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Abstract

In business-cycle, macro models the elasticity of intertemporal substitution (EIS) governs the economy's response to demand shocks and policy changes ("multipliers"). With general non-separable preferences, the EIS is determined by consumption-hours complementarity and the income effect on hours. Complementarity helps generate business-cycle co-movement following demand shocks, fiscal multipliers, and allows reconciling low EIS with low income-wealth effects. Yet existing utility functions restrict either complementarity, or income effects---or both---and artificially imply that EIS is exclusively a function of either. I propose a novel utility function where both complementarity and the income effect are arbitrary and can be calibrated separately.

JEL Classification: D11, E21, E62, H31

Keywords: consumption-hours complementarity, business-cycle co-movement, income and wealth effects, elasticity of intertemporal substitution, Fiscal multipliers, news shocks

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Complementarity, Income, and Substitution: A $U(C, N)$ Utility for Macro[I]

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Abstract

In business-cycle, macro models the elasticity of intertemporal substitution (EIS) governs the economy's response to demand shocks and policy changes ("multipliers"). With general non-separable preferences, the EIS is determined by consumption-hours complementarity and the income effect on hours. Complementarity helps generate business-cycle co-movement following demand shocks, fiscal multipliers, and allows reconciling low EIS with low income-wealth effects. Yet existing utility functions restrict either complementarity, or income effects—or both—and artificially imply that EIS is exclusively a function of either. I propose a novel utility function where both complementarity and the income effect are arbitrary and can be calibrated separately.

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1 Introduction

If you are reading this, chances are you are a macroeconomist who at some point had to, and most likely will still have to, choose what utility function to use. Most likely, you had and have to specify the preferences of households over consumption and hours worked: how to do this, i.e. what utility function $U(C, N)$ should a macroeconomist use?

The answer depends on the topic being explored. Somewhat paradoxically but for good reasons, the analysis of *business cycles* followed for the most part King, Plosser, and Rebelo (1988, hereinafter KPR) to impose preference restrictions that deliver *long-run balanced growth*. Prominent exceptions from the business-cycle literature that introduced widely-used functional forms that are not consistent with balanced growth are MaCurdy's (1981) analysis of labor supply with constant relative risk aversion (CRRA) preferences; and Greenwood, Hercowitz, and Huffman (1988, hereinafter GHH). Jaimovich and Rebelo (2009) propose a time-nonseparable functional form that nests KPR and GHH and still delivers balanced growth except in the latter limit case.[1] Given these restrictions on preferences, business cycle analysis consists of deriving moments, co-movements and the like, all of which depend on deep parameters that are a direct consequence of the assumed functional form.

In this paper, I tackle this problem somewhat backwards: given a general $U(C, N)$ utility function that I characterize by a restricted set of parameters (elasticities), I study how some key macro business-cycle moments and co-movements depend on these utility parameters. Then, I propose a novel utility function that can generate those co-movements and has enough degrees of freedom to allow calibrating the necessary parameters; this function can be used in quantitative macro analyses of the business cycle, including models with heterogeneous agents.

The general preferences that I consider feature nonseparability between consumption and hours worked, with most of the (time-separable) functional forms mentioned above as special cases. As an existing literature has emphasized, such preferences are useful for fiscal multipliers (Bilbiie, 2009, 2011; Monacelli and Perotti, 2008), and for co-movement in response to investment-specific and news shocks (Beaudry and Portier, 2009, 2014; Jaimovich and Rebelo, 2009; Eusepi and Preston, 2009, 2015; Furlanetto and Seneca, 2015). While more recently, such preferences have also been used in incomplete-market models (see e.g. Kaplan, Moll, and Violante, 2017; Auclert and Ronglie, 2017). The paper will show how under nonseparable preferences one composite parameter—the elasticity of intertemporal substitution EIS—is key for all these issues, and how it is determined by the degree of

[1] More recently, Boppart and Krusell (2016) argue that there has been a secular downward trend in hours worked in the data, and propose a class of utility functions that are consistent with that stilized fact (they also provide an excellent review of the literature through the prism of balanced-growth restrictions)

complementarity between consumption and hours and by income effects on labor supply.

Yet functional forms that are currently used in the literature restrict this key part of the model artificially. In particular, MaCurdy's CRRA utility has arbitrary income effect, but no complementarity—so the EIS is pinned down by the income effect only; KPR's nonseparable utility has arbitrary complementarity but unitary income effect (as required for balanced-growth); while GHH preferences have zero income/wealth effect on labor supply—so the EIS is pinned down by consumption-hours complementarity only.[2]

What I propose here, instead of focusing on growth restriction, is to be mindful of restrictions for business cycle co-movement (and, with heterogeneous agents, for cross-section facts such as the existence and magnitude of income/wealth effect on hours worked). That is to consider that in the data income effects do exist, although they are perhaps not very strong: micro evidence such as i.a. Imbens, Rubin and Sacerdote (2001) and Kimball and Shapiro (2008) finds a range of wealth effects of unearned income on labor supply. Whereas with existing utility functions they are either 1 for KPR, zero for GHH (which is blatantly contradicted by the evidence), and free only for MaCurdy preferences that are separable and lack complementarity (intertemporal substitution is pinned down by the income effect, aka relative risk aversion)—which brings us to the second ingredient.

Complementarity between consumption and hours is supported by the data from a variety of sources.[3] Yet in business-cycle models, when it is present (and it is a key ingredient to fit several business cycle facts), it is often restricted and artificially related to other parameters, and thus artificially pins down uniquely the elasticity of intertemporal substitution. Indeed, as will become clear below the EIS goes to infinity with GHH preferences (by virtue of linearity, so of zero income effect), and goes to the inverse of the income effect (which is unitary) with KPR.

Here, I thus propose a utility function where both complementarity ξ and the income effect γ are free parameters that can be calibrated, and they jointly determine the elasticity

[2] This is also counterfactual, let alone that, as will become clear below, in an efficient equilibrium it implies infinite EIS—just like linear utility does.

[3] While direct evidence for C-N complementarity is alas not available (athough the functional form I propose here may help to eventually obtain it), Hall (2009a) argues that it implies that consumption rises with the wage at constant marginal utility—which further implies that marginal utility rises when an individual moves from unemployed to employed or when the individual works more hours. Hall reviews evidence supporting this, one of which is the "consumption retirement puzzle" studied i.a. by Aguiar and Hurst (2005). See also Kimball and Shapiro (2008, section 6.1) for a review of the literature supporting C-N complementarity in this context. Hall and Milgrom (2008) use C-N complementarity in a model with credible wage bargaining where unemployment has little influence on the wage, thus generating plausible unemployment fluctuationst.

of intertemporal substitution. The utility function takes the disarmingly simple form:

$$U(C,N)=\frac{1}{1-\frac{\xi}{1-\gamma}}\left(\frac{C^{1-\gamma}}{1-\gamma}-h\frac{N^{1+\varphi}}{1+\varphi}\right)^{1-\frac{\xi}{1-\gamma}},$$

where φ will still stand for constant-consumption labor supply. To those seasoned in business-cycle research, although this is novel, it will look very familiar: it applies a GHH-type operator (with a twist) to a MaCurdy-style CRRA function of consumption C and hours N, rather than to a function that is affine in consumption (which is GHH *stricto sensu*). The remainder of the paper is devoted to formalizing the points mentioned above, thus explaining the virtues of this simple utility function for business-cycle macro research.

