DISCUSSION PAPER SERIES

DP12478

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PUBLIC ECONOMICS

ISSN 0265-8003

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Discussion Paper DP12478
Published 02 December 2017
Submitted 02 December 2017

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Abstract

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JEL Classification: D10

Keywords: Conspicuous consumption, inequality, Happiness, rat race, reference group, Status, utility, welfare

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A Pure Hedonic Theory of Utility and Status: Unhappy but Efficient Invidious Comparisons*

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November 24, 2017

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JEL: D10, D60.

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*We thank Michael Tsatsomeros for his help and Rob Cowan-Douglas and Alex Lam for research assistance. Pascal Courty acknowledges funding from SSHRC grant 410-2011-1256. The analysis and opinions presented are exclusively those of the authors and errors remain our sole responsibility.

Our envy always lasts longer than the happiness of those we envy.

duc de la Rochefoucauld (1665)

Apart from economic payoffs, social status seems to be the most important incentive and motivating force of social behavior.

Harsanyi (1976, p. 204)

1 Introduction

There is now a substantial body of economic theory that incorporates interpersonal consumption comparisons into economic models and examines allocation distortions that reduce welfare. This body of theory is motivated by the classic works of Veblen (1899) and Duesenberry (1949) as well as empirical studies. Most studies in the subjective well-being literature find that an increase in happiness from an increase in own income is substantially, if not completely, offset by an equal increase in the income of the relevant reference group.[1] For example, Luttmer (2005, p. 990) concludes: "... negative effect of neighbors' earnings on well-being is real and that it is most likely caused by a psychological externality, that is, people having utility functions that depend on relative consumption in addition to absolute consumption. Frank (1985, 2010), Layard (1980, 2010) and Oswald (1983) have long urged us to accept the sad reality that relative consumption is a major negative psychological externality that, like pollution, could be taxed to public benefit.

Arrow and Dasgupta (2009) accept that status motivated comparisons may generate much unhappiness, but are skeptical that such comparisons generate large allocative inefficiencies because of an absence of direct empirical evidence. We support this skeptical assessment in our updated review of the efficiency evidence, while finding ongoing strong evidence for unhappy comparisons. This poses an "invidious comparison - efficiency paradox": Why do status motivated comparisons appear to induce so much unhappiness without much evidence of their generating large allocative inefficiencies? The term "invidious comparison" is used in Veblen (1899) to describe pervasive status comparisons that induce chronic dissatisfaction for the relatively poor and restless straining for the relatively rich. Like Arrow and Dasgupta (2009), our response is to go back to theory to reconcile status comparisons with allocative efficiency. At the same time, we reconcile status comparisons with widespread dissatisfaction and welfare losses.

[1] Clark et al. (2008) review the empirical and theoretical literature, and we explore the wide ranging literature in Section 4. Easterlin (1974, 2013) argues that relative comparisons are a key culprit that explain his controversial claim that income growth has not improved the human lot in developed nations.

Our theory is grounded on a hedonic foundation and investigates whether allocation and utility distortions occur when the status function involves comparisons of cardinal utilities across individuals. We name this class of status function "status utility" because utilities are being compared (versus income, consumption or leisure as is done in the literature). Status utility captures how well an individual is doing in utility terms relative to their reference group. An individual's utility affects their status utility and vice versa. For status conscious individuals, where maximizing their own utility is their end goal, utility comparison is the ultimate basis for comparing individuals. Equating cardinal utility with measures of happiness from the subjective well-being being literature operationalizes the theory. Happier individuals have higher status and are happier for it. We motivate and operationalize this methodology in Section 2. This introduction describes the theory as providing a useful and tractable framework.

In the model, each agent's utility is a function of their primary utility and their status utility. Primary utility is the standard cardinal utility function defined on consumption goods. Status utility is a linear function of the difference between own utility and the weighted sum of utilities of other agents in the reference group. The utility functions are therefore implicit functions of each other. With this system, and assuming status comparisons are not too intense, we derive the following key results: (i) The set of Pareto efficient allocations is unaffected by the introduction of status utility. (ii) The competitive equilibrium is unaffected by status utility. (iii) Survey evidence finding positional concerns suggests the importance of status utility, without resort to positional preferences over goods. (iv) Status utility generates network effects that typically increase utility inequality. (v) In the most relevant cases, status utility is negative-sum and equity policy increases average utility. (vi) Utilitarian allocations reduce negative status externalities by allocating more to those who compare themselves more intensely to others. (vii) The very rich may pursue status in ways that resemble descriptions of conspicuous consumption in Veblen (1899). (viii) Competitive allocations are not in the core of the economy when the rich benefit by making gifts to the "enemy of my enemy".

Including relative utility comparisons into the standard model need not alter key positive and normative results. Neutrality results (i) and (ii) boil down to each agent's utility ultimately depending on their own primary utility and not being able to manipulate others' primary utilities. In contrast, the literature departs from the standard model by examining status good comparisons where a subset of goods, termed "positional goods" by Hirsch (1976), generate negative consumption externalities. The more positional is the good, the greater is the externality and distortion. To this conclusion, Arrow and Dasgupta (2009) provide a major qualification. They show that if all goods are equally positional then the externalities cancel out.[2] In their econ-

[2]Layard (1980) briefly discusses offsetting externalities. For recent work on offsetting externalities see Aronsson and Johansson-Stenman (2013), Johansson-Stenman and Sterner (2015), Wendner (2011).

omy, each good is separable in the utility function and there is a negative externality from the absolute level of average consumption of each good. The market equilibrium has no allocation distortions when good externalities are proportionally symmetric across consumption goods. In our model, the balance is naturally struck as agents in effect maximize primary utility. Allocations and primary utilities are the same as in the standard model. This neutrality result survives recursive utility comparisons and all specifications of reference groups.

Our model offers other contrasting perspectives to the literature. Solnick and Hemenway (1998, 2005) and others find dramatic survey evidence for positional concerns. For example, about half of those surveyed prefer a world where they get a lower absolute income when it improves their relative income position. However, for vacations only a few respondents choose less vacation time. The result that income is far more positional than leisure in the utility function is at odds with the requirement of Arrow and Dasgupta (2009) that goods be equally positional for externalities to cancel out. In contrast, we show that the survey evidence is consistent with standard primary utility functions when status utility is important (result iii). From this perspective, utility is positional and goods need not be positional. Another contrast with the literature is that we specify reference groups quite generally. The theoretical status literature has narrowly concentrated on two references standards, either the average consumption of the positional good or rank in the income distribution. Notable exceptions are Ghiglino and Goyal (2010) and Immorlica et al. (2017); they specify local positional good comparisons and work out network effects. We explore a wide range of reference groups and show that the social network increases the dispersion of utilities (result iv).

Status utility can affect average utility and utility inequality. When agents have identical status preferences, utility inequality rapidly increases in the weight placed on status utility. However, in this benchmark case, status utility is zero sum and average utility is unaffected, a property we call "utility conservation". This property reveals that primary utility plays the same underlying absolute role as it does in the standard model without status. With asymmetric status preferences, utility conservation fails and status utility may be either positive sum or negative sum. If it is negative sum, then average utility falls.

Status utility is negative sum in empirically relevant cases. The literature finds that poorer individuals tend to compare more intensely to others. In the model, this implies that less well off agents place greater weight on status utility. Also, individuals tend to compare upward, towards those richer than themselves. This implies that agents place greater weight in their reference group on those who are better off than themselves. We show that either of these patterns of status comparisons can generate negative-sum status utility (result v). For example, this will be the case when income growth accrues only to the richest. The utility gain to them can induce an offsetting utility loss on others that is negative-sum overall. Consequently, the growth in average utility may be negligible or even negative in extreme cases. Also, utility inequality increases. These results arise solely from the intensity of upward status comparisons and not from diminishing marginal utility and relative income and good comparisons (e.g. Johansson-Stenman et al. 2002, Arrow and Dasgupta 2009). The literature has neglected this explanation in trying to rationalize the "Easterlin paradox" (e.g. Clark et al. 2008, Easterlin 2013) evidence that economic income growth may not have increased average long-term happiness. As our model does not generate allocative distortions, it also provides an explanation for the invidious comparison – efficiency paradox.

Welfare improving policies are affected by status utility when utility conservation fails. When status utility is negative sum, average utility is increased by policies, like progressive taxation, which reduce utility inequality. An Utilitarian planner, who respects status preferences, allocates more resources to individuals who feel status comparisons more intensely so that they end up comparing downward to others who are made less better off (result vi). The optimal policy generates positive-sum status utility by creating new inequalities.

Interestingly, behavior resembling Veblen's description of the leisure class emerges when we allow satiation in primary utility. Those rich enough to be satiated in consumption compete for status through "conspicuous consumption" —in the sense of consuming goods obviously beyond the point of consumption satiation rather than give to the poor (result vii). Here, the efficiency equivalence between the status model and the standard model fails, because willful waste is efficient in a world with insatiable status.

Finally, a positive implication of our model is that the competitive equilibrium is not always in the core of the economy. In particular, voluntary "spiteful transfers" may occur if giving to the less fortunate benefits the "enemy of my enemy"(result viii). We consider two mutually exclusive groups in which intergroup comparisons are far more intense than intragroup comparisons. It is in the interest of the rich agents of a group to coordinate on transfers to their poorest group members. But, this charity is not for the love of one's brethren but rather to diminish others outside the group. Neither spiteful transfers nor conspicuous consumption arise in more egalitarian societies.

The paper proceeds to describe the model and methodology in Section 2. Section 3 characterizes the utility system and derives the neutrality and welfare results. Section 4 develops six applications of the theory which shed light on aspects of the literature. Section 5 argues that the theory provides a consistent hedonic approach which is promising for further inquiry.

2 Agents, Interdependent Utility and Methodology

Agents are indexed $i = 1, ..., N$ for $N \geq 2$. Each agent i's cardinal utility u_i depends on two components, "primary utility" U_i and "status utility" S_i:

$$u_i = F_i(U_i, S_i), \tag{1}$$

where the utility function F_i is continuously differentiable and strictly increasing in primary utility, $\frac{\partial F_i}{\partial U_i} > 0$, and weakly increasing in status utility, $\frac{\partial F_i}{\partial S_i} \geq 0$, with strict inequality for at least one agent i. Later, we limit the absolute intensity of status comparisons by bounding $\frac{\partial F_i}{\partial S_i}$.

Primary utility $U_i = U_i(x_i)$ is the standard cardinal utility function defined on a vector x_i of own consumption goods. It does not depend on comparison with others and has the usual properties. For example, primary utility could be an increasing strictly concave function of own consumption expenditures and leisure to highlight the trade-off studied in the rat race literature. It is convenient to assume a finite minimum for primary utility and normalize it $U_i^{\min} \equiv U_i = 0$.

Our innovation is that status utility S_i is determined by agent i comparing their own utility, u_i, relative to their perception of the utility of their reference group, v_i^g, as follows

$$S_i = u_i - v_i^g, \;\; with \;\; v_i^g = \sum_{j \neq i} \omega_{i,j} v_{i,j}, \tag{2}$$

where $v_{i,j}$ is agent i's perception of agent j's utility and $\omega_{i,j} \in [0, 1]$ is the relative status intensity with which agent i compares to agent j. The weights implicitly define the reference group and sum to one, $\sum_{j \neq i} \omega_{i,j} = 1$. Status utility is a hedonic construct because it involves utility comparisons. We follow Bergstrom (1999) and complete the theory by assuming that perceptions of utility ("apparent happiness" in Bergstrom) are accurate:

$$v_i^g = u_i^g \;\; with \;\; u_i^g \equiv \sum_{j \neq i} \omega_{i,j} u_j. \tag{3}$$

Thus, we start with a pure hedonic version of the theory. Discussion of how the theory can be operationalized is left to Section 2.2, and the motivation for the methodology is in Section 2.3.

If agent i compares to a reference group with higher weighted average utility, then agent i receives a negative status payoff ($S_i < 0$). This suffering, by comparing predominantly with those who are better off, is associated in the status literature with feelings of envy, inferiority and/or low self esteem. On the other hand, if agent i compares predominantly downward, to a reference group with lower utility, then agent i receives a positive payoff ($S_i > 0$). Benefiting is associated with feelings of superiority, high self esteem and/or "counting your blessings". Finally

Table 1: Examples of Reference Groups

	u_i^g
Identical Relative Status	$\frac{1}{N-1}\sum_{j\neq i} u_j$
Two Agents	$u_{j\neq i}$ for $N=2$
Two Mutually Envious Groups	$\frac{1}{N-n}\sum_{j=n+1} u_j$ for $i=1,...,n<N$
One-up Status	u_{i+1} for $i<N$, $S_N=0$
All-up Status	$\frac{1}{N-i}\sum_{j>i} u_j$ for $i<N$, $S_N=0$

note that if utility is the same for all agents, $u_i = u_j$ for all $i \neq j$, then $u_i = U_i$ for all i. Here status utility is $S_i = 0$ for each agent and zero sum, $\sum_i S_i = 0$, in aggregate.

Table 1 introduces benchmark reference groups. Under Identical Relative Status, all agents have symmetric other regarding preferences. In the Two Agents case, agents have identical relative status intensities, $\omega_{1,2} = \omega_{2,1} = 1$. Two Mutually Envious Groups divides the society into groups of sizes n and $N-n$, where $n < N$. Members of a group are not envious of each other, but are equally envious of each member of the other group (their reference group). The last two examples assume that primary utilities are ordered $U_1 < U_2 < ... < U_N$ from the least to the most "affluent". With One-up Status each agent is envious of the person with the next highest utility. With All-up Status each agent is envious of all persons with higher utility. In both cases agent N does not compare to anyone. These different reference groups yield quite different results as explored later in the paper in Figures 1-3 and Tables 2-3.

The system of equations (1)-(3) can be solved when status and primary utility are linearly separable. Equation (1) can be written $u_i = U_i + s_i S_i$, where the parameter $s_i = \frac{\partial F_i}{\partial S_i}$ is agent i's absolute status intensity. Substituting (2) and (3), linear separable utility becomes

$$u_i = U_i + s_i \left(u_i - \sum_{j\neq i} \omega_{i,j} u_j \right). \tag{4}$$

The leading case is Identical Status $u_i = U_i + s\left(u_i - \frac{1}{N-1}\sum_{j\neq i} u_j\right)$ where all agents have identical absolute and relative status intensities. It is analogous to the rat race formulation (Frank (1985), Arrow and Dasgupta (2009)) and to most of the empirical literature where agents compare their own consumption to average consumption.

More generally, the utility system in the separable case can be written in matrix notation

$$\left(\mathbf{I} - \mathbf{s}(\mathbf{I} - \omega)\right) u = U \tag{5}$$

where u and U are vectors, $\mathbf{s}$ is a diagonal matrix with elements s_i, $\mathbf{I}$ is the identity matrix and ω

is a matrix with diagonal elements zero and off diagonal elements $\omega_{i,j}$. Define $\mathbf{A} \equiv \mathbf{I} - \mathbf{s}(\mathbf{I} - \omega)$, and assume for now that the inverse matrix $\mathbf{B} = \mathbf{A}^{-1}$ exists. Utilities are explicitly related to primary utilities according to $u = \mathbf{B}U$, or equivalently

$$u_i = b_{i,i}U_i + \sum_{j \neq i} b_{i,j}U_j \tag{6}$$

for all i, where $b_{i,i} \equiv \frac{\partial u_i}{\partial U_i}$ and $b_{i,j} \equiv \frac{\partial u_i}{\partial U_j}$ are the own and cross marginal utilities from changes in primary utilities.

