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When exposed to similar migration flows, countries with different institutional systems may respond with different levels of openness. We study in particular the different responses determined by different electoral systems. We find that Winner Take All countries would tend to be more open than countries with PR when all other policies are kept constant, but, crucially, if we consider the endogenous differences in redistribution levels across systems, then the openness ranking may switch.

JEL Classification: D72, F22

Keywords: Proportional representation, Median voter, taxation, Occupational choice, migration, Walls

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Electoral Systems, Taxation and Immigration Policies *

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September 25, 2018

Abstract

When exposed to similar migration flows, countries with different institutional systems may respond with different levels of openness. We study in particular the different responses determined by different electoral systems. We find that Winner Take All countries would tend to be more open than countries with PR when all other policies are kept constant, but, crucially, if we consider the endogenous differences in redistribution levels across systems, then the openness ranking may switch.

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1 Introduction

What role do institutions play for the interpretation of the different responses that different countries seem to have to the threat of increasing migration flows? When there is a perception that migrants could be a threat for employment or income levels, politicians' electoral incentives may push them to display increasing hostility to open borders, but such electoral concerns could have different intensities and/or implications depending on the electoral system. We analyze this question using a political economy model previously used to study the implications of electoral systems for the level of redistribution, with the additional goal of studying the interplay between immigration and redistribution policies.

We use a model of policy making with endogenous occupational choice, an extension of Austen-Smith (2000). In that paper the population size is fixed, while in this paper we assume that entry of immigrants is a constant flow as long as the institutional system is such that leaving the doors open is preferred to building a wall by the majority of members of parliament. The main insight

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of the paper is that the predictions about immigration policies chosen in countries with different electoral systems may be completely reversed when redistribution levels are made endogenous to the electoral system as well. We show that openness would be more likely in winner take all systems than in PR systems for any given same level of redistribution of income, but, once one takes into account the endogenous redistribution levels, the relative openness result switches.

As an intuition, in the absence of endogenous taxation differences, PR is weakly more closed because the average worker's preferences are the ones that matter (especially in a capital intensive and productive country), while in a WTA system the decisive agent is the median voter in the distribution of preferences over immigration policies, which happens to be an agent with lower talent, deriving relative greater utility from the increased aggregate income from migration. Endogenous taxation is crucial, and can reverse the prediction: PR induces higher taxes, and this can induce the set of native agents who self-select into an employee occoupation to be less than 1/2 of the total native population, which implies that the decisive agent cannot remain the average worker, but rather an unemployed individual, whose primary concern are the redistributive benefits. Given that total benefits increase in aggregate income under reasonable conditions, especially at the beginning of the migration flow, PR countries can be more open than WTA countries.

Let us briefly discuss the relationship of this paper to the empirical literature. In our model, immigration affects the native population both through wages and through welfare transfers. The report produced by the National Academy of Sciences, Engineering, and Medicine (2017) contains an extensive review of theoretical and empirical results of the effect of immigration on employment and wages as well as on its fiscal impact. The empirical evidence about the impact of immigration on natives's wages is mixed. Some papers found that immigration decreases wages in the receiving country (among others, Altonji and Card (1991), Monras (2015), Borjas (2003, 2016)), others that the effect is negligible (among others Card (2001, 2009)) and some others that the effect is positive (Ottaviano and Peri (2012)). Dustman, Frattini, and Preston (2013) estimate the effect of immigration along the distribution of wages and show that the effect is negative for lower parts of the distribution and positive at the top. Important factors affecting the outcome of the analysis are the degree of substitutability between natives and immigrants, the degree of substitutability among different groups of workers and whether the analysis takes a short-run or long run perspective (i.e. whether capital is allowed to adjust to the inflow of migrants or not).[1] Our model assumes perfect substitutability between natives and immigrants, it considers only one labour market and disregards the adjustment of capital. Given these assumptions, the negative effect of immigration on equilibrium wage that our model displays seems in line with empirical findings.

Immigration impacts welfare transfers to natives in two ways. On the one hand, when working, immigrants increase tax revenues and therefore transfers. On the other hand, they increase the

[1]The different results are also due to different estimation techniques and possible misallocation of migrants in the relevant experience and skills groups (downgrading). See Dustmann, Schönberg and Stuhler (2016).

number of people among which tax revenues must be redistributed. Whether immigrants are net contributors or receivers depends on the share of resources they are entitled to receive. This is a key parameter in our analysis (the parameter α) and is a key determinant of natives' attitudes towards immigrants (see e.g. Facchini and Mayda, 2009, and Preston, 2014).

Given that in our model migration is described as a flow that keeps modifying endogenous variables as it continues, the paper offers a stylized dynamics also of natives' preferences. The economic consequences of immigration can indeed affect the natives' preference over immigration (see e.g. Scheve and Slaughter, 2001). Barone et al (2016) show that these effects can also affect voting decisions by the native population: immigration leads natives to vote more for center-right parties.

As far as the political economy literature is concerned, to our knowledge we are the first to compare winner take all and proportional representation electoral systems in terms of endogenous immigration policies. We have chosen to use mainly the modeling insights of Austen-Smith (2000) because endogenous occupational choice seems to be an important part of the dynamic phenomenon we wanted to describe. Morelli (2004) displays other important contrasts between winner take all and proportional systems, in terms of party formation and policy outcomes, and hence some more future results could be obtained also from that framework.

The paper is organized as follows: Section 2 describes the model of political and economic choices, namely occupational choice by citizens and the consequent class and party formation that determines, through the political institutions, the taxation and immigration policies. Section 3 describes the equilibrium results when the tax rate is kept equal across countries with different electoral systems. Section 4 displays the results when redistribution levels differ endogenously across systems. Section 5 shows by simulation the reversal result, and section 6 concludes.

2 Model

We consider two countries that are identical in every aspect, except for the electoral system they use (see description below). Both countries have a mass one of native individuals. Moreover, there is a mass one of potential entrants in each country. At the beginning of the game all potential migrants are out, and, if a country leaves the borders open, they enter at a constant rate. Formally, consider any country and let $Q_t \in [0, 1)$ be the share of immigrants that have already entered in the country at time t, with $Q_0 = 0$. The assumption of constant flow if borders are kept open implies that $Q_{t+1} = Q_t + \delta$, $\delta > 0$, until either $Q = 1$ is reached or until the country's government decides to *build a wall* to stop the flow, whichever comes first.

Each individual (native or immigrant) is characterised by a type $\theta \in (0, \bar{\theta})$. We denote by $g(\theta)$ the distribution of types in the population of natives. We assume $g(\cdot)$ symmetric, with mean and median denoted by $\tilde{\theta}$. The set of immigrants entering each of the two countries in each period is sampled from a distribution $h(\theta)$. Let both $\theta g(\theta)$ and $\theta h(\theta)$ be non decreasing in θ. We will be more precise about the characteristics of $h(\theta)$ later in the paper.

Individuals can select one of three possible occupations: becoming an employer (e), becoming an employee (l) or being unemployed (d). An employer of type θ can employ L units of labor to produce an amount $F(L, \theta)$ of consumption good, which is assumed to be the only good consumed in the economy and whose price is normalized to one. The function $F(\cdot, \cdot)$ is at least twice differentiable, strictly increasing in both arguments, strictly concave in L and strictly convex in θ. Furthermore, it is also assumed that $\partial^2 F/\partial\theta\partial L > 0$ for all $\theta > 0$.

Letting w be the wage paid for each unit of labor, the employer's gross income is

$$y_e(L, w, \theta) = F(L, \theta) - wL.$$

If an individual chooses to become an employee, she inelastically provides θ units of labor and receives a gross income

$$y_l(w, \theta) = \theta w.$$

Both employers and employees pay a cost of working $c > 0$ and their income is taxed at a rate $\tau \in [0, 1]$. Taxes are redistributed to the whole population in the form of lump-sum transfers. For any stock Q_t of immigrants having entered the country at a given date t, and for any tax level τ and wage w, let $\lambda_j(\tau, w, Q_t)$ be the set of types choosing occupation $j \in \{e, l, d\}$. The total aggregate income in the country is

$$Y(\tau, w, Q_t) = \int_{\lambda_e(\tau,w,Q_t)} y_e(L, w, \theta)[g(\theta) + Q_t h(\theta)]d\theta + \int_{\lambda_l(\tau,w,Q_t)} y_l(L, w, \theta)[g(\theta) + Q_t h(\theta)]d\theta \quad (1)$$

so that tax revenues are $\tau Y(\tau, w, Q_t)$. We assume that no debt can be accumulated and that each immigrant obtains a fraction $\alpha \in (0, 1)$ of the tax revenues. The remaining amount is redistributed equally among natives. Let $b_I(\tau, w, Q_t, \alpha) = \alpha\tau Y(\tau, w, Q_t)$ be the benefits received by each immigrant and $b(\tau, w, Q_t, \alpha) = (1 - \alpha Q_t)\tau Y(\tau, w, Q_t)$ be those received by each native. The net income $x_j(\cdot, \theta)$ of a native individual of type θ in occupation $j \in \{e, l, d\}$ is

$$x_e(L, \tau, w, Q_t, \alpha, \theta) = (1 - \tau)y_e(L, w, \theta) + b(\tau, w, Q_t, \alpha) - c$$

$$x_l(\tau, w, Q_t, \alpha, \theta) = (1 - \tau)y_l(w, \theta) + b(\tau, w, Q_t, \alpha) - c$$

$$x_d(\tau, w, Q_t, \alpha, \theta) = b(\tau, w, Q_t, \alpha)$$

The corresponding net incomes for immigrants are obtained by replacing $b(\tau, w, Q_t, \alpha)$ with $b_I(\tau, w, Q_t, \alpha)$ in the expressions above.

For any wage level w and any type θ, let $L(w, \theta)$ denote the amount of labour that maximizes an employer's net income. Given the assumptions on the production function, $L(w, \theta)$ is strictly decreasing in w and strictly increasing in θ. Since from now on we will only consider the optimal amount of labour demanded by employers, we will sometimes simplify notation by using L instead of $L(w,\theta)$. Definition 1 extends the concept of *sorting equilibrium* contained in Austen-Smith (2000) (AS henceforth) to our framework.

