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Abstract

This paper shows that vertical foreclosure can have a dynamic rationale. By refusing to supply an efficient downstream rival, a vertically integrated incumbent sacrifices current profits but can exclude the rival by depriving it of the critical profits (or sales) it needs to be successful. In turn, monopolising the downstream market may prevent the incumbent from losing its future profits because: (a) it allows the incumbent to extract rents from an efficient upstream rival if future upstream entry cannot be discouraged; or (b) it also deters future upstream entry by weakening competition for the input and reducing the post-entry profits of the prospective upstream competitor.

JEL Classification: K21, L41

Keywords: Inefficient foreclosure, Refusal to supply, Exclusion, Monopolisation

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Dynamic Vertical Foreclosure*

Chiara Fumagalli[†] Massimo Motta[‡]

December 11, 2017

Abstract

This paper shows that vertical foreclosure can have a dynamic rationale. By refusing to supply an efficient downstream rival, a vertically integrated incumbent sacrifices *current* profits but can exclude the rival by depriving it of the critical profits (or sales) it needs to be successful. In turn, monopolising the downstream market may prevent the incumbent from losing its future profits because: (a) it allows the incumbent to extract rents from an efficient upstream rival if future upstream entry cannot be discouraged; or (b) it also deters *future* upstream entry by weakening competition for the input and reducing the post-entry profits of the prospective upstream competitor.

Keywords: Inefficient foreclosure, Refusal to supply, Scale economies, Exclusion, Monopolisation.

JEL Classification: K21, L41

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1 Introduction

Vertical foreclosure refers to situations in which a vertically integrated firm which dominates one market acts in such a way to exclude (or marginalize) rivals in vertically related markets. For example, a monopoly owner of a necessary input may refuse to sell it to the downstream competitors and reserve it all for its own downstream affiliate. The upstream monopolist may also resort to more subtle ways to foreclose the activity of the downstream rivals, for instance by reducing the quality of the input supplied to rivals, by degrading interconnection, or by delaying the input provision.[1]

However, the rationale for vertical foreclosure has long been contested. In particular, the so-called Chicago School critique pointed out that while the owner of an essential input may have the *ability* to exclude downstream rivals, it would rarely have the *incentive* to do so, especially in the presence of more efficient downstream rivals: this is because the control of the bottleneck input enables the upstream monopolist to earn higher profits by serving efficient downstream rivals, and extracting rents from them, rather than excluding them.

Modern industrial organization and antitrust scholars have been struggling to find a rationale for anti-competitive vertical foreclosure. The common explanation behind these theories, that we will discuss more in detail at the end of this Section, is that they all rely on *imperfect* rents extraction: they have identified some circumstances under which the upstream monopolist is able to extract *too little* from the downstream rivals and for this reason it may find it more profitable to foreclose them and to monopolise the final market through the own, though less efficient, affiliate.

These theories are not always satisfactory, and either require strong assumptions or apply only to very particular (e.g., regulated) sectors. They also share a static perspective. This paper identifies instead a dynamic rationale for anti-competitive vertical foreclosure. We consider a vertically integrated incumbent that faces the threat of entry in the downstream market in the current period and in the upstream market in the future period. (However, the same mechanism would apply if the scope for current entry is upstream and future entry may take place in the downstream market.) In this setting, we show that the upstream monopolist may have an incentive to foreclose a more efficient downstream rival even though it sacrifices current profits.

In the current period, in which only downstream entry can take place, the incumbent would find it more profitable to accommodate downstream entry and to supply the more efficient downstream competitor, because it would be able to extract sufficient rents from it. However, if downstream entry occurs in the current period, also upstream entry will occur in the future, and with entry in both markets the incumbent will make little (or zero) profits. Instead, if the incumbent engages in refusal to supply, it will affect the future market structure and will earn higher future profits. Lack of suitable access to the input may deprive the rival of the critical profits (or, more generally, of the critical scale, customer base, or reputation) it needs in order to be successful in the downstream market. Then refusal to supply excludes (or marginalises) the independent rival from the downstream market. Monopolisation of the downstream market will in turn allow the vertically integrated incumbent to increase its future profits, via either of the following mechanisms.

If future entry cannot be discouraged (for instance because upstream entry entails very low fixed setup costs relative to the profits the entrant could make even in the worst case scenario), the

[1] Alternatively, the vertically integrated firm could set a combination of high upstream (or wholesale) prices and low downstream (or retail) prices such that competitors cannot profitably operate in the downstream market, a practice known as margin squeeze.

incentives to engage in vertical foreclosure turn out to be very strong. The incumbent knows that it will lose the upstream monopoly in any event, but it finds it optimal to foreclose the downstream competitor in the current period so as to obtain a downstream monopoly in the future and use such position to extract rents from the more efficient upstream entrant. (If a more efficient downstream rival was in the market, the incumbent would be able to extract fewer - or no - rents from the upstream entrant.) In this case foreclosure is motivated by the incumbent's intent to build monopoly power downstream so as to gain a better position when contracting with the upstream rival.

If upstream entry costs are not that small, then foreclosure of the downstream rival will weaken competition for input procurement, and reduce the post-entry profits of the prospective upstream competitor, thereby discouraging also *future* upstream entry. In this case foreclosure *protects* the incumbent's monopoly position in *both* the vertically related market. Note that the incumbent's incentive to engage in vertical foreclosure is weaker in this latter case: once the incumbent dominates the downstream market, its future profits are higher when the independent upstream firm – rather than the own less efficient affiliate – operates in the upstream market, because some rents can be extracted from it. However, monopolising both markets is more profitable than facing competition in both of them, and the incumbent benefits from vertical foreclosure also in this latter case, although to a lower extent.

The reader familiar with the literature on exclusionary practices[2] will have noticed that the latter mechanism (but not the former) is reminiscent of Carlton and Waldman (2002)'s model of exclusionary tying between a primary product and a complementary one. We will discuss in Section 2.3 the additional insights that our analysis of vertical foreclosure brings as compared to Carlton and Waldman (2002).

We study the dynamic rationale for anti-competitive vertical foreclosure in a baseline model in which the rival in the downstream market is a potential entrant (Section 2). In this environment the decision to engage in refusal to supply needs to have commitment value to lead to foreclosure. We extend the baseline model considering the case in which the decision to engage in refusal to supply is reversible (Section 3). We show that refusal to supply can nonetheless lead to foreclosure because it allows a weak (or inefficient) incumbent to build a reputation of being tough (or very efficient), thereby discouraging entry in other downstream markets. In Section 5 we consider the case where the commitment to refusal to supply is short-lived (i.e. the incumbent can supply the rival in the second period) and that where the downstream and upstream entrants belong to the same company. We also modify the baseline model considering a downstream market characterised by learning effects and we show that the incumbent can engage in refusal to supply to exclude a rival that is already in the market (Section 4). Finally, we will discuss in the Conclusions (6) the crucial ingredients for our theory of harm to be applied as well as some recent cases in which our theory might rationalize the conduct of the dominant firm.

We close the Introduction with a brief discussion of the related literature on vertical foreclosure. As we mentioned earlier, in this literature it is the inability of the upstream monopolist to extract sufficient rents from the more efficient downstream competitor that may generate an incentive to foreclose its activity. This inability to extract rents may be due to the presence of sectoral regulation which restricts the upstream monopolist's freedom to contract with downstream rivals.[3]

[2] See Fumagalli et al (2018) for an extensive discussion.

[3] See Jullien, Rey and Saavedra (2014) and Fumagalli et al. (2018), for models that study the conditions under which regulation of the wholesale price induces a vertically integrated incumbent to engage in refusal to supply and in

Another source of imperfect rent extraction is the so called 'commitment problem', first proposed by Hart and Tirole (1990) and recently applied by Reisinger and Tarantino (2014) to a context in which a vertically integrated incumbent faces a more efficient downstream rival.[4] The vertically integrated incumbent would like to extract the entire rents produced by the more efficient downstream competitor, through a suitable choice of contracts. For instance, it may want the more efficient downstream rival to set all or most of the production; to set (industry) profit maximizing prices and to earn high profits. Such profits would then be extracted by the upstream monopolist by requiring a large (fixed) payment for the input in exchange. However, if the downstream rival feared that the incumbent might use its downstream affiliate to compete for consumers (i.e. that it would behave opportunistically, rather than being inactive in the downstream market) its willingness to pay for the input would decrease, since expected competition from the incumbent's downstream affiliate would decrease the rival's expected profits. In turn, this would limit the ability of the incumbent to extract profits from the independent downstream rival and may make a foreclosure strategy potentially more profitable.[5]

Finally, if the incumbent faces some competition in the provision of the input, then the incentive to deny the input to independent downstream firm may come from the so-called *raising rivals' cost* argument, due to Ordover et al. (1990): the incumbent's withdrawal from the wholesale market will make the downstream rival pay a higher price for its input requirements, because such inputs will be bought from the independent upstream firm, which will enjoy stronger market power over the independent downstream firm once the integrated incumbent commits not to serve the input to it. In this case the downstream competitor is not completely excluded from the market, but it faces higher input costs, which makes it less competitive and aggressive in the downstream market, to the benefit of the incumbent's downstream profits.[6] The credibility of the commitment is also a delicate issue in this theory, since after the upstream independent rival raises its input price, the vertically integrated affiliate would be tempted to serve the independent downstream rival.

As we emphasized earlier, we depart from this literature because in our paper the incentive to engage in vertical foreclosure does not stem from imperfect rent extraction. Indeed, in the *current* period, when entry occurs only in one of the vertically related market, the incumbent sacrifices profits by engaging in refusal to deal. However, vertical foreclosure affects the future market structure and allows the incumbent to make larger profits in the *future*.

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margin squeeze.

[4]See also the work by O'Brien and Shaffer (1992), McAfee and Schwartz (1994), Rey and Vergé (2004). See also Rey and Tirole (2007) for an insightful review of this literature.

[5]Interestingly, Reisinger and Tarantino (2014) shows that the inability not to operate the downstream affiliate does not necessarily lead to complete foreclosure of the independent rival. If the efficiency gap between the incumbent's affiliate and the independent rival is not too large, the incumbent engages in partial foreclosure: it does supply the independent rival, but at less favourable terms (i.e. at a higher wholesale price than the own affiliate. Instead, if the efficiency gap is large enough, the incumbent finds it optimal to offer to the independent rival a wholesale price which is even lower than the one paid by the own affiliate.

[6]In both cases, there is a reduction in the competition for the input. However, in Ordover and al.'s paper the market structure is given (neither downstream nor upstream entry can be deterred), and the aim of refusal to supply is to relax downstream competition; in doing so, however, the upstream rival actually benefits from it. In our paper instead, due to the lack of downstream independent entry the upstream rival can actually be harmed - and its entry may be deterred - from refusal to supply.

2 The Baseline Model

An indispensable input is sold by a monopolist seller, U_I, which is the upstream affiliate of the vertically integrated firm I. Firm I also operates in a downstream market through its downstream affiliate D_I which uses one unit of the input to produce one unit of a final product. Upstream and downstream production is characterised by constant marginal costs. Market demand is given by $Q = 1 - p$.[7]

We analyse a two-period game. In period 1 a rival firm D_E considers entry in the downstream market, while an upstream competitor U_E can enter at a subsequent period, i.e. in period 2. The two entrants are not vertically integrated.

Upstream firms and (respectively) downstream firms sell perfectly homogeneous inputs and (respectively) outputs. We also assume that potential entrants (both upstream and downstream) are more efficient than the incumbent: $c_{UE} = c_{DE} = 0 < c = c_{UI} = c_{DI} \leq \bar{c}$.[8] Upstream and downstream entry entails fixed costs F_U and F_D respectively, which satisfy the following restrictions:

$$0 \leq F_U < \frac{\pi^d(0, 2c)}{2} \equiv \overline{F}_U \tag{A1}$$

$$0 < F_D < \frac{1}{2}\pi^d(0, 2c) + \frac{1}{2}[\pi^m(c) - \pi^m(2c)] \equiv \overline{F}_U \tag{A2}$$

where $\pi^m(c_i)$ indicates the monopoly profits of a firm with marginal cost c_i and facing market demand $Q(p)$, while $\pi^d(c_i, c_j)$ indicates the total duopoly profits obtained by a firm with marginal cost c_i competing à la Bertrand in the final market with a firm with marginal cost $c_j > c_i$ and $p^m(c_i) > c_j$.

The timing of the game is as follows:

1. Period 0: The incumbent decides whether to commit to 'refusal to supply' or, alternatively, to supply the downstream rival.

2. Period 1, stage 1: Firm D_E, after observing the incumbent's decision, decides whether to enter (and pay the fixed sunk cost F_D) or not;

3. Period 1, stage 2: If D_E is active, with probability 1/2, the incumbent makes a take-it-or-leave-it offer to D_E. It offers the contract $T_I(q) = w_I q + FF$ where q is the amount of input that D_E purchases from the incumbent. With probability 1/2, it is D_E that makes a take-it-or-leave-it offer to the incumbent. We assume that the incumbent can credibly commit not to operate the downstream unit.

4. Period 1, stage 3: If D_E is active, the contract offer is accepted/rejected. Then active downstream firms choose final prices p_E and p_I, firm D_E orders the input to satisfy demand, paying accordingly, and transforms one unit of the input into one unit of the final product.

5. Period 2, stage 1: Firm U_E decides whether it wants to enter the upstream market; D_E can still enter if it did not enter in period 1.

[7]Many of the results can be obtained assuming that market demand is given by a generic function $Q = Q(p)$ with $Q(p)$ continuous, decreasing in p, concave and twice differentiable. We explicitly mention in the proofs when we need to parametrise the model.

[8]The assumption that the incumbent's affiliates have equal marginal costs simplifies the exposition. The assumption $c \leq \bar{c}$ ensures that $p^m(0) \geq 2c$: the monopoly price at zero marginal costs is higher than the marginal cost of the (less efficient) vertical integrated firm, so that $p_I = p_2 = 2c$ is an equilibrium in the final market when one firm has zero marginal cost and the other has marginal cost equal to $2c$.

6. Period 2, stage 2: With probability 1/2 active upstream firms make take-it-or-leave-it offers; with probability 1/2 active downstream firms do.

7. Period 2, stage 3: Contract offers are accepted/rejected. The active downstream firms set final prices p_I and p_E, orders are made, payments take place and payoffs are realized.

We also assume that the factor at which firms discount future profits is $\delta = 1$. The timing of the game is summarised by Figure 1.

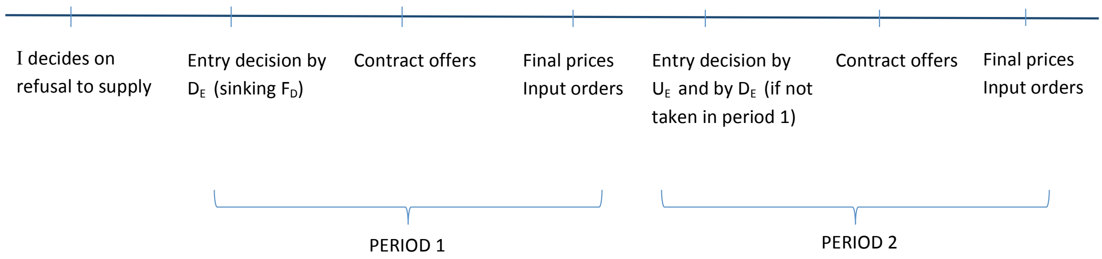

Figure 1. Timeline

Discussion of the assumptions. Before solving the model by backward induction, let us discuss the role of some assumptions.

The upper bounds on the (downstream and upstream) fixed costs ensure that, absent refusal to supply, entry (in the downstream and upstream market respectively) is profitable. This makes the analysis meaningful.

In the baseline model the downstream independent rival is a potential entrant. In this environment refusal to supply discourages downstream entry because it deprives the downstream entrant of the key profits necessary to cover the entry costs. Lack of downstream entry will allow the incumbent to extract more rents from the second period contracting with the upstream entrant. In Section 4 we will consider a variant of the model in which the downstream rival is already in the market and in which there exist inter-temporal scale economies. In this case refusal to supply deprives the rival of the key first-period sales, thereby preventing learning. For this reason, the incumbent will gain a favourable position in the second period contracting. One would reach a similar conclusion if the downstream market was characterised by network externalities, i.e. demand side scale economies.

We also assume throughout that the incumbent can credibly commit not to operate the downstream unit, thereby ruling out the possibility for the incumbent to engage in opportunistic behavior.[9] We do so because we want to highlight a new rational for vertical foreclosure, other than imperfect rents extraction.

In the baseline model we also assume that the incumbent is able to (publicly and irreversibly) commit not to serve the independent downstream firm, at least for one period. In that model this is crucial for foreclosure to arise. One possible way to credibly commit to refusal to supply may be to design the input in such a way that it is compatible with the downstream affiliate only (see Choi and

[9]See Hart and Tirole (1990) and Reisinger and Tarantino (2015).

Yi, 2000 and Church and Gandal, 2000).[10] In Section 3 we will assume that the decision to engage in refusal to supply is reversible and that the incumbent can discourage downstream entry because of reputation building effects. In Section 4, in which there are inter-temporal scale economies in the downstream market, the independent rival is already in the market and the decision to engage in refusal to supply does not need to be irreversible to exclude the rival. The same qualitative results would arise in a model with network externalities.

Finally, we assume that upstream and downstream firms have equal probability to make take-it-or-leave-it offers when contracting the terms of trade. This simplifies the analysis without loss of generality.

We now derive the equilibria of the game, moving by backward induction.

2.1 The incumbent did not commit to refusal to supply

Let us start with the subgame in which the incumbent decided not to engage in refusal to supply. The following Lemma summarises the post-entry payoffs in period 2, depending on the configurations of active firms.

Lemma 1. ***Post-entry payoffs (No Refusal to Supply)***
The following table indicates the (period-2) payoffs of the incumbent and of the independent firms (gross of the entry costs), in the different configurations of active firms, when the incumbent did not commit to refusal to supply:

Table 1. Post-entry payoffs in period 2 without refusal to supply.