2.1 Examples

With Identical Status, agent i's utility can be expressed in terms of primary utilities:

$$u_i = U_i + b\left(U_i - \bar{U}_{-i}\right), \tag{7}$$

where $\bar{U}_{-i} \equiv \frac{1}{N-1}\sum_{j \neq i}^{N} U_j$ is the average primary utility of others, and $b \equiv \frac{s(N-1)}{(1-s)(N-1)-s} > 0$ provided that the absolute status intensity $s < \frac{N-1}{N}$. In terms of equation (6), the own marginal primary utility effect is greater than one, $b_{i,i} = 1 + b > 1$. The cross primary utility effect is negative $b_{i,j} = -\frac{b}{N-1} < 0$ for all $j \neq i$ (see Appendix C). With identical status, utility conservation obtains, $\sum_i^N u_i = \sum_i^N U_i$. Utility inequality is magnified by status, and the rank ordering of utility is the same as primary utility. If agent i is less endowed than others, $U_i < \bar{U}_{-i}$, then agent i is worse off with status comparisons, $u_i < U_i$. For this agent, as s increases, u_i decreases. In the limit as $s \to \frac{1}{N-1}$ then $b \to \infty$ and $u_i \to -\infty$. Though status utility is zero sum in the aggregate, it can be the dominant factor in determining utility for individuals.

A predominantly upward comparison is simply analyzed with Two Agents. Suppose agent 1 is less affluent, $U_1 < U_2$, and more status conscious, $s_1 \epsilon \left(s_2, \frac{1}{2}\right)$. The utility of agent 1 is

$$u_1 = \frac{1 - s_2}{1 - s_1 - s_2}U_1 - \frac{s_1}{1 - s_1 - s_2}U_2, \tag{8}$$

and the utility of agent 2 involves switching the subscripts 1 and 2. Utility inequality is greater than for primary utility, $u_1 < U_1 < U_2 \leq u_2$, while preserving the rank ordering of utility. Also, the sum of the utilities is smaller, $u_1 + u_2 < U_1 + U_2$, so that status utility is negative sum, $S_1 + S_2 < 0$. These results generalize to $N > 2$. In Section 3.2, we show that utility conservation fails with asymmetric status preferences across agents, but the ranking of utilities is usually preserved. Finally, note that if $s_i \epsilon \left(\frac{1}{2}, 1\right)$, then we have the bizarre case where utilities are decreasing in own primary utility and increasing in the primary utility of the other agent.

2.2 Operationalizing Status Utility

Before developing the theory further, we address its causal plausibility. Whether individuals can process utility comparisons inherently involves whether individuals can assess their own and others' overall subjective well-being (SWB), also referred to as "happiness". Recently, a SWB literature has emerged in which many argue that cardinal indicators of happiness are useful empirical proxies for cardinal utility. Clark et al. (2008) and Frey and Stutzer (2010) survey this literature as well as the evidence that self-reported happiness scores are often accurately predicted by others.[3] Below we discuss direct and indirect proximate evidence and approaches for happiness comparisons. In the next subsection, the theoretical methodology and distal evidence are briefly reviewed. Overall, we believe that there is a reasonable case for considering invidious utility comparisons seriously.

An indirect approach to operationalizing the theory is to use equation (6), which involves deriving status utility from individual primary utilities. Consider the benchmark case of Identical Status, equation (7), where all individuals have the same consumption preferences. Since $U_i = U(x_i)$ can be replaced with the indirect primary utility function which only depends on own income, individuals and researchers only need to know the distribution of income in the population in order to estimate the average primary utility. This is the same information that would be required if individuals cared about relative income as is assumed in the rat race literature, and our model is essentially observationally equivalent to the literature on relative income that we review in Section 4.1.

More generally, with preference heterogeneity, status utility can be solved using equation (6) with knowledge of the status parameters of others and unbiased estimates of the primary utilites others. Of course, an individual i's assessment of the happiness of a particular individual j may very well error. But, if their assessments are unbiased for all j, $v_{i,j} = u_j + \epsilon_{i,j}$ where $\epsilon_{i,j}$ is an unbiased *iid* error term, then individual i's expectation of the apparent happiness of the reference group is unbias, $Ev_i^g = u_i^g$. This is consistent with our assumption that perceptions are accurate in equation (3) so that agents are not systematically fooled. With unbiased errors in observations of apparent happiness and rational expectations, equation (5) becomes $U = (\mathbf{I} - \mathbf{s}(\mathbf{I} - \omega))\, u + \mathbf{s}\omega\epsilon$ where ϵ is the matrix of $\epsilon_{i,j}$ with $\epsilon_{i,i} = 0$. We obtain $u = \mathbf{B}U - \mathbf{B}\mathbf{s}\omega\epsilon$. The term $\mathbf{B}\mathbf{s}\omega\epsilon$ is an unbiased error term that would not change agents' behavior or the welfare analysis developed

[3]Happiness refers to SWB measures ranging from emotional affect to life satisfaction to eudaimonia. We use the term "hedonic" in this paper in the original Bethamite sense of utility as quantifiable happiness. Clark (2016) shows that the different happiness measures are usually strongly correlated and that the discrete data from surveys can be considered essentially cardinal. Also, Clark (2016) reviews that using SWB measures is promising for decision making. Benjamin et al. (2012) report: "On average, SWB and choice coincide 83 percent of the time in our data" (p. 2085). Zou et al. (2013) find that self-reported happiness and informant ratings are equally valid.

in Section 3.1.[4] It would not change the distribution of utility if errors cancel out, which will be the case with large groups as studied in Section 3.3.

Only a few studies have direct evidence from individuals identifying the relevant reference groups. Clark and Senik (2010) examine an European Social Survey of 18 countries which asks respondents: "How Important Is It For You to Compare Your Income With Other Peoples Income?" and "Whose income would you be most likely to compare your own with?". In principle, similar questions with respect to happiness could be used to infer information about the relative status weights ($\omega_{i,j}$) and the correlates for the intensity of status comparisons by individual (e.g. the s_i). Clark and Senik (2010) find that comparisons are mainly with people in proximity, family, friends and colleagues. They find strong evidence that individuals predominantly compare upwards towards those richer than themselves and that the intensity of comparisons decreases with own income. In Knight et al. (2009), Chinese villagers report comparing intensely to neighbors and but not much beyond their village. The intensity of comparisons decreases with own income. In terms of our model, this evidence suggests that those with a higher income rank have smaller absolute weights s_i. It also suggests that each agent i places a greater relative weight $\omega_{i,j}$ on an agent j the greater is j in the income rank. We assume that these are the most relevant status specifications in the rest of the paper.[5]

A direct approach is to operationalize status utility equation (2) through its components. An individual i who is able to directly apprehend the apparent happiness of their own reference group v_i^g can directly process their own utility, equations (1)-(2), without needing to know the preferences of others. Similarly, individual i can directly process their own utility by constructing v_i^g from their own direct perceptions of the apparent happiness of each member $v_{i,j}$ of their own reference group. If perceptions are unbiased (see above), equation (4) implies $Eu_i = \frac{1}{1-s_i}U_i - \frac{s_i}{1-s_i}u_i^g$, and the benchmark of Identical Status is readily estimated in the case of a large number of agents using average reported happiness to proxy u_i^g. Own primary utility could be proxied by a measure of financial or consumption satisfaction. More objectively, own primary utility could be represented by the maximum value function of income.

We were only able to find a few studies which could be interpreted as directly testing the

[4]Individuals do not act strategically because they each have a dominate strategy to maximize their own primary utility in our basic model, see Proposition 3. A separate issue is that agents may take costly actions to try to fool others by strategically influencing the perceptions of others (e.g. Ireland 1994, Bagwell and Bernheim 1996). We leave for future research the strategic possibilities of agents either using resources in a way that succeeds in misrepresenting SWB (with systematic discrepancies between u_i and v_i) or choosing identities to achieve higher social group status and endogenously altering group formation (e.g. Bernard et al. 2016).

[5]They fit with anecdotal accounts going back to Veblen (1899) and Duesenberry (1949) as well as the success of using neighbors or similar socio-economics reference groups in empirical work (see Section 4.3). Corazzini et al. (2012) find that comparisons in positional experiments are not only made upwards but also downwards. Relative income concerns are far greater in high-income countries than low-income countries.

specification of preferences in equation (1) using SWB measures for each of utility, primary utility and status utility. Shin and Johnson (1978) use a numerical measure where "respondents compare themselves with the people whom they know in terms of life enjoyment" (p. 481). This measure proxies status utility equation (2) and enters positively and significantly as the strongest predictor of happiness in their regressions. It largely offsets the happiness-increasing effects from own income and satisfaction with standard of living, indicating intense invidious hedonic comparisons. Michalos (1991, 2012) develops multiple discrepancy theory to generalize research on happiness, and asks respondents: "Consider your life as a whole. How does it measure up to the average for most people your own age and sex in this area?" (e.g. Michalos (1991, p.79)). Using extensive surveys of students across 39 countries, he concludes that comparisons with others is consistently the second most important domain in determining own happiness after self wants. This early work is consistent with invidious happiness comparisons.[6]

2.3 Methodology and Invidious Comparisons

Methodologically, our approach is related to the non-paternalistic altruism literature, where individual utility (termed social utility) is specified as a function of own and others' primary utility functions.[7] Bergstrom (1989) describes altruistic preferences more generally as depending on the cardinal utilities of others, and Bergstrom (1999) shows that the utility system has a recursive solution when utility is additively separable. The informational and computational for requirements to implement altruistic utility comparisons is very similar to that for invidious utility comparisons. With altruism the reference group may be more intimate, and hence direct observation may be more informative in inferring apparent happiness. Yet, the arguments and evidence, presented above and below, point to intense invidious comparisons. Our study of status utility is definitely on the dismal side but is not the converse of altruism because status utility involves the explicit comparison of utilities.[8]

Of course, cardinal utility and utility comparisons have been discussed in economics since

[6] It also points the way to designing appropriate survey questions which elicit apparent happiness comparisons relative to whom individuals know and compare. Constructing the status utility difference using own happiness and the apparent happiness of others has been found to be more reliable in the psychology literature that finds self enhancement biases relative to group averages (see Ong et al. (2017) for references). However, group averages may be quite misleading of the relevant reference groups the members of whom assessors may know quite well.

[7] Kolm (2006) traces this approach back to Vilfredo Pareto who distinguished what we call primary utility (his "ophelimity" function) from utility which depended on primary utility functions of others. Archibald and Donaldson (1976) discuss nonpaternalistic altruism and derive the welfare implications. Bergstrom (2006) illustrates economic problems involving recursive utility; see Hori (2009) for further work and references. Bourlès et al. (2017) describe social utility as a network and solve the Nash equilibrium of a gift giving game.

[8] The different implications of the approaches is discussed following Proposition 2 in Section 3.1. Interpersonal utilities that enter positively (altruism) and negatively (status or envy) are not necessarily contradictory if they influence different preference domains and govern non-overlapping choices.

Bentham along with notions of happiness. On the theoretical side, Harsanyi (1987) argues that cardinal utility is both a necessary and defensible assumption for interpersonal utility comparisons. In his analysis, everybody has consistent empathetic preferences that satisfy the Von Neumann and Mortenstern rationality requirements.[9] Binmore (2007, Section 19.5) extends the analysis to (the anti-sympathetic motivation) envy as requiring knowledge of not just others' possessions but also their preferences and gives examples. Binmore (2009) describes the accuracy in empathetic identification with others as guided by an evolution of similar consistent preferences and norms that serve the important function of equilibrium coordination. People are not necessarily sympathetic to others but rather have developed the facility to conjecture the behavior of others.

The key importance of envy in human society is described in Elster's (1989) formidable book "The Cement of Society". He surveys human motivations and ironically concludes that envy, opportunism and honor are the preeminent motivations that "without which chaos and anarchy would prevail" (p.251). Consistent with our formulation of utility, he describes how the target of envy could include another person's utility function (ability to enjoy goods) and even their good fortune in not having to experience intense envy. Elster (1991, p. 52) describes happiness as the object of envy and, anticipating our results, notes that this can generate a vicious spiral that diminishes happiness.

The importance of having a reference group and knowing it well is also suggested by recent research which finds that envy intensity is positively correlated with interpersonal knowledge and counterfactual thinking (e.g. van de Ven and Zeelenberg 2014). Lebreton et al. (2012) find that coveting what others want may be hardwired into the brain. It is debated to what extent interpersonal comparisons are the expression of hardwired preferences, which could result from evolutionary selection (e.g. Eaton and Eswaran 2003, Rayo and Becker 2007, Binmore 2009), or play a complex instrumental role (e.g. Ireland 1994, Bagwell and Bernheim 1996).

Much of the applied theory literature, while noting that positional goods may be playing an instrumental role, model a positional status externality as a preference. For example, Hopkins and Kornienko (2004) referring to Veblen (1899) and Duesenberry (1949) write: "There is no need for incomplete information to produce conspicuous consumption. For example, the Smiths may know the Joneses are richer than the Smiths, however, they may not envy them unless they visibly see the Joneses enjoying a more lavish lifestyle." This suggests that the Smiths are envying the enjoyment of the Joneses and consumption could be proxying happiness or utility, consistent with Binmore (2007, Section 19.5).

[9]Ng (1997) argues that cardinal utility and interpersonal comparisons follow from the Von-Neuman expected utility axioms together with the assumption that individuals are unable to infinitely distinguish between gradations of pleasure.

In his complex description, Veblen (1899) describes the evolution of invidious comparisons where, in different epochs, each of ability, consumption, leisure and manners play a predominant role in asserting pecuniary respectability and superiority. Pecuniary respectability is quite different than maximizing wealth. Of his own era, Veblen stressed that norms for cultural refinement and good living were needed for successful emulation of respectability. Veblen (1899, p.84) writes: "None the less, while manners have this intrinsic utility, in the apprehension of the performer and the beholder alike, this sense of the intrinsic rightness of decorum is only the proximate ground of the vogue of manners and breeding." He explains after this passage, as elsewhere, that the respectability norms for asserting superiority evolve, even though individuals believe they are maximizing intrinsic utility in comparison with their peers in any given epoch. To the extent that respectability norms circumscribe primary utility less in the current era, perhaps the less is the distance between Veblen's approach and ours. We reconcile the structural case for status utility as a deep preference with the empirical literature on invidious pecuniary comparisons in Section 4.1 and positional goods in Section 4.2.

3 Analysis

The utility system (1)-(3) can be more compactly expressed

$$u_i = F_i\left(U_i, u_i - \sum_{j \neq i} \omega_{i,j} u_{i,j}\right) \tag{9}$$

for all $i = 1, ..., N$, where status utility $S_i = u_i - \sum_{j \neq i} \omega_{i,j} u_{i,j}$. In an example, we have already found that $\frac{\partial F_i}{\partial S_i} < \frac{1}{2}$ is needed to ensure that the utility system is well defined and utility is increasing in own primary utility. In the general case, we need a slightly stronger sufficient condition which we maintain throughout the paper.

Assumption 1. $\frac{\partial F_i}{\partial S_i} \leq \sigma < \frac{1}{2}$ *for all* i*, where* $\sigma > 0$ *is a positive constant.*

Note that σ can be arbitrarily close to $\frac{1}{2}$. Assumption 1 limits the intensity of status comparisons and is sufficient to ensure that each agent's utility is a well-defined function and increasing in their own primary utility.

Proposition 1. *(i) For any vector of primary utilities* $U = (U_1, ..., U_N)^T$*, there exists a unique solution* $u(U) = (u_1(U), ..., u_N(U))^T$ *to utility system (9) where each agent* i*'s utility* $u_i = u_i(U)$ *is a well-defined function of primary utilities. (ii) Each agent* i*'s utility is strictly increasing in their own primary utility,* $\frac{du_i}{dU_i} > 0$*.*

All proofs are collected in Appendix B.

