DISCUSSION PAPER SERIES

DP12689

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FINANCIAL ECONOMICS

ISSN 0265-8003

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Discussion Paper DP12689
Published 05 February 2018
Submitted 05 February 2018

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MARKET DISCIPLINE AND SYSTEMIC RISK

Abstract

We analyze a general equilibrium model in which financial institutions generate endogenous systemic risk, even in the absence of any government support. Banks optimally select correlated investments and thereby expose themselves to fire sale risk so as to sharpen their incentives. Systemic risk is therefore a natural consequence of banks' fundamental role as delegated monitors. Our model sheds light on recent and historical trends in measured systemic risk. Technological innovations and government-directed lending can cause surges in systemic risk. Strict capital requirements and well-designed government asset purchase programs can combat systemic risk.

JEL Classification: G01, G21

Keywords: systemic risk, market discipline, return correlation, macro-prudential regulation

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Acknowledgements
We thank Max Bruche, Alessandro Fontana, Martin Hellwig, Enisse Kharroubi, Frederic Malherbe, Pascal Paul, Uday Rajan, Tarun Ramadorai, and Joel Shapiro for helpful comments, and also seminar audiences at the Max Plank Institute in Bonn, at Nottingham University, the Sa\"id Business School, Southampton University, Essex University, the Vienna 2015 EFA meetings, the Mannheim 2015 EEA meetings, the Venice 2015 CREDIT conference and the Bank of England Financial Intermediation Workshop 2017.

Market Discipline and Systemic Risk*

Alan D. Morrison† and Ansgar Walther‡

January 2018

Abstract

We analyze a general equilibrium model in which financial institutions generate endogenous systemic risk, even in the absence of any government support. Banks optimally select correlated investments and thereby expose themselves to fire sale risk so as to sharpen their incentives. Systemic risk is therefore a natural consequence of banks' fundamental role as delegated monitors. Our model sheds light on recent and historical trends in measured systemic risk. Technological innovations and government-directed lending can cause surges in systemic risk. Strict capital requirements and well-designed government asset purchase programs can combat systemic risk.

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1. Introduction

The 2007-8 financial crisis was triggered by systemic bank liquidity freezes, which resulted from a belief that banks had a common exposure to problematic mortgage and securitization markets (Gorton and Metrick, 2012). An important question in this context is whether banks have a general tendency to gravitate towards common, correlated exposures during credit booms, and therefore to generate systemic liquidity risk. Acharya and Yorulmazer (2007) and Farhi and Tirole (2012) articulate the "too many to fail" hypothesis: When governments respond to systemic crises with bail-outs or liquidity support, bankers can increase the value of their implicit subsidy by assuming correlated risks.

However, while bail-outs and other implicit subsidies are undoubtedly distortive, recent and historical data suggest that government policies may not be the only driver of systemic risk. Figure 1a shows that measures of global systemic risk have remained fairly stable since 2008, despite robust

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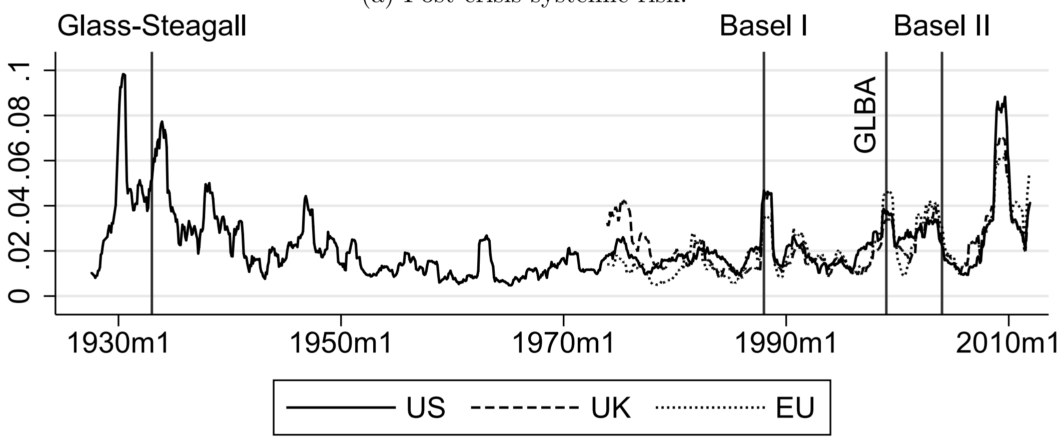
(a) Post-crisis systemic risk.



(b) Long-run systemic risk.

Figure 1: Figure (a) plots the SRISK systemic risk from the 2009 crisis to the present. SRISK is the total expected capital shortfall amongst financial firms conditional upon a financial crisis due to Acharya, Pedersen, Philippon, and Richardson (2017). Figure (b) plots Acharya *et al.*'s Marginal Expected Shortfall (MES) from 1929 to the present. MES is the fifth percentile of losses in the system's loss distribution. Some significant regulatory events are illustrated; GLBA is the Gramm-Leach-Bliley Act of 1999, which repealed the Glass-Steagall Act. Data for Figure (a) were extracted from the NYU VLAB website (`https://vlab.stern.nyu.edu`); data for Figure (b) are reported by Giglio, Kelly, and Pruitt (2016) and are available at Stefano Giglio's website (`https://sites.google.com/view/stefanogiglio/`).

regulatory efforts to remove implicit subsidies.[1] Figure 1b plots US systemic risk measures over a longer horizon, and there are no immediately visible structural breaks around major changes in financial policy. More generally, the historical record indicates that banking crises and correlated failures pre-date the regulatory state and the systematic provision of government support during times of distress.[2]

[1]Regulatory capital requirements have increased (Basel Committee on Banking Supervision, 2011), new rules have been introduced to ease lender lender bail-ins, and the political climate in Europe and the USA is now much less sympathetic towards government support for failing banks. Recent experience is that these changes have rendered bail-outs less likely: support for the Italian bank Monte dei Paschi was provided only after lengthy negotiations and after lenders experienced losses. See, for example: Alex Barker and Rachel Sanderson, "Brussels and Rome seal rescue deal for Monte dei Paschi", *Financial Times*, 1 June 2017; Jim Brunsden and Patrick Jenkins, "Bank rescues: putting bondholders on the hook," *Financial Times*, 4 January 2016

[2]Reinhart and Rogoff (2009) provide an account of historical financial crises. Indeed, Calomiris and Gorton (1991) argue that the modern financial regulatory state evolved in response to this type of crisis.

In this paper, we study banks' portfolio choices in a general equilibrium model with endogenous market liquidity. Our main result is that banks may optimally expose themselves to correlated shocks even in the absence of government support, and even when banks have access to equally profitable investments that are not exposed to aggregate risk.

In our model, correlated investments and systemic risk are driven by banks' desire for market discipline: By selecting investments that are correlated with those of its peers, a bank exposes itself to aggregate liquidity shocks and fire sales. Exposure to fire sales is valuable because it creates sharp incentives for bankers to select high-quality investments. Correlated investments can therefore be optimal because they alleviate managerial moral hazard. This insight is new not only to the literature on the origins of systemic risk, but also to the literature on market discipline (e.g. Calomiris and Kahn (1991), Diamond and Rajan (2011)), which abstracts from endogenous correlation.

Our formal model is based on two widely acknowledged features of the banking sector. First, bank managers face a moral hazard problem (Holmström and Tirole, 1997), and need to be given incentives to select high-quality investments. Second, banks use short-term liabilities to finance the purchase of long-term assets that require specialist management skills that only banks possess (see, e.g., Allen and Gale (1998) and Diamond and Rajan (2011)). This can give rise to endogenous fire sales. Specifically, banks that experience adverse shocks to their investment cannot roll over short-term debt, because creditors cannot enforce the repayment of debt backed by fragile assets. Fragile banks must then sell their assets to other banks in order to repay maturing short-term debt. However, after a common adverse shock to the banking sector, the universe of potential buyers is restricted so that the price of bank assets drops below their fair value.

Intermediaries in our model choose between investments in a local sector, which is uncorrelated with the local sectors available to other banks, and investments in an aggregate sector, which is subject to common shocks.[3] They also choose whether to finance themselves with short-term debt issued to households, or with equity provided by insiders. Equity finance is privately costly.

When choosing their portfolio, banks trade off the increased fire sale risk associated with aggregate sector investment against the tighter borrowing constraints associated with local investment. If a bank invests in the local sector, the diversification implied by this investment provides protection from fire sale risk, but incentive constraints imply a borrowing limit: Too much leverage erodes bankers' "skin in the game" and, consequently, weakens their incentives to choose high-quality investments. If the moral hazard problem is severe, the bank must fund a large proportion of any local investment with equity. If the bank invests in the aggregate sector, by contrast, it is directly exposed to costly fire sales. However, the threat of wasteful liquidation during fire sales sharpens managers' incentives and therefore relaxes the borrowing constraint.

This trade-off underpins our main result: If the private costs of bank equity finance are high enough, then the benefit of relaxing the borrowing constraint exceeds the cost of increased fire sale risk, and banks optimally invest in the aggregate sector.

[3] Throughout the paper, we use the term "aggregate sector investment" as shorthand for investments that are exposed to aggregate shocks. More precisely, the overall aggregate investment in our economy is a weighted average of investments in the aggregate sector and local sectors, with the weights determined by banks' endogenous portfolio choice.

In general equilibrium, the disciplining effect of aggregate sector investment introduces a strategic complementarity between banks. If all banks opt for local sector investment, then the banking sector is not exposed to aggregate risk and, consequently, there are no fire sales. As a result, there is no incentive to invest in the aggregate sector. In contrast, if enough banks opt for aggregate sector investment, then adverse aggregate shocks trigger fire sales, and a banker can achieve market discipline by also investing in the aggregate sector. Our model therefore admits two equilibria: a *diversified* equilibrium, in which all banks invest locally; and a *correlated* equilibrium, in which all banks invest in the aggregate sector.

To complement our positive analysis, we analyze the consequences of these outcomes for social welfare. The social trade-off between the diversified and correlated equilibrium boils down to a comparison between, on the one hand, the social costs of raising additional equity for local sector investment in the absence of market discipline and, on the other hand, the inefficient liquidation that occurs when all banks invest in the aggregate sector. In case equity is privately but not socially costly, the latter effect is clearly dominant, so that the diversified equilibrium results in strictly higher welfare than the correlated equilibrium. In the case where equity is socially as well as privately costly, we are able to show that the former effect dominates, so that the correlated equilibrium is socially preferred.

If the costs of equity are entirely private, for example because they derive from the tax code, then there is a role for policies that encourage local sector investment and, hence, guarantee diversified equilibria.[4] We identify two such policies. First, we show that a capital requirement, which forces banks to invest sufficient inside equity, resolves the moral hazard problem without the need for market discipline and therefore reduces systemic risk in our model. Second, we consider a government program like the US Troubled Asset Relief Program to purchase fragile assets during aggregate crises. This type of program prevents fire sales and, hence, eliminates the market discipline benefits of aggregate sector investment. It follows that an appropriately designed asset purchase programme enhances banking sector stability by discouraging correlated investment.

We extend our analysis to demonstrate that, if a critical mass of banks invests in the aggregate sector by default (for example, because they have no local knowledge), it is impossible to prevent fire sales in the event of an aggregate crisis. Then, as a result of the strategic complementarities that drive our results, only the correlated equilibrium can occur.