2 A General Characterization of $U(C,N)$: Complementarity, Income, and Substitution

Consider a household who maximizes the expected discounted value of future utility of the general non-separable form:

$$U(C_t,N_t),$$

where C_t is consumption and N_t are hours worked. We assume that utility is increasing in both arguments $U_C>0, U_N<0, U_{CN}=U_{NC}\neq 0$ and concave.[4]. The household works and receives labor income, saves by buying riskless, one-period, discount government bonds B_{t+1} that cost $(1+I_t)^{-1}$ and deliver one unit of account next period, receive profits D_t from the firms and pays lump-sum taxes T_t; the budget constraint is:

$$(1+I_t)^{-1}B_{t+1}+P_tC_t=B_t+W_tN_t+D_t.$$

Maximizing lifetime utility subject to the budget constraint, taking prices as given, we obtain the labor supply, intratemporal optimality condition

$$-U_N(C_t,N_t)=U_C(C_t,N_t)\frac{W_t}{P_t} \tag{1}$$

[4]This implies $U_{CC}\leq 0; U_{NN}\leq 0$ and $U_{CC}U_{NN}-(U_{CN})^2\geq 0$ which insures that the Hessian of $U(.)$ is negative semidefinite. Strict concavity requires that the Hessian be negative definite, i.e. only the first and third inequality be satisfied with strong inequality. Quasi-concavity (which is implied by concavity) requires that the *bordered* Hessian be negative semidefinite, i.e. $2U_CU_NU_{CN}-U_{CC}(U_N)^2-U_{NN}(U_C)^2\geq 0$.

and the Euler equation governing intertemporal substitution, a manifestation of the fundamental asset pricing theorem:

$$\begin{aligned} Q_{t,t+1} &= \beta \frac{U_C\left(C_{t+1}, N_{t+1}\right)}{U_C\left(C_t, N_t\right)} \\ \frac{1}{1+R_t} &= E_t Q_{t,t+1} \end{aligned} \tag{2}$$

The consumption basket consists of a constant-elasticity of substitution aggregator of individual goods j, $C_t^{\frac{\theta-1}{\theta}} = \int_0^1 C_t(j)^{\frac{\theta-1}{\theta}} dj$, where $\theta > 1$ is the elasticity of substitution.Each of the differentiated goods $j \in [0,1]$ is produced by one monopolistic firm who employs labor using a decreasing returns to scale technology: $Y_t(j) = F(N_t) = Z_t N_t^{\alpha}(j)$ and receives a sales subsidy s. Each firm maximizes profits $D_t(j) = (1+s) P_t(j) Y_t(j) - W_t N_t(j)$ by choosing its own price $P_t(j)$, subject to the demand constraint $Y_t(j) = \left(P_t(j)/P_t\right)^{-\theta} C_t$, where P_t is the welfare price index $P_t^{1-\theta} = \int_0^1 P_t(\omega)^{1-\theta} dj$. Regardless of firms' ability to set their prices, we can define average gross markup (the inverse of real marginal cost) as:

$$\frac{\mu_t}{1+s} = \frac{F_N(N_t)}{W_t/P_t}, \tag{3}$$

where the dynamics of markup μ_t depend on the specific price-setting scheme and will be discussed in the loglinearized version of the model below. In equilibrium the goods market clears: $Y_t = C_t + G_t$. The model is closed by specifying price-setting, and a monetary policy rule; for simplicity, we choose a Taylor rule that includes a response to expected inflation:

$$1 + I_t = \beta^{-1}\left(1 + E_t \Pi_{t+1}\right)^{\phi_\pi}.$$

In a flexible-price equilibrium which we shall denote by a star superscript (whereby all firms are able to adjust their prices), the only optimality condition of firms is the constant-markup rule:

$$\frac{\mu}{1+s} = \frac{F_N(N_t^*)}{(W_t/P_t)^*},$$

where $\mu = \theta/(\theta-1)$ is the desired markup.

Consider a long-run equilibrium ("steady-state") whereby all variables are constant. Focus for simplicity on the case where government spending is zero $G = 0 \rightarrow C = Y$. The intratemporal optimality condition (1) combined with labor demand (3) delivers the labor share:

$$\frac{-U_N}{U_C}\frac{N}{C} = \frac{WN}{PY} = \alpha(1-\Phi), \tag{4}$$

where I defined the *steady-state distortion* following Woodford, 2003, Chapter 6:

$$\Phi = 1 - \frac{1+s}{\mu} \in [0,1]. \tag{5}$$

A value of 0 corresponds to an undistorted steady-state equilibrium (such as under an optimal subsidy $s = \mu - 1$) and the maximum of 1 obtains when the markup tends to infinity and the subsidy s is fixed. Several other distortions such as imperfect labor markets and a wage markup, or distortionary taxation could be added; they would change the exact expression for Φ but neither its interpretation nor the limit result that under appropriately designed Pigouvian taxes it can be brought down to zero to restore efficiency. The reader can thus safely regard Φ as a general index of supply distortions in this economy from now onwards.

2.1 Complementarity, Income Effects, and Intertemporal Substitution

The following definitions outline the key parameters describing preferences. A complete nomenclature of preferences in the context of intertemporal substitution in consumption and hours worked is already outlined in Barro and King (1984) and extended in some dimensions by Bilbiie (2009, 2011). Rather than reiterate those definitions, I focus here on the minimal set of ingredients needed for this paper's point.

Definition 1 ***Complementarity*** *between consumption and hours occurs when:*

$$\kappa \equiv \frac{U_{CN}C}{-U_N} > 0.$$

According to this definition C and N are complements in the Edgeworth sense ($U_{CN} > 0$) when $\kappa > 0$, and substitutes otherwise.

Definition 2 *The* ***income effect*** *on labor supply is:*

$$\gamma \equiv -\frac{U_{CC}C}{U_C} + \frac{U_{CN}C}{U_N}.$$

The income effect is most clearly seen by loglinearizing[5] (1) to obtain the *constant-consumption labor supply*:

$$\begin{aligned} \varphi n_t &= w_t - \gamma c_t, \\ \text{where } \varphi &\equiv \frac{U_{NN}N}{U_N} - \frac{U_{CN}N}{U_C} \end{aligned} \tag{6}$$

is the constant-consumption labor supply (inverse) elasticity. γ measures the income effect because it tells how, at given relative price of consumption and leisure, the labor supply schedule shifts when income changes (a more detailed description is in the Appendix). There is a one-to-one mapping between the 'original' derivatives of the utility function and the newly introduced φ, γ and κ since $U_{NN}N/U_N = \varphi + \kappa\alpha(1-\Phi)$ and $-U_{CC}C/U_C = \gamma + \kappa$.