Definition 1. *At any fixed tax rate $\tau \in [0,1]$ and immigration level $Q_t \in [0,1]$, a sorting equilibrium is a wage rate $w_t = w(\tau, Q_t)$ such that*

$$\int_{\lambda_e(\tau,w_t,Q_t)} L(w_t,\theta)[g(\theta)+Q_t h(\theta)]d\theta = \int_{\lambda_l(\tau,w_t,Q_t)} \theta[g(\theta)+Q_t h(\theta)]d\theta$$

and for all $\theta \in (0,\bar{\theta})$, for all $j,j' \in \{e,l,d\}$, $\theta \in \lambda_j(\tau,w_t,Q_t)$ implies $x_j(\cdot,\theta) \geq x_{j'}(\cdot,\theta)$.

By Proposition 1 in AS, a sorting equilibrium always exists and is characterised by pairs of types $\theta_t^1 = \theta^1(\tau,w_t,Q_t)$ and $\theta_t^2 = \theta^2(\tau,w_t,Q_t))$, with $\theta_t^1 < \theta_t^2$, such that

$$\lambda_d(\tau,w_t,Q_t) = (0,\theta_t^1) \qquad \lambda_l(\tau,w_t,Q_t) = [\theta_t^1,\theta_t^2] \qquad \lambda_e(\tau,w_t,Q_t) = (\theta_t^2,\bar{\theta})$$

Type θ_t^1 is the type who is indifferent between becoming unemployed and working as an employee. Given the definition of net income for the two types,

$$\theta_t^1 = \frac{c}{(1-\tau)w_t} \tag{2}$$

Type θ_t^2 is the type who is indifferent between becoming an employee or an employer and is implicitly defined by

$$F(L(w_t,\theta_t^2),\theta_t^2) - w_t L(w_t,\theta_t^2) = w_t\theta_t^2 \tag{3}$$

From Definition 1, then, the wage rate w_t satisfies

$$\int_{\theta_t^2}^{\bar{\theta}} L(w_t,\theta)[g(\theta)+Q_t h(\theta)]d\theta = \int_{\theta_t^1}^{\theta_t^2} \theta[g(\theta)+Q_t h(\theta)]d\theta \tag{4}$$

From now on, we assume that the distribution $h(\theta)$ is such that immigrants contribute more to the supply side of the labour market.

Assumption 1. *The distribution of immigrant types $h(\theta)$ is such that*

$$\int_{\theta_t^1}^{\theta_t^2} \theta h(\theta)d\theta - \int_{\theta_t^2}^{\bar{\theta}} L(\tilde{w},\theta)h(\theta)d\theta \geq 0$$

where $\tilde{w} = w(\tau,1)$.

In each period, each country can decide to stop the inflow of migrants. We will sometimes refer to this decision as *building a wall* against immigration. We assume that if in period t the option of building the wall can win the majority in parliament, a party that supports it will propose a wall bill. With this assumption the analysis simply needs to focus on the time when the possibility of building a wall becomes a winning option.

In one of the two countries, the composition of parliament is determined by a winner take all system. We assume that the majority of parliament members has preferences over immigration that are identical to those of the median voter in the population (in the next section, we show that the median voter is well defined in this framework). In this country, the wall will be build at a given time t if and only if the median type θ_t^m is in favour of it.

The other country uses a proportional representation system. We assume that there exist three parties, each representing a different occupation. We denote by $\mathcal{E}$ the party of employers, by $\mathcal{L}$ the party of employees and by $\mathcal{D}$ the one of unemployed. Each party wants to maximise the average utility of the native individuals in the occupation it represents. That is,

$$u_{\mathcal{E}}(\tau, Q_t) = b(\tau, w_t, Q_t, \alpha) + (1-\tau)\hat{y}_e(L, w_t, Q_t) - c$$

$$u_{\mathcal{L}}(\tau, Q_t) = b(\tau, w_t, Q_t, \alpha) + (1-\tau)\hat{\theta}_l(\tau, Q_t)w_t - c$$

$$u_{\mathcal{D}}(\tau, Q_t) = b(\tau, w_t, Q_t, \alpha)$$

where

$$\hat{\theta}_l(\tau, Q_t) = \frac{\int_{\theta_t^1}^{\theta_t^2} \theta g(\theta) d\theta}{\int_{\theta_t^1}^{\theta_t^2} g(\theta) d\theta}$$

and

$$\hat{y}_e(L, w_t, Q_t) = \frac{\int_{\theta_t^2}^{\bar{\theta}} y_e(L, w_t, \theta) g(\theta) d\theta}{\int_{\theta_t^2}^{\bar{\theta}} g(\theta) d\theta}$$

Each party's share of parliament seats corresponds to the share of native individuals in the occupation it represents. When a party has the majority of parliament seats, it unilaterally decides about the construction of the wall. If no party has the majority in parliament, coalition governments will be formed and the wall will be built when at least two parties agree about it.

In what follows, we will refer to the country using the winner take all system as country W and to the one using PR as country P.

3 Results for a fixed tax rate

We begin by assuming that the tax rate τ does not differ across the two countries. Our first goal is to establish the effect of immigration on wages and occupational choices. Under Assumption 1, immigrants contribute more to the supply side of the labour market. Then,

Lemma 1. *The equilibrium wage rate w_t is differentiable, strictly decreasing and nonlinear in Q.*

For any level of immigration Q_t, then, $w_t > w_{t+1}$. When wages decrease, being an employee becomes less attractive. Indeed, employees' gross income is strictly increasing in w, while the envelope theorem implies

$$\frac{\partial y_e(L, w, \theta)}{\partial w} = -L(w, \theta) < 0. \tag{5}$$

For both occupations, the magnitude of the effect increases with θ. Since benefits are equally distributed across the population, then, the entrance of migrants modifies optimal labour decisions. More formally, from (2) and (3), one gets $\partial\theta_t^1/\partial w < 0$ and $\partial\theta_t^2/\partial w > 0$[2]. Then, $\theta_t^1 < \theta_{t+1}^1 < \theta_{t+1}^2 < \theta_t^2$.

In order to avoid trivial cases, in what follows we will maintain the following assumption:

Assumption 2. *Let $\tilde{w} = w(\tau, 1)$. Then,*

$$\int_{\theta^2(\tau,\tilde{w},1)}^{\bar{\theta}} g(\theta)d\theta < 1/2$$

In words, we simply assume that even in the extreme situation of full openness, where all potential migrants enter, the set of endogenous employers can never be an absolute majority of the population. Given that $\theta^2(\tau, w, Q_t)$ decreases as migrants keep entering, this assumption is a sufficient condition to guarantee that the set of employers is never an absolute majority throughout the whole entry process.

3.1 Immigration under winner take all

Immigration affects the native population through three channels. First, by decreasing wages, it reduces employees' gross income and increases employers' profits. Secondly, it might increase or decrease all natives' net income by changing the amount of benefits they receive. This second effect can be positive or negative, depending on whether migrants' contribution to aggregate income is higher or lower than the share of resources they divert from natives through their participation to the welfare system. The third, indirect, effect arises through occupational choice. The interplay between changes in gross income and benefits changes the attractiveness of different occupations after immigration has occurred.

The third effect implies that the population of natives can be partitioned in five different sets, depending on the occupation chosen before and after immigration. All individuals with a type $\theta \in (0, \theta_t^1)$ are unemployed at time t and remain unemployed at time $t+1$, after immigration has taken place. The preferences of these individuals over the construction of the wall only depend on the effect of immigration on benefits. More precisely, these individuals will prefer to keep the

[2]The first result immediately follows by differentiating (2) with respect to w. For the second, we refer to equation (A5) in the proof of Proposition 1 in AS (p. 1258).

borders open only if

$$x_d(\tau, w_{t+1}, Q_{t+1}, \alpha, \theta) - x_d(\tau, w_t, Q_t, \alpha, \theta) = b(\tau, w_{t+1}, Q_{t+1}, \alpha) - b(\tau, w_t, Q_t, \alpha) \geq 0$$

Individuals of type $\theta \in [\theta_t^1, \theta_{t+1}^1)$ are employees at time t but prefer to switch to unemployment at time $t+1$. These individuals will be in favor of keeping the borders open only if immigration increases benefits by an amount that is large enough to compensate for the loss of labor income. That is,

$$x_d(\tau, w_{t+1}, Q_{t+1}, \alpha, \theta) - x_l(\tau, w_t, Q_t, \alpha, \theta) = b(\tau, w_{t+1}, Q_{t+1}, \alpha) - b(\tau, w_t, Q_t, \alpha) - [(1-\tau)\theta w_t - c] \geq 0$$

where the last term in square brackets is non-negative for all $\theta \geq \theta_t^1$. Then, the change in net income due to immigration for these individuals linearly decreases in type at a rate $(1-\tau)w_t$. Notice that if unemployed individuals are in favor of building the wall, these individuals will be in favor too.

The types $\theta \in [\theta_{t+1}^1, \theta_{t+1}^2]$ are employees in both periods t and $t+1$. For these individuals to be in favor of keeping the borders open, the (positive) effect of immigration on benefits must be large enough to compensate for the decrease in wage. That is

$$x_l(\tau, w_{t+1}, Q_{t+1}, \alpha, \theta) - x_l(\tau, w_t, Q_t, \alpha, \theta) = b(\tau, w_{t+1}, Q_{t+1}, \alpha) - b(\tau, w_t, Q_t, \alpha) - (1-\tau)(w_t - w_{t+1})\theta \geq 0$$

Clearly, when $\theta = \theta_{t+1}^1$

$$x_l(\tau, w_{t+1}, Q_{t+1}, \alpha, \theta_{t+1}^1) - x_l(\tau, w_t, Q_t, \alpha, \theta_{t+1}^1) = x_d(\tau, w_{t+1}, Q_{t+1}, \alpha, \theta_{t+1}^1) - x_l(\tau, w_t, Q_t, \alpha, \theta_{t+1}^1)$$

The change in net income due to immigration for these individuals is also linearly decreasing in type, at a rate $(1-\tau)(w_t - w_{t+1})$. As for the previous subset of types, these individuals will prefer to keep the borders open only if unemployed individuals are in favor too.