This observation generates several empirical implications. First, technological innovations that lower the cost of aggregate sector investment serve to lower the investment threshold above which correlated equilibria, and system risk, are inevitable. Second, any program of government-directed credit could push aggregate sector investment over the critical threshold and so encourage correlated lending. It follows that government-directed lending in the housing market may have sown the seeds for the 2008–09 financial crisis.[5]

[4]The simplest intervention would directly require banks to make fewer correlated investments. Despite recent advances in measuring correlation (see, e.g., Acharya, Pedersen, Philippon, and Richardson (2017) and Adrian and Brunnermeier (2016)), it seems that regulators are reluctant to implement direct restrictions on correlation between banks. We therefore focus on indirect mechanisms that combat banks' incentives to make correlated investments.

[5]Of course, technological change and government intervention would affect investment decisions in any portfolio model. However, our model exhibits a novel amplification effect: a relatively small increase in the attractiveness of aggregate sector investment can have a significant impact upon systemic risk due to strategic complementarities.

Our model also implies, first, that any increase to the relative cost of equity should render correlated investment more attractive; and, second, that when there are more specialist non-bank investors in fragile bank projects, correlated investment is less attractive. These implications serve to distinguish our work from the literature on herding, which generates correlated investment because investors believe either their own information to be inferior (Graham (1999); Bikchandani, Hirshleifer, and Welch (1992)), or that non-herding will have adverse reputational effects (Scharfstein and Stein, 1990).

Finally, we extend the model to allow for richer contracting possibilities. In our baseline model, households cannot verify the results of bank investments and, as a result, banks are forced to rely upon non state-contingent debt contracts. When banks are able to contract upon aggregate states, we show that the incentive to correlate remains, and that it may even be enhanced. We also argue that, while complete contracting would remove the incentive to correlate in our base set-up, a slight modification to our assumptions would re-introduce it.

Literature review

Our work contributes to a large literature on correlation in the banking system. Acharya and Yorulmazer (2007) and Farhi and Tirole (2012) examine models in which the government has incentives to help troubled banks whenever sufficiently many fail at the same time; banks therefore choose correlated portfolios so as to benefit from bailouts when they occur. Martinez-Miera and Suarez (2014) consider a model where banks with limited liability have incentives to make systemic investments, because they are more profitable in good times; correlated investments are also driven by asset return distributions in Wagner (2010), which shows that correlation can occur when banks have a common preference for a diversified portfolio.

In a recent working paper, Zetlin-Jones (2014) shows that correlated bank returns can prevent banks' contracts with depositors from being renegotiated. In common with our work, correlation in Zetlin-Jones' model therefore sharpens incentives but, in contrast to us, Zetlin-Jones treats correlation as exogenously determined. We focus in depth on the endogenous determination of correlation in general equilibrium, and highlight the fact that strategic complementarities among banks can result in systemic risk. In line with legal practice, we consider deposit contracts to be hard claims that cannot be renegotiated. In our model, correlation endogenously sharpens incentives during asset fire sales.

Even when the distribution of returns on bank assets is given exogenously, Rochet and Tirole (1996) and Allen and Gale (2000) show that interbank trading intended to share liquidity risks can cause bank returns to be correlated. All banks in our model face the same liquidity conditions, so there is no role for an interbank market. An interbank market in our set-up could generate additional incentives to create systemic risk, by allowing banks to expose themselves to aggregate risk not only by making aggregate sector investments, but also by lending to other banks.

In summary, our work is distinct from analyzes that rely use government support, the desire to manipulate return distributions, or liquidity risk as the rationale for correlated investment. There are no government subsidies in our economy, the marginal distribution of returns on aggregate and local investments is the same and we assume in line with market practice that deposit dilution is impossible. Banks need to find a source of discipline, and systemic risk emerges in our analysis as the cheapest source of discipline. This reasoning provides an explanation for the prevalence of systemic risk before either explicit or implicit government support existed.

We further contribute to the literature on market discipline. Calomiris and Kahn (1991) and Diamond and Rajan (2000) argue that the threat of liquidation by short-term debt holders can discipline banks funding exposes banks to market discipline. Martinez Peria and Schmukler (2001) present evidence that depositors exert market discipline, and that this effect is most pronounced during aggregate crises. Eisenbach (2017) considers the disciplining effect of rollover risk in a general equilibrium setting, and demonstrates that it is only effective when intermediary risks are uncorrelated; when, for exogenous reasons, they are positively correlated, a feedback effect undermines discipline in good states and renders discipline too severe in bad states. We extend this literature by considering the correlation of bank portfolios as well as their maturity.

Correlated bank failures are also optimal in Allen and Gale (1998), because financial crises facilitate risk-sharing among depositors. In practice, a large proportion of bank liabilities are held by large wholesale investors, who have sufficient expertise and scale to hedge risks without relying on deposit contracts and, hence, like the depositors in our model, do not have a risk-sharing motive. We show that correlated equilibria still occur, as long as there is moral hazard, and can be socially optimal if the social costs of bank equity finance are large.

Banks with local investments profit in our model by buying distressed assets at firesale prices after aggregate sector failure. Similar incentive effects are identified by Perotti and Suarez (2002), who argue that the "last bank standing" after an aggregate shock can earn scarcity rents on its capital.[6] In our analysis, however, the last bank standing effect is dominated, when moral hazard problems are sufficiently severe, by banks' desire to generate market discipline via aggregate sector investments.

2. Model

We consider a three-period model in which banks borrow from households in order to invest at the initial date, and face potential liquidity shortages at the interim date. We first outline the formal setup of our model, the timing of the game and the definition of equilibrium. Then, we discuss the practical relevance of our setup in a series of remarks.

2.1. Timing and players

The economy that evolves over three dates $t \in \{0, 1, 2\}$. There is a single consumption good. Two types of agent are active in the economy. First, there is a continuum of *bankers*, who are active at all three dates. Second, there are two overlapping generations of *households*. *Early* households live at dates 0 and 1, while *late* households live at dates 1 and 2.

All agents are risk-neutral, and all are born with a large endowment of the consumption good. A

[6] Martinez-Miera and Suarez (2014) argue that this effect is weakened during credit booms, when bank equity is abundant and expected scarcity rents are low.

household's utility is $c_t + c_{t+1}$, where t denotes the first period of the household's life. Each bankers has utility $\phi c_0 + c_1 + c_2$, where $\phi > 1$. Bankers' preference for early consumption, measured by the parameter ϕ, creates gains from trade. Specifically, bankers wish to borrow from households to invest in productive projects, which we describe in detail below.

In our environment, ϕ captures the costs of bank equity finance. A contentious issue is whether the costs of bank equity are social costs or private costs. Since we do not wish to take a stand on this debate, we analyze two cases separately. In the *social cost* case, agents' final utility is exactly as described above, and equity finance is socially costly. This could occur, for example, in situations where the advantages of debt finance reflect the value of safe, information-insensitive securities (see, e.g., Gorton and Pennacchi, 1990). In the *private costs* case, by contrast, the net benefit of early banker consumption $(\phi - 1)c_0$ is deducted from early households' consumption as a lump-sum tax. The only costs of equity finance in this case are private. The private costs case would obtain if the only benefits of debt finance derived from the associated tax shield. The analysis of competitive equilibrium is identical in both cases. We return to the distinction between private and social costs in our discussion of welfare in Section 4.

2.2. Bank projects

Each banker can invest in one project at date 0. Projects can be originated either in a bank-specific local sector, or in an aggregate sector. The required date 0 investment is 1 in each local sector, and $1 + c$ in the aggregate sector. We introduce the differential cost c so that we can conduct comparative statics on the attractiveness of local sector investment; all of our main results go through unchanged with $c = 0$.

At date 1, each sector enters a *boom* state with probability σ and a *crisis* state with probability $1 - \sigma$. State realisations are independent across sectors. Hence, if banks i and j both make local investments, or if one of them invests locally and the other in the aggregate sector, then bank i faces booms and crises independently of bank j. If both i and j invest in the aggregate sector, then either both banks face a crisis or both face a boom.

If a given sector enters a crisis, each bank project in that sector has one of three types: *strong*, *fragile*, and *dead*, which we denote by $\tau \in \{\mathbb{S}, \mathbb{F}, \mathbb{D}\}$, respectively. Conditional on a crisis, project types are drawn independently for all banks that have invested in that sector, according to a probability distribution that we specify below. In contrast, all banks projects in a booming sector are strong.

Strong projects yield R at time $t = 2$, irrespective of who owns them. Fragile projects require specialist recovery skills: they yield $r < R$ at time 2 if owned by any banker, but 0 if owned by a household.[7] Fragile projects can be liquidated early at time 1 for return $\lambda r < r$.[8] Dead projects yield 0 at time 2.

All projects are tradeable on a competitive secondary market at date 1. Project types are

[7] When modelling time 1 continuation values, we do not distinguish between projects that were originated locally or in the aggregate sector. We therefore assume that, while local projects require the skills of a specific bank for their origination at time 0, all bank possess the specialist skills required to bring them to fruition after origination.

[8] We can further allow strong projects to be liquidated early. However, since we will show that strong project can always be sold at fair value, this option would not be exercised in equilibrium.

publicly observable, so that there is a separate secondary market for each project type.

2.3. Moral hazard

Before investing in a project at date 0, bankers select a screening effort $e \in \{L, H\}$, which is not observed by households. Effort $e = L$ yields private benefit $B > 0$ for the banker; effort $e = H$ generates no private benefit. Screening effort influences the distribution of project quality in a crisis. Specifically, if a sector enters a crisis, then projects in that sector are strong with probability $p_e \in \{p_L, p_H\}$ and fragile with probability $q_e \in \{q_L, q_H\}$. We write $\Delta_p = p_H - p_L$ and $\Delta_q = q_H - q_L$, and we assume that

$$\Delta_p > 0 > \Delta_q; \tag{1}$$

$$\Delta_p + \Delta_q > 0. \tag{2}$$

Together, Equations (1) and (2) guarantee that high effort $e = H$ renders strong projects more likely, and fragile as well as dead projects less likely.

We denote by $\hat{p}_e$ and $\hat{q}_e$ the respective time 0 probabilities that projects are strong and fragile, conditional on banker effort $e \in \{L, H\}$:

$$\hat{p}_e = \sigma + (1 - \sigma)p_e. \tag{3}$$

$$\hat{q}_e = (1 - \sigma)q_e. \tag{4}$$

2.4. Contracting

Bankers make take-it-or-leave-it offers financing contract offers to early households at date 0, and to late households at date 1. These contracts are subject to two sets of frictions.

First, we assume that households have limited ability to enforce payments, as in Hart and Moore (1994). Specifically, when repayments to early or late households are due (at dates 1 and 2, respectively), bankers can attempt to renegotiate by offering a lower repayment than initially promised. If households reject this offer, then the bank is in default, and households are entitled to seize the bank's project and obtain cash flows from it. If the bank defaults at date 1, early households can seize projects and either liquidate them early, or sell them in the secondary market. If the bank defaults at date 2, late households can seize projects and extract the final payoff. Anticipating equilibrium contracts, we note that the latter option is viable only if the bank's project is strong, since a fragile project owned by households yields no payoff.

