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On the Optimal Majority Rule

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November 2017

Abstract

We develop a simple model that rationalizes why less stringent majority rules are preferable to unanimity in large committees. Proposals are randomly generated and the running proposal is adopted whenever it is approved by a sufficiently large share of voters. Unanimity induces excessive delays while too weak majority requirements induce the adoption of suboptimal proposals. The optimal majority rule balances these two inefficiencies: it requires the approval by a share equal to the probability (assumed to be constant across proposals) that a given member gets more than the average welfare associated with the running proposal. Various extensions are considered.

1 Introduction

It is widely accepted that when a committee is large, unanimity is a source of inaction and immobility.[1] As the EU grew larger for example, it became clear that maintaining the requirement that decisions should be approved by unanimity would lead to much inaction, and the Lisbon treaty has been an attempt to correct for this deficiency, by lowering the majority requirement for a number of decisions.

Despite the wide acceptance of such a claim, the bargaining literature has had difficulties providing a rationale for it. For example, following the legislative bargaining literature (pioneered by Baron and Ferejohn (1989)) and viewing committees as bargaining over the division of a pie of fixed size, unanimity is found to be no worse than any other majority rule (in fact all majority

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[1] This view is expressed in various ways in a number of classic writings. For example, Black (1958, page 99) writes:

"The larger the size of majority needed to arrive at a new decision on a topic, the smaller will be the likelihood of the committee reaching a decision that alters the existing state of affairs."

And Buchanan and Tullock (1962) express related concerns about the cost of unanimity rules.

rules are welfare equivalent).[2] Even more striking: If one extends the basic Baron-Ferejohn's setup to allow for the size of the pie to evolve according to a stochastic process, then the unanimity rule provides a welfare efficient outcome (see Merlo and Wilson (1998)) and other majority rules typically lead to welfare inferior outcomes (see Eraslan and Merlo (2002)): Coalitions agree too quickly on inferior outcomes, for fear of later being excluded from a winning coalition.

We revisit this question by applying a collective search framework developed in Compte and Jehiel (2004-2010).[3] In the collective search model, the members of the committee do not control the proposals put to a vote. Their strategic decision consists in voting on whether they are in favor of the proposal or whether they prefer waiting for a better alternative. If the proposal put to a vote receives the support of the required majority, it is implemented. Otherwise, the search process continues until a proposal is adopted.[4]

A key difference between collective bargaining and collective search is that under the collective bargaining approach all possible partitions of the pie are simultaneously available, while under the collective search approach, a proposal at a given date determines both the size of the pie and how the pie would be divided among the committee members: Some other proposals for partitioning the pie will eventually arise, but only through later draws. With patient agents, this difference between the two approaches would not seem to matter a great deal, as in principle each member could at little cost wait for a division he would like to see proposed. This paper shows that this intuition appears to be incorrect when the number of agents grows large. The reason is that one may wait for a proposal that one prefers at little cost, and this mere option exerts a negative externality on those who would be happy completing a deal. As a result waiting for a proposal that every one prefers can be very costly.[5]

To illustrate our basic ideas, we consider a setup with a large number of agents. We structure preferences so that (i) the welfare varies across proposals, and (ii) individuals are heterogenous.[6]

[2] This is because an agreement is reached immediately in all cases.

[3] See Sakaguchi (1973), Kurano et al. (1980), Yasuda et al. (1982) and Ferguson (2005) for percursors of the collective search model developed in Operations Research and that focus on issues of existence and uniqueness of equilibrium (as opposed to welfare and other economic related properties). See also Albrecht et al. (2010) for another paper making use of this model, which is further discussed below.

[4] The view that committee members have no control at all on the proposals put to a vote may sound extreme in some applications (outside say hiring decisions), as some transfers between members can often be suggested together with the proposal. By contrast, assuming that any transfer scheme can freely be proposed (as typically considered in the bargaining literature) sounds extreme to us too, in light of the practical complications of implementing side-payments between the bargaining parties (in part due to the difficulty in assessing how the parties would assess the proposal). As we note later, our main insights carry over to intermediate situations in which some limited transfers can be made.

[5] For example, with a proposal that may be favorable or unfavorable to each with equal probability, the chance that a proposal turns out to be favorable to all is $1/2^n$, which can be a quite small number even for rather small commitees: $1/2^{10} = 10^{-4}$.

[6] Heterogeneity is essential. If individual preferences are perfectly homogeneous, the collective search problem is identical to a single agent search problem no matter what the majority rule is. That welfare varies is also important. If all proposals are welfare equivalent, the best

Formally, for each member i, the value u_i of adopting a particular proposal can be decomposed into two components: A common component x that affects all members in the same way, and an idiosyncratic element θ_i (assumed without loss generality to have mean 0) that describes how that particular proposal affects member i, that is,

$$u_i = x + \theta_i,$$

and in each period there is a new draw of $(x, \theta_1, .., \theta_n)$ corresponding to a new proposal put to a vote. Draws are independent.

The idiosyncratic part θ_i is meant to reflect the idea that any proposal inevitably generates heterogenous benefits across individuals, or at least that it is hard to define transfers that would correct for this heterogeneity. The independence across periods means that benefits are not systematically biased in favor of some members.[7]

A statistic that will play a key role in our result is the probability γ that θ_i is no smaller than 0 (the mean value of θ_i):

$$\gamma \equiv \Pr(\theta_i > E\theta_i).$$

γ measures the chance that a given committee member gets a draw above the average draw (from an ex ante perspective). When n is large, γ measures the proportion of committee members that get a draw more favorable than the average one. We shall refer to it as a *popularity index.* When payoffs are uniformly spread across members, γ is equal to 1/2. When payoffs are more unequally distributed with positive skewness (i.e. the mean is above the median – i.e. γ smaller than 1/2), then there is a smaller share of members who are better off than the "average member".[8]

We use this simple collective search setup to characterize simply the welfare (measured as the discounted sum of utilities) associated with the various majority rules, in the limit of committees of arbitrarily large size.

In general, the expected welfare associated with a particular majority rule depends on whether proposals with high common components x are selected, and on how long it takes to reach a decision. The optimal majority rule precisely solves the tradeoff between selecting proposals with high common component and reducing the expected delay in reaching a decision. As one considers more stringent majority rules, higher welfare levels are obtained upon reaching a decision, but at the cost of longer delays. And as one contemplates less stringent majority rules, decisions are reached faster but at the cost of agreeing on proposals with lower welfare ex post.

In the simple model outlined above, the optimal majority rule is related to the dispersion of the idiosyncratic part only (and not to the distribution of the

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for expected welfare is to reach a decision as quickly as possible, which is achieved for the lowest majority requirement.

[7] This assumption simplifies the analysis by making all individuals symmetric. Our results however do not hinge on that assumption, so long as θ_i is not fully persistent (See Section 4.2.).

