit; Contestable share.

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Price-cost tests and loyalty discounts*

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Abstract

We analyze, by means of a formal economic model, the use of price-cost tests to assess the competitive effects of loyalty discounts. In the model, a dominant firm enjoys a competitive advantage over its rivals and uses loyalty discounts as a means to boost the demand for its product. We show that in this framework price-cost tests are misleading or, at best, completely uninformative. Our results cast doubts on the applicability of price-tests to loyalty discount cases.

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*We are grateful to Philippe Choné and seminar participants at the European Commission (DGComp) for useful comments. We are, however, responsible for any remaining errors. E-mail addresses: giacomo.calzolari@unibo.it, vincenzo.denicolo@unibo.it.

1 Introduction

This paper analyzes the application of price-cost tests to antitrust cases involving *loyalty discounts*. We use the term "loyalty discounts" to refer to contracts that reference rivals' volumes. These include discounts conditional on exclusivity (exclusivity discounts), those conditional on purchasing from the seller at least a certain percentage of one's total requirements (market-share discounts), and other similar practices.[1] These strategies impose on buyers an opportunity cost of purchasing "abroad," and as such are often regarded as potentially anticompetitive.

After the recent decision of the European Court of Justice on the *Intel* case,[2] loyalty discounts are subject to the rule of reason on both sides of the Atlantic. However, there is wide disagreement on what factors should be considered by agencies and the courts. Much of the debate turns on whether some variant of the *price-cost tests*, originally devised for predatory pricing cases,[3] should be applied to loyalty discounts. These tests compare the dominant firm's price to its own cost, on the grounds that only those practices that are capable of foreclosing an *as-efficient competitor* are to be regarded as anticompetitive.

In this respect, the case law is divided both in the US and in Europe. In the US, price-cost tests have been adopted by several Circuit Courts of Appeals,[4] with the notable exception, however, of the Third Circuit.[5] The Supreme Court has not taken a stance on the matter yet. In Europe, the Commission has endorsed the use of price-cost tests in its 2008 *Article 102 Guidance Paper* [6] and has applied a version of the test in the *Intel* decision.[7] But the EU Court of Justice has taken a more cautious approach so far, stating that, in any case, the test cannot be dispositive.[8]

The academic debate is also far from decided. The prevailing view of loyalty discounts is that these practices entail a *sacrifice of profit* and thus can be

[1] The loyalty discounts category is sometimes taken to include also bundled discounts and retroactive (all units) quantity discounts. While such discounts are also of antitrust concern, they differ from loyalty discounts as defined in this paper in that they are conditioned only on own volumes and not on rivals' volumes.

[2] This decision has overturned previous case law, according to which loyalty discounts were regarded as presumptively illegal. See Case C-413/14 P *Intel Corp. v. European Commission* EU:C:2017:632

[3] The tests were advocated for predatory pricing by Areeda and Turner (1975) and adopted by the US Supreme Court in the *Brooke Group* case: see *Brooke Grp. Ltd. v. Brown & Williamson Tobacco Corp.*, 509 U.S. 209 (1993).

[4] See e.g. *Concord Boat Corp. v. Brunswick Corp.*, 207 F.3d 1039, 1062 (8th Cir. 2000)), *Cascade Health Solutions v. PeaceHealth*, 515 F.3d 883 (9th Cir. 2008), and *Ortho Diagnostic Systems v. Abbott Laboratories*, 920 F. Supp. 455 (S.D.N.Y. 1996).

[5] See *Le Page's Inc. v. 3M*, 324 F 3d 141 (3rd Cir 2003) (en banc), *ZF Meritor LLC and Meritor Transmission Corporation v. Eaton Corporation*, 696 F.3d 254 (3d Cir.2012), *Eisai, Inc. v. Sanofi-Aventis U.S.*, LLC, 821 F.3d 394, 403 (3d Cir. 2016).

[6] See the "Guidance on the Commission Enforcement Priorities in Applying Article 82 of the EC Treaty to Abusive Exclusionanry Conduct by Dominant Undertakings" at 2009 O.J. (C45) 7.

[7] Case T-457/08, Intel v Commission, [2009] E.C.R. II-12*.

[8] Besides the recent decision on the *Intel* case referred to in footnote 3, see also Case C-23/14, *Post Danmark A/S v. Konkurrencerådet*, ECLI:EU:C:2015:343 (May 21, 2015).

profitable only indirectly, by weakening the dominant firm's rivals and impairing their ability to compete elsewhere – e.g. in the future, or in adjacent markets.[9] According to this view, it is only in the recoupment stage that the possible anticompetitive effects materialize.

But even scholars who share the profit-sacrifice view arrive at different policy conclusions. Some argue that the sacrifice-recoupment logic is clearly reminiscent of the mechanism of predatory pricing and thus calls for the adoption of the same sort of policies. According to these authors, the use of the same legal standard in predation and loyalty discounts cases is convenient and reduces legal uncertainty.[10]

Others counter that in loyalty discounts cases the sacrifice and recoupment phases may be intertwined, that they may not be easy to discern, and that, as a result, application of the test is likely to lead to many mistakes. These latter authors also argue that loyalty discounts have a greater anticompetitive potential than predatory pricing, implying that false negatives should be given a greater weight, and false positives a lower one, than in predation cases. From all this, they conclude that loyalty discounts should be governed by more stringent legal standards.[11]

The debate has seldom called into question the notion that loyalty discounts must entail, at least in principle, profit sacrifice and recoupment.[12] The point of disagreement is whether or not the sacrifice of profit should be proved by means of some kind of price-cost test, given the difficulties of administering the test in practice.

This paper presents a more radical critique of price-cost tests in loyalty discounts cases. We contend that price-cost tests are not just impractical, they are conceptually flawed. These tests would produce systematic errors (both

[9] The view that profit sacrifice is crucial for most theories of loyalty discounts is articulated in Bernheim and Heeb (2015) and, indeed, reflects the conclusions of a number of economic models. The indirect gain may take a variety of forms, such as entry deterrence (e.g. Rasmusen et al., 1991), the exploitation of a future entrant (e.g. Aghion and Bolton, 1987), the protection of non-contractible investments (e.g. Marvel, 1982), and so on. See Whinston (2008) and Fumagalli, Motta and Calcagno (2018) for excellent surveys of the literature.

[10] See for instance Hovenkamp (2006), Crane (2013), Amici Curiae (2013) and Klein and Lerner (2016).

[11] See for instance Bernheim and Heeb (2015) Moore and Wright (2015), and Salop (2017). A more nuanced view maintains that price-cost tests provide useful information which however should be supplemented by a more complete analysis of the relevant factors: see, for instance, Fumagalli and Motta (2017).

[12] Some authors, including for instance Moore and Wright (2015) and Salop (2017), have argued that loyalty discounts should be conceptualized as a "raising rivals' costs" (RRC) strategy. One interpretation of this view is that loyalty discounts serve to deny rivals economies of scale, impeding entry or driving existing competitors out of the market. In this interpretation, the RRC view is just another incarnation of the sacrifice-recoupment approach. But an alternative interpretation of the RRC view is that loyalty discounts make it more costly for rivals to get to buyers and hence increase the demand for the dominant firm's products. This latter interpretation is fully consistent with the demand-boost approach followed in this paper. Literally speaking, however, loyalty discounts do not really raise rivals' costs of distribution but simply imply that rivals will have to reduce their prices to compete effectively. In fact, in our model loyalty discounts decrease rivals' prices, whereas true RRC strategies would increase such prices.

false positives and false negatives) even if they were applied in ideal conditions, in which all relevant variables are exactly measurable.

The reason for this is that loyalty discounts are, more often than not, directly profitable and do not entail any sacrifice of profit. In fact, competitors' ability to compete in the future, or in adjacent markets, is often impregnable to the dominant firm's attacks. What loyalty discounts always do, however, is to boost the demand for the dominant firm's products. When marginal prices exceed marginal costs, the upwards jump in demand may increase the dominant firm's profit, even accounting for the discount. This demand-boost mechanism is both more general and straightforward than the sacrifice-recoupment one, and as such it has received considerable attention in the economics literature.[13]

This paper develops a formal economic model that captures the demand-boost mechanism and uses the model to assess different variants of the price-cost test. Plainly, if loyalty discounts are directly profitable the dominant firm must be pricing above cost, so the price-cost test in its simplest form is always passed. However, the test which is often applied in loyalty discount cases (which is sometimes referred to as the "incremental test," or the "discount-attribution test") does not simply compare the dominant firm's average price to its incremental cost. The test in fact relies on a weaker notion of as-efficiency, which allows for the possibility that a competitor may be as efficient in supplying one extra unit of the product but less efficient as the buyer's sole supplier. In this spirit, the discount is not attributed to the dominant firm's entire output but only to the share that is regarded as effectively *contestable*. Even discounts that are directly profitable need not always pass this variant of the test, or others similar to it. The question then is whether or not the test is passed when the discounts are procompetitive and failed when they are anticompetitive.

To abstract from practical issues, we address that question assuming that all the variables used in the test – i.e. costs, the contestable share of the market, and the magnitude of the discounts – can be measured exactly. Even so, we show that the test does not screen out or tend to screen out anticompetitive cases. *The test is actually misleading or, at best, completely uninformative even in the absence of measurement errors.*

The intuitive reason for this is that price-cost tests are designed to detect low prices, whereas demand-boost theories imply that loyalty discounts are anticompetitive only to the extent that they result in high prices. The straightforward conclusion we draw from our analysis is that antitrust authorities and the courts should not apply price-cost tests in loyalty discounts cases. An alternative, sounder approach is set forth in our companion paper (Calzolari and Denicolò, 2018).