Second, we assume that, while project quality (strong, fragile or dead) and sectoral states (crisis or boom) are publicly observable at date 1, they cannot be verified in court. As a result, all contracts between households and banks are single-period loans with a fixed end-of-period promised repayment. We make this assumption for expositional reasons, and in order to match the empirical reality that banks are largely financed with fixed income securities. We argue in Section 7 that our main results remain valid in a setting where contracts can be made contingent on sectoral and project outcomes.

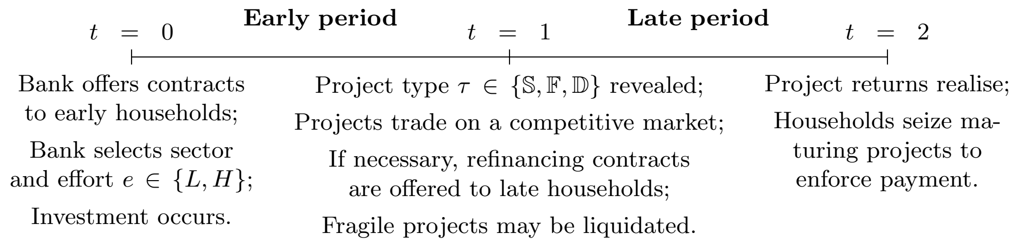


Figure 2: **Timing of the game between households and banks**. Banks offer single period loans to early and late households. Households can enforce payment only by seizing maturing projects.

2.5. Game timing and equilibrium definition

The game between one bank and its household investors is illustrated in Figure 2. At time 0, each bank offers short-term debt contracts to early households contracts. We write D for the face value of short-term debt issued to early households at date 0: that is, D is the promised repayment due at date 1. In return, households contribute to the bank's investment at date 0;[9] the remaining funds required for investment are contributed by the banker as inside equity, denoted A. The bank then selects its investment sector, makes a screening effort $e \in \{L, H\}$, and invests in a project.

At time 1, the state of each sector and each project's type $\tau \in \{\mathbb{S}, \mathbb{F}, \mathbb{D}\}$ is revealed. Banks and households trade projects on secondary markets. Each bank offers short-term debt contracts to late households, and may liquidate a fraction of a fragile project. The bank then repays D to early households. If it cannot repay, the bank is in default; in that case, the bank's creditors (early households) seize its project and extract the largest possible repayment, either by selling the project on the appropriate secondary market or by liquidating it early.

At time 2, project returns realise. Late households may seize maturing projects to force repayment.

An *equilibrium* comprises (i) time 0 contract offers from banks to households; (ii) time 1 contract offers from each bank to households; (iii) date 1 secondary market prices for each project type τ; and (iv) time 1 secondary market trading decisions for banks and late households. As discussed above, all time 1 decisions, but not time 0 contracts, can be made contingent on the realization of bank project types τ and sectoral states (boom or crisis). Banks choose contracts to maximize their expected utility subject to household participation constraints and incentive compatibility constraints, which we describe in detail in Sections 3.3 and 3.5, respectively. Households decide whether to accept or reject contracts, and whether to trade in the secondary market, so as to maximize their expected utility.

[9]Note that this is a risky debt contract, since no repayment is possible when the bank holds a dead project at date 0. Therefore, households' date 0 contribution will equal the fair (risk-neutral) price of risky debt in equilibrium.

2.6. Remarks on the model set-up

Remark 1: Portfolio choices and competitive forces

Banks in our model make a binary choice between local and aggregate investments. This assumption simplifies our presentation, but is not required: the same results hold if the bank is free to invest any fraction of its assets in the aggregate and local sectors respectively. Indeed, our analysis below demonstrates that each bank's maximization problem is linear in its portfolio choice. Allowing for continuous portfolio choices would therefore lead us to "bang-bang" solutions that are equivalent to the optimal choices we identify below.

We abstract from the competitive effects of banks' portfolio choice so as to clarify the exposition. Banks compete for lending opportunities in a more complex model. With this competition, aggregate lending rates in a given sector would fall as banks entered in that sector. With quantity competition, portfolio choices would be strategic substitutes: As more banks entered the aggregate sector, loan rates in that sector would fall, thus increasing banks' incentives to invest in their local sector, where they enjoy greater market power. This effect would counteract, but not eliminate, the strategic complementarities that we emphasize below.

Remark 2: Screening effort and the business cycle

We rely in our analysis on the assumption that banker effort is more important in crisis states. In support of this hypothesis, see, for example, Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012), who find a wider dispersion of firm productivities in bad times than bad; Bloom *et al.'s* findings are consistent with a greater role for intermediary screening in bad times. To isolate the key strategic effects, we make the stark assumption that bank screening does not matter in booms, but our qualitative conclusions remain true whenever screening is less important in booms.

Remark 3: Bank capital structure

We focus our attention on the case where the only sources of finance available to banks are (i) short-term contracts with either early or late households, and (ii) inside equity. We could also introduce (subordinated) long-term debt, for example, by assuming that some households are long-lived, or that early and late households can trade long-term contracts among each other at time 1. The effects we emphasize would arise in this extension only if short-term debt was sufficiently cheap relative to long-term debt. This could occur for several reasons. First, short-term debt in our environment offers superior market discipline and so better addresses the moral hazard problem. Moreover, short-term debt can be cheaper, because it better insures risk-averse households against individual-level liquidity shocks as in Diamond and Dybvig (1983), or serves a signal of quality as in Diamond (1991). Given these trade-offs, and the fact that real banks are financed to a large extent with short-term wholesale debt, we believe that our qualitative results would remain valid in empirically relevant parametric regions, even if alternative debt funding was available.

2.7. Parametric assumptions

We make three sets of assumptions which clarify the exposition of our main results, and which could be relaxed at the cost of additional notation. First, we consider the non-trivial case where bank projects have positive Net Present Value (NPV) when they are funded with equity:

$$\hat{p}_H R + \hat{q}_H r > (1+c)(1+\phi), \tag{5}$$

and we assume that screening effort increases NPV, even when it necessitates a shift from debt to equity funding:

$$(1-\sigma)(\Delta_p R + \Delta_q r) > B + \phi(1+c). \tag{6}$$

Second, we restrict attention to the empirically relevant region of the parameter space where banks use a mixture of debt and equity finance. This amounts to assuming that (i) the cost ϕ or inside equity is sufficiently high, so that banks wish to finance themselves with debt; and (ii) the moral hazard problem as measured by the private benefit B is sufficiently severe, so that incentives impose a meaningful limit on debt finance at date 0:

$$\phi > (1-\sigma)\frac{1-\lambda}{\lambda}; \tag{7}$$

$$B > \frac{1-\lambda}{\lambda}\Delta_p(R-1). \tag{8}$$

Finally, we impose a restriction that guarantees that fire sales occur only following adverse aggregate shocks. We assume that a perfectly diversified economy, where all banks invest in their local sectors, cannot experience liquidity shortages; specifically, that banks with strong projects always have sufficient debt capacity to buy all fragile projects at their fair value. Moreover, we assume that a perfectly correlated economy, in which every bank invests in the aggregate sector, experiences a liquidity shortage if and only if the aggregate sector is in a crisis. We therefore bound the ex-ante likelihood ratio $\hat{p}_H/\hat{q}_H$ of strong and fragile projects, as well as the ex-post likelihood ratio p_H/q_H conditional on entering a crisis, as follows:

$$\frac{\hat{p}_H}{\hat{q}_H} > \frac{r}{R-1}; \tag{9}$$

$$\frac{p_H}{q_H} < \frac{r}{R}. \tag{10}$$

3. Model solution

We solve the model by backward induction. First, we consider the secondary market for projects and banks' optimal choices at time 1. Second, we characterize the incentive compatibility and participation constraints facing banks at date 0. Finally, we solve the optimal contracting problem subject to these constraints, and characterize banks' equilibrium choices.

3.1. Time 1 *market for projects*

At time 1, there is a separate secondary market for each type $\tau \in \{\mathbb{S}, \mathbb{F}, \mathbb{D}\}$ of projects.

We begin by noting that strong ($\mathbb{S}$) and dead ($\mathbb{D}$) projects must be priced at their fair values, which are R and 0 respectively. This follows for strong projects because they require no specialist skills, so that late households can purchase them at date 1 and extract R at date 2. Households have deep pockets and, hence, any deviation of market prices from R would constitute an arbitrage opportunity. Dead ($\mathbb{D}$) projects have a market price of zero because they are guaranteed to return 0 at date 2.

Fragile ($\mathbb{F}$) projects, by contrast, may trade at a discount because only banks have the skills required to extract a return r from a fragile project at time 2. As we will see in the next Subsection, banks' demand for fragile projects is constrained by the availability of high-quality collateral. As a result, "cash-in-the-market" effect may cause fragile projects to trade for strictly less than r in the time 1 secondary market. We conjecture that, when the aggregate sector is in a boom, there is sufficient liquidity for fragile projects to trade at their fair value r; we will verify later that this conjecture is correct in equilibrium. We then write m for the price of fragile projects in an aggregate crisis, and we define κ to be the return that banks earn from the time 1 purchase of fragile projects in an aggregate crisis:

$$\kappa \triangleq \frac{r - m}{m}. \tag{11}$$

We refer to aggregate crises for which $\kappa > 0$ as *financial crises.* In a financial crisis, bank illiquidity causes fragile assets to trade at a discount to their fair value. *Systemic risk* is the danger that a financial crisis realises.

Note that the market price of fragile projects can never fall below their immediate liquidation value, so that $m \geq \lambda r$. In terms of the fire sales return κ, we then obtain $\kappa = r/m - 1 \leq 1/\lambda - 1 = (1 - \lambda)/\lambda$. Assumption (7) therefore yields and upper bound on the fire sales return κ:

$$\phi > (1 - \sigma)\kappa. \tag{12}$$

3.2. Time 1 *optimal choices*

We consider a bank's date 1 decision given the type τ of its project and the state of the aggregate sector; possible continuation values are illustrated in Figure 3. For now, we take as given the repayment D promised to early households in the bank's time 0 financing contract. In order to repay D, the bank must either sell its projects on the secondary market, or re-finance itself by offering contracts to late households. Refinancing is constrained by limited contract enforcement: The bank can renegotiate any promised repayment at date 2 by threatening to default.

If the bank holds a strong ($\mathbb{S}$) project, it can credibly promise late households a guaranteed repayment of R at time 2; it follows that, because households do not discount the future, the bank can borrow up to R at date 1. An optimal strategy in an aggregate boom is to borrow D in order to repay early households.[10] In an aggregate crisis, the bank can make additional profits by buying

[10] We can restrict attention to the case $D \leq R$ without loss of generality. Any promised repayment $D > R$ would lead to default at time 1 with probability 1, and cannot be part of an optimal contract at date 0.

