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## **BUYER GROUP AND BUYER POWER WHEN SELLERS COMPETE**

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## Abstract

We study how buyer group affects buyer power when sellers compete with non-linear tariffs and buyers operate in separate markets. In the baseline model of two symmetric sellers and two symmetric buyers, we characterize the set of equilibria under buyer group, the set without buyer group and compare both. We find that the interval of each buyer's equilibrium payoffs without buyer group is a strict subset of the interval under buyer group if each seller's cost function is strictly convex, whereas the two intervals are identical if the cost function is concave. Our result implies that buyer group has no effect when the cost function is concave. When it is strictly convex, buyer group strictly reduces the buyers' payoff as long as, under buyer group, we select the Pareto-dominant equilibrium in terms of the sellers' payoffs. We extend this result to asymmetric settings with an arbitrary number of buyers.

JEL Classification: D4, K21, L41, L82

Keywords: Buyer Group, Buyer Power, Competition in Non-linear Tariffs, Discriminatory Offers, Common Agency

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# Buyer Group and Buyer Power When Sellers Compete

Doh-Shin Jeon\* and Domenico Menicucci†

November 2, 2017

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We study how buyer group affects buyer power when sellers compete with non-linear tariffs and buyers operate in separate markets. In the baseline model of two symmetric sellers and two symmetric buyers, we characterize the set of equilibria under buyer group, the set without buyer group and compare both. We find that the interval of each buyer's equilibrium payoffs without buyer group is a strict subset of the interval under buyer group if each seller's cost function is strictly convex, whereas the two intervals are identical if the cost function is concave. Our result implies that buyer group has no effect when the cost function is concave. When it is strictly convex, buyer group strictly reduces the buyers' payoff as long as, under buyer group, we select the Pareto-dominant equilibrium in terms of the sellers' payoffs. We extend this result to asymmetric settings with an arbitrary number of buyers.

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# 1 Introduction

Does buyer group lead to buyer power? Do large buyers obtain size-related discounts from their suppliers? Examples of buyer groups abound in retailing (Inderst and Shaffer, 2007, Caprice and Rey, 2015), health care (Marvel and Yang, 2008), cable TV (Chipty and Snyder, 1999), academic journals (Jeon and Menicucci, 2017) etc. Understanding how buyer group or buyer size affects buyer power is very important as policy makers in Europe and in U.S. are concerned about buyer power due to increasing buyer market concentration (European Commission, 1999 and OECD, 2008).<sup>1</sup>

Even if there is a large literature addressing the above questions both theoretically and empirically, the literature provides nuanced answers such that large-buyer discounts do not arise under all circumstances but only under certain conditions. For instance, Chipty and Snyder (1999) and Inderst and Wey (2007) consider bargaining between a monopolist seller and each among multiple buyers and find that buyer group increases (reduces) the total payoff of the group members if the seller's cost function is convex (concave). Inderst and Wey (2003) study a two-seller-two-buyer game which results in each agent earning his Shapley value, and find the same result.<sup>2</sup> Although the finding of Chipty and Snyder (1999) is confirmed in a laboratory experiment (Normann, Ruffle, B. and Snyder, 2007), it is not confirmed by real world data as Ellison and Snyder (2010) find that in the U.S. wholesale pharmaceutical industry, large-buyer discounts can exist only if buyers face competing sellers.

In this paper, we consider a setting in which sellers producing substitutes compete by offering personalized non-linear tariffs and study how buyer group affects each player's payoff. As in Chipty and Snyder (1999) and Inderst and Wey (2003, 2007), we do not consider competition among buyers: each buyer operates in a separate market. After showing that all equilibria are efficient regardless of whether or not the buyers form a group (Proposition 1), we consider a baseline model with two symmetric sellers and two symmetric buyers. We characterize the set of equilibria that arise under buyer group (Section 3), the set of equilibria that arise without buyer group (Section 4), and compare these sets (Section 5). All equilibria can be ranked according to the payoff of each buyer, and we find that the interval of each buyer's equilibrium payoffs without buyer group is a strict subset of the interval under buyer group if each seller's cost function is strictly convex, whereas the two intervals are identical if the cost function is concave.

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<sup>1</sup>In US, the Federal Trade Commission organized in 2000 a workshop on slotting allowances, a major buyer power issue in grocery retailing. See Chen (2007) for a survey of the literature on buyer power and antitrust policy implications.

<sup>2</sup>More broadly, in cooperative game theory there is a literature on joint bargaining paradox (Harsanyi, 1977) or collusion neutrality (van den Brink, 2012), which studies when a coalition formation increases or reduces the joint payoff of its members from bargaining.

This finding suggests that buyer group has no effect on the buyers' payoff when the sellers' cost function is concave. When it is strictly convex, buyer group can increase or decrease the buyers' payoff depending on the equilibrium selection. If we select, under buyer group, the equilibrium which is Pareto-dominant in terms of the sellers' payoffs as in Bernheim and Whinston (1986, 1998),<sup>3</sup> our result implies that buyer group strictly reduces the buyers' payoff no matter the equilibrium played without buyer group. Therefore, we can conclude that buyer group does not increase the buyers' payoff regardless of whether the sellers' cost function is convex or concave. Then we generalize the result obtained from the baseline model with convex cost: we consider two asymmetric sellers and  $n(\geq 2)$  asymmetric buyers and show that as long as a seller's cost function is strictly convex, in any equilibrium without buyer group, the buyers obtain a strictly higher total payoff (and each seller obtains a strictly lower payoff) than in the Pareto-dominant equilibrium for sellers under buyer group (Proposition 5). We notice that the Pareto-dominant equilibrium for sellers under buyer group is also called a "sell-out equilibrium" by Bernheim and Whinston (1998).<sup>4</sup>

We below provide the intuition for our result in the baseline model regarding the equilibrium which is Pareto-dominant in terms of the sellers' payoffs. It is well-known from Bernheim and Whinston (1998) that in the equilibrium under buyer group, each seller  $j$  is indifferent between inducing the group to buy from both sellers (as in equilibrium) and inducing the group to buy exclusively from seller  $j$ . The latter strategy is equivalent to the strategy, without the group, that induces both buyers to buy exclusively from seller  $j$ . However, when there is no group, seller  $j$  can also deviate by inducing only one buyer to buy exclusively from himself while inducing the other buyer to keep buying the equilibrium quantity from the rival seller. No buyer group reduces the sellers' (best) payoff (and hence increases the buyers' (worst) payoff)<sup>5</sup> if and only if the deviation inducing both buyers to buy exclusively is less powerful than the deviation inducing only one buyer to buy exclusively. Note that inducing both buyers to buy exclusively requires a larger increase in output of  $j$  than inducing only one buyer to buy exclusively. Therefore, when the cost function is strictly convex, because of increasing marginal costs, if the deviation inducing only one buyer to buy exclusively is not profitable, the deviation inducing both buyers to buy exclusively is not profitable either. However, the reverse does not hold and if a seller is indifferent between no deviation and the deviation inducing both buyers to buy exclusively, the deviation inducing only one buyer to buy exclusively becomes profitable

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<sup>3</sup>The equilibrium is also the truthful equilibrium under buyer group.

<sup>4</sup>In the sell-out equilibrium, each seller  $j$  uses a sell-out strategy such that the payment of the buyer group for purchasing quantity  $q_j$  takes the form of  $F_j + C_j(q_j)$ , where  $F_j$  is a fixed fee and  $C_j$  is  $j$ 's cost function.

<sup>5</sup>Since all equilibria are efficient, a decrease in the sellers' payoff implies an increase in the buyers' payoff.

since the marginal cost increases less due to smaller output expansion. A similar reasoning implies that when the cost function is concave, if the deviation inducing both buyers to buy exclusively is not profitable, the deviation inducing only one buyer to buy exclusively is not profitable. Therefore, buyer group reduces the buyers' (worst) payoff if the cost function is strictly convex while it does not affect it if the cost function is concave.<sup>6</sup>

A big picture consistent with some empirical findings emerges when our results are combined with the findings of Inderst and Shaffer (2007) and Dana (2012).<sup>7</sup> Their models are similar to ours as they assume that sellers have complete information about buyers' preferences and hence compete by offering personalized tariffs, regardless of whether or not buyers form a group. The crucial difference is that they assume that the buyer group makes an exclusive purchase commitment while we do not consider any such commitment. They find that a buyer group never decreases the total payoff of its members and strictly increases it unless the members have identical preferences. This is because group formation among heterogenous buyers makes the buyers more homogenous (as a group) and thereby intensifies competition for exclusivity between sellers.

Combining their result with ours leads to the following prediction: when sellers producing substitutes compete, buyer group increases the total payoff of its members only if the group can pre-commit to limit its purchases to a subset of sellers. This prediction is consistent with the empirical findings of Ellison and Snyder (2010), Competition Commission of U.K. (2008) and Sorensen (2003). Their common findings are: buyer size alone does not explain discounts but it is a buyer's credible threat to exclude certain products from purchase that explains discounts. For instance, Ellison and Snyder (2010) find that large buyers (chain drugstores) receive either very small or no statistically significant discounts relative to small buyers (independent drugstores) for off-patent antibiotics which have one or more generic substitutes available but that hospitals and health-maintenance

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<sup>6</sup>We can also provide the intuition for our result in the baseline model regarding the equilibrium which generates the worst payoff for the sellers. It turns out that in the equilibrium under buyer group, each seller  $j$  is indifferent between inducing the group to buy the equilibrium quantity from each seller and inducing the group to buy a larger quantity from the rival, which implies that seller  $j$  sells less upon the deviation. The latter strategy is equivalent to the strategy, without the group, that induces both buyers to buy more from the rival seller. However, when there is no group, seller  $j$  can also deviate by inducing only one buyer to buy more from the rival. No buyer group increases the sellers' (worst) payoff and hence decreases the buyers' (best) payoff if and only if the deviation inducing both buyers to buy more from the rival is less powerful than the deviation inducing only one buyer to buy more from the rival. When the cost function is convex (concave), inducing a first buyer to buy more from the rival generates a larger (smaller) cost saving than inducing a second buyer to buy more from the rival. Therefore, no buyer group decreases the buyers' (best) payoff if the cost function is strictly convex while it does not affect it if the cost function is concave.

<sup>7</sup>See also Caprice and Rey (2015) who provide a mechanism through which buyer group among (competing) retailers leads to buyer power through joint delisting decision.

organizations (HMOs) receive substantial discounts relative to drugstores. They explain this finding by the fact that chain drugstores and independent drugstores do not differ much in the substitution opportunities whereas hospitals and HMOs can and do commit to limit their purchases to certain drugs by issuing restrictive formularies as they can control which drugs their affiliated doctors prescribe.<sup>8</sup>

Our analysis of the symmetric setting in Sections 3 and 4 has some technical contribution as well. We show that any symmetric equilibrium can be replicated by an equilibrium obtained from allowing each seller to offer only two pairs of quantity and price. We use this approach to characterize the whole set of equilibria depending on whether or not the buyers formed a group. The approach is useful because it reduces the problem of finding all the equilibria to a relatively simple problem.<sup>9</sup>

In Section 7, we apply our insights to a situation in which one seller's entry is endogenous and the buyers decide whether or not to form a group before the entry decision.<sup>10</sup> We assume that the sellers' cost functions are strictly convex. The entrant has to incur a fixed cost of entry, which is a random draw from a (commonly known) distribution as in Aghion and Bolton (1987), Innes and Sexton (1994) and Chen and Shaffer (2014). Upon entry, both sellers simultaneously offer non-linear tariffs and we assume that they play the Pareto-dominant equilibrium. Therefore, when the buyers decide to form or not a group, they face a trade-off: forming the buyer group increases the probability of entry but reduces the total payoff of the buyers conditional on entry. In particular, conditional on buyer group formation, the private entry decision coincides with the socially optimal one as in the sell-out equilibrium, each firm's profit is equal to its social marginal contribution. This implies that no formation of group leads to a socially suboptimal entry. The existing literature on naked exclusion explains suboptimal entry by the incumbent's taking advantage of coordination failure among buyers (Rasmusen, Ramseyer and Wiley 1991, Segal and Whinston, 2000, Fumagalli and Motta, 2006, 2008, Chen and Shaffer,

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<sup>8</sup>Similarly, the study of the competition commission (2008) of the U.K. grocery industry finds significant buyer-size discounts for non-primary brand goods (for which the grocer can freely substitute among different suppliers) but not for primary brand goods (for which grocers have limited substitution opportunities). Sorensen (2003) also find that size alone cannot explain why some insurers get much better deal from hospitals than other insurers and that an insurer's ability to channel its patients to selected hospitals can explain why small managed care organizations are often able to extract deeper discounts from hospitals than very large indemnity insurers.

<sup>9</sup>This approach is adopted partly because the sell-out equilibrium (Bernheim and Whinston, 1998), which exists under buyer group, does not exist without group if the costs are strictly convex or strictly concave. Hence, even if we want to focus on the Pareto-dominant equilibrium, the approach is useful for the analysis of no group.

<sup>10</sup>For instance, state-wide (or multi-state) large pharmaceutical purchasing alliances (Ellison and Snyder, 2010) can affect entry decision of generic drug producers or a buying alliance among large chains of supermarkets (Caprice and Rey, 2015) can have an impact on entry of national brand manufacturers.

2014). While the literature typically assumes that the incumbent makes offers to buyers before the entry, Fumagalli and Motta (2008) show that the coordination failure survives even if both sellers make simultaneous offers. However, in these papers, buyers have no reason not to form a group (at least among those operating in separate markets) as this would remove the coordination failure. Our application shows that buyers may choose not to form a group even if this leads to a suboptimal entry.

In Section 8, we consider the case in which the two products can be substitutes or complements while the cost functions are linear. We focus on sell-out equilibria which exist regardless of whether or not the buyers form a group. Then, buyer group has no effect on any player's payoff if the products sold by the sellers are complements to both buyers (or substitutes to both buyers). But if the products are strict substitutes for buyer 1 and strict complements for buyer 2, then buyer group reduces the buyers' total payoff. Note that the sellers have some residual market power with respect to buyer 2 as each seller charges less than the incremental value of his product because charging the incremental value leads to a strictly negative payoff to buyer 2. Buyer group allows the sellers to transfer the residual market power to buyer 1 in the same way as multi-market contact facilitates collusion by transferring residual collusive power from one market to another (Bernheim and Whinston, 1990).

## 1.1 Literature review

The literature has used various ways to generate size-related discounts. Katz (1987), Scheffman and Spiller (1992) and Inderst and Valletti (2011) model buyer power as downstream firms' ability to integrate backwards by paying a fixed cost. Buyer group makes this outside option stronger as the group can share the fixed cost among its members, which in turn allows the group to obtain a larger discount from a seller. The papers which do not consider such size-related outside option can be regrouped into two different categories. On the one hand, there are papers studying buyer group among competing downstream firms. Hence, buyer group not only allows them to join force as buyers but also eliminates competition between them in the downstream market: see von Ungern-Sternberg (1996), Dobson and Waterson (1997), Chen (2003), Erutku (2005) and Gaudin (2017).<sup>11</sup> On the other hand, there are papers which focus on 'pure' buyer power in the sense that group members only interact on the buying side as they do not compete. The second branch of literature can be further distinguished depending on whether they consider a monopoly seller (Chipty and Snyder, 1999, and Inderst and Wey, 2007) or competing sellers (Inderst and Shaffer, 2007, Marvel and Yang, 2008, Dana, 2012, Chen

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<sup>11</sup>Caprice and Rey (2015) is an exception as they consider a buyer group among retailers which maintain downstream competition.

and Li, 2013). As we consider upstream competition but no downstream competition, our paper is close to the last subcategory, in particular to Inderst and Shaffer, (2007) and Dana (2012). However, our paper is also related to Chipty and Snyder (1999) and Inderst and Wey (2003, 2007) in terms of prediction based on the curvature of the seller(s)' cost function. Namely, they predict that the buyer group increases (reduces) the total payoff of the group members if the seller's cost function is convex (concave). This occurs because buyers are assumed to have some bargaining power and the incremental costs to serve the group is smaller (larger) than the sum of the incremental cost to serve each buyer if sellers' costs are convex (concave).<sup>12</sup> In contrast, we predict that the buyer group reduces (does not affect) the total payoff of the group members if the seller's cost function is convex (concave).<sup>13</sup>

To the best of our knowledge, we are the first to analyze the effect of buyer group on buyer power in a framework that extends the common agency (Bernheim and Whinston, 1986, 1998) to multiple buyers. For the setting without buyer group, Prat and Rustichini (2003) analyze a general setup with multiple sellers and multiple buyers in which buyers' utility functions are concave and sellers' cost functions are convex. They allow sellers to use more complex tariffs than ours: seller  $j$  can make buyer  $i$ 's payment depend on the whole vector of  $i$ 's purchases such that the payment depends on  $q_j^i$  and also on  $q_k^i$  for each seller  $k$  different from  $j$ . They prove the existence of an efficient equilibrium, but do not specify the equilibrium strategies and do not compare the case of buyer group with the case of no group. We consider tariffs such that the payment of each buyer  $i$  to each seller  $j$  only depends on  $q_j^i$ , the quantity buyer  $i$  buys from seller  $j$ <sup>14</sup> and compare buyer group with no group.<sup>15</sup>

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<sup>12</sup>In our setting, using the Shapley value to compare the buyers' payoffs under buyer group with the payoffs under no group, yields the same results as in Inderst and Wey (2003).