[13] After the so-called Chicago critique, the demand-boost mechanism was dismissed on the grounds that it does not work when firms price efficiently, setting marginal prices at cost and extracting the profit by means of non-distortionary, lump-sum payments. But it is now widely recognized that efficient pricing is the exception rather than the rule, and that firms often optimally distort their prices upwards. See, for instance, Mathewson and Winter (1987), Yong (1996), Bernheim and Whinston (1998), Majumdar and Shaffer (2008) and Calzolari and Denicolò (2013, 2015). Calzolari et al. (2018) show that all these seemingly different models in fact share the demand-boost mechanism.

The rest of the paper is organized as follows. Section 2 sets out a model of two firms, a dominant firm and its rival, which supply differentiated products. Initially we focus on exclusivity discounts under linear pricing. The model's equilibrium under these assumptions is characterized in section 3. This section analyzes also the welfare effects of exclusivity discounts, showing that the discounts are anticompetitive when the dominant firm's competitive advantage is large and procompetitive when it is small. Section 4 analyzes the application of price-cost tests in this setting, showing that these tests are at best uninformative and at worst totally misleading. Section 5 extends the analysis to the case in which firms can offer two-part tariffs and market-share discounts. Section 6 concludes the paper by discussing the policy implications of our results.

2 The model

In this section, we present a simple model that captures the demand-boost effect of loyalty discounts mentioned above: by creating an opportunity cost of buying abroad, loyalty discounts decrease rivals' volumes and hence increase the demand for the dominant firm's products. The increase in demand may, but need not to, come at the cost of a price reduction. Even when it does, the resulting price-volume trade-off may be favorable for the dominant firm.

To highlight the demand-boost effect, we rule out other, more roundabout mechanisms that may be at work in more complex models. We do this by considering a one-stage model of price competition where two profit-maximizing firms, a dominant firm and its rival, supply substitute products. The firms interact only in this one market. Marginal costs are constant, and we abstract from fixed costs. In this setting, there is no room for profit sacrifice and recoupment, or for strategies that raise rivals' costs. Loyalty discounts are used simply to increase the demand for the dominant firm's product.

The increase in volumes could not however be profitable if firms priced efficiently, setting marginal prices at cost and extracting buyers' surplus efficiently by means of fixed fees only.[14] A key ingredient for our explanation is that firms distort marginal prices above marginal costs.

This property is, in our opinion, quite realistic. Price distortions naturally arise in various settings, such as for instance in the presence of moral hazard (Bernheim and Whinston 1998, section V) or adverse selection (Calzolari and Denicolò 2013, 2015). To simplify the analysis, however, we start from a simple model with no asymmetric information where firms are restricted to linear pricing, as in Mathewson and Winter (1987). The assumption of linear pricing is admittedly far-fetched as lump-sum payments must be feasible when trade is non-anonymous – a necessary condition for contracts that reference rivals' volumes to be enforceable. However, in section 5 we extend the analysis to the case

[14] In fact, in this case firms engaging in loyalty discounts would have to reduce the fixed fee so as to compensate the buyer for the reduced variety. As a result, loyalty discounts would be unprofitable. See O'Brien and Shaffer (1997) and Bernheim and Whinston (1998, section II and III) for a rigorous proof.

in which firms use two-part tariffs. Using a reduced-form model that captures in a simplified way moral hazard, adverse selection, and other possible sources of price distortions, we show that our results are robust to this extension.

2.1 Demand and cost

We denote the goods by $i = 1, 2$; the same indexes are used also for the firms. The constant marginal costs are denoted by c_i. There are no fixed costs.

As for demand, we assume that buyers (which we shall sometimes refer to as retailers but could be, more generally, downstream firms),[15] do not interact strategically with each other. Thus, we can focus on the firms' relationships with a single retailer. The retailer's demand is derived from the payoff function $v(q_1, q_2)$, which represents the gross profit of a retailer who buys q_1 units of good 1 and q_2 units of good 2. The retailer's net profit is $v(q_1, q_2)$ minus the payments he makes to the upstream firms. The function $v(q_1, q_2)$ is assumed to be symmetric, which implies symmetry of demand.

The goods are differentiated. To obtain closed-form solutions, we assume that the payoff function is quadratic:[16]

$$v(q_1, q_2) = q_1 + q_2 - \frac{1}{2}(q_1^2 + q_2^2) - \gamma q_1 q_2, \tag{1}$$

which implies that demand functions are linear.[17] The parameter γ captures the degree of substitutability between the products: it ranges from 1 (perfect substitutes) to 0 (independent goods).

2.2 Competitive advantage

We now introduce asymmetries between the firms by considering various possible sources of competitive advantage for the dominant firm (firm 1): lower cost, better quality, and a limited ability on the part of the rival to satisfy the buyer's demand. This latter factor allows us to precisely identify the contestable share

[15] In principle, buyers could also be final consumers. In practice, however, the enforcement of contracts that reference rivals' volumes is easier when buyers are sufficiently "large" that it is possible to observe their purchases from rivals.

[16] Our results extend to more general demand structures, at the cost of a more complex analysis. For example, Calzolari et al. (2018) use a more general specification where the function $v(q_1, q_2)$ is simply assumed to be increasing, concave, and smooth.

[17] When buyers are final consumers, (1) implies that under linear pricing the demand functions would be:

$$p_i = 1 - q_i - \gamma q_j.$$

If, on the contrary, buyers are downstream firms, assuming that such firms are local monopolies and are restricted to linear pricing, and normalising their costs to zero, (1) is the downstream profit function associated with the following demand functions for the final products:

$$p_i = 1 - \frac{1}{2}q_i - \frac{\gamma}{2}q_j.$$

of the market, which is a key ingredient of the price-cost tests as typically employed in loyalty discounts cases.

Firstly, the dominant firm may enjoy a cost advantage: $c_1 \leq c_2$. Secondly, product 1 may be of better quality, and hence in greater demand, than product 2. This can be captured by assuming that the retailer's payoff is in fact $v(q_1, q_2) - \delta q_2$, where the parameter $\delta \geq 0$ can be interpreted as an index of vertical product differentiation.

The first two sources of competitive advantage can be merged into a single index of superior efficiency, the quality-adjusted cost gap $\Delta = \delta + (c_2 - c_1)$. Given the analytical equivalence of cost and quality advantages, to save on notation we shall henceforth set $\delta = 0$, interpreting $\Delta \geq 0$ as a pure cost advantage.

Thirdly, firm 2 may not be able to fully replace the dominant firm as a supplier. We capture this possibility simply by positing that

$$q_2 \leq K. \tag{2}$$

One possible interpretation of this constraint is that the buyer's willingness to pay for product 2 is $v_{q_2}(q_1, q_2)$ for $q_2 \leq K$ but vanishes when $q_2 > K$. Another interpretation is that firm 2 may be unable to supply more than K units of the good to the particular retailer under consideration, as in Yong (1996). In any case, we assume that

$$K \leq 1 - c_2 \tag{3}$$

so that the constraint potentially bites.[18]

Even if firm 2 is less efficient than the dominant firm, we assume that exclusion is inefficient as firm 2 produces a differentiated good for which there is positive demand. To guarantee this, costs must be sufficiently small that the efficient output levels are strictly positive.[19] This condition, also known as "positive primary outputs," (Amir and Jin, 2001) places an upper bound on c_1 and Δ: to be precise, $c_1 < 1$ and

$$\Delta < \Delta_{PPO} = (1 - c_1)(1 - \gamma). \tag{4}$$

2.3 Pricing

Initially, we assume that firms compete in linear prices; the extension to two-part tariffs is considered later.

When loyalty discounts are prohibited, each firm just names a price p_i. When loyalty discounts are permitted, in contrast, the dominant firm may condition its price on the rival's volumes. Specifically, we assume that the dominant firm may offer contracts of the type:

$$p_1 = \begin{cases} p_1^H & \text{if } \dfrac{q_1}{q_1 + q_2} < s \\ \\ p_1^L & \text{if } \dfrac{q_1}{q_1 + q_2} \geq s. \end{cases} \tag{5}$$

[18] Expression $1 - c_2$ is the largest possible demand for product 2. It is attained when product 1 is not purchased and product 2 is priced at cost.

[19] The efficient output levels.are $\arg\max_{q_1, q_2} [v(q_1, q_2) - c_1 q_1 - c_2 q_2]$.

That is, the dominant firm names both a reference price p_1^H, which applies if the retailer buys from the dominant firm less that a prescribed share s of his total purchases, and a reduced price p_1^L, which applies if the retailer buys at least the prescribed share. The difference $p_1^H - p_1^L$ is the *loyalty discount* offered by the dominant firm. It is a *market-share discount* if $s < 1$, an *exclusivity discount* if $s = 1$.[20] Initially, we focus on exclusivity discounts but later (section 5) we extend the analysis to the case of market-share discounts.