All individuals with type $\theta \in (\theta_{t+1}^2, \theta_t^2]$ are employees in period t and become employers in period $t+1$. They will be in favor of keeping the borders open only if

$$\begin{aligned} x_e(L, \tau, w_{t+1}, Q_{t+1}, \alpha, \theta) - x_l(\tau, w_t, Q_t, \alpha, \theta) &= b(\tau, w_{t+1}, Q_{t+1}, \alpha) - b(\tau, w_t, Q_t, \alpha) \\ &\quad + (1-\tau)[y_e(L, w_{t+1}, \theta) - \theta w_t] \geq 0 \end{aligned}$$

The last term measures the change in gross (and net) income for this type of individuals and is increasing and convex in type because of the properties of the production function. However, it is not necessarily positive. For all these types $y_e(L, w_t, \theta) \leq \theta w_t$. Then, change in gross income is positive only if the immigration-driven increase in profits is large enough. As a matter of fact, notice that by the definition of θ_t^2 and θ_{t+1}^2

$$y_e(L, w_{t+1}, \theta_t^2) - \theta_t^2 w_t = y_e(L, w_{t+1}, \theta_t^2) - y_e(L, w_t, \theta_t^2) > 0$$

and

$$y_e(L, w_{t+1}, \theta_{t+1}^2) - \theta_{t+1}^2 w_t = w_{t+1}\theta_{t+1}^2 - w_t\theta_{t+1}^2 < 0$$

Finally, all types $\theta \in (\theta_{t+1}^2, \bar{\theta}]$ are and remain employers after immigration occurs. These are the individuals that are most likely to oppose the construction of the wall. Indeed, given the decrease in wage generated by migrants, they always experience an increase in gross income. If this increase is larger than a potential decrease in benefits, they will always be in favor of keeping the borders open. More precisely, a type $\theta \in (\theta_{t+1}^2, \bar{\theta}]$ will not support the construction of the wall only if

$$\begin{aligned} x_e(L, \tau, w_{t+1}, Q_{t+1}, \alpha, \theta) - x_e(L, \tau, w_t, Q_t, \alpha, \theta) = b(\tau, w_{t+1}, Q_{t+1}, \alpha) - b(\tau, w_t, Q_t, \alpha) \\ + (1-\tau)[y_e(L, w_{t+1}, \theta) - y_e(L, w_t, \theta)] \geq 0 \end{aligned}$$

Notice that the last term is always positive. Using (5) and the properties of the production function, we get

$$\frac{\partial^2 y_e(L, w, \theta)}{\partial w \partial \theta} = -\frac{\partial L(w, \theta)}{\partial \theta} < 0$$

Then, the change in profits due to immigration is increasing in type.

Figure 1 shows the change in net income from period t to period $t+1$ as a function of types. Given our discussion above, the function is flat for all types $\theta \in (0, \theta_t^1)$, it decreases at a rate $(1-\tau)w_t$ in between θ_t^1 and θ_{t+1}^1 and at a rate $(1-\tau)(w_t - w_{t+1})$ in between θ_{t+1}^1 and θ_{t+1}^2. From θ_{t+1}^2 on, the function starts increasing again. Different changes in benefits due to immigration shift the curve upwards or downwards. The three cases shown in Figure 1 correspond to a positive increase in benefits (+), no change in benefits (=) and negative change in benefits (−).

Under the assumption that some individuals in the native population must be opposed to immigration, then, we can identify two types $\tilde{\theta}_t^1 = \tilde{\theta}^1(\tau, w_t, Q_t)$ and $\tilde{\theta}_t^2 = \tilde{\theta}^2(\tau, w_t, Q_t)$ such that all types $\theta \in (\tilde{\theta}_t^1, \tilde{\theta}_t^2)$ strictly prefer to build the wall and all types $\theta \in (0, \tilde{\theta}_t^1] \cup [\tilde{\theta}_2^2, \bar{\theta})$ are either indifferent or strictly prefer to keep the borders open. Type $\tilde{\theta}_t^1 = 0$ if $b(\tau, w_{t+1}, Q_{t+1}, \alpha) - b(\tau, w_t, Q_t, \alpha) < 0$. It is equal to θ_t^1 if $b(\tau, w_{t+1}, Q_{t+1}, \alpha) - b(\tau, w_t, Q_t, \alpha) = 0$, since the change in income for type θ_t^1 coincides with the change in benefits. When $b(\tau, w_{t+1}, Q_{t+1}, \alpha) - b(\tau, w_t, Q_t, \alpha) > 0$, then $\tilde{\theta}_t^1 > \theta_t^1$. Moreover, $\tilde{\theta}_t^1 < \theta_{t+1}^2$, because otherwise no native would support the construction of the wall. In what follows, we will maintain the assumption that $\tilde{\theta}_t^1 < \theta_{t+1}^1$. The assumption implies that individuals that are employees at time t and remain employees at time $t+1$ are strictly harmed by immigration, and it amounts to assuming that type θ_{t+1}^1 (weakly) prefers to build the wall. The following is a necessary and sufficient condition for this.

Assumption 3. *In all periods t,*

$$b(\tau, w_{t+1}, Q_{t+1}, \alpha) - b(\tau, w_t, Q_t, \alpha) \leq \frac{w_t - w_{t+1}}{w_{t+1}} c$$

Assumption 3 simply states that the increase in benefits due to immigration cannot overcome the decrease in wage. Under this assumption then, $\tilde{\theta}_t^1$ must be an employee that decides to switch to unemployment (when benefits increase with immigration). Summing up everything,

$$\tilde{\theta}_t^1 = \begin{cases} 0 & \text{if } b(\tau, w_{t+1}, Q_{t+1}, \alpha) - b(\tau, w_t, Q_t, \alpha) < 0 \\ \theta_t^1 & \text{if } b(\tau, w_{t+1}, Q_{t+1}, \alpha) - b(\tau, w_t, Q_t, \alpha) = 0 \\ \frac{b(\tau, w_{t+1}, Q_{t+1}, \alpha) - b(\tau, w_t, Q_t, \alpha) + c}{(1-\tau) w_t} & \text{if } b(\tau, w_{t+1}, Q_{t+1}, \alpha) - b(\tau, w_t, Q_t, \alpha) > 0 \end{cases} \tag{6}$$

where the last row is the solution of $x_d(\tau, w_{t+1}, Q_{t+1}, \alpha, \tilde{\theta}_t^1) - x_l(\tau, w_t, Q_t, \alpha, \tilde{\theta}_t^1) = 0$.

Let us turn to type $\tilde{\theta}_t^2$ now. Clearly, $\tilde{\theta}_t^2 > \theta_{t+1}^2$, because otherwise the entire native population would be in favor of open borders. If benefits are non-decreasing in immigration, then $\tilde{\theta}_t^2 < \theta_t^2$ since

$$\begin{aligned} x_e(L, \tau, w_{t+1}, Q_{t+1}, \alpha, \theta_t^2) - x_l(\tau, w_t, Q_t, \alpha, \theta_t^2) &= x_e(L, \tau, w_{t+1}, Q_{t+1}, \alpha, \theta_t^2) - x_e(L, \tau, w_t, Q_t, \alpha, \theta_t^2) \\ &= b(\tau, w_{t+1}, Q_{t+1}, \alpha) - b(\tau, w_t, Q_t, \alpha) + (1-\tau)[y_e(L, w_{t+1}, \theta_t^2) - y_e(L, w_t, \theta_t^2)] > 0 \end{aligned}$$

In this case, type $\tilde{\theta}_t^2$ is implicitly defined by

$$b(\tau, w_{t+1}, Q_{t+1}, \alpha) - b(\tau, w_t, Q_t, \alpha) + (1-\tau)[y_e(L, w_{t+1}, \tilde{\theta}_2) - \tilde{\theta}_2 w_t] = 0 \tag{7}$$

When benefits are decreasing in immigration, $\tilde{\theta}_t^2 > \theta_t^2$ if and only if

$$b(\tau, w_{t+1}, Q_{t+1}, \alpha) - b(\tau, w_t, Q_t, \alpha) + (1-\tau)[y_e(L, w_{t+1}, \theta_t^2) - y_e(L, w_t, \theta_t^2)] < 0$$

when this holds, type $\tilde{\theta}_t^2$ is implicitly defined by

$$b(\tau, w_{t+1}, Q_{t+1}, \alpha) - b(\tau, w_t, Q_t, \alpha) + (1-\tau)[y_e(L, w_{t+1}, \tilde{\theta}_2) - y_e(L, w_t, \tilde{\theta}_2)] = 0 \tag{8}$$

The following proposition follows immediately from our discussion.

Proposition 1. *A sufficient condition for country W to build the wall is*

$$\int_{\tilde{\theta}_t^1}^{\tilde{\theta}_t^2} g(\theta) d\theta > \frac{1}{2} \tag{9}$$

where $\tilde{\theta}_t^1$ is as defined in (6) *and $\tilde{\theta}_t^2$ is as defined in* (7) *and* (8). *When $b(\tau, w_{t+1}, Q_{t+1}, \alpha) -$*