<sup>13</sup>The literature on buyer group is large. Snyder (1996, 1998) apply the idea of Rotemberg and Saloner (1986) to explain why large buyers get discounts. Namely, he considers a model of tacit collusion among sellers and shows that sellers charge lower prices to large buyers while charging monopoly prices to small buyers. Ruffle (2013) test experimentally the theory and find support for it. Chae and Heidhues (2004) (DeGraba, 2005) generate large-buyer discounts from buyers' risk aversion (seller's risk aversion). Loertscher and Marx (2017) study competitive effects of upstream merger in a market with buyer power.

<sup>14</sup>There are two reasons for restricting attention to such tariff. First, a seller  $j$  may not observe the quantity that a buyer buys from the other seller. Second, market-share contracts that provide price rebates conditional on buying a large share of quantity from the seller making the offer are often prohibited by antitrust authorities when practiced by dominant firms. For instance, on May 13, 2009, European Commission imposed a fine of 1.06 billion euros on Intel for such behavior.

<sup>15</sup>Jeon and Menicucci (2012) extends the common agency to competition between portfolios in the presence of the buyer's slot constraint and provide conditions to make the "sell-out equilibrium" the unique equilibrium. Contrary to what happens under slot constraint, Jeon and Menicucci (2006) show that in the presence of the buyer's budget constraint, the well-known result in the common agency literature that competition between sellers achieves the outcome that maximizes all parties' joint payoff

Our paper is distantly related to our companion paper (Jeon and Menicucci, 2017) which studies library consortium in the market for academic journals. Each seller (i.e., publisher) is a monopolist of his journals and sells a bundle of his electronic academic journals at personalized price(s). However, we assume that each seller's marginal cost is zero such that in the absence of buyer group (i.e., library consortium), each library's market can be studied in isolation. Competition among sellers occurs because of the budget constraint of each buyer. We find that depending on the sign and degree of correlation between each member library's preferences, building a library consortium can increase or reduce the total payoffs' of the buyers.<sup>16</sup>

## 2 The model and a preliminary result

There are two sellers ( $A$  and  $B$ ) and two buyers (1 and 2). Let  $q_j^i \geq 0$  represent the quantity of the product that buyer  $i$  ( $= 1, 2$ ) buys from seller  $j$  ( $= A, B$ ). For each buyer  $i$ , the gross utility from consuming  $(q_A^i, q_B^i)$  is given by  $U^i(q_A^i, q_B^i)$ , with  $U^i$  strictly increasing and strictly concave in  $(q_A^i, q_B^i)$ . Precisely, we let  $U_j^i = \partial U^i / \partial q_j^i$ ,  $U_{jk}^i = \partial U^i / \partial q_j^i \partial q_k^i$ , and assume that for each buyer  $i$  (i)  $U_j^i > 0$ , but  $U_j^i \rightarrow 0$  as  $q_j^i \rightarrow +\infty$ ; (ii) the Hessian matrix of  $U^i$  is negative definite, hence  $U^i$  is strictly concave; (iii)  $U_{AB}^i < 0$ , hence the goods are strict substitutes.<sup>17</sup> For each seller  $j$ , the cost of serving the buyers is  $C_j(q_j^1 + q_j^2)$ . We assume that  $C_j(0) = 0$ ,  $C_j$  is strictly increasing, but  $C_j$  can be convex or concave. However, if  $C_j$  is concave, then we assume that  $U^1, U^2$  are concave enough such that social welfare  $U^1(q_A^1, q_B^1) + U^2(q_A^2, q_B^2) - C_A(q_A^1 + q_A^2) - C_B(q_B^1 + q_B^2)$  is a concave function.

Each seller offers a non-linear tariff to each buyer (or to the buyer group). In particular, we consider tariffs such that what buyer  $i$  pays to seller  $j$  depends only on the quantity that the buyer purchases from seller  $j$ , and not on the quantity she purchases from the other seller. When there is no buyer group, we allow each seller to offer personalized non-linear tariffs: the tariff offered by seller  $j$  to buyer  $i$  is denoted by  $T_j^i(q)$  (with  $T_j^i(0) = 0$ ), and can be different from the tariff seller  $j$  offers to buyer  $h$  ( $\neq i$ ). After seeing the tariffs, buyer  $i$  chooses  $(q_A^i, q_B^i)$  in order to maximize  $U^i(q_A^i, q_B^i) - T_A^i(q_A^i) - T_B^i(q_B^i)$ .

When the buyers form a buyer group, the sellers compete for serving the group. Let  $G$  denote the buyer group,  $q_j^G$  the quantity  $G$  buys from seller  $j$ , and  $T_j^G(q)$  the non-linear tariff that seller  $j$  offers to  $G$  (with  $T_j^G(0) = 0$ ). We define  $U^G(q_A^G, q_B^G)$  as follows:

$$U^G(q_A^G, q_B^G) \equiv \max_{q_A^1, q_B^1, q_A^2, q_B^2} U^1(q_A^1, q_B^1) + U^2(q_A^2, q_B^2) \quad (1)$$

fails to hold.

<sup>16</sup>Therefore, in terms of the results, the companion paper is situated between Inderst and Shaffer (2007) and Dana (2012) on the one hand, and the current paper on the other hand.

<sup>17</sup>We consider the case of complementary goods in Proposition 7.

$$\text{subject to} \quad q_A^1 + q_A^2 = q_A^G, \quad q_B^1 + q_B^2 = q_B^G. \quad (2)$$

Thus  $U^G(q_A^G, q_B^G)$  is the group's gross utility from buying  $(q_A^G, q_B^G)$ , as it results from the optimal allocation of  $(q_A^G, q_B^G)$  between the two buyers.<sup>18</sup> Hence, after seeing the tariffs,  $G$  chooses  $(q_A^G, q_B^G)$  in order to maximize  $U^G(q_A^G, q_B^G) - T_A^G(q_A^G) - T_B^G(q_B^G)$ .

Define  $\mathbf{q}^* \equiv (q_A^{1*}, q_B^{1*}, q_A^{2*}, q_B^{2*})$  as the unique allocation vector that maximizes social welfare,<sup>19</sup> and  $V_{AB}^G$  as the social welfare in the first-best allocation  $\mathbf{q}^*$ :

$$\mathbf{q}^* \equiv \arg \max_{(q_A^1, q_B^1, q_A^2, q_B^2)} U^1(q_A^1, q_B^1) + U^2(q_A^2, q_B^2) - C_A(q_A^1 + q_A^2) - C_B(q_B^1 + q_B^2), \quad (3)$$

$$V_{AB}^G \equiv U^1(q_A^{1*}, q_B^{1*}) + U^2(q_A^{2*}, q_B^{2*}) - C_A(q_A^{1*} + q_A^{2*}) - C_B(q_B^{1*} + q_B^{2*}) \quad (4)$$

We say that an equilibrium is *efficient* if the equilibrium allocation is  $\mathbf{q}^*$ . From Proposition 2 to Proposition 7 below, we assume that each buyer buys a positive quantity from each seller in the first best allocation:  $q_j^{i*} > 0$  for  $i = 1, 2$  and  $j = A, B$ .<sup>20</sup>

We consider a game with the following timing:

- Stage 0: The buyers decide whether they will form a group or not.
- Stage 1: When there is no buyer group, each seller  $j$  ( $= A, B$ ) simultaneously chooses  $T_j^i(q)$  for  $i = 1, 2$ . When the buyer group is formed, each seller  $j$  ( $= A, B$ ) simultaneously chooses  $T_j^G(q)$ .
- Stage 2: Each buyer  $i$ , or the buyer group, makes purchase decisions.

As we are mainly interested in comparing the outcome of competition under buyer group with the one without group, when we refer to an equilibrium, we mean an equilibrium of the game that starts from Stage 1 with or without buyer group.

We start by proving that each equilibrium is efficient both in the case of buyer group and in the case of no group. This

**Proposition 1** (*efficiency*) *Any equilibrium is efficient regardless of whether or not the buyers form a group.*

This result implies that the total quantity that the buyers buy from each seller does not depend on whether or not the buyers form a group. In other words, the total payoff of the buyers is completely determined by the payments they make to the sellers. Hence,

<sup>18</sup>In principle, the constraints in (2) should be written as  $q_A^1 + q_A^2 \leq q_A^G$ ,  $q_B^1 + q_B^2 \leq q_B^G$ , but since  $U^1, U^2$  are strictly increasing, equality holds in the optimum.

<sup>19</sup>Our assumptions imply that for each maximization problem below, a maximizer exists and is unique since the objective function is strictly concave.

<sup>20</sup>This assumption is not needed in Proposition 1.

if buyer group increases (reduces) the total payoff of the buyers, it is because the buyers pay less (more) for the same total quantity. This creates a natural connection to the literature that studies buyer power defined as size-related discounts.

The result for the case of buyer group is a consequence of Proposition 1 in O'Brien and Shaffer (1997), which implies that in any equilibrium under buyer group, the equilibrium quantities, denoted by  $(q_A^{Ge}, q_B^{Ge})$ , are such that

$$q_A^{Ge} \in \arg \max_{q_A^G} (U^G(q_A^G, q_B^{Ge}) - C_A(q_A^G)), \quad q_B^{Ge} \in \arg \max_{q_B^G} (U^G(q_A^{Ge}, q_B^G) - C_B(q_B^G)).$$

This means that  $q_A^{Ge}$  maximizes the sum of  $G$ 's payoff and firm  $A$ 's profit, given  $q_B^G = q_B^{Ge}$ , and an analogous property holds for  $q_B^{Ge}$ . Therefore,

$$\begin{cases} U_A^G(q_A^{Ge}, q_B^{Ge}) \leq C_A'(q_A^{Ge}), & \text{with equality if } q_A^{Ge} > 0; \\ U_B^G(q_A^{Ge}, q_B^{Ge}) \leq C_B'(q_B^{Ge}), & \text{with equality if } q_B^{Ge} > 0. \end{cases} \quad (5)$$

Since  $U^1$  and  $U^2$  are concave, it follows that  $U^G$  is concave and social welfare  $U^G(q_A^G, q_B^G) - C_A(q_A^G) - C_B(q_B^G)$  is a concave function of  $(q_A^G, q_B^G)$  as we assumed above. The conditions in (5) are the necessary first order conditions for maximization of social welfare, and moreover they are sufficient since social welfare is concave. Hence,  $(q_A^{Ge}, q_B^{Ge})$  maximizes social welfare, that is  $q_A^{Ge} = q_A^{1*} + q_A^{2*}$ ,  $q_B^{Ge} = q_B^{1*} + q_B^{2*}$ .<sup>21</sup>

A similar intuition applies also to the case of no buyer group: We can prove that in any equilibrium, the equilibrium quantities  $(q_A^{1e}, q_B^{1e}, q_A^{2e}, q_B^{2e})$  are such that

$$\begin{cases} (q_A^{1e}, q_B^{2e}) \in \arg \max_{q_A^1, q_B^2} (U^1(q_A^1, q_B^{1e}) + U^2(q_A^2, q_B^{2e}) - C_A(q_A^1 + q_A^2)); \\ (q_B^{1e}, q_B^{2e}) \in \arg \max_{q_B^1, q_B^2} (U^1(q_A^{1e}, q_B^1) + U^2(q_A^{2e}, q_B^2) - C_B(q_B^1 + q_B^2)). \end{cases} \quad (6)$$

This means that  $(q_A^{1e}, q_B^{2e})$  maximizes the sum of the buyers' total payoff and firm  $A$ 's profit, given  $(q_B^1, q_B^2) = (q_B^{1e}, q_B^{2e})$ , and an analogous property holds for  $(q_B^{1e}, q_B^{2e})$ . Since social welfare  $U^1(q_A^1, q_B^1) + U^2(q_A^2, q_B^2) - C_A(q_A^1 + q_A^2) - C_B(q_B^1 + q_B^2)$  is a concave function, it follows from (6) that  $(q_A^{1e}, q_B^{1e}, q_A^{2e}, q_B^{2e})$  maximizes social welfare, that is  $(q_A^{1e}, q_B^{1e}, q_A^{2e}, q_B^{2e}) = (q_A^{1*}, q_B^{1*}, q_A^{2*}, q_B^{2*})$ .

In the next three sections we consider a setting in which both buyers and sellers are symmetric, that is  $U^1(\cdot) = U^2(\cdot) \equiv U(\cdot)$ ,  $U(q_A, q_B) = U(q_B, q_A)$ , and  $C_A(\cdot) = C_B(\cdot) \equiv C(\cdot)$ . In this setting  $q_A^{1*} = q_B^{1*} = q_A^{2*} = q_B^{2*} \equiv q^*$  such that

$$U_j(q^*, q^*) = C'(2q^*) \quad \text{for } j = A, B \quad (7)$$

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<sup>21</sup>O'Brien and Shaffer (1997) exhibit a setting in which an inefficient equilibrium exists, but that setting has discontinuous cost functions for sellers (because of a fixed cost each seller bears for each positive quantity produced). Therefore, the social welfare function is not concave.

and

$$U^G(q_A^G, q_B^G) = 2U\left(\frac{1}{2}q_A^G, \frac{1}{2}q_B^G\right) \quad (8)$$

We call this the *symmetric setting*.

In the symmetric setting, we will illustrate our results through the following example:

$$U(q_A, q_B) = (q_A^{1/2} + q_B^{1/2})^{1/2} \text{ and } C(q) = \frac{1}{2}q^2.$$

This implies

$$U_A(q_A, q_B) = \frac{1}{4q_A^{1/2}(q_A^{1/2} + q_B^{1/2})^{1/2}}$$

and  $q^*$  solves

$$\frac{1}{4q^{1/2}(q^{1/2} + q^{1/2})^{1/2}} = 2q$$

Therefore  $q^* = \frac{1}{4}$ ,  $U(q^*, q^*) = 1$ ,  $C(2q^*) = \frac{1}{8}$ , and  $V_{AB}^G = \frac{7}{4}$ . From (8), we have.

$$U^G(q_A^G, q_B^G) = 2^{3/4}((q_A^G)^{1/2} + (q_B^G)^{1/2})^{1/2}.$$

### 3 Equilibria under buyer group

In this section, we consider the symmetric setting and study the case in which buyers have formed a group  $G$ . Given the symmetric setting, we focus on symmetric equilibria such that each seller offers  $G$  the same tariff  $T_A^G(\cdot) = T_B^G(\cdot) \equiv T^G(\cdot)$ .

Since each equilibrium is efficient (see Proposition 1), we know that  $G$  buys quantity  $2q^*$  [see (7)] from each seller in each equilibrium, and  $G$ 's payoff is  $u^G \equiv U^G(2q^*, 2q^*) - 2T^G(2q^*)$ . In fact,  $u^G$  is also equal to  $\max_{q \geq 0}[U^G(0, q) - T^G(q)]$ , that is the highest payoff  $G$  can obtain by trading only with one seller. This equality holds because if  $u^G < \max_{q \geq 0}[U^G(0, q) - T^G(q)]$ , then buying quantity  $2q^*$  from each seller is not  $G$ 's best choice. If  $u^G > \max_{q \geq 0}[U^G(0, q) - T^G(q)]$ , then seller  $A$  (to fix the ideas) for instance can increase his profit as follows: He makes a take-it-or-leave-it offer to  $G$  with quantity  $2q^*$  and payment  $T^G(2q^*) + \varepsilon$  (with  $\varepsilon > 0$  and small), and if  $G$  buys  $2q^*$  from each seller then his payoff is  $U^G(2q^*, 2q^*) - 2T^G(2q^*) - \varepsilon = u^G - \varepsilon$  which is still larger than the highest payoff  $G$  can make by trading only with seller  $B$ .

For future reference, we let

$$\hat{q} \in \arg \max_{q \geq 0}[U^G(0, q) - T^G(q)], \quad \hat{t}^G \equiv T^G(\hat{q}), \quad t^{G*} \equiv T^G(2q^*). \quad (9)$$

Hence,

$$u^G = U^G(2q^*, 2q^*) - 2t^{G*} = U^G(0, \hat{q}) - \hat{t}^G. \quad (10)$$

Note that  $\hat{q} \geq 2q^*$  should hold since  $2q^* \in \arg \max_{q \geq 0} [U^G(2q^*, q) - T^G(q)]$  and the products are substitutes. In fact, we can prove that  $\hat{q} > 2q^*$  in each equilibrium: see the proof of Lemma 1 below.