When Assumption 1 fails for at least two agents, there exist a matrix ω of relative status weights such that $\frac{du_i}{dU_i} < 0$.[10] Here agent i prefers to deny themselves consumption in order to deny other agents additional utility. This bizarre behavior is reminiscent of the expression "cut off your nose to spite your face". We do not believe that such extreme behavior is at work in most societies, and therefore concentrate on interdependent utilities where the own effect is positive, $\frac{du_i}{dU_i} > 0$ for all i.

Under Assumption 1, utility comprising own primary utility and relative utility comparisons can always be unraveled to be express utility as a function of primary utilities, $u_i = u_i(U_1(x_i), ..., U_i(x_i), ..., U_N(x_N))$. This transformation reveals that own consumption x_i is separable from the consumption of other agents. It is this feature (which does not obtain in models where utility contains both own consumption and relative consumption comparisons) that delivers the equivalence results we develop below.

3.1 Equivalence Results: Pareto Efficiency and Competitive Equilibrium

The following result shows that that the utility solution $u(U)$ described in Proposition 1 preserves the underlying primary utility Pareto ordering. As is standard, we suppose that the primary utility and production are such that the primary utility possibility set is convex and that no agent is ever satiated in consumption.

Proposition 2. *The set of Pareto efficient allocations is the same in an economy in which some agents consider status utility and in an otherwise identical economy where agents do not consider status utility.*

This equivalence result relies on the own effects being positive, i.e. $\frac{du_i}{dU_i} > 0$ for all i. Interestingly, though utility is rivalrous, the result does not require that all cross effects $\frac{du_i}{dU_j}$ be negative. Indeed, it is possible that all but one of the cross effects may be positive, as we show in an example for Two Mutually Envious Groups in Appendix C.

When primary utilities are interdependent and possibly spiteful, Rader (1980) finds that with the assumption of "no local Pareto satiation" the set of Pareto efficient allocations is a subset of the Pareto efficient allocations for a classical economy with no preference externalities. This result implies the Second Welfare Theorem. More generally, Dufwenberg et al. (2011) derive the same result when the utility of agents may be affected by both the consumption of other

[10] Formally, if Assumption 1 is violated such that $\frac{\partial F_k}{\partial S_k} + \frac{\partial F_l}{\partial S_l} \geq 1$ for any two agents k and l, then there is a matrix ω such that $\frac{du_i}{dU_i} < 0$ for at least one i.

agents as well as their budget sets. Our relative utility specification implies the equivalence of the Pareto optimal sets which, in turn, yield the stronger welfare results we develop below.

Now consider a perfectly competitive market economy where each agent i takes prices and the allocations of others as given. With standard preferences, where status is not considered ($S_i = 0$ for all i), each agent i maximizes their primary $U_i^* = U_i(x_i^*)$, where x_i^* is the unique own consumption allocation that maximizes their primary utility subject to their budget constraint. Nonsatiation in consumption ensures that agent i's budget constraint binds. Similarly, with status preferences, each agent i maximizes $u_i^* = u_i(U_i(x_i^*), U_{-i})$ where U_{-i} is the vector of primary utilities of all other agents $j \neq i$. The positive own effect, $\frac{du_i}{dU_i} > 0$, ensures that agent i's budget constraint binds. Thus, each agent i's consumption optimal choice x_i^* is unaffected by status utility comparisons. Assuming a competitive equilibrium exists, we immediately have the next result.

Proposition 3. *The set of competitive equilibrium allocations are the same in an economy in which some agents consider status utility and in an otherwise identical economy where agents do not consider status utility.*

This result seems counter-intuitive from the perspective of our recursive relative utility specification (9): agent i's choice x_i influences the utilities of other agents through relative utility comparsions which in turn influences agent i's utility. It seems that status utility should affect agent i's decisions. And it does, but without generating allocative distortions, since Proposition 1 implies that own consumption is separable from the consumption of other agents. As agent i's utility is increasing in their primary utility, the best they can do is to maximize their own primary utility. Dubey and Shubik (1985) show that if interdependent consumption preferences do not alter preferences over own goods (their condition b) then the Nash equilibrium is unaffected by interlinked preferences in a model with a continuum of traders: "In other words, benevolence or malevolence towards others is washed out by virtue of perfect competition; i.e. by the presence of a continuum of traders" (p3). Dufwenberg et al. (2011) review the literature on competitive equilibrium equivalence and find that agents act *as-if-classical* (i.e. demand functions depend only on prices and own income) when own consumption is separable from the consumption and budget sets of other agents. Our utility representation $u_i = u_i(U_1(x_i), ..., U_i(x_i), ..., U_N(x_N))$ satisfies their Definition 2 for separability.

Together Propositions 2 and 3 imply that competitive equilibrium economies with and without status utility are not only observationally equivalent but also have the same efficiency implications. We immediately have the following corollary.

Corollary 1. *When both the First and Second Fundamental Theorems of Welfare Economics are satisfied in the economy without status utility, they hold in an economy with status utility.*

As with standard policy analysis, lump sum taxation can be used to shift the competitive equilibrium efficient outcome to address equity concerns. Implications for the distribution of utility are discussed in Section 3.4. Efficiency equivalence may fail when we allow primary utility satiation or spiteful transfers as discussed in Sections 4.5 and 4.6.

The neutrality result in Corollary 1 is distinct from other papers finding that the competitive equilibrium is efficient. Becker et al. (2005) examine a world where status is a good that can be traded but not produced. When the status good is a complement with consumption, it will be efficiently traded. Kolm (1995) characterizes conditions on initial allocations that neutralize the impact of envy on the final allocation. Of course, in static and dynamic models with one composite good there is no inefficiency induced by contemporaneous consumption comparisons. For this reason, Layard (1980) and Arrow and Dasgupta (2009) model contemporaneous substitution between consumption and leisure.

3.2 Linear Separable Status

In the rest of the paper we investigate the explicit implications of status utility when utility is linearly separable in primary utility and status as specified in equations (4) – (6). Under Assumption 1, or less restrictively $\frac{\partial F_i}{\partial S_i} = s_i < \frac{1}{2}$, the mapping from primary utilities to utilities has the following properties.[11]

Proposition 4. *With linear separable status, the marginal utilities from changes in primary utilities are as follows: (i) Own effect, $b_{i,i} > 1$ for $s_i > 0$ (and $b_{i,i} = 1$ for $s_i = 0$); (ii) Own effect dominates effect on others, $b_{i,i} > |b_{j,i}|$ for $j \neq i$; (iii) Total effect invariance, $\sum_j b_{i,j} = 1$.*

For status conscious agents, effects (i) and (iii) imply $\sum_{j \neq i} b_{i,j} < 0$ so that the total status comparison effect is always negative. In many cases, $b_{i,j} < 0$ for all $j \neq i$ (e.g. Two Agents and Identical Status economies). However, $\sum_{j \neq i} b_{i,j} < 0$ does not rule out $b_{i,j} > 0$ for a subset of j. For example, $b_{i,j} > 0$ for up to $n = N - 1$ agents in the Two Mutually Envious Groups economy. The intuition for $b_{i,j} > 0$ is that agent i does not compare intensely to agent j but agent i does compare intensely to others who in turn compare intensely to agent j. Thus, agent i indirectly benefits when agent j is better off.[12]

Proposition 4(*iii*) and equation (6) imply two simple benchmark results. First, if primary

[11] When $s_i < \frac{1}{2}$, matrix $\mathbf{A} = \mathbf{I} - \mathbf{s}(\mathbf{I} - \omega)$ in equation (5) is row diagonally dominant, $|a_{i,i}| > \sum_{j \neq i} |a_{i,j}|$, with positive diagonal entries $a_{i,i} > 0$. Hence, the solution matrix $\mathbf{B}$ is well-defined (Horn and Johnson 1991).

[12] For example, with $N = 3$ we have $b_{1,2} = s_1[s_3\omega_{1,3}\omega_{3,2} - (1 - s_3)\omega_{1,2}]/\det A$, where $\det A > 0$. Then $b_{1,2} > 0$ when agent 1 does not compare intensely to agent 2 ($\omega_{1,2}$ small) but compares intensely to agent 3 ($\omega_{1,3} > 0$) who in turn compares intensely to agent 2 ($\omega_{3,2} > 0$ and $s_3 > 0$). In Section 4.6, we explore when agent i gives to agent j as a way of spiting agent i's enemies.

utilities are equal for all agents, $U_i = U_j$ for all $i \neq j$, then utility is equal to primary utility, $u_i = U_i$, and status utility is zero, $S_i = 0$. Second, if the primary utilities of all agents change by the same amount, $\Delta U_i = \Delta U_j$ for all $i \neq j$, then so does utility, $\Delta u_i = \Delta u_j$ for all $i \neq j$, and there is no change in status utility, $\Delta S_i = 0$ for all i.

To get a different perspective on patterns of interdependent relationships, we solve the matrix system recursively. Rearranging equation (5) gives $u = [\mathbf{I}-\mathbf{s}]^{-1} U - \widetilde{\mathbf{s}}\ \omega u$**,** where $\widetilde{\mathbf{s}}$ is a diagonal matrix with elements $\frac{s_i}{1-s_i} \in [0,1)$, and ω is a matrix with diagonal elements 0 and off diagonal elements $\omega_{i,j}$. Since $u = \mathbf{B}U$ it follows that $u = \left[[\mathbf{I}-\mathbf{s}]^{-1} - \widetilde{\mathbf{s}}\ \omega\mathbf{B}\right] U$, and we obtain the recursive expression $\mathbf{B} = [\mathbf{I}-\mathbf{s}]^{-1} - \widetilde{\mathbf{s}}\ \omega\mathbf{B}$**.** Substituting this equation into itself gives the series[13]

$$\mathbf{B} = \left[\mathbf{I} - \widetilde{\mathbf{s}}\ \omega + [\widetilde{\mathbf{s}}\ \omega]^2 - [\widetilde{\mathbf{s}}\ \omega]^3 + [\widetilde{\mathbf{s}}\ \omega]^4 - ...\right] [\mathbf{I}-\mathbf{s}]^{-1}.$$

The component matrices in the first bracket alternate in sign. The first matrix $\mathbf{I}$, corresponds to the own primary utility effect. It affects $b_{i,i}$ but not $b_{i,j}$. The second matrix, $-\widetilde{\mathbf{s}}\ \omega$**,** involves direct status comparisons where each agent i compares to those in their reference group. This direct negative feedback affects $b_{i,j}$ but not $b_{i,i}$. The third matrix, $[\widetilde{\mathbf{s}}\ \omega]^2$, captures the indirect positive feedback from all utilities negatively impinging on the utilities of agents j in i's reference group. This indirect positive effect is loosely described by the adage "the enemy of my enemy is my friend". The next matrix, $-[\widetilde{\mathbf{s}}\ \omega]^3$, is negative as it describes "the enemy of the enemy of my enemy is my foe". Similarly, we get alternate positive and negative feedbacks for higher order terms. Agent i benefits from agent j being better off, $b_{i,j} > 0$, only if the indirect positive feedback effects dominate both the direct and indirect negative feedback effects.

Finally, consider that cardinality is often meant to imply that affine transformations of the utility functions are permissible. Consider an identical affine transformation of utilities $u_i^a = c + du_i$ for all i, where $c \geq 0$ and $d > 0$ are constants. Applying the transform to equations (2) and (3) requires that $S_i^a = dS_i$. Applying the transformation to equation (4) preserves the preference relation $u_i^a = U_i^a + s_i S_i^a$ when primary utility is equivalently transformed $U_i^a = c + dU_i$. Since this transformation does not alter preference tradeoffs and utility orderings, it follows that all our general result are unaffected.[14] A common affine transformation may be useful when

[13] It can be readily verified that $\mathbf{BA} = \mathbf{I}$, where $\mathbf{A} = [\mathbf{I}-\mathbf{s}][\mathbf{I}+\widetilde{\mathbf{s}}\ \omega]$. The series omits the transversality term, $\lim_{K\to\infty} [-\widetilde{\mathbf{s}}\ \omega]^K \mathbf{B}$, because it is equal to zero when $s_i < 1/2$. Consider $[-\widetilde{\mathbf{s}}\ \omega]^K = (-1)^K \widetilde{\mathbf{s}}^K\ \omega^K$. Using the sub-multiplicative property of the max norm (defined as $\|\mathbf{X}\|_{max} = max_{i,j}\mathbf{X}_{i,j}$ for matrix $\mathbf{X}$), we obtain $\left\|[-\widetilde{\mathbf{s}}\ \omega]^K\right\|_{max} \leq \left\|\widetilde{\mathbf{s}}^K\right\|_{max} \left\|\ \omega^K\right\|_{max}$, where $\left\|\widetilde{\mathbf{s}}^K\right\|_{max} = \left(\frac{s^*}{1-s^*}\right)^K$ and $\left\|\ \omega^K\right\|_{max} \leq 1$. When $s_i < 1/2$, $\lim_{K\to\infty} [-\widetilde{\mathbf{s}}\ \omega]^K \mathbf{B} = 0$ and the series $\sum_{k=1}^{\infty} [-\widetilde{\mathbf{s}}\ \omega]^k$ converges to a well-defined matrix.

[14] The specific utility distributions illustrated in Figures 1 and 2 would have different slopes and intercepts. Of course, different affine transformations across agents may affect the ordering of utilities and marginal trade-offs between agents. Nevertheless, Propositions 1-4 hold provided that Assumption 1 is satisfied. These results are

interpreting numerical responses to SWB happiness questions as affine measures of cardinal utility. For example, it could be used to compare a survey where respondents answer on a scale, say 0 to 10, with another survey where respondents answer an otherwise identical happiness question but on another scale, say 1 to 4.

3.3 Status and Average Utility

There is a major policy interest in understanding how status comparisons impact the average well-being in society. Here we examine how status utility affects average utility. Denote average utility $\bar{u} \equiv \frac{1}{N}\sum_i u_i$. Average utility can be written $\bar{u} = \bar{U} + \overline{S}$, where $\bar{U} \equiv \frac{1}{N}\sum_i U_i$ is average primary utility and $\overline{S} \equiv \frac{1}{N}\sum_i s_i S_i$ is average status utility. Status parameters $(\mathbf{s}, \omega)$ only affect $\bar{u}$ through $\overline{S}$. For example, "negative-sum status" $\overline{S} < 0$, implies a loss of utility compared to primary utility $\bar{u} < \bar{U}$. A useful benchmark is when zero-sum status $\overline{S} = 0$ obtains for any distribution of primary utility.

Definition 1. *Utility conservation holds if $\bar{u} = \bar{U}$, or equivalently $\bar{S} = 0$, for any vector U.*

To identify zero-sum status situations, denote by $\lambda_i \equiv \sum_k b_{k,i}$ the column-sum of matrix $\mathbf{B}$. It is agent i's "marginal contribution to total utility" since $\lambda_i = \sum_k \frac{\partial u_k}{\partial U_i} = N\frac{\partial \bar{u}}{\partial U_i}$. Similarly, denote $\tilde{w}_i \equiv \sum_{k \neq i} \omega_{k,i} s_k - s_i$. It is related to the column-sum of matrix $\mathbf{A}$. When s_i is constant, $\tilde{w}_i = s\,(w_i - 1)$ where $w_i \equiv \sum_{k \neq i} \omega_{k,i}$ is a measure of the "cumulative relative status envy" that society feels toward agent i. Agents with high w_i attract more status envy from others *ceteris paribus*. Hence, $\tilde{w}_i$ can be thought of as the "weighted cumulative status envy" that society feels towards agent i. Though there is no uncertainty in the model, covariances are useful to characterize average status utility.

Lemma 1. *$\bar{S} = -Cov\,(\tilde{w}_i, u_i) = Cov(\lambda_i, U_i)$.*

Zero-sum status, $\overline{S} = 0$, obtains if either $\tilde{w}_i$ or λ_i are constant for all i. The following proposition describes the specific relationship.