[8] By "average member" we mean the one who would get a draw that coincides with the ex ante expected welfare.

common part nor to the impatience of agents assumed to be homogeneous). More precisely, with a large number of agents, the socially efficient decision depends on the value of the common component only: It calls for agreeing if and only if that component x exceeds a given threshold x^*, as if there were no heterogeneity among agents. We find that when the majority rule exceeds the popularity index γ, then players are subject to inefficient rejections (hence delays), failing to agree on socially efficient proposals, and that when the majority requirement is below γ, then players agree (too fast) on socially inefficient proposals. The optimal majority requirement coincides with the popularity index γ. The intuition is that in that case, the pivotal member in the current vote has his preferences aligned with the preference of the average member: his preference is aligned with the common component only, hence with the welfare of the group (for large n).

The inefficiency obtained for majority requirements less stringent than the optimal majority rule bears some similarity with the inefficiency derived in the collective bargaining model of Eraslan and Merlo (2002) (for rules other than unanimity). In our case as in Eraslan and Merlo (2002), a decision is reached too early (as compared with the first-best) because there is a sufficiently large share of members who fear that, later on, other members might agree on an outcome less favorable to them.[9]

The inefficiency obtained under majority requirements more stringent than the optimal one is specific to the collective search approach [of our model], and does not arise in the collective bargaining approach, unless one introduces high transfer costs as in Aghion and Bolton (2003).[10] In our case, a decision is reached too late because of the difficulty in finding a proposal that would suit a sufficiently large share of members.

Toward the end of the paper, we propose several extensions. First we consider one in which members exert some control over the proposals. The model proposed is neither a search model (there is control), nor a collective bargaining one (control is imperfect). Control typically increases the optimal majority requirement, yet the same qualitative insights hold in that model, with too stringent (weak) majority requirement inducing inefficient rejections (acceptance).

Second, we consider the case in which the idiosyncratic part θ_i in agent i's payoff is somewhat persistent across proposals and we observe that the optimal majority rule still coincides with the popularity index.

Finally, we allow the idiosyncratic part to be drawn from several distinct distributions with distinct popularity indices, say $\gamma \in \{\gamma_1, \gamma_2\}$, and we note that the first best can no longer be achieved: The majority rule cannot adjust

[9] This insight is related to Strulovici (2010)'s insight that there is insufficient experimentation when decided by simple majority.

[10] Aghion and Bolton (2003) consider a two-period collective bargaining model in which transfers are costly. Aghion and Bolton find that when transfers are sufficiently costly, the unanimity rule is not optimal because of the transfer costs required to get unanimous consent. With less stringent majority rules, one may get a deal without the consent of agents who request large compensations. Compared to their model, our collective search approach allows us to derive the inefficiency of unanimity even in the limit as search frictions get small, and to relate the inefficiency of the various majority rules to the dispersion of individuals' preferences.

to the specific popularity index of each proposal, so the optimal majority γ^* has to tradeoff the two types of inefficiencies described above, inefficient rejections when $\gamma < \gamma^*$, and inefficient acceptance when $\gamma > \gamma^*$.

The rest of the paper is organized as follows. In Section 2 we present the general collective search model. In Section 3 we analyze the various majority rules in the simple setup outlined above. We also compare the welfare so obtained with that obtained in an analogous bargaining framework. In Section 4 we suggest some extensions of the basic model. Section 5 concludes.

2 The Model

We consider a committee consisting of n members, labeled $i = 1, ..., n$. At any date $t = 1, ...,$ if a decision has not been made yet, a new proposal is drawn and examined. A proposal is denoted u, where $u = (u_i)_{i\in\{1,...,n\}}$ is a vector in R^n that describes the utility u_i that member i gets if the proposal u is adopted.

Distribution over proposals. The set of possible proposals is denoted U, and it is assumed to be a compact convex subset of $[\underline{u}, \overline{u}]^n$. We also assume that proposals at the various dates $t = 1, ..$ are drawn *independently* from the same distribution with continuous density $f(\cdot) \in \Delta(U)$.

In most of the paper, we put the following structure on proposals. We assume that for each member i, the value u_i of adopting a particular proposal u can be decomposed into two components: A common component x that affects all members in the same way, and an idiosyncratic element θ_i:

$$u_i = x + \theta_i,$$

and that in each period, the common component x is drawn according to a density $g(\cdot)$ on $[\underline{x}, \overline{x}]$, while the idiosyncratic parts θ_i are drawn independently of one another and of x, according to a smooth density $h(\cdot)$ on $\left[-\underline{\theta}, \bar{\theta}\right]$. Besides, we also let

$$w(z) \equiv E(x \mid x > z)$$

and assume that $w(.)$ is a smoothly differentiable function with slope no greater than 1,[11] and we normalize θ_i so that $E(\theta_i) = 0$.[12]

Decision rules. Upon arrival of a new proposal u, each member decides whether to accept that proposal. We consider various majority rules. Under the k-majority rule, the game stops whenever at least k out of the n members vote in favor of the proposal.

We normalize to 0 the payoff that members obtain under perpetual disagreement, and we let δ denote the common discount factor of the committee members. That is, if the proposal u is accepted at date t, the date 0 payoff of member i is $\delta^t u_i$.[13]

[11] This holds true for the uniform distribution and for many more densities $g(\cdot)$ with bounded variations. This assumption will ensure the uniqueness of a stationary equilibrium.

[12] This is just a normalization because if $E(\theta_i) \neq 0$ we can add $E(\theta_i)$ to x.

[13] Observe that we allow that $\underline{u}$ be negative, that is, we do not impose that proposals deliver payoffs above the status quo payoffs to all members.

Strategies and equilibrium. In principle, a strategy specifies an acceptance rule that may at each date be any function of the history of the game. We will however restrict our attention to *stationary* equilibria of this game, where each member adopts the same acceptance rule at all dates.[14]

Given any stationary acceptance rule σ_{-i} followed by members j, $j \neq i$, we may define the expected payoff $\bar{v}_i(\sigma_{-i})$ that member i derives given σ_{-i} from following his (best) strategy. An optimal acceptance rule for member i is thus to accept the proposal u if and only if

$$u_i \geq \delta \bar{v}_i(\sigma_{-i}),$$

which is stationary as well (this defines the best-response of member i to σ_{-i}).