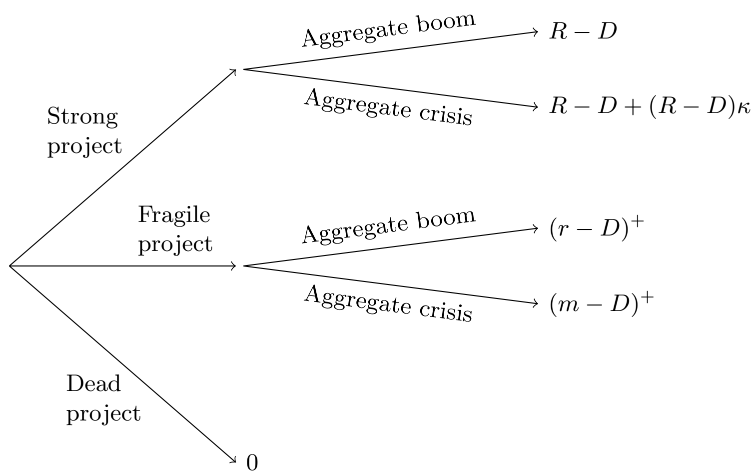


Figure 3: **Income of a bank with debt level D.** A bank with a strong project can use its unpledged income of $R - D$ from that project as collateral for the purchase of fragile projects from other banks. Those projects earn a return of 0 in aggregate booms, and κ in aggregate crises. A bank with a weak project is obliged to sell it; it earns the fair price r in aggregate booms and the reduced price m in aggregate crises. Banks with dead projects return 0.

fragile projects in the secondary market, provided those projects trade at a discount $\kappa > 0$; the optimal strategy in this situation is to borrow R, repay D to early households, and spend $R - D$ on fragile projects. Since the excess return on fragile projects is κ, a strong bank's continuation value at date 1 is therefore $R - D$ in an aggregate boom, and $R - D + (R - D)\kappa$ in an aggregate crisis.

If the bank holds a fragile ($\mathbb{F}$) project, it cannot borrow from late households. To see why, note that, were the bank to promise a positive repayment at date 2 to late households, it could renegotiate at time 2 to an arbitrarily small repayment $\varepsilon > 0$, because households would be unable to extract any cash flows in default. It follows that late households would reject the date 1 contract and, hence, that the bank must sell or liquidate its project to repay depositors. In an aggregate boom, the bank is able to sell its project at the fair value r. In an aggregate crisis, it can liquidate its project for λr, or sell it at the market price m. Since $m \geq \lambda r$, a (weakly) dominant strategy is to sell the project. The bank defaults if the proceeds are insufficient to repay D to early households. Accounting for limited liability, a fragile bank's continuation value is therefore $(m - D)^+$ in an aggregate crisis, and $(r - D)^+$ in an aggregate boom, where we write $x^+ = \max\{x, 0\}$ for the positive part of x.

Finally, if the bank holds a dead ($\mathbb{D}$) project, default is inevitable, and its continuation value is 0.

3.3. Time 0 *constraints*

Suppose that a bank exerts screening effort $e \in \{L, H\}$ and invests in sector $s \in \{a, l\}$ (corresponding to aggregate and local, respectively). We use the following shorthand notation: We write AB and AC for the respective events "Aggregate Boom," "Aggregate Crisis" and SB_e^s and FB_e^s for the events "Strong Bank project," and "Fragile Bank project" in case the bank makes screening effort

$e \in \{L, H\}$ and invests in sector $s \in \{l, a\}$.

Recall that bankers invest inside equity A at date 0, and that the remaining investment is financed by the contribution of early households as debt investors, in return for a promised repayment D at date 1. If early households expect effort e, then the date 0 contract must satisfy the following investor participation constraint:

$$1 + c\mathbb{1}_{s=a} - A \leq \hat{p}_e D + \hat{q}_e \left(\min(r, D)\, \mathbb{P}[AB|FB_e^s] + \min(m, D)\, \mathbb{P}[AC|FB_e^s]\right). \tag{13}$$

We can use Figure 3 to write down an expression for the bank's expected profit. Let $U_e^s(A, D)$ be the bank's profit in terms of contracting choices (A, D) if it invests in sector s and selects effort level e:

$$\begin{aligned} U_e^s(A, D) \triangleq\, & \hat{p}_e(R - D) + \hat{p}_e\, \mathbb{P}[AC|SB_e^s](R - D)\kappa + B\mathbb{1}_{e=L} \\ & + \hat{q}_e \left(\mathbb{P}[AB|FB_e^s](r - D)^+ + \mathbb{P}[AC|FB_e^s](m - D)^+\right) - (1 + \phi)A. \end{aligned} \tag{14}$$

Equation (14) can be re-written as follows:

$$\begin{aligned} U_e^s(A, D) = &\, \hat{p}_e(R - D) + \hat{q}_e(r - D)^+ - (1 + \phi)A + B\mathbb{1}_{e=L} \\ & + \hat{p}_e(R - D)\kappa\, \mathbb{P}[AC|SB_e^s] \\ & - \hat{q}_e\, \mathbb{P}[AC|FB_e^s] \left((r - D)^+ - (m - D)^+\right). \end{aligned} \tag{15}$$

The first line of Equation (15) is the expected profit that the bank derives from its investment in the absence of fire sale effects. The second line represents the expected *fire sale profit* that the bank makes from time 1 secondary market purchases of fragile projects: it earns $(R - D)\kappa$ if it is strong and the aggregate sector is in crisis; the bank is strong with probability $\hat{p}_e$, and the aggregate sector experiences a crisis with probability $\mathbb{P}[AC|SB_e^s]$. The third line is the bank's expected *fire sale loss* from forced sales performed if it is fragile at time 1: the bank is fragile with probability $\hat{q}_e$, in which case a fire sale occurs if, with probability $\mathbb{P}[AC|FB_e^s]$, there is an aggregate crisis; the bank's loss if there is a fire sale is $(r - D)^+ - (m - D)^+$.

The aggregate sector state is independent of local sector investments, but not of aggregate sector investments. We have

$$\mathbb{P}[AB|SB_e^l] = \mathbb{P}[AB|FB_e^l] = \sigma; \tag{16}$$

$$\mathbb{P}[AB|SB_e^a] = \sigma/\hat{p}_e; \mathbb{P}[AB|FB_e^a] = 0. \tag{17}$$

It is easy to see that, as a consequence of Assumption (6), it is optimal to incentivize high screening effort $e = H$ in the optimal contract. The screening incentive compatibility constraint in sector s is therefore $U_H^s(A,D) - U_L^s(A,D) \geq B$, which reduces to the following requirement:

$$\begin{aligned} IC^s(D) \triangleq\ & \Delta_p(R-D)\left(1+\kappa(1-\sigma)+\kappa\sigma\mathbb{1}_{s=a}\right) \\ & + \Delta_q\left(\sigma(r-D)^+ + (1-\sigma)(m-D)^+\right. \\ & \left. -\sigma((r-D)^+ - (m-D)^+)\mathbb{1}_{s=a}\right) \geq \frac{B}{1-\sigma}. \end{aligned} \tag{18}$$

3.4. Time 0 trade-off between aggregate and local investment

We now show that the time 0 fire sales have a more positive effect upon the profitability of banks that invest locally than upon those that invest in the aggregate sector. Fire sale profits and losses are given by the second and third line of Equation (15), respectively.

LEMMA 1. *For banks with a given level of indebtedness D and screening effort e:*

1. *Expected fire sale profits are higher from local than from aggregate sector investments;*
2. *Expected fire sale losses are lower from local than from aggregate sector investments.*

Proof: For part 1, note that $\mathbb{P}[AC|SB_e^a] = 1 - \sigma/\hat{p}_e < 1 - \sigma = \mathbb{P}[AC|SB_e^l]$ so that the second line of Equation (15) is higher when $s = l$ than when $s = a$. For part 2, note that $\mathbb{P}[AC|FB_e^a] = 1 > 1 - \sigma = \mathbb{P}[AC|FB_e^l]$ so that the third line of Equation (15) is more negative when $s = a$ than when $s = l$. □

Lemma 1 demonstrates that, for a given screening effort e, fire sales have a better effect upon the value of local sector investments than upon aggregate sector investments. Hence, ceteris paribus, bankers should prefer local to aggregate sector investment. But the choice between the two is complicated in practice by the banker's time 0 screening decision. Screening effort is not contractible. Hence, if bankers are to convince investors that they will screen investments, they must ensure that screening is incentive compatible. We now show that this is easier to accomplish with aggregate sector than local investment.

It is immediate from Equations (16), (17) and (18) that the difference between incentives under aggregate and local investments is given by Equation (19):

$$IC^a(D) - IC^l(D) = \sigma\Delta_p(R-D)\kappa - \Delta_q\left((r-D)^+ - (m-D)^+\right); \tag{19}$$

this expression is positive because $r \geq m$ and $\Delta_p > 0 > \Delta_q$. We therefore have the following result:

LEMMA 2. *For a given debt level D, if the screening incentive compatibility constraint is satisfied for local sector investment then so is the corresponding constraint for aggregate sector investment: $IC^a(D) > IC^l(D)$.*

When banks undertake screening effort, they are rewarded by a higher probability of a fire sale profit, and by a lower probability of a fire sale loss. The sensitivity of these effects to screening effort is higher for banks with aggregate sector investment, which renders the incentive constraint slacker for aggregate than for local investment. The difference in sensitivities is apparent in Equation

(15). One the one hand, increasing e from L to H increases $\mathbb{P}[AC|SB_e^a]$ but has no effect upon $\mathbb{P}[AC|SB_e^l]$, so that the profits from time 1 purchases of fire sale assets generate stronger incentives for aggregate than for local investments. On the other hand, because $\Delta_q < 0$, raising e from L to H serves to lower the likelihood that fire sale losses are incurred: the scale of the reduction depends upon $\mathbb{P}[AC|FB_e^s]$, which is higher for aggregate sector than for local investments.

To summarise, then, the choice between local and aggregate sector investment boils down to a trade-off between the efficiency costs that fire sale effects impose upon aggregate sector investors and the improved incentive effects of aggregate sector investments, which allow banks to take on more leverage and, hence, to reduce their cost ϕA of equity. In Section 3.5 we see how these trade-offs are resolved in the optimal bank contract.

3.5. Optimal time 0 *contracts*

Lemma 3 establishes that the optimal contract induces high effort, and that incentive and participation constraints bind.

LEMMA 3. *Under the optimal contract, bankers make high effort irrespective of the sector in which they invest, and banker incentive compatibility and creditor participation constraints both bind.*

The optimality of high effort follows from Equation (6). The incentive compatibility constraint binds for high enough debt D. Bankers face a trade-off when deciding whether to increase D. On the one hand, a higher level of debt exposes the bank to higher potential fire sale losses, and also reduces its ability to earn fire sale profits. On the other hand, bankers with higher debt levels use fewer of their own funds and, hence, incur a lower cost of equity. Equation (7) guarantees that the cost ϕ of equity is sufficiently high for the second of these effects to outweigh the first.

Lemma 3 implies that the optimal contract is obtained by solving the banker incentive compatibility constraint and the household participation constraint simultaneously to yield respective debt and equity levels $D^s(\kappa)$ and $A^s(\kappa)$. Lemma 2 implies that $D^a(\kappa) \geq D^l(\kappa)$; it is immediate from Equation (19) that $D^a(\kappa) = D^l(\kappa)$ precisely when $\kappa = 0$ (so that $(r - D)^+ = (m - D)^+$), in which case there are no fire sale profits or losses.

We define the expected profit of a bank that uses an optimal contract and invests in sector $s \in \{l, a\}$ as follows:

$$V^s(\kappa) \triangleq U^s(A^s(\kappa), D^s(\kappa)). \tag{20}$$

A bank chooses to invest in the aggregate sector precisely when $V^a(\kappa) > V^l(\kappa)$. Lemma 4 establishes the condition under which this is the case.