When $s = 1$, the case initially we focus on, we assume that firm 2 names just one price, p_2. This price applies when the target market share is not reached (the only case in which $q_2 > 0$). When instead $s < 1$, firm 2, too, may set different prices, denoted p_2^H (if $\frac{q_1}{q_1+q_2} \geq s$) and p_2^L (if $\frac{q_1}{q_1+q_2} < s$).[21]

3 Competitive effects

In this section, we analyze the effects of loyalty discounts on prices, output, profits and social welfare. We focus on exclusivity discounts, assuming that $s = 1$. Our main results is that loyalty discounts are anticompetitive if the dominant firm enjoys a large competitive advantage over its rivals, procompetitive if the competitive advantage is small. In the next section, we shall ask whether price-cost tests may help screen out the anticompetitive cases.

3.1 Benchmark

When exclusivity discounts are prohibited, we have a standard Bertrand equilibrium. The only twist is that one firm is "capacity constrained." If K is large, however, the constraint is not binding. The equilibrium then is entirely standard and is in pure strategies. Each firm exploits the market power it enjoys thanks to production differentiation and sets its price above its marginal cost. Due to its greater efficiency, however, the dominant firm sets a lower price and produces a greater output than the rival.

For lower values of K, the equilibrium is in mixed strategies. To be precise, firm 2 prices deterministically but firm 1 randomizes between a high and a low price. When the price is low (lower than the rival's), constraint (2) does not bind, but when the price is high the constraint does bind. The dominant firm then takes advantage of the rival's limited capacity to supply the buyer to sell relatively large volumes even if its price is high.

The explicit formulas for equilibrium prices and outputs are not particularly informative and thus are relegated to the Appendix.

[20] The discounts are assumed to be all-units, as they often are in reality. The analysis would not change significantly if the discounts were framed as incremental, as in Calzolari and Denicolò (2015).

[21] The presumption here is that firm 2 offers a discounts when the target market share set by the dominant firm is not met. Firm 2 might also set its own target market share, but allowing for this possibility would not change equilibrium outcomes.

3.2 Exclusivity discounts

Now suppose that firms can use exclusivity discounts. First of all, we show that the dominant firm has always an incentive to offer such discounts.

Lemma 1 *If exclusivity discounts are permitted, the dominant firm will always offer such discounts in equilibrium.*

Proof. To prove the claim, it suffices to show that the dominant firm has a profitable deviation from the equilibrium (reported in the Appendix) which arises when exclusivity discounts are prohibited. The simplest such deviation is to impose an exclusivity clause without changing the price. Formally, the dominant firm sets p_1^L exactly as in the benchmark equilibrium, and p_1^H arbitrarily large. (If the benchmark is a mixed strategy equilibrium, where the dominant firm is indifferent between a low and a high price, the deviation involves setting p_1^L at the low level.)

Faced with the choice of buying from either firm, but not both, the retailer will choose to buy from the dominant firm, which offers the better deal. Since the products are substitutes, exclusive dealing increases the demand for the dominant firm's product, and hence its profit. The deviation from the original equilibrium is therefore profitable. ■

In fact, the deviation considered in the proof of Lemma 1 need not be the most profitable one. The dominant firm may further raise its profits by adjusting its price p_1^L. In particular, it is possible to show that the optimal deviation involves raising the price.[22]

At any rate, Lemma 1 implies that exclusivity discounts will always be offered and accepted in equilibrium. We next characterize the equilibrium with exclusivity discounts. In equilibrium, the weaker firm makes no sales but stands ready to supply its product at cost. The dominant firm charges a reference price p_1^H high enough that the buyers prefers to buy one product only and wins the competition for exclusives by undercutting its rival *in profit space.*[23]

Lemma 2 *If exclusivity discounts are permitted, there is a unique Nash equilibrium. In this equilibrium,*

$$p_1^L = \min\left[1 - \sqrt{2(1-c_2)K - K^2}, \frac{1+c_1}{2}\right] \tag{6}$$

[22]The direction of the price adjustment depends on whether the exclusive demand curve is more or less elastic than the non-exclusive one. With the payoff function (1), exclusive demand is $p_1^E = 1 - q_1$, whereas non-exclusive demand is $p_1^{NE} = 1 - q_1 - \gamma q_2$. Clearly, exclusive demand is more rigid, which implies that after having set $s = 1$, the dominant firm has an incentive to increase the price. The price raise cannot be too large, however, for otherwise the retailer might prefer to switch to firm 2 as his sole supplier. In other words, the optimal deviation must guarantee the retailer's "participation."

[23]To avoid issues of equilibrium existence, we assume that ties are broken in favour of the dominant firm.

and

$$p_2 = c_2.^{24} \tag{7}$$

Proof. Lemma 1 implies that there cannot exist any equilibrium where exclusivity discounts are not offered and accepted. The reason for this is that any such equilibrium must coincide with the benchmark equilibrium described in the Appendix, but we have already noted that the dominant firm has a profitable deviation.

Next, observe that under exclusive representation firms compete in profit space, where their products are effectively homogeneous. In other words, firms may be thought of as offering to the retailer not specific products, but levels of profit. The standard Bertrand logic then implies that the dominant firm wins the competition for exclusives by undercutting its rival in profit space. The weaker firm, which is foreclosed, must stand ready to supply its product at competitive terms.

To undercut the rival in profit space, the dominant firm must offer a greater net payoff to the retailer. With $p_2 = c_2$, the net payoff that the retailer can obtain by dealing with firm 2 only is

$$v^R = v(0, K) - c_2 K,$$

as by assumption constraint (2) binds if the retailer does not purchase from 1. To induce the retailer to deal exclusively with the dominant firm, the discounted price p_1^L must guarantee a net payoff of at least v^R. Therefore, the dominant firm charges either the monopoly price, $\frac{1+c_1}{2}$, or the price that just guarantees participation (i.e., $1 - \sqrt{2(1-c_2)K - K^2}$), whichever is lower. ■

The dominant firm charges either the monopoly price, $\frac{1+c_1}{2}$, or the price that makes the buyer just indifferent between buying from the dominant firm or from the rival. The monopoly price applies when the dominant firm's competitive advantage is sufficiently large, i.e. when

$$\Delta \geq 1 - \frac{K}{2} - \frac{(1-c_1)^2}{8K} - c_1. \tag{8}$$

The equilibrium output is

$$q_1^E = \max\left[\sqrt{2(1-c_2)K - K^2}, \frac{1-c_1}{2}\right]. \tag{9}$$

It is important to note that while the weaker firm always lowers its price compared to the benchmark equilibrium without loyalty discounts, the dominant firm may increase or decrease its own price, depending on the size of its competitive advantage. If the competitive advantage is large, i.e. Δ is large

[24] The reference price p_1^H is not fully determined. As noted above, it must be large enough that the retailer prefers exclusive to common representation. This implies a lower bound on p_1^H, but apart from this p_1^H may be arbitrarily large. The lower bound will be derived precisely in the next section.

and/or K is small, the net payoff offered by the rival, $v^R = v(0, K) - c_2K$, is small and thus can be matched easily. If this is so, then the dominant firm may raise its price, possibly up to the monopoly level. If the competitive advantage is small, in contrast, v^R is large and thus the dominant firm will have to decrease its price. The price reduction required is greater, the lower is the competitive advantage.

3.3 Profit and welfare

Having characterized the equilibrium that arises when exclusivity discounts are permitted, consider now the effects of such discounts on the different players. Quite obviously, exclusivity discounts harm the weaker firm, leading invariably to its foreclosure. Less obviously, exclusivity discounts do not always benefit the dominant firm, even if it engages in this practice voluntarily. Relatedly, exclusivity discounts may either benefit or harm the retailer and hence, ultimately, final consumers.

Proposition 1 *If the dominant firm's competitive advantage is small (that is, Δ is small and K is large), exclusivity discounts are procompetitive, decreasing profits and increasing consumer surplus. If instead the competitive advantage is large, exclusivity discounts are anticompetitive, increasing the dominant firm's profit and decreasing consumer surplus.*

Proposition 1 can be proved algebraically but the proof is tedious and cumbersome. Details can be found in a *Mathematica* file available on line.[25] Here, we just provide some intuitive insights.

The reason for the possible procompetitive effects is the aggressive defense of the rival, which in equilibrium always cuts its price down to its cost. As we have seen above, such reaction may force the dominant firm to reduce its price as well. If the price reduction is very large, it may more than offset, on the one hand, the loss of product variety (thereby benefiting the retailer and final consumers), and on the other hand the increase in demand (thereby harming the dominant firm). In this case, firms are trapped in a sort of prisoners' dilemma: the possibility of using exclusivity discounts induces them to compete fiercely, even if the final outcome is bad for both.

These procompetitive outcomes are obtained when the dominant firm's competitive advantage is small. When instead the competitive advantage is large, the price reduction required is small: in fact, the dominant firm may even increase its price. In this case, the dominant firm's profit increases, whereas the retailer and final consumers are harmed.

The exact frontier between cases in which exclusivity discounts are pro or anticompetitive is reported in the *Mathematica* file mentioned above. Formulas are cumbersome, but the frontiers are depicted in Figure 1. The picture represents both the *profit frontier* (exclusivity discounts increase the dominant firm's profit above the frontier, decrease it below), and the *welfare frontier* (exclusivity

[25] https://sites.google.com/view/giacomo-calzolari/research/exclusivity

discounts increase welfare below the frontier, decrease it above). The exact welfare frontier depends, of course, on the specific welfare criterion adopted. Figure 1 uses a consumer-surplus criterion and proxies the surplus of final consumers by the retailer's net payoff[26]

$$\pi_R = v(q_1, q_2) - p_1 q_1 - p_2 q_2. \tag{10}$$

In any case, the competitive effects of exclusivity discounts depend on the overall level of the dominant firm's competitive advantage. The competitive advantage increases with Δ and decreases with K, reflecting the fact that the two sources of competitive advantage are substitutes. Both frontiers are therefore increasing.