Stationary equilibrium acceptance rules are thus characterized by a vector $v = (v_1, .., v_n)$ such that member i votes in favor of u if $u_i \geq \delta v_i$ and votes against it otherwise. For any k-majority rule and value vector v**,** we let $A_{v,k}$ be the corresponding *acceptance set*, that is, the set of proposals that get support from at least k members when failing to agree today yields member i a continuation payoff of v_i (from the viewpoint of next period):[15]

$$A_{v,k} = \{u \in U, \exists K \subset \{1, ..., n\}, \ | K |= k, \, u_i \geq \delta v_i \text{ for all } i \in K\}. \tag{1}$$

Equilibrium consistency then requires that

$$v_i = \Pr(u \in A_{v,k}) E[u_i \mid u \in A_{v,k}] + [1 - \Pr(u \in A_{v,k})] \, \delta v_i \tag{2}$$

or equivalently

$$v_i = \frac{\Pr(u \in A_{v,k})}{1 - \delta + \delta \Pr(u \in A_{v,k})} E[u_i \mid u \in A_{v,k}]. \tag{3}$$

A stationary equilibrium is characterized by a vector v and an acceptance set $A_{v,k}$ that satisfy (1)-(2). It always exists, as shown in Compte and Jehiel (2004-2010). In the more structured setting described above, given the symmetry, v_i is identical across members, and the fixed point characterization implicit in (3) takes a simpler form. In the sequel, we shall be interested here in properties of the limit equilibrium value (denoted v^* below) when the number of players gets arbitrarily large, and how v^* varies with the majority rule.

3 Optimal majority rule in large committees

We wish to analyze how the various majority rules compare in terms of expected welfare as the number of members grows large. Specifically, we will compute

[14] To avoid coordination problems that are common in voting (for example, all players always voting "no"), we will also restrict attention to equilibria that employ no weakly dominated strategies (in the stage game). These coordination problems could alternatively be avoided by assuming that votes are sequential.

[15] For any finite set B, $| B |$ denotes the cardinality of B.

the ex ante payoff obtained by every member in equilibrium under the various majority rules, for any given discount factor (possibly set close to 1 but not necessarily), and make the comparison taking the limit as the number of members grows *arbitrarily large.*

As we shall see, when the number of members grows large, whether a proposal is accepted or not depends almost exclusively on the realization of x (this will be due to the law of large numbers). In subsequent results we refer to $\alpha = \frac{k}{n}$ as the majority rule where k is the majority requirement defined in Section 2. For every α and δ, there will be an equilibrium threshold x^* such that, as n grows large, only proposals such that $x > x^*$ are accepted. Our first objective is to characterize the *equilibrium acceptance threshold* $x^*(\delta, \alpha)$.

3.1 Optimal acceptance threshold

Before characterizing the equilibrium acceptance threshold, we derive the acceptance threshold that would maximize expected welfare if there were no heterogeneity, or equivalently if one could condition the acceptance decision on x only, rather than on the vector of realizations (x, θ). We denote that threshold by x^{**} and refer to it as the *optimal threshold.*

Observe that, as the size of the committee increases without bound, $\frac{1}{n}\sum \theta_i$ vanishes to 0 with probability arbitrarily close to 1, so the optimal threshold x^{**} acceptance policy must yield a welfare that approximates the first-best (defined to be the acceptance policy over (x, θ) that maximizes the average payoff obtained by members).

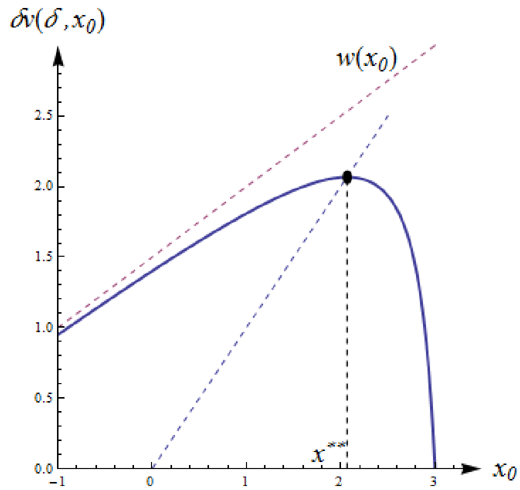
Let us define $v(x_0, \delta)$ as the expected payoff that any member receives if all proposals such that $x > x_0$ are accepted and only such proposals are accepted. We have:[16]

$$\delta v(x_0, \delta) \equiv Q(\delta, x_0) w(x_0) \text{ where } Q(\delta, x_0) \equiv \frac{\delta \Pr(x > x_0)}{1 - \delta + \delta \Pr(x > x_0)} \tag{4}$$

The following figure draws $\delta v(., \delta)$ for a discount factor $\delta = 0.95$, assuming

[16] This is because $v(x_0, \delta) = \Pr(x > x_0) w(x_0) + (1 - \Pr(x > x_0))\delta v(x_0, \delta)$

that x is uniformly distributed on $[-1,3]$.



The figure illustrates that starting from $\underline{x}$, a more stringent acceptance threshold x_0 increases welfare, up to the point where this would reduce so much the probability of acceptance that welfare starts decreasing.[17]

The optimal acceptance threshold is the point $x^{**}(\delta)$ where $\delta v(\delta, x)$ is maximum. Since rejecting a proposal x yields $\delta v(\delta, x^{**})$, and since $\bar{x} > 0$, the optimal acceptance threshold is positive and characterized, as in a standard one person search problem by the condition:[18]

$$x = \delta v(\delta, x), \tag{5}$$

and it is determined graphically by the point at which $\delta v(\delta, .)$ and the $45°$ line cross.

3.2 Equilibrium acceptance threshold

We now turn to the equilibrium acceptance threshold. For every α, define $\theta^*(\alpha)$ as the threshold that solves:[19]

$$\Pr(\theta_i > \theta^*(\alpha)) = \alpha.$$

The threshold $\theta^*(\alpha)$ is thus set so that each member i has a probability α to have his idiosyncratic part θ_i exceed $\theta^*(\alpha)$. Note that as α increases, the threshold $\theta^*(\alpha)$ decreases.[20]

[17] At the limit where x_0 approaches $\bar{x}$, no proposals are accepted so $v(\delta, x_0)$ approaches 0.

[18] The optimal threshold exists and is uniquely defined: since w has slope less than 1 (by assumption), and since Q is decreasing in x_0, $x_0 \to \delta v(x_0, \delta)$ has slope less than 1; also $\delta v(0, \delta) > 0$ and $\delta v(\bar{x}, \delta) = 0 < \bar{x}$.

[19] Equivalently, $\theta^*(\alpha)$ solves $1 - H(\theta^*(\alpha)) = \alpha$, where $H(.)$ denotes the cumulative of $h(\cdot)$.

[20] When α gets close to 1, $\theta^*(\alpha)$ gets close to $\underline{\theta}$, while when α gets close to 0, $\theta^*(\alpha)$ gets close to $\bar{\theta}$. For the uniform distribution on $(-1, 1)$ for example, $\Pr(\theta_i > \theta) = (1-\theta)/2$, so $\theta^*(\alpha) = 1 - 2\alpha$.