Notice that with nonseparable preferences the *Frisch* (constant-marginal-utility) elasticity of labor supply is different; in particular (see Bilbiie 2011 and the Appendix for a derivation) Frisch labor supply is, denoting by mu_t the log-deviation of $U_C(.)$:

$$\left[\varphi + \frac{\kappa\gamma}{\gamma+\kappa}\alpha(1-\Phi)\right] n_t = w_t - \frac{\gamma}{\gamma+\kappa} mu_t. \tag{7}$$

The Frisch elasticity is the (inverse of the) term in square brackets, and it is always positive when U is concave. Frisch elasticity is lower than constant-consumption elasticity *if and only if* $\kappa > 0$, i.e. when consumption and hours are Edgeworth complements: when facing a higher wage, the household is willing to work relatively more hours to keep the level of consumption constant than it is to keep marginal utility of consumption constant. This distinction turns out to be crucial for the effects of shocks and co-movement.

As shown formally in Bilbiie (2009, 2011) the conditions for utility to be concave and quasiconcave and for consumption and leisure to be normal goods can be compactly summarized in terms of these parameters as:

$$\varphi \geq 0;\ \gamma \geq 0;\ \kappa \geq -\frac{\gamma\varphi}{\varphi + \gamma\alpha(1-\Phi)}. \tag{8}$$

In other words, labor supply should be upward-sloping, the income effect should be positive, and the degree of substitutability of consumption and leisure cannot be too high.

[5] To write the model using elasticities around the long-run equilibrium I take a first-order approximation to all optimality conditions, letting small case letters denote percentage deviations from steady state, e.g. for any variable X_t we have $x_t \equiv (X_t - X)/X$. The only exceptions from this are inflation and interest rates (which are already in percentage points), and government spending, whose steady-state value is assumed to be zero $G = 0$, therefore $g_t \equiv (G_t - G)/Y$.

The key parameter for business cycle analysis is the **elasticity of intertemporal substitution EIS**, defined formally as the partial derivative of relative demands of the consumption good at two different times with respect to the marginal rate of (intertemporal) substitution between the two goods, formally:

Definition 3 *The elasticity of intertemporal substitution EIS denoted by η is given by:*

$$\eta = \frac{d \ln \left(C_{t+1}/C_t \right)}{d \ln Q_{t,t+1}}$$

Since the marginal rate of intertemporal substitution $Q_{t,t+1}$ is equal to the (inverse) interest rate, the **EIS** is generally equal to the elasticity of consumption (growth) to real interest rates; this, in turn, is fully determined by the income effect on labor supply and complementarity, as emphasized in our main Theorem.

Theorem 1 *The elasticity of intertemporal substitution EIS is determined by the income effect γ and complementarity κ as follows:*

$$\eta = \left(\gamma + \kappa \Phi \right)^{-1} > 0,$$

where Φ is the index of supply-side distortions.

The proof follows immediately by taking a local approximation of the Euler equation (2):

$$\frac{U_{CC}C}{U_C} c_t + \frac{U_{CN}N}{U_C} n_t = \frac{U_{CC}C}{U_C} E_t c_{t+1} + \frac{U_{CN}N}{U_C} E_t n_{t+1} + r_t. \tag{9}$$

The resource constraint $y_t = c_t + g_t$ and production function $n_t = \frac{1}{\alpha} \left(y_t - z_t \right)$ respectively substituted in the Euler equation (9) deliver:

$$y_t = E_t y_{t+1} - \eta r_t + \eta \left(\gamma + \kappa \right) \left(g_t - E_t g_{t+1} \right) + \eta \kappa \left(1 - \Phi \right) \left(E_t z_{t+1} - z_t \right), \tag{10}$$

which illustrates that η in the Theorem governs intertemporal substitution in aggregate demand, or the inverse elasticity of aggregate demand to interest rate. Its being positive $\eta > 0$ follows from the restrictions (8), since $-\kappa < \gamma$ and $\Phi < 1$. A similar expression is derived for the first time in Bilbiie, 2011—but focusing on the topic of fiscal multipliers and co-movement.

As we shall see below, EIS η is the main determinant of the effects of shocks, so it is important to understand its determinants. Start with the simplest case, of separable utility, i.e. no complementarity $\kappa = 0$: the EIS is the standard, inverse of income effect on labor supply, γ^{-1} (likewise, in a non-distorted economy $\Phi = 0$). The intuition is straightforward: a given change in the marginal rate of intertemporal substitution (or interest rate) corresponds to a larger change in consumption today relative to tomorrow when γ is smaller. A smaller income effect means that equilibrium income needs to increase by relatively more to deliver a same shift in constant-consumption labor supply. In other words, intertemporal substitution and the income/wealth effect on labor supply are related one-to-one with separable preferences.

With complementarity (nonseparability) this is no longer the case, although as shown in the Theorem the EIS still depends negatively on the income effect, for the same reasons outlined above. But complementarity weakens this link, and the intuition why is simple.[6] The "income effect" γ now comprises a part beyond mere curvature in consumption, that is due to complementarity: whenever income/consumption changes, at given marginal utility and with consumption a function of hours worked, the cross-derivative consumption-hours also matters. That is, the income effect depends on complementarity. But the key element is that this dependence is not one-to-one, except in a special case. Indeed, the transformation of hours into consumption on the production side is not efficient: there is a wedge between the marginal rate of substitution and the marginal rate of transformation, captured by Φ. When this wedge is nil (such as when a system of Pigouvian taxes and subsidies restores efficiency) complementarity does not affect the EIS beyond the income effect; but when it is positive, complementarity needs to compensate for this inefficiency and induce the variation in hours that is necessary to deliver the consumption variation called for by intertemporal substitution.

Let us dissipate this potential source of confusion: complementarity does **not** imply more intertemporal substitution, and thus more effect of any shocks (such as monetary policy) that work almost exclusively through intertemporal substitution in this model. On the contrary—at given income effect, complementarity implies *less* elasticity of intertemporal substitution. But, as we shall study next, complementarity does imply more effects of some shocks, *despite* implying lower EIS!

Another set of important implications concerns empirical estimates of the EIS. The empirical literature estimating aggregate Euler equations finds very small value for this parameter, from Hall (1988) to e.g. Yogo (2004) and Bilbiie and Straub (2012). And since with separable preferences the EIS is the inverse of the income effect parameter (aka relative risk aversion) these low-EIS estimates are inconsistent with a wealth of micro evidence suggesting that income (wealth) effects on labor supply are typically small—and thus that γ is small;

[6] Epstein and Zin (1989) disentangle intertemporal elasticity of substitution from *risk aversion* by using a very different, (recursive) utility that is non-separable over time.

independent evidence on risk aversion (the other interpretation of this parameter) is in line with this.