4 Price-cost tests

In the preceding section, we have seen that exclusivity discounts may be either pro or anticompetitive, depending on the size of the dominant firm's competitive advantage. We now ask whether price-cost tests may help antitrust authorities screen out anticompetitive cases.

Price-cost tests follow the "as-efficient competitor" logic. The question asked is not whether the dominant firm can foreclose its actual competitor, but whether it can foreclose an hypothetical, equally efficient one.

More precisely, the test asks the following question: Suppose that the hypothetical as-efficient competitor prices at cost; given the dominant firm's price schedule, can the retailer reduce his total expenditure by diverting his purchases away from the dominant firm and towards the rival? If the answer is yes, then the test is passed, competition is regarded as viable, and the discounts are presumed to be procompetitive. Should foreclosure nevertheless occur, it must be because the excluded firm is in fact less efficient than the dominant one. If the answer is no, however, then even an equally efficient competitor would be foreclosed by the dominant firm's pricing strategy. The test is then failed, and the discounts are presumed to be anticompetitive.

4.1 Global as-efficiency

The simplest version of the test assumes that the competitor must be as efficient as the dominant firm in all relevant respects. This implies not only that the hypothetical competitor has the same marginal cost as the dominant firm but also that it can fully replace the dominant firm as the retailer's sole supplier. Denoting variables pertaining to the hypothetical competitor by a tilde, the

[26] Qualitatively, the frontier would have the same shape if the welfare criterion were the social surplus

$$S = v(q_1, q_2) - c_1 q_1 - c_2 q_2.$$

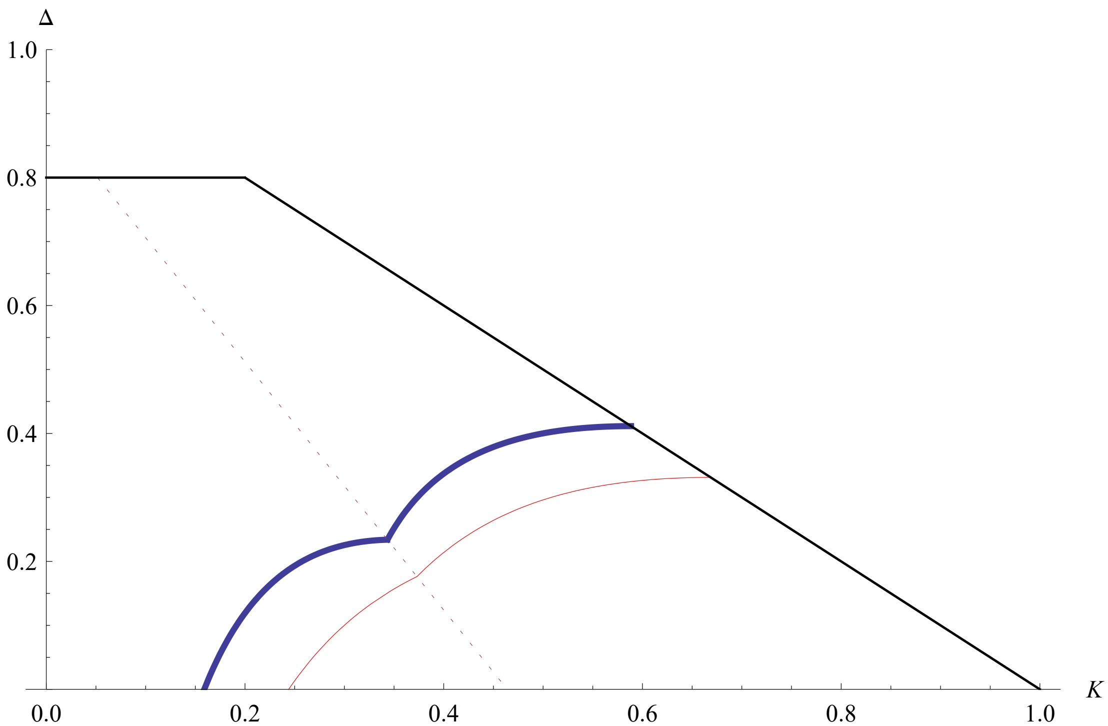

Figure 1: Exclusivity discounts are anticompetitive above the *welfare frontier* (blue thick line), procompetitive below it. Likewise, the discounts are profitable above the *profit frontier* (red thin line), unprofitable below it. The cuspids occur when there is a switch from pure to mixed strategies in the benchmark where exclusivity discounts are prohibited. The figure is drawn for $c_1 = 0$.

assumption is $\tilde{c}_2 = c_1$ and $\tilde{K} \geq 1 - \tilde{c}_2$. In this case, the price-cost is passed if and only if

$$p_1^L \geq c_1. \tag{11}$$

It is immediate to verify that the inequality always holds in equilibrium. In fact, it is quite obvious that the dominant firm always prices above cost, as we have ruled out by assumption any possibility of recoupment.

The test then gives the correct result only when the competitive advantage is small. When the competitive advantage is large, the test delivers false negatives.

4.2 Local as efficiency

The notion of global as efficiency may, however, be called into question. If the hypothetical competitor was really as efficient *in all relevant respects* as the dominant firm, it could never be foreclosed with certainty.[27] Explicitly or implicitly, antitrust concerns arise when the "as efficient" competitor is actually, in some sense, weaker than the dominant firm.

In particular, one can distinguish between the ability to compete for marginal units (*local* as efficiency) and the ability to compete for the entire market (*global* as efficiency). In loyalty discounts cases, the antitrust concern is precisely that superior *global* efficiency could allow a dominant firm to foreclose a competitor that should not be excluded, as it is *locally* equally efficient as the dominant firm (if not more).

Following this logic, it is often argued that in loyalty discounts cases the price-cost test should account for the possibility that the competitor may not be able to replace the dominant firm as the buyer's sole supplier and that, as a result, output diversion cannot be complete. In our model, for instance, output diversion has an upper bound of K. Accounting for this factor, the price-cost test becomes

$$Kc_1 + (q_1^E - K)p_1^H \leq q_1^E p_1^L. \tag{12}$$

The right-hand side of this inequality is the actual expenditure, given that in equilibrium the retailer purchases only from the dominant firm at the discounted price p_1^L. The left-hand side is what the retailer would spend in the counterfactual by purchasing as much as he can from an hypothetical locally as-efficient competitor that prices at cost, and the rest from the dominant firm at the non-exclusive price p_1^H.[28] This variant of the test is sometimes referred to as the "incremental test" or the "discount-attribution test," and it is the variant applied, for instance, by the European Commission in the *Intel* case.

4.2.1 Mechanics of the test

Before proceeding, it may be useful to restate the test in two equivalent ways that are often used in practice. Firstly, inequality (12) may be rewritten as

$$p_1^H - \frac{p_1^H - p_1^L}{\frac{K}{q_1^E}} \geq c_1. \tag{13}$$

The interpretation of this expression is as follows. Any discounted price may be viewed as the difference between a reference price and the discount. For the purposes of the test, the discount $(p_1^H - p_1^L)$ is however attributed only to the contestable share of the market,

$$s_C = \frac{K}{q_1^E}. \tag{14}$$

[27] The reason for this is that such a competitor could exactly replicate any strategy of the dominant firm. The resulting equilibrium may still exhibit foreclosure. However, if the equilibrium is in mixed strategies, both firms must have the same probability of being foreclosed. If instead the equilibrium is asymmetric, and a firm is foreclosed with probability one, either firm could be foreclosed.

[28] The retailer purchases as much as he can from the dominant firm's competitor since $p_1^H > c_1$.

The discounted price is therefore obtained by subtracting from p_1^H the effective discount $\frac{p_1^H - p_1^L}{s_C}$. The test then simply compares such fictitious price to the cost. If the fictitious price is greater than the cost, then a hypothetical as-efficient rival can survive. If it is lower, then an as-efficient competitor would be foreclosed.

Secondly, another equivalent way of stating the test is as a comparison between what in the jargon of the test is called the "required" share, s_R, and the contestable share, s_C. The required share is defined as

$$s_R = \frac{p_1^H - p_1^L}{p_1^H - c_1}. \tag{15}$$

The test then is passed if and only if the contestable share is at least as large as the required share, i.e. if and only if

$$s_C \geq s_R. \tag{16}$$

4.2.2 Measurement issues

In this version of the test, five variables are involved: the dominant firm's volume, the contestable share, the dominant firm's marginal cost, the actual (discounted) price, and the hypothetical price that the dominant firm would apply if the buyer did not qualify for the discount. Actual volume and price are typically easy to observe. The dominant firm's cost must be estimated, but this poses the same problems as in standard predatory pricing tests. Over the years, antitrust authorities and the courts have come to cope with these problems.

The estimation of the contestable share, in contrast, is an issue that arises specifically in loyalty discounts cases. Clearly, the lower is the contestable share, the harder it is for the rival to replace the dominant firm, and hence the more likely the test is failed. This variable is therefore crucial for the outcome of the test but is also very difficult to observe. These difficulties have been highlighted by many critics, so we need not repeat their arguments here.[29]

An issue which has attracted less attention but may also be important is the estimation of the price that would apply if the buyer did not qualify for the discount. In practice, this is often taken to be the list price. But this may overestimate the size of the discount. In fact, list prices are often inflated, and even buyers who do not reach the target market share may get a discount over the list price.[30]

In practice, measurement problems may raise formidable difficulties. But in this paper we abstract from these practical issues and assume that agencies and the courts can measure all relevant variables perfectly. In our model, the dominant firm's marginal cost, c_1, and the contestable output, K, are just

[29] See for instance Steuer (2017) and the literature cited therein.