Now assume that v^* is the equilibrium expected payoff received by each member (computed from date 0). A member votes in favor of a proposal with common component x whenever

$$x + \theta_i > \delta v^*. \tag{6}$$

Define $\phi_\alpha(\theta)$ as the $(\alpha n)^{th}$ largest element of $\theta = (\theta_1, ..., \theta_n)$, or equivalently, the lowest realization among the (αn) most favorable ones. For a given (x, θ), if

$$x + \phi_\alpha(\theta) > \delta v^*$$

the proposal passes because at least αn members are in favor of it, and if $x + \phi_\alpha(\theta) < \delta v^*$, then the proposal does not get enough support to pass.

For a fixed n, $\phi_\alpha(\theta)$ is a random variable. However, as n grows large, by the law of large number, $\phi_\alpha(\theta)$ gets concentrated on $\theta^*(\alpha)$. As a consequence, defining x^* so that

$$x^* + \theta^*(\alpha) = \delta v^*$$

we conclude that any proposal with common component $x > x^*$ is accepted, and that any proposal with common component $x < x^*$ is rejected. Thus when n is arbitrarily large, acceptance is solely determined by the realization of x, and by definition of $v(., \delta)$ we must thus have:

$$v^* = v(x^*, \delta),$$

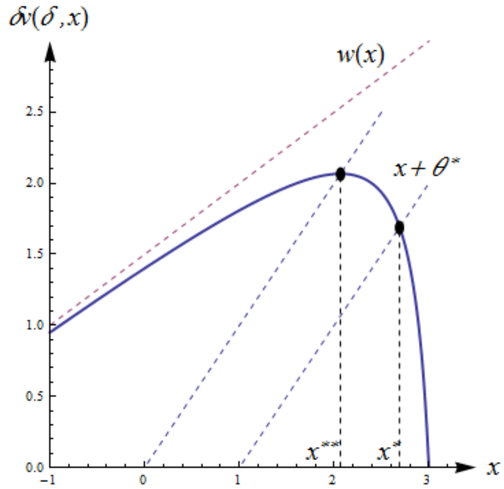
implying that the threshold x^* solves:[21]

$$x^* + \theta^*(\alpha) = \delta v(x^*, \delta) \tag{7}$$

Going back to the uniform distribution case above, and assuming that θ_i is uniformly distributed over $[-1, 1]$, the following figure explains graphically how the threshold x^* is obtained for the unanimity rule $\alpha = 1$ (in which case

[21] Equation (7) has a unique solution because $z \to \delta v(z, \delta) - z$ is decreasing. Note that (i) if $\bar{x} + \theta^*(\alpha) \leq 0$, then $x^* \geq \bar{x}$, and there is perpetual disagreement; and (ii) if $\underline{x} - \delta E(x) + \theta^*(\alpha) \geq 0$, then $x^* \leq \underline{x}$ and there is immediate agreement on the first proposal. In all other cases, the threashold x^* belongs to $(\underline{x}, \bar{x})$.

$\theta^*(\alpha) = -1$), and for the majority rule $\alpha = 1/2$ (in which case $\theta^*(\alpha) = 0$).



As one decreases the majority requirement α, $\theta^*(\alpha)$ increases, hence so does the line $x + \theta^*(\alpha)$. As the figure above illustrates, the threshold x^* shifts to the left, and reaches x^{**} when $\alpha = 1/2$ (and $\theta^*(\alpha) = 0$), at which point welfare is maximum. Further decrease in the majority requirement decreases x^* below x^{**}, and welfare decreases as well.

Note that the analysis above has been made assuming that n **is** arbitrarily large. The next proposition makes a formal statement for cases where n is large but fixed. It is proven in Appendix.

Proposition 1. *Fix α and for every n consider the majority requirement $k(n) = Int(\alpha n)$ where $Int(\alpha n)$ denotes the integer that is closest to αn. For every $\varepsilon > 0$, there exists $\overline{n}$ such that for every $n > \overline{n}$, the expected equilibrium payoff obtained by every member under the $k(n)$ majority requirement $w(\delta, n)$ satisfies $|w(\delta, n) - v(x^*, \delta)| < \varepsilon$ where x^* is the threshold defined in* (7).

3.3 Comparative statics

As the above Figure illustrates, Proposition 1 can be used to compare the welfare obtained under different majority scenarios. It will be convenient to define γ as the probability that the idiosyncratic part is positive:[22]

$$\gamma \equiv \Pr(\theta_i > 0).$$

With a large number of members, the parameter γ corresponds to the share of members who get a draw θ_i above the average θ_i. It may be interpreted as an *intrinsic popularity index.*

[22] Note that when θ_i is symmetric around 0, we have $\gamma = \frac{1}{2}$.

We first observe that when the majority rule α coincides with γ, then $\theta^*(\alpha) = 0$, so the equation (5) that determines the optimal acceptance threshold coincides with the equation (7) that determines the equilibrium acceptance threshold. In other words, we have:

$$x^{**}(\delta) = x^*(\delta, \gamma).$$

As the majority requirement is modified away from γ, the equilibrium acceptance threshold (which is determined by (7)) moves away from the optimal acceptance threshold $x^{**}(\delta)$, so welfare is reduced. Besides, since $z \to z - \delta v(z, \delta)$ is increasing and since $-\theta^*(\alpha)$ is increasing, the solution to $z - \delta v(z, \delta) = -\theta^*(\alpha)$ increases when α increases, implying that the equilibrium acceptance threshold $x^*(\delta, \alpha)$ increases with α. So we have:

Proposition 2. *Let $\gamma = \Pr(\theta_i > 0)$. As the number of members gets large, expected welfare is maximized for the γ majority rule. Besides, the equilibrium acceptance threshold $x^*(\delta, \alpha)$ increases with α.*

This Proposition illustrates the general insight made in introduction that as the majority requirement becomes more stringent, acceptance only obtains for higher realizations of x, which implies that higher welfare levels are obtained upon reaching a decision. Of course, effective expected welfare also considers the effect of discounting so that a too stringent majority rule is not optimal.

The intuition as to why the γ-majority rule is optimal is as follows. When the majority rule is the popularity index, the pivotal member in the current vote is precisely the member having preferences aligned with current welfare, thereby explaining that the same outcome as in the first-best arises under this majority rule.

When the majority requirement increases above γ, $\theta^*(\alpha)$ decreases (below 0), and proposals pass only if the difference $x - \delta v^*$ is sufficiently large (by an amount at least equal to $-\theta^*(\alpha)$). The interpretation is that proposals need to get stronger support to pass, and that support is obtained when the common component is sufficiently high compared to the continuation value δv^*. Welfare is reduced because compared to the first best, only excellent proposals are accepted: socially efficient proposals are rejected.

When the majority requirement decreases below γ, $\theta^*(\alpha)$ increases (above 0), and proposals pass even if x is below δv^* (so long as $x - \delta v^* \geq -\theta^*(\alpha)$). Intuitively, proposals pass even when they get weak support, and such weak support obtains even when x is below δv^*. Welfare is reduced because socially inefficient proposals are accepted.