Clearly, complementarity offers a way out of of this conundrum through the mechanism described above: a low EIS is consistent with a low income effect γ, if there is enough consumption-hours complementarity (high enough κ). An important caveat is that, as discussed above, this depends on the size of the supply–side distortion Φ: the lower this parameter (the closer the economy is to the efficient benchmark), the higher the degree of complementarity necessary to bring in line a low EIS with low income effects. In the efficient limit $\Phi = 0$, there is evidently no way out of the conundrum. In other words, small income effects necessarily imply high elasticity of intertemporal substitution (and, as we shall see, high multipliers); in the limit, the case of GHH preferences $\gamma = 0$ artificially implies infinite EIS (just like under linear separable utility)—a point also raised by Auclert and Ronglie (2017), cautioning that the use of GHH preferences has consequences that are often not immediately obvious in quantitative models.

3 Complementarity and co-movement: demand shocks, policies, and business cycles

The foregoing clarifies how one key parameter in any macro model, the elasticity of intertemporal substitution, depends on income effects and complementarity under general non-separable utility functions. To solve for the full equilibrium and study the implication of this for business cycles, we need to go back to the supply side and specify how firms set prices. It is by now well understood that a model with flexible prices cannot yield (absent other assumptions) realistic business cycles—understood as co-movements between the key macro variables consumption, output, and hours worked as well as, in a version with physical capital, investment—in response to demand shocks. The label "demand shocks" includes a large class of disturbances: interest-rate changes (be they driven by monetary policy or originating in the financial sector), changes in government spending, news shocks, or investment-specific technology shocks. This has been studied in turn, starting with Barro and King's seminal 1984 contribution, in Bilbiie (2009, 2011), Monacelli and Perotti (2008), and Auclert and Ronglie (2017) for government spending shocks; Beaudry and Portier (2006, 2014) and Jaimovich and Rebelo (2009) for news shocks; Justiniano, Primiceri and Tambalotti (2010, 2011) and Furlanetto and Seneca (2015) for investment-specific technology shocks; and Eusepi and Preston (2009, 2015) for generic demand (non-TFP) shocks.

Lets us restate the co-movement problem briefly here and refer the reader to the above papers (in particular Bilbiie, 2009; Beaudry and Portier, 2014; and Eusepi and Preston,

2015) for a detailed treatment and discussion of other solutions. The local approximation to the labor demand condition of firms (3) yields:

$$w_t = -(1-\alpha) n_t + mc_t + z_t, \tag{11}$$

where $\alpha = F_{NN}N/F_N < 1$ is the degree of returns to scale in labor and mc_t is the log-deviation of real marginal cost (the inverse of average markups)—the dynamics of which will depend on the specific price-setting scheme. Replacing this in constant-consumption labor supply we obtain:

$$\gamma c_t = -(\varphi + 1 - \alpha) n_t + mc_t + z_t,$$

which is a key equation for the argument. Clearly, absent movements in either TFP z_t or real marginal cost mc_t consumption and hours will co-move negatively—insofar as conditions of concavity and non-inferiority on preferences are satisfied.

Conversely, consumption and hours worked can co-move positively if and only if—absent TFP shocks—real marginal costs mc_t also move sufficiently. One example delivering the latter is, as is by now well understood in the literature, the assumption of sticky prices (labor demand may shift for other reasons, such as variable utilization). Adding the rest of the model (the approximated resource constraint and production function which combined deliver $n_t = \alpha^{-1}(c_t + g_t - z_t)$), we have the marginal cost schedule:

$$mc_t = \left(\gamma - 1 + \frac{\varphi + 1}{\alpha}\right) y_t - \gamma g_t - \frac{\varphi + 1}{\alpha} z_t. \tag{12}$$

Notice that while complementarity does not influence at all these conditions directly, it does influence via (10) the intertemporal substitution channel, which plays through marginal cost: Higher complementarity (more positive κ) means a larger effect on demand, so larger inflation which feeds back through supply. To understand this we need to complete the model: in addition to (10) and (12), we have to specify a price-setting scheme—which leads to an equation linking real marginal cost to inflation, a "Phillips curve"—and a Taylor rule specifying how the interest rate r_t responds to endogenous variables.

One issue with sticky price models is that they imply inflation variations that are often unrealistic, or require some degree of monetary accommodation for certain shocks to have the desired effects (see Beaudry and Portier, 2014). I first isolate the effect of complementarity by assuming that prices are completely fixed—as a consequence, I treat the interest rate r_t in (9) or (10) as exogenous. Under this simplifying assumption—first used by Bilbiie (2011) to look at fiscal multipliers—output is entirely demand-determined and the model boils down to one equation, (10).[7] The effects of "demand" (including policy) shocks are then immediately derived as described in the following Proposition.

Proposition 1 *The effects of interest rate cuts ($-r_t$), government spending increases (g_t) and news about future productivity improvements $\varepsilon_t \equiv E_t z_{t+1} - z_t$ (with given z_t) are given by, respectively:*

$$\begin{aligned}
&\textit{monetary:} && \frac{dy_t}{d(-r_t)} = \eta = (\gamma + \kappa\Phi)^{-1} \\
&\textit{fiscal:} && \frac{dy_t}{dg_t} = \eta(\gamma + \kappa) = \frac{\kappa + \gamma}{\kappa\Phi + \gamma} \\
&\textit{news:} && \frac{d(y_t - E_t y_{t+1})}{d\varepsilon_t} = \eta\kappa(1 - \Phi) \ \ \textit{and} \ \frac{d(n_t - E_t n_{t+1})}{d\varepsilon_t} = \frac{1 + \eta\kappa(1 - \Phi)}{\alpha}
\end{aligned}$$

As the Proposition makes clear, the EIS and its determinants, in particular the degree of complementarity, are key for *all* these effects; the rest of this Section discusses the intuition for each finding in turn.

Monetary policy and intertemporal substitution

The first part is very easy and follows immediately from our discussion in the previous section: the effect of a monetary expansion under fixed prices is given precisely by η, the elasticity of intertemporal substitution EIS discussed at length above. This is particularly useful in the context of heterogeneous-agent NK (HANK) models such as Kaplan, Moll, and Violante (2017); indeed, a working-paper draft of that paper and several other papers in the literature (see Auclert and Ronglie, 2017 for a related discussion and references) assume GHH preferences in order to do away with income effects in the cross-section. As obvious from our discussion above, that choice has drastic consequences regarding the effects of shocks and policies, as captured through the EIS. In particular, the EIS (and hence the effect of monetary policy) is *amplified* by the reduction of income effects on labor supply, at given complementarity. Whereas the EIS and the effect of monetary policy are in fact **dampened***, not amplified, by more complementarity*—at given income effect. GHH preferences mix the two because they eliminate income effects and feature a fortiori complementarity: but it is thus not the complementarity that delivers high EIS, it is the lack of income effect. This is important to understand, because it is specific to monetary policy shocks: as we shall see next, the effects of other demand shocks are indeed amplified by complementarity in and of itself.

[7] This amounts to assuming that prices are completemy fixed, or a Taylor rule of the form $i_t = E_t\pi_{t+1} + r_t$ where r_t is the exogenous part.