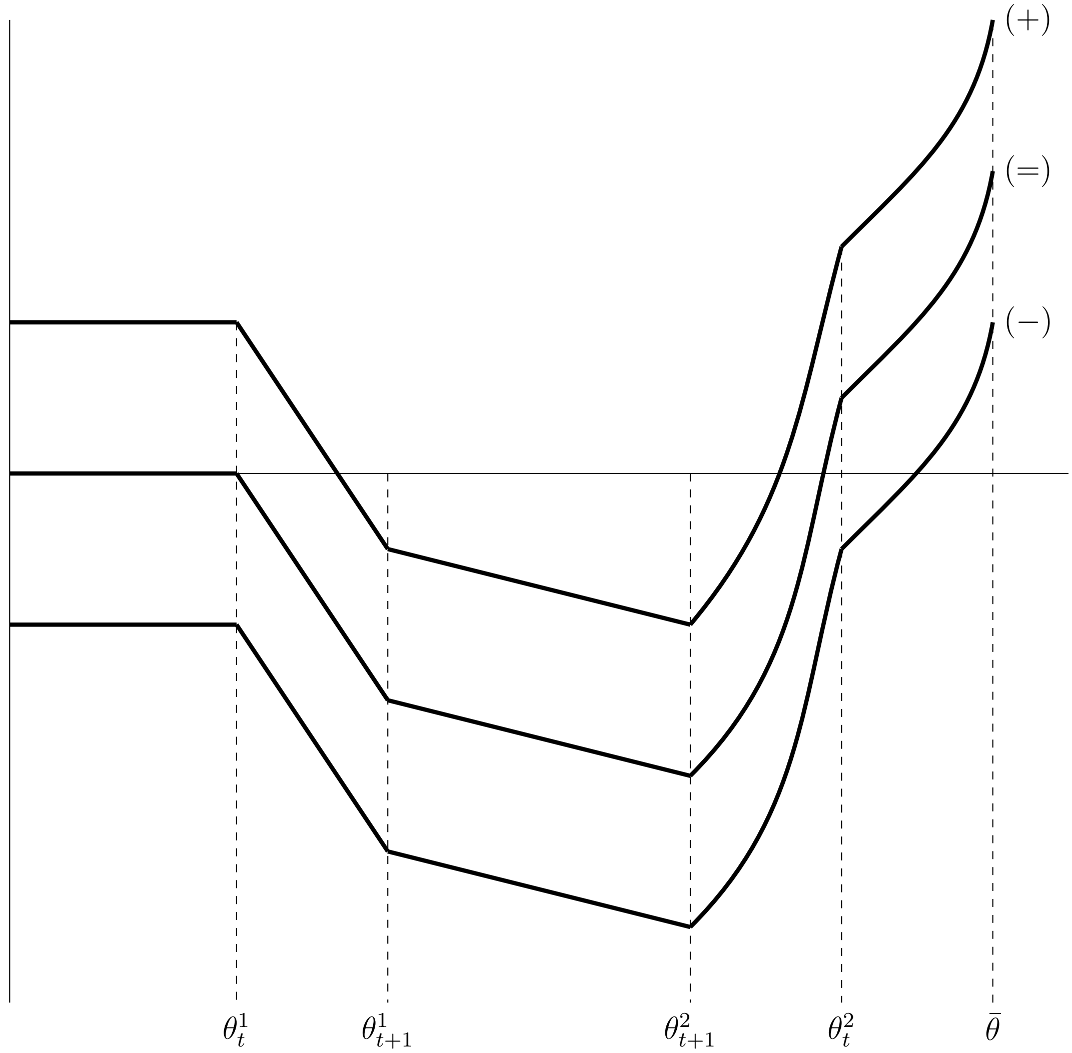

Figure 1: Change in net income from period t to period $t+1$ as a function of types, for the case of $b(\tau, w_{t+1}, Q_{t+1}, \alpha) - b(\tau, w_t, Q_t, \alpha) > 0$ (+), $b(\tau, w_{t+1}, Q_{t+1}, \alpha) - b(\tau, w_t, Q_t, \alpha) = 0$ (=) and $b(\tau, w_{t+1}, Q_{t+1}, \alpha) - b(\tau, w_t, Q_t, \alpha) < 0$ (−).

$b(\tau, w_t, Q_t, \alpha) \neq 0$, *the condition is also necessary.*

The reason why condition (9) is only sufficient when benefits do not change with immigration is that, in this case, unemployed individuals are indifferent between closing the borders or not. Suppose that non-economic factors affect the choice of closing the borders. If these factors induce unemployed individuals to favor the construction of the wall, the bill will have the support of the majority of the population even if the set of types that are strictly against immigration (for economic reasons) is lower than 1/2.

3.2 Immigration under PR

Under PR, the decision to build the wall will be taken by one party if this party has the majority of votes in parliament. Given Assumption 2, this party can only be $\mathcal{L}$ or $\mathcal{D}$. Then, if at a given time t,

$$\int_{\theta_t^1}^{\theta_t^2} g(\theta)d\theta \geq \frac{1}{2} \tag{10}$$

employees constitute the majority in the population, and borders will be kept open only if $u_{\mathcal{L}}(\tau, Q_{t+1}) \geq u_{\mathcal{L}}(\tau, Q_t)$, or equivalently only if

$$x_l(\tau, w_{t+1}, Q_{t+1}, \alpha, \hat{\theta}_l(\tau, Q_{t+1})) \geq x_l(\tau, w_t, Q_t, \alpha, \hat{\theta}_l(\tau, Q_t))$$

If instead

$$\int_{0}^{\theta_t^1} g(\theta)d\theta \geq \frac{1}{2} \tag{11}$$

then unemployed individuals will be the majority and borders will be kept open only if $u_{\mathcal{D}}(\tau, Q_{t+1}) \geq u_{\mathcal{D}}(\tau, Q_t)$, or equivalently only if

$$b(\tau, w_{t+1}, Q_{t+1}, \alpha) - b(\tau, w_t, Q_t, \alpha) \geq 0$$

Now suppose that no party has the majority in Parliament, so that (10) and (11) do not hold. In this case, borders will be closed if the decision is supported by a coalition of at least two parties. Our next lemma focuses on this scenario. Suppose that party $\mathcal{D}$ prefers to build the wall. Party $\mathcal{L}$ will support the wall bill if employees' average net income is decreasing in immigration. Given the non-positive effect on benefits, a sufficient condition for this to hold is that employees' average gross income $\hat{\theta}_l(\tau, Q_t)w_t$ decreases with Q. Using the definition of $\hat{\theta}_l(\tau, Q_t)$, we can write

$$\frac{d\hat{\theta}_l(\tau, Q_t)w_t}{dQ} = \left\{ \hat{\theta}_l(\tau, Q_t) + \frac{w_t(\theta_t^2 - \hat{\theta}_l(\tau, Q_t))}{\int_{\theta_t^1}^{\theta_t^2} g(\theta)d\theta} \frac{\partial \theta_t^2}{\partial w} g(\theta_t^2) - \frac{w_t(\theta_t^1 - \hat{\theta}_l(\tau, Q_t))}{\int_{\theta_t^1}^{\theta_t^2} g(\theta)d\theta} \frac{\partial \theta_t^1}{\partial w} g(\theta_t^1) \right\} \frac{\partial w_t}{\partial Q} \tag{12}$$

By reducing the wage rate, immigration affects employees' average gross income both directly and indirectly. The direct effect is due to a decrease in the income of the average worker and corresponds to the first term in (12). The second and third term measure the indirect effect of immigration. Immigration shrinks the set of types choosing occupation l: high-type employees will become employers (second term), while lower-type employees will switch to unemployment (third term). The first component decreases the average, the second affects it positively. In what follows, we assume that the two negative effects overcome the positive effect, so that employees' average gross income is decreasing in immigration. This is always the case if the distribution $g(\theta)$ is uniform.

Assumption 4. $\frac{d\hat{\theta}_l(\tau,Q_t)w_t}{dQ} < 0$.

Under Assumption 4, if party $\mathcal{D}$ wants to close the borders, it can pass the wall bill by forming a coalition with party $\mathcal{L}$.

Now suppose party $\mathcal{D}$ wants to keep the borders open. This happens only if benefits are non-decreasing in immigration. In this case, party $\mathcal{D}$ can form a coalition with party $\mathcal{E}$ if employers' average net income is increasing in Q. A sufficient condition for this is that employers' average gross income $\hat{y}_e(L, w_t, Q_t)$ increases with immigration. Using the definition of $\hat{y}_e(L, w_t, Q_t)$, we get

$$\frac{d\hat{y}_e(L, w_t, Q_t)}{dQ} = \Bigg\{ \left(\int_{\theta_t^2}^{\bar{\theta}} g(\theta) d\theta \right)^{-1} \int_{\theta_t^2}^{\bar{\theta}} \frac{\partial y_e(L, w_t, \theta)}{\partial w} g(\theta) d\theta \\ - (y_e(L, w_t, \theta_t^2) - \hat{y}_e(L, w_t, Q_t)) \frac{\partial \theta_t^2}{\partial w} g(\theta_t^2) \Bigg\} \left(\int_{\theta_t^2}^{\bar{\theta}} g(\theta) d\theta \right)^{-1} \frac{\partial w_t}{\partial Q} \quad (13)$$

The first term represents the increase in average gross income due to an increase in income for all the individuals that are employers at time t. The second component measures the indirect effect of immigration through occupational choice. A decrease in wage induces more individuals to become employers. Furthermore, these individuals have an income that is lower than the average employers' income at time t. This second component therefore reduces the average. In what follows, we assume that the direct effect on gross income is larger than the indirect effect, so that employers' average gross income is increasing in immigration.

Assumption 5. $\frac{d\hat{y}_e(L,w_t,Q_t)}{dQ} > 0$.

Lemma 2. *If, at some period t, no party has the majority in parliament, country P will pass a wall bill if and only if the bill has the support of party $\mathcal{D}$.*

Lemma 2 and the discussion above directly imply the following proposition.

Proposition 2. *The sufficient condition for the wall to be build in country P is $u_{\mathcal{L}}(\tau, Q_{t+1}) < u_{\mathcal{L}}(\tau, Q_t)$ if* (10) *holds, $u_{\mathcal{D}}(\tau, Q_{t+1}) < u_{\mathcal{D}}(\tau, Q_t)$ otherwise.*

3.3 Comparison between the two systems

In this section, we compare the degree of openness of the two countries. As a preliminary remark, notice that when benefits are decreasing in immigration, both countries will decide to close their borders. Indeed, when $b(\tau, w_{t+1}, Q_{t+1}, \alpha) - b(\tau, w_t, Q_t, \alpha) < 0$, all types in between 0 and $\tilde{\theta}_t^2$ will be negatively affected by immigration. By Assumption 2, these types are the majority in the population, and the wall bill will be passed in country W. Furthermore, $u_{\mathcal{D}}(\tau, Q_{t+1}) < u_{\mathcal{D}}(\tau, Q_t)$ and, by Assumption 4, $u_{\mathcal{L}}(\tau, Q_{t+1}) < u_{\mathcal{L}}(\tau, Q_t)$. Then, the wall bill must be passed in country P too.

