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Abstract

I discuss two alternative notions of social welfare (utilitarian and self-enforcing) in a dynastic model with heterogeneous and persistent degrees of parental altruism and evaluate the implied levels of consumption inequality. Then, I study a decentralization of planning optima in a competitive equilibrium where the only source of inequality arises from intergenerational wealth transmission and I show that the self-enforcing criterion implies a negative tax rate on the less altruistic individuals' capital income.

JEL Classification: D31, E21, H21, J62

Keywords: Wealth, inequality, Capital taxation

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1. Introduction

Inequality across individuals is largely accounted for by the unequal distribution of wealth and the latter is, in turn, significantly affected by intentional bequests, *i.e.*, motivated by parental altruism[1]. This observation raises many normative questions. Do altruistic intergenerational linkages be

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[1]Kotlikoff and Summers (1981) argue that as much as 46% of household wealth is accounted for by bequests and Kopczuk and Lupton (2007) find that roughly 75% of the elderly population has a bequest motive. Using Danish administrative data, Boserup et al. (2016) find that bequests account for about 26% of the average post-bequest wealth 1-3 years after parental death and that they increase absolute wealth inequality (variance), although they reduce relative inequality (top wealth shares).

ignored or undone by the social planner[2]? Is the optimal allocation of resources in economies with heterogeneous degrees of parental altruism more or less unevenly distributed than in the market economy?

In this paper I review these questions in a simple overlapping generations economy where individuals' degrees of parental altruism are inherited, *i.e.*, persistent across individuals belonging to the same dynasty. In particular, I consider two notions of social welfare and compare the "optimal" amount of wealth inequality (*i.e.*, the one produced by the planning optimum) with that generated through the market system in the equivalent economy where the only source of consumption inequality may arise from intergenerational wealth transmission. Finally, I discuss how and whether optimal allocations can be decentralized. The first notion of social welfare is derived from a planning optimum generated by an utilitarian social welfare function where the utilities of all individuals belonging to the same generation get positive equal weights. This criterion has been proposed by Bernheim (1989) (later applied by Phelan (2006), Fahri and Werning (2007), Soares (2015) and others in different contests), and I call it *Utilitarian Social Welfare* (USW). The second notion is based on the maximization of a simple average of the individuals' utilities of the initial generation subject to a set of conditions guaranteeing that all generations (at all stages of their life) receive some minimum level of utility. This criterion is based on a set of exogenous individual *reservation utilities* and I call it *Self-Enforcing Social Welfare* (SESW).

Utilitarian welfare criteria with intergenerational wealth transmission have been studied extensively. In particular, Bernheim (1989) uses a framework very similar to the one considered in this paper and shows that a USW is

[2]For instance, estate taxes and a global tax on wealth, have been famously advocated in Piketty (2014), although some authors have shown that such policies may have unintended consequences for inequality (see Becker and Tomes (1979)).

characterized by a key condition that makes social optima hardly implementable in a competitive equilibrium with bequests. The latter says that, for all old individuals, the marginal benefit from transferring some extra resources to their offsprings is lower than the marginal cost in terms of the old's forgone consumption. Hence, implementation of a USW in a market economy is only possible if the government imposes a lower bound on the amount of bequests[3]. A zero lower bound is a natural requirement arising from one-sided parental altruism. However, suppose that consumption inequality across individuals in the market economy can only arise through intergenerational wealth transmission. Then, implementation of the USW through zero bequests would generate perfect equality of consumption across individuals endowed with different degrees of parental altruism, but this is not compatible with the USW, where more altruistic individuals are assigned more consumption. Hence, implementation of the USW may require the government to impose positive individual specific lower bounds on bequests. In other words, richer individuals should be forced to leave positive and larger bequests than they would like to if they were allowed to chose based on their free will. These lower limits can be hardly justified and implementable in a market economy. A further problem with the USW program is that, although the Planner may assign positive and arbitrarily large weights on future generations' utilities (subject to an upper limit guaranteeing boundedness of the social welfare function), this leads, most likely, to higher inequality than the one generated in the market economy. Finally, the USW is not a time-consistent program (as highlighted in Bernheim (1989)). In particular, in my model, the solution with pre-commitment implies more consumption inequality and smaller lower bounds on bequests (in the decentralized economy) than it is implied in a solution without commitment.

The notion of SESW excludes the undesirable characteristics described above. In this case, the optimal allocation is characterized by a minimum

[3]Similarly, Fahri and Werning (2010) show that a negative estate tax may be optimal.

level of welfare for the less altruistic individuals (*reservation utilities*), to be taken as exogenous parameters by the Planner, defining a set of participation constraints for incoming generations. The underlying assumption is that future generations can opt away from the Planner's program. The typical solution to this planning optimum is characterized by binding participation constraints for the less altruistic dynasties. Interestingly, the within generation marginal rate of substitution between young and old age consumption equals the marginal product of capital for the more altruistic individuals only, and it exceeds the marginal product of capital for the remaining class of individuals. Hence, the SESW program lends itself to a natural decentralization as an equilibrium with one-sided altruism with (possibly) a lump-sum transfer (a *basic income*[4]) coupled with a capital income subsidy for the less altruistic individuals. In other words, the (more) selfish individuals save too little at a competitive equilibrium. This contradict a natural intuition that we may derive from the famous Chamley-Judd result[5], *i.e.*, that the optimal long-run tax rate on capital should be zero. The result is similar to the the one found by Soares (2015) in an olg model with parental altruism (and identical individuals per generation). It provides a social welfare base to the idea that borrowing constraints may be useful to enhance capital accumulation (see Jappelli and Pagano (1994)). Whether the optimal amount of inequality is, in this case, greater or smaller than the one generated through the market mechanism depends on how large are the reservation utilities. The fact that the latter are taken parametrically by the Planner introduces some indeterminacy in the SESW criterion. What is the meaning of these reservation utilities? And why should the Planner take them as given? This indeterminacy can be understood as an unresolved trade-off between total

[4]The possibility of decentralizing the SESW with a basic income was suggested to me by Joydeep Bhattacharya.