Fiscal multipliers

Fiscal multipliers have been derived previously in Bilbiie (2011, Proposition 3); rewriting the expression from the Proposition we obtain:

$$\frac{dy_t}{dg_t} = 1 + \eta\kappa\left(1 - \Phi\right) > 1 \text{ iff } \kappa > 0.$$

With fixed prices, a positive multiplier of consumption occurs if and only if consumption and hours worked are complements. The same caveat as for monetary policy applies: as discussed also by Auclert and Ronglie, with zero income effects $\gamma = 0$ such as for GHH preferences (studied as a special case in Bilbiie, 2011 and in Monacelli and Perotti, 2008) the multiplier is simply given by Φ^{-1} and is artificially driven to infinity in the efficient limit. More generally, the multiplier is increasing with the degree of complementarity (its derivative with respect to κ is proportional to $\gamma\left(1 - \Phi\right) \geq 0$) and decreasing with the income effect (the derivative with respect to γ is proportional to $\kappa\left(\Phi - 1\right) \leq 0$).

One constructive implication is that complementarity thus delivers a way out of the "Catch-22" for heterogeneous-agent NK models discussed in Bilbiie (2017b): on the one hand, it delivers the fiscal multipliers found in some of the empirical literature, from Blanchard and Perotti's (2002) VAR study to Nakamura and Steinsson's (2014) "local multipliers" evidence. While on the other, it is consistent with an elasticity of the income of constrained hand-to-mouth agents to aggregate income less than unity, which as shown in Bilbiie (2017b) is the necessary condition to rule out a series of puzzling predictions in heterogeneous-agent NK models; in short, with complementarity we can have both multipliers and a puzzle-free NK model.

News shocks

Consider news about tomorrow's productivity $\varepsilon_t \equiv E_t z_{t+1} - z_t$; this can be thought of as a proxy for shocks to tomorrow's capital (or investment-specific technology)—the point is thus more general, as discussed at length in the papers cited above. Here we take the simplest example that allows to clarify the roles of complementarity and income/wealth effects, through EIS. Proposition 1 derives the effect of such news shocks on expected output growth (which, conditional on productivity only, is the same as consumption growth) $y_t - E_t y_{t+1}$ and on the growth of hours worked (which using production function are given by $n_t - E_t n_{t+1} = \alpha^{-1}\left(y_t - E_t y_{t+1} + \varepsilon_t\right)$).

Take first the case of C-N **substitutability** $\kappa < 0$, which clearly implies that output falls; is co-movement possible, i.e. do hours worked also fall? No, because this requires $\gamma + \kappa < 0$, which violates concavity. While with separability $\kappa = 0$ news shocks evidently have no effect on output (a manifestation of Barro and King's critique), and hours increase. This is where **complementarity** comes into play: with $\kappa > 0$, output goes up. To have hours also go up we need $\gamma > -\kappa$ which is always satisfied. Notice that while the size of the income/wealth effect does affect the size of the response to news shocks (the smaller it is, the larger the effect), news shocks generate business cycles *if and only if* there is complementarity.[8] Furthermore, the effect of news shocks is increasing with complementarity (the derivative with respect to κ is proportional to $(1 - \Phi)\,\gamma$) even though the elasticity of intertemporal substitution EIS is decreasing with κ. In other words, the direct effect of complementarity dominates.

Two points are important to emphasize the differences with other mechanisms generating co-movement. Thus, the mechanism of this paper is different from Eusepi and Preston (2015) who use a heterogeneous-agent structure that changes *intratemporal* optimality conditions: de facto, it allows an aggregate utility function to violate non-inferiority and/or concavity while individual utility functions do not. Here, I focus on changes to the *intertemporal* optimality conditions while maintaining the representative-agent assumption.

The mechanism here does rely on sticky prices, but it does *not* rely on monetary accommodation; an important critique of stick-price-based co-movement solutions is formulated by Beaudry and Portier (2014), who show that in sticky-price models (*without C-N complementarity*) co-movement and news-driven business cycles occur because of a concomitant monetary accommodation. I rule that out on purpose: the interest rate is fixed, so there is no monetary accommodation in the background. Everything is driven here by complementarity instead and could in fact be triggered by another shift in labor demand (a separate literature, such as GHH 1988 and Jaimovich and Rebelo, 2009 focused on variable capacity utilization).

Adding a supply side: inflation and monetary accommodation

Evidently, adding monetary accommodation by adding a Phillips curve and a Taylor rule changes the quantitative properties of the model—but not the main insight. To see this, let us add a supply side in the simplest possible way: a standard Phillips curve $\pi_t = \beta_f E_t \pi_{t+1} + \psi mc_t$, but with a twist to obtain clear-cut closed-form results (see Bilbiie, 2011 for a full analysis with the above Phillips curve in the context of fiscal multipliers). Thus, consider the simpler setup whereby each period a fraction of firms f keep their price fixed, while the rest

[8] Jaimovich and Rebelo (2009) in an important contribution emphasize the role of preferences with low wealth effects for news-driven business cycles in a model with capital and adjustment costs; I emphasize the (complementary, pun intended) complementarity channel here, that holds even with time-separable preferences.

can re-optimize their price freely *but* ignoring that this price affects future demand. This delivers the Phillips curve with $\beta_f = 0$:

$$\begin{aligned} \pi_t &= \psi mc_t, \\ \text{where } \psi &= \frac{1-f}{f}\frac{\alpha}{\alpha+\theta(1-\alpha)}. \end{aligned} \tag{13}$$

also used in Bilbiie (2016, 2017b) in different contexts. While clearly over-simplified, this setup nevertheless captures a key mechanism of the NK model, i.e. a trade-off between inflation and real activity; results are conceptually very similar when considering a more standard Phillips curve adding future expected inflation.[9] Replacing the marginal cost schedule into (13) delivers the Phillips curve in terms of output:

$$\pi_t = \psi\left(\gamma - 1 + \frac{\varphi+1}{\alpha}\right) y_t - \psi\gamma g_t - \psi\frac{\varphi+1}{\alpha} z_t. \tag{14}$$

The model is closed by a simple Taylor rule responding to expected inflation:

$$r_t = (\phi_\pi - 1) E_t \pi_{t+1}. \tag{15}$$

Substituting this together with (14) into (10) delivers one first-order difference equation that fully determines the dynamics of output, inflation, and real interest rates in response to shocks in this model:

$$\begin{aligned} y_t = & \left[1 - \eta(\phi_\pi - 1)\psi\left(\gamma + \frac{\varphi+1-\alpha}{\alpha}\right)\right] E_t y_{t+1} \\ & + \eta(\kappa+\gamma)(g_t - E_t g_{t+1}) + \eta(\phi_\pi - 1)\psi\gamma E_t g_{t+1} \\ & + \eta\kappa(1-\Phi)(E_t z_{t+1} - z_t) + \eta(\phi_\pi - 1)\psi\frac{\varphi+1}{\alpha} E_t z_{t+1} \end{aligned} \tag{16}$$

Two differences are worth emphasizing, pertaining to the effects of government spending and news shocks respectively. The multiplier of government spending shocks with persistence p ($E_t g_{t+1} = p g_t$) on consumption is now:

$$\frac{dc_t}{dg_t} = \frac{(1-p)[\eta(\kappa+\gamma) - 1] - p\eta(\phi_\pi - 1)\psi\frac{\varphi+1-\alpha}{\alpha}}{1 - p + p\eta(\phi_\pi - 1)\psi\left(\gamma + \frac{\varphi+1-\alpha}{\alpha}\right)} \tag{17}$$

[9] Essentially, such a setup reduces to assuming $\beta_f = 0$ *only* in the firms' problem (they do not recognize that today's reset price prevails with some probability in future periods). See Bilbiie (2016) for an extension to the case $\beta_f > 0$, and a comparison in the context of optimal forward guidance in the baseline NK model.

which is lower that under fixed prices because of a mechanism well-understood in the NK literature: persistent G leads to future inflation (because of future demand increasing), which under an active monetary policy $\phi_\pi > 1$ leads to an increase in real interest rates and thus less demand today by intertemporal substitution. Complementarity influences this "supply-feedback" channel only insofar as it modifies intertemporal substitution, so there is not much novelty here.