The most interesting scenario is the one where $b(\tau, w_{t+1}, Q_{t+1}, \alpha) - b(\tau, w_t, Q_t, \alpha) > 0$. If country W wants to build the wall, Proposition 1 implies

$$\int_{\tilde{\theta}_t^1}^{\tilde{\theta}_t^2} g(\theta) d\theta > \frac{1}{2}$$

with $\tilde{\theta}_t^1 \in (\theta_t^1, \theta_{t+1}^1)$ and $\tilde{\theta}_t^2 \in (\theta_{t+1}^2, \theta_t^2)$. Then, employees must be the majority in the population and, by Proposition 2, country P will close the borders if $u_{\mathcal{L}}(\tau, Q_{t+1}) < u_{\mathcal{L}}(\tau, Q_t)$. For any two types θ_A, θ_B such that $\theta_t^1 \leq \theta_A < \theta_B \leq \theta_t^2$, define the average income for types $\theta \in [\theta_A, \theta_B]$ given some immigration level Q_t as

$$\hat{x}_l(\theta_A, \theta_B, Q_t) = \frac{\int_{\theta_A}^{\theta_B} x_l(\tau, w, Q_t, \alpha, \theta) g(\theta) d\theta}{\int_{\theta_A}^{\theta_B} g(\theta) d\theta}$$

Notice that $\hat{x}_l(\theta_t^1, \theta_t^2, Q_t) = u_{\mathcal{L}}(\tau, Q_t)$ for all t. Using the definition of $u_{\mathcal{L}}(\tau, Q_t)$, then, the change in party $\mathcal{L}$'s utility due to immigration can be decomposed as a weighted average of three terms

$$\begin{aligned} u_{\mathcal{L}}(\tau, Q_{t+1}) - u_{\mathcal{L}}(\tau, Q_t) = & \frac{\int_{\theta_{t+1}^1}^{\theta_{t+1}^2} g(\theta) d\theta}{\int_{\theta_t^1}^{\theta_t^2} g(\theta) d\theta} [\hat{x}_l(\theta_{t+1}^1, \theta_{t+1}^2, Q_{t+1}) - \hat{x}_l(\theta_{t+1}^1, \theta_{t+1}^2, Q_t)] \\ & + \frac{\int_{\theta_{t+1}^2}^{\theta_t^2} g(\theta) d\theta}{\int_{\theta_t^1}^{\theta_t^2} g(\theta) d\theta} [\hat{x}_l(\theta_{t+1}^1, \theta_{t+1}^2, Q_{t+1}) - \hat{x}_l(\theta_{t+1}^2, \theta_t^2, Q_t)] \\ & + \frac{\int_{\theta_t^1}^{\theta_{t+1}^1} g(\theta) d\theta}{\int_{\theta_t^1}^{\theta_t^2} g(\theta) d\theta} [\hat{x}_l(\theta_{t+1}^1, \theta_{t+1}^2, Q_{t+1}) - \hat{x}_l(\theta_t^1, \theta_{t+1}^1, Q_t)] \end{aligned}$$

The first term measures the change in average income for types $\theta \in (\theta_{t+1}^1, \theta_{t+1}^2)$. These types are employees at time t and remain employees after immigration. If country W wants to build the wall, they must all be strictly harmed by immigration, implying that the first term must be negative. The second term measures the difference between average income at time $t+1$ for types $\theta \in (\theta_{t+1}^1, \theta_{t+1}^2)$ and the average income at time t for types $\theta \in (\theta_{t+1}^2, \theta_t^2)$. This term must be negative, since

$$\hat{x}_l(\theta^1_{t+1}, \theta^2_{t+1}, Q_{t+1}) < \hat{x}_l(\theta^1_{t+1}, \theta^2_{t+1}, Q_t) < \hat{x}_l(\theta^2_{t+1}, \theta^2_t, Q_t)$$

Finally, the third term measures the difference between average income at time $t+1$ for types $\theta \in (\theta^1_{t+1}, \theta^2_{t+1})$ and the average income at time t for types $\theta \in (\theta^1_t, \theta^1_{t+1})$. The term could be positive or negative depending on whether the decrease in average income for types $\theta \in (\theta^1_{t+1}, \theta^2_{t+1})$ is large enough to compensate the initial difference in average incomes between the two sets of types. That is, $\hat{x}_l(\theta^1_{t+1}, \theta^2_{t+1}, Q_{t+1}) - \hat{x}_l(\theta^1_t, \theta^1_{t+1}, Q_t) \leq 0$ if and only if

$$\hat{x}_l(\theta^1_{t+1}, \theta^2_{t+1}, Q_{t+1}) - \hat{x}_l(\theta^1_{t+1}, \theta^2_{t+1}, Q_t) \leq \hat{x}_l(\theta^1_t, \theta^1_{t+1}, Q_t) - \hat{x}_l(\theta^1_{t+1}, \theta^2_{t+1}, Q_t) \tag{14}$$

Then, if (14) holds, country P will build the wall whenever country W does.

Finally, consider the case of $b(\tau, w_{t+1}, Q_{t+1}, \alpha) - b(\tau, w_t, Q_t, \alpha) = 0$. If the types in between $\tilde{\theta}^1_t$ and $\tilde{\theta}^2_t$ are the majority in the population, this case is identical to the one we just analyzed. If instead these types are not the majority in the population, then the construction of the wall depends on non-economic factors that might affect the opinion of unemployed individuals. If these individuals decide to close the borders in country W, so will party $\mathcal{D}$ in country P.

Let t^W denote the smallest t at which country W decides to build the wall and denote by t^P the equivalent date country P. Then

Proposition 3. *Let the common level of taxation in the two countries be exogenously fixed. If* (14) *holds,* $t^W \geq t^P$.

4 Results with different endogenous tax levels

Let us now fix the level of immigration Q_t and examine the optimal choice of tax τ under the two electoral systems.[3] For winner take all, we assume that the implemented tax level is the one preferred by the majority of the population of natives. For PR, we consider a legislative bargaining process as follows.

Denote by τ_0 the given status quo level of taxation. After elections, if a party $\mathcal{P} \in \{\mathcal{E}, \mathcal{L}, \mathcal{D}\}$ obtains the majority of the votes, it will implement the tax level maximizing $u_{\mathcal{P}}(\tau, Q_t)$. If no party obtains the majority, one party will be selected randomly to propose a tax rate. We assume each party is selected as a proposer with a probability equal to the share of native individuals in the occupation it represents. The proposed tax rate is then put to a vote against the status quo τ_0. If at least another party agrees with the proposal, the new tax rate is implemented. The tax rate

[3]The conclusions in this section are practically identical to those discussed by AS. The results in AS rely on single-peakedness of individuals' preferences over taxation. However, as noted by Morelli and Negri (2017), the argument proving single-peakedness contains a mistake and the property cannot be established. Morelli and Negri (2017) provide an alternative proof of the results, based on the property of single-crossing preferences (Gans and Smart (1996)). This section complements the results in AS and Morelli and Negri (2017) with additional results for PR.

remains τ_0 otherwise. Denote by $p(\tau|\tau_0)$ the probability that tax rate τ is chosen by the legislative bargaining process when τ_0 is the status quo. Our focus is on *stable* tax rates.[4]

Definition 2. *A tax rate τ is stable if $p(\tau|\tau) = 1$.*

A stable tax rate is a status quo tax rate that is never changed by the legislative bargaining process and could be interpreted as a long-run tax rate. Focusing on stable tax rates allows to not make the comparison between winner take all and PR dependent on the status quo tax rate.

Before introducing the results for the two systems, notice that

Lemma 3 (Lemma 1 in AS)**.** *For a fixed level of immigration Q_t, the equilibrium wage rate w_t is differentiable, strictly increasing and nonlinear in τ.*

We refer to AS for the proof of the lemma and simply note that

$$\frac{\partial w_t}{\partial \tau} = \frac{w_t(\theta_t^1)^2[g(\theta_t^1) + Q_t h(\theta_t^1)]}{(1-\tau)A(\tau, w_t, Q_t)}$$

with $A(\tau, w_t, Q_t)$ as defined in (21). Let $\epsilon(\tau)$ and $\tilde{\epsilon}(\tau)$ denote the tax elasticity of the equilibrium wage rate and the tax elasticity of the marginal equilibrium wage rate,

$$\epsilon(\tau) = \frac{\partial w_t}{\partial \tau}\frac{\tau}{w_t}$$

$$\tilde{\epsilon}(\tau) = \frac{\partial^2 w_t}{\partial \tau^2}\frac{\tau}{\frac{\partial w_t}{\partial \tau}}$$

In what follows, we assume

Assumption 6.

$$(1-\tau)\tilde{\epsilon}(\tau) \leq (1-\tau)\epsilon(\tau) + \tau$$

Assumption 6 is identical to condition (5) in AS[5] and allows us to use some of the results contained in the paper.

4.1 Taxation under winner take all

For any given immigration level Q_t, let $\xi(\tau, Q_t, \theta)$ denote a type θ's maximum consumption level at a given tax rate τ and sorting equilibrium $w_t = w(\tau, Q_t)$. That is

$$\xi(\tau, Q_t, \theta) = \max_{j \in \{e,l,d\}} x_j(\cdot, \theta)$$

Suppose the median type in the distribution $g(\theta)$, $\tilde{\theta}$, is an employee, i.e. $\theta_t^1 < \tilde{\theta} < \theta_t^2$.

[4]Our definition of stable tax rate is a simplified version of the *PRPE-stable* equilibrium in AS (p.1251).

[5]Condition (5) in AS also includes a lower bound for $(1-\tau)\tilde{\epsilon}(\tau)$. As shown in Morelli and Negri (2017), this lower bound is not necessary.

Lemma 4. *For any two tax levels τ, τ' such that $\tau < \tau'$,*

1. $\xi(\tau, Q_t, \tilde{\theta}) \geq \xi(\tau', Q_t, \tilde{\theta}) \Rightarrow \xi(\tau, Q_t, \theta) \geq \xi(\tau', Q_t, \theta)$ *for all* $\theta > \tilde{\theta}$
2. $\xi(\tau', Q_t, \tilde{\theta}) \geq \xi(\tau, Q_t, \tilde{\theta}) \Rightarrow \xi(\tau', Q_t, \theta) \geq \xi(\tau, Q_t, \theta)$ *for all* $\theta < \tilde{\theta}$

Lemma 4 proves that individuals' preferences over taxation satisfy a weak version of the single-crossing condition (Gans Smart (1996)). The result was proven by Morelli and Negri (2017) and the proof we provide in the appendix is just an adaptation of that proof to our framework. A direct implication of the lemma is that $\tilde{\theta}$ is the median type in the distribution of preferences over taxation. Let τ^W be the tax level implemented by country W. Then

$$\tau^W = \arg\max_{\tau} (1-\tau)\tilde{\theta} w_t + b(\tau, w_t, Q_t, \alpha) - c$$

4.2 Taxation under PR

In order to identify the stable tax rate, we first need to understand parties' behavior in the legislative bargaining process. In the following lemma, we show that parties' preferences over taxation also satisfy a weak version of the single-crossing condition. More precisely, the lemma shows that party $\mathcal{L}$ is the median party. The first part of the lemma is a direct consequence of Lemma 2 and Lemma 5 in AS. Lemma 2 states that, under Assumption 6, benefits (and therefore $u_{\mathcal{D}}(\tau, Q_t)$) are strictly concave in τ, with interior $\arg\max$. Define

$$V(\tau) \equiv 1 - \frac{(1-\tau)}{w_t} \frac{\partial w_t}{\partial \tau} \tag{15}$$