[5]See Judd (1985) and Chamley (1981).

social welfare (efficiency) and inequality. The higher are the reservation utilities, the lower is the degree of consumption inequality between the members of the different dynasties and the lower is social welfare. I argue that a possible solution to this indeterminacy is to assume that the reservation utilities are equal to the indirect utilities that individuals would achieve when they start their life with zero bequests and have access to a competitive labor and capital market. In a stationary allocation, these reservation utilities are uniquely determined by the subsidy on the selfish individuals' capital income. Hence, this subsidy defines the desired Planner's (or government's) solution to the efficiency-inequality trade-off.

The issue of how should resources be distributed across altruistically linked individuals with unobservable heterogeneous preferences (taste shocks) or with costly and unobservable actions have been studied by Fahri and Werning (2007) and Phelan (2006), among others. These authors extend the framework introduced in Atkeson and Lucas (1992), who have studied the case of incentive compatible Pareto efficient allocations of resources with heterogeneous taste shocks and have shown that the degree of inequality continually increases, with a diminishing fraction of the population receiving an increasing fraction of total resources (*immiseration*). Like Bernheim (1989), both Fahri and Werning (2007) and Phelan (2006) show that immiseration may not hold in the model with altruistic finitely lived generations when the planner's social welfare function places positive and sufficiently large weights on future generations' welfare. More generally, there is a large literature investigating the effects of bequests on long run wealth distribution and the effects of estate taxes. However, most of this literature assumes that bequests arise out of joy of giving instead of non-paternalistic parental altruism and it concentrates on the interplay between stochastic labor earnings and wealth transmission. A central result is that bequests compensate differences in random labour income (see, among others, Becker and Tomes

(1979), Davies (1986) and Bossmann et al. (2007)). In particular, Bossmann et al. (2007) argues that bequests diminish the inequality of wealth since they raise private savings and, hence, average wealth holdings more than the variance of wealth. This compensation effect is inoperative in my model, since I assume that there is no wage income heterogeneity. The idea that inheritance taxes or any alternative limit on wealth transmission through bequests may be effective in reducing wealth inequality is controversial. Becker and Tomes (1979) and Davies (1986) claim that inheritance taxes may increase inequality when these compensate differences in random income shocks, whereas Bossmann et al. (2007) finds the opposite result. Finally, using a framework very similar to the one used in this paper, Soares (2015) argues that borrowing constraints on the young may be optimal and Andersen and Bhattacharya (2015) show that, using a Pareto criterion, a combination of taxed financed public spending on education and unfunded social security improve upon the *laissez faire* equilibrium even when market failures are absent.

This paper is organized as follows. Section 2 defines the economy under consideration and characterizes the market equilibrium. Section 3 investigates the property of the utilitarian social welfare program and discusses the long run inequality and the problems with decentralization. Section 4 introduces the notion of self-enforcing social welfare, section 5 shows how is it possible to decentralize it, section 6 discusses the key assumptions and section 7 concludes.

2. The Economy

The economy corresponds to an overlapping generations model with bequests motivated by parental altruism. At any period $t = 0, 1, ..$ there is a mass of young and old individuals belonging to types $i = a, s$ with mass m_a, m_s, respectively. Each type belongs to one of two dynasties whose preferences are described by

$$V_{-1}^i = \delta u(c_0^{o,i}) + \beta_i V_0^i, \tag{1}$$

$$V_t^i = u(c_t^{y,i}) + \delta u(c_{t+1}^{o,i}) + \beta_i V_t^i, \tag{2}$$

for all integers $t \geq 0$. The parameters $\delta > 0$ and $\beta_i \in [0, 1)$ define the *within generation* and the *inter-generations* discount rates, respectively. The per period utility function, $u(.)$, satisfies standard properties, *i.e.*, it is increasing, strictly concave, and it verifies the boundary (Inada) conditions

$$\lim_{c \to 0} u'(c) = \infty, \qquad \lim_{c \to \infty} u'(c) = 0.$$

To complete the model, I assume that output is produced with a Neoclassical technology described by a CRTS production function using capital and labor as inputs. In particular, denoting with k_t the time-t stock of capital, and with $f(k)$ the intensive form (per capita) production function, aggregate output is $y = f(k)$. As usual, we let f be increasing and strictly concave, and such that $\lim_{k \to 0} f'(k) = \infty$, $\lim_{k \to \infty} f'(k) = 0$. For simplicity, I assume that capital depreciates in one period. Then, at all periods, $t \geq 0$, the following feasibility constraints hold

$$\sum_i m_i (c_t^{o,i} + c_t^{y,i}) = f(k_t) - k_{t+1}. \tag{3}$$

The market economy is characterized by private ownership of capital, perfectly competitive firms and optimal consumption decisions subject to budget constraints. In particular, all type-i individuals born at $t \geq 0$ maximize utility as defined by (1) subject to the period-by-period budget constraints

$$c_t^{y,i} + a_{t+1}^i = w_t + b_t^i, \tag{4}$$

$$c_{t+1}^{o,i} + b_{t+1}^i = R_{t+1} a_{t+1}^i, \tag{5}$$

with respect to $(c_t^{y,i}, c_{t+1}^{o,i}, a_{t+1}^i, b_{t+1}^i)$, where

$$w_t = f(k_t) - k_t f'(k_t), \qquad R_{t+1} = f'(k_{t+1}),$$

are the wage rate and the gross interest rates at time t and $t+1$. Evidently, the initial generation maximizes the utility function defined by (1) with respect to $(c_0^{o,i}, b_0^i)$, subject to

$$c_0^{o,i} + b_0^i = R_0 a_0^i, \tag{6}$$

for given a_0^i. Since altruism is one-sided, utility maximization is subject to the non-negativity constraint $b_{t+1}^i \geq 0$.

