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## Abstract

A common criticism of behavioral economics is that it has not shown that the psychological biases of individual investors lead to aggregate long-run effects on both asset prices and macroeconomic quantities. Our objective is to address this criticism by providing a simple example of a production economy where individual portfolio biases cancel when summed across investors, but still have an effect on aggregate quantities that does not vanish in the long-run. Specifically, we solve in closed form a model of a stochastic general-equilibrium production economy with a large number of heterogeneous firms and investors. Investors in our model are averse to ambiguity and so hold portfolios biased toward familiar assets. We specify this bias to be unsystematic so it cancels out when aggregated across investors. However, because of holding underdiversified portfolios, investors bear more risk than necessary, which distorts the consumption of all investors in the same direction. Hence, distortions in consumption do not cancel out in the aggregate and therefore increase the price of risk and distort aggregate investment and growth. The increased risk from holding biased portfolios, which increases the demand for the risk-free asset, leading to a higher equity risk premium and a lower risk-free rate that match the values observed empirically. Furthermore, all investors survive in the long-run, and so the effects of their biases never vanish. Our analysis illustrates that idiosyncratic behavioral biases can have long-run distortionary effects on both financial markets and the macroeconomy.



JEL Classification: G02, E03, E44, G11

Keywords: behavioral finance, ambiguity aversion, underdiversification, aggregate growth, investment



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# Do Individual Behavioral Biases Affect Financial Markets and the Macroeconomy?*

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October 30, 2017

**Abstract**

A common criticism of behavioral economics is that it has not shown that the psychological biases of individual investors lead to aggregate long-run effects on *both* asset prices and macroeconomic quantities. Our objective is to address this criticism by providing a simple example of a production economy where individual portfolio biases cancel when summed across investors, but still have an effect on aggregate quantities that does not vanish in the long-run. Specifically, we solve in closed form a model of a stochastic general-equilibrium production economy with a large number of heterogeneous firms and investors. Investors in our model are averse to ambiguity and so hold portfolios biased toward familiar assets. We specify this bias to be *unsystematic* so it cancels out when aggregated across investors. However, because of holding underdiversified portfolios, investors bear more risk than necessary, which distorts the consumption of all investors in the *same* direction. Hence, distortions in consumption do *not* cancel out in the aggregate and therefore increase the price of risk and distort aggregate investment and growth. The increased risk from holding biased portfolios, which increases the demand for the risk-free asset, leading to a higher equity risk premium and a lower risk-free rate that match the values observed empirically. Furthermore, all investors survive in the long-run, and so the effects of their biases never vanish. Our analysis illustrates that idiosyncratic behavioral biases can have long-run distortionary effects on both financial markets and the macroeconomy.

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# 1 Introduction and Motivation

The rational-expectations paradigm is the traditional approach to explaining phenomena in financial markets and the macroeconomy. It assumes that investors are risk-averse utility optimizers with unbiased Bayesian forecasts. However, the rational paradigm has been criticized because these assumptions are descriptively false and its predictions fail to explain the data. An alternative behavioral paradigm has been developed, which relaxes the assumptions of rational expectations,[1] and consequently, is much more successful in explaining the observed behavior of investors. But the behavioral paradigm has been criticized on the grounds that it has not shown that the psychological biases exhibited by individual investors lead to aggregate effects. For instance, while Fama (2012) agrees that behavioral finance is very good at describing individual behavior, and concedes that some sorts of professionals are inclined toward the same sort of biases as others, he argues that the "jumps that [behaviorists] make from there to markets aren't validated by the data." Similarly, Scholes (2009) says that the trouble with behavioral economics is that "it really hasn't shown in aggregate how it affects prices."

The main contribution of our paper is to provide a simple and transparent example of a general-equilibrium production economy with multiple heterogeneous investors whose behavioral biases are idiosyncratic in the sense that they cancel out when summing portfolio weights across investors, but still distort *both* aggregate financial variables and macroeconomic quantities. Furthermore, the distortions to aggregate financial variables and macroeconomic quantities do not vanish in the long-run.[2] Thus, our work illustrates that idiosyncratic behavioral biases exhibited in financial markets distort not just asset prices, but also firm-level and hence aggregate growth and investment.

In behavioral finance, agents deviate from at least one of the two foundational assumptions of rational decision making: Bayes' law and Subjective Expected Utility. Shleifer and Summers (1990) describe the psychological biases exhibited by people when forming beliefs

[1] For instance, the behavioral paradigm allows for overconfidence (Daniel, Hirshleifer, and Subrahmanyam, 1998, Barber and Odean, 2000, 2013), decision framing (Kahneman and Tversky, 1979), narrow framing (Kahneman and Lovallo, 1991, Barberis, Huang, and Thaler, 2006), loss aversion (Kahneman and Tversky, 1979, Barberis, Huang, and Santos, 2001), bounded rationality (Hong and Stein, 1999, Berrada, 2009, Gabaix, 2014) and non-Bayesian forecasting (Kahneman and Tversky, 1974, Lakonishok, Shleifer, and Vishny, 1994, Barberis, Shleifer, and Vishny, 1998, Brunnermeier and Parker, 2005).

[2] Abandoning the representative-agent assumption and building in explicit heterogeneity across individual agents is an important challenge for finance and macroeconomics. For instance, Hansen (2007, p. 27) in his Ely lecture says: "While introducing heterogeneity among investors will complicate model solution, it has intriguing possibilities. ... There is much more to be done."

and making decisions based on these beliefs. Moreover, "limits to arbitrage" make it difficult for rational traders to take advantage of traders who deviate from rational behavior, and so "natural selection" (through the forces of competition, learning, and evolution) may not eliminate investors with behavioral biases. Barberis and Thaler (2003), Hirshleifer (2001), Shefrin (2007, 2010), Shleifer (2000), and Statman (2010, 2011) provide excellent surveys of these two building blocks of behavioral finance—psychological biases of investors and limits to arbitrage.

Our model relies on the first building block of behavioral finance, psychological biases in decision making. Motivated by empirical evidence, we assume individual investors have a behavioral bias toward familiar assets (or equivalently, they are averse to the ambiguity regarding unfamiliar stocks).[3] We construct the model so that biases in portfolios toward familiar assets cancel out when aggregated across all investors. That is, the cross-sectional average portfolio across all investors is unbiased. However, we show that these idiosyncratic portfolio biases have an effect on individual consumption, which does not cancel out in aggregate; instead, aggregation *amplifies* the bias in consumption choices and impacts asset prices with spillover effects on macroeconomic quantities such as investment and growth. In particular, the increased risk from holding biased portfolios, which increases the demand for the risk-free asset, leads to a higher equity risk premium and a lower risk-free rate that match the values observed empirically. For instance, if we calibrate the model to U.S. stock-market data, the familiarity bias reduces the interest rate level from 3.51% to 0.56%, increases the equity risk premium from 4% to 6.94%, increases the Sharpe ratio from 19.82% to 34.43%, changes the growth rate by 0.74%, and changes the investment-to-output ratio by 9.82%. Moreover, these macro-finance effects driven by the idiosyncratic biases do not vanish in the long-run.

In order to study the effect of behavioral biases not solely on asset prices, but also on macroeconomic quantities, we consider a model with production. As in Cox, Ingersoll, and Ross (1985), we consider a model with a finite number of heterogeneous firms whose physical capital is subject to exogenous shocks. But, in contrast with Cox, Ingersoll, and Ross, we have heterogeneous investors with Epstein and Zin (1989) and Weil (1990) preferences coupled with familiarity bias. Consistent with empirical evidence described below, each

[3]The mathematical machinery behind our model of behavioral biases is related to the work on robustness by Hansen and Sargent (2009).

individual investor is more familiar with a small subset of firms, which are specific to her.[4] Familiarity creates a desire to concentrate investments in a few familiar firms at the expense of holding a portfolio that is well diversified across all firms. When forming the average cross-sectional familiarity bias across investors, the portfolio biases cancel out in aggregate because each investor is familiar with a different subset of firms.

The idea of greater familiarity with certain assets is developed in the novel and important paper by Huberman (2001). We conceptualize this idea via *ambiguity* in the sense of Knight (1921); the lower the level of ambiguity about an asset, the more "familiar" is that asset.[5] To allow for *differences* in familiarity across assets, we build on the modeling approach in Uppal and Wang (2003), extending it along three dimensions: one, we distinguish between risk across states of nature and over time by giving investors Epstein-Zin-Weil preferences as opposed to time-separable preferences; two, we consider a production economy instead of an endowment economy; three, we consider a general-equilibrium rather than a partial equilibrium framework.

We now explain how firm-specific ambiguity aversion tilts an individual investor's portfolio toward assets which she perceives as more familiar. The optimal portfolio is similar to the standard mean-variance portfolio, but with one difference: firm-level expected returns are adjusted downward for lack of familiarity. The adjustment is downward because investors are averse to ambiguity. Moreover, the downward adjustment is greater for less familiar assets, thereby creating a portfolio tilted toward more familiar assets.

While our assumptions about individual investors are behavioral, the overarching aim of our work is the same as that of the rational paradigm. As described by Cochrane (2000, p. 455), "The central task of financial economics is to figure out what are the real risks that drive asset prices ...." As in the rational paradigm, the asset prices and macroeconomic

[4] In contrast with our model, where heterogeneity across investors arises because of differences in the subset of firms with which investors are familiar, other sources of heterogeneity are differences in preferences and beliefs. For a review of the literature on differences in beliefs, see Basak (2005), Jouini and Napp (2007), and Yan (2008). For a model with differences in both beliefs and preferences, see Shefrin (2008, Ch. 14) and Bhamra and Uppal (2014). In addition to having a different source of heterogeneity, our paper differs from the work cited above in two other ways: our model has a real sector and all investors survive in the long run.

[5] For a survey of ambiguity in decision making, see Camerer and Weber (1992); for a discussion of ambiguity aversion in the context of portfolio choice and the equity risk premium, see Barberis and Thaler (2003, Sections 3.2.2 and 4.1.2), and for a more recent review of different ways of modeling ambiguity aversion in static and dynamic models, see Epstein and Schneider (2010). For a general discussion of the importance of allowing for ambiguity, see Hansen (2014). For a survey that focuses on the behavior of individual investors and their trading of individual stocks, see Barber and Odean (2013), Shefrin (2007, 2010), and Statman (2011). For evidence of ambiguity aversion in individuals' portfolio choices see Ahn, Choi, Gale, and Kariv (2014).

quantities in our model are driven by risk. But, because of the tilt toward familiar assets, the portfolio of each individual investor is excessively risky relative to the unbiased portfolio. This extra financial risk distorts the intertemporal consumption-saving decision of each investor. The resulting consumption decision of each investor is more volatile than in the absence of familiarity bias. In equilibrium, the excessively volatile consumption of individual investors increases the price of risk, reduces asset prices, and distorts growth. We find that for reasonable parameter values these effects are substantial.

The single key assumption in our model, that investors hold poorly-diversified portfolios, is one for which economists have gathered a great deal of empirical evidence. Guiso, Haliassos, and Jappelli (2002), Haliassos (2002), Campbell (2006), and Guiso and Sodini (2013) highlight underdiversification in their surveys of portfolio characteristics of individual investors. Polkovnichenko (2005), using data from the Survey of Consumer Finances, finds that for investors that invest in individual stocks directly, the median number of stocks held was two from 1983 until 2001, when it increased to three, and that poor diversification is often attributable to investments in employer stock, which is a significant part of equity portfolios. Barber and Odean (2000) and Goetzman and Kumar (2008) report similar findings of underdiversification based on data for individual investors at a U.S. brokerage firm. In an influential paper, Calvet, Campbell, and Sodini (2007) undertake a detailed and comprehensive analysis of household-level records covering the entire Swedish population. They find that of the investors who participate in the equity market, many are poorly diversified and bear significant idiosyncratic risk.[6]

We now describe evidence showing that underdiversification is intimately linked with the idea of familiarity. Typically, the few assets that investors hold are ones with which they are "familiar." Huberman (2001) introduces the idea that people invest in familiar assets and provides evidence of this in a multitude of contexts; for example, investors in the United States prefer to hold the stock of their local telephone company. Grinblatt and Keloharju (2001), based on data on Finnish investors, find that investors are more likely to hold stocks of Finnish firms that are "familiar;" that is, firms that are located close to the investor, that communicate in the investor's native language, and that have

[6]Lack of diversification is a phenomenon that is present not just in a few countries, but across the world. Countries for which there is evidence of lack of diversification include: Australia (Worthington (2009)), France (Arrondel and Lefebvre (2001)), Germany (Börsch-Supan and Eymann (2002) and Barasinska, Schäfer, and Stephan (2008)), India (Campbell, Ramadorai, and Ranish (2012)), Italy (Guiso and Jappelli (2002)), Netherlands (Alessie and Van Soest (2002)), and the United Kingdom (Banks and Smith (2002)).

a chief executive of the same cultural background. Massa and Simonov (2006) also find that investors tilt their portfolios toward stocks that are geographically and professionally close to the investor. Keloharju, Knüpfer, and Linnainmaa (2012) find that people tend to invest in firms they know through their product-market experiences, and that this bias is linked to preferences as opposed to information. French and Poterba (1990) and Cooper and Kaplanis (1994) document that investors bias their portfolios toward "home equity" rather than diversifying internationally. The most striking example of investing in familiar assets is the investment in "own-company stock," that is, stock of the company where the person is employed; Haliassos (2002) reports extensive evidence of limited diversification based on the tendency of investors to hold stock in the employer's firm.[7]

Dimmock, Kouwenberg, Mitchell, and Peijnenburg (2014), in a paper rich with insights, test the relation between familiarity bias and several portfolio-choice puzzles. Based on a survey of U.S. investors, they find that the familiarity bias is related to stock-market participation, the fraction of financial assets in stocks, foreign-stock ownership, own-company-stock ownership, and underdiversification. They also show that these results cannot be explained by risk aversion. Important empirical work by Korniotis and Kumar (2011) shows that risk sharing is lower in U.S. states where investors are less sophisticated and exhibit greater behavioral biases.

Finally, we explain how our work is related to the existing literature. Influential papers in the behavioral economics literature, such as Barberis, Shleifer, and Vishny (1998) and Daniel, Hirshleifer, and Subrahmanyam (1998), focus on the psychology of the representative investor. Other papers in behavioral economics eschew the representative agent and focus on psychological biases that arise in the context of social interaction between agents. See, for example the two innovative papers by Hong and Stein (1999) and Hong, Kubik, and Stein (2005); the first paper examines the interaction between two types of boundedly-rational agents ("newswatchers" and "momentum traders"), while the second investigates how stock-market participation is influenced by social interaction within a group, such as a church or residential neighborhood. We, too, do not rely on the representative-agent setting. Instead, we model heterogeneous agents who have psychological biases. However, rather than social interaction, our emphasis is on the aggregation of heterogeneous investors, each

[7] Mitchell and Utkus (2004) report that five million Americans have over sixty percent of their retirement savings invested in company stock and that about eleven million participants in 401(k) plans invest more than twenty percent of their retirement savings in their employer's stock.

with their own familiarity biases. Importantly, in our model the deviations of individual portfolios from the fully diversified portfolio cancel out when averaged across investors. This also distinguishes our paper from recent work by Hassan and Mertens (2011), who find that when investors make small correlated errors in forming expectations, the errors are amplified with significant distortive effects on growth and social welfare. In contrast, we show that such effects can arise even when investors make uncorrelated errors in portfolio choices.