AS shows that $V(\tau) > 0$.[6] Lemma 5 in AS states that, when

$$\frac{\partial \hat{\theta}_l(\tau, Q_t)}{\partial \tau} \geq \frac{\partial^2 \hat{\theta}_l(\tau, Q_t)}{\partial \tau^2} \left[\frac{1-\tau}{1+V(\tau)} \right] \tag{16}$$

party $\mathcal{L}$'s utility is strictly quasiconcave in τ. Furthermore, denoting by $\tau_{\mathcal{P}}$ the maximizer of $u_{\mathcal{P}}(\tau, Q_t)$, the lemma proves that $\tau_{\mathcal{L}} < \tau_{\mathcal{D}}$. This immediately implies that for all $\tau' > \tau$, if party $\mathcal{L}$ prefers τ' to τ, then party $\mathcal{D}$ also prefers τ', which is the first statement in our Lemma 5. The second statement in our lemma states that, when party $\mathcal{L}$ prefers a lower tax rate, party $\mathcal{E}$ must prefer lower taxes too. A sufficient condition for this to hold is that average employers' income is decreasing in τ. Higher tax rates imply higher wages. On the one hand, this decreases the

[6] This is shown in the proof of Lemma 2 in AS. Using the formula for $\partial w_t / \partial \tau$, one gets

$$V(\tau) = 1 - \frac{(\theta_t^1)^2 [g(\theta_t^1) + Q_t h(\theta_t^1)]}{A(\tau, w_t, Q_t)}$$

and since $A(\tau, w_t, Q_t) > (\theta_t^1)^2 [g(\theta_t^1) + Q_t h(\theta_t^1)]$, $V(\tau) > 0$.

income of every employer, therefore decreasing the average. On the other hand, it induces low-type employers to become employees, therefore increasing the average. The net effect is negative when

$$\int_{\theta_t^2}^{\bar{\theta}} L(w_t,\theta)g(\theta)d\theta > \frac{\partial \theta_t^2}{\partial w} g(\theta_t^2)[\hat{y}_e(L,w_t,Q_t) - y_e(L,w_t,\theta_t^2)] \tag{17}$$

The left-hand-side of (17) measures the total increase in the cost of labor due to an increase in wages. The right-hand-side corresponds to the increase in the average employers' income due to the endogenous occupational decisions.

Lemma 5. *If* (16) *and* (17) *hold,*

1. $u_{\mathcal{L}}(\tau,Q_t) \leq u_{\mathcal{L}}(\tau',Q_t) \Rightarrow u_{\mathcal{D}}(\tau,Q_t) \leq u_{\mathcal{D}}(\tau',Q_t)$

2. $u_{\mathcal{L}}(\tau,Q_t) \geq u_{\mathcal{L}}(\tau',Q_t) \Rightarrow u_{\mathcal{E}}(\tau,Q_t) \geq u_{\mathcal{E}}(\tau',Q_t)$

for all $\tau < \tau'$*.*

Lemma 5 directly implies the following proposition.

Proposition 4. *If no party has the absolute majority of seats in parliament and* (16) *and* (17) *hold, the unique stable tax rate in a PR country is*

$$\tau^P = \tau_{\mathcal{L}} = \arg\max_{\tau} u_{\mathcal{L}}(\tau,Q_t)$$

4.3 Immigration decisions under different endogenous tax rates

From now on, we assume that conditions (16) and (17) in Proposition 4 are satisfied. One of the most important results in AS, which holds in our model too, is the following conclusion about the tax rates in the two countries:

Proposition 5 (Proposition 6 in AS)**.** *There exists a cost of working* $\bar{c}$ *such that, for all* $c \leq \bar{c}$*,* $\tau^P > \tau^W$*.*

We refer to AS for the proof. Proposition 5 becomes very important for our purposes when combined with the following lemma

Lemma 6. *The share of employees in the native population is decreasing in* τ*,*

$$\frac{\partial}{\partial \tau}\left[\int_{\theta_t^1}^{\theta_t^2} g(\theta)d\theta\right] < 0$$

Assume by contradiction that the share of employees in the native population was increasing in τ. Then the labor supply would increase with τ too. Since $\partial w/\partial \tau > 0$, the increase in labor supply must be associated with an even larger increase in labor demand. However, $\partial\theta_t^2/\partial w > 0$ implies that labor demand must decrease. Then, the share of employees in the population must be decreasing in τ.

Consider time $t = 0$ before the inflow of migrants begins and let τ_0^W and τ_0^P denote the tax levels implemented by the two countries at this date. By Proposition 5 and Lemma 6, the share of natives choosing occupation l in country P is strictly lower than the one in country W. In particular, it is possible to have

$$\int_{\theta_0^1(\tau_0^P)}^{\theta_0^2(\tau_0^P)} g(\theta)d\theta < \frac{1}{2} < \int_{\theta_0^1(\tau_0^W)}^{\theta_0^2(\tau_0^W)} g(\theta)d\theta \tag{18}$$

where $\theta_0^1(\tau) \equiv \theta^1(\tau, w_0, 0)$ and $\theta_0^2(\tau) \equiv \theta^2(\tau, w_0, 0)$. The second inequality in (18), combined with Proposition 2, implies that country P will build the wall whenever party $\mathcal{D}$ supports the bill, or equivalently when $b(\tau_0^P, w_{t+1}, Q_{t+1}, \alpha) - b(\tau_0^P, w_t, Q_t, \alpha) < 0$. In every period t, whether the condition holds or not depends on the value of α. Indeed, for $\alpha = 0$, $b(\tau_0^P, w_{t+1}, Q_{t+1}, \alpha) - b(\tau_0^P, w_t, Q_t, \alpha) > 0$, since immigrants would positively contribute to tax revenues without receiving any welfare benefits. When α increases, so do the chances that benefits decrease with immigration. For any tax rate τ and level of immigration Q_t, let $\alpha_d(\tau, Q_t)$ denote the maximum share of resources transferred to migrants that would keep benefits non-decreasing in immigration. That is, $\alpha_d(\tau, Q_t)$ is such that

$$b(\tau, w_{t+1}, Q_{t+1}, \alpha) - b(\tau, w_t, Q_t, \alpha) \geq 0 \Leftrightarrow \alpha \leq \alpha_d(\tau, Q_t)$$

Using the definition of benefits $b(\tau, w, Q_t, \alpha) = (1 - \alpha Q_t)\tau Y(\tau, w, Q_t)$, $\alpha_d(\tau, Q_t)$ can be explicitly computed as

$$\alpha_d(\tau, Q_t) \equiv \frac{Y(\tau, w_{t+1}, Q_{t+1}) - Y(\tau, w_t, Q_t)}{Q_{t+1}Y(\tau, w_{t+1}, Q_{t+1}) - Q_t Y(\tau, w_t, Q_t)} \tag{19}$$

For any tax rate τ and level of immigration Q_t, define

$$\eta(\tau, Q_t) \equiv \frac{dY(\tau, w_t, Q_t)}{d\tau} \frac{\tau}{Y(\tau, w_t, Q_t)}$$

$$\tilde{\eta}(\tau, Q_t) \equiv \frac{d^2Y(\tau, w_t, Q_t)}{dQ_t d\tau} \frac{\tau}{\frac{dY(\tau, w_t, Q_t)}{dQ}}$$

as the tax elasticity of aggregate income and the tax elasticity of the marginal effect of immigration on aggregate income, respectively.

Lemma 7. *If*

$$\tilde{\eta}(\tau, Q_t) > \eta(\tau, Q_t) \tag{20}$$

then

$$\frac{\partial \alpha_d(\tau, Q_t)}{\partial \tau} > 0$$

Given that $\eta(\tau, Q_t) < 0$, condition (20) is equivalent to requiring that higher taxes do not reduce too much the positive effect that immigration has on aggregate income. Lemma 7 implies that, for any Q_t, $\alpha_d(\tau_0^W, Q_t) < \alpha_d(\tau_0^P, Q_t)$. Then,

Proposition 6. *Let the level of taxation in the two countries be endogenous. If* (18) *and* (20) *hold, then* $t^W \leq t^P$.

The proof of Proposition 6 is straightforward. If at any time t country P wants to close its borders, it must be that $\alpha_d(\tau_0^P, Q_t) < \alpha$. But then, by Lemma 7, $\alpha_d(\tau_0^W, Q_t) < \alpha$. Given the definition of $\alpha_d(\tau_0^W, Q_t)$, this is equivalent to $b(\tau_0^W, w_{t+1}, Q_{t+1}, \alpha) - b(\tau_0^W, w_t, Q_t, \alpha) < 0$ and, by our discussion in Section 3.1, a majority of individuals in country W will support the construction of the wall. Now suppose that at some period t, $\alpha_d(\tau_0^W, 0) < \alpha \leq \alpha_d(\tau_0^P, 0)$. Then, country W will close the borders, while country P will keep them open for at least one more period.

It is interesting to notice that country W might build the wall before country P even if $\alpha < \alpha_d(\tau_0^W, 0)$, i.e. even if benefits increase with immigration in both countries. For country P, indeed, $b(\tau_0^W, w_{t+1}, Q_{t+1}, \alpha) - b(\tau_0^W, w_t, Q_t, \alpha) > 0$ is a sufficient condition for borders to be kept open. As we noted in Section 2, whenever party $\mathcal{D}$ is in favor of receiving more migrants, it will be able to form a coalition with party $\mathcal{E}$ to prevent the construction of the wall (under Assumption 5). For country W instead, $b(\tau_0^W, w_{t+1}, Q_{t+1}, \alpha) - b(\tau_0^W, w_t, Q_t, \alpha) > 0$ is only necessary. If the set of employees that are harmed by immigration constitutes the majority in the population, i.e. if

$$\int_{\tilde{\theta}_t^1}^{\tilde{\theta}_t^2} g(\theta) d\theta > \frac{1}{2}$$

(notice that this is possible when (18) holds) country W will close its borders even if benefits unemployed individuals strictly benefit from immigration.

Clearly, Proposition 6 does not imply that countries using a PR system are always more open than those using winner take all. However, it has an important implication on the analysis of migration policies. The main conclusion of Section 3 (Proposition 3) was that, *ceteris paribus*, winner take all systems are relatively more open than PR. The results in this section show that the *ceteris paribus* assumption is not innocuous. When the tax levels are determined endogenously, countries using PR systems can be strictly more open to immigration than countries using winner take all systems.

5 Conclusions and Future Research

We have shown that different electoral systems may induce countries to choose different immigration policies, and that the predictions depend crucially on the implications that electoral systems have also for the determination of redistribution policies.

In the analysis, we assumed that the share of resources transferred to migrants is fixed to some value α. One could instead assume that natives' benefits are fixed and migrants only receive a fraction of the resources left after redistribution to natives has occurred. With a fixed and exogenous supply of migrants, this is just a particular case of our model and corresponds to the situation where natives' benefits do not change with immigration (the line marked by the = sign in Figure 1). Our conclusions are therefore be robust to this alternative assumption.