<table>
<tr><th>D_E, U_E</th><th>Active</th><th>Not Active</th></tr>
<tr><td>Active</td><td>$\pi_I = \begin{cases} 0 & \text{if } c \in (\widehat{c}, \overline{c}] \\ \pi^m(c) - \pi^d(0,2c) & \text{otherwise} \end{cases}$
$\Pi_{D_E} = \frac{1}{2}\pi^d(0,2c)$
$\Pi_{U_E} = \frac{1}{2}\pi^d(0,2c)$</td><td>$\pi_I = \frac{1}{2}\pi^m(c) + \frac{1}{2}\pi^m(2c)$
$\Pi_{D_E} = \frac{1}{2}[\pi^m(c) - \pi^m(2c)]$
$\Pi_{U_E} = 0$</td></tr>
<tr><td>Not Active</td><td>$\pi_I = \frac{1}{2}\pi^m(c) + \frac{1}{2}\pi^m(2c)$
$\Pi_{D_E} = 0$
$\Pi_{U_E} = \frac{1}{2}[\pi^m(c) - \pi^m(2c)]$</td><td>$\pi_I = \pi^m(2c)$
$\Pi_{D_E} = 0$
$\Pi_{U_E} = 0$</td></tr>
</table>

Proof. See Appendix A.1.1. □

There are three aspects to discuss concerning Table 1. First, the incumbent is better off when only one independent firm is active (either in the upstream or downstream market) as opposed to the case in which no independent firm is active. In the latter case the incumbent monopolises the final market by using its own *less efficient* upstream and downstream technologies. Hence, the incumbent makes the monopoly profits associated to a marginal cost equal to $2c$: $\pi^m(2c)$. When only one independent firm is active, say the independent firm in the downstream market, the incumbent is left with $\pi^m(2c)$ when it does not make the take-it-or-leave-it offer (which occurs with probability 1/2); however, when it does make the offer, the incumbent extracts the entire monopoly profits associated with the more efficient downstream technology, i.e. $\pi^m(c) > \pi^m(2c)$.

[10]In the baseline model we assume that the commitment to refusal to supply lasts forever. Section 5.1 will discuss the case in which the commitment to refusal to supply is reversible after one period. In that case refusal to supply is less likely to arise at the equilibrium because it is less likely that it discourages downstream entry. Moreover, important scale economies need to characterise the downstream market for refusal to supply to emerge.

Second, each independent firm makes higher profits when the other independent firm is also active and competition in that market intensifies. Consider the independent downstream firm. When the independent upstream firm is not active, the profits of firm D_E are entirely extracted by the incumbent, when the incumbent makes the take-it-or-leave-it offer; firm D_E is able to appropriate the increase in monopoly profits due to the use of its more efficient downstream technology when, with probability 1/2, it makes the take-it-or-leave-it offer. Instead, when firm U_E is active, each independent firm obtains half of the duopoly profits produced in the final market when a firm with marginal cost equal to zero competes with a less efficient rival having marginal cost equal to $2c$. Since $\pi^d(0, 2c) > \pi^m(c) - \pi^m(2c)$ (by the Arrow's replacement effect), increased competition in the vertically related market benefits independent firms. An implication of this result is that an independent firm being active in a market facilitates entry in the vertically related market, as Lemma 2 indicates.

Third, when both independent firms are active, the incumbent makes zero profits when the marginal cost of its own affiliates is large enough, i.e. when they are sufficiently inefficient relative to the independent entrants. Instead, the incumbent makes positive profits when the inefficiency of its own affiliate is not too large. Indeed, when the inefficiency of the own affiliates is not too large ($c \leq \widehat{c}$), competition both in the upstream and downstream market transfers most of the benefits of the presence of the more efficient independent firms to final consumers, making industry profits when both independent firms participate to trade (the duopoly profits obtained when the vertical chain formed by the independent firms competes with the vertically integrated incumbent) lower than industry profits when one independent firm does not sell, i.e. the monopoly profits obtained when the other independent firm trades with the incumbent's affiliate. This implies that at the equilibrium one independent firm does not sell and the incumbent manages to extract the increase in industry profits due to the lessening of (downstream/upstream) competition. Instead, when the inefficiency of the incumbent's affiliates is large ($c > \widehat{c}$), industry profits when both independent firms participate to trade are larger than industry profits when one independent firm does not sell. This ensures that, at the equilibrium, the independent downstream firm will be supplied by the independent upstream firm and will prevail in the final market, while the incumbent's vertically integrated firm will not sell.

Lemma 2. ***Entry decisions in Period 2 (No Refusal to Supply).***
If the incumbent did not commit to refusal to supply:

> *(i) If firm D_E entered in period 1, then firm U_E enters in period 2 for any feasible value of the entry cost F_U.*
>
> *(ii) If firm D_E did not enter in period 1, then depending on the configurations of fixed costs F_U and F_D, the continuation of the game may exhibit a unique equilibrium in which both independent firms enter the market; a unique equilibrium in which only firm U_E enters the upstream market; an equilibrium in which no independent firm enters the market; multiple equilibria in which either both upstream firms enter the market or neither of them does.*

Proof. (i) If firm D_E entered the downstream market in period 1, then firm U_E will earn $\pi_{U_E} = \frac{1}{2}\pi^d(0, 2c) - F_U$ if it enters and $\pi_{U_E} = 0$ otherwise. By assumption $A1$, it decides to enter.
(ii) See Appendix A.1.2. □

We can now study the entry decision taken by firm D_E in the first period. As shown by the following Proposition, by assumption A2 the total post-entry profits earned by firm D_E when it enters in period 1 are large enough to cover the entry cost. Then, period-1 entry is more profitable than no entry (both in period 1 and in period 2). Moreover, entry in period 1 is more profitable than entry in period 2 because it allows firm D_E to earn positive profits for an additional period. Then, if the incumbent does not engage in refusal to supply, the downstream independent firm always enters in period 1.

Lemma 3. ***Entry decision at period*** 1 ***(No Refusal to Supply)****:*
If the incumbent did not commit to refusal to supply, firm D_E enters downstream in the first period.

Proof. Firm D_E anticipates that, if it enters in period 1, then firm U_E will enter in period 2 and its total profit is $\pi^{1+2}_{D_E|Entry} = \frac{1}{2}[\pi^m(c) - \pi^m(2c)] + \frac{1}{2}\pi^d(0, 2c) - F_D$. $\pi^{1+2}_{D_E|Entry} > 0$ by assumption A2. Firm D_E also anticipates that, if it decides not to enter in period 1, then as established by Lemma 2 the continuation equilibrium may be such that either firm D_E does not enter in period 2 (because post-entry profits earned in period 2 alone are insufficient to cover the entry cost), and its continuation profits are zero; or such that firm D_E enters in period 2; in that case continuation profits are $\pi^d(0, 2c) - F_D$. In both cases, it is more profitable for firm D_E to enter in period 1:

$$\frac{1}{2}[\pi^m(c) - \pi^m(2c)] + \frac{1}{2}\pi^d(0, 2c) - F_D > max\{0, \frac{1}{2}\pi^d(0, 2c) - F_D\}$$

□

2.2 The incumbent committed to refusal to supply

Let us analyse now the subgame in which the incumbent decided to engage in refusal to supply, starting from the post-entry payoffs in period 2.

Lemma 4. ***Post-entry payoffs (Refusal to Supply)***
The following table indicates the (period-2) payoff of the incumbent and of the independent firm (gross of the entry cost), in the different configurations of active firms, when the incumbent committed to refusal to supply:

Table 2. Post-entry payoffs in period 2 with refusal to supply.

D_E, U_E	Active	Not Active
Active	$\pi_I = \begin{cases} 0 & \text{if } c \in (\widehat{c}, \overline{c}] \\ \frac{1}{2}[\pi^m(c) - \pi^d(0, 2c)] & \text{otherwise} \end{cases}$ $\Pi_{D_E} = \begin{cases} \frac{1}{2}[\pi^d(0, 2c) - \pi^m(c)] & \text{if } c \in (\widehat{c}, \overline{c}] \\ 0 & \text{otherwise} \end{cases}$ $\Pi_{U_E} = \begin{cases} \frac{1}{2}[\pi^d(0, 2c)] + \frac{1}{2}[\pi^m(c)] & \text{if } c \in (\widehat{c}, \overline{c}] \\ \pi^d(0, 2c) & \text{otherwise} \end{cases}$	$\pi_I = \pi^m(2c)$ $\Pi_{D_E} = 0$ $\Pi_{U_E} = 0$
Not Active	$\pi_I = \frac{1}{2}\pi^m(c) + \frac{1}{2}\pi^m(2c)$ $\Pi_{D_E} = 0$ $\Pi_{U_E} = \frac{1}{2}[\pi^m(c) - \pi^m(2c)]$	$\pi_I = \pi^m(2c)$ $\Pi_{D_E} = 0$ $\Pi_{U_E} = 0$

Proof. See Appendix A.1.3. □

By comparing Table 2 above with Table 1, one can note that the decision of the incumbent to engage in refusal to deal limits the profits that firm D_E earns if it enters the market as compared to the case in which there is no commitment to refusal to supply. Trivially, when the independent firm in the upstream market is not active, under refusal to deal firm D_E cannot obtain the input and makes zero profits. However firm D_E's profits are reduced also when the independent firm in the upstream market is active. Under refusal to supply the independent downstream firm cannot receive any offer from the incumbent's upstream affiliate, which removes competition between the upstream suppliers and weakens firm D_E's ability to extract rents when contracting for the input.[11] For this same reason, entering the upstream market when firm D_E is also active downstream is more profitable for firm U_E under refusal to supply than when refusal to supply is not in place. Then, if firm D_E entered the downstream market in period 1, it would *a fortiori* be true that upstream entry will occur in period 2 (see Lemma 5 point (i)).

From the above discussion it follows that refusal to supply reduces (or cancels out) the post-entry profits that firm D_E can earn in the second period and prevents firm D_E from earning profits in the first period, when the upstream independent firm cannot be active. Then, refusal to supply makes downstream entry in period 2 (see Lemma 5, point (ii)) as well as in period 1 (see Lemma 6) less likely than in the case in which there is no refusal to supply. Moreover, since the presence of the downstream independent firm is beneficial for the upstream independent firm, when upstream entry costs are large enough lack of downstream entry discourages also upstream entry (see Lemma 5, point (ii)).

The decision to engage in refusal to supply also reduces the incumbent's profits. When only the downstream firm is active, refusal to supply prevents the incumbent from extracting from the independent firm the increase in monopoly profits that the use of its more efficient technology can produce. Then the decision to engage in refusal to supply determines a short-term sacrifice of profits for the incumbent, as we will discuss in Section 2.3. When both independent firms are active, and the incumbent's affiliates are sufficiently inefficient, (i.e. when $c > \widehat{c}$), refusal to supply is irrelevant because the incumbent makes zero profits irrespective of it. Instead, when the incumbents' subsidiaries are not very inefficient, the incumbent makes positive profits with and without refusal to supply. However, those profits are lower in the former case. The reason is that, when offers are made by upstream firms, refusal to supply limits the incumbent's ability to react to U_E's offers and the entire industry profits are appropriated by U_E. Instead, absent refusal to supply, when upstream firms make the offers the incumbent would appropriate the increase in industry profits that exclusion of one independent firm (U_E in this scenario), and the consequent lessening of competition, produces.

Lemma 5. ***Entry decisions in period 2. (Refusal to Supply)***
If the incumbent committed to refusal to supply:

(i) If firm D_E entered in period 1, then firm U_E enters in period 2.

(ii) If firm D_E did not enter in period 1 then there is less scope for downstream and upstream entry in period 2 relative to the case of no refusal to supply.

Proof. See Appendix A.1.4. □

[11] This is the same effect that arises Ordover et al. (1990). However, differently from their model, here refusal to deal benefits firm U_E but not the incumbent. This is because there is no product differentiation in the final market.

Lemma 6. ***Entry decision at period*** 1 ***(Refusal to Supply)***
If the incumbent committed to refusal to supply downstream entry in period 1 *occurs for a narrower range of parameters' values relative to the case of no refusal to supply.*

> *(i) The downstream independent firm enters the downstream market iff $c > \widehat{c}$ and $F_D \leq F_D^{RtoS}$. In this case upstream entry follows in period* 2*.*
>
> *(ii) Otherwise, firm D_E does not enter in either period. In this case, upstream entry follows iff $F_U \leq F_U^{RtoS}$.*

Proof. Firm D_E anticipates that firm U_E will enter for sure in period 2 if firm D_E enters in period 1. In this case the total post-entry profits of firm D_E are $\pi_{D_E} = \frac{\pi^d(0,2c)-\pi^m(c)}{2} - F_D$ when $c > \widehat{c}$ and $0 - F_D$ otherwise. Downstream entry is profitable iff $c > \widehat{c}$ and $F_D \leq \frac{\pi^d(0,2c)-\pi^m(c)}{2} \equiv F_D^{RtoS}$. Firm D_E makes the same profits by postponing entry to period 2, when in period 2 an equilibrium with both upstream and downstream entry occurs, i.e. when either $c > \widehat{c}$, $F_D \leq F_D^{RtoS}$ and $F_U \leq \frac{\pi^m(c)-\pi^m(2c)}{2} \equiv F_U^{RtoS}$ (when entry in both markets is the unique equilibrium) or $c > \widehat{c}$, $F_D \leq F_D^{RtoS}$, $F_U > F_U^{RtoS}$ and independent firms coordinate on the entry equilibrium. In these cases firm D_E is indifferent between entry in period 1 and entry in period 2. If instead, $c > \widehat{c}$, $F_D \leq F_D^{RtoS}$, $F_U > F_U^{RtoS}$ and independent firms coordinate on the no-entry equilibrium, firm D_E would obtain $0 - F_D$ by entering in period 2. In this case it strictly prefers to enter in period 1 which avoids coordination failures. □

2.3 Refusal to supply in equilibrium

Lemma 1 highlights that refusal to supply sacrifices the incumbent's profits in the first period as the incumbent would extract some of the downstream entrant's efficiency rents if it dealt with it in the first period. Moreover, when the upstream and downstream rivals are active, refusal to supply (weakly) decreases the incumbent's profits also in the second period, by limiting the incumbent's ability to make counteroffers. Then, if downstream entry occurs irrespective of refusal to supply, the incumbent will not have an incentive to engage in it. When instead refusal to supply discourages downstream entry, there is a trade-off: in the short-run, refusing to supply is costly, but in the long-run it is beneficial.

Refusal to supply increases long-run profits for two reasons. When upstream entry costs are sufficiently low (i.e. $F_U \leq F_U^{RtoS}$), upstream entry occurs in period 2 even in the absence of downstream entry. In this case, by discouraging downstream entry, refusal to supply allows the incumbent to build monopoly power downstream, thereby gaining a more favorable position in the second-period contracting for the input. Indeed, when the incumbent engages in refusal to supply, it will be the unique buyer of the input in the second period, and it will manage to extract some of the efficiency rents produced by the more efficient upstream producer. Instead, when it does not engage in refusal to supply and downstream entry occurs, the incumbent will face competition from the downstream independent firm when contracting for the input and will lose all of its second period profits when the own affiliates are sufficiently inefficient, i.e. $c > \widehat{c}$; when the own affiliates are not that inefficient (when $c < \widehat{c}$), not all the profits are lost, but competition exerted by the independent downstream firm still decreases them.

When, instead, upstream costs are sufficiently large (i.e. $F_U > F_U^{RtoS}$), lack of downstream entry, by reducing the post-entry profits of the upstream independent firm, discourages also future upstream entry. In this case, refusal to supply allows the incumbent to increase future profits because it *protects* its monopoly power in *both* vertically related markets. Note that once downstream entry is discouraged, the incumbent's profits would be higher if upstream entry occurred, since the incumbent could extract some rents from the more efficient independent upstream firm. However, entry in neither market is more profitable for the incumbent than entry in both of them. Then, refusal to supply is beneficial in the long-run also in this case, even though to a lower extent than in the case in which upstream entry occurs anyway.

As we show below the long-term increase in profits dominates the short-term cost, and the incumbent engages in refusal to supply at the equilibrium.

When the incumbent engages in refusal to supply consumers suffer a loss: since entry of the more efficient downstream firm is discouraged, they end up paying a higher price relative to the case in which there is no refusal to supply. Refusal to supply also harms the independent firms. The downstream firm is harmed because it is excluded from the market. The upstream firm is either excluded or it enters anyway, but post-entry profits are lower under refusal to supply because lack of downstream entry softens competition for the provision of the input (see Lemma 1). Then, refusal to supply reduces total welfare if the loss of consumers and of the independent firms dominates the incumbent's gain.

As stated by Proposition 1, when the own affiliates are relatively inefficient ($c > \widehat{c}$), refusal to supply decreases total welfare whenever the incumbent decides to engage in it. When instead the incumbent's own affiliates are relative efficient ($c < \widehat{c}$), refusal to supply decreases total welfare if the entry costs F_D and F_U are sufficiently low. Instead, when the incumbent's own affiliates are relative efficient and entry costs are large enough, refusal to supply is welfare beneficial because it discourages inefficient entry.

Proposition 1. ***Profitability of refusal to supply and welfare effects***

(i) The incumbent engages in refusal to supply if either $c \leq \widehat{c}$ or $c > \widehat{c}$ and $F_D > F_D^{RtoS}$.

(i.a) Refusal to supply discourages only downstream entry when $F_U \leq F_U^{RtoS}$.

(i.b) Refusal to supply discourages also upstream entry when $F_U > F_U^{RtoS}$.

(ii) Refusal to supply decreases consumer surplus.

(iii) When $c > \widehat{c}$ and $F_D > F_D^{RtoS}$ refusal to supply lowers total welfare. When $c \leq \widehat{c}$ refusal to supply lowers total welfare if either $F_U \leq F_U^{RtoS}$ and $F_D \leq F_D^W$ or $F_U > F_U^{RtoS}$ and $F_D + F_U \leq F^W$.

Proof. See Appendix A.1.5 □

Discussion Proposition 1 has shown that vertical foreclosure can have a dynamic rationale. When future upstream entry cannot be deterred (i.e. when the incumbent cannot protect its upstream monopoly), then refusing the input today to the downstream rival may allow the vertically integrated dominant firm to *'transfer'* or *'create'* a downstream monopoly in the future, and use such position to extract rents from the more efficient upstream entrant when contracting with it. Instead, when a downstream rival's success is a pre-condition for an upstream rival's entry, a vertically integrated dominant firm may deny the input to the downstream rival today with the objective of maintaining its monopoly power in *both* vertically related markets tomorrow.