LEMMA 4. *The difference between banker profit with aggregate and local sector investments is given* *by the following expression:*

$$\begin{aligned} V^a(\kappa) - V^l(\kappa) = &-c(1+\phi) \\ &- \sigma\hat{q}_H(1+\phi)(r-m) \\ &- \kappa\left(\hat{p}_H(1-\sigma)(D^a(\kappa)-D^l(\kappa)) + (1-\hat{p}_H)\sigma(R-D^a(\kappa))\right) \\ &+ \phi\Big[\hat{p}_H(D^a(\kappa)-D^l(\kappa)) + \hat{q}_H\Big(\sigma(r-D^l(\kappa))\mathbb{1}_{D^l(\kappa)<r} \\ &\qquad + (1-\sigma)(m-D^l(\kappa))\mathbb{1}_{D^l(\kappa)<m} - (m-D^a(\kappa))\mathbb{1}_{D^a(\kappa)<m}\Big)\Big]. \end{aligned} \tag{21}$$

A bank therefore chooses aggregate sector investment over local sector investment precisely when this expression is positive.

Equation (21) has a simple intuitive relationship to Equation (15). The first line in Equation (21) reflects the fact that the cost of investment in the aggregate sector exceeds the cost of local sector investment by c; that cost is born by equity holders. The second line reflects the higher expected fire sale losses that accrue to banks with aggregate sector investments, and which result in higher ex ante equity funding. The third line reflects the fact that banks with aggregate sector investments make lower expected profits than those with local sector investments; those profits are debt-funded and, hence, are not multiplied by the cost ϕ of equity. The final line of the expression is the benefit of higher leverage (and, hence, of lower equity financing) for banks with aggregate sector investments.

3.6. Equilibrium

Section 3.5 establishes that κ determines each bank's investment decisions. In equilibrium, individual bank investment decisions determine the aggregate level of bank capital available to purchase fragile assets at time 1, and so imply a market fire sale discount κ. We therefore face a fixed-point problem; equilibrium investment decisions and the discount κ must be consistent.

PROPOSITION 1.

1. *There exists* a diversified equilibrium *in which all banks invest locally. In the diversified equilibrium, $\kappa = 0$ and no fragile projects are liquidated;*
2. *If the cost ϕ of equity is high enough and the cost advantage c of local investment is low enough, then there exists an additional* correlated equilibrium *in which all banks invest in the aggregate sector. In this equilibrium, $\kappa = (1-\lambda)/\lambda$ and some fragile projects are liquidated in an aggregate crisis.*

Proposition 1 shows that our model can exhibit two equilibria: one with diversified, local, investments; and one with correlated, aggregate sector, investments. Financial crises are an equilibrium phenomenon in the latter case: that is, as discussed in Section 3.1, when an aggregate crisis arises in a correlated equilibrium, bank illiquidity causes fragile project fire sales. The co-existence of equilibria with and without financial crises follows from a critical feature of our model: namely, that moral hazard introduces strategic complementarities. Banks prefer to invest locally if most

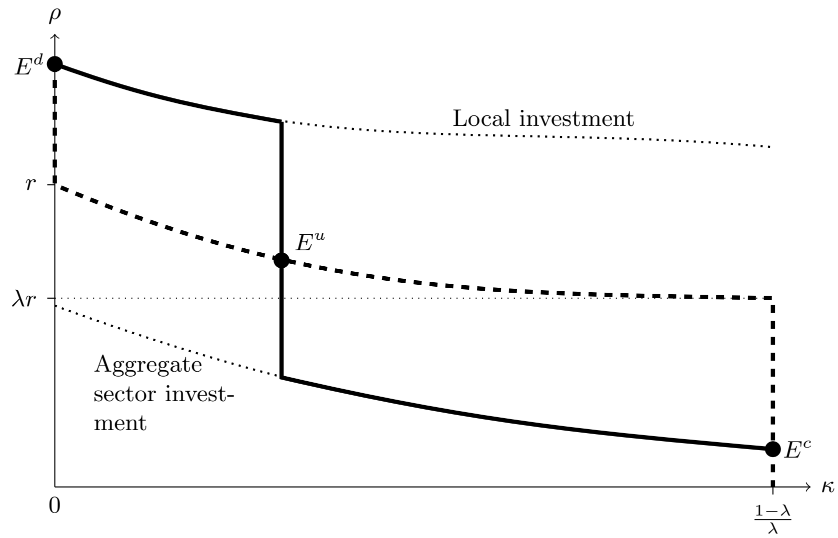


Figure 4: Equilibrium conditions

of their peers are doing the same, and they prefer aggregate sector investment if their peers are exposed to the aggregate sector.

To understand the intuition for this result, recall that a bank's investment decision boils down to a trade-off between the efficiency benefits of local investment and the incentive benefits of aggregate investment. The incentive benefits realise only when there are time 1 fire sales.

If all banks opt for local sector investment then, regardless of the aggregate state, time 1 fragile projects amount to $\hat{q}_H r$ and banks with strong projects can raise at least $\hat{p}_H(R-1)$ against their collateral; Equation (9) guarantees that the latter is sufficient to absorb the former so that there are no fire sales. Since $\kappa = 0$, aggregate sector investment yields no incentive benefits. Each bank therefore prefers local investment because it is cheaper.

If all banks invest in the aggregate sector, then, in an aggregate crisis, time 1 fragile projects amount to $q_H r$ and banks with strong projects can raise at most $p_H R$ against their collateral; Equation (10) guarantees that the latter cannot absorb the former, so that fire sales occur. Since $\kappa > 0$, aggregate sector investment yields incentive benefits that enable banks to borrow more. Aggregate sector investment is therefore sustainable in equilibrium if the benefits of higher debt outweigh the efficiency costs of aggregate sector investment.

Figure 4 illustrates the equilibria of Proposition 1. The vertical axis of the Figure is ρ, which we define to be the amount of capital that strong banks are able to deploy at time 1 for each fragile project.

The thick dashed line illustrates for each value of ρ the fire sale discount κ that clears the market for time 1 fragile projects. If $\rho \geq r$ then banks with strong projects have sufficient capital to purchase all of the fragile projects at their fair value r; if $\rho \leq \lambda r$ then there is insufficient capital to avoid some fragile project liquidation, so that $\kappa = (1-\lambda)/\lambda$; for intermediate values of ρ the market price of fragile projects adjusts so that there is just enough strong bank capital to purchase all fragile projects; in this case, $\rho = m$, or, equivalently, $\kappa = (r-\rho)/\rho$.

The upper dotted line in Figure 4 illustrates the ρ that obtains if all banks invest in their local sector. In this case there is always sufficient capital in strong banks to purchase all fragile projects at their fair price, so that the dotted line lies above $\rho = r$. The lower dotted line illustrates the ρ that obtains if all banks invest in the aggregate sector, so that their returns are perfectly correlated. In this case, there is always a fire sale in an aggregate crisis.

The thick unbroken line in Figure 4 illustrates a bank's preferred investment strategy as a function of the fire sale discount κ. As in Proposition 1, banks strictly prefer to perform cheap local investments when κ is low; if c is low and ϕ is high then, as in the Figure, banks strictly prefer to make aggregate sector investments when κ is sufficiently high.

Equilibria occur where the thick unbroken and dashed lines intersect. E^d and E^c are the diversified and correlated equilibria identified in Proposition 1. Both are stable equilibria. The equilibrium labelled E^u requires specific proportions of local and aggregate sector investments and is unstable: if any positive measure of households with the same equilibrium strategy deviates to the other strategy then all other households follow them. More generally, there is an odd number of unstable equilibria, but the precise number is not fixed by our assumptions.

Our analysis demonstrates that systemic risk can arise endogenously in our model as a result of strategic complementarities created by moral hazard problems in banking. We now turn to a discussion of the welfare consequences of our analysis.

4. Welfare implications

We now consider the welfare implications of aggregate and local sector investment. The most important consideration is whether the cost of equity ϕ is a transfer or a social cost; we do not take a view on this question, but we derive implications in both cases.

PROPOSITION 2. *Suppose that diversified and correlated equilibria both exist. Then:*

1. *If the cost ϕ of equity is a private cost then the correlated equilibrium has lower welfare than the diversified equilibrium;*
2. *If the cost ϕ of equity is a social cost then the correlated equilibrium has higher welfare than the diversified equilibrium.*

The proof of Proposition 2 appears in the appendix. Its intuition is as follows.

First, suppose that ϕ is a private cost that has no effect upon aggregate surplus. In this case, the inefficient liquidation that occurs in the correlated equilibrium of Proposition 1 and Figure 4 is uncompensated from a social point of view. In other words, the correlated equilibrium generates strictly lower welfare in this case than the diversified equilibrium, in which no liquidation occurs.

Second, suppose that ϕ is a social cost. Then, because only the banker extracts any surplus in our model, ex ante welfare is exactly equal to the banker's expected profit. The difference in welfares between the aggregate and local investment equilibria is therefore given by the following expression:

$$V^a(\bar{\kappa}) - V^l(0) = \left(V^a(\bar{\kappa}) - V^l(\bar{\kappa})\right) + \left(V^l(\bar{\kappa}) - V^l(0)\right), \tag{22}$$

where $\bar{\kappa} \triangleq (1-\lambda)/\lambda$.

By Proposition 1, the first bracketed term on the right had side of Equation (22) is positive whenever the correlated equilibrium exists. Two effects determine the sign of the second bracketed term. First, Assumption (9) guarantees that a local bank expects to experience higher fire sale profits than fire sale losses and, hence, that it prefers a higher κ. Second, bankers are able to assume more debt when κ is high because fire sale effects strengthen their incentives. Both effects work in the same direction so that $V^l(\kappa) > V^l(0)$. Since both bracketed terms in Equation (22) are positive, welfare is higher in the correlated than the diversified equilibrium.

Proposition 2 identifies two situations in which a suboptimal equilibrium could arise. First, if the cost ϕ of equity is a social cost then diversified equilibria in which all banks invest locally are suboptimal; regulators in this case should attempt to encourage correlated investment, which leaves the system exposed to inefficient liquidation in case of an aggregate sector crisis. This set-up is reminiscent of Allen and Gale's (1998) analysis of optimal financial crises. While this case is interesting, we concentrate in our policy analysis on the alternative case in which ϕ is a private cost. In this case, the correlated equilibrium is inefficient and should be discouraged by the regulator. As discussed in the Introduction, this type of intervention lies at the heart of recent developments in macro prudential regulation. We discuss policy approaches in Section 5.

5. Policy Implications

We have shown that our economy can have both diversified equilibria and correlated equilibria with systemic risk. In this Section, we examine policy interventions in the case where ϕ is a private cost. If banks choices were perfectly observable, governments could make it illegal for banks to make correlated investments. Casual empiricism suggests that this is a practical impossibility. We consider the more challenging case where regulators cannot observe the correlation between bank portfolio returns, and so are forced to rely on indirect strategies. We consider policies relating to capital adequacy rules, and to asset purchase programs.

5.1. Capital requirements

Proposition 3 demonstrates that a minimum capital requirement can rule out correlated equilibria.