[30] For example, Intel's informal contract with Dell specified the amount of the discount Dell was entitled to in case of exclusivity, but there was considerable uncertainty about what price would apply in case Dell had breached the exclusivity clause. Dell was confident that it would still obtain a discount over the list price (as indeed it turned out to be the case), but the exact amount of the discount was a matter of considerable speculation.

parameters. The actual quantity, q_1^E, and the discounted price, p_1^L, are fully pinned down in equilibrium (see equations (9) and (6), respectively). What is not pinned down uniquely is the non-exclusive price p_1^H. However, we can determine a lower bound for p_1^H as this price must be sufficiently high that the retailer prefers to take the discount. The condition writes as

$$\max_{q_1, q_2 \leq K} [v(q_1, q_2) - p_1^H q_1 - c_2 q_2] \leq v(q_1^E, 0) - p_1^L q_1^E, \tag{17}$$

and it uniquely defines a lower bound $\underline{p_1^H}$.[31] For the purposes of the test, we set p_1^H at its lower bound. The justification for this is simple: given that p_1^H does not affect equilibrium outcomes, if the dominant firm is concerned about the possibility of antitrust intervention it will choose the price that minimizes the risk that the test is failed and the discount is found anticompetitive.[32]

4.2.3 Results

With all the necessary ingredients at hand, we can now calculate the frontier between cases where the test is passed or failed. Once again the explicit formula is cumbersome and is reported only in the *Mathematica* file, but the frontier is depicted in Figure 2 – the decreasing curve.

Ideally, the frontier for the test should coincide with the welfare frontier – the increasing curve depicted in Figure 1, which is reproduced in Figure 2 to facilitate the comparison. In practice, small differences between the two frontiers could be tolerated. But in fact the two frontiers are nearly orthogonal to each other. The areas of disagreement are depicted in grey in Figure 2. In these areas, the test delivers either type I or type II errors. Evidently, errors are so likely that the price-cost test is totally uninformative.

The reason for this may be explained intuitively as follows. As we have seen above, whether exclusivity discounts are pro or anticompetitive depends on the overall level of the dominant firm's competitive advantage. Whether the test is passed or failed, in contrast, depends on the source of the competitive advantage, rather than on its total level.

To see this, consider the extreme cases in which the competitive advantage is due only to a lower quality-adjusted cost (i.e., only to the Δ), or only to a limited

[31] The lower bound for p_1^H may take on two values. When the dominant firm cannot charge the monopoly price, p_1^L is such that the right-hand side of (17) is equal to $v^R = v(0, K) - c_2 K$. In this case, the lower bound is

$$\underline{p_1^H} = 1 - \gamma K.$$

Intuitively, the price must be so high that the retailer prefers not to buy any amount of product 1. When instead the optimal price is unconstrained, i.e. $p_1^L = \frac{1+c_1}{2}$, then the price p_1^H can be somewhat reduced. The lower bound becomes

$$\underline{p_1^H} = 1 - K\gamma - \frac{1}{2}\sqrt{(1-c)^2 - 8(1-c-d)K + 4K^2}.$$

[32] If the price is higher than this lower bound, then the discount is greater and hence the test is more difficult to pass. The frontier derived below would therefore shift upwards.

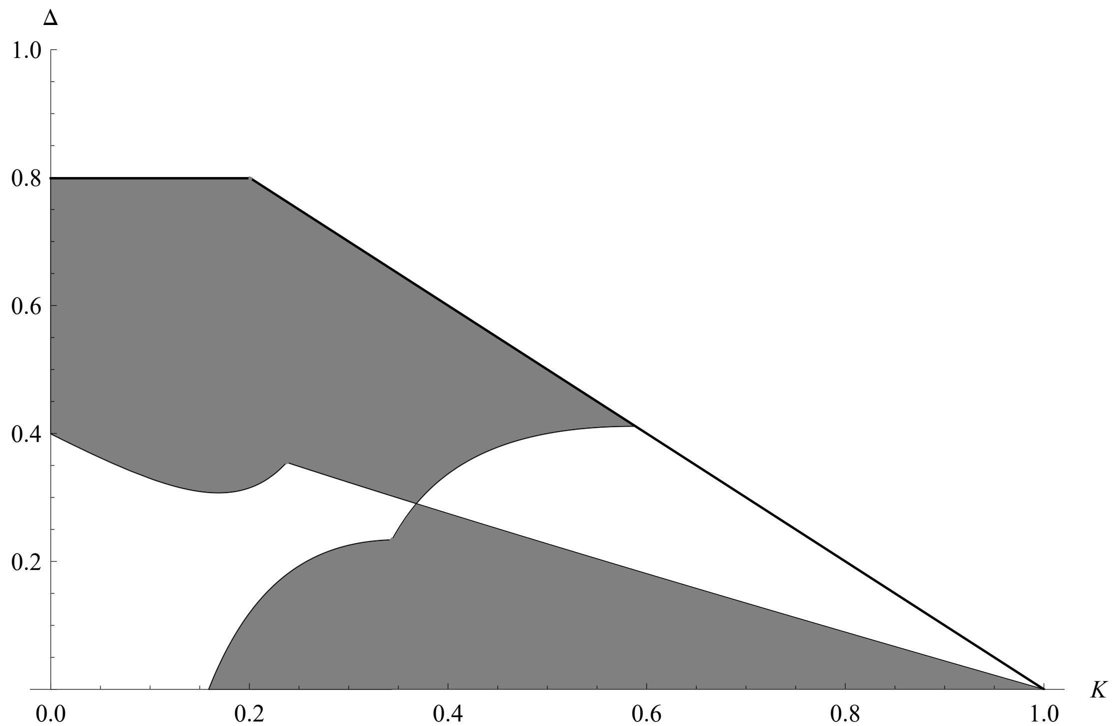

Figure 2: The grey areas are when the test is mistaken. Since the test fails below the test frontier (the thin curve) and exclusivity discounts are anti-competitive above the welfare frontier (the thick curves), in the upper grey region the test delivers false negatives, in the lower one false positives. The figure is for $c_1 = 0$.

ability to supply the retailer (i.e., only to the K). The first case corresponds to $K = 1 - \tilde{c}_2$, which implies that constraint (2) is never binding. In this case, the contestable share is one, so the test boils down to the standard price-cost comparison. Since $p_1^L = \min\left[c_2, \frac{1+c_1}{2}\right] \geq c_1$, the test is always passed. But we know that exclusivity discounts are anticompetitive if Δ is sufficiently large. This means that in this case the test delivers false negatives.

The opposite extreme case is obtained when the competitive advantage is due entirely to the rival's limited ability to supply the retailer: $c_2 = c_1$, or $\Delta = 0$. In this case, the test is always failed.[33] But we know that exclusivity discounts

[33] This can be easily checked algebraically. The intuitive explanation is as follows. Suppose that condition (12) holds as an equality, meaning that p_1^L is a weighted average of p_1^H and c_1. Since the payoff function $v(q_1, q_2)$ exhibits preference for variety, a buyer who is given the option of purchasing both products at prices $p_1 = p_1^H$ and $p_2 = c_1$ would obtain a greater payoff than a buyer who can only purchase one good at a price of p_1^L. Therefore, the lower bound on p_1^H implied by condition (17) must violate inequality (12). To put it differently, the test is biased because it treats the products as if they were homogeneous, whereas in reality they are differentiated.

are procompetitive if the competitive advantage is sufficiently small, i.e., if K is sufficiently large. Therefore, in this case the test delivers false positives.

When the competitive advantage is due to a combination of both factors, whether the test is passed or failed depends on which factor has the greater weight. In general, for given capacity, increasing Δ tends to make the test easier to pass; for a given cost gap, decreasing capacity tends to make the test easier to fail.[34] But both moves increase the dominant firm's competitive advantage. Hence, the test frontier tends to be decreasing in the (K, Δ) space, whereas the welfare frontier is increasing.

4.3 Product differentiation

Figure 2 is striking. It shows that even if all relevant variables can be measured with perfect accuracy, applying the price-cost test to loyalty discounts cases has the same chances of delivering the correct decision as tossing a coin. But in fact, things are even worse than that. The logic of the test is flawed in a more fundamental way.

The reason for this is that the test treats the products as if they were homogeneous, but in fact they are differentiated. With differentiated products, local and global efficiency may diverge even if the dominant firm's rival can supply whatever output the retailer may demand.[35] To see why, it may be useful to present some formal definitions. Let

$$S(q_1, q_2) = v(q_1, q_2) - c_1 q_1 - c_2 q_2 \tag{18}$$

denote the social surplus. We may say that firm 1 and 2 are *locally as efficient* at $(\bar{q}_1, \bar{q}_2)$ if and only if

$$\frac{\partial S}{\partial q_1} \Big|_{(\bar{q}_1, \bar{q}_2)} = \frac{\partial S}{\partial q_2} \Big|_{(\bar{q}_1, \bar{q}_2)} . \tag{19}$$

In other words, local as efficiency means that the social value of one extra unit of product 2 is as large as that of product 1. In contrast, firm 1 and 2 are *globally as efficient* if and only if

$$\max_{q_1 \geq 0} [v(q_1, 0) - c_1 q_1] = \max_{K \geq q_2 \geq 0} [v(0, q_2) - c_2 q_2]. \tag{20}$$

In other words, global as efficiency means that the two firms would be equally good as the retailer's sole supplier.