Note that the profitability of vertical foreclosure is *weaker* in the latter case. Once the incumbent dominates the downstream market, its future profits are higher when the upstream independent firm – rather than the own less efficient affiliate – operates in the upstream market, precisely because some rents can be extracted from it. However, monopolising both markets is more profitable than facing competition in both of them, and the incumbent has an incentive to engage in vertical foreclosure also in the latter case, although to a lower extent.

Note also that scale economies in the downstream market need *not* to be important for the incentive to engage in vertical foreclosure to arise. Indeed, when the incumbent's own affiliates are relative efficient refusal to supply prevents the independent downstream firm from earning profits also in the second period in the presence of the independent upstream firm. Then, for refusal to supply to discourage downstream entry downstream entry costs need not to be sufficiently large so as to ensure that second-period profits are insufficient to cover them. Indeed, refusal to supply discourages downstream entry even if F_D is quite small.

Finally, the underlying mechanism in the case in which refusal to supply discourages entry *both* in the downstream and in the upstream market is similar to the one proposed by Carlton and Waldman (2002) in the context of exclusionary tying. In their model tying a primary product and a complementary one discourages current entry in the complementary market which in turn discourages future entry in the primary market.

An important difference between our model and theirs, though, is that in the setting proposed by Carlton and Waldman (2002) vertical foreclosure motivated by the intent to 'transfer' or 'create' a downstream monopoly in the future cannot arise. In their setting the entrant and the incumbent are equally efficient in the primary market and compete à la Bertrand offering homogeneous products. Then, *stand-alone* entry in the primary market is never profitable, even in the absence of tying and even if entry has occurred in the complementary market. However, entry in the primary market increases profitability of the complementary market.[12] Then a firm that has entered the complementary market has an incentive to enter also the primary market, as long as the same company sells both the primary and the complementary product.

Then our model unveils a *new rationale* for vertical foreclosure, for which the incentives to exclude the rival are indeed stronger than in the case in which vertical foreclosure protects the incumbent's dominant position in both the vertically related markets. A further implication of this analysis is that it is not necessary that downstream entry opens the way to upstream entry to build a theory of harm for vertical foreclosure, as focusing on the latter case would suggest. Indeed we show that a crucial ingredient for a theory of harm is that future entry in the upstream market is conceivable, irrespective of whether upstream entry would occur anyway or it depends on the success of entry in the vertically related market.

A second important difference is that in our model the entrants do not need to be *vertically integrated*. Indeed the baseline model focuses on the case of stand-alone entrants. We consider the case of vertically integrated entrants in Section A.2, which allows us to highlight that the incumbent's incentives to engage in vertical foreclosure are weaker when it faces vertically integrated entrants. In other words, being vertically integrated protects the entrants from the incumbent's exclusionary practices.

[12]This occurs because entry in the primary market prevents the incumbent from engaging in price-squeeze, i.e. in a below-cost price of the complementary product compensated by a high price for the primary product.

3 Reputation can explain foreclosure when the incumbent is unable to credibly commit

In the base model the commitment to refuse to supply the downstream entrant is crucial for the exclusionary effect. Indeed, if the decision to engage in refusal to supply were reversible, then the independent firm D_E would decide to enter the downstream market in period 1 despite the incumbent's threat not to supply it. D_E would correctly anticipate that, once entry has taken place, the incumbent would renege on its previous decision and would supply D_E instead, because not trading with the more efficient downstream firm reduces the profits that the incumbent can extract from the negotiation for the input.

In this section we relax the assumption that refusal to supply has a commitment value and we show that, in a modified version of the game, the incumbent may find it optimal to engage in it once entry has occurred to build a reputation of being "tough" and discourage subsequent entry. The logic is very similar to the reputational model of predation of Kreps and Wilson (1982).

Description of the game We consider a situation where in period 1 (i) the incumbent faces successive downstream entry in two separate geographic markets, market A and B, and (ii) there is incomplete information: the downstream entrants do not know whether the incumbent's affiliates have marginal cost $c_{DI} = c_{UI} = c$ as in the base model, or they are very efficient, and hence marginal costs are $c_{DI} = c_{UI} = 0$. We call the inefficient incumbent "weak", and the efficient incumbent "tough". At the beginning of the game, the probability that the incumbent is tough is x. Downstream entrants have marginal cost $c_{DE} = 0$. In period 2, like in the baseline model, entry can occur in the upstream market, both in geographical market A and B. The upstream entrants also have marginal cost $c_{UE} = 0$. The timing of the game is the following:

1. Period 1, stage 1: Firm D_{EA} decides whether to enter (and pay fixed sunk cost F_D) or not in market A;
2. Period 1, stage 2: The incumbent decides whether to 'refuse (R) to supply' or, alternatively, to supply (S) the downstream rival D_{EA};
3. Period 1, stage 3: Firm D_{EB} decides whether to enter (and pay fixed sunk cost F_D) or not in market B;
4. Period 1, stage 4: The incumbent decides whether to 'refuse to supply' or, alternatively, to supply the downstream rival D_{EB};
5. Period 1, stage 5: Contracts are offered in each market where entry has occurred. In each market, with probability 1/2 the incumbent's upstream affiliate makes a take-it-or-leave-it offer, with probability 1/2 the downstream firm (if active) makes a take-it-or-leave-it offer. Like in the baseline model we continue to assume that the incumbent can credibly commit not to operate the downstream unit.
6. Period 1, stage 6: If D_{Ei} (for $i = A, B$) is active, the contract offer is accepted/rejected. Then active downstream firms choose final prices p_i and p_I, firm D_{Ei} orders the input to satisfy demand, paying accordingly, and transforms one unit of the input into one unit of the final product.

7. Period 2, stage 1: Firm U_{Ei} enters the upstream market i (and pays $F_U = 0$), with $i = A, B$.

8. Period 2, stage 2: In each market separately, with probability 1/2 active upstream firms make take-it-or-leave-it offers; with probability 1/2 active downstream firms do. The decision taken in period 1 concerning refusal to supply is reversible.

9. Period 2, stage 3: Contract offers are accepted/rejected. The active downstream firms set final prices p_i and p_I, orders are made, payments take place and payoffs are realized.

Note that, consistently with the assumption that refusal to supply has no commitment value, we allow the incumbent to modify its supply strategy even in period 2.

Moreover, we make a number of simplifying assumptions to avoid a proliferation of cases. Notably we assume that $F_{Ui} = 0$, so that entry occurs for sure in the upstream market is period 2. This allows us to disregard the upstream entrant's entry decision and hence simplifies the analysis. Similarly, we do not allow for the downstream entrant D_{Ei} to enter in the second period if it has not done so in period 1. We also assume that $c \in [\widehat{c}, \overline{c}]$ and that, absent refusal to supply, entry in period 1 is profitable for a downstream entrant facing a weak incumbent; instead, second period profits alone are insufficient for the downstream entrants to cover the entry cost. As a consequence, F_D is subject to the following restrictions:

$$\frac{1}{2}\pi^d(0, 2c) < F_D < \frac{1}{2}\pi^d(0, 2c) + \frac{1}{2}[\pi^m(c) - \pi^m(2c)] \tag{A}$$

Finally, market demand is given by $Q = 1 - p$.

As the timing of the game indicates, in period 1 contracts in market A are offered after entry decisions and refusal to supply decisions are taken in both downstream markets. If contracts in market A were offered before entry in market B, contractual terms would be an additional signal of the incumbent's marginal costs.

In period 1, when indifferent between refusal to supply and offering the input, the incumbent is assumed to choose the former. This can be rationalised through the existence of a negligible but positive cost that the vertically integrated incumbent bears when trading with an external buyer.

We solve the game using the concept of Perfect Bayesian Nash Equilibrium. Before discussing the equilibria of the game, in what follows we indicate the total profits obtained by the incumbent and by the independent downstream firm D_i in market i, with $i = A, B$ when one considers market i in isolation, thereby abstracting from possible reputation building effects. Those profits depend on whether entry occurred in period 1, on the incumbent's type and on whether the incumbent engages or not in refusal to supply.

Firm D_{Ei} did not enter and the incumbent is tough. In this case the incumbent monopolises the final market through its own affiliate in period 1 and is indifferent between doing the same also in period 2 and trading with U_{Ei} (since the upstream affiliate is equally efficient). Then, total profits across the two periods are given by:

$$\pi_{I,t}^{NoEntry} = \pi^m(0) + \pi^m(0); \quad \Pi_{D_E,t}^{NoEntry} = 0.$$

Firm D_{Ei} did not enter and the incumbent is weak. In this case the incumbent monopolises the final market through its own affiliate in period 1, while in period 2 it trades with the upstream independent firm, partially appropriating the increase in monopoly profits that the latter's more efficient technology produces. Then total profits over the two periods are given by:

$$\pi_{I,w}^{NoEntry} = \pi^m(2c) + \frac{\pi^m(c)}{2} + \frac{\pi^m(2c)}{2}; \quad \Pi_{D_E,w}^{NoEntry} = 0$$

Firm D_{Ei} entered and the incumbent is tough. Let us consider first period 2, when both independent firms are active and contract offers are made. In this case in which $c = 0$, there exist multiple equilibria. For instance when upstream firms make the offer there exists an equilibrium in which both U_I and U_{Ei} make offers to D_{Ei} and an equilibrium in which firm U_{Ei} makes an offer to D_I and U_I does not make any offer. The logic is similar when downstream firms make the offers. In all of these equilibria the incumbent obtains the entire industry profits $\pi^m(0)$.

Let us consider the first period, in which only firm D_{Ei} is active. The incumbent makes profits $\pi^m(0)$ both if it monopolises the final market through its own affiliate, and if it trades with D_{Ei}. A negligible cost to trade with an external buyer is enough to make the incumbent prefer not to trade with D_{Ei}. Total profits over the two periods are given by:

$$\pi_{I,t}^{Entry} = \pi^m(0) + \pi^m(0); \quad \Pi_{D_E,t}^{Entry} = 0$$

Firm D_{Ei}'s profits are gross of the entry cost.

Firm D_{Ei} entered and the incumbent is weak. In period 2, when both independent firms are active, the incumbent does not sell and obtains zero profits at the equilibrium of the contracting game (see Table 1). It has never an incentive to refrain from making offers to the independent firms. The independent firms share evenly the duopoly profits.

In period 1 only firm D_{Ei} is active. When the incumbent supplies the independent firm total profits over the two periods are given by:

$$\pi_{I,w}^{Entry,S} = \frac{\pi^m(c)}{2} + \frac{\pi^m(2c)}{2} + 0; \quad \Pi_{D_E,w}^{Entry,S} = \frac{1}{2}[\pi^m(c) - \pi^m(2c)] + \frac{\pi^d(0,2c)}{2}$$

If the incumbent decides not to trade with firm D_{Ei} then total profits over the two periods are given by:

$$\pi_{I,w}^{Entry,R} = \pi^m(2c) + 0; \quad \Pi_{D_E,w}^{Entry,R} = \frac{\pi^d(0,2c)}{2}$$

Note that if there was only one market and reputation-building did not play any role, then the weak incumbent would find it more profitable to trade with the independent downstream firm because it would partially appropriate the increase in monopoly profits produced by D_{Ei}'s more efficient technology. Note also that Firm D_{Ei}'s profits are gross of the entry cost. Given assumption A, downstream entry is profitable if the incumbent does not engage in refusal to supply.

Looking at the above profits one can conclude the following:

Lemma 7.

(i) A tough incumbent always engages in refusal to supply in period 1 *after downstream entry in market i.*

(ii) Absent a reputation building effect, a weak incumbent never engages in refusal to supply in period 1*.*

(iii) An independent firm that assigns probability 1 *to the incumbent being tough does not enter the downstream market* i *in period* 1.

3.1 The equilibria of the game

The equilibria of the game are described by the following Proposition.

Proposition 2. ***(Perfect Bayesian equilibria of the game)***

(i) The game does not admit a separating equilibrium in pure strategies in which a tough incumbent refuses to supply the input and a weak incumbent supplies it.

(ii) If $F_D \geq F_D^{Rep}$*, there exists a pooling equilibrium in which both the weak and the tough incumbent refuse the input to the downstream entrant in market* A*.*

(iii) If $x < \widehat{x}$ *and* $F_D < \widehat{F}_D^{Rep} < F_D^{Rep}$*, there exist semi-separating equilibria in which:*

(1.i) Firm D_{EA} *enters the downstream market* A*.*

(1.ii) The tough incumbent engages in refusal to supply in market A*. The weak incumbent engages in refusal to supply with probability* $\frac{xF_D}{(1-x)(\Pi_{D_{E,w}}^{Entry,S}-F_D)}$.

(1.iii) Firm D_{EB} *enters the downstream market* B *with probability* $\frac{\pi_{I,w}^{Entry,S}-\pi_{I,w}^{Entry,R}}{\pi_{I,w}^{NoEntry}-\pi_{I,w}^{Entry,S}}$ *if* D_{EA} *was refused the input. Firm* D_{EB} *enters the downstream market* B *with probability* 1 *if* D_{EA} *was supplied the input.*

(1.iv) If firm D_{EB} *enters the downstream market, the tough incumbent engages in refusal to supply, while the weak incumbent supplies it.*

(2.i) In period 2*, if entry occurred in downstream market* i*, for the weak incumbent it is never profitable to refrain from making offers to the independent firms. There are equilibria in which the tough incumbent does not make offers to the independent firms.*

Proof. Recall that for a candidate equilibrium to be a PBE each player's strategy must be optimal given the other player's strategies, given the player's beliefs, and with beliefs satisfying Bayes' rule.

Separating equilibrium.
If entry has occurred in market A, firm D_{EB} observes the behaviour of the incumbent towards the entrant in market A and infers that the incumbent is weak if it observes supply of the input, while it infers that the incumbent is tough if it observes refusal to supply. In market B no reputation effect can arise. Then, by Lemma 7, firm D_{EB} expects the weak incumbent not to engage in refusal to supply and enters the downstream market only if it observes supply in market A. A necessary condition for a separating equilibrium to exist is that, when facing entry in market A, the weak incumbent finds it more profitable to supply rather than mimic the tough one and refusing the input:

$$\pi_{I,w}^{Entry,S} + \pi_{I,w}^{Entry,S} \geq \pi_{I,w}^{Entry,R} + \pi_{I,w}^{NoEntry}. \tag{1}$$

By replacing the payoff values in this expression, one obtains:

$$2[\frac{\pi^m(c)}{2} + \frac{\pi^m(2c)}{2}] \geq \pi^m(2c) + \pi^m(2c) + \frac{\pi^m(c)}{2} + \frac{\pi^m(2c)}{2}$$

which becomes

$$3\pi^m(2c) \leq \pi^m(c).$$

Under our assumption of demand $Q = 1 - p$, $\pi^m(z) = (1-z)^2/4$ and $\bar{c} = 1/4$. The above condition becomes $3(1-2c)^2 \leq (1-c)^2$ and is never satisfied for $c \leq \bar{c}$. Hence, no separating equilibrium exists.

Pooling equilibrium.
If entry has occurred in market A, firm D_{EB} does not revise its beliefs when observing a refusal to supply decision. The posterior belief (i.e. the probability that the incumbent is tough conditional on the observation of refusal to supply) coincides with the prior: $\Pr(t \mid R) = x$. For this to be an equilibrium, D_{EB} must prefer not to enter after observing a refusal to supply. By Lemma 7, the downstream entrant in market B expects the incumbent not to engage in refusal to supply when weak, and expects to earn zero profits when facing a tough incumbent. Then, the no entry condition is:

$$x\Pi^{Entry,R}_{D_E,t} + (1-x)\Pi^{Entry,S}_{D_E,w} - F_D \leq 0.$$

After substitution this condition becomes:

$$F_D \geq (1-x)\left\{\frac{1}{2}[\pi^m(c) - \pi^m(2c)] + \frac{\pi^d(0,2c)}{2}\right\} \equiv F_D^{Rep}, \tag{2}$$

a condition which is compatible with assumption A. Note that as x tends to zero, this equilibrium tends to disappear: if it is unlikely that the incumbent is efficient (= tough), then after observing a refusal to supply the downstream entrant in market B would still hold a high ex-post probability it faces a weak incumbent, and hence will enter (which in turn would discourage a weak incumbent from engaging in refusal to supply with the downstream entrant in market A).

Note also that the fact that condition 1 is never satisfied ensures that a weak incumbent finds it profitable to engage in refusal to supply when facing entry in market A so as to mimic a tough incumbent and discourage future downstream entry in market B. Since the first entrant anticipates that any incumbent would refuse to supply, it will not enter. But since the prior probability x is sufficiently high for equation 2 to hold, the second downstream entrant will not enter either. Hence, refusal to supply will never be observed at equilibrium.

Semi-separating equilibrium.
Let us consider the case in which condition 2 is not satisfied and $F_D < F_D^{Rep}$. Note that as $x \to 0$, F_D^{Rep} tends to the upperbound of the downstream entry cost as defined by assumption A. Morevoer, $F_D^{Rep} = 0$ if $x = 1$; since assumption A requires that $F_D > \frac{\pi^d(0,2c)}{2}$, if $x = 1$ $F_D < F_D^{Rep}$ cannot be satisfied. Then a necessary condition for a semi-separating equilibrium to exist is that $x < \bar{x}$ and $F_D < F_D^{Rep}$ where $\bar{x} \in [0, 1)$ is such that:

$$F_D^{Rep} = (1-\bar{x})\left[\frac{\pi^m(c) - \pi^m(2c)}{2} + \frac{\pi^d(0,2c)}{2}\right] = \frac{\pi^d(0,2c)}{2} \tag{3}$$

Period 2 does not involve any reputation building. As shown above, following entry in the downstream and upstream market, the weak incumbent will always want to make offers to the independent firms; instead, there exist equilibria in which the tough incumbent refrains from making offers. (This proves 2.i.)

No reputation can be built in market B in period 1. Hence, by Lemma 7, following entry in downstream market B, the tough incumbent engages in refusal to supply and the weak incumbent does not.(This proves 1.iv.)