In future research, we plan to complement the model with an empirical analysis. First of all, we want to verify whether the sufficient conditions for our results on the effect of endogenous taxation (conditions (18) and (20)) are supported by the data. If this is the case, then our model unambiguously predicts that PR countries are more open to immigration than WTA ones. We also plan to test this result by exploiting differences in electoral systems used at the local level within a country. France is a good candidate for our purposes, as its municipalities with less than 3500 inhabitants use a WTA system, while those with more use a PR system.

We have conducted the analysis keeping constant and equal the supply of migrants across countries, because our focus was exclusively on the *demand* side. In future research we plan to complemment these results with a supply or selection analysis, and we plan to answer a number of important questions:

- first, it can be shown that borders remaining open is the more politically feasible the more selection is possible in terms of enfranchisement, i.e., giving the right to vote to agents with θ above a certain threshold actually helps the possibility of endogenous open borders, especially in WTA systems.[7]

- Second, an interesting question could be the attractiveness of different immigration policies across systems, in the sense that one system could favor changes in welfare extensions or enfranchisement rules whereas the other could be more likely to build the wall or choose selection policies at the entry point.

- Third, we plan to address endogenous selection of types on the supply side: for similar economic structure and perspectives in two countries, migrants would prefer one to the other if the conditions on institutional insurance or expectations of integration (or even voting) differ substantially. PR, having higher wages and taxes, could induce negative selection, in the sense that the most talented individuals could prefer to supply themselves to WTA countries. The conjecture is that such selection effects may make it comparatively more likely that borders would be closed first in PR systems.

Can a destination ranking be sustainable and under what conditions? In a world of equal growth rate across destination countries, it seems likely that the expected payoffs of migrants should equalize across destinations, and hence there should be a frontier of immigration policies. For example two countries offer the same expected utility to migrants of a given type if either all

[7] The comparison in terms of enfranchisement between the two systems can be done in terms of the $\underline{\theta}$ above which the majority of parliamentarians is in favor of having them vote.

variables are the same or one has higher α but the other has more generous enfranchisement.

Answering all these questions will further increase the heuristic power of the model we have chosen to propose for the study of immigration policies, which is an increasingly important topic in political economy.

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Appendix

Proof of Lemma 1. By Proposition 1 in AS, $w(\tau, Q_t)$ is unique and implicitly defined by (4). Differentiating the condition with respect to Q, we get

$$\frac{\partial w_t}{\partial Q} = -\frac{w_t X(\tau, w_t, Q_t)}{A(\tau, w_t, Q_t)} < 0.$$

with

$$A(\tau, w_t, Q_t) = F(L(w_t, \theta_t^2), \theta_t^2)[g(\theta_t^2) + Q_t h(\theta_t^2)]\frac{\partial \theta_t^2}{\partial w} + (\theta_t^1)^2[g(\theta_1) + Q_t h(\theta_1)] \\ - w_t \int_{\theta_t^2}^{\bar{\theta}} L_w(w_t, \theta)[g(\theta) + Q_t h(\theta)]d\theta \quad (21)$$

where we used (3) to obtain the first term in $A(\tau, w_t, Q_t)$ and we subsituted for $\partial\theta_t^1/\partial w$ using (2) in the second term. Using (3), we get $\partial\theta_t^2/\partial w > 0$ (see Footnote 3) and since labour demand is decreasing in w, we get $A(\tau, w_t, Q_t) > 0$. □

Proof of Lemma **??**. Suppose

$$\int_{\theta_{t+1}^1}^{\theta_{t+1}^2} g(\theta)d\theta \geq \frac{1}{2}$$

and let θ_t^l satisfy (**??**). Since $w_t > w_{t+1}$, the function

$$x_l(\tau, w_t, Q_t, \alpha, \theta) - x_l(\tau, w_{t+1}, Q_{t+1}, \alpha, \theta) = (1-\tau)\theta(w_t - w_{t+1}) + b(\tau, w_t, Q_t, \alpha) - b(\tau, w_{t+1}, Q_{t+1}, \alpha) \quad (22)$$

must be increasing in θ. Then, $x_l(\tau, w_t, Q_t, \alpha, \theta_t^l) - x_l(\tau, w_{t+1}, Q_{t+1}, \alpha, \theta_t^l) \geq 0$, implies $x_l(\tau, w_t, Q_t, \alpha, \theta) - x_l(\tau, w_{t+1}, Q_{t+1}, \alpha, \theta) > 0$ for all $\theta \in (\theta_t^l, \theta_{t+1}^2]$. By the definition of θ_t^l, these voters constitute the majority in the native population.

Now let $x_l(\tau, w_t, Q_t, \alpha, \theta_t^l) - x_l(\tau, w_{t+1}, Q_{t+1}, \alpha, \theta_t^l) \leq 0$. By the same reasoning, $x_l(\tau, w_t, Q_t, \alpha, \theta) - x_l(\tau, w_{t+1}, Q_{t+1}, \alpha, \theta) < 0$ for all $\theta \in (\theta_{t+1}^1, \theta_t^l]$. Furthermore, since the first term in (22) is positive, it must be that $b(\tau, w_{t+1}, Q_{t+1}, \alpha) > b(\tau, w_t, Q_t, \alpha)$. This immediately implies $x_d(\tau, w_t, Q_t, \alpha, \theta) - x_d(\tau, w_{t+1}, Q_{t+1}, \alpha, \theta) < 0$ for all $\theta \in (0, \theta_t^1]$. Since

$$\frac{\partial y_e(L, w, \theta)}{\partial w} = -L(w, \theta) < 0$$

for all w, we have that

$$x_e(L, \tau, w_t, Q_t, \alpha, \theta_t^l) - x_e(L, \tau, w_{t+1}, Q_{t+1}, \alpha, \theta_t^l) \\ = (1-\tau)[y_e(L, w_t, \theta) - y_e(L, w_{t+1}, \theta')] + b(\tau, w_t, Q_t, \alpha) - b(\tau, w_{t+1}, Q_{t+1}, \alpha) < 0 \quad (23)$$

for all $\theta \in (\theta_t^2, \bar{\theta})$. Now consider all $\theta \in (\theta_t^1, \theta_{t+1}^1)$. For all these types

$$(1-\tau)\theta w_t - c < (1-\tau)\theta(w_t - w_{t+1}) \leq b(\tau, w_{t+1}, Q_{t+1}, \alpha) - b(\tau, w_t, Q_t, \alpha)$$

which directly implies $x_l(\tau, w_t, Q_t, \alpha, \theta) - x_d(\tau, w_{t+1}, Q_{t+1}, \alpha, \theta) < 0$. Finally, consider any $\theta \in (\theta_{t+1}^2, \theta_t^2]$. For these types

$$(1-\tau)[w_t\theta - y_e(L, w_{t+1}, \theta)] < (1-\tau)\theta(w_t - w_{t+1}) \leq b(\tau, w_{t+1}, Q_{t+1}, \alpha) - b(\tau, w_t, Q_t, \alpha) \quad (24)$$

so that $x_l(\tau, w_t, Q_t, \alpha, \theta) - x_e(L, \tau, w_{t+1}, Q_{t+1}, \alpha, \theta) < 0$. Combining everything, we have that if θ_t^l prefers Q_{t+1} to Q_t, then all $\theta \notin (\theta_t^l, \theta_{t+1}^2)$ also prefer Q_{t+1} to Q_t. As before, by the way θ_t^l was defined, these people constitute the majority in the population of natives.

Now suppose

$$\int_{\theta_{t+1}^1}^{\theta_{t+1}^2} g(\theta)d\theta < \frac{1}{2} \qquad \int_{\theta_{t+1}^2}^{\bar{\theta}} g(\theta)d\theta < \frac{1}{2} \quad (25)$$

Whenever $x_d(\tau, w_t, Q_t, \alpha, \theta) - x_d(\tau, w_{t+1}, Q_{t+1}, \alpha, \theta) = b(\tau, w_t, Q_t, \alpha) - b(\tau, w_{t+1}, Q_{t+1}, \alpha) > 0$, then

$$x_l(\tau, w_t, Q_t, \alpha, \theta) - x_d(\tau, w_{t+1}, Q_{t+1}, \alpha, \theta) = (1-\tau)\theta w_t - c + b(\tau, w_t, Q_t, \alpha) - b(\tau, w_{t+1}, Q_{t+1}, \alpha) > 0$$

for all $\theta \in (\theta_t^1, \theta_{t+1}^1)$. Furthermore, by (22), $x_l(\tau, w_t, Q_t, \alpha, \theta_t^l) - x_l(\tau, w_{t+1}, Q_{t+1}, \alpha, \theta_t^l) > 0$ for all $\theta \in [\theta_{t+1}^1, \theta_{t+1}^2)$. Thus, if an unemployed individual prefers Q_t to Q_{t+1}, all types $\theta \notin [\theta_{t+1}^2, \bar{\theta})$ also prefer Q_t. By the second inequality in (25), these types constitute the majority. Whenever $x_d(\tau, w_t, Q_t, \alpha, \theta) - x_d(\tau, w_{t+1}, Q_{t+1}, \alpha, \theta) \leq 0$, so that unemployed individuals prefer Q_{t+1} to Q_t, then by (23) and (24) all types $\theta \in [\theta_{t+1}^2, \bar{\theta})$ also prefer Q_{t+1}. By the first inequality in (25), Q_{t+1} will have the support of the majority of the native population.