Next lemma proves that each equilibrium is characterized by  $(\hat{q}, \hat{t}^G)$ , that is  $G$ 's best choice when trading only with a seller, such that there exists an equivalent equilibrium (in terms of outcome) in which each seller offers only two contracts  $(2q^*, t^{G*})$  and  $(\hat{q}, \hat{t}^G)$ .<sup>22</sup> Note that once  $(\hat{q}, \hat{t}^G)$  is determined,  $t^{G*}$  is determined from (10). This allows to characterize all the equilibrium outcomes by restricting attention to a simple class of tariffs.

**Lemma 1** *Consider the symmetric setting with buyer group. Given any symmetric equilibrium in which each seller offers the same tariff  $T^G$ , there exists an equilibrium in which each seller offers only two contracts  $(2q^*, t^{G*})$  and  $(\hat{q}, \hat{t}^G)$  and of which the outcome is the same as that of the original equilibrium.*

Therefore, from now on we consider the equilibrium in which each seller offers only the two contracts  $(2q^*, t^{G*})$  and  $(\hat{q}, \hat{t}^G)$ . Suppose that seller  $B$  offers the two contracts  $(2q^*, t^{G*})$  and  $(\hat{q}, \hat{t}^G)$  but that seller  $A$  considers deviating with a take-it-or-leave-it offer  $(q_A, t_A)$  to  $G$ . Then  $G$  accepts the offer if and only if  $U^G(q_A, q_B) - t_A - T^G(q_B) \geq u^G$  for at least one  $q_B \in \{0, 2q^*, \hat{q}\}$ . Thus  $U^G(q_A, q_B) - T^G(q_B) - u^G$  is the highest payment seller  $A$  can obtain from  $G$ . Therefore, given  $q_B \in \{0, 2q^*, \hat{q}\}$ , seller  $A$  chooses  $q_A \in \arg \max_{x \geq 0} [U^G(x, q_B) - T^G(q_B) - u^G - C(x)]$ . For this reason, for each  $b \geq 0$  we define

$$f^G(b) \equiv \max_{x \geq 0} (U^G(x, b) - C(x)). \quad (11)$$

Conditional on that seller  $A$  induces  $G$  to buy  $q_B = b$  from seller  $B$ , the deviation generates a joint payoff of  $f^G(b) - T^G(b)$  for  $G$  and seller  $A$  and  $f^G(b) - T^G(b) - u^G$  is the profit of seller  $A$ .

Next proposition identifies the set of equilibrium payoffs for  $G$ .

**Proposition 2** *(Equilibria with Buyer Group) Consider the symmetric setting and suppose that the buyers formed a group. There exists a symmetric equilibrium in which the group's payoff is  $u^G$  if and only if*

$$2f^G(0) - V_{AB}^G \leq u^G \leq V_{AB}^G - 2 \lim_{b \rightarrow +\infty} [f^G(b) - U^G(0, b)]. \quad (12)$$

We below explain how the left hand side and the right hand side in (12) are obtained. First, we notice that  $f^G(0) = \max_{x \geq 0} (U^G(x, 0) - C(x))$  is the maximal social welfare when  $G$  trades only with seller  $A$ , and in any equilibrium the sum of  $G$ 's payoff  $u^G$  and

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<sup>22</sup>This means that each seller chooses a tariff  $\bar{T}^G$  such that  $\bar{T}^G(q)$  is very high if  $q > 0$ ,  $q \neq 2q^*$ ,  $q \neq \hat{q}$ .

$A$ 's profit  $\pi^A$  is not smaller than  $f^G(0)$ . This is so because if  $u^G + \pi^A < f^G(0)$ , then seller  $A$  can deviate by offering to  $G$  a suitable trade in which  $G$  buys only from  $A$  – in what follows, we call this the *deviation with  $q_B^G = 0$*  – and their joint payoff increases to  $f^G(0)$ . Therefore, in each equilibrium the profit of seller  $B$ ,  $\pi^B$ , is not larger than the maximal social welfare  $V_{AB}^G$  (when  $G$  trades with both sellers, defined in (4)) minus  $f^G(0)$ , that is  $\pi^B \leq V_{AB}^G - f^G(0)$ . Likewise,  $\pi^A \leq V_{AB}^G - f^G(0)$  and these upper bounds for  $\pi^A, \pi^B$  yield a lower bound for  $u^G$  equal to  $V_{AB}^G - (V_{AB}^G - f^G(0)) - (V_{AB}^G - f^G(0))$ , which is the left hand side in (12).

Precisely, the lower bound  $u^G = 2f^G(0) - V_{AB}^G$  is achieved if  $\hat{q} = \arg \max_{x \geq 0} [U^G(0, x) - C(x)]$ ,  $\hat{t}^G = C(\hat{q}) + V_{AB}^G - f^G(0)$ , and  $t^{G*} = C(2q^*) + V_{AB}^G - f^G(0)$ . The agents' payoffs in this equilibrium are the same as in the sell-out equilibrium (Bernheim and Whinston, 1998). In fact, there is another possible deviation for seller  $A$ , which induces  $G$  to buy quantity  $\hat{q}$  ( $> 2q^*$ ) from seller  $B$  and  $q$  from  $A$ , with  $q$  suitably chosen – in the following, we call this the *deviation with  $q_B^G = \hat{q}$* . This generates a total payoff of  $f^G(\hat{q}) - \hat{t}^G$  for  $G$  and seller  $A$ , hence  $u^G + \pi^A \geq f^G(\hat{q}) - \hat{t}^G$  must hold in any equilibrium, which implies that  $\pi^B \leq V_{AB}^G - (f^G(\hat{q}) - \hat{t}^G)$  must hold. But the latter is necessarily a weaker restriction on  $\pi^B$  than  $\pi^B \leq V_{AB}^G - f^G(0)$ , otherwise we would conclude that in any equilibrium  $u^G$  is strictly greater than  $2f^G(0) - V_{AB}^G$ , which contradicts the existence of the sell-out equilibrium. Therefore, the lower bound for  $u^G$  is determined by the deviation with  $q_B^G = 0$ .

About the right hand side in (12), the deviation of seller  $A$  with  $q_B^G = \hat{q}$  yields  $A$  the profit  $f^G(\hat{q}) - \hat{t}^G - u^G = f^G(\hat{q}) - U^G(0, \hat{q})$ , where the equality is from  $\hat{t}^G = U^G(0, \hat{q}) - u^G$  in (10), hence  $\pi^A \geq f^G(\hat{q}) - U^G(0, \hat{q})$  and  $\pi^B \geq f^G(\hat{q}) - U^G(0, \hat{q})$ . Since  $\pi^A + \pi^B + u^G = V_{AB}^G$ , we obtain  $u^G \leq V_{AB}^G - 2[f^G(\hat{q}) - U^G(0, \hat{q})]$ , which generates an upper bound. Moreover, substitute goods imply that  $f^G(\hat{q}) - U^G(0, \hat{q})$  is decreasing in  $\hat{q}$ : As  $\hat{q}$  (bought from seller  $B$ ) increases, the goods offered by seller  $A$  are less valuable to  $G$  and  $A$ 's profit from the deviation is reduced.<sup>23</sup> Hence, for this upper bound the supremum is obtained by letting  $\hat{q}$  tend to infinity. In fact, seller  $A$  may deviate also with  $q_B^G = 0$  and earn the profit  $f^G(0) - u^G$ , but this is smaller than  $f^G(\hat{q}) - U^G(0, \hat{q})$  when  $u^G = V_{AB}^G - 2[f^G(\hat{q}) - U^G(0, \hat{q})]$ , that is

$$f^G(0) - V_{AB}^G + 2[f^G(\hat{q}) - U^G(0, \hat{q})] < f^G(\hat{q}) - U^G(0, \hat{q}). \quad (13)$$

Therefore, the upper bound for  $u^G$  is determined by the deviation with  $q_B^G = \hat{q}$ , as  $\hat{q} \rightarrow +\infty$ .<sup>24</sup>

<sup>23</sup>Formally,  $U^G(x, b) - U^G(0, b) - C(x) = \int_0^x (U_A^G(z, b) - C'(z)) dz$  is decreasing in  $b$ , given that goods are substitutes. Hence,  $\max_{x \geq 0} [U^G(x, b) - U^G(0, b) - C(x)]$  is decreasing in  $b$ .

<sup>24</sup>It is simple to verify that (13) is equivalent to  $f^G(\hat{q}) + f^G(0) < V_{AB}^G + U^G(0, \hat{q})$ , which coincides with  $\max_{x, y} [U^G(x, \hat{q}) + U^G(0, y) - C(x) - C(y)] < \max_{x, y} [U^G(x, y) + U^G(0, \hat{q}) - C(x) - C(y)]$ . The latter inequality holds as long as  $\hat{q}$  is larger than  $\arg \max_y [U^G(0, y) - C(y)]$ , since then strict substitute goods imply  $U^G(x, \hat{q}) - U^G(0, \hat{q}) < U^G(x, y) - U^G(0, y)$ , that is  $U^G(x, \hat{q}) + U^G(0, y) < U^G(x, y) + U^G(0, \hat{q})$ .

**Remark** It is interesting to notice that even if we consider competition between differentiated products, it may result in the outcome of zero profit for the sellers. Namely, in some cases we find that

$$\lim_{b \rightarrow +\infty} [f^G(b) - U^G(0, b)] = 0 \quad (14)$$

and then, for each  $\varepsilon > 0$ , there exists an equilibrium such that each seller's profit is smaller than  $\varepsilon$ , and  $G$ 's payoff is larger than  $V_{AB}^G - 2\varepsilon$ . For instance if

$$\lim_{b \rightarrow +\infty} U_A^G(0, b) < \inf_{x \geq 0} C'(x) \quad (15)$$

then the effect of substitute goods is sufficiently strong to make the optimal  $x$  in the maximization problem in (11) equal to 0 for large values of  $b$ . Therefore  $f^G(b) = U^G(0, b)$  for each large  $b$  and (14) holds; hence the right hand side in (12) is  $V_{AB}^G$ .

**Example** Consider the example of  $U(q_A, q_B) = (q_A^{1/2} + q_B^{1/2})^{1/2}$  and  $C(q) = \frac{1}{2}q^2$ , with  $q^* = \frac{1}{4}$ ,  $U(q^*, q^*) = 1$ ,  $V_{AB}^G = \frac{7}{4} = 1.75$ . Since  $U^G(q_A^G, q_B^G) = 2^{3/4}((q_A^G)^{1/2} + (q_B^G)^{1/2})^{1/2}$ , we have  $f^G(0) = \max_{x \geq 0} (2^{3/4}x^{1/4} - \frac{1}{2}x^2) = 1.300245$ ; hence the lower bound for  $u^G$  is  $2f^G(0) - V_{AB}^G = 0.8505$ . Regarding the upper bound, we find

$$f^G(b) - U^G(0, b) = \max_{x \geq 0} [2^{3/4}(x^{1/2} + b^{1/2})^{1/2} - \frac{1}{2}x^2 - 2^{3/4}b^{1/4}] = \max_{x \geq 0} [\frac{2^{3/4}x^{1/2}}{(x^{1/2} + b^{1/2})^{1/2} + b^{1/4}} - \frac{1}{2}x^2]$$

For each fixed  $x \geq 0$ , we have that  $\frac{2^{3/4}x^{1/2}}{(x^{1/2} + b^{1/2})^{1/2} + b^{1/4}}$  tends to 0 as  $b \rightarrow +\infty$ , and we can use this to prove that  $\lim_{b \rightarrow +\infty} (f^G(b) - U^G(0, b)) = 0$ . Therefore, the interval for  $u^G$  is

$$[0.8505, 1.75].^{25}$$

For each  $u^G$  in this interval, we find  $t^{G*}$  from  $2 - 2t^{G*} = u^G$ , that is  $t^{G*} = 1 - u^G/2$ , and  $\pi^G = [V_{AB}^G - u^G]/2 = 7/8 - u^G/2$ . For the contract  $(\hat{q}, \hat{t}^G)$  we need to determine  $\hat{q}$  such that  $f^G(\hat{q}) - U^G(0, \hat{q}) \leq \pi^G$ , in order to make unprofitable a seller's deviation which induces  $G$  to buy  $\hat{q}$  from the other seller.<sup>26</sup> Such a  $\hat{q}$  exists since  $\pi^G > 0$  and  $\lim_{b \rightarrow +\infty} (f^G(b) - U^G(0, b)) = 0$ . For instance, if  $u^G = 1.2$  then  $t^{G*} = 0.4$ ,  $\pi^G = 0.275$  and a suitable  $\hat{q}$  is  $\hat{q} = 4$ , since  $f^G(4) - U^G(0, 4) = 0.2703 < 0.275$ .<sup>27</sup> Hence,  $\hat{t}^G = U^G(0, 4) - 1.2 = 1.1784$  and the pair of strategies in which each seller offers the two contracts  $(2q^*, t^{G*}) = (\frac{1}{2}, 0.4)$  and  $(\hat{q}, \hat{t}^G) = (4, 1.1784)$  is an equilibrium in which  $G$ 's payoff is equal to 1.2.

<sup>25</sup>In Proposition 2 we have written  $u^G \leq V_{AB}^G - 2 \lim_{b \rightarrow +\infty} [f^G(b) - U^G(0, b)]$ , with a weak inequality, because for a fixed  $b$ ,  $V_{AB}^G - 2[f^G(b) - U^G(0, b)]$  is the highest possible payoff for  $G$ . But in the example,  $\lim_{b \rightarrow +\infty} (V_{AB}^G - 2[f^G(b) - U^G(0, b)]) = 1.75$  is greater than  $V_{AB}^G - 2[f^G(b) - U^G(0, b)]$  for each  $b$ . Hence 1.75 is not an equilibrium value for  $u^G$ .

<sup>26</sup>The deviation of a seller which induces  $G$  not to buy from the other seller yields a profit of  $f^G(0) - u^G = 1.300245 - u^G$ , which is smaller than  $\frac{7}{8} - \frac{1}{2}u^G$  since  $u^G \geq 0.8505$ .

<sup>27</sup>Also any  $\hat{q}$  greater than 4 is suitable since  $f^G(\hat{q}) - U^G(0, \hat{q})$  is decreasing in  $\hat{q}$ .

## 4 Equilibria without buyer group

In this section, we keep considering the symmetric setting and study the case in which buyers have not formed any group. We still focus on symmetric equilibria in which each seller offers the same tariff  $T(q)$  to each buyer.

Before we characterize the set of equilibria, we note an important difference between the case of buyer group and the case of no group. Under buyer group, every quantity  $G$  buys from either seller is split equally between the buyers: see (8). In contrast, when there is no group, seller  $A$  can make different offers to different buyers: we call them *discriminatory offers*. These offers may induce buyer 1 to buy from seller  $B$  a quantity which is different from the quantity buyer 2 buys from seller  $B$ . At an extreme, discriminatory offers of seller  $A$  may induce buyer 1 to buy exclusively from seller  $A$ , and buyer 2 to buy a positive quantity from seller  $B$ . In equilibrium, no seller makes any discriminatory offer, but the possibility to deviate using discriminatory offers is the main difference with respect to the setting in which the buyers formed a group. We prove below that this affects the set of equilibria under no group if the cost functions are strictly convex, but not if the cost functions are concave (which includes the case of linear costs).

Since each equilibrium is efficient (see Proposition 1), we know that each buyer buys quantity  $q^*$  [defined in (7)] from each seller in equilibrium, and the payoff of each buyer is  $u \equiv U(q^*, q^*) - 2T(q^*)$ . In fact, we can argue like in the previous section to prove that  $u$  is also equal to  $\max_{q \geq 0} [U(0, q) - T(q)]$ , the highest payoff a buyer can make when trading only with one seller. For future reference we let

$$\tilde{q} \in \arg \max_{q \geq 0} [U(0, q) - T(q)], \quad \tilde{t} \equiv T(\tilde{q}), \quad t^* \equiv T(q^*). \quad (16)$$

Hence, we have

$$u = U(q^*, q^*) - 2t^* = U(0, \tilde{q}) - \tilde{t} \quad (17)$$

Note that  $\tilde{q} \geq q^*$  should hold since  $q^* \in \arg \max_{q \geq 0} [U(q^*, q) - T(q)]$  and the products are substitutes. In fact, we can prove that  $\tilde{q} > q^*$  must hold in any equilibrium. As in the case of buyer group, it is useful to show that without loss of generality, we can restrict our attention to equilibria in which each seller offers only two contracts to each buyer. This allows to restrict attention to a class of simple tariffs.