A *competitive equilibrium* is a sequence of consumptions and capital stocks, $\{c_t^{o,i}, c_t^{o,i}, k_{t+1}; i = a, s\}_{t=0}^{\infty}$, verifying the budget constraints, the feasibility constraints (3) and the first order conditions

$$\begin{aligned} \delta u'(c_t^{o,i}) &\geq \beta_i u'(c_t^{y,i}), &(7)\\ u'(c_t^{y,i})/\delta u'(c_{t+1}^{o,i}) &= R_{t+1} = f'(k_{t+1}), &(8)\end{aligned}$$

for all i and $t \geq 0$ and with (7) being verified with inequality only if $b^i = 0$. An equilibrium steady state is a time invariant allocation of age and individual specific consumptions and capital such that

$$f'(k) = R = 1/\beta_a, \qquad w = f(k) - kf'(k) \equiv w(k), \qquad b^i \geq 0$$

and, for $i = a, s$,

$$\begin{aligned} \delta u'(c^{o,i}) &\geq \beta_i u'(c^{y,i}), &(9)\\ u'(c^{y,i})/\delta u'(c^{o,i}) &= R, &(10)\\ c^{y,i} + c^{o,i}/R &= w(k) + b^i(1 - 1/R), &(11)\\ \sum_i m_i(c^{o,i} + c^{y,i}) &= f(k) - k. &(12)\end{aligned}$$

Notice that, by the Inada conditions, a steady state equilibrium generates a positive level of consumption for both types of individuals at all ages.

Steady state equilibria may be of two alternative types, depending on the parameter values. The first corresponds to the canonical equilibrium of the overlapping generations economy such that bequests are zero for all individuals. This implies perfect equality of consumption across individuals and a capital stock, k^*, such that

$$u'(w^* - k^*) = \delta R^* u'(R^* k^*), \qquad w^* = f(k^*) - k^* f'(k^*), \qquad R^* = f'(k^*),$$

if and only if $\beta_a \leq 1/f'(k^*)$.

The second type implies separation between altruistic and selfish individuals, with the former leaving positive and the latter zero bequests. This steady state holds when $\beta_a > 1/f'(k^*)$ and it is characterized by a capital stock k^o and a gross interest rate, $R^o > R^*$, such that

$$R^o = f'(k^o) = 1/\beta_a.$$

Evidently, since $1/f'(k^o) = \beta_a > 1/f'(k^*)$, we have $f'(k^*) > f'(k^o)$ and, then, $k^o > k^*$. A steady state equilibrium with positive bequests exists under mild conditions. In particular, from now on I impose that the canonical overlapping generations steady state equilibrium delivers a unique $k^* > 0$, *i.e.*, there is a unique $k^* > 0$ solving $k = s(w(k), f'(k))$, where $w(k) = f(k) - kf'(k)$ and $s(w, R)$ is the unique solution to

$$u'(w - s(w, R)) = \delta R u'(Rs(w, R)).$$

Proposition 1. *If $\beta_a > 1/f'(k^*)$, there exists a unique steady state equilibrium with positive bequests.*

Notice that, by the Inada conditions, a steady state equilibrium generates a positive level of consumption for both types of individuals at all ages, a higher lifetime utility and present value of income for the altruistic individuals compared with the selfish. The latter property comes about because $R = 1/\beta_a > 1$. In fact, since $c^y = w + b - s(w, b)$ and $c^o = Rs(w, b) - b$, by the properties of the savings function, we derive

$$c^y_b = 1 - s_b = (1 - s_w)(1 - \beta_a), \qquad c^o_b = s_b/\beta_a - 1 = s_w(1 - \beta_a)/\beta_a.$$

Hence, for all $b > 0$,

$$w + b - s(w, b) > w - s(w, 0), \qquad Rs(w, b) - b > Rs(w, 0).$$

Now assume

$$u(c) = \log c, \qquad f(k) = k^\alpha,$$

for some $\alpha \in (0, 1)$. Then, it is readily verified that, at a positive bequests steady state, the equilibrium individual bequests verify

$$m_a b^a = \frac{w^o}{\beta_a + s_w(1-\beta_a)} \left(\frac{\alpha}{1-\alpha}\beta_a - s_w \right),$$

where

$$w^o = (1-\alpha)(\alpha\beta_a)^{\frac{\alpha}{1-\alpha}}, \qquad s_w = \delta/(1+\delta).$$

The above steady state holds for

$$\beta_a > \left(\frac{1-\alpha}{\alpha} \right)(1-s_w),$$

and it generates the following young age consumptions:

$$c^{y,a} = (1-s_w)(w^o + (1-\beta_a)b^a), \qquad c^{y,s} = (1-s_w)w^o.$$

It follows that, in this example, consumption inequality at young age can be measured by

$$\text{(13)} \qquad \frac{c^{y,a}}{c^{y,s}} = 1 + \frac{(1-\beta_a)}{m_a[\beta_a + s_w(1-\beta_a)]} \left(\frac{\alpha}{1-\alpha}\beta_a - s_w \right).$$

3. Utilitarian Social welfare

I assume throughout that the intergenerational discount rates, β_i, are public information and, for some $\gamma \in (\beta_a, 1)$, I define the *Utilitarian Social Welfare* (USW) as

$$\text{(14)} \qquad \mathcal{V} = \sum_i m_i \left(V_{-1}^i + \sum_{t=0}^{\infty} \gamma^{t+1} V_t^i \right).$$

The Planner's time discount rate γ plays an important role. In particular, I assume that the latter is smaller than one in order to make $\mathcal{V}$ bounded and well defined, and to be greater than β_i for all $i = a, s$ in order to make the Planner as much equalitarian as possible (*i.e.*, as it is allowed by the requirement that $\mathcal{V}$ is well defined).