If the payoff function v is symmetric and the capacity constraint is not binding, global as efficiency requires $c_1 = c_2$, which is what the test assumes. But

[34] To be precise, this is true only when the dominant firm's equilibrium exclusive price is constrained. When it is unconstrained, things are more comlicated but in that case exclusivity discounts are always anticompetitive.

[35] Global and local as efficiency may differ also for other reasons. For example, the dominant firm might offer a broader product line than the rival. In this case, the rival may be as (or more) efficient in supplying the varieties that he offers, but less efficient as a global supplier.

local as efficiency entails this condition only if the products are homogeneous. If the products are differentiated, firm 2 can be locally as efficient even if $c_2 > c_1$, provided that $q_2 < q_1$. Since in a foreclosure equilibrium q_2 vanishes, the weaker firm could be locally as efficient even if c_2 were substantially higher than c_1.

One can therefore think of still another version of the test, which accounts for product differentiation by assuming that the hypothetical competitor is locally as efficient as the dominant firm at $(q_1^E, 0)$. To keep things simple, assume that constraint (2) is not binding, so that the only source of competitive advantage is the cost gap Δ. In other words, the contestable share s_C is 1. Using the notion of local as efficiency defined by (19), the hypothetical as-efficient competitor must have a cost $\tilde{c}_2$ which is the solution to

$$v_{q_1}(q_1^E, 0) - c_1 = v_{q_2}(q_1^E, 0) - \tilde{c}_2, \tag{21}$$

that is

$$\tilde{c}_2 = c_1 + \left[v_{q_2}(q_1^E, 0) - p_1^L\right]. \tag{22}$$

The term inside square brackets is positive as the payoff function is concave. This implies that, as noted above, local as efficiency is fully consistent with $\tilde{c}_2 > c_1$.

This new variant of the price cost-test is passed if

$$p_1^L \geq c_1 + \left[v_{q_2}(q_1^E, 0) - p_1^L\right], \tag{23}$$

or, equivalently,

$$2p_1^L \geq v_{q_2}(q_1^E, 0) + c_1. \tag{24}$$

Figure 3 represents, in the (γ, Δ) space, the welfare frontier and the test frontier for this new variant of the test. Note that K is no longer relevant here, as constraint (2) by assumption does not bind. The variable plotted on the x-axis is the degree of product differentiation γ. The thick curve is the welfare frontier, the thin curve the test frontier. Exclusivity discounts are procompetitive below the welfare frontier, anticompetitive above. The test is passed above the test frontier, failed below.

The two curves almost overlap, but the test points in the opposite direction to the welfare comparison. This means that the result of the test is almost always mistaken. In particular, we have false negatives above both curves, false positives below. The picture is now almost entirely colored in grey.

We can therefore conclude that if one properly accounts for product differentiation, the test becomes not just uninformative but completely misleading. The intuitive reason for this is very simple. Exclusivity discounts are anticompetitive when they either increase prices, or do not lead to significant price reductions. But price-cost tests are designed precisely in order to detect low prices, not high ones. In other words, price-cost tests regards as pathological what is, in our framework, beneficial, and vice versa.

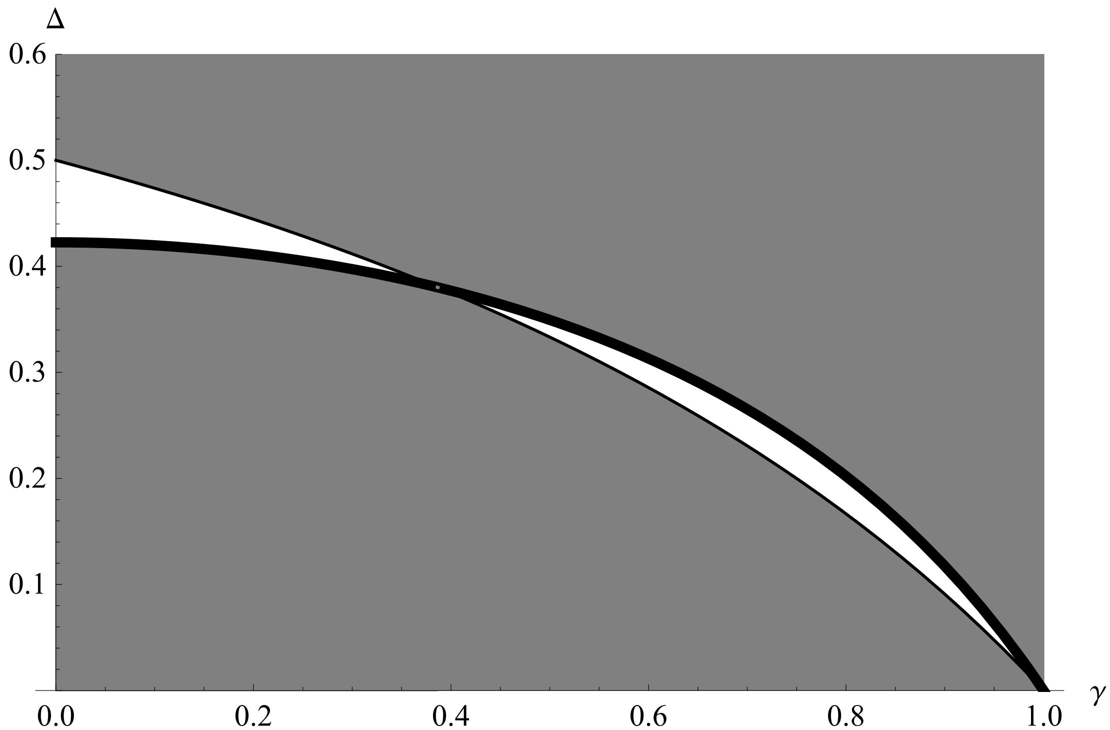

Figure 3: The local-as-efficient competitor test is mistaken in the grey areas: the test fails below the thin curve and exclusivity discounts are anticompetitive above the thick curve. The Figure has been drawn for $c_1 = 0$ and $\gamma = 0.3$.

5 Extensions

In this section, we extend our analysis in two ways. Firstly, we allow firms to offer two-part tariffs rather than linear prices. Secondly, we allow the target market share s to be less than 100%.

5.1 Competition in two-part tariffs

In this subsection we assume that upstream firms compete in two-part tariffs $P_i = F_i + p_i q_i$, where F_i is the fixed fee and p_i is the marginal price.

When loyalty discounts are prohibited, the firms may offer only one tariff, which applies irrespective of the rival's sales. When instead loyalty discounts are permitted, the dominant firm can offer two different tariffs:

$$P_1 = \begin{cases} F_1^H + p_1^H q_1 & \text{if } \frac{q_1}{q_1+q_2} < s \\ \\ F_1^L + p_1^L q_1 & \text{if } \frac{q_1}{q_1+q_2} \geq s. \end{cases} \tag{25}$$

The higher tariff applies if the retailer purchases less that the prescribed share s, the lower, discounted tariff applies if the retailer buys at least the prescribed share. In this subsection we still focus on exclusivity discounts, where $s = 1$. Firm 2 then offers a single tariff, $P_2 = F_2 + p_2 q_2$, which becomes relevant only if the retailer breaches exclusivity.

As discussed above, the demand-boost theory of loyalty discounts requires that marginal prices are distorted upwards. To produce price distortions with two-part tariffs, we use a reduced-form model, proposed by Calzolari et al. (2018), that simply assumes that fixed fees entail a cost for the retailer in excess of the payment accruing to the upstream firm. Calzolari et al. show that the reduced-form model captures in a stylized fashion moral hazard, adverse selection and other similar factors.

We take the cost to be proportional to the size of the fixed fee. In other words, we assume that by charging a fixed fee $F_i > 0$ the firm gains F_i but the retailer loses $(1 + \mu)F_i$, with $\mu \geq 0$. We assume that the cost μ appears only when $F_i > 0$. As a consequence, lump-sum subsidies do not entail any gain and hence will never be used.

5.1.1 Benchmark

The new benchmark equilibrium, which arises when loyalty discounts are prohibited, is reported in the Appendix. In the limiting case $\mu = 0$, marginal prices equal marginal costs and the equilibrium delivers the efficient output levels. As soon as $\mu > 0$, however, marginal prices are distorted upwards and equilibrium quantities are distorted downwards. The equilibrium with linear pricing is re-obtained as a limiting case for $\mu \to \infty$.

When $\mu > 0$, the equilibrium is in pure strategies if K is large, in mixed strategies if K is small. In both cases, the equilibrium is qualitatively similar to that arising under linear pricing. Both firms price above cost, exploiting the market power they have thanks to product differentiation. The dominant firm, being more efficient, produces more and sets a lower price than its competitor.

5.1.2 Exclusivity discounts

Now suppose that exclusivity discounts are permitted. In the limiting case $\mu = 0$, efficient pricing would prevail and exclusivity discounts would not be offered. As soon as $\mu > 0$, however, exclusivity discounts are offered by the dominant firm and accepted by the retailer. As a result, at equilibrium only the dominant firm is active, but the rival stands ready to supply its product at cost. Equilibrium tariffs are characterized in the following lemma.