The entrant in market B infers that the incumbent is weak if it observes that firm D_{EA} was supplied the input. Since it anticipates that the weak incumbent will not engage in refusal to supply in market B, it is profitable for D_{EB} to enter the downstream market if it observes that firm D_{EA} was supplied the input. If D_{EB} observes that firm D_{EA} was refused the input, then it updates its belief about the incumbent being weak:

$$\Pr(w \mid R) = \frac{(1-x)\Pr(R \mid w)}{(1-x)\Pr(R \mid w) + x\Pr(R \mid t)} = \frac{(1-x)\Pr(R \mid w)}{(1-x)\Pr(R \mid w) + x} \tag{4}$$

since $\Pr(R \mid t) = 1$. It is optimal for D_{EB} to randomize between entry and not entry if the two alternatives are equally profitable. Then, given its revised beliefs, the profits that it expects to make by entering market B must be equal to zero:

$$0 = \Pr(w \mid R)\Pi_{D_E,w}^{Entry,S} - F_D.$$

Substituting the revised beliefs from equation 4, one obtains that the probability that a weak incumbent engages in refusal to supply after downstream entry in market 1 must be:

$$\Pr(R \mid w) = \frac{xF_D}{(1-x)(\Pi_{D_E,w}^{Entry,S} - F_D)}. \tag{5}$$

Note that $\Pr(R \mid w) \geq 0$ by assumption A. Moreover, $F_D < F_D^{Rep}$ ensures that $\Pr(R \mid w) < 1$. (This proves 1.iii)

It is optimal for the weak incumbent to randomize between refusal to supply and supply following downstream entry in market A if the two alternatives are equally profitable. If the weak incumbent supplies the input to D_{EA}, then firm D_{EB} will infer that it is weak and will enter market B. The incumbent's profits (across both periods and both markets) are $2\pi_{I,w}^{Entry,S}$. If the weak incumbent refuses to supply D_{EA}, then firm D_{EB} will enter with probability $\Pr(Entry \mid R)$. In this case the weak incumbent's expected profits (across the two market and the two periods) are given by:

$$\pi_{I,w}^{Entry,R} + \Pr(Entry \mid R)\pi_{I,w}^{Entry,S} + [1 - \Pr(Entry \mid R)]\pi_{I,w}^{NoEntry}$$

Then, the indifference condition is:

$$2\pi_{I,w}^{Entry,S} = \pi_{I,w}^{Entry,R} + \Pr(Entry \mid R)\pi_{I,w}^{Entry,S} + [1 - \Pr(Entry \mid R)]\pi_{I,w}^{NoEntry}$$

from which one obtains:

$$\Pr(Entry \mid R) = 1 - \frac{\pi_{I,w}^{Entry,S} - \pi_{I,w}^{Entry,R}}{\pi_{I,w}^{NoEntry} - \pi_{I,w}^{Entry,S}}.$$

Note that by Lemma 7 $\pi_{I,w}^{Entry,S} > \pi_{I,w}^{Entry,R}$. Moreover, by Proposition 1 $\pi_{I,w}^{NoEntry} > \pi_{I,w}^{Entry,S}$. Then, $\Pr(Entry \mid R) < 1$. Since we have verified above that condition (1) does not hold, $\Pr(Entry \mid R) > 0$. (This proves 1.ii.)

For firm D_{EA} it is optimal to enter downstream market A if its expected payoff is higher than staying out:

$$-F_D + x\Pi^{Entry}_{D_E,t} + (1-x)\left[\Pr(R \mid w)\Pi^{Entry,R}_{D_E,w} + (1-\Pr(R \mid w))\,\Pi^{Entry,S}_{D_E,w}\right] > 0.$$

Substituting the payoffs obtained above, the condition becomes:

$$F_D < (1-x)\left[\frac{\pi^d(0,2c)}{2} + (1-\Pr(R \mid w))\left(\frac{\pi^m(c)-\pi^m(2c)}{2}\right)\right] \equiv \widehat{F}^{Rep}_D. \tag{6}$$

Note that $\Pr(R \mid w) = 0$ if $x = 0$. In that case $\widehat{F}^{Rep}_D = F^{Rep}_D$ and condition 6 is always satisfied. Moreover, $F_D < F^{Rep}_D$ ensures that $\Pr(R \mid w) < 1$. This implies that when $x = \overline{x}$ $\widehat{F}^{Rep}_D < F^{Rep}_D = \frac{\pi^d(0,2c)}{2}$. By assumption A condition 6 cannot be satisfied. Since $\Pr(R \mid w)$ is increasing in x, there exists a threshold level of x, $\widehat{x} < \overline{x}$ such that:

$$(1-\overline{x})\left[\frac{\pi^d(0,2c)}{2} + (1-\Pr(R \mid w)|_{x=\overline{x}}\frac{\pi^m(c)-\pi^m(2c)}{2}\right] = \frac{\pi^d(0,2c)}{2} \tag{7}$$

The semi-separating equilibrium exists iff $x < \widehat{x}$ and $F_D < \widehat{F}^{Rep}_D$. □

Comment Given the similarity with the Kreps and Wilson (1982)'s predation game, it is reasonable to conjecture that if there were $T > 2$ downstream entrants a similar equilibrium would arise: in the earlier periods of the game even a weak incumbent engages in refusal to supply with certainty, and anticipating this, early downstream entrants stay out. As the game proceeds, the optimal strategies are mixed ones, like in the $T = 2$ case.

The main insight of this model is that even a small departure from the perfect information setting gives rise to refusal to supply even if an incumbent were unable to credibly commit to such a strategy before downstream rivals enter.

4 A model with learning effects

In the baseline model the downstream rival is a potential entrant. In that setting refusal committing to supply prevents the downstream rival from earning the key *profits* necessary to cover the entry cost and discourages entry. Lack of downstream entry benefits the incumbent in the later period, when upstream entry will occur, because the incumbent will be in the position to extract more rents when contracting for the input. In this Section we assume instead that the downstream rival is already active and that the downstream market is characterised by learning effects: sales in the early period increase efficiency in the later period. We will show that also in this environment the incumbent has an incentive to engage in refusal to supply to deprive the rival of the critical first-period *sales*, thereby extracting more rents in the second period, when contracting with the more efficient upstream entrant. Differently from the baseline model, however, the foreclosure effect arises even though the decision not to deal with the downstream rival has no commitment value. Also, the mechanism illustrated here for the case of learning applies to other settings in which early sales affect later profitability, for instance due to the presence of network externalities.

The set-up of the model is similar to the one analysed in Section 2. In period 1 the incumbent's affiliates have marginal cost $c_{UI_1} = c_{DI_1} = c < 1/2$; the rival downstream firm, D_R, is more efficient and has marginal cost $c_{R_1} < c$.[13] With probability 1/2 the incumbent upstream affiliate makes a take-it-or-leave-it offer to the downstream rival; with probability 1/2 the downstream rival makes a take-it-or-leave-it offer to the incumbent's upstream affiliate. Then, contract offers are accepted or rejected. Within this contracting game, the incumbent can decide not to trade with the downstream rival. The decision not to trade with the independent downstream rival has no commitment value. Finally, active downstream firms set final prices, orders are made, payments take place and payoffs are realised.

In period 2, firm U_E considers entering the upstream market. The upstream entrant is more efficient than the incumbent's upstream affiliate: $c_{UE} = 0 < c$. For simplicity, we focus here on the case in which upstream entry occurs for sure in the second period and we assume that upstream entry costs are $F_U = 0$. After the entry decision has been taken, with probability 1/2 upstream firms make take-it-or-leave-it offers to the downstream firms; with probability 1/2 downstream firms make the offers. Then, contract offers are accepted/rejected. Also in this case the incumbent can decide not to trade with the downstream rival. Finally, active downstream firms set final prices, orders are made, payments take place and payoffs are realised.

Due to the presence of learning effects, second period marginal costs of downstream firms depend on first-period sales: $c_{i_2} = c_{i_1} - \lambda$, with $i = R, I$. To simplify the analysis we assume that in each period there is one unit of the final product for sale, that is demanded as long as the final price is $p \leq 1$.[14] The parameter $\lambda > 0$ captures the strength of the learning effect. We assume that the learning effect is strong enough to make the incumbent's affiliate more efficient than the rival when it is the incumbent's affiliate that benefits from learning: $c - \lambda < c_{R_1}$. Moreover to ensure that second-period marginal costs are non-negative, we impose $c_{R_1} - \lambda \geq 0$.

4.1 Second-period contracting

The second-period equilibrium contract offers and the corresponding firms' payoffs depend on whether the incumbent's own downstream affiliate or the independent rival sell in period 1. They are displayed in Table 3:

Lemma 8. ***Second-period payoffs.***
The following table indicates the second period payoffs of the incumbent and of the independent firm:

Proof. See Appendix A.3. □

Table 3 and Appendix A.3 show that the incumbent's second-period profits are larger when the incumbent's downstream affiliate sells in period 1 relative to the case in which the rival sells in period 1. This occurs because selling in period 1 allows the incumbent's affiliate to benefit from learning and to deny the rival from benefiting from it. Then the own affiliate will be more efficient than the rival in the second period, and will be able to extract large rents when contracting for the input. Instead, it is the rival that benefits from learning if it sells in period 1, thereby being in a strong position in

[13]In this Section we slightly change the notation and denote the downstream firm as D_R to highlight that it is a rival already in the market and not a potential entrant.

[14]Absent this assumption second period costs would depend on how much has been sold in the first period, instead of just on whether a firm sold. In turn, in the first period there would be an incentive to sell more to strategically affect second period outcomes. Assuming inelastic demand allows us to abstract from such 'demand' effects which would complicate the analysis without adding much insight.

Table 3. Second-period payoffs

	D_R sold in period 1	D_I sold in period 1
$c < \frac{1}{3}$	$\pi_I = \frac{1}{2}\pi^m(c) + \frac{1}{2}\pi^m(c + c_{R_1} - \lambda) - \pi^d(c_{R_1} - \lambda, 2c)$ $\pi_{D_R} = \frac{1}{2}\pi^d(c_{R_1} - \lambda, 2c)$ $\Pi_{U_E} = \frac{1}{2}\pi^d(c_{R_1} - \lambda, 2c)$	$\pi_I = \frac{1}{2}\pi^m(2c - \lambda) + \frac{1}{2}[\pi^m(c - \lambda) - \pi^d(c_{R_1}, 2c - \lambda)]$ $\pi_{D_R} = 0$ $\Pi_{U_E} = \frac{1}{2}[\pi^m(c - \lambda) - \pi^m(2c - \lambda)] + \frac{1}{2}[\pi^d(c_{R_1}, 2c - \lambda)]$
$c \in [\frac{1}{3}, \frac{1}{3} + \frac{c_{R_1} - \lambda}{3})$	$\pi_I = \frac{1}{2}[\pi^m(c) - \pi^d(c_{R_1} - \lambda, 2c)]$ $\pi_{D_R} = \frac{1}{2}\pi^m(c + c_{R_1} - \lambda)$ $\Pi_{U_E} = \pi^d(c_{R_1} - \lambda, 2c) - \frac{1}{2}\pi^m(c + c_{R_1} - \lambda)$	$\pi_I = \frac{1}{2}\pi^m(2c - \lambda) + \frac{1}{2}[\pi^m(c - \lambda) - \pi^d(c_{R_1}, 2c - \lambda)]$ $\pi_{D_R} = 0$ $\Pi_{U_E} = \frac{1}{2}[\pi^m(c - \lambda) - \pi^m(2c - \lambda)] + \frac{1}{2}[\pi^d(c_{R_1}, 2c - \lambda)]$
$c \geq \frac{1}{3} + \frac{c_{R_1} - \lambda}{3}$	$\pi_I = 0$ $\pi_{D_R} = \frac{1}{2}\pi^m(c + c_{R_1} - \lambda) + \frac{1}{2}[\pi^d(c_{R_1} - \lambda, 2c) - \pi^m(c)]$ $\Pi_{U_E} = \frac{1}{2}[\pi^d(c_{R_1} - \lambda, 2c) - \pi^m(c + c_{R_1} - \lambda)] + \frac{1}{2}\pi^m(c)$	$\pi_I = \frac{1}{2}\pi^m(2c - \lambda) + \frac{1}{2}[\pi^m(c - \lambda) - \pi^d(c_{R_1}, 2c - \lambda)]$ $\pi_{D_R} = 0$ $\Pi_{U_E} = \frac{1}{2}[\pi^m(c - \lambda) - \pi^m(2c - \lambda)] + \frac{1}{2}[\pi^d(c_{R_1}, 2c - \lambda)]$

the second period contracting and reducing or canceling out the profits that the incumbent manages to extract. Appendix A.3 also shows that in the second period the incumbent cannot increase its profits by deciding not to trade with the independent rival, as it would deprive itself of the possibility to make profitable offers.

4.2 First-period contracting

We now study the contracting game in period 1. We do so under two possible scenarios. One in which in period 1 firms cannot contract on period-2 payoffs (see Section 4.2.1). In Section 4.2.2 we relax this assumption and we allow firms to contract on period-2 payoffs in period 1.

4.2.1 Contracting on period-2 payoffs is not feasible.

In the first period the independent rival is more efficient than the incumbent's own affiliate. As a consequence, the incumbent sacrifices profits by deciding not to supply the more efficient rival in period 1, as it loses the possibility to partially appropriate the increase in industry profits that the rival's more efficient technology produces in period 1. The incumbent's loss in period 1 amounts to:

$$\Delta\pi_I^1 = \frac{1}{2}[\pi^m(c + c_{R_1}) - \pi^m(2c)]$$

However, refusal to supply prevents the independent rival from learning, thereby increasing the incumbent's second-period profits, as discussed above:

$$\Delta\pi_I^2 = \pi^2_{I|I\ in\ 1} - \pi^2_{I|D_R\ in\ 1} > 0.$$

Appendix A.3 shows that the second-period gain always dominates the first-period loss, so that the incumbent finds it profitable not to supply the rival in period 1. Moreover, refusal to supply is welfare detrimental because the more efficient downstream rival does not produce both in period 1 and 2.

Proposition 3. ***First-period contracting***
When it is not possible to contract on period-2 payoffs, in period 1 the incumbent decides not to supply the independent downstream rival. Refusal to supply is welfare detrimental.

Proof. See Appendix A.3. □

4.2.2 Contracting on period-2 payoffs is feasible.

In this case through the fixed fee the firm that makes the offer in period 1 (i.e. either the incumbent or the downstream rival) internalises the change in the rival's second-period profits that occur when the independent rival sells in period 1 relative to the case in which the incumbent's affiliate sells in period 1: when the incumbent makes the offer, it appropriates the increase in the second-period profits of the downstream rival; when the downstream firm makes the offer, it takes into account the decrease in the incumbent's second period profits. Then, in period 1, the incumbent and the downstream rival do not trade if this increases their joint profits over the two periods relative to the case in which trade occurs.

This scenario arises if not trading with firm D_R in period 1 increases so much the incumbent's second-period profits to dominate the decrease in period-2 profits suffered by firm D_R and the decrease in period-1 total profits. For this reason, refusal to supply is less likely to arise than in the case in which second-period payoff cannot be contracted upon in period 1, because in that case the incumbent decides based on the the variation of own profits only.

Finally, since the incumbent and the downstream rival do not internalize the payoff of the agents not involved in period-1 contracting – i.e. final consumers and the upstream entrant – a decision not to trade is privately optimal for them but it turns out to be welfare detrimental.

Proposition 4. ***First-period contracting***
When it is possible to contract on period-2 payoffs:

(i) The incumbent and the independent downstream firm do not trade in period 1 if this increases their joint payoff over the two periods relative to the case in which trade occurs.

(ii) The incumbent and the independent downstream firm do not trade in period 1 if $c < \frac{1}{3} + \frac{c_{R_1}-\lambda}{3}$*; or* $\lambda \geq 1/6$ *and* $c \in [\frac{1}{3} + \frac{c_{R_1}-\lambda}{3}, \overline{c})$*; or* $\lambda < 1/6$, $c_{R1} \geq \underline{c}_{R_1}$ *and* $c \in [\frac{1}{3} + \frac{c_{R_1}-\lambda}{3}, \overline{c})$.

(iii) The decision not to trade in period 1 *decreases total welfare.*

Proof. (i) *Upstream offers.*
If firm D_R rejects the contract offered by U_I, it sells neither in period 1 nor in period 2 (see Table 3) and its total payoff is zero. Then, the best contract that the incumbent can offer in period 1 and that D_R will accept is $(w = c, FF = \pi^m(c + c_{R_1}) + \pi^2_{D_R|D_R\ in\ 1})$: the fixed fee allows the incumbent to extract not only the monopoly profits that firm D_R earns in the final market in period 1, but also the profits that D_R makes in period 2 given that it sold in period 1. Then, the incumbent's total profits in this case amount to the joint profits of D_R and I over the two periods given that D_R sells in period 1:

$$\pi_I^{1+2,S} = \pi^m(c + c_{R_1}) + \pi^2_{D_R|D_R\ in\ 1} + \pi^2_{I|D_R\ in\ 1}.$$

If the incumbent does not offer a contract to D_R then its total profits over the two periods amount to:

$$\pi_I^{1+2,R} = \pi^m(2c) + \pi^2_{I|D_I\ in\ 1}$$

which coincides with the joint profits of D_R and I over the two periods, since D_R's total two-period profits are zero in this case.

Then, in period 1, the incumbent decides not to supply firm D_R if this decision increases the joint profits of I and D_R over the two periods:

$$\underbrace{\pi^m(2c) - \pi^m(c + c_{R_1})}_{<0} + \underbrace{\pi^2_{I|D_I\ in\ 1} - \pi^2_{I|D_R\ in\ 1}}_{>0} + \underbrace{\underbrace{\pi^2_{D_R|D_I\ in\ 1}}_{=0} - \pi^2_{D_R|D_R\ in\ 1}}_{<0} > 0 \tag{8}$$

This condition is satisfied if not trading with firm D_R increases so much the incumbent's profits in period 2 to dominate the decrease in period-2 profits suffered by firm D_R and the decrease in period-1 total profits.