Proposition 5. *The following three statements are equivalent: (i) Utility conservation, (ii) $\tilde{w}_i = 0$ for all i, and (iii) $\lambda_i = 1$ for all i.*

Utility conservation follows for Identical Status societies as $s_i = s$ and $\omega_{i,j} = \frac{1}{N-1}$ give $\tilde{w}_i = 0$. More generally, when $s_i = s$, utility conservation requires the same total envy be directed toward all agents, which is implied, for example, by ω symmetric. When the absolute status intensities

about the existence and equivalence and do not directly describe utility distributions.

vary, utility conservation requires $s_i = \sum_{k \neq i} \omega_{k,i} s_k$, so that agents with high s_i put a greater relative status weight on other agents who also have high absolute status intensities.

Without loss of generality, we assume that an initial allocation is given and order the primary utility of agents $U_1 \leq U_2 \leq ... \leq U_N$. We describe agents $i = 1, ..., N$ as ordered from the least to the most affluent, to loosely make the connection to the well-being literature where the challenge is to explain happiness and income. Lemma 1 allows us to identify patterns for $\tilde{w}_i$ and λ_i that determine the sign of $\overline{S}$. Negative-sum status ($\overline{S} < 0$) obtains if agents' marginal contributions to total utility (λ_i) decrease as their primary utilities (U_i) increase or if the weighted cumulative status envy that society feels towards agents ($\tilde{w}_i$) increases with the utilities of agents (u_i). Whereas primary utilities are assumed to be weakly increasing in i, utilities are endogenous and $u_{i+1} \geq u_i$ does not necessarily obtain. The following proposition shows that, when agents have identical relative status intensities ($\omega_{i,j} = \frac{1}{N-1}$ for $i \neq j$), the sign of $\overline{S}$ can still be determined even if the ranking of u_i is not preserved.[15]

Proposition 6. $\overline{S} \lesseqgtr 0$ *if any of the following three statement holds for all* $i < N$*: (i)* $\tilde{w}_{i+1} \gtreqless \tilde{w}_i$ *and* $u_{i+1} \geq u_i$*, (ii)* $s_{i+1} \lesseqgtr s_i$ *and* $\omega_{i,j} = \frac{1}{N-1}$ *for* $i \neq j$*, or (iii)* $\lambda_{i+1} \lesseqgtr \lambda_i$ *and* $U_{i+1} \geq U_i$.

The literature on reference groups reviewed in Section 2.2 suggests that the evidence about absolute and relative status intensity is at odds with utility conservation. First, the less affluent tend to compare more intensely which suggests that absolute status intensities s_i are decreasing with i. Then $\tilde{w}_i$ are increasing in i *ceteris paribus* (i.e. for identical relative status intensities, $\omega_{i,j} = \frac{1}{N-1}$). Second, individuals tend to compare themselves more intensely to those who are more affluent ($w_{i,j}$ increases with j). This pattern, which suggests that richer agents attract greater cumulative relative status envy, implies that $\tilde{w}_i$ are increasing in i *ceteris paribus* (i.e. holding absolute status weights constant across agents, $s_i = s$). Since $\tilde{w}_i$ are increasing when s_i are decreasing or w_i are increasing *ceteris paribus*, status is negative sum in societies where poorer individuals envy more or where richer individuals are the predominant focus of envy.

3.4 Status-Induced Inequality

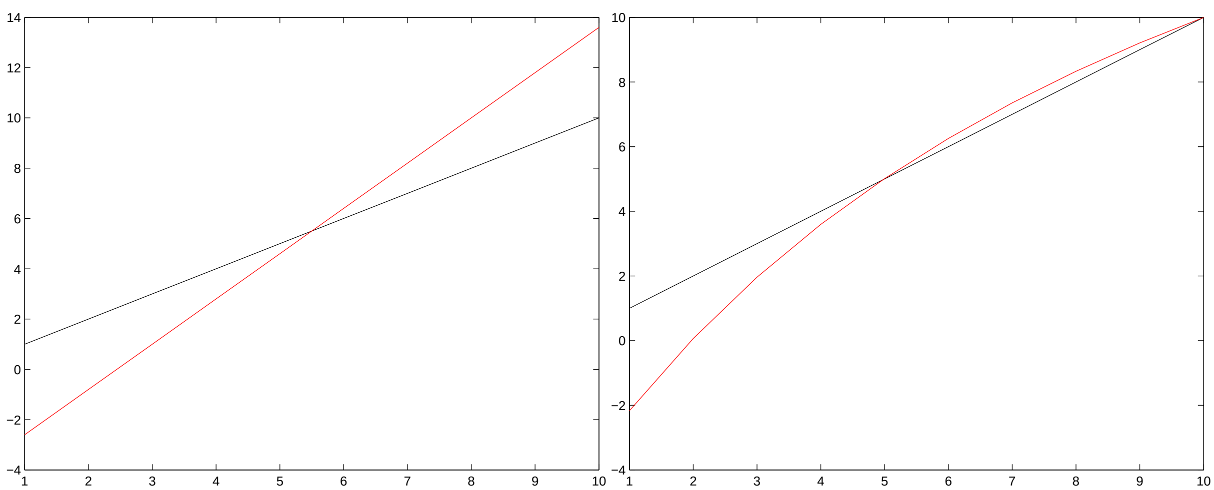
This section explores welfare implications of status that are not due to allocative inefficiencies, an area that has been largely overlooked by the literature. Consumption allocations are unaffected by status utility according to Proposition 3. Thus, status parameters $(\mathbf{s}, \omega)$ do not affect primary utility and can only affect utility through status utility, as $u_i = U_i + s_i S_i$. We focus on two

[15] See Appendix C for an example with identical relative status intensities where the ranking is not preserved. We prove that the utility ranking is always preserved assuming a condition, Assumption 2, which is slightly stronger than $\tilde{w}_i$ increasing; see Lemma 2 in Section 3.4.

questions: (1) When does status decrease average utility in society? and (2) When does status increase inequalities amongst individuals?

Figures 1 and 2 plot the distribution of utility for $s = 0.4$, for a given sets of weights ω and for given vectors of primary utilities. Table 2 summarizes some of the properties displayed in these figures that hold more generally. Taken together, the table and figures show that the distribution of primary utility can be transformed into a rich set of utility distributions.

Figure 1: Primary Utility (U) and Utility (u)



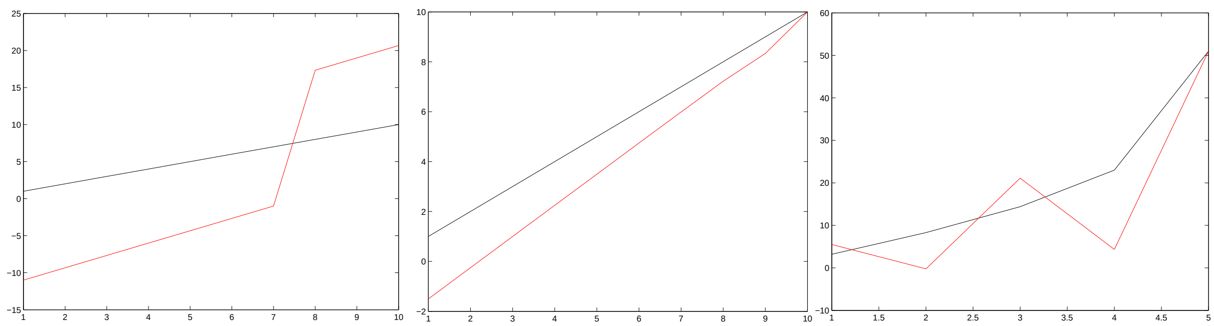
(a) Identical Status: Magnified Inequality and Order Preserved

(b) Heterogenous Weights ($s_i \neq s_j$): Order Preserved

Note: The blue line plots the values of primary utilities and the green line plots the values of utilities. These figures assume $N = 10$ and that primary utility takes values $U_i = i$ for $i = 1, ..., 10$. Figure (a) assumes $s = .4$ and figure (b) assumes $s_1 = .4$, $s_{10} = 0$ and status weights decrease by equal increments.

Figure 1(a) plots the distribution of utility for Identical Agents. Two interesting patterns emerge: (A) The ranking of utilities is preserved (u_i is increasing in i); and (B) Status magnifies utility inequalities (the utility curve is steeper than the primary utility curve). Properties (A) and (B) holds more generally when the weights $(\omega_{i,j})_{i,j}$ satisfy a monotonicity property that is defined shortly. The remaining figures illustrate violations of properties (A) and/or (B). Figure 1(b) assumes that s_i is decreasing with affluence. This attenuates the impact of status and the attenuation effect increases with affluence; property (B) is violated. This property is also violated for One-up Status and All-up Status in Figure 2. In Figure 2 status magnifies inequalities in the case of Two Mutually Envious Groups and All-up Status. Property (A) is violated for One-up Status (right panel). Interestingly, that figure displays a saw tooth pattern where some agents move up in ranking while their neighbors move down.

Figure 2: Primary Utility (U) and Utility (u)



(a) Two Mutually Envious Groups　(b) All-up Status　(c) One-up Status

Note: The blue line plots the values of primary utilities and the green line plots the values of utilities. $s = .4$ in all figures. Figures (a) and (b) set $N = 10$ and primary utility takes values $U_i = i$ for $i = 1, ..., 10$. In figure (a), there are two groups of 3 and 7, respectively. Figure (c) sets $N = 5$ and primary utility takes values corresponding to the 5 quintiles of the U.S. distribution of money income (2012 Census Bureau).

Table 2: Properties of (U, u)

	Magnify Inequalities	Order Preserved
Two Agents	Yes	Yes
Identical Status	Yes	Yes
2-Mutually Envious Groups	Yes within groups	Yes
All-up Status	Not Always	Not Always

In order to examine the role of relative status intensities, we hold the absolute intensity weights fixed $s_i = s > 0$ for all i . Let $W_{i,j} \equiv \sum_{k \geq j} \omega_{i,k}$ denote the cumulated envy that agent i feels toward those who are at least as affluent as agent j. Note that $W_{i,1} = 1$ and that a higher value of $W_{i,j}$ means that agent i's reference group puts more weight on agents who are at least as affluent as agent j. The following monotonicity assumption orders cumulative envy.

Assumption 2. *$s_i = s$ and $W_{i-1,j} \geq W_{i,j}$ for all $(i, j) > 1$.*

This assumption is satisfied by societies with Two Agents, Identical Status, and Two Mutually Envious Groups for $n > N/2$.

Lemma 2. *Under Assumption 2 the ranking of utilities is the same as the ranking of primary utilities: $u_1 \leq u_2 \leq ... \leq u_N$.*

The inequalities are strict when $U_{i-1} < U_i$. The preserved ranking generalizes beyond Assumption 2. Say we compare two agents $i > j$ with identical reference groups $\omega_{i,k} = \omega_{j,k}$ for

$k \neq (i, j)$ and $\omega_{i,j} = \omega_{j,i}$. For these two agents, we have $b_{i,k} = b_{j,k}$ and $b_{i,j} = b_{j,i}$. By Proposition 4 (ii) we have $b_{i,i} > b_{j,i}$, and the utility order is preserved for these two agents

$$u_i - u_j = (b_{i,i} - b_{j,i})(U_i - U_j) \geq 0. \tag{10}$$

The utility order can be reversed when Assumption 2 is violated such that some relatively poor agents envy poorer agents more. For example, in Figure 2(c) the utility order is reversed with One-up Status and when the top agent is significantly above the rest of society.

Corollary 2. *Under Assumption 2, utility inequalities are magnified:*

$$u_h - u_i \geq \frac{1}{1-s}(U_h - U_i) > (U_h - U_i) \ \ for\ U_h > U_i.$$

Identical Status yields the difference: $u_h - u_i = \frac{N-1}{(N-1)(1-s)-s}(U_h - U_i)$. For $N = 2$ the multiplication factor, $\frac{1}{1-2s}$, can be arbitrarily large. Interestingly, we get the same large multiplication factor $\frac{1}{1-2s}$ with Two Mutually Envious Groups when all agents within each group have the same level of primary utility.[16] The proof of Corollary 2, shows that $u_h - u_i$ is composed of a "direct effect", $\frac{1}{1-s}(U_h - U_i)$, which always magnifies utility differences and a "reference group effect", $-\sum_{j=2}^{j=N}(W_{h,j} - W_{i,j})v_j$, which is non-negative under Assumption 2. However, the reference group effect could be negative when Assumption 2 is violated as with One-up Status.

Finally, consider when an agent benefits from status comparisons. The utility definition $u_i - U_i = s\left(u_i - \sum_{j\neq i}\omega_{i,j}u_j\right)$ implies that an agent i benefits from status utility if and only if $u_i > \sum_{j\neq i}\omega_{i,j}u_j$. This occurs when agent i compares predominantly downward so that the relative status weights for agents with higher utility than agent i are sufficiently small. The following statement directly follows from Lemma 2.

Corollary 3. *Under Assumption 2, individual i is better off, $u_i > U_i$, if their reference group is less affluent than individual i, $W_{i,i} = 0$. Conversely, individual i is worse off $u_i < U_i$, if their reference group is more affluent than individual i, $W_{i,i} = 1$.*

The Two Agents economy is a special case where agent 1 loses and agent 2 gains. In the case of Two Mutually Envious Groups, the less affluent group loses and the affluent group gains. The corollary holds even when Assumption 2 fails as long as the utility order is preserved. In the All-up Status example, all but the top agent are absolute losers.

[16]The results in the above corollary generalize beyond Assumption 2. Return to equation (10) which compares two individuals with identical reference groups. As long as $b_{j,i}$ is not too positive, we have $b_{i,i} - b_{j,i} > 1$ and the difference in utility is magnified.

3.5 Maximizing Welfare

Status utility in general affects social optimal allocations. First consider the Utilitarian objective of maximizing the sum of utilities. Without status utility, the Utilitarian planner maximizes $\sum_{i=1}^{N} u_i = \sum_{i=1}^{N} U_i$ subject to being on the Pareto frontier in primary utility PF^U. The planner chooses a point on PF^U which trades off primary utility at rate 1 between agents such that $MRT^U_{j,i} = -\frac{dU_i}{dU_j}\Big|_{PF^U} = 1$ for all pairs of agents i and j.[17] In contrast, in an economy with status utility, the planner maximizes $\sum_{i=1} u_i = \sum_{i=1} \lambda_i U_i$ subject to PF^U. Recall, $\lambda_i \equiv \sum_k \frac{\partial u_k}{\partial U_i}$ is individual i's marginal contribution to total utility. Thus, the planner chooses a point on PF^U that considers the external effects of each agent's utility on other agents:

$$MRT^U_{j,i} = \frac{\lambda_j}{\lambda_i}$$

for all pairs of agents i and j. Unless the marginal contributions of all agents to total utility are equal, $\frac{\lambda_j}{\lambda_i} = 1$ for all i and j, status utility will affect the optimal allocation. This requirement of equal marginal contributions holds under utility conservation in Proposition 5. The following proposition shows that without utility conservation status utility matters.

Proposition 7. *The Utilitarian optimal allocation is unaffected by status utility if and only if utility conservation holds.*

When utility conservation does not hold, a Utilitarian planner allocates more to agent j the greater is their marginal contribution λ_j. Consider the Two Agents economy with identical primary utility functions. The Pareto frontier PF^U is symmetric around the 45 degree line. Without status utility, the Utilitarian planner picks the egalitarian outcome $U_j = U_i = U^*$ on PF^U. Now suppose agent j compares to i ($s_j > 0$) but agent i does not compare to j ($s_i = 0$). Then $\frac{\lambda_j}{\lambda_i} = \frac{1}{1-2s_j} > 1$, and the planner allocates $u_j > U_j > U^* > U_i = u_i > 0$. Ironically, it is the status conscious agent j that is allocated more and achieves higher utility. Agent j has a larger marginal contribution because they imposes less of a negative externality. The planner puts them on top so they can look down on the poorer agent i and "count their blessings". This generates positive-sum status utility, and average utility exceeds the egalitarian utility, $\overline{u} = \frac{u_i+u_j}{2} > U^*$.