The effects of news shocks allow an understanding of the very different implications of sticky prices and complementarity in generating co-movement. The last line of equation (14) has indeed two components: the first one is the one we studied before, that occurs even with fixed prices—the effect of good news is $\eta\kappa(1-\Phi)$ and comes from complementarity, the mechanism emphasized here. The second component is $\eta(\phi_\pi - 1)\psi\frac{\varphi+1}{\alpha}$ and occurs through a supply feedback and a concomitant monetary accommodation: expectations of future productivity improvements create expectations of deflation (as some firms cut prices and others who cannot do not) which with active monetary policy imply an interest rate cut today, and intertemporal substitution towards today. This mechanism delivering an expansion (and co-movement too) in response to news is thus orthogonal to complementarity—it holds for separable preferences, and it holds for substitutability too.

Complementarity does of course affect the "monetary accommodation" effect of news through changing the intertemporal substitution channel; but more complementarity implies *less* effect of news (because it implies less intertemporal substitution); whereas through the direct "complementarity" channel emphasized here, more complementarity implies *more* effect of news. The foregoing is thus an alternative way to formalize the mechanism of "monetary accommodation" forcefully criticized by Beaudry and Portier (2014), and serves the purpose of illustrating that the "complementarity" channel emphasized here does not similarly rely on such monetary accommodation; it is in fact orthogonal to it, even though it does rely on price stickiness—but only as a vehicle to deliver a shift in labor demand.

4 A Simple $U(C, N)$ with Complementarity for Quantitative Business-Cycle Models

The previous sections emphasized the role of C-N complementarity generally for generating realistic business cycles; we now move to a specific utility function featuring such complementarity that can be used in quantitative macro models. Several utility functions with C-N complementarity do exist and have been extensively used in the macro literature; but as we shall see, they each restrict somewhat artificially some other parameter that is key for business cycles—in particular, the income effect on labor supply. Among the most prominent examples in the literature: GHH implies zero income/wealth effect (with the additional artificial implication of very large intertemporal elasticity of substitution—indeed infinite around an efficient equilibrium). While the non-separable utility function of KPR has unitary income effect—needed, of course, to induce balanced growth.

There is thus to the best of my knowledge no utility function with both flexible income effect on labor supply and consumption-hours complementarity (MaCurdy CRRA has the former but restricts the latter to zero; GHH and KPR always have the latter but restrict former to zero or one, respectively; and Jaimovich and Rebelo consists of a time-nonseparable flexible form that nests these last two).

Here, I introduce a (to the best of my knowledge) new class of utility functions that restricts none of this but still satisfies the regularity conditions outlined in Bilbiie (2009, 2011) and (8) above (concavity, and non-inferiority). This is not *the* most general family of functions that satisfies those restrictions, although in principle that could be found too, by solving a system of partial differential equations formed by the restrictions; it is instead one example that:

1. nests standard utility functions: indeed, MaCurdy CRRA and GHH are special cases, and KPR is a limit case.

2. is time-separable which differentiates it from Jaimovich and Rebelo.

3. we can keep thinking of γ as income effect on labor and φ as inverse (constant-consumption) labor elasticity, but we now have a new free parameter that fully determines complementarity.

The function is so disarmingly simple that I confess being astonished that nobody has come up with it before:

$$U\left(C_t, N_t\right) = \frac{1}{1-\frac{\xi}{1-\gamma}}\left(\frac{C_t^{1-\gamma}}{1-\gamma} - h\frac{N_t^{1+\varphi}}{1+\varphi}\right)^{1-\frac{\xi}{1-\gamma}} \tag{18}$$

defined for arbitrary income effect $\gamma \neq 1$ (whereas when $\gamma = 1$ one can use $U\left(C_t, N_t\right) = \left(\ln C_t - h\frac{N_t^{1+\varphi}}{1+\varphi}\right)^{1-\xi} / \left(1-\xi\right)$, see discussion below).

As announced, the function nests some of the most widely used functions in macro, business-cycle analysis: $\xi = 0$ is MaCurdy's CRRA, and $\gamma = 0$ is GHH. And it can be easily shown that γ is the income effect while φ is the constant-consumption labor elasticity, as defined above (see Appendix for the derivations).

The function is thus defined (with $1-\gamma$ dividing ξ) in order for the free parameter ξ's sign to entirely determine complementarity. Indeed, complementarity as defined above is given by:

$$\kappa = \xi\Upsilon$$

where $\Upsilon = \left[1 - \frac{1-\gamma}{1+\varphi}\alpha\left(1-\Phi\right)\right]^{-1} > 0$ and we used the expressions derived in the Appendix. It then follows immediately that the EIS is $\eta = \left(\gamma + \xi\Phi\Upsilon\right)^{-1}$ and our previous general discussion still applies.

The income effect γ can be calibrated to match cross-section facts, which would presumably require a low γ. Or perhaps one would want to match long-run growth macro facts, which requires something different. Indeed, as mentioned in the Introduction, Boppart and Krusell (2016) recently argued that there is a downward long-run trend in hours worked and proposed a general class of utility functions that delivers that. Since it has arbitrary income effect, my functional form can also deliver that, for $\gamma > 1$. Indeed, just as MaCurdy utility, when it comes to long-run growth facts my function also implies (intratemporal optimality) $hN_t^{1+\varphi} = \alpha\left(1-\Phi\right)C_t^{1-\gamma}$. From this it follows immediately that with an upward trend in C there can only be a downward trend in N if the income effect is strong enough, i.e. $\gamma > 1$.[10]

5 Conclusions

One of the very primitives of any macroeconomic model used for business-cycle and policy analysis is the utility function. In this paper, I first showed (and to some extent reviewed) that some key macro facts pertaining to business cycles depend crucially on a restricted set of parameters that can be defined for very general utility functions $U\left(C,N\right)$ of consumption and hours worked. In particular, the elasticity of intertemporal substitution, a key parameter in any dynamic model, shapes the response of the economy to demand shocks and to changes in policy. With general nonseparable preferences, this elasticity depends on the income effect on labor supply, and on the degree of complementarity of consumption and hours worked (as well as on some supply-side parameters).