Comment 1. In more general terms, one could say that members having a positive θ_i exert a negative externality on others by voting too often in favor of proposals, whereas members having a negative θ_i exert a negative externality on others by delaying too much the adoption of proposals. While transfers that would allow to align individual preferences with welfare are not available, a well chosen majority rule (i.e. equal to the popularity index) allows to ensure that the collective search process works as if the search process was delegated to the member having preferences aligned with welfare.

Comment 2. The majority requirement, if set away from γ, generates inefficiencies. The magnitude of these inefficiencies depend on the heterogeneity of preferences among members: when preferences are more dispersed (as resulting, for example, from an expansion of the support of the idiosyncratic part), $\mid \theta^*(\alpha) \mid$ is higher, and reduction in welfare is larger.

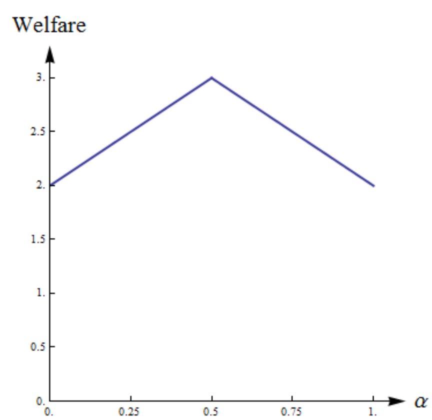
Comment 3. Proposition 2 holds even when members are patient, and it can be expressed as follows:

$$Q(\delta, x^*) = \frac{x^* + \theta^*(\alpha)}{w(x^*)} \tag{8}$$

If $\alpha < \gamma$, then $\theta^*(\alpha) > 0$. Since Q cannot exceed 1, and since $w(x) - x$ tends to 0 when x tends to $\overline{x}$, the condition above requires x^* to be bounded away from $\overline{x}$, thereby implying that $Q(\delta, x^*)$ be close to 1 when δ is close to 1. This means that socially inefficient proposals are accepted, and that delay does not generate further inefficiencies.

If $\alpha > \gamma$, then $\theta^*(\alpha) < 0$, and the right hand side of (8) must be strictly below 1, which implies that x^* must be close to $\overline{x}$ with patient players. This means that only very good proposals are accepted, but with substantial delay. This delay generates an expected cost for each member equal to $\mid \theta^*(\alpha) \mid$.

The following figure summarizes how welfare varies as a function of α in our uniform distribution example.[23]



Expected (per member) welfare as
a function of α

Comment 4: The conclusion of Proposition 2 may seem at odds with the observation made in Albrecht et al. (2010) that as members get very patient,

[23] For uniform distributions, $\theta^*(\alpha) = 1 - 2\alpha$. When $\alpha > 1/2$, welfare is equal to $\bar{x} + \theta^*(\alpha)$. When $\alpha < 1/2$, welfare is equal to $x^* + \theta^*(\alpha)$, where $x^* + \theta^*(\alpha) = w(x^*) = (\bar{x} + x^*)/2$, implying that $x^* + \theta^*(\alpha) = \bar{x} - \theta^*(\alpha)$.

the best rule is the unanimity rule (see also Compte and Jehiel (2004) for an early statement of the same insight in a less general class of examples). Yet, our conclusion holds for any fixed δ (possibly close to 1), in the limit as n goes to infinity. For a fixed n, unanimity would be optimal in the limit as members become arbitrarily patient.

3.4 Collective search vs collective bargaining

In our collective search framework, the best majority rule is interior, as we have just shown. For comparative purpose, we now consider a collective bargaining framework. By this, we mean a bargaining framework in which members may propose targeted transfers in the hope of getting adequate support. We will show that in this scenario the more stringent the majority requirement the better (thereby confirming insights from Merlo and Wilson (1998) and Eraslan and Merlo (2002)).

Specifically, in every period, (x, θ) is drawn as before, and a randomly selected member may propose a transfer schedule $t = (t_1, ..., t_n)$, under the constraint that $x + \theta_i + t_i$ is non-negative for all i, and $\sum_i t_i = 0$. Each member has equal chance of being selected as a proposer. After the proposal is made, there is a vote. If the proposal receives the support of at least $k = Int(\alpha n)$ members, it is implemented and bargaining stops. Otherwise, one moves to the next period, which has the same structure.[24]

We consider stationary equilibria of the above collective bargaining game obtained under the α-majority rule. As in the case of collective search, we consider the case of arbitrarily large n.

Call w^b the expected equilibrium welfare obtained by members. In equilibrium, the draw x gets implemented whenever there is enough surplus to be shared so that a fraction α of the members can *each* be allocated a payoff equal to $\delta w^b/n$, that is, taking the limit as n grows large, whenever:

$$x > \delta \alpha w^b.$$

The equilibrium acceptance threshold x_b^* thus satisfies:

$$\frac{x_b^*}{\alpha} = \delta v(x_b^*, \delta), \tag{9}$$

and the equilibrium acceptance threshold yields the first-best if and only if $\alpha = 1$. Besides, since $z \to z - \delta v(z, \delta)$ is strictly increasing,[25] expected equilibrium welfare is an increasing function of α. To summarize,

Proposition 3. *In the collective bargaining model, expected welfare is maximized for the unanimity rule. The equilibrium acceptance threshold* $x_b^*(\alpha, \delta)$

[24] Note that payoff structure has not been modified, so the rule induced by the optimal threshold x^{**} still approximates the first best when n is large.

[25] This is because $x_0 \to E[x \mid x > x_0]$ is increasing with slope no greater than 1.

increases with α, and expected welfare is an increasing function of the majority rule α.

In other words, majority rules less stringent than unanimity do not maximize welfare because agreement obtains even for pies of low size. The contrast between Propositions 2 and 3 is striking. The collective search model explains why unanimity is undesirable in large committees, and the collective bargaining model does not.

4 Discussion

4.1 Partial control

Collective search and collective bargaining are two polar cases, and in practice, one expects that proposals put to a vote are never completely random, nor fully targeted to the preferences of the members. On the one hand, one expects influence groups to promote their own interests, and yet take into account the majority requirement in formulating demands or exerting influence. On the other hand, targeting members is a difficult task: The framework for formulating proposals may not allow for detailed or agent-specific targeting; agents who formulate proposals may not be fully knowledgeable of each member's preferences; and agents themselves may have noisy perceptions of their own value for the proposal made.

It is not our purpose here to deal with all these possible extensions, but to suggest that more (lesser) control calls for more stringent (weaker) majority rules. We sketch below a simple extension of our model that captures that idea.