Downstream offers.
Firm D_R offers a contract to U_I that makes it indifferent between accepting and rejecting. If the incumbent rejects the contract then its total profits over the two periods amount to $\pi^m(2c)+\pi^2_{I|D_I\ in\ 1}$. If the incumbent accepts the contract it obtains $FF+\pi^2_{I|D_R\ in\ 1}$, i.e. the fixed fee and the incumbent's second-period payoff given that firm D_R sold in period 1. Then, in order to accept that contract, the incumbent requires a fee that allows it to appropriate also the increase in its second-period profits that would occur if the own affiliate sold in period 1:

$$FF = \pi^m(2c) + \pi^2_{I|D_I\ in\ 1} - \pi^2_{I|D_R\ in\ 1}$$

Offering such a contract is unprofitable for D_R if:

$$\pi^m(c + c_{R_1}) - FF + \pi^2_{D_R|D_R\ in\ 1} < 0$$

By substituting FF from equation 4.2.2, one obtains that D_R does not offer any contract to U_I (or it offers a contract that the incumbent does not accept), if the two-period joint payoff of the incumbent and firm D_R is higher when they do not trade in period 1 relative to the case in which they do trade:

$$\pi^m(c + c_{R_1}) - \pi^m(2c) - [\pi^2_{I|D_I\ in\ 1} - \pi^2_{I|D_R\ in\ 1}] + \pi^2_{D_R|D_R\ in\ 1} - \underbrace{\pi^2_{D_R|D_I\ in\ 1}}_{=0} < 0.$$

Note that this inequality is exactly the same as in the expression 8 above which was obtained for the case of upstream offers. (ii) Appendix A.3 will identify under which conditions inequality 8 is satisfied.

(iii) When the incumbent and firm D_R decide not to trade, total welfare decreases as compared to the case in which they do trade because the incumbent's less efficient technology prevails in period 1 and 2. □

5 Other extensions

5.1 Short term commitment to refusal to supply

In the baseline model the decision to engage in refusal to supply discourages downstream entry for two reasons: first it prevents the downstream rival from earning profits in period 1; second, by making the downstream rival more dependent on the upstream firm, it reduces (or cancels out) its second-period profits. Indeed, when the incumbent's own affiliates are relatively efficient (i.e. when $c < \widehat{c}$) the independent firm makes zero profits also in the second period. In that case it is not necessary that scale economies are important in the downstream market for refusal to supply to discourage entry.

Assume now that the commitment to refusal to supply does not last forever, but is reversible after one period. If downstream entry has occurred, the incumbent has no incentive to reiterate refusal to supply to period 2: upstream entry would occur and refusal to supply would be (weakly) detrimental to the incumbent because it would limit its ability to react to U_E's offers (see Table 4). Then, the independent downstream firm makes larger profits in the second period relative to the case in which refusal to supply lasts forever and it is less likely that refusal to supply discourages downstream entry: important scale economies in the downstream market are necessary for entry to be deterred and for the incumbent to engage in refusal to supply at the equilibrium.

Proposition 5. *Short-term commitment to refusal to supply*
If the commitment to refusal to supply lasts one period, the incumbent engages in refusal to supply iff $F_D > F_D'^{RtoS}$ *(for any feasible value of c), where* $F_D'^{RtoS} \equiv \frac{\pi^d(0,2c)}{2} > F_D^{RtoS} \equiv \frac{\pi^d(0,2c)-\pi^m(c)}{2}$.

5.2 Vertically Integrated Entrants

So far we have considered entrant firms that are independent firms. In this Section, instead, we assume that the entrants are subsidiaries of the same firm/group. When this is the case, the vertically integrated firm fully internalises the positive effect that entry in one market exerts on the profitability of the unit active in the vertically related market. This makes the incentive to enter both markets stronger as compared to the case of independent entrants and exclusion less likely.

In fact, as the following proposition shows, the incumbent engages in refusal to supply if the total entry costs are sufficiently large. Moreover, refusal to supply discourages entry in both the vertically related markets. The case in which upstream entry occurs anyway and the incumbent engages in refusal to supply to shift its dominant position from the upstream to the downstream market does not arise with vertically integrated entrants. Imagine that upstream entry is profitable, even for a stand-alone firm. Then, the vertically integrated firm finds it profitable to enter also downstream, because the entry decision is driven by the increase in *joint* profits produced by downstream entry, not only by the profits generated in the downstream market.

Proposition 6. *Vertically Integrated Entrants*

(i) When the entrants are vertically integrated, the incumbent engages in refusal to supply if (and only if) the following conditions on entry costs are satisfied:

(a) $F_U > \frac{\pi^d(0,2c)}{2} - \frac{\pi^m(c)-\pi^m(2c)}{2}$;

(b) $F_D > \frac{\pi^d(0,2c)}{2}$;

(c) $F_U + F_D > \pi^d(0,2c)$.

(ii) Refusal to supply discourages entry in both markets and is welfare detrimental.

Proof. See Appendix A.2. □

6 Conclusions and competition policy implications

In this paper we provide a dynamic rationale for vertical foreclosure. We consider a situation where a vertically integrated incumbent faces current potential (or actual) competition in the downstream market, and future competition in the upstream market. (But we would arrive at identical conclusions if we considered current competition in the upstream market, and future competition downstream.) In a static perspective (that is, if future market conditions did not change, and upstream entry were not a concern), and in line with the Chicago School insights - the incumbent would prefer to deal with the more efficient downstream rival and extract its rents. However, dealing with the downstream entrant today may imply that the incumbent will end up facing efficient rivals both downstream and upstream tomorrow, thereby losing (all or most of) its future market profits. More particularly, we have identified two circumstances in which the vertically integrated incumbent may prefer to engage in refusal to supply the downstream rival with its input.

In a situation where future upstream entry cannot be deterred (that is, the incumbent cannot protect its upstream monopoly) then refusing the input to the downstream rival now may allow the vertically integrated dominant firm to 'transfer' or 'create' a downstream monopoly in the future, and use such position to extract rents from the more efficient upstream entrant when contracting with it. (If a more efficient downstream rival was also in the market, the incumbent would be able to extract fewer - or no - rents from the upstream entrant.)

Instead, in a situation where a downstream rival's success is a pre-condition for an upstream rival's entry, a vertically integrated dominant firm may deny the input to the downstream rival today with the objective of maintaining its monopoly power in both vertically related markets tomorrow. (If a more efficient downstream rival was in the market, the upstream rival would make more profits and its entry would be more likely. In turn, the incumbent would lose all or most of its profits when facing two efficient upstream and downstream rivals.)

Note that the mechanism whereby refusal to supply harms the prospects of the downstream rival may hinge either upon a reduction of expected profits (which would make it more difficult to cover fixed entry costs) or a decrease in actual sales (which would deny the downstream rival the opportunity to go down its learning curve).

Further, note that in the latter case (learning cost model), we do not rely on the assumption that the incumbent can credibly commit to refusal to supply before entry decisions are taken, whereas in the former case (fixed entry cost model), we show that the incumbent may be able to exclude by building a reputation for engaging in refusal to supply.

We also show that vertical foreclosure may also take place when the upstream and downstream entrants belong to the same company.

Cases This paper suggests that it is important to consider the expected evolution of a market when analysing incentives for vertical foreclosure in competition cases. However, it is worth asking more in detail which sort of cases may fit the dynamic vertical foreclosure theory of harm presented in this paper. In general, they must concern markets where a vertically integrated firm is facing *potential* competition both upstream and downstream (in one of the segments, competition might also be already there - like in the variant of the model where downstream competition is characterised by either learning effects or network externalities).

As for the situation where foreclosure takes place in order to "create" a downstream monopoly, possible candidates for this theory of harm might include industries where a vertically integrated incumbent derives most of its market power either from a patent which is about to expire or from some assets whose monopoly is about to lose, due for instance to technological or regulatory changes which makes it easier for an upstream rival to successfully enter the market. As the upstream monopoly becomes closer to the end, there may be an incentive not to sell through downstream rivals, so as to enjoy a downstream monopoly - and be able to extract more rents - when upstream rivals will be in the market.

For instance, a firm which holds a monopolistic position of broadcasting rights of sports events and packages them into a sports TV channel, may anticipate that in the future it will not be able to continue to monopolise such rights (say, because regulation prevents them from being bundled in a single package and sold to the same company). In such circumstances, it may have the incentive not to supply its sports broadcasting rights to a potential competing TV channel, so as to prevent it from being more competitive, in order to enjoy a stronger contracting position with upstream competitors in the future. Similarly, a vertically integrated media company which owns "must-have" content such as TV channels and distributes them through a downstream affiliate - say a cable operator - may refuse to license its channels to competing TV distributors, if it expects that changes in demand pattern or successful introduction of competing content will jeopardise its upstream market position.[15]

Another vertical market where our theory of harm may conceivably be invoked to rationalise anti-competitive refusal to supply consists of services and spare parts of some primary equipment. For instance, suppose that there is an Original Equipment Manufacturer (OEM) which sells not only the primary product (say, a piece of hardware, or a machine) but also - and these are the vertically related markets we refer to in the model - spare parts (the upstream market) and Maintenance and Repair (MR) services (the downstream market). From a static perspective, the OEM would have an incentive to offer spare parts to independent providers of MR services, but if it felt that over time other manufacturers will be able to supply substitute spare parts in an efficient way, it may find it convenient to deny spare parts to MR competitors, so as to use its downstream service monopoly to appropriate rents from future competing providers of spare parts.[16]

The other situation covered in this paper deals with vertical foreclosure which maintains (or protect) overall monopoly power. This theory of harm may apply to industries where success in downstream activities is a necessary condition for entering the upstream market successfully. A case in point may have been *Telefónica*, where the European Commission (EC) found that the eponymous Spanish telecoms incumbent abused its dominant position by excluding downstream competitors (through a margin squeeze) in the Spanish broadband market.[17]

Telefónica was the unique operator having a local access network, i.e. a network that reaches final users. Alternative operators wishing to provide services throughout Spain had no other option than buying wholesale services from Telefónica. According to the EC, "[...]alternative operators [i.e. the entrants in the broadband market] are likely to follow a step-by-step approach to continuously expanding their customer base and infrastructure investments. In particular, when climbing up the "investment ladder", alternative operators seek to obtain a minimum critical mass, in order to be able to make further investments. (Para. 392 of the Decision)."

"[...] The first step of the "investment ladder"is occupied by an operator whose strategy consists

[15] There have been several cases where competition authorities have investigated vertical foreclosure concerns in the case of media mergers. See for instance the Liberty Global/De Vijver Media merger, EC Decision of 24 February 2015.

[16] This hypothetical example shares some similarity with the complaint by IATA (the association of the airline industry) that some engine OEMs are withholding repair information from qualified third-party maintenance shops. See Financial Times, "Airlines complain to Brussels over parts and maintenance contracts", March 23, 2016.

[17] Decision 2008/C 83/05 [2007] OJ C 83/06, upheld by the General Court and then the Court of Justice (Cases T-336/07 and C-295/12 P. Note that in many EU member states there were similar cases against the national telecom incumbent, accused of exclusionary practices against broadband rivals.

in targeting a mass market (thus involving considerable marketing and advertising expenditure), but who is merely acting as a reseller of the ADSL access product of the vertically integrated provider (the incumbent). As its customer base increases, then the alternative operator makes further investment. In a further step, it may even seek to connect its customers directly (local loop unbundling). Thus the progressive investments take the alternative operator progressively closer to the customer, reduce the reliance on the wholesale product of the incumbent, and increasingly enable it to add more value to the product offered to the end-user and to differentiate its service from that of the incumbent." (Para. 178 of the Decision)

The dynamic theory proposed here seems well aligned with the story proposed by the EC, and provides a possible rationale for Telefónica's vertical foreclosure strategy. Only if it obtained a critical size in the retail market, an alternative producer would be able to make the investment necessary to reach customers directly (through local loop unbundling in this case) and to gain independence from the services provided by the incumbent. By engaging in vertical foreclosure (here taking the form of margin squeeze)[18], the incumbent is preventing alternative operators from achieving the critical size that would justify investment in their own infrastructure, thereby discouraging them from investing upstream. Vertical foreclosure can therefore be interpreted as a defensive strategy adopted by the incumbent to protect its dominant position in the upstream market.

Arguably, our dynamic model also fits the facts of *Genzyme*,[19] a well-known UK abuse of dominance case. Genzyme was the only producer of Cerezyme, which at the time was the only drug available for the treatment of Gaucher disease (a rare metabolic disorder).[20] Another company, TKT, may have entered the market with a competing drug, although not in the short-run.

For home patients, the drug needed to be administered by specialised nurses or doctors. Initially, Genzyme used Healthcare at Home as its exclusive distributor and provider of home-care services for Cerezyme, but it later opened its own home-care service. After the contract was terminated, Healthcare at Home, in order to continue to offer the delivery/home-care service, had to purchase Cerezyme from Genzyme first, and Genzyme sold the drug to it at a price identical to its final downstream price. The OFT concluded that Genzyme had engaged in an anti-competitive margin squeeze, leaving no scope for downstream competition (i.e. in home delivery service).[21]

The OFT noted that in addition to restricting the extent of competition in Cerezyme delivery/homecare services, Genzyme's behaviour - by preventing viable independent provision of delivery/homecare services for Cerezyme (and potentially other drugs) - also raised barriers to entry into the (upstream) market for the supply of drugs for the treatment of Gaucher disease: "As a result of Genzyme's conduct it is more difficult for competitors to enter the upstream market for the supply of drugs for the treatment of Gaucher disease. Since the supply of homecare services is effectively tied to Genzyme Homecare, a new competitor would face the additional hurdle of persuading the patient to switch not only to a new drug, but also to a new homecare services provider." (Paragraph 331 of the OFT decision)[22]

[18]Telefónica could not flatly engage into a refusal to supply, since it was subject to regulatory obligations.

[19]Decision No. CA98/3/03 - Exclusionary behaviour by Genzyme Limited.

[20]One drug, Zavesca, had just received marketing authorisation but, according to the Office of Fair Trading (OFT, the UK competition authority at the time) would likely have provided only limited competition to Cerezyme.

[21]The Competition Appeal Tribunal (CAT) confirmed the finding of margin squeeze by the Office of Fair Trading. Case No: 1016/1/1/03, [2004] CAT 4.

[22]Experts are reported to explain that the presence in the downstream market is key for upstream success: "Professor Cox [...] expresses the view that changing homecare provider in circumstances where he was considering switching treatment could definitely affect the choice of treatment, especially in the case of vulnerable patients requiring infusion

The OFT decision might have provided more information about the real chances of successful upstream entry, but the narrative of the case does appear to be consistent with the dynamic leveraging model illustrated in this paper.

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A Appendix

A.1 Proofs of the baseline model

A.1.1 Proof of Lemma 1: Post-entry payoffs in period 2 absent Refusal to Supply

In what follows we will indicate with $\pi^m(c_i)$ the monopoly profits of a firm with marginal cost c_i and facing market demand $Q(p)$, while we will indicate with $\pi^d(c_i, c_j)$ the total duopoly profits obtained by a firm with marginal cost c_i competing à la Bertrand in the final market with a firm with marginal cost $c_j > c_i$ and $p^m(c_i) > c_j$.

One can show that $\pi^m(c) - \pi^m(2c) < \pi^d(c, 2c) < \pi^d(0, 2c)$, the first inequality due to the Arrow's replacement effect.

Note that when $c = 0$, $\pi^d(0, 2c) = 0 < \pi^m(c)$; when $c = \overline{c}$, $\pi^d(0, 2c) = \pi^m(0) > \pi^m(c)$. Since $\pi^d(0, 2c)$ is strictly increasing in c when $c \in [0, \overline{c}]$ and $\pi^m(c)$ is decreasing in c, then there exists a threshold level of $\widehat{c} \in (0, \overline{c})$ such that $\pi^d(0, 2c) > \pi^m(c)$ iff $c > \widehat{c}$. When this is the case, industry profits obtained when the vertical chain formed by the independent firms competes with the vertically integrated incumbent, i.e. $\pi^d(0, 2c)$, are larger than the industry profits obtained when one of the independent firms does not sell and the other independent firm trades with the incumbent's subsidiaries, i.e. $\pi^m(c)$. The equilibrium that emerges in the second period, when all firms are active, will depend on whether that condition is satisfied.

When market demand is given by $Q(p) = 1-p$, $\pi^m(c_i) = \frac{(1-c_i)^2}{4}$ and $\pi^d(c_i, c_j) = (c_j - c_i)(1-c_j)$.

(1) Only downstream independent firm is active.

Upstream offers. The incumbent offers firm D_E the contract involving $w = c$ and $FF = \pi^m(c)$ and commits not to sell through the downstream affiliate. Since there is no scope for opportunistic behavior, firm D_E accepts the contract and the incumbent extracts all the rents from the more efficient downstream competitor.

Downstream offers. Firm D_E offers the incumbent to pay the wholesale price $w = c$ for the input and to pay the fee $FF = \pi^m(2c)$ under the commitment that the incumbent will not sell through the own downstream affiliate. The incumbent accepts the offer. Firm D_E extracts the increase in monopoly profits due to its more efficient production process.

Expected profits of the incumbent and the downstream rival are the following:

$$\pi_I = \frac{1}{2}\left[\pi^m(c)\right] + \frac{1}{2}\left[\pi^m(2c)\right]; \Pi_{D_E} = \frac{1}{2}\left[\pi^m(c) - \pi^m(2c)\right]; \Pi_{U_E} = 0 \tag{9}$$

(2) Only independent upstream firm is active.

Upstream offers. Firm U_E offers the incumbent the contract involving $w = 0$ and $FF = \pi^m(c) - \pi^m(2c)$. The incumbent accepts the offer. Firm U_E extracts the increase in monopoly profits due to the use of its cheaper input.

Downstream offers. The incumbent offers firm U_E to pay the wholesale price $w = 0$ for the input. U_E accepts.

Expected profits of the incumbent and the upstream rival are the following:

$$\pi_I = \frac{1}{2}\left[\pi^m(2c)\right] + \frac{1}{2}\left[\pi^m(c)\right]; \Pi_{U_E} = \frac{1}{2}\left[\pi^m(c) - \pi^m(2c)\right]; \Pi_{D_E} = 0 \tag{10}$$

(3) Both upstream and downstream independent firms are active.

Case (i): $c > \widehat{c}$

Upstream offers. Equilibrium offers are such that the incumbent offers D_E the contract $w = c$ and $FF = 0$, under the commitment not to operate the downstream affiliate. This would allow firm D_E to earn $\Pi_{D_E} = \pi^m(c)$, if accepted. Firm U_E offers D_E the contract $w = 0$ and $FF = \pi^d(0, 2c) - \pi^m(c)$ (or a contract involving a slightly lower fee). Firm D_E accepts the contract offered by the upstream independent firm. It will compete in the final market having marginal cost equal to zero vis-à-vis the incumbent's affiliate having marginal cost $2c$. By $c > \widehat{c}$, the upstream independent firm makes positive profits. The incumbent's upstream affiliate cannot profitably offer a better deal. Firm U_E cannot increase its profits by making a different offer. Imagine firm U_E deviates and offers D_I the contract $w = 0$ and $FF = \pi^m(c) - \varepsilon$. The offers on the table are U_E's deviation offer to D_I and U_I's initial offer to D_E. Following such offers, D_E's dominant strategy is to accept U_I's offer, which involves the incumbent's commitment not to operate the downstream affiliate. Then D_E anticipates that the incumbent will not accept its deviation offer and such a deviation is not profitable.