The paper provides a general discussion of the role of all of these three parameters: complementarity, income effect, and substitution (a function of the first two) in determining key macroeconomic effects of demand shocks and policies, and business-cycle co-movement. Complementarity between consumption and hours is key to reconcile business-cycle models with: evidence on the elasticity of intertemporal substitution and on income effects, both of which are small in the data—which, absent complementarity, cannot happen in the model; multipliers of government spending shocks; co-movement in response to news shocks.

[10] The utility function proposed in (18) is not defined for $\gamma = 1$. But that is the case that the macro literature has hitherto focused on because it makes income and substitution effects on labor cancel out, thus delivering constant hours along a growth path. In other words we already have an utility function with arbitrary complementarity—but with restricted income effect $\gamma = 1$, the KPR utility function: $U\left(C,N\right) = \xi^{-1}\left\{\left[C\exp\left(-\frac{N^{1+\varphi}}{1+\varphi}\right)\right]^{\xi} - 1\right\}$.While this is not strictly speaking a limit case of (18), it can be obtained as the limit of an equivalent function, defined over leisure instead of labor, and restricted to the CES form.

The above results are purposefully derived for a very general (arbitrary) utility function defined over consumption and hours, $U(C, N)$. Lastly, I propose a parametric class of utility functions that, with a minimal departure from already-used and familiar functions, allows parameterizing the degree of complementarity freely—while also allowing simultaneously for arbitrary income effects on labor supply, and constant-consumption labor supply elasticity. This "MaCurdy meets GHH" function can be employed in quantitative studies and for estimation in any business-cycle model, including models with heterogeneous agents where income and wealth effects can be calibrated based on cross-sectional data.

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Appendix

A Intertemporal utility maximization

An agent j chooses consumption, asset holdings and hours solving the standard intertemporal problem: $\max E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t)$, subject to the sequence of constraints:

$$B_t \leq Z_t + P_t D_t + W_t N_t - P_t C_t.$$

C_t, N_t are consumption and hours worked, B_t is the nominal value at *end of period* t of a portfolio of all state-contingent assets held (including shares in firms)—likewise for Z_t, beginning of period wealth. D_t is the real dividend payoff. Absence of arbitrage implies that there exists a stochastic discount factor $Q_{t,t+1}$ such that the price at t of a portfolio with uncertain payoff at $t+1$ is for state-contingent assets:

$$\frac{B_t}{P_t} = E_t \left[Q_{t,t+1} \frac{Z_{t+1}}{P_{t+1}} \right] , \tag{19}$$

which iterated forward gives the fundamental pricing equation. The riskless gross short-term REAL interest rate R_t is a solution to:

$$\frac{1}{R_t} = E_t Q_{t,t+1} \tag{20}$$

Note for nominal assets we have the nominal interest rate $\frac{1}{I_t} = E_t \frac{P_t}{P_{t+1}} Q_{t,t+1}$

Substituting the no-arbitrage conditions (19) into the wealth dynamics equation gives the flow budget constraint. Together with the usual 'natural' no-borrowing limit for *each* state this implies the usual intertemporal budget constraint:

$$E_t \left[Q_{t,t+1} X_{t+1} \right] \leq X_t + W_t N_t - P_t C_t.$$

$$X_t = Z_t + P_t D_t$$

$$\begin{aligned} E_t \sum_{i=0}^{\infty} Q_{t,t+i} C_{t+i} &\leq \frac{X_t}{P_t} + E_t \sum_{i=0}^{\infty} Q_{t,t+i} \frac{W_{t+i}}{P_{t+i}} N_{j,t+i} \\ &= \frac{Z_t}{P_t} + E_t \sum_{i=0}^{\infty} Q_{t,t+i} Y_{j,t+i} \end{aligned} \tag{21}$$

where

$$Y_{t+i} = D_{t+i} + \frac{W_{t+i}}{P_{t+i}} N_{t+i} \tag{22}$$

is income. Maximizing utility subject to this constraint gives the first-order necessary and sufficient conditions at each date and in each state:

$$\beta \frac{U_C\left(C_{j,t+1}\right)}{U_C\left(C_{j,t}\right)} = Q_{t,t+1}$$

along with (21) holding with equality (or alternatively flow budget constraint holding with equality and transversality conditions ruling out Ponzi games be satisfied: $\lim_{i\to\infty} E_t\left[Q_{t,t+i}Z_{t+i}\right] = 0$). Using (21) and the functional form of the utility function the short-term real interest rate must obey:

$$\frac{1}{R_t} = \beta E_t\left[\frac{U_C\left(C_{j,t+1}\right)}{U_C\left(C_{j,t}\right)}\right].$$

A.1 Frisch elasticity and labor supply

One distinguishing feature of non-separable preferences which turns out to be crucial for understanding the issue under study here is that the Frisch labor supply, i.e. labor supply for a constant level of marginal utility $U_C\left(C_t, L_t\right)$, is different from the constant-consumption labor supply (6). Frisch labor supply is obtained by treating marginal utility as a variable (MU, which is indeed the Lagrange multiplier on the household budget constraint): $U_C\left(C_t, N_t\right) = MU_t$, or in log-linearized form:

$$\frac{U_{CC}C}{U_C}c_t + \frac{U_{CN}N}{U_C}n_t = mu_t$$

Substituted in (6) to eliminate consumption $c_t = \left(\frac{U_{CC}C}{U_C}\right)^{-1} mu_t - \left(\frac{U_{CC}C}{U_C}\right)^{-1} \frac{U_{CN}N}{U_C} n_t$, this delivers

$$\frac{U_{CN}C}{U_N}\left(\frac{U_{NN}N}{U_{CN}C} - \frac{U_{CN}N}{U_{CC}C}\right) n_t = w_t + \left(1 - \frac{U_{CN}}{U_N}\frac{U_C}{U_{CC}}\right) mu_t. \tag{23}$$

or, using the parameters defined above and the newly defined (inverse) Frisch elasticity of labor supply φ_F replacing $\frac{U_{NN}N}{U_N} = \varphi + \kappa\frac{\alpha}{\mu}$ $U_{CC}C/U_C = -\kappa - \gamma$ we obtain the expression in text.

A.2 Concavity and non-inferiority restrictions

This section is just a reiteration of the conditions derived in Bilbiie (2011), with the slight modification that utility is here defined over hours rather than leisure. The conditions may be of practical use to researchers using such an utility function in applied work. The restrictions implied by concavity and non-inferiority given in text (8) are obtained as follows. The three concavity requirements are equivalent to:$U_{CC} \leq 0 \leftrightarrow \kappa \geq -\gamma; U_{NN} \leq 0 \leftrightarrow \kappa\alpha(1-\Phi) \geq -\varphi$ and $U_{CC}U_{NN} - (U_{CN})^2 \geq 0 \leftrightarrow \kappa \geq -\frac{\gamma\varphi}{\varphi+\gamma\alpha(1-\Phi)}$. Quasi-concavity (which is implied by concavity) requires that the bordered Hessian be negative semidefinite, i.e. $2U_C U_N U_{CN} - U_{CC}(U_N)^2 - U_{NN}(U_C)^2 \geq 0 \leftrightarrow \gamma + \frac{\varphi}{\alpha(1-\Phi)} \geq 0$.