Definition 1. *A Utilitarian Social Welfare (USW) program is an array, $\{c_t^{o,i}, c_t^{y,i}, k_{t+1}; i = a, s\}$, that maximizes $\mathcal{V}$, as defined in (14), subject to the sequence of feasibility constraints (3) for $t \geq 0$ and some given $k_0 > 0$.*

By rearranging the terms in $\mathcal{V}$, it is readily verified that

$$\mathcal{V} = \sum_{t=0}^{\infty} \gamma^t \sum_i m_i \left(\rho_t^i \delta u(c_t^{o,i}) + (\gamma + \beta_i \rho_t^i) u(c_t^{y,i}) \right),$$

where

$$\rho_t^i = 1 + (\beta_i/\gamma) + .. + (\beta_i/\gamma)^t = \frac{1 - (\beta_i/\gamma)^{t+1}}{1 - (\beta_i/\gamma)}.$$

Hence, individuals' per-period utilities are weighted differently at all time periods, with $\rho_t^s < \rho_t^a$ for all $t \geq 1$. Furthermore, observe that

$$\rho_t^i \geq \gamma + \beta_i \rho_t^i \quad \Leftrightarrow \quad \frac{1-\gamma}{1-\beta_i} \geq \left(\frac{\beta_i}{\gamma} \right)^{t+1},$$

i.e., the Planner's weight on the type-i old individuals's utility is greater than the one on the contemporaneous type-i young's utility for any large enough t. These observations are useful for understanding the characteristics of a USW.

Proposition 2. *A USW program is a sequence $\{c_t^{o,i}, c_t^{y,i}\}_{t=0}^{\infty}$ such that*

$$\delta u'(c_t^{o,i}) = \beta_i u'(c_t^{y,i}) \left(1 + \frac{\gamma - \beta_i}{\beta_i - (\beta_i/\gamma)^{t+1}} \right), \tag{15}$$

$$u'(c_t^{y,i})/\delta u'(c_{t+1}^{o,i}) = f'(k_{t+1}), \tag{16}$$

$$u'(c_t^{y,s})/u'(c_t^{y,a}) = \left(\frac{\gamma - \beta_s}{\gamma - \beta_a} \right) \left(\frac{1 - (\beta_a/\gamma)^{t+2}}{1 - (\beta_s/\gamma)^{t+2}} \right), \tag{17}$$

for all $i = a, s$ and $t \geq 0$.

Observe that, by the above proposition, the asymptotic USW consumption allocations satisfy

$$\lim_{t \to \infty} \delta u'(c_t^{o,i})/\beta_i u'(c_t^{y,i}) = \gamma/\beta_i > 1, \tag{18}$$

$$\lim_{t \to \infty} u'(c_t^{y,s})/u'(c_t^{y,a}) = (\gamma - \beta_s)/(\gamma - \beta_a) > 1. \tag{19}$$

In particular, for $u(c) = \log c$, we derive that the asymptotic ratio between the young age consumptions is

$$\lim_{t\to\infty} \frac{c^{y,a}}{c^{y,s}} = 1 + \frac{\beta_a - \beta_s}{\gamma - \beta_a}. \tag{20}$$

Hence, proposition 2 make two predictions. First of all, by (19), the selfish individuals get lower consumption than the altruistic and they approach starvation (irrespective of the size of their intergenerational discount rate) as the Planner's discount rate approaches β_a. This looks like a race: your descendants may have very low consumption no matter how much you care about your offsprings, as long as some other dynasties are more caring. The utilitarian criterion tolerates this outcome for the seek of total welfare maximization. These observations imply that the degree of inequality at the USW may very well be asymptotically higher than the one at a competitive equilibrium for low values of γ whatever is the initial allocation of capital ownership, since in a competitive equilibrium with one-sided altruism bequests are bounded below by zero (which implies a lower bound on the selfish individuals' consumption).

The second important result, highlighted in equation (18), is that the inter-generations allocation of consumption at the USW and at equilibrium (cf. (15) and (7)) are consistent only if, at equilibrium, the non-negativity condition $b_t^i \geq 0$ is binding for all i and $t \geq 0$. The intuition is that, when bequests are positive, the Planner takes into account the young benefit from receiving additional bequests, whereas the old are just indifferent about any additional marginal transfer to their children. Then, condition (18) is compatible with voluntary bequests only if they are at the zero lower bound. However, in this model (where wages are not individual specific) this would generate consumption equalization across different types, a condition that cannot be verified at the USW. In showing this result, Bernheim (1989) claims that a "first best policy entails a redistribution of resources that gives rise to an equilibrium with no bequests (Bernheim (1989), p. 119)." However, I should add that, in my model, no competitive equilibrium with zero bequests can be a welfare optimum because, with zero bequests, a competitive equilibrium implies consumption equalization, which contradicts condition (19). In particular, suppose that the government implements the USW program through (binding) lower limits on bequests, $\bar{b}^i \geq 0$, for $i = a, s$. Then, with log utility, a possible implementation provides

$$\bar{b}^a = \frac{w(\beta_a - \beta_s)}{(1-\gamma)(\gamma - \beta_a)}, \qquad \bar{b}^s = 0,$$

where $w = f(k) - kf'(k)$, $f'(k) = 1/\gamma$.

Observe that the degree of inequality implied by the USW program is very likely to be larger than the one implied by the competitive equilibrium based on back of the envelope calculations. This can be seen by comparing the asymptotic measures of inequality at equilibrium and at the USW plan, expressed in equations (13) and (20), respectively, and under the assumptions $u(c) = \log c$, $f(k) = k^{\alpha}$. I consider the following values

$$\beta_s = 0.5,\ \gamma = 0.98,\ s_w = 0.2,\ \alpha = 0.3,\ m_a = 0.4,$$

and consider any value of $\beta_a \in (0.5, 1)$. The values for s_w and α are in line with the available empirical evidence and the value 0.98 for γ implies that the Planner is extremely patient and equalitarian across generations of individuals. Taking 0.5 as the lower bound for β_i guarantees the existence of an equilibrium steady state with positive bequests. Based on these assumptions one can verify that the degree of consumption inequality at equilibrium is greater than the one at the USW plan only if $\beta_a \leq 0.52$, *i.e.*, only 0.02 points above β_s. For all $\beta_a > 0.52$ the market generates less inequality than the social optimum.