Downstream offers. The incumbent offers to firm U_E the contract $w = 0$ and $FF = \pi^m(c)$. Firm

D_E can match this offer with the contract $w = 0$ and $FF = \pi^m(c)$ (or a contract involving a slightly higher fee). Firm U_E accepts the contract offered by the downstream independent firm. The latter will compete in the final market having marginal cost equal to zero vis-á-vis the incumbent's affiliate having marginal cost $2c$ and will earn $\pi^d(0, 2c) - FF = \pi^d(0, 2c) - \pi^m(c) > 0$ by $c > \widehat{c}$. Firm D_E cannot profitably deviate and make an offer to U_I. Given the contract offered by D_I to U_E (and that U_E will accept), D_E should offer the incumbent a contract involving a fixed fee at least as large as $FF = \pi^m(c)$. The deviation offer is therefore unprofitable.

Expected profits of the incumbent and the rivals are the following:

$$\pi_I = 0; \Pi_{U_E} = \frac{1}{2}[\pi^d(0, 2c) - \pi^m(c)] + \frac{1}{2}[\pi^m(c)] = \frac{1}{2}[\pi^d(0, 2c)] = \Pi_{D_E} \tag{11}$$

Case (ii): $c < \widehat{c}$

Upstream offers. Equilibrium offers are such that the upstream independent firm offers to D_E the contract $w = 0$ and $FF = 0$. This offer would allow D_E to make profits $\pi^d(0, 2c)$, if accepted. The incumbent offers the contract $w = c$ and $FF = \pi^m(c) - \pi^d(0, 2c)$, or a slightly lower fee (under the commitment not to operate the downstream affiliate). The independent downstream firm accepts the incumbent's offer. It will monopolise the downstream market making profits $\pi^m(c)$. Net of the fixed fee, it makes profits $\pi^d(0, 2c)$ (or slightly more), while the incumbent obtains $\pi^m(c) - \pi^d(0, 2c) > 0$ by $c < \widehat{c}$. The independent upstream firm makes 0 profits. However it cannot profitably deviate making an offer to D_I, for instance offering the contract $w = 0$ and $FF = \varepsilon$. U_E anticipates that D_E will accept the incumbent's offer on the table, which involves the commitment not to operate the downstream affiliate. Hence, U_E anticipates that the incumbent will not accept its deviation offer and such a deviation is not profitable. Finally, an equilibrium in which U_E makes an offer to D_I and the incumbent does not make any offer does not exist. If the candidate equilibrium offer involves a fixed fee $FF > 0$, the incumbent could profitably deviate making an offer to D_E. If the candidate equilibrium offer involves a fixed fee $FF = 0$, firm U_E could profitably deviate and make an offer to D_E.

Downstream offers. The downstream independent firm offers to U_E the contract $w = 0$ and $FF = \pi^d(0, 2c)$. The incumbent offers the same contract (or a slightly higher fee). The upstream independent firm accepts the incumbent's offer. The incumbent makes profits equal to $\pi^m(c) - \pi^d(0, 2c)$. The independent downstream firm makes zero profits. It cannot profitably deviate by making an offer to U_I. Given the contract offered by D_I to U_E (and that U_E will accept), D_E should offer the incumbent a contract involving a fixed fee at least as large as $FF = \pi^m(c)$, which would allow the incumbent to pay the fee $FF = \pi^d(0, 2c)$ to U_E and to be as well off. The deviation offer is therefore unprofitable. Similarly to the case in which the upstream firms make take-it-or-leave-it offers, an equilibrium in which D_E makes an offer to U_I and the incumbent does not make any offer does not exist. If the candidate equilibrium offer involves a fixed fee $FF < \pi^m(c)$, the incumbent could profitably deviate making an offer to U_E. If the candidate equilibrium offer involves a fixed fee $FF = \pi^m(c)$, firm D_E could profitably deviate and make an offer to U_E.

Hence, expected profits of the incumbent and the rivals are the following:

$$\pi_I = [\pi^m(c) - \pi^d(0, 2c)]; \Pi_{U_E} = \frac{1}{2}[0] + \frac{1}{2}[\pi^d(0, 2c)] = \Pi_{D_E} \tag{12}$$

Differently from case (i), in case (ii) the incumbent does not make zero profits. Indeed it appropriates the difference between the monopoly profit (given the less efficient technology in one of the related markets) and the duopoly profits. Essentially it appropriates the increase in profits due to the exclusion from trade of one independent firm (and the consequent lessening of competition). As we will discuss later, this will affect the incumbent's incentives to engage in refusal to supply.

Instead the expected payoff of the independent firms is the same in the two cases: each of them obtain half of the asymmetric Bertrand duopoly profits. However, in case (ii) profits are distributed differently across the two states of the world: with probability 1/2 the independent firm makes zero profits, with probability 1/2 it obtains the entire duopoly profits.

(4) No independent firm is active.

In this case,

$$\pi_I = \pi^m(2c); \Pi_{U_E} = 0 = \Pi_{D_E} \tag{13}$$

A.1.2 Proof of Lemma 2: Entry decision in period 2 under No Refusal to Supply

(ii) Let us consider the case in which firm D_E decided not to enter in period 1.

Case I: Either $F_U \leq \frac{1}{2}[\pi^m(c) - \pi^m(2c)]$ and $0 \leq F_D \leq \frac{1}{2}\pi^d(0, 2c) \equiv F_D^{NoRtoS}$ or $F_U \leq \frac{1}{2}\pi^d(0, 2c)$ and $0 \leq F_D \leq \frac{1}{2}[\pi^m(c) - \pi^m(2c)]$. In this case entering the market is a dominant strategy either for firm U_E or for firm D_E, or for both. The best reply of the independent firm in the vertically related market is to enter. Then, the unique continuation equilibrium is such that the upstream and the downstream firm enter the market.

Case II: $\frac{1}{2}[\pi^m(c) - \pi^m(2c)] < F_U \leq \frac{1}{2}\pi^d(0, 2c)$ and $\frac{1}{2}[\pi^m(c) - \pi^m(2c)] < F_D \leq \frac{1}{2}\pi^d(0, 2c)$. In this case entering the market is a best reply to entry in the vertically related market. Not entering is a best reply to lack of entry in the vertically related market. Then, there exist multiple equilibria. One in which both firms enter the market; another in which no firm does.

Case III: $F_U < \frac{1}{2}[\pi^m(c) - \pi^m(2c)]$ and $F_D > \frac{1}{2}\pi^d(0, 2c)$. In this case entering the market is a dominant strategy for firm U_E. Not entering is a dominant strategy for firm D_E. Then, the unique continuation equilibrium is such that firm D_E does not enter while firm U_E enters.

Case IV: $F_U > \frac{1}{2}[\pi^m(c) - \pi^m(2c)]$ and $F_D > \frac{1}{2}\pi^d(0, 2c)$. In this case not entering the downstream market is a dominant strategy for firm D_E. The best reply of firm U_E is not to enter. Then the unique continuation equilibrium is such that neither firm enters the market.

A.1.3 Proof of Lemma 4: Post-entry payoffs with Refusal to Supply

(1) Only the independent downstream firm is active.

Since D_E cannot trade with the incumbent, its payoff is always zero.

$$\pi_I = \pi^m(2c); \Pi_{D_E} = 0; \Pi_{U_E} = 0 \tag{14}$$

(2) Only the independent upstream firm is active.

In this configuration, firms' expected payoffs are the same as in the case in the case in which the incumbent does not engage in refusal to deal.

$$\pi_I = \frac{1}{2}\left[\pi^m(2c)\right] + \frac{1}{2}\left[\pi^m(c)\right]; \Pi_{U_E} = \frac{1}{2}\left[\pi^m(c) - \pi^m(2c)\right]; \Pi_{D_E} = 0 \tag{15}$$

(3) Both independent firms are active.

Case (i): $c > \widehat{c}$

Upstream offers. The incumbent cannot trade with D_E. Hence, firm U_E will offer D_E the contract $w = 0$ and $FF = \pi^d(0, 2c)$ (or a contract involving a slightly lower fee). Firm D_E accepts. The upstream independent firm extracts, through the fixed fee, the entire duopoly profits. Firm U_E cannot profitably deviate an make an offer to D_I because it would obtain at most $\pi^m(c) < \pi^d(0, 2c)$ by $c > \widehat{c}$.

Downstream offers. The incumbent offers U_E a contract $w = 0$ and $FF = \pi^m(c)$. Firm D_E can match this offer with the contract $w = 0$ and $FF = \pi^m(c)$ (or a contract involving a slightly higher fee). Firm U_E accepts the contract offered by the downstream entrant. Firm D_E cannot trade with U_I because of the commitment to refusal to supply and cannot make any deviation offer to U_I.

Expected profits of the incumbent and the rivals are the following:

$$\pi_I = 0; \Pi_{U_E} = \frac{1}{2}\left[\pi^d(0, 2c)\right] + \frac{1}{2}\left[\pi^m(c)\right]; \Pi_{D_E} = \frac{1}{2}\left[\pi^d(0, 2c) - \pi^m(c)\right] \tag{16}$$

Case (ii): $c < \widehat{c}$

Upstream offers. The incumbent cannot trade with D_E. Firm U_E offers D_E a contract $w = 0$ and $FF = \pi^d(0, 2c)$ (or a contract involving a slightly lower fee). Firm D_E accepts. Firm U_E extracts entire duopoly profits. It cannot profitably deviate and make an offer to D_I. In that case it should offer D_I at least $\pi^m(2c)$ to have the offer accepted (if D_I rejects the deviation offer, the independent firms would not have any contract in place and would not trade with each other; then the incumbent monopolises the final market with the own affiliates). Hence, firm U_E's deviation profit would be $\pi^m(c) - \pi^m(2c) < \pi^d(0, 2c)$. For a similar reason, an equilibrium in which U_E makes an offer to D_I and the incumbent does not make any offer does not exist.

Downstream offers. D_E offers U_E the contract $w = 0$ and $FF = \pi^d(0, 2c)$. Firm D_I offers the same contract (or a slightly higher fee). Firm U_E accepts the incumbent's offer. The incumbent makes profits equal to $\pi^m(c) - \pi^d(0, 2c) > 0$ by $c < \widehat{c}$. The independent downstream firm makes zero profits. It cannot profitably deviate because it cannot trade with D_I. For the same reason an equilibrium in which D_E makes an offer to U_I and the incumbent does not make any offer does not exist.

Expected profits of the incumbent and the rivals are the following:

$$\pi_I = \frac{1}{2}[0] + \frac{1}{2}\left[\pi^m(c) - \pi^d(0, 2c)\right]; \Pi_{U_E} = \frac{1}{2}\left[\pi^d(0, 2c)\right] + \frac{1}{2}\left[\pi^d(0, 2c)\right]; \Pi_{D_E} = 0 \tag{17}$$

(4) No independent firm is active.

In this case,

$$\pi_I = \pi^m(2c); \quad \Pi_{U_E} = 0 = \Pi_{D_E} \tag{18}$$

A.1.4 Proof of Lemma 5

(i) If firm D_E entered the downstream market in period 1, then firm U_E will earn $\pi_{U_E} = \frac{1}{2}\pi^d(0, 2c) + \frac{1}{2}\min\left\{\pi^d(0, 2c), \pi^m(c)\right\} - F_U$ if it enters and $\pi_{U_E} = 0$ otherwise. By assumption $A1$, it decides to enter.

(ii) Let us consider now the case in which firm D_E decided not to enter in period 1.

Case I: $c > \widehat{c}$, $F_U \leq \frac{1}{2}[\pi^m(c) - \pi^m(2c)]$ and $0 < F_D \leq \frac{1}{2}[\pi^d(0, 2c) - \pi^m(c)] \equiv F_D^{RtoS}$. In this case entering the market is a dominant strategy for firm U_E. The best reply of firm D_E is to enter. Then, the unique continuation equilibrium is such that the upstream and the downstream firm enter the market.

Case II: $c > \widehat{c}$, $\frac{1}{2}[\pi^m(c) - \pi^m(2c)] < F_U \leq \frac{1}{2}\pi^d(0, 2c)$ and $0 < F_D \leq \frac{1}{2}[\pi^d(0, 2c) - \pi^m(c)]$. In this case entering the market is a best reply to entry in the vertically related market. Not entering is a best reply to lack of entry in the vertically related market. Then, there exist multiple equilibria. One in which both firms enter the market; another in which no firm does.

Case III: $F_U < \frac{1}{2}[\pi^m(c) - \pi^m(2c)]$ and either $c \leq \widehat{c}$ or $c > \widehat{c}$ and $F_D > \frac{1}{2}[\pi^d(0, 2c) - \pi^m(c)]$. In this case entering the market is a dominant strategy for firm U_E. Not entering is a dominant strategy for firm D_E. Then, the unique continuation equilibrium is such that firm D_E does not enter while firm U_E enters.

Case IV: $F_U > \frac{1}{2}[\pi^m(c) - \pi^m(2c)]$ and either $c \leq \widehat{c}$ or $c > \widehat{c}$ and $F_D > \frac{1}{2}[\pi^d(0, 2c) - \pi^m(c)]$. In this case not entering the downstream market is a dominant strategy for firm D_E. The best reply of firm U_E is not to enter. Then the unique continuation equilibrium is such that neither firm enters the market.

Note that with refusal to supply the set of the parameters' values that sustain an equilibrium in which no independent firm enters the market is wider relative to the case in which the incumbent did not engage in refusal to supply. With refusal to supply an equilibrium with no independent entry (as unique equilibrium or as the miscoordination equilibrium when equilibria are multiple) exists iff $F_U > \frac{1}{2}[\pi^m(c) - \pi^m(2c)]$, while without refusal to supply it exists iff $F_U > \frac{1}{2}[\pi^m(c) - \pi^m(2c)]$ and $F_D > \frac{1}{2}[\pi^m(c) - \pi^m(2c)]$.

Moreover, with refusal to supply the set of the parameters' values that sustain an equilibrium in only the upstream independent firm enters the market is wider relative to the case in which the incumbent did not engage in refusal to supply. With refusal to supply an equilibrium with entry only upstream exists iff $F_U \leq \frac{1}{2}[\pi^m(c) - \pi^m(2c)]$, and either $c \leq \widehat{c}$ or $c > \widehat{c}$ and $F_D > F_D^{RtoS}$; instead, without refusal to supply it exists iff $F_U \leq \frac{1}{2}[\pi^m(c) - \pi^m(2c)]$ and $F_D > F_D^{NoRtoS}$ with $F_D^{NoRtoS} \equiv \frac{\pi^d(0,2c)}{2} > F_D^{RtoS} \equiv \frac{1}{2}[\pi^d(0, 2c) - \pi^m(c)]$.

Then, downstream entry is less likely with refusal to supply. Note that when $F_U \leq \frac{\pi^m(c)-\pi^m(2c)}{2}$ and $\frac{1}{2}[\pi^d(0, 2c) - \pi^m(c)] < F_D < \frac{1}{2}\pi^d(0, 2c)$, refusal to supply turns an equilibrium in which entry occurs both upstream and downstream into an equilibrium in which entry occurs only upstream. In this respect, upstream entry is not discouraged. Instead when $F_U > \frac{\pi^m(c)-\pi^m(2c)}{2}$ refusal to supply turns an equilibrium in which both independent firms enter the market into an equilibrium in which no independent entry occurs. In this case, upstream entry is discouraged.

A.1.5 Proof of Proposition 1

(i) If $c > \widehat{c}$ and $F_D \leq F_D^{RtoS}$, downstream entry occurs with and without refusal to supply. Then, the incumbent's total profits are:

$$\begin{aligned}\pi_I^{1+2,RtoS} &= \pi^m(2c) + \max\left\{0, \frac{1}{2}[\pi^m(c) - \pi^d(0, 2c)]\right\} \\ &< \frac{1}{2}\pi^m(c) + \frac{1}{2}\pi^m(2c) + \max\left\{0, \pi^m(c) - \pi^d(0, 2c)\right\} = \pi_I^{1+2,NoRtoS}\end{aligned}$$

The incumbent has no incentive to engage in refusal to supply.

If either $c \leq \widehat{c}$ or $c > \widehat{c}$ and $F_D > F_D^{RtoS}$, downstream entry does not occur with refusal to supply, while it does occur without refusal to supply.

If $F_U \leq F_U^{RtoS}$, upstream entry occurs irrespective of the incumbent's decision to engage in refusal to supply. Then with refusal to supply the incumbent's total profits are:

$$\pi_I^{1+2,RtoS} = \pi^m(2c) + \frac{1}{2}\pi^m(c) + \frac{1}{2}\pi^m(2c)$$

Without refusal to supply, the incumbent's total profits are:

$$\pi_I^{1+2,NoRtoS} = \frac{1}{2}\pi^m(c) + \frac{1}{2}\pi^m(2c) + \max\left\{0, \pi^m(c) - \pi^d(0,2c)\right\}$$

Recall that, by the Arrow replacement effect, $\pi^m(c) - \pi^m(2c) < \pi^d(c,2c) < \pi^d(0,2c)$. It follows that $\frac{1}{2}\pi^m(c) + \frac{1}{2}\pi^m(2c) > \pi^m(c) - \pi^d(0,2c)$: refusal to supply, by discouraging downstream entry, always increases the incumbent's second-period profits. However, lack of downstream entry decreases the incumbent's first-period profits. By $\pi^m(c) - \pi^m(2c) < \pi^d(0,2c)$ it also follows that the increase in second-period profits, which amounts to:

$$\Delta\pi_I^2 = \frac{1}{2}\pi^m(c) + \frac{1}{2}\pi^m(2c) - \max\left\{0, \pi^m(c) - \pi^d(0,2c)\right\}$$

is always larger than the decrease in first-period profits, which amounts to:

$$\Delta\pi_I^1 = \frac{1}{2}\pi^m(c) + \frac{1}{2}\pi^m(2c) - \pi^m(2c)$$

so that refusal to supply is profitable for the incumbent: $\pi_I^{1+2,RtoS} > \pi_I^{1+2,NoRtoS}$.