Proposition 3. *Let D^0 and A^0 solve Equations (23) and (24):*

$$\frac{B}{1-\sigma} = \Delta_p(R - D^0) + \Delta_q(r - D^0)^+; \tag{23}$$

$$A^0 = 1 - \hat{p}_H D^0 - \hat{q}_H \min(D^0, r). \tag{24}$$

If banks are subject to a minimum capital requirement that obliges them to set $A \geq A^0$, then the diversified equilibrium without fire sales is the unique equilibrium.

Proposition 3 is a consequence of the following intuition. Banker effort can be incentivised in two ways: either they can expose themselves to fire sale risks by investing in the aggregate sector, or they can give themselves sufficient "skin in the game" by using equity instead of debt to finance their projects. Absent any form of regulation, bankers prefer the former approach when the private cost

ϕ of equity is high enough. The capital requirement A^0 of Proposition 3 forces bankers to deploy just enough equity to satisfy their incentive compatibility constraint at any $\kappa \geq 0$. Exposure to fire sale losses would then be wasteful, because it would generate efficiency losses with no concomitant incentive effect; bankers that are subject to minimum capital requirements therefore opt for local sector investment.

5.2. Asset purchase programs

In our model, the only time 1 buyers to purchase fragile bank assets are strong banks; this is because we assume that only banks have the specialist skills required to extract a return from fragile projects. We now consider an extension of our model in which the government is prepared to purchase fragile assets at time 1 in the event of an aggregate crisis. This extension allows us to consider the implications for financial stability of asset purchase schemes like the Troubled Asset Relief Program (TARP) in the United States and the European Asset Purchase Program (ARP). Proposition 4 demonstrates that financial stability increases in our model when banks anticipate this type of scheme.

Proposition 4. *Suppose that the government purchases fragile bank assets for their fair value, r, in an aggregate crisis. Then the diversified equilibrium without fire sales is the unique equilibrium.*

Proposition 4 is an immediate consequence of the fact that an asset repurchase program rules out fire sales and, hence, eliminates the incentive benefits of investment in the aggregate sector. In short, when bankers anticipate government support in the event of an aggregate crisis, our model predicts that they will opt for uncorrelated local investments so that financial crises cannot occur.

Many commentators argued in the wake of the 2008–09 financial crisis that the anticipation of government support had a destabilising effect upon the banking sector, because it shielded bankers from the consequences of bad lending decisions. Our analysis identifies a countervailing effect. In our model, it is cheaper for bankers to incentivize themselves by creating aggregate sector risks than by maintaining high equity capital levels. Shielding bankers from the effects of their aggregate risk taking forces them to use equity capital to incentivise themselves and, hence, eliminates the risk of financial crises.

Traditional model hazard concerns can be addressed in our set-up. Proposition 4 relies upon the assumption that bankers expect government asset purchases to occur at r. This is a critical assumption. If the government were to pay more than r, it would effectively reward banks for failing during aggregate crisis; standard moral hazard problems would then arise, because bankers would therefore have a strong incentive to select the correlated investments that cause financial crises. Moreover, if fragile assets were purchased for much less than r, then fire sale risk would remain and, hence, so would the incentive to make aggregate sector investments.

6. Empirical Implications

In this Section we identify circumstances under which correlated equilibria are more likely to arise than diversified equilibria. To do we assume a parametrisation under which stable diversified and correlated equilibria both exist in the baseline model. We consider an extension of the basic model in which a fraction α of banks can only invest in the aggregate sector. Lemma 5 demonstrates that correlated equilibria are inevitable in this extension when α is high enough.

LEMMA 5. *Suppose that a fraction α of banks have no local knowledge. Then there exists a threshold fraction $\alpha^\star \in (0,1)$ such that:*

1. *The correlated equilibrium is the unique equilibrium if $\alpha > \alpha^\star$;*
2. *α^* is weakly increasing in the cost c of aggregate investment.*

If α is high enough, then the strategic complementarities that generate Proposition 1 lead even banks with local knowledge to opt for aggregate sector investment, too, so as to exploit the incentive benefits of fire sale risks. This raises the number of banks that make correlated investments to a sufficient level to guarantee a fire sale in the event of an aggregate crisis. Aggregate sector investment is less attractive when its cost c is higher and, hence, the effect of the strategic complementarities is dampened. As a result, the threshold value α^* increases.

Lemma 5 shows that bank asset correlation and systemic risk will both increase in response to shocks to policy or to technologies that serve either to increase the fraction α of banks that is constrained to invest in the aggregate sector, or to lower the cost c of aggregate sector investment. This point is developed below.

6.1. Technological advances

Kindleberger (2005) argues that technological innovations have caused many financial booms by catalysing surges of optimism about the prospects for a particular sector such as railroads, or the internet, and so stimulating high levels of investment. Kindleberger notes that such booms frequently end in systemic crises.

The optimism upon which Kindleberger's story rests is irrational and, hence, so too is the investment that gives rise to financial booms and ultimately to busts. Our model suggests an alternative explanation of Kindleberger's data. A technological shock that reduces the cost c of entering an aggregate sector also lowers the entry threshold α^* above which strategic complementarities kick in and an investment boom occurs. The ensuing boom, as in our model, is the result of individually rational actions. Indeed, a relatively small innovation can lead to a discontinuous shift towards equilibria with perfect correlation and systemic risk.

6.2. Government-sponsored credit

Rajan (2011) argues that the subprime mortgage boom of the 2000s is largely attributable to a government decision to promote home ownership. The government-sponsored agencies Fannie Mae and Freddie Mac were directed to guarantee loans to low-income houses; it is nevertheless unclear a priori why the majority of private commercial banks should also have elected to assume large exposures to the subprime mortgage sector.

The strategic complementarities at the heart of our model provide a candidate explanation. As in Lemma 5, once a sufficient level of lending occurs in an aggregate sector, it is rational for other lenders also to move into that sector. Systemic risk can therefore be an unintended consequence of any policy that directs lending in a specific sector.

6.3. Cost of equity and specialist investors

Proposition 1 states that correlated equilibria arise when the cost ϕ of equity is sufficiently high. Ceteris paribus, therefore, anything that increases the relative cost of equity finance could result in more correlated investment and, hence, a greater degree of systemic risk. It follows that increases in corporate tax and heightened information asymmetry should be associated with heightened systemic risk. Note that, in our model, systemic risk is created in order that banks can raise more debt: it is not a consequence of an exogenous increase in indebtedness.

Proposition 4 notes that correlated equilibria do not occur when the government stands ready to acquire fragile bank assets for their fair value. A similar effect should occur when any other specialist buyer of distressed assets is available. Hence, for example, when vulture investment funds that are able to manage distressed bank assets emerge, they should serve to diminish correlated investment. For evidence that vulture funds realise value, see Hotchkiss and Mooradian (1997) and Guo, Hotchkiss, and Song (2011).

Note that the implications of the preceding two paragraphs distinguish our model from the literature on herding. In that literature, economic actors discard their own private information and instead copy the investment decisions of their peers, so that aggregate sector investment returns are correlated. It is rational for actors in herding models to copy others either because they believe that others investments reveal positive information (Scharfstein and Stein, 1990), or because they fear that others will interpret their deviation from the herd as revealing bad information about themselves (Bikchandani, Hirshleifer, and Welch, 1992). Like us, herding models generate rational correlated investment in the absence of government action. But herding models have nothing to say either about the relationship between the cost of equity and the incidence of herding, or about the importance of expert investors. These effects can therefore be used to distinguish our work from the herding literature.

7. Robustness: Contracting extensions

We assume in Section 2.4 that project quality and sectoral state cannot be identified in court. We now examine the effect upon our intuitive results of expanding the contract space. Section 7.1 analyzes contracts that can depend upon the aggregate state, and Section 7.2 considers complete state-contingent contracts.

7.1. Contracts on aggregate state

Suppose that it is possible to contract upon the realisation of the aggregate state, so that the bank can promise different payments D_{AB} and D_{AC} in aggregate sector booms and crises, respectively. In practice a bank could achieve such contracts either by promising debt repayments contingent upon the level of a stock market index, or by combining a standard debt contract with an option on that index.

Recall from Lemma 2 that, when state-contingent contracts are impossible, aggregate sector investment is attractive because it renders the incentive constraint easier to satisfy and, hence, allows the bank to borrow more. We wish to determine whether this effect still arises when it is possible to contract upon the aggregate state.

Following Section 3.3 we have the following expression for the expected profit of a bank with state-contingent debt contract (D_{AB}, D_{AC}):

$$\begin{aligned} U_e^s(A, D_{AB}, D_{AC}) = \hat{p}_e \left(\mathbb{P}[AB|SD_e^s](R - D_{AB}) + \mathbb{P}[AC|SB_e^s](R - D_{AC})(1+\kappa)\right) \\ + \hat{q}_e \left(\mathbb{P}[AB|FB_e^s](r - D_{AB})^+ + \mathbb{P}[AC|FB_e^s](m - D_{AC})+\right) \\ - c\mathbb{1}_{s=a} + B\mathbb{1}_{e=L} - (1+\phi)A. \end{aligned} \tag{25}$$

As before, the screening incentive compatibility constraint in sector s with contract (D_{AB}, D_{AC}) is $U_H^s(A, D_{AB}, D_{AC}) - U_L^s(A, D_{AB}, D_{AC}) \geq B$. It is easier for expositional purposes to consider the constraint separately for each sector. In the aggregate sector, Equations (17) and (25) yield the following incentive contract:

$$IC_c^a(D_{AB}, D_{AC}) \triangleq \Delta_p(R - D_{AC})(1+\kappa) + \Delta_q(m - D_{AC})^+ \geq \frac{B}{1-\sigma}, \tag{26}$$

where we use the c subscript in IC_c^a to indicate that the IC constraint arises with state-contingent contracts.

Conditional upon a decision to invest in the aggregate sector, the bank generates the strongest possible incentives by selecting D_{AB} and D_{AC} so as to maximize IC_c^a subject to the requirement that it at least meets the depositor's participation constraint given a fixed investment level $\bar{D}$:

$$\sigma D_{AB} + (\hat{p}_H - r)D_{AC} + \hat{q}_H \min(D_{AC}, m) \geq \bar{D}. \tag{27}$$

The following result follows from inspection of Equations (41) and (27):

LEMMA 6. *Suppose that the bank elects for aggregate sector investment. Then its optimal aggregate sector-contingent debt contract (D_{AB}, D_{AC}) sets $D_{AC} = 0$ and $D_{AB} = \bar{D}/\sigma$. With this debt contract, $Ic_c^a = \Delta_p R(1+\kappa) + \Delta_q m$.*

Effort is irrelevant for aggregate sector investments in an aggregate boom, so the optimal contract is obtained by selecting D_{AC} to generate the strongest possible incentives, and then setting D_{AB} so as to satisfy Condition (27) as cheaply as possible. Lowering D_{AC} has two effects. First, it increases the fire sale profits that a strong bank can expect to earn in an aggregate crisis; second, it increases the marginal difference between success and failure in crisis states. Both effects serve to strengthen incentives and, hence, the bank optimally sets D_{AC} to its minimum level of 0.[11]

With $D_{AC} = 0$, a bank with aggregate sector investments is exposed to the whole of the marginal consequences of its effort decision; it behaves as if it were entirely equity financed. Consider instead a

[11]Formally, we have $\frac{dD_{AC}}{dIC_c^a} = -(\Delta_p + \Delta_q \mathbb{1}_{D_{AC}<m}) - \kappa\Delta_p$, which is negative by Equation (2).

bank with local sector investments. Allowing for aggregate sector-contingent debt financing expands its contract set but, because it is still unable to focus its contracts entirely upon the sectors in which its effort decision matters, it cannot achieve the same level of incentives as the aggregate sector bank. This reasoning yields Lemma 7, which we prove formally in the Appendix:

LEMMA 7. *Suppose that it is possible to condition debt repayments upon the state of the aggregate sector. Then it is easier to satisfy the screening incentive constraint for aggregate sector investments than for local sector investments.*

Lemma 7 demonstrates that our basic intuition survives the introduction of contracts that are contingent upon the aggregate sector. Such contracts allow the bank to expose itself to the full marginal effect of its effort decisions in the aggregate sector, but not in the local sector. They therefore generate stronger incentives for aggregate sector investments than for local sector investments. This effect, which is an example of Holmström's (1979) informativeness principle, applies even in the absence of any fire sale effects. Hence, when the cost advantage c of local investment is low enough, contracting on the aggregate sector may dominate local sector contracting without fire sale effects. If this happens, aggregate sector contracting removes the strategic complementarity that drives the rest of our results, and also has a deleterious effect upon systemic risk.