A final remark is that, as highlighted in Bernheim (1989), the USW program is not time consistent. In particular, let $\mathcal{V}_T$ the utilitarian social welfare function for a Planner that evaluates the optimal program starting from time $T \geq 0$ and, for all $t \geq T$ and $i = a, s$, let $(c_t^{o,i}(T), c_t^{y,i}(T))$ be the optimal consumption allocation maximizing $\mathcal{V}_T$ subject to feasibility. Then, this verifies the following two key conditions:

$$\begin{aligned}\delta u'(c_t^{o,i}(T))/u'(c_t^{y,i}(T)) &= 1 + (\alpha/\beta_i)\left(\frac{1-\beta_i/\alpha}{1-(\beta_i/\alpha)^{t+1-T}}\right),\\ u'(c_t^{y,s}(T))/u'(c_t^{y,a}(T)) &= \left(\frac{\alpha-\beta_s}{\alpha-\beta_a}\right)\left(\frac{1-(\beta_a/\alpha)^{t+2-T}}{1-(\beta_s/\alpha)^{t+2-T}}\right).\end{aligned}$$

The first condition is a measure of how large must be the implied lower bound on bequests if the USW program was decentralized as a competitive equilibrium with "regulated" bequests. The second condition is a measure of consumption inequality. Using the above we can compare the t-period consumption allocation with commitment, $(c_t^{o,i}(0), c_t^{y,i}(0))$, *i.e.*, the one studied above in this section, and the one that follows if the Planner re-evaluates the program at time t, *i.e.*, $(c_t^{o,i}(t), c_t^{y,i}(t))$. This comparison leads to

$$\frac{\delta u'(c_t^{o,i}(0))}{u'(c_t^{y,i}(0))} = 1 + \left(\frac{\alpha}{\beta_i}\right)\left(\frac{1-\beta_i/\alpha}{1-(\beta_i/\alpha)^{t+1}}\right) < 1 + \left(\frac{\alpha}{\beta_i}\right) = \frac{\delta u'(c_t^{o,i}(t))}{u'(c_t^{y,i}(t))}$$

and

$$\frac{u'(c_t^{y,s}(0))}{u'(c_t^{y,a}(0))} = 1 + \left(\frac{\alpha-\beta_s}{\alpha-\beta_a}\right)\left(\frac{1-(\beta_a/\alpha)^{t+2}}{1-(\beta_s/\alpha)^{t+2}}\right) > \left(\frac{\alpha+\beta_a}{\alpha+\beta_s}\right) = \frac{u'(c_t^{y,s}(t))}{u'(c_t^{y,a}(t))}.$$

Then, the solution with commitment implies smaller lower bounds on bequests (*i.e.*, relatively higher old age consumption) and more consumption inequality than the solution obtained when the Planner is allowed to re-optimize at time t.

4. Self-Enforcing Social Welfare

Now I assume that individuals, whenever they are born, have some outside option, as an alternative to the level of consumption that they are assigned by the Planner. Namely, they can reject the Planner's "offer" and insure, for them self and their offsprings, some "minimum" levels of utility at young and old age respectively. This implies that the planning optimum is subject to a set of reservation utilities to be assigned to each individual at all stages of their life,

$$\{U_t^{o,i}, U_t^{y,i}; i = a, s\}_{t=0}^{\infty},$$

where $U_t^{o,i}$ represents the reservation utility of type-i old individual born at any time $t-1$ and $U_t^{y,i}$ the reservation utility of the type-i young individual born at any time t, for all $t \geq 0$.

A possible way to generate each pair $(U_t^{o,i}, U_t^{y,i})$ is by defining a given stream of consumption levels, $\{e_t^{y,i}, e_t^{o,i}\}_{t=0}^{\infty}$, to which individuals are able to fall back to when rejecting the Planner's offer. In this case, let

$$\bar{V}_{-1}^i = \delta u(e_0^{o,i} - b_0) + \beta_i \bar{V}_0^i$$

and, for all $t \geq 0$,

$$\bar{V}_t^i(b_t) = u(e_t^{y,i} + b_t) + \delta u(e_{t+1}^{o,i} - b_{t+1}) + \beta_i \bar{V}_{t+1}^i(b_{t+1}).$$

The above represent the utilities of the initial old and of all the young individuals when the Planner's offer is rejected, and b_t is the bequest to the type-i individual born at t from the type-i individual born in the previous period. Furthermore, define the value function from the recursive maximization:

$$W_t^i(b_t) = \max_{b \geq 0} \left\{ u(e_t^{y,i} + b_t) + \delta u(e_{t+1}^{o,i} - b) + \beta_i W_{t+1}^i(b) \right\}.$$

The above problems generate an optimal amount of bequests, $\{\bar{b}_t^i\}_{t=0}^{\infty}$, such that

$$\delta u'(e_t^{o,i} - \bar{b}_t^i) \geq \beta_i u'(e_t^{y,i} + \bar{b}_t^i), \quad (\delta u'(e_t^{o,i} - \bar{b}_t^i) - \beta_i u'(e_t^{y,i} + \bar{b}_t^i))\bar{b}_t^i = 0.$$

In particular, $\bar{b}_t^i = 0$ for all $t \geq 0$ when $\delta u'(e_t^{o,i}) \geq \beta^i(e_t^{y,i})$ for all $t \geq 0$. Then, we can define

$$U_t^{y,i} \equiv W_t^i(\bar{b}_t^i), \qquad U_t^{o,i} = \delta u(e_t^{o,i} - \bar{b}_t^i) + \beta_i W_t^i(\bar{b}_t^i).$$