Lemma 3 *If exclusivity discounts are permitted and $\mu > 0$, there is a unique Nash equilibrium. In this equilibrium,*

$$p_1^L = \min\left[\frac{c_1 + (1 + c_1)\mu}{1 + 2\mu}, 1 - \sqrt{2(1 - c_2)K - K^2}\right] \tag{26}$$

$$F_1 = \max\left[\frac{(1-c_1)^2(1+\mu)^2 - \left[2(1-c_2)K - K^2\right](1+2\mu)^2}{2(1+\mu)(1+2\mu)^2}, 0\right]$$

and

$$p_2 = c_2 \qquad F_2 = 0. \tag{27}$$

The proof is similar to that of Lemma 2 and is not repeated here.

The dominant firm always charges a tariff that maximizes its profit under the constraint that the buyer is just indifferent between buying from the dominant firm or from the rival. The solution to this problem may be interior, with $F_1 > 0$, or it may be a corner solution with $F_1 = 0$. In this latter case, the marginal price must be the same as in the case of linear pricing, and the equilibrium outcome is exactly the same. The interior solution applies when the dominant firm's competitive advantage is sufficiently large, i.e. when

$$\Delta \geq c_1 + \frac{\left[2(1-c_1)K + K^2\right](1+2\mu)^2 + (1-c_1)^2(1+\mu)^2}{2K(1+2\mu)^2}. \tag{28}$$

The corner solution, in contrast, applies when the competitive advantage is relatively small.

As son as $\mu > 0$, the conclusion of Proposition 1 continues to hold even with two-part tariffs. That is, exclusivity discounts are anticompetitive if the dominant firm's competitive advantage is large, procompetitive if it is small. However, the region where exclusivity discounts are anticompetitive gets larger as μ decreases, and for $\mu \to 0$ exclusivity discounts tend to become always anticompetitive (however, the magnitude of the anticompetitive effects becomes smaller and smaller).

5.1.3 The test

Now consider the application of the test to this model. To fix ideas, we focus on the version of the test which assumes that the as-efficient competitor has a marginal cost $\tilde{c}_2 = c_1$ but can produce at most K. In the counterfactual, such a hypothetical as-efficient competitor prices at cost, $F = 0$ and $p = c_1$. Therefore, the test now requires that

$$Kc_1 + (q_1^E - K)p_1^H + F_1^H \leq q_1^E p_1^L + F_1^L. \tag{29}$$

As under linear pricing, the higher tariff (p_1^H, F_1^H) is not fully pinned down in equilibrium. The tariff must be high enough that the buyer prefers to buy exclusively from the dominant firm at the lower tariff (p_1^L, F_1^L). But this still leaves one degree of freedom as two variables, p_1^H and F_1^H, are now to be determined. The analysis is simplest if one exploits this degree of freedom by setting $F_1^H = F_1^L$.[36] In this case, the test reduces to that of the linear pricing case, i.e.

$$Kc_1 + (q_1^E - K)p_1^H \leq q_1^E p_1^L. \tag{12}$$

[36] This need not be the tariff that minimizes the risk that the test is failed, however. To

When the constrained solution with $F_1^L = 0$ applies, equilibrium outcomes are exactly the same as under linear pricing and therefore the test delivers the same results. When instead the competitive advantage is larger, so that the interior solution becomes relevant, the dominant firm now prices more efficiently. Therefore, q_1^E is greater than in the linear pricing case, and p_1^L lower, and the more so the lower is μ. This implies that the left-hand side of the inequality increases, and the right-hand side decreases. Thus, the test is now failed more often.

Summarizing, as μ decreases the welfare frontier shifts downwards while the lower branch of the test frontier shifts upwards. The downwards shift of the welfare frontier implies that there are fewer false positives but more false negatives. The upwards shift of the test frontier just reduces the likelihood of false negatives, as exclusivity discounts are always anticompetitive when the solution is interior.

Even if there are fewer false negatives and more false positives, the broad picture does not change substantially. The conclusion that the price-cost test is uninformative is robust to the possibility that firms may use two-part tariffs.

5.2 Market share discounts

In the preceding subsection, we have shown that allowing for two part-tariffs does not change the analysis substantially. To explore the case of market-share discounts, where $s < 1$, we therefore go back to the simpler assumption of linear pricing.

For the sake of simplicity, we take s as an exogenous parameter.[37] To keep the analysis interesting, we assume that s is sufficiently large that constraint (2) does not bind in equilibrium.[38]

Like firm 1, firm 2 now sets two prices, p_2^H and p_2^L. The former is intended for cases where $\frac{q_1}{q_1+q_2} \geq s$, i.e. when the target market share is met, the latter for $\frac{q_1}{q_1+q_2} < s$. No trade takes place at the off-equilibrium prices p_1^H and p_2^L. However, unlike p_1^H, which does not affect equilibrium outcomes, the price p_2^L affects the retailer's reservation payoff and hence the prices that will indeed apply at equilibrium. This opens the possibility of multiple equilibria. For ease of comparison with the case of exclusivity discounts, in what follows we shall

find such a tariff, one would have to solve the problem of minimizing the expenditure

$$(q_1^E - K)p_1^H + F_1^H$$

under the constraint

$$\max_{q_1,q_2 \leq K} [v(q_1,q_2) - p_1^H q_1 - F_1^H - c_2 q_2] \leq v(q_1^E, 0) - p_1^L q_1^E - F_1^L.$$

[37] Our aim here is not to provide a full theory of market-share discounts, but just to test the robustness of our results. A proper model of market-share discount would require endogenizing s and would probably require also a more highly structured model, which is outside of the scope of the present paper

[38] Incidentally, this guarantees that the equilibrium with market-share discounts is always in pure strategies.

focus on the equilibrium that is least profitable for the dominant firm. In this equilibrium, p_2^L is set at cost: $p_2^L = c_2$. That is, firm 2 prices, off equilibrium, as aggressively as possible. This implies that the retailer's reservation payoff is $v^R = v(0, K) - c_2 K$, as under exclusivity discounts.

The prices that are destined to be accepted in equilibrium are determined as follows. Firstly, firm 2 sets p_2^H at the lowest level such that the constraint $\frac{q_1}{q_1+q_2} \geq s$ is met:

$$p_2^H = \frac{(1-\gamma)(2s-1) + [1 - s(1-\gamma)]\, p_1^L}{(1-\gamma)s + \gamma}. \tag{30}$$

In the absence of the market-share discount, firm 2's best response to p_1^L would be lower. However, with the market-share discount in place the retailer would not increase his demand for product 2 in order not to lose the discount, so firm 2 has no further incentive reduce its price.

Firm 1 maximizes its profit anticipating that in the presence of the market-share discount firm 2 prices according to (30). In doing so, it must guarantee the retailer's participation, which requires that

$$\max_{q_1, q_2} [v(q_1, q_2) - p_1 q_1 - p_2 q_2] \geq v(0, k) - p_2 k. \tag{31}$$

Thus, firm 1's optimal price is

$$p_1^L = \min \left[1 - \frac{[(1-\gamma)s + \gamma] \sqrt{2(1-c_2)K - K^2}}{\sqrt{1 - 2(1-\gamma)s(1-s)}}, \frac{1+c_1}{2} \right] \tag{32}$$

When the participation constraint does not bind, the optimal price for the dominant firm is simply the monopoly price. Substituting (32) into (30) one finally gets firm 2's equilibrium price. It can be easily checked that as $s \to 1$, the equilibrium outcome converges to the one with exclusivity discounts.

Under our assumptions, with market-share discounts the welfare frontier is exactly the same as in the case of exclusivity discounts. The reason for this is twofold. On the one hand, when the participation constraint is binding, the retailer's equilibrium payoff coincides with his reservation payoff $v^R = v(0, K) - c_2 K$ which is independent of s. On the other hand, when the dominant firm's pricing is unconstrained, loyalty discounts are always anticompetitive.

The incremental price-cost test now requires that

$$(K - q_2)c_1 + [q_1 - (K - q_2)]\, p_1^H \leq q_1 p_1^L \tag{33}$$

as firm 2 can supply only the additional quantity $(K - q_2)$. The analysis reveals that decreasing s makes it more difficult to pass the test. In other words, as s decreases the test frontier shifts downwards. There are various effects at work, but the driving force is that decreasing s increases the equilibrium output of firm 2 and thus decreases the "contestable" output $(K - q_2)$. As a result, the effective discount increases, which makes the test more likely to fail.

Since the test frontier shifts downwards, there are both fewer false positives and more false negatives than in the case of exclusivity discounts. Even if the type of errors may change, however, there is no significant improvement in the precision of the test. On average, running the test is no more informative than tossing a coin.

6 Conclusions

In this paper, we have analyzed by means of a formal economic model the use of price-cost tests to assess the competitive effects of loyalty discounts. In the model, a dominant firm enjoys a competitive advantage over its rival and uses loyalty discounts as a means to boost the demand for its product. Loyalty discounts may then be directly profitable, without necessarily entailing an immediate sacrifice of profit, and hence the need of recoupment. In this framework, we have shown that price-cost tests are misleading or, at best, completely uninformative.

The implications of our analysis for competition policy are clear. If the theory of harm invoked by plaintiffs or adopted by the court is the demand-boost theory, then price-cost tests should not be used. Things may be different if the relevant theory of harm is of the more traditional sacrifice-recoupment type. Even in this case, however, practical considerations may militate against the use of price-cost tests, as discussed in the introduction.