We take the collective bargaining version, putting further constraints on the transfers that can be made. We suppose that a share λ of $x + \theta_i$ is monetary, and that only that share can be used to make a (positive) transfer schedule $t = (t_i)_i$ with $t_i \geq 0$ for all i. A proposal (x, θ, t) is thus valued as $v_i(x, \theta, t) = (1 - \lambda)(x + \theta_i) + t_i$ by member i, and our constraint means that the surplus $\lambda \sum_i (x + \theta_i)$ is available for transfers, or equivalently that the set of feasible transfers is

$$T(\lambda, x, \theta) = \{t_i, t_i \geq 0 \text{ for all } i \text{ and } \sum_i t_i = \lambda \sum_i (x + \theta_i)\}$$

When $\lambda = 0$, no transfers are possible: This is our collective search model. When $\lambda = 1$, the whole surplus may be reassigned, under the constraint that each one gets a non-negative payoff. This is our collective bargaining model.

Assume that we wish to give at least Y to a share α of members. For a member with draw θ_i, the transfer needed is $\max(Y - (1 - \lambda)(x + \theta_i), 0)$. So let us define:

$$S(Y, x, \alpha, \lambda) = \int_{\theta_i > \theta^*(\alpha)} \max(Y - (1 - \lambda)(x + \theta_i), 0) h(\theta_i) d\theta_i$$

S represents the (normalized) surplus needed to give at least Y to a share α of members, at the limit when n is arbitrarily large. Given that for a draw x, the total (normalized) surplus available is (approximately) λx, let us also define

$$\phi(x,\alpha,\lambda) = \sup\{Y, S(Y,x,\alpha,\lambda) < \lambda x\}$$

To interpret $\phi(x,\alpha,\lambda)$, define, for each proposal and each coalition/subgroup of size α, the weak member as the one who gets the lowest payoff among coalition members. Then $\phi(x,\alpha,\lambda)$ can be interpreted as the largest payoff obtained by the weak member of a coalition of size α, across all feasible proposals.[26]

The payoff $\phi(x,\alpha,\lambda)$ plays a key role, as global acceptance requires that the weaker player prefers accepting than rejecting. Calling v^*_λ the equilibrium payoff, we observe than any draw x such that $\phi(x,\alpha,\lambda) > \delta v^*_\lambda$ must lead to a proposal that is accepted, while any draw x such that $\phi(x,\alpha,\lambda) < \delta v^*_\lambda$ cannot generate a proposal that gathers sufficient support. So in equilibrium, only proposals above some acceptance threshold are accepted, and that equilibrium acceptance threshold $x^*_{\alpha,\lambda}$ must thus satisfy:

$$\phi(x,\alpha,\lambda) = \delta v(x,\delta)$$

This condition generalizes equations (7) and (9): It is easy to check that when $\lambda = 0$, $\phi(x,\alpha,\lambda) = x + \theta^*(\alpha)$, and when $\lambda = 1$, $\phi(x,\alpha,\lambda) = x/\alpha$, so we get back Propositions 2 and 3, respectively.

Observe next that for a given λ, the first best x^{**} can be achieved at some majority rule α that satisfies

$$\phi(x^{**},\alpha,\lambda) = x^{**}$$

Finally, it is also easy to see that when control is weakened (smaller λ) or when the support required is increased (higher α), then ϕ shifts down (the weaker member is worse off). Since $x \to \phi(x,\alpha,\lambda) - \delta v(x,\delta)$ is increasing,[27] the equilibrium threshold $x^*_{\alpha,\lambda}$ increases above x^{**}: Socially efficient proposals are rejected. To keep up efficiency, weaker control should thus go along with less stringent majority rules.

Comment. The above formulation assumes that transfers even if limited by λ can be requested from members opposed to the deal (this is in line with the bargaining models considered by Baron and Ferejohn (1989)). If only transfers from approving members can be used, then the amount of money that can be redistributed to facilitate approval is reduced, leading to a different specification of the threshold equilibrium.

[26] This largest payoff in general depends on the realization θ, but when the number of members grows arbitratrily large, it does not. Formally, we may define $\phi_\theta(x,\alpha,\lambda)$ as :

$$\phi_\theta(x,\alpha,\lambda) = \max\{Y, v_i(x,\theta_i,t) \geq Y \text{ for a coalition of size } \alpha\text{, for some } t \in T(\lambda,x,\theta)\}$$

By the law of large numbers, when n gets large, $\phi_\theta(x,\alpha,\lambda)$ gets concentrated on $\phi(x,\alpha,\lambda)$.

[27] It is easy to see that $\phi'_x \geq 1$. This is because $S(Y+\Delta, x+\Delta,\alpha,\lambda) \leq S(Y,x,\alpha,\lambda) + \lambda\Delta$

4.2 Persistence of the idiosyncratic component

In our baseline collective search model, the idiosyncratic component θ_i in i 's assessment of a proposal is assumed to be drawn independently across periods. A natural alternative is one in which θ_i has some persistence. Such persistence might arise when there are costs associated with implementing proposals, and when there are similarities over time in how these costs are shared among members.

To formalize persistence, one may assume that with probability $\mu > 0$, all θ_i's remain the same, and that with probability $(1-\mu)$, each player i gets a new draw. The case $\mu = 1$ corresponds to the fully persistent case.

The persistence in heterogeneity implies that all members do not have the same incentives to accept a proposal, and those with higher realization θ_i are more impatient to terminate bargaining. With a large number of members, this implies that the pivotal member in the α-majority rule has type $\theta^*(\alpha)$ defined by $\Pr(\theta_i > \theta^*(\alpha)) = \alpha$. From this, it follows that the γ-majority rule with $\gamma = \Pr(\theta_i > 0)$ is the welfare-maximizing rule, since it results in the same outcome as the one described in subsection 3.1.

Concerning the properties of equilibrium when $\alpha \neq \gamma$, observe that the least favorable draw x accepted by the pivotal player should coincide with what he obtains by rejecting the proposal, hence the following condition:

$$\begin{aligned} x + \theta^*(\alpha) &= \delta v^\mu(\delta, x) \text{ where} \\ v^\mu(\delta, x_0) &\equiv \mu[\Pr(x > x_0)(w(x_0) + \theta^*(\alpha)) + \delta \Pr(x < x_0) v^\mu(\delta, x_0)] \\ &\quad + (1-\mu)\delta v(\delta, x_0) \end{aligned}$$

Simple computations yield:[28]

$$x + \theta^*(\alpha) = Q(\delta, x) w(x) + Q(\mu\delta, x)\theta^*(\alpha)$$

where $Q(\cdot,\cdot)$ was defined in (8). Consider an increase in persistence starting from $\mu = 0$.

If $\alpha < \gamma$ (low majority requirement) then $\theta^*(\alpha) > 0$ and the equilibrium threshold $x^*(\delta, \alpha)$ obtained without persistence is below the optimal threshold $x^{**}(\delta)$: Socially inefficient proposals are accepted. When persistence increases, the right hand side shifts up, so the equilibrium threshold increases, and inefficiencies are reduced.