Definition 2. *A Self-Enforcing Social Welfare (SESW) allocation is an array,* $\{c_t^{o,i}, c_t^{y,i}, k_{t+1}; i = a, s\}$*, that maximizes*

$$\mathcal{U} = \sum_i m_i V_{-1}^i$$

subject to

$$\begin{aligned}
\delta u(c_t^{o,i}) + \beta_i V_t^i &\geq U_t^{o,i}, && (21)\\
V_t^i &\geq U_t^{y,i}, && (22)\\
\sum_i m_i(c_t^{y,i} + c_t^{o,i}) &\leq f(k_t) - k_{t+1}, && (23)
\end{aligned}$$

for all $t \geq 0$ *and some given* k_0*, where (21), (22) define the individuals' participation constraint and (23) the feasibility constraint.*

To find the planning optimum I use a lagrangean method, assuming that the lagrangean function is well defined. Namely, for some non-negative sequence of lagrange multipliers, $\{\eta_t^{y,i}, \eta_t^{o,i}, \lambda_t\}_{t=0}^\infty$, associated to the constraints (21), (22) and (23), let

$$\mathcal{L}^i = V_{-1}^i + \sum_{t=0}^\infty \eta_t^{o,i}\beta_i^t[\delta u(c_t^{o,i}) + \beta_i V_t^i - U_t^{o,i}] + \sum_{t=0}^\infty \eta_{t+1}^{y,i}\beta_i^{t+1}[V_t^i - U_t^{y,i}].$$

Then, the lagrangean function for the planning optimum is defined as

$$\mathcal{L} = \sum_i m_i \mathcal{L}^i - \sum_{t=0}^\infty \lambda_t \left(\sum_i m_i(c_t^{y,i} + c_t^{o,i}) + k_{t+1} - f(k_t) \right).$$

Using the above expressions, we can state the first order conditions associated to the SESW plan as follows:

$$\begin{aligned}
\delta u'(c_t^{o,i}) &= \beta_i u'(c_t^{y,i}) \left(1 + \frac{\eta_{t+1}^{y,i}}{1 + \xi_t^i} \right), && (24)\\
u'(c_t^{y,i})/\delta u'(c_{t+1}^{o,i}) &= f'(k_{t+1}) \left(1 + \frac{\eta_{t+1}^{o,i}}{1 + \xi_t^i + \eta_{t+1}^{y,i}} \right). && (25)
\end{aligned}$$

where

$$\xi_t^i = \eta_0^{o,i} + \sum_{j=1}^t \left(\eta_j^{o,i} + \eta_j^{y,i} \right)$$

and

$$\eta_t^{o,i}[\delta u(c_t^{y,i}) + \beta_i V_t^i - U_t^{o,i}] = \eta_{t+1}^{y,i}[V_t^i - U_t^{y,i}] = 0$$

for all $t \geq 0$.

One can easily show that, if the participation constraints were not binding at all $t \geq 0$ for the selfish individual (*i.e.*, if $\eta_t^{o,s} = \eta_{t+1}^{y,s} = 0$ for all $t \geq 0$), the planning optimum would imply that she gets zero age contingent consumptions asymptotically. However, this would violate the participation constraints. Hence, at a SESW, the participation constraint of the selfish individuals are eventually binding. Following this argument, a time-invariant (or stationary) SESW program is characterized by the following conditions

$$\delta u'(c^{o,a}) = \beta_a u'(c^{y,a}), \tag{26}$$

$$\delta u'(c^{o,s}) > \beta_s u'(c^{y,s}), \tag{27}$$

$$f'(k) = u'(c^{y,a})/\delta u'(c^{o,a}) = 1/\beta_a, \tag{28}$$

$$f'(k) < u'(c^{y,s})/\delta u'(c^{o,s}), \tag{29}$$

together with

$$u(c^{y,a}) > U^{y,a} - U^{o,a}, \quad \delta u(c^{o,a}) > U^{o,a} - \beta_a U^{y,a}, \tag{30}$$

$$u(c^{y,s}) = U^{y,s} - U^{o,s}, \quad \delta u(c^{o,s}) = U^{o,s} - \beta_s U^{y,s}. \tag{31}$$

The first order conditions (26)-(29) suggest that a SESW could be decentralized as a competitive equilibrium when the non-negativity constraint on bequests is binding and when capital income is subsidized. In particular, (27) corresponds to the first order condition defining the optimal bequest at a competitive equilibrium when the non-negative bequest constraint is binding and (29) can be interpreted as a first order condition defining the optimal savings when the selfish individuals are forced to save more than they would in the absence of a subsidy.

5. Decentralizing the Self-Enforcing Social Welfare

One problem with the notion of SESW is that the reservation utilities, $U^{o,i}$, $U^{y,i}$, are arbitrary. By changing them, the Planner faces a trade-off between total welfare and inequality. In fact, the Planner's solution implies that the participation constraints of the individuals born in the selfish dynasty is binding whenever their reservation utilities are larger than some given thresholds, $\hat{U} = (\hat{U}^{o,s}, \hat{U}^{y,s})$, as a function of the parameters of the model. Hence, when reservation utilities are lower than $\hat{U}$, we obtain the non binding solution and total welfare, $\mathcal{U} = \sum_i m_i V^i_{-1}$, is independent of $\hat{U}$. If, on the other hand, reservation utilities are larger than $\hat{U}$, the Planner's objective function, $\mathcal{U}$, decreases with $\hat{U}$. In other words, if the Planner can modify $\hat{U}$, she has to strike a balance between total welfare and the degree of end-generation inequality.

A natural way to resolve this dilemma is to assume that the reservation utilities correspond to the indirect utilities that the selfish individuals would get by renouncing to any intergenerational transfer, selling their labor services at the market wage and save as much as they would like by purchasing securities in a competitive capital market. I call this the *self-made-man* (SMM) reservation utilities. Evidently, the Planner (the government) may be tempted to manipulate market prices or introduce taxes and transfers so as to affect the values of the SMM reservation utilities for the sake of rising total welfare $\mathcal{U}$. But this would increase inequality. I assume that a given balance between total social welfare and welfare inequality is part of the Planner's objective and that these objectives cannot be modified.