Extreme inequality can be the consequence of incorporating status preferences into the Utilitarian calculus. Suppose all agents have identical primary utility functions, and consider Two Mutually Envious Groups where agent 1 faces a group of all other agents $k \geq 2$. Then

[17] This allocation of primary utility can be achieved by an allocation of goods that is not explicitly modeled here. Recall that PF^U is assumed to be downward sloping.

$\lambda_1 = \frac{1-Ns}{(1-2s)}$. Let $s \geq \frac{1}{N}$, so that and $\lambda_1 \leq 0$, to focus on the corner solution where the planner allocates agent 1 zero consumption and hence zero primary utility $U_1 = U(0,0) = 0$. The impoverished agent 1 suffers negative utility as they compare to the majority; $u_1 = S_1 = -\frac{s}{(1-2s)}U_k < 0$. Ironically, maximizing intergroup inequality gives positive-sum status utility, $\overline{S} > 0$, and average utility exceeds the egalitarian utility, $\overline{u} > U^*$. The affluent majority is better described as not counting their blessings so much as spitefully enjoying the poverty and enmity of the minority. This draconian prescription for inequality shows vividly that status preferences can lead to the classic equity critique of Utilitarianism (Sen 1973), where the Utilitarian criterion is criticized for unfairly allocating less resources to those whom are poorer (primary) utility generators. However, in our model, the inequality arises from the asymmetry in other regarding behavior.

We now consider a welfare criterion which places extra weight on the utilities of the less well off. In particular, consider the extreme case of a maximin planner who maximizes the minimum utility amongst all agents. The following proposition holds even when individuals do not have the same utility functions. As before, U^* is the egalitarian primary utility on PF^U.

Proposition 8. *Maximizing the minimum utility amongst all agents implies an egalitarian utility allocation $u_i = U_i = U^*$ for all i.*

Unlike the Utilitarian allocation, the maximin allocation precludes status differences. This is optimal because the worst off agent does not benefit from status comparisons. Compared with allocations in economies which generate negative-sum status utility, the egalitarian utility allocation boosts average utility in the status component. It also weakly maintains the rank ordering in primary utility between any pair of agents. Moreover, it preclude conspicuous consumption and spiteful transfer behavior. These arguments for progressive taxation are different from, although complementary to, the case based on decreasing marginal utility of wealth.

4 Applications

This section develops six applications of our pure theory of status utility which offer new perspectives on some central features of the status and happiness literatures. Sections 4.1 and 4.2 marshal evidence on relative income and good comparisons from the econometric and experimental status literatures. Sections 4.3 and 4.4 resolve paradoxes. Section 4.3 argues that empirical basis for the invidious comparison-efficiency paradox still remains since Arrow and Dasgupta (2009) review of the literature. Section 4.4 shows that extreme utility distributions in our model can generate the Easterlin paradox without requiring inefficiency or satiation. Sections 4.5 and 4.6 generate behavior reminiscent of Veblen; non-satiation in status utility results in the very rich destroying or transferring resources to spite others.

4.1 Invidious Pecuniary Comparisons

The term ("invidious") is used in a technical sense as describing a comparison of persons with a view to rating and grading them in respect of their relative worth of value ... with which they may legitimately be contemplated by themselves and by others.

Veblen (1899, p. 34)

Proposition 4's implication that people compare themselves to some reference group and do so negatively in some average sense ($\sum_{j\neq i} b_{i,j} < 0$) goes a long way back in social science. In Veblen (1899)'s rich sociological description, invidious comparison is the mechanism that drives an evolutionary dynamic of differentiation and emulation. Duesenberry (1949) also invokes the maintenance of self-esteem as a basic drive which is achieved through what he coined a "demonstration effect" to describe the greater frequency of exposure to higher living standards as leading to greater consumption. More recently, the role of relative income has been widely examined at the individual level by explaining reported happiness (subjective well-being) with own income and the income of a reference group. Starting from a utility function that is separable in absolute and relative income, Clark et al. (2008) motivate the following log-linear structural specification that they use in their survey:

$$u_{i,t} = \beta_1 \ln(y_{i,t}) + \beta_2 \left[\ln(y_{i,t}) - \ln(y_{-i,t})\right], \tag{11}$$

where $u_{i,t}$ is a measure of the "happiness" of agent i at time t, $y_{i,t}$ is the income of agent i, $y_{-i,t}$ is average income of i's reference group. Equation (11) matches the Identical Status equation (7) once one associates primary utility to the logarithm of income and utility to happiness, $U = \ln(y)$ and set $\beta_1 + \beta_2 = 1 - b$ and $\beta_2 = b$.

The reduced form estimate of own income usually enters significantly positively, $\beta_1 + \beta_2 > 0$, in explaining happiness. Most studies in developed countries find that the coefficient on reference income is significantly negative, $-\beta_2 < 0$. Whereas this literature does not establish causality, the presumption is that intense comparisons reduce well-being. A number of studies find that the coefficient on reference income is of the same magnitude as own income, that is, $\frac{\beta_2}{\beta_1+\beta_2}$ is close to one.[18] Other studies find that relative income partially offsets income, $\frac{\beta_2}{\beta_1+\beta_2} \in (0,1)$, which is interpreted as evidence that income is positional.[19] We argue that the evidence is also

[18] Luttmer (2005), Ferrer-i Carbonell (2005), Knight et al. (2009), Senik (2009), Helliwell and Huang (2010), Layard et al. (2010).

[19] Arrow and Dasgupta (2009) use $\frac{\beta_2}{\beta_1+\beta_2} = \frac{1}{3}$, citing Blanchflower and Oswald (2004) analysis of American GSS data. Clark et al. (2008, p. 111) settle on the higher figure of $\frac{2}{3}$. Praag and Ferrer-i Carbonell (2004) summarize several studies associated with the Leyden School in which income comparisons offset up to 80% of the individual

consistent with our model of status utility where it is utility that is positional. This distinction is important as allocations are efficient with Identical Status. The ratio of the reference group to own effect, $\frac{b}{b+1} = \frac{(N-1)s}{N-1-s}$, can take any value in $(0,1)$ as $s < \frac{N-1}{N}$. This covers the entire range of values estimated for $\frac{\beta_2}{\beta_1+\beta_2}$ from equation (11).[20]

4.2 Positional Concerns

The existence of positional concerns has been vividly revealed by experimental surveys. In their widely cited experiment, Solnick and Hemenway (1998) ask individuals to choose between two worlds, one where "[y]our current yearly income is $50,000; others earn $25,000 ", and another where "[y]our current yearly income is $100,000; others earn $200,000." An individual is said to have a positional preference for income if they prefer the former world, which is often the case. However, when the same questions are about leisure, offering the choice between 2 weeks of vacation when others have 1 week versus 4 weeks when others have 8 weeks, individuals typically prefer the latter non-positional world.

Johansson-Stenman et al. (2002) measure positional concerns using utility specification $\tilde{U}(x,\bar{x}) = x^{1-\gamma}\left(\frac{x}{\bar{x}}\right)^{\gamma}$, where x is own consumption, $\bar{x}$ is others' per capita consumption, and the exponent $\gamma \in [0,1]$ is interpreted as good x's level of positionality. Good x is non-positional when $\gamma = 0$ and positional with full offset when $\gamma = 1$. Denote the positional and absolute distributions used in surveys, $((x_p,\bar{x}_p),(x_a,\bar{x}_a))$, with $\bar{x}_a > x_a > x_p > \bar{x}_p$. A consumer is said to have "level of positionality" γ_x for good x if the consumer is indifferent between the two distributions, $\tilde{U}(x_a,\bar{x}_a) = \tilde{U}(x_p,\bar{x}_p)$. The median range for the level of positionality for income reported in Johansson-Stenman et al. (2002) is $\gamma_x \in [.2,.5]$, which has been confirmed in subsequent studies (Alpizar et al. 2005, Carlsson et al. 2007). These studies also estimate γ_x over a variety of goods and find significant differences in the levels of positionality across goods. Carlsson et al. (2007) cannot reject $\gamma_x = 0$ for leisure, and Solnick and Hemenway (2005) conclude that leisure is a much less positional good than income.

Our model can explain what appear to be different positional concerns without requiring different positional coefficients (γ) for each good. Primary utilities depend only on the level of x, and the positional and absolute distributions respectively are $(U(x_p),U(\bar{x}_p))$ and $(U(x_a),U(\bar{x}_a))$. Using these primary utilities in the utility system, $u(x,\bar{x}) = b_{i,i}U(x) - (b_{i,i}-1)U(\bar{x})$, an agent

effect. On the opposite end of the debate, Deaton and Stone (2013) examine Gallup surveys and find evidence for relative income negatively influencing happiness, but only for a measure of hedonic well-being, and not for a measure of evaluative well-being.

[20] In equation (11) the reference group benchmark is the logarithm of average income, $\ln y^*$; whereas, in our model with log income it is the average of log incomes, $U_i^* = \frac{1}{N-1}\sum_{j\neq i}\ln y_j$. Only if $y_j = y^*$ for all j can we directly back out an estimate of s. In this case, a ratio $\frac{\beta_2}{\beta_1+\beta_2} = \frac{2}{3}$ would imply $s = .4$ for N large.

prefers the positional distribution if and only if $u(x_p, \bar{x}_p) > u(x_a, \bar{x}_a)$ or

$$\frac{b_{i,i}-1}{b_{i,i}} > \frac{U(x_a)-U(x_p)}{U(\bar{x}_a)-U(\bar{x}_p)}.$$

Here $\frac{b_{i,i}-1}{b_{i,i}} \in (0,1)$ as $b_{i,i} > 1$. Thus, the positional over the absolute distribution is preferred when $U(x_a) - U(x_p)$ is small relative to $U(\bar{x}_a) - U(\bar{x}_p)$; that is, the own primary utility gain is small relative to others' primary utility gain. The condition is satisfied as x_p approaches x_a, and it fails as $x_a - x_p$ approaches $\bar{x}_a - \bar{x}_p$.

To explain the example from Solnick and Hemenway (1998), we show that the above condition can be satisfied by income while at the same time failing for vacations. An individual prefers the positional distribution for income (interpreted as consumption on the RHS) and the absolute distribution for vacations (interpreted as leisure on the LHS) if

$$\frac{U(c,4)-U(c,2)}{U(c,8)-U(c,1)} > \frac{b_{i,i}-1}{b_{i,i}} > \frac{U(100000,l)-U(50000,l)}{U(200000,l)-U(25000,l)},$$

where the primary utility function is $U(c,l)$. Consistent with the literature, assume that $U(c,l)$ is additively separable so that the endowment of the other good, c or l, does not affect the choice over the quantity of the good that is being varied.[21] Using the benchmark Identical Status case gives the specific threshold $\frac{b_{i,i}-1}{b_{i,i}} = \frac{(N-1)s}{N-1-s}$ where $s < \frac{N-1}{N}$. Now the inequalities hold for many values of s and N when $U(c,l)$ has relatively rapid diminishing marginal utility to vacationing.[22] Here the survey evidence is consistent with invidious utility comparisons without resort to asymmetrical positional preferences over goods.

4.3 The Invidious Comparison - Efficiency Paradox

The efficiency consequences of invidious comparisons are difficult to pin down on theoretical or empirical grounds. As discussed in the introduction, the status literature largely presumes that intense status comparison should inevitably lead to large allocative distortions. Yet, Arrow and Dasgupta (2009) do not find that the empirical literature directly supports this presumption.

[21] In the literature respondents are asked to choose between worlds where only one good is varied without mentioning other goods or mentioning whether others in the hypothetical world know the respondents relative position. When utility is additively separable, the the endowment of other goods does not affect the choice over the quantity of the good being varied.

[22] For example, the inequalities hold when $N = 1000$ for $U(c,l) = c + ln(l)$ with $s \in (0.286, \frac{1}{3})$, or for $U(c,l) = \frac{c^{1-\eta}-1}{1-\eta} + \min[4-l, 0]$ with $s \in (\frac{1}{3}, \frac{2}{3})$ and $\eta \in (0,4)$. The labour supply is increasing and large at a high wage in these examples. We are not aware of positional experiments directly on the labour leisure trade off. Interestingly, Alvarez-Cuadrado (2007) calibrates a model where consumption is positional and finds that when positionality is removed labour hours per week fall from 34 to 21 in his baseline case.

Their explanation is that allocative distortions cancel out when all goods are positional with symmetric offsetting externalities. In contrast, our model does not rely on positional preferences over goods while allowing substantial asymmetries in positional concerns.

Of course, very strong positional concerns on some visible goods (e.g. Charles et al. 2009, Heffetz 2011, Friehe and Mechtel 2014) such as luxury goods (Frank 2010) and cars (e.g. Grinblatt et al. 2008, Kuhn et al. 2011, Winkelmann 2012) is hard to comprehend without appealing to positional preferences on select goods. We acknowledge that our model is not the right one for these goods and that future work incorporating asymmetric information into our model is needed to capture these specific positional concerns. However, we believe that the major margins for potentially large distortions are around labor-leisure and inter-temporal savings choices. Clark et al. (2008) indicate that leisure or labor should be examined, as a third argument in an extension of equation (11), as evidence for losses from excess labor supply. However, they do not cite, nor were we able to find, any estimates of welfare losses in that literature.[23] It is difficult to estimate such losses given the complexity of labor supply (Clark et al. 2012).

The case for saving distortions is stronger, although not definite. After taking into account endogenous reference group effects, Maurer and Meier (2008) find modest externalities in their lifecycle analysis of PSID data. Alvarez-Cuadrado et al. (2015) also estimate a lifecycle model. Using Spanish data, they find a substantial consumption externality when the reference group is the census tract but find no externality with a socio-economic reference group (as in Maurer and Meier (2008)). Bertrand and Morse (2016) and Frank et al. (2014) examine US data and find evidence for "expenditure cascades", also called "trickle down effects", whereby increases in expenditures (or earnings) of those at the top of the income distribution increases the expenditures of those below. Direct evidence for widespread allocation distortions along important dimensions is sensitive to the specification of reference groups and the difficulty of establishing causation and ruling out other hypotheses. Without more analysis, the puzzle of intense invidious comparisons without necessarily large allocation distortions remains.