The good inferiority conditions can be obtained starting with the static budget constraint rewritten as:

$$C_t + W_t(1-N_t) \leq E_t,$$

where E_t is 'full income' given by $E_t = W_t + B_t + D_t - T_t - (1+R_t)^{-1} B_{t+1}$. A good is inferior if its income elasticity of demand is negative at given prices. We find demands for leisure and consumption for a given level of full income E by solving the static optimization problem: $\max U(C,N)$ s.t. $C + W(1-N) \leq E$, which leads to:

$$\begin{aligned} WU_C(C,N) + U_N(C,N) &= 0 \\ C + W(1-N) - E &= 0 \end{aligned} \tag{24}$$

We apply the implicit function theorem to this system to study variations in consumption and leisure demand to changes in income E.The solution is (using $W = U_L/U_C$, the definitions of φ and $\tilde{\gamma}$ and the steady-state restriction (4)):

$$\begin{aligned} \frac{dC}{dE} &= \frac{1}{\alpha(1-\Phi)\frac{\gamma}{\varphi}+1} \\ \frac{dN}{dE} &= -\frac{\gamma}{\varphi}\frac{\alpha(1-\Phi)}{\alpha(1-\Phi)\frac{\gamma}{\varphi}+1}\frac{1}{W} \end{aligned}$$

Non-inferiority of consumption requires $\frac{\partial C}{\partial E} \geq 0$, i.e.:

$$-\frac{\gamma\alpha(1-\Phi)}{\varphi} \leq 1. \tag{25}$$

Non-inferiority of leisure requires $\frac{\partial N}{\partial E} \leq 0$, so:

$$\frac{\frac{\gamma}{\varphi}\alpha(1-\Phi)}{\alpha(1-\Phi)\frac{\gamma}{\varphi}+1} \geq 0$$

$$-\frac{\varphi}{\gamma\alpha(1-\Phi)} \leq 1. \tag{26}$$

Neither good is inferior if and only if (25) and (26) hold *simultaneously*, i.e.:

$$\frac{\gamma\alpha\left(1-\Phi\right)}{\varphi}\geq 0. \tag{27}$$

Non-inferiority and quasi-concavity ($\gamma\alpha\left(1-\Phi\right)+\varphi\geq 0$) imply the first two restrictions in (8).

The concavity requirements outlined above imply $\kappa\geq-\min\left(\gamma,\frac{\varphi}{\alpha(1-\Phi)},\frac{\gamma\varphi}{\varphi+\gamma\alpha(1-\Phi)}\right)$, while these non-inferiority restrictions imply the third condition on complementarity in (8), because $\gamma>\frac{\gamma}{1+\varphi^{-1}\gamma\alpha(1-\Phi)}$ and $\frac{\varphi}{\alpha(1-\Phi)}>\frac{\gamma\varphi}{\varphi+\gamma\alpha(1-\Phi)}$.

B Utility derivations

Marginal utilities for the nonseparable utility (18) are:

$$U_C=C^{-\gamma}\left(\frac{C^{1-\gamma}}{1-\gamma}-h\frac{N^{1+\varphi}}{1+\varphi}\right)^{-\frac{\xi}{1-\gamma}};U_N=-hN^{\varphi}\left(\frac{C^{1-\gamma}}{1-\gamma}-h\frac{N^{1+\varphi}}{1+\varphi}\right)^{-\frac{\xi}{1-\gamma}}$$

The intratemporal optimality condition thus implies

$$\frac{-U_N}{U_C}=hC^{\gamma}N^{\varphi}=\frac{\left(1+s\right)F_N}{\mu};h\frac{N^{1+\varphi}}{C^{1-\gamma}}=\alpha\left(1-\Phi\right)$$

Cross-derivatives are hence:

$$\begin{aligned}
U_{CC} &= -\gamma C^{-\gamma-1}\left(\frac{C^{1-\gamma}}{1-\gamma}-h\frac{N^{1+\varphi}}{1+\varphi}\right)^{-\frac{\xi}{1-\gamma}}-\frac{\xi}{1-\gamma}\left(C^{-\gamma}\right)^2\left(\frac{C^{1-\gamma}}{1-\gamma}-h\frac{N^{1+\varphi}}{1+\varphi}\right)^{-\frac{\xi}{1-\gamma}-1}\\
U_{CN} &= h\frac{\xi}{1-\gamma}C^{-\gamma}N^{\varphi}\left(\frac{C^{1-\gamma}}{1-\gamma}-h\frac{N^{1+\varphi}}{1+\varphi}\right)^{-\frac{\xi}{1-\gamma}-1}\\
U_{NN} &= -h\varphi N^{\varphi-1}\left(\frac{C^{1-\gamma}}{1-\gamma}-h\frac{N^{1+\varphi}}{1+\varphi}\right)^{-\frac{\xi}{1-\gamma}}+\frac{\xi}{1-\gamma}\left(hN^{\varphi}\right)^2\left(\frac{C^{1-\gamma}}{1-\gamma}-h\frac{N^{1+\varphi}}{1+\varphi}\right)^{-\frac{\xi}{1-\gamma}-1}
\end{aligned}$$

Using these expressions, it can be shown directly that φ and γ correspond to constant-consumption labor supply and, respectively, income effect on labor supply as defined in text; furthermore, we obtain complementarity as:

$$\kappa=\frac{U_{CN}C}{-U_N}=\frac{\xi}{1-\frac{1-\gamma}{1+\varphi}h\frac{N^{1+\varphi}}{C^{1-\gamma}}}.$$

Concavity requires $\kappa\geq-\frac{\gamma\varphi}{\varphi+\gamma\alpha(1-\Phi)}$ so $\xi\geq-\frac{\gamma\varphi}{1+\varphi}\left(1+\frac{1-\alpha(1-\Phi)}{\varphi+\gamma\alpha(1-\Phi)}\right)$.

With log utility in consumption we have instead:

$$U_C = \frac{1}{C}\left(\ln C - h\frac{N^{1+\varphi}}{1+\varphi}\right)^{-\xi};\ U_N = -hN^{\varphi}\left(\ln C - h\frac{N^{1+\varphi}}{1+\varphi}\right)^{-\xi}$$

and $U_{CN} = hN^{\varphi}\xi\frac{1}{C}\left(\ln C - h\frac{N^{1+\varphi}}{1+\varphi}\right)^{-\xi-1}$ implying complementarity $\kappa = \frac{U_{CN}C}{-U_N} = \frac{\xi}{U}$ where $U = \ln C - h\frac{N^{1+\varphi}}{1+\varphi}$ and $hCN^{\varphi} = \frac{(1+s)F_N}{\mu}$ implies $N^{1+\varphi} = \frac{\alpha(1-\Phi)}{h}$.