I show now that a stationary SESW program can be decentralized as a stationary competitive equilibrium with a capital subsidy for the selfish individuals financed by the young altruistic individuals.

Assume that $\beta_a > 1/f'(k^*)$, where $k^* > 0$ is the unique equilibrium capital stock of the canonical overlapping generations economy (with zero bequests), and consider a steady state equilibrium with positive bequests where the government sets a per unit of capital subsidy, τ, financed by a lump-sum tax, T, on the young altruistic individuals. Then, this equilibrium is characterized by an array, $(\tilde{s}^i, b^i, k^o; i = a, s)$, of type-contingent savings, bequests and a stock of capital satisfying the following set of equations,

$$
\begin{aligned}
&(32) \qquad & u'(w^o + b^a - T - \tilde{s}^a) &= (\delta/\beta_a)u'(\tilde{s}^a/\beta_a - b^a), \\
&(33) \qquad & u'(w^o - \tilde{s}^s) &= (\delta/\beta_a)(1+\tau)u'((1+\tau)\tilde{s}^s/\beta_a), \\
&(34) \qquad & k^o &= m_a\tilde{s}^a + m_s\tilde{s}^s, \\
&(35) \qquad & f'(k^o) &= 1/\beta_a,
\end{aligned}
$$

where $w^o = f(k^o) - k^o/\beta_a$ and

$$T = (m_s/m_a)\tau\tilde{s}^s/\beta_a.$$

It is evident that, for all $\tau \leq (\beta_a - \beta_s)/\beta_s$, the above conditions imply

$$
\begin{aligned}
\delta u'(\tilde{s}^a/\beta_a - b^a) &= \beta_a u(w^o + b^a - T - \tilde{s}^a), \\
\delta u'(\tilde{s}^s/\beta_a) &\geq \beta_s u(w^o - \tilde{s}^s),
\end{aligned}
$$

so that $b^s = 0$. The array $(\tilde{s}^i, b^i, k^o; i = a, s)$ satisfying the above equations is called a τ-*equilibrium steady state with positive bequests.*

Proposition 3. *For $\beta_a > 1/f'(k^*)$ and any small enough capital subsidy, $\tau < (\beta_a - \beta_s)/\beta_s$, there exists a τ-equilibrium steady state with positive bequests.*

Now define the indirect utility functions

$$
\begin{aligned}
U^y(\tau) &= [u(w^o - s(w^o, 0)) + \delta u((1+\tau)s(w^o, 0)/\beta_a)]/(1-\beta_s), \\
U^o(\tau) &= [\beta_s u(w^o - s(w^o, 0)) + \delta u((1+\tau)s(w^o, 0)/\beta_a)]/(1-\beta_a).
\end{aligned}
$$

The above are the reservation utilities for a SESW stationary plan decentralized as a τ-equilibrium steady state.

Proposition 4. *Let $\beta_a > 1/f'(k^*)$ and $\tau < (\beta_a - \beta_s)/\beta_s$ small enough. Then, a stationary SESW plan with reservation utilities $(U^y(\tau), U^o(\tau))$ can be decentralized as τ-equilibrium steady state.*

Observe that, by the envelope theorem, $U^y(\tau)$ and $U^o(\tau)$ are both increasing in τ. Then, by increasing τ, the Planner (government) trades off less total social welfare (efficiency) for more equality. With log utility and Cobb-Douglas production function, $f(k) = k^{\alpha}$, the market economy delivers

$$b(\tau) = \frac{(\alpha/(1-\alpha))\beta_a - s_w + m_s s_w^2 \tau/\beta_a}{m_a(s_w(1-\beta_a) + \beta_a)},$$

and

$$\frac{c^{y,a}}{c^{y,s}} = 1 + \frac{(\alpha/(1-\alpha))\beta_a - s_w - m_s s_w \tau}{m_a(s_w(1-\beta_a) + \beta_a)}.$$

Hence, somewhat counterintuitively, a higher τ reduces inequality but increases bequests. The letter effect comes about because, with log utility, total saving decreases with τ, as the altruistic individual's saving falls with τ and the selfish individuals's is unaffected. Then, a higher τ generates an excess demand of loanable funds. Since the interest rate is fixed at the altruistic discount rate, this imbalance can only be eliminated through a rise in bequests, which, in turn, increase total savings.

6. Discussion of the Key Assumptions

The heterogeneity and persistence of the degree of parental altruism across generations born from the same ancestors is a key feature of my model and, certainly, a controversial assumption. I offer here some motivations as to why I think this is a useful albeit simplistic starting point.

First, there have been attempts to model increasing dispersion in individuals' wealth levels based on market imperfections (eg., incomplete markets or debt limits). It is well known, however, that, even accounting for these imperfections, the existing level of wealth inequality across citizens and nations over the long run is not easily replicated in the standard Neoclassical growth model, unless one assumes a somewhat unrealistic level of income inequality (see Castaneda et al. (2003)). If we impose a large degree of heterogeneity in households' initial asset positions coupled with identical time discount rates and realistic income shocks, we run into the problem that (initially) rich individuals tend to save much less than the (initially) poor over their lifespan, so that large differences in wealth accumulation will eventually narrow down[6]. As shown in Cordoba (2008), debt limits are an even less plausible explanation. Then, a recent and growing literature has tried to explain the existing high level of wealth inequality by assuming that individuals' savings rates are increasing in wealth, and this feature can be obtained by assuming joy of giving or heterogeneous subjective time discount rates[7]. In particular, Krusell and Smith (1998), Sun (2013) and Hubmer et al. (2016) assume that discount rates are stochastic with a high degree of persistence[8]. Some of the results obtained in this paper may survive under this more general assumption.