If $F_U > F_U^{RtoS}$, upstream entry is discouraged by refusal to supply. Then with refusal to supply the incumbent's total profits are:

$$\pi_I^{1+2,RtoS} = \pi^m(2c) + \pi^m(2c)$$

Note that refusal to supply is less profitable than in the previous case. Since upstream entry is discouraged, in the second period the incumbent cannot extract some of the rents that the more efficient upstream firm produces and supplies the final market through the own, less efficient, subsidiary. However, it is still true that refusal to supply increases the incumbent's second-period profits: by $\pi^m(c) - \pi^m(2c) < \pi^d(0,2c)$ it follows that $\pi^m(2c) > \max\left\{0, \pi^m(c) - \pi^d(0,2c)\right\}$. In this case it cannot be established in general terms that the increase in the incumbent's second-period profits, which now amounts to

$$\Delta\pi_I^2 = \pi^m(2c) - \max\left\{0, \pi^m(c) - \pi^d(0,2c)\right\}$$

is always larger than the decrease in first-period profits. Specifically, when $c > \widehat{c}$, this requires $\frac{3}{2}\pi^m(2c) > \frac{1}{2}\pi^m(c)$; when $c \in (0, \widehat{c}]$, this requires $\pi^d(0,2c) > \frac{3}{2}[\pi^m(c) - \pi^m(2c)]$. If we parametrise the model and assume market demand $Q = 1 - p$, it turns out that also in this case refusal to supply is profitable for the incumbent for $c \in (0, \overline{c}]$: $\pi_I^{1+2,RtoS} > \pi_I^{1+2,NoRtoS}$.

(ii) Consumers suffer from refusal to supply because they pay a higher price. Lack of downstream entry implies that in period 1 they pay the monopoly price $p^m(2c)$ determined by the incumbent's technology rather than the monopoly price $p^m(c)$ determined by the activity of the more efficient independent firm in the downstream market. In period 2, when $F_U \leq F_U^{RtoS}$, consumers suffer a loss when the incumbent's affiliates are sufficiently inefficient (i.e. $c > \widehat{c}$): in that case, absent refusal to supply the duopoly price $2c$ prevails in the final market, while consumers pay the higher monopoly price $p^m(c)$ when the incumbent engages in refusal to supply and only upstream entry takes place. Instead when the incumbent's affiliates are efficient enough (i.e. $c \leq \widehat{c}$), in period 2 consumers pay the same price $p^m(c)$ irrespective of refusal to supply. In that case industry profits when both independent firms participate to trade are lower than industry profits when one independent firm does not sell. Then, even absent refusal to supply the price prevailing in the final market is $p^m(c)$. Consumers always suffer a loss when $F_U > F_U^{RtoS}$. In this case lack of downstream *and* upstream entry implies that consumer pay the monopoly price $p^m(2c)$ also in period 2, under refusal to supply, which is higher than the price that prevails absent refusal to supply (either the duopoly price $2c$ or the monopoly price $\pi^m(c)$.

(iii) Total welfare when the incumbent does not engage in refusal to supply is given by:

$$\begin{aligned}
W^{NoRtoS} &= \pi_I^{1+2,NoRtoS} = \begin{cases} \frac{1}{2}\pi^m(c) + \frac{1}{2}\pi^m(2c) + 0 & \text{if } c \in (\widehat{c}, \overline{c}] \\ \frac{1}{2}\pi^m(c) + \frac{1}{2}\pi^m(2c) + \pi^m(c) - \pi^d(0,2c) & otherwise \end{cases} \\
&+ \pi_{D_E}^{1+2,NoRtoS} = \frac{1}{2}[\pi^m(c) - \pi^m(2c)] + \frac{1}{2}\pi^d(0,2c) - F_D \\
&+ \pi_{U_E}^{2,NoRtoS} = \frac{1}{2}\pi^d(0,2c) - F_U + \\
&+ CS^{1+2,NoRtoS} = \begin{cases} CS(p^m(c)) + CS(2c) & \text{if } c \in (\widehat{c}, \overline{c}] \\ CS(p^m(c)) + CS(p^m(c)) & otherwise \end{cases}
\end{aligned}$$

If $F_U > F_U^{RtoS}$ total welfare when the incumbent engages in refusal to supply is given by:

$$W^{RtoS} = \underbrace{\pi^m(2c) + \pi^m(2c)}_{\pi_I^{1+2,RtoS}} + \underbrace{CS(p^m(2c)) + CS(p^m(2c))}_{CS^{1+2,RtoS}}.$$

If $F_U \leq F_U^{RtoS}$ total welfare under refusal to supply is given by:

$$W^{RtoS} = \underbrace{\pi^m(2c) + \frac{1}{2}\pi^m(c) + \frac{1}{2}\pi^m(2c)}_{\pi_I^{1+2,RtoS}} + \underbrace{\frac{1}{2}[\pi^m(c) - \pi^m(2c)] - F_U}_{\pi_{U_E}^{2,RtoS}} + \underbrace{CS(p^m(2c)) + CS(p^m(c))}_{CS^{1+2,RtoS}}.$$

By point (i), the incumbent gains from refusal to supply. Instead, independent firms suffer a loss from refusal to supply (see above discussion). By point (ii) consumers are harmed by refusal to supply. Then, refusal to supply is welfare detrimental if the loss suffered by consumers and independent firms dominates the incumbent's gain.

Let us start from the case in which the incumbent's affiliates are relatively inefficient: $c > \widehat{c}$. If $F_U \leq F_U^{RtoS}$, the following inequality must be satisfied:

$$\begin{aligned}
W^{NoRtoS} - W^{RtoS} = \Delta W > 0 \Longleftrightarrow \underbrace{CS(p^m(c)) + CS(2c) - CS(p^m(2c)) - CS(p^m(c))}_{\Delta CS} + \\
+ \underbrace{\frac{1}{2}[\pi^m(c) - \pi^m(2c)] + \frac{1}{2}\pi^d(0,2c) - F_D}_{\Delta\pi_{D_E}} + \underbrace{\frac{1}{2}[\pi^d(0,2c) - (\pi^m(c) - \pi^m(2c))]}_{\Delta\pi_{U_E}} > \underbrace{\pi^m(2c)}_{\Delta\pi_I}
\end{aligned}$$

which boils down to:

$$CS(2c) + \pi^d(0, 2c) - F_D > \pi^m(2c) + CS(p^m(2c))$$

Assumption $A2$ implies that $F_D < \frac{1}{2}\pi^d(0, 2c) + \frac{1}{2}[\pi^m(c) - \pi^m(2c)] < \pi^d(0, 2c)$. Moreover, by the monopoly deadweight loss $CS(2c) > CS(p^m(2c)) + \pi^m(2c)$. Then the previous inequality is always satisfied and refusal to supply is welfare detrimental.

If $F_U > F_U^{RtoS}$, the inequality to be satisfied becomes:

$$W^{NoRtoS} - W^{RtoS} = \Delta W > 0 \iff \underbrace{CS(p^m(c)) + CS(2c) - CS(p^m(2c)) - CS(p^m(2c))}_{\Delta CS} +$$

$$+ \underbrace{\frac{1}{2}[\pi^m(c) - \pi^m(2c)] + \frac{1}{2}\pi^d(0, 2c) - F_D}_{\Delta\pi_{D_E}} + \underbrace{\frac{1}{2}\pi^d(0, 2c) - F_U}_{\Delta\pi_{U_E}} > \underbrace{\pi^m(2c) - [\frac{1}{2}\pi^m(c) - \frac{1}{2}\pi^m(2c)]}_{\Delta\pi_I}$$

By the monopoly deadweight loss $CS(2c) > CS(p^m(2c)) + \pi^m(2c)$. Moreover $CS(p^m(c)) - CS(p^m(2c))$, and by assumption $A2$ $\Delta\pi_{D_E}$ and $\Delta\pi_{U_E}$ are both positive. Then the previous inequality is always satisfied and refusal to supply is welfare detrimental.

Let us consider now the case in which the incumbent's affiliates are sufficiently efficient: $c \leq \widehat{c}$. If $F_U \leq F_U^{RtoS}$, total welfare decreases iff:

$$W^{NoRtoS} - W^{RtoS} = \Delta W > 0 \iff \underbrace{CS(p^m(c)) + CS(p^m(c)) - CS(p^m(2c)) - CS(p^m(c))}_{\Delta CS} +$$

$$+ \underbrace{\frac{1}{2}[\pi^m(c) - \pi^m(2c)] + \frac{1}{2}\pi^d(0, 2c) - F_D}_{\Delta\pi_{D_E}} + \underbrace{\frac{1}{2}[\pi^d(0, 2c) - (\pi^m(c) - \pi^m(2c))]}_{\Delta\pi_{U_E}} > \underbrace{\pi^m(2c) - [\pi^m(c) - \pi^d(0, 2c)]}_{\Delta\pi_I}$$

which boils down to:

$$CS(p^m(c)) + \pi^m(c) - F_D > CS(p^m(2c)) + \pi^m(2c)$$

The above condition is always satisfied when $F_D = 0$. With market demand $Q = 1 - p$ the above condition is not satisfied when F_D equals the highest feasible value identified by assumption $A2$, i.e. $F_D = \overline{F}_D$. Then, in that case, there exists a threshold level of the downstream entry cost, $F_D^W \in (0, \bar{F}_D)$, such that refusal to supply is welfare beneficial if $c \leq \widehat{c}$ and $F_D > F_D^W$.

If $F_U > F_U^{RtoS}$ refusal to supply is welfare detrimental iff:

$$W^{NoRtoS} - W^{RtoS} = \Delta W > 0 \iff \underbrace{CS(p^m(c)) + CS(p^m(c)) - CS(p^m(2c)) - CS(p^m(2c))}_{\Delta CS} +$$

$$+ \underbrace{\frac{1}{2}[\pi^m(c) - \pi^m(2c)] + \frac{1}{2}\pi^d(0, 2c) - F_D}_{\Delta\pi_{D_E}} + \underbrace{\frac{1}{2}\pi^d(0, 2c) - F_U}_{\Delta\pi_{U_E}} > \underbrace{\pi^d(0, 2c) - \frac{3}{2}[\pi^m(c) - \pi^m(2c)]}_{\Delta\pi_I}$$

which boils down to:

$$2[CS(p^m(c)) + \pi^m(c)] - F_D - F_U > 2[CS(p^m(2c)) + \pi^m(2c)]$$

With market demand $Q = 1 - p$, the above condition is satisfied for any feasible value of c when

$F_D = 0$ and $F_U = F_U^{RtoS}$. Instead it is satisfied for no feasible value of c when $F_D = \overline{F}_D$ and $F_U = \overline{F}_U$. Then, there exists a threshold level of $F_D + F_U$, $F^W \in (0 + F_U^{RtoS}, \overline{F}_D + \overline{F}_U$, such that refusal to supply is welfare beneficial $c \leq \widehat{c}$ and $F_D + F_U > F^W$.

A.2 Vertically integrated entrants

In this Section we extend the baseline model considering vertically integrated entrants, i.e. the upstream independent firm and the downstream independent firm belong to the same company. For simplicity, we focus on the case in which the incumbent's affiliates are relatively inefficient: $c > \widehat{c}$.

A.2.1 The incumbent did not commit to refusal to supply

When the entrants are vertically integrated, the entry decisions in period 2 are as follows:

Lemma 9. ***Entry decision in Period 2 (No refusal to supply)***
If the incumbent engaged in refusal to supply:

(i) If firm D_E entered in period 1, then firm U_E enters in period 2.

(ii) If firm D_E did not enter in period 1, then in period 2 entry occurs in neither market if (and only if) $F_D + F_U \geq \pi^d(0, 2c)$. Otherwise, entry occurs in both markets.

Proof. If firm D_E did enter the downstream market in period 1, then the decision to enter upstream in the following period is driven by the comparison between the upstream entry cost and the increase in *total* profits caused by upstream entry:

$$\pi^d(0, 2c) - \frac{\pi^m(c) - \pi^m(2c)}{2} \geq F_U \tag{19}$$

Since $F_U \leq \frac{\pi^d(0,2c)}{2}$ by assumption $A1$ and $\frac{\pi^d(0,2c)}{2} > \frac{\pi^m(c) - \pi^m(2c)}{2}$, then the above condition is always satisfied.

If firm D_E did not enter the downstream market in period 1, then four possible outcomes may arise in period 2: (i) no entry takes place; (ii) entry takes place only downstream; (iii) entry takes place only upstream; (iv) entry takes place in both markets. However, one can show that outcomes (ii) and (iii) cannot arise. To see this, imagine that entry alone is profitable in one market. For the vertically integrated firm it is profitable to enter also the other market iff the increase in total profits dominates the entry cost. Then, if:

$$\pi^d(0, 2c) - \frac{\pi^m(c) - \pi^m(2c)}{2} \geq \max\{F_D, F_U\} \tag{20}$$

entry in one market will always go with entry in the vertically related market. Since $F_U \leq \frac{\pi^d(0,2c)}{2}$ by assumption $A1$, $F_D \leq \frac{\pi^d(0,2c)}{2} + \frac{\pi^m(c) - \pi^m(2c)}{2}$ by assumption $A2$ and $\frac{\pi^d(0,2c)}{2} = \pi^d(c, 2c) > \pi^m(c) - \pi^m(2c)$, the above condition is always satisfied.

Then, either the vertically integrated firm enters both market – thereby earning total profits equal to $\pi^d(0, 2c) - F_U - F_D$ – or it enters no market, thereby making 0 profits. Entry in no market is the optimal choice iff:

$$\pi^d(0, 2c) \leq F_U + F_D. \tag{21}$$

Note that, given the upper bounds to the feasible values of F_U and F_D established by assumption $A1$ and $A2$, for condition 21 to be satisfied it is necessary that $F_D > \frac{\pi^d(0,2c)}{2}$ and that $F_U > \frac{\pi^d(0,2c)}{2} - \frac{\pi^m(c)-\pi^m(2c)}{2}$. □

We can now move to the entry decision in period 1.

Lemma 10. ***Entry decision in Period 1 (No Refusal to Supply)***
If the incumbent did not engage in refusal to supply, then downstream entry always occurs in period 1*, followed by upstream entry in period* 2*.*

Proof. If downstream entry occurs in period 1, followed by upstream entry in period 2, total profits of the vertically integrated entrants amount to:

$$\pi_{U+E} = \frac{\pi^m(c) - \pi^m(2c)}{2} + \pi^d(0,2c) - F_U - F_D > max\left\{0, \pi^d(0,2c) - F_U - F_D\right\} \tag{22}$$

By assumptions $A1$ and $A2$, $F_U + F_D < \frac{\pi^m(c)-\pi^m(2c)}{2} + \pi^d(0,2c)$. Hence, entering downstream in period 1 and upstream in period 2 is more profitable than not entering at all; it is also more profitable than entering in both markets in period 2, because period-1 entry allows to earn profits for one more period. □

A.2.2 The incumbent committed to refusal to supply

When the entrants are vertically integrated, the entry decisions in period 2 are as follows:

Lemma 11. ***Entry decision in Period 2 (Refusal to Supply)***
If incumbent engaged in refusal to supply, then:

(i) If firm D_E entered in period 1*, then firm U_E enters in period* 2*.*

(ii) If firm D_E did not enter in period 1*, then in period* 2 *entry occurs in neither market iff $F_U + F_D \geq \pi^d(0,2c)$. Otherwise entry takes place in both markets.*

Proof. If firm D_E did enter the downstream market in period 1, then entering the upstream market in the following period is the only way for the vertically integrated entrant to make positive profits. Upstream entry occurs if (and only if):

$$\pi^d(0,2c) \geq F_U \tag{23}$$

The above condition is always satisfied by assumption $A1$. If firm D_E did not enter the downstream market in period 1, then the argument follows the line of the proof of Lemma 9 with minor differences. □

We can now move to the entry decision in period 1.

Lemma 12. ***Entry decision in Period 1 (Refusal to Supply)***
When the incumbent did commit to refusal to supply, then entry occurs in neither market if (and only if) $F_U + F_D \geq \pi^d(0,2c)$.

Proof. If downstream entry occurs in period 1, firm D_E makes zero profits in that period. Entry will occur in the upstream market in period 2 and the vertically integrated entrant will earn total profits $\pi_{U+E}^{1+2} = \pi^d(0, 2c) - F_U - F_D$. Hence, if $F_U + F_D \leq \pi^d(0, 2c)$, entry will occur in both markets. It is irrelevant whether entry occurs downstream in period 1 followed by upstream entry in period 2, or whether it occurs in both markets in period 2 (by assumption the discount factor equals 1). If $F_U + F_D > \pi^d(0, 2c)$, then entry occurs in neither market. □

A.2.3 Decision to engage in refusal to supply. Proof of Proposition A.2

Proof. If $F_U + F_D \leq \pi^d(0, 2c)$, then refusal to supply does not discourage entry and the incumbent sacrifices profits in period 1 without increasing profits in period 2. Refusal to supply does not arise at the equilibrium. If $F_U + F_D > \pi^d(0, 2c)$ then refusal to supply discourages both upstream and downstream entry, while entry in both markets would occur absent refusal to supply. We have already showed in the Proof of Proposition 1 that refusal to supply is profitable for the incumbent. Also the welfare analysis is the same as the one developed in the proof of Proposition 1 and leads to the conclusion that vertical foreclosure is welfare detrimental. □

A.3 Model with learning effects

A.3.1 Proof of Lemma 8: Equilibrium offers and second-period payoffs

Let us start from the case in which firm D_R sold in period 1. Then, in period 2 the incumbent's downstream affiliate has marginal cost c, while the rival's marginal cost is $c_{R_1} - \lambda < c$.