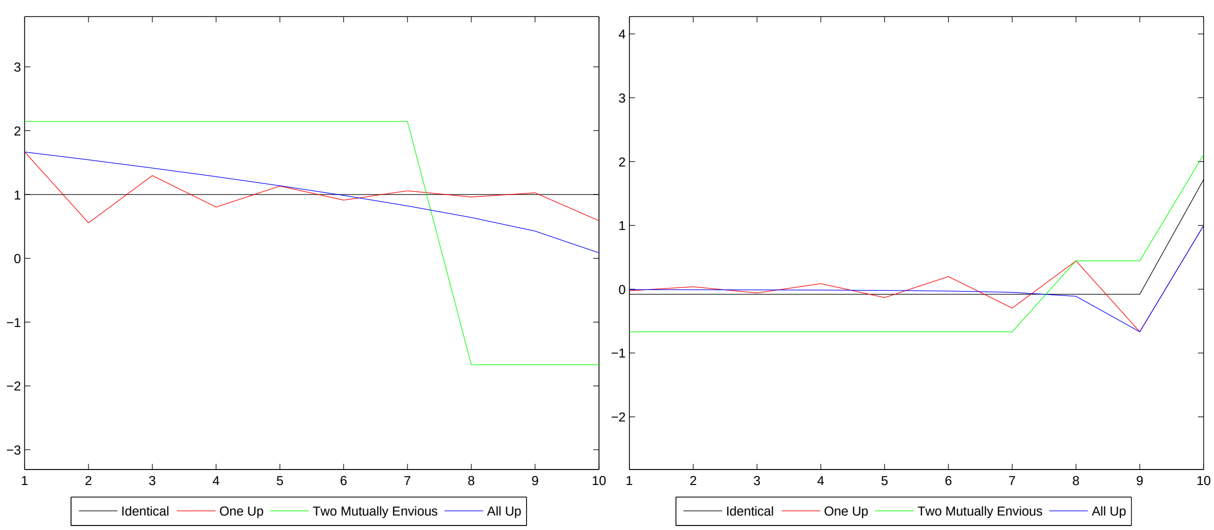
4.4 Pro-Rich Growth, Negative-Sum Status and the Easterlin Paradox

In examining international evidence, Easterlin (1974, 2013) finds that happiness increases with income in a cross section of individuals at a point in time in a country, but paradoxically income growth does not increase average happiness over time within the country. In our model, pro-rich growth, which increases income more at the top of the income distribution, can be associated with a negligible or negative impact on average happiness. For example, an increase

[23] Alvarez-Cuadrado (2007) offers calibration evidence of substantial inefficiencies assuming that one-half of consumption has no effect because it is status related.

in everyone's primary utility in the sense of stochastic dominance may have a negligible or even negative impact on average utility. This is illustrated in Figure 3a, which plots the values of λ_i for the four parametrized economies detailed in Table 5. (Recall $\lambda_i = \sum_k \frac{\partial u_k}{\partial U_i}$ is agent i's marginal contribution to total utility.) In the case of Two Mutually Envious Groups, increasing the primary utility of any of the top three agents reduces average utility ($\lambda_i < 0$ for $i \geq 8$)! Pro-rich growth can also be negative sum when status intensity decreases with affluence (see Figure 1(b)). With All-up Status, increasing the primary utility of the most affluent agent has a negligible impact on average utility ($\lambda_{10} = .08 \ll 1$). These examples with negative-sum status utility suggest a"pro-rich growth" explanation for the Easterlin Paradox (Clark et al. 2008) that does not rely on features like adaption, aspirations, concave utility, or allocative inefficiencies.[24] Unlike the relative income hypothesis, our explanation requires status preference asymmetries where, consistent with the evidence (see Sections 2.2 and 3.3), the rich on average are envied more than others. Without status preference asymmetries, the model delivers utility conservation so that the average increase in utility is equal to the average increase in primary utility. Our analysis is a caution against the common practice of averaging income and happiness within a group or country which ignores potentially important heterogeneous interactions.

Figure 3: Marginal Contributions to Total Utility (λ_i) and Agent N's Marginal Effect on Agent i ($b_{i,N}$) for Four Status Functions



(a) λ_i for $i = 1, ..., 10$

(b) $b_{i,N}$ for $i = 1, ..., 10$

Note: The figures assume $N = 10$, $s = .4$.

To illustrate the cross-sectional impact of increasing the affluence of the top cohort on the

[24] In contrast with pro-rich growth, equal increases in primary utilities, implies that each agent's utility increases one-for-one with primary utility (see discussion following Proposition 4(iii)).

Table 3: Marginal Contributions to Total Utility (λ_i) and Average Status Utility ($\overline{S}$)

	λ_i	$\overline{S} < 0; \overline{S} = 0; \overline{S} > 0$
Identical Status	1	Zero Sum
Two Agents	$\frac{1-2s_i}{(1-s_1)(1-s_2)-s_1 s_2}$	$s_1 > s_2$; $s_1 = s_2$; $s_1 < s_2$
2-Mutually Envious Groups	$(1-2s)^{-1}\left(1 - \frac{s}{1-n/N}\right)$ for $i > n$	$n > \frac{1}{2}$; $n = \frac{1}{2}$; $n < \frac{1}{2}$
All-up Status	$\lambda_n = \lambda_{n-1}\left(1 - \frac{s/(1-s)}{N-n-1}\right)$	Negative Sum

happiness of each cohort, Figure 3b plots $b_{i,N} \equiv \frac{du_i}{dU_N}$ for all i, where the top agent is $N = 10$. Consistent with Proposition 4, the impact on the top agent is positive (own effect $b_{N,N} \geq 1$) and strongest in magnitude ($b_{N,N} > |b_{i,N}|$ for $i < N$). Also, the utility inequality between the rich and the rest increases ($b_{N,N} - b_{i,N} > 1$ typically). Beyond this, the figure reveals complex patterns on the happiness in the rest of society. With Identical Status all other agents are hurt a little. The One-up Status and All-up Status cases display ripple effects. With Two Mutually Envious Groups there is a trickle down pattern (Bertrand and Morse 2016). These patterns display magnified utility inequality and utility losses for the non-rich. They suggest why income growth in the past few decades, which have largely benefited the richest in society, appear to have given rise to widespread discontent among the non-rich.

4.5 "Waste" from Insatiable Status

A major insight in economics is that selfish agents interacting in competitive markets may achieve an efficient outcome. Propositions 2 and 3 imply that this result generalizes to status conscious agents. Here we show that efficient outcomes may continue to obtain even if a subset of agents are sufficiently rich to be satiated in own primary utility, i.e. satiated in both consumption and leisure. Efficiency requires behavior that seems wasteful because it involves either consumption beyond satiation or the disposal of resources.[25] This miserly behavior is the consequence of the maintained assumption that agents are never satiated in status utility. Now, the equivalence between models as described in Proposition 2 fails, because efficiency in the standard model without status involves distributing some goods to those agents who are not satiated.

Consider the Two Agents example, but where agent 2 is satiated in primary utility but not status utility. Then, the frontier of the primary utility possibility set, in the positive quadrant, is flat up to the point where agent 2 stops being satiated. With $s_2 > 0$ we have $b_{2,1} < 0$ so that the frontier of the utilities possibilities set is downward sloping. Any allocation that achieves a point on the flat segment of the primary utility frontier corresponds to a point on the

[25] Brennan (1973) finds that waste may be efficient with envy, although never unilaterally.

downward sloping utility frontier. Thus, consuming beyond the threshold for primary utility satiation, increases the utility of agent 2 as it deprives agent 1 of consumption. The example generalizes to costly disposal and many agents. Agent 2 has a strong incentive to deprive agent 1 from consuming. With diminishing marginal primary utility, the marginal negative effect of agent 1 on agent 2 is greatest when agent 1 is consuming least. Thus, agent 2 would be willing to incur at least a small cost to dispose of extra resources to deprive agent 1 of them. But, such willful waste by standard sensibilities can be thought of as "conspicuous consumption" and "conspicuous waste". Willful waste could take the form of lavish gift-giving and partying as long as the activity were strictly in the company of the consumption satiated.

Gift giving becomes spiteful when a consumption satiated agent i benefits from a subset of other agents j doing well, i.e. $b_{i,j} > 0$. At the margin there is zero cost to giving to agents j. Thus, some resources are gifted to at least one non-satiated agent j. It is easy to imagine that gifts to the enemy-of-my-enemy might resemble Veblen's "vicarious consumption" where, for example, paying associates and underlings extravagantly invokes the envy of rivals and their entourages. Whereas satiable consumption is sufficient for gift giving to indirectly spite others, such behavior can also occur even with nonsatiation as we analyze next.

4.6 Spiteful Transfers

In the standard model, without status utility, the core of the economy is equivalent to the set of competitive allocations. However, this core-equivalence result may fail in our model when a subset of agents can increase all members utilities through a redistribution among themselves. Since no Pareto improvement is possible, the utility of at least one agent in the rest of the economy has to decrease. We assume nonsatiation so that the marginal rate of transformation along the primary utility Pareto frontier is positive, $MRT^U_{i,j} = -\frac{dU_i}{dU_j}\Big|_{PF^U} > 0$.

Consider a transfer from one agent, agent i, to another agent, agent j. Equation 6 identifies the cross primary utility benefit to agent i of agent j receiving a unit primary utility transfer, $dU_j = 1$, to be $b_{i,j}$. Agent i gives only if $b_{i,j} > 0$; agent i is able to spite others by making agent j better off (see Section 3.2 and footnote 12). The cost for agent i to make such a transfer is $b_{i,i}MRT^U_{i,j}$. Thus, agent i is better off giving when $b_{i,j} > b_{i,i}MRT^U_{j,i}$ or, equivalently, $\frac{b_{i,j}}{b_{i,i}} > MRT^U_{j,i}$. The ratio $\frac{b_{i,j}}{b_{i,i}}$ has the interpretation (from equation 6) as agent's i marginal rate of substitution between own and agent j's primary utility, $MRS_{j,i} = -\frac{dU_i}{dU_j}\Big|_{du_i=0} = \frac{b_{i,j}}{b_{i,i}}$. A mutually agreeable transfer of resources from agent i to agent j exists if and only if $MRT^U_{j,i} < MRS_{j,i}$.[26]

[26]The recipient, agent j, benefits since $du_j = b_{j,j}dU_j + b_{j,i}dU_i > 0$ or, equivalently, $1 > \frac{b_{j,i}}{b_{j,j}}(-\frac{dU_i}{dU_j})$; this inequality holds because $\frac{b_{j,i}}{b_{j,j}} < 1$, by Proposition 4(ii), and the transfer rate is $(-\frac{dU_i}{dU_j}) \leq MRS_{j,i} < 1$.

The inequality $MRT^U_{j,i} < MRS_{j,i}$ says that agent i will give a gift when the feasible rate of transferring primary utility is less than the subjective threshold rate that makes agent i no worse off. The inequality is satisfied when $MRT^U_{j,i}$ is sufficiently small and $b_{i,j} > 0$ so that agent i is able to spite others outside the group by agent j doing better. The inequality is only satisfied if $MRT^U_{j,i} < 1$, since $MRS_{j,i} < 1$ from $b_{i,i} > b_{i,j}$ in Proposition 4(ii). Thus, if all agents have the same primary utility functions, agent i would have to be richer than agent j. There would be no bilateral transfers in an egalitarian society.

We now show that multilateral redistributions can readily occur between agents within a group of any size. Consider the Two Mutually Envious Groups economy in Table 1, where the size of the first group is $n < N$. Let n be an even number so that we can divide $\frac{n}{2}$ with i's in one subgroup and j's in the other subgroup. Suppose that each pair (i, j) has the same $MRT^U_{j,i} < 1$. Then an equal transfer $dU_i = -MRT^U_{j,i} dU_j < 0$ from i to j for all pairs (i, j), increases the representative agent i's utility if and only if the benefit exceeds the loss. The benefit is $\frac{n}{2} MRS_{j,i}$, where $MRS_{j,i} = \frac{s^2}{s^2+n(1-2s)}$. There is a direct loss from i giving of $MRT^U_{j,i}$ and an indirect loss associated with the other $\frac{n}{2} - 1$ givers. The representative giver benefits when $\frac{n}{2} MRS_{j,i} > MRT^U_{j,i} \left(1 + (\frac{n}{2} - 1) MRS_{j,i}\right)$, which implies

$$\frac{MRT^U_{j,i}}{1 - MRT^U_{j,i}} < \frac{s^2}{2(1-2s)}. \tag{12}$$

This condition obtains for $MRT^U_{j,i}$ small enough, independent of group size. Though the redistribution reduces primary utility inequality within the first group, it increases their overall primary utilities in a way that draws greater envy from the members of the second group. Enjoying the hurt induced by this envy makes all members of the first group better off.

5 Conclusion

This paper develops a hedonic theory that combines consumer utility, representing economic payoffs, with what we call status utility, capturing the social status payoff. An individual derives status utility from comparing their cardinal utility to others. Utility itself is taken as the logical metric for social status comparisons, because the hedonic approach takes maximizing utility as the ultimate end. In this paper, we have examined the pure version of theory, where individuals accurately infer the utilities of those in their reference group.

The pure hedonic theory can be simply operationalized in straightforward cases, in which errors in economic data and subjective well-being perceptions are unbiased. The resulting reduced form analyses are observationally equivalent to benchmarks studied in the applied literature on relative income and positional concerns. Though status utility is consistent with these benchmarks, it does not imply the presence of status goods. In contrast, the status literature is quite scattered in locating status good preferences as interpersonal comparisons over different actions and objects.

Our model is consistent with standard economic theory in ways that help explain puzzles associated with status, interpersonal comparisons, conspicuous and positional consumption, and relative income. Comparisons of utility in our model do not change competitive equilibrium allocations or generate allocative distortions. As we have reviewed, the empirical evidence is weak on status comparisons inducing large allocation inefficiencies. In contrast, there is strong evidence that status comparisons have a very large negative impact on subjective well-being. Our theory provides an explanation for this invidious comparison - efficiency paradox. This paradox challenges economists to either reject one of its two empirical premises or subscribe to a theory that reconciles them as we do here.

We attribute the discontent with status comparisons to the magnification of utility inequality and to the reduction in average utility. The increase in utility inequality is a general feature of our model, whereas average utility declines only when upward comparisons prevail as suggested by surveys we reviewed. This demonstrates that the standard assumption of identical status made in the literature on interpersonal comparisons is not innocuous. For example, we show that an increase in average income that accrues to the most affluent, does not have to be associated with an increase in average happiness as is commonly argued in the controversial debate on the Easterlin paradox. More generally, we show that status comparisons can have complex dispersion effects with trickle down and ripples across society. When comparisons are mostly upwards, policies that reduce inequality are welfare increasing. Our analysis is counterpoint to the focus in the status literature on the distortionary consequences of consumption externalities, and on the assumption that status comparisons are identical across agents.

In this paper, utility is upheld as the ultimate hedonic comparison. There is some direct evidence for hedonic comparisons but more investigation is needed. If nature or culture instrumentally pick status tournaments to minimize the allocative cost of status comparisons, then relative utility is a leading contender. The signaling of status through relative or positional consumption remains to be explored within asymmetric information extensions of our model. As culture and information change, so do individual responses and social norms. Facebook "friends" and Twitter "hearts" are examples of new media technologies that enable self-promotion and social comparison. Such social media appear to be leading rapidly changing social norms. Modeling cultures of conspicuous sociability as signaling "conspicuous happiness", if you will, remains for future research.

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Appendices

A Notation

Table 4: Notation

$i = 1, ..., N$	agents in the economy
u_i	agent i's utility
U_i	agent i's primary utility
S_i	agent i's status utility
s_i	agent i's absolute status intensity
$\omega_{i,j}$	agent i's relative status intensity for agent j
$\mathbf{A} = [a_{i,j}]$	matrix that transforms $U = \mathbf{A}u$
$\mathbf{B} = [b_{i,j}]$	matrix that transforms $u = \mathbf{B}U$
PF^U	Pareto frontier in primary utility (same for PF^u)
$MRT^U_{j,i}$	marginal rate of transformation $-\frac{dU_i}{dU_j}$ along PF^U (same for $MRT^u_{j,i}$)
$MRS_{j,i}$	$-\frac{dU_i}{dU_j}\Big\vert_{du_i=0} = \frac{b_{i,j}}{b_{i,i}}$
$\bar{S}$	$\bar{u} - \bar{U} = \frac{1}{N}\sum_i S_i$
ω_i	$\sum_{j \neq i} \omega_{j,i}$
λ_i	$\sum_k b_{k,i}$
$W_{i,j}$	$\sum_{k \geq j} \omega_{i,k}$

B Proofs

Proof of Proposition 1: (i) From utility system equations (9), define the functions

$$f_i(u) \equiv u_i - F_i\left(U_i, u_i - \sum_{j \neq i} w_{i,j} u_j\right)$$

for all $i = 1, ..., N$, where $f_i(u)$ is a C^1 function because $F_i(U_i, S_i)$ is a C^1 function and $S_i = u_i - \sum_{j \neq i} w_{i,j} u_j$. For any exogenous vector U, a solution to system (9) must satisfy system $f_i(u) = 0$ for all $i = 1, ..., N$. According to Hadamard's theorem (Ortega and Rheinboldt 1970), a unique solution to this system exists if there are positive numbers (h, κ) such that

$$|\det(\mathbf{J}^f(u))| \geq h > 0 \;\; and \;\; \left|\frac{\partial f_i(u)}{\partial u_j}\right| < \kappa,$$

where $\mathbf{J}^f(u)$ is the Jacobian matrix of partial derivatives with components $\mathbf{J}^f_{i,j} = \frac{\partial f_i(u)}{\partial u_j}$. Here $\mathbf{J}^f(u) = \mathbf{I} - \mathbf{F_S}(u)(\mathbf{I} - \omega)$ where $\mathbf{I}$ is the identity matrix, $\mathbf{F_S}(u)$ is the diagonal matrix with elements $\frac{\partial F_i}{\partial S_i}$, and ω is a matrix with diagonal elements 0 and off diagonal elements $\omega_{i,j}$. (Note that $\mathbf{J}^f(u) = \mathbf{A}$ for the linear case $u_i = U_i + s_i S_i$.) For notational simplicity let $F_S^i \equiv \frac{\partial F_i}{\partial S_i}$. Under assumption $F_S^i \leq \sigma$ for all $i = 1, ..., N$, where $\sigma \in \left(0, \frac{1}{2}\right)$, each diagonal element of matrix $\mathbf{J}^f(u)$ is greater than the row sum of the off diagonal elements, $1 - F_S^i > F_S^i \sum_{j \neq i} \omega_{i,j} = F_S^i$. Thus, matrix $\mathbf{J}^f(u)$ is strictly row diagonally dominant, so that $1 - 2F_S^i \geq 1 - 2\sigma > 0$ for all i. Equation (2) in Ostrowski (1952) gives a positive bound for its determinant:

$$|\det(\mathbf{J}^f(u))| \geq \Pi_{i=1,..,.N}(1 - 2F_S^i) \geq (1 - 2\sigma)^N \geq h > 0.$$

Take the second condition. For all partial derivatives, we have

$$\left|\frac{\partial f_i(u)}{\partial u_j}\right| \leq F_S^i \leq \sigma < \frac{1}{2} \leq \kappa.$$

Thus, Hadamard's theorem applies, and for any vector of primary utilities U there exists a unique solution to the system $f_i(u) = 0$ for all $i = 1, ..., N$. It follows that each agent i's utility is a well-defined function of primary utilities $u_i = u_i(U)$.

(ii) The above implies $f_i(u(U)) = 0$ for all $i = 1, ..., N$, where $u(U) = (u_1(U), ..., u_N(U))$ is the vector of utilities. Taking the total differential of each function, assuming for now that $u(U)$ is C^1, and arranging in matrix form gives

$$\mathbf{J}^f(u)\mathbf{J}^u(U) = \mathbf{F}_U,$$

where $\mathbf{J}^u(U)$ is a matrix with elements $\mathbf{J}^u_{i,j} = \frac{du_i}{dU_j}$, and $\mathbf{F}_U$ is a diagonal matrix with elements $[\mathbf{F}_U]_{i,i} = F_U^i \equiv \frac{\partial F_i}{\partial U_i}$. As described above, $\mathbf{J}^f(u)$ is a non-negative (Jacobian) matrix that is strictly row diagonally dominant with positive diagonal terms. Such matrices are part of the class of P-matrices which have well-defined inverses and positive principal minors (see Generating Method 4.1 in Tsatsomeros (2002)). Thus, $\mathbf{J}^u(U) = (\mathbf{J}^f(u))^{-1}\mathbf{F}_U$, where $\mathbf{J}^u(U)$ is continuous because both $(\mathbf{J}^f(u))^{-1}$ and $\mathbf{F}_U$ are continuous. Isolating the total own effect for agent i gives

$$\frac{du_i}{dU_i} = \left[(\mathbf{J}^f(u))^{-1}\right]_{i,i} F_U^i = \frac{\det \mathbf{J}^f_{-i,-i}}{\det \mathbf{J}^f(u)} F_U^i,$$

where $\mathbf{J}^f_{-i,-i}$ denotes the principal submatrix obtained by removing the i^{th} row and column from $\mathbf{J}^f(u)$. As the principal minors are positive and agents are never satiated in primary utility, the total own effect is unambiguously positive $\frac{du_i}{dU_i} > 0$ for all $i = 1, ..., N$. $\square$

Proof of Proposition 2: We first show that primary utility allocations that are efficient in an economy in which agents do not consider status are also efficient in an otherwise identical economy where some agents consider status. Suppose not; i.e. vector U^0 is a Pareto efficient in terms of primary utility but that the corresponding vector u^0 is not Pareto efficient in utility. Then there is a feasible u^1 which Pareto dominates u^0: $u_j^1 \geq u_j^0$ for all $j = 1, .., N$ with at least one strict inequality. This implies $u_i^1 \geq u_i^0$ and $u_i^{g1} \geq u_i^{g0}$ (since $u_j^1 \geq u_j^0$ for all $j \neq i$). Feasibility requires that $U_i^1 < U_i^0$ for some agent i, because U^0 is a Pareto efficient. For this agent i, we have $u_i^1 < F_i(U_i^0, u_i^1 - u_i^{g0})$, where $u_i^1 = F_i(U_i^1, u_i^1 - u_i^{g1})$. Subtracting $u_i^0 = F_i(U_i^0, u_i^0 - u_i^{g0})$ from both sides, we obtain

$$u_i^1 - u_i^0 < F_i(U_i^0, u_i^1 - u_i^{g0}) - F_i(U_i^0, u_i^0 - u_i^{g0}),$$

which can be written as

$$\int_{u_i^0}^{u_i^1} \left(1 - F_S^i(U_i^0, u_i - u_i^{g0})\right) du_i < 0.$$

As $1 - F_S^i > \frac{1}{2}$, we have $u_i^1 < u_i^0$. This contradicts the assumption that u^1 Pareto dominates u^0.

Now consider the converse claim: suppose U^0 is inefficient in primary utility but u^0 is efficient in utility. Since the primary utility possibility set is convex and no agent is satiated, the primary Pareto frontier intercepts the coordinate hyperplanes and is strictly decreasing everywhere. Whether U^0 lies in the interior of the primary utility possibility set or on a coordinate hyperplane below the Pareto frontier, it is possible to increase all U_i by arbitrary small amounts. Positive increments $dU_i = \frac{\epsilon}{F_U^i} > 0$ for all $i = 1, ..., N$ are within the primary utility possibility set for $\epsilon > 0$ sufficiently small (recall that $F_U^i \equiv \frac{\partial F_i}{\partial U_i} > 0$ for $i = 1, ..., N$). To find the corresponding changes in utilities, we take the total differential of each utility function

$$du_i = F_U^i dU_i + F_S^i \left(du_i - \sum_{j \neq i} \omega_{i,j} du_j \right)$$

for all $i = 1, ..., N$. It is readily verified that $du_i = \epsilon > 0$ for all $i = 1, ..., N$ is a solution to this system, given $dU_i = \frac{\epsilon}{F_U^i} > 0$ for all $i = 1, ..., N$. Further, this is the unique solution to the system, because Proposition 1 establishes that utility system (1)-(3) is well-defined. Thus, the allocation which generates u^0 is inefficient, a contraction. □

Proof of Proposition 4: (i) Equation (10) in Ostrowski (1952) gives bounds

$$b_{i,i} \in \left[\frac{1}{|a_{i,i}| + t_i \sum_{j \neq i} |a_{i,j}|}, \frac{1}{|a_{i,i}| - t_i \sum_{j \neq i} |a_{i,j}|} \right],$$

where $t_i = \max\limits_{k \neq i} \frac{\sum_{j \neq k} |a_{k,j}|}{|a_{k,k}|}$. Since $a_{i,i} = 1 - s_i$ and $\sum_{j \neq i} |a_{i,j}| = s_i$, we obtain $t_i = \max\limits_{k \neq i} \frac{s_k}{1 - s_k}$ and $b_{i,i} \in \left[\frac{1}{1 - s_i + t_i s_i}, \frac{1}{1 - s_i - t_i s_i} \right]$. $t_i \in (0, 1)$ under assumption $s_i < 1/2$. If $s_i > 0$, $b_{i,i} \geq \frac{1}{1 - s_i + t_i s_i} > 1$. If $s_i = 0$, equation (2) gives $u_i = U_i$.

(ii) Theorem 2.5.12, in Horn and Johnson (1991), states that if $\mathbf{A}$ is strictly row diagonal dominant then the inverse matrix $\mathbf{B}$ is strictly diagonally dominant of its column entries, so that $b_{i,i} > |b_{j,i}|$ for all i. (Note the different definitions for strictly row diagonally dominant and strictly diagonally dominant of its column entries).

(iii) We show that $J = (1, ..., 1)'$ is an eigenvector of $\mathbf{A}$ and $\mathbf{B}$ with eigenvalue one. For

$u = J$, we have $\mathbf{A}u = (\mathbf{I} - \mathbf{s} + \mathbf{s}\omega)J = J - \mathbf{s}J + \mathbf{s}\omega J = J$, since $\omega J = J$. Thus, we obtain $AJ = J$. For $U = J$, we have $\mathbf{B}U = \mathbf{B}J = \mathbf{B}\mathbf{A}J = J$. $\square$

Proof of Lemma 1: $\bar{S} = \frac{1}{N}J'(u - U) = \frac{1}{N}J'(u - ABU) = \frac{1}{N}J'(I - A)u = -\frac{1}{N}J'\mathbf{s}(\omega - I)u$ where $J = (1, ..., 1)'$. Vector $J'\mathbf{s}(\omega - I)$ has i^{th} element $\tilde{w}_i = \sum_{k \neq i} \omega_{k,i} s_k - s_i$ such that $\sum_i \tilde{w}_i = \sum_i \left(s_i - \sum_{k \neq i} \omega_{k,i} s_k \right) = 0$. Thus, we obtain $\frac{1}{N}J'\mathbf{s}(\omega - I)u = Cov(\tilde{w}_i, u_i)$.

$\bar{S} = \frac{1}{N}J'(u - U) = \frac{1}{N}J'(B - I)U = \frac{1}{N}(\lambda - J)'U = Cov(\lambda_i, U_i)$ where the last equality holds since $\frac{1}{N}\sum_i \lambda_i = 1$ ($\mathbf{B}J = J$ gives $J'\mathbf{B}J = J'J$ or $\lambda' J = N$). $\square$

Proof of Proposition 5: To prove the equivalence between utility conservation and $\lambda_i = 1$, note that $\bar{S} = 0$ for any vector U is equivalent to $J'(I - B)U = 0$ for any U which implies that $J'(I - B) = 0$. J is an eigenvector of B', that is, $B'J = J$ or $\lambda = J$.

Next, we show that $\lambda_i = 1$ for all i is equivalent to $\tilde{w}_i = 0$ for all i. $\lambda = J$ is equivalent to $(J'B)' = J$ or $B'J = J$. J is an eigenvector of B'. Then J has to be an eigenvector of $(B')^{-1} = A'$. $A'J = J$ implies $1 - s_i + \sum_{k \neq i} \omega_{k,i} s_k = 1$ for all i so that $\tilde{w}_i = 0$ for all i. $\square$

Proof of Proposition 6: Statements (i) and (iii) follow directly from Lemma 1. We prove statement (ii). The matrix A has elements $a_{i,i} = 1 - s_i$ and $a_{i,j} = \frac{s_i}{N-1}$. Then $\lambda_i = \sum_j b_{j,i}$ with

$$b_{j,i} = (-1)^{i+j} \frac{|A_{-i,-j}|}{|A|},$$

where $A_{-i,-j}$ is the matrix obtained after removing line i and column j from A. To simplify the exposition, we use notation $det(\mathbf{M}) = |\mathbf{M}|$. After replacing the formula for $b_{j,i}$, we obtain

$$\lambda_i |A| = |A_{-i,-i}| + \sum_{j \neq i} (-1)^{i+j} |A_{-i,-j}|.$$

For $i \neq j$, denote by $\tilde{A}_{-i,-j}$ the matrix obtained from matrix $A_{-i,-j}$ as follows: (a) If $j < i$, column $i - 1$ is moved after column $j - 1$. (b) If $j > i$, column i is moved after column $j - 1$. Matrices $\tilde{A}_{-i,-j}$ have the same columns as matrices $A_{-i,-j}$ and each line has identical non-diagonal entries. We have $\tilde{A}_{-i,-i+1} = A_{-i,-i+1}$ and $\tilde{A}_{-i,-i-1} = A_{-i,-i-1}$. Moreover, matrix $\tilde{A}_{-i,-j}$ satisfies the following property:

$$|\tilde{A}_{-i,-j}| = (-1)^{|i-j|-1} |A_{-i,-j}|. \tag{13}$$

This property is obtained by applying the swap rule for determinants and noting that matrix $\tilde{A}_{-i,-j}$ is obtained from matrix $A_{-i,-j}$ after making $|i - j| - 1$ column swaps. Replace the expressions $|A_{-i,-j}| = (-1)^{|i-j|-1} |\tilde{A}_{-i,-j}|$ in the equation for $\lambda_i |A|$:

$$\lambda_i |A| = |A_{-i,-i}| - \sum_{j \neq i} |\tilde{A}_{-i,-j}|.$$

To evaluate the determinants in the above expression, we use the following result:

Lemma 3. *(a) Let M_k^N be a matrix such that $M_{i,i}^N = m_i$ for $i \neq k$ and $M_{i,j}^N = n_i$ for $i \neq j$ and for $i = j = k$. We have $|M_k^N| = n_k \Pi_{i \neq k}(m_i - n_i)$. (b) Let M^N be a matrix such that $M_{i,i}^N = m_i$ and $M_{i,j}^N = n_i$ for $i \neq j$. We have $|M^N| = \Pi_i(m_i - n_i) + \sum_i n_i \Pi_{j \neq i}(m_j - n_j)$.*

Proof: (a) Any line $l \neq k$ of M_k^N can be written as

$$(n_l, ..., n_l, m_l, n_l, ..., n_l) = (n_l, ..., n_l) + (0, ..., 0, m_l - n_l, 0, ..., 0),$$

where m_l and $m_l - n_l$ are on the l^{th} column. Applying the line linearity rule of determinants, we obtain that the determinant of M_k^N is equal to the sum of the determinants obtained by replacing line l in M_k by each of the two lines on the right hand side in the above expression. The first determinant is equal to zero because line l is equal to $n_l(1, ..., 1)$ and line k is equal to $n_k(1, ..., 1)$. Thus, $|M_k^N|$ is equal to the determinant of the matrix obtained by replacing line l with $(0, ..., 0, m_l - n_l, 0, ..., 0)$ in matrix M_k^N. We can repeat this operation for all lines but line k and obtain that $|M_k^N|$ is equal to the determinant of Q^N defined as: (a) $Q_{k,j}^N = n_k$ for $j = 1, ..., N$, (b) for $i \neq k$, (b1) $Q_{i,i}^N = m_i - n_i$ and (b2) for $j \neq i$ $Q_{i,j}^N = 0$. Since $Q_{i,j}^N = 0$ for $i \neq j$ and $i \neq k$, Q^N's determinant is equal to the product of its diagonal elements: $|M_k^N| = |Q^N| = n_k \Pi_{i \neq k}(m_i - n_i)$.

(b) Line one of M^N can be expressed as

$$(m_1, n_1, ..., n_1) = (m_1 - n_1, 0, ..., 0) + (n_1, ..., n_1).$$

Applying the line linearity rule of determinants to M^N, we obtain

$$|M^N| = (m_1 - n_1)|M^{N-1}| + |M_1^N|,$$

where M^{N-1} is obtained by deleting the first line and first column in matrix M^N. Applying result (a) we have $|M_1^N| = n_1 \Pi_{i \neq 1}(m_i - n_i)$. We can repeat the same decomposition to matrix M^{N-1} to obtain

$$|M^{N-1}| = (m_2 - n_2)|M^{N-2}| + |M_1^{N-1}|,$$

where M^{N-2} is obtained by deleting the first two lines columns in matrix M^N. Applying result (a) we have $|M_1^{N-1}| = n_2 \Pi_{i \neq 1,2}(m_i - n_i)$. After replacement, we obtain the equation for $|M^N|$

$$|M^N| = (m_1 - n_1)(m_2 - n_2)|M^{N-2}| + n_2 \Pi_{i \neq 2}(m_i - n_i) + n_1 \Pi_{i \neq 1}(m_i - n_i).$$

Repeat the same operation iteratively to obtain the expression $|M^N| = \Pi_i(m_i - n_i) + \sum_i n_i \Pi_{j \neq i}(m_j - n_j)$. $\square$

In order to simplify the expressions, we factor out $\frac{1}{N-1}$ in all matrices. For example, we write matrix A as $\frac{1}{N-1}$ times a matrix with diagonal elements $(N-1)(1-s_i)$ and non-diagonal element (i, j) equal to s_i. Applying Lemma 3 (a) to $|A_{-i,-i}|$, we obtain

$$|A_{-i,-i}| = (N-1)^{-(N-1)} \left(\Pi_{j \neq i} (N - 1 + N s_j) + \sum_{k \neq i} s_k \Pi_{l \neq k,i}(N - 1 - N s_l) \right)$$

and applying Lemma 3 (b) to $|\tilde{A}_{-i,-j}|$, we obtain

$$|\tilde{A}_{-i,-j}| = s_j (N-1)^{-(N-1)} \ \Pi_{l \neq j,i}(N - 1 - N s_l).$$

These two expressions give the following identity

$$|A_{-i,-i}| = (N-1)^{-(N-1)} \left(\Pi_{j \neq i} (N - 1 - N s_i) + \sum_{j \neq i} |\tilde{A}_{-i,-j}| \right)$$

and substituting for $|A_{-i,-i}|$ in equation (13) we obtain the new expression for $\lambda_i|A|$:

$$\lambda_i|A| = (N-1)^{-(N-1)}\Pi_{j\neq i}(N-1-Ns_j).$$

The first difference in λ can thus be expressed as

$$\lambda_i - \lambda_{i+1} = \frac{N(N-1)^{-(N-1)}}{|A|}(s_i - s_{i+1})\Pi_{j\neq i,i+1}\left(N-1-Ns_j\right).$$

Since $s_j < \frac{N-1}{N}$, when s_i is decreasing (increasing) we obtain that λ_i is decreasing (increasing) and deduct that $\bar{S} < 0$ (> 0) by applying Lemma 2.[27] □

Proof of Lemma 2: Define the difference in adjacent utilities $v_i \equiv u_i - u_{i-1}$ for $i > 1$ and let $v_1 \equiv u_1$. We first want to prove that $v_i \geq 0$ for $i > 1$. Note that $u_i = \sum_{k=1}^{k=i} v_k$. Subtracting $u_i = U_i + s(u_i - \sum_j \omega_{i,j}u_j)$ for agent i and the same expression for agent $h > i$ gives

$$\sum_{j=1}^{j=N}(\omega_{h,j} - \omega_{i,j})u_j + \frac{1-s}{s}(u_h - u_i) = \frac{1}{s}(U_h - U_i), \tag{14}$$

where the first term on the LHS can be expressed

$$\sum_j(\omega_{h,j} - \omega_{i,j})u_j = \sum_j(\omega_{h,j} - \omega_{i,j})\sum_{k=1}^{k=j} v_k = \sum_{j=2}^{j=N}(W_{h,j} - W_{i,j})v_j.$$

Note that the first term in the sum for v_1 drops because $W_{i,1} = 1$ for any i. For $h = i+1$ equation (14) simplifies to

$$\sum_{j=2}^{j=N}\Delta W_{i+1,j}v_j + \frac{1-s}{s}v_{i+1} = \frac{1}{s}\Delta U_{i+1}; \quad for\ i = 1, ..., N-1. \tag{15}$$

where $\Delta U_{i+1} \equiv U_{i+1} - U_i$, $\Delta W_{i+1,j} \equiv W_{i+1,j} - W_{i,j} \in [-1, 0]$ by Assumption 2. In matrix notation we have

$$\left(\mathbf{\Delta W} + \frac{1-s}{s}\mathbf{I}\right)v = \frac{1}{s}\Delta U. \tag{16}$$

Here $\mathbf{\Delta W}$ is a $N-1$ square matrix with generic element $\Delta W_{i+1,j} \in [-1, 0]$. The matrix $\mathbf{\Delta W} + \frac{1-s}{s}\mathbf{I}$ has the following properties: (a) non-positive off-diagonal elements $\Delta W_{i+1,j}$ for $j \neq i+1$, (b) positive diagonal elements $\Delta W_{i+1,i+1} + \frac{1-s}{s} > 0$ since $\frac{1-s}{s} > 1$, and (c) positive

[27] Note that the coefficients for the utility function are:

$$b_{i,i} = (N-1)\frac{\Pi_{j\neq i}\left(N-1-Ns_j\right) + \sum_{k\neq i} s_k\Pi_{l\neq k,i}(N-1-Ns_l)}{\Pi_j\left(N-1-Ns_j\right) + \sum_k s_k\Pi_{l\neq k}(N-1-Ns_l)},$$

$$b_{i,j} = -(N-1)\frac{s_i\Pi_{l\neq i,j}(N-1-Ns_l)}{\Pi_j\left(N-1-Ns_j\right) + \sum_k s_k\Pi_{l\neq k}(N-1-Ns_l)}.$$

column sums $\sum_{i=1}^{i=N-1} \Delta W_{i+1,j} + \frac{1-s}{s} > 0$ where $\sum_{i=1}^{i=N-1} \Delta W_{i+1,j} = W_{N,j} - W_{1,j} \in [-1, 0]$. Matrix $\mathbf{\Delta W} + \frac{1-s}{s}\mathbf{I}$ is column diagonally dominant with positive diagonal entries and negative off-diagonal entries.

Lemma 4. *Let z be a non-negative vector and $\mathbf{C}$ a square matrix with (a) non-positive off-diagonal elements, (b) positive diagonal elements, and (c) columns that sum to positive numbers. The solution v to system $\mathbf{C}v = z$ is a non-negative vector.*

Proof: Assume not; i.e. some elements $v_i < 0$ for $i \in \Omega \subseteq \{1, 2, ..., N-1\}$. Sum all the lines $i \in \Omega$:

$$\sum_{i\in\Omega} v_i \sum_{j\in\Omega} c_{i,j} + \sum_{i\notin\Omega} v_i \sum_{j\in\Omega} c_{i,j} = \sum_{i\in\Omega} z_i.$$

The properties of $\mathbf{C}$ imply that $\sum_{j\in\Omega} c_{i,j} > 0$ for $i \in S$, as it includes diagonal elements. Since $v_i < 0$ for $i \in S$, the first sum on the LHS is negative. Regarding the second sum, $\sum_{j\in\Omega} c_{i,j} \leq 0$ for $i \notin S$, as it does not include diagonal elements. Since $v_i \geq 0$ for $i \notin \Omega$, the second sum is non-positive, making the $LHS < 0$. The $RHS \geq 0$, a contradiction. □

Applying Lemma 4 to equation (16) gives $v \geq 0$ or $u_i \leq u_{i+1}$ for $i < N$. □

Proof of Corollary 2: Rewriting equation (14):

$$u_h - u_i = \frac{1}{1-s}(U_h - U_i) - \frac{s}{1-s} \sum_{j=2}^{j=N} (W_{h,j} - W_{i,j}) v_j.$$

Assumption 2 requires $W_{h,j} \leq W_{i,j}$ for $h > i$. Thus, the second term in the above equation is non-positive and $u_h - u_i \geq \frac{1}{1-s}(U_h - U_i) > U_h - U_i$ as $\frac{1}{1-s} > 1$ by Assumption $s < 1/2$. □

Proof of Proposition 7. The optimality conditions from both planners' problems are the same when $MRT^U_{j,i} = \frac{\lambda_j}{\lambda_i} = 1$ for all pairs i and j. Together $\lambda_i = \lambda_j$ for all i and j and $\sum_{i=1}^{N} \lambda_i = N$ implies $\lambda_i = 1$ for all i. By Proposition 5, $\lambda_i = 1$ for all i if and only if $\bar{S} = 0$. □

Proof of Proposition 8. The optimal allocation of a maximin planner is the point on PF^u that intercepts the 45 degree line. The proof of Proposition 4 Point (iii) shows that J is an eigenvector of $\mathbf{A}$ and $\mathbf{B}$. Thus, the allocation on PF^u that crosses the 45 degree line is also the allocation on PF^U that crosses the 45 degree line. Proposition 5 implies $u_i = U_i$. □

C Examples

Identical Status ($s_i = s$ and $\omega_{i,j} = \frac{1}{N-1}$): Matrix equation 5 simplifies to $u_i = U_i + s\left(u_i - \frac{1}{N-1}\sum_{j\neq i} u_j\right)$ for $i = 1, ..., N$. Solving for u_i gives

$$u_i = \frac{N-1-s}{N-1-Ns}U_i - \frac{s}{N-1-Ns}\sum_{j\neq i} U_j.$$

A necessary and sufficient condition for $b_{i,i} > 0$ is $s < \frac{N-1}{N}$. Then, all the cross effects are equal and negative: $b_{i,j} < 0$.

Non-preservation of utility rankings under heterogeneous s_i: Reconsider the 10-agent example in Figure 2(b) keeping $s_1 = .4$ and $s_{10} = 0$ but replacing equal declining increments with $s_i = .41 - .01i$ for $i \leq 9$. We obtain $u_9 = 11.01 > u_{10} = 10.00 > u_8 = 9.62 > ... > u_1 = -2.36$. Here agent 9 not only enjoys great benefits from counting their blessing over the less affluent but they also enjoys the utility comparison with the more affluent agent 10.

Two Agents ($N = 2$): Equation 5 implies $u_i = U_i + s_i(u_i - u_j)$ for $i \neq j \in \{1, 2\}$. We obtain $b_{i,i} = \frac{1-s_j}{\det \mathbf{A}}$, $b_{i,j} = -\frac{s_i}{\det \mathbf{A}}$, and $\det \mathbf{A} = (1-s_i)(1-s_j) - s_i s_j$. When $s_i \in [0, \frac{1}{2})$ for $i = 1, 2$, $\det \mathbf{A} > \mathbf{0}$, $b_{i,i} > 0$ and $b_{i,j} < 0$.

Two Mutually Envious Groups: $N \geq 3$ agents are divided into two groups. Assume $s_i = s$ for all i. The first group consists of agents $i \leq n$, where $2 < n < N$; members of this group are not envious of each other, $\omega_{i,j} = 0$ for $j \leq n$, but are equally envious of each member of the other group (their reference group), $\omega_{i,j} = \frac{1}{N-n}$ for $j > n$. Conversely, for the other group. For $i \leq n$, equation 5 simplifies to

$$u_i = U_i + s\left(u_i - \frac{1}{N-n}\sum_{j=n+1}^{N} u_j\right)$$

and we obtain $b_{i,i} = \frac{s^2+n(1-2s)}{n(1-2s)(1-s)}$, $b_{i,j\leq n} = \frac{s^2}{n(1-2s)(1-s)}$, $b_{i,j>n} = -\frac{s}{(N-n)(1-2s)}$. Consistent with Proposition 4, if $s < \frac{1}{2}$, then $b_{i,i} > 0$, $b_{i,j\leq n} > 0$, and $b_{i,j>n} < 0$. That is, agent i benefits by other agents in their group ($j \leq n$) doing well. When others in their group are doing well, this hurts agents in the other group ($j > n$), which indirectly benefit's agent i. This result obtains even if $n = N - 1$, so that there is only one agent outside the group, agent N. In this case, only one of the cross effects is negative. In general, for any pair of agents who are part of the first group, $(i, j) \leq n$ and $i \neq j$, we have $MRS_{j,i} = \frac{b_{i,j}}{b_{i,i}} = \frac{s^2}{s^2+n(1-2s)} > 0$.

One-up Status: Primary utilities are ordered such that $U_1 < U_2 < ...U_{N-1} < U_N$. Each agent $i < N$ only compares to the next richest agent, $\omega_{i,i+1} = 1$ and $s_i = s$. Equation (5) simplifies to $u_i = U_i + s(u_i - u_{i+1})$ for $i < N$. Agent N is the richest agent and does not compare to others, $u_N = U_N$. Solving recursively, one obtains that the utility of agent $i < N$ depends only on the

Table 5: Characterization of Utility Functions

	$b_{i,i}$	$b_{i,j}$
Identical Status	$\frac{(N-1)(1-s)+(N-2)s}{(N-1)(1-s)-s}$	$\frac{-s}{(N-1)(1-s)-s}$
Two Agents	$\frac{1-s_j}{(1-s_1)(1-s_2)-s_1 s_2}$	$\frac{-s_i}{(1-s_1)(1-s_2)-s_1 s_2}$
Two Mutually Envious Groups	$\frac{s^2+n(1-2s)}{n(1-2s)(1-s)}$	$b_{i,j\leq n} = \frac{s^2}{n(1-2s)(1-s)}$, $b_{i,j>n} = \frac{-s}{(N-n)(1-2s)}$ for $i = 1, ..., n$
One-up Status	$\frac{1}{1-s}$	$b_{i,j} = \frac{1}{1-s}(\frac{-s}{1-s})^{k-i}$, $b_{i,N} = \left(\frac{-s}{1-s}\right)^{N-i}$ for $i < N$

Note: $u_i = b_{i,i}U_i + \sum_{j\neq i}^{N} b_{i,j}U_j$.

primary utilities of more affluent agents $k \geq i$:

$$u_i = \frac{U_i}{1-s} + \frac{1}{1-s}\sum_{k=i+1}^{N-1}\left(\frac{-s}{1-s}\right)^{k-i} U_k + \left(\frac{-s}{1-s}\right)^{N-i} U_N.$$

Here $b_{i,i} > 0$ obtains for $s < 1$. Notice that when $s \in (\frac{1}{2}, 1)$ the indirect effects increase geometrically in $k-i$, whereas they decrease when $s < \frac{1}{2}$. The influence from the primary utilities of more affluent agents alternates in sign: $\frac{\partial u_i}{dU_k} < 0$ for $k - i > 0$ odd and $\frac{\partial u_i}{dU_{i+j}} > 0$ for $k - i > 0$ even. Agent i directly compares with agent $i + 1$ and thus is made directly worse off with an increase in U_{i+1}. However, agent i is made better off with an increase in U_{i+2}. This is because agent $i + 1$ directly suffers by comparing with agent $i + 2$. Thus, agent i indirectly benefits from from an increase in U_{i+2} as that hurts agent $i + 1$.

This example violates Assumption 2 (as $W_{i-1,i+1} = 0 < W_{i,i+1} = 1$). We have $u_{N-1} < u_N$ and the ranking is preserved for other agents as well, $u_i < u_{i+1}$, when $s < \frac{1}{2+\Delta_i}$ where $\Delta_i = \frac{U_{i+2}-U_{i+1}}{U_{i+1}-U_i}$. For equal increments ($U_{i+1} - U_i$ constant), we have $\Delta_i = 1$ for $i = 1, ..., N - 2$ and the ranking is preserved when $s < \frac{1}{3}$.