DISCUSSION PAPER SERIES

DP12685

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FINANCIAL ECONOMICS

ISSN 0265-8003

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Discussion Paper DP12685
Published 05 February 2018
Submitted 05 February 2018

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Abstract

We define “deep value” as episodes where the valuation spread between cheap and expensive securities is wide relative to its history. Examining deep value across global individual equities, equity index futures, currencies, and global bonds provides new evidence on competing theories for the value premium. Following these episodes, the value strategy has (1) high average returns; (2) low market betas, but high betas to a global value factor; (3) deteriorating fundamentals; (4) negative news sentiment; (5) selling pressure; (6) increased limits to arbitrage; and (7) increased arbitrage activity. Lastly, we find that deep value episodes tend to cluster and a deep value trading strategy generates excess returns not explained by traditional risk factors.

JEL Classification: G02, G11, G12, G14, G15

Keywords: value investing, Market Efficiency, bubbles, behavioral finance, over-reaction, demand pressure, arbitrage, noise

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Acknowledgements
Asness, Liew and Thapar are at AQR Capital Management, Two Greenwich Plaza, Greenwich, CT 06830. Pedersen is at AQR Capital Management, Copenhagen Business School, NYU, and CEPR. We grateful for helpful comments from Kobi Boudoukh, Robin Greenwood, Antti Ilmanen, Ronen Israel, David Kupersmith, Scott Richardson, Andrei Shleifer, and seminar participants at AQR, and especially to Nick Barberis for detailed suggestions and Tarun Chordia for data on order flow. AQR Capital Management is a global investment management firm, which may or may not apply similar investment techniques or methods of analysis as described herein. The views expressed here are those of the authors and not necessarily those of AQR.

Deep Value

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First version: December 2016. This draft: December 2017.

Abstract.

We define "deep value" as episodes where the valuation spread between cheap and expensive securities is wide relative to its history. Examining deep value across global individual equities, equity index futures, currencies, and global bonds provides new evidence on competing theories for the value premium. Following these episodes, the value strategy has (1) high average returns; (2) low market betas, but high betas to a global value factor; (3) deteriorating fundamentals; (4) negative news sentiment; (5) selling pressure; (6) increased limits to arbitrage; and (7) increased arbitrage activity. Lastly, we find that deep value episodes tend to cluster and a deep value trading strategy generates excess returns not explained by traditional risk factors.

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1. Introduction: deep value – risk, (anti)bubbles, or noise?

We seek to shed new light on the source and variation of the returns to value investing[1] by studying deep value, that is, periods when the valuation spread between cheap and expensive securities (value spread) is high. By examining these deep value periods, we address the following questions: Are value returns higher when the value spread is large? If so, what happens at these times? Do value returns reflect rational compensation for risk, behavioral over-reaction, or pure noise in prices? These questions are important for the academic debate on market efficiency,[2] for value investors, for risk managers, and for corporate issuance and merger decisions. A central element in our analysis is that we can see more clearly the drivers of value investing by studying what happens at the extremes.

One obvious challenge with a study focused on extremes is a lack of data points with which to draw conclusions. To help address this concern, we study a broad set of test strategies over a long time series: we examine "stock selection" strategies within equity markets across four global regions and three "asset allocation" strategies within global equity index futures, currencies, and bond futures. To create even more deep value episodes, we examine two types of value strategies in each of our seven test markets: "standard" and "intra." Our 7 "standard value strategies" are long-short portfolios formed over the full set of securities in each market, consistent with most studies in the literature. Our "intra value strategies" are long-short portfolios formed within *subsets* of securities in each market. For equities, we create intra value strategies by forming long-short portfolios within each industry and region, e.g., a value strategy for US auto stocks. For asset allocation, we create intra value strategies by exploiting valuation differentials between every pair of instruments within each asset class, e.g., a dollar-vs.-yen strategy. The result is 515 intra value strategies across markets. Overall, we therefore have 7+515=522 value strategies across our seven markets, yielding about 3000 instances of deep value in our century-long sample.[3]

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[1] Value strategies have been found to deliver positive returns in almost all global asset classes where they have been examined (Asness, Moskowitz, and Pedersen 2013), including U.S. equities (Stattman 1980; Rosenberg, Reid, and Lanstein 1985; and Fama and French 1992), global equities (Fama and French 1998; Liew and Vassalou 2000) in country equity indices (Asness, Liew, and Stevens 1997), and other global asset classes such as currencies (Asness, Moskowitz, and Pedersen 2013).

[2] The importance of understanding value returns is emphasized in the presidential address of Cochrane (2011).

[3] Since standard strategies are based on much more data than each individual intra strategy, simply adding up their numbers is a bit like adding apples and oranges. The point is that we provide different perspectives on deep value: standard strategies are very broad (but, even using global data across asset classes, we only have 7 of those), whereas the intra strategies are numerous (but each is more narrow).

To motivate our study, we first lay out the predictions of the competing theories, which give rise to our empirical strategy summarized in Table 1. The table shows predictions of each of three theories for the value premium, namely rational risk-based theories,[4] behavioral theories of over-reaction and limited arbitrage,[5] and the notion that price changes are driven by noise (e.g., demand pressure unrelated to information about fundamentals).[6] Taking these theories to our extensive data, we find seven key results (enumerated in the abstract), which we discuss in turn. First:

1. Value spreads predict returns to value strategies. Therefore, the returns to value strategies are particularly high during "deep value episodes," which we define here as top quintile value spreads.[7]

While the finding of time-varying expected returns can be consistent with all of the competing theories, we can start to differentiate the theories based on their predictions for the source of the time-varying returns. According to rational theories, the expected return of value securities should depend on their risk. Analyzing the risk of our many value portfolios, we find mixed results:

2. Value is not compensation for market risk, but may reflect another risk factor.
    a. Value portfolios have average market betas close to zero, and their betas are lower, indeed negative, during deep value episodes.
    b. Value portfolios obviously load on a global value factor, and, interestingly, this value factor loading is larger during deep value episodes.

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[4] There are several rational risk-based theories that could potentially explain the value effect. The standard capital asset pricing model (CAPM) cannot explain the value effect as cheap stocks have, if anything, lower market betas (Fama and French 1992). Some papers suggest that the conditional CAPM perform better and that conditional betas may depend on valuation ratios (Gomes, Kogan, and Zhang (2003)) although the conditional CAPM has been rejected by Lewellen and Nagel (2006). See also Berk, Green, and Naik (1999), Zhang (2005), Liu, Whited, and Zhang (2009), and Garleanu, Kogan, and Panageas (2012).

[5] Sentiment theories of over-reaction include Barberis, Shleifer, and Vishny (1998), Daniel, Hirshleifer, and Subrahmanyam (1998), and Hong and Stein (1999), and evidence is provided by Lakonishok, Shleifer, and Vishny (1994). Arbitrage is limited due to the risk of outflows from asset managers (Shleifer and Vishny 1997), market liquidity risk (Pastor and Stambaugh 2003; Acharya and Pedersen 2005), or funding liquidity risk (Brunnermeier and Pedersen 2009). Prices can drop-and-rebound when everyone runs for the exit and capital is slow moving (Mitchell, Pedersen, and Pulvino 2007; Pedersen 2009).

[6] E.g., Black (1986) states that "noise is contrasted with information."

[7] This finding complements the evidence that the value spread for US stocks predicts returns to the standard value strategy (Asness, Friedman, Krail, and Liew (2000), Cohen, Polk, and Vuolteenaho (2003)). To the best of our knowledge, this finding is novel for the "intra" equity strategies and for the standard and "intra" strategies in the other asset classes. See also Liu and Zhang (2008) who consider whether the value spread predicts the return of the overall equity market.

These findings suggest that value returns are not compensation for market risk, but may be compensation for exposure to a value risk factor.

We next turn to the predictions for economic fundamentals. We focus on stocks for this component of the analysis given greater data availability. While we lay out a more realistic theoretical motivation in Section 2, let us consider here a simpler framework based on Gordon's growth model for any stock:

$$D/P_i = E\left(r^i\right) - g^i \ = (\text{risk}^i) \times (\text{risk premium}) - g^i$$

where D/P_i represents the dividend yield, $E\left(r^i\right)$ represents the expected return, and g^i represents the expected future growth in the dividends of stock i. This equation shows that, if a stock is relatively cheap (has high D/P_i), then this must be due to a high expected return or a low expected growth, or some combination of these.

In a rational model, the expected return depends on the stock's risk exposure and the risk premium. Thus, a cheap stock must be risky or have low growth. Further, during deep value events (when these stocks are particularly cheap vs. the market), such stocks must have a particularly high risk or particularly low growth or a combination of these.

Behavioral theories of over-reaction also predict that stocks become cheap when their expected growth is low,[8] and, like the rational theories, they also suggest that when value spreads are wide, the expected growth differential between cheap and rich stocks should be even starker. These predictions run counter to the idea that price changes are driven by pure noise in which case, we would expect no relation between current cheapness and future growth. We find:

3. Value stocks experience weak growth, especially during deep value episodes.
    a. The earnings of cheap stocks are generally lower than the earnings of expensive stocks (as shown by Fama and French (1995) and Cohen, Polk, and Vuolteenaho (2003)). Further, during deep value episodes, the earnings of cheap stocks have been particularly weak vs. that of expensive stocks leading up to the event, and this relative earnings weakness continues after the event.

[8] The difference is that behavioral theories predict price over-reaction, while rational theories predict a price reaction consistent with the above formulation.

b. Analysts' earnings forecasts for value stocks, versus those of growth stocks tend to be revised downwards.[9] Further, this deterioration of analyst forecasts is greater leading into deep value events and continues about a year, after which it starts to partly reverse.

These findings lend support to the rational and behavioral theories, but present a rejection of the notion that price changes are pure noise.

We next turn to more predictions of the behavioral theories. Behavioral theories predict that value stocks become cheap due to negative sentiment and selling pressure. The behavioral theories come in two flavors (as seen in Table 1 and in our theory section): one is based on the idea that investors over-react to fundamentals, while the other is based on investors who over-extrapolate past returns.[10] To examine these theories, we use news sentiment data from Ravenpack, and signed order flow data based on the Lee and Ready (1991) method as in Chordia, Roll and Subrahmanyam (2002, 2005, 2008) and Chordia and Subrahmanyam (2004). We find:

4. The average sentiment, as measured by the tone of news stories covering a given stock, is worse for value stocks than for growth stocks. This differential in news sentiment is particularly strong during deep value events and mean-reverts over the following year.
5. Value stocks face less buying pressure than growth stocks, or said differently; value stocks face more selling pressure. This selling pressure is particularly strong leading into deep value events and continues for five years, although it subsides after a year.
    a. To differentiate the behavioral theories, we regress signed order flow on past returns and past fundamentals, finding that order flow loads significantly on the former, but not on the latter, consistent with theories of over-extrapolation of past returns.
    b. Changing the dependent variable to be the future return, we find that past fundamentals predict returns positively while past returns beyond 1 year predict returns negatively. This evidence is also consistent with over-extrapolation of past returns and inconsistent with over-reaction to fundamentals on average.[11]

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[9] Analyst forecasts are from the Thompson Reuters I/B/E/S database.

[10] Theories of over-reaction to fundamentals include Barberis, Shleifer, and Vishny (1998) and Daniel, Hirshleifer, and Subrahmanyam (1998) while theories of over-extrapolation of past returns include Hong and Stein (1999), Barberis and Shleifer (2003), and Barberis, Greenwood, Jin, and Shleifer (2015).

[11] In a similar spirit, the well-known post-earnings announcement drift (Ball and Brown 1968) is inconsistent with over-reaction to fundamentals on average, and so is the high return to quality stocks (see Asness, Frazzini, and

In other words, the evidence is consistent with the idea that sentiment and demand pressure driven by over-extrapolation of past returns make value stocks too cheap and growth stocks too expensive, especially during deep value episodes. Said differently, deep value can be interpreted as a systematic way to identify episodes where growth stocks exhibit "bubbles" while distressed value stocks exhibit "antibubbles." See Greenwood, Shleifer, and You (2017) for a recent study of bubbles.[12]

In order for "bubbles" to persist, the "arbitrage" of value investors would need to be limited, as we study next using bid-ask spreads from CRSP and the short-selling costs from Data Explorers. We find:

6. Limits to arbitrage increase during deep value episodes:
    a. Bid-ask spreads for the value portfolio tend to widen.
    b. The cost of short-selling growth stocks (as is necessary for a long-short value investor) is higher than normal.
    c. The volatility of the long-short value portfolio is higher than normal.

Taking this one step further, if securities are mispriced (rather than earning a rational risk premium) and arbitrageurs recognize this, then we should see evidence that arbitrageurs try to take advantage of the opportunity (even if they can only do so to a limited extent). While we do not observe arbitrage activity directly, we can consider short interest as a measure of one side of this activity. Further, we can examine whether firms act as "arbitrageurs of last resort" in extreme situations, based on the ideas of Dong et al. (2006), Hong, Wang, and Yu (2008), Baker and Wurgler (2012), Edmans, Goldstein and Jiang (2012) and references therein. We find:

7. Value "arbitrage" increases around deep value episodes:
    a. Short interest of growth stocks is higher during deep value episodes.

Pedersen 2017). Indeed, quality stocks have good fundamentals and, if investors over-reacted to such fundamentals on average, then the risk-adjusted returns of these stocks should be negative.

[12] Greenwood, Shleifer, and You (2017) consider US industries that have increased by over 100% over two years while we consider US and international equities as well as other asset classes, value spreads to take fundamentals into account in addition to prices, examine numerous economic characteristics, and long-short portfolios. Greenwood et al. cite Fama's comment that "For bubbles, I want a systematic way of identifying them. It's a simple proposition. You have to be able to predict that there is some end to it. All the tests people have done trying to do that don't work. Statistically, people have not come up with ways of identifying bubbles." Our definition of deep value can be viewed as such a systematic way to identify bubbles and we consider an implementable strategy below. Goetzmann and Kim (2017) consider "negative bubbles", that is, stock market crashes.

b. Firms' net issuance is greater for growth stocks than value stocks, consistent with the idea that managers of over-valued stocks are more likely to issue shares and less likely to repurchase shares. This effect is stronger after deep value events (but not before).
c. Value stocks are more likely to be acquired in a merger than growth stocks and this is especially the case in the years following deep value events, consistent with the notion that acquirers finds these stocks cheap.

Put together, all of these observations provide insight into the source of the value premium. The evidence rejects the notion that price changes are pure noise. In contrast, stock prices and deep value episodes appear to be driven by mix of rational and behavioral factors.

The economic effects that we study seek to explain the high returns of value during deep value episodes, but we note that, so far, we have only documented this predictability in-sample. This does not necessarily translate into out-of-sample predictability (Campbell and Thompson (2008), Welch and Goyal (2008)) and, even when it does, weak out-of-sample predictability does not necessarily translate into a significantly improved trading strategy (Asness, Ilmanen, and Maloney (2015)). Hence, for the behavioral theory to be convincing, we need to examine whether the return predictability is useful out-of-sample.

As a simple test of this, we consider a "deep value" strategy that is fully implementable and out-of-sample. The strategy invests in value when we observe the value spread exceeding its 80th percentile (i.e., top quintile) relative to the history known up until that time, and exits when the value spread declines to a normal (median) level. We start by implementing this deep value strategy over just our seven standard value strategies, and find that, while it has positive returns, it is highly related to regular value strategies. The alphas of these deep values strategies are positive on average (consistent with the results of Arnott et al. (2016) and Asness et al. (2017)), but the signs are mixed and are generally not significant within stock selection and only marginally significant in asset allocation, suggesting a very limited efficacy of timing any one individual value strategy (consistent with Asness et al. (2017)).[13]

However, the power of combining many deep value strategies is shown when we consider our more granular deep value strategy based on our 515 intra value strategies in the seven test markets (as defined

[13] Asness et al. (2017) consider value timing of non-value factors (momentum and quality, in contrast to value timing of value, as we consider here), finding that, while the univariate results appear promising, the performance of the timed strategies is disappointing in a portfolio context since timing these other factors based on their value spreads resembles simply increasing the portfolio weight on an (untimed) value strategy, resulting in sub-optimal varying amounts of value exposure.

above). These intra-deep-value strategies have access to a much greater opportunity set by being able to identify deep value in subsets of our stock selection and asset allocation universes. Empirically, we see that the result of this increased diversification is a highly statistically significant alpha over other known factors for the overall stock-selection, asset-allocation, and all-asset intra strategies. Moreover, this significant out-of-sample alpha is robust to a variety of different specifications of the deep value strategy.[14] In essence, it is very difficult to increase the performance of a portfolio using value timing based on factors constructed the standard way as shown by Asness et al. (2017), but, when we substantially richen the set of comparisons being made, we find a much larger and more successful role for the value timing of value.

Lastly, we find that deep value opportunities cluster. That is, wide value spreads tend to occur around the same time across many of the asset classes and securities that we study. Perhaps not surprisingly, two notable clustering's of deep value opportunities occur around the tech bubble in the late 1990's and around the 2008 global financial crisis. This clustering could be explained by either a time-varying global risk premium to value or bouts of market-wide irrationality together with widening limits to arbitrage. Interestingly, the global deep value strategy performs better following times of abundant deep value events.

Our findings are related to several papers in addition to the citations above. A large literature studies how aggregate valuation ratios predict the return on the overall stock market (Campbell and Shiller (1988), Cochrane (1992)), but this time series evidence has been questioned in terms of its econometrics and out-of-sample applicability (Stambaugh 1999; Welch and Goyal 2008). We focus on long-short strategies as Asness et al. (2000) and Cohen, Polk, and Vuolteenaho (2003) and complement these papers by studying the economics of deep value across many asset classes. Thus, we also complement the growing literature on multi-asset class global asset pricing, including studies of discount rates (Cochrane 2011), value and momentum in international equity and across asset classes (Fama and French 2012; Asness, Moskowitz, and Pedersen 2013), time series momentum across asset classes (Moskowitz, Ooi, and Pedersen 2012), betting against beta effects across asset classes (Frazzini and Pedersen 2014), and global carry returns (Koijen, Moskowitz, Pedersen, and Vrugt 2012).

The rest of the paper is organized as follows. The next section more formally motivates our focus on deep value episodes and presents rational, behavioral, and noise model predictions. Section 3 discusses the data that we use and presents a detailed description of our methodology for both constructing our

[14] Robustness checks include changing the entry threshold from the 80th percentile to a more extreme "two standard deviation" cutoff, and using a simple linear timing function.

value portfolios and identifying deep value episodes. Section 4 presents results on the ability of value spreads to forecast value returns across all the securities and asset classes that we examine. To examine the economics of value at the extremes, Section 5 investigates the characteristics of the cheap and expensive securities during deep value episodes. Section 6 presents the results of our out-of-sample deep value trading strategy and the clustering of deep value opportunities. Section 7 concludes.

2. Theoretical Motivation

2.1. Rational Asset Pricing Theory

To understand the motivation for considering value spreads and deep value opportunities, consider the following simple model. For any security *i* at time *t*, we denote its price by P_t^i and its relevant scaling variable by B_t^i. For instance, a natural scaling variable B_t^i for stocks and stock indices is the book value, for currencies it is the exchange rate consistent with Purchasing Power Parity, and for bonds it is the present value based on inflation (see Section 3). For simplicity, we refer to B_t^i as the "book value" in all cases. Section 2.4 explains in more detail how we define the "book value" for bonds and currencies and Section 3 explains how we implement this definition empirically.

We are interested in each security's book-to-price ratio, denoted by $V_t^i := \frac{B_t^i}{P_t^i}$, corresponding to a price-to-book ratio of $\frac{P_t^i}{B_t^i} = \frac{1}{V_t^i}$. Price-to-book ratios are mean-reverting. Specifically, for simplicity we consider just two dates, t and $t+1$, and assume that the mean-reversion in price-to-book (the inverse of V_t^i) happens over this single time period such that:

$$\frac{1}{V_{t+1}^i} = \frac{1}{\bar{V}} + \varepsilon_{t+1}^i \tag{1}$$

where ε_{t+1}^i is a mean-zero noise, $E_t\left(\varepsilon_{t+1}^i\right) = 0$, and $\bar{V}$ is a constant (e.g., the cross-sectional average book-to-price ratio). Further, book values grow at rate g_t^i such that

$$B_{t+1}^i = B_t^i(1 + g_t^i) \tag{2}$$

Security *i* does not pay dividends and has an exposure β_t^i to the pricing kernel, which carries a risk premium of λ_t, such that the required return is $\lambda_t \beta_t^i$. Under the CAPM, we have that $\lambda_t = E_t(r_{t+1}^{MKT}) - r^f$ and $\beta_t^i = \frac{cov_t(r_{t+1}^i, r_{t+1}^{MKT})}{\sigma_t^2(r_{t+1}^{MKT})}$, but a similar expression holds for any rational model. We can now compute the rational price at time *t* as:

$$P_t^i = \frac{E_t(P_{t+1}^i)}{1+\lambda_t\beta_t^i} = \frac{E_t(B_{t+1}^i/V_{t+1}^i)}{1+\lambda_t\beta_t^i} = \frac{B_t^i(1+g_t^i)/\bar{V}}{1+\lambda_t\beta_t^i} \tag{3}$$

We are interested in the value strategy that goes long a "value" portfolio *H* of securities with high valuation ratios while shorting the "growth" portfolio *L* of securities with low values. In other words, the value strategy is characterized by going long assets that are cheaper than those shorted, $V_t^H > V_t^L$. We define the value spread as the difference in valuation ratios for the *H* versus *L* portfolios:

$$value\ spread_t := \log(V_t^H) - \log(V_t^L) \cong \frac{V_t^H - V_t^L}{\bar{V}} \tag{4}$$

Clearly, the value spread is positive by definition for the value portfolio. Empirically, we compute the valuation spread as the difference in log-valuation-ratios for simplicity. Specifically, for each of our strategies, we first compute the aggregate valuation ratio for the long and short portfolios. We then compute the value spread as the difference in the logarithm of these valuation ratios. Using the logarithm rather than the fraction that is approximately equal to the same thing is helpful because it does not require us to take a stance on what the denominator should be. In any event, these are very similar empirically, and here we ignore second order terms to derive some simple intuitive expressions.

Using equations (3) and (4), we can write the value spread as[15]

$$value\ spread_t \cong \frac{V_t^H - V_t^L}{\bar{V}} = \frac{1+\lambda_t\beta_t^H}{1+g_t^H} - \frac{1+\lambda_t\beta_t^L}{1+g_t^L} \cong \lambda_t(\beta_t^H - \beta_t^L) - (g_t^H - g_t^L) \tag{5}$$

We see that, according to the rational theory of asset pricing, the value spread should be higher if the value portfolio has a particularly high risk exposure relative to the growth portfolio (high $\beta_t^H - \beta_t^L$), or if value stocks have lower growth rates than growth stocks (low $g_t^H - g_t^L$), or some combination. However, the value spread should only be linked to the expected return on the value portfolio to the extent that the spread arises from risk differences:

$$E_t\left(r_{t+1}^{value}\right) = \lambda_t(\beta_t^H - \beta_t^L) = \lambda_t\beta_t^{value}$$

[15] It is well-known that the logarithm captures relative differences, e.g., $\frac{V_t^i}{\bar{V}} \cong \log\left(\frac{V_t^i}{\bar{V}}\right) + 1 = \log\left(V_t^i\right) - \log(\bar{V}) - 1$ when the ratio $\frac{V_t^i}{\bar{V}}$ is close to 1. Using this approximation for $i = l, s$ and taking the difference gives the result. Cohen, Polk, and Vuolteenaho (2003) also define the value spread in this way.

2.2. Behavioral Over-Reaction

We consider the same model as above, with two exceptions. First, all securities have the same systematic risk, $\beta_t^i = \beta$, such that the value portfolio should deliver a zero expected return. Second, people a subject to one (or both) of the following behavioral biases.

Over-reaction to fundamentals

Suppose that people over-react to growth rates, behaving as if the growth rate is $(1 + z_t)g_t^i$ rather than its true value g_t^i, where $z_t \geq 0$ is the degree of over-reaction in this asset class at time t.

The price is $P_t^i = \frac{B_t^i(1+\,(1+z_t)g_t^i)/\bar{V}}{1+\,\lambda_t\beta}$ and the value spread is only driven by growth rates:

$$value\ spread_t \cong -(1 + z_t)(g_t^H - g_t^L)$$

Nevertheless, the value spread is linked to expected return, where the expectation is taken from the perspective of the empirical researcher, that is, the rational expectation:

$$E_t\left(r_{t+1}^{value}\right) = E_t\left(\frac{P_{t+1}^H}{P_t^H} - \frac{P_{t+1}^L}{P_t^L}\right)$$

$$= E_t\left(\frac{V_t^H}{V_{t+1}^H}\frac{B_{t+1}^H}{B_t^H} - \frac{V_t^L}{V_{t+1}^L}\frac{B_{t+1}^L}{B_t^L}\right)$$

$$\cong \frac{V_t^H - V_t^L}{\bar{V}} + g_t^H - g_t^L$$

$$\cong z_t(g_t^L - g_t^H)$$

We see that under this behavioral theory, the value portfolio has a positive expected return to the extent that investors over-react, $z_t > 0$. Further, the expected return is increasing in the degree of over-reaction, z_t, multiplied by the spread in growth rates (as seen from the last expression), or, equivalently, in the valuation spread adjusted for the growth-rate spread (the second-to-last expression). It would be straightforward to introduce limited arbitrage in the model, but we refrain from this, referring instead the reader to the papers cited in the introduction.

Over-extrapolation of past returns

A related, but slightly different, behavioral view is that people over-extrapolate past returns, behaving as if the growth rate is $g_t^i + z_t r_{t-k,t}^i$ (rather than its true value g_t^i), where $r_{t-k,t}^i$ is the past return and $z_t \geq 0$ is the degree of over-reaction in this asset class at time t.[16]

The price is $P_t^i = \frac{B_t^i(1+\, g_t^i + z_t r_{t-k,t}^i)/\overline{V}}{1+\, \lambda_t \beta}$ and the value spread is now driven by growth rates and past returns:

$$value\ spread_t \cong -(g_t^H - g_t^L) + z_t(r_{t-k,t}^L - r_{t-k,t}^H)$$

The rational expected return of the value portfolio is:

$$E_t\left(r_{t+1}^{value}\right) = E_t\left(\frac{P_{t+1}^H}{P_t^H} - \frac{P_{t+1}^L}{P_t^L}\right) \cong z_t(r_{t-k,t}^L - r_{t-k,t}^H)$$

This expected value spreads tends to be positive since more expensive stocks typically have higher past returns than cheap stocks. Under this hypothesis, the return on the value portfolio is driven by the degree of over-extrapolation and the difference in past returns.

2.3. Pure Noise in Prices

Lastly, suppose that all securities have the same systematic risk, $\beta_t^i = \beta$, and growth $g_t^i = g$. However, prices at time t include a noise term, ε_t^i, such that

$$P_t^i = \frac{B_t^i(1+\mathrm{g})/\overline{V}}{1+\, \lambda_t \beta}(1 + \varepsilon_t^i)$$

In this case, the value spread is given by

$$value\ spread_t \cong \frac{1+\, \lambda_t \beta}{1+\mathrm{g}}\left(\frac{1}{1+\varepsilon_t^H} - \frac{1}{1+\varepsilon_t^L}\right) \cong -\frac{1+\, \lambda_t \beta}{1+\mathrm{g}}(\varepsilon_t^H - \varepsilon_t^L)$$

We see that the value spread is purely driven by differences in price noise. Hence, in this case, the value spread is not related to systematic risk, nor is it related to growth of fundamentals. Still, the value spread predicts returns:

$$E_t\left(r_{t+1}^{value}\right) = E_t\left(\frac{P_{t+1}^H}{P_t^H} - \frac{P_{t+1}^L}{P_t^L}\right)$$

[16] Here we make the simple assumption that past returns affect investors' expectations about future fundamental growth rates while Barberis, Greenwood, Jin, and Shleifer (2015) consider investors who forecast future returns by extrapolating past returns.

$$= E_t\left(\frac{V_t^H}{V_{t+1}^H}\frac{B_{t+1}^H}{B_t^H} - \frac{V_t^L}{V_{t+1}^L}\frac{B_{t+1}^L}{B_t^L}\right)$$

$$\cong (1+g)\frac{V_t^H - V_t^L}{\bar{V}}$$

3. Data and Methodology

This section first lays out our data sources stocks and non-stock assets and describes the methodology used to test our hypotheses.

3.1. Asset Classes and Standard vs. Intra

An important goal of this paper is to study value strategies as extensively as possible across many regions and asset classes. As a result, we use the above data to construct a variety of different long-short value strategies across multiple asset classes in the following way. First, we focus on seven major markets, grouped in global "stock selection" and "asset allocation":

I. Stock selection (SS)
   a. US equities (US)
   b. UK equities (UK)
   c. Continental Europe equities (EU)
   d. Japan equities (JP)

II. Asset allocation (AA)
   a. Equity index futures (EQ)
   b. Global bond futures (FI)
   c. Currencies (FX)

Within each of the seven markets outlined above, we consider both standard and intra test assets. Standard is meant to capture cross-sectional differences in valuation across the entire universe of a given investment category, consistent with the approach typically used in the academic literature. For example, standard US equities, within stock selection, selects among all stocks in the CRSP dataset when determining which to go long and short.

The intra tests assets are meant to capture differences within subsets of securities within each investment category. For stock selection, we consider intra-industry portfolios. For example, within U.S. equities, we have a value portfolio within the group of U.S. auto stocks, and so on for other industries. The

benefit of doing sorts within each industry is that it leads to many value portfolios, each of which with its own value spread, increasing our statistical power. Similarly, for the asset allocation samples, each pair of assets represents an intra value strategy, i.e., an additional test asset. For example, for currencies, euro-vs-yen is an intra test asset and we consider the valuation spread of euro-vs-yen over time.

In summary, we construct 522 value strategies, of which 7 are standard value strategies (one for each major asset class and region) and 515 are intra value strategies. The intra value strategies consists of 272 strategies that go long and short stocks within each industry in each region, and 243 strategies that are pairs of equity index futures, bond futures, and currencies.

3.2. Empirical Value Measures

For stock selection, we use the classical value measure, namely their ratio of book value to market capitalization or, equivalently, the ratio of the book value per share to the stock price, denoted B/P. Naturally, stocks with high B/P are considered "cheap," while those with low B/P are considered "expensive."

Within asset allocation, for equity indices, we also use B/P. Specifically, for each equity index, we use the B/P of the constituent stocks aggregated to the index level. Said differently, the index level B/P ratio is an average of the B/P ratios of underlying stocks, averaged by their index weights. These B/P ratios are provided by MSCI.

In currencies and fixed income, the concept of "book value" does not apply, but we compute corresponding metrics. In currencies, our value metric is the real exchange rate corresponding to a "book value" based on Purchasing Power Parity (PPP). The idea is that the fair value of the exchange rate between countries A and B is equal to the ratio of their price levels. If this relationship fails to hold, it indicates that goods in country A are more expensive (cheaper) than those in country B and therefore that country A's currency is expensive (cheap). PPP calculations are provided by Penn World Tables, and exchange rates derive from a proprietary AQR capital data set that combines data from MSCI with various other sources.

For global bonds, the value metric used is the real bond yield, which is computed as the difference between the nominal bond yield and forecasted inflation. If the real bond yield of country A is higher (lower) than that of country B, it indicates that an investors expected real returns to a long term investment in country A is higher (lower) than those to a long term investment in country B, and therefore the bonds of country A are cheap (expensive). Government yield data is sourced from Datastream, and long term inflation forecasts are provided by Consensus Economics.

3.3. Measurement of Value Spreads for Non-Equity Asset Classes

We previously discussed the computation of value spreads based on Book to Price ratios, which is relevant for stocks and stock indices, but we need a corresponding definition for the other asset classes. For currencies, a natural "book value" is the exchange rate consistent with the Purchasing Power Parity. For bonds, we measure value spreads as follows. We start with bond yields denoted by Y_t^i for each country i and time t. We translate these yields into hypothetical zero-coupon bond prices $P_t^i = e^{-T(Y_t^i)}$, where T is the time to maturity. The "book" value based on the inflation forecast π_t^i and a constant real interest of k is then given by $B_t^i = e^{-T(\pi_t^i + k)}$. Hence, the value spread for bonds is

$$\log(V_t^H) - \log(V_t^L) = \log\left(\frac{B_t^H}{P_t^H}\right) - \log\left(\frac{B_t^L}{P_t^L}\right) = T(\underbrace{(Y_t^H - \pi_t^H)}_{\text{real bond yield } H} - \underbrace{(Y_t^L - \pi_t^L)}_{\text{real bond yield } L})$$

In other words, the value spread for bonds is simply the difference in real bond yields, setting $T = 1$ for simplicity.

3.4. Value Portfolios and Strategies

Having described the security universes and value measures, we now lay out the methodology for creating long-short value portfolios in each case. Beginning with the standard U.S. stock selection strategy, at each point in time, we rank all U.S. stocks based on their B/P ratio and take long positions in the top one third (i.e., the cheap) and short positions in the bottom one third (i.e., the expensive). Data for B/P ratios of U.S. stocks is from Compustat where available (starting in the early 1980s), and from CRSP prior to that. Within the long and short portfolios, stocks are weighted by their winsorized market capitalization. Specifically, for stocks with market capitalization below the 99th cross sectional percentile of global market capitalizations on each date are simply weighted by their market capitalization, and stocks above this threshold are weighted by the threshold itself. As of our most recent date, this threshold was roughly USD 60 billion. This method avoids taking large positions in small companies, thus ensuring implementability of the portfolio, while at the same time ensuring that "mega cap" stocks do not overly influence the results. Winsorizing large capitalizations is especially helpful in the intra-industry portfolios where certain industry portfolios would otherwise be dominated by a single stock (and not winsorizing small capitalizations

ensures realism). This approach therefore produces a U.S. stock value strategy that is similar to traditional U.S. stock value strategies used in the literature.

We repeat this process on broad universes of stocks in the U.K., continental Europe and Japan to form standard value portfolios in each of those regions. Data for international stocks comes from Compustat global. Finally, we repeat the process within each of the 68 industries within each of the four regions. The process for forming intra-industry (intra) value portfolios is identical to that of forming standard value portfolios, other than starting by filtering to stocks falling within the relevant industry. We use GICS industry classifications for non-CRSP data sets, and Ken French 49 industry classifications for CRSP data. By definition, industries are non-overlapping, and therefore each industry portfolio in each country consists of a different set of securities. We note that the aggregation of industry-level value portfolios within a country is not exactly the same as the corresponding standard value portfolio in that country because the intra-industry portfolios are industry dollar-neutral by construction (i.e., long and short an equal amount of stocks within each industry), whereas the standard portfolio is not.

For asset allocation strategies, our process for creating standard value strategies is largely consistent with that used for stock selection. Once again, we start by ranking all securities within each asset class according to the value metric relevant to each asset class. The value portfolio then goes long the top one third of assets, and shorts the bottom one third, consistent with our approach in stock selection. Here, we equal weight rather than value weight the portfolios, as the concept of market value is not well defined in all asset allocation asset classes, and there is less of a concern of taking large weights in illiquid securities.

Finally, within asset allocation, given the narrow cross sections, we create pairs-trading strategies, which trade value strategies within each pair of assets in each asset class. We label these pairs strategies intra for consistency with stock selection. If a pair is comprised of assets A and B, in a given time period, the value strategy would take a position of long asset A and short asset B if asset A were cheaper than asset B, and the opposite if asset B were cheaper than asset A. Note that unlike in stock selection, pairs are not disjoint (e.g. EUR vs. USD, EUR vs. JPY and JPY vs. USD would be 3 distinct strategies), and therefore we would expect the standard macro strategies to closely resemble the aggregation of all underlying intra pairs strategies. Nonetheless, studying the pairs individually provides us with meaningfully more data points of "deep value" episodes.

In addition to the long-short value strategies described above, we also construct long-only quintile portfolios by sorting assets in each market into five groups based on their valuation metrics at the start of each period. Within each quintile portfolio, stocks are weighted by their winsorized market capitalization,

while asset allocation portfolios are equal-weighted. Within stock selection, reported results are based on an average of quintile sorts within each industry (matching our intra portfolio construction) and across all stocks (matching our standard portfolio construction). In asset allocation, quintile sorts are always conducted across all assets (matching our standard portfolio construction), as a quintile sort would be impossible in our two asset pairs universe.

3.5. Summary Statistics

Table 2 presents summary information regarding our samples and value strategies. Let's start with individual stock selection value strategies which are shown in the top row of the table. Our standard U.S. equity sample has an average universe of 3949 stocks, while there are 309 stocks in the typical industry cross-section (though this varies across industries and over time). The Sharpe ratio from Jan-1926 to Sep-2015 is 0.2 for the standard stock selection value strategy and 0.6 for the portfolio of all the intra-industry strategies. These results are consistent with Asness, Porter, Stevens (2000) who show that value strategies produce higher risk adjusted returns within industries than across industries (or the whole universe of stocks).

In the next 3 rows of Table 2 we repeat the exact same exercise for stock selection within Japan, Europe, and the U.K. Thus, we have one standard strategy and 68 intra strategies for each of the four equity regions in our study. Just like in the U.S., in all four regions, the Sharpe ratio of the portfolio of intra-industry value strategies is consistently higher than that for the standard value strategy.

The bottom panel of Table 2 presents summary information for asset allocation strategies within global equity index futures, bond futures, and currencies. Again we create standard value strategies, which sort over the full universe, and intra strategies which correspond to pairs trading strategies. For instance, within our set of 10 currencies, we make all 45 pairwise comparisons (i.e., EUR-USD, EUR-JPY, USD-JPY, etc.). We construct long-short value strategies by simply going long the cheaper side of each pair and short the more expensive side. This process results in 153 intra value strategies for stock index futures, 45 for bond futures, and 45 for currencies. Unlike what we saw for individual stocks, the portfolios of intra value strategies do not produce higher Sharpe ratios than their standard counterparts.

4. Predicting Value Returns Everywhere

We begin this section by addressing the question of whether the variation in value returns can be predicted using value spreads. As we will see, we find strong evidence of such variation across our global

asset classes, which sets the stage for our examination of deep value opportunities. Indeed, it is precisely the fact that wider value spreads correspond with greater long term returns to value strategies that makes the analysis of deep value so interesting and helpful for distinguishing theories of the source of value.

4.1. Value Returns and Value Spreads: Portfolio Sorts

Table 3 utilizes quintile sorts to provide evidence on this relation across all of our asset classes and securities. Let's start with standard portfolio quintiles (the left side of the table) for U.S. stock selection, (top row of the "Global Stock Selection" panel). At each point in time, we group the value spread for the standard U.S. stock selection value strategy into one of 5 quintile buckets based on the size of the spread. The smallest spreads go into quintile 1 and the largest into quintile 5 (and the other value spreads are grouped into quintiles 2-3-4 accordingly). The table reports the mean return in the following month for all the data points that fall within the given quintile. The table shows that when the standard U.S. stock selection value spread is in the 5th quintile, the standard U.S. stock selection value strategy returns an average of 1.2% per month. Correspondingly, when the value spread is in the bottom 1st quintile, the strategy returns an average of 0.0%. In other words, we see the expected result that value performs better when the value spread is wide within U.S. stock selection, although we note that the result is weaker when looking at Sharpe ratios and weaker when sorting based on out-of-sample value spread quintiles (Section 6).

Table 3 also reports the corresponding results for the intra-industry portfolios (on the right side of the table). For U.S. stock selection, this means grouping the value return for each industry based on the value spread of that within-industry portfolio relative to its own history. The results for the intra portfolios are also consistent with the idea that a wider value spread is associated with a higher future return.

The remainder of the "Global Stock Selection" panel repeats this exercise across all four regions in our study. In general the results are mixed. Like the U.S., when standard values spreads are in the 5th quintile, value produces higher average returns than when spreads are in the 1st quintile in Japan and Europe, but not so in the U.K. Furthermore, unlike in the U.S. the results for standard Japan and Europe are not monotonic. The results for the intra portfolios are almost monotonic in all four regions. The evidence suggests that there may be a positive relationship, but the relationship in any one region alone can be weak.

With that said, the panel labeled "All Asset Portfolio" groups all 4 regions (U.S., Japan, Europe, and U.K.) together in the row labeled "Pooled Global Stocks" and finds together the results improve. The relation

between value spreads and next month's value returns is positive and standard value produces a 2.4 *t*-statistic when spreads are in the 5th quintile, and a 6.7 *t*-statistic for intra value.

We next turn to the asset allocation portfolios (stock index futures, bond futures, and currencies). The panel labeled "Global Asset Allocation Portfolios" shows that the relation between value spreads and next month value returns are positive and generally weak within any one standard value strategy, but stronger among the intra portfolios. To gain more statistical power, in the "All Asset Portfolio" panel we pool the asset classes together.

Furthermore, when we go the next step of pooling all the stocks and asset classes together, the results significantly improve. If we believe that the relation between value spreads and future value returns is a more general phenomenon and not unique to one or just a few asset classes, then the pooled result is our statistically most powerful answer to the question of whether the relation exists. Note that a quantile sort inherently embeds a "look ahead" bias in that the full sample of data is used when evaluating breakpoints for splitting the sample into quantiles. As such, these results do not necessitate the existence of an implementable trading strategy - rather, they merely indicate the presence of a positive relation between value spreads and returns for the pooled sample.

4.2. Value Returns and Value Spreads: Regression Analysis

We next address the predictability of value returns by performing a regression analysis rather than portfolio sorts. We consider the following regressions of the long-short value strategy return VAL_{t+1}^{i} in asset class i on the corresponding ex ante value spread, based on the theoretical insights from section 1.4:

$$VAL_{t+1}^{i} = \alpha + \beta\, VALUE\ SPREAD_{t}^{i} + \varepsilon_{t+1}^{i}$$

We run the regression on a monthly basis, using as dependent variable the next 12-month return to the corresponding value strategy. Our *t*-statistics account for the overlapping data by clustering standard errors for correlation in both the time series and cross section, according to the method of Hansen and Hodrick (1980). Using 12-month future returns is helpful since it is a simple way to partially mitigate the countervailing momentum effect and, indeed, we get similar regression results using 1-month return controlling for momentum as a right-hand side variable. Also, using 12-month returns may better resemble the experience of actual value investors.

Table 4 reports the results. Let's start with stock selection. In each of the four regions (U.S., Japan, Europe, and U.K.) the slope coefficient is positive for both standard and intra portfolios, and the coefficient is statistically significant in 7 of the 8 cases at a 1% significance level. Also, like in table 3, if you pool the data points the coefficient is significantly positive.

Similarly, each of the asset allocation strategies (equity indices, fixed income, currencies), the predictive regressions show a positive and significant relation between current value spreads and the next 12-month excess return to the standard and intra value strategies. Also, when we pool all three of the asset classes together, the results remain strong. Finally, as expected, when we pool across the four stock regions and the three asset classes, we get our strongest results that support a positive predictive relationship when using value spreads to predict value returns. These regressions provide a robust alternative check on the in-sample predictability, while out-of-sample predictability is studied in Section 6.

5. The Economics of (Deep) Value

We next seek to analyze in more depth the economics of value portfolios with a particular focus on how the portfolio characteristics evolve around deep value events. For this analysis, we focus on intra strategies in stock selection and asset allocation due to the presence of many more deep value events.

5.1. Value Returns in Event Time

We first consider how value returns evolve as seen in Figure 1, whose structure is repeated in the following six figures. The bar plot on the left shows properties (in this case, returns) for securities sorted on value (B/P), whereas the line plot on the right shows properties of long-short value portfolios in their evolution in event time, given different value spread environments as of event time zero.

More specifically, the bar plot on the top left shows the return on quintile portfolios sorted on their value characteristic, B/P. In other words, in each month, we form five portfolios each comprised of stocks belonging to the five full sample quintiles of B/P ratio, with stocks being weighted according to their winsorized market capitalizations. We then compute the portfolio level characteristic (in this case returns) for each quintile. For characteristics based on returns (including the level of returns and various measures of risk of returns), the reported value is the full sample realization of the returns stream of the quintile portfolio. These measures are reported for both stock selection and asset allocation. Within stock selection, in each region, we do quintile sorts both within each industry and over the full universe of stocks. This matches the event study methodology which also considers both types of portfolio construction. Within

asset allocation, we only do sorts within the full universe, as a quintile sort would be infeasible in the two asset intra portfolios. For other characteristics, reported in stock selection only, the portfolio level characteristic is computed each month for each quintile, and the time series average is reported. Again, this is conducted within each industry within each region, as well as over the full universe in each region with the overall average result being reported.

We see that returns are higher for value stocks (the left-most bar) vs. growth stocks (the right-most bar). This evidence simply means that value investing works on average, a well-known result, but we include it here for consistency and to help the understanding of the next figures. Indeed, the following figures have similar bar plots where we see other characteristics (rather than returns) of stocks sorting on their B/P.

The line-plot of the right is the corresponding event-time evolution of returns for the long-short value portfolio depending on the starting value spread. Specifically, for each intra-industry value strategy, we compute quintile break points for valuation spreads. We then perform event studies for each time period *t* falling into each quintile portfolio, to track the evolution of returns before and after time t. We report an average of the results. For example, the blue line, corresponding to "deep value" time periods, represents the average of all value strategies before and after time periods when their valuation spreads were in their top quintile. For all such *t,* we freeze the portfolio as of time *t*, and track characteristics 24 months before and after time *t.* In other words, when studying deep value portfolios, we learn how characteristics evolved in order for the portfolio in question to become a deep value portfolio, and how they evolve afterwards. In the case of "flow" like variables (including returns), we cumulate results and normalize such that the cumulative is zero at event time zero. We do the same for each of the other value spread quintiles to produce the remaining 4 lines on the event study. Hence, the return at event time 1 corresponds to the results in Table 3. Indeed, these are the average 1-month value returns sorted on the ex-ante value spread.

The fact that the cumulative returns are falling before event time 0, for all lines, simply means that value portfolios tend to go long securities that have performed worse than those that it tends to go short. In other words, stocks become cheap by falling in price, on average (see DeBondt and Thaler, 1985). Not surprisingly, this fall in price is more pronounced when the value spread is wider, as losses to valuation portfolios are one mechanism via which value spreads widen.

The increasing cumulative returns to the right of event time zero, for all lines, reflect that value investing works. The fact that the slope is greater for the deep value portfolios than for the intra value-spread portfolios means that value investing works better when the value spread is wide. This result is consistent with previously shown quantile sort and predictive regression: there appears to be a relationship between

in sample valuation spreads and future returns to value strategies. Additionally, the event study demonstrates that the difference persists over several years, on average (and no sign that the effect ends at 24 months).

The bottom set of plots in figure 1 show that similar conclusions hold up when looking at asset allocation. Here, results were computed within each pair within each asset class and averaged across asset classes (quintile sorts are computed at the standard universe level only, as we cannot do a quintile sort among the two assets forming a pair). Again, the bar plot shows that global assets that look cheaper based on their valuation ratio performs better on average than expensive assets. The line plot shows that value portfolios goes long assets that have underperformed the short over the past two years, on average, and that these portfolios profit over the following two years, especially the deep value portfolio. We see both for stocks and, especially for other assets, that the return after portfolio formation is less positive than the cumulative loss of these assets before portfolio formation, reflecting that the initial decline in these assets was partly justified by deteriorating fundamentals as we study further below.

5.2. The Risk Dynamics of Value Investing

Figure 2 considers the risk of value portfolios (instead of the return shown figure 1). The top-left bar plot in panel A measures the full sample beta of stocks returns for each quintile with market returns, using 3-day overlapping daily returns. We see that while there is not a clear, linear relationship, betas tend to be higher for value stocks on average. As expected, betas are roughly 1 on average (the average is not exactly 1, as we do a simple average of the results from industry level sorts, rather than weighting by market capitalizations of different industries). The corresponding bar plot for asset allocation strategies in panel B shows no relationship between beta and valuations.

The top-right line plot shows how market betas of long-short value portfolios vary with the value spread and over event time, to try to help discern time variation in the relationship. Betas are once again computed using three-day overlapping daily returns. We see that the market beta of daily returns is slightly negative for all groups and is, if anything, slightly more negative for deep value. Hence, deep value investing in stocks appears to hedge market risk, which makes the returns to deep value all the more puzzling from the perspective of the CAPM model. Panel B shows a similar finding for other assets, although to a lesser extent.

The second bar plot shows the loading of stock quintiles on a global value factor. The global value factor is defined separately for each stock selection region and asset class, and is computed as the simple average return of all value strategies within that region or asset class. For example, the U.S. stock selection value

factor is the average of the return to value traded within each of the 68 industries in the U.S.. Of course, value stocks have a positive loading on the value factor and growth stocks have a negative loading, and we see the same phenomenon for other assets in panel B. The second line plot shows the corresponding value loadings for the long-short value strategy. These value loadings are naturally all positive, but, more interesting, the loading is highest for deep value, especially around the time of portfolio formation. Hence, to the extent that value investing is exposed to a rational risk premium, this risk is most severe for deep value, which can potentially explain the high returns to deep value.

5.3. The Changing Fundamentals of Value

We next consider the economic fundamentals for the different portfolios. Here, we focus on equities since this is where we have in depth fundamental data about earnings, which captures how book values evolve over time. Figure 3 shows results for both realized earnings, and forecasted revisions to earnings.

Starting with the realized earnings, we consider firms' return on equity (income before extraordinary items divided by book value of equity, based on data from Compustat). We see from the bar plot that value stocks are less profitable than growth stocks, as is well known (see, e.g., Cohen, Polk, and Vuolteenaho, 2003). The line plot considers how cumulative earnings for value portfolios, falling into different quintiles of value spread, evolve over time. In each month, we compute the sumproduct of the value portfolio weights, and the stock level returns on equity, to derive a portfolio level return on equity. Finally, we cumulate this figure, and rescale to a level of zero at event time zero.

We see that the earnings of the value firms (the long positions in the value strategy) minus that of the growth firms (the short positions) deteriorate more than two years before portfolio formation and this deterioration continues for more than two years after. Importantly, this deterioration of earnings is stronger for the deep value events and is monotonic across the value spread buckets. Hence, part of the explanation for the large price discrepancy between value and growth stocks during deep value events is that prices rationally predict the future evolution of earnings.

We next turn to analyst forecast revisions. These are sourced from equity analyst earnings forecasts in the Thompson Reuters Institutional Brokers' Estimate System (IBES), and we track a standard earnings "revisions ratio" metric. This is a measure which tracks trends in analyst forecasts on a stock: positive numbers indicate analysts becoming more bullish, and a negative number that analysts are becoming more bearish. Specifically, it is computed as a 3-month moving average of the number of upwards revisions in earnings forecasts minus the number of downwards revisions in earnings forecasts divided by the total number of forecasters. The bar plot shows that value stocks face negative revisions while growth stocks

experience positive revisions, on average, though on average, forecasters tend to revise down forecasts more often than they revise them up. The corresponding event study shows that the value portfolio faces negative revisions before portfolio formation (that is, the revisions of the long positions minus those of the short positions is negative), and these negative revisions continue for about a year after portfolio formation. Interestingly, for deep value, these effects are stronger and, additionally, we see a reversal a year after portfolio formation where the revisions start to turn positive.

5.4. Sentiment for Value vs. Growth Stocks: The Tone of News Stories

We next try to consider how investors feel about value vs. growth stocks, that is, investor sentiment of value portfolios. Of course, sentiment is notoriously difficult to measure, but we can at least look at some proxies while acknowledging their limitations. First, note that analyst revisions themselves can be driven by a mixture of fundamental changes and sentiment. Hence, the negative analyst revisions for value stocks mentioned in the previous section may also partly reflect negative sentiment.

As a more novel measure of sentiment, we consider the tone of news stories about value vs. growth stocks. Here, data is sourced from RavenPack, and we track a metric of "Event Sentiment Score". This is a score from 0 to 100 that is intended to capture the average sentiment (positive vs negative) of news stories about companies. A score of 50 denotes a neutral sentiment, one greater than 50 denotes a positive sentiment and one below 50 denotes a negative sentiment. The bar plot in figure 4 shows that the tone of news regarding value stocks tends to be less favorable than the tone in stories about growth stocks, on average.

The line plot in figure 4 shows the evolution of the difference in tone of news about value vs. growth stocks. We see that this measure of sentiment turns particularly negative leading into the time of portfolio formation, especially for deep value, and recovers to a more normal level in the year thereafter.

5.5. Demand Pressure: Do Investors Run when Stocks Get Cheap?

We next consider whether investors in fact act on this sentiment in terms of their buying and selling decisions. Figure 5 shows net buying of stocks: a measure of which types of stocks face buying or selling pressure.

The bar plot shows the net buying for each stock, defined as dollar buys minus dollar sells divided by buys plus sells. Here, buys and sells classified based on tick-level data using the Lee and Ready (1991)

methodology as implemented by Chordia, Roll and Subrahmanyam (2002, 2005, 2008).[17] We see that all groups of stocks experience more buying than selling on average, but the buying pressure is much stronger for growth stocks than for value stocks.

The line plots shows how the demand pressure for the long-short value portfolio evolves over time. We see that the value portfolio experiences net selling pressure in the sense that the net buying of the longs is smaller than the net buying of the shorts, which explains why all of the lines are decreasing in event time. Interestingly, the selling pressure in strongest for the deep value strategy, starting more than two years before portfolio formation and continuing for more than two years after. Hence, when value stocks become really cheap, some investors appear to run for the exits, consistent with the behavioral models.

5.6. What Do Investors (Over-)React to?

We have seen that value stocks face selling pressure relative to growth stocks, but what do selling investors react to? Are they reacting to past fundamentals or past returns? Answering this question will help us differentiate between two competing behavioral theories as seen in Table 1. To address this question, we run regress demand pressure (measured as signed order flow, defined in section 5.5) on past returns and past changes in fundamentals (measured as changes in return on equity, defined in section 5.3). Due to data availability for our demand pressure measure, these regressions are run for U.S. stocks only.

The results are reported in the first three columns of Table 5. We see in column 3 that demand pressure is driven recent returns (within the past year) and long-term returns (over the past 5 years), but, controlling for these effects, demand pressure is not driven by past fundamentals. In the regression in column 2, demand pressure is related to past fundamentals because past fundamentals and returns are correlated. This evidence is consistent with the theories of over-extrapolation of past returns, but not over-reaction to fundamentals.

Table 5 also reports the evidence for how past returns and fundamentals predict returns (rather than demand pressure) in columns 4-6. Columns 4 and 6 show that recent returns predict future returns positively (the momentum effect) and long-term returns predict future returns negatively (as in DeBondt and Thaler, 1985). In column 5, changes in fundamentals do not predict future returns, but, when controlling for past price changes in column 6, short-term and, especially, long-term changes in

[17] We are grateful to Tarun Chordia for this data.

fundamentals predict returns positively. This evidence is consistent with the idea that investors on average under-react to fundamentals (so good fundamentals predict positive returns, controlling for past returns) and over-react to past returns, leading to short-term momentum and eventual return reversal, thus creating a value effect.

5.7. The Limits of Value Arbitrage: Transaction Costs, Shorting Costs, and Arbitrage Risk

Of course, for every seller there is a buyer and behavioral models suggest that arbitrageurs take the other side when behavioral investors run for the exits. However, arbitrageurs only do so to a limited extent if there are costs and risks associated with the trade, which would provide further evidence that behavioral price effects can explain the high returns to deep value. Figure 6 presents evidence consistent with this hypothesis.

The top-left bar plot shows that bid-ask spreads are greater for value stocks than growth stocks, on average. Hence, models of liquidity and liquidity risk could potentially explain part of the value effect (Amihud and Mendelson (1986) and Acharya and Pedersen (2005)). Bid-ask data is sourced from CRSP, and therefore this analysis is computed for U.S. stock selection value only.

Further, transaction cost pose a limit to arbitrage and we are interested in whether transaction costs are particularly severe during deep value events as we study in the top-right line plot of figure 6. This event study shows the average bid-ask spreads across the long and short sides of the value strategy (rather than the difference in longs vs. shorts as in the other event studies), reflecting the costs incurred by an arbitrageur trading on the value strategy. We see that the bid-ask spread is much higher during deep value events, especially around the time of portfolio formation, consistent with the idea of limits of arbitrage.

The next set of plots in figure 6 show the short-selling fees, for which we use the simple average short fee from Data Explorers. The bar plot shows that, on average, shorting costs are similar for value and growth stocks, but both sides face higher shorting costs than for average stocks (those with B/P ratios in the second through fourth quintile). Since the value arbitrageur only shorts the expensive growth stocks, the event study shows plots the evolution of this relevant cost of arbitrage. We see that the shorting cost for growth stocks is particularly high during deep value event: both leading into the event and for more than a year after portfolio formation, a period during which the shorts are typically maintained. Hence, shorting costs present another limit of arbitrage for (deep) value investing.

Finally, the third row of plots considers volatility, measured simply as the annualized standard deviation of returns. The bar plot, which tracks the realized volatility of 3-day overlapping daily returns for stocks in

different value quintiles, shows that value stocks tend to realize higher volatility on average than do growth stocks. The line chart tracks the realized volatility of 3-day overlapping daily returns of long short-value portfolios. Here, we see that value portfolios experience significantly greater volatility during deep value episodes than on average. In particular, volatility increases meaningfully into the portfolio formation period, and then persists at a high level for up to two years after. In other words, investors looking to take advantage of deep value opportunities must bear greater volatility risk, presenting another potential limit to arbitrage. Said differently, deep value is not as attractive when considering its Sharpe ratio rather than its expected return.

5.8. Short-Sellers and Firms Arbitrageurs of Last Resort

Finally, we look at whether arbitrageurs appear to trade on value and whether they do so to a larger extent during deep value events. Figure 7 first considers short interest, reflecting an element of arbitrage that is more easily observable in the data. The specific metric plotted is the short interest divided by the number of shares outstanding, which is provided for U.S. stocks by FT interactive. The bar plot shows that, perhaps surprisingly, the short interest is actually slightly higher for value stocks than growth stocks, although both value and growth stocks have higher short interest than other stocks. For the event study, we focus on the short interest of growth stocks, as that should track the shorting activity of value-focused arbitrageurs. The findings are intuitive: short interest for growth stocks is larger during deep value events, and is elevated for several years before and after the portfolio formation time. Interestingly, we also see a small dip in short interest just after event time zero, albeit to a level which is still high, and this dip could reflect that some arbitrageurs are forced to reduce their positions due to risk management, lack of capital due to losses, shorting costs, lack of short availability (perhaps even forced closure of certain short positions), or other effects.

We next consider whether firms act as arbitrageurs of last resort in their decisions to issue, repurchase, or perform take-overs. We first consider net buybacks, defined as the negated percentage change in shares outstanding. The bar plot shows that all values are negative on average, indicating that companies tend to issue shares on average, and that growth companies tend to issue more shares than value companies. Moreover, after a deep value event, we see that the cheap value firms have much larger net buybacks than the growth stocks. Over the two years after portfolio formation, the difference is 3%, which means that the cheap firms have repurchased 3% of their own shares, assuming that the growth firms have zero net buybacks. In reality, we see that while the management of both value stocks and growth stocks tend to issue stocks, the issuance is much more aggressive for growth stocks. Over a two year horizon, we see roughly 6% net issuance for growth stocks, as compared to 3% for value stocks.

Lastly, we look at a firm's propensity to being bought depending on its valuation. We use the CRSP data set here, so the analysis is done for U.S. stocks only, and a buyback is a binary event at a stock level. Specifically, we look for cases when a stock is delisted with a delisting code of 300 or 400. For the purpose of the bar chart, we lag the valuations used to form quintiles by 6 months, in an attempt to capture "pre announcement" valuations. We believe this adjustment is prudent, given the tendency of takeover targets to dramatically change price after the announcement of the takeover, potentially altering their valuation profile relative to when the takeover was announced. The bar plot shows that buyouts are much more likely for value stocks than for growth stocks. Specifically, the chance that a value stock is bought any given month is 0.26% whereas the corresponding number for growth stock is only 0.15%. The event study shows the cumulative probability of being taken over after the time of portfolio formation for value stocks, minus the cumulative probability for the growth stocks (here we don't lag as the time dynamics of future buyouts of current value stocks is naturally captured by an event study). The lines start at zero because any firms included in the portfolio at formation time must necessarily not have been bought prior to that time. We see that, over a two-year period, all the lines have increased, consistent with the evidence from the bar plot that value stocks are more likely to be taken over. Interestingly, the total increase is largest for the deep value events and smallest for the narrow spread portfolios, suggesting that acquirers act as arbitrageurs of last resort during deep value events. Interestingly, the lines tend to decrease initially, for a period of roughly 6 months, before increasing, indicating a propensity for growth stocks to be bought more than value stocks immediately following portfolio formation. In all likelihood, these are stocks for which the takeover announcement was made prior to portfolio formation, and therefore for which the takeover premium has already been reflected in prices. In other words, if a stock was the target of a merger announced the month before the portfolio formation, it might be expensive relative to its own book value as of time zero, and hence included in the growth portfolio. If the takeover subsequently completes three months after time zero, it would reflect as a negative in the line chart.

6. Deep Value around the World: Out-of-Sample Tests

Having studied the in-sample returns and economics of deep value, we finally consider the strategy's out-of-sample performance. Since behavioral theories predict that arbitrageurs can profit from deep value investing (at least to a limited extent), these profits are only meaningful if they can be a realized out of sample. That is, it remains to be seen whether one can profit from deep value when deep value events are identified without the benefits of hindsight.

To conduct a simple and realistic out-of-sample test, we start by constructing deep value trading strategies based on our previously defined standard value factors. The strategy is intended to simulate a trader opportunistically entering value trades after observing wide value spreads and exiting after observing convergence. Specifically, in each of the seven test markets, when we observe value spreads crossing the 80th percentile filter, we add that market's value portfolio, as of that point in time, to the deep value portfolio.[18] After being added, these opportunistic trades remain in the deep value portfolio until valuation spreads have declined below their historical median without being rebalanced. Having thus formed the deep value strategy in each stock selection region and in each asset class, we form our combined deep value strategies as an equal risk weighted average over stock selection strategies, asset allocation strategies, and all strategies. For simplicity, we use a measure of ex-post realized risk for this averaging.

Table 6 reports the alphas of these global deep value strategies from 1976 to present, when we have data available for both stocks and macro asset classes. In particular, for each deep value strategy, we run a regression of the excess returns on both the market, as measured by MSCI world, and "untimed" value and momentum factors. Here, "untimed" value and momentum factors means factors that are constructed to exactly match the investment universe and portfolio construction of those used in the deep value strategy, except that we trade all underlying value portfolios without filtering for a wide valuation spread. In other words, for the intra deep value strategies, the untimed value and momentum strategies on the right hand side of the regression are also formed on an intra basis.

Panel A shows the deep value alphas for the seven standard value strategies, while Panel B considers the intra strategies. We report results individually for each market, combined for all stock selection strategies (SS), combined across all asset allocation strategies (AA), and combined across all strategies (ALL).

[18] Each trade that is added to the portfolio is scaled to target a fixed level of annualized volatility, so that trades in different asset classes have comparable risk allocations (due to later rescaling at the portfolio level, the actual level of the risk target at this intermediate step is unimportant). To do this, we measure the expected volatility of each trade portfolio on an unlevered basis, and then scale leverage to achieve the target volatility. In stock selection strategies, the measure of expected volatility is the volatility of trailing 1-year daily returns of the value strategy in the given region and asset class. Conversely, in asset allocation strategies, where different assets can have very different volatilities, we measure the trailing 1-year daily volatility of the specific trade portfolio.

We generally see the intuitive result that each deep value strategy has a significant loading on the corresponding untimed value strategy.[19] We also see that deep value has a significant negative loading on momentum, even controlling for value, indicating that the momentum of deep value assets is even more negative than that of untimed value. This is intuitive since deep value assets tend to have particularly negative past returns as seen in figure 1. Indeed, the strategy buys value portfolios after value spreads have risen (typically associated with the value portfolio losing money), and sells value portfolios when valuations spreads have fallen (typically associated with a profitable period for value). The loadings on the market are mixed, but generally negative.

The alphas of the deep value strategies are positive in most cases, but mixed in terms of magnitude and statistical significance. In stock selection in particular, all the alphas are insignificant. This indicates that, while the standard deep value strategy may be profitable stand alone, its profitability is derived largely from its loading on a regular value strategy with a limited benefit of using the information contained in any one value spread to time the amount of exposure. In other words, timing any one value strategy would not be meaningfully additive in the context of a portfolio with optimal allocations to untimed value and momentum, consistent with Asness at al. (2017). However, when we put all seven strategies together (the column in the table labelled ALL), we see a significant alpha.

The power of diversifying across many value strategies is far greater when we turn to our 515 intra value strategies within the 7 test markets. Specifically, the intra strategies separately tracks value spreads of value within each industry in each stock market and of each pair of assets in asset allocation. Looking at each intra sub-strategy, we continue to enter a trade when the value spread is at its 80th percentile and exit at the 50th percentile.[20] In other words, in each of the seven test markets, rather than the deep value trade simply being "on" or "off" for the entire market (as in the standard specification above), the intra deep value strategy may be "on" for auto stocks and biotech stocks while being "off" for other industries.

[19] Asness et al (2017) find that using value to time non-value factors leads to a strategy that is correlated to value and investors not accounting for this may end up with a (suboptimal) increase in their value exposure if they do so. Deep value is also a value-based timing strategy, however the correlation to value that we see here is more trivially obvious as deep value is long value (when it is cheap) or has no position. It can never be short value. As before, investors should account for this correlation when allocating to deep value, as explored later in this section.

[20] For practicality purposes, we cap each strategy to target at most 20 times the individual trade level risk target (if more than 20 trades exist in the portfolio, we start to proportionally reduce the risk target per trade), though this capping does not meaningfully impact results.

Further, the intra deep value strategy granularly varies risk according to the number of deep value trades that exist, taking more risk when more sectors experience deep value events.

The performance of the intra deep value strategies is presented in Panel B of Table 6. The intra deep value strategy shows significant alphas in three of the four equity markets (where we had seen weak results for the standard deep value strategy) and also in FX. This higher alpha is particularly impressive given that, in stock selection, performance for the intra value strategy used as a control variable in this regression is much stronger than for the standard value strategy (used in the regression with the standard deep value strategy). As seen in the right columns, the diversified strategies are significant in stock selection, asset allocation, and overall. The overall strategy has an alpha of over 6.4% with a *t*-statistic exceeding 5. In other words, we see that while there is little information content in any one value spread, the aggregate information across many value spreads is significant.

Note that, even though the intra deep value strategy is profitable stand alone and has alpha to other known trading signals, its correlation to regular value strategies should be considered when adding it to a diversified portfolio that already includes value. For example, suppose that we start with an equal-weighted combination of untimed intra value and momentum strategies, which delivers a Sharpe ratio of 1.8 in our historical simulation. Then, this simulated portfolio Sharpe ratio can be increased to 2.0 by moving half of the value allocation from untimed value to deep value (i.e., with final portfolio weights of 50% momentum, 25% value, and 25% deep value). However, if instead the deep value allocation is funded by proportionally reducing weight on both value and momentum (i.e., 37.5% momentum, 37.5% value, and 25% deep value), then there is no benefit to the Sharpe ratio since the weight on value strategies has been increased beyond an optimal level.

As a final set of robustness checks, we consider three additional approaches to deep value. Said differently, we consider three alternative ways to use the value spread to time our intra value strategies in order to ensure that the details of the entry and exit points are not essential. All of the value timing strategies are out-of-sample in the sense that they are designed to be implementable by using information known at each point in time, either using the expanding value spread percentile or the expanding value spread STD (defined as the value spread minus its median observed until that time, divided by the standard deviation of the value spread observed until that time). As seen in Table 7, we consider the following strategies:

- "Deep value": as described above (which enters value trades when the expanding value spread percentile exceeds 80th, and exits when it falls below 50th).

- "Deeper value": enters when the expanding value spread STD exceeds 2 and exits when it falls below 1, i.e., a more extreme version of deep value.
- "Threshold": trades value in all periods when the expanding value spread percentile is greater than its 80th, i.e., a simpler version of deep value with symmetric entry/exit points.
- "Linear timing": varies risk in proportion to the value spread STD, taking no risk when the STD falls below negative two and increasing risk linearly in the STD until it reaches its maximum when the STD exceeds two. This strategy takes more risk when value spreads are wider (like the other deep value strategies), but in a continuous way that explicitly avoids focusing on the extremes (by capping the exposure at two STD).

The results of these robustness checks are showed in Panel A of Table 7. In all cases, the overall timed intra value strategies have statistically significant alpha over the known factors. In other words, regardless of the timing function, a diversified intra deep value strategy that takes additional risk in value trades in sub-markets with wide value spreads appears to complement the other factors. Additionally, given that a central result from Table 6 was meaningfully better performance of the diversified intra deep value strategies compared to the more concentrated standard ones, we explicitly test the alpha of intra deep value over standard value timing in Panel B. As expected, while intra deep value strategies have a significant loading on standard value timing, the alpha of intra deep value does not materially change when standard value timing is included in the regression.

6.1. Deep Value Opportunities: Absent or Abound

Finally, we look at how the total number of deep value opportunities varies over time. In particular, Figure 8 shows the number of intra deep value trades that are triggered at each time. We see that the number of deep value events tends to cluster, with large peaks around the internet bubble in 2000 and the global financial crisis of 2008, and smaller peaks around the Volcker experiment in the early 1980s, the first invasion of Iraq and other events in the early 1990s, and the European crisis in 2012. This interesting clustering may be related to deep value's large loading on a global value factor that we documented in Table 6 and the event study of Figure 2. In other words, deep value events across markets and asset classes appear to share commonalities in terms of timing and risk characteristics, which could be driven by common rational risks, joint behavioral effects, shocks to arbitrageurs overall level of funding, or some combination of those. This clustering also constitutes its own limit to arbitrage by making it more difficult to trade on deep value.

Figure 8 also shows the cumulative out-of-sample return to the overall deep value strategy. We see that the strategy appears to do well during periods of abounded deep value events. This intuitive result is confirmed in Table 8, where we regress the monthly return to deep value, $DVAL_{t+1}^{i}$, on the percent of available trades meeting the deep value filter at the end of the previous month, $EVENTS_{t}^{i}$:

$$DVAL_{t+1}^{i} = \alpha + \beta\, EVENTS_{t}^{i} + \varepsilon_{t+1}^{i}$$

where i indicates either deep value among stocks, asset allocation, or all assets. We see that the slope coefficient is significantly positive for all assets and stock selection, indicating that the deep value strategies earns higher average returns when there are many deep value events.

Table 8 also considers how the volatility and Sharpe ratio (SR) or returns vary with the number of events. Specifically, we regress one year ahead annualized returns, one year ahead volatility of returns, and one year ahead annualized SR of the deep value strategy on the percentage of included value strategies. By design, the volatility rises with the number of events, given that our strategy deploys a greater risk budget in these instances, and it is not surprising that returns also rise as a result. However, since SR is the ratio of average excess returns to volatility, it is not obvious how SR changes when both the numerator and denominator rise. However, we see that the slope coefficient for the SR is significantly positive for all assets and for stocks for the deep value strategy. This finding indicates that, when many deep value events occur, the strategies have a high return even relative to its high risk. This is also somewhat intuitive, given that the strategies are more diversified in these periods given a greater number of trades included (though this greater diversification is partially offset by value trades being more correlated to each other during this periods of wider value spreads).

7. Conclusion: A Deep Look at Value Investing

We show how deep value investing complements regular value strategies, in-sample and out-of-sample, and test competing economic theories of value based on the evolution of a host of characteristics. The evidence for the rational theories is mixed. Consistent with rational explanations, value stocks have cheap prices for a reason since their earnings fundamentals deteriorate over time, especially during deep value events. The return patterns are more puzzling from a rational perspective since value strategies have negative market betas, especially during deep value events when the returns are particularly high.

However, consistent with the existence of a non-market risk factor, deep value strategies have particularly large loadings on a global value factor. Interestingly, these findings hold consistently among stocks (across all regions), stock indices, global bonds, and currencies.

Our findings reject the notion that price changes are driven by pure noise. The data does not support this theory because deep value opportunities are clearly linked to changes in fundamentals and because we find that deep value opportunities cluster globally across market and asset classes.

The data is largely consistent with behavioral theories of over-extrapolation of past returns, but not with theories of over-reaction to fundamentals. Indeed, based on the idea that sentiment-driven investors sell cheap value stocks, we document a negative sentiment for value stocks based on their tone of news coverage and, consistent with the idea that some investors act on this negative sentiment, we see selling pressure of value stocks relative to growth stocks. We find that this selling pressure is driven by past returns, not past fundamentals, supporting the theories of over-extrapolation of past returns. Further, we find that arbitrage is limited for deep value portfolios by their high transaction costs and short-sale costs, but nevertheless arbitrage appears to exist in the sense of elevated short interest and firms appear to act as “arbitrageurs of last resort” in their decisions to conduct new issues, share repurchases, and corporate take-overs.

Finally, studying-out-of-sample predictability, we find that a concentrated deep value approach focused on timing a single standard value strategy often has an insignificant alpha, but a diversified approach that accesses a broad set of deep value opportunities can robustly capture the aggregate information in many value spreads.

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Table 1. Testing theories of value.

This table summarizes our strategy to test three different theories of why value strategies have historically delivered positive average returns and why these returns have been especially large after deep value episodes in which the value spread was large.

		Theories			
		Fully rational	**Behavioral**		**Pure noise**
			Over-reaction to fundamentals	Over-extrapolation of past returns	
Prediction	(Empirical measures)				
Positive returns to value, especially deep value	(Returns, value spreads based on book-to-price)	Yes	Yes	Yes	Yes
High risk of cheap stocks, especially deep value	(Market beta, beta to value factor)	Yes			
Deteriorating fundamentals, especially deep value	(Earnings , analyst revisions)	Yes	Yes	Potentially via past returns	No
Negative sentiment, especially deep value	(Tone of news stories)		Yes	Yes	
Selling pressure, especially deep value	(Signed order flow)		Yes	Yes	Yes
Selling pressure driven by:	(Signed order flow)		Past fundamentals	Past returns	
Limited arbitrage, especially deep value	(Transaction costs, shorting costs, volatility of long-short portfolio)		Yes	Yes	Yes
Arbitrageurs come in, especially deep value	(Short interest, issuance, mergers)		Yes	Yes	Yes
Deep value opportunities comove across asset classes	(clustering)	Yes	Yes	Yes	No

Table 2. Summary statistics and value performance.

This table contains descriptive statistics about the assets and universes included in our study. We describe four regions in which we do stock selection, and three asset classes in which we do asset allocation based on valuation signals. Within each stock selection region, we do a Standard valuation sort on all stocks in the region, and we do a set of Intra valuation sorts within each industry in the region. Correspondingly, we also do Standard sorts in each asset allocation universe, but also look at Intra valuation strategies trading each pair of assets within the asset class. For each type of strategy, we report backtest periods, universe size, valuation signal considered and Sharpe ratio for Standard and Intra sorts. Standard sort Sharpe ratios are computed by aggregating trading strategies across all Intra subsets in the universe; for example, the 0.6 Sharpe ratio for "Intra Value" in U.S. Stock Selection is the performance of a strategy that sorts U.S. stock within each industry, and then aggregates to a single portfolio.

Stock Selection Portfolios							
Region	Start Date	Average Universe Size ("Standard")	Number Industries ("Intra")	Average Industry Universe Size ("Intra")	Value Factor	Standard Value Factor Sharpe Ratio	Combined Intra Value Factor Sharpe Ratio
United States (US)	31-Jan-26	3949	68	77	Book-to-Price Ratio	0.2	0.6
Japan (JP)	31-Jan-86	2768	68	44	Book-to-Price Ratio	0.9	1.1
Europe (EU)	31-Jan-88	2489	68	39	Book-to-Price Ratio	0.4	0.6
United Kingdom (UK)	31-Jan-86	773	68	12	Book-to-Price Ratio	0.0	0.4

Asset Allocation Portfolios							
Asset Class	Start Date	Average Universe Size ("Standard")	Number Pairs ("Intra")	Average Pair Size ("Intra")	Value Factor	Standard Value Factor Sharpe Ratio	Combined Intra Value Factor Sharpe Ratio
Equity Indices (EQ)	31-Jan-75	18	153	2	Book-to-Price Ratio	0.3	0.1
Fixed Income (FI)	31-Jan-80	10	45	2	Real Bond Yield	0.4	0.4
Currencies (FX)	28-Feb-75	10	45	2	Real Exchange	0.3	0.3

Table 3. Value strategy returns by value spread.

This table shows the result of a bucketing exercise in which value returns for each value strategy in each month are bucketed by quintiles of the full sample value spread at the start of each month. We report results within each underlying stock selection region and each asset allocation asset class, as well as aggregated at various levels. Results are reported for both Standard and Intra sorts. For Intra sorts, returns at the region level are the aggregate of all industries within that region (stock selection), or all asset pairs within the asset class (asset allocation), though break points for bucketing are computed relative to each individual time series.

		All Asset Portfolios									
		Standard Portfolio Quintiles					Intra Portfolio Quintiles				
		1	2	3	4	5	1	2	3	4	5
All Assets	Mean	0.1%	0.2%	0.2%	0.3%	0.9%	0.0%	0.1%	0.2%	0.4%	0.9%
01/1926 to 09/2015	(t-stat)	(1.9)	(2.3)	(1.6)	(2.0)	(3.2)	(-0.6)	(2.7)	(4.1)	(5.3)	(7.6)
Stock Selection (SS)	Mean	0.2%	0.3%	0.3%	0.4%	1.1%	0.0%	0.3%	0.4%	0.6%	1.3%
01/1926 to 09/2015	(t-stat)	(1.6)	(2.1)	(1.5)	(1.9)	(2.4)	(-0.2)	(3.4)	(5.3)	(6.1)	(6.7)
Asset Allocation (AA)	Mean	0.1%	0.1%	0.1%	0.1%	0.7%	0.0%	0.0%	0.0%	0.1%	0.5%
01/1975 to 09/2015	(t-stat)	(1.2)	(1.2)	(0.7)	(0.8)	(4.1)	(-1.0)	(-0.3)	(0.1)	(1.6)	(4.5)
		Stock Selection Portfolios									
		Standard Portfolio Quintiles					Intra Portfolio Quintiles				
		1	2	3	4	5	1	2	3	4	5
United States (US)	Mean	0.0%	0.2%	0.2%	0.3%	1.2%	0.1%	0.2%	0.4%	0.7%	1.4%
01/1926 to 09/2015	(t-stat)	(0.2)	(0.9)	(1.0)	(1.1)	(1.9)	(0.8)	(1.8)	(4.1)	(5.0)	(4.9)
Japan (JP)	Mean	0.5%	0.4%	1.5%	1.1%	1.7%	0.1%	0.6%	0.7%	1.0%	1.5%
01/1986 to 09/2015	(t-stat)	(1.8)	(1.0)	(2.7)	(2.7)	(2.8)	(0.9)	(3.6)	(3.7)	(5.0)	(5.1)
Europe (EU)	Mean	0.2%	0.6%	-0.2%	0.4%	1.3%	-0.1%	0.3%	0.5%	0.4%	1.0%
01/1988 to 09/2015	(t-stat)	(0.8)	(2.1)	(-0.4)	(0.7)	(1.9)	(-0.3)	(1.9)	(3.1)	(2.5)	(3.9)
United Kingdom (UK)	Mean	0.3%	0.4%	-0.4%	0.3%	-0.1%	-0.3%	0.2%	0.1%	0.3%	1.1%
01/1986 to 09/2015	(t-stat)	(1.3)	(0.9)	(-0.9)	(0.6)	(-0.1)	(-2.3)	(0.9)	(0.4)	(1.5)	(4.6)
		Asset Allocation Portfolios									
		Standard Portfolio Quintiles					Intra Portfolio Quintiles				
		1	2	3	4	5	1	2	3	4	5
Equity Indices (EQ)	Mean	0.4%	0.3%	0.1%	-0.2%	1.0%	0.0%	0.0%	0.0%	0.1%	0.6%
01/1975 to 09/2015	(t-stat)	(2.0)	(1.1)	(0.5)	(-0.6)	(2.7)	(-0.8)	(-0.5)	(-0.4)	(1.0)	(3.4)
Fixed Income (FI)	Mean	0.1%	0.0%	0.1%	0.0%	0.6%	-0.1%	0.1%	0.0%	0.1%	0.3%
01/1980 to 09/2015	(t-stat)	(1.2)	(-0.0)	(0.6)	(-0.2)	(2.5)	(-1.5)	(1.4)	(1.2)	(1.8)	(3.7)
Currencies (FX)	Mean	-0.1%	0.1%	0.1%	0.5%	0.4%	0.0%	0.0%	0.1%	0.2%	0.5%
01/1975 to 09/2015	(t-stat)	(-0.9)	(0.6)	(0.3)	(2.4)	(1.7)	(0.0)	(-0.1)	(1.5)	(1.7)	(3.8)

Table 4. Value strategy returns regressed on value spreads.

This table shows the results of regressing 12-month ahead value strategy returns on value spreads at the start of each period. Standard regressions are individual time series regressions run within each stock selection region or asset class. Intra regressions are pooled regressions, pooling across all industries within a region (stock selection) or all pairs within an asset class (asset allocation). We also report pooled regression results at the overall stock selection, overall asset allocaiton and all asset level. All pooled regressions are run with entity fixed effects and standard errors used in t-statistics are corrected for correlation the time series and cross section using Hansen and Hodrick (1980).

		All Asset Portfolios			
		Standard		Intra	
		Beta	R2	Beta	R2
All Assets	Estimate	0.2	15.5%	0.1	6.1%
01/1926 to 09/2015	(t -statistic)	(4.0)		(10.7)	
Stock Selection (SS)	Estimate	0.3	15.9%	0.1	5.8%
01/1926 to 09/2015	(t -statistic)	(3.6)		(8.5)	
Asset Allocation (AA)	Estimate	0.2	9.6%	0.1	4.0%
01/1975 to 09/2015	(t -statistic)	(2.9)		(7.5)	

		Stock Selection Portfolios			
		Standard		Intra	
		Beta	R2	Beta	R2
United States (US)	Estimate	0.3	15.6%	0.1	5.4%
01/1926 to 09/2015	(t -statistic)	(2.7)		(7.2)	
Japan (JP)	Estimate	0.4	12.8%	0.1	6.0%
01/1986 to 09/2015	(t -statistic)	(2.6)		(2.9)	
Europe (EU)	Estimate	0.4	16.5%	0.1	6.3%
01/1988 to 09/2015	(t -statistic)	(2.7)		(3.4)	
United Kingdom (UK)	Estimate	0.14	5.6%	0.05	5.6%
01/1986 to 09/2015	(t -statistic)	(1.3)		(3.3)	

		Asset Allocation Portfolios			
		Standard		Intra	
		Beta	R2	Beta	R2
Equity Indices (EQ)	Estimate	0.3	11.5%	0.1	3.8%
01/1975 to 09/2015	(t -statistic)	(2.1)		(6.6)	
Fixed Income (FI)	Estimate	0.1	12.1%	0.1	5.1%
01/1980 to 09/2015	(t -statistic)	(2.6)		(6.0)	
Currencies (FX)	Estimate	0.4	14.7%	0.2	6.3%
01/1975 to 09/2015	(t -statistic)	(2.9)		(8.5)	

Table 5. What do Investors (Over-)React to?

This table shows the results of regressing the monthly signed order flow (the column labelled "1-Month Ahead Demand Pressure") or the future 1-month return ("1-Month Ahead Return") on past returns and past fundamentals. Specifically, the independent variables are the past returns over the last 2-to-12 months ("Ret(2,12)"), the returns over the past month 13 through 60 months ("Ret(13,60)"), and changes in fundamentals measured as changes in return-on-equity over the same time horizons ("△ROE(2,12)" and "△ROE(13,60)"). Demand pressure is capture by signed order flow defined as dollar buys minus dollar sells divided by buys plus sells per stock based on the Lee and Ready (1991) methodology. Regressions are run with time fixed effects, with standard errors adjusted using Newey and West (1987).

	1-Month Ahead Demand Pressure			1-Month Ahead Returns		
	Returns Only	ROE Only	Both	Returns Only	ROE Only	Both
Ret (2,12)	0.019		0.019	0.002		0.1%
(t -statistic)	(14.3)		(13.2)	(1.5)		(1.2)
Ret (13,60)	0.018		0.018	-0.003		-0.003
(t -statistic)	(17.4)		(14.4)	(-3.9)		(-3.9)
ΔROE (2,12)		0.002	0.000		0.000	0.000
(t -statistic)		(7.4)	(0.1)		(0.1)	(1.3)
ΔROE (13,60)		0.012	0.001		0.000	0.002
(t -statistic)		(13.1)	(1.0)		(-0.4)	(2.6)
R2	1.67%	0.12%	1.58%	0.03%	0.00%	0.03%

Table 6. The alpha of deep value out-of-sample.

This table shows the returns to our out-of-sample deep value trading strategies regressed on known factors. The deep value strategy buys value portfolios when the value spread exceeds the 80th percentile (using only known data at each time) and sells when value spreads revert to its median level. We regress the returns to this strategy on excess returns to the market (MSCI World) and value and momentum factors that are constructed as those used in the deep value strategy, but without filtering for wide valuation spreads (e.g., for the intra deep value strategy, the right-hand-side value and momentum strategies are also formed on an intra basis). We run these regressions for the standard full cross-section (Panel A) and for the intra strategies (Panel B).

Panel A: Deep value constructed on standard value strategies

	Stock Selection Portfolios				Asset Allocation Portfolios			Aggregated Portfolios		
	US	JP	EU	UK	FX	FI	EQ	SS	AA	ALL
Annualized Alpha	0.5%	0.5%	0.9%	-0.5%	1.5%	1.0%	1.3%	0.7%	3.3%	3.5%
(t -statistic)	(0.6)	(0.5)	(0.9)	(-0.5)	(2.0)	(2.1)	(2.1)	(0.9)	(3.9)	(3.0)
Market	-0.02	-0.02	-0.03	-0.04	-0.04	0.01	-0.01	0.00	-0.05	-0.05
(t -statistic)	(-1.2)	(-0.8)	(-1.2)	(-1.4)	(-2.2)	(0.7)	(-0.9)	(-0.1)	(-2.4)	(-1.7)
Value	0.24	0.19	0.22	0.33	0.31	0.07	0.19	0.27	0.31	0.47
(t -statistic)	(11.3)	(8.5)	(8.5)	(11.7)	(17.1)	(5.0)	(10.0)	(11.2)	(13.1)	(13.8)
Momentum	-0.09	-0.05	-0.06	-0.01	-0.05	0.02	-0.02	-0.09	-0.07	-0.10
(t -statistic)	(-4.4)	(-1.8)	(-2.2)	(-0.5)	(-2.8)	(1.7)	(-1.3)	(-3.6)	(-2.9)	(-2.9)
R2	37%	27%	31%	35%	41%	7%	21%	33%	30%	37%

Panel B: Deep value constructed on intra value strategies

	Stock Selection Portfolios				Asset Allocation Portfolios			Aggregated Portfolios		
	USA	JPN	ROE	UKI	FX	FI	EQ	GSS	GAA	ALL
Annualized Alpha	3.0%	-0.5%	4.5%	5.2%	1.9%	2.6%	2.1%	3.5%	2.5%	6.4%
(t -statistic)	(2.7)	(-0.4)	(3.0)	(2.9)	(1.8)	(1.8)	(1.8)	(4.2)	(3.0)	(5.1)
Market	-0.03	-0.02	0.02	-0.06	-0.04	0.02	0.03	-0.05	-0.02	-0.07
(t -statistic)	(-1.0)	(-0.5)	(0.5)	(-1.3)	(-1.5)	(0.5)	(1.0)	(-2.5)	(-0.9)	(-2.4)
Value	0.52	0.55	0.57	0.23	0.50	0.37	0.45	0.41	0.35	0.61
(t -statistic)	(19.2)	(15.7)	(15.3)	(6.8)	(21.8)	(10.3)	(15.1)	(17.7)	(14.7)	(18.2)
Momentum	-0.15	-0.11	-0.06	-0.20	-0.14	-0.14	-0.20	-0.14	-0.15	-0.27
(t -statistic)	(-5.4)	(-3.1)	(-1.6)	(-4.5)	(-5.5)	(-3.9)	(-6.5)	(-6.1)	(-6.5)	(-8.0)
R2	57%	52%	54%	24%	54%	26%	43%	52%	39%	53%

Table 7. The alpha of deep value out-of-sample: Robustness.

This table shows the alpha to various different intra deep value strategies. We repeat the results for Deep Value from Table 3 and add three new versions: "Deeper Value" has an identical methodology except with a more extreme entry threshold of two standard deviations and exit of one standard deviation (rather than entering at 80th percentile and exiting at 50th percentile); "Threshold" trades each value strategy with a value spreads above its 80th percentile (i.e., symmetric rather than asymmetric entry and exit points); and "Linear" trades each value strategy, allocating risk to each component in linear proportion to its value spread. We control for the excess returns to the market (MSCI World) and untimed value and momentum factors that are constructed as those used in the deep value strategies. In Panel B, we also control for a linear timing strategy conducted on standard value factors.

Panel A: Different intra deep value strategies regressed on known factors

	Deep Value			Deeper Value			Threshold			Linear		
	SS	AA	ALL	SS	AA	ALL	SS	AA	ALL	SS	AA	ALL
Annualized Alpha	3.5%	2.5%	6.4%	3.6%	3.8%	7.5%	3.0%	4.2%	5.5%	4.6%	3.0%	4.4%
(t -statistic)	(4.2)	(3.0)	(5.1)	(3.9)	(3.7)	(5.2)	(1.8)	(2.7)	(3.5)	(4.7)	(2.6)	(4.0)
Market	0.0	0.0	-0.1	-0.1	0.0	-0.1	-0.1	0.0	-0.1	-0.1	0.0	0.0
(t -statistic)	(-2.5)	(-0.9)	(-2.4)	(-2.8)	(-1.7)	(-3.1)	(-2.3)	(0.0)	(-1.9)	(-2.3)	(-0.0)	(-1.2)
Match Value	0.4	0.3	0.6	0.3	0.3	0.5	0.3	0.3	0.3	0.8	0.6	0.7
(t -statistic)	(17.7)	(14.7)	(18.2)	(10.7)	(9.4)	(11.9)	(8.0)	(6.2)	(7.7)	(28.9)	(19.5)	(24.2)
Match Momentum	-0.1	-0.2	-0.3	-0.1	-0.2	-0.2	-0.2	-0.3	-0.3	-0.1	-0.2	-0.2
(t -statistic)	(-6.1)	(-6.5)	(-8.0)	(-3.3)	(-5.6)	(-5.7)	(-4.3)	(-6.3)	(-6.6)	(-4.9)	(-7.3)	(-6.2)
R2	52%	39%	53%	29%	24%	34%	22%	17%	24%	71%	52%	62%

Panel B: Different intra deep value strategies regressed on known factors and standard value timing

	Deep Value			Deeper Value			Threshold			Linear		
	SS	AA	ALL	SS	AA	ALL	SS	AA	ALL	SS	AA	ALL
Annualized Alpha	3.1%	2.0%	5.8%	3.2%	3.0%	6.7%	2.2%	3.4%	4.8%	4.0%	1.5%	3.3%
(t -statistic)	(4.0)	(2.5)	(4.8)	(3.6)	(3.1)	(4.9)	(1.5)	(2.3)	(3.1)	(4.6)	(1.8)	(3.8)
Market	0.0	0.0	0.0	0.0	0.0	-0.1	0.0	0.0	0.0	0.0	0.0	0.0
(t -statistic)	(-0.4)	(-0.2)	(-0.9)	(-1.2)	(-0.9)	(-1.6)	(-0.5)	(0.6)	(-0.6)	(0.2)	(1.9)	(2.2)
Match Value	0.3	0.2	0.4	0.2	0.1	0.2	0.1	0.1	0.1	0.6	0.2	0.4
(t -statistic)	(10.8)	(6.7)	(9.8)	(5.5)	(1.6)	(4.4)	(2.4)	(1.1)	(1.9)	(20.7)	(7.2)	(12.1)
Match Momentum	-0.1	-0.1	-0.2	0.0	-0.1	-0.2	-0.1	-0.2	-0.2	-0.1	-0.2	-0.1
(t -statistic)	(-4.1)	(-5.7)	(-6.6)	(-1.6)	(-4.6)	(-4.2)	(-2.4)	(-5.6)	(-5.3)	(-2.5)	(-6.8)	(-3.9)
Standard Value Timing	0.2	0.2	0.3	0.2	0.3	0.4	0.4	0.3	0.4	0.6	0.6	0.6
(t -statistic)	(9.4)	(8.0)	(7.2)	(6.9)	(9.6)	(7.9)	(8.4)	(5.8)	(6.4)	(18.0)	(20.3)	(18.0)
R2	60%	47%	58%	35%	36%	42%	32%	22%	30%	78%	74%	78%

Table 8. Deep value returns vs. the number of deep value opportunities.

This table shows the results of regressing the level, volatility and Sharpe ratio of 12-month ahead returns to deep value on the size of the opportunity set at the start of each period. The size of the opportunity set is measured by the number of included trades (stock selection industries or asset allocation pairs) as a percent of available trades (those meeting the 80th percentile filter for value spreads).

	All Asset Portfolios			Stock Selection			Asset Allocation		
	Returns	Volatility	Sharpe	Returns	Volatility	Sharpe	Returns	Volatility	Sharpe
Estimate	0.5	0.1	4.3	0.2	0.0	2.1	0.2	0.2	2.5
(t -statistic)	(3.2)	(1.3)	(2.4)	(2.9)	(3.3)	(2.3)	(2.0)	(4.1)	(1.1)

Figure 1. The Returns to Value Investing

This figure has two components: on the left, a bucket sort of the level of returns for assets falling into different quintiles of valuations, and on the right an event study tracking historical and future returns to value portfolios having different levels of valuation spreads.

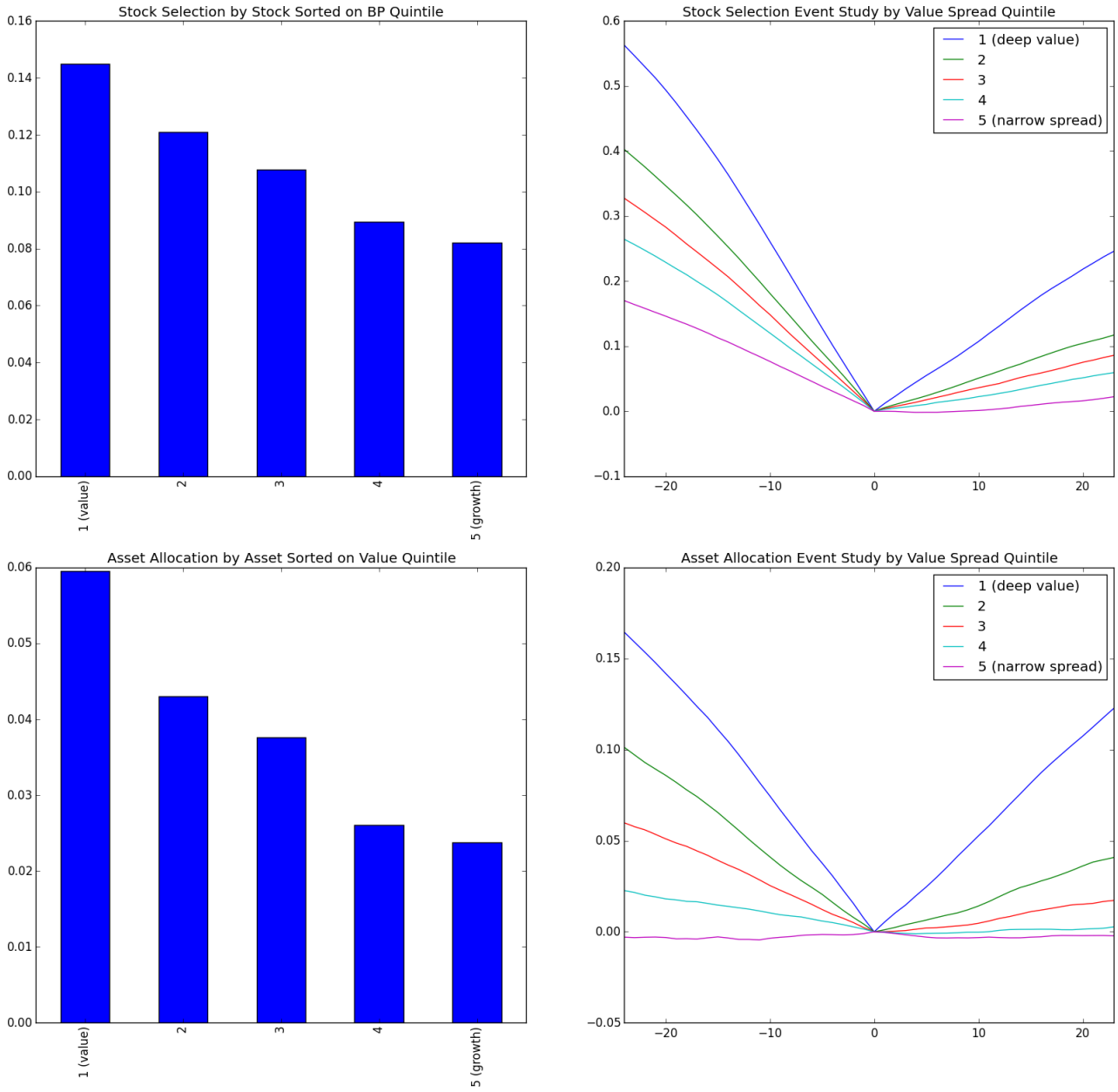
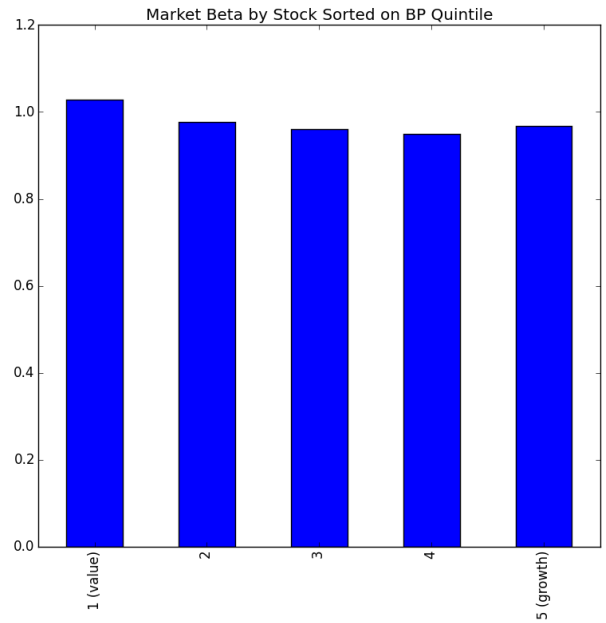
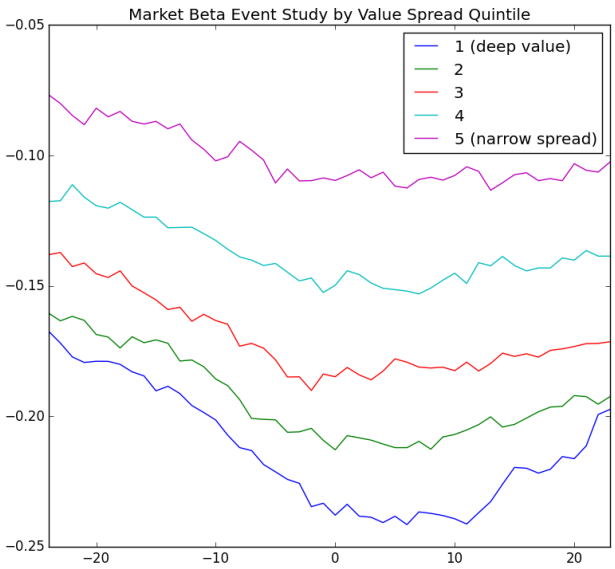


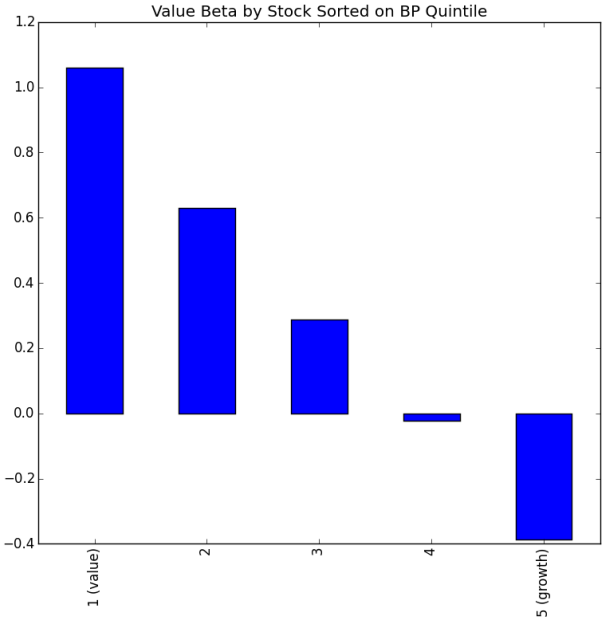
Figure 2. Risk Dynamics of Value Investing

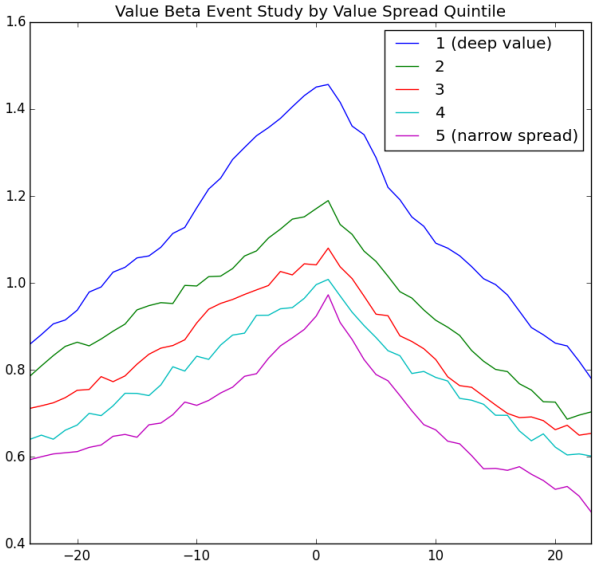
This figure has two panels, which each contain two columns of graphs: on the left, bucket sorts of measures of risk of stocks of different valuation quintiles, and on the right an event study tracking historical and future risk of value portfolios having different levels of valuation spreads. Event studies are computed as in Figure 1. The two panels correspond to results for stock selection (Panel A) and asset allocation (Panel B) value strategies. Two metrics of risk are considered: beta of returns to the market, and beta of returns to a value benchmark. In stock selection, the value benchmark for each industry value strategy is the average return of all industry value strategies industries within the given region. For asset allocation, the value benchmark for each asset pair value strategy is the average return for all pairs value strategies within the given asset class.

Panel A: Stock Selection

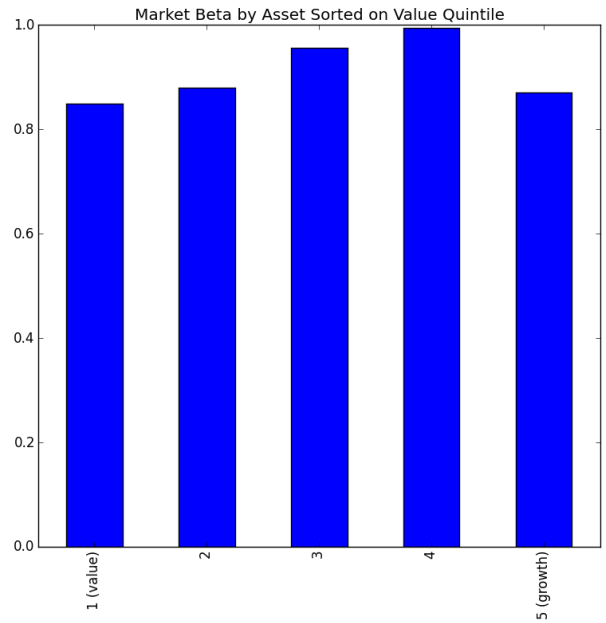


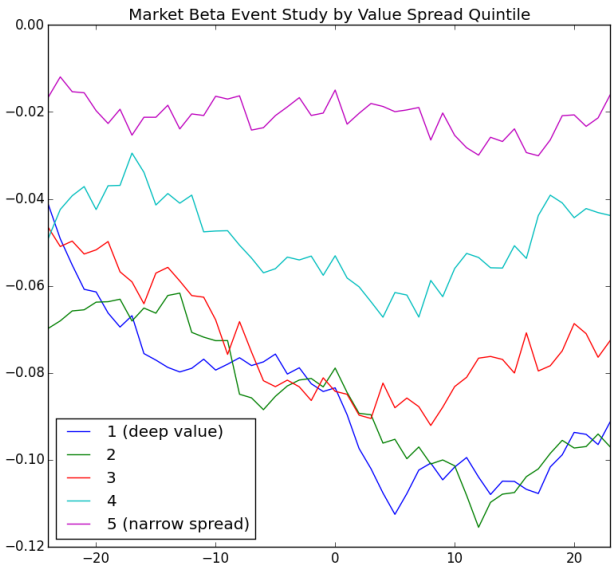


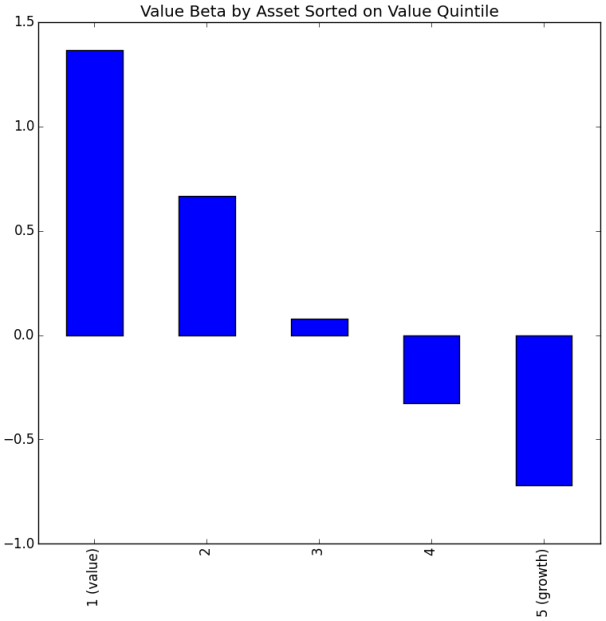




Panel B: Asset Allocation







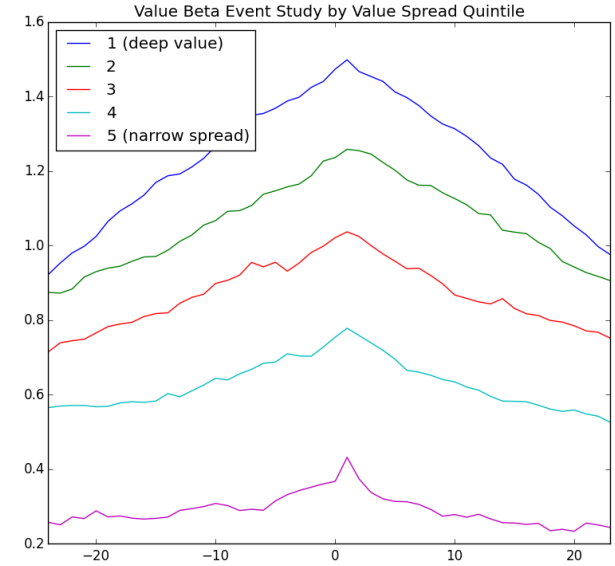


Figure 3: Earnings Fundamentals of Value

We show earnings fundamentals for stocks sorted by value quintile on the left, and event studies of earnings fundamentals for value portfolios having different valuation spread quintiles on the right. The first row of graphics shows the annual return on equity (income before extraordinary items divided by book value of equity). The second is the rolling three month analyst earnings forecast "revision ratio" (up revisions minus down revisions divided by number of forecasters) from Thompson Reuters I/B/E/S. Both event studies are cumulated.

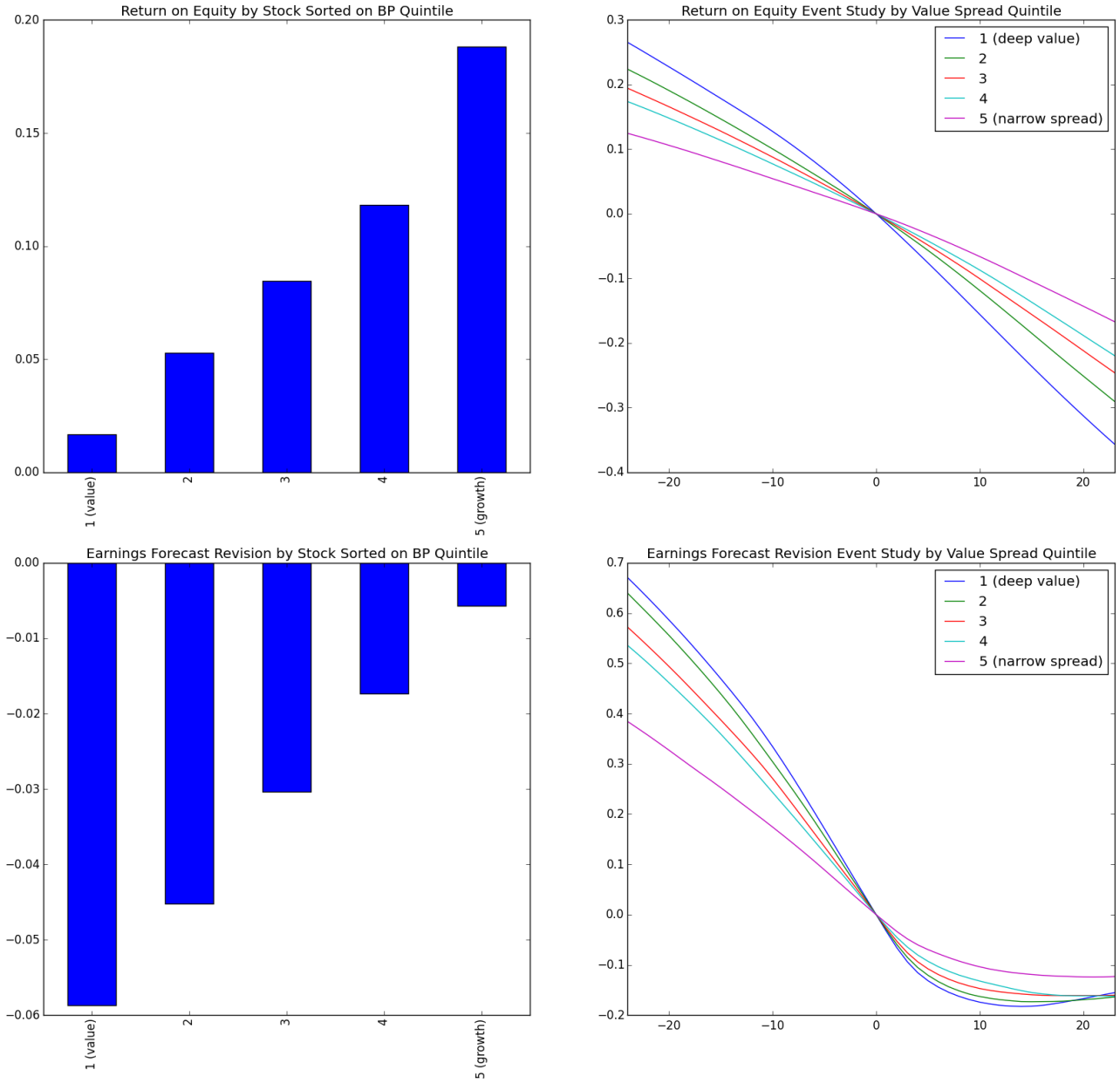
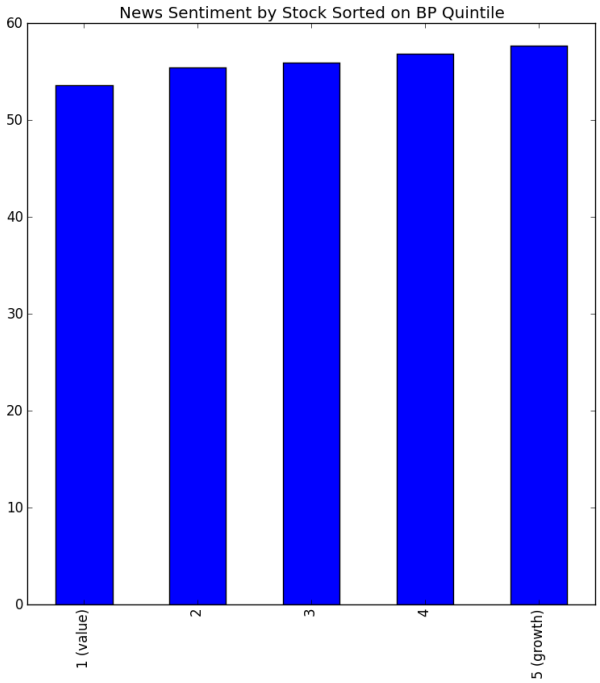


Figure 4: News Sentiment for Value

We show news sentiment for stocks sorted by value quintile on the left, and event studies of news sentiment for value portfolios having different valuation spread quintiles on the right. The measure of news sentiment is the Event Sentiment Score (ESS), provided by RavenPack. The ESS is a score between 0 and 100 that represents the average sentiment of news stories related to earnings, dividends or revenues for a given company. The event study is cumulated.



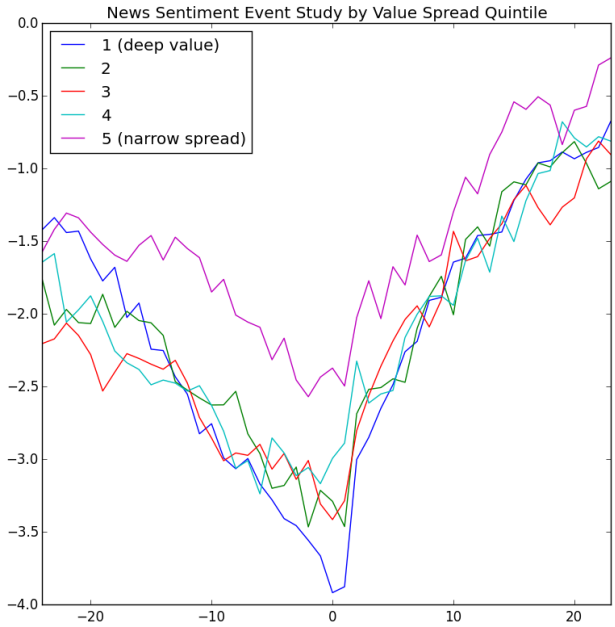


Figure 5: Demand Pressure

We show demand pressure for stocks sorted by value quintile on the left, and event studies of demand pressure for value portfolios having different valuation spread quintiles on the right. The measure of demand pressure is dollar buys minus dollar sells divided by buys plus sells per stock based on the Lee and Ready (1991) methodology. In both cases, event studies are on the difference in the characteristic between the long and short side of the portfolio. The event study is cumulated.

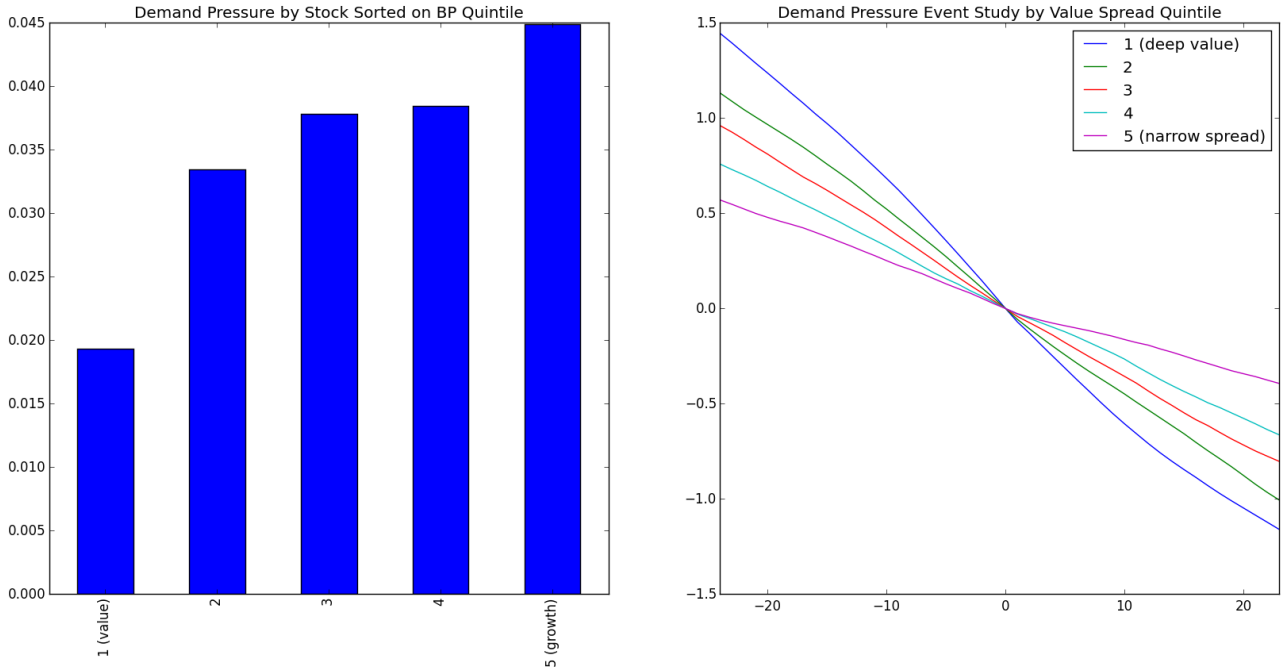
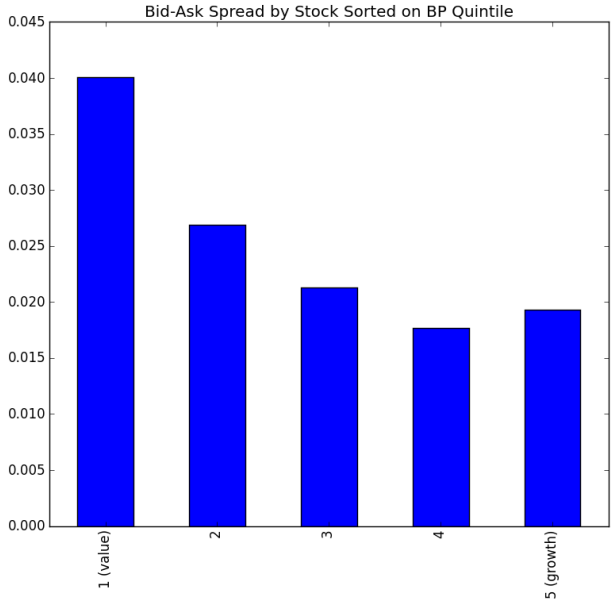
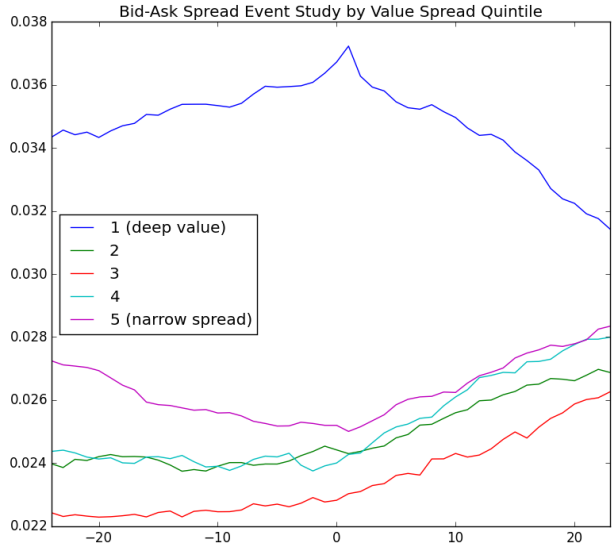
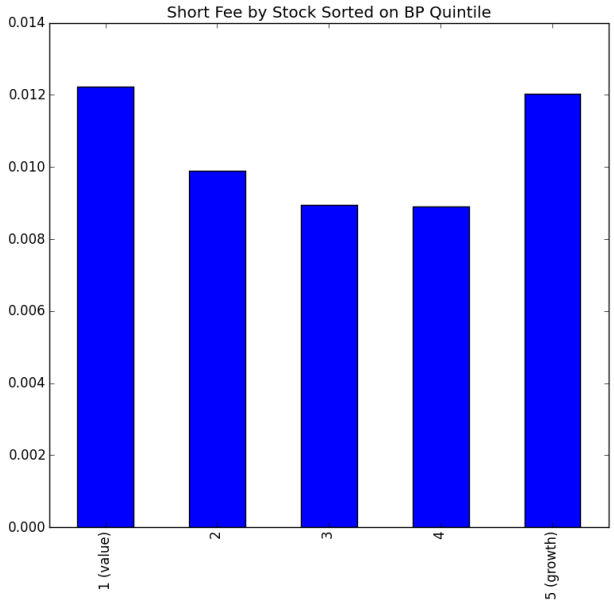


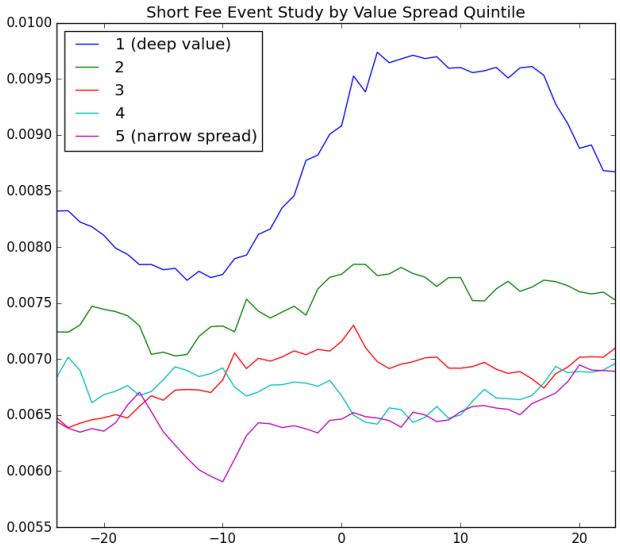
Figure 6: The Limits of Value Arbitrage

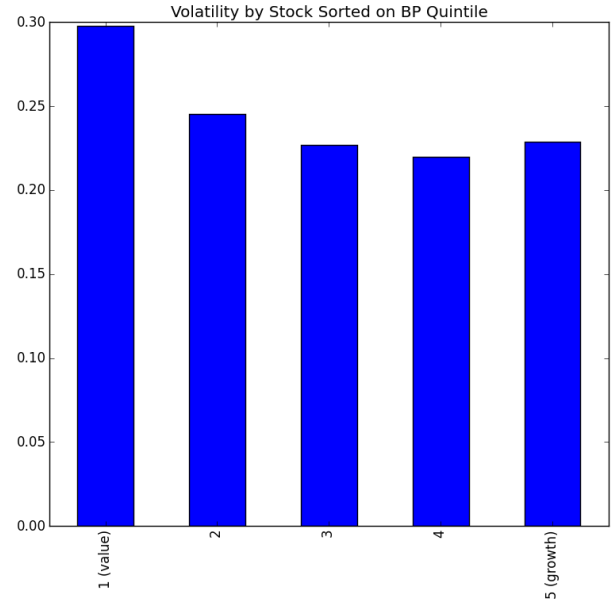
We show measures of limits of arbitrage for stocks sorted by value quintile on the left, and event studies of measures of limits of arbitrage for value portfolios having different valuation spread quintiles on the right. The first row of graphics shows the "simple average fee" of monthly stock borrows from hedge funds in each stock, provided by Data Explorers. The event study represents the short costs of the short (expensive) side of the portfolio only. The second row of figures has the bid-ask spread as a percent of the price of stocks from CRSP. The event study shows the average of bid-ask spreads for the long and short side of the portfolio. The third row shows the volatility of returns of stocks, computed from daily returns of stocks on a rolling monthly basis. The event study shows the volatility of the long-short value portfolio.


Bid-Ask Spread by Stock Sorted on BP Quintile
0.045
0.040
0.035
0.030
0.025
0.020
0.015
0.010
0.005
0.000
1 (value)
2
3
4
5 (growth)


Bid-Ask Spread Event Study by Value Spread Quintile
0.038
0.036
0.034
0.032
0.030
0.028
0.026
0.024
0.022
−20
−10
0
10
20
1 (deep value)
2
3
4
5 (narrow spread)


Short Fee by Stock Sorted on BP Quintile
0.014
0.012
0.010
0.008
0.006
0.004
0.002
0.000
1 (value)
2
3
4
5 (growth)


Short Fee Event Study by Value Spread Quintile
0.0100
0.0095
0.0090
0.0085
0.0080
0.0075
0.0070
0.0065
0.0060
0.0055
−20
−10
0
10
20
1 (deep value)
2
3
4
5 (narrow spread)


Volatility by Stock Sorted on BP Quintile
0.30
0.25
0.20
0.15
0.10
0.05
0.00
1 (value)
2
3
4
5 (growth)

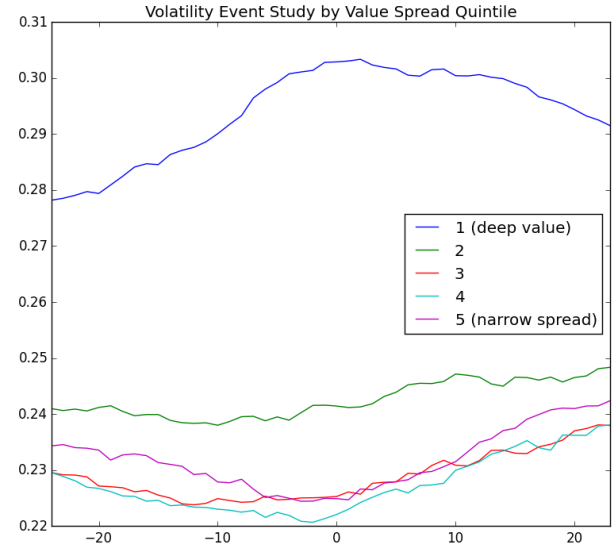
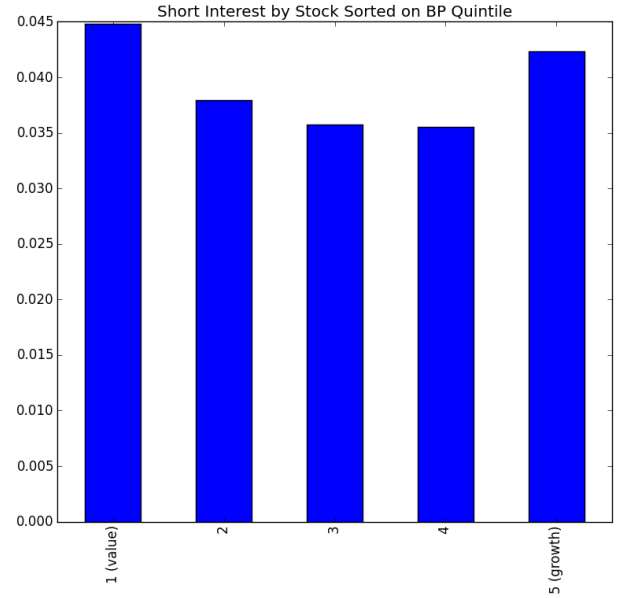
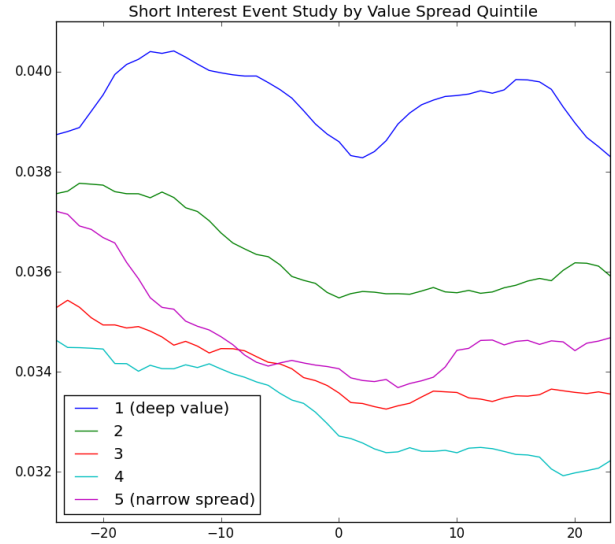
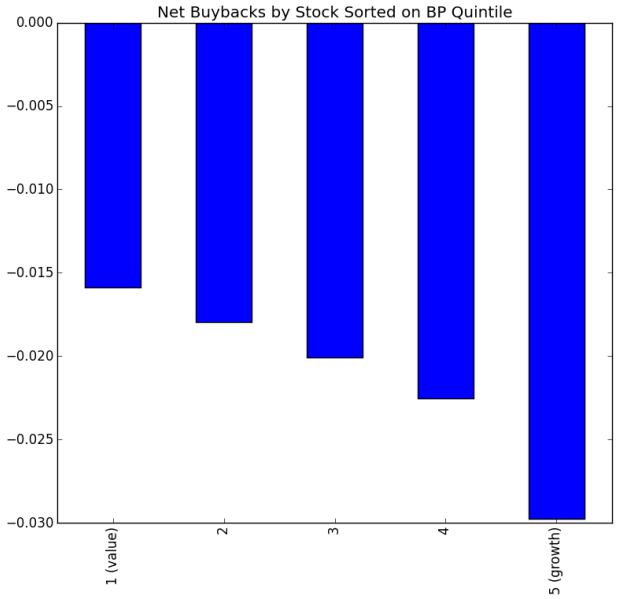

Volatility Event Study by Value Spread Quintile
0.31
0.30
0.29
0.28
0.27
0.26
0.25
0.24
0.23
0.22
−20
−10
0
10
20
1 (deep value)
2
3
4
5 (narrow spread)

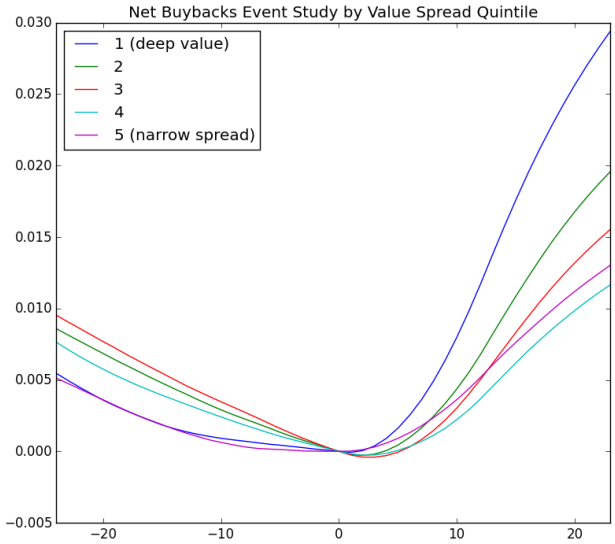
Figure 7: Value Arbitrage Activity

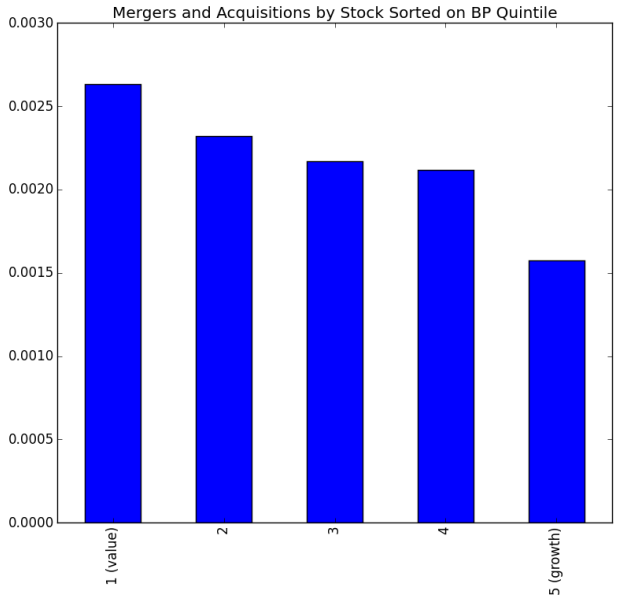
We show measures of arbitrage acitivity for stocks sorted by value quintile on the left, and event studies of measures of arbitrage activity for value portfolios having different valuation spread quintiles on the right. In the first row, we show short interest data from Compustat normalized by the number of shares outstanding. The event study is shown for the short side of the portfolio only. In the second row, we show net buybacks, as measured by the negated monthly change in shares outstanding, provided by Compustat. The bar chart, which shows the average rate of buybacks for different valuation quintiles, has negative values, consistent with issuance on average. The event study tracks cumulative buybacks for the valuation portfolio (difference between long side and short side). The third row of graphics tracks acquisitions of stocks using CRSP delisting codes (in the left graphic, we lag book-to-price ratios by 6 months before sorting in an attempt to capture pre-announcement valuations). In the event study, we show the cumulative acquisitions in the long short value portfolio (difference between long side and short side).


Short Interest by Stock Sorted on BP Quintile
0.045
0.040
0.035
0.030
0.025
0.020
0.015
0.010
0.005
0.000
1 (value)
2
3
4
5 (growth)


Short Interest Event Study by Value Spread Quintile
0.040
0.038
0.036
0.034
0.032
−20
−10
0
10
20
1 (deep value)
2
3
4
5 (narrow spread)


Net Buybacks by Stock Sorted on BP Quintile
0.000
−0.005
−0.010
−0.015
−0.020
−0.025
−0.030
1 (value)
2
3
4
5 (growth)


Net Buybacks Event Study by Value Spread Quintile
0.030
0.025
0.020
0.015
0.010
0.005
0.000
−0.005
−20
−10
0
10
20
1 (deep value)
2
3
4
5 (narrow spread)


Mergers and Acquisitions by Stock Sorted on BP Quintile
0.0030
0.0025
0.0020
0.0015
0.0010
0.0005
0.0000
1 (value)
2
3
4
5 (growth)

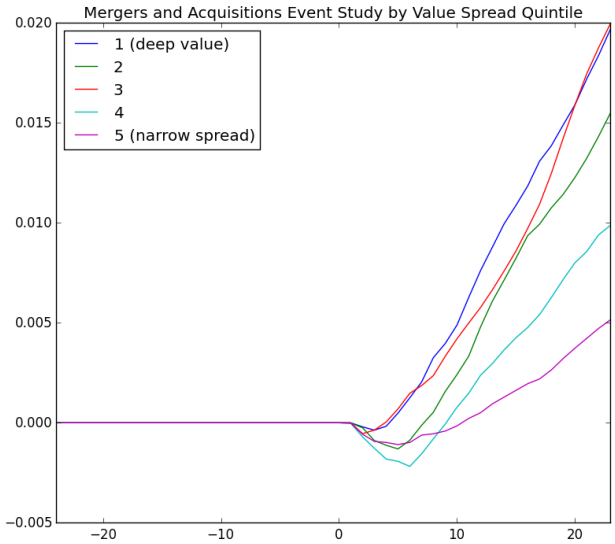

Mergers and Acquisitions Event Study by Value Spread Quintile
0.020
0.015
0.010
0.005
0.000
−0.005
−20
−10
0
10
20
1 (deep value)
2
3
4
5 (narrow spread)

Figure 8. Deep value strategy: cumulative returns and opportunity set

This graphic depicts the cumulative returns of the deep value trading strategy overlaid against the percent of available value trades included (stock selection industries and asset allocation pairs).