**Lemma 2** *Consider the symmetric setting without buyer group. Given any symmetric equilibrium in which each seller offers the same tariff  $T$  to each buyer, there exists an equilibrium in which each seller offers only two contracts  $(q^*, t^*)$  and  $(\tilde{q}, \tilde{t})$  to each buyer and which generates the same outcome as the original equilibrium.*

Suppose now that seller  $B$  offers two contracts  $(q^*, t^*)$  and  $(\tilde{q}, \tilde{t})$  to each buyer, but seller  $A$  considers a deviation which consists of making a take-it-or-leave-it offer  $(q_A^i, t_A^i)$

to buyer  $i$ , for  $i = 1, 2$ . Then we can argue as in the case of buyer group to show that given that seller A induces buyer 1 to buy  $q_B^1 = b^1 \in \{0, q^*, \tilde{q}\}$  and buyer 2 to buy  $q_B^2 = b^2 \in \{0, q^*, \tilde{q}\}$  from seller B, seller A chooses  $(q_A^1, q_A^2)$  in  $\arg \max_{x^1 \geq 0, x^2 \geq 0} [U(x^1, b^1) + U(x^2, b^2) - C(x^1 + x^2)]$ . For this reason, for each  $b^1 \geq 0, b^2 \geq 0$  we define

$$f(b^1, b^2) = \max_{x^1 \geq 0, x^2 \geq 0} [U(x^1, b^1) + U(x^2, b^2) - C(x^1 + x^2)] \quad (18)$$

and  $f(b^1, b^2) - T(b^1) - T(b^2) - 2u$  is the profit of seller A given that he induces the buyers to buy  $(q_B^1, q_B^2) = (b^1, b^2)$  from seller B. Notice that  $f$  is symmetric in its arguments, that is  $f(b^1, b^2) = f(b^2, b^1)$ , and it satisfies the following property.

**Lemma 3** *Given  $\underline{b}, \bar{b}$  such that  $0 \leq \underline{b} < \bar{b}$ , inequality (19) holds if  $C$  is strictly convex, whereas inequality (20) holds if  $C$  is concave:*

$$f(\bar{b}, \bar{b}) - f(\bar{b}, \underline{b}) < f(\underline{b}, \bar{b}) - f(\underline{b}, \underline{b}); \quad (19)$$

$$f(\bar{b}, \bar{b}) - f(\bar{b}, \underline{b}) \geq f(\underline{b}, \bar{b}) - f(\underline{b}, \underline{b}). \quad (20)$$

In order to explain Lemma 3, we first focus on the inequality (19), which is established in the case of strictly convex  $C$ . When  $b^2$  increases from  $\underline{b}$  to  $\bar{b}$ , the value of  $f$  increases, but the magnitude of increase depends on whether  $b^1 = \bar{b}$  or  $b^1 = \underline{b}$ . Precisely, (19) reveals that the increase is smaller if  $b^1 = \bar{b}$  than if  $b^1 = \underline{b}$ , that is the return from increasing  $b^2$  is smaller the greater is  $b^1$ . This result is determined by the marginal utility of good B for buyer 2, as we now illustrate.

Regarding  $f(\underline{b}, \underline{b})$  on the right hand side of (19), notice that when  $(b^1, b^2) = (\underline{b}, \underline{b})$ , the optimal  $x^1$  and  $x^2$  in the maximization problem (18) have the same value, denoted by  $\underline{x}$ . Regarding  $f(\bar{b}, \underline{b})$  on the left hand side of (19), notice that when  $(b^1, b^2) = (\bar{b}, \underline{b})$ , the marginal utility of good A for buyer 1 is lower than when  $(b^1, b^2) = (\underline{b}, \underline{b})$ , given substitutes. This implies that A sells a lower quantity to buyer 1 and this output contraction together with  $C$  strictly convex implies that the marginal cost for the choice of  $x^2$  becomes lower; hence the optimal  $x^1, x^2$ , denoted by  $\bar{x}^1, \bar{x}^2$ , are such that  $\bar{x}^1 < \underline{x} < \bar{x}^2$ . From  $\underline{x} < \bar{x}^2$ , it follows that the marginal utility of good B for buyer 2 when  $(b^1, b^2) = (\bar{b}, \underline{b})$  is smaller than when  $(b^1, b^2) = (\underline{b}, \underline{b})$ , hence (19) is proved by applying the Envelope Theorem.

A similar argument applies when  $C$  is concave, because then the output contraction relative to  $x^1$  together with  $C$  concave implies that the marginal cost for the choice of  $x^2$  becomes (at least weakly) higher, which results in  $\bar{x}^1 < \underline{x}, \bar{x}^2 \leq \underline{x}$ . Therefore, the marginal utility of good B for buyer 2 when  $(b^1, b^2) = (\bar{b}, \underline{b})$  is at least as large as when  $(b^1, b^2) = (\underline{b}, \underline{b})$ , which implies (20).

Lemma 4 and 5 below rely on Lemma 3 to prove that some deviation is less powerful or more powerful than other deviations depending on the curvature of  $C$ . It is also useful

to know that (19) is equivalent to

$$f(\bar{b}, \bar{b}) - f(\underline{b}, \underline{b}) < 2 [f(\underline{b}, \bar{b}) - f(\underline{b}, \underline{b})];$$

and (20) is equivalent to

$$f(\bar{b}, \bar{b}) - f(\underline{b}, \underline{b}) \geq 2 [f(\underline{b}, \bar{b}) - f(\underline{b}, \underline{b})].$$

Given  $u$ , each seller's profit is  $2t^* - C(2q^*)$ , which is equal to  $f(q^*, q^*) - U(q^*, q^*) - u$  [from (17) and (18)]. In equilibrium, this profit should be higher than any deviation profit. Let (IC ( $b^1, b^2$ )) represent the incentive constraint that makes it unprofitable for seller  $j$  ( $= A, B$ ) to deviate by inducing the buyers to buy  $(q_k^1, q_k^2) = (b^1, b^2)$  from seller  $k$  ( $= A, B$ ) with  $j \neq k$ ,  $b^1 \in \{0, q^*, \tilde{q}\}$ ,  $b^2 \in \{0, q^*, \tilde{q}\}$  and  $(b^1, b^2) \neq (q^*, q^*)$ :

$$\text{(IC } (b^1, b^2)) \quad f(q^*, q^*) - U(q^*, q^*) - u \geq f(b^1, b^2) - T(b^1) - T(b^2) - 2u.$$

We start by considering two deviations that determine a lower bound of  $u$ . First, the deviation which induces both buyers to buy exclusively from seller  $j$  yields a profit of  $f(0, 0) - 2u$ . Therefore,  $f(q^*, q^*) - U(q^*, q^*) - u \geq f(0, 0) - 2u$  must be satisfied, which is equivalent to

$$\text{(IC } (0, 0)) \quad f(0, 0) - f(q^*, q^*) + U(q^*, q^*) \leq u. \quad (21)$$

Second, the deviation which induces one buyer to buy exclusively from  $j$  and the other buyer to buy  $q^*$  from  $k$  yields  $j$  a profit of  $f(0, q^*) - 2u - t^*$ . Therefore,  $f(q^*, q^*) - U(q^*, q^*) - u \geq f(0, q^*) - 2u - t^*$  must be satisfied, which is equivalent to

$$\text{(IC } (0, q^*)) \quad 2f(0, q^*) - 2f(q^*, q^*) + U(q^*, q^*) \leq u. \quad (22)$$

The second deviation involves a discriminatory offer, and now we inquire when this strategy has a bite, in the sense that it is stronger than the deviation inducing both buyers to buy exclusively from  $j$ . To this purpose, we use Lemma 3 with  $\underline{b} = 0 < \bar{b} = q^*$ :

**Lemma 4** *Consider the symmetric setting without buyer group.*

(i) *Suppose that  $C$  is strictly convex. Then, if (IC ( $0, q^*$ )) is satisfied, (IC ( $0, 0$ )) is satisfied as well:  $f(q^*, q^*) - f(0, 0) < 2 [f(0, q^*) - f(0, 0)]$*

(ii) *Suppose that  $C$  is concave. Then, if (IC ( $0, 0$ )) is satisfied, (IC ( $0, q^*$ )) is satisfied as well:  $f(q^*, q^*) - f(0, 0) \geq 2 [f(0, q^*) - f(0, 0)]$ .*

Let us consider seller  $A$ 's deviations. Both deviations we are focusing on require seller  $A$  to produce more output than without the deviations but the deviation which induces both buyers to make exclusive purchases from  $A$  requires a greater increase in output than the other deviation. When costs are convex (concave), marginal cost is increasing

(decreasing) and hence inducing a first buyer to buy exclusively involves a smaller (larger) increase in cost per output than inducing a second buyer to buy exclusively. Therefore, when costs are strictly convex, inducing both buyers to make exclusive purchases is not profitable if inducing only one buyer to make exclusive purchase is not profitable. Symmetrically, when costs are concave, inducing only one buyer to make exclusive purchase is not profitable if inducing both buyers to make exclusive purchase is not profitable.

Lemma 4 implies the following lower bound on  $u$  depending on whether  $C$  is convex or concave:

$$\begin{aligned} 2f(0, q^*) - 2f(q^*, q^*) + U(q^*, q^*) &\leq u && \text{if } C \text{ is strictly convex;} \\ f(0, 0) - f(q^*, q^*) + U(q^*, q^*) &\leq u && \text{if } C \text{ is concave.} \end{aligned} \quad (23)$$

Now we consider two different deviations which generate an upper bound for  $u$ . First, the deviation which induces buyer 1 to buy  $q^*$  and buyer 2 to buy  $\tilde{q} (> q^*)$  from  $B$  yields seller  $A$  the profit  $f(q^*, \tilde{q}) - t^* - \tilde{t} - 2u$ . Therefore,  $f(q^*, q^*) - U(q^*, q^*) - u \geq f(q^*, \tilde{q}) - t^* - \tilde{t} - 2u$  must be satisfied, which is equivalent to

$$(\text{IC } (q^*, \tilde{q})) \quad u \leq 2f(q^*, q^*) - U(q^*, q^*) - 2[f(q^*, \tilde{q}) - U(0, \tilde{q})].$$

Second, the deviation which induces each buyer to buy  $\tilde{q}$  from  $B$  yields seller  $A$  profit  $f(\tilde{q}, \tilde{q}) - 2\tilde{t} - 2u$ . Therefore,  $f(q^*, q^*) - U(q^*, q^*) - u \geq f(\tilde{q}, \tilde{q}) - 2\tilde{t} - 2u$  must be satisfied, which is equivalent to

$$(\text{IC } (\tilde{q}, \tilde{q})) \quad u \leq f(q^*, q^*) - U(q^*, q^*) - [f(\tilde{q}, \tilde{q}) - 2U(0, \tilde{q})].$$

In order to inquire which deviation is stronger, we use again Lemma 3, this time with  $\underline{b} = q^* < \bar{b} = \tilde{q}$ :

**Lemma 5** *Consider the symmetric setting without buyer group.*

(i) *Suppose that  $C$  is strictly convex. Then, if  $(\text{IC } (q^*, \tilde{q}))$  is satisfied,  $(\text{IC } (\tilde{q}, \tilde{q}))$  is satisfied as well:  $f(\tilde{q}, \tilde{q}) - f(q^*, q^*) < 2[f(q^*, \tilde{q}) - f(q^*, q^*)]$ .*

(ii) *Suppose that  $C$  is concave. Then, if  $(\text{IC } (\tilde{q}, \tilde{q}))$  is satisfied,  $(\text{IC } (q^*, \tilde{q}))$  is satisfied as well:  $f(\tilde{q}, \tilde{q}) - f(q^*, q^*) \geq 2[f(q^*, \tilde{q}) - f(q^*, q^*)]$ .*

Both deviations we are considering require seller  $A$  to produce less output than without the deviations, and the deviation which induces each buyer to buy quantity  $\tilde{q}$  from seller  $B$  is the deviation involving a higher reduction in output than the other deviation. When costs are convex (concave), the marginal saving in cost from output contraction decreases (increases) as the amount of output reduction increases. Therefore, when costs are convex, the deviation inducing  $(q_B^1, q_B^2) = (\tilde{q}, \tilde{q})$  generates a lower saving per reduced output than the deviation inducing  $(q_B^1, q_B^2) = (q^*, \tilde{q})$ , which implies that if the latter deviation is

unprofitable, then the former deviation is unprofitable as well. A symmetric argument applies when costs are concave, which implies that if the deviation inducing  $(q_B^1, q_B^2) = (\tilde{q}, \tilde{q})$  is unprofitable, the deviation inducing  $(q_B^1, q_B^2) = (q^*, \tilde{q})$  is unprofitable.

Lemma 5 implies the following upper bound on  $u$  depending on whether  $C$  is convex or concave:

$$\begin{aligned} u &\leq 2f(q^*, q^*) - U(q^*, q^*) - 2[f(q^*, \tilde{q}) - U(0, \tilde{q})] && \text{if } C \text{ is strictly convex;} \\ u &\leq f(q^*, q^*) - U(q^*, q^*) - [f(\tilde{q}, \tilde{q}) - 2U(0, \tilde{q})] && \text{if } C \text{ is concave.} \end{aligned} \quad (24)$$

From (23) and (24) we derive an interval of values for  $u$  which depends on the curvature of  $C$ . In fact, before we can conclude that this interval is the set of equilibrium values for each buyer's payoff, we need to verify that, given  $(t^*, \hat{t})$  satisfying (17), each buyer wants to buy quantity  $q^*$  from each seller and we must establish the unprofitability of any deviation of each seller. The proof of the next proposition takes care of these issues and, in addition, establishes that each of the two right hand sides in (24) is increasing in  $\tilde{q}$ .

**Proposition 3** (*Equilibrium without buyer group*) Consider the symmetric setting without buyer group.

(a) Suppose that  $C$  is strictly convex. There exists a symmetric equilibrium in which each buyer's payoff is  $u$  if and only if

$$U(q^*, q^*) - 2f(q^*, q^*) + 2f(0, q^*) \leq u \leq 2f(q^*, q^*) - U(q^*, q^*) - 2 \lim_{b \rightarrow +\infty} [f(q^*, b) - U(0, b)] \quad (25)$$

(b) Suppose that  $C$  is concave. There exists a symmetric equilibrium in which each buyer's payoff is  $u$  if and only if

$$f(0, 0) - f(q^*, q^*) + U(q^*, q^*) \leq u \leq f(q^*, q^*) - U(q^*, q^*) - \lim_{b \rightarrow +\infty} [f(b, b) - 2U(0, b)] \quad (26)$$

**Example** Consider again the example of  $U(q_A, q_B) = (q_A^{1/2} + q_B^{1/2})^{1/2}$  and  $C(q) = \frac{1}{2}q^2$ , with  $U(q^*, q^*) = 1$ ,  $C(2q^*) = \frac{1}{8}$ . Since  $C$  is strictly convex, the interval of values for  $u$  is (25). Since  $f(q^*, q^*) = \frac{15}{8}$  and  $f(0, q^*) = \max_{x \geq 0, y \geq 0} [x^{1/4} + (y^{1/2} + \frac{1}{2})^{1/2} - \frac{1}{2}(x + y)^2] = 1.5913$ , the lower bound for  $u$  is 0.4326.

Regarding the upper bound for  $u$ , we find that

$$\begin{aligned} f(q^*, b) - U(0, b) &= \max_{x \geq 0, y \geq 0} [(x^{1/2} + \frac{1}{2})^{1/2} + (y^{1/2} + b^{1/2})^{1/2} - \frac{1}{2}(x + y)^2 - b^{1/4}] \\ &= \max_{x \geq 0, y \geq 0} [(x^{1/2} + \frac{1}{2})^{1/2} - \frac{1}{2}(x + y)^2 + \frac{y^{1/2}}{(y^{1/2} + b^{1/2})^{1/2} + b^{1/4}}] \end{aligned} \quad (27)$$

and  $\lim_{b \rightarrow +\infty} (f(q^*, b) - U(0, b)) = \max_{x \geq 0} [(x^{1/2} + \frac{1}{2})^{1/2} - \frac{1}{2}x^2] = 0.98442$ .

Hence the interval for  $u$  is

$$[0.4326, 0.78116)$$

## 5 Comparison between buyer group and no group

In this section, we still consider the symmetric setting and compare the equilibria under buyer group and those without group. In the case of buyer group, if  $u^G$  is the group's payoff then each buyer's payoff is  $u^G/2$ , hence the bounds in (12) below for the case of group can be expressed in terms of  $u$  as in (28), where we exploit the equalities  $f^G(0) = f(0, 0)$  and  $\lim_{b \rightarrow +\infty} [f^G(b) - U^G(0, b)] = \lim_{b \rightarrow +\infty} [f(b, b) - 2U(0, b)]$

$$f(0, 0) - \frac{1}{2}V_{AB}^G \leq u \leq \frac{1}{2}V_{AB}^G - \lim_{b \rightarrow +\infty} [f(b, b) - 2U(0, b)] \quad (28)$$

Since  $\frac{1}{2}V_{AB}^G = U(q^*, q^*) - C(2q^*) = f(q^*, q^*) - U(q^*, q^*)$ , we see that the interval of payoff in (28) is the same as the interval for no group we previously identified in (26). Therefore, when  $C$  is concave (which includes the case of linear  $C$ ), the set of equilibrium outcomes is unaffected by group formation. Matters are different when  $C$  is strictly convex, as next proposition establishes

**Proposition 4** *Consider the symmetric setting.*

(a) *If  $C$  is concave, then the interval of equilibrium payoffs for each buyer is the same regardless of whether or not buyers form a group.*

(b) *If  $C$  is strictly convex, then the interval of equilibrium payoffs for each buyer under no group is a strict subset of the interval of equilibrium payoffs for each buyer under buyer group. Formally,*

$$f(0, 0) - \frac{1}{2}V_{AB}^G < U(q^*, q^*) - 2f(q^*, q^*) + 2f(0, q^*) \quad (29)$$

$$2f(q^*, q^*) - U(q^*, q^*) - 2 \lim_{b \rightarrow +\infty} [f(q^*, b) - U(0, b)] < \frac{1}{2}V_{AB}^G - \lim_{b \rightarrow +\infty} [f(b, b) - 2U(0, b)] \quad (30)$$

Buyer group has no effect on the interval of the equilibrium payoffs for the buyers when  $C$  is concave, but if  $C$  is strictly convex, from the buyers' point of view, the payoff of the worst equilibrium without group is higher than the payoff of the worst equilibrium under buyer group, while the payoff of the best equilibrium without group is lower than the payoff of the best equilibrium under buyer group.