Similarly, if $\alpha > \gamma$, $x^*(\delta, \alpha)$ is above the optimal threshold $x^{**}(\delta)$: Socially efficient proposals are rejected. When persistence increases however, the right hand side shifts down (because $\theta^*(\alpha) < 0$), hence the equilibrium threshold decreases, thereby limiting inefficiencies.

Our conclusion regarding the optimal majority rule and the nature of the inefficiencies when $\alpha \neq \gamma$ remain valid when there is persistence. The magnitude of the effects however is reduced when there is more persistence.

[28] Observe that $\delta v^\mu(\delta, x_0) - \delta v(\delta, x_0) = \frac{\mu\delta(\Pr(x>x_0))}{1-\mu\delta\Pr(x<x_0)}\theta^*(\alpha)$

4.3 Stochastic popularity index

Our baseline model is special in that the distribution of θ_i is assumed to be deterministically the same in every period. Moreover, it assumes that the distribution of θ_i is independent of x. The suboptimality of unanimity and the general trade-off we identified about the effect of increasing the majority requirement would still hold in the more general case in which these two assumptions are relaxed. However the optimal majority rule may no longer induce the first-best acceptance decision and the optimal majority rule may depend on the impatience of members unlike in the baseline model studied above.

A complete analysis of these extensions is beyond the scope of the present paper. Yet, to illustrate the possible effects, consider the following scenario.

Assume that a proposal is characterized as before by a common component x, and idiosyncratic elements θ_i. In a given period, these idiosyncratic elements are all drawn from the same distribution with density $h(.)$, with $E\theta_i = 0$, but we now assume that $h(.)$ is one of two possible densities $h_1(.)$ or $h_2(.)$. A random variable $\nu \in \{1, 2\}$ determines in each period (independently across periods) which of these two densities applies, say with equal probability. We also let $\gamma_\nu = \Pr_\nu(\theta_i > 0)$ and assume that $\gamma_1 < \gamma_2$.

The proposed model thus extends the previous one in assuming that proposals do not all have the same popularity index: when a proposal gets idiosyncratic elements drawn from h_2 rather than h_1, there is a larger share of members who get a positive θ_i.

In that model, the optimal acceptance threshold x^{**} is determined as before. Implementing such an acceptance threshold however would require to set a majority rule γ_1 in periods where the idiosyncratic elements are drawn from h_1, and a majority rule γ_2 in periods where the idiosyncratic elements are drawn from h_2. When the majority rule is fixed throughout the decision process, welfare is necessarily below that obtained under the optimal acceptance threshold.[29]

An optimal majority rule α^* still exists, and it can be shown that $\alpha^* \in (\gamma_1, \gamma_2)$. However, in periods when the θ_i's are drawn from h_1, the optimal majority rule α^* induces inefficient rejections, and in periods when the θ_i's are drawn from h_2, α^* induces inefficient approvals.

Finally, and contrary to our baseline model, the optimal majority rule now depends on δ. When players are patient, the cost of inefficient rejections gets small. One should thus try to reduce the cost of inefficient approvals, hence α^* should be set closer to γ_2. In that model, patience calls for more stringent majority rules.

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[29] Formally, call v^* the equilibrium payoff, and denote by $\theta^*_{(\nu)}(\alpha)$ the threshold θ for which $\Pr_\nu(\theta_i > \theta) = \alpha$. The difference with our basic search model is that the threshold acceptance for x depends on the realization of ν. We denote by x_1^* and x_2^* these acceptance thresholds. They must satisfy:

$$x_1^* + \theta^*_{(1)}(\alpha) = \delta v^* = x_2^* + \theta^*_{(2)}(\alpha)$$

Efficiency would require $x_1^* = x_2^* = x^{**}$, hence $\theta^*_{(1)}(\alpha) = \theta^*_{(2)}(\alpha) = 0$ or equivalently $\alpha = \gamma_1 = \gamma_2$.

4.4 Delegation

Instead of letting the members vote on the approval of proposals, an alternative decision mechanism would be to delegate the decision making to one (or few) members in the committee. Our basic model suggests that if the majority rule is adequately fixed (i.e. $\alpha = \gamma$), then delegation cannot improve efficiency.

With a delegation to a single agent for example, the outcome would be that of a search problem with a single agent having the preference of the chosen delegate. Clearly, if one could ensure that the delegate has preferences aligned with the welfare of the group that would be optimal. Yet, if the delegate is chosen once for all to be member i, such a decision process would inevitably lead to inefficiencies, given that the search process would be governed by the utility $x+\theta_i$ and thus would not screen proposals according to x only. A similar insight would hold if decision making is delegated to few members, as there is no hope that the decision rule within a small group could mimic the deterministic criterion for efficiency, accept if and only if $x > x^*$.

Still, if the majority rule is not set right, and if for example, the majority rule is overly stringent, then, delegation may perform better by reducing the cost of inefficient rejections.

4.5 Fixed majority rules

In many contexts, modifying a majority rule is difficult, and it may be preferable to think of α as exogenously set, or at least not easily adjustable to the distributive characteristics of proposals. Then, inefficiencies may be inevitable, either because proposals are socially efficient but have a small popularity index, or because proposals are socially inefficient but have a high popularity index.

The later source of inefficiency may even be reinforced when members exert some control or can have some influence over the distributive properties of proposals. Then, the possibility to extract benefits from a minority and spread it across remaining members facilitates approvals, and this also facilitates agreements on inefficient outcomes.

We have mentioned earlier a strenghtening of the majority rule as a possible response to this source of inefficiency. Another response is minority protection, taking the form of a ban on proposals that hurt too harshly a share of the members: Such policies potentially make it more difficult for members or subgroups exerting some control over the distributive characteristics of proposals put to a vote to artificially increase the popularity index of inefficient proposals.

More generally, rather than examining adjustments of the majority requirement, one may examine policies or rules that modify the proposals that can be put to a vote, relying on broad distributive characteristics of these proposals. Of course, if one required equal treatment for any proposal (and if this could be verified), one would entirely eliminate the second source of inefficiency, but this might be at the expense of much delay in finding a suitable agreement. What we point out here is a tradeoff between minority protection (that may limit the chance of agreeing on inefficient proposals – through limits on the strategic influence exerted by some), and possible delays in reaching an agreement.

5 Conclusion

This paper has provided a very simple model that accounts for the widely spread intuition that as committees get large, (well chosen) majority rules are preferable to unanimity. Unlike the well developed models of collective bargaining (with transferable utility) which would unambiguously favor unanimity, our model assumes that members do not control the proposal put to a vote. The main drawback of unanimity in such collective search settings is that finding a proposal acceptable by all is often too difficult, and this in turn induces extra delay costs in comparison with less stringent majority rules. The majority rule should not be too low though, as it would result in the acceptance of too inefficient proposals. The best majority rule is the one that solves best this trade-off.[30]

Our analysis also suggests that stronger transfer mechanisms or control over proposals should go along with tighter majority requirements.