Case (i): $c > \frac{1}{3} + \frac{c_{R_1} - \lambda}{3}$. In this case, $\pi^d(c_{R_1} - \lambda, 2c) > \pi^m(c) > \pi^m(c + c_{R_1} - \lambda)$.
Upstream offers. Firm U_I offers D_R the contract $w = c$, under the commitment not to operate the own affiliate. Firm U_E offers D_R the contract $w = 0$, $FF = \pi^d(c_{R_1} - \lambda, 2c) - \pi^m(c + c_{R_1} - \lambda)$. Firm D_R accepts U_E's offer. The incumbent's payoff is zero. Firm D_R earns $\pi_{D_R} = \pi^m(c + c_{R_1} - \lambda)$, and firm U_E earns $\pi_{U_E} = \pi^d(c_{R_1} - \lambda, 2c) - \pi^m(c + c_{R_1} - \lambda) > 0$. The incumbent's affiliate cannot make positive profits by offering a different contract. Given U_I's offer on the table (and the commitment not to operate the own downstream affiliate), the upstream independent firm cannot profitably deviate making an offer to D_I. Finally, an equilibrium in which firm U_E makes an offer to D_I and the incumbent does not make any offer does not exist. Since $\pi^m(c) < \pi^d(c_{R_1} - \lambda)$, firm U_E could profitably deviate making an offer to D_R.
Downstream offers. Firm D_I offers U_E the contract $w = 0$, $FF = \pi^m(c)$. Firm D_R offers U_E the same contract. Firm U_E accepts D_R's contract. The incumbent's payoff is zero. Firm D_R earns $\pi_{D_R} = \pi^d(c_{R_1} - \lambda, 2c) - \pi^m(c) > 0$, and firm U_E earns $\pi_{U_E} = \pi^m(c)$. Firm D_R cannot profitably deviate and make an offer to U_I. Given the contract offered by D_I to U_E (and that U_E will accept), D_R should offer the incumbent a contract involving a fixed fee at least as large as $FF = \pi^m(c) > \pi^m(c + c_{R_1} - \lambda)$. The deviation offer is unprofitable. An equilibrium in which firm D_R makes an offer to U_I and the incumbent does not make any offer does not exist. D_R could profitably deviate making an offer to U_E.
In both cases the incumbent would not increase its profits by deciding not to trade with firm D_R.

Expected profits of the incumbent and the rivals are the following:

$$\pi_I = 0; \pi_{U_E} = \frac{1}{2}\left[\pi^d(c_{R_1} - \lambda, 2c) - \pi^m(c + c_{R_1} - \lambda)\right] + \frac{1}{2}\left[\pi^m(c)\right];$$
$$\pi_{D_R} = \frac{1}{2}[\pi^m(c + c_{R_1} - \lambda)] + \frac{1}{2}\left[\pi^d(c_{R_1} - \lambda, 2c) - \pi^m(c)\right]$$

Case (ii): $c \in (\frac{1}{3}, \frac{1}{3} + \frac{c_{R_1} - \lambda}{3}]$. In this case, $\pi^m(c) > \pi^d(c_{R_1} - \lambda, 2c) > \pi^m(c + c_{R_1} - \lambda)$.
Upstream offers. Firm U_I offers D_R the contract $w = c$, under the commitment not to operate the own affiliate. Firm U_E offers D_R the contract $w = 0$, $FF = \pi^d(c_{R_1} - \lambda, 2c) - \pi^m(c + c_{R_1} - \lambda)$. Firm D_R accepts U_E's offer. The incumbent's payoff is zero. Firm D_R earns $\pi_{D_R} = \pi^m(c + c_{R_1} - \lambda)$, and firm U_E earns $\pi_{U_E} = \pi^d(c_{R_1} - \lambda, 2c) - \pi^m(c + c_{R_1} - \lambda) > 0$. The incumbent's affiliate cannot make positive profits by offering a different contract. The upstream independent firm cannot profitably deviate making an offer to D_I. Finally, an equilibrium in which firm U_E makes an offer to D_I and the incumbent does not make any offer does not exist. If firm U_E offers a contract involving a fee $FF > \pi^m(c) - \pi^m(c + c_{R_1} - \lambda)$, then the incumbent would have an incentive to deviate and make an offer to D_R. If the contract involves a fee $FF < \pi^m(c) - \pi^m(c + c_{R_1} - \lambda)$, then firm U_E could profitably deviate making an offer to D_R (recall that $\pi^d(c_{R_1} - \lambda, 2c) > \pi^m(c) - \pi^m(c + c_{R_1} - \lambda)$.
Downstream offers. Firm D_R offers U_E the contract $w = 0$, $FF = \pi^d(c_{R_1} - \lambda, 2c)$. Firm D_I offers U_E the same contract. Firm U_E accepts D_I's contract. The incumbent makes profits equal to $\pi^m(c) - \pi^d(c_{R_1} - \lambda, 2c)$. Firm D_R makes zero profits and firm U_E earns $\pi_{U_E} = \pi^d(c_{R_1} - \lambda, 2c)$. Firm D_R cannot profitably deviate and make an offer to U_I. Given the contract offered by D_I to U_E (and that U_E will accept), D_R should offer the incumbent a contract involving a fixed fee at least as large as $FF = \pi^m(c)$. The deviation offer is unprofitable. An equilibrium in which firm D_R makes an offer to U_I and the incumbent does not make any offer does not exist. Since $\pi^m(c + c_{R_1} - \lambda) < \pi^m(c)$, whatever fee $FF \leq \pi^m(c + c_{R_1} - \lambda)$ involved in the contract, D_I could profitably deviate making an offer to U_E.
In both cases the incumbent would not increase its profits by deciding not to trade with firm D_R. Expected profits of the incumbent and the rivals are the following:

$$\pi_I = 0 + \frac{1}{2}[\pi^m(c) - \pi^d(c_{R_1} - \lambda, 2c)]; \pi_{D_R} = \frac{1}{2}\left[\pi^m(c + c_{R_1} - \lambda)\right]$$
$$\pi_{U_E} = \frac{1}{2}\left[\pi^d(c_{R_1} - \lambda, 2c) - \pi^m(c + c_{R_1} - \lambda)\right] + \frac{1}{2}\left[\pi^d(c_{R_1} - \lambda, 2c)\right]$$

Case (iii): $c \leq \frac{1}{3}$. In this case, $\pi^m(c) > \pi^m(c + c_{R_1} - \lambda) > \pi^d(c_{R_1} - \lambda, 2c)$.
Upstream offers. Firm U_E offers D_R the contract $w = 0$, $FF = 0$. Firm U_I offers the contract $w = c$, $FF = \pi^m(c + c_{R_1} - \lambda) - \pi^d(c_{R_1} - \lambda, 2c)$, under the commitment not to operate the own affiliate. Firm D_R accepts U_I's offer. The incumbent's payoff is $\pi^m(c + c_{R_1} - \lambda) - \pi^d(c_{R_1} - \lambda, 2c) > 0$. Firm U_E earns zero profits and firm D_R earns $\pi_{D_R} = \pi^d(c_{R_1} - \lambda, 2c)$. The upstream independent firm cannot profitably deviate making an offer to D_I. Finally, an equilibrium in which firm U_E makes an offer to D_I and the incumbent does not make any offer does not exist. If firm U_E offers a contract involving a fee $FF > \pi^m(c) - \pi^m(c + c_{R_1} - \lambda)$, then the incumbent would have an incentive to deviate and make an offer to D_R. If the contract involves a fee $FF < \pi^m(c) - \pi^m(c + c_{R_1} - \lambda)$, then firm U_E could profitably deviate making an offer to D_R.
Downstream offers. Firm D_R offers U_E the contract $w = 0$, $FF = \pi^d(c_{R_1} - \lambda, 2c)$. Firm D_I offers U_E the same contract. Firm U_E accepts D_I's contract. The incumbent makes profits equal to

$\pi^m(c) - \pi^d(c_{R_1} - \lambda, 2c)$. Firm D_R makes zero profits and firm U_E earns $\pi_{U_E} = \pi^d(c_{R_1} - \lambda, 2c)$. Firm D_R cannot profitably deviate and make an offer to U_I. Given the contract offered by D_I to U_E (and that U_E will accept), D_R should offer the incumbent a contract involving a fixed fee at least as large as $FF = \pi^m(c)$. The deviation offer is unprofitable. An equilibrium in which firm D_R makes an offer to U_I and the incumbent does not make any offer does not exist. Whatever fee $FF \leq \pi^m(c+c_{R_1}-\lambda)$ involved in the contract, D_I could profitably deviate making an offer to U_E.
In both cases the incumbent would not increase its profits by deciding not to trade with firm D_R. Expected profits of the incumbent and the rivals are the following:

$$\pi_I = \frac{1}{2}[\pi^m(c + c_{R_1} - \lambda) - \pi^d(c_{R_1} - \lambda, 2c)] + \frac{1}{2}[\pi^m(c) - \pi^d(c_{R_1} - \lambda, 2c)];$$
$$\pi_{D_R} = \frac{1}{2}\left[\pi^d(c_{R_1} - \lambda, 2c)\right]; \pi_{U_E} = \frac{1}{2}\left[\pi^d(c_{R_1} - \lambda, 2c)\right]$$

Let us consider now the case in which firm D_I sold in period 1. Then, in period 2 the incumbent's downstream affiliate has marginal cost $c - \lambda$ and is more efficient than the rival, whose marginal cost is $c_{R_1} > c - \lambda$.

Upstream offers. Firm U_E offers D_I the contract $w = 0$, $FF = \pi^m(c-\lambda) - \pi^m(2c-\lambda)$ and makes no offer to D_R, and the incumbent does not make any offer. Firm U_E earns $\pi_{U_E} = \pi^m(c-\lambda) - \pi^m(2c-\lambda) = c$, firm D_R earns zero profits, and the incumbent earns $\pi_I = \pi^m(2c - \lambda)$.

Firm U_I cannot profitably deviate making an offer to D_R: at most it could obtain $\pi^m(c+c_{R_1}) < \pi^m(2c-\lambda)$. Firm U_E cannot profitably deviate making an offer to D_R: it would earn $\pi^d(c_{R_1}, 2c-\lambda) = 2c - \lambda - c_{R_1}$ (note that $2c - \lambda > c_{R_1}$, by the assumption $c_{R_1} < c$ and $\lambda < c$), which is lower than the candidate equilibrium payoff $\pi^m(c - \lambda) - \pi^m(2c - \lambda) = c$ since $c - \lambda < c_{R_1}$.
Downstream offers. Firm D_R offers firm U_E the contract $w = 0$, $FF = \pi^d(c_{R_1}, 2c-\lambda)$, firm D_I offers the same contract. Firm U_E accepts the incumbent's offer. Firm U_E earns $\pi_{U_E} = \pi^d(c_{R_1}, 2c - \lambda)$, firm D_R earns zero profits, and the incumbent earns $\pi_I = \pi^m(c - \lambda) - \pi^d(c_{R_1}, 2c - \lambda) > 0$. Firm D_R cannot profitably deviate making an offer to U_I, because it could pay at most a fee $FF = \pi^m(c-c_{R_1})$ which is not enough to make the incumbent accept the offer.
In both cases the incumbent would not increase its profits by deciding not to trade with firm D_R. Expected profits of the incumbent and the rivals are the following:

$$\pi_I = \frac{1}{2}\pi^m(2c - \lambda) + \frac{1}{2}[\pi^m(c - \lambda) - \pi^d(c_{R_1}, 2c - \lambda)]$$
$$\pi_{D_R} = 0; \pi_{U_E} = \frac{1}{2}[\pi^m(c - \lambda) - \pi^m(2c - \lambda)] + \frac{1}{2}[\pi^d(c_{R_1}, 2c - \lambda)]$$

The incumbent's profits are larger when D_I sold in period 1. When $c \geq \frac{1}{3} + \frac{c_{R_1}-\lambda}{3}$ the incumbent obtains positive profits when D_I sold in period 1 while it earns zero profits when D_R did. When $c \in [\frac{1}{3}, \frac{1}{3} + \frac{c_{R_1}-\lambda}{3})$, $\pi^2_{I|D_I\text{sold}} > \pi^2_{I|D_R\text{sold}}$ because $\pi^m(c-\lambda) > \pi^m(c)$ and $\pi^d(c_{R_1} - \lambda, 2c) > \pi^d(c_{R_1}, 2c-\lambda)$. When $c < 1/3$,

$$\pi^2_{I|D_I\text{sold}} - \pi^2_{I|D_R\text{sold}} = \frac{c}{2} + 2\lambda > 0.$$

A.3.2 Proof of Proposition 3

In period 1 the downstream rival is more efficient than the incumbent's own affiliate: $c_{R1} < c$. Then the incumbent suffers a loss if it does not trade with the rival, due to the fact that when offers are made by the upstream affiliate it loses the possibility to extract the increase in industry profits that occurs in period 1 due to the use of the rival's more efficient technology. The incumbent's loss in period 1 amounts to:

$$\Delta\pi_I^1 = \frac{1}{2}[\pi^m(c + c_{R_1}) - \pi^m(2c)].$$

However, by Lemma 8, refusal to supply benefits the incumbent in the second period. When $c \geq \frac{1}{3} + \frac{c_{R_1}-\lambda}{3}$, the incumbent's second period gain amounts to:

$$\Delta\pi_I^2 = \frac{1}{2}\pi^m(2c - \lambda) + \frac{1}{2}[\pi^m(c - \lambda) - \pi^d(c_{R_1}, 2c - \lambda)] - 0.$$

$\Delta\pi_I^2 > \Delta\pi_I^1$ because $\pi^m(2c - \lambda) > \pi^m(c + c_{R_1})$ (by $c - \lambda < c_{R_1}$) and $\pi^m(c - \lambda) - \pi^d(c_{R_1}, 2c - \lambda) > 0$.

When $c \in [\frac{1}{3}, \frac{1}{3} + \frac{c_{R_1}-\lambda}{3})$, the incumbent's second period gain amounts to:

$$\Delta\pi_I^2 = \frac{1}{2}\pi^m(2c - \lambda) + \frac{1}{2}[\pi^m(c - \lambda) - \pi^d(c_{R_1}, 2c - \lambda)] - \frac{1}{2}[\pi^m(c) - \pi^d(c_{R_1} - \lambda, 2c)].$$

$\Delta\pi_I^2 > \Delta\pi_I^1$ because $\pi^m(2c - \lambda) > \pi^m(c + c_{R_1})$ (by $c - \lambda < c_{R_1}$), $\pi^m(c - \lambda) > \pi^m(c)$ and $\pi^d(c_{R_1} - \lambda, 2c) > \pi^d(c_{R_1}, 2c - \lambda)$.

Finally, when $c < 1/3$, the incumbent's second period gain amounts to:

$$\Delta\pi_I^2 = \frac{1}{2}\pi^m(2c-\lambda)+\frac{1}{2}[\pi^m(c-\lambda)-\pi^d(c_{R_1}, 2c-\lambda)]-\frac{1}{2}[\pi^m(c)-\pi^d(c_{R_1}, 2c-\lambda)]-\frac{1}{2}[\pi^m(c+c_{R_1}-\lambda)-\pi^d(c_{R_1}, 2c-\lambda)]$$

$\Delta\pi_I^2 > \Delta\pi_I^1$ because $\pi^m(2c - \lambda) > \pi^m(c + c_{R_1})$ (by $c - \lambda < c_{R_1}$), $\pi^m(c - \lambda) > \pi^m(c + c_{R_1} - \lambda)$, $\pi^d(c_{R_1} - \lambda, 2c) > \pi^d(c_{R_1}, 2c - \lambda)$ and $\pi^m(2c) > \pi^m(c) - \pi^d(c_{R_1} - \lambda, 2c)$.

A.3.3 Proof of Proposition 4.

In period 1 the incumbent and the downstream firm do not trade if the following condition is satisfied:

$$\pi^m(c + c_{R_1}) - \pi^m(2c) - [\pi^2_{I|D_I\ in\ 1} - \pi^2_{I|D_R\ in\ 1}] + \pi^2_{D_R|D_R\ in\ 1} - \underbrace{\pi^2_{D_R|D_I\ in\ 1}}_{=0} < 0 \qquad (24)$$

From Table 3, when $c < \frac{1}{3} + \frac{c_{R1}-\lambda}{3}$ the joint profit that the incumbent and firm D_R earn in the second period given that D_R sold in period 1 amount to:

$$\pi^2_{I|D_R\ in\ 1} + \pi^2_{D_R|D_R\ in\ 1} = \frac{1}{2}\pi^m(c) + \frac{1}{2}\pi^m(c + c_{R_1}) - \frac{1}{2}\pi^d(c_{R_1} - \lambda, 2c) = 1 - 2c.$$

The joint profit that the incumbent and firm D_R earn in the second period given that I sold in period 1 amount to:

$$\pi^2_{I|I\ in\ 1} + \pi^2_{D_R|I\ in\ 1} = \frac{1}{2}\pi^m(2c - \lambda) + \frac{1}{2}\pi^m(c - \lambda) - \frac{1}{2}\pi^d(c_{R_1}, 2c - \lambda) = 1 + \frac{3\lambda - 5c + c_{R_1}}{2}.$$

Finally,

$$\pi^m(c + c_{R_1}) - \pi^m(2c) = c - c_{R_1}.$$

Then, condition 24 is satisfied iff $c < \lambda + c_{R_1}$, which is always true by assumption. Then, when $c < \frac{1}{3} + \frac{c_{R1}-\lambda}{3}$, then condition 24 is satisfied for any feasible value of the parameters.

From Table 3, when $c \geq \frac{1}{3} + \frac{c_{R1}-\lambda}{3}$, the joint profit that the incumbent and firm D_R earn in the second period given that D_R sold in period 1 amount to:

$$\pi^2_{I|D_R\ in\ 1} + \pi^2_{D_R|D_R\ in\ 1} = \frac{1}{2}\pi^m(c + c_{R_1}) + \frac{1}{2}\pi^d(c_{R_1} - \lambda, 2c) - \frac{1}{2}\pi^m(c) = c - c_{R_1} + \lambda.$$

Condition 24 is satisfied iff $c < \frac{\lambda+2+5c_{R_1}}{9} \equiv \bar{c}$. Note that $\frac{\lambda+2+5c_{R_1}}{9} > c_{R_1}$ from $\lambda < 1/2$. Moreover, $\frac{\lambda+2+5c_{R_1}}{9} > \frac{1+c_{R_1}-\lambda}{3}$ iff $c_{R_1} > \frac{1}{2} - 2\lambda \equiv \underline{c}_{R_1}$. Finally, when $\lambda > 1/6$ then $\frac{1}{2} - 2\lambda < \lambda$ and $c_{R_1} > \frac{1}{2} - 2\lambda$ for any feasible value of c_{R_1}.