In order to see why these differences arise, recall from Section 3 that the lower bound for  $u^G$  is determined by seller  $A$ 's deviation with  $q_B^G = 0$ , which is equivalent to the deviation of seller  $A$  under no group with  $q_B^1 = 0, q_B^2 = 0$ . The latter deviation implies  $2u + \pi^A \geq f(0, 0)$  in any equilibrium, hence  $\pi^B \leq V_{AB}^G - f(0, 0)$ . But under no group, seller  $A$  can also deviate with  $q_B^1 = 0, q_B^2 = q^*$ . This generates a total payoff for buyers and seller  $A$  equal to  $f(0, q^*) - t^*$ , hence  $2u + \pi^A \geq f(0, q^*) - t^*$ . When  $u = f(0, 0) - \frac{1}{2}V_{AB}^G$ , we find that  $f(0, 0) \geq f(0, q^*) - t^*$  if  $C$  is concave, but  $f(0, q^*) - t^* > f(0, 0)$  if  $C$  is strictly

convex. Therefore, for concave  $C$  the lower bound in (26) is determined by the deviation which induces both buyers to make exclusive purchase from the deviating seller, as under buyer group. Conversely, when  $C$  is strictly convex the inequality  $f(0, q^*) - t^* > f(0, 0)$  implies for  $\pi^B$  (and for  $\pi^A$ ) that  $\pi^B \leq V_{AB}^G - (f(0, q^*) - t^*) < V_{AB}^G - f(0, 0)$ , therefore the lower bound in (25) is greater than the lower bound in (28), as (29) establishes.

In Section 3 we have also explained that under buyer group, the best equilibrium for buyers is determined by seller  $A$ 's deviation with  $q_B^G = \hat{q}$  (for  $\hat{q} \rightarrow +\infty$ ). This is analogous to seller  $A$ 's deviation under no group with  $q_B^1 = \tilde{q}$ ,  $q_B^2 = \tilde{q}$  (for  $\tilde{q} \rightarrow +\infty$ ) which yields seller  $A$  the profit  $f(\tilde{q}, \tilde{q}) - 2\tilde{t} - 2u$ . But under no group, seller  $A$  can also deviate with  $q_B^1 = \tilde{q}$ ,  $q_B^2 = q^*$ , which yields  $A$  the profit  $f(\tilde{q}, q^*) - \tilde{t} - t^* - 2u$ . When  $u = \frac{1}{2}V_{AB}^G - [f(\tilde{q}, \tilde{q}) - 2U(0, \tilde{q})]$ , we have  $f(\tilde{q}, \tilde{q}) - 2\tilde{t} - 2u \geq f(\tilde{q}, q^*) - \tilde{t} - t^* - 2u$  if  $C$  is concave, but  $f(\tilde{q}, \tilde{q}) - 2\tilde{t} - 2u < f(\tilde{q}, q^*) - \tilde{t} - t^* - 2u$  if  $C$  is strictly convex. Therefore, for concave  $C$  the upper bound in (26) is determined by the deviation which induces each buyer to buy from the non-deviating seller a quantity higher than the equilibrium quantity, just like in the deviation which identifies the upper bound in (28) under buyer group. Conversely, when  $C$  is strictly convex the inequality  $f(\tilde{q}, \tilde{q}) - 2\tilde{t} - 2u < f(\tilde{q}, q^*) - \tilde{t} - t^* - 2u$  increases the lower bound for  $\pi^A$ , hence decreases the upper bound for  $u$  as (30) establishes.

As we remarked in Section 4, the difference between buyer group and no group lies in the possibility for sellers to use discriminatory offers when there is no group. But when  $C$  is concave that has no effect as the deviations that determine the bounds for the equilibrium payoffs do not involve discriminatory offers. Conversely, when  $C$  is strictly convex, discriminatory offers have multiple effects: (i) they intensify competition between sellers at their most favorable equilibrium, thus increasing  $u$  in the worst equilibrium for buyers; (ii) they make more effective the deviation which determines the sellers' lowest equilibrium profit, thus reducing the buyers' highest payoff. As a result, under no buyer group the bounds for  $u$  are tighter. Notice that if discriminatory offers were infeasible, then the interval of equilibrium payoffs for each buyer would be given by (28), that is (26), even when  $C$  is strictly convex.<sup>28</sup>

**Example** Consider the example of  $U(q_A, q_B) = (q_A^{1/2} + q_B^{1/2})^{1/2}$  and  $C(q) = \frac{1}{2}q^2$ . Under the buyer group,  $u^G/2$  belongs to  $[0.4252, 0.875)$ , whereas without buyer group,  $u$  belongs to  $[0.4326, 0.78116)$ . Consistently with Proposition 4(b), given that  $C$  is strictly convex, buyer group reduces the lower bound of each buyer's payoff and increases the upper bound.

<sup>28</sup>Our results depend on the assumption that goods are *strict* substitutes. If instead products have independent values (i.e., there exists a  $\bar{U} : [0, +\infty) \rightarrow \mathbb{R}$  such that  $U(q_A, q_B) = \bar{U}(q_A) + \bar{U}(q_B)$ ), then the lower and upper bounds in (25), (26), (28) are all equal to 0, independently of the curvature of  $C$ . This occurs because each seller can act like a perfectly discriminating monopolist, as each buyer's valuation for seller  $j$ 's product does not depend on the quantity the buyer buys from the other seller. Therefore, seller  $j$  can fully extract each buyer's surplus regardless of whether or not buyers form a group.

In terms of the percentage with respect to the payoff without group, buyer group reduces the lower bound by 1.7 percent and increases the upper bound by 12 percent.

**Remark** The multiplicity of equilibria, both with and without buyer group, makes it not straightforward to derive clear predictions about the effect of forming a buyer group. When  $C$  is concave, buyer group has no effect on the set of equilibrium payoffs and hence we can say that buyer group has no effect no matter the equilibrium selection. When  $C$  is strictly convex, we notice that one natural and widely applied selection rule among multiple equilibria is the one which focuses on Pareto optimal equilibria for the players. In the game which starts at stage one (after either decision of the buyers about group formation at stage zero), the players are sellers  $A$  and  $B$ , and both in the case of group formation and of no group, there exists a unique Pareto optimal equilibrium for sellers. This is the equilibrium which corresponds to the lowest  $u^G$  in (12), and to the lowest  $u$  in (25). Under the Pareto optimal equilibrium selection rule, Proposition 4 implies that forming a group is harmful for buyers because of (29). In fact, when  $C$  is strictly convex, this prediction holds as long as the Pareto-dominant equilibrium is selected under buyer group, no matter the equilibrium selected without buyer group.

## 6 Extension to a general setting

In this section, we extend a main result from the symmetric setting to a general setting with two asymmetric sellers and  $n \geq 2$  asymmetric buyers. Namely, we extend the result in Proposition 4(b) that buyer group reduces the worst payoff of the buyers when sellers' cost functions are strictly convex. In the generalization, we compare the payoffs without buyer group with the payoffs in the sell-out equilibrium under buyer group, which is the Pareto-dominant equilibrium for sellers. We show that every equilibrium without group yields the buyers a strictly higher total payoff (and each seller a strictly lower payoff) than the payoff in the sell-out equilibrium under buyer group.

**Proposition 5** *Suppose that there are  $n \geq 2$  buyers (with no assumption of symmetry for buyers, nor for sellers), and the cost function of at least one seller is strictly convex. Then, in any equilibrium without buyer group, the buyers obtain a strictly higher total payoff (and each seller obtains a strictly lower payoff) than in the sell-out equilibrium under buyer group.*

The proof of Proposition 5 relies on the following generalization of (19) in Lemma 3 to  $n \geq 2$  asymmetric buyers and two asymmetric sellers, which is proved in the proof of Lemma 3 in Appendix. Let us consider seller  $A$  for instance. Given  $b^1 \geq 0, b^2 \geq 0, \dots, b^n \geq 0$ ,  $f_A(b^1, b^2, \dots, b^n)$  is defined as  $\max_{x^1, x^2, \dots, x^n} [U^1(x^1, b^1) + U^2(x^2, b^2) + \dots + U^n(x^n, b^n) -$

$C_A(x^1 + x^2 + \dots + x^n)$ . Then, given  $\underline{b}^1, \dots, \underline{b}^n$  and  $\bar{b}^1, \dots, \bar{b}^n$  such that  $\underline{b}^h < \bar{b}^h$  for  $h = 1, \dots, n$ , we prove that

$$f_A(\bar{b}^1, \dots, \bar{b}^n) - f_A(\underline{b}^1, \dots, \underline{b}^n) < \sum_{i=1}^n [f_A(\underline{b}^1, \dots, \bar{b}^i, \dots, \underline{b}^n) - f_A(\underline{b}^1, \dots, \underline{b}^n)]$$

if  $C_A$  is strictly convex. This generalizes the inequality  $f(\bar{b}, \bar{b}) - f(\underline{b}, \underline{b}) < 2 [f(\underline{b}, \bar{b}) - f(\underline{b}, \underline{b})]$  in Section 4.

## 7 Application: entry and buyer group

In this section, we use our main insight to study how the possibility of a seller's entry affects the buyers' decision to form a group. For instance, a buying alliance among large chains of supermarkets can have an impact on the entry of national brand manufacturers.<sup>29</sup> Innes and Sexton (1993, 1994) allow the possibility for the buyers to form a coalition in order to contract with an outside entrant (or vertically integrate into the upstream market). In their papers, the incumbent has the first mover advantage and employs divide-and-conquer strategies to disrupt the coalition building. In contrast, we assume that the buyers move first by deciding whether to form or not a group and also assume that the incumbent and the entrant compete by making simultaneous offers as in Fumagalli and Motta (2008).<sup>30</sup>

We consider a setting with an incumbent, seller  $A$ , and a potential entrant, seller  $B$ , assuming that the cost functions are strictly convex and the products are strict substitutes. The buyers decide whether to form a group or not before the realization of the fixed cost of entry  $f$  of seller  $B$ . Let  $V_j^G \equiv \max_{x \geq 0} [U^G(x, 0) - C_j(x)]$  denote the welfare generated when the group trades only with seller  $j$  ( $= A, B$ ). We assume that  $f$  is distributed over  $[0, \bar{f}]$  with  $\bar{f} > V_{AB}^G - V_A^G (> 0)$  with a cumulative distribution function  $H$  with a strictly positive density  $h$  in  $[0, \bar{f}]$ . We study the following game:

- Stage 0: The buyers decide whether or not to form a group.
- Stage 0.5: The fixed cost of entry  $f \geq 0$  of the entrant is realized. The entrant decides whether to incur the cost or not.

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<sup>29</sup>Caprice and Rey (2015) describe the buying alliance between Leclerc and Système U, each having 17% and 9% of sales in French grocery and daily goods retail markets in 2009.

<sup>30</sup>Fumagalli and Motta (2008) assume that the entrant can make his offer before incurring a fixed cost of entry, which seems less natural than our timing that the entrant makes an offer after incurring the entry cost.

- Stage 1: If the entrant entered, then the incumbent and the entrant compete by simultaneously proposing non-linear tariffs to each buyer (or to the buyer group). If the entrant did not enter, the incumbent becomes monopolist.
- Stage 2: Each buyer  $i$ , or the buyer group, makes purchase decisions.

If the entrant enters, given the multiplicity of equilibria, we select the equilibrium which is Pareto-dominant in terms of the sellers' payoffs. Hence, the sellers play the sell-out equilibrium under buyer group and the sellers' payoffs are  $V_{AB}^G - V_B^G$  for seller  $A$ ,  $V_{AB}^G - V_A^G$  for seller  $B$  (Bernheim and Whinston, 1998). The assumption  $V_{AB}^G - V_A^G \in (0, \bar{f})$  implies that when the buyer group is formed, seller  $B$  enters with a positive probability  $H(V_{AB}^G - V_A^G) \in (0, 1)$ . Proposition 5 suggests that the buyers face a clear trade-off. Conditional on entry, the total payoff of the buyers is lower under buyer group than under no group. However, the fact that the entrant's payoff is higher under buyer group than without group makes the probability of entry higher under buyer group than under no group. Furthermore, in the sell-out equilibrium under buyer group, each seller obtains the social incremental contribution of his product, implying that the entry decision is always socially optimal. This in turn suggests that if buyers remain separate, then the entry is suboptimal. Therefore, Proposition 5 suggests that the buyers may deliberately induce a suboptimal entry by remaining separate in order to benefit from more intense competition upon entry.

**Proposition 6** (*entry*) *Suppose that the cost functions are strictly convex and the products are strict substitutes. Consider the game of buyer group formation followed by the entry. In case the entrant enters, we select the equilibrium which is Pareto-dominant in terms of the sellers' payoffs.*

(i) *If the buyers form a group, then the private entry decision is socially efficient. But if the buyers remain separate, then the entry is suboptimal.*

(ii) *The buyers may deliberately induce a suboptimal entry by remaining separate in order to benefit from more intense competition upon entry.*

The existing literature on naked exclusion explains suboptimal entry by the incumbent's taking advantage of coordination failure among buyers (Rasmusen, Ramseyer and Wiley 1991, Segal and Whinston, 2000, Fumagalli and Motta, 2006, 2008, Chen and Shaffer, 2014). While the literature typically assumes that the incumbent makes offers to buyers before the entry, Fumagalli and Motta (2008) show that the coordination failure survives even if both sellers make simultaneous offers. However, in these papers, buyers have no reason not to form a group (at least among those operating in separate markets) as this would remove the coordination failure. We show that buyers may choose not to form a group even if this leads to a suboptimal entry.

## 8 Linear costs with both complements and substitutes

In this section, we consider a setting with linear costs, but we relax the assumption of substitute goods and assume that goods are complements for at least one buyer. We discover another mechanism which makes buyers lose from building a group, and is different from the one identified previously based on strictly convex costs.

Given linear cost functions, with constant marginal cost  $c_j$  for each seller  $j$ , there exists sell-out equilibria both if the buyers form a group and if they remain separate. In what follows, we focus on this class of equilibria. More precisely, in the case of no group, let  $V_{AB}^i \equiv U^i(q_A^{i*}, q_B^{i*}) - c_A q_A^{i*} - c_B q_B^{i*}$  denote the welfare generated when buyer  $i$  purchases  $(q_A^{i*}, q_B^{i*})$  from both sellers. Let  $V_j^i \equiv \max_{q_j^i} [U^i(q_j^i, 0) - c_j q_j^i]$  denote the welfare generated when buyer  $i$  trades only with seller  $j$ . If the two products are substitutes for buyer  $i$ , then it is necessary that  $V_{AB}^i - F_A^i - F_B^i = V_A^i - F_A^i = V_B^i - F_B^i$  in the sell-out equilibrium. From these equalities we obtain the unique equilibrium fees:

$$F_A^{i*} = V_{AB}^i - V_B^i, \quad F_B^{i*} = V_{AB}^i - V_A^i, \quad \text{for } i = 1, 2. \quad (31)$$

and the payoff of buyer  $i$  is  $V_A^i + V_B^i - V_{AB}^i \geq 0$ .

Suppose now that products are complements for buyer 2. This implies  $V_A^2 + V_B^2 < V_{AB}^2$ , hence the payoff of buyer 2 is negative if the fixed fees are given as in (31). As the payoff cannot be negative, the equilibrium fixed fees must be lower than in (31), but in fact there exist infinitely many equilibrium fixed fees: they are such that  $F_A^2 + F_B^2 = V_{AB}^2$  and  $V_A^2 \leq F_A^2$ ,  $V_B^2 \leq F_B^2$ .<sup>31</sup> The first equality implies that buyer 2's payoff is zero. The multiplicity is about how the sellers share the surplus of  $V_{AB}^2 - V_A^2 - V_B^2$  generated by complementarity between the two products, but this is not very relevant for us, as in every equilibrium buyer 2's payoff is zero.