Said differently, while reducing the majority rule preserves from inefficient delays, our analysis points toward a potential weakness of majority rules that would be too permissive, in particular when agents would have control over proposals and set them in ways that too easily redistribute welfare away from a minority. To mitigate this risk, it may be desirable to strengthen the majority requirement (even if sometimes this leads to inefficient delays), or to put a priori limits on how harshly a minority can be hurt.

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[30] A very different argument in favor of majority as opposed to unanimity follows the line of the Condorcet jury theorem by suggesting that majority rules may better aggregate information (this has been formalized by Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1996)).

Our setup offers a different perspective by emphasizing the delay costs attached to unanimity (and by removing the common value uncertainty present in such models - in our setting unlike in those settings, every member knows how much he values the proposal put to a vote, hence our setting falls in the category of private value uncertainty).

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Appendix

Proof of Claim: Observe that by symmetry, the acceptance threshold δv_i is the same for all members and depends only on the majority requirement k. Denote by v_k^*, the per-member welfare obtained in equilibrium under the $k-$majority rule, and by π_k^* the equilibrium probability of agreement. Assume by contradiction that one can have $k_1 > k_2$ and $v_{k_1}^* \geq v_{k_2}^*$. Given that $\Pr(u \in A_{v,k})$ is decreasing with k and v, we would have $\pi_{k_1}^* = \Pr(u \in A_{v_{k_1}^*,k_1}) < \pi_{k_2}^* = \Pr(u \in A_{v_{k_2}^*,k_2})$. Since the expected welfare v_k^* is an increasing function of the probability of agreement π_k^*,[31] we must have $v_{k_2}^* > v_{k_1}^*$, contradicting the premise that $v_{k_1}^* \geq v_{k_2}^*$.

Proof of Proposition 1. For any w we define $\pi(w) \equiv \Pr(x > -\theta^*(\alpha)+\delta w)$, and $u(w) = E[x \mid x > -\theta^*(\alpha) + \delta w]$. Also define w^* as the value satisfies:

$$w = \frac{\pi(w)}{1-\delta+\delta\pi(w)}u(w) \tag{10}$$

Note that by construction, we have $w^* = v^*(x^*, \delta)$ and $x^* = -\theta^*(\alpha) + \delta w^*$.

We show below that the equilibrium value must be close to w^* when n gets large. The argument makes use of the following lemma (some form of the law of large numbers). Define the events $B_\varepsilon, C_\varepsilon, D_\varepsilon$ as:

$$\begin{aligned} B_\varepsilon &= \{\#\{i,\ \theta_i > \theta^*(\alpha) + \varepsilon\}/n > \alpha\} \\ C_\varepsilon &= \{\#\{i,\ \theta_i < \theta^*(\alpha) - \varepsilon\}/n > 1-\alpha\} && (11) \\ D_\varepsilon &= \{\frac{1}{n}\sum_i \theta_i \notin [-\varepsilon, +\varepsilon]\} && (12) \end{aligned}$$

and let E_ε denote the event complement to $B_\varepsilon \cup C_\varepsilon \cup D_\varepsilon$.

Lemma. $\forall \varepsilon, \exists \overline{n}$ such that for all $n > \overline{n}, \Pr\{E_\varepsilon\} > 1 - \varepsilon$.

Now choose ε small, and n large enough so that the inequality of the Lemma holds ($n > \bar{n}$).

Assume now that the equilibrium value is w. We are going to establish bounds on w. To this end, it is convenient to denote by A the event where the current proposal passes. It is also convenient to denote by $F_{w,\varepsilon}^+$ the event $\{x > -\theta^*(\alpha) + \delta w + \varepsilon\}$, by $F_{w,\varepsilon}^-$ the event $\{x < -\theta^*(\alpha) + \delta w - \varepsilon\}$, by $F_{w,\varepsilon}^0$ the event complement to $F_{w,\varepsilon}^+ \cup F_{w,\varepsilon}^-$. Note that since the distribution over proposals has a continuously differentiable density, there exists h such that $\Pr(F_{w,\varepsilon}^+) < h\varepsilon$

Observe that under $F_{w,\varepsilon}^+$, member i accepts x if $\theta_i + x > \delta w$, hence a fortiori if $\theta_i > \theta^*(\alpha) - \varepsilon$. Under event E_ε, there is a fraction α of members for which this is true, hence such proposals x must pass. Similarly, under event $E_\varepsilon \cap F_{w,\varepsilon}^-$,

[31] This is because $E[u_i \mid u \in A_{v,k}] = w/n$ so (3) implies $v_k^* = \frac{\pi_k^*}{1-\delta+\delta\pi_k^*}\frac{w}{n}$.

proposals cannot pass. It follows that in any period, a proposal passes with probability at least

$$\pi^- \equiv \Pr(E_\varepsilon \cap F^+_{w,\varepsilon}) = (1-\varepsilon)\pi(w+\varepsilon/\delta)$$

and at most

$$\pi^+ \equiv 1 - \Pr(E_\varepsilon \cap F^-_{w,\varepsilon}) = 1-(1-\varepsilon)(1-\pi(w-\varepsilon/\delta)).$$

We now derive bounds on the expected payoff that any given member i obtains, conditional on the event A where the current proposal passes.

Observe that by symmetry, for all x,

$$E[\theta_i \mid A] = E[\frac{1}{n}\sum_i \theta_i \mid A_x]$$

which implies that

$$\mid E[\theta_i \mid A \cap E_\varepsilon] \mid < \varepsilon.$$

Finally we have seen that $E_\varepsilon \cap F^+_{w,\varepsilon} \subset A$ and that $E_\varepsilon \cap F^-_{w,\varepsilon} \cap A = \emptyset$, thus:

$$\Pr(E_\varepsilon \cap F^+_{w,\varepsilon} \mid A) > 1 - \Pr E^c_\varepsilon - \Pr F^0_{w,\varepsilon} > 1-(1+h)\varepsilon$$

It follows that $E[x+\theta_i \mid A]$ is bounded above by

$$u^+ \equiv (1-(1+h)\varepsilon)E[x \mid F^+_{w,\varepsilon}] + (1+h)\varepsilon\bar{u}$$

and bounded below by:

$$u^- \equiv (1-(1+h)\varepsilon)E[x \mid F^+_{w,\varepsilon}] + (1+h)\varepsilon\underline{u}$$

where $\bar{u}$ and $\underline{u}$ are bounds on the payoff that any member may get. The equilibrium value must satisfy:

$$\frac{\pi^-}{1-\delta+\delta\pi^-}u^- < w < \frac{\pi^+}{1-\delta+\delta\pi^+}u^+$$

As ε get small, π^+ and π^- converge to $\pi(w)$, and u^+ and u^- converge to $u(w)$. Hence w must converge to the (unique) solution of (10), that is, w^*. **Q. E. D.**