Our paper is related also to other theoretical models where investors choose not to invest in all available assets. For example, Merton (1987) develops a static mean-variance model where each investor is aware only of a subset of the available securities. Shapiro (2002) extends this model to a dynamic setting. Shefrin and Statman (1994) study a model with noise traders with logarithmic utility who commit cognitive errors in processing information and show the effect of these noise traders on asset prices. Yan (2010) also studies a model with noise traders with logarithmic utility in an exchange economy and shows that because the effect of "noise" on asset prices is nonlinear and also because the wealth-distribution fluctuates over time, the effect of noise traders on asset prices do not cancel out in the aggregate. Garleânu, Panageas, and Yu (2014) use distance-dependent participation costs to generate differences in portfolio holdings across investors and study several interesting implications for crashes and contagion in financial markets. Our paper differs from these papers both in terms of the research question and modeling framework. In contrast to these papers, the focus of our work is to show that idiosyncratic behavioral biases can have aggregate effects not only on asset prices but also on macroeconomic quantities. To achieve this objective, our modeling framework allows for production, whereas the above papers study exchange economies. Moreover, to distinguish between the effects of risk preferences and timing preferences on financial and real quantities, we use Epstein-Zin-Weil utility functions, whereas the papers above use time-separable utility functions.

The rest of this paper is organized as follows. We describe the main features of our model in Section 2. The choice problem of an individual investor who exhibits a bias toward familiar assets is solved in Section 3, and the equilibrium implications of aggregating these choices across all investors are described in Section 4. In Section 5, we evaluate the quantitative magnitude of the effects of familiarity bias on asset prices and macroeconomic quantities. We conclude in Section 6. Proofs for all results are collected in the appendix.

# 2 The Model

In this section, we develop a parsimonious model of a stochastic dynamic general equilibrium economy with a finite number of production sectors and investor types. Growth occurs endogenously in this model via capital accumulation. When defining the preferences of investors, we show how to extend Epstein and Zin (1989) and Weil (1990) preferences to allow for familiarity biases, where the level of the bias differs across risky assets.

## 2.1 Firms

There are $N$ firms indexed by $n \in \{1, \ldots, N\}$. The value of the capital stock in each firm at date $t$ is denoted by $K_{n,t}$ and the output flow by

$$Y_{n,t} = \alpha K_{n,t}, \tag{1}$$

for some constant technology level $\alpha > 0$. The level of a firm's capital stock can be increased by investment at the rate $I_{n,t}$. We thus have the following capital accumulation equation for an individual firm:

$$dK_{n,t} = I_{n,t}\, dt + \sigma\, K_{n,t}\, dZ_{n,t},$$

where $\sigma$, the volatility of the exogenous shock to a firm's capital stock, is constant. The term $dZ_{n,t}$ is the increment in a standard Brownian motion and is firm-specific; the correlation between $dZ_{n,t}$ and $dZ_{m,t}$ for $n \neq m$ is given by $0 < \rho < 1$, which is also assumed to be constant over time and the same for all pairs $n \neq m$. Firm-specific shocks create heterogeneity across firms. The $N \times N$ correlation matrix of returns on firms' capital stocks is given by $\Omega = [\Omega_{nm}]$, where the elements of the matrix are

$$\Omega_{nm} = \begin{cases} 1, & n = m, \\ \rho, & n \neq m. \end{cases}$$

Firm-level heterogeneity creates benefits from diversifying investments across firms. We assume the expected rate of return is the same across the $N$ firms. Thus, diversification benefits manifest themselves solely through a reduction in risk—expected returns do not change with the level of diversification.

A firm's output flow is divided between its investment flow and dividend flow:

$$Y_{n,t} = I_{n,t} + D_{n,t}.$$

We can therefore rewrite the capital accumulation equation as

$$dK_{n,t} = \big(\alpha K_{n,t} - D_{n,t}\big)dt + \sigma\, K_{n,t}\, dZ_{n,t}. \tag{2}$$

In the Cox, Ingersoll, and Ross (1985) model, the return on a firm's physical capital, $\alpha$, equals the return on its stock. Similarly, the volatility of the return on a firm's capital, $\sigma$, equals the volatility of the return on its stock.

## 2.2 The Investment Opportunities of Investors

There are $H$ investors, indexed by $h \in \{1, \ldots, H\}$. Investors can invest their wealth in two classes of assets. The first is a risk-free asset, which has an interest rate $i$ that we assume for now is constant over time—and we show below, in Section 4.2, that this is indeed the case in equilibrium. Let $B_{h,t}$ denote the stock of wealth invested by investor $h$ in the risk-free asset at date $t$. Then, the change in $B_{h,t}$ is given by

$$\frac{dB_{h,t}}{B_{h,t}} = i\, dt.$$

Additionally, investors can invest in $N$ risky firms, or equivalently, in the stocks of these $N$ firms. We denote by $K_{hn,t}$ the stock of investor $h$'s wealth invested in the $n$'th risky firm. Given that the investor's wealth, $W_{h,t}$, is held in either the risk-free asset or invested in a risky firm, we have that:

$$W_{h,t} = B_{h,t} + \sum_{n=1}^{N} K_{hn,t}.$$

The proportion of an investor's wealth invested in firm $n$ is denoted by $\omega_{hn}$, and so

$$K_{hn,t} = \omega_{hn} W_{h,t}.$$

The amount of investor $h$'s wealth invested in the risk-free asset is

$$B_{h,t} = \Big(1 - \sum_{h=1}^{N} \omega_{hn}\Big) W_{h,t}.$$

The dividends distributed by firm $n$ are consumed by investor $h$:

$$C_{hn,t} = D_{hn,t} = \frac{K_{hn,t}}{K_{n,t}} D_{n,t},$$

where $C_{hn,t}$ is the consumption rate of investor $h$ from the dividend flow of firm $n$. Hence, the dynamic budget constraint for investor $h$ is given by

$$\frac{dW_{h,t}}{W_{h,t}} = \Big(1 - \sum_{n=1}^{N} \omega_{hn,t}\Big) idt + \sum_{n=1}^{N} \omega_{hn,t}\Big(\alpha dt + \sigma dZ_{n,t}\Big) - \frac{C_{h,t}}{W_{h,t}} dt,$$

where $C_{h,t}$ is the consumption rate of investor $h$ and $C_{h,t} = \sum_{n=1}^{N} C_{hn,t}$.

### 2.3 Preferences and Familiarity Biases of Investors

In the absence of any familiarity bias, each investor maximizes her date-$t$ utility level, $U_{h,t}$, defined as in Epstein and Zin (1989) by an intertemporal aggregation of date-$t$ consumption flow, $C_{h,t}$, and the date-$t$ certainty-equivalent of date $t + dt$ utility:

$$U_{h,t} = \mathcal{A}(C_{h,t}, \mu_t[U_{h,t+dt}]),$$

where $\mathcal{A}(\cdot,\cdot)$ is the time aggregator, defined by

$$\mathcal{A}(x,y) = \Big[(1 - e^{-\delta dt})x^{1-\frac{1}{\psi}} + e^{-\delta dt} y^{1-\frac{1}{\psi}}\Big]^{\frac{1}{1-\frac{1}{\psi}}}, \tag{3}$$

in which $\delta > 0$ is the rate of time preference, $\psi > 0$ is the elasticity of intertemporal substitution, and $\mu_t[U_{h,t+dt}]$ is the date-$t$ certainty equivalent of $U_{h,t+dt}$.[8]

The standard definition of a certainty equivalent amount of a risky quantity is the equivalent risk-free amount in static utility terms, and so the certainty equivalent $\mu_t\left[U_{h,t+dt}\right]$ satisfies

$$u_\gamma\left(\mu_t\left[U_{h,t+dt}\right]\right) = E_t[u_\gamma(U_{h,t+dt})], \tag{4}$$

where $u_\gamma(\cdot)$ is the static utility index defined by the power utility function[9]

$$u_\gamma(x) = \begin{cases} \frac{x^{1-\gamma}}{1-\gamma}, & \gamma > 0,\ \gamma \neq 1 \\ \ln x, & \gamma = 1, \end{cases} \tag{5}$$

and the conditional expectation $E_t[\cdot]$ is defined relative to a reference probability measure $\mathbb{P}$, which we discuss below. The preference parameters, $\delta$, $\psi$, and $\gamma$ are common across investors.

[8] The only difference with Epstein and Zin (1989) is that we work in continuous time, whereas they work in discrete time. The continuous-time version of recursive preferences is known as stochastic differential utility (SDU), and is derived formally in Duffie and Epstein (1992). Schroder and Skiadas (1999) provide a proof of existence and uniqueness.

[9] In continuous time the more usual representation for utility is given by $J_{h,t}$, where $J_{h,t} = u_\gamma(U_{h,t})$, with the function $u_\gamma$ defined in (5).

We can exploit our continuous-time formulation to write the certainty equivalent of investor utility an instant from now in a more intuitive fashion, as shown in the Lemma below.

**Lemma 1.** *The date-t certainty equivalent of investor h's date-t + dt utility is given by*

$$\mu_t[U_{h,t+dt}] = E_t[U_{h,t+dt}] - \frac{1}{2}\,\gamma\, U_{h,t}\, E_t\left[\left(\frac{dU_{h,t}}{U_{h,t}}\right)^2\right]. \tag{6}$$

Equation (6) reveals that the certainty equivalent of utility an instant from now is simply the expected value of utility an instant from now adjusted downward for risk. Naturally, the size of the risk adjustment depends on how risk averse the investor is, that is, $\gamma$. The risk adjustment depends also on the volatility of the proportional change in investor utility, which is given by $E_t\left[\left(\frac{dU_{h,t}}{U_{h,t}}\right)^2\right]$. Additionally, the risk adjustment is scaled by the current utility of the investor, $U_{h,t}$.[10]

Typically, standard models of portfolio choice assume that investors know the true expected return $\alpha$ on the value of each stock or firm. Such perfect knowledge would make each investor fully familiar with every firm and the probability measure $\mathbb{P}$ would then be the true objective probability measure.[11] However, in practice investors do not know the true expected returns, so they do not view $\mathbb{P}$ as the true objective probability measure—they treat it merely as a common reference measure. The name "reference measure" is chosen to capture the idea that even though investors do not observe true expected returns, they do observe the same data and use it to obtain identical point estimates for expected returns.

We now explain how investors are impacted by familiarity biases and how such biases differ across investors, creating investor heterogeneity. We assume investors are averse to their lack of knowledge about the true expected return and respond by reducing their point estimates. For example, investor $h$ will change the empirically estimated return on capital for firm $n$ from $\alpha$ to $\alpha + \nu_{hn,t}$, thereby reducing the magnitude of the firm's expected risk premium ($\nu_{hn,t} \leq 0$ if $\alpha > i$ and $\nu_{hn,t} \geq 0$ if $\alpha < i$). The size of the reduction depends on each investor's familiarity with a particular firm—the reduction is smaller for firms with

[10] The scaling ensures that if the expected proportional change in investor utility and its volatility are kept fixed, doubling current investor utility also doubles the certainty equivalent. For a further discussion, see Skiadas (2009, p. 213).

[11] In continuous time when the source of uncertainty is a Brownian motion, one can always determine the true volatility of the return on the capital stock by observing its value for a finite amount of time; therefore, an investor can be uncertain only about the expected return.

which the investor is more familiar. Thus, differences in familiarity across investors lead them to use different estimates of expected returns in their decision making, despite having observed the same data. From Girsanov's Theorem, we know this is equivalent to an investor changing the reference measure to a new measure, denoted by $\mathbb{Q}^{\nu_h}$.[12]

We can get a sense for how this plays out in portfolio decisions by observing that in the presence of familiarity, the contribution of risky portfolio investment to an investor's expected return on wealth changes from $\sum_{n=1}^{N} \omega_{hn,t}\alpha dt$ to $\sum_{n=1}^{N} \omega_{hn,t}(\alpha + \nu_{hn,t})dt$. The adjustment to the expected return on an investor's wealth stemming from familiarity bias is thus

$$\sum_{n=1}^{N} \omega_{hn,t}\nu_{hn,t}dt, \tag{7}$$

which we can write more succinctly as

$$\boldsymbol{\omega}_{h,t}^{\top}\boldsymbol{\nu}_{h,t}dt,$$

where $\boldsymbol{\omega}_{h,t} = (\omega_{h1,t}, \ldots, \omega_{hN,t})^{\top}$ is the column vector of portfolio weights and $\boldsymbol{\nu}_{h,t} = (\nu_{h1,t}, \ldots, \nu_{hN,t})^{\top}$ is the column vector of adjustments of expected returns.

Without familiarity bias, the decision of an investor on how much to invest in a particular firm depends solely on the certainty equivalent. Therefore, to allow for familiarity bias it is natural to generalize the concept of the certainty equivalent.

**Theorem 1.** *The date-t familiarity-biased certainty equivalent of date-t+dt investor utility is given by*

$$\mu_{h,t}^{\boldsymbol{\nu}}[U_{h,t+dt}] = \widehat{\mu}_{h,t}^{\boldsymbol{\nu}}[U_{h,t+dt}] + U_{h,t}L_{h,t}dt, \tag{8}$$

*where $\widehat{\mu}_{h,t}^{\boldsymbol{\nu}}[U_{h,t+dt}]$ is defined by*

$$u_{\gamma}\left(\widehat{\mu}_{h,t}^{\boldsymbol{\nu}}[U_{h,t+dt}]\right) = E_t^{\mathbb{Q}^{\boldsymbol{\nu}_h}}[u_{\gamma}\left(U_{h,t+dt}\right)], \tag{9}$$

*and*

$$L_{h,t} = \frac{1}{2\gamma}\frac{\boldsymbol{\nu}_{h,t}^{\top}(\Gamma_h\Omega)^{-1}\boldsymbol{\nu}_{h,t}}{\sigma^2}, \tag{10}$$

[12] We define $\mathbb{Q}^{\nu_h}$ formally in Definition A2 of the Appendix.

*where* $\boldsymbol{\nu}_{h,t} = (\nu_{h1,t}, \ldots, \nu_{hN,t})^\top$ *is the column vector of adjustments to expected returns, and* $\Gamma_h = [\Gamma_{h,nm}]$ *is the* $N \times N$ *diagonal matrix defined by*

$$\Gamma_{h,nm} = \begin{cases} \frac{1-f_{hn}}{f_{hn}}, & n = m, \\ 0, & n \neq m, \end{cases}$$

*and* $f_{hn} \in [0,1]$ *is a measure of how familiar the investor is with firm,* $n$*, with* $f_{hn} = 1$ *implying perfect familiarity, and* $f_{hn} = 0$ *indicating no familiarity at all.*

The matrix $\Gamma_h$ encodes the differing levels of familiarity investor $h$ has with each firm in the economy. We can see that $\widehat{\mu}^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}]$ is like a certainty equivalent, but with the expectation taken under $\mathbb{Q}^{\boldsymbol{\nu}_h}$ in order to adjust for familiarity bias. The additional term $U_{h,t}L_{h,t}dt$ depends on $L_{h,t}$, which is a penalty function for using the measure $\mathbb{Q}^{\boldsymbol{\nu}_h}$ instead of $\mathbb{P}$. The intuition behind the expression for $L_{h,t}$ in (10) is that it measures the familiarity-weighted distance between the reference measure and the measure $\mathbb{Q}^{\boldsymbol{\nu}_h}$, where the distance between them is the conditional Kullback-Leibler divergence between $\mathbb{P}$ and $\mathbb{Q}^{\boldsymbol{\nu}_h}$. The penalty function is also a familiarity-weighted measure of the information lost by using $\mathbb{Q}^{\boldsymbol{\nu}_h}$ instead of $\mathbb{P}$. The factor $U_{h,t}$ is for scaling as explained in footnote (10).

Importantly, investors are sufficiently rational to realize that familiarity with a particular firm implies familiarity with firms which have correlated returns – this is why, in the penalty function, the matrix, $\Gamma_h$, encoding investor $h$'s familiarity biases with respect to firms is postmultiplied by the correlation matrix, $\Omega$. Consequently, our modelling framework deviates from full rationality along only one dimension.

We can write the date-$t$ familiarity-biased certainty equivalent of date-$t + dt$ utility in a more intuitive form as shown in the following Corollary.