Linear costs imply  $V_{AB}^G = V_{AB}^1 + V_{AB}^2$  and  $V_A^G = V_A^1 + V_A^2$ ,  $V_B^G = V_B^1 + V_B^2$ , where we recall that  $V_A^G$  for instance represents the welfare generated when the group trades only with seller A. Therefore, if products are complements for each buyer, that is if  $V_A^1 + V_B^1 \leq V_{AB}^1$  and  $V_A^2 + V_B^2 \leq V_{AB}^2$ , then both buyers have zero payoff under no group, and also the group has zero payoff since  $V_A^G + V_B^G \leq V_{AB}^G$ . Buyer group has no effect in this case.

Suppose now that the products are strict substitutes for buyer 1 and strict complements for buyer 2:  $V_A^1 + V_B^1 > V_{AB}^1$  and  $V_A^2 + V_B^2 < V_{AB}^2$ . Then, if the group is not formed, the sum of the buyers' payoffs coincides with the payoff of buyer 1, which is equal to  $V_A^1 + V_B^1 - V_{AB}^1 > 0$ . If the group is formed, then its payoff is given by

$$\max \{0, V_A^1 + V_B^1 + V_A^2 + V_B^2 - V_{AB}^1 - V_{AB}^2\},$$

<sup>31</sup>These inequalities imply  $F_A^2 \leq V_{AB}^2 - V_B^2$  and  $F_B^2 \leq V_{AB}^2 - V_A^2$ .

which is always strictly less than  $V_A^1 + V_B^1 - V_{AB}^1$  because  $V_A^2 + V_B^2 < V_{AB}^2$ . Therefore, in this case the group strictly reduces the sum of the payoffs of the buyers. The intuition is pretty simple. When the products are complements for buyer 2, each seller  $j$  has some residual market power with respect to buyer 2 in the sense that because of the constraint that buyer 2's payoff cannot be negative, seller  $j$  charges less than the marginal value created by his product. Integrating the two markets through buyer group allows the sellers to transfer their residual market power to the market of buyer 1. This mechanism is similar to the mechanism through which multi-market contact facilitates collusion by transferring residual collusive power from one market to another (Bernheim and Whinston, 1990).

**Proposition 7** (*complements and substitutes*) *Assume linear cost functions. Then the formation of the buyer group*

- (i) *has no effect on any player's payoff if the products are complements to both buyers;*
- (ii) *strictly reduces the joint payoff of the buyers if the products are strict complements for one buyer but strict substitutes for the other.*

## 9 Conclusion

We considered buyer group which cannot pre-commit to exclusive purchase and found that buyer group does not generate buyer power (i.e., larger discounts than no buyer group), no matter the curvature of the sellers' cost function. Combining our result with those of Inderst and Shaffer (2007) and Dana (2012) generates a clear message: when multiple sellers producing substitutes compete, buyer group creates buyer power only if the group can pre-commit to limit its purchase to a subset of sellers. This has policy implications, for instance, to large healthcare procurement alliances. It is not the mere size of the alliance, but the credible threat to limit its purchase to a subset of sellers which increases its buyer power.

It would be interesting to extend our framework by considering a linear contract between an upstream firm and a downstream firm instead of a non-linear contract. The linear contract is prominent in relations between TV channels and cable TV distributors (Crawford and Yurukoglu, 2012), between hospitals and medical device suppliers (Grennan, 2013, 2014), and between book publishers and resellers (Gilbert, 2015). With such simple but realistic contract, the equilibrium outcome is in general inefficient since each upstream firm adds a positive markup. Therefore, if buyer fragmentation intensifies competition between sellers as in this paper, it will increase welfare by reducing the dead-weight loss. In addition, the framework with the linear contract may be tractable enough to incorporate competition among downstream firms.

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# 11 Appendix

## 11.1 Proof of Proposition 1

**The case of buyer group** Here we only prove that  $U^G$  is concave, since the rest of the proof is in the main text.

Consider  $(q_A^G, q_B^G) = (\bar{q}_A, \bar{q}_B)$ , and suppose that  $(\bar{q}_A^1, \bar{q}_B^1, \bar{q}_A^2, \bar{q}_B^2)$  is the optimal allocation (i.e., it maximizes  $G$ 's payoff<sup>32</sup>) given  $(q_A^G, q_B^G) = (\bar{q}_A, \bar{q}_B)$ , that is  $U^G(\bar{q}_A, \bar{q}_B) = U^1(\bar{q}_A^1, \bar{q}_B^1) + U^2(\bar{q}_A^2, \bar{q}_B^2)$ . Likewise, consider  $(\check{q}_A, \check{q}_B)$  and suppose that  $(\check{q}_A^1, \check{q}_B^1, \check{q}_A^2, \check{q}_B^2)$  is the optimal allocation given  $(q_A^G, q_B^G) = (\check{q}_A, \check{q}_B)$ , that is  $U^G(\check{q}_A, \check{q}_B) = U^1(\check{q}_A^1, \check{q}_B^1) + U^2(\check{q}_A^2, \check{q}_B^2)$ . Then consider  $(q_A^G, q_B^G) = \lambda(\bar{q}_A, \bar{q}_B) + (1 - \lambda)(\check{q}_A, \check{q}_B)$ , and notice that  $(\lambda\bar{q}_A^1 + (1 - \lambda)\check{q}_A^1, \lambda\bar{q}_B^1 + (1 - \lambda)\check{q}_B^1, \lambda\bar{q}_A^2 + (1 - \lambda)\check{q}_A^2, \lambda\bar{q}_B^2 + (1 - \lambda)\check{q}_B^2)$  is a feasible allocation. Hence, we have

$$\begin{aligned} & U^G(\lambda(\bar{q}_A, \bar{q}_B) + (1 - \lambda)(\check{q}_A, \check{q}_B)) \\ & \geq U^1(\lambda\bar{q}_A^1 + (1 - \lambda)\check{q}_A^1, \lambda\bar{q}_B^1 + (1 - \lambda)\check{q}_B^1) + U^2(\lambda\bar{q}_A^2 + (1 - \lambda)\check{q}_A^2, \lambda\bar{q}_B^2 + (1 - \lambda)\check{q}_B^2) \\ & \geq \lambda U^1(\bar{q}_A^1, \bar{q}_B^1) + (1 - \lambda)U^1(\check{q}_A^1, \check{q}_B^1) + \lambda U^2(\bar{q}_A^2, \bar{q}_B^2) + (1 - \lambda)U^2(\check{q}_A^2, \check{q}_B^2) = \lambda U^G(\bar{q}_A, \bar{q}_B) + (1 - \lambda)U^G(\check{q}_A, \check{q}_B) \end{aligned}$$

in which the first inequality follows from the definition of  $U^G$  and the second inequality follows from concavity of  $U^1$  and of  $U^2$ . The resulting inequality  $U^G(\lambda(\bar{q}_A, \bar{q}_B) + (1 - \lambda)(\check{q}_A, \check{q}_B)) \geq \lambda U^G(\bar{q}_A, \bar{q}_B) + (1 - \lambda)U^G(\check{q}_A, \check{q}_B)$  establishes that  $U^G$  is concave.

**The case of no group** Here we provide the proof for the case of  $n \geq 2$  buyers as we cover the case of  $n$  buyers in Proposition 5. For  $i = 1, \dots, n$ , we use  $(q_j^{ie}, q_k^{ie})$  to denote the equilibrium quantities purchased by buyer  $i$ , and in order to shorten notation in this proof, we define  $U^{ie} \equiv U^i(q_j^{ie}, q_k^{ie})$ ,  $T_j^{ie} \equiv T_j^i(q_j^{ie})$ , and  $C_j^e \equiv C_j(\sum_{i=1}^n q_j^{ie})$  for  $i = 1, \dots, n$  and  $j = A, B$ .

The equilibrium payoff of buyer  $i$  is  $u^i \equiv U^{ie} - T_j^{ie} - T_k^{ie}$ , whereas if  $i$  trades only with seller  $j$ , then  $i$ 's payoff is  $u_j^i \equiv \max_{q_j^i}(U^i(q_j^i, 0) - T_j^i(q_j^i))$ . In any equilibrium it is necessary that  $u^{ie} = u_j^i = u_k^i$ , therefore we can use  $U^{ie} - T_j^{ie} - T_k^{ie} = u_j^i$  to obtain

$$T_k^{ie} = U^{ie} - T_j^{ie} - u_j^i = U^{ie} - w_j^i, \quad \text{with} \quad w_j^i \equiv \max_{q_j^i}(U^i(q_j^i, 0) - T_j^i(q_j^i)) + T_j^{ie} \quad (32)$$

Likewise,  $T_j^{ie} = U^{ie} - w_k^i$  and buyer  $i$ 's payoff can be written as

$$u^{ie} = w_j^i + w_k^i - U^{ie}. \quad (33)$$

---

<sup>32</sup>Given  $(q_A^G, q_B^G)$ , the maximization problem in (1)-(2) has a unique solution because the objective function is strictly concave and the feasible set is convex.

The profit of seller  $j$  is

$$\pi_j^{nG} \equiv \sum_{i=1}^n T_j^{ie} - C_j^e = \sum_{i=1}^n (U^{ie} - w_k^i) - C_j^e. \quad (34)$$

Now we consider deviations of seller  $j$  in the form of take-it-or-leave-it offers  $(q_j^i, t_j^i)$  to each buyer  $i = 1, \dots, n$  such that

$$U^i(q_j^i, q_k^{ie}) - t_j^i - T_k^{ie} \geq u_k^i, \quad \text{for } i = 1, \dots, n.$$

These inequalities state that buyer  $i$  weakly prefers to accept  $j$ 's take-it-or-leave-it offer and continue to buy  $q_k^{ie}$  from seller  $k$ , rather than to trade only with  $k$ . Choosing  $t_j^i$  to satisfy the above inequalities as equalities for  $i = 1, \dots, n$ , we infer that from this kind of deviation, seller  $j$ 's profit  $\pi_j^d$  is

$$\pi_j^d(q_j^1, \dots, q_j^n) = \sum_{i=1}^n (U^i(q_j^i, q_k^{ie}) - T_k^{ie} - u_k^i) - C_j(\sum_{i=1}^n q_j^i).$$

Since no profitable deviation exists, we infer that  $\pi_j^d(q_j^1, \dots, q_j^n) \leq \pi_j^{nG}$  for each  $(q_j^1, \dots, q_j^n)$ . The equality holds if  $j$  chooses  $q_j^i = q_j^{ie}$  for  $i = 1, \dots, n$ , since  $T_k^{ie} + u_k^i = w_k^i$  for each  $i$ . Therefore  $(q_j^1, \dots, q_j^n) = (q_j^{1e}, \dots, q_j^{ne})$  is a maximizer of  $\pi_j^d$  (see (6) for the case of  $n = 2$ ), and this implies that  $U_j(q_j^{ie}, q_k^{ie}) \leq C_j'(\sum_{i=1}^n q_j^{ie})$ , with equality if  $q_j^{ie} > 0$ , for  $i = 1, \dots, n$ . Since social welfare is concave, and the conditions above are first order conditions for maximization of social welfare, we conclude that  $(q_j^{ie}, q_k^{ie}) = (q_j^{i*}, q_k^{i*})$  for  $i = 1, \dots, n$ .

## 11.2 Proof of Lemma 1

Given any symmetric equilibrium in which each seller offers the same tariff  $T^G$  to  $G$ , we know from Proposition 1 that  $G$  buys quantity  $2q^*$  from each seller, that is

$$(2q^*, 2q^*) \in \arg \max_{q_A^G \geq 0, q_B^G \geq 0} (U^G(q_A^G, q_B^G) - T^G(q_A^G) - T^G(q_B^G))$$

This implies that

$$(2q^*, 2q^*) \in \arg \max_{q_A^G \in \{0, 2q^*, \hat{q}\}, q_B^G \in \{0, 2q^*, \hat{q}\}} (U^G(q_A^G, q_B^G) - T^G(q_A^G) - T^G(q_B^G))$$

[recall that  $\hat{q}$  is defined in (9)]. Since  $t^{G*} = T^G(2q^*)$ ,  $\hat{t}^G = T^G(\hat{q})$  it follows that  $G$  still buys  $2q^*$  from each seller when each seller offers only the two contracts  $(2q^*, t^{G*})$ ,  $(\hat{q}, \hat{t}^G)$ , essentially because the set of opportunities for  $G$  has shrank, but the best alternative of the group is still available.

Regarding the existence of profitable deviations, consider seller  $A$  (to fix ideas), and notice that when seller  $B$  offers the tariff  $T^G$ , the profit seller  $A$  can make with a take-it-or-leave-it offer  $(q^G, t^G)$  is not larger than  $\sup_{q^G \geq 0, q_B^G \geq 0} (U^G(q^G, q_B^G) - T^G(q_B^G) - u^G - C(q^G))$ , in which  $u^G = U^G(0, \hat{q}) - \hat{t}^G$ . Since no profitable deviation exists, we infer that

$$\sup_{q^G \geq 0, q_B^G \geq 0} (U^G(q^G, q_B^G) - T^G(q_B^G) - u^G - C(q^G)) \leq t^{G*} - C(2q^*)$$

This implies that

$$\sup_{q^G \geq 0, q_B^G \in \{0, 2q^*, \hat{q}\}} (U^G(q^G, q_B^G) - T^G(q_B^G) - u^G - C(q^G)) \leq t^{G*} - C(2q^*)$$

hence, no deviation is profitable for seller  $A$  when  $B$  offers only the contracts  $(2q^*, t^{G*})$ ,  $(\hat{q}, \hat{t}^G)$ , because the feasible set for  $q_B^G$  has shrunk.

Here we prove that in any equilibrium under buyer group the inequality  $\hat{q} > 2q^*$  holds. In each equilibrium,  $(2q^*, t^{G*})$  and  $(\hat{q}, \hat{t}^G)$  need to satisfy

$$U^G(2q^*, 2q^*) - 2t^{G*} \geq U^G(\hat{q}, 2q^*) - \hat{t}^G - t^{G*} \quad (35)$$

$$U^G(\hat{q}, 0) - \hat{t}^G \geq U^G(2q^*, 0) - t^{G*} \quad (36)$$

hence

$$U^G(2q^*, 2q^*) - U^G(2q^*, 0) \geq U^G(\hat{q}, 2q^*) - U^G(\hat{q}, 0)$$

or

$$\int_0^{2q^*} \int_{\hat{q}}^{2q^*} U_{BA}^G(w, z) dw dz \geq 0$$

and this implies  $\hat{q} \geq 2q^*$  since  $U_{BA}^G < 0$ . Now suppose that  $\hat{q} = 2q^*$ . Then both (35) and (36) are equalities and  $t^{G*} = \hat{t}^G$ , hence from (10) we obtain  $U^G(2q^*, 2q^*) - 2t^{G*} = U^G(2q^*, 0) - t^{G*}$ , thus  $t^{G*} = U^G(2q^*, 2q^*) - U^G(2q^*, 0)$ . Then the payoff for  $G$  is  $U^G(2q^*, 2q^*) - 2t^{G*} = 2U^G(2q^*, 0) - U^G(2q^*, 2q^*) \equiv u^G$ . Then suppose that seller  $A$  deviates with  $(q, t)$  such that  $U^G(q, 0) - t = u^G$ , hence  $A$ 's profit is  $U^G(q, 0) - u^G - C(q)$  and its maximum with respect to  $q$  has value  $f^G(0) - 2U^G(2q^*, 0) + U^G(2q^*, 2q^*)$ . This is greater than  $A$ 's profit without deviation  $t^{G*} - C(2q^*) = U^G(2q^*, 2q^*) - U^G(2q^*, 0) - C(2q^*)$ , that is  $f^G(0) > U^G(2q^*, 0) - C(2q^*)$  since  $2q^*$  is not a solution to the problem  $\max_{x \geq 0} [U^G(x, 0) - C(x)]$ .

### 11.3 Proof of Proposition 2

Given that each seller offers only the contracts  $(2q^*, t^{G*})$  and  $(\hat{q}, \hat{t}^G)$ , the group chooses a pair  $(q_A^G, q_B^G) \in \{0, 2q^*, \hat{q}\} \times \{0, 2q^*, \hat{q}\}$ . In order to shorten notation, we define  $U^{G*} \equiv U^G(2q^*, 0)$ ,  $\hat{U}^G \equiv U^G(\hat{q}, 0)$ ,  $U_t^{G*} \equiv U^G(2q^*, 2q^*)$  (the subscript  $t$  means that  $G$  buys

quantity  $2q^*$  twice, that is from both sellers),  $\hat{U}^{G*} \equiv U^G(2q^*, \hat{q})$ ,  $\hat{U}_t^G \equiv U^G(\hat{q}, \hat{q})$ . Recalling that  $t^{G*} = \frac{1}{2}U_t^{G*} - \frac{1}{2}u^G$ ,  $\hat{t}^G = \hat{U}^G - u^G$ , we compute  $G$ 's payoffs from the various purchase alternatives:

$$\left\{ \begin{array}{lll} \text{alternatives} & q_A^G = 0, q_B^G = 2q^* & q_A^G = 0, q_B^G = \hat{q} & q_A^G = q_B^G = 2q^* \\ \text{payoff} & U^{G*} - \frac{1}{2}U_t^{G*} + \frac{1}{2}u^G & u^G & u^G \\ \\ \text{alternatives} & q_A^G = 2q^*, q_B^G = \hat{q} & q_A^G = q_B^G = \hat{q} & \\ \text{payoff} & \hat{U}^{G*} - \hat{U}^G - \frac{1}{2}U_t^{G*} + \frac{3}{2}u^G & \hat{U}_t^G - 2\hat{U}^G + 2u^G & \end{array} \right. \quad (37)$$

Below, Step 1 determines the conditions under which  $G$  buys quantity  $2q^*$  from each seller. Step 2 determines the conditions under which no seller wants to deviate. Step 3 proves that the latter conditions are more restrictive than the former, and that the whole set of values for  $u^G$  is given by (12).