$\square$

Proof of Lemma 2. Suppose $u_{\mathcal{D}}(Q_t) - u_{\mathcal{D}}(Q_{t+1}) = b(\tau, w_t, Q_t, \alpha) - b(\tau, w_{t+1}, Q_{t+1}, \alpha) \geq 0$, so that party $\mathcal{D}$ is in favour of building the wall. The party will be able to form a coalition with party $\mathcal{L}$ if and only if

$$u_{\mathcal{L}}(Q_t) - u_{\mathcal{L}}(Q_{t+1}) = b(\tau, w_t, Q_t, \alpha) - b(\tau, w_{t+1}, Q_{t+1}, \alpha) + (1-\tau)[\hat{\theta}_l(\tau, Q_t)w_t - \hat{\theta}_l(Q_{t+1})w_{t+1}] \geq 0$$

A sufficient condition for this to hold is

$$\frac{d\hat{\theta}_l(\tau, Q_t)w_t}{dQ} = w_t\frac{d\hat{\theta}_l(\tau, Q_t)}{dQ} + \hat{\theta}_l(\tau, Q_t)\frac{\partial w_t}{\partial Q} < 0 \quad (26)$$

Using the definition of $\hat{\theta}_l(\tau, Q_t)$, we get

$$\frac{d\hat{\theta}_l(\tau, Q_t)}{dQ} = \left(\int_{\theta_t^1}^{\theta_t^2} g(\theta)d\theta\right)^{-2} \frac{\partial w_t}{\partial Q} \left[\frac{\partial \theta_t^2}{\partial w} g(\theta_t^2) \int_{\theta_t^1}^{\theta_t^2} (\theta_t^2 - \theta)g(\theta)d\theta - \frac{\partial \theta_t^1}{\partial w} g(\theta_t^1) \int_{\theta_t^1}^{\theta_t^2} (\theta_t^1 - \theta)g(\theta)d\theta\right]$$

Then,

$$\frac{d\hat{\theta}_l(\tau, Q_t)w_t}{dQ} = \frac{\partial w_t}{\partial Q} \left[\left(\int_{\theta_t^1}^{\theta_t^2} g(\theta)d\theta\right)^{-1} \left(\frac{\partial \theta_t^2}{\partial w} g(\theta_t^2) w_t \int_{\theta_t^1}^{\theta_t^2} (\theta_t^2 - \theta)g(\theta)d\theta - \frac{\partial \theta_t^1}{\partial w} g(\theta_t^1) w_t \int_{\theta_t^1}^{\theta_t^2} (\theta_t^1 - \theta)g(\theta)d\theta\right) + \int_{\theta_t^1}^{\theta_t^2} \theta g(\theta)d\theta\right] \left(\int_{\theta_t^1}^{\theta_t^2} g(\theta)d\theta\right)^{-1}$$

When (12) holds,

$$\int_{\theta_t^1}^{\theta_t^2} \theta g(\theta)d\theta > \left(\int_{\theta_t^1}^{\theta_t^2} g(\theta)d\theta\right)^{-1} \left(\frac{\partial \theta_t^1}{\partial w} g(\theta_t^1) w_t \int_{\theta_t^1}^{\theta_t^2} (\theta_t^1 - \theta)g(\theta)d\theta\right)$$

and $d\hat{\theta}_l(\tau, Q_t)w_t/dQ > 0$.

□

Proof of Lemma 4 (from [14]). Let $\theta > \tilde{\theta}$ first. A sufficient condition for 1. to hold is

$$\xi(\tau, Q_t, \theta) - \xi(\tau', Q_t, \theta) \geq \xi(\tau, Q_t, \tilde{\theta}) - \xi(\tau', Q_t, \tilde{\theta})$$

for all $\tau < \tau'$. Rearranging terms, we get

$$\xi(\tau, Q_t, \theta) - \xi(\tau, Q_t, \tilde{\theta}) \geq \xi(\tau', Q_t, \theta) - \xi(\tau', Q_t, \tilde{\theta})$$

Thus, 1. holds if the function $\Delta\xi(\tau) \equiv \xi(\tau, Q_t, \theta) - \xi(\tau, Q_t, \tilde{\theta})$ is decreasing in τ. By assumption, the median type is an employee, so that $\xi(\tau, Q_t, \tilde{\theta}) = x_l(\tau, w, Q_t, \alpha, \tilde{\theta})$. For all $\theta \in (\tilde{\theta}, \theta_t^2)$, $\xi(\tau, Q_t, \theta) = x_l(\tau, w, Q_t, \alpha, \theta)$. Then, $\Delta\xi(\tau) = (1 - \tau)(\theta - \tilde{\theta})w_t$. Deriving it with respect to τ and rearranging terms, we get

$$\frac{d\Delta\xi(\tau)}{d\tau} = -(\theta - \tilde{\theta})w_t V(\tau)$$

where $V(\tau) > 0$ is as defined in (15). For all $\theta \in [\theta_t^2, \bar{\theta})$, $\xi(\tau, Q_t, \theta) = x_e(L, \tau, w, Q_t, \alpha, \theta)$. Then, $\Delta\xi(\tau) = (1 - \tau)[y_e(L, w_t, \theta) - w_t\tilde{\theta}]$ and

$$\frac{d\Delta\xi(\tau)}{d\tau} = -[y_e(L, w_t, \theta) - w_t\tilde{\theta}] + (1 - \tau)\left[\frac{\partial y_e(L, w_t, \theta)}{\partial \tau} - \tilde{\theta}\frac{\partial w_t}{\partial \tau}\right]$$

By the envelope theorem,

$$\frac{\partial y_e(L, w_t, \theta)}{\partial \tau} = -L(w_t, \theta)\frac{\partial w_t}{\partial \tau}$$

Then,

$$\frac{d\Delta\xi(\tau)}{d\tau} = -[y_e(L, w_t, \theta) - w_t\tilde{\theta}] - (1-\tau)\left[L(w_t, \theta) + \tilde{\theta}\right]\frac{\partial w_t}{\partial \tau} < 0$$

as gross income is increasing in θ and wage is increasing in τ.

By a similar reasoning, a sufficient condition for 2. to hold is $d\Delta\xi(\tau)/d\tau > 0$, whenever $\theta < \tilde{\theta}$. For all $\theta \in [\theta_t^1, \tilde{\theta})$, $\xi(\tau, Q_t, \tilde{\theta}) = x_l(\tau, w, Q_t, \alpha, \tilde{\theta})$ and $\Delta\xi(\tau) = (1-\tau)(\theta - \tilde{\theta})w_t$. For all types $\theta \in (0, \theta_t^1)$, $\xi(\tau, \theta) = x_d(\tau, w, Q_t, \alpha, \theta)$ and $\Delta\xi(\tau) = -(1-\tau)\tilde{\theta}w_t + c$. In both cases, $V(\tau) > 0$ implies $d\Delta\xi(\tau)/d\tau > 0$. □

Proof of Lemma 5, point 1. Consider the first item in the statement of the lemma. A sufficient condition for it to hold is that

$$u_{\mathcal{E}}(\tau, Q_t) - u_{\mathcal{E}}(\tau', Q_t) \geq u_{\mathcal{L}}(\tau, Q_t) - u_{\mathcal{L}}(\tau', Q_t)$$

or, rearranging terms,

$$u_{\mathcal{E}}(\tau, Q_t) - u_{\mathcal{L}}(\tau, Q_t) \geq u_{\mathcal{E}}(\tau', Q_t) - u_{\mathcal{L}}(\tau', Q_t). \tag{27}$$

Substituting for $u_{\mathcal{E}}(\tau, Q_t)$ and $u_{\mathcal{L}}(\tau, Q_t)$, (27) becomes

$$(1-\tau)[\hat{y}_e(L, w_t, Q_t) - \hat{\theta}_l(\tau, Q_t)w_t] \geq (1-\tau')[\hat{y}_e(L, w_t', Q_t) - \hat{\theta}_l(\tau', Q_t)w_t']$$

where $w_t' \equiv w(\tau', Q_t)$. Define the function $\Delta(\tau) \equiv (1-\tau)[\hat{y}_e(L, w_t, Q_t) - \hat{\theta}_l(\tau, Q_t)w_t]$. Then

$$\frac{\partial \Delta(\tau)}{\partial \tau} = -[\hat{y}_e(L, w_t, Q_t) - \hat{\theta}_l(\tau, Q_t)w_t] + (1-\tau)\left[\frac{\partial \hat{y}_e(L, w_t, Q_t)}{\partial \tau} - \frac{\partial \hat{\theta}_l(\tau, Q_t)w_t}{\partial \tau}\right]$$

The first term in $\partial\Delta(\tau)/\partial\tau$ is always negative since

$$\hat{y}_e(L, w_t, Q_t) > y_e(L, w_t, \theta_t^2) = \theta_t^2 w_t > \hat{\theta}_l(\tau, Q_t)w_t$$

Furthermore,

$$\frac{\partial \hat{y}_e(L, w_t, Q_t)}{\partial \tau} = -\left(\int_{\theta_t^2}^{\bar{\theta}} g(\theta)d\theta\right)^{-1}\frac{\partial w}{\partial \tau}\left\{\int_{\theta_t^2}^{\bar{\theta}} L(w_t, \theta)g(\theta)d\theta - \frac{\partial \theta_t^2}{\partial w}g(\theta_t^2)[\hat{y}_e(L, w_t, Q_t) - y_e(L, w_t, \theta_t^2)]\right\} < 0$$

when (17) holds. Finally,

$$\frac{\partial \hat{\theta}_l(\tau, Q_t) w_t}{\partial \tau} = w_t \frac{\partial \hat{\theta}_l(\tau, Q_t)}{\partial \tau} + \hat{\theta}_l(\tau, Q_t) \frac{\partial w_t}{\partial \tau} > 0$$

since

$$\frac{\partial \hat{\theta}_l(\tau, Q_t)}{\partial \tau} = \left(\int_{\theta_t^1}^{\theta_t^2} g(\theta) d\theta \right)^{-1} \left\{ \frac{\partial \theta_t^2}{\partial w} \frac{\partial w}{\partial \tau} g(\theta_t^2) \int_{\theta_t^1}^{\theta_t^2} (\theta_t^2 - \theta) g(\theta) d\theta + \frac{\theta_t^1 g(\theta_t^1)}{1 - \tau} V(\tau) \int_{\theta_t^1}^{\theta_t^2} (\theta - \theta_t^1) g(\theta) d\theta \right\} > 0$$

with $V(\tau)$ as defined in (15). Combining everything, we get $\partial \Delta(\tau)/\partial \tau < 0$, implying that (27) holds.

$\square$

Proof of Proposition 4. Suppose $\tau_{\mathcal{L}}$ is the status quo tax rate. By Lemma 5, a coalition of two parties always prefers $\tau_{\mathcal{L}}$ to any other proposed tax rate τ: when $\tau < \tau_{\mathcal{L}}$, the coalition includes parties $\mathcal{L}$ and $\mathcal{D}$; when $\tau > \tau_{\mathcal{L}}$, it includes parties $\mathcal{L}$ and $\mathcal{E}$. This proves that $\tau_{\mathcal{L}}$ is stable.

Now consider any other status quo tax rate $\tau_0 \neq \tau_{\mathcal{L}}$. With some positive probability

$$\pi_{\mathcal{L}} = \int_{\theta_t^1}^{\theta_t^2} g(\theta) d\theta$$

party $\mathcal{L}$ will be the proposer and will always be able to form a coalition to replace τ_0 with $\tau_{\mathcal{L}}$. Then, for any $\tau_0 \neq \tau_{\mathcal{L}}$, $p(\tau_0|\tau_0) < 1$. $\square$