7.2. Complete state-contingent contracts

We now allow the banks in our model to contract on aggregate and local sector states. In this case, banks that invest in the local sector can achieve the same exposure to the marginal effect of their actions as those with aggregate sector investments in Section 7.1, without resorting to any inefficient liquidation. It follows that correlated investment does not occur when we allow for complete contracting in our baseline model. However, a small modification to the model assumptions restores our qualitative results.[12]

In the spirit of Diamond and Rajan (2000) and Rochet and Tirole (1996), assume that the originating bank has critical skills required to extract any value from a fragile project that is not liquidated. Hence, the originator always earns rent from unliquidated fragile projects, even if they are sold to another bank. Ex ante incentives might then be optimally created by writing a contract that prevents originators from earning ex post rents from fragile projects. These rents undermine ex ante incentives since they increase the bank's continuation value whenever it is fragile. Consequently, the optimal incentive contract mandates that fragile projects are liquidated at date 1, as long as the cost ϕ of inside equity is sufficiently large.

Crucially, this optimal contract is credible only if there is no liquid market in which fragile projects can be sold at a price that exceeds the liquidation value. If such a market exists, then banks and depositors prefer date 1 renegotiation to liquidation. The only way to prevent such markets from emerging may be to perform correlated investment, so that systemic risk re-emerges even in a complete contracting framework if we require that contracts in that framework be renegotiation-proof.

[12]We present an informal outline of the extension. A complete treatment appears in an earlier working paper.

8. Conclusion

This paper starts from the observation that modern measures of systemic risk appear to have exhibited fairly constant time series properties since the market crash of 1929. More recently, a range of regulatory and legislative changes intended to reduce the likelihood of state support for failing banks have had little effect upon systemic risk. These data suggest that systemic risk is driven by economic fundamentals that are relatively invariant to legal and institutional change.

We demonstrate that systemic risk can arise as a natural consequence of the need to incentivize bankers who act as delegated monitors. Bankers can incentivize themselves in a competitive equilibrium either by retaining inside equity, or by exposing themselves to the risk of fire sale losses upon financial fragility. Fire sales occur in our model when financial distress is sufficiently widespread, so banks can only achieve this type of incentive when other banks have systemic investments. This fact generates strategic complementarities: when one bank selects systemic investments, others follow it so as to achieve fire sale incentives.

Our model therefore admits equilibria with and without systemic risk. When systemic risk arises, it is an endogenous response to incentive problems, and not an attempt to earn bailout rents. While regulation does not cause systemic risk in our model, it can attempt to address it. We identify two effective forms of regulation. First, the regulator can force banks to retain sufficient inside equity to ensure that they have strong incentives even without fire sale effects; banks then achieve no economies by exposing themselves to fire sale effects, and so opt instead for local sector investment. Second, the regulator can render fire sale effects ineffective. For example, it can communicate its willingness to sponsor asset purchase programs like TARP. Such programs prevent fire sale effects, and so remove the incentive advantages of aggregate sector investment so that, once again, the bank opts for local investment and so avoids systemic risks. We show that the design of TARP-style programs is critical: if they generate subsidies for failing banks then they can actually serve to enhance systemic risk.

Our analysis generates insights into some current policy debates. The correlated investments that cause systemic risk generate incentive benefits only when they are adopted by sufficiently many banks. Any policy that renders correlated investment more likely thus has the potential to destabilize the banking sector. Hence, a system of government-sponsored credit, as for example in the U.S. mortgage markets, could generate systemic risks. Such institutions inevitably lend in the aggregate sector and, hence, may cause an amplification effect that substantially increases systemic risk.

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APPENDIX

Proof of Lemma 3

Note that the incentive compatibility constraint (18) is satisfied when $D = 0$; a bank that invests in sector s and makes high effort therefore earns at least the income $U(\hat{A}, 0)$ it earns without debt, where $\hat{A}$ is 1 for local investment and $1 + c$ for aggregate sector investment. A firm that exerts low effort can fund itself entirely with debt; we write $\hat{D}$ for its indebtedness so that its income is $U(0, \hat{D})$. Then

$$\begin{aligned} U(\hat{A}, 0) - U(0, \hat{D}) = {} & (\Delta_p R + \Delta_q r)(1 - \sigma) + D(\hat{p}_L + \hat{q}_L) - (1 + \phi)\hat{A} - B \\ & + \mathbb{P}[AC|SB_e^s](\kappa R \Delta_p (1 - \sigma) + D\hat{p}_L) \\ & - \mathbb{P}[AC|FB_e^s]\left(\mathbb{1}_{D \leq m} \Delta_q (1 - \sigma)(r - m)\right. \\ & \qquad \left. + \mathbb{1}_{m < D \leq r}(\hat{q}_H (r - m)\mathbb{1}_{m < D \leq r} - \hat{q}_H (r - D)\right) \end{aligned}$$

The last line of this expression is greater than or equal to $\mathbb{1}_{m<D\leq r}\Delta_q(1 - \sigma)(r - m)$. Then, as $\Delta_p > 0 > \Delta_q$, we have

$$U(\hat{A}, 0) - U(0, \hat{D}) \geq (\Delta_p R + \Delta_q r)(1 - \sigma) - (1 + \phi) - B,$$

which is positive by Equation (6). It follows that, irrespective of sector, banks opt for high effort.

Writing $x^+ = x\mathbb{1}_{x\geq 0}$ and $\mathbb{1}_{D\leq m} = \mathbb{1}_{D\leq r} - \mathbb{1}_{m<D\leq r}$ we can differentiate Equation (18) to get

$$\frac{dIC^s}{dD}(D) = -(\Delta_p + \Delta_q \mathbb{1}_{D\leq r} - (1 - \sigma(1 - \mathbb{1}_{s=a}))(\kappa\Delta_p - \Delta_q \mathbb{1}_{m<D\leq r}), \tag{28}$$

which is negative because, by Equations (1) and (2), $\Delta_p + \Delta_q > 0$ and $\Delta_p > 0 > \Delta_q$. Hence, for a given A, lowering D slackens the Banker IC constraint (18). It follows that, if the participation constraint (13) is slack at an equilibrium (A, D), then the bank is able to reduce D slightly; because $\frac{\partial U_e^s}{\partial D}(A, D) < 0$ the bank will choose to do so. The bank's participation constraint therefore binds in equilibrium.

Assume that the incentive constraint (18) is satisfied. Then the identity $\min(x - y) = x - (x - y)\mathbb{1}_{y<x}$ allows us to write the participation constraint (13) in the respective cases $s = l$ and $s = a$ as follows:

$$A \geq 1 - \hat{p}_H D - \hat{q}_H \left(\sigma(r - (r - D)\mathbb{1}_{D<r}) + (1 - \sigma)(m - (m - D)\mathbb{1}_{D<m}\right); \tag{29}$$

$$A \geq 1 + c - \hat{p}_H D - \hat{q}_H \left(m - (m - D)\mathbb{1}_{D<m}\right). \tag{30}$$

Allowing these expressions to bind and substituting for A in Equation (14) yields the following expressions for the expected profit of banks with local and aggregate sector investments:

$$\begin{aligned} U^l(D) &= \hat{p}_H(R-D)(1+(1-\sigma)\kappa) - (1+\phi) + \hat{p}_H(1+\phi)D \\ &\quad - \phi\hat{q}_H\left(\sigma(r-D)\mathbb{1}_{D<r} + (1-\sigma)(m-D)\mathbb{1}_{D<m}\right) \\ &\quad + \hat{q}_H(1+\phi)(\sigma r + (1-\sigma)m); \qquad (31) \\ U^a(D) &= \hat{p}_H(R+D\phi) + (\hat{p}_H - \sigma)(R-D)\kappa + \hat{q}_H m(1+\phi) \\ &\quad - (1+c)(1+\phi) - -\hat{q}_H\phi(m-D)\mathbb{1}_{D<m}. \qquad (32) \end{aligned}$$

Direct differentiation of these expressions then yields

$$\begin{aligned} \frac{dU^l}{dD} &= \hat{p}_H\left(\phi - (1-\sigma)\kappa\right) + \hat{q}_H\left(\sigma\mathbb{1}_{D<r} + (1-\sigma)\mathbb{1}_{D<m}\right) \\ &\geq \hat{p}_H\left(\phi - (1-\sigma)\kappa\right); \\ \frac{dU^a}{dD} &= \hat{p}_H(\phi-\kappa) + \sigma\kappa + \hat{q}_H\phi\mathbb{1}_{D<m} \\ &= \hat{p}_H\left[\phi - \left(1 - \frac{\sigma}{\hat{p}_H}\kappa\right)\right] + \hat{q}_H\phi\mathbb{1}_{D<m}, \end{aligned}$$

both of which are positive by Equation (12). If the bank's incentive constraint is slack then the banker will therefore raise D, until it binds.

Proof of Lemma 4

Expressions for $V^a(\kappa)$ and $V^l(\kappa)$ are obtained by setting $D = D^l(\kappa)$ and $D = D^a(\kappa)$ in Equations (31) and (32), respectively. Straightforward manipulation yields Equation (21).

Proof of Proposition 1

Note that, by Equation (19), if $\kappa = 0$, then $D^a(\kappa) = D^l(\kappa)$, so that, setting $r = m$ in Equation (21), we have $V^a(\kappa) - V^l(\kappa) = -c(1+\phi) \leq 0$ so that every bank opts for local sector investment. Now suppose that all banks are local; if $\kappa = 0$ then part 1 of the Proposition is proved.

If all banks are local then, regardless of the aggregate state, at time 1 there are $\hat{p}_H$ strong projects and $\hat{p}_L$ weak projects. Banks with strong projects are able to absorb all fragile projects at their fair value, so that $\kappa = 0$, precisely when $\hat{p}_H(R - D^l(0)) > \hat{q}_H r$, or

$$\frac{\hat{p}_H}{\hat{q}_H} > \frac{r}{R - D^l(0)}. \qquad (33)$$

But, because $D^l(0) < 1$, we have $r/(R - D^l(0)) < r/(R-1)$, so that, by Equation (9), Condition (33) is satisfied and, hence, $\kappa = 0$.