**Step 1** Given the alternatives in (37),  $G$  buys quantity  $2q^*$  from each seller if and only if

$$2U^{G*} - U_t^{G*} \leq u^G \leq U_t^{G*} + 2\hat{U}^G - 2\hat{U}^{G*} \quad (38)$$

PROOF From (37) we see that the following inequalities need to hold:

$$u^G \geq U^{G*} - \frac{1}{2}U_t^{G*} + \frac{1}{2}u^G, \quad u^G \geq \hat{U}^{G*} - \hat{U}^G - \frac{1}{2}U_t^{G*} + \frac{3}{2}u^G, \quad u^G \geq \hat{U}_t^G - 2\hat{U}^G + 2u^G \quad (39)$$

which are equivalent, respectively, to  $2U^{G*} - U_t^{G*} \leq u^G$ , to  $u^G \leq U_t^{G*} + 2\hat{U}^G - 2\hat{U}^{G*}$ , and to  $u^G \leq 2\hat{U}^G - \hat{U}_t^G$ . Finally, we obtain (38) by proving that  $U_t^{G*} + 2\hat{U}^G - 2\hat{U}^{G*} < 2\hat{U}^G - \hat{U}_t^G$ . This inequality is equivalent to  $\hat{U}_t^G - \hat{U}^{G*} < \hat{U}^{G*} - U_t^{G*}$ , that is  $\int_{2q^*}^{\hat{q}} U_A^G(z, \hat{q}) dz < \int_{2q^*}^{\hat{q}} U_A^G(z, 2q^*) dz$ , which holds true because  $\hat{q} > 2q^*$  and goods are strict substitutes.

**Step 2** No profitable deviation exists for a seller if and only if

$$U_t^{G*} - 2f^G(2q^*) + 2f^G(0) \leq u^G \leq 2f^G(2q^*) - U_t^{G*} - 2[f^G(\hat{q}) - \hat{U}^G] \quad (40)$$

PROOF The equilibrium profit for each seller is  $t^{G*} - C(2q^*) = f^G(2q^*) - \frac{1}{2}U_t^{G*} - \frac{1}{2}u^G$ , and now we examine the conditions under which no profitable deviation for seller  $A$  exists. The deviation which induces  $G$  to buy only from  $A$  yields seller  $A$  profit  $f^G(0) - u^G$ , hence  $U_t^{G*} - 2f^G(2q^*) + 2f^G(0) \leq u^G$  is necessary. The deviation in which  $G$  buys  $2q^*$  from seller  $B$  yields  $A$  profit  $f^G(2q^*) - t^{G*} - u^G$ , which is just equal to  $f^G(2q^*) - \frac{1}{2}U_t^{G*} - \frac{1}{2}u^G$ . The deviation in which  $G$  buys  $\hat{q}$  from seller  $B$  yields  $A$  profit  $f^G(\hat{q}) - \hat{t}^G - u^G = f^G(\hat{q}) - \hat{U}^G$ , hence  $u^G \leq 2f^G(2q^*) - U_t^{G*} - 2[f^G(\hat{q}) - \hat{U}^G]$  is necessary.

**Step 3** If  $u^G$  satisfies (40), then it satisfies (38) and the whole set of  $G$ 's equilibrium payoffs is obtained from (40) as  $\hat{q} \rightarrow +\infty$ .

PROOF The inequality  $2U^{G*} - U_t^{G*} \leq U_t^{G*} - 2f^G(2q^*) + 2f^G(0)$  reduces to  $U^{G*} - C(2q^*) \leq f^G(0)$ , which holds by definition of  $f^G(0)$ . The inequality  $2f^G(2q^*) - U_t^{G*} - 2[f^G(\hat{q}) - \hat{U}^G] \leq U_t^{G*} + 2\hat{U}^G - 2\hat{U}^{G*}$  reduces to  $\hat{U}^{G*} - C(2q^*) \leq f^G(\hat{q})$ , which holds by definition of  $f^G(\hat{q})$ .

Since the right hand side in (40) is increasing in  $\hat{q}$ , it follows that (12) describes the set of values for  $u^G$ .

## 11.4 Proof of Lemma 2

This proof is omitted since it is very similar to the proof of Lemma 1.

## 11.5 Proof of Lemma 3

### Proof of inequality (19) for the case of strictly convex $C$

Here we prove a generalization of (19) for the case of  $n \geq 2$  asymmetric buyers and two asymmetric sellers. Precisely, in this proof, buyers may have different utility functions and sellers can be asymmetric.

Given  $b^1 \geq 0, \dots, b^n \geq 0$ , we define

$$f_A(b^1, \dots, b^n) = \max_{x^1, \dots, x^n} [U^1(x^1, b^1) + \dots + U^n(x^n, b^n) - C_A(x^1 + \dots + x^n)]. \quad (41)$$

We prove that if  $C_A$  is strictly convex and  $\underline{\mathbf{b}} = (\underline{b}^1, \dots, \underline{b}^n)$  and  $\bar{\mathbf{b}} = (\bar{b}^1, \dots, \bar{b}^n)$  are such that  $\underline{b}^h < \bar{b}^h$  for  $h = 1, \dots, n$ , then

$$f_A(\bar{b}^1, \bar{b}^2, \dots, \bar{b}^n) - f_A(\bar{b}^1, \underline{b}^2, \dots, \underline{b}^n) < \sum_{i=2}^n (f_A(\underline{b}^1, \dots, \bar{b}^i, \dots, \underline{b}^n) - f_A(\underline{b}^1, \dots, \underline{b}^i, \dots, \underline{b}^n)), \quad (42)$$

which is equivalent to

$$f_A(\bar{b}^1, \bar{b}^2, \dots, \bar{b}^n) - f_A(\underline{b}^1, \dots, \underline{b}^n) < \sum_{i=1}^n (f_A(\underline{b}^1, \dots, \bar{b}^i, \dots, \underline{b}^n) - f_A(\underline{b}^1, \dots, \underline{b}^i, \dots, \underline{b}^n)).$$

In the special case of  $n = 2$ , with  $\underline{\mathbf{b}} = (\underline{b}, \underline{b})$  and  $\bar{\mathbf{b}} = (\bar{b}, \bar{b})$ , (42) reduces to  $f_A(\bar{b}, \bar{b}) - f_A(\bar{b}, \underline{b}) < f_A(\underline{b}, \bar{b}) - f_A(\underline{b}, \underline{b})$ , which is (19).

**Step 1** Consider  $\underline{\mathbf{b}}$  and  $\bar{\mathbf{b}}$  such that  $\underline{b}^i = \bar{b}^i$  for one  $i$ ,  $\underline{b}^h < \bar{b}^h$  for each  $h \neq i$ , and denote with  $\underline{\mathbf{x}}$  (with  $\bar{\mathbf{x}}$ ) the optimal  $(x^1, \dots, x^n)$  given  $\underline{\mathbf{b}}$  (given  $\bar{\mathbf{b}}$ ) in problem (41). Then  $\bar{x}^i > \underline{x}^i$  and  $\sum_{h \neq i} \bar{x}^h < \sum_{h \neq i} \underline{x}^h$ .

PROOF The first order conditions for  $x^1, \dots, x^n$  given  $\mathbf{b} = \underline{\mathbf{b}}$  and given  $\mathbf{b} = \bar{\mathbf{b}}$  are, respectively,

$$U_A^h(\underline{x}^h, \underline{b}^h) - C'_A(\underline{x}^1 + \dots + \underline{x}^n) = 0 \text{ for } h \neq i, \quad U_A^i(\underline{x}^i, \underline{b}^i) - C'_A(\underline{x}^1 + \dots + \underline{x}^n) = 0 \quad (43)$$

$$U_A^h(\bar{x}^h, \bar{b}^h) - C'_A(\bar{x}^1 + \dots + \bar{x}^n) = 0 \text{ for } h \neq i, \quad U_A^i(\bar{x}^i, \bar{b}^i) - C'_A(\bar{x}^1 + \dots + \bar{x}^n) = 0 \quad (44)$$

First we prove that  $\bar{x}^1 + \dots + \bar{x}^n < \underline{x}^1 + \dots + \underline{x}^n$ . In view of a contradiction, we suppose that  $\bar{x}^1 + \dots + \bar{x}^n \geq \underline{x}^1 + \dots + \underline{x}^n$ . Then  $U_A^h(\bar{x}^h, \bar{b}^h) - C'_A(\bar{x}^1 + \dots + \bar{x}^n) < U_A^h(\underline{x}^h, \underline{b}^h) -$

$C'_A(\underline{x}^1 + \dots + \underline{x}^n) = 0$  for  $h \neq i$  (as  $\bar{b}^h > \underline{b}^h$  and goods are strict substitutes), which implies  $\bar{x}^h < \underline{x}^h$  for  $h \neq i$ . Moreover,  $U_A^i(\underline{x}^i, \underline{b}^i) - C'_A(\bar{x}^1 + \dots + \bar{x}^n) \leq 0$ , which implies  $\bar{x}^i \leq \underline{x}^i$ . The inequalities  $\bar{x}^h < \underline{x}^h$  for  $h \neq i$  and  $\bar{x}^i \leq \underline{x}^i$  contradict  $\bar{x}^1 + \dots + \bar{x}^n \geq \underline{x}^1 + \dots + \underline{x}^n$ . Using  $\bar{x}^1 + \dots + \bar{x}^n < \underline{x}^1 + \dots + \underline{x}^n$  and the equality for  $\bar{x}^i$  in (44), we conclude that  $\bar{x}^i > \underline{x}^i$ . These two inequalities jointly lead to  $\sum_{h \neq i} \bar{x}^h < \sum_{h \neq i} \underline{x}^h$ . ■

**Step 2** Consider  $\underline{\mathbf{b}}$  and  $\bar{\mathbf{b}}$  such that  $\underline{b}^h < \bar{b}^h$  for  $h = 1, \dots, n$ . Then (42) holds.

PROOF Define the  $n-1$  vectors  $\underline{\mathbf{b}}^i(t) = (\underline{b}^1, \dots, \underline{b}^i + (\bar{b}^i - \underline{b}^i)t, \dots, \underline{b}^n)$  for  $i = 2, \dots, n$ , the vector  $\bar{\mathbf{b}}(t) = (\bar{b}^1, \underline{b}^2 + (\bar{b}^2 - \underline{b}^2)t, \dots, \underline{b}^n + (\bar{b}^n - \underline{b}^n)t)$ , and the two functions  $g(t) = \sum_{i=2}^n f_A(\underline{\mathbf{b}}^i(t))$ ,  $h(t) = f_A(\bar{\mathbf{b}}(t))$ . Then (42) is equivalent to

$$g(1) - g(0) > h(1) - h(0)$$

We have that  $g'(t) = \sum_{i=2}^n f_{A_i}(\underline{\mathbf{b}}^i(t))(\bar{b}^i - \underline{b}^i)$  and  $f_{A_i}(\underline{\mathbf{b}}^i(t)) = U_B^i(\underline{x}^i(t), \underline{b}^i + (\bar{b}^i - \underline{b}^i)t)$ , in

which  $\underline{x}^i(t)$  is the optimal  $x^i$  given  $\mathbf{b} = \underline{\mathbf{b}}^i(t)$ ; hence  $g(1) - g(0) = \int_0^1 \sum_{i=2}^n U_B^i(\underline{x}^i(t), \underline{b}^i + (\bar{b}^i - \underline{b}^i)t)$

$(\bar{b}^i - \underline{b}^i)dt$ . Likewise,  $h'(t) = \sum_{i=2}^n f_{A_i}(\bar{\mathbf{b}}(t))(\bar{b}^i - \underline{b}^i)$  and  $f_{A_i}(\bar{\mathbf{b}}(t)) = U_B^i(\bar{x}^i(t), \underline{b}^i + (\bar{b}^i - \underline{b}^i)t)$ , in

which  $\bar{x}^i(t)$  is the optimal  $x^i$  given  $\mathbf{b} = \bar{\mathbf{b}}(t)$ ; hence  $h(1) - h(0) = \int_0^1 \sum_{i=2}^n U_B^i(\bar{x}^i(t), \underline{b}^i + (\bar{b}^i - \underline{b}^i)t)$

$(\bar{b}^i - \underline{b}^i)dt$ . From Step 1 we know that  $\bar{x}^i(t) > \underline{x}^i(t)$  for  $i = 2, \dots, n$ , for each  $t \in (0, 1)$ ,<sup>33</sup> hence  $U_B^i(\underline{x}^i(t), \underline{b}^i + (\bar{b}^i - \underline{b}^i)t) > U_B^i(\bar{x}^i(t), \underline{b}^i + (\bar{b}^i - \underline{b}^i)t)$ , which implies  $g(1) - g(0) > h(1) - h(0)$ . ■

### Proof of inequality (20) for the case of concave $C$

Here we prove a generalization of (20) for the case of  $C_A$  concave and  $n = 2$  (but without assumption of symmetric buyers or symmetric sellers). Given  $(\underline{b}^1, \bar{b}^1)$ ,  $(\underline{b}^2, \bar{b}^2)$  such that  $\underline{b}^1 < \bar{b}^1$ ,  $\underline{b}^2 < \bar{b}^2$ , we show that

$$f_A(\bar{b}^1, \bar{b}^2) - f_A(\bar{b}^1, \underline{b}^2) \geq f_A(\underline{b}^1, \bar{b}^2) - f_A(\underline{b}^1, \underline{b}^2). \quad (45)$$

Hence, if  $\underline{b}^1 = \underline{b}^2 \equiv \underline{b} < \bar{b}^1 = \bar{b}^2 \equiv \bar{b}$  then (45) reduces to  $f_A(\bar{b}, \bar{b}) - f_A(\bar{b}, \underline{b}) \geq f_A(\underline{b}, \bar{b}) - f_A(\underline{b}, \underline{b})$ , which is (20).

We can argue like in the proof of Step 2 above to find that the inequality boils down to  $\int_0^1 U_B^2(\underline{x}^2(t), \underline{b}^2 + (\bar{b}^2 - \underline{b}^2)t)dt \leq \int_0^1 U_B^2(\bar{x}^2(t), \underline{b}^2 + (\bar{b}^2 - \underline{b}^2)t)dt$ , in which  $\underline{x}^2(t)$  is the optimal  $x^2$  given  $(\underline{b}^1, \underline{b}^2 + (\bar{b}^2 - \underline{b}^2)t)$  and  $\bar{x}^2(t)$  is the optimal  $x^2$  given  $(\bar{b}^1, \underline{b}^2 + (\bar{b}^2 - \underline{b}^2)t)$ .

<sup>33</sup>In Step 1, set  $\underline{\mathbf{b}} = \underline{\mathbf{b}}^i(t)$ ,  $\bar{\mathbf{b}} = \bar{\mathbf{b}}(t)$  and notice that  $\underline{b}^i = \bar{b}^i$  but  $\underline{b}^h < \bar{b}^h$  for each  $h \neq i$ .