Alternatively, one may follow the idea that the subjective discount rate is inherited or determined by cultural transmission within the family. In fact, there is some evidence that the propensity of leaving bequests depends on cultural traits. For instance, using survey data, Laitner and Juster (1996) find evidence of a surprising heterogeneity of values and preference for intentional bequests despite the homogeneity of earnings, occupation, and education and, to explain why the 1995 wealth ratio of blacks in the US was much lower than the corresponding income ratio (0.48 versus 0.17), Wolff (1999) documents that only 11% of blacks receive inheritance, whereas 24%

[6]See De Nardi and Fang (2015), De Nardi (2004) and Benhabib and Bisin (2017) for surveys of the literature trying to reconcile the observed high concentration of wealth with standard models of savings-consumptions decisions.

[7]An alternative route not followed in this paper is to assume that rates of return are increasing in wealth.

[8]In Hubmer et al. (2016), the discount rate of an individual follows an AR(1) process, *i.e.*, $\beta_t = \rho\beta_{t-1} + (1-\rho)\mu + \sigma\epsilon_t$, where ρ is calibrated to 0.992 and ϵ_t is a white noise normally distributed.

of white do. A possible formalization of the cultural transmission process may be based on Bisin and Verdier (2001) and Jellal and Wolff (2002), *i.e.*, offsprings are subject to a *vertical transmission* process, whereby they learn the degree of altruism from their parents, and an *oblique transmission* process, whereby they learn it from members of the parents' generation. In a simple representation of this model, the distribution of the population converges to a degenerate distribution comprising the single type (selfish or altruistic) that have the higher ability (or make the higher effort) to transmit their own values (or traits), and a limit distribution with heterogeneous types is only possible if types have identical abilities. However, Bisin and Verdier (2001) show that, when the ability or effort to transmit one own's trait to the offsprings is decreasing in the share of the population with that trait, the limit distribution is non-degenerate. On an empirical ground, Jellal and Wolff (2002) finds empirical support of the cultural transmission of altruism.

7. Conclusion

This paper shows that, when intergenerational wealth transmission is non paternalistic and generated by heterogeneous degrees of parental altruism, the notion of social welfare based on purely utilitarian arguments is problematic. This problem has been raised by other authors in the past (cf. Bernheim (1989)). In particular, socially optimal inequality is likely to exceed the level achieved in a market economy when parental altruism is one-sided, and decentralization is essentially impossible by using standard economic policies. Hence, I have advanced an alternative notion of social optimality based on participation constraints and reservation utilities. The central result of the paper is that this notion of social optimum can be decentralized through capital subsidies.

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Appendix

Proof of proposition 1. Let $\beta_a = 1/f'(k^o) > 1/f'(k^*)$ and define the saving function $\tilde{s}(b)$ from

$$u'(w(k^o) + b - \tilde{s}(b)) = (\delta/\beta_a)u'(\tilde{s}(b)/\beta_a - b),$$

i.e., the optimal savings implicitly defined by equation (10) for $i = a, s$. Observe that $\tilde{s}'(b) = s_w + \beta_a(1 - s_w) > 0$, where $s_w = s_w(w(k^o), 1/\beta^a) \in (0,1)$ is the marginal propensity to save out of wage income, and $\lim_{b\to\infty} \tilde{s}(b) = \infty$. Since $\beta_s < \beta_a$, we have

$$\delta u'(\tilde{s}/\beta_a - b)/\beta_s u(w + b - \tilde{s}) = \beta_a u'(w + b - \tilde{s})/\beta_s u'(w + b - \tilde{s}) > 1.$$

Then, the optimal savings of the altruistic and selfish individuals are $s^a = \tilde{s}(b^a)$, $s^s = \tilde{s}(0)$, respectively. To determine the equilibrium value of bequests, it is sufficient to impose the capital market equilibrium condition

$$k^o = m_a\tilde{s}(b^a) + m_s\tilde{s}(0). \tag{A1}$$

Now observe that, by definition, $\tilde{s}(0) = s(w(k^o), f'(k^o))$, and by the uniqueness of $k^* > 0$, we have $k > s(w(k), f'(k))$ for all $k > k^* > 0$. Since we are assuming $\beta_a = 1/f'(k^o) > 1/f'(k^*)$, it is $k^o > k^*$. Now define the continuous function

$$\phi(b) = m_a\tilde{s}(b) + m_s\tilde{s}(0) - k^o$$

and notice that

$$\phi(0) = \tilde{s}(0) - k^o = s(w(k^o), f'(k^o)) - k^o < 0, \quad \phi(\infty) > 0.$$

Since $\phi'(b) > 0$, equation (A1) has a unique solution $b^a > 0$.

Proof of proposition 3. Let the function $\tilde{s}(b - T, \tau)$ be the unique solution to

$$u'(w^o + b - T - \tilde{s}) = (1 + \tau)(\delta/\beta_a)u'(\tilde{s}/\beta_a - b).$$

Evidently, $\tilde{s}(.)$ is increasing in $b - T$ and the first partial derivative of $\tilde{s}(.)$ is $\tilde{s}_1 = s_w + \beta_a(1 - s_w) > 0$, where $s_w \in (0,1)$ is the marginal propensity to save out of wage income. Moreover, $\lim_{b \to \infty} \tilde{s}(b - T, \tau) = \infty$. Now let $T(\tau) = (m_s/m_a)\tau\tilde{s}(0, \tau)/\beta_a$ and consider the function

$$\psi(b, \tau) = m_a\tilde{s}(b - T(\tau)) + m_s\tilde{s}(0, \tau).$$

Evidently, any value $b(\tau) > 0$ such that $\psi(b(\tau), \tau) = 0$ satisfies the capital market equilibrium condition and characterizes a τ-equilibrium steady state. Since $\psi_1(b, \tau) > 0$ and

$$\psi(0, 0) < 0 < \psi(b, 0)$$

for any large enough $b > 0$, there exists $b(0) > 0$ such that $\psi(b(0), 0) = 0$. By continuity and since $w(k^o) > 0$, there exists a small enough value τ such that $\psi(b(\tau), \tau) = 0$.