We now prove the second part of the Proposition. First, note that $IC^a(D^a) = B/(1-\sigma) =$

$IC^l(D^l)$. Using this expression and Equation (18) yields

$$
\begin{aligned}
\Delta_p(D^a - D^l)(1+\kappa(1-\sigma)) &= \kappa\sigma\Delta_p(R - D^a) + \Delta_q\Big(\sigma((r-D^a)^+ - (r-D^l)^+) \\
&\quad +(1-\sigma)((m-D^a)^+ - (m-D^l)^+)\Big) \\
&\quad - \sigma((r-D^a)^+ - (m-D^a)^+) \\
&\geq \kappa\sigma\Delta_p(R-D^a),
\end{aligned}
$$

where the inequality follows from the facts that $\Delta_q < 0$ and, by Lemma 2, that $D^a \geq D^l$. Hence we have

$$
D^a - D^l \geq \frac{\sigma\kappa}{1+\kappa(1-\sigma}(R-D^a). \tag{34}
$$

Substituting for $D^a - D^l$ in Equation (21) and rearranging yields

$$
\begin{aligned}
V^a - V^l \geq (1+\phi)&\left(-c + \sigma\kappa\left(\hat{p}_H\frac{R-D^a}{1+\kappa(1-\sigma)} - \hat{q}_H\frac{r}{1+\kappa}\right)\right) \\
&- (R - D^a) + \Theta\sigma\kappa, \qquad (35)
\end{aligned}
$$

where we use the fact that $r - m = r\kappa/(1+\kappa)$ and we write

$$
\begin{aligned}
\Theta &= \sigma(r - D^l(\kappa))\mathbb{1}_{D^l(\kappa)<r} + (1-\sigma)(m - D^l(\kappa))\mathbb{1}_{D^l(\kappa)<m} - (m - D^a(\kappa))\mathbb{1}_{D^a(\kappa)<m} \\
&= \mathbb{1}_{D^l<m<D^a}(\sigma r + (1-\sigma)m - D^l) + \mathbb{1}_{m>D^a}(\sigma(r-m) + D^a - D^l) + \mathbb{1}_{m<D^l<r}\sigma(r - D^l) \\
&\geq 0.
\end{aligned}
$$

Using Equation (9) and the fact that $D^a < 1$ we can establish the following inequality:

$$
\sigma\kappa\left(\hat{p}_H\frac{R-D^a}{1+\kappa(1-\sigma)} - \hat{q}_H\frac{r}{1+\kappa}\right) \geq \hat{p}_H(R-1)\sigma\kappa\frac{\sigma\kappa}{1+\kappa(1-\sigma)},
$$

which is positive. It follows that $V^a - V^l$ is positive for low c and high enough ϕ; when this is the case a correlated equilibrium exists.

Proof of Proposition 2

The correlated equilibrium includes inefficient fire sales that are compensated by lower costs of equity. Hence, if ϕ is a private cost, then the correlated equilibrium has lower welfare than the diversified equilibrium.

As all of the surplus is captured by the bank, when ϕ is a social cost, the welfare difference between the correlated and diversified equilibria is

$$
\Delta_W \triangleq V^a(\kappa) - V^l(0) = \Big(V^a(\kappa) - V^l(\kappa)\Big) + \Big(V^l(\kappa) - V^l(0)\Big).
$$

For an aggregate equilibrium to exist we must have $V^a(\kappa) > V^l(\kappa)$, so it is sufficient to demonstrate that $V^l(\kappa) - V^l(0) > 0$.

Allowing the participation constraint (13) to bind and substituting for A in Equation (14) for U_H^l yields

$$\begin{aligned} V^l(\kappa) &= \hat{p}_H(R - D^l(\kappa))(1 + (1-\sigma)\kappa) - (1+\phi) + \hat{p}_H(1+\phi)D \\ &\quad - \phi\hat{q}_H\left(\sigma(r - D^l(\kappa))\mathbb{1}_{D^l(\kappa)<r} + (1-\sigma)(m - D^l(\kappa))\mathbb{1}_{D^l(\kappa)<m}\right) \\ &\quad + \hat{q}_H(1+\phi)(\sigma r + (1-\sigma)m). \end{aligned} \tag{36}$$

After some manipulation, Equation (36) yields

$$\begin{aligned} V^l(\kappa) - V^l(0) = {}& \hat{p}_H(R - D^l(\kappa))(1-\sigma)\kappa + \hat{p}_H\phi(D^l(\kappa) - D^l(0)) \\ & + \hat{q}_H(1+\phi)(1-\sigma)(m-r) \\ & + \phi\hat{p}_H\Big[\left((r-m)(1-\sigma) + D^l(\kappa) - D^l(0)\right)\mathbb{1}_{D^l(\kappa)<m} \\ & \qquad + \left(\sigma D^l(\kappa) + (1-\sigma)r - D^l(0)\right)\mathbb{1}_{m<D^l(\kappa)<r} \\ & \qquad + \left(r - D^l(0)\right)\Big]. \end{aligned} \tag{37}$$

The square-bracketed term in Equation (37) is non-negative. By Lemma 3, the incentive compatibility constraint (18) binds: setting $IC^l(D^l(\kappa)) = IC^l(0)$ and rearranging gives us the following expression for $D^l(\kappa) - D^l(0)$:

$$\begin{aligned} \Delta_p\left(D^l(\kappa) - D^l(0)\right) = {}& \Delta_p\left(R - D^l(\kappa)\right)(1-\sigma)\kappa \\ & + \Delta_q\Big[\sigma\left((r - D^l(\kappa))^+ - (r - D^l(0))^+\right) \\ & \qquad +(1-\sigma)\left((m - D^l(\kappa))^+ - (m - D^l(0))^+\right)\Big]. \end{aligned} \tag{38}$$

The equare-bracketed term in Equation (38) is non-positive and $\Delta_q < 0$, so we have

$$D^l(\kappa) - D^l(0) \geq (1-\sigma)\kappa\left(R - D^l(\bar{\kappa})\right), \tag{39}$$

where $\bar{\kappa} \triangleq ((1-\lambda)/\lambda$ is the highest possible κ.

Setting the square bracketed term in Equation (37) to zero and using Equation (38) to substitute for $D^l(\kappa) - D^l(0)$ gives us the following:

$$\begin{aligned} V^l(\kappa) - V^l(0) &\geq (1-\sigma)\kappa(1+\phi)\left(\hat{p}_H(R - D^l(\bar{\kappa})) - \hat{q}_H\frac{r}{1+\kappa}\right) \\ &\geq \hat{q}_H r(1-\sigma)\kappa(1+\phi)\left(\frac{R - D^l(\bar{\kappa})}{R-1} - \frac{1}{1+\kappa}\right) \\ &\geq \hat{q}_H r(1-\sigma)\kappa(1+\phi)\frac{\kappa}{1+\kappa} \\ &\geq 0. \end{aligned}$$

where the second inequality follows from Equation (9) and the third from the fact that $D^l(\kappa) \geq 1$. The final inequality establishes our result.

Proof of Proposition 3

Note that $IC^l(D)$ is $\Delta_p(R - D^0) + \Delta_q(r - D^0)^+$ when $\kappa = 0$ and is increasing in κ. Hence, if $A \geq A^0$, it is possible to satisfy the creditor's participation constraint at a debt level that satisfies the bank's incentive constraint at any $k \geq 0$. At every κ the bank therefore generates no incentive benefits from aggregate sector investment. Hence, by Lemma 1, the bank has no reason to perform aggregate sector investments. The result follows immediately.

Proof of Lemma 5

If $\kappa = \bar{\kappa}$, then all banks, including those with local knowledge, strictly prefer aggregate sector investment. This follows because we have assumed that a correlated equilibrium exists in the baseline model. A parallel argument to Proposition 1 establishes that a correlated equilibrium exists for all values of α.

Any other equilibrium must have $\kappa < \bar{\kappa}$, so that there is no costly liquidation. By Equation (10), such an equilibrium can exist only if some banks invest locally.

Let $K^l = \{\kappa | V^l(\kappa) \geq V^a(\kappa)\}$ be the set of prices for which banks with local knowledge prefer local investment. Note that $0 \in K^l$, because banks prefer local investment if there is no fire sale. If banks with local knowledge invest locally, bank capital ρ per fragile project in an aggregate crisis is

$$\rho^l(\kappa, \alpha) = \frac{(1-\alpha)\hat{p}_H(R - D^l) + \alpha p_H(R - D^a)}{(1-\alpha)\hat{q}_H + \alpha q_H}.$$

Let $\rho^n(\kappa) = r/(1+\kappa)$ denote the demand for bank capital in the absence of liquidation and define:

$$\psi(\alpha) = \max_{\kappa \in K^l}(\rho^l(\kappa, \alpha) - \rho^n(\kappa)) \tag{40}$$

It is easy to see that an equilibrium without liquidation exists if and only if $\psi(\alpha) \geq 0$.

Equation (9) implies that $\rho^l(0,0) > \rho^n(0)$, which gives $\psi(0) > 0$. Equation (10) implies $\rho^l(\kappa, 1) < \rho(\kappa)$ for all κ, which gives $\psi(1) < 0$. Applying the envelope theorem,

$$\psi'(\alpha) = \frac{\partial \rho^l(\kappa, \alpha)}{\partial \alpha} < 0.$$

It follows that $\psi(\alpha) < 0$, or equivalently that the correlated equilibrium is uniqe, if and only if $\alpha > \alpha^\star$ for an appropriately defined $\alpha^\star \in (0, 1)$.

Finally, consider a decrease in c. This increases the value of aggregate sector investment $V^a(\kappa)$, and hence shrinks the constrained set K^l in problem (40). Therefore, $\psi(\alpha)$ weakly decreases for all α, and the critical value which solves $\psi(\alpha^\star) = 0$ also weakly decreases.

Proof of Lemma 6

Equations (16) and (25) yield the following incentive constraint for local sector investment:

$$
\begin{aligned}
IC_c^l(D_{AB}, D_{AC}) \triangleq\ & \Delta_p \left(\delta(R - D_{AB}) + (1-\sigma)(R - D_{AC})(1+\kappa)\right) \\
& + \Delta_q \left(\sigma(r - D_{AB})^+ + (1-\sigma)(m - D_{AC})^+\right) \geq \frac{B}{1-\sigma}, \qquad (41)
\end{aligned}
$$

We can write

$$
\begin{aligned}
IC_a^l(D_{AB}, D_{AC}) - & IC_c^a(D_{AB} = \bar{D}/\sigma, D_{AC} = 0) \\
& = -\Delta_p \sigma \kappa R + \Delta_q \sigma (r - m) \\
& \quad - (\Delta_p + \Delta_q)\left(\sigma \min(r D_{AB}) + (1-\sigma)\min(m, D_{AC})\right) \\
& \quad + \Delta_p \left(\sigma(D_{AB} - \min(r, D_{AB})) + (1-\sigma)(D_{AC}(1+\kappa) - \min(m, D_{AC})\right), \qquad (42)
\end{aligned}
$$

which is negative because $\Delta_q < 0$ and $\Delta_p + \Delta_q > 0$.