Thus it suffices to prove that  $\underline{x}^2(t) \geq \bar{x}^2(t)$  for each  $t \in [0, 1]$ . The first order conditions for  $\underline{x}^1(t)$ ,  $\underline{x}^2(t)$ , and for  $\bar{x}^1(t)$ ,  $\bar{x}^2(t)$  are (for the sake of brevity, here we write  $\bar{x}^i$ ,  $\underline{x}^i$  instead of  $\bar{x}^i(t)$ ,  $\underline{x}^i(t)$ ):

$$U_A^1(\underline{x}^1, \underline{b}^1) - C'_A(\underline{x}^1 + \underline{x}^2) = 0 \quad \text{and} \quad U_A^2(\underline{x}^2, \underline{b}^2 + (\bar{b}^2 - \underline{b}^2)t) - C'_A(\underline{x}^1 + \underline{x}^2) = 0 \quad (46)$$

$$U_A^1(\bar{x}^1, \bar{b}^1) - C'_A(\bar{x}^1 + \bar{x}^2) = 0 \quad \text{and} \quad U_A^2(\bar{x}^2, \bar{b}^2 + (\bar{b}^2 - \underline{b}^2)t) - C'_A(\bar{x}^1 + \bar{x}^2) = 0 \quad (47)$$

First we prove that  $\bar{x}^1 < \underline{x}^1$ . Consider  $F(x^1, x^2) \equiv U^1(x^1, \bar{b}^1) + U^2(x^2, \underline{b}^2 + (\bar{b}^2 - \underline{b}^2)t) - C_A(x^1 + x^2)$ , which is maximized by  $(\bar{x}^1, \bar{x}^2)$ . Since  $F$  is concave, we have that

$$\begin{aligned} F(t\bar{x}^1 + (1-t)\underline{x}^1, t\bar{x}^2 + (1-t)\underline{x}^2) &= F(\underline{x}^1 + t(\bar{x}^1 - \underline{x}^1), \underline{x}^2 + t(\bar{x}^2 - \underline{x}^2)) \\ &\geq tF(\bar{x}^1, \bar{x}^2) + (1-t)F(\underline{x}^1, \underline{x}^2) \quad \text{for each } t \in [0, 1]. \end{aligned} \quad (48)$$

Let  $\ell(t)$  denote the left hand side in (48), and  $r(t)$  the right hand side in (48). From (48) we see that  $\ell(t) \geq r(t)$  for each  $t \in [0, 1]$ , and since  $\ell(0) = r(0)$  it is necessary that  $\ell'(0) \geq r'(0)$ . It is immediate that  $r'(t) = F(\bar{x}^1, \bar{x}^2) - F(\underline{x}^1, \underline{x}^2) > 0$ ,<sup>34</sup> and  $\ell'(t) = F_1(\underline{x}^1 + t(\bar{x}^1 - \underline{x}^1), \underline{x}^2 + t(\bar{x}^2 - \underline{x}^2))(\bar{x}^1 - \underline{x}^1) + F_2(\underline{x}^1 + t(\bar{x}^1 - \underline{x}^1), \underline{x}^2 + t(\bar{x}^2 - \underline{x}^2))(\bar{x}^2 - \underline{x}^2)$ . Thus  $\ell'(0) = F_1(\underline{x}^1, \underline{x}^2)(\bar{x}^1 - \underline{x}^1) + F_2(\underline{x}^1, \underline{x}^2)(\bar{x}^2 - \underline{x}^2)$ , and  $F_1(\underline{x}^1, \underline{x}^2) = U_A^1(\underline{x}^1, \bar{b}^1) - C'_A(\underline{x}^1 + \underline{x}^2)$  is negative because of (46) and goods are strict substitutes,  $F_2(\underline{x}^1, \underline{x}^2) = U_A^2(\underline{x}^2, \underline{b}^2 + (\bar{b}^2 - \underline{b}^2)t) - C'_A(\underline{x}^1 + \underline{x}^2)$  is zero because of (46). Therefore,  $\bar{x}^1 < \underline{x}^1$  is necessary otherwise  $\ell'(0) \leq 0 < r'(0)$ .

Using  $\bar{x}^1 < \underline{x}^1$ , we compare  $U_A^2(\underline{x}^2, \underline{b}^2 + (\bar{b}^2 - \underline{b}^2)t) - C'_A(\underline{x}^1 + \underline{x}^2)$  in (46) with  $U_A^2(\bar{x}^2, \bar{b}^2 + (\bar{b}^2 - \underline{b}^2)t) - C'_A(\bar{x}^1 + \bar{x}^2)$  in (47), and since  $C_A$  is concave we conclude that  $\bar{x}^2 \leq \underline{x}^2$ .

## 11.6 Proof of Proposition 3

Given that each seller offers only the contracts  $(q^*, t^*)$ ,  $(\tilde{q}, \tilde{t})$ , each buyer chooses a pair  $(q_A, q_B) \in \{0, q^*, \tilde{q}\} \times \{0, q^*, \tilde{q}\}$ . In order to shorten notation, we define  $U^* \equiv U(q^*, 0)$ ,  $\tilde{U} \equiv U(\tilde{q}, 0)$ ,  $U_t^* \equiv U(q^*, q^*)$  (the subscript  $t$  means that the buyer buys quantity  $q^*$  twice, that is from both sellers),  $\tilde{U}^* \equiv U(q^*, \tilde{q})$ ,  $\tilde{U}_t \equiv U(\tilde{q}, \tilde{q})$ . Moreover, recall that  $t^* = \frac{1}{2}U_t^* - \frac{1}{2}u$ ,  $\tilde{t} = \tilde{U} - u$ .

**Step 1** Each buyer buys quantity  $q^*$  from each seller if and only if

$$2U^* - U_t^* \leq u \leq U_t^* + 2\tilde{U} - 2\tilde{U}^* \quad (49)$$

PROOF We can derive (49) by arguing as in Step 1 of the proof of Proposition 2.

**Step 2** No profitable deviation exists for a seller if and only if both (50) and (51) hold:

$$f(0, \tilde{q}) - \tilde{U} \leq f(q^*, q^*) - U_t^* \quad (50)$$

<sup>34</sup>Notice that since  $F$  is strictly concave,  $(\bar{x}^1, \bar{x}^2)$  is the *unique* maximum point for  $F$ ; hence  $F(\bar{x}^1, \bar{x}^2) > F(\underline{x}^1, \underline{x}^2)$ .

$$\left\{ \begin{array}{l} \max\{U_t^* - 2f(q^*, q^*) + 2f(0, q^*), U_t^* - f(q^*, q^*) + f(0, 0)\} \leq u \\ \text{and} \\ u \leq \min\{2f(q^*, q^*) - U_t^* - 2[f(q^*, \tilde{q}) - \tilde{U}], f(q^*, q^*) - U_t^* - [f(\tilde{q}, \tilde{q}) - 2\tilde{U}]\} \end{array} \right. \quad (51)$$

PROOF For each seller, the equilibrium profit is  $\pi^* \equiv 2t^* - C^* = f(q^*, q^*) - U_t^* - u$ , in which  $C^* \equiv C(2q^*)$ . We consider the possible deviations of seller  $A$ .

(i) The deviation which induces both buyers to buy only from  $A$  yields  $A$  the profit  $f(0, 0) - 2u$ , and the inequality  $f(0, 0) - 2u \leq \pi^*$  reduces to  $U_t^* - f(q^*, q^*) + f(0, 0) \leq u$ .

(ii) The deviation which induces buyer 1 to buy only from  $A$  and buyer 2 to buy  $q^*$  from  $B$  yields  $A$  the profit  $f(0, q^*) - t^* - 2u$ , and the inequality  $f(0, q^*) - t^* - 2u \leq \pi^*$  reduces to  $U_t^* - 2f(q^*, q^*) + 2f(0, q^*) \leq u$ .

(iii) The deviation which induces buyer 1 to buy only from  $A$  and buyer 2 to buy  $\tilde{q}$  from  $B$  yields  $A$  the profit  $f(0, \tilde{q}) - \tilde{t} - 2u$ , and the inequality  $f(0, \tilde{q}) - \tilde{t} - 2u \leq \pi^*$  reduces to (50).

(iv) The deviation which induces both buyers to buy  $q^*$  from  $B$  yields  $A$  the profit  $f(q^*, q^*) - 2t^* - 2u = f(q^*, q^*) - U_t^* - u$ , which is just the equilibrium profit.

(v) The deviation which induces buyer 1 to buy  $q^*$  from  $B$  and buyer 2 to buy  $\tilde{q}$  from  $B$  yields  $A$  the profit  $f(q^*, \tilde{q}) - t^* - \tilde{t} - 2u$ , and the inequality  $f(q^*, \tilde{q}) - t^* - \tilde{t} - 2u \leq \pi^*$  reduces to  $u \leq 2f(q^*, q^*) - U_t^* - 2[f(q^*, \tilde{q}) - \tilde{U}]$ .

(vi) The deviation which induces both buyers to buy  $\tilde{q}$  from  $B$  yields  $A$  the profit  $f(\tilde{q}, \tilde{q}) - 2\tilde{t} - 2u$ , and the inequality  $f(\tilde{q}, \tilde{q}) - 2\tilde{t} - 2u \leq \pi^*$  reduces to  $u \leq f(q^*, q^*) - U_t^* - [f(\tilde{q}, \tilde{q}) - 2\tilde{U}]$ .

The inequalities obtained from (i)-(ii) and (v)-(vi) are summarized by (51).

**Step 3** If  $u$  satisfies (51), then it satisfies (49).

PROOF The inequality  $2U^* - U_t^* \leq U_t^* - f(q^*, q^*) + f(0, 0)$  is equivalent to  $2U^* - C^* \leq f(0, 0)$ , which holds by definition of  $f(0, 0)$ . The inequality  $f(q^*, q^*) - U_t^* - [f(\tilde{q}, \tilde{q}) - 2\tilde{U}] \leq U_t^* + 2\tilde{U} - 2\tilde{U}^*$  is equivalent to  $2\tilde{U}^* - C^* \leq f(\tilde{q}, \tilde{q})$ , which holds by definition of  $f(\tilde{q}, \tilde{q})$ .

**Step 4** If  $C$  is strictly convex, then no profitable deviation exists for a seller if and only if (50) and (52) both hold, with

$$U_t^* - 2f(q^*, q^*) + 2f(0, q^*) \leq u \leq 2f(q^*, q^*) - U_t^* - 2[f(q^*, \tilde{q}) - \tilde{U}] \quad (52)$$

If  $C$  is concave, then no profitable deviation exists for a seller if and only if (50) and (53) both hold, with

$$U_t^* - f(q^*, q^*) + f(0, 0) \leq u \leq f(q^*, q^*) - U_t^* - [f(\tilde{q}, \tilde{q}) - 2\tilde{U}] \quad (53)$$

PROOF We prove that (51) reduces to (52) if  $C$  is strictly convex, and that (51) reduces to (53) if  $C$  is concave.

The inequality  $U_t^* - f(q^*, q^*) + f(0, 0) < U_t^* - 2f(q^*, q^*) + 2f(0, q^*)$  is equivalent to  $f(q^*, q^*) - f(q^*, 0) < f(0, q^*) - f(0, 0)$ , which holds if  $C$  is strictly convex, by (19).

Conversely, if  $C$  is concave then (20) applies and it follows that  $U_t^* - f(q^*, q^*) + f(0, 0) \geq U_t^* - 2f(q^*, q^*) + 2f(0, q^*)$  holds. The inequality  $2f(q^*, q^*) - U_t^* - 2[f(q^*, \tilde{q}) - \tilde{U}] < f(q^*, q^*) - U_t^* - [f(\tilde{q}, \tilde{q}) - 2\tilde{U}]$  is equivalent to  $f(\tilde{q}, \tilde{q}) - f(\tilde{q}, q^*) < f(q^*, \tilde{q}) - f(q^*, q^*)$ , which holds if  $C$  is strictly convex, by (19). Conversely, if  $C$  is concave then (20) applies and it follows that  $2f(q^*, q^*) - U_t^* - 2[f(q^*, \tilde{q}) - \tilde{U}] \geq f(q^*, q^*) - U_t^* - [f(\tilde{q}, \tilde{q}) - 2\tilde{U}]$ .

**Step 5** Increasing  $\tilde{q}$  relaxes (50), (52), and (53).

PROOF In the left hand side in (50),  $f(0, \tilde{q}) - \tilde{U} = \max_{x,y}[U(x, 0) + \int_0^y U_A(z, \tilde{q})dz - C(x+y)]$  is decreasing in  $\tilde{q}$  since goods are substitutes. The same property implies that the right hand side in (52),  $2\tilde{U} - 2f(q^*, \tilde{q}) = -2\max_{x,y}[U(x, q^*) + \int_0^y U_A(z, \tilde{q})dz - C(x+y)]$ , is increasing in  $\tilde{q}$  as well as the right hand side in (53),  $2\tilde{U} - f(\tilde{q}, \tilde{q}) = -\max_{x,y}[\int_0^x U_A(z, \tilde{q})dz + \int_0^y U_A(z, \tilde{q})dz - C(x+y)]$ .

**Step 6** Inequality (50) holds if  $\tilde{q}$  is sufficiently large, and the whole set of values for  $u$  is obtained from (52) and (53) as  $\tilde{q} \rightarrow +\infty$ .

PROOF The left hand side in (50) is equal to  $\max_{x,y}[U(x, 0) + \int_0^y U_A(z, \tilde{q})dz - C(x+y)]$ , whereas the right hand side in (50) is equal to  $U_t^* - C^* = \max_{x,y}(U(x, y) - C(x+y))$ . We prove that  $\lim_{\tilde{q} \rightarrow +\infty}[U(x, 0) + \int_0^y U_A(z, \tilde{q})dz] < U(x, y)$ , which implies that (50) holds for a large  $\tilde{q}$ . Precisely, the previous inequality is equivalent to  $\lim_{\tilde{q} \rightarrow +\infty} \int_0^y U_A(z, \tilde{q})dz < \int_0^y U_B(x, z)dz = \int_0^y U_A(z, x)dz$  (since  $U$  is symmetric), and the latter inequality holds since goods are strict substitutes.

## 11.7 Proof of Proposition 5

For  $j = A, B$ , let  $V_j^G \equiv \max_{q_j^1, \dots, q_j^n}(U^1(q_j^1, 0) + \dots + U^n(q_j^n, 0) - C_j(q_j^1 + \dots + q_j^n))$ . Then in each equilibrium under no group we have  $\pi^A + u^1 + \dots + u^n \geq V_A^G$  because we can argue as when  $n = 2$  (see the argument just after Proposition 2, and notice that this argument does not depend on whether or not a group is formed): if  $\pi^A + u^1 + \dots + u^n < V_A^G$ , then seller  $A$  can deviate by offering to the buyers a suitable trade in which each buyer buys only from  $A$ , and their joint payoffs increase to  $V_A^G$ .

Hence, we have  $\pi^B \leq V_{AB}^G - V_A^G$  and  $\pi^A \leq V_{AB}^G - V_B^G$ , which implies  $u^1 + \dots + u^n \geq V_A^G + V_B^G - V_{AB}^G$ , where  $V_A^G + V_B^G - V_{AB}^G$  is just the total payoff of the buyers in the sell-out equilibrium under buyer group.

Now we prove that without group, there exists no equilibrium satisfying  $u^1 + \dots + u^n = V_A^G + V_B^G - V_{AB}^G$ , hence  $u^1 + \dots + u^n > V_A^G + V_B^G - V_{AB}^G$  in each equilibrium. We argue by contradiction and suppose that an equilibrium exists such that  $u^1 + \dots + u^n = V_A^G + V_B^G - V_{AB}^G$ . Then  $\pi^A + \pi^B = 2V_{AB}^G - V_A^G - V_B^G$ . This together with  $\pi^A \leq V_{AB}^G - V_B^G$  and  $\pi^B \leq V_{AB}^G - V_A^G$  implies  $\pi^A = V_{AB}^G - V_B^G$  and  $\pi^B = V_{AB}^G - V_A^G$ .

Then we suppose that  $C_A$  (to fix the ideas) is strictly convex, and consider the deviation of seller  $A$  with  $b^i = q_B^{i*}$  for one  $i$ ,  $b^h = 0$  for  $h \neq i$ . This yields  $A$  the profit

$f_A(0, \dots, q_B^{i^*}, \dots, 0) - t_B^{i^*} - (V_A^G + V_B^G - V_{AB}^G)$ ,<sup>35</sup> hence this deviation is unprofitable if and only if  $f_A(0, \dots, q_B^{i^*}, \dots, 0) - V_A^G \leq t_B^{i^*}$ . The existence of no profitable deviation for  $A$  requires that this inequality holds for  $i = 1, \dots, n$ . Hence, we have

$$\sum_{i=1}^n f_A(0, \dots, q_B^{i^*}, \dots, 0) - nV_A^G \leq t_B^{1^*} + \dots + t_B^{n^*}. \quad (54)$$

Then notice that  $\pi^B = V_{AB}^G - V_A^G$  is equivalent to  $t_B^{1^*} + \dots + t_B^{n^*} = V_{AB}^G - V_A^G + C_B(q_B^{1^*} + \dots + q_B^{n^*})$ . In addition, we use (54),  $V_A^G = f_A(0, \dots, 0)$ ,  $V_{AB}^G + C_B(q_B^{1^*} + \dots + q_B^{n^*}) = f_A(q_B^{1^*}, \dots, q_B^{n^*})$  to obtain the inequality

$$\sum_{i=2}^n (f_A(0, \dots, q_B^{i^*}, \dots, 0) - f_A(0, \dots, 0)) \leq f_A(q_B^{1^*}, \dots, q_B^{n^*}) - f_A(q_B^{1^*}, \dots, 0). \quad (55)$$

Finally, we use (42) to show that (55) cannot hold: setting  $\underline{b}^i = 0$ ,  $\bar{b}^i = q_B^{i^*}$  for  $i = 1, \dots, n$  in (42) yields the opposite of (55).

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<sup>35</sup>The function  $f_A$  is defined in (41).