We believe that the demand-boost theory of loyalty discounts is more natural, straightforward and robust than sacrifice-recoupment theories. Therefore, in many cases the demand-boost theory appears to be the elective theory of harm. If this is so, then the scope for using price-cost test in loyalty discounts cases seems very limited.

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Appendix

In this Appendix we derive the benchmark equilibrium in which loyalty discounts are prohibited, both for the case in which firms are constrained to linear pricing and the case in which they can use two-part tariffs.

Linear pricing. The retailer's demand functions are obtained by maximizing the net payoff $v(q_1, q_2) - p_1 q_1 - p_2 q_2$ and are

$$q_i = \frac{1 - \gamma - p_i + \gamma p_j.}{1 - \gamma^2} \tag{A1}$$

To calculate the equilibrium we first derive the firms' best response functions. Firm 2's best response comprises two branches, depending on whether it is optimal to price so that $q_2 < K$ or $q_2 = K$. The former is optimal when p_1 is low,[39] the latter when it is high:

$$p_2 = \begin{cases} \frac{1+c_2-\gamma(1-p_1)}{2} & \text{if } p_1 < \hat{p}_1 \\ 1 - \Upsilon^2 K - \gamma(1-p_1) & \text{if } p_1 \geq \hat{p}_1, \end{cases} \tag{A2}$$

where $\hat{p}_1 = \frac{1-c_2-\gamma-\Upsilon^2 K}{\gamma}$ and $\Upsilon = \sqrt{1-\gamma^2}$. Firm 2's best response is continuous at $\hat{p}_1$.

Firm 1's best response also comprises two branches:

$$p_1 = \begin{cases} \frac{1+c_1-\gamma K}{2} & \text{if } p_2 < \hat{p}_2 \\ \frac{1+c_2-\gamma(1-p_2)}{2} & \text{if } p_2 \geq \hat{p}_2, \end{cases} \tag{A3}$$

where

$$\hat{p}_2 = \frac{(1-c_1-K\gamma)\Upsilon - (1-c_1-\gamma)}{\gamma}. \tag{A4}$$

On the lower branch, firm 1's price does not depend on p_2. The reason for this is that when p_2 is low, firm 1 realizes that firm 2 is capacity constrained: $q_2 = K$. In this case, the demand for product 1 is given by

$$p_1 = 1 - q_1 - \gamma K, \tag{A5}$$

so the optimal price is $\frac{1+c_1-\gamma K}{2}$. When instead p_2 is high, the constraint $q_2 < K$ does not bind and the best response is the standard one. Unlike firm 2, however, firm 1's best response exhibits a jump. At $p_2 = \hat{p}_2$, firm 1 is just indifferent between charging the low and the high price, but the two prices are different.

[39] Here we drop the index H as there is no risk of confusion.

Two cases may then arise. When $K > \hat{K}$, where

$$\hat{K} = \frac{\gamma\left[2(1-\Delta)+\gamma\left(2-\Upsilon\right)\right]+c_1(2+\gamma)\left[2\left(1-\Upsilon\right)-\gamma\left(2-\Upsilon\right)\right]-4\left(1-\Upsilon\right)}{\Upsilon\gamma\left(4-\gamma^2\right)}, \tag{A6}$$

the firms' best responses intersect. More precisely, the lower branch of firm 2's best response cuts the upper branch of firm 1's one. In this case, we have the standard Bertrand equilibrium where

$$p_i = \frac{2+2c_i-\gamma c_j-\gamma(1+\gamma)}{4-\gamma^2}. \tag{A7}$$

When instead $K \leq \hat{K}$, there is no intersection point: firm 2's best response passes through the hole in firm 1's best response. In this case, the equilibrium is in mixed strategies. Firm 2 sets $p_2 = \hat{p}_2$ and firm 1 randomizes between the high price $\bar{p}_1 = \frac{1+c_1-\gamma K}{2}$ and the low price $\underline{p}_1 = \frac{1+c_2-\gamma(1-\hat{p}_2)}{2}$. The probability of setting the high price,

$$x = \frac{\Phi}{\Phi+2\gamma K\Upsilon^2}, \tag{A8}$$

where $\Phi = 4(1-c_1)+2\gamma\left[\Delta-(1+\gamma)(1-c_1)\right]-\Upsilon\left[4(1-c_1-\gamma K)-\gamma^2(1-c_1)+\gamma^3 K\right]$, is such that it is indeed optimal for firm 2 to charge $\hat{p}_2$.

Two-part tariffs. As in the case of linear pricing, we first derive the firms' best response functions.

First of all, it must be noticed that each firm will always set its fixed fee in such a way that the retailer is just indifferent between purchasing from both firm or exclusively from the rival. This implies that

$$F_i = \frac{\left(1-\gamma+p_i+\gamma p_j\right)^2}{2(1-\gamma^2)(1+\mu)} \tag{A9}$$

if constraint (2) is slack. If the constraint binds, in contrast, (A9) continues to hold for firm 2 but the dominant firm's fixed fee is

$$F_1 = \frac{\frac{(1-p_1)^2+(1-p_2)^2-2\gamma(1-p_1)(1-p_2)}{1-\gamma^2}-K([2(1-p_2)-K]}{2(1+\mu)}. \tag{A10}$$

Substituting (A9) and (A10) into the profit functions, we can then focus on the marginal prices only.

In the space of marginal prices, best responses have a similar structure to the case of linear pricing. In particular, firm 2's best response comprises two branches, depending on whether it is optimal to price so that $q_2 < K$ or $q_2 = K$. The former is optimal when p_1 is low, the latter when it is high:

$$p_2 = \begin{cases} \frac{c_2(1+\mu)+\mu\left[1-\gamma(1-p_1^L)\right]}{1+2\mu} & \text{if } p_1 < \hat{p}_1 \\ \\ 1-\Upsilon^2 K-\gamma(1-p_1) & \text{if } p_1 \geq \hat{p}_1, \end{cases} \tag{A11}$$

where

$$\hat{p}_1 = \frac{c_2(1+\mu) - (1-\gamma)\left[(1+\mu) - (1-\gamma)(1+2\mu)K\right]}{\gamma(1+\mu)}. \tag{A12}$$

Firm 2's best response is continuous at $p_1 = \hat{p}_1$.

Firm 1's best response also comprises two branches:

$$p_1 = \begin{cases} \frac{c_1(1+\mu)+\mu(1-\gamma K)}{1+2\mu} & \text{if } p_2 < \hat{p}_2 \\ \\ \frac{c_1(1+\mu)+\mu[1-\gamma(1-p_2)]}{1+2\mu} & \text{if } p_2 \geq \hat{p}_2, \end{cases} \tag{A13}$$

where $\hat{p}_2$ is still given by (A4). On the lower branch, where the constraint $q_2 \leq K$ binds, firm 1's price does not depend on p_2, for the same reason as under linear pricing. When instead p_2 is high, the constraint $q_2 \leq K$ does not bind and the best response is increasing in p_2. Unlike firm 2, however, firm 1's best response exhibits a jump. At $p_2 = \hat{p}_2$, firm 1 is just indifferent between charging the low and the high price, but the two prices are different.

The structure of the equilibrium is also similar to the case of linear pricing. When $K > \hat{K}$, where

$$\hat{K} = \frac{\gamma c_1\left[1+2\mu+\gamma\mu\right]\left[1+2\mu+\mu\Upsilon\right] + \left[2\mu(1+2\mu) - \gamma^2\mu^2\right](1+\Upsilon) - (1-\Delta)\gamma(1+\mu)(1+2\mu) - \gamma^2\mu(1+\mu)}{\Upsilon\gamma\left[(1+2\mu)^2 - \gamma^2\mu^2\right]}, \tag{A14}$$

the equilibrium is in pure strategies. Equilibrium marginal prices are

$$p_i = \frac{c_i(1+\mu)(1+2\mu) - \gamma c_j\mu(1+\mu) + \mu(1+2\mu) - \gamma\mu[1+\mu(1+\gamma)]}{1+\mu\left[4+\left(4-\gamma^2\right)\mu\right]}. \tag{A15}$$

The corresponding fixed fees can be obtained by replacing these expressions into (A9). It can be easily checked that as $\mu \to \infty$ the marginal converge to the standard Bertrand prices and the fixed fees converge to zero.

When instead $K \leq \hat{K}$, the equilibrium is in mixed strategies. Firm 2 sets $p_2 = \hat{p}_2$ and firm 1 randomizes between the high price $\bar{p}_1 = \frac{c_1(1+\mu)+\mu(1-\gamma K)}{1+2\mu}$ and the low price $\underline{p}_1 = \frac{c_1(1+\mu)+\mu\left[1-\gamma(1-p_2^H)\right]}{1+2\mu}$. The probability of setting the high price,

$$x = \frac{\Phi}{\Phi + \gamma\Upsilon^2 K\mu(1+2\mu)}, \tag{A16}$$

where $\Phi = c_1(1-\gamma)(1+2\mu)[1+\mu(2+\gamma)] - \gamma(1-\Delta)(1+\mu)(1+2\mu) + [(1+2\mu)^2 + \gamma^2\mu^2]\left[1+(1-c_1-\gamma K)\Upsilon\right] - \gamma^2\mu^2$, is such that it is indeed optimal for firm 2 to charge the price $\hat{p}_2$.