**Corollary 1.** *The date-t familiarity-biased certainty equivalent of date-t+dt investor utility is given by*

$$\mu^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}] = \mu_t[U_{h,t+dt}] + U_{h,t} \times \left( \frac{W_{h,t}U_{W_{h,t}}}{U_{h,t}} \boldsymbol{\nu}^\top_{h,t}\boldsymbol{\omega}_{h,t} + L_{h,t} \right) dt, \tag{11}$$

*where* $U_{W_{h,t}} = \frac{\partial U_{h,t}}{\partial W_{h,t}}$ *is the partial derivative of the utility of investor* $h$ *with respect to her wealth.*

The first term in (11), the pure certainty equivalent $\mu_t[U_{h,t+dt}]$, does not depend directly on the familiarity-bias adjustments. As before, we introduce the scaling factor $U_{h,t}$ (see

footnote 10 for the role of the scaling factor). The next term, $\frac{W_{h,t}U_{W_{h,t}}}{U_{h,t}}\boldsymbol{\nu}_{h,t}^{\top}\boldsymbol{\omega}_{h,t}$, is the adjustment to the expected change in investor utility. It is the product of the elasticity of investor utility with respect to wealth, $\frac{W_{h,t}U_{W_{h,t}}}{U_{h,t}}$, and the change in the expected return on investor wealth arising from the adjustment made to returns, $\boldsymbol{\nu}_{h,t}^{\top}\boldsymbol{\omega}_{h,t}$, which is given in (7).

The tendency to make adjustments to expected returns is tempered by the penalty term, $L_{h,t}$, defined in (10), which captures two distinct features of investor decision making. The first pertains to the idea that when an investor has more accurate estimates of expected returns, she will be less willing to adjust them. The accuracy of investor expected return estimates is measured by their standard errors, which are proportional to $\sigma$.[13] With smaller standard errors, there is a stiffer penalty for adjusting returns away from their empirical estimates. The second feature pertains to familiarity—when an investor is more familiar with a particular firm, she is less willing to adjust its expected return.

# 3 Portfolio and Consumption of an Individual Investor

We solve the model described above in two steps. First, we solve in partial equilibrium the problem of an individual investor who suffers from familiarity bias. This gives us the investor's portfolio, which is biased toward a few assets, and also the consumption policy that is financed by this portfolio. Then, in the next section, we aggregate over all investors to obtain in general equilibrium the interest rate, stock prices, aggregate growth, and investment.

## 3.1 The Intertemporal Choice Problem of an Individual Investor

In the absence of familiarity-bias, an individual investor would choose her consumption rate, $C_{h,t}$, and portfolio policy, $\boldsymbol{\omega}_{h,t}$, according to the standard choice problem:

$$\sup_{C_{h,t}} \mathcal{A}\Big(C_{h,t}, \sup_{\boldsymbol{\omega}_{h,t}} \mu_{h,t}[U_{h,t+dt}]\Big). \tag{12}$$

With familiarity bias, the time aggregator $\mathcal{A}(\cdot)$ in (3) is unchanged—all we need to do is to replace the maximization of the certainty-equivalent, $\sup_{\boldsymbol{\omega}_{h,t}} \mu_t[U_{h,t+dt}]$, with the

[13] In our continuous-time framework, an infinite number of observations are possible in finite time, so standard errors equal the volatility of proportional changes in the capital stock, $\sigma$, divided by the square root of the length of the observation window.

combined maximization and minimization of the familiarity-biased certainty equivalent, $\sup_{\boldsymbol{\omega}_{h,t}} \inf_{\boldsymbol{\nu}_t} \mu^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}]$ to obtain

$$\sup_{C_{h,t}} \mathcal{A}\Big(C_{h,t}, \sup_{\boldsymbol{\omega}_{h,t}} \inf_{\boldsymbol{\nu}_{h,t}} \mu^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}]\Big). \tag{13}$$

An investor, because of her aversion to ambiguity, chooses $\boldsymbol{\nu}_{h,t}$ to *minimize* her familiarity-biased certainty equivalent; that is, the investor adjusts expected returns more for firms with which she is less familiar, which acts to reduce the familiarity-biased certainty equivalent.[14] By comparing (12) and (13), we can see that once an investor has chosen the vector $\boldsymbol{\nu}_{h,t}$ to adjust the expected returns of each firm for familiarity bias, she makes consumption and portfolio choices in the standard way.

Given any portfolio choice $\boldsymbol{\omega}_{h,t}$ for an investor, finding the adjustments to firm-level expected returns is a matter of minimizing the familiarity-biased certainty equivalent in (11). The solution is given in the proposition below.

**Proposition 1.** *For a given portfolio, $\boldsymbol{\omega}_{h,t}$, adjustments to firm $n$'s expected return are given by*

$$\nu_{hn,t} = -\frac{W_{h,t} U_{W_{h,t}}}{U_{h,t}} \Big(\frac{1}{f_{hn}} - 1\Big) \sigma^2 \gamma \left(\omega_{hn,t} + \rho \sum_{m \neq n} \omega_{hm,t}\right), \; n \in \{1, \ldots, N\}. \tag{14}$$

The above expression shows that if an investor is fully familiar with firm $n$, then $f_{hn} = 1$ and $\nu_{hn,t} = 0$, so she makes no adjustment to the firm's expected return. For the case when $\rho = 0$, when she is less than fully familiar, $f_{hn} \in [0, 1)$, one can see that $\nu_{hn,t}$ is negative (positive) when $\omega_{hn,t}$ is positive (negative), reflecting the idea that lack of familiarity leads an investor to moderate her portfolio choices, shrinking both long and short positions toward zero.

To solve an investor's consumption-portfolio choice problem under familiarity bias we use Ito's Lemma to derive the continuous-time limit of (13), which leads to the Hamilton-Jacobi-Bellman equation shown in the proposition below.

[14] In the language of decision theory, investors are averse to ambiguity and so they minimize their familiarity-biased certainty equivalents.

**Proposition 2.** *The utility function of an investor with familiarity biases is given by the following Hamilton-Jacobi-Bellman equation:*

$$0 = \sup_{C_{h,t}} \left( \delta\, u_\psi \left( \frac{C_{ht}}{U_{ht}} \right) + \sup_{\omega_t} \inf_{\nu_{h,t}} \frac{1}{U_{h,t}} \mu^{\boldsymbol{\nu}}_{h,t} \left[ \frac{dU_{h,t}}{dt} \right] \right), \tag{15}$$

*where the function*

$$u_\psi(x) = \frac{x^{1-\frac{1}{\psi}} - 1}{1 - \frac{1}{\psi}}, \psi > 0,$$

*and*

$$\mu^{\boldsymbol{\nu}}_{h,t}\left[dU_{h,t}\right] = \mu^{\boldsymbol{\nu}}_{h,t}\left[U_{h,t+dt} - U_{h,t}\right] = \mu^{\boldsymbol{\nu}}_{h,t}\left[U_{h,t+dt}\right] - U_{h,t},$$

*with* $\mu^{\boldsymbol{\nu}}_{h,t}\left[U_{h,t+dt}\right]$ *given in* (11).

The Hamilton-Jacobi-Bellman equation can be decomposed into a portfolio-optimization problem and an intertemporal consumption choice problem. Given the assumption of a constant risk-free rate, homotheticity of preferences combined with constant returns to scale for production lead to an investment opportunity set that is constant over time. This implies that maximized investor utility is a constant multiple of the investor's wealth. In this case, the Hamilton-Jacobi-Bellman equation can be decomposed into two parts: *a single-period mean-variance* optimization problem for an investor with familiarity bias and an intertemporal consumption choice problem, as shown in the proposition below.

**Proposition 3.** *The investor's optimization problem consists of two parts, a mean-variance optimization*

$$\sup_{\boldsymbol{\omega}_{h,t}} \inf_{\boldsymbol{\nu}_{h,t}} MV(\boldsymbol{\omega}_{h,t}, \boldsymbol{\nu}_{h,t}),$$

*and an intertemporal consumption choice problem*

$$0 = \sup_{C_{h,t}} \left( \delta\, u_\psi \left( \frac{C_{ht}}{U_{ht}} \right) - \frac{C_{ht}}{W_{ht}} + \sup_{\boldsymbol{\omega}_t} \inf_{\boldsymbol{\nu}_{h,t}} MV(\boldsymbol{\omega}_{h,t}, \boldsymbol{\nu}_{h,t}) \right), \tag{16}$$

*where*

$$MV(\boldsymbol{\omega}_{h,t}, \boldsymbol{\nu}_{h,t}) = i + \big(\alpha - i\big)\mathbf{1}^\top \boldsymbol{\omega}_{h,t} - \frac{1}{2}\gamma\sigma^2 \boldsymbol{\omega}_{h,t}^\top \Omega \boldsymbol{\omega}_{h,t} + \boldsymbol{\nu}_{h,t}^\top \boldsymbol{\omega}_{h,t} + \frac{1}{2\gamma} \frac{\boldsymbol{\nu}_{h,t}^\top (\Gamma_h \Omega)^{-1} \boldsymbol{\nu}_{h,t}}{\sigma^2}, \tag{17}$$

*and* $\mathbf{1}$ *denotes the* $N \times 1$ *unit vector.*

In the above proposition, $MV(\boldsymbol{\omega}_{h,t}, \boldsymbol{\nu}_{h,t})$ is the objective function of a single-period mean-variance investor with familiarity bias: $i + \big(\alpha - i\big)\mathbf{1}^\top \boldsymbol{\omega}_{h,t}$ is the expected portfolio return, $-\frac{1}{2}\gamma\sigma^2 \boldsymbol{\omega}_{h,t}^\top \Omega \boldsymbol{\omega}_{h,t}$ is the penalty for portfolio variance, $\boldsymbol{\nu}_{h,t}^\top \boldsymbol{\omega}_{h,t}$ is the adjustment to the portfolio's expected return arising from familiarity bias, and $\frac{1}{2}\frac{1}{\gamma}\frac{\boldsymbol{\nu}_{h,t}^\top (\Gamma_h \Omega)^{-1} \boldsymbol{\nu}_{h,t}}{\sigma^2}$ is the penalty for adjusting expected returns.[15]

In the mean-variance problem with familiarity bias, the firm-level expected returns are optimally adjusted downward because of lack of familiarity. Because each investor's utility is a constant multiple of wealth, the expression for the adjustment to expected returns in equation (14) simplifies to:

$$\boldsymbol{\nu}_{h,t} = -\gamma\sigma^2 \left(\Gamma_h \Omega\right) \boldsymbol{\omega}_{h,t}. \tag{18}$$

Substituting the above expression into (17), we see that the investor faces the following mean-variance portfolio problem:

$$\sup_{\boldsymbol{\omega}_{h,t}} MV(\boldsymbol{\omega}_{h,t}) = \left( i + \big(\alpha \mathbf{1} + \frac{1}{2}\boldsymbol{\nu}_{h,t} - i\mathbf{1}\big)^\top \boldsymbol{\omega}_{h,t} \right) - \frac{1}{2}\gamma\sigma^2 \boldsymbol{\omega}_{h,t}^\top \Omega \boldsymbol{\omega}_{h,t},$$

where $\boldsymbol{\nu}_{h,t}$ is given by (18). When the investor is fully familiar with all firms, then $\Gamma_h$ is the zero matrix, and from (18) we can see the adjustment to expected returns is zero and the portfolio weights are exactly the standard mean-variance portfolio weights. For the case where the investor is completely unfamiliar with all firms, then each $\Gamma_{h,nn}$ becomes infinitely large and $\boldsymbol{\omega}_h = \mathbf{0}$: complete unfamiliarity leads the investor to avoid any investment in risky firms, in which case one would get non-participation in the stock market in this partial-equilibrium setting.

## 3.2 Solution to the Choice Problem of an Individual Investor

In this section, we present the solution to the choice problem of an individual investor, and in the next section, we impose market clearing to get the equilibrium solutions.

**Proposition 4.** *The optimal adjustment to expected returns is:*

$$\boldsymbol{\nu}_h = -(\alpha - i)(\mathbf{1} - \boldsymbol{f}_h),$$

[15] The familiarity-bias adjustment is obtained from a minimization problem, so the associated penalty is positive, in contrast with the penalty for return variance.

*where $\boldsymbol{f}_h$ is the vector of familiarity coefficients*

$$\boldsymbol{f}_h = (f_{h1}, \ldots, f_{hN})^\top.$$

*The vector of optimal portfolio weights is*

$$\boldsymbol{\omega}_h = \frac{1}{\gamma}\frac{\alpha - i}{\sigma^2}\,\Omega^{-1}\boldsymbol{f}_h. \tag{19}$$

*We can write the $n$'th element of the above vector of portfolio weights as*

$$\omega_{hn} = \frac{1}{\gamma}\frac{\alpha - i}{\sigma^2} q_{hn},$$

*where $q_{hn}$ is the correlation-adjusted familiarity of investor $h$ with respect to firm $n$, defined by*

$$q_{hn} = \boldsymbol{e}_n^\top \Omega^{-1}\boldsymbol{f}_h, \tag{20}$$

*where $\boldsymbol{e}_n$ is the $N \times 1$ column vector, with a one in the $n$'th entry and zeros everywhere else.*

*If we denote by $\boldsymbol{x}_h = \frac{\boldsymbol{\omega}_h}{\mathbf{1}^\top \boldsymbol{\omega}_h}$ the weights in the risky assets normalized by the total investment in all $N$ risky assets, then the optimal portfolio of risky assets is*

$$x_{hn} = \frac{q_{hn}}{\sum_{n=1}^{N} q_{hn}}. \tag{21}$$

*With familiarity bias, the* optimized *portfolio-choice objective function can be expressed as:*

$$\sup_{\boldsymbol{\omega}_{h,t}} \inf_{\boldsymbol{\nu}_{h,t}} MV(\boldsymbol{\omega}_{h,t}, \boldsymbol{\nu}_{h,t}) = i + \frac{1}{2}\frac{1}{\gamma}\left(\frac{\alpha - i}{\sigma_{\boldsymbol{x}_h}}\right)^2, \tag{22}$$

*where*

$$\sigma^2_{\boldsymbol{x}_h} = \sigma^2 \boldsymbol{x}_h^\top (I + \Gamma_h)\Omega\,\boldsymbol{x}_h. \tag{23}$$

To see the intuition, we first note that the optimal adjustment made by investor $h$ to firm $n$'s expected returns is

$$\nu_{hn} = -(\alpha - i)(1 - f_{hn}). \tag{24}$$

This adjustment impacts optimal portfolio weights for all firms that have returns correlated with firm $n$, as seen in (19). For the special case when firm-level returns are mutually

orthogonal, i.e. $\rho = 0$, the correlation matrix is no longer needed, and we obtain

$$\omega_{hn} = \frac{1}{\gamma}\frac{\alpha - i}{\sigma^2} f_{hn}, \tag{25}$$

From (24), we can see that the size of an investor's adjustment to a firm's return is smaller when the level of familiarity, $f_{hn}$, is larger; if $f_{hn} = 1$, then the adjustment vanishes altogether. From (25), we see that the standard mean-variance portfolio weight for firm $n$, $\frac{\alpha - i}{\gamma\sigma^2}$, is scaled by the level of investor $h$'s familiarity with firm $n$, $f_{hn}$. As an investor's level of familiarity with a particular firm decreases, the proportion of her wealth that she chooses to invest in that firm also decreases.

We can interpret Equation (23) as the variance of the investor's portfolio of risky assets with an additional penalty for familiarity bias, which is reflected by the presence of the diagonal matrix $I + \Gamma_h$, which postmultiplies $\Omega$. Equation (22) thus makes it clear that the familiarity-biased portfolio of only risky assets, $\boldsymbol{x}_h$, is the minimum-variance portfolio with a familiarity-biased adjustment.[16] Given that all risky assets have the same volatility and correlation, the minimum-variance portfolio with no familiarity bias is given by $x_{hn} = \frac{1}{N}$. Familiarity bias tilts the weights of the portfolio of only risky assets away from $\frac{1}{N}$.

Finally, we solve for optimal consumption, which is given in the following proposition.

**Proposition 5.** *An investor's optimal consumption-to-wealth ratio is given by*

$$\frac{C_{h,t}}{W_{h,t}} = \psi\delta + (1-\psi)\left(\left[i + \left(\alpha\mathbf{1} + \frac{1}{2}\boldsymbol{\nu}_{h,t} - i\mathbf{1}\right)^\top \boldsymbol{\omega}_{h,t}\right] - \frac{1}{2}\gamma\sigma^2\boldsymbol{\omega}_{h,t}^\top\Omega\boldsymbol{\omega}_{h,t}\right) \tag{26}$$

$$= \psi\delta + (1-\psi)\left(i + \frac{1}{2}\frac{1}{\gamma}\left(\frac{\alpha - i}{\sigma_{\boldsymbol{x}_h}}\right)^2\right). \tag{27}$$

The above expressions show that an investor's portfolio choice impacts her intertemporal consumption choice. We see from (27) that the optimal consumption-wealth ratio is a weighted average of the impatience parameter $\delta$ and the *optimized* single-period, mean-variance objective function, defined in (22). If the investor holds a portfolio that is underdiversified, then $\sigma_{\boldsymbol{x}_h}$, is higher than it needs to be, which distorts consumption. If the substitution effect dominates ($\psi > 1$), choosing a portfolio subject to familiarity bias increases consumption and reduces savings. In contrast, when the income effect dominates ($\psi < 1$), familiarity bias decreases consumption and increases savings.

[16] If $\rho = 0$, then $x_{hn} = \frac{f_{hn}}{\sum_{n=1}^{N} f_{hn}}$.

# 4 Aggregation: The Impact of Familiarity Bias on Asset Prices and Macroeconomic Quantities

In the previous section, we examined how a bias toward familiar assets increases the risk of an investor's portfolio and distorts her consumption choice. In this section, we aggregate across investors to determine how the distortion in individual consumption choices impacts asset prices, investment, growth, and investment, obtaining all results in closed form. In the subsequent section, we provide a quantitative assessment of the effects of familiarity bias.

## 4.1 No Aggregate Familiarity Bias Across Investors

In this section, we explain how the familiarity bias is specified for each investor and how it "cancels out in aggregate."

We start by defining the "no-aggregate bias condition."

**Definition 1.** *Suppose investor $h$'s risky portfolio weight for firm $n$ is given by*

$$x_{hn} = \frac{1}{N} + \epsilon_{hn}, \tag{28}$$

*where $\frac{1}{N}$ is the unbiased portfolio weight and $\epsilon_{hn}$ is the bias of investor $h$'s portfolio when investing in firm $n$. The biases $\epsilon_{hn}$ "cancel out in aggregate" if*

$$\forall n, \ \frac{1}{H}\sum_{h=1}^{H} \epsilon_{hn} = 0. \tag{29}$$

The above definition starts by building on Equation (21), which implies that (28) does indeed hold with $\epsilon_{hn} = \frac{q_{hn}}{\sum_{n=1}^{N} q_{hn}} - \frac{1}{N}$. The definition tells us that by "canceling out in aggregate" we mean that the bias in the cross-sectional average risky portfolio across investors is zero. In other words, while it is possible for an individual investor's portfolio to be biased, that is, to deviate from the unbiased $\frac{1}{N}$ portfolio, this bias must cancel out when forming the average portfolio across all investors.[17]

[17] An equivalent way of expressing (29) is that the mean risky portfolio equals the $\frac{1}{N}$ portfolio:

$$\forall n, \ \frac{1}{H}\sum_{h=1}^{H} x_{hn} = \frac{1}{N}.$$

The following proposition gives a symmetry condition, which implies that the no-aggregate bias condition holds.

**Proposition 6.** *For every investor $h \in \{1, \ldots, H\}$, define the adjusted-familiarity vector $(q_{h1}, \ldots, q_{hN})$, where $q_{hn}$ is defined in* (20)*. If the following symmetry condition holds:*

1. *given an investor $h \in \{1, \ldots, H\}$, for all investors $h' \in \{1, \ldots, H\}$, there exists a permutation $\tau_{h'}$ such that $\tau_{h'}(q_{h'1}, \ldots, q_{h'N}) = (q_{h1}, \ldots, q_{hN})$; and,*
2. *given a firm $n \in \{1, \ldots, N\}$, for all firms $n' \in \{1, \ldots, N\}$, there exists a permutation $\tau_{n'}$ such that $\tau_{n'}(q_{1n'}, \ldots, q_{Hn'}) = (q_{1n}, \ldots, q_{Hn})$,*

*then there is no aggregate bias.*

To understand the symmetry condition note that if one were to define a $H \times N$ familiarity matrix,

$$Q = [q_{hn}]$$

then the permutations described in the above symmetry condition imply that one can obtain all the rows of the matrix by rearranging any particular row, and one can obtain all the columns of the matrix by rearranging any particular column.

To interpret the symmetry condition further, observe that it implies that

$$\forall n, \forall h, \ \frac{1}{H} \sum_{h=1}^{H} q_{hn} = \frac{1}{N} \sum_{n=1}^{N} q_{hn}. \tag{30}$$

Intuitively, the condition in (30) says that the mean correlation-adjusted familiarity of an investor across all firms, $\frac{1}{N} \sum_{n=1}^{N} q_{hn}$, is equal to the mean correlation-adjusted familiarity toward a firm from all investors, $\frac{1}{H} \sum_{h=1}^{H} q_{hn}$; furthermore, the total correlation-adjusted familiarity of an investor is the same across all investors and the total correlation-adjusted familiarity toward a firm is the same across all firms.

Observe also that the condition in equation (30) is equivalent to

$$\forall n, \forall h, \ \frac{1}{H} \hat{q}_n = \frac{1}{N} \hat{q}_h, \tag{31}$$

where

$$\hat{q}_n = \sum_{h=1}^{H} q_{hn}, \qquad \text{and} \qquad \hat{q}_h = \sum_{n=1}^{N} q_{hn}.$$

From (31) we can also see that $\hat{q}_n$ and $\hat{q}_h$ must be independent of $n$ and $h$, respectively. The condition in (31) tells us that in addition to the mean risky portfolio being unbiased, and hence equal to $1/N$, the mean proportion of aggregate wealth invested in firm $n$ is $\frac{1}{N}$.

We illustrate the symmetry condition for the case $\rho = 0$ with two examples. For the special case in which the asset returns are uncorrelated, $\rho = 0$, the symmetry condition simplifies to

$$\frac{1}{N}\sum_{n=1}^{N} f_{hn} = \frac{1}{H}\sum_{h=1}^{H} f_{hn}, \quad \forall\, h \text{ and } n,$$

and the $H \times N$ familiarity matrix, $Q$, reduces to

$$F = [f_{hn}].$$

In both examples, we set the number of firms to be equal to the number of investors, $N = H$, and assume that each investor is equally familiar with a different subset of firms, and the investor is unfamiliar with the remaining firms. The number of firms in the familiar subset is the same for each investor.

In the first example, illustrated in Figure 1, we assume that investor 1 is familiar with firm 1, investor 2 is familiar with firm 2, and so on, with each investor investing only in the firm with which it is familiar; that is, $f_{1,1} = f_{2,2} = f_{3,3} = \ldots = f_{N,N} = f$, where $f \in (0, 1]$, while $f_{hn} = 0$ for $h \neq n$. Thus, the familiarity matrix in this case is:

$$F = \begin{bmatrix} f & 0 & \cdots & 0 \\ 0 & f & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & f \end{bmatrix}.$$

In the second example, illustrated in Figure 2, we assume that each investor is familiar with 2 firms. Let the firms be arranged in a circle, and let each investor $h$ be equally familiar with the two firms nearest to it on either side. Thus, in this case the familiarity matrix is:

$$F = \begin{bmatrix} f & f & 0 & \cdots & \cdots \\ 0 & f & f & 0 & \cdots \\ \vdots & \cdots & \cdots & \cdots & \vdots \\ f & 0 & \cdots & 0 & f \end{bmatrix}.$$

**Figure 1: First example of symmetry condition**

In this figure, we illustrate the first example of the symmetry condition for familiarity of investors with certain firms. We set the number of firms to be equal to the number of investors, $N = H$, and assume that investor 1 is familiar with firm 1, investor 2 is familiar with firm 2, and so on, with each investor investing only in the firm with which it is familiar; that is, $f_{1,1} = f_{2,2} = f_{3,3} = \ldots = f_{N,N} = f$, where $f \in (0, 1]$, while $f_{hn} = 0$ for $h \neq n$.

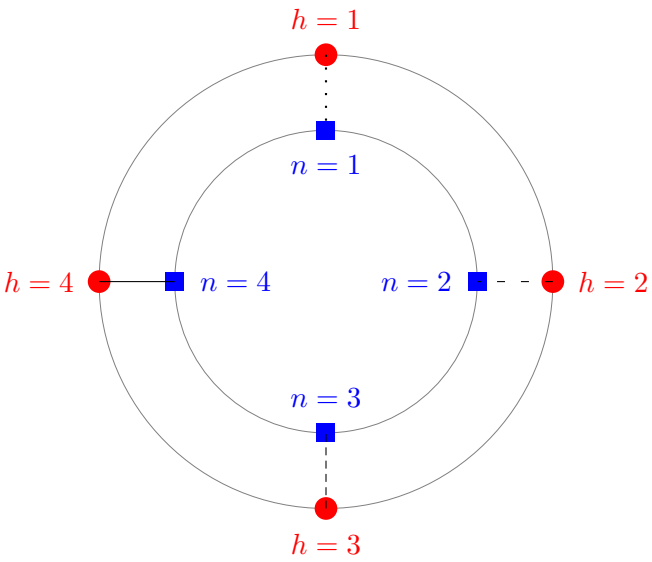
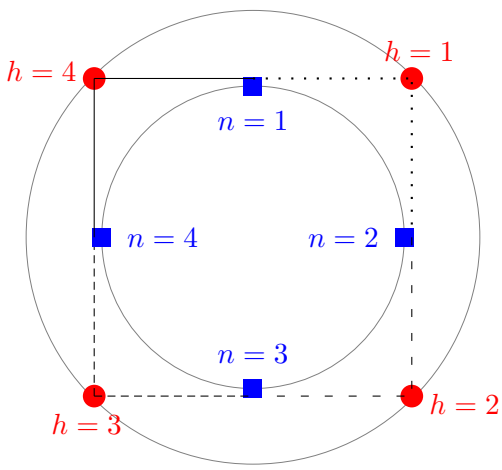



**Figure 2: Second example of symmetry condition**

In this figure, we illustrate the second example of the symmetry condition for familiarity of investors with certain firms. We set the number of firms to be equal to the number of investors, $N = H$, and assume that each investor is familiar with two firms. Let the firms be arranged in a circle, and let each investor $h$ be equally familiar with the two firms nearest to it on either side.



## 4.2 The Equilibrium Risk-free Interest Rate

By imposing market clearing in the risk-free bond market, we obtain the equilibrium risk-free interest rate shown in the following Proposition

**Proposition 7.** *The equilibrium risk-free interest rate is given by the constant*

$$i = \alpha - \gamma\,\sigma_p^2, \tag{32}$$

*where*

$$\sigma_p^2 = \frac{\sigma^2}{\widehat{q}}$$

*is the variance of the portfolio held by each investor and $\hat{q}$ is defined by*

$$\forall h,\ \hat{q} = \hat{q}_h,$$

*where $\hat{q}_h = \sum_{n=1}^{N} q_{hn}$.*

We can see immediately that reducing familiarity (that is, a reduction in $\widehat{q}$) increases the riskiness of each investor's portfolios, $\sigma_p$, leading to a greater demand for the risk-free asset, and hence, a decrease in the risk-free interest rate. So, clearly, the behavioral bias has an effect on the price of the single-period bond

## 4.3 The Price of the Aggregate Stock Market

From (32), we see that the aggregate stock market equity premium, the expected return on stocks in excess of the risk-free return, is given by:

$$\begin{aligned} \alpha - i &= \gamma\,\frac{\sigma^2}{\widehat{q}} \\ &= \gamma\,\sigma_p^2. \end{aligned} \tag{33}$$

From the right-hand side of the above expression, we see straight away that an increase in familiarity bias (decrease in $\widehat{q}$) will lead to an increase in the equity risk premium.

We denote by $p_t^{\text{agg}}$ the price-dividend ratio of the aggregate capital stock, or equivalently, the aggregate wealth-consumption ratio:

$$p_t^{\text{agg}} = \frac{K_t^{\text{agg}}}{C_t^{\text{agg}}} = \frac{W_t^{\text{agg}}}{C_t^{\text{agg}}}.$$

The following proposition provides closed-form expressions for the aggregate price-dividend ratio and the aggregate consumption-wealth ratio.

**Proposition 8.** *The aggregate price-dividend ratio is given in terms of the endogenous expected growth rate of aggregate output, $g$, and the perceived volatility of investor portfolios, $\sigma_p$ by*

$$p_t^{agg} = \frac{1}{i + \gamma\sigma_p^2 - g} \tag{34}$$

$$= \frac{1}{\alpha - g}, \tag{35}$$

*where $i$ is the risk-free interest rate given in (32), $\gamma$ is the risk aversion of investors in this economy, $\sigma_p^2$ is the variance of the portfolio held by each investor, $\alpha$ is the risk-adjusted discount rate, and $g$ is the expected growth rate of the dividend flow paid out by an individual firm, which is also common across all firms, and hence equal to the endogenous expected growth rate of aggregate output, given by*

$$g = \psi(\alpha - c) + (1 - \psi)\frac{1}{2}\gamma\sigma_p^2.$$

*The general equilibrium economy-wide consumption-wealth ratio is given by*

$$\frac{C_t^{agg}}{W_t^{agg}} = c = \alpha - g, \tag{36}$$

*where*

$$c = \psi\delta + (1 - \psi)\Big(\alpha - \frac{1}{2}\gamma\sigma_p^2\Big). \tag{37}$$

Observe that (34) is just the well-known Gordon-growth formula according to which the price-dividend ratio of an asset with constant dividend growth and volatility is the inverse of the constant risk-free rate plus the risk premium less the growth rate of dividends. The equation in (35) follows from the definition of the equity risk premium in (33).

From (34), we see that the effect of the familiarity bias on the aggregate price-dividend ratio, depends on the growth rate, $g$. We determine $g$ in two steps. In the first step, we derive an expression for the aggregate consumption-wealth ratio in equilibrium, denoted by $c$. In the second step, we derive the aggregate growth rate, $g$, in terms of $c$.

From (35), (36), and (37), we can see that the aggregate price-dividend ratio is

$$p_t^{\text{agg}} = \frac{1}{\psi\delta + (1-\psi)\left(\alpha - \frac{1}{2}\gamma\sigma_p^2\right)}.$$

Interpreting $\alpha - \frac{1}{2}\gamma\sigma_p^2$ as the expected return on the aggregate stock market adjusted for risk and familiarity bias, we see that the denominator is a weighted sum of the rate of time preference and the risk-adjusted return, with the weights depending on the elasticity of intertemporal substitution. A decrease in an individual investor's average correlation-adjusted familiarity, $\widehat{q}$, makes her portfolio riskier, that is, $\sigma_p^2 = \frac{\sigma^2}{\widehat{q}}$ increases. This reduces the equilibrium expected return adjusted for risk and familiarity bias, given by $\alpha - \frac{1}{2}\gamma\sigma_p^2$. The effect of this reduction in expected returns on the aggregate price-dividend ratio will depend on whether $\psi$ is greater or less than unity, which determines whether the substitution or income effect dominates.

### 4.4 Aggregate Investment and Growth

Above, we have examined the effect of the familiarity bias on asset prices. We now study how familiarity bias impacts aggregate investment and growth, starting with the following proposition.

**Proposition 9.** *The aggregate growth rate of the economy is the aggregate investment-capital ratio,*

$$g = \frac{I_t^{agg}}{K_t^{agg}},$$

*which is given by*

$$\frac{I_t^{agg}}{K_t^{agg}} = \alpha - c = \psi(\alpha - \delta) - \frac{1}{2}(\psi - 1)\gamma\sigma_p^2. \tag{38}$$

A decrease in an individual investor's average correlation-adjusted familiarity makes her portfolio riskier, that is, $\sigma_p^2 = \frac{\sigma^2}{\widehat{q}}$ increases. There is then a reduction in the equilibrium expected return adjusted for risk and familiarity bias, given by $\alpha - \frac{1}{2}\gamma\sigma_p^2$. When the substitution effect dominates ($\psi > 1$), the aggregate investment-capital ratio falls because investors will consume more of their wealth. We can also see that a decrease in the aggregate investment-capital ratio reduces output growth.

# 5 Quantitative Implications of the Model

To assess the magnitude of the effects of familiarity bias on asset prices and macroeconomic quantities, in this section we evaluate the closed-form results derived above using a reasonable set of parameter values. In our model, we have two sets of exogenous parameters. One set of parameters are for the stock return processes, while the second set govern agents' preferences and familiarity biases. In our analysis, we specify the parameters for the stock-return processes based on empirical estimates reported in Beeler and Campbell (2012, Table 2).[18] We then choose the values for the preference parameters based on a method-of-moments exercise, where we match moments from both the real and financial sectors.

When implementing the method of moments, we assume that the empirical data is drawn from an economy with familiarity bias. After estimating these parameter values, we then ask what asset prices and microeconomic quantities would be if investors did not suffer from this behavioral bias; that is, if the volatility of their portfolio was that of a fully-diversified portfolio, $\sigma_{1/N}$, as opposed to $\sigma_p$, where $\sigma_{1/N} < \sigma_p$ because of the gains from diversification.

## 5.1 Specification of Values for Non-preference-related Parameters

The first stock-return parameter we specify is $N$, the number of firms in which an investor can invest. In the United States, according to the World Federation of Exchanges, the number of companies traded on major U.S. stock exchanges at the end of 2013 was 5,008. To be conservative, we will assume that $N = 100$.[19] Choosing the number of investable stocks to be 100 rather than some larger number is conservative because the effect of the familiarity bias, which leads to underdiversified portfolios, decreases as $N$ decreases. All $N$ firms are assumed to have the same parameters driving their stock returns, and are heterogeneous only in terms of the shocks to their capital stocks.

Next, we specify the parameter $\alpha$, which is the expected rate of return on stocks. Based on the estimate in Beeler and Campbell (2012, Table 2), we specify that $\alpha = 7.50\%$ per

[18] We also undertook the analysis using parameters reported in Guvenen (2009), and the results are similar to the ones reported here.

[19] Alternatively, one can interpret these firms as ones in the S&P 100 index. Constituents of the S&P 100 are selected for sector balance and they represent about 45% of the market capitalization of the U.S. equity markets and about 57% of the market capitalization of the S&P 500.

annum, which is the historical average return on the U.S. equity market over the period 1930-2008.[20]

We now explain the values we choose for the volatility of individual firms, denoted by $\sigma$, and the correlation between firms, denoted by $\rho$. Herskovic, Kelly, and Van Nieuwerburgh (2014) find that the average of pairwise correlations over the period 1926-2010 for monthly stock returns in the United States to be 30%, so we set the correlation to be $\rho = 30\%$. The volatility of returns for a portfolio with equal amounts invested in $N_f$ familiar assets is given by the following expression:

$$\sigma_p = \left( \frac{1}{N_f}\sigma^2 + \left(1 - \frac{1}{N_f}\right)\rho\sigma^2 \right)^{\frac{1}{2}}. \tag{39}$$

In the expression above, if we set $N_f = N = 100$, we get the volatility of the fully-diversified portfolio, which in our model corresponds to the market portfolio. Beeler and Campbell (2012) report that the volatility of the stock-market return over the period 1930–2008 is 20.17%. Above, we have already specified $N = 100$ and $\rho = 30\%$. Hence, the only free parameter in (39) is the volatility of individual stock returns, $\sigma$. Therefore, we choose $\sigma$ to match the aggregate stock-market volatility of 20.17%. This leads to an estimate of $\sigma = 36.4\%$ for the volatility of individual stocks.

On the other hand, if investor's hold only 3 familiar assets, as reported in Polkovnichenko (2005), the volatility of the investor's portfolio is obtained by setting $N_f = 3$ in equation (39), with $\rho = 0.30$ and $\sigma = 36.4\%$. This leads to a volatility of the familiarity-biased portfolio of $\sigma_p = 26.58\%$. Thus, the volatility of the portfolio of individual investors is 26.58%, relative to that of the fully-diversified portfolio 20.17%, is about a third higher.[21]

## 5.2 Choice of Values for Preference-related Parameters

In this section, we explain how we use the method of moments to estimate the values for the preference-related parameters. On the real side, the moments (in per annum terms) we would like to match are the investment-to-output ratio, which is about 25.7% (as reported in Uhlig (2006, Table 2)), and the expected growth rate of output in the economy, which is

[20] We adjust the numbers reported in Beeler and Campbell (2012, Table 2) for continuous compounding by adding half the variance to the estimated mean.

[21] In the expressions of interest, the quantity that appears is the variance of the portfolio of individual investors rather that the volatility; and, the variance of the underdiversified portfolios is about 70% higher than the variance of the fully-diversified portfolio.

**Table 1: Financial and Real Moments**

This table reports the values of moments from financial markets and the real sector. All values are reported in per annum terms. Other than the investment-output ratio, which is from Uhlig (2006, Table 2), the other moments are based on the numbers reported in Beeler and Campbell (2012, Table 2) for the period 1930–2008, after being corrected for continuous compounding by the addition of one-half of the variance.

| Moment | Value (%) |
|---|---|
| Mean of stock-market return | 7.50 |
| Volatility of stock-market return | 20.17 |
| Real Interest rate | 0.56 |
| Equity-market risk premium | 6.94 |
| Market Sharpe ratio | 34.40 |
| Dividend yield | 3.84 |
| Aggregate growth rate | 1.95 |
| Investment-output ratio | 25.70 |

reported in Beeler and Campbell (2012, Table 2) to be $g = 1.95\%$. On the financial side, we use the same values as those reported in Beeler and Campbell (2012, Table 2) for the period 1930–2008. The six moment conditions we use, along with the values for these moments, are given below.

$$\text{Real interest rate} \qquad 0.56\% = i = \alpha - \gamma\sigma_p^2, \tag{40}$$

$$\text{Equity risk premium:} \qquad 6.94\% = \alpha - i = \gamma\sigma_p^2, \tag{41}$$

$$\text{Market Sharpe ratio:} \qquad 34.40\% = \frac{\alpha - i}{\sigma_{1/N}} = \frac{\gamma\sigma_p^2}{\sigma_{1/N}}, \tag{42}$$

$$\text{Aggregate output growth rate:} \qquad 1.95\% = g, \tag{43}$$

$$\text{Dividend yield} \qquad 3.84\% = 1/p_t^{\text{agg}} = \alpha - g, \tag{44}$$

$$\text{Investment-output ratio:} \qquad 25.70\% = \frac{I^{\text{agg}}}{Y^{\text{agg}}} = 1 - \frac{\alpha - g}{\alpha}, \tag{45}$$

where $g = \psi(\alpha - \delta) - \frac{1}{2}(\psi - 1)\gamma\sigma_p^2$ and $\sigma_p$ is defined in equation (39).

To impose discipline on our choice of parameter values, we restrict the choice of the familiarity parameters $f_{hn}$ to be either 0 or 1. A value of $f_{hn} = 0$ implies that investor does not invest at all in asset $n$, while a value of $f_{hn} = 1$ implies that the investor is fully familiar with asset $n$. To interpret this restriction, note that if the correlation across returns of different firms were zero, then the familiarity parameter $\widehat{f}$ could be interpreted as the number of firms with which an investor is fully familiar. For example, if $\widehat{f} = 3$, it would

imply that each investor invests in only 3 firms; of course, the set of three firms differs across investors.

We have already specified the mean stock-market return, $\alpha = 7.50\%$, and we have used the volatility of the stock-market return, 20.17%, to choose the value for the volatility of individual stocks, $\sigma = 36.4\%$. Based on the evidence in Polkovnichenko (2005), we assume that investors hold only three risky assets, $N_f = 3$. From (39), this implies that the volatility of an investor's portfolio is $\sigma_p = 26.58\%$.

We now use the remaining six moment conditions listed in equations (40)–(45) to pin down the three preference parameters, $\delta$, $\gamma$, and $\psi$. The values for the mean and volatility of the stock-market return, along with the values for the moment conditions are listed in Table 1.

A careful examination of the first three moment conditions, (40), (41), and (42), shows that they depend on only the preference parameter $\gamma$, which appears linearly in these three conditions. Thus, the objective function in the method-of-moments minimization is a U-shaped quadratic equation in $\gamma$, and hence, one can identify the unique value of $\gamma$ that minimizes the sum of the squared errors of these three moment conditions.

Observe also that in the six moment conditions, the preference parameters $\delta$ and $\psi$ appear only through $g$. Furthermore, note that $g$ appears in only the last three moment conditions listed above: (43), (44), and (45). Thus, we pin down $g$ by choosing it to minimize the squared errors for these three moment conditions, where the errors are defined to be the difference between the values given on the left-hand side and the moment conditions given on the right-hand side. Moreover, these moment conditions are linear in $g$, and thus, minimizing the squared errors for these conditions again leads to a unique solution for $g$.

For the case where the number of familiar assets is $N_f = 3$, the volatility of the familiarity biased portfolio is $\sigma_p = 26.58\%$. With this value of $\sigma_p$, the moment-matching exercise leads to estimates of $\gamma = 0.9825$ and $g = 1.9423\%$. Having pinned down $\gamma$ and $g$ from the case *with* familiarity bias, there is a family of $\delta$ and $\psi$ that satisfies the moment condition for $g$. Below, we report three pairs of $(\psi, \delta)$ that satisfy this value of $g$. The three values we choose for $\psi$ are picked so that in one case $\psi > 1$, in the second case $\psi = 1$, and in the last case $\psi < 1$. The values of $\gamma$ and $g$, along with the family of $(\psi, \delta)$ are reported in Table 2.

Using the parameter values for the stock return process based on moments reported in Beeler and Campbell (2012), along with the values for the preference parameters estimated above, we report in Table 3 the values of the moments from financial and real data. We see from this table that the parameters estimated using the method of moments do a reasonably good job of satisfying the six moment conditions.

We now compare asset prices and macroeconomic quantities with and without familiarity bias. Note that in the absence of familiarity bias, the volatility of an individual investor's portfolio would be reduced, and it is this reduction in volatility that drives the differences in the quantities of interest.

The numbers reported in Table 4 show the values for six quantities from the model with and without familiarity bias. The six quantities are the interest rate, the equity risk premium, the Sharpe ratio, the dividend yield, the growth rate of aggregate output, and the aggregate investment-output ratio. The second column in Table 4 reports numbers for the case where investors have a behavioral bias toward familiar assets. The last three columns give the values for asset prices and real quantities for the case where the behavioral bias is absent. These three columns give results for three pairs of $(\psi, \delta)$ that correspond to the same level of the aggregate growth rate with familiarity bias; that is, $g = 1.94\%$.

Comparing the case with familiarity bias and without this bias, we have three observations. First, we focus on the case where $\psi = 1.5$. Comparing the numbers in the second and third column of Table 4, we see that the changes in asset prices and real quantities are substantial for the case with familiarity bias compared to the case without. For instance, the interest rate, $i$, increases from 0.56% to 3.51% because the fully-diversified portfolio is

**Table 2: Estimated Values of Preference Parameters**

This table reports the results of the moment-matching exercise, where we choose $\gamma$ and $g$ to match six moments from financial markets and the real sector. There is a family of $\delta$ and $\psi$ that satisfies the moment condition for $g$. Below, we report three pairs of $(\psi, \delta)$ that satisfy this value of $g$. The three values we choose for $\psi$ are picked so that in one case $\psi > 1$, in the second case $\psi = 1$, and in the last case $\psi < 1$.

| $\gamma$ | $g(\%)$ | $\psi$ | $\delta$ |
|---|---|---|---|
| 0.9825 | 1.9423 | 1.5000 | 0.0505 |
| 0.9825 | 1.9423 | 1.0000 | 0.0556 |
| 0.9825 | 1.9423 | 0.5000 | 0.0709 |

**Table 3: Matching Empirical Moments to Model Moments**

This table compares the moments of asset returns and macroeconomic quantities from the data and from the model.

| Variable | Data (%) | Model (%) |
|---|---|---|
| Interest rate | 0.56 | 0.56 |
| Equity risk premium | 6.94 | 6.94 |
| Sharpe ratio | 34.43 | 34.43 |
| Dividend yield | 3.14 | 5.56 |
| Aggregate growth rate | 1.95 | 1.94 |
| Investment-to-output ratio | 25.70 | 25.88 |

**Table 4: Effect of Familiarity Bias on Asset Prices and Real Quantities**

In this table, we report the financial prices and real quantities listed in the first column, with and without familiarity bias. The quantities without familiarity bias are reported for three pairs of values for $(\psi, \delta)$: $(1.5, 0.0505)$, $(1.0, 0.0556)$, and $(0.5, 0.0709)$.

| | | $\psi = 1.5$ $\delta = 0.0505$ | $\psi = 1.0$ $\delta = 0.0556$ | $\psi = 0.5$ $\delta = 0.0709$ |
|---|---|---|---|---|
| Quantity | With familiarity bias (%) | Without familiarity bias (%) | | |
| Interest rate | 0.56 | 3.51 | 3.51 | 3.51 |
| Equity risk premium | 6.94 | 4.00 | 4.00 | 4.00 |
| Sharpe ratio | 34.43 | 19.82 | 19.82 | 19.82 |
| Dividend yield | 5.57 | 4.83 | 5.57 | 6.30 |
| Aggregate growth rate | 1.94 | 2.68 | 1.94 | 1.20 |
| Investment-to-output ratio | 25.88 | 35.65 | 25.83 | 16.01 |

less risky than the familiarity-biased portfolio, and hence, there is a decrease in the demand for the risk-free asset. The change in the volatility of investors' portfolios leads to a change in also the equity risk premium, $\alpha - i$: the equity risk premium drops from 6.94% to 4.00%. Similarly, there is a change in the Sharpe ratio from 34.43% to 19.82%. The three quantities we have described above do not depend on $\delta$ and $\psi$, and hence, the values of these quantities in the last two columns of the table (for $\psi = 1.0$ and $\psi = 0.5$) are identical to their values in the column for $\psi = 1.5$.

Next, we compare the aggregate dividend yield, $1/p_t^{\text{agg}}$. We see that with familiarity bias the aggregate dividend yield is 5.57%, whereas without the bias it changes to 4.83% if $\psi = 1.5$ and to 6.30% if $\psi = 0.5$; for $\psi = 1.0$, the income and substitution effects offset each other exactly, and hence, there is no change. Similarly, the aggregate investment-to-output

ratio, $\frac{I^{\text{agg}}}{Y^{\text{agg}}}$, is 25.88% with the familiarity bias while it is 35.65% in the absence of the bias if $\psi = 1.5$ and 16.01% if $\psi = 0.5$. The growth rate $g$ is 1.94% with familiarity bias while it is 2.68% without the bias if $\psi = 1.5$ and 1.20% if $\psi = 0.5$. Observe that whether holding a fully-diversified portfolio leads to an increase or decrease in the growth rate depends on whether or not $\psi$ is larger or smaller than one, which determines whether it is the substitution or income effect resulting from the change in portfolio risk that dominates.[22] The results described above illustrate that the behavioral bias toward familiar assets has a substantial effect on both financial markets and macroeconomic aggregates.

The quantities reported in the above experiment are driven by the change in the volatility of the investor's portfolio when it is biased toward familiar assets ($\sigma_p = 26.58\%$) relative to when it is fully diversified ($\sigma_m = 20.17\%$). In this experiment, we assumed that the investor is familiar with only $N_f = 3$ risky assets. One may wonder how the results change if the investor is assumed to be familiar with more than three assets. In Table 5 we report the effect of the familiarity bias for three additional cases. In the first case, we study the situation where out of the $N = 100$ assets available, the investor is familiar with $N_f = 6$ assets; in the second case, $N_f = 9$; and, in the third case, $N_f = 12$. As the number of familiar assets increases, the volatility of the investor's portfolio, $\sigma_p$, declines and approaches the volatility of the fully-diversified portfolio where the investor holds all available risky assets: $N_f = N = 100$. The volatility of the portfolio corresponding to different number of familiar assets held is displayed below.

| $N_f$ | $\sigma_p(\%)$ |
|---|---|
| 3 | 26.58 |
| 6 | 23.50 |
| 9 | 22.37 |
| 12 | 21.79 |
| 100 | 20.17 |

Based on the estimates of return volatility of the familiarity-biased portfolios for $N_f \in \{3, 6, 9\}$, we repeat the estimation of $\gamma$. The estimates of $\gamma$ that correspond to the portfolio volatilities of 23.5%, 22.37%, and 21.79% are 1.2576, 1.3870, and 1.4623, respectively. We see from these estimates of relative risk aversion, $\gamma$, that in order to match the financial moments, as the volatility of the portfolio declines, it is offset by an increase in the investor's

[22] Of course, if all investors switched to holding a fully-diversified portfolio, social welfare would increase irrespective of whether there was an increase or a decrease in growth.

risk aversion. The estimate of $g$ does not depend on the volatility of the portfolio return, and hence, it stay the same, $g = 1.94$, and the values for the pairs of $(\psi, \delta)$ that correspond to $g$ also do not change (because in the expression for $g$ the change in $\sigma_p$ is fully offset by the change in $\gamma$). As before, we look at three pairs of $(\psi, \delta)$ that correspond to this level of $g$, for each case of portfolio volatility.

Based on estimates of $\gamma$ and $g$, we compute the financial and microeconomic quantities that correspond to the portfolio volatilities when $N_f \in \{3, 6, 9\}$. These results are reported in Panels A, B, and C of Table 5. We see from Table 5 that even for the case where $N_f = 12$, the effects on asset prices and macroeconomic quantities are not small. For example, the interest rate is more than double in a world without familiarity bias, 1.55%, compared to the interest rate of 0.56% with familiarity bias. The difference in the equity risk premium is 1%, while the difference in the Sharpe ratio is about 5%. Similarly, the difference in the investment-to-output ratio is more than 3% relative to the cases of $\psi = 1.5$ and $\psi = 0.5$.

Above, the base case we considered was one where each investor held a portfolio with only three familiar assets. Then, for robustness, we considered the cases where the number of familiar assets ranged from 3 to 12. But, one may wish to consider a different situation in which, besides holding personal portfolios biased toward a few familiar assets, investors also have wealth invested in professionally-managed mutual funds, which are well diversified. In this case, the volatility of the familiarity-biased portfolio will be lower than $\sigma_p = 26.58\%$ we have used for the base case analyzed in Table 4, where the investor was holding only three familiar assets. To investigate the effect of the familiarity bias in this kind of a setting, we repeat our analysis assuming now that the investor has varying proportions of her wealth invested in a fully-diversified market portfolio, and only the balance of her wealth invested in the portfolio biased toward three familiar assets.

In this analysis, where part of the investor's wealth is held in a fully-diversified portfolio, we consider three cases. In the first case, the investor has 30% of her wealth invested in a fully-diversified portfolio, with the remaining proportion invested in a portfolio biased toward three familiar assets; in the second, 45% of her wealth is invested in a fully-diversified portfolio; and, in the third, the investor has 52.5% of her wealth invested in a fully-diversified portfolio. It turns out that these three cases correspond to the same level of volatility of the biased portfolio, $\sigma_p$, as if the investor had invested in 6, 9, and 12 familiar assets, respectively, instead of just 3. Thus, the results for the case where the investor holds $\{30\%, 45\%, 52.5\%\}$

**Table 5: Effect of Familiarity Bias for Different Number of Familiar Assets**
In this table, we report the financial prices and real quantities listed in the first column, with and without familiarity bias, but for the case where the investor is biased toward $N_f$ familiar assets, where in Panel A the number of familiar assets is $N_f = 6$, in Panel B we have $N_f = 9$, and in Panel C we have $N_f = 12$. The quantities without familiarity bias are reported for three pairs of values for $(\psi, \delta, )$: $(1.5, 0.0505)$, $(1.0, 0.0556)$, and $(0.5, 0.0709)$.

| Quantity | With familiarity bias (%) | $\psi = 1.5$ $\delta = 0.0505$ | $\psi = 1.0$ $\delta = 0.0556$ | $\psi = 0.5$ $\delta = 0.0709$ |
|---|---|---|---|---|
| | | Without familiarity bias (%) | | |
| *Panel A:* $N_f = 6$ | | | | |
| Interest rate | 0.56 | 2.39 | 2.39 | 2.39 |
| Equity risk premium | 6.94 | 5.12 | 5.12 | 5.12 |
| Sharpe ratio | 34.43 | 25.37 | 25.37 | 25.37 |
| Dividend yield | 5.57 | 5.11 | 5.57 | 6.02 |
| Aggregate growth rate | 1.94 | 2.40 | 1.94 | 1.48 |
| Investment-to-output ratio | 25.83 | 31.92 | 25.83 | 19.74 |
| *Panel B:* $N_f = 9$ | | | | |
| Interest rate | 0.56 | 1.86 | 1.86 | 1.86 |
| Equity risk premium | 6.94 | 5.64 | 5.64 | 5.64 |
| Sharpe ratio | 34.43 | 27.98 | 27.98 | 27.98 |
| Dividend yield | 5.57 | 5.24 | 5.57 | 5.89 |
| Aggregate growth rate | 1.94 | 2.26 | 1.94 | 1.61 |
| Investment-to-output ratio | 25.83 | 30.16 | 25.83 | 21.50 |
| *Panel C:* $N_f = 12$ | | | | |
| Interest rate | 0.56 | 1.55 | 1.55 | 1.55 |
| Equity risk premium | 6.94 | 5.95 | 5.95 | 5.95 |
| Sharpe ratio | 34.43 | 29.50 | 29.50 | 29.50 |
| Dividend yield | 5.57 | 5.32 | 5.57 | 5.81 |
| Aggregate growth rate | 1.94 | 2.19 | 1.94 | 1.69 |
| Investment-to-output ratio | 25.83 | 29.14 | 25.83 | 22.52 |

of her wealth in a fully-diversified portfolio, with only the balance invested in three familiar assets, correspond to the results reported in the three panels of Table 5. Thus, we conclude that even in this setting the effects of familiarity bias on asset prices and macroeconomic aggregates are substantial.

# 6 Conclusion

In response to the question posed in the title of this paper, we have provided a simple example of an economy where biases in portfolios of individual investors wash out in the aggregate, but the implications of these biases on asset prices and macroeconomic quantities do not. Investors in our model have a behavioral bias toward familiar assets. We construct this familiarity bias so that it is symmetric across all investors. Consequently, the portfolio bias cancels out when aggregated across investors. However, the effect of the familiarity bias does *not* cancel out in the aggregate. Instead, investor-level distortions to individual consumption stemming from excessive financial risk taking are *amplified* by aggregation. Consequently, the behavioral bias of individual investors toward familiar assets impacts the prices of these assets and distorts aggregate growth and investment. In particular, the increased risk from holding biased portfolios, which increases the demand for the risk-free asset, leads to a higher equity risk premium and a lower risk-free rate that match the values observed empirically. For instance, if we calibrate the model to U.S. stock-market data, the familiarity bias reduces the interest rate by about 3%, increases the equity risk premium by 3%, increases the Sharpe ratio by 15%, changes the growth rate by 0.74% and the investment-to-output ratio by 9.82%.

In this paper, we have constructed a specific example to illustrate that the effects of behavioral biases at the individual level do not wash out in aggregate. But, there is nothing special about this example. The only key aspect of this example is that the idiosyncratic behavioral bias we consider impacts the second moment—an increase in the variance of each investor's portfolio. Because the variance of the portfolio of each investor *increases*, the aggregate impact of the idiosyncratic portfolio arising from familiarity bias does not cancel out. This will be true also of other behavioral biases, such as overconfidence about particular assets relative to others, that impact the risk of the investor's portfolio.[23]

[23] See Brenner, Izhakian, and Sade (2011) for the relation between familiarity and overconfidence.

# A Appendix

In this appendix, we provide all derivations for the results in the main text.

### Proof of Lemma 1

The definition of the certainty equivalent in (4) implies that

$$\mu_t[U_{h,t+dt}] = E_t\left[U_{h,t+dt}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}.$$

Therefore

$$\mu_t[U_{h,t+dt}] = E_t\left[U_{h,t+dt}^{1-\gamma}\right]^{\frac{1}{1-\gamma}} = E_t\left[U_{h,t}^{1-\gamma} + d(U_{h,t}^{1-\gamma})\right]^{\frac{1}{1-\gamma}}.$$

Applying Ito's Lemma, we obtain

$$\begin{aligned} d(U_{h,t}^{1-\gamma}) &= (1-\gamma)U_{h,t}^{-\gamma}dU_{h,t} - \frac{1}{2}(1-\gamma)\gamma U_{h,t}^{-\gamma-1}(dU_{h,t})^2 \\ &= (1-\gamma)U_{h,t}^{1-\gamma}\left[\frac{dU_{h,t}}{U_{h,t}} - \frac{1}{2}\gamma\left(\frac{dU_{h,t}}{U_{h,t}}\right)^2\right]. \end{aligned}$$

Therefore

$$\begin{aligned} \mu_t[U_{h,t+dt}] = E_t\left[U_{h,t+dt}^{1-\gamma}\right]^{\frac{1}{1-\gamma}} &= U_{h,t}\left(E_t\left[1 + (1-\gamma)\left[\frac{dU_{h,t}}{U_{h,t}} - \frac{1}{2}\gamma\left(\frac{dU_{h,t}}{U_{h,t}}\right)^2\right]\right]\right)^{\frac{1}{1-\gamma}} \\ &= U_{h,t}\left(1 + (1-\gamma)\left[E_t\left[\frac{dU_{h,t}}{U_{h,t}}\right] - \frac{1}{2}\gamma E_t\left[\left(\frac{dU_{h,t}}{U_{h,t}}\right)^2\right]\right]\right)^{\frac{1}{1-\gamma}}. \end{aligned}$$

Hence,

$$\mu_t[U_{h,t+dt}] = U_{h,t}\left(1 + E_t\left[\frac{dU_{h,t}}{U_{h,t}}\right] - \frac{1}{2}\gamma E_t\left[\left(\frac{dU_{h,t}}{U_{h,t}}\right)^2\right]\right) + o(dt).$$

Therefore, in the continuous time limit, we obtain

$$\frac{\mu_t[dU_{h,t+dt}]}{dt} = \frac{\mu_t[U_{h,t+dt}] - U_{h,t}}{dt} = U_{h,t}\left(E_t\left[\frac{dU_{h,t}}{U_{h,t}}\right] - \frac{1}{2}\gamma E_t\left[\left(\frac{dU_{h,t}}{U_{h,t}}\right)^2\right]\right).$$

□

## Auxiliary Lemmas for Proof of Theorem 1

In order to prove Theorem 1, giving the familiarity-biased certainty equivalent, we shall need some additional definitions and lemmas.

Existing work (Uppal and Wang (2003)) considers familiarity biases with respect to orthogonal factors. In contrast, we assume investors have varying degrees of familiarity with respect to firms, rather than the orthogonal factor structure underlying firm-level returns. In general, firms have returns which are not mutually orthogonal, so we cannot directly use the results in Uppal and Wang (2003). We must transform the matrix $\Gamma_h$, which encodes familiarity biases with respect to firms into a matrix $\overline{\Gamma}_h$, which encodes familiarity biases with respect to orthogonal factors. Having changed to the orthogonal factor basis, we use the derivation of the penalty function for deviations from the reference probability measure given in Theorem 1 of Uppal and Wang (2003). We can then obtain the correct form of the penalty function with respect to the original non-orthogonal basis of shocks to firm-level returns.

We begin by defining the orthogonal *factor basis.*

**Definition A1.** *The factor basis is a vector Brownian motion,* $\overline{\mathbf{Z}}$ *(under the common reference measure* $\mathbb{P}$*):*

$$\overline{\mathbf{Z}} = (\overline{Z}_1, \ldots, \overline{Z}_N)^\top,$$

*where* $\overline{Z}_n$, $n \in \{1, \ldots, N\}$ *is a set of mutually orthogonal standard Brownian motions under* $\mathbb{P}$ *such that*

$$\overline{\mathbf{Z}} = M^{-1}(Z_1, \ldots, Z_N)^\top,$$

*and*

$$M = \Omega^{\frac{1}{2}}. \tag{A1}$$

To see how the above definition is constructed, observe that the matrix $M^\top$ maps the factor basis to the original non-orthogonal basis:

$$M^\top \overline{\mathbf{Z}} = (Z_1, \ldots, Z_N)^\top.$$

We now uncover the properties of $M$. We know that

$$(dZ_1, \ldots, dZ_N)^\top (dZ_1, \ldots, dZ_N) = \Omega$$

and so

$$M^\top d\overline{\mathbf{Z}}\, d\overline{\mathbf{Z}}^\top M = \Omega.$$

We know that $d\overline{Z}_i d\overline{Z}_j = \delta_{ij} dt$, where $\delta_{ij}$ is the Kronecker delta, and so

$$d\overline{\mathbf{Z}}\, d\overline{\mathbf{Z}}^\top = I,$$

where $I$ is the $N \times N$ identity matrix. Therefore

$$M^\top M = \Omega.$$

We have some degree of freedom with how we define $M$, so for parsimony we assume that $M$ is symmetric, i.e. $M^\top = M$. Hence

$$M^2 = \Omega.$$

The matrix $\Omega$ is real and symmetric. Therefore, $\Omega$ has $N$ real eigenvalues, $d_n \in \mathbb{R}$, $n \in \{1, \ldots, N\}$ and the corresponding (column) eigenvectors $\underline{e}_n$, $n \in \{1, \ldots, N\}$ are mutually orthogonal with real elements. We can therefore define the $N \times N$ real matrix

$$S = [\underline{e}_1, \ldots, \underline{e}_N]$$

and use $S$ to diagonalize $\Omega$:

$$\Omega = SDS^{-1},$$

where $D = diag(d_1, \ldots, d_N)$. It follows that the square root of the matrix $\Omega$ is given by

$$\Omega^{\frac{1}{2}} = SD^{\frac{1}{2}}S^{-1},$$

so we obtain

$$M = \Omega^{\frac{1}{2}}.$$

Intuitively, the factor basis $\overline{\mathbf{Z}}$ represents a set of mutually orthogonal factors underlying the returns on firms' capital and we can use it to rewrite the stochastic differential equations for the evolution of firms' capital stocks as

$$\left(\frac{dK_1 + D_1 dt}{K_1}, \ldots, \frac{dK_N + D_N dt}{K_N}\right)^\top = \alpha \mathbf{1} dt + \sigma M d\overline{\mathbf{Z}}.$$

We now exploit the factor basis representation to define the measure $\mathbb{Q}^{\boldsymbol{\nu}_h}$.

**Definition A2.** *The probability measure $\mathbb{Q}^{\boldsymbol{\nu}_h}$ is defined by*

$$\mathbb{Q}^{\boldsymbol{\nu}_h}(A) = \mathbb{E}[1_A \xi_{h,T}],$$

*where $\mathbb{E}$ is the expectation under $\mathbb{P}$, $A$ is an event and $\xi_{h,t}$ is the exponential martingale (under the reference probability measure $\mathbb{P}$)*

$$\frac{d\xi_{h,t}}{\xi_{h,t}} = \frac{1}{\sigma}\overline{\boldsymbol{\nu}}_{h,t}^\top d\overline{\boldsymbol{Z}}_t,$$

*where the $N \times 1$ vector $\overline{\boldsymbol{\nu}}_{h,t}$ is the factor basis representation of the $N \times 1$ vector $\boldsymbol{\nu}_{h,t}$:*

$$\overline{\boldsymbol{\nu}}_{h,t} = M^{-1}\boldsymbol{\nu}_{h,t}. \tag{A2}$$

Recall that when an investor is less familiar with a particular firm, she adjusts its expected return, which is equivalent to changing the reference measure to a new measure, denoted by $\mathbb{Q}^{\nu_h}$. Applying Girsanov's Theorem, we see that under the new measure $\mathbb{Q}^{\boldsymbol{\nu}_h}$, the evolution of firm $n$'s capital stock is given by

$$dK_{n,t} = [(\alpha + \nu_{hn,t})K_{n,t} - D_{n,t}]dt + \sigma K_{n,t} dZ^{\boldsymbol{\nu}_h}_{n,t},$$

where $Z^{\boldsymbol{\nu}_h}_{n,t}$ is a standard Brownian motion under $\mathbb{Q}^{\boldsymbol{\nu}_h}$, such that

$$dZ^{\boldsymbol{\nu}_h}_{n,t} dZ^{\boldsymbol{\nu}_h}_{m,t} = \begin{cases} dt, & n = m. \\ \rho dt, & n \neq m. \end{cases}$$

**Lemma A1.** *The matrix, $\overline{\Gamma}_h$, which encodes familiarity biases with respect to the factor basis is given by*

$$\overline{\Gamma}_h = \Omega^{-\frac{1}{2}} \Gamma_h \Omega^{\frac{1}{2}}. \tag{A3}$$

**Proof of Lemma A1**

Consider a linear map which acts on $\boldsymbol{Z}$ and is represented with respect to the original basis via the matrix $A$. Suppose the same linear map is represented with respect to the factor basis via the matrix $\overline{A}$. It is well known that

$$\overline{A} = M^{-1} A M.$$

Therefore, under the factor basis, the matrix $\overline{\Gamma}_h$, which encodes familiarity biases with respect to the factor basis is given by

$$\overline{\Gamma}_h = M^{-1} \Gamma_h M = \Omega^{-\frac{1}{2}} \Gamma_h \Omega^{\frac{1}{2}}.$$

We now define a penalty function for using the measure $\mathbb{Q}^{\boldsymbol{\nu}_h}$ instead of $\mathbb{P}$. Since the factor basis is orthogonal, we can use Theorem 1 in Uppal and Wang (2003) to define the penalty function with respect to the factor basis as shown below.

**Definition A3.** *The penalty function for investor $h$ associated with her familiarity biases is given by*

$$\hat{L}_{h,t} = \frac{1}{\sigma^2} \overline{\boldsymbol{\nu}}^{\top}_{h,t} \overline{\Gamma}^{-1}_h \overline{\boldsymbol{\nu}}_{h,t}.$$

The following additional definition will aid in understanding the role of the penalty function.

**Definition A4.** *The probability measure $\mathbb{Q}^{\boldsymbol{\nu}_{h,n}}$ is defined by*

$$\mathbb{Q}^{\boldsymbol{\nu}_{h,n}}(A) = \mathbb{E}[1_A \xi_{h,n,T}],$$

*where $\mathbb{E}$ is the expectation under $\mathbb{P}$, $A$ is an event and $\xi_{h,n,t}$ is the exponential martingale (under the reference probability measure $\mathbb{P}$)*

$$\frac{d\xi_{h,n,t}}{\xi_{h,n,t}} = \frac{1}{\sigma}\overline{\nu}_{h,n,t}d\overline{Z}_{n,t}.$$

The probability measure $\mathbb{Q}^{\boldsymbol{\nu}_{h,n}}$ is just the measure associated with familiarity bias along the $n$'th orthogonal factor. Familiarity bias along this factor is equivalent to using $\mathbb{Q}^{\boldsymbol{\nu}_{h,n}}$ instead of $\mathbb{P}$, which leads to a loss in information. The information loss stemming from familiarity bias along the $n$'th orthogonal factor can be quantified via the date-$t$ conditional Kullback-Leibler divergence between $\mathbb{P}$ and $\mathbb{Q}^{\boldsymbol{\nu}_{h,n}}$, given by

$$D_{t,u}^{KL}[\mathbb{P}|\mathbb{Q}^{\boldsymbol{\nu}_{h,n}}] = E_t^{\mathbb{Q}^{\boldsymbol{\nu}_{h,n}}}\left[\ln\left(\frac{\xi_{h,n,u}}{\xi_{h,n,t}}\right)\right].$$

Now observe that the penalty function can be written in terms of the information losses along each of the orthogonal factors, i.e.

$$\hat{L}_{h,t} = D_{t,u}^{KL}[\mathbb{P}|\mathbb{Q}^{\boldsymbol{\nu}_{h,n}}][\overline{\Gamma}_h^{-1}]_{nm}D_{t,u}^{KL}[\mathbb{P}|\mathbb{Q}^{\boldsymbol{\nu}_{h,m}}],\ n,m \in \{1,\ldots,N\}, \tag{A4}$$

where we employ the Einstein summation convention.

We now unravel the intuition embedded within (A4). We can think of $\overline{\Gamma}_h^{-1}$ as a weighting matrix for information losses, analogous to the weighting matrix in the generalized method of moments. Suppose, for simplicity that $\rho = 0$, so the shocks to firm-level returns are mutually orthogonal, rendering the original basis equal to the orthogonal factor basis. Suppose further that investor $h$ is completely unfamiliar with all the orthogonal factors save factor 1. In this case,

$$\overline{\Gamma}_h^{-1} = diag\left(\frac{f_1}{1-f_1}, \mathbf{0}_{N-1}^{\top}\right),$$

where $\mathbf{0}_{N-1}$ is the $(N-1)\times 1$ vector of zeros. The penalty function reduces to

$$\hat{L}_{h,t} = \frac{f_1}{1-f_1}\left(D_{t,u}^{KL}[\mathbb{P}|\mathbb{Q}^{\boldsymbol{\nu}_{h,1}}]\right)^2,$$

so the information losses along the factors with which the investor is totally unfamiliar are not penalized in the penalty function. The investor is penalized only for deviating from $\mathbb{P}$ along a particular factor if she has some level of familiarity with that factor. If she has full familiarity with a factor, the associated penalty becomes infinitely large, so when making decisions involving this factor, she will not deviate at all from the reference probability measure $\mathbb{P}$.

We end with the main lemma of this section, which shows how to write the penalty function when using the original non-orthogonal basis of shocks to firm-level returns.

**Lemma A2.** *The penalty function for investor $h$ associated with her familiarity biases can be written in terms of the original non-orthogonal basis*

$$\hat{L}_{h,t} = \frac{1}{\sigma^2}\boldsymbol{\nu}_{h,t}^\top(\Gamma_h\Omega)^{-1}\boldsymbol{\nu}_{h,t}.$$

**Proof of Lemma A2**

From Definition A3, we know

$$\hat{L}_{h,t} = \frac{1}{\sigma^2}\overline{\boldsymbol{\nu}}_{h,t}^\top\overline{\Gamma}_h^{-1}\overline{\boldsymbol{\nu}}_{h,t}.$$

From Equations (A1), (A2), and (A3), we obtain

$$\begin{aligned}
\hat{L}_{h,t} &= \frac{1}{\sigma^2}(\Omega^{-\frac{1}{2}}\boldsymbol{\nu}_{h,t})^\top(\Omega^{-\frac{1}{2}}\Gamma_h\Omega^{\frac{1}{2}})^{-1}(\Omega^{-\frac{1}{2}}\boldsymbol{\nu}_{h,t}) \\
&= \frac{1}{\sigma^2}\boldsymbol{\nu}_{h,t}^\top\Omega^{-\frac{1}{2}}\Omega^{-\frac{1}{2}}\Gamma_h^{-1}\Omega^{\frac{1}{2}}\Omega^{-\frac{1}{2}}\boldsymbol{\nu}_{h,t} \\
&= \frac{1}{\sigma^2}\boldsymbol{\nu}_{h,t}^\top\Omega^{-1}\Gamma_h^{-1}\boldsymbol{\nu}_{h,t} \\
&= \frac{1}{\sigma^2}\boldsymbol{\nu}_{h,t}^\top(\Gamma_h\Omega)^{-1}\boldsymbol{\nu}_{h,t}.
\end{aligned}$$

□

The above lemma tells us that we do not obtain the correct penalty function merely through replacing $\overline{\boldsymbol{\nu}}_{h,t}$ and $\overline{\Gamma}_h$ by $\boldsymbol{\nu}_{h,t}$ and $\Gamma_h$, respectively. The matrix $\Gamma_h$ must be post-multiplied by the correlation matrix, $\Omega$, to reflect the fact that a given level of familiarity with respect to a particular firm translates into a familiarity with respect to firms with correlated returns. We can see this explicitly by noting that

$$[\Gamma_h\Omega]_{nm} = \begin{cases} \frac{1-f_n}{f_n}, & n = m \\ \rho\frac{1-f_n}{f_n}, & n \neq m \end{cases}. \tag{A5}$$

While the matrix $\Gamma_h$ is diagonal, the matrix $\Gamma_h\Omega$, which appears in the penalty function is not diagonal – familiarity bias with respect to a particular firm translates into familiarity bias with respect to correlated firms – the level of translated familiarity bias depends directly on the correlation coefficient.

## Proof of Theorem 1

Using the penalty function given in Lemma A2, the construction of the familiarity-biased certainty equivalent of date-$t + dt$ utility is straightforward – it is merely the certainty-equivalent of date-$t+dt$ utility computed using the probability measure $\mathbb{Q}^{\boldsymbol{\nu}_h}$ plus a penalty.

The investor will choose her adjustment to expected returns by minimizing the familiarity-biased certainty equivalent of her date-$t+dt$ utility — the penalty stops her from making the adjustment arbitarily large by penalizing her for larger adjustments. The size of the penalty is a measure of the information she loses by deviating from the common reference measure, adjusted by her familiarity preferences, and so

$$\mu^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}] = \widehat{\mu}^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}] + U_{h,t}L_{h,t}dt,$$

where $\widehat{\mu}^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}]$ is defined by

$$u_\gamma\left(\widehat{\mu}^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}]\right) = E_t^{\mathbb{Q}^{\boldsymbol{\nu}_h}}[u_\gamma\left(U_{h,t+dt}\right)],$$

and

$$L_{h,t} = \frac{1}{2\gamma}\hat{L}_{h,t}.$$

□

## Proof of Corollary 1

The date-$t$ familiarity-biased certainty equivalent of date-$t+dt$ investor utility is given by (8), (9), and (10). We can see that $\widehat{\mu}^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}]$ is like a certainty equivalent, but with the expectation taken under $\mathbb{Q}^{\boldsymbol{\nu}_h}$ in order to adjust for familiarity bias. From Lemma 1 , we know that

$$\widehat{\mu}^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}] = U_{h,t}\left(1 + E_t^{\mathbb{Q}^{\nu_h}}\left[\frac{dU_{h,t}}{U_{h,t}}\right] - \frac{1}{2}\gamma E_t\left[\left(\frac{dU_{h,t}}{U_{h,t}}\right)^2\right]\right) + o(dt).$$

We therefore obtain from (1)

$$\mu^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}] = U_{h,t}\left(1 + E_t^{\mathbb{Q}^{\nu_h}}\left[\frac{dU_{h,t}}{U_{h,t}}\right] - \frac{1}{2}\gamma E_t\left[\left(\frac{dU_{h,t}}{U_{h,t}}\right)^2\right] + L_{h,t}dt\right) + o(dt). \tag{A6}$$

Applying Ito's Lemma, we see that under $\mathbb{Q}^{\boldsymbol{\nu}_h}$

$$dU_{h,t} = W_{h,t}\frac{\partial U_{h,t}}{\partial W_{h,t}}\frac{dW_{h,t}}{W_{h,t}} + \frac{1}{2}W_{h,t}^2\frac{\partial^2 U_{h,t}}{\partial W_{h,t}^2}\left(\frac{dW_{h,t}}{W_{h,t}}\right)^2,$$

where

$$\frac{dW_{h,t}}{W_{h,t}} = \left(1 - \sum_{n=1}^{N}\omega_{hn,t}\right)idt + \sum_{n=1}^{N}\omega_{hn,t}\left((\alpha + \nu_{h,t})dt + \sigma dZ_{n,t}^{\mathbb{Q}^{\nu_h}}\right) - \frac{C_{h,t}}{W_{h,t}}dt.$$

Hence, from Girsanov's Theorem, we have

$$E_t^{\mathbb{Q}^{\nu_h}}\left[\frac{dU_{h,t}}{U_{h,t}}\right] = E_t\left[\frac{dU_{h,t}}{U_{h,t}}\right] + \frac{W_{h,t}}{U_{h,t}}\frac{\partial U_{h,t}}{\partial W_{h,t}}\boldsymbol{\omega}_{h,t}^\top\boldsymbol{\nu}_{h,t}dt.$$

We can therefore rewrite (A6) as

$$\mu^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}] = U_{h,t}\left(1 + E_t\left[\frac{dU_{h,t}}{U_{h,t}}\right] - \frac{1}{2}\gamma E_t\left[\left(\frac{dU_{h,t}}{U_{h,t}}\right)^2\right] + L_{h,t}dt + \frac{W_{h,t}}{U_{h,t}}\frac{\partial U_{h,t}}{\partial W_{h,t}}\boldsymbol{\omega}^\top_{h,t}\boldsymbol{\nu}_{h,t}dt\right) + o(dt).$$

Using (6) we obtain

$$\mu^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}] = \mu_t[U_{h,t+dt}] + U_{h,t}\left(L_{h,t}dt + \frac{W_{h,t}}{U_{h,t}}\frac{\partial U_{h,t}}{\partial W_{h,t}}\boldsymbol{\omega}^\top_{h,t}\boldsymbol{\nu}_{h,t}dt\right) + o(dt).$$

and hence (11).

□

## Proof of Proposition 1

From (11), we can see that

$$\inf_{\boldsymbol{\nu}_{h,t}} \mu^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}]$$

is equivalent to

$$\inf_{\boldsymbol{\nu}_{h,t}} \frac{W_{h,t}U_{W_{h,t}}}{U_{h,t}}\boldsymbol{\nu}^\top_{h,t}\boldsymbol{\omega}_{h,t} + \frac{1}{2\gamma\sigma^2}\boldsymbol{\nu}^\top_{h,t}(\Gamma_h\Omega)^{-1}\boldsymbol{\nu}_{h,t}.$$

The minimum exists and is given by the FOC

$$\frac{\partial}{\partial\boldsymbol{\nu}_{h,t}}\left[\frac{W_{h,t}U_{W_{h,t}}}{U_{h,t}}\boldsymbol{\nu}^\top_{h,t}\boldsymbol{\omega}_{h,t} + \frac{1}{2\gamma\sigma^2}\boldsymbol{\nu}^\top_{h,t}(\Gamma_h\Omega)^{-1}\boldsymbol{\nu}_{h,t}\right] = 0$$

Carrying out the differentiation and exploiting the fact that $(\Gamma_h\Omega)^{-1}$ is symmetric, we obtain

$$0 = \frac{W_{h,t}U_{W_{h,t}}}{U_{h,t}}\boldsymbol{\omega}_{h,t} + \frac{1}{\gamma\sigma^2}(\Gamma_h\Omega)^{-1}\boldsymbol{\nu}_{h,t}.$$

Hence

$$\boldsymbol{\nu}_{h,t} = -\gamma\sigma^2\frac{W_{h,t}U_{W_{h,t}}}{U_{h,t}}\Gamma_h\Omega\,\boldsymbol{\omega}_{h,t}.$$

Using (A5), we obtain

$$\nu_{h,n,t} = -\gamma\sigma^2\frac{W_{h,t}U_{W_{h,t}}}{U_{h,t}}\frac{1-f_n}{f_n}\left(\omega_{h,n,t} + \rho\sum_{m\neq n}\omega_{h,m,t}\right).$$

□

## Proof of Proposition 2

Writing out (13) explicitly gives

$$U_{h,t}^{1-\frac{1}{\psi}} = (1-e^{-\delta dt})C_{h,t}^{1-\frac{1}{\psi}} + e^{-\delta dt}\left(\mu_{h,t}^{\boldsymbol{\nu}}[U_{h,t+dt}]\right)^{1-\frac{1}{\psi}},$$

where for ease of notation sup and inf have been suppressed. Now

$$\begin{aligned}\left(\mu_{h,t}^{\boldsymbol{\nu}}[U_{h,t+dt}]\right)^{1-\frac{1}{\psi}} &= \left(U_{h,t} + \mu_{h,t}^{\boldsymbol{\nu}}[dU_{h,t}]\right)^{1-\frac{1}{\psi}} \\ &= U_{h,t}^{1-\frac{1}{\psi}}\left(1+\mu_{h,t}^{\boldsymbol{\nu}}\left[\frac{dU_{h,t}}{U_{h,t}}\right]\right)^{1-\frac{1}{\psi}} \\ &= U_{h,t}^{1-\frac{1}{\psi}}\left(1+\left(1-\frac{1}{\psi}\right)\mu_{h,t}^{\boldsymbol{\nu}}\left[\frac{dU_{h,t}}{U_{h,t}}\right]\right) + o(dt).\end{aligned}$$

Hence

$$U_{h,t}^{1-\frac{1}{\psi}} = \delta C_{h,t}^{1-\frac{1}{\psi}}dt + U_{h,t}^{1-\frac{1}{\psi}}\left(1+\left(1-\frac{1}{\psi}\right)\mu_{h,t}^{\boldsymbol{\nu}}\left[\frac{dU_{h,t}}{U_{h,t}}\right]\right) - \delta U_{h,t}^{1-\frac{1}{\psi}}dt + o(dt),$$

from which we obtain (15). □

## Proof of Proposition 3

Assuming a constant risk-free rate, homotheticity of preferences combined with constant returns to scale for production implies that we have $U_{h,t} = \kappa_h W_{h,t}$, for some constant $\kappa_h$. Equations (17) and (16) are then direct consequences of (11) and (15).

□

## Proof of Proposition 4

Minimizing (17) with respect to $\nu_{h,t}$ gives (18). Substituting (18) into (17) and simplifying gives

$$MV_h = i + (\alpha - i)\pi_h - \frac{1}{2}\gamma\pi_h^2\sigma^2\boldsymbol{x}_h^\top(I+\Gamma_h)\Omega\,\boldsymbol{x}_h,$$

where $\pi_h$ denotes the proportion of investor $h$'s wealth held in risky assets,

$$\pi_h = \mathbf{1}^\top\boldsymbol{\omega}_h,$$

and $\boldsymbol{x}_h$ is the vector of risky asset weights,

$$\boldsymbol{x}_h = \frac{\boldsymbol{\omega}_h}{\pi_h}.$$

We find $\boldsymbol{x}_h$ by minimizing $\sigma^2 \boldsymbol{x}_h^\top (I+\Gamma_h)\Omega\, \boldsymbol{x}_h$, so we can see that $\boldsymbol{x}_h$ is investor $h$'s minimum-variance portfolio adjusted for familiarity bias. The minimization we wish to perform is

$$\min \frac{1}{2} \boldsymbol{x}_h^\top (I+\Gamma_h)\Omega\, \boldsymbol{x}_h$$

subject to the constraint

$$\mathbf{1}^\top \boldsymbol{x}_h = 1.$$

The Lagrangian for this problem is

$$L_h = \frac{1}{2} \boldsymbol{x}_h^\top (I+\Gamma_h)\Omega\, \boldsymbol{x}_h + \lambda_h (1 - \mathbf{1}^\top \boldsymbol{x}_h),$$

where $\lambda_h$ is the Lagrange multiplier. The first order condition with respect to $\boldsymbol{x}_h$ is

$$(I+\Gamma_h)\Omega\, \boldsymbol{x}_h = \lambda_h \mathbf{1}.$$

Hence

$$\boldsymbol{x}_h = \lambda_h \Omega^{-1} (I+\Gamma_h)^{-1} \mathbf{1} = \lambda_h \Omega^{-1} \boldsymbol{f}_h,$$

where

$$\boldsymbol{f}_h = (f_{h1}, \ldots, f_{hN})^\top.$$

The first order condition with respect to $\lambda_h$ gives us the constraint

$$\mathbf{1}^\top \boldsymbol{x}_h = 1,$$

which implies that

$$\lambda_h = \left[ \mathbf{1}^\top \Omega^{-1} \boldsymbol{f}_h \right]^{-1}.$$

Therefore, we have

$$\boldsymbol{x}_h = \frac{\Omega^{-1} \boldsymbol{f}_h}{\mathbf{1}^\top \Omega^{-1} \boldsymbol{f}_h}.$$

Substituting the optimal choice of $\boldsymbol{x}_h$ back into $\boldsymbol{x}_h^\top (I+\Gamma_h)\Omega \boldsymbol{x}_h$ gives

$$\boldsymbol{x}_h^\top (I+\Gamma_h)\Omega \boldsymbol{x}_h = \frac{\boldsymbol{f}_h \Omega^{-1} (I+\Gamma_h)\Omega\Omega^{-1} \boldsymbol{f}}{[\mathbf{1}^\top \Omega^{-1} \boldsymbol{f}]^2} = \frac{\boldsymbol{f}_h \Omega^{-1} (I+\Gamma_h) \boldsymbol{f}}{[\mathbf{1}^\top \Omega^{-1} \boldsymbol{f}]^2} = \frac{\boldsymbol{f}_h^\top \Omega^{-1} \mathbf{1}}{[\mathbf{1}^\top \Omega^{-1} \boldsymbol{f}_h]^2} = \lambda_h. \quad \text{(A7)}$$

Therefore, to find the optimal $\pi$, we need to minimize

$$MV_h = i + (\alpha - i)\pi_h - \frac{1}{2} \gamma_h \pi_h^2 \sigma^2 \lambda_h.$$

Hence

$$\pi_h = \frac{1}{\lambda_h} \frac{1}{\gamma} \frac{\alpha - i}{\sigma^2},$$

which implies that

$$\boldsymbol{\omega}_h = \pi_h \boldsymbol{x}_h = \frac{1}{\lambda_h}\frac{1}{\gamma}\frac{\alpha - i}{\sigma^2}\lambda_h \Omega^{-1}\boldsymbol{f}_h = \frac{1}{\gamma}\frac{\alpha - i}{\sigma^2}\Omega^{-1}\boldsymbol{f}_h.$$

We can rewrite the expression for $\boldsymbol{\omega}_h$ in (19) in terms of the familiarity-biased adjustment made to expected returns:

$$\boldsymbol{\omega}_h = \frac{1}{\gamma}\Omega^{-1}\frac{\alpha\mathbf{1} + \boldsymbol{\nu}_h - i\mathbf{1}}{\sigma^2},$$

where

$$\boldsymbol{\nu}_h = -(\alpha - i)\boldsymbol{a}_h,$$
$$\boldsymbol{a}_h = \mathbf{1} - \boldsymbol{f}_h.$$

We now use (19) to derive an expression for $\omega_{hn}$, that is, the $n$'th element of $\boldsymbol{\omega}_n$. For all $n \in \{1, \ldots, N\}$, define $\boldsymbol{e}_n$, the $N \times 1$ column vector, with a one in the $n$'th entry and zeros everywhere else. Clearly $\{\boldsymbol{e}_1, \ldots, \boldsymbol{e}_N\}$ is the standard basis for $\mathbb{R}^N$ and the proportion of investor $h$'s wealth invested in firm $n$ is given by

$$\omega_{hn} = \boldsymbol{e}_n^\top \boldsymbol{\omega}_h.$$

We define

$$q_{hn} = \boldsymbol{e}_n^\top \Omega^{-1}\boldsymbol{f}_h,$$

and so

$$\omega_{hn} = q_{hn}\frac{1}{\gamma}\frac{\alpha - i}{\sigma^2}, \tag{A8}$$

and

$$\pi_h = \frac{1}{\gamma}\frac{\alpha - i}{\sigma^2}\sum_{n=1}^{N} q_{hn}. \tag{A9}$$

It follows from (A8) that the $n$'th element of investor $h$'s portfolio of risky assets is given by

$$x_{hn} = \frac{\omega_{hn}}{\sum_{n=1}^{N}\omega_{hn}} = \frac{q_{hn}}{\sum_{n=1}^{N} q_{hn}}.$$

Substituting the expression for the optimal portfolio weight into the mean-variance objective function gives the optimized mean-variance objective function:

$$MV = i + \frac{1}{2}\frac{1}{\lambda_h \gamma}\left(\frac{\alpha - i}{\sigma}\right)^2.$$

Using (A7) we thus obtain the result in the proposition. □

## Proof of Proposition 5

From the Hamilton-Jacobi-Bellman equation in (15), the first-order condition with respect to consumption is

$$\delta \left( \frac{C_{ht}}{U_{ht}} \right)^{-\frac{1}{\psi}} = \frac{U_{ht}}{W_{h,t}}.$$

Substituting the above first-order condition into the Hamilton-Jacobi-Bellman equation allows us to solve for investor utility, and hence, optimal consumption. We obtain (26). □

## Proof of Proposition 6

Observe that the no aggregate bias condition is equivalent to

$$\frac{1}{H} \sum_{h=1}^{H} q_{hn} = \frac{1}{N} \sum_{n=1}^{N} q_{hn}. \tag{A10}$$

Now define a $H \times N$ familiarity matrix,

$$Q = [q_{hn}].$$

The permutations described in the symmetry condition imply that one can obtain all the rows of the matrix by rearranging any particular row, and one can obtain all the columns of the matrix by rearranging any particular column, which implies that (A10) is satisfied.

□

## Proof of Proposition 7

We start by observing the symmetry condition in Theorem 6 implies that

$$\forall n, \forall h, \ \frac{1}{H} \sum_{h=1}^{H} q_{hn} = \frac{1}{N} \sum_{n=1}^{N} q_{hn},$$

which is equivalent to

$$\forall n, \forall h, \ \frac{1}{H} \hat{q}_n = \frac{1}{N} \hat{q}_h, \tag{A11}$$

where

$$\hat{q}_n = \sum_{h=1}^{H} q_{hn}, \ \hat{q}_h = \sum_{n=1}^{N} q_{hn}.$$

From (A11) we can also see that $\hat{q}_n$ and $\hat{q}_h$ must be independent of $n$ and $h$, respectively.

We now prove that the condition that $\hat{q}_h$ is independent of $h$ implies that the risk-free interest rate is the constant given by (32). Market clearing in the bond market implies that

$$\sum_{h=1}^{H} B_{h,t} = 0., \tag{A12}$$

where the amount of wealth held in the bond by investor $h$ is given by

$$B_{h,t} = (1 - \pi_{h,t}) W_{h,t}.$$

Using the expression for $\pi_{h,t}$ given in (A9), we can rewrite the market clearing condition (A12) as

$$\sum_{h=1}^{H} \left(1 - \frac{1}{\gamma} \frac{\alpha - i}{\sigma^2} \left(\sum_{n=1}^{N} q_{hn}\right)\right) W_{h,t} = 0.$$

Hence,

$$\begin{aligned}
0 &= \sum_{h=1}^{H} \left(W_{h,t} - \hat{q}_h \frac{1}{\gamma} \frac{\alpha - i}{\sigma^2} W_{h,t}\right) \\
\sum_{h=1}^{H} W_{h,t} &= \frac{1}{\gamma} \frac{\alpha - i}{\sigma^2} \sum_{h=1}^{H} \hat{q}_h W_{h,t} \\
i &= \alpha - \frac{\sum_{h=1}^{H} W_{h,t}}{\sum_{h=1}^{H} \hat{q}_h W_{h,t}} \gamma \sigma^2 \\
&= \alpha - \frac{1}{\hat{q}} \gamma \sigma^2.
\end{aligned}$$

Therefore, in equilibrium

$$B_{h,t} = 0.$$

We thus conclude that in equilibrium, each investor invests solely in risky firms.

□

## Proof of Proposition 8

Substituting the equilibrium interest rate in (32) into the expression in (27) for the consumption-wealth ratio for each individual gives the general equilibrium consumption-wealth ratio:

$$\frac{C_{h,t}}{W_{h,t}} = c,$$

where

$$c = \psi\delta + (1 - \psi)\Big(\alpha - \frac{1}{2}\gamma\sigma_p^2\Big).$$

Observe that in the expression above, all the terms on the right-hand side are constants, implying that the consumption-wealth ratio is the same across investors. Exploiting the fact that the consumption-wealth ratio is constant across investors allows us to obtain the ratio of aggregate consumption-to-wealth ratio, where aggregate consumption is $C_t^{\text{agg}} = \sum_{h=1}^{H} C_{h,t}$ and aggregate wealth is $W_t^{\text{agg}} = \sum_{h=1}^{H} W_{h,t}$:

$$\frac{C_t^{\text{agg}}}{W_t^{\text{agg}}} = c.$$

Equation (1) implies

$$\sum_{n=1}^{N} Y_{n,t} = \alpha \sum_{n=1}^{N} K_{n,t},$$

and Equation (2) implies

$$dE_t \left[ \sum_{n=1}^{N} K_{n,t} \right] = E_t \left[ d \sum_{n=1}^{N} K_{n,t} \right] = \alpha \sum_{n=1}^{N} K_{n,t} - \sum_{n=1}^{N} D_{n,t} dt.$$

In equilibrium $\sum_{n=1}^{N} K_{n,t} = W_t^{\text{agg}}$ and $\sum_{n=1}^{N} D_{n,t} = C_t^{\text{agg}}$. Therefore,

$$\frac{dW_t^{\text{agg}}}{W_t^{\text{agg}}} = \left( \alpha - \frac{C_t^{\text{agg}}}{W_t^{\text{agg}}} \right) dt.$$

We also know that

$$\frac{dW_t^{\text{agg}}}{W_t^{\text{agg}}} = \frac{dY_t^{\text{agg}}}{Y_t^{\text{agg}}}$$

and so

$$g\, dt = E_t \left[ \frac{dY_t^{\text{agg}}}{Y_t^{\text{agg}}} \right] = \left( \alpha - \frac{C_t^{\text{agg}}}{W_t^{\text{agg}}} \right) dt.$$

Therefore

$$c = \alpha - g.$$

From (33) it follows that

$$c = i + \gamma \sigma_p^2 - g.$$

□

## Proof of Proposition 9

We start by deriving the aggregate investment-capital ratio. The aggregate investment flow must be equal to aggregate output flow less the aggregate consumption flow:

$$I_t^{\text{agg}} = \alpha K_t^{\text{agg}} - C_t^{\text{agg}}.$$

It follows that the aggregate investment-capital ratio is given by (38).

Trend output growth is given by $E_t\left[\frac{dY_t^{\text{agg}}}{Y_t^{\text{agg}}}\right]$. Observe that $Y_t^{\text{agg}} = \alpha K_t^{\text{agg}} = \alpha W_t^{\text{agg}} = \frac{\alpha}{c} C_t^{\text{agg}}$. It follows that trend output growth equals the growth rate of aggregate consumption:

$$g = E_t\left[\frac{dY_t^{\text{agg}}}{Y_t^{\text{agg}}}\right].$$

We now relate trend output growth to aggregate investment. Firms all have constant returns to scale and differ only because of shocks to their capital stocks. Therefore, the aggregate growth rate of the economy is the aggregate investment-capital ratio:

$$g = \frac{I_t^{\text{agg}}}{K_t^{\text{agg}}}.$$

□

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