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JEL Classification: E31, E49, E52, E58

Keywords: forward guidance, zero lower bound, central banks, transparency, global robustness, Sobol' Indices, polynomial chaos expansion

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Versatile Forward Guidance: Escaping or Switching?*

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1 Introduction

"[...] the most logical way to make such commitment achievable and credible is by publicly stating the commitment, in a way that is sufficiently unambiguous to make it embarrassing for policymakers to simply ignore the existence of the commitment when making decisions at a later time." (Woodford, 2012)

1.1 Motivation

Forward guidance is the attempt by central banks to affect expectations about the path of inflation and output by announcing the interest rate policy they will follow in the near future. In the last decade, central banks have used forward guidance frequently when attempting to ease the zero lower bound (henceforth ZLB) constraint on interest rates. Generally, three types of forward guidance have been pursued: open-ended announcements, time-contingent announcements, and state-contingent announcements. As to the last type, the period for which the announcement is supposed to hold depends on macroeconomic conditions such as the inflation or the unemployment rate.

Experience with forward guidance reveals three insights. First, forward guidance can influence the market participants' expectations, as shown e.g. in Woodford (2012) for the Bank of Canada. Second, when announcements about future monetary policies are made in vague terms or indicate measures that are hard to pin down, the central bank can abandon such announcements rather easily, and the impact of such forward guidance may be negligible. An example of "vague" terms is the announcement made by the Bank of England in August 2013, stating that the bank rate would stay at 0.5% at least until the unemployment rate fell below 7%. Simultaneously, three criteria were set that enabled the bank to break this commitment: *a*) the consumer price index (CPI) inflation eighteen to twenty-four months ahead was, in the Monetary Policy Committee's view *"more likely than not to be"* 0.5% above the 2% inflation target, *b*) inflation expectations became poorly anchored, and *c*) the policy imposed potential threats on financial stability (Bank of England, 2013). All criteria require interpretation and thus permit discretion as to their application. In May 2014, the Bank of England continued its low bank rate policy, despite the fact that the unemployment rate had fallen below the announced threshold of 7%. Third, when central banks engage in state-contingent forward guidance with objectively measurable criteria such as the unemployment rate, they can change their commitment over time. An example is the Federal Reserve. In December 2012, it made its low policy rate dependent on the level of unemployment and set a critical threshold of 6.5% for that level. In December 2013 the Federal Reserve then rephrased its statements and announced it would keep the federal funds rate low as long as the projected inflation rate stayed below 2%, even if the unemployment rate fell below 6.5%.

The fundamental problem of forward guidance is to make credible, time-consistent announcements while retaining elbow room for reacting to new shocks. While the literature reviewed in Subsection 1.3 examines optimal forward guidance in the presence of a particular type of shock, in this paper we examine how forward guidance can be designed when the economy is hit by a sequence of different shocks, i.e. first a negative natural real interest-rate shock and then a supply shock. We compare two promising designs for forward guidance: escaping and switching.

With *escaping* forward guidance, the central banker partially commits to low future interest rates after a negative natural real interest-rate shock. At the same time, he announces a threshold inflation rate. As soon as inflation oversteps this threshold, the central banker is freed from his announcement to keep interest rates low and regains flexibility. With *switching* forward guidance, the central banker switches from interest-rate forecasts to inflation forecasts when the supply shock hits the economy. That is, he switches from a partial commitment to *interest rates* to a partial commitment to *inflation rates*.

1.2 Approach, results and implications

We examine *escaping* and *switching* forward guidance and compare them to discretionary monetary policy, to standard unconditional interest-rate forward guidance, and to each other. We perform these analyses and comparisons in the New Keynesian Framework, with negative natural real interest-rate shocks and a subsequent supply shock. Monetary policy is performed by a scrupulous central banker who faces intrinsic losses if he deviates from his own forecasts.[1]

Our main results are as follows: First, a scrupulous central banker only applies standard forward guidance with zero interest-rate forecasts in a severe downturn. Otherwise, he uses either *escaping* or *switching* forward guidance.

Second, with *escaping* forward guidance, announcing zero interest-rate forecasts in the downturn becomes attractive for any negative natural real interest-rate shock. It matches or lowers social losses at any natural real interest-rate shock level, compared to social losses under standard forward guidance or without forward guidance. The reason is that this avoids the risk of having an excessively low interest rate connected with high inflation in a subsequent major boom. The inflation threshold above which the central banker can abandon his announcement without facing costs increases with the severity of the negative natural real interest-rate shock.

Third, *switching* forward guidance offers a further prospect for decreasing social losses, and it dominates *escaping* forward guidance for medium-sized negative natural real interest-rate shocks. The reason is that in a medium range, *switching* forward guidance is better at balancing gains and costs from partially committing to low future interest rates through the switch to inflation forecasts, since such forecasts moderate inflation immediately when positive supply shocks occur. This does not work efficiently for small natural real interest-rate shocks, since the excessive inflation expectations created by the zero interest-rate forecast in downturns will not be lowered sufficiently by inflation forecasts in normal times. This leads to higher expected losses in downturns under *switching* forward guidance compared to *escaping* forward guidance. Furthermore, for large natural real interest-rate shocks, *switching* forward guidance is unable to elevate inflation expectations as strongly as *escaping* forward guidance due to the moderating effect of inflation forecasts. Under *escaping* forward guidance, the central banker simply chooses a threshold which signals that he will never escape without cost, which in our setting maximally increases expectations. To sum up, *escaping* forward guidance provides desirable levels of inflation expectation in downturns for every level of the natural real interest-rate shock, since an adequate inflation threshold will be chosen.

[1] We provide a detailed discussion of the origins of scrupulosity in Subsection 1.3.

Switching forward guidance produces the same inflation expectation in downturns for different natural real interest-rate shocks. This is appropriate only for the medium range of natural real interest-rate shocks. Compared to *escaping* forward guidance for this range of shocks, *switching* forward guidance leads to similar losses in downturns but lower losses in normal times due to the use of inflation forecasting in normal times.

Finally, we use Sobol' Indices and the polynomial chaos expansion methodology (henceforth PCE) to assess the global robustness of the results with respect to ranges of admissible parameters. This is a method borrowed from engineering (see Sudret (2008)) and goes beyond standard local sensitivity analysis. Given plausible parameter spaces it enables us to draw a more complete picture of the sensitivity of a model. In particular, PCE helps to efficiently identify those structural parameters that contribute most to the variance of the model's output, in our setup, expected social losses. The application of this method to the New Keynesian Model with a scrupulous central banker reveals that, typically, the slope of the Phillips Curve turns out to be the parameter to which social losses react the most. Most importantly, the analysis reveals that the areas for which *escaping* forward guidance and *switching* forward guidance dominate other monetary policy approaches are robust to parameter uncertainty. That is, *escaping* forward guidance remains the optimal approach for substantial or small negative natural real interest-rate shocks, while *switching* forward guidance is preferred for intermediate negative natural real interest-rate shocks under parameter uncertainty. We also show that the principal gains of applying forward guidance will materialize even for central bankers with a low degree of scrupulosity.

Our results allow for two broader conclusions. First, *escaping* forward guidance is a promising policy approach, and our results rationalize recent attempts to apply this type of forward guidance: On December 12, 2012, the Federal Reserve announced *"[...] to keep the target range for the federal funds rate at 0 to 1/4 percent and [it] currently anticipates that this exceptionally low range for the federal funds rate will be appropriate at least as long as [...] inflation between one and two years ahead is projected to be no more than a half percentage point above the Committee's 2 percent longer run goal [...]"*(Federal Reserve, 2012). Second, *switching* forward guidance tends to be superior for natural real interest-rate shocks of an intermediate size. Thus central banks might want to use the switching approach, as it enables them to switch credibly from one type of forward guidance to another.

1.3 Literature

The literature on forward guidance and the degree of scrupulosity of central bankers can be divided into three parts. First, there is a considerable body of literature on the pros and cons of forward guidance. In a recent article, Svensson (2014) finds that applying forward guidance in the form of a published policy rate path for the countries Sweden, New Zealand, and the U.S. has met with mixed success. Gersbach and Hahn (2011), Woodford (2012), and the survey by Moessner et al. (2016) provide detailed accounts both of what forward guidance can achieve and of its limitations.

Second, the potential and limitations of forward guidance have been analytically and numerically assessed for specific shock scenarios in Eggertsson and Woodford (2003), Rudebusch and Williams (2008), Gersbach and Hahn (2014), and Gersbach et al. (2015), with Boneva et al. (2015) and Florez-Jimenez and Parra-Polania (2016) focusing on threshold-based forward guidance or forward guidance with an escape clause. In our paper, we examine how forward guidance should be performed when the economy is hit by a series of different shocks. Moreover, we introduce and compare *escaping* and *switching* as promising approaches to forward guidance in such circumstances.

Third, the way in which central banks can increase the commitment power of their announcements—or equivalently, deviations from announcements generate material or immaterial costs for central bankers—has been discussed for several approaches to forward guidance. In this paper, we adopt the view that the central banker displays a certain degree of scrupulosity and will face intrinsic costs if he deviates from his previous announcements.[2] Such costs have been specified in various cases. Svensson (2009) reports from experience as a Deputy Governor at the Sveriges Riksbank: *"[...] any signal might pre-commit some members and distort the final decision [...]"*. The former Bank of England Governor Mervyn King faced such costs when handling the

[2] Other interesting approaches focus on inertia in revising plans of central bankers. Roberds (1987) ("stochastic replanning"), Schaumburg and Tambalotti (2007) ("quasi-commitment"), and Debortoli and Lakdawala (2016) ("loose commitment") introduce the concept of a central banker who revises his previously announced plans with a certain probability. The latter authors estimate the Federal Reserve's probability of fulfilling the announcement to be 80%.

Northern Rock bailout. His initial announcement not to bail out the bank and the subsequent reversal of this announcement led to public attacks.

At a deeper level, there are three potential causes for the costs of broken promises.[3] First, a central banker may have reputational concerns: breaking a promise may harm future payoffs (see Blinder (2000)). The anecdotal evidence discussed above can be seen under the heading of reputational concerns. Second, a central banker who makes a promise wants to avoid guilt after disappointing the expectations he has generated.[4] Third, the promisor may have a preference for keeping his word, no matter what expectations others have. The latter two sources of costs from breaking promises have been documented by experimental research.[5] To sum up, anecdotal, empirical, and experimental evidence supports the assumption that central bankers face intrinsic costs when they break promises in the context of forward guidance. This, in turn, creates some—albeit weak—commitment to stick to the announcements. In this paper, we assume that central bankers have some degree of scrupulosity and thus face intrinsic costs if they deviate from announcements. We also note that such costs could be actively generated by governments or central banks themselves by appointing scrupulous central bankers, as discussed in Gersbach and Hahn (2013), or with incentive pay or particular asset holdings for central banks, as discussed in Gersbach et al. (2015). In this article, we take an agnostic view on generating costs for central bankers when they deviate from announcements.

1.4 Organization of the paper

The paper is organized as follows: The model under standard forward guidance is presented in the next section. In Section 3 we investigate *escaping* forward guidance. In Section 4 we introduce *switching* forward guidance and study its welfare implications. In Section 5 we calibrate the model and provide intuition for our results. The global

[3]See Ederer and Stremitzer (2016).

[4]Ederer and Stremitzer (2016) provide a detailed account of the forces for keeping promises and experimental evidence for expectation-based promise-keeping.

[5]See Charness and Dufwenberg (2006), Vanberg (2008), Charness and Dufwenberg (2010), Ellingsen et al. (2010), and Ederer and Stremitzer (2016), for expectation-based forces for keeping promises. See Ostrom et al. (1992), Ellingsen and Johannesson (2004), Vanberg (2008), or Ismayilov and Potters (2012), for the commitment-based force.

sensitivity analysis is presented in Section 6. Finally, a discussion and the conclusion make up Section 7.

2 Model

2.1 The macroeconomic environment

We start from the standard New Keynesian Framework as described in Clarida et al. (1999). The dynamics of the economy are governed by the IS Curve and the Phillips Curve. The IS Curve is

$$x_t = \mathbb{E}_t[x_{t+1}] - \frac{1}{\sigma}(i_t - \mathbb{E}_t[\pi_{t+1}] - r_t), \tag{1}$$

where x_t is the output gap in period t, $\mathbb{E}_t[\pi_{t+1}]$ and $\mathbb{E}_t[x_{t+1}]$ are the inflation rate and the output gap in period $t+1$ expected in period t. i_t is the nominal interest rate set by the central banker, and r_t is the natural real interest rate. $\sigma > 0$ denotes the inverse inter-temporal elasticity of substitution.

The Phillips Curve is

$$\pi_t = \kappa x_t + \beta \mathbb{E}_t[\pi_{t+1}] + \xi_t, \tag{2}$$

where $\kappa > 0$ and the discount factor is $\beta \in (0,1)$. ξ_t denotes the supply shock at time t, which follows the AR(1) process

$$\xi_t = \rho \xi_{t-1} + \epsilon_t, \tag{3}$$

where $\rho \in [0,1)$ and ϵ_t represents i.i.d. disturbance with zero mean.

We consider a sequence of shocks, first a negative natural real interest-rate shock and second, upon recovery, a supply shock. The earlier shock causes a ZLB problem, as due to the constraint $i_t \geq 0$ the central bank cannot do enough to counteract this shock.[6] The later shock may cause inflation and involves standard output/inflation trade-offs.

[6] The ZLB problem has been addressed in many papers. Several articles are relevant for our purpose. Eggertsson (2003) has outlined a convenient framework for assessing the optimal dynamic linkages between policies in the downturn and upon recovery. Adam (2007) shows that the existence of the ZLB on nominal interest rates makes it beneficial to have a central banker acting under commitment rater than a discretionary central banker. Orphanides and Wieland (2000) find that monetary policy should be asymmetric and that central banks should embark on a more aggressive and expansionary path when inflation declines and when they face a ZLB problem.

More specifically, as in Eggertsson (2003) and Gersbach et al. (2015), the economy starts in a downturn (Phase D) with a negative natural real interest rate $r_t = r_D < 0$, and ends up in normal times (Phase H), where $r_t = r_H > 0$. In each period in the downturn, there is a certain probability $1 - \delta \in (0, 1)$ that the economy will extricate itself from this downturn. Once the economy reverts to normal times, the natural real interest rate will stay at r_H forever. However, the supply shock occurs after the economy has recovered.[7] The initial size of the shock is evenly distributed within the range $[-\xi, \xi]$. The sequence of events is illustrated in Figure 1.

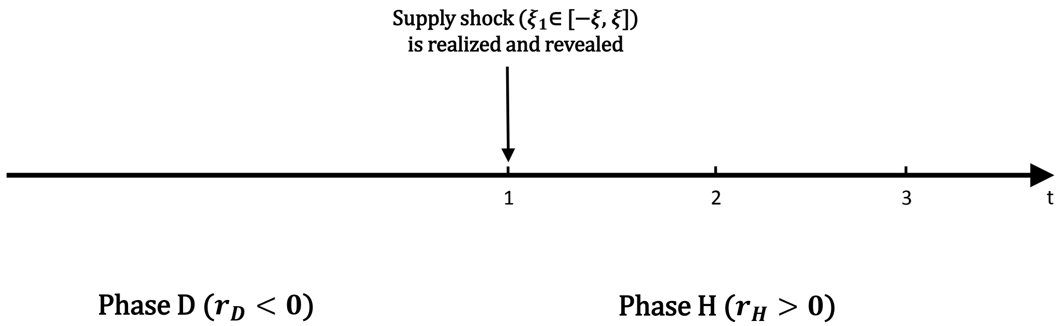

Figure 1: Sequence of events.

It is convenient to start the time index $t = 1$ with the period in which the economy enters Phase H. The stochastic return of the natural real interest rate to $r_H > 0$ during the downturn means that the situation in the downturn is identical in each period. Therefore we denote variables in a typical period in Phase D by subscript "D" without specifying for how many periods the economy has already been trapped in the downturn.

The instantaneous social loss function in period $t = D, 1, 2, ...$ is

$$l_t = \frac{1}{2}\left(\pi_t^2 + \lambda x_t^2\right), \tag{4}$$

where $\lambda > 0$ and future losses are discounted by β.

We consider a scrupulous central banker who is reluctant to deviate from the forecasts—interest rate or inflation forecast—he made in the previous period, if any. Therefore,

[7]For the sake of simplicity, we assume that the supply shock is zero in the downturn, since in the presence of a natural real interest-rate shock, the impact of the supply shock on the economy is only secondary.

apart from the social loss in Equation (4), the central banker incurs an additional intrinsic loss if he deviates from the forecast. Accordingly, the central banker's instantaneous loss function in period t is

$$\tilde{l}_t = \frac{1}{2}\left(\pi_t^2 + \lambda x_t^2\right) + \frac{1}{2} b(q_t - q_t^f)^2, \tag{5}$$

where q_t is either the interest rate, i.e. $q_t = i_t$ (correspondingly, $q_t^f = i_t^f$ is the interest-rate forecast), or the inflation rate, i.e. $q_t = \pi_t$ (correspondingly, $q_t^f = \pi_t^f$ is the inflation forecast). If no forecasts were made in the previous period, the central banker would have the same loss function as society. We will compare different types of forecasts, including the absence of forecasts, in the remainder of the paper. Parameter b measures the intrinsic costs the central banker incurs when he deviates from his forecasts, relative to the social losses. b thus stands for the central banker's degree of scrupulosity. With a larger value of b, the central banker has a higher willingness to stick to his forecast. We focus on values $b > 0$ and perform a robustness analysis for the range of values of b in $(0, 1]$. The polar case $b = 0$ stands for a central banker who acts in a purely discretionary manner in each period.

It is useful to introduce specific loss functions. The instantaneous social loss in a period in Phase D is given by

$$l_D = \frac{1}{2}(\pi_D^2 + \lambda x_D^2).$$

Once the natural real interest rate returns to r_H and the supply shock manifests itself, the latter follows the dynamic process in Equation (3), and we denote variables in Phase H by the respective time-subscript "$t = 1, 2, 3, ...$". The expected inter-temporal social losses in Phase H are denoted by an "H" subscript and are defined as

$$l_H = \frac{1}{2} \sum_{t=1}^{\infty} \beta^{t-1} \int_{-\rho^{t-1}\xi}^{\rho^{t-1}\xi} (\pi_t^2 + \lambda x_t^2) ds.$$

Since in the downturn there is a certain probability δ in each period that the economy will stay in this downturn, and the future losses are discounted by β, the expected cumulative social loss in a particular period in the downturn is

$$L = l_D + \beta(\delta l_D + (1-\delta)l_H)\sum_{t=0}^{\infty}\delta^t\beta^t \tag{6}$$

$$= l_D\sum_{t=0}^{\infty}\delta^t\beta^t + \beta(1-\delta)l_H\sum_{t=0}^{\infty}\delta^t\beta^t \tag{7}$$

$$= \frac{l_D + \beta(1-\delta)l_H}{1-\beta\delta}. \tag{8}$$

Analogously, the central banker's expected cumulative loss in a particular period in the downturn is

$$\tilde{L} = \frac{\tilde{l}_D + \beta(1-\delta)\tilde{l}_H}{1-\beta\delta}, \tag{9}$$

where[8]

$$\tilde{l}_D = \frac{1}{2}(\pi_D^2 + \lambda x_D^2 + b(q_D - q_D^f)^2),$$
$$\tilde{l}_H = \frac{1}{2}\sum_{t=1}^{\infty}\beta^{t-1}\int_{-\rho^{t-1}\xi}^{\rho^{t-1}\xi}(\pi_t^2 + \lambda x_t^2 + b(q_t - q_t^f)^2)ds.$$

In Subsection 2.2, we first consider a design where the scrupulous central banker does not make any forecast at all, either in Phase D or in Phase H. We denote variables and loss functions in this discretionary design by superscript "N" and refer to it as no forward guidance (NFG). In Subsection 2.3, we consider a scenario where in the downturn the scrupulous central banker makes interest-rate forecasts only. We denote variables in this rigid forward guidance design by superscript "F" and refer to it as interest-rate forward guidance (IFG) in the remainder of the paper. At the end of this section, we compare the two designs and establish the central banker's optimal behavior. This will then serve as a basis for examining *escaping* and *switching* forward guidance.

2.2 No forecast in the downturn

In this design the central banker does not make any forecasts. Thus, his loss function in Equation (5) coincides with the social loss function in Equation (4). We derive the

[8]Since the situation is identical in all periods of the downturn, we assume that the central banker's behavior is the same in every period of Phase D.

variables of interest by backward induction.[9] First, we consider Phase H. The central banker selects i_t optimally to minimize the loss function in Equation (4) in each period subject to the IS Curve (1) and the Phillips Curve (2). Inflation and the output gap in Phase H evolve according to

$$\pi_t^N = \frac{\lambda}{\lambda(1-\rho\beta)+\kappa^2}\xi_t, \tag{10}$$

$$x_t^N = -\frac{\kappa}{\lambda(1-\rho\beta)+\kappa^2}\xi_t. \tag{11}$$

Inserting Equations (10) and (11) into the IS Curve (1) yields[10]

$$i_t^N = r_H + \frac{\sigma\kappa(1-\rho)+\lambda\rho}{\lambda(1-\beta\rho)+\kappa^2}\xi_t. \tag{12}$$

We observe that in Phase H, inflation and the output gap merely depend on the supply shock and have opposite signs. Because of the inflationary (deflationary) pressure induced by a positive (negative) supply shock, the nominal interest rate is set above (below) the natural real interest rate r_H.

In a second step, we derive the dynamics in Phase D. Note that since the size of the supply shock in the first period of Phase H is symmetrically distributed, i.e. $\xi_1 \in [-\xi, \xi]$, expected inflation and the output gap in the downturn are

$$\mathbb{E}_D[\pi_{t+1}^N] = \delta\pi_D^N + (1-\delta)\mathbb{E}_D[\pi_1^N] = \delta\pi_D^N \tag{13}$$

and

$$\mathbb{E}_D[x_{t+1}^N] = \delta x_D^N + (1-\delta)\mathbb{E}_D[x_1^N] = \delta x_D^N. \tag{14}$$

Combining Equations (1), (2), (13), (14), and using $i_D^N = 0$ yields

$$\pi_D^N = \frac{\kappa}{h} r_D < 0 \tag{15}$$

and

$$x_D^N = \frac{1-\beta\delta}{h} r_D < 0, \tag{16}$$

where

$$h := \sigma(1-\delta)(1-\beta\delta) - \kappa\delta > 0. \tag{17}$$

That is, in the downturn, the central banker lowers the nominal interest rate to the ZLB, and the economy incurs deflation and an output collapse.

[9] The detailed derivation of the economic dynamics can be found in Appendix A.

[10] Note that the forward-looking property of the IS Curve yields $\mathbb{E}_t[\pi_{t+1}^N] = \frac{\lambda}{\lambda(1-\rho\beta)+\kappa^2}\mathbb{E}_t[\xi_{t+1}]$ and $\mathbb{E}_t[x_{t+1}^N] = -\frac{\kappa}{\lambda(1-\rho\beta)+\kappa^2}\mathbb{E}_t[\xi_{t+1}]$, where $\mathbb{E}_t[\xi_{t+1}] = \rho\xi_t$.

2.3 Interest-rate forecast in the downturn

In the presence of a negative natural real interest-rate shock, the central banker's policy tool is constrained by the ZLB of the nominal interest rate As the previous subsection has illustrated. In this subsection, we consider the situation where, in each period of the downturn, the central banker makes a zero interest-rate forecast[11] for the next period to create inflationary expectations and stops making zero interest-rate forecasts in Phase H.

Again, we use backward induction.[12] In a first step, we derive the dynamics in Phase H. In periods $t \geq 2$, the dynamics of π_t, x_t, and i_t are the same as in Equations (10), (11), and (12), since the central banker does not make any zero interest-rate forecasts in Phase H. Hence, inflation and output-gap expectations in the first period of Phase H are

$$\mathbb{E}_1[\pi_2^F] = \frac{\lambda\rho}{\lambda(1-\rho\beta)+\kappa^2}\xi_1, \tag{18}$$

$$\mathbb{E}_1[x_2^F] = -\frac{\kappa\rho}{\lambda(1-\rho\beta)+\kappa^2}\xi_1. \tag{19}$$

In $t = 1$, the central banker is still constrained by the zero interest-rate forecast made in the downturn. He thus minimizes his loss function (5) by appropriately selecting the nominal interest rate i_1^F subject to the zero interest-rate forecast $i_1^f = 0$ and the IS Curve (1) and the Phillips Curve (2). In Appendix B, we calculate the interest rate, the inflation, and the output gap in Phase H, $t = 1$ and obtain

$$i_1^F = \frac{\lambda+\kappa^2}{\lambda+\kappa^2+b\sigma^2}r_H + g_1(b)\xi_1 \leq i_1^N, \tag{20}$$

$$\pi_1^F = \frac{b\kappa\sigma}{\lambda+\kappa^2+b\sigma^2}r_H + g_2(b)\xi_1 \geq \pi_1^N, \tag{21}$$

$$x_1^F = \frac{b\sigma}{\lambda+\kappa^2+b\sigma^2}r_H + g_3(b)\xi_1 \geq x_1^N, \tag{22}$$

where $g_1(b)$, $g_2(b)$, and $g_3(b)$ are functions of b and given in Appendix B. We note that when $b = 0$, Equations (20), (21), and (22) are the same as Equations (10), (11), and (12) for $t = 1$. That is, if the central banker does not incur an intrinsic loss from

[11] Forecasting a positive interest rate would dampen the economic variables of interest, π_D and x_D, and would generate additional losses because there is a certain probability that the economy will remain in the downturn in the next period. The Federal Reserve, for instance, has adopted zero interest-rate forward guidance since 2008.

[12] The detailed derivation of the economic dynamics is given in Appendix B.

deviating from his own forecasts, he will ignore the zero interest-rate forecast and set an interest rate that minimizes the social loss function (4).

Also, we note that Equation (20) strictly decreases with b. In other words, the higher the central banker's utility loss from the forecast deviation—i.e. the larger the value of b or the higher the central banker's degree of scrupulosity—, the closer the interest rate is set to the zero forecast. This accommodative monetary policy stance leads to higher inflation and a higher output gap, i.e. $\pi_1^F \geq \pi_1^N$ and $x_1^F \geq x_1^N$.

Next, we derive the dynamics of the economy in the downturn. Expected inflation and output gap in the downturn are

$$\mathbb{E}_D[\pi_{t+1}^F] = \delta\pi_D^F + (1-\delta)\mathbb{E}_D[\pi_1^F] \tag{23}$$

and

$$\mathbb{E}_D[x_{t+1}^F] = \delta x_D^F + (1-\delta)\mathbb{E}_D[x_1^F]. \tag{24}$$

Note that due to the zero interest-rate forecast made by the scrupulous central banker, the expectations about inflation and the output gap in the initial period of Phase H, i.e. $\mathbb{E}_D[\pi_1^F]$ and $\mathbb{E}_D[x_1^F]$, are no longer zero as in Equations (13) and (14). This shows how forward guidance affects the economy both in the downturn and in normal times: Forward guidance lowers the real interest rate in a downturn at the expense of inflation and an output boom in normal times.

Using Equations (1), (2), (21), (22), (23), (24) and $\mathbb{E}_D[\xi_1] = 0$, we obtain

$$\pi_D^F = \frac{(1-\delta)b\kappa\sigma(\sigma+\kappa+\sigma\beta(1-\delta))}{(\lambda+\kappa^2+b\sigma^2)h} r_H + \frac{\kappa}{h}(r_D - i_D^F) \geq \pi_D^N, \tag{25}$$

$$x_D^F = \frac{(1-\delta)b\sigma(\sigma(1-\beta\delta)+\kappa)}{(\lambda+\kappa^2+b\sigma^2)h} r_H + \frac{1-\beta\delta}{h}(r_D - i_D^F) \geq x_D^N. \tag{26}$$

From these dynamics, we obtain the following lemma:

Lemma 1

With zero interest-rate forecasts in the downturn, the inflation and the output gap in the downturn are higher than without forecasts, and they increase with the central banker's degree of scrupulosity, i.e. π_D^F *and* x_D^F *increase with* b.

Note that when $b = 0$, Equations (25) and (26) are the same as Equations (15) and (16). A zero interest-rate forecast in such circumstances has no impact on public expectations, since it will be abandoned without cost once the economy returns to normal.

Lemma 1 shows that the high inflation and output gap expectations induced by the zero interest-rate forecast lower the real interest rate in the downturn and thus alleviate deflation and output collapse.

Using Equations (1), (2), and the zero interest-rate forecast $i_D^f = 0$, the first-order condition of the loss function (5) with respect to the nominal interest rate i_D^F yields

$$\kappa \pi_D^F + \lambda x_D^F - b\sigma i_D^F = 0. \tag{27}$$

Combining Equations (25), (26) and (27) leads to the following proposition:

Proposition 1
Under a zero interest-rate forecast in the downturn, the central banker will set the interest rate at zero in the downturn if

$$r_D \leq r_D^c, \tag{28}$$

where

$$r_D^c := -\frac{b\sigma(1-\delta)[\kappa^2(\kappa + \sigma(1+\beta(1-\delta))) + \lambda(\kappa + \sigma(1-\beta\delta))]}{(\lambda + \kappa^2 + b\sigma^2)(\kappa^2 + \lambda(1-\beta\delta))} r_H. \tag{29}$$

Otherwise, he will set the interest rate to

$$i_D^F = \frac{r_D - r_D^c}{1 + \frac{b\sigma h}{\kappa^2 + \lambda(1-\beta\delta)}} > 0. \tag{30}$$

Equation (30) implies that the central banker will set the nominal interest rate above zero in the downturn when the natural real interest rate is larger than r_D^c. The reason is that in the presence of small natural real interest-rate shock, the zero interest-rate forecast will generate excessive inflation expectations. This may even lead to inflation and an output boom in the downturn.

In the next two sections we introduce two more sophisticated designs of forward guidance and explore whether they can further improve social welfare.

3 Escaping Forward Guidance

We introduce forward guidance with a self-chosen escaping clause. In the downturn, the central banker promises to keep the interest rate at zero in the next period, as long as the inflation in that period remains below a critical threshold π^c chosen by the central banker himself.[13] The idea is to find a way for central bankers to abandon their forecasts without cost when due to the supply shock inflation is particularly high after recovery, and when stabilizing such a shock has high priority. The subtleties are twofold. First, the escape has to be chosen endogenously by the central banker, which, in turn, may encourage strategic interest-rate setting once the threshold π^c has been established.[14] Second, *escaping* forward guidance should allow inflationary pressure for a positive supply shock to be alleviated without waiving the necessity of future booms lifting output and inflation in the downturn.

We call the *escaping* forward guidance design "EFG" for short and denote variables by the superscript "E".

We use backward induction to study the dynamics of the economy under EFG and thus start in Phase H. In $t \geq 2$, the dynamics of the economy follow Equations (10), (11), and (12) since there are no interest-rate forecasts. In $t = 1$, the central banker's loss function is

$$\tilde{l}_1^E = \frac{1}{2}[(\pi_1^E)^2 + \lambda(x_1^E)^2 + b(i_1^E)^2], \tag{31}$$

if $\pi_1^E < \pi^c$, and

$$\tilde{l}_1^E = \frac{1}{2}[(\pi_1^E)^2 + \lambda(x_1^E)^2], \tag{32}$$

otherwise. If the inflation realized in the first period of Phase H is below the critical threshold π^c, the central banker will still be subject to the zero interest-rate forecast and will thus bear the additional loss $b(i_1^E)^2$. Otherwise, the central banker can discard the

[13] One could use a critical interest rate as a threshold, i.e. the central banker could discard the zero interest-rate forecast if the interest rate is above the critical interest rate he has chosen in the downturn. A critical interest rate is more easily observable and verifiable than a critical inflation rate. However, with a critical interest rate, the central banker has incentives to strategically raise the interest rate above the critical threshold to avoid deviation costs. This strategical interest rate hike induces deflation expectation in the downturn and thus worsens the situation.

[14] Note that in our setup setting a critical output gap is equivalent to setting a critical inflation rate. We take π^c as the critical threshold because inflation is easily measurable and quickly available. The output gap is often revised, sometimes long after its first publication, and it is only available on a quarterly or yearly basis. In addition, there are many uncertainties about the measurement of potential output.

zero interest-rate forecast made in the previous period without incurring any deviation costs. In other words, depending on the inflation realized in $t = 1$, there are three regimes under EFG:

- $\pi_1 < \pi^c$, the central banker behaves as under IFG, i.e. $i_1^E = i_1^F, \pi_1^E = \pi_1^F$ and $x_1^E = x_1^F$;
- $\pi_1 > \pi^c$, the central banker behaves as under NFG, i.e. $i_1^E = i_1^N, \pi_1^E = \pi_1^N$ and $x_1^E = x_1^N$;
- $\pi_1 = \pi^c$, the central banker behaves as follows:

Proposition 2

Under EFG with a self-chosen critical inflation π^c, the interest rate that just allows the central banker to escape is[15]

$$i_1^c = r_H - \frac{\sigma}{\kappa}\pi^c + \frac{\lambda(\kappa\rho + \sigma) + \sigma\kappa^2(1-\rho)}{\kappa[\lambda(1-\rho\beta) + \kappa^2]}\xi_1, \tag{33}$$

and the corresponding output gap is

$$x_1^c = \frac{\pi^c - \frac{\lambda+\kappa^2}{\lambda(1-\rho\beta)+\kappa^2}\xi_1}{\kappa}. \tag{34}$$

The formulas in Proposition 2 are obtained by using $\pi_1 = \pi^c$, Equations (1), (2), (10), and (11).

We note that i_1^c increases with the size of the supply shock and decreases with the choice of π^c. Hence, by his choice of π^c the central banker can influence how he will act when the economy enters Phase H.

The dynamics of π_D^E, x_D^E, and i_D^E in the downturn can be derived similarly to Equations (25), (26) and (30) (see Appendix C).

For a given natural real interest-rate shock, the central banker chooses the critical threshold π^c to minimize his expected losses (9). The respective loss functions $\tilde{l}_D^E$ and $\tilde{l}_H^E$ are given by

$$\tilde{l}_D^E = \frac{1}{2}[(\pi_D^E)^2 + \lambda(x_D^E)^2] \tag{35}$$

[15]We use the superscript "c" in combination with the time subscript to denote the critical values in Period 1.

and

$$\tilde{l}_H^E = \tilde{l}_1^E + \sum_{t=2}^{\infty} l_t^N. \tag{36}$$

We provide a more intuitive discussion of EFG's properties in Subsection 5.2.

4 Switching Forward Guidance

In this section, we study the alternative design *switching* forward guidance (SFG). In the downturn, the central banker makes a zero interest-rate forecast without an escaping clause.[16] Once the economy recovers, i.e. in the initial period of Phase H, the central banker switches to issuing an inflation forecast. That is, in $t = 1$, the central banker is still subject to the zero interest-rate forecast made in the downturn. At the same time, the central banker makes an inflation forecast for the next period to anchor the inflation in the current period. In other words, the central banker makes zero interest-rate forecasts in the downturn to raise inflation expectations and switches to inflation forecasts in normal times to anchor the inflation. We denote variables in this section by superscript "S". The idea of SFG is to let the economy create a short-lived inflation and output boom as soon as the economy returns to normal both to lift the economy in the downturn and also to start fighting inflation by inflation forecasts once the supply shock is seen to be pushing up inflation strongly.

We first examine the dynamics of the economy under SFG in Phase H. In $t = 1$, the central banker is still subject to the zero interest-rate forecast made in the previous period. Thus, his instantaneous loss function is

$$\tilde{l}_1^S = \frac{1}{2}[(\pi_1^S)^2 + \lambda(x_1^S)^2 + b(i_1^S)^2]. \tag{37}$$

The first-order condition with respect to i_1^S subject to the IS Curve (1) and the Phillips Curve (2) is

$$\kappa\pi_1^S + \lambda x_1^S - b\sigma i_1^S = 0. \tag{38}$$

[16]We do not intend to introduce more sophisticated designs of forward guidance, such as *switching* forward guidance with escaping clauses. The reasons are as follows: On the one hand, our intention is to study simple designs of forward guidance. On the other, sophisticated forward guidance comes at the cost of clarity, which may undermine its efficacy. Nevertheless, with the setup described in this paper, one could indeed study different combinations of *escaping* and *switching* forward guidance.

Apart from setting the nominal interest rate i_1^S, the central banker also forecasts the next period's inflation π_2^f. The inflation forecast π_2^f affects the current inflation expectation, as the precision of the inflation forecast enters the central banker's loss function in $t = 2$.

In $t \geq 2$, the central banker's loss function is

$$\tilde{l}_t^S = \frac{1}{2}[(\pi_t^S)^2 + \lambda(x_t^S)^2 + b(\pi_t^S - \pi_t^f)^2], \tag{39}$$

where π_t^f is the inflation forecast made in $t - 1$.

Thus, in each period $t \geq 2$, the central banker sets i_t^S and π_{t+1}^f to minimize

$$\mathbb{E}_t\left[\sum_{j=t}^{\infty} \beta^{j-t}\tilde{l}_j^S\right]. \tag{40}$$

Given the central banker's own policy path $\{i_t, \pi_{t+1}^f\}_{t=2}^{\infty}$, in $t = 1$, he will set the nominal interest rate i_1^S and the inflation forecast π_2^f to minimize the expected intertemporal loss function

$$\mathbb{E}_1\left[\sum_{j=1}^{\infty} \beta^{j-1}\tilde{l}_j^S\right]. \tag{41}$$

We use the algorithm presented by Söderlind (1999) to compute the solution.[17]

5 Numerical Results

We illustrate the results obtained above by calibrating the model. We follow Woodford (2003) and use the quarterly values of Table 1 for the structural parameters λ, κ, and σ. The parameter values β and ρ are taken from Gersbach and Hahn (2014). We assume the probability of staying in the downturn to be $\delta = 0.5$. The natural real interest rate is assumed to be $r_H = 0.02$ in Phase H and $r_D \in (-\infty, 0)$ in the downturn. For the supply shock, we assume a symmetric range around zero $\xi_1 \in [-0.004, 0.004]$, where the lower bound is chosen in such a way that, when the economy enters Phase H, the ZLB is no longer binding. We set the degree of scrupulosity to $b = 1$, which is a comparatively high level. With $b = 1$, the central banker is indifferent between deviating from his forecast by one percentage point and incurring one percent inflation. In Subsection 6.2 we explore the robustness of our findings regarding the desirability of different forward guidance designs for $b \in [0, 1]$.

[17]Details are provided in Appendix D.

$\beta = 0.99$	Discount factor
$\lambda = 0.003$	Weight of output gap in social loss function
$\kappa = 0.024$	Slope of Phillips Curve
$\sigma = 0.16$	Inverse inter-temporal elasticity of substitution
$r_H = 0.02$	Natural real interest rate in Phase H
$r_D \in (-\infty, 0)$	Natural real interest rate in Phase D
$\xi_1 \in [-0.004, 0.004]$	Supply shock in Phase H, $t = 1$
$\delta = 0.5$	Probability of being trapped in Phase D
$\rho = 0.9$	Persistence of supply shock
$b = 1$	Scrupulous central banker's intrinsic weight on his forecast

Table 1: Quarterly parameter values used in the calibration.

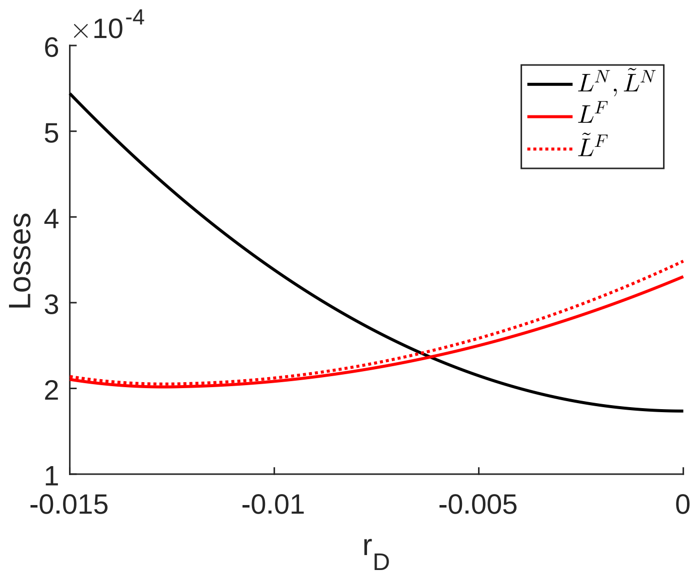

Figure 2: Expected social losses and central banker's losses.

5.1 Benchmark results

In Figure 2 we show the expected social losses and the expected central banker's losses in both NFG (black line) and IFG (red lines) as a function of the size of the natural real interest-rate shock. The expected social losses are represented by solid lines, the expected central banker's losses by the dotted line. Note that for NFG, the central banker and the society incur the same loss. For large shocks, the red dotted line is below the black line, i.e. the central banker incurs lower losses under IFG. Thus, the central banker will make zero interest-rate forecast in such circumstances. At $r_D = -0.64\%$, $\tilde{L}^N$ and $\tilde{L}^F$ intersect, i.e. the central banker is indifferent between NFG and IFG at this point. For $r_D > -0.64\%$, it is more beneficial for the central banker to abstain from forecasting. Thus, given that the central banker can decide between applying forward guidance in the downturn or abstaining from forecasting, the realized social loss is represented by the red solid line for $r_D < -0.64\%$ and jumps to the solid black line for $r_D \geq 0.64\%$.

We observe that without forecasts the expected social losses, i.e. L^N, increase with the size of the r_D shock. However, under zero interest-rate forecasts, i.e. L^F, the expected social losses first decrease, then increase with r_D. The reason is that with the zero interest-rate forecast, the inflation expectations are excessively high for small natural real interest-rate shocks. These excessively high inflation expectations lead to an output boom and push up inflation in the downturn. Thus, forecasting is costly both in the downturn and in normal times. Therefore, for small shocks it is socially desirable to abstain from forecasts in the downturn, and the central banker will not make any forecasts in such circumstances. As the size of the shock increases, high inflationary expectations induced by the zero interest-rate forecast become more and more socially beneficial, since they alleviate deflation and output decline in the downturn.

Note that the social desirability and the central banker's preference for making forecasts are not aligned, since the intersections of L^N with L^F and L^N with $\tilde{L}^F$ are different due to the additional deviation costs $b(i_t - i_t^f)^2$. The central banker starts making zero interest-rate forecasts when the shock is more severe ($r_D = -0.64\%$) than socially desirable (at $r_D = -0.62\%$, where the two solid lines intersect).

We summarize the figure in the following observation:

Numerical Finding 1

A zero interest-rate forecast in the downturn is only socially beneficial if the natural real interest-rate shock is sufficiently severe.

In the following subsections, we provide intuition about the dynamics of the inflation, the output gap, the interest rate, and the expected losses under EFG and SFG, and compare them to the benchmark case and to each other.

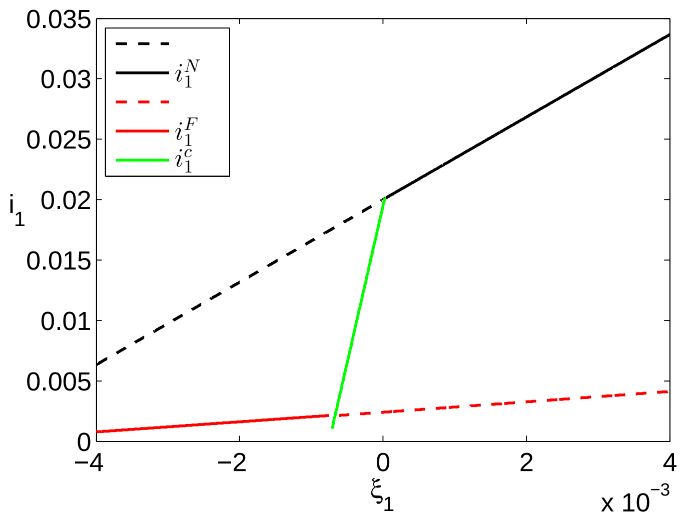

Figure 3: The interest rate in $t = 1$ under different designs.

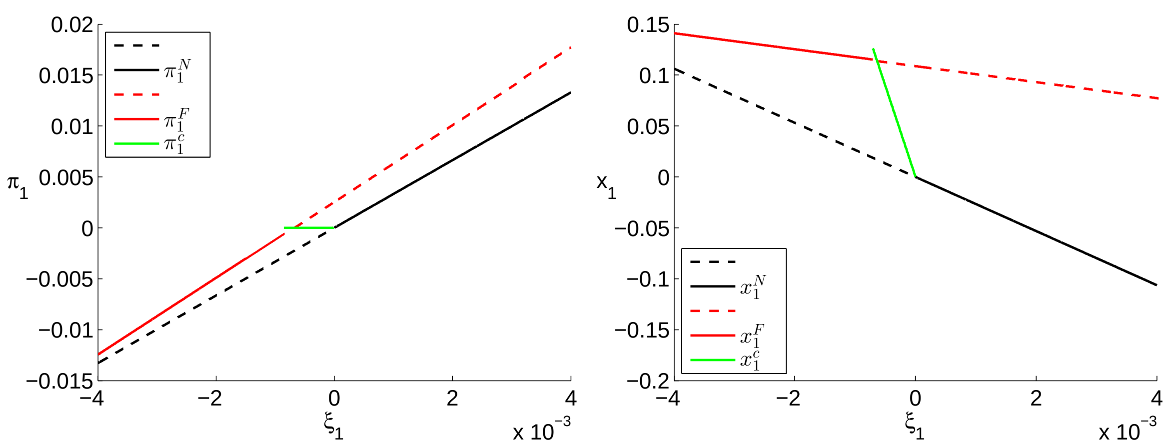

Figure 4: Inflation and the output gap in $t = 1$ under different designs.

5.2 EFG results

Figures 3 and 4 plot the variables of interest in a single period $t = 1$ in Phase H for the whole range of $\xi_1 \in [-\xi, \xi]$ with (red lines) and without (black lines) forward guidance. The interest rates i_1^N and i_1^F are plotted as functions of the supply shock in Figure 3 and confirm Inequality (20), which states $i_1^F \leq i_1^N$. Lower interest rates under IFG lead to higher inflation and a higher output gap (see Figure 4), i.e. $\pi_1^F \geq \pi_1^N$ and $x_1^F \geq x_1^N$ (see Inequalities (21) and (22)).

Furthermore, Figures 3 and 4 illustrate the working of EFG for the polar case $\pi^c = 0$. The interest rate in Phase H, $t = 1$, set by the central banker under EFG with $\pi^c = 0$, is represented by the segmented solid lines in Figure 3 and the inflation and the output gap in Figure 4. In the presence of a large negative supply shock, the central banker sets the interest rate as he would set it under IFG (solid red line). When the supply shock is large and positive, the central banker sets the interest rate discretionarily (solid black line), which leads to inflation above the critical threshold $\pi^c = 0$, as shown in Figure 4. For a supply shock of a size close to zero, the central banker sets the interest rate higher than he would set it under IFG and lower than he would set it discretionarily (see the green line in Figure 3), so that inflation is maintained at π^c (see the green line in Figure 4). Thus inflation arrives at $\pi^c = 0$ and the central banker's loss function becomes Equation (32). Note that in the illustrative example in Figure 3, for supply shocks around $\xi_1 = -0.0007$, the central banker will even find it beneficial to lower the interest rate strategically below the value of i_1^F to escape and avoid the term $b(i_1^E)^2$ in his loss function (31).

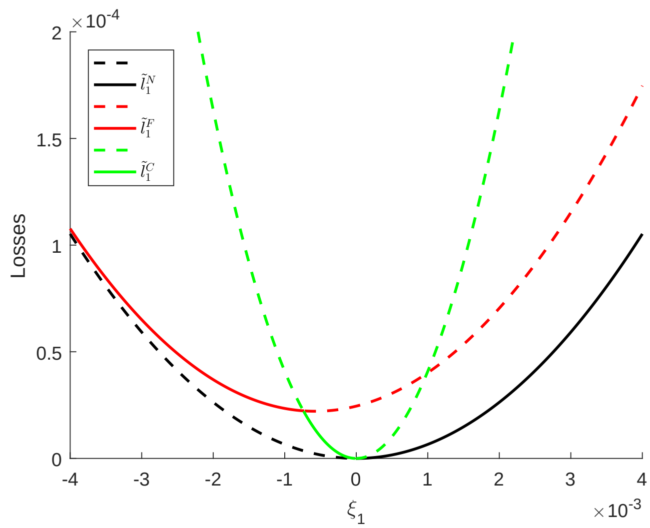

Figure 5: The central banker's losses in $t = 1$.

Figure 5 depicts the central banker's losses in $t = 1$ under designs NFG, IFG, and EFG as a function of the size of the supply shock. The solid and dashed red (black) curves represent the central banker's loss if he is (is not) subject to the zero interest-rate forecast. The solid and dashed green curves represent the central banker's loss if he keeps inflation at $\pi^c = 0$. Figure 5 displays three situations the central banker faces when he engages in EFG, indicated by the three colors of the solid lines. For a supply shock within the range $\xi_1 \in [-0.004, -0.0007)$, it is too costly for the central banker to escape, and he remains fettered by the forecast made in the downturn. From

$\xi_1 = -0.0007$ to $\xi_1 = 0$, the central banker escapes strategically by setting the interest rate at i_1^c so that the realized inflation reaches the critical threshold, i.e. $\pi_1^E = \pi^c$. In the range $\xi_1 \in [-0.004, 0)$, if the interest rate was set optimally according to a discretionary approach, the inflation rate would not reach the critical threshold (see the dashed black line in Figure 4). For all positive supply shocks, the central banker can escape his own forward guidance announcement without acting strategically. Hence, the segmented solid lines in Figure 5 depict the central banker's losses in Phase H, $t = 1$ under EFG with $\pi^c = 0$.

To sum up, under EFG with $\pi^c = 0$, the central banker faces three cases in $t = 1$, depending on the size of the supply shock:

(i) *No escape*: It is not beneficial to escape—thus we obtain $i_1^E = i_1^F$ and $\pi_1^E = \pi_1^F$.

(ii) *Strategic escape*: It is beneficial to escape by setting the interest rate strategically, just low enough to escape—i.e. $i_1^E = i_1^c$ and $\pi_1^E = \pi^c$. Hence, the central banker does not suffer a utility loss from the deviation of the zero interest-rate forecast, i.e. $b(i_1^E)^2$ falls off.

(iii) *Unconstrained escape*: Escaping by setting the interest rate as at the banker's full discretion is beneficial—$i_1^E = i_1^N$ and $\pi_1^E = \pi_1^N$.

We note that over the whole range of supply shocks, the discretionary solution dominates the other two. However, EFG has a favorable impact on the economy in the downturn. This, together with the optimal choice of π^c, will be addressed in the remainder of this subsection.

We first turn our attention to the optimal choice of π^c by the central banker under EFG. Figure 6 displays the inflation in $t = 1$ expected in the downturn. The output gap exhibits a similar pattern. With larger π^c, there is a smaller probability that the central banker can escape and set the interest rate discretionarily. That is, a higher critical threshold chosen by the central banker will lead to a higher inflation expectation in the downturn. Thus, contingent on the size of the natural real interest-rate shock, the central banker can manage the expected inflation in the downturn by choosing the critical threshold π^c in addition to making the zero interest-rate forecast.

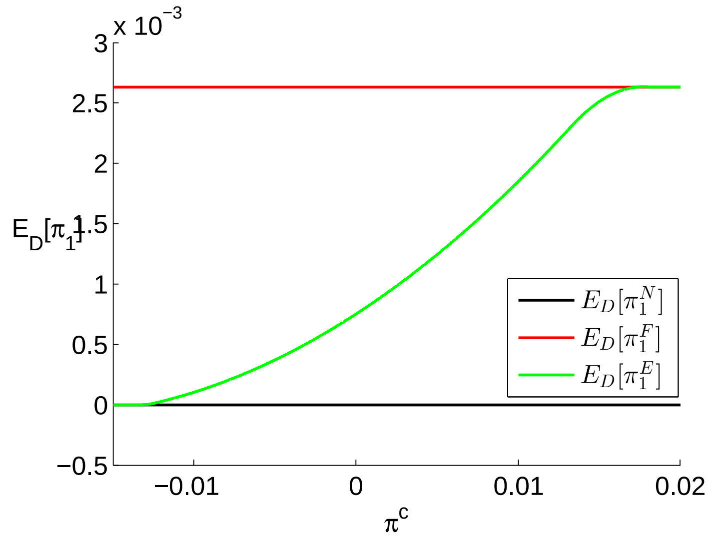

Figure 6: The inflation in $t = 1$ expected in Phase D under different designs.

Figure 6 confirms the intuition that NFG and IFG are two polar cases of EFG. That is, NFG (IFG) corresponds to EFG with very low (high) value of π^c such that the central banker will always (never) escape from the zero interest-rate forecast. Hence, the expected social losses under EFG dominate those under NFG and IFG, since the central banker can tune the desired levels of inflation expectation by setting the critical threshold π^c at proper levels. In the presence of small shocks, the central banker would set a low value of critical threshold so that when the economy enters the normal time there is a large chance that the central banker will be able to discard the zero interest-rate forecast and act discretionarily. In such circumstances this yields a lower but welcome level of inflation expectation in downturns. Vice versa for large shocks. We summarize the numerical findings as follows:

Numerical Finding 2

It is socially beneficial to make a zero interest-rate forecast with a self-chosen escaping clause in the downturn. The critical threshold chosen by the central banker increases with the size of the natural real interest-rate shock.

5.3 SFG results

Figures 7 and 8 plot the dynamics of the economy under NFG, IFG, and SFG in Phase H for two polar cases $\xi_1 = \xi$ (left panels) and $\xi_1 = -\xi$ (right panels).

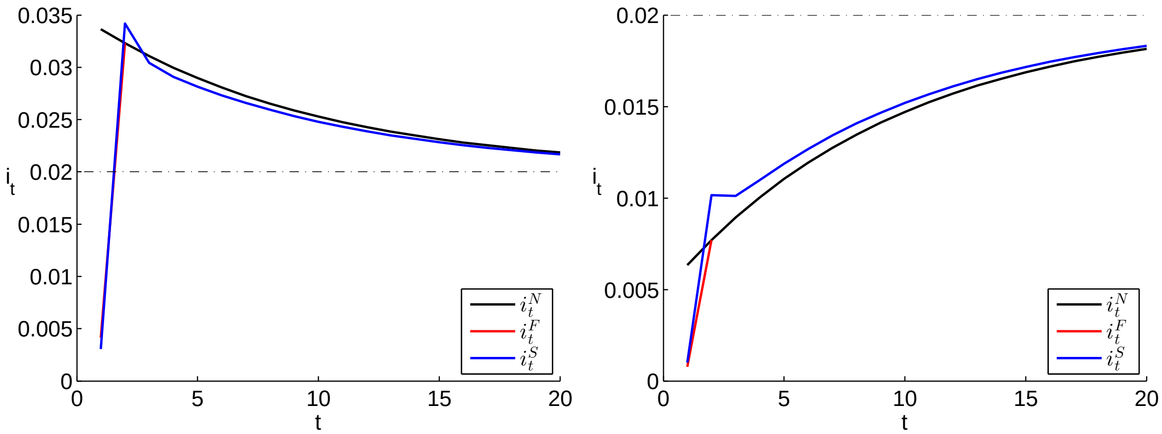

Figure 7: The dynamics of the interest rate in Phase H under different designs when $\xi_1 = \xi$ (left) and $\xi_1 = -\xi$ (right).

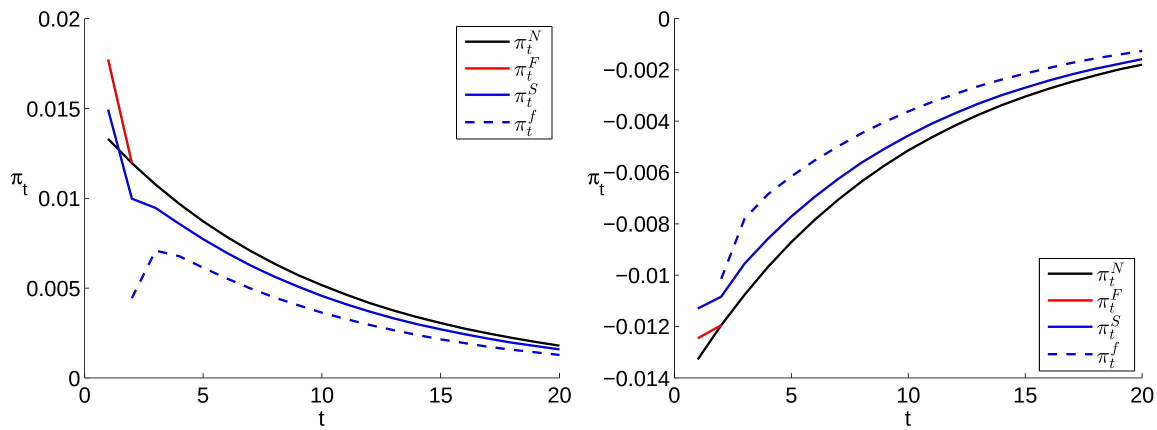

Figure 8: The dynamics of inflation in Phase H under different designs when $\xi_1 = \xi$ (left) and $\xi_1 = -\xi$ (right).

Figure 7 shows the dynamics of the interest rates i_t^N and i_t^F in Phase H when $\xi_1 = \xi$ and $\xi_1 = -\xi$. In $t = 1$, the interest rate i_1^F is set closer to zero due to the zero interest-rate forecast, which leads to a higher inflation rate (see the red line in the left panel of Figure 8). In $t \geq 2$, the interest rate is set equal to i_t^N, since no zero interest-rate forecast is made in Phase H.

Figure 8 shows that inflation π_t^N converges to zero as the supply shock dies out. The dynamics of the output gap show a similar pattern. Further, Figure 8 shows that π_t^F is higher than π_t^N in the initial period of Phase H—since $i_1^F < i_1^N$—and then the two coincide.

Under SFG, the central banker is still subject to the zero interest-rate forecast in $t = 1$. This is why he sets interest rates lower than he would set them without any forecast, i.e. $i_1^S < i_1^N$. Note that the nominal interest rate under SFG is even lower than i_1^F when $\xi_1 = \xi$. The reason is that under SFG, the central banker can forecast low inflation π_2^f to lower the inflation expectations. The intention to keep future inflation low is illustrated by the low π_2^f value at the origin of the blue dashed line in the left panel of Figure 8. Lower inflation expectations lead to lower current inflation $\pi_1^S < \pi_1^F$ and output gap $x_1^S < x_1^F$, although $i_1^S < i_1^F$. The former two inequalities suggest that the central banker can balance out the harmful effects of a low interest rate i_1^S. Together with Equation (38), the inequalities imply that the central banker will set a lower interest rate, i.e. $i_1^S < i_1^F$. In $t = 2$, due to the low inflation forecast π_2^f made in the previous period, the central banker will set a relatively high interest rate to achieve a low inflation rate π_2^S—see the peak of the blue line in the left panel in Figure 7.

In the presence of a negative supply shock, e.g. $\xi_1 = -\xi$, the central banker will set the interest rate i_1^S slightly higher than i_1^F. The inflation forecast π_2^f yields increased inflation expectations $\mathbb{E}_1[\pi_2^S] > \mathbb{E}_1[\pi_2^F]$, which leads to higher current inflation and output gap, i.e. $\pi_1^S > \pi_1^F$ and $x_1^S > x_1^F$. These two inequalities, together with Equation (38), imply $i_1^S > i_1^F$.

5.4 Loss comparison

In this subsection we compare the expected social losses under NFG, IFG, EFG, and SFG. The expected social losses under different forward guidance designs are plotted in Figure 9. Figure 9 confirms the intuition that EFG dominates NFG and IFG. One main feature of SFG is that the expected social losses in Phase H are reduced since the central banker can effectively manage the public's inflation expectations. However, the ability to manage inflation expectations weakens the power of the zero interest-rate forecast in the downturn. Thus, we have

Numerical Finding 3

Given the parameter values in Table 1, switching forward guidance is socially more beneficial than all other forward guidance designs for the natural real interest-rate range $r_D \in (-1.74\%, -0.46\%)$.

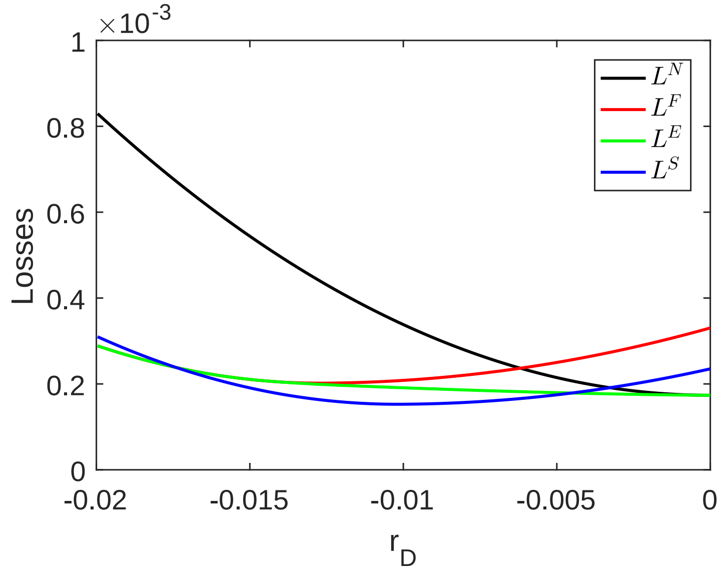

Figure 9: Expected social losses under different designs.

The intuition of this finding is as follows: Unlike EFG, the inflation expectation created by SFG in downturns is at a fixed level, as for situations under NFG and IFG (see the black and red lines in Figure 6). For small natural real interest-rate shocks, the inflation expectations under SFG are excessively high, while under EFG, the central banker can choose the inflation expectation at the desired levels. For large natural real interest-rate shocks, SFG is not as effective as EFG in raising inflation expectations or as standard forward guidance because it reduces inflation expectations in $t = 1$ through inflation forecasts. Thus, for large and small natural real interest-rate shocks, EFG is preferable to SFG. Nevertheless, when the size of the shock lies within a medium range, the inflation expectation created under SFG is at an adequate level, and the unduly large social losses caused by supply shocks in normal times can be reduced significantly by inflation forecasts. This explains why SFG is socially more beneficial in the medium range of natural real interest-rate shocks.

6 Global Sensitivity Analysis

So far, we have followed Rotemberg and Woodford (1997) and Woodford (2003) in setting the parameter values (see Table 1). We now examine whether changes in the parameter values qualitatively affect our results, and if so, which specific parameters have a strong impact on our analysis. In other words, we examine whether our findings regarding the relative desirability of EFG and SFG are robust to parameter uncertainty.

In economics, such issues are usually addressed by means of comparative statics routines—one value is varied while all others remain constant—or by a robustness check, where different parameter scenarios are considered, i.e. parameter constellations that can be either optimistic or pessimistic.[18] But, such approaches produce only local results and thus do not allow for non-linearities and interactions of the parameters in the entire parameter space. Moreover, the results also depend on the chosen combination of parameters. Therefore, these measures generally yield an incomplete picture.

Instead of using a traditional local approach, we apply a global sensitivity analysis (henceforth GSA) in the form of Sobol' Indices.[19] Sobol' Indices break down the variance of the output variable into variance contributions from each input parameter and the interactions of these input parameters. Using the Sobol' Indices, we can thus rank the parameters according to their importance. This enables us, first, to identify the parameters that contribute most to the variance of the output variable and should, therefore, receive particular attention. Second, the method enables us to determine the non-influential input parameters that can be fixed at a constant value without significantly affecting model output, i.e. the expected social losses in our model. Third, the GSA is independent of the chosen evaluation points and disentangles the direct effect of a parameter from its interactions with other parameters. Fourth, through the investigation of the interaction of the input parameters the method deepens our understanding of the drivers of model output.

In a methodological paper, Ratto (2008) first discusses Sobol' Indices as an analytical tool to analyze the properties of the structural parameters in a DSGE model. In the context of a standard RBC model, Harenberg et al. (2017) conclude that the sensitivity measures typically used in economics can be highly misleading and that the findings

[18]In the monetary policy literature, Giannoni (2007), for example, tackles the problem of uncertainty about structural parameters in such a *pessimistic*-scenario-based approach. Giannoni specifies a vector of structural parameters which lie in a given compact set. The structural parameter values are the ones that maximize social losses, given the central bank's optimal behavior. Then, subject to this worst-case parameter constellation the central bank sets the nominal interest rate to minimize social losses.

[19]For a brief introduction to Sobol' Indices, see Appendix E.1.

in the respective analysis vary substantially, depending on the values chosen for the analysis. They advocate the application of a GSA in the form of Sobol' Indices in combination with PCE.

Sobol' Indices are calculated by Monte Carlo Simulations with sample draws from underlying input parameter distributions. At each draw, the model is evaluated, which makes the analysis extremely power- and time-consuming due to the slow convergence properties of the Monte Carlo Method. Hence the main drawback is the cost of analysis if the model is expensive to run (Saltelli et al., 2008). In the engineering sciences, computationally efficient methods have been developed that lower the computational burden significantly. We use the PCE proposed by Sudret (2008) to address this particular problem. In Appendix E.2, the PCE is introduced, and the derivation of Sobol' Indices from PCE is explained in Appendix E.3.[20]

To apply this method, we assume that the four standard New Keynesian structural parameters, β, λ, κ, and σ, are uniformly[21] distributed between the smallest and the largest values found in the literature.[22] Table 2 summarizes the distributional assumptions.

$\beta \sim \mathcal{U}(0.99, 0.995)$
$\lambda \sim \mathcal{U}(0.003, 0.007)$
$\kappa \sim \mathcal{U}(0.024, 0.057)$
$\sigma \sim \mathcal{U}(0.16, 0.26)$

Table 2: Parameter ranges in the GSA.

6.1 GSA results

We investigate how much the parameters listed in Table 2 and their interactions contribute to the variances of the output variables in the four designs we addressed in

[20] We use the Matlab-based software UQLab for the calculations. The software is available on http://www.uqlab.com and Marelli et al. (2017) provide a user manual and introduction to the methods.

[21] *"[...] we express complete ignorance by assigning a uniform prior probability density [...]"* (Jaynes, 2003, p. 377). This distributional assumption is consistent with the maximum entropy principle. A compact overview of maximum entropy distributions is given in Park and Bera (2009).

[22] Core papers that provide parameter estimates are Rotemberg and Woodford (1997), Woodford (2003), Amato and Laubach (2003), Fuhrer (2006), and Adam (2007). Estimates from Rotemberg and Woodford (1997) are heavily used in the literature.

Sections 2 through 4, i.e. we derive Sobol' Indices calculated with PCE of the expected social losses $\mathbb{E}[L^p]$, where $p \in \{N, F, E, S\}$ and perform a GSA in the form of Sobol' Indices for a given size, $r_D = -0.01$, of the natural real interest-rate shock. This analysis provides the main insights on the parameters that drive the output. In Section 6.4, we then consider the robustness of the welfare analysis for the range of natural real interest-rate shocks $r_D \in [-0.02, 0]$.

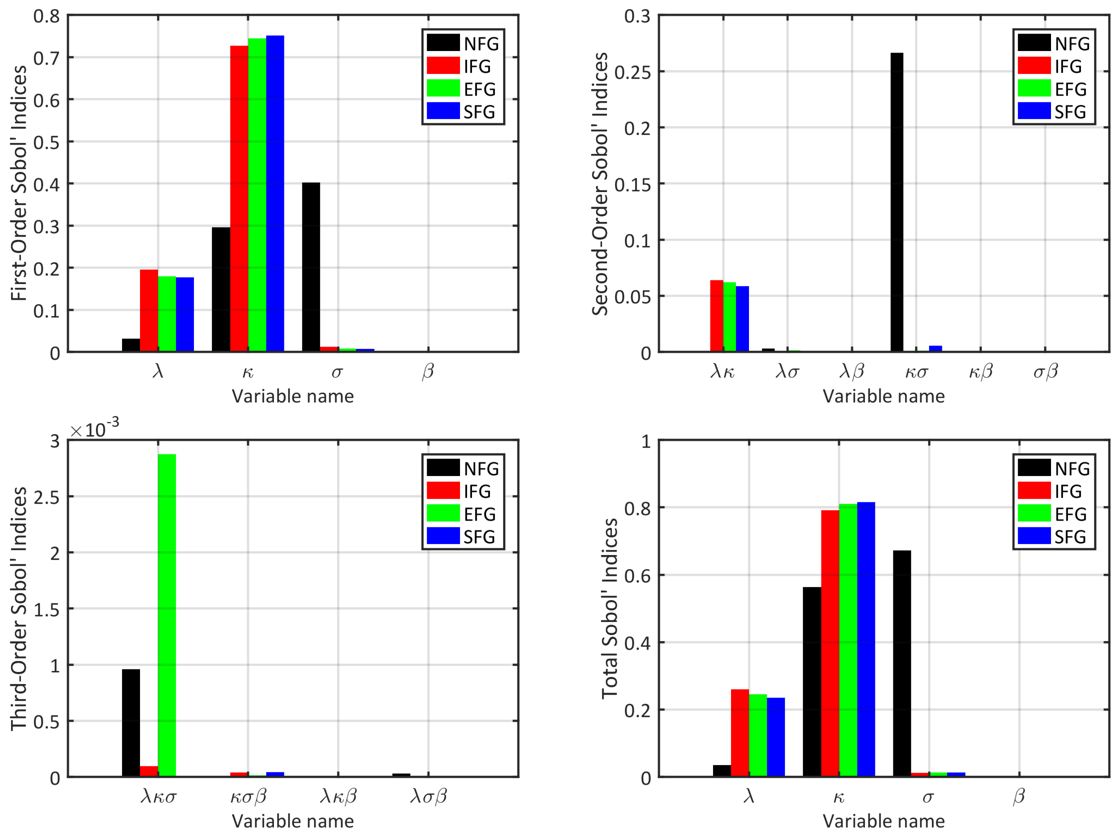

Figure 10: PCE-based Sobol' Indices.

In Figure 10, we present the result of the GSA for the different designs.[23] To obtain intuition on how to interpret the GSA results, we start by describing the GSA results on NFG in more detail. Figure 10 shows the first-order, second-order, third-order, and total Sobol' Indices. The magnitudes of the third-order Sobol' Indices are 10^{-3} and smaller, and thus negligible in their contribution to the output variation. We abbreviate the total Sobol' Indices to S_i^{tot}, where i indicates the parameter we are referring to, and all other indices to S_i^j, where j denotes the order. We note that S_σ^1 and S_κ^1—which

[23]An overview of the PCE specifications and the respective accuracy is provided in Appendix E.2.3.

can be interpreted as the linear, additive effect of each single parameter—contribute most to output variability under NFG. 40% of total variability can be attributed to σ and 30% to κ. The interaction term of κ and σ contributes 27% and is hence relatively important. $S^2_{\kappa\sigma}$ is the joint effect of κ and σ, and implies that the underlying model is non-additive and non-linear in the input parameters. Intuitively, two parameters interact when their effect on the output variable cannot be expressed as a sum of their single effects (Saltelli et al., 2008). Accordingly, the total impact of σ ($S^{tot}_{\sigma} = 67\%$) dominates, followed by κ ($S^{tot}_{\kappa} = 56\%$), λ ($S^{tot}_{\lambda} = 2\%$), and β ($S^{tot}_{\beta} = 0\%$). Therefore, since β does not visibly contribute to total variability, it can be set at a constant value within its range without influencing the output variation in any significant way. κ and σ, on the other hand, greatly affect output variation. Thus it is important to calibrate these two parameters accurately.

Among the different forward guidance designs we obtain qualitatively the same results. The most striking difference compared to the discretionary central banker is that the inverse inter-temporal elasticity of substitution, σ, does not play a significant role in contributing to the output variance for IFG, EFG, and SFG. The weight of the output gap, λ, and the slope of the Phillips Curve, κ, become more important. For all designs, the first-, and second-order indices are the most important ones. Higher-order interactions make no significant difference.

In Table 3 we summarize the total Sobol' Indices for all designs NFG, IFG, EFG, and SFG. Our main insight is that in all cases β is an unimportant parameter, with a total Sobol' Index below 1%. This means we can fix β at a constant value within the parameter range. λ's importance varies from being close to negligible in NFG to being the second most important parameter in IFG, EFG, and SFG. The importance of σ depends on the design. κ is either the most important or the second most important parameter.[24]

[24]The importance ranking of the parameters depends on the size of r_D. When the natural real interest-rate shock becomes very severe, the ranking of the S^{tot}-indices for all designs converges to $\kappa(\approx 64\%) > \sigma(\approx 59\%) > \lambda(\approx 1\%) > \beta(\approx 0\%)$. When r_D approaches zero, the ranking and the size of the total indices become more heterogeneous. In general, we can state that for designs NFG, EFG, and SFG, the ranking is $\kappa > \lambda > \sigma > \beta$, where contributions of σ and β to the output variation are negligible. For IFG, the ranking is $\kappa > \sigma > \lambda > \beta$. Here only β's contribution is negligible.

Total Sobol' Indices				
S_N^{tot}	σ	κ	λ	β
	(0.67)	(0.56)	(0.04)	(0)
S_F^{tot}	κ	λ	σ	β
	(0.79)	(0.26)	(0.01)	(0)
S_E^{tot}	κ	λ	σ	β
	(0.81)	(0.25)	(0.01)	(0)
S_S^{tot}	κ	λ	σ	β
	(0.81)	(0.24)	(0.01)	(0)

Table 3: Total Sobol' Indices for $\mathbb{E}[L^N]$, $\mathbb{E}[L^F]$, $\mathbb{E}[L^E]$, and $\mathbb{E}[L^S]$ for $r_D = -0.01$.

6.2 Univariate effects

Apart from enabling us to break down the contribution of each parameter to the variation of the output variable, PCE also enables us to study the univariate effects of each input parameter. Specifically, we analyze *(a)* whether the effect of changing a parameter on the output variable is positive or negative, *(b)* whether the relationship is linear or non-linear, and *(c)* the regions of the parameter space in which the output sensitivity is most pronounced. We follow Harenberg et al. (2017) and Deman et al. (2016) and use conditional expectations to investigate the univariate effects. Formally, we use

$$\mathcal{M}_i(x_i) = \mathbb{E}[\mathcal{M}(\mathbf{X}|X_i = x_i)] - \mathbb{E}[\mathcal{M}(\mathbf{X})], \tag{42}$$

where $\mathcal{M}$ is the model we investigate, i.e. the loss functions L^p, and $\mathbf{X}$ is a vector of input parameters (in our case $\mathbf{X} = [\lambda, \kappa, \sigma, \beta]$).

Figure 11 shows the univariate effects of the four structural parameter values. The graphs display how a structural parameter moves the conditional mean compared to its unconditional mean level. If a parameter's line plot is flat or is small in magnitude on the y-axis relative to other parameters, then it has a negligible effect on the output variable. Notably β, for example, exerts relatively small effects. This confirms our finding that the β parameter is of minor importance. λ has a positive and linear influence indicated by the positive linear slope in Figure 11. κ (σ) has a non-linear and positive (negative) effect. The non-linearities show that the conditionally-expected social loss $\mathbb{E}[L^N|\kappa = \kappa_i]$ is very sensitive to high κ values and $\mathbb{E}[L^N|\sigma = \sigma_i]$ is very sensitive to low σ values. The steep slope of these curves indicates that a small shift in

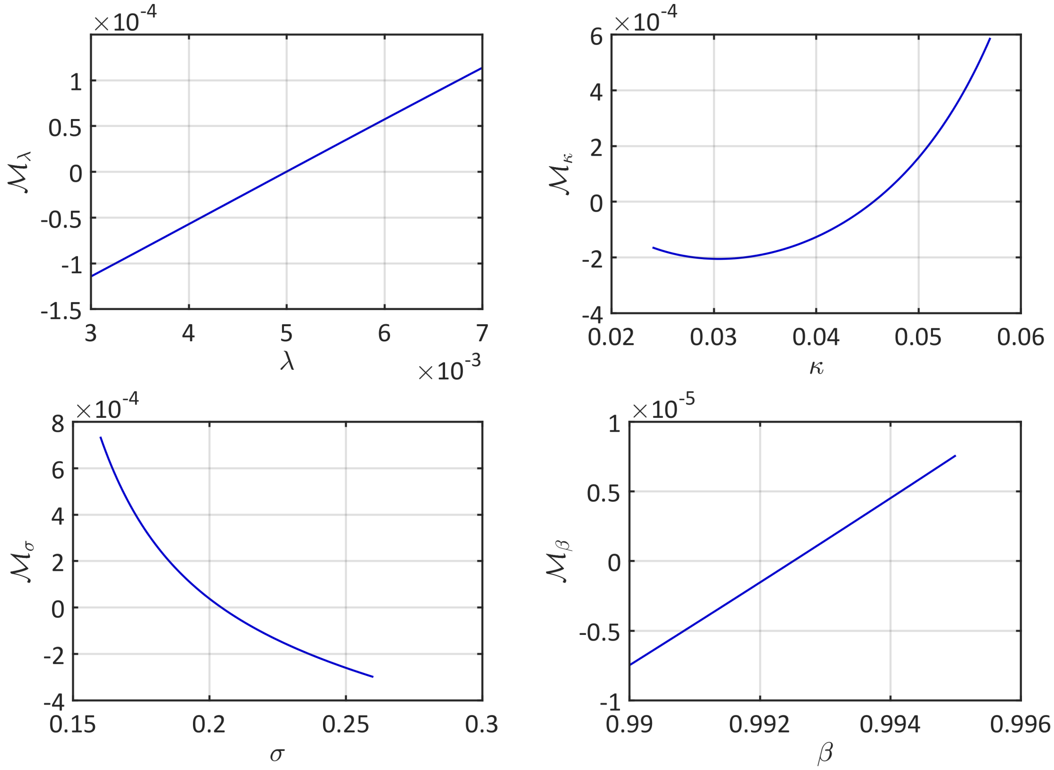

Figure 11: Univariate effects for design NFG at $r_D = -0.01$.

the parameter value will lead to large variations in the output. For designs IFG, EFG, and SFG we have similar results.[25]

We now extend the univariate analysis to parameter b. For the main part of the paper, we have assumed a scrupulous central banker with an intrinsic value of $b = 1$. Figure 12 shows the univariate effects of parameter b for designs IFG, EFG, and SFG.

For all designs, the effects of b are almost identical at $r_D = -0.01$.[26] The magnitudes are nearly identical, ranging from roughly -10^{-4} to $5 * 10^{-4}$. This implies that a particular value of b will yield the same improvement for all designs. Moreover, all functions are strictly downward-sloping in b. This means that introducing forward guidance is an improvement over NFG, represented by $b = 0$. Furthermore, the steep slope for small values of b indicates that a small degree of scrupulosity will already generate a large part of the gains achieved through forward guidance.

[25] Available upon request.

[26] These results are robust with respect to other sizes of r_D shocks.

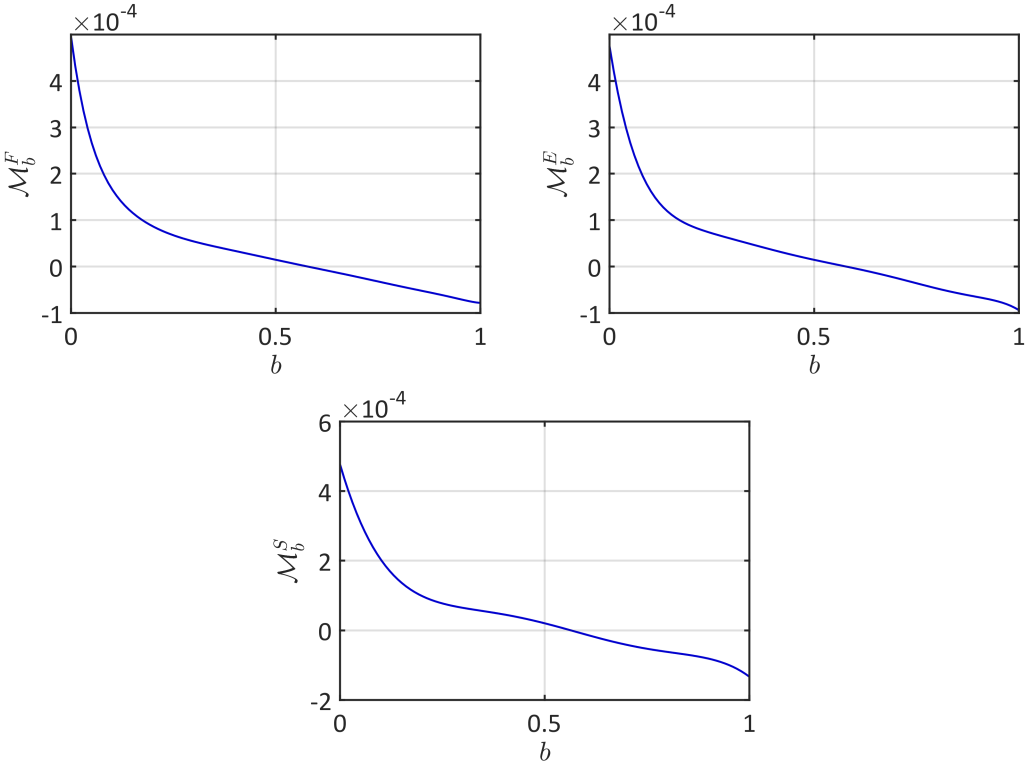

Figure 12: Robust policy analysis of parameter b, at $r_D = -0.01$.

6.3 Welfare comparison

We now apply the GSA to welfare gains. Table 4 shows the ranking of parameters that contribute to the variations of $L^N - L^F$, $L^N - L^E$, $L^N - L^S$, $L^F - L^E$, $L^F - L^S$, and $L^E - L^S$. The results in Table 4 show that σ and κ are crucial for the determination of

Total Sobol' Indices				
S^{tot}_{N-F}	κ	σ	λ	β
	(0.65)	(0.58)	(0.02)	(0)
S^{tot}_{N-E}	κ	σ	λ	β
	(0.65)	(0.56)	(0.01)	(0)
S^{tot}_{N-S}	κ	σ	λ	β
	(0.64)	(0.57)	(0.01)	(0)
S^{tot}_{F-E}	κ	σ	λ	β
	(0.85)	(0.10)	(0.09)	(0)
S^{tot}_{F-S}	κ	σ	λ	β
	(0.81)	(0.70)	(0.09)	(0.01)
S^{tot}_{E-S}	σ	κ	λ	β
	(0.82)	(0.69)	(0.65)	(0.01)

Table 4: Total Sobol' Indices for welfare comparisons at $r_D = -0.01$.

welfare gains. The importance ranking, as well as the magnitudes of the Sobol' Indices, stay approximately the same for smaller and larger values of r_D.

In Figure 13, we then plot the output of the PCE model for $100'000$ sample draws from the parameter distributions. In the header of each histogram we report the mean value of the distribution ("Mean"), the standard deviation ("Sd"), and the difference of the losses under the benchmark parameters ("BM") in Table 1. The histograms show that at $r_D = -0.01$, no parameter constellation will render discretionary monetary policy superior.[27] The red dashed line indicates where the benchmark calibration lies. The benchmark is close to the mode of the heavily right-skewed distributions in all cases.

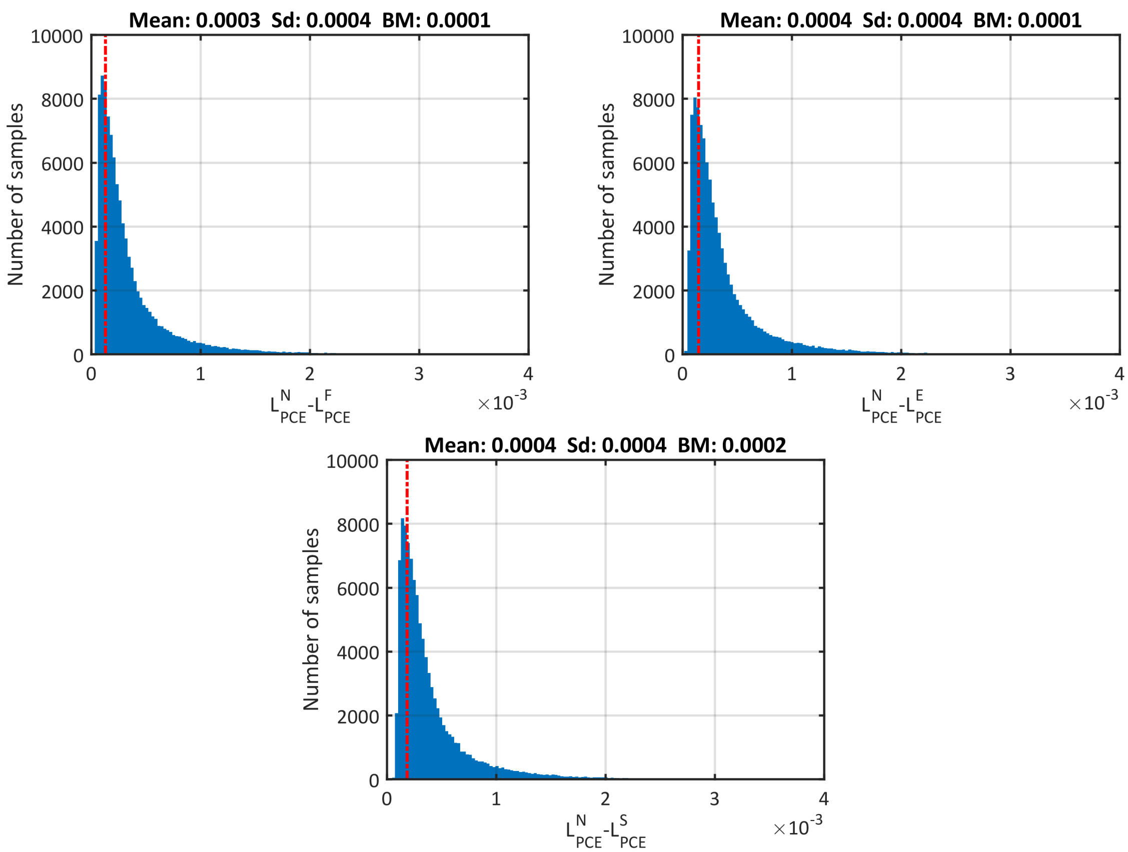

Figure 13: Distribution of PCE-based output $L^N_{PCE} - L^F_{PCE}$, $L^N_{PCE} - L^F_{PCE}$, and $L^N_{PCE} - L^F_{PCE}$ at $r_D = -0.01$.

[27] As a robustness check we plotted the histograms of EFG and SFG for $r_D = -0.5\%$ and -3%. The qualitative results stayed the same.

6.4 Robust forward guidance

In this subsection we address the most important question, i.e. whether our results regarding the social desirability of EFG and SFG are robust over a range of r_D shocks when we only know the parameter distributions in Table 2. That is, we reproduce Figure 9 under parameter uncertainty.

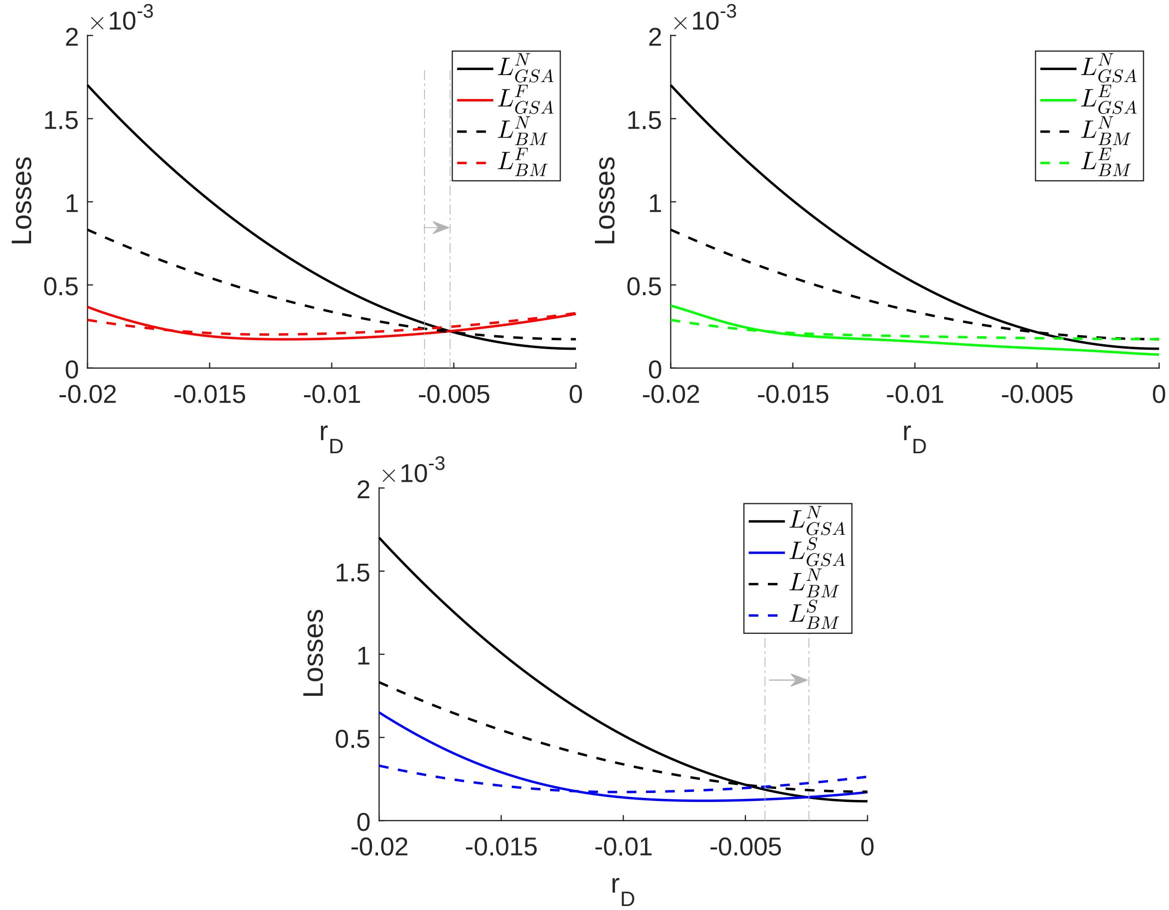

Figure 14: Conditionally-expected social losses under NFG, IFG, EFG and SFG.

Figure 14 shows the social losses, given the benchmark parameterization indicated by the subscript BM, and the conditionally-expected social losses, given the parameter distributions in Table 2 indicated by the subscript GSA. When we compare designs, not only the absolute level is important but also the point of intersection. In the upper left corner of Figure 14, we observe that the intersection point of L^N_{GSA} and L^F_{GSA}—the intersection of the two solid lines—is to the right of the intersection point of L^N_{BM} and L^F_{BM}—the intersection of the two dashed lines. The same holds for the comparison between NFG and SFG in Figure 14. Hence, by using the benchmark parameters we tend to underestimate the benefits of applying forward guidance.

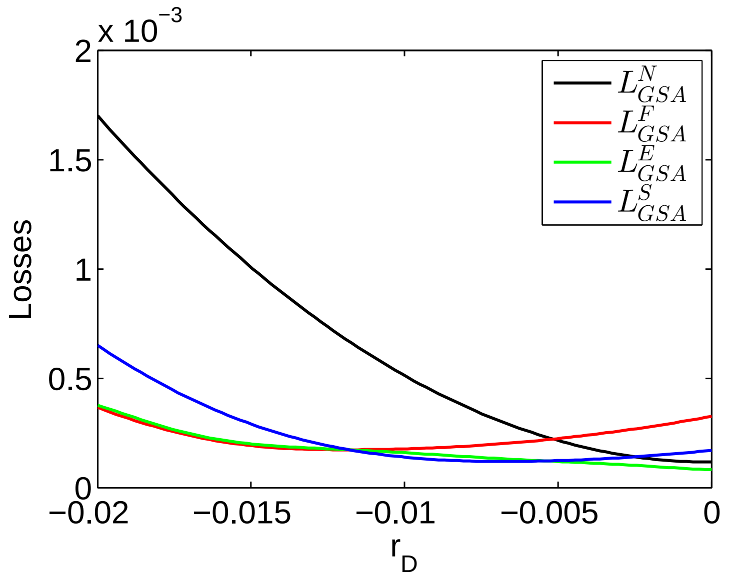

Figure 15: Conditionally-expected social losses for different designs under GSA.

Figure 15 combines the conditionally-expected social losses under NFG, IFG, EFG, and SFG, given the uncertainty calibration in Table 2. The qualitative results remain unchanged compared to the benchmark calibration in Figure 9. EFG dominates for small and large natural real interest-rate shocks, while in the intermediate range SFG yields lower expected social losses. However, the range for which SFG dominates EFG is narrower than under the benchmark calibration in Figure 9: Under the benchmark calibration, SFG dominates for $r_D \in (-1.74\%, -0.46\%)$, while under the uncertainty calibration, SFG dominates for $r_D \in (-1.17\%, -0.55\%)$.[28]

7 Conclusions

We have studied two promising forward guidance designs in the presence of sequential shocks—a natural real interest-rate shock followed by a supply shock of unknown size. We have demonstrated that *escaping* forward guidance is preferable to discretionary monetary policy and rigid forward guidance, while *switching* forward guidance further

[28]This result is robust to the introduction of ρ to the set of parameters in the GSA.

reduces welfare losses for medium-sized natural real interest-rate shocks. In this particular range, *switching* forward guidance is better at balancing the marginal benefits and costs of creating inflation expectations in the downturn and suppressing unduly high social losses (caused by the zero interest-rate forecast) in the presence of the supply shock. We have complemented the numerical evaluations with a global sensitivity analysis. It turns out that the slope parameter in the Phillips Curve (κ), the inverse inter-temporal elasticity of substitution (σ), and the weight on output gap in the loss function (λ) are crucial for calibrating the model. The discount factor (β) has proved to be of negligible importance. Moreover, we have shown that substantial benefits from applying versatile forward guidance already materialize for central bankers with small degrees of scrupulosity. Finally, the global sensitivity analysis shows that the conclusion that both *escaping* forward guidance and *switching* forward guidance are socially welfare-improving is robust to parameter uncertainty.

A No Forecast in the Downturn

We derive the variables of interest by backward induction and thus first consider Phase H. The central banker selects i_t optimally to minimize the loss function in (4) in each period subject to the IS Curve and Phillips Curve:

$$\frac{\partial l_t}{\partial i_t} = \frac{1}{2}\frac{\partial \pi_t^2}{\partial i_t} + \frac{\lambda}{2}\frac{\partial x_t^2}{\partial i_t}, \text{ s. t.} \quad \begin{aligned} x_t &= \mathbb{E}_t[x_{t+1}] - \frac{1}{\sigma}(i_t - \mathbb{E}_t[\pi_{t+1}] - r_H), \\ \pi_t &= \kappa x_t + \beta \mathbb{E}_t[\pi_{t+1}] + \xi_t. \end{aligned}$$

The first-order condition of the loss function with respect to i_t yields

$$\kappa \pi_t + \lambda x_t = 0. \tag{43}$$

Using Equation (43) to eliminate x_t in the Phillips Curve (2) yields

$$\pi_t^N = \frac{\lambda\beta}{\lambda+\kappa^2}\mathbb{E}_t[\pi_{t+1}^N] + \frac{\lambda}{\lambda+\kappa^2}\xi_t. \tag{44}$$

Inserting Equation (44) into itself recursively forms the sum

$$\begin{aligned} \pi_t^N &= \left[\frac{\lambda\beta}{\lambda+\kappa^2}\right]^2 \mathbb{E}_t[\pi_{t+2}^N] + \beta\left[\frac{\lambda}{\lambda+\kappa^2}\right]^2 \mathbb{E}_t[\xi_{t+1}] + \frac{\lambda}{\lambda+\kappa^2}\xi_t & (45) \\ &= \ldots & (46) \\ &= \left[\frac{\lambda\beta}{\lambda+\kappa^2}\right]^n \mathbb{E}_t[\pi_{t+n}^N] + \beta^{n-1}\left[\frac{\lambda}{\lambda+\kappa^2}\right]^n \mathbb{E}_t[\xi_{t+n-1}] + \\ &\quad \beta^{n-2}\left[\frac{\lambda}{\lambda+\kappa^2}\right]^{n-1} \mathbb{E}_t[\xi_{t+n-2}] + \ldots + \frac{\lambda}{\lambda+\kappa^2}\xi_t. & (47) \end{aligned}$$

After noting that $\mathbb{E}_t[\xi_{t+1}] = \rho\xi_t$ and letting $n \to \infty$, we obtain

$$\begin{aligned} \pi_t^N &= \left[1 + \rho\beta\frac{\lambda}{\lambda+\kappa^2} + \rho^2\beta^2\left(\frac{\lambda}{\lambda+\kappa^2}\right)^2 + \ldots\right]\frac{\lambda}{\lambda+\kappa^2}\xi_t & (48) \\ &= \frac{1}{1-\rho\beta\frac{\lambda}{\lambda+\kappa^2}}\frac{\lambda}{\lambda+\kappa^2}\xi_t & (49) \\ &= \frac{\lambda}{\lambda(1-\rho\beta)+\kappa^2}\xi_t. & (50) \end{aligned}$$

Substituting Equation (50) into Equation (43), we obtain

$$x_t^N = -\frac{\kappa}{\lambda(1-\rho\beta)+\kappa^2}\xi_t. \tag{51}$$

Combining Equations (50) and (51) with the IS Curve in (1), we conclude that

$$i_t^N = r_H + \frac{\sigma\kappa(1-\rho)+\lambda\rho}{\lambda(1-\beta\rho)+\kappa^2}\xi_t. \tag{52}$$

By definition, we can write $\mathbb{E}_1[\xi_t] = \rho^{t-1}\xi_1$, which results in

$$i_t^N = r_H + \frac{\sigma\kappa(1-\rho)+\lambda\rho}{\lambda(1-\beta\rho)+\kappa^2}\xi_1\rho^{t-1}, \tag{53}$$

$$\pi_t^N = \frac{\lambda\xi_1}{\lambda(1-\beta\rho)+\kappa^2}\rho^{t-1}, \tag{54}$$

$$x_t^N = -\frac{\kappa\xi_1}{\lambda(1-\beta\rho)+\kappa^2}\rho^{t-1}. \tag{55}$$

In a second step, we derive the dynamics in Phase D. Note that if the economy is in downturn in two subsequent periods, the central banker will face the same optimization problem in both instances. Therefore we drop the time subscript and mark variables in downturn by a D subscript. We combine the IS Curve (1) and the Phillips Curve (2) and resolve the forward-looking property of these equations by noting that, since $\xi_1 \in [-\xi, \xi]$ is a symmetric supply shock,

$$\mathbb{E}_D^N[\pi_{t+1}] = \delta\pi_D^N + (1-\delta)\mathbb{E}_D[\pi_1^N] = \delta\pi_D^N \tag{56}$$

and

$$\mathbb{E}_D^N[x_{t+1}] = \delta x_D^N + (1-\delta)\mathbb{E}_D[x_1^N] = \delta x_D^N. \tag{57}$$

Combining Equations (1), (2), (56), and (57) and using $\xi_D = 0$ yields

$$\pi_D^N = \frac{\kappa}{h}(r_D - i_D^N) \tag{58}$$

and

$$x_D^N = \frac{1-\beta\delta}{h}(r_D - i_D^N), \tag{59}$$

where[29]

$$h := \sigma(1-\delta)(1-\beta\delta) - \kappa\delta > 0. \tag{60}$$

As $r_D < 0$, the optimal interest rate in the downturn is constrained at the ZLB, i.e. $i_D^N = 0$.

[29]In our calibration the inequality is satisfied for $\delta < 0.68$.

B Interest Rate Forecast in the Downturn

We now turn to the case where the central banker makes a zero interest-rate forecast in Phase D. As in the case without interest-rate forecasts, we use backward induction and, in a first step, derive the dynamics in Phase H. In periods $t \geq 2$, the dynamics of i_t, π_t, and x_t are the same as in Equations (53), (54), and (55), since the central banker does not provide interest-rate forecasts in Phase H. For this reason, inflation and output gap expectations in the first period of Phase H are

$$\mathbb{E}_1[\pi_2^F] = \mathbb{E}_1[\pi_2^N] = \frac{\lambda\rho}{\lambda(1-\rho\beta)+\kappa^2}\xi_1, \tag{61}$$

$$\mathbb{E}_1[x_2^F] = \mathbb{E}_1[x_2^N] = -\frac{\kappa\rho}{\lambda(1-\rho\beta)+\kappa^2}\xi_1. \tag{62}$$

In $t = 1$, the central banker is still subject to the zero interest-rate forecast made in the downturn. The optimal interest rate in the first period can be attained by using the first-order condition:

$$\frac{\partial \tilde{l}_1}{\partial i_1^F} = \frac{1}{2}\frac{\partial(\pi_1^F)^2}{\partial i_1^F} + \frac{\lambda}{2}\frac{\partial(x_1^F)^2}{\partial i_1^F} + \frac{b}{2}\frac{\partial(i_1^F)^2}{\partial i_1^F}, \text{ s. t. } x_1^F = \mathbb{E}_1[x_2^N] - \frac{1}{\sigma}(i_1^F - \mathbb{E}_1[\pi_2^N] - r_H),$$
$$\pi_1^F = \kappa x_1^F + \beta\mathbb{E}_1[\pi_2^N] + \xi_1.$$

Simplifying the first-order condition yields

$$-\frac{1}{\sigma}[\kappa\pi_1^F + \lambda x_1^F - b\sigma i_1^F] = 0. \tag{63}$$

Combining Equations (1), (2), (61), (62), and (63), we obtain

$$i_1^F = \frac{\lambda+\kappa^2}{\lambda+\kappa^2+b\sigma^2}r_H + g_1(b)\xi_1, \tag{64}$$

$$\pi_1^F = \frac{b\kappa\sigma}{\lambda+\kappa^2+b\sigma^2}r_H + g_2(b)\xi_1, \tag{65}$$

$$x_1^F = \frac{b\sigma}{\lambda+\kappa^2+b\sigma^2}r_H + g_3(b)\xi_1, \tag{66}$$

where

$$g_1(b) = \frac{(\kappa^2+\lambda)[\sigma\kappa(1-\rho)+\rho\lambda]}{(\lambda+\kappa^2+b\sigma^2)(\lambda(1-\beta\rho)+\kappa^2)},$$
$$g_2(b) = \frac{b\kappa^2\sigma^2(1-\rho)+b\sigma\lambda(\sigma+\kappa\rho)+\lambda(\kappa^2+\lambda)}{(\lambda+\kappa^2+b\sigma^2)(\lambda(1-\beta\rho)+\kappa^2)},$$
$$g_3(b) = \frac{b\sigma\rho(\lambda-\sigma\kappa)-\kappa(\kappa^2+\lambda)}{(\lambda+\kappa^2+b\sigma^2)(\lambda(1-\beta\rho)+\kappa^2)}.$$

We now proceed to derive the dynamics in the downturn. The central banker minimizes his loss function in (5) subject to the IS Curve (1) and the Phillips Curve (2) by choosing i_D^F appropriately. The first-order condition has the same form as (63):

$$\frac{\partial \tilde{l}_D^F}{\partial i_D^F} = -\frac{1}{\sigma}[\kappa \pi_D^F + \lambda x_D^F - b\sigma i_D^F] = 0. \tag{67}$$

Combining Equation (67) with (1) and (2) and using the forward-looking nature of these equations with

$$\mathbb{E}_D^F[\pi_{t+1}] = \delta \pi_D^F + (1-\delta)\mathbb{E}_D[\pi_1^F] \tag{68}$$

and

$$\mathbb{E}_D^F[x_{t+1}] = \delta x_D^F + (1-\delta)\mathbb{E}_D[x_1^F], \tag{69}$$

we obtain in an intermediary step

$$\pi_D^F = \frac{\sigma\kappa(1-\delta)\mathbb{E}_D[x_1^F] + (1-\delta)(\kappa + \sigma\beta - \sigma\beta\delta)\mathbb{E}_D[\pi_1^F] + \kappa(r_D - i_D^F)}{h}, \tag{70}$$

$$x_D^F = \frac{\sigma(1-\delta)(1-\beta\delta)\mathbb{E}_D[x_1^F] + (1-\delta)\mathbb{E}_D[\pi_1^F] + (1-\beta\delta)(r_D - i_D^F)}{h}, \tag{71}$$

$$i_D^F = \frac{\kappa\pi_D^F + \lambda x_D^F}{b\sigma}. \tag{72}$$

Using Equations (65) and (66), and $\mathbb{E}_D[\xi_1] = 0$ yields

$$\pi_D^F = \frac{(1-\delta)b\kappa\sigma(\sigma + \kappa + \sigma\beta(1-\delta))}{(\lambda + \kappa^2 + b\sigma^2)h} r_H + \frac{\kappa}{h}(r_D - i_D^F), \tag{73}$$

$$x_D^F = \frac{(1-\delta)b\sigma(\sigma(1-\beta\delta) + \kappa)}{(\lambda + \kappa^2 + b\sigma^2)h} r_H + \frac{1-\beta\delta}{h}(r_D - i_D^F), \tag{74}$$

$$i_D^F = \max\Bigg\{0, \tag{75}$$
$$\frac{(1-\delta)b\sigma[(\kappa^2+\lambda)(1-\beta\delta) + \kappa^4 + \kappa^2\sigma\beta + \kappa\lambda]}{(\lambda + \kappa^2 + b\sigma^2)(b\sigma h + \kappa^2 + \lambda(1-\beta\delta))} r_H + \frac{\kappa^2 + \lambda(1-\beta\delta)}{b\sigma h + \kappa^2 + \lambda(1-\delta\beta)} r_D\Bigg\}.$$

C Escaping Forward Guidance

For *escaping* forward guidance, we first derive the inflation and the output gap in $t = 1$ that are expected in the downturn for a given value of the inflation threshold π^c.

In a first step, we define two auxiliary functions $\underline{\xi}_1$ and $\bar{\xi}_1$. $\underline{\xi}_1$ is the value of ξ_1 at which the central banker's loss functions $\tilde{l}_1^F$ and $\tilde{l}_1^C$ intersect[30]—see the intersection point of

[30] $\tilde{l}_1^C = \frac{1}{2}[(\pi_1^c)^2 + \lambda(x_1^c)^2]$ denotes the central banker's loss when inflation in $t = 1$ just reaches the critical threshold, i.e. when $i_1 = i_1^c, \pi_1 = \pi_1^c$ and $x_1 = x_1^c$.

the solid green and solid red lines in Figure 5. $\bar{\xi}_1$ is the value of ξ_1 at which the central banker's loss functions $\tilde{l}_1^N$ and $\tilde{l}_1^C$ intersect—see the intersection point of the solid green and solid black lines in Figure 2.[31] That is, if the realized supply shock in $t = 1$ is lower (higher) than $\underline{\xi}_1$ ($\bar{\xi}_1$), the central banker will set $i_1^E = i_1^F$ ($i_1^E = i_1^N$) and the realized inflation and the output gap will be $\pi_1^E = \pi_1^F$ ($\pi_1^E = \pi_1^N$), and $x_1^E = x_1^F$ ($x_1^E = x_1^N$). If $\xi_1 \in [\underline{\xi}_1, \bar{\xi}_1]$, the central banker will set $i_1^E = i_1^c$, and the inflation and the output gap will be π^c and x_1^c. Therefore we have

$$\mathbb{E}_D[\pi_1^E] = \begin{cases} \int_{-\xi}^{\xi} \frac{\pi_1^N(\xi_1)}{2\xi} d\xi_1 & \text{if } \bar{\xi}_1 < -\xi \\ \frac{(\bar{\xi}_1+\xi)\pi^c}{2\xi} + \int_{\bar{\xi}_1}^{\xi} \frac{\pi_1^N(\xi_1)}{2\xi} d\xi_1 & \text{if } \underline{\xi}_1 < -\xi \le \bar{\xi}_1 < \xi \\ \int_{-\xi}^{\underline{\xi}_1} \frac{\pi_1^F(\xi_1)}{2\xi} d\xi_1 + \frac{(\bar{\xi}_1-\underline{\xi}_1)\pi^c}{2\xi} + \int_{\bar{\xi}_1}^{\xi} \frac{\pi_1^N(\xi_1)}{2\xi} d\xi_1 & \text{if } -\xi \le \underline{\xi}_1 < \bar{\xi}_1 < \xi \\ \int_{-\xi}^{\underline{\xi}_1} \frac{\pi_1^F(\xi_1)}{2\xi} d\xi_1 + \frac{(\xi-\underline{\xi}_1)\pi^c}{2\xi} & \text{if } -\xi < \underline{\xi}_1 < \xi \le \bar{\xi}_1 \\ \int_{-\xi}^{\xi} \frac{\pi_1^F(\xi_1)}{2\xi} d\xi_1 & \text{if } \xi \le \underline{\xi}_1 \end{cases} \tag{76}$$

and

$$\mathbb{E}_D[x_1^E] = \begin{cases} \int_{-\xi}^{\xi} \frac{x_1^N(\xi_1)}{2\xi} d\xi_1 & \text{if } \bar{\xi}_1 < -\xi \\ \int_{-\xi}^{\bar{\xi}_1} \frac{x_1^c(\xi_1)}{2\xi} d\xi_1 + \int_{\bar{\xi}_1}^{\xi} \frac{x_1^N(\xi_1)}{2\xi} d\xi_1 & \text{if } \underline{\xi}_1 < -\xi \le \bar{\xi}_1 < \xi \\ \int_{-\xi}^{\underline{\xi}_1} \frac{x_1^F(\xi_1)}{2\xi} d\xi_1 + \int_{\underline{\xi}_1}^{\bar{\xi}_1} \frac{x_1^c(\xi_1)}{2\xi} d\xi_1 + \int_{\bar{\xi}_1}^{\xi} \frac{x_1^N(\xi_1)}{2\xi} d\xi_1 & \text{if } -\xi \le \underline{\xi}_1 < \bar{\xi}_1 < \xi \\ \int_{-\xi}^{\underline{\xi}_1} \frac{x_1^F(\xi_1)}{2\xi} d\xi_1 + \int_{\underline{\xi}_1}^{\xi} \frac{x_1^c(\xi_1)}{2\xi} d\xi_1 & \text{if } -\xi < \underline{\xi}_1 < \xi \le \bar{\xi}_1 \\ \int_{-\xi}^{\xi} \frac{x_1^F(\xi_1)}{2\xi} d\xi_1 & \text{if } \xi \le \underline{\xi}_1. \end{cases} \tag{77}$$

Note that, as in Equations (23) and (24), we have

$$\mathbb{E}_D[\pi_{t+1}^E] = \delta\pi_D^E + (1-\delta)\mathbb{E}_D[\pi_1^E] \tag{78}$$

and

$$\mathbb{E}_D[x_{t+1}^E] = \delta x_D^E + (1-\delta)\mathbb{E}_D[x_1^E]. \tag{79}$$

Hence, using Equations (1), (2), (78), (79), and (27)—which also applies to EFG,—we obtain the inflation, the output gap, and the interest rate in the downturn

$$\pi_D^E = \frac{\sigma\kappa(1-\delta)\mathbb{E}_D[x_1^E] + (1-\delta)(\kappa+\sigma\beta-\sigma\beta\delta)\mathbb{E}_D[\pi_1^E] + \kappa(r_D - i_D^E)}{h}, \tag{80}$$

$$x_D^E = \frac{\sigma(1-\delta)(1-\beta\delta)\mathbb{E}_D[x_1^E] + (1-\delta)\mathbb{E}_D[\pi_1^E] + (1-\beta\delta)(r_D - i_D^E)}{h}, \tag{81}$$

$$i_D^E = \max\{0, \hat{i}_D^E\}, \tag{82}$$

[31]Note that $\underline{\xi}_1$ and $\bar{\xi}_1$ depend on the choice of π^c.

where

$$\hat{i}_D^E = \frac{(1-\delta)\sigma(\kappa^2+\lambda(1-\beta\delta))\mathbb{E}_D[x_1^E] + (1-\delta)(\kappa(\kappa+\sigma\beta-\sigma\beta\delta)+\lambda)\mathbb{E}_D[\pi_1^E] + (\kappa^2+\lambda(1-\beta\delta))r_D}{b\sigma h+\kappa^2+\lambda(1-\beta\delta)}.$$

D Switching Forward Guidance

We now show how to derive the dynamics of the economy in the downturn and Phase H under SFG. As before, we proceed by backward induction. We first derive the dynamics of the economy in $t \geq 2$ for each given ξ_2 and π_2^f. Given the dynamics of the economy in $t \geq 2$, we can then derive the central banker's optimal inflation forecast π_2^f in $t = 1$ for each realized ξ_1. In the last step, with the inflation and the output gap in $t = 1$ for each supply shock, we can derive the inflation, the output gap, and the interest rate in the downturn.

Using the notation of Söderlind (1999), we have $y_t := (\xi_t, \pi_t^f, r_H, \pi_t, x_t)'$, where $y_{1,t} := (\xi_t, \pi_t^f, r_H)'$ are predetermined and $y_{2,t} := (\pi_t, x_t)'$ are non-predetermined entries. The vector of policy instruments is $u_t := (i_t, \pi_{t+1}^f)'$.

In our model, the dynamics of y_t for $t \geq 2$ are given by

$$\begin{pmatrix} y_{1,t+1} \\ \mathbb{E}_t[y_{2,t+1}] \end{pmatrix} = A \begin{pmatrix} y_{1,t} \\ y_{2,t} \end{pmatrix} + Bu_t, \tag{83}$$

where

$$A := \begin{pmatrix} \rho & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ -\frac{1}{\beta} & 0 & 0 & \frac{1}{\beta} & -\frac{\kappa}{\beta} \\ \frac{1}{\beta\sigma} & 0 & -\frac{1}{\sigma} & -\frac{1}{\beta\sigma} & 1+\frac{\kappa}{\beta\sigma} \end{pmatrix}, B := \begin{pmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{\sigma} & 0 \end{pmatrix}. \tag{84}$$

The central banker's loss function

$$\tilde{l}_t^S = \frac{1}{2}[(\pi_t^S)^2 + \lambda(x_t^S)^2 + b(\pi_t^S - \pi_t^f)^2] \tag{85}$$

can be written in the form used by Söderlind:

$$\tilde{l}_t^S = y_t' Q y_t + 2y_t' U u_t + u_t' R u_t, \tag{86}$$

with

$$Q := \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{b}{2} & 0 & \frac{-b}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-b}{2} & 0 & \frac{1+b}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{\lambda}{2} \end{pmatrix}, U := \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, R := \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}. \tag{87}$$

As in Söderlind (1999), the central banker's optimization problem can then be formulated in a Bellman Equation with the value function $v(y_t) = y'_{1,t}V_t y_{1,t} + v_t$. The value function incorporates the predetermined variables $y_{1,t}$, the matrix V_t, which is assumed to be independent of exogenous shocks, and the term v_t that includes shocks:

$$v(y_t) = \min_{u_t}\Big\{y'_t Q y_t + 2y'_t U u_t + u'_t R u_t + \beta\mathbb{E}_t[v(y_{t+1})]\Big\}, \tag{88}$$

s.t. $\mathbb{E}_t[y_{2,t+1}] = C_{t+1}\mathbb{E}_t[y_{1,t+1}]$, (83), and given $y_{1,t}$, where C_{t+1} is a 2×3-matrix.

We use the Matlab algorithm provided by Söderlind (1999) to recursively solve this Bellman Equation. Thus, for each given ξ_2 and π_2^f, we obtain the dynamics of the economy for $t \geq 2$. Using Equations (1), (2) and (38), we obtain the dynamics in $t = 1$:

$$x_1^S = \frac{b\sigma^2\mathbb{E}_1[x_2^S] - (\kappa\beta - b\sigma)\mathbb{E}_1[\pi_2^S] - \kappa\xi_1 + b\sigma r_H}{b\sigma^2 + \kappa^2 + \lambda}, \tag{89}$$

$$\pi_1^S = \frac{(\beta(b\sigma^2 + \lambda) + \kappa\sigma b)\mathbb{E}_1[\pi_2^S] + \kappa\sigma^2 b\mathbb{E}_1[x_2^S] + (b\sigma^2 + \lambda)\xi_1 + \kappa\sigma b r_H}{b\sigma^2 + \kappa^2 + \lambda}, \tag{90}$$

$$i_1^S = \frac{(\kappa^2 + \beta\kappa\sigma + \lambda)\mathbb{E}_1[\pi_2^S] + \sigma(\kappa^2 + \lambda)\mathbb{E}_1[x_2^S] + \sigma\kappa\xi_1 + (\kappa^2 + \lambda)r_H}{b\sigma^2 + \kappa^2 + \lambda}. \tag{91}$$

Therefore, the dynamics of the economy for a given π_2^f in normal times are determined, and the central banker chooses the inflation forecast π_2^f so that the loss function in (41) is minimized. Similar to Equations (80)–(82), we could obtain the inflation, output gap, and interest rate under SFG in the downturn.

E Sobol' Indices and Polynomial Chaos Expansion

Sobol' Indices belong to the class of analysis of variance techniques (ANOVA) that aim to decompose the variance of the output variable—in our case, the expected social losses L^p with $p \in \{N, F, E, S\}$—into a sum of variances of each input variable and the variances of interaction terms of these input variables. Sobol' Indices allow to identify to what extent each input parameter and the interaction of the input parameters contribute to the variation of the output variable. Sobol' Indices are calculated by using Monte Carlo Simulation. This turns out to be a drawback if the underlying model requires a lot of computational effort for the calculation of these indices. For this reason, we use the so-called polynomial chaos expansion (PCE) method. We outline the concept of Sobol' Indices in Section E.1. The theoretical foundation behind PCE and its connection to Sobol' Indices is provided by Sudret (2008) and will be addressed in Sections E.2 and E.3. For more detailed information about this global sensitivity approach the reader is referred to Le Gratiet et al. (2016).

E.1 Sobol' Indices

Consider a computational model $\mathcal{M} : \mathbf{x} \in \mathcal{D}_{\mathbf{X}} \subset \mathbb{R}^d \mapsto Y = \mathcal{M}(\mathbf{x}) \in \mathbb{R}$, where $\mathbf{x}$ is a d-dimensional vector containing the input parameters. Denote by $w(\mathbf{x})$ the density function of the input $\mathbf{x}$. The Sobol' decomposition then reads

$$\begin{aligned}\mathcal{M}(\mathbf{x}) =& \mathcal{M}_0 + \sum_{i=1}^{d} \mathcal{M}_i(x_i) + \sum_{1 \leq i < j \leq d} \mathcal{M}_{ij}(x_i, x_j) + ... \\ &+ \sum_{1 \leq i_1 < ... < i_s \leq d} \mathcal{M}_{i_1 ... i_s}(x_{i_1}, ..., x_{i_s}) + ... + \mathcal{M}_{12 \cdots d}(\mathbf{x}),\end{aligned} \tag{92}$$

where we have the mean of the function

$$\mathcal{M}_0 = \int_{\mathcal{D}_{\mathbf{X}}} \mathcal{M}(\mathbf{x}) w(\mathbf{x}) \mathbf{dx} = \mathbb{E}[\mathcal{M}(\mathbf{X})],$$

the univariate functions[32]

$$\mathcal{M}_i(x_i) = \int_{\mathcal{D}_{\mathbf{X}\sim i}} \mathcal{M}(\mathbf{x}) w(\mathbf{x}) d\mathbf{x} - \mathcal{M}_0 = \mathbb{E}[\mathcal{M}(\mathbf{X}) | X_i = x_i] - \mathcal{M}_0,$$

the bivariate functions

$$\begin{aligned}\mathcal{M}_{ij}(x_i, x_j) &= \int_{\mathcal{D}_{\mathbf{X}\sim ij}} \mathcal{M}(\mathbf{x}) w(\mathbf{x}) d\mathbf{x} - \mathcal{M}_i(x_i) - \mathcal{M}_j(x_j) - \mathcal{M}_0 \\ &= \mathbb{E}[\mathcal{M}(\mathbf{X}) | X_i, X_j = x_i, x_j] - \mathcal{M}_i(x_i) - \mathcal{M}_j(x_j) - \mathcal{M}_0,\end{aligned}$$

and so on.

[32]We denote by X_i a given component of $\mathbf{X}$ and by $\mathbf{X}_{\sim i}$ the remaining components, such that $\mathbf{X} \equiv (X_i, \mathbf{X}_{\sim i})$.

Using the set notation $A \equiv \{i_1, ..., i_s\} \subset \{1, ..., d\}$, we can rewrite Equation (92) as

$$\mathcal{M}(\mathbf{x}) = \mathcal{M}_0 + \sum_{\substack{A \subset \{1,...,d\} \\ A \neq \emptyset}} \mathcal{M}_A(\mathbf{x}_A), \tag{93}$$

where $\mathbf{x}_A$ is a subvector of $\mathbf{x}$ and only contains parameters that belong to the index set A. Due to the orthogonality property of the summands, the total variance of the model output Y can be written as

$$Var[Y] = \sum_{\substack{A \subset \{1,...,d\} \\ A \neq \emptyset}} \mathbb{E}[\mathcal{M}_A^2(\mathbf{x}_A)]. \tag{94}$$

Therefore we can define the partial variance associated to $\{i_1, ..., i_s\}$ by

$$V_{i_1...i_s} := \int \mathcal{M}^2_{i_1...i_s}(x_{i_1}, ..., x_{i_s}) w(x_{i_1}, ..., x_{i_s}) dx_{i_1} ... dx_{i_s}. \tag{95}$$

The total variance of the model output (94) can thus be split up into parts:

$$Var[Y] = V = \sum_{i=1}^{d} V_i + \sum_{1 \leq i < j \leq d}^{d} V_{ij} + ... + V_{12...d}. \tag{96}$$

Finally, the s-order Sobol' Indices are defined as

$$S^s_{i_1...i_s} = \frac{V_{i_1...i_s}}{V}. \tag{97}$$

The total Sobol' Index of a given parameter x_i is defined as

$$S_i^{tot} := S_i + \sum_{j=1, j \neq i}^{d} S_{ij} + \sum_{1 \leq j < k \leq d, j,k \neq i} S_{ijk} + ... + S_{12...d}. \tag{98}$$

The total Sobol' Index quantifies the total effect of parameter x_i on the variance of output variable.

In our setting, Sobol' Indices mainly serve two purposes (see Saltelli et al. (2008)). First, they can be used for *factor prioritization*. Total Sobol' Indices allow an importance ranking of the input variables and subsequently indicate which input parameters are worth investigating. Second, they are used for *factor fixing*. We can identify those parameters that do not significantly contribute to output variation. That is, we can set these parameters at deterministic values. In general, if $S_i^{tot} < 1\%$, the respective parameter x_i can be set to a constant value within the respective range.

E.2 Polynomial Chaos Expansion

As in the previous section, we consider a model[33] $\mathcal{M} : x \in D_X \subset \mathbb{R} \to \mathbb{R}$, where $\mathcal{M}(x)$ has a finite variance: $\mathbb{E}[\mathcal{M}^2(x)] < \infty$ and the input variable x with support D_X has a probability density function $w(x)$. Using PCE, $\mathcal{M}(x)$ can be represented by a series expansion (*metamodel*)

$$\mathcal{M}(x) = \sum_{k=0}^{\infty} y_k \psi_k(x), \tag{99}$$

where $\{\psi_k\}_{k=0}^{\infty}$ forms the orthonormal polynomials basis of a suitable space and $\{y_k\}_{k=0}^{\infty}$ is the set of coordinates of $\mathcal{M}(x)$ in this basis. These are the PCE coefficients to be computed. Let us now see how we can construct the orthonormal polynomials basis and determine the coefficients.

E.2.1 Construction of the orthonormal basis

We first explain how the orthonormal basis $\{\psi_k\}_{k=0}^{\infty}$ is constructed in PCE.

For any two functions $\phi_k, \phi_l : x \in D_X \to \mathbb{R}$, we define the functional inner product as

$$< \phi_k, \phi_l >_w \equiv \mathbb{E}[\phi_k(x)\phi_l(x)] = \int_{D_X} \phi_k(x)\phi_l(x)w(x)dx. \tag{100}$$

Moreover, ϕ_k and ϕ_l are said to be orthogonal if their inner product is zero.

Given the notation above and some algebra[34], we can define the sequence of *orthogonal polynomials* $\{P_k, k \in \mathbb{N}\}$ with respect to a weight function w as

$$< P_k, P_l >_w = a_k \delta_{kl}, \tag{101}$$

where δ_{kl} is the Kronecker Symbol, i.e. $\delta_{kl} = 0$ if $k \neq l$ and $\delta_{kl} = 1$ if $k = l$, and k is the degree of polynomial P_k. Hence, the functional inner product $< P_k, P_l >_w$ is equal to the squared norm $a_k \equiv ||P_k||^2$ if $k = l$ and zero otherwise. The polynomials can then be normalized:

$$\psi_k = \frac{P_k}{\sqrt{a_k}}.$$

[33] Note that for illustrative reasons we consider the univariate case. We later apply a multivariate version where we assume the input variables to be statistically independent.

[34] For example, one can use the Gram-Schmit Orthogonalization Procedure of $\{1, x, x^2, \cdots\}$ to build a family of orthogonal polynomials. For the generic method of constructing an orthonormal basis we refer the reader to Abramowitz and Stegun (1970).

Classical families of orthonormal polynomials are known analytically. For instance, Legendre Polynomials are the orthonormal basis for the uniform distribution over $[-1, 1]$ and Hermite Polynomials are the orthonormal basis for the Gaussian distribution.

To exactly replicate the model $\mathcal{M}(x)$ in Equation (99) we need a metamodel made up of an infinite series. In practice, we truncate the series to a reasonable number of terms, so that it is possible to estimate the PCE coefficients. By using polynomials up to degree p, we achieve the approximation

$$\hat{\mathcal{M}}(x) = \sum_{k=0}^{p} y_k \psi_k(x). \tag{102}$$

The corresponding approximation error is

$$\mathcal{M}(x) - \hat{\mathcal{M}}(x) = \sum_{k=p+1}^{\infty} y_k \psi_k(x). \tag{103}$$

Later, we will want to use multivariate functions and thus are interested in multivariate orthonormal polynomials that can be constructed from the univariate orthonormal polynomials using tensor products. We define $\alpha \in \mathbb{N}^d$, which are ordered lists of integers $\alpha = (\alpha_1, ..., \alpha_d)$, where d is the number of input variables. Then we can write the multivariate polynomial $\mathbf{\Psi}_\alpha$ to a multi-index α as

$$\mathbf{\Psi}_\alpha(\mathbf{x}) \equiv \prod_{i=1}^{d} \psi_{\alpha_i}^{(i)}(x_i), \tag{104}$$

where $\psi_{\alpha_i}^{(i)}(x_i)$ is the univariate polynomial of degree α_i from the orthonormal family associated with variable x_i.

It can be proved that the set of all multivariate polynomials in the random input vector $\mathbf{x}$ form a basis of the Hilbert space in which $Y = \mathcal{M}(\mathbf{x})$ is represented[35] by

$$Y = \sum_{\alpha \in \mathbb{N}^d} y_\alpha \mathbf{\Psi}_\alpha(\mathbf{x}). \tag{105}$$

[35] See Soize and Ghanem (2004).

E.2.2 Computation of PCE coefficients by least-square minimization

In the preceding subsection we derived the orthonormal basis $\mathbf{\Psi}_\alpha$. We now turn to the computation of the PCE coefficients y_α. As shown above, we need to introduce a *truncation scheme* $\mathcal{A}$ to reduce the infinite series to a finite number of terms, so that the coefficients can be computed. Once the truncation scheme is selected[36], a variety of approaches to computing the coefficients is available in the literature. Following Harenberg et al. (2017) and Le Gratiet et al. (2016), we use the least-square estimation, which we address in the following.

Equation (105) in a truncated form is

$$Y = \mathcal{M}(\mathbf{x}) = \sum_{\alpha \in \mathcal{A}} y_\alpha \mathbf{\Psi}_\alpha(\mathbf{x}) + \epsilon, \tag{106}$$

where ϵ is the residual that contains all the PCE polynomials not in the truncation set $\mathcal{A}$ (see Equation (103) for the univariate case). The set of coefficients $\mathbf{y} = \{y_\alpha, \alpha \in \mathcal{A}\}$ is selected such that the mean squared error $\mathbb{E}[\epsilon^2]$ is minimized:

$$\mathbf{y} = \arg \min_{\mathbf{y} \in \mathbb{R}^{card\mathcal{A}}} \mathbb{E}\left[\left(\mathcal{M}(\mathbf{x}) - \sum_{\alpha \in \mathcal{A}} y_\alpha \mathbf{\Psi}_\alpha(\mathbf{x})\right)^2\right]. \tag{107}$$

To obtain an estimate of $\mathbf{y}$, we draw N samples[37] of the input parameters $\mathcal{X}_{ed} = \{\mathbf{X}^{(i)}, i = 1, ..., N\}$, where *ed* stands for *experimental design*. The above equation can then be written as

$$\hat{\mathbf{y}} = \arg \min_{\hat{\mathbf{y}} \in \mathbb{R}^{card\mathcal{A}}} \frac{1}{N} \sum_{i=1}^{N} \left(\mathcal{M}(\mathbf{X}^{(i)}) - \sum_{\alpha \in \mathcal{A}} \hat{y}_\alpha \mathbf{\Psi}_\alpha(\mathbf{X}^{(i)})\right)^2. \tag{108}$$

Hence, in a first step, the model $\mathcal{M}$ has to be run N times to obtain a vector $\mathcal{Y} = \{\mathcal{M}(\mathbf{X}^{(1)}), ..., \mathcal{M}(\mathbf{X}^{(N)})\}^T$. Second, the basis polynomials have to be evaluated at each point of the experimental design to create the information matrix $\mathbf{A}$:

$$\mathbf{A} = \{\mathbf{A}_{ij} \equiv \mathbf{\Psi}_j(\mathbf{X}^{(i)}), \quad i = 1, ..., N, \quad j = 1, ..., card\mathcal{A}\}. \tag{109}$$

[36] The detailed truncation scheme is presented in Section E.2.3.

[37] N should be larger than the number of unknowns ($card\mathcal{A}$), but at the same time not too large, so that the model $\mathcal{M}(\mathbf{x})$ does not have to be evaluated too many times. Le Gratiet et al. (2016) propose a rule of thumb $N \approx 2\binom{d+p}{p}$ or $3\binom{d+p}{p}$, where p represents the total degree of the truncation scheme, i.e. $|\alpha| = \alpha_1 + \cdots + \alpha_d = p$.

Therefore the mean-square error can be written as

$$\Delta = \sum_{i=1}^{N} \epsilon_i^2 = (\mathcal{Y} - \mathbf{A}\hat{\mathbf{y}})^T \cdot (\mathcal{Y} - \mathbf{A}\hat{\mathbf{y}}). \tag{110}$$

The error is minimized when

$$\frac{\partial \Delta}{\partial \hat{\mathbf{y}}^{\mathbf{T}}} = -2\mathbf{A}^T\mathcal{Y} + 2(\mathbf{A}^T\mathbf{A})\hat{\mathbf{y}} = \mathbf{0}. \tag{111}$$

Thus the solution to the minimization problem takes the form

$$\hat{\mathbf{y}} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathcal{Y}. \tag{112}$$

Finally, the truncated PCE can be written as

$$Y^{PC} = \mathcal{M}^{PC}(\mathbf{x}) = \sum_{\alpha \in \mathcal{A}} \hat{y}_\alpha \mathbf{\Psi}_\alpha(\mathbf{x}). \tag{113}$$

To avoid over-fitting, we reduce the number of PCE coefficients by employing the least-angle regression (LAR) algorithm to select only the significant coefficients in the PCE. That is, we apply the LAR algorithm to the candidate basis $\mathcal{A}$, which contains all possible coefficients, i.e. $\binom{d+p}{p}$, and we select the most significant coefficients to form a sparse basis.[38] Figure 16 enumerates the ordered lists of integers α on the x-axis and shows the size of the estimated PCE coefficients y_α on the y-axis for the PCE approximation of L^N with $p = 10$ and $d = 4$. The mean value and the coefficients of the linear polynomial terms are largest in size and are hence the most influential. As the degree of the polynomial terms increases, the size of the coefficients decreases. The total candidate basis consists of $\binom{d+p}{p} = 1001$ coefficients and is reduced to the sparse basis of 559 non-zero coefficients (NNZ) using the LAR algorithm.

[38] This algorithm was initially proposed by Efron et al. (2004). Blatman and Sudret (2011) introduced LAR into the PCE literature. For a formal discussion of the algorithm, we refer to these papers. Note that the LAR algorithm is only defined for non-constant regressors. Hence, after selecting the sparse basis, we perform an ordinary least-square regression, which includes a constant regressor, and calculate the coefficients we ultimately use in the PCE. Marelli and Sudret (2017) call this approach *"hybrid LAR"*.

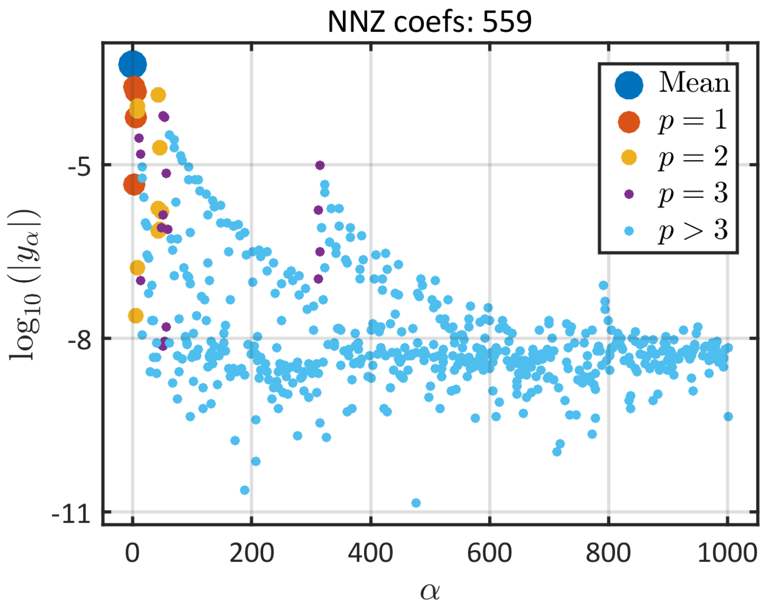

Figure 16: Sparse basis selected using LAR for the model L^N_{PCE}.

E.2.3 Truncation scheme of PCE coefficients

We now turn to the question of how the truncation set should be chosen. Once the coefficients are obtained (see Equation (112)), the approximation error of the truncated PCE can be computed ex post by comparing the fitted output responses (see Equation (113)) to the true output response (see Equation (106)). As suggested in Le Gratiet et al. (2016), we use the *leave-one-out* (LOO) error estimator to find an appropriate degree of truncation that yields an accurate approximation. A brief sketch of the procedure follows. First, an experimental design $\mathcal{X}_{ed} \backslash X^{(i)} \equiv \{X^1, ..., X^{(i-1)}, X^{(i+1)}, ..., X^N\}$ is set up. Second, a PCE model $\mathcal{M}^{PC\backslash i}$ is estimated and the error at the point that was left out is computed. Then the average over the sum of the squared errors is calculated:

$$err_{LOO} \equiv \frac{1}{N} \sum_{i=1}^{N} \Big(\mathcal{M}(\mathbf{X}^{(i)}) - \mathcal{M}^{PC\backslash i}(\mathbf{X}^{(i)}) \Big)^2. \tag{114}$$

Le Gratiet et al. (2016) show that, after some algebra, this expression reduces to

$$err_{LOO} = \frac{1}{N} \sum_{i=1}^{N} \left(\frac{\mathcal{M}(\mathbf{X}^{(i)}) - \mathcal{M}^{PC}(\mathbf{X}^{(i)})}{1 - h_i} \right)^2, \tag{115}$$

where h_i is the i-th diagonal term of $\mathbf{A}(\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T$ and $\mathcal{M}^{PC}$ is the PCE model of the full experimental design $\mathcal{X}_{ed}$. We follow Harenberg et al. (2017) and assume that an $err_{LOO} \leq 10^{-2}$ yields sufficient accuracy for a sensitivity analysis.

For example, for the expected social losses $\mathbb{E}[L^N]$ and $\mathbb{E}[L^F]$ in NFG and IFG, we truncate the PCE at $p = 10$ and use 3100 samples in the experimental design. The corresponding errors are of a magnitude $err_{LOO} \approx 10^{-7}$ and $err_{LOO} \approx 10^{-12}$, respectively. Due to the computationally more involved approaches in EFG and SFG, we truncate the PCE at $p = 7$ and use 990 sample draws. The resulting errors are of a magnitude $err_{LOO} \approx 10^{-4}$ and $err_{LOO} \approx 10^{-6}$, respectively. Figure 17 shows how well the metamodel approximates the true model by plotting the true model response (Y) on the x-axis and the truncated PCE model response on the y-axis (Y^{PC}). The figure shows that all 10'000 sample points lie on a straight 45-degree line. Therefore the PCE model of degree 10 manages to capture the true model very well.

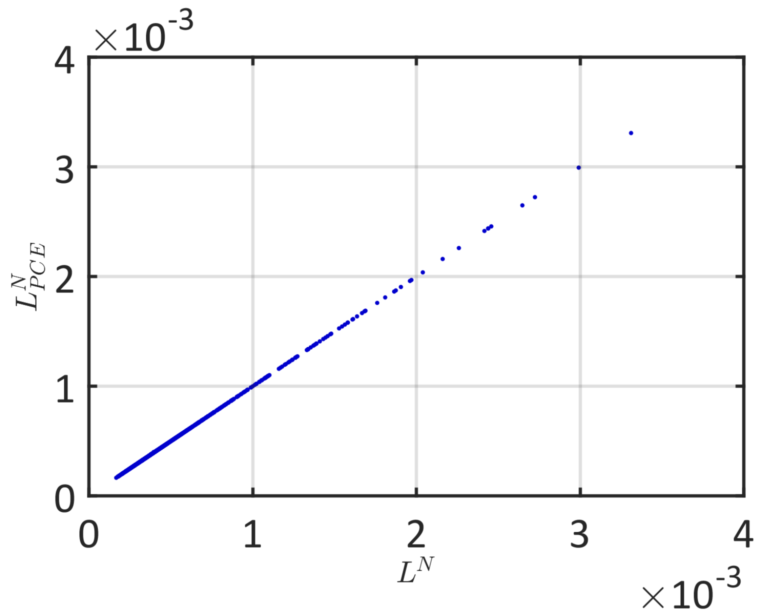

Figure 17: Metamodel output ($Y^{PC} = L^N_{PCE}$) vs true model output ($Y = L^N$) for NFG.

E.3 Sobol' Indices and PCE

Sudret (2008) shows that the sum of orthogonal functions in the Sobol' decomposition in Equation (93) can be analytically derived from the sum of orthogonal functions in the truncated PCE in Equation (113). First, note that due to the orthogonality[39] of

[39] The orthogonality property of the polynomial basis implies that $\mathbb{E}[\Psi_\alpha(\mathbf{x})] = 0$ and $\mathbb{E}[\Psi_\alpha(\mathbf{x})\Psi_\beta(\mathbf{x})] = \delta_{\alpha\beta}$.

the PCE basis, mean and variance of the output variable read

$$\mathbb{E}[\hat{Y}] = \mathbb{E}\left[\sum_{\alpha\in\mathcal{A}} \hat{y}_\alpha \Psi_\alpha(\mathbf{x})\right] = \hat{y}_0, \tag{116}$$

$$Var[\hat{Y}] = \mathbb{E}\left[(\hat{Y} - \hat{y}_0)^2\right] = \sum_{\substack{\alpha\in\mathcal{A}\\ \alpha\neq 0}} \hat{y}_\alpha^2. \tag{117}$$

That is, the mean value is the first coefficient of the series, and the variance is the sum of the squares of the remaining coefficients.

Second, given the PCE coefficients, Sobol' Indices of any order may be obtained by combining the squares of the respective coefficients, i.e. first-order Sobol' Indices read

$$\hat{S}_i^1 = \frac{\sum_{\alpha\in\mathcal{A}_i} \hat{y}_\alpha^2}{\sum_{\alpha\in\mathcal{A},\alpha\neq 0} \hat{y}_\alpha^2}, \quad \text{where} \quad \mathcal{A}_i = \{\alpha \in \mathcal{A} : \alpha_i > 0, \alpha_{j\neq i} = 0\}. \tag{118}$$

In general, the polynomials can be gathered according to the parameters they depend on, and the Sobol' Indices are written as

$$\hat{S}^s_{i_1\dots i_s} = \frac{\sum_{\alpha\in\mathcal{A}_{i_1\dots i_s}} \hat{y}_\alpha^2}{\sum_{\alpha\in\mathcal{A},\alpha\neq 0} \hat{y}_\alpha^2}, \quad \text{where} \quad \mathcal{A}_{i_1,\dots,i_s} = \{\alpha \in \mathcal{A} : \alpha_k > 0, \ \text{iff } k \in \{i_1, \dots, i_s\}\}. \tag{119}$$

The PCE-based total Sobol' Indices read

$$\hat{S}_i^{tot} = \frac{\sum_{\alpha\in\mathcal{A}_i^{tot}} \hat{y}_\alpha^2}{\sum_{\alpha\in\mathcal{A},\alpha\neq 0} \hat{y}_\alpha^2}, \quad \text{where} \quad \mathcal{A}_i^{tot} = \{\alpha \in \mathcal{A} : \alpha_i > 0\}. \tag{120}$$

Note that the univariate function of parameter x_i can be written as

$$\mathcal{M}_i(x_i) = \mathbb{E}[\mathcal{M}(\mathbf{X})|X_i = x_i] - \mathcal{M}_0 = \sum_{\alpha\in\mathcal{A}_i} y_\alpha \Psi_\alpha(\mathbf{x}). \tag{121}$$

$\mathcal{M}_i(x_i)$ represents the deviation of the model's expected output for a given x_i from the mean value of the model's output. While the total Sobol' Indices in Equation (120) can be used to identify the importance ranking of input parameters, univariate function provides further information about the parameter's impact on the output variable. For example, whether the parameter's effect on the output variable is positive or negative, whether the relationship is linear or non-linear, and the regions of the parameter space in which output sensitivity